

# axiom<sup>TM</sup>



## The 30 Year Horizon

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| 28.5 INT.lsp BOOTSTRAP . . . . .                   | 2990        |
| 28.6 ISTRING.lsp BOOTSTRAP . . . . .               | 3001        |
| 28.7 LIST.lsp BOOTSTRAP . . . . .                  | 3019        |
| 28.8 NNI.lsp BOOTSTRAP . . . . .                   | 3025        |
| 28.9 OUTFORM.lsp BOOTSTRAP . . . . .               | 3028        |
| 28.10PI.lsp BOOTSTRAP . . . . .                    | 3042        |
| 28.11PRIMARR.lsp BOOTSTRAP . . . . .               | 3044        |
| 28.12REF.lsp BOOTSTRAP . . . . .                   | 3047        |
| 28.13SINT.lsp BOOTSTRAP . . . . .                  | 3050        |
| 28.14SYMBOL.lsp BOOTSTRAP . . . . .                | 3063        |
| 28.15VECTOR.lsp BOOTSTRAP . . . . .                | 3079        |
| <b>29 Chunk collections</b>                        | <b>3083</b> |
| <b>30 Index</b>                                    | <b>3093</b> |

## New Foreword

On October 1, 2001 Axiom was withdrawn from the market and ended life as a commercial product. On September 3, 2002 Axiom was released under the Modified BSD license, including this document. On August 27, 2003 Axiom was released as free and open source software available for download from the Free Software Foundation's website, Savannah.

Work on Axiom has had the generous support of the Center for Algorithms and Interactive Scientific Computation (CAISS) at City College of New York. Special thanks go to Dr. Gilbert Baumslag for his support of the long term goal.

The online version of this documentation is roughly 1000 pages. In order to make printed versions we've broken it up into three volumes. The first volume is tutorial in nature. The second volume is for programmers. The third volume is reference material. We've also added a fourth volume for developers. All of these changes represent an experiment in print-on-demand delivery of documentation. Time will tell whether the experiment succeeded.

Axiom has been in existence for over thirty years. It is estimated to contain about three hundred man-years of research and has, as of September 3, 2003, 143 people listed in the credits. All of these people have contributed directly or indirectly to making Axiom available. Axiom is being passed to the next generation. I'm looking forward to future milestones.

With that in mind I've introduced the theme of the "30 year horizon". We must invent the tools that support the Computational Mathematician working 30 years from now. How will research be done when every bit of mathematical knowledge is online and instantly available? What happens when we scale Axiom by a factor of 100, giving us 1.1 million domains? How can we integrate theory with code? How will we integrate theorems and proofs of the mathematics with space-time complexity proofs and running code? What visualization tools are needed? How do we support the conceptual structures and semantics of mathematics in effective ways? How do we support results from the sciences? How do we teach the next generation to be effective Computational Mathematicians?

The "30 year horizon" is much nearer than it appears.

Tim Daly  
CAISS, City College of New York  
November 10, 2003 ((iHy))

# Chapter 1

## Chapter Overview

This book contains the domains in Axiom, in alphabetical order.

Each domain has an associated 'dotpic' chunk which only lists the domains, categories, and packages that are in the layer immediately below in the build order. For the full list see the algebra Makefile where this information is maintained.

Each domain is preceded by a picture. The picture indicates several things. The colors indicate whether the name refers to a category, domain, or package. An ellipse means that the name refers to something in the bootstrap set. Thus,





## Chapter 2

# Chapter A

### 2.1 domain AFFPL AffinePlane

— AffinePlane.input —

```
)set break resume
)sys rm -f AffinePlane.output
)spool AffinePlane.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show AffinePlane
--R AffinePlane K: Field is a domain constructor
--R Abbreviation for AffinePlane is AFFPL
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for AFFPL
--R
--R----- Operations -----
--R ?=? : (%,%) -> Boolean          affinePoint : List K -> %
--R coerce : List K -> %           coerce : % -> List K
--R coerce : % -> OutputForm       conjugate : % -> %
--R definingField : % -> K         degree : % -> PositiveInteger
--R ?.? : (%,Integer) -> K         hash : % -> SingleInteger
--R latex : % -> String            list : % -> List K
--R orbit : % -> List %            origin : () -> %
--R pointValue : % -> List K       rational? : % -> Boolean
--R setelt : (%,Integer,K) -> K    ?~=? : (%,%) -> Boolean
--R conjugate : (%,NonNegativeInteger) -> %
--R orbit : (%,NonNegativeInteger) -> List %
--R rational? : (%,NonNegativeInteger) -> Boolean
```



```

--R removeConjugate : List % -> List %
--R removeConjugate : (List %,NonNegativeInteger) -> List %
--R
--E 1

)spool
)lisp (bye)

```

---

— AffinePlane.help —

```

=====
AffinePlane examples
=====

```

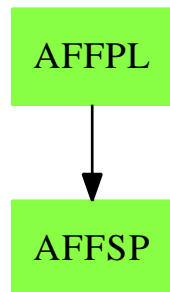
```

See Also:
o )show AffinePlane

```

---

### 2.1.1 AffinePlane (AFFPL)



Exports:

— domain AFFPL AffinePlane —

```

)abbrev domain AFFPL AffinePlane
++ Author: Gaetan Hache
++ Date Created: 17 nov 1992
++ Date Last Updated: May 2010 by Tim Daly

```

## 2.2. DOMAIN AFFPLPS AFFINEPLANEOVERPSEUDOALGEBRAICCLOSUREOFFINITEFIELD5

```

++ Description:
++ The following is all the categories and domains related to projective
++ space and part of the PAFF package
AffinePlane(K):Exports == Implementation where
  K:Field

  Exports ==> AffineSpaceCategory(K)

  Implementation ==> AffineSpace(2,K)

```

— AFFPL.dotabb —

```

"AFFPL" [color="#88FF44",href="bookvol10.3.pdf#nameddest=AFFPL"];
"AFFSP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=AFFSP"];
"AFFPL" -> "AFFSP"

```

## 2.2 domain AFFPLPS AffinePlaneOverPseudoAlgebraic-ClosureOfFiniteField

— AffinePlaneOverPseudoAlgebraicClosureOfFiniteField.input —

```

)set break resume
)sys rm -f AffinePlaneOverPseudoAlgebraicClosureOfFiniteField.output
)spool AffinePlaneOverPseudoAlgebraicClosureOfFiniteField.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show AffinePlaneOverPseudoAlgebraicClosureOfFiniteField
--R AffinePlaneOverPseudoAlgebraicClosureOfFiniteField K: FiniteFieldCategory is a domain constructor
--R Abbreviation for AffinePlaneOverPseudoAlgebraicClosureOfFiniteField is AFFPLPS
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for AFFPLPS
--R
--R----- Operations -----
--R ==? : (% ,%) -> Boolean          coerce : % -> OutputForm
--R conjugate : % -> %              degree : % -> PositiveInteger

```

```

--R hash : % -> SingleInteger          latex : % -> String
--R orbit : % -> List %                origin : () -> %
--R rational? : % -> Boolean           ?~=? : (%,% ) -> Boolean
--R affinePoint : List PseudoAlgebraicClosureOfFiniteField K -> %
--R coerce : List PseudoAlgebraicClosureOfFiniteField K -> %
--R coerce : % -> List PseudoAlgebraicClosureOfFiniteField K
--R conjugate : (% ,NonNegativeInteger) -> %
--R definingField : % -> PseudoAlgebraicClosureOfFiniteField K
--R ?.? : (% ,Integer) -> PseudoAlgebraicClosureOfFiniteField K
--R list : % -> List PseudoAlgebraicClosureOfFiniteField K
--R orbit : (% ,NonNegativeInteger) -> List %
--R pointValue : % -> List PseudoAlgebraicClosureOfFiniteField K
--R rational? : (% ,NonNegativeInteger) -> Boolean
--R removeConjugate : List % -> List %
--R removeConjugate : (List % ,NonNegativeInteger) -> List %
--R setelt : (% ,Integer,PseudoAlgebraicClosureOfFiniteField K) -> PseudoAlgebraicClosureOfFiniteField K
--R
--E 1

)spool
)lisp (bye)

```

---

— AffinePlaneOverPseudoAlgebraicClosureOfFiniteField.help —

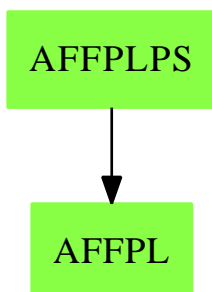
```

oooo
=====
AffinePlaneOverPseudoAlgebraicClosureOfFiniteField examples
=====

See Also:
o )show AffinePlaneOverPseudoAlgebraicClosureOfFiniteField

```

### 2.2.1 AffinePlaneOverPseudoAlgebraicClosureOfFiniteField (AFF-PLPS)



#### Exports:

|                 |               |        |             |           |
|-----------------|---------------|--------|-------------|-----------|
| ??              | ==?           | ?~=?   | affinePoint | coerce    |
| conjugate       | definingField | degree | hash        | latex     |
| list            | orbit         | origin | pointValue  | rational? |
| removeConjugate | setelt        |        |             |           |

— domain AFFPLPS AffinePlaneOverPseudoAlgebraicClosureOfFiniteField —

```

)abbrev domain AFFPLPS AffinePlaneOverPseudoAlgebraicClosureOfFiniteField
++ Author: Gaetan Hache
++ Date Created: 17 nov 1992
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
++ The following is all the categories and domains related to projective
++ space and part of the PAFF package
AffinePlaneOverPseudoAlgebraicClosureOfFiniteField(K):Exports == Impl where
  K:FiniteFieldCategory

  KK ==> PseudoAlgebraicClosureOfFiniteField(K)

  Exports ==> AffineSpaceCategory(KK)

  Impl ==> AffinePlane(KK)

```

— AFFPLPS.dotabb —

```

"AFFPLPS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=AFFPLPS"];
"AFFPL" [color="#88FF44",href="bookvol10.3.pdf#nameddest=AFFPL"];
"AFFPLPS" -> "AFFPL"

```

## 2.3 domain AFFSP AffineSpace

— AffineSpace.input —

```

)set break resume
)sys rm -f AffineSpace.output
)spool AffineSpace.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show AffineSpace
--R AffineSpace(dim: NonNegativeInteger,K: Field) is a domain constructor
--R Abbreviation for AffineSpace is AFFSP
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for AFFSP
--R
--R----- Operations -----
--R ?=? : (%,% ) -> Boolean          affinePoint : List K -> %
--R coerce : List K -> %             coerce : % -> List K
--R coerce : % -> OutputForm         conjugate : % -> %
--R definingField : % -> K           degree : % -> PositiveInteger
--R ?.? : (%,Integer) -> K           hash : % -> SingleInteger
--R latex : % -> String              list : % -> List K
--R orbit : % -> List %              origin : () -> %
--R pointValue : % -> List K         rational? : % -> Boolean
--R setelt : (%,Integer,K) -> K      ?~=? : (%,% ) -> Boolean
--R conjugate : (%NonNegativeInteger) -> %
--R orbit : (%NonNegativeInteger) -> List %
--R rational? : (%NonNegativeInteger) -> Boolean
--R removeConjugate : List % -> List %
--R removeConjugate : (List %,NonNegativeInteger) -> List %
--R
--E 1

)spool
)lisp (bye)

```

— AffineSpace.help —

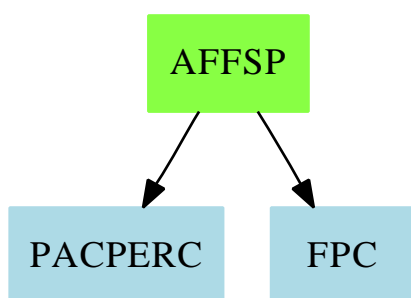
=====

AffineSpace examples

See Also:

o )show AffineSpace

### 2.3.1 AffineSpace (AFFSP)



#### Exports:

|                 |               |        |             |           |
|-----------------|---------------|--------|-------------|-----------|
| ??              | ?=?           | ?~=?   | affinePoint | coerce    |
| conjugate       | definingField | degree | hash        | latex     |
| list            | orbit         | origin | pointValue  | rational? |
| removeConjugate | setelt        |        |             |           |

— domain AFFSP AffineSpace —

```

)abbrev domain AFFSP AffineSpace
++ Author: Gaetan Hache
++ Date Created: 17 nov 1992
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
++ The following is all the categories and domains related to projective
++ space and part of the PAFF package
AffineSpace(dim,K):Exports == Implementation where
  dim:NonNegativeInteger
  K:Field

LIST ==> List
NNI ==> NonNegativeInteger

Exports ==> AffineSpaceCategory(K)
  
```

```

Implementation ==> List(K)  add

Rep:= List(K)

origin== new(dim,0$K)$List(K)

coerce(pt:%)::OutputForm ==
  dd:OutputForm:= ":" :: OutputForm
  llout:List(OutputForm):=[ hconcat(dd, a::OutputForm) for a in rest pt]
  lout:= cons( (first pt)::OutputForm , llout)
  out:= hconcat lout
  oo:=paren(out)
  ee:OutputForm:= degree(pt) :: OutputForm
  oo**ee

definingField(pt)==
  K has PseudoAlgebraicClosureOfPerfectFieldCategory => _
    maxTower(pt pretend Rep)
  1$K

degree(pt)==
  K has PseudoAlgebraicClosureOfPerfectFieldCategory => _
    extDegree definingField pt
  1

coerce(pt:%)::List(K) == pt pretend Rep

affinePoint(pt:LIST(K))==
  pt :: %

list(ptt)==
  ptt pretend Rep

pointValue(ptt)==
  ptt pretend Rep

conjugate(p,e)==
  lp:Rep:=p
  pc:List(K):=[c**e for c in lp]
  affinePoint(pc)

rational?(p,n)== p=conjugate(p,n)

rational?(p)==rational?(p,characteristic()$K)

removeConjugate(l)==removeConjugate(l,characteristic()$K)

removeConjugate(l:LIST(%),n:NNI):LIST(%)==
  if K has FiniteFieldCategory then
    allconj:LIST(%):=empty()

```

```

    conjrem:LIST(%):=empty()
    for p in l repeat
        if ^member?(p,allconj) then
            conjrem:=cons(p,conjrem)
            allconj:=concat(allconj,orbit(p,n))
        conjrem
    else
        error "The field is not finite"

conjugate(p)==conjugate(p,characteristic())$K

orbit(p)==orbit(p,characteristic())$K

orbit(p,e)==
    if K has FiniteFieldCategory then
        l:LIST(%):=[p]
        np:%=conjugate(p,e)
        flag:=^(np=p)::Boolean
        while flag repeat
            l:=concat(np,l)
            np:=conjugate(np,e)
            flag:=not (np=p)::Boolean
        l
    else
        error "Cannot compute the conjugate"

aa:% = bb:% ==
    aa =$Rep bb

coerce(pt:LIST(K))==
    ^(dim=#pt) => error "Le point n'a pas la bonne dimension"
    ptt:%= pt
    ptt

```

---

— AFFSP.dotabb —

```

"AFFSP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=AFFSP"];
"PACPERC" [color=lightblue,href="bookvol10.2.pdf#nameddest=PACPERC"];
"FPC" [color=lightblue,href="bookvol10.2.pdf#nameddest=FPC"];
"AFFSP" -> "FPC"
"AFFSP" -> "PACPERC"

```

---



## 2.4 domain ALGSC AlgebraGivenByStructuralConstants

— AlgebraGivenByStructuralConstants.input —

```

)set break resume
)sys rm -f AlgebraGivenByStructuralConstants.output
)spool AlgebraGivenByStructuralConstants.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show AlgebraGivenByStructuralConstants
--R AlgebraGivenByStructuralConstants(R: Field,n: PositiveInteger,ls: List Symbol,gamma: Vec
--R Abbreviation for AlgebraGivenByStructuralConstants is ALGSC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ALGSC
--R
--R----- Operations -----
--R ?? : (SquareMatrix(n,R),%) -> %      ?? : (R,%) -> %
--R ?? : (% ,R) -> %                      ?? : (% ,%) -> %
--R ?? : (Integer,%) -> %                 ?? : (PositiveInteger,%) -> %
--R ??? : (% ,PositiveInteger) -> %      ?+? : (% ,%) -> %
--R ?-? : (% ,%) -> %                    -? : % -> %
--R ?=? : (% ,%) -> Boolean              0 : () -> %
--R alternative? : () -> Boolean          antiAssociative? : () -> Boolean
--R antiCommutative? : () -> Boolean      antiCommutator : (% ,%) -> %
--R apply : (Matrix R,%) -> %             associative? : () -> Boolean
--R associator : (% ,% ,%) -> %           basis : () -> Vector %
--R coerce : Vector R -> %               coerce : % -> OutputForm
--R commutative? : () -> Boolean           commutator : (% ,%) -> %
--R convert : Vector R -> %               convert : % -> Vector R
--R coordinates : % -> Vector R           ?.? : (% ,Integer) -> R
--R flexible? : () -> Boolean             hash : % -> SingleInteger
--R jacobiIdentity? : () -> Boolean        jordanAdmissible? : () -> Boolean
--R jordanAlgebra? : () -> Boolean        latex : % -> String
--R leftAlternative? : () -> Boolean       leftDiscriminant : () -> R
--R leftDiscriminant : Vector % -> R      leftNorm : % -> R
--R leftTrace : % -> R                   leftTraceMatrix : () -> Matrix R
--R lieAdmissible? : () -> Boolean         lieAlgebra? : () -> Boolean
--R powerAssociative? : () -> Boolean      rank : () -> PositiveInteger
--R represents : Vector R -> %            rightAlternative? : () -> Boolean
--R rightDiscriminant : () -> R           rightDiscriminant : Vector % -> R
--R rightNorm : % -> R                   rightTrace : % -> R
--R rightTraceMatrix : () -> Matrix R     sample : () -> %
--R someBasis : () -> Vector %            zero? : % -> Boolean
--R ?~=? : (% ,%) -> Boolean
--R ?? : (NonNegativeInteger,%) -> %

```

```

--R associatorDependence : () -> List Vector R if R has INTDOM
--R conditionsForIdempotents : () -> List Polynomial R
--R conditionsForIdempotents : Vector % -> List Polynomial R
--R coordinates : Vector % -> Matrix R
--R coordinates : (Vector %,Vector %) -> Matrix R
--R coordinates : (% ,Vector %) -> Vector R
--R leftCharacteristicPolynomial : % -> SparseUnivariatePolynomial R
--R leftMinimalPolynomial : % -> SparseUnivariatePolynomial R if R has INTDOM
--R leftPower : (% ,PositiveInteger) -> %
--R leftRankPolynomial : () -> SparseUnivariatePolynomial Polynomial R if R has FIELD
--R leftRecip : % -> Union(%,"failed") if R has INTDOM
--R leftRegularRepresentation : % -> Matrix R
--R leftRegularRepresentation : (% ,Vector %) -> Matrix R
--R leftTraceMatrix : Vector % -> Matrix R
--R leftUnit : () -> Union(%,"failed") if R has INTDOM
--R leftUnits : () -> Union(Record(particular: %,basis: List %),"failed") if R has INTDOM
--R noncommutativeJordanAlgebra? : () -> Boolean
--R plenaryPower : (% ,PositiveInteger) -> %
--R recip : % -> Union(%,"failed") if R has INTDOM
--R represents : (Vector R,Vector %) -> %
--R rightCharacteristicPolynomial : % -> SparseUnivariatePolynomial R
--R rightMinimalPolynomial : % -> SparseUnivariatePolynomial R if R has INTDOM
--R rightPower : (% ,PositiveInteger) -> %
--R rightRankPolynomial : () -> SparseUnivariatePolynomial Polynomial R if R has FIELD
--R rightRecip : % -> Union(%,"failed") if R has INTDOM
--R rightRegularRepresentation : % -> Matrix R
--R rightRegularRepresentation : (% ,Vector %) -> Matrix R
--R rightTraceMatrix : Vector % -> Matrix R
--R rightUnit : () -> Union(%,"failed") if R has INTDOM
--R rightUnits : () -> Union(Record(particular: %,basis: List %),"failed") if R has INTDOM
--R structuralConstants : () -> Vector Matrix R
--R structuralConstants : Vector % -> Vector Matrix R
--R subtractIfCan : (% ,%) -> Union(%,"failed")
--R unit : () -> Union(%,"failed") if R has INTDOM
--R
--E 1

)spool
)lisp (bye)

```

---

— AlgebraGivenByStructuralConstants.help —

```

=====
AlgebraGivenByStructuralConstants examples
=====

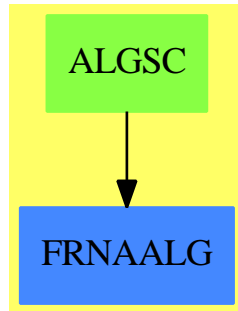
```

See Also:

o )show AlgebraGivenByStructuralConstants

—————

#### 2.4.1 AlgebraGivenByStructuralConstants (ALGSC)



Exports:

|                               |                              |
|-------------------------------|------------------------------|
| 0                             | alternative?                 |
| antiAssociative?              | antiCommutative?             |
| antiCommutator                | apply                        |
| associative?                  | associator                   |
| associatorDependence          | basis                        |
| coerce                        | commutative?                 |
| commutator                    | conditionsForIdempotents     |
| convert                       | coordinates                  |
| flexible?                     | hash                         |
| jacobiIdentity?               | jordanAdmissible?            |
| jordanAlgebra?                | latex                        |
| leftAlternative?              | leftCharacteristicPolynomial |
| leftDiscriminant              | leftMinimalPolynomial        |
| leftNorm                      | leftPower                    |
| leftRankPolynomial            | leftRecip                    |
| leftRegularRepresentation     | leftTrace                    |
| leftTraceMatrix               | leftUnit                     |
| leftUnits                     | lieAdmissible?               |
| lieAlgebra?                   | noncommutativeJordanAlgebra? |
| plenaryPower                  | powerAssociative?            |
| rank                          | recip                        |
| represents                    | rightAlternative?            |
| rightCharacteristicPolynomial | rightDiscriminant            |
| rightMinimalPolynomial        | rightNorm                    |
| rightPower                    | rightRankPolynomial          |
| rightRecip                    | rightRegularRepresentation   |
| rightTrace                    | rightTraceMatrix             |
| rightUnit                     | rightUnits                   |
| sample                        | someBasis                    |
| structuralConstants           | subtractIfCan                |
| unit                          | zero?                        |
| ?*?                           | ?**?                         |
| ?+?                           | ?-?                          |
| -?                            | ?=?                          |
| ?..?                          | ?~=?                         |
| ?*?                           |                              |

— domain ALGSC AlgebraGivenByStructuralConstants —

```

)abbrev domain ALGSC AlgebraGivenByStructuralConstants
++ Authors: J. Grabmeier, R. Wisbauer
++ Date Created: 01 March 1991
++ Date Last Updated: 22 January 1992
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:

```

```

++ Keywords: algebra, structural constants
++ Reference:
++ R.D. Schafer: An Introduction to Nonassociative Algebras
++ Academic Press, New York, 1966
++ Description:
++ AlgebraGivenByStructuralConstants implements finite rank algebras
++ over a commutative ring, given by the structural constants \spad{gamma}
++ with respect to a fixed basis \spad{[a1,..,an]}, where
++ \spad{gamma} is an \spad{n}-vector of n by n matrices
++ \spad{[(gammaijk) for k in 1..rank()] } defined by
++ \spad{ai * aj = gammaij1 * a1 + ... + gammaijn * an}.
++ The symbols for the fixed basis
++ have to be given as a list of symbols.

AlgebraGivenByStructuralConstants(R:Field, n : PositiveInteger, _
  ls : List Symbol, gamma: Vector Matrix R ): public == private where

V ==> Vector
M ==> Matrix
I ==> Integer
NNI ==> NonNegativeInteger
REC ==> Record(particular: Union(V R,"failed"),basis: List V R)
LSMP ==> LinearSystemMatrixPackage(R,V R,V R, M R)

--public ==> FramedNonAssociativeAlgebra(R) with
public ==> Join(FramedNonAssociativeAlgebra(R), _
  LeftModule(SquareMatrix(n,R)) ) with

coerce : Vector R -> %
  ++ coerce(v) converts a vector to a member of the algebra
  ++ by forming a linear combination with the basis element.
  ++ Note: the vector is assumed to have length equal to the
  ++ dimension of the algebra.

private ==> DirectProduct(n,R) add

Rep := DirectProduct(n,R)

x,y : %
dp : DirectProduct(n,R)
v : V R

recip(x) == recip(x)$FiniteRankNonAssociativeAlgebra_&(% ,R)

(m:SquareMatrix(n,R))*(x:%) == apply((m :: Matrix R),x)
coerce v == directProduct(v) :: %

structuralConstants() == gamma

```

```

coordinates(x) == vector(entries(x :: Rep)$Rep)$Vector(R)

coordinates(x,b) ==
  --not (maxIndex b = n) =>
  -- error("coordinates: your 'basis' has not the right length")
  m : NonNegativeInteger := (maxIndex b) :: NonNegativeInteger
  transitionMatrix : Matrix R := new(n,m,0$R)$Matrix(R)
  for i in 1..m repeat
    setColumn_!(transitionMatrix,i,coordinates(b.i))
  res : REC := solve(transitionMatrix,coordinates(x))$LSMP
  if (not every?(zero?$R,first res.basis)) then
    error("coordinates: warning your 'basis' is linearly dependent")
  (res.particular case "failed") =>
    error("coordinates: first argument is not in linear span of second argument")
  (res.particular) :: (Vector R)

basis() == [unitVector(i::PositiveInteger)::% for i in 1..n]

someBasis() == basis()$%

rank() == n

elt(x,i) == elt(x:Rep,i)$Rep

coerce(x:%):OutputForm ==
  zero?(x::Rep)$Rep => (0$R) :: OutputForm
  le : List OutputForm := nil
  for i in 1..n repeat
    coef : R := elt(x::Rep,i)
    not zero?(coef)$R =>
      --
      one?(coef)$R =>
        ((coef) = 1)$R =>
          -- sy : OutputForm := elt(ls,i)$(List Symbol) :: OutputForm
          le := cons(elt(ls,i)$(List Symbol) :: OutputForm, le)
          le := cons(coef :: OutputForm * elt(ls,i)$(List Symbol)_
            :: OutputForm, le)
  reduce("+",le)

x * y ==
  v : Vector R := new(n,0)
  for k in 1..n repeat
    h : R := 0
    for i in 1..n repeat
      for j in 1..n repeat
        h := h +$R elt(x,i) *$R elt(y,j) *$R elt(gamma.k,i,j )
    v.k := h
  directProduct v

```

```

alternative?() ==
  for i in 1..n repeat
    -- expression for check of left alternative is symmetric in i and j:
    -- expression for check of right alternative is symmetric in j and k:
    for j in 1..i-1 repeat
      for k in j..n repeat
        -- right check
        for r in 1..n repeat
          res := 0$R
          for l in 1..n repeat
            res := res - _
              (elt(gamma.l,j,k)+elt(gamma.l,k,j))*elt(gamma.r,i,l)+_
              (elt(gamma.l,i,j)*elt(gamma.r,l,k) + elt(gamma.l,i,k)*_
              elt(gamma.r,l,j) )
          not zero? res =>
            messagePrint("algebra is not right alternative")$OutputForm
            return false
    for j in i..n repeat
      for k in 1..j-1 repeat
        -- left check
        for r in 1..n repeat
          res := 0$R
          for l in 1..n repeat
            res := res + _
              (elt(gamma.l,i,j)+elt(gamma.l,j,i))*elt(gamma.r,l,k)-_
              (elt(gamma.l,j,k)*elt(gamma.r,i,l) + elt(gamma.l,i,k)*_
              elt(gamma.r,j,l) )
          not (zero? res) =>
            messagePrint("algebra is not left alternative")$OutputForm
            return false

    for k in j..n repeat
      -- left check
      for r in 1..n repeat
        res := 0$R
        for l in 1..n repeat
          res := res + _
            (elt(gamma.l,i,j)+elt(gamma.l,j,i))*elt(gamma.r,l,k)-_
            (elt(gamma.l,j,k)*elt(gamma.r,i,l) + elt(gamma.l,i,k)*_
            elt(gamma.r,j,l) )
        not (zero? res) =>
          messagePrint("algebra is not left alternative")$OutputForm
          return false
      -- right check
      for r in 1..n repeat
        res := 0$R
        for l in 1..n repeat
          res := res - _
            (elt(gamma.l,j,k)+elt(gamma.l,k,j))*elt(gamma.r,i,l)+_
            (elt(gamma.l,i,j)*elt(gamma.r,l,k) + elt(gamma.l,i,k)*_

```

```

        elt(gamma.r,l,j) )
    not (zero? res) =>
        messagePrint("algebra is not right alternative")$OutputForm
    return false

    messagePrint("algebra satisfies 2*associator(a,b,b) = 0 = 2*associator(a,a,b) = 0")$OutputForm
    true

-- should be in the category, but is not exported
-- conditionsForIdempotents b ==
--   n := rank()
--   --gamma : Vector Matrix R := structuralConstants b
--   listOfNumbers : List String := [STRINGIMAGE(q)$Lisp for q in 1..n]
--   symbolsForCoef : Vector Symbol :=
--     [concat("%", concat("x", i))::Symbol for i in listOfNumbers]
--   conditions : List Polynomial R := []
--   for k in 1..n repeat
--     xk := symbolsForCoef.k
--     p : Polynomial R := monomial( - 1$Polynomial(R), [xk], [1] )
--     for i in 1..n repeat
--       for j in 1..n repeat
--         xi := symbolsForCoef.i
--         xj := symbolsForCoef.j
--         p := p + monomial(
--           elt((gamma.k),i,j) :: Polynomial(R), [xi,xj], [1,1])
--         conditions := cons(p,conditions)
--   conditions

associative?() ==
  for i in 1..n repeat
    for j in 1..n repeat
      for k in 1..n repeat
        for r in 1..n repeat
          res := 0$R
          for l in 1..n repeat
            res := res + elt(gamma.l,i,j)*elt(gamma.r,l,k)-_
              elt(gamma.l,j,k)*elt(gamma.r,i,l)
          not (zero? res) =>
            messagePrint("algebra is not associative")$OutputForm
          return false
  messagePrint("algebra is associative")$OutputForm
  true

antiAssociative?() ==
  for i in 1..n repeat
    for j in 1..n repeat
      for k in 1..n repeat
        for r in 1..n repeat
          res := 0$R

```



```

    for l in 1..n repeat
      res := res + elt(gamma.l,i,j)*elt(gamma.r,l,k)+_
        elt(gamma.l,j,k)*elt(gamma.r,i,l)
    not (zero? res) =>
      messagePrint("algebra is not anti-associative")$OutputForm
      return false
messagePrint("algebra is anti-associative")$OutputForm
true

commutative?() ==
  for i in 1..n repeat
    for j in (i+1)..n repeat
      for k in 1..n repeat
        not ( elt(gamma.k,i,j)=elt(gamma.k,j,i) ) =>
          messagePrint("algebra is not commutative")$OutputForm
          return false
      messagePrint("algebra is commutative")$OutputForm
      true

antiCommutative?() ==
  for i in 1..n repeat
    for j in i..n repeat
      for k in 1..n repeat
        not zero? (i=j => elt(gamma.k,i,i); elt(gamma.k,i,j)+elt(gamma.k,j,i) ) =>
          messagePrint("algebra is not anti-commutative")$OutputForm
          return false
      messagePrint("algebra is anti-commutative")$OutputForm
      true

leftAlternative?() ==
  for i in 1..n repeat
    -- expression is symmetric in i and j:
    for j in i..n repeat
      for k in 1..n repeat
        for r in 1..n repeat
          res := 0$R
          for l in 1..n repeat
            res := res + (elt(gamma.l,i,j)+elt(gamma.l,j,i))*elt(gamma.r,l,k)-_
              (elt(gamma.l,j,k)*elt(gamma.r,i,l) + elt(gamma.l,i,k)*elt(gamma.r,j,l) )
          not (zero? res) =>
            messagePrint("algebra is not left alternative")$OutputForm
            return false
          messagePrint("algebra is left alternative")$OutputForm
          true

rightAlternative?() ==
  for i in 1..n repeat
    for j in 1..n repeat
      -- expression is symmetric in j and k:

```

```

    for k in j..n repeat
      for r in 1..n repeat
        res := 0$R
        for l in 1..n repeat
          res := res - (elt(gamma.l,j,k)+elt(gamma.l,k,j))*elt(gamma.r,i,l)+_
            (elt(gamma.l,i,j)*elt(gamma.r,l,k) + elt(gamma.l,i,k)*elt(gamma.r,l,j) )
        not (zero? res) =>
          messagePrint("algebra is not right alternative")$OutputForm
          return false
      messagePrint("algebra is right alternative")$OutputForm
    true

flexible?() ==
  for i in 1..n repeat
    for j in 1..n repeat
      -- expression is symmetric in i and k:
      for k in i..n repeat
        for r in 1..n repeat
          res := 0$R
          for l in 1..n repeat
            res := res + elt(gamma.l,i,j)*elt(gamma.r,l,k)-_
              elt(gamma.l,j,k)*elt(gamma.r,i,l)+_
              elt(gamma.l,k,j)*elt(gamma.r,l,i)-_
              elt(gamma.l,j,i)*elt(gamma.r,k,l)
          not (zero? res) =>
            messagePrint("algebra is not flexible")$OutputForm
            return false
          messagePrint("algebra is flexible")$OutputForm
        true

lieAdmissible?() ==
  for i in 1..n repeat
    for j in 1..n repeat
      for k in 1..n repeat
        for r in 1..n repeat
          res := 0$R
          for l in 1..n repeat
            res := res_
              + (elt(gamma.l,i,j)-elt(gamma.l,j,i))*(elt(gamma.r,l,k)-elt(gamma.r,k,l)) _
              + (elt(gamma.l,j,k)-elt(gamma.l,k,j))*(elt(gamma.r,l,i)-elt(gamma.r,i,l)) _
              + (elt(gamma.l,k,i)-elt(gamma.l,i,k))*(elt(gamma.r,l,j)-elt(gamma.r,j,l))
          not (zero? res) =>
            messagePrint("algebra is not Lie admissible")$OutputForm
            return false
          messagePrint("algebra is Lie admissible")$OutputForm
        true

jordanAdmissible?() ==
  recip(2 * 1$R) case "failed" =>

```

```

    messagePrint("this algebra is not Jordan admissible, as 2 is not invertible in the g
    false
  for i in 1..n repeat
    for j in 1..n repeat
      for k in 1..n repeat
        for w in 1..n repeat
          for t in 1..n repeat
            res := 0$R
            for l in 1..n repeat
              for r in 1..n repeat
                res := res_
                + (elt(gamma.l,i,j)+elt(gamma.l,j,i))_
                  * (elt(gamma.r,w,k)+elt(gamma.r,k,w))_
                  * (elt(gamma.t,l,r)+elt(gamma.t,r,l))_
                - (elt(gamma.r,w,k)+elt(gamma.r,k,w))_
                  * (elt(gamma.l,j,r)+elt(gamma.l,r,j))_
                  * (elt(gamma.t,i,l)+elt(gamma.t,l,i))_
                + (elt(gamma.l,w,j)+elt(gamma.l,j,w))_
                  * (elt(gamma.r,k,i)+elt(gamma.r,i,k))_
                  * (elt(gamma.t,l,r)+elt(gamma.t,r,l))_
                - (elt(gamma.r,k,i)+elt(gamma.r,k,i))_
                  * (elt(gamma.l,j,r)+elt(gamma.l,r,j))_
                  * (elt(gamma.t,w,l)+elt(gamma.t,l,w))_
                + (elt(gamma.l,k,j)+elt(gamma.l,j,k))_
                  * (elt(gamma.r,i,w)+elt(gamma.r,w,i))_
                  * (elt(gamma.t,l,r)+elt(gamma.t,r,l))_
                - (elt(gamma.r,i,w)+elt(gamma.r,w,i))_
                  * (elt(gamma.l,j,r)+elt(gamma.l,r,j))_
                  * (elt(gamma.t,k,l)+elt(gamma.t,l,k))
              not (zero? res) =>
                messagePrint("algebra is not Jordan admissible")$OutputForm
            return false
          messagePrint("algebra is Jordan admissible")$OutputForm
        true

jordanAlgebra?() ==
  recip(2 * 1$R) case "failed" =>
    messagePrint("this is not a Jordan algebra, as 2 is not invertible in the ground ring
    false
  not commutative?() =>
    messagePrint("this is not a Jordan algebra")$OutputForm
    false
  for i in 1..n repeat
    for j in 1..n repeat
      for k in 1..n repeat
        for l in 1..n repeat
          for t in 1..n repeat
            res := 0$R
            for r in 1..n repeat
              for s in 1..n repeat

```

```

res := res + _
elt(gamma.r,i,j)*elt(gamma.s,l,k)*elt(gamma.t,r,s) - _
elt(gamma.r,l,k)*elt(gamma.s,j,r)*elt(gamma.t,i,s) + _
elt(gamma.r,l,j)*elt(gamma.s,k,k)*elt(gamma.t,r,s) - _
elt(gamma.r,k,i)*elt(gamma.s,j,r)*elt(gamma.t,l,s) + _
elt(gamma.r,k,j)*elt(gamma.s,i,k)*elt(gamma.t,r,s) - _
elt(gamma.r,i,l)*elt(gamma.s,j,r)*elt(gamma.t,k,s)
not zero? res =>
  messagePrint("this is not a Jordan algebra")$OutputForm
  return false
messagePrint("this is a Jordan algebra")$OutputForm
true

jacobiIdentity() ==
for i in 1..n repeat
  for j in 1..n repeat
    for k in 1..n repeat
      for r in 1..n repeat
        res := 0$R
        for s in 1..n repeat
          res := res + elt(gamma.r,i,j)*elt(gamma.s,j,k) +_
elt(gamma.r,j,k)*elt(gamma.s,k,i) +_
elt(gamma.r,k,i)*elt(gamma.s,i,j)
        not zero? res =>
          messagePrint("Jacobi identity does not hold")$OutputForm
          return false
        messagePrint("Jacobi identity holds")$OutputForm
      true

```

---

— ALGSC.dotabb —

"ALGSC" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALGSC"]  
"FRNAALG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FRNAALG"]  
"ALGSC" -> "FRNAALG"

---

## 2.5 domain ALGFF AlgebraicFunctionField

— AlgebraicFunctionField.input —

)set break resume

```

)sys rm -f AlgebraicFunctionField.output
)spool AlgebraicFunctionField.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show AlgebraicFunctionField
--R AlgebraicFunctionField(F: Field,UP: UnivariatePolynomialCategory F,UPUP: UnivariatePolynomialCategory F)
--R Abbreviation for AlgebraicFunctionField is ALGFF
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ALGFF
--R
--R----- Operations -----
--R ??? : (Fraction UP,%) -> %          ??? : (%,Fraction UP) -> %
--R ??? : (%,%) -> %                    ??? : (Integer,%) -> %
--R ??? : (PositiveInteger,%) -> %      ??? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                    ?-? : (%,%) -> %
--R -? : % -> %                         ?=? : (%,%) -> Boolean
--R 1 : () -> %                         0 : () -> %
--R ?? : (%,PositiveInteger) -> %       basis : () -> Vector %
--R branchPoint? : UP -> Boolean         branchPoint? : F -> Boolean
--R coerce : Fraction UP -> %            coerce : Integer -> %
--R coerce : % -> OutputForm             convert : UPUP -> %
--R convert : % -> UPUP                  convert : Vector Fraction UP -> %
--R convert : % -> Vector Fraction UP    definingPolynomial : () -> UPUP
--R discriminant : () -> Fraction UP     elt : (%,F,F) -> F
--R generator : () -> %                  genus : () -> NonNegativeInteger
--R hash : % -> SingleInteger             integral? : (%,UP) -> Boolean
--R integral? : (%,F) -> Boolean          integral? : % -> Boolean
--R integralBasis : () -> Vector %        latex : % -> String
--R lift : % -> UPUP                     norm : % -> Fraction UP
--R one? : % -> Boolean                  primitivePart : % -> %
--R ramified? : UP -> Boolean             ramified? : F -> Boolean
--R rank : () -> PositiveInteger           rationalPoint? : (F,F) -> Boolean
--R recip : % -> Union(%, "failed")       reduce : UPUP -> %
--R represents : (Vector UP,UP) -> %      retract : % -> Fraction UP
--R sample : () -> %                     singular? : UP -> Boolean
--R singular? : F -> Boolean              trace : % -> Fraction UP
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ??? : (%,Fraction Integer) -> % if Fraction UP has FIELD
--R ??? : (Fraction Integer,%) -> % if Fraction UP has FIELD
--R ??? : (NonNegativeInteger,%) -> %
--R ??? : (%,Integer) -> % if Fraction UP has FIELD
--R ??? : (%,NonNegativeInteger) -> %
--R ?/? : (%,%) -> % if Fraction UP has FIELD
--R D : % -> % if Fraction UP has DIFRING and Fraction UP has FIELD or Fraction UP has FIELD
--R D : (%,NonNegativeInteger) -> % if Fraction UP has DIFRING and Fraction UP has FIELD or Fraction UP has FIELD
--R D : (%,Symbol) -> % if Fraction UP has FIELD and Fraction UP has PDRING SYMBOL
--R D : (%,List Symbol) -> % if Fraction UP has FIELD and Fraction UP has PDRING SYMBOL

```

```

--R D : (% , Symbol, NonNegativeInteger) -> % if Fraction UP has FIELD and Fraction UP has PDRING SYMBOL
--R D : (% , List Symbol, List NonNegativeInteger) -> % if Fraction UP has FIELD and Fraction UP has PDRING SYMBOL
--R D : (% , (Fraction UP -> Fraction UP)) -> % if Fraction UP has FIELD
--R D : (% , (Fraction UP -> Fraction UP), NonNegativeInteger) -> % if Fraction UP has FIELD
--R ?? : (% , Integer) -> % if Fraction UP has FIELD
--R ?? : (% , NonNegativeInteger) -> %
--R absolutelyIrreducible? : () -> Boolean
--R algSplitSimple : (% , (UP -> UP)) -> Record(num: % , den: UP , derivden: UP , gd: UP)
--R associates? : (% , %) -> Boolean if Fraction UP has FIELD
--R branchPointAtInfinity? : () -> Boolean
--R characteristic : () -> NonNegativeInteger
--R characteristicPolynomial : % -> UPUP
--R charthRoot : % -> Union(% , "failed") if Fraction UP has CHARNZ
--R charthRoot : % -> % if Fraction UP has FFIELDC
--R coerce : % -> % if Fraction UP has FIELD
--R coerce : Fraction Integer -> % if Fraction UP has FIELD or Fraction UP has RETRACT FRAC INT
--R complementaryBasis : Vector % -> Vector %
--R conditionP : Matrix % -> Union(Vector % , "failed") if Fraction UP has FFIELDC
--R coordinates : Vector % -> Matrix Fraction UP
--R coordinates : % -> Vector Fraction UP
--R coordinates : (Vector % , Vector %) -> Matrix Fraction UP
--R coordinates : (% , Vector %) -> Vector Fraction UP
--R createPrimitiveElement : () -> % if Fraction UP has FFIELDC
--R derivationCoordinates : (Vector % , (Fraction UP -> Fraction UP)) -> Matrix Fraction UP if Fraction UP has FFIELDC
--R differentiate : % -> % if Fraction UP has DIFRING and Fraction UP has FIELD or Fraction UP has FFIELDC
--R differentiate : (% , NonNegativeInteger) -> % if Fraction UP has DIFRING and Fraction UP has FIELD or Fraction UP has FFIELDC
--R differentiate : (% , Symbol) -> % if Fraction UP has FIELD and Fraction UP has PDRING SYMBOL
--R differentiate : (% , List Symbol) -> % if Fraction UP has FIELD and Fraction UP has PDRING SYMBOL
--R differentiate : (% , Symbol, NonNegativeInteger) -> % if Fraction UP has FIELD and Fraction UP has PDRING SYMBOL
--R differentiate : (% , List Symbol, List NonNegativeInteger) -> % if Fraction UP has FIELD and Fraction UP has PDRING SYMBOL
--R differentiate : (% , (UP -> UP)) -> %
--R differentiate : (% , (Fraction UP -> Fraction UP)) -> % if Fraction UP has FIELD
--R differentiate : (% , (Fraction UP -> Fraction UP), NonNegativeInteger) -> % if Fraction UP has FIELD
--R discreteLog : (% , %) -> Union(NonNegativeInteger, "failed") if Fraction UP has FFIELDC
--R discreteLog : % -> NonNegativeInteger if Fraction UP has FFIELDC
--R discriminant : Vector % -> Fraction UP
--R divide : (% , %) -> Record(quotient: % , remainder: %) if Fraction UP has FIELD
--R elliptic : () -> Union(UP, "failed")
--R euclideanSize : % -> NonNegativeInteger if Fraction UP has FIELD
--R expressIdealMember : (List % , %) -> Union(List % , "failed") if Fraction UP has FIELD
--R exquo : (% , %) -> Union(% , "failed") if Fraction UP has FIELD
--R extendedEuclidean : (% , %) -> Record(coef1: % , coef2: % , generator: %) if Fraction UP has FIELD
--R extendedEuclidean : (% , % , %) -> Union(Record(coef1: % , coef2: % , "failed") if Fraction UP has FIELD
--R factor : % -> Factored % if Fraction UP has FIELD
--R factorsOfCyclicGroupSize : () -> List Record(factor: Integer, exponent: Integer) if Fraction UP has FIELD
--R gcd : (% , %) -> % if Fraction UP has FIELD
--R gcd : List % -> % if Fraction UP has FIELD
--R gcdPolynomial : (SparseUnivariatePolynomial % , SparseUnivariatePolynomial %) -> SparseUnivariatePolynomial %
--R hyperelliptic : () -> Union(UP, "failed")
--R index : PositiveInteger -> % if Fraction UP has FINITE

```

```

--R init : () -> % if Fraction UP has FFIELDC
--R integralAtInfinity? : % -> Boolean
--R integralBasisAtInfinity : () -> Vector %
--R integralCoordinates : % -> Record(num: Vector UP,den: UP)
--R integralDerivationMatrix : (UP -> UP) -> Record(num: Matrix UP,den: UP)
--R integralMatrix : () -> Matrix Fraction UP
--R integralMatrixAtInfinity : () -> Matrix Fraction UP
--R integralRepresents : (Vector UP,UP) -> %
--R inv : % -> % if Fraction UP has FIELD
--R inverseIntegralMatrix : () -> Matrix Fraction UP
--R inverseIntegralMatrixAtInfinity : () -> Matrix Fraction UP
--R knownInfBasis : NonNegativeInteger -> Void
--R lcm : (% ,%) -> % if Fraction UP has FIELD
--R lcm : List % -> % if Fraction UP has FIELD
--R lookup : % -> PositiveInteger if Fraction UP has FINITE
--R minimalPolynomial : % -> UPUP if Fraction UP has FIELD
--R multiEuclidean : (List % ,%) -> Union(List % ,"failed") if Fraction UP has FIELD
--R nextItem : % -> Union(% ,"failed") if Fraction UP has FFIELDC
--R nonSingularModel : Symbol -> List Polynomial F if F has FIELD
--R normalizeAtInfinity : Vector % -> Vector %
--R numberOfComponents : () -> NonNegativeInteger
--R order : % -> OnePointCompletion PositiveInteger if Fraction UP has FFIELDC
--R order : % -> PositiveInteger if Fraction UP has FFIELDC
--R prime? : % -> Boolean if Fraction UP has FIELD
--R primeFrobenius : % -> % if Fraction UP has FFIELDC
--R primeFrobenius : (% ,NonNegativeInteger) -> % if Fraction UP has FFIELDC
--R primitive? : % -> Boolean if Fraction UP has FFIELDC
--R primitiveElement : () -> % if Fraction UP has FFIELDC
--R principalIdeal : List % -> Record(coef: List % ,generator: %) if Fraction UP has FIELD
--R ?quo? : (% ,%) -> % if Fraction UP has FIELD
--R ramifiedAtInfinity? : () -> Boolean
--R random : () -> % if Fraction UP has FINITE
--R rationalPoints : () -> List List F if F has FINITE
--R reduce : Fraction UPUP -> Union(% ,"failed") if Fraction UP has FIELD
--R reduceBasisAtInfinity : Vector % -> Vector %
--R reducedSystem : Matrix % -> Matrix Fraction UP
--R reducedSystem : (Matrix % ,Vector %) -> Record(mat: Matrix Fraction UP,vec: Vector Fraction UP)
--R reducedSystem : (Matrix % ,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) if Fraction UP has FIELD
--R reducedSystem : Matrix % -> Matrix Integer if Fraction UP has LINEP INT
--R regularRepresentation : % -> Matrix Fraction UP
--R regularRepresentation : (% ,Vector %) -> Matrix Fraction UP
--R ?rem? : (% ,%) -> % if Fraction UP has FIELD
--R representationType : () -> Union("prime",polynomial,normal,cyclic) if Fraction UP has FFIELDC
--R represents : Vector Fraction UP -> %
--R represents : (Vector Fraction UP,Vector %) -> %
--R retract : % -> Fraction Integer if Fraction UP has RETRACT FRAC INT
--R retract : % -> Integer if Fraction UP has RETRACT INT
--R retractIfCan : % -> Union(Fraction UP ,"failed")
--R retractIfCan : % -> Union(Fraction Integer ,"failed") if Fraction UP has RETRACT FRAC INT
--R retractIfCan : % -> Union(Integer ,"failed") if Fraction UP has RETRACT INT

```

```

--R singularAtInfinity? : () -> Boolean
--R size : () -> NonNegativeInteger if Fraction UP has FINITE
--R sizeLess? : (%,%) -> Boolean if Fraction UP has FIELD
--R squareFree : % -> Factored % if Fraction UP has FIELD
--R squareFreePart : % -> % if Fraction UP has FIELD
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger,NonNegativeInteger) if Fraction UP has
--R traceMatrix : () -> Matrix Fraction UP
--R traceMatrix : Vector % -> Matrix Fraction UP
--R unit? : % -> Boolean if Fraction UP has FIELD
--R unitCanonical : % -> % if Fraction UP has FIELD
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if Fraction UP has FIELD
--R yCoordinates : % -> Record(num: Vector UP,den: UP)
--R
--E 1

```

```

)spool
)lisp (bye)

```

---

— AlgebraicFunctionField.help —

```

=====
AlgebraicFunctionField examples
=====

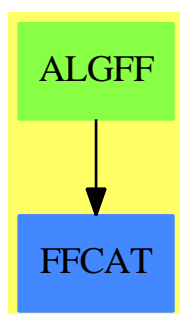
```

```

See Also:
o )show AlgebraicFunctionField

```

### 2.5.1 AlgebraicFunctionField (ALGFF)



See



⇒ “RadicalFunctionField” (RADFF) 19.1.1 on page 2153

**Exports:**

|                                 |                          |                           |
|---------------------------------|--------------------------|---------------------------|
| 1                               | 0                        | absolutelyIrreducible?    |
| algSplitSimple                  | associates?              | basis                     |
| branchPoint?                    | branchPointAtInfinity?   | characteristic            |
| characteristicPolynomial        | charthRoot               | coerce                    |
| complementaryBasis              | conditionP               | convert                   |
| coordinates                     | createPrimitiveElement   | D                         |
| definingPolynomial              | derivationCoordinates    | differentiate             |
| discreteLog                     | discriminant             | divide                    |
| elliptic                        | elt                      | euclideanSize             |
| expressIdealMember              | exquo                    | extendedEuclidean         |
| factor                          | factorsOfCyclicGroupSize | gcd                       |
| gcdPolynomial                   | generator                | genus                     |
| hash                            | hyperelliptic            | index                     |
| init                            | integral?                | integralAtInfinity?       |
| integralBasis                   | integralBasisAtInfinity  | integralCoordinates       |
| integralDerivationMatrix        | integralMatrix           | integralMatrixAtInfinity  |
| integralRepresents              | inv                      | inverseIntegralMatrix     |
| inverseIntegralMatrixAtInfinity | knownInfBasis            | latex                     |
| lcm                             | lift                     | lookup                    |
| minimalPolynomial               | multiEuclidean           | nextItem                  |
| nonSingularModel                | norm                     | normalizeAtInfinity       |
| numberOfComponents              | one?                     | order                     |
| prime?                          | primeFrobenius           | primitive?                |
| primitiveElement                | primitivePart            | principalIdeal            |
| ramified?                       | ramifiedAtInfinity?      | random                    |
| rank                            | rationalPoint?           | rationalPoints            |
| recip                           | reduce                   | reduceBasisAtInfinity     |
| reducedSystem                   | regularRepresentation    | representationType        |
| represents                      | retract                  | retractIfCan              |
| sample                          | singular?                | singularAtInfinity?       |
| size                            | sizeLess?                | squareFree                |
| squareFreePart                  | subtractIfCan            | tableForDiscreteLogarithm |
| trace                           | traceMatrix              | unit?                     |
| unitCanonical                   | unitNormal               | yCoordinates              |
| zero?                           | ?*?                      | ?**?                      |
| ?+?                             | ?-?                      | -?                        |
| ?=?                             | ?^?                      | ?~=?                      |
| ?/?                             | ?quo?                    | ?rem?                     |

— domain ALGFF AlgebraicFunctionField —

```
)abbrev domain ALGFF AlgebraicFunctionField
++ Author: Manuel Bronstein
```

```

++ Date Created: 3 May 1988
++ Date Last Updated: 24 Jul 1990
++ Keywords: algebraic, curve, function, field.
++ Description:
++ Function field defined by  $f(x, y) = 0$ .

AlgebraicFunctionField(F, UP, UPUP, modulus): Exports == Impl where
  F      : Field
  UP      : UnivariatePolynomialCategory F
  UPUP    : UnivariatePolynomialCategory Fraction UP
  modulus: UPUP

  N ==> NonNegativeInteger
  Z ==> Integer
  RF ==> Fraction UP
  QF ==> Fraction UPUP
  UP2 ==> SparseUnivariatePolynomial UP
  SAE ==> SimpleAlgebraicExtension(RF, UPUP, modulus)
  INIT ==> if (deref brandNew?) then startUp false

Exports ==> FunctionFieldCategory(F, UP, UPUP) with
  knownInfBasis: N -> Void
  ++ knownInfBasis(n) is not documented

Impl ==> SAE add
  import ChangeOfVariable(F, UP, UPUP)
  import InnerCommonDenominator(UP, RF, Vector UP, Vector RF)
  import MatrixCommonDenominator(UP, RF)
  import UnivariatePolynomialCategoryFunctions2(RF, UPUP, UP, UP2)

  startUp      : Boolean -> Void
  vect         : Matrix RF -> Vector $
  getInfBasis: () -> Void

  brandNew?:Reference(Boolean) := ref true
  infBr?:Reference(Boolean) := ref true
  discPoly:Reference(RF) := ref 0
  n := degree modulus
  n1 := (n - 1)::N
  ibasis:Matrix(RF) := zero(n, n)
  invibasis:Matrix(RF) := copy ibasis
  infbasis:Matrix(RF) := copy ibasis
  invinfbasis:Matrix(RF) := copy ibasis

  branchPointAtInfinity?() == (INIT; infBr?())
  discriminant() == (INIT; discPoly())
  integralBasis() == (INIT; vect ibasis)
  integralBasisAtInfinity() == (INIT; vect infbasis)
  integralMatrix() == (INIT; ibasis)
  inverseIntegralMatrix() == (INIT; invibasis)

```

```

integralMatrixAtInfinity() == (INIT; infbasis)
branchPoint?(a:F)          == zero?((retract(discriminant())@UP) a)
definingPolynomial()       == modulus
inverseIntegralMatrixAtInfinity() == (INIT; invinfbasis)

vect m ==
  [represents row(m, i) for i in minRowIndex m .. maxRowIndex m]

integralCoordinates f ==
  splitDenominator(coordinates(f) * inverseIntegralMatrix())

knownInfBasis d ==
  if deref brandNew? then
    alpha := [monomial(1, d * i)$UP :: RF for i in 0..n1]$Vector(RF)
    ib := diagonalMatrix
      [inv qelt(alpha, i) for i in minIndex alpha .. maxIndex alpha]
    invib := diagonalMatrix alpha
    for i in minRowIndex ib .. maxRowIndex ib repeat
      for j in minColIndex ib .. maxColIndex ib repeat
        infbasis(i, j) := qelt(ib, i, j)
        invinfbasis(i, j) := invib(i, j)
  void

getInfBasis() ==
  x := inv(monomial(1, 1)$UP :: RF)
  invmod := map(s +-> s(x), modulus)
  r := mkIntegral invmod
  degree(r.poly) ^= n => error "Should not happen"
  ninvmmod:UP2 := map(s +-> retract(s@UP, r.poly)
  alpha := [(r.coef ** i) x for i in 0..n1]$Vector(RF)
  invalpha := [inv qelt(alpha, i)
    for i in minIndex alpha .. maxIndex alpha]$Vector(RF)
  invib := integralBasis()$FunctionFieldIntegralBasis(UP, UP2,
    SimpleAlgebraicExtension(UP, UP2, ninvmmod))
  for i in minRowIndex ibasis .. maxRowIndex ibasis repeat
    for j in minColIndex ibasis .. maxColIndex ibasis repeat
      infbasis(i, j) := ((invib.basis)(i,j) / invib.basisDen) x
      invinfbasis(i, j) := ((invib.basisInv)(i, j)) x
  ib2 := infbasis * diagonalMatrix alpha
  invib2 := diagonalMatrix(invalpha) * invinfbasis
  for i in minRowIndex ib2 .. maxRowIndex ib2 repeat
    for j in minColIndex ibasis .. maxColIndex ibasis repeat
      infbasis(i, j) := qelt(ib2, i, j)
      invinfbasis(i, j) := invib2(i, j)
  void

startUp b ==
  brandNew?() := b
  nmod:UP2 := map(retract, modulus)
  ib := integralBasis()$FunctionFieldIntegralBasis(UP, UP2,

```

```

SimpleAlgebraicExtension(UP, UP2, nmod))
for i in minRowIndex ibasis .. maxRowIndex ibasis repeat
  for j in minColIndex ibasis .. maxColIndex ibasis repeat
    qsetelt_!(ibasis, i, j, (ib.basis)(i, j) / ib.basisDen)
    invibasis(i, j) := ((ib.basisInv) (i, j))::RF
  if zero?(infbasis(minRowIndex infbasis, minColIndex infbasis))
    then getInfBasis()
  ib2 := coordinates normalizeAtInfinity vect ibasis
  invib2 := inverse(ib2)::Matrix(RF)
  for i in minRowIndex ib2 .. maxRowIndex ib2 repeat
    for j in minColIndex ib2 .. maxColIndex ib2 repeat
      ibasis(i, j) := qelt(ib2, i, j)
      invibasis(i, j) := invib2(i, j)
  dsc := resultant(modulus, differentiate modulus)
  dsc0 := dsc * determinant(infbasis) ** 2
  degree(number dsc0) > degree(denom dsc0) =>error "Shouldn't happen"
  infBr?() := degree(number dsc0) < degree(denom dsc0)
  dsc := dsc * determinant(ibasis) ** 2
  discPoly() := primitivePart(number dsc) / denom(dsc)
  void

integralDerivationMatrix d ==
  w := integralBasis()
  splitDenominator(coordinates([differentiate(w.i, d)
    for i in minIndex w .. maxIndex w]$Vector($))
    * inverseIntegralMatrix())

integralRepresents(v, d) ==
  represents(coordinates(represents(v, d)) * integralMatrix())

branchPoint?(p:UP) ==
  INIT
  (r:=retractIfCan(p)@Union(F,"failed")) case F =>branchPoint?(r::F)
  not ground? gcd(retract(discriminant())@UP, p)

```

---

— ALGFF.dotabb —

```

"ALGFF" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALGFF"]
"FFCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FFCAT"]
"ALGFF" -> "FFCAT"

```

---

## 2.6 domain AN AlgebraicNumber

— AlgebraicNumber.input —

```

)set break resume
)sys rm -f AlgebraicNumber.output
)spool AlgebraicNumber.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show AlgebraicNumber
--R AlgebraicNumber is a domain constructor
--R Abbreviation for AlgebraicNumber is AN
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for AN
--R
--R----- Operations -----
--R ??? : (PositiveInteger,%) -> %      ??? : (Integer,%) -> %
--R ??? : (%,%) -> %                  ??? : (%,Fraction Integer) -> %
--R ??? : (Fraction Integer,%) -> %    ??? : (%,PositiveInteger) -> %
--R ??? : (%,Integer) -> %           ??? : (%,Fraction Integer) -> %
--R ?+? : (%,%) -> %                 -? : % -> %
--R ?-? : (%,%) -> %                 ?/? : (%,%) -> %
--R ?<? : (%,%) -> Boolean           ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean           ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean          D : % -> %
--R D : (%,NonNegativeInteger) -> %  1 : () -> %
--R 0 : () -> %                      ?^? : (%,PositiveInteger) -> %
--R ?^? : (%,Integer) -> %           associates? : (%,%) -> Boolean
--R belong? : BasicOperator -> Boolean box : List % -> %
--R box : % -> %                     coerce : Integer -> %
--R coerce : % -> %                  coerce : Fraction Integer -> %
--R coerce : Kernel % -> %           coerce : % -> OutputForm
--R convert : % -> Complex Float      convert : % -> DoubleFloat
--R convert : % -> Float              differentiate : % -> %
--R distribute : (%,%) -> %           distribute : % -> %
--R elt : (BasicOperator,%,%) -> %    elt : (BasicOperator,%) -> %
--R eval : (%,List %,List %) -> %     eval : (%,%,%) -> %
--R eval : (%,Equation %) -> %        eval : (%,List Equation %) -> %
--R eval : (%,Kernel %,%) -> %        factor : % -> Factored %
--R freeOf? : (%,Symbol) -> Boolean   freeOf? : (%,%) -> Boolean
--R gcd : (%,%) -> %                  gcd : List % -> %
--R hash : % -> SingleInteger          height : % -> NonNegativeInteger
--R inv : % -> %                       is? : (%,Symbol) -> Boolean
--R kernel : (BasicOperator,%) -> %   kernels : % -> List Kernel %
--R latex : % -> String                lcm : (%,%) -> %

```

```

--R lcm : List % -> %
--R max : (% , %) -> %
--R norm : (% , List Kernel %) -> %
--R nthRoot : (% , Integer) -> %
--R paren : List % -> %
--R prime? : % -> Boolean
--R recip : % -> Union(% , "failed")
--R ?rem? : (% , %) -> %
--R retract : % -> Integer
--R rootOf : Polynomial % -> %
--R sample : () -> %
--R sqrt : % -> %
--R squareFreePart : % -> %
--R tower : % -> List Kernel %
--R unitCanonical : % -> %
--R zeroOf : Polynomial % -> %
--R ?~=?: (% , %) -> Boolean
--R ?*?: (NonNegativeInteger , %) -> %
--R ?**?: (% , NonNegativeInteger) -> %
--R ?^?: (% , NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R coerce : SparseMultivariatePolynomial(Integer , Kernel %) -> %
--R definingPolynomial : % -> % if $ has RING
--R denom : % -> SparseMultivariatePolynomial(Integer , Kernel %)
--R differentiate : (% , NonNegativeInteger) -> %
--R divide : (% , %) -> Record(quotient: % , remainder: %)
--R elt : (BasicOperator , List %) -> %
--R elt : (BasicOperator , % , % , % , %) -> %
--R elt : (BasicOperator , % , % , %) -> %
--R euclideanSize : % -> NonNegativeInteger
--R eval : (% , BasicOperator , (% -> %)) -> %
--R eval : (% , BasicOperator , (List % -> %)) -> %
--R eval : (% , List BasicOperator , List (List % -> %)) -> %
--R eval : (% , List BasicOperator , List (% -> %)) -> %
--R eval : (% , Symbol , (% -> %)) -> %
--R eval : (% , Symbol , (List % -> %)) -> %
--R eval : (% , List Symbol , List (List % -> %)) -> %
--R eval : (% , List Symbol , List (% -> %)) -> %
--R eval : (% , List Kernel % , List %) -> %
--R even? : % -> Boolean if $ has RETRACT INT
--R expressIdealMember : (List % , %) -> Union(List % , "failed")
--R exquo : (% , %) -> Union(% , "failed")
--R extendedEuclidean : (% , %) -> Record(coef1: % , coef2: % , generator: %)
--R extendedEuclidean : (% , % , %) -> Union(Record(coef1: % , coef2: % ) , "failed")
--R gcdPolynomial : (SparseUnivariatePolynomial % , SparseUnivariatePolynomial %) -> SparseUnivariatePolynomial %
--R is? : (% , BasicOperator) -> Boolean
--R kernel : (BasicOperator , List %) -> %
--R mainKernel : % -> Union(Kernel % , "failed")
--R minPoly : Kernel % -> SparseUnivariatePolynomial % if $ has RING
--R multiEuclidean : (List % , %) -> Union(List % , "failed")
--R map : ((% -> % ) , Kernel %) -> %
--R min : (% , %) -> %
--R norm : (% , Kernel %) -> %
--R one? : % -> Boolean
--R paren : % -> %
--R ?quo? : (% , %) -> %
--R reduce : % -> %
--R retract : % -> Fraction Integer
--R retract : % -> Kernel %
--R rootsOf : Polynomial % -> List %
--R sizeLess? : (% , %) -> Boolean
--R squareFree : % -> Factored %
--R subst : (% , Equation %) -> %
--R unit? : % -> Boolean
--R zero? : % -> Boolean
--R zerosOf : Polynomial % -> List %

```

```

--R norm : (SparseUnivariatePolynomial %,List Kernel %) -> SparseUnivariatePolynomial %
--R norm : (SparseUnivariatePolynomial %,Kernel %) -> SparseUnivariatePolynomial %
--R numer : % -> SparseMultivariatePolynomial(Integer,Kernel %)
--R odd? : % -> Boolean if $ has RETRACT INT
--R operator : BasicOperator -> BasicOperator
--R operators : % -> List BasicOperator
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R reducedSystem : Matrix % -> Matrix Fraction Integer
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Fraction Integer,vec: Vector F
--R reducedSystem : Matrix % -> Matrix Integer
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer)
--R retractIfCan : % -> Union(Fraction Integer,"failed")
--R retractIfCan : % -> Union(Integer,"failed")
--R retractIfCan : % -> Union(Kernel %,"failed")
--R rootOf : SparseUnivariatePolynomial % -> %
--R rootOf : (SparseUnivariatePolynomial %,Symbol) -> %
--R rootsOf : SparseUnivariatePolynomial % -> List %
--R rootsOf : (SparseUnivariatePolynomial %,Symbol) -> List %
--R subst : (%,List Kernel %,List %) -> %
--R subst : (%,List Equation %) -> %
--R subtractIfCan : (%,%) -> Union(%, "failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R zeroOf : SparseUnivariatePolynomial % -> %
--R zeroOf : (SparseUnivariatePolynomial %,Symbol) -> %
--R zerosOf : SparseUnivariatePolynomial % -> List %
--R zerosOf : (SparseUnivariatePolynomial %,Symbol) -> List %
--R
--E 1

)spool
)lisp (bye)

```

---

— AlgebraicNumber.help —

```

=====
AlgebraicNumber examples
=====

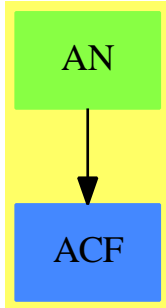
```

```

See Also:
o )show AlgebraicNumber

```

## 2.6.1 AlgebraicNumber (AN)



See

⇒ “InnerAlgebraicNumber” (IAN) 10.20.1 on page 1240

**Exports:**

|                   |                    |                    |               |
|-------------------|--------------------|--------------------|---------------|
| 1                 | 0                  | associates?        | belong?       |
| box               | characteristic     | coerce             | convert       |
| D                 | definingPolynomial | denom              | differentiate |
| distribute        | divide             | elt                | euclideanSize |
| eval              | even?              | expressIdealMember | exquo         |
| extendedEuclidean | factor             | freeOf?            | gcd           |
| gcdPolynomial     | hash               | height             | inv           |
| is?               | kernel             | kernels            | latex         |
| lcm               | mainKernel         | map                | max           |
| min               | minPoly            | multiEuclidean     | norm          |
| nthRoot           | numer              | odd?               | one?          |
| operator          | operators          | paren              | prime?        |
| principalIdeal    | recip              | reduce             | reducedSystem |
| retract           | retractIfCan       | rootOf             | rootsOf       |
| sample            | sizeLess?          | sqrt               | squareFree    |
| squareFreePart    | subst              | subtractIfCan      | tower         |
| unit?             | unitCanonical      | unitNormal         | zero?         |
| zeroOf            | zerosOf            | ?*?                | ?**?          |
| ?+?               | -?                 | ?-?                | ?/?           |
| ?<?               | ?<=?               | ?=?                | ?>?           |
| ?>=?              | ?^?                | ?quo?              | ?rem?         |
| ?~=?              |                    |                    |               |

— domain AN AlgebraicNumber —

```
)abbrev domain AN AlgebraicNumber
++ Author: James Davenport
++ Date Created: 9 October 1995
++ Date Last Updated: 10 October 1995 (JHD)
++ Keywords: algebraic, number.
```



```

++ Description:
++ Algebraic closure of the rational numbers, with mathematical =

AlgebraicNumber(): Exports == Implementation where
  Z ==> Integer
  P ==> SparseMultivariatePolynomial(Z, Kernel %)
  SUP ==> SparseUnivariatePolynomial

Exports ==> Join(ExpressionSpace, AlgebraicallyClosedField,
  RetractableTo Z, RetractableTo Fraction Z,
  LinearlyExplicitRingOver Z, RealConstant,
  LinearlyExplicitRingOver Fraction Z,
  CharacteristicZero,
  ConvertibleTo Complex Float, DifferentialRing) with

coerce : P -> %
  ++ coerce(p) returns p viewed as an algebraic number.
numer : % -> P
  ++ numer(f) returns the numerator of f viewed as a
  ++ polynomial in the kernels over Z.
denom : % -> P
  ++ denom(f) returns the denominator of f viewed as a
  ++ polynomial in the kernels over Z.
reduce : % -> %
  ++ reduce(f) simplifies all the unreduced algebraic numbers
  ++ present in f by applying their defining relations.
norm : (SUP(%),Kernel %) -> SUP(%)
  ++ norm(p,k) computes the norm of the polynomial p
  ++ with respect to the extension generated by kernel k
norm : (SUP(%),List Kernel %) -> SUP(%)
  ++ norm(p,l) computes the norm of the polynomial p
  ++ with respect to the extension generated by kernels l
norm : (% ,Kernel %) -> %
  ++ norm(f,k) computes the norm of the algebraic number f
  ++ with respect to the extension generated by kernel k
norm : (% ,List Kernel %) -> %
  ++ norm(f,l) computes the norm of the algebraic number f
  ++ with respect to the extension generated by kernels l
Implementation ==> InnerAlgebraicNumber add
Rep:=InnerAlgebraicNumber
a,b:%
zero? a == trueEqual(a::Rep,0::Rep)
one? a == trueEqual(a::Rep,1::Rep)
a=b == trueEqual((a-b)::Rep,0::Rep)

```

```
"AN" [color="#88FF44",href="bookvol10.3.pdf#nameddest=AN"]
"ACF" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ACF"]
"AN" -> "ACF"
```

## 2.7 domain ANON AnonymousFunction

— AnonymousFunction.input —

```
)set break resume
)sys rm -f AnonymousFunction.output
)spool AnonymousFunction.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show AnonymousFunction
--R AnonymousFunction is a domain constructor
--R Abbreviation for AnonymousFunction is ANON
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ANON
--R
--R----- Operations -----
--R ?=? : (% ,%) -> Boolean          coerce : % -> OutputForm
--R hash : % -> SingleInteger       latex : % -> String
--R ?~=? : (% ,%) -> Boolean
--R
--E 1

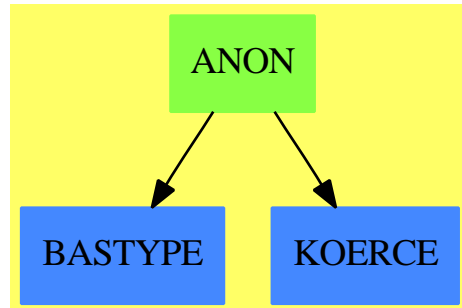
)spool
)lisp (bye)
```

— AnonymousFunction.help —

```
=====
AnonymousFunction examples
=====
```

```
See Also:
o )show AnonymousFunction
```

### 2.7.1 AnonymousFunction (ANON)



#### Exports:

coerce hash latex  $?=?$   $?^{\sim}=?$

— domain ANON AnonymousFunction —

```

)abbrev domain ANON AnonymousFunction
++ Author: Mark Botch
++ Description:
++ This domain implements anonymous functions

AnonymousFunction():SetCategory == add
  coerce(x:%):OutputForm == x pretend OutputForm
  
```

— ANON.dotabb —

```

"ANON" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ANON"]
"BASTYPE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=BASTYPE"]
"KOERCE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=KOERCE"]
"ANON" -> "BASTYPE"
"ANON" -> "KOERCE"
  
```

## 2.8 domain ANTISYM AntiSymm

## — AntiSymm.input —

```

)set break resume
)sys rm -f AntiSymm.output
)spool AntiSymm.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show AntiSymm
--R AntiSymm(R: Ring,lVar: List Symbol) is a domain constructor
--R Abbreviation for AntiSymm is ANTISYM
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ANTISYM
--R
--R----- Operations -----
--R ?? : (R,%) -> %           ?? : (%,%) -> %
--R ?? : (Integer,%) -> %     ?? : (PositiveInteger,%) -> %
--R ***? : (%,PositiveInteger) -> %  ?+? : (%,%) -> %
--R ?-? : (%,%) -> %         -? : % -> %
--R ?? : (%,%) -> Boolean     1 : () -> %
--R 0 : () -> %              ?? : (%,PositiveInteger) -> %
--R coefficient : (%,%) -> R   coerce : R -> %
--R coerce : Integer -> %     coerce : % -> OutputForm
--R degree : % -> NonNegativeInteger  exp : List Integer -> %
--R hash : % -> SingleInteger  homogeneous? : % -> Boolean
--R latex : % -> String       leadingBasisTerm : % -> %
--R leadingCoefficient : % -> R  map : ((R -> R),%) -> %
--R one? : % -> Boolean        recip : % -> Union(%, "failed")
--R reductum : % -> %          retract : % -> R
--R retractable? : % -> Boolean  sample : () -> %
--R zero? : % -> Boolean       ~=? : (%,%) -> Boolean
--R ?? : (NonNegativeInteger,%) -> %
--R ***? : (%,NonNegativeInteger) -> %
--R ?? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R generator : NonNegativeInteger -> %
--R retractIfCan : % -> Union(R, "failed")
--R subtractIfCan : (%,%) -> Union(%, "failed")
--R
--E 1

)spool
)lisp (bye)

```

## — AntiSymm.help —

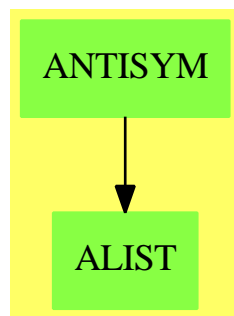
```
=====
AntiSymm examples
=====
```

See Also:

```
o )show AntiSymm
```

—————

### 2.8.1 AntiSymm (ANTISYM)



See

⇒ “ExtAlgBasis” (EAB) 6.8.1 on page 711

⇒ “DeRhamComplex” (DERHAM) 5.6.1 on page 515

#### Exports:

|        |                  |                    |              |               |
|--------|------------------|--------------------|--------------|---------------|
| 1      | 0                | coefficient        | coerce       | coerce        |
| coerce | degree           | exp                | hash         | homogeneous?  |
| latex  | leadingBasisTerm | leadingCoefficient | map          | one?          |
| recip  | reductum         | retract            | retractable? | sample        |
| zero?  | characteristic   | generator          | retractIfCan | subtractIfCan |
| ?*?    | ?**?             | ?+?                | ?-?          | -?            |
| ?=?    | ?^?              | ?~=?               |              |               |

— domain ANTISYM AntiSymm —

```
)abbrev domain ANTISYM AntiSymm
```

```
++ Author: Larry A. Lambe
```

```
++ Date : 01/26/91.
```

```
++ Revised : 30 Nov 94
```

```
++ Description:
```

```
++ The domain of antisymmetric polynomials.
```

```
AntiSymm(R:Ring, lVar:List Symbol): Export == Implement where
```

```
  LALG ==> LeftAlgebra
```

```

FMR ==> FM(R,EAB)
FM   ==> FreeModule
I    ==> Integer
L    ==> List
EAB  ==> ExtAlgBasis      -- these are exponents of basis elements in order
NNI  ==> NonNegativeInteger
O    ==> OutputForm
base ==> k
coef ==> c
Term ==> Record(k:EAB,c:R)

Export == Join(LALG(R), RetractableTo(R)) with
  leadingCoefficient : %          -> R
  ++ leadingCoefficient(p) returns the leading
  ++ coefficient of antisymmetric polynomial p.
-- leadingSupport    : %          -> EAB
  leadingBasisTerm   : %          -> %
  ++ leadingBasisTerm(p) returns the leading
  ++ basis term of antisymmetric polynomial p.
  reductum           : %          -> %
  ++ reductum(p), where p is an antisymmetric polynomial,
  ++ returns p minus the leading
  ++ term of p if p has at least two terms, and 0 otherwise.
  coefficient         : (%,% )    -> R
  ++ coefficient(p,u) returns the coefficient of
  ++ the term in p containing the basis term u if such
  ++ a term exists, and 0 otherwise.
  ++ Error: if the second argument u is not a basis element.
  generator           : NNI        -> %
  ++ generator(n) returns the nth multiplicative generator,
  ++ a basis term.
  exp                 : L I        -> %
  ++ exp([i1,...in]) returns \spad{u_1\^{i_1} ... u_n\^{i_n}}
  homogeneous?        : %          -> Boolean
  ++ homogeneous?(p) tests if all of the terms of
  ++ p have the same degree.
  retractable?        : %          -> Boolean
  ++ retractable?(p) tests if p is a 0-form,
  ++ i.e., if degree(p) = 0.
  degree              : %          -> NNI
  ++ degree(p) returns the homogeneous degree of p.
  map                  : (R -> R, %) -> %
  ++ map(f,p) changes each coefficient of p by the
  ++ application of f.

-- 1 corresponds to the empty monomial Nul = [0,...,0]
-- from EAB. In terms of the exterior algebra on X,
-- it corresponds to the identity element which lives
-- in homogeneous degree 0.

```

```

Implement == FMR add
  Rep := L Term
  x,y : EAB
  a,b : %
  r : R
  m : I

  dim := #lVar

  1 == [[ Nul(dim)$EAB, 1$R ]]

  coefficient(a,u) ==
    not null u.rest => error "2nd argument must be a basis element"
    x := u.first.base
    for t in a repeat
      if t.base = x then return t.coef
      if t.base < x then return 0
    0

  retractable?(a) ==
    null a or (a.first.k = Nul(dim))

  retractIfCan(a):Union(R,"failed") ==
    null a => 0$R
    a.first.k = Nul(dim) => leadingCoefficient a
    "failed"

  retract(a):R ==
    null a => 0$R
    leadingCoefficient a

  homogeneous? a ==
    null a => true
    siz := _+/exponents(a.first.base)
    for ta in reductum a repeat
      _+/exponents(ta.base) ^= siz => return false
    true

  degree a ==
    null a => 0$NNI
    homogeneous? a => (_+/exponents(a.first.base)) :: NNI
    error "not a homogeneous element"

  zo : (I,I) -> L I
  zo(p,q) ==
    p = 0 => [1,q]
    q = 0 => [1,1]
    [0,0]

```

```

getsgn : (EAB,EAB) -> I
getsgn(x,y) ==
  sgn:I := 0
  xx:L I := exponents x
  yy:L I := exponents y
  for i in 1 .. (dim-1) repeat
    xx := rest xx
    sgn := sgn + (_+/xx)*yy.i
  sgn rem 2 = 0 => 1
  -1

Nalpha: (EAB,EAB) -> L I
Nalpha(x,y) ==
  i:I := 1
  dum2:L I := [0 for i in 1..dim]
  for j in 1..dim repeat
    dum:=zo((exponents x).j,(exponents y).j)
    (i:= i*dum.1) = 0 => leave
    dum2.j := dum.2
  i = 0 => cons(i, dum2)
  cons(getsgn(x,y), dum2)

a * b ==
  null a => 0
  null b => 0
  ((null a.rest) and (a.first.k = Nul(dim))) => a.first.c * b
  ((null b.rest) and (b.first.k = Nul(dim))) => b.first.c * a
  z:% := 0
  for tb in b repeat
    for ta in a repeat
      stuff:=Nalpha(ta.base,tb.base)
      r:=first(stuff)*ta.coef*tb.coef
      if r ^= 0 then z := z + [[rest(stuff)::EAB, r]]
  z

coerce(r):% ==
  r = 0 => 0
  [ [Nul(dim), r] ]

coerce(m):% ==
  m = 0 => 0
  [ [Nul(dim), m::R] ]

characteristic() == characteristic()$R

generator(j) ==
  -- j < 1 or j > dim => error "your subscript is out of range"
  -- error will be generated by dum.j if out of range
  dum:L I := [0 for i in 1..dim]
  dum.j:=1

```



```

[[dum::EAB, 1::R]]

exp(li:(L I)) == [[li::EAB, 1]]

leadingBasisTerm a ==
  [[a.first.k, 1]]

displayList:EAB -> 0
displayList(x):0 ==
  le: L I := exponents(x)$EAB
-- reduce(*, [(lVar.i)::0 for i in 1..dim | le.i = 1])$L(0)
-- reduce(*, [(lVar.i)::0 for i in 1..dim | one?(le.i)])$L(0)
  reduce(*, [(lVar.i)::0 for i in 1..dim | ((le.i) = 1)])$L(0)

makeTerm:(R,EAB) -> 0
makeTerm(r,x) ==
-- we know that r ^= 0
  x = Nul(dim)$EAB => r::0
-- one? r => displayList(x)
  (r = 1) => displayList(x)
-- r = 1 => displayList(x)
-- r = 0 => 0$I::0
-- x = Nul(dim)$EAB => r::0
  r::0 * displayList(x)

coerce(a):0 ==
  zero? a      => 0$I::0
  null rest(a @ Rep) =>
    t := first(a @ Rep)
    makeTerm(t.coef,t.base)
  reduce(+,[makeTerm(t.coef,t.base) for t in (a @ Rep)])$L(0)

-----

— ANTISYM.dotabb —

"ANTISYM" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ANTISYM"]
"ALIST"   [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"ANTISYM" -> "ALIST"

-----

```

## 2.9 domain ANY Any

— Any.input —

```

)set break resume
)sys rm -f Any.output
)spool Any.output
)set message test on
)set message auto off
)clear all

--S 1 of 18
a:Any := [1,2]
--R
--R
--R (1) [1,2]
--R
--R                                         Type: List PositiveInteger
--E 1

--S 2 of 18
b:Any := [1,2]
--R
--R
--R (2) [1,2]
--R
--R                                         Type: List PositiveInteger
--E 2

--S 3 of 18
(a = b)@Boolean
--R
--R
--R (3) true
--R
--R                                         Type: Boolean
--E 3

--S 4 of 18
c := [1,2]
--R
--R
--R (4) [1,2]
--R
--R                                         Type: List PositiveInteger
--E 4

--S 5 of 18
typeOf a
--R
--R
--R (5) Any
--R
--R                                         Type: Domain
--E 5

--S 6 of 18
typeOf c
--R

```

```

--R
--R (6) List PositiveInteger
--R
--R                                         Type: Domain
--E 6

--S 7 of 18
(a = c)@Boolean
--R
--R
--R (7) true
--R
--R                                         Type: Boolean
--E 7

--S 8 of 18
b := [1,3]
--R
--R
--R (8) [1,3]
--R
--R                                         Type: List PositiveInteger
--E 8

--S 9 of 18
(a = b)@Boolean
--R
--R
--R (9) false
--R
--R                                         Type: Boolean
--E 9

--S 10 of 18
a := "A"
--R
--R
--R (10) "A"
--R
--R                                         Type: String
--E 10

--S 11 of 18
(a = b)@Boolean
--R
--R
--R (11) false
--R
--R                                         Type: Boolean
--E 11

--S 12 of 18
b := "A"
--R
--R
--R (12) "A"

```

```

--R                                                    Type: String
--E 12

--S 13 of 18
(a = b)@Boolean
--R
--R
--R (13) true
--R
--R                                                    Type: Boolean
--E 13

--S 14 of 18
Sae := SAE(FRAC INT, UP(x, FRAC INT), x^2-3)
--R
--R
--R (14)
--R SimpleAlgebraicExtension(Fraction Integer,UnivariatePolynomial(x,Fraction Int
--R eger),x*x-3)
--R
--R                                                    Type: Domain
--E 14

--S 15 of 18
a := generator()$Sae
--R
--R
--R (15) x
--RType: SimpleAlgebraicExtension(Fraction Integer,UnivariatePolynomial(x,Fraction Integer),x*x-3)
--E 15

--S 16 of 18
b := generator()$Sae
--R
--R
--R (16) x
--RType: SimpleAlgebraicExtension(Fraction Integer,UnivariatePolynomial(x,Fraction Integer),x*x-3)
--E 16

--S 17 of 18
(a = b)@Boolean
--R
--R
--R (17) true
--R
--R                                                    Type: Boolean
--E 17

--S 18 of 18
)show Any
--R
--R Any is a domain constructor
--R Abbreviation for Any is ANY

```

```

--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ANY
--R
--R----- Operations -----
--R ?=? : (%,% ) -> Boolean          any : (SExpression,None) -> %
--R coerce : % -> OutputForm        dom : % -> SExpression
--R domainOf : % -> OutputForm      hash : % -> SingleInteger
--R latex : % -> String             obj : % -> None
--R objectOf : % -> OutputForm      ?~=? : (%,% ) -> Boolean
--R showTypeInOut : Boolean -> String
--R
--E 18

)spool
)lisp (bye)

```

---

— Any.help —

=====  
Any examples  
=====

Any implements a type that packages up objects and their types in objects of Any. Roughly speaking that means that if  $s : S$  then when converted to Any, the new object will include both the original object and its type. This is a way of converting arbitrary objects into a single type without losing any of the original information. Any object can be converted to one of Any.

So we can convert a list to type Any

```

a:Any := [1,2]
      [1,2]

```

and another list to type Any

```

b:Any := [1,2]
      [1,2]

```

Equality works

```

(a = b)@Boolean
      true

```

We can compare the Any type with other types:

```

c := [1,2]

```

```

typeOf a
typeOf c
(a = c)@Boolean

```

If the values are different than we see the difference:

```

b := [1,3]
    [1,3]
(a = b)@Boolean
    false

```

The Any type works with many types:

```

a := "A"
    "A"
(a = b)@Boolean
    false
b := "A"
    "A"
(a = b)@Boolean
    true

```

This is true for more complex types:

```

Sae := SAE(FRAC INT, UP(x, FRAC INT), x^2-3)
a := generator()$Sae
    x
b := generator()$Sae
    x
(a = b)@Boolean
    true

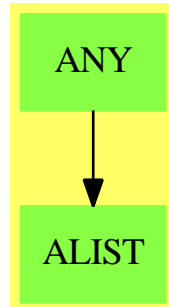
```

See Also:

o )show Any

---

### 2.9.1 Any (ANY)



See

⇒ “None” (NONE) 15.4.1 on page 1700

#### Exports:

|       |        |          |                  |      |
|-------|--------|----------|------------------|------|
| any   | coerce | dom      | domainOf         | hash |
| latex | obj    | objectOf | showTypeInOutput | ?=?  |
| ?~=?  |        |          |                  |      |

— domain ANY Any —

```

)abbrev domain ANY Any
++ Author: Robert S. Sutor
++ Date Created:
++ Change History:
++ Basic Functions: any, domainOf, objectOf, dom, obj, showTypeInOutput
++ Related Constructors: AnyFunctions1
++ Also See: None
++ AMS Classification:
++ Keywords:
++ Description:
++ \spadtype{Any} implements a type that packages up objects and their
++ types in objects of \spadtype{Any}. Roughly speaking that means
++ that if \spad{s : S} then when converted to \spadtype{Any}, the new
++ object will include both the original object and its type. This is
++ a way of converting arbitrary objects into a single type without
++ losing any of the original information. Any object can be converted
++ to one of \spadtype{Any}.
  
```

```

Any(): SetCategory with
  any      : (SExpression, None) -> %
  ++ any(type,object) is a technical function for creating
  ++ an object of \spadtype{Any}. Arugment \spad{type} is a
  ++ \spadgloss{LISP} form for the type of \spad{object}.
  domainOf : % -> OutputForm
  ++ domainOf(a) returns a printable form of the type of the
  ++ original object that was converted to \spadtype{Any}.
  
```

```

objectOf      : % -> OutputForm
  ++ objectOf(a) returns a printable form of the
  ++ original object that was converted to \spadtype{Any}.
dom           : % -> SExpression
  ++ dom(a) returns a \spadgloss{LISP} form of the type of the
  ++ original object that was converted to \spadtype{Any}.
obj           : % -> None
  ++ obj(a) essentially returns the original object that was
  ++ converted to \spadtype{Any} except that the type is forced
  ++ to be \spadtype{None}.
showTypeInOutput: Boolean -> String
  ++ showTypeInOutput(bool) affects the way objects of
  ++ \spadtype{Any} are displayed. If \spad{bool} is true
  ++ then the type of the original object that was converted
  ++ to \spadtype{Any} will be printed. If \spad{bool} is
  ++ false, it will not be printed.

== add
Rep := Record(dm: SExpression, ob: None)

printTypeInOutputP:Reference(Boolean) := ref false

obj x      == x.ob
dom x      == x.dm
domainOf x == x.dm pretend OutputForm
x = y      == (x.dm = y.dm) and EQ(x.ob, y.ob)$Lisp
x = y      ==
(x.dm = y.dm) and EQUAL(x.ob, y.ob)$Lisp

objectOf(x : %) : OutputForm ==
  spad2BootCoerce(x.ob, x.dm,
    list("OutputForm"::Symbol)$List(Symbol))$Lisp

showTypeInOutput(b : Boolean) : String ==
  printTypeInOutputP := ref b
  b=> "Type of object will be displayed in output of a member of Any"
  "Type of object will not be displayed in output of a member of Any"

coerce(x):OutputForm ==
  obj1 : OutputForm := objectOf x
  not deref printTypeInOutputP => obj1
  dom1 :=
    p:Symbol := prefix2String(devaluate(x.dm)$Lisp)$Lisp
    atom?(p pretend SExpression) => list(p)$List(Symbol)
    list(p)$Symbol
  hconcat cons(obj1,
    cons(":"::OutputForm, [a::OutputForm for a in dom1]))

any(domain, object) ==
  (isValidType(domain)$Lisp)@Boolean => [domain, object]

```



```

domain := devaluate(domain)$Lisp
(isValidType(domain)$Lisp)@Boolean => [domain, object]
error "function any must have a domain as first argument"

```

---

— ANY.dotabb —

```

"ANY" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ANY"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"ANY" -> "ALIST"

```

---

## 2.10 domain ASTACK ArrayStack

— ArrayStack.input —

```

)set break resume
)sys rm -f ArrayStack.output
)spool ArrayStack.output
)set message test on
)set message auto off
)clear all

--S 1 of 44
a:ArrayStack INT:= arrayStack [1,2,3,4,5]
--R
--R
--R (1) [1,2,3,4,5]
--R
--R                                          Type: ArrayStack Integer
--E 1

--S 2 of 44
pop! a
--R
--R
--R (2) 1
--R
--R                                          Type: PositiveInteger
--E 2

--S 3 of 44
a
--R

```

```

--R
--R (3) [2,3,4,5]
--R
--R                                         Type: ArrayStack Integer
--E 3

--S 4 of 44
extract! a
--R
--R
--R (4) 2
--R
--R                                         Type: PositiveInteger
--E 4

--S 5 of 44
a
--R
--R
--R (5) [3,4,5]
--R
--R                                         Type: ArrayStack Integer
--E 5

--S 6 of 44
push!(9,a)
--R
--R
--R (6) 9
--R
--R                                         Type: PositiveInteger
--E 6

--S 7 of 44
a
--R
--R
--R (7) [9,3,4,5]
--R
--R                                         Type: ArrayStack Integer
--E 7

--S 8 of 44
insert!(8,a)
--R
--R
--R (8) [8,9,3,4,5]
--R
--R                                         Type: ArrayStack Integer
--E 8

--S 9 of 44
a
--R
--R
--R (9) [8,9,3,4,5]

```

[illegible]

```

--S 16 of 44
more?(a,9)
--R
--R
--R (16) false
--R
--R                                          Type: Boolean
--E 16

--S 17 of 44
size?(a,#a)
--R
--R
--R (17) true
--R
--R                                          Type: Boolean
--E 17

--S 18 of 44
size?(a,9)
--R
--R
--R (18) false
--R
--R                                          Type: Boolean
--E 18

--S 19 of 44
parts a
--R
--R
--R (19) [8,9,3,4,5]
--R
--R                                          Type: List Integer
--E 19

--S 20 of 44
bag([1,2,3,4,5])$ArrayStack(INT)
--R
--R
--R (20) [5,4,3,2,1]
--R
--R                                          Type: ArrayStack Integer
--E 20

--S 21 of 44
b:=empty()$(ArrayStack INT)
--R
--R
--R (21) []
--R
--R                                          Type: ArrayStack Integer
--E 21

--S 22 of 44

```

```

empty? b
--R
--R
--R (22) true
--R
--R                                          Type: Boolean
--E 22

--S 23 of 44
sample()$ArrayStack(INT)
--R
--R
--R (23) []
--R
--R                                          Type: ArrayStack Integer
--E 23

--S 24 of 44
c:=copy a
--R
--R
--R (24) [8,9,3,4,5]
--R
--R                                          Type: ArrayStack Integer
--E 24

--S 25 of 44
eq?(a,c)
--R
--R
--R (25) false
--R
--R                                          Type: Boolean
--E 25

--S 26 of 44
eq?(a,a)
--R
--R
--R (26) true
--R
--R                                          Type: Boolean
--E 26

--S 27 of 44
(a=c)@Boolean
--R
--R
--R (27) true
--R
--R                                          Type: Boolean
--E 27

--S 28 of 44
(a=a)@Boolean
--R

```

```

--R
--R (28) true
--R
--R                                          Type: Boolean
--E 28

--S 29 of 44
a:=c
--R
--R
--R (29) false
--R
--R                                          Type: Boolean
--E 29

--S 30 of 44
any?(x+-(x=4),a)
--R
--R
--R (30) true
--R
--R                                          Type: Boolean
--E 30

--S 31 of 44
any?(x+-(x=11),a)
--R
--R
--R (31) false
--R
--R                                          Type: Boolean
--E 31

--S 32 of 44
every?(x+-(x=11),a)
--R
--R
--R (32) false
--R
--R                                          Type: Boolean
--E 32

--S 33 of 44
count(4,a)
--R
--R
--R (33) 1
--R
--R                                          Type: PositiveInteger
--E 33

--S 34 of 44
count(x+-(x>2),a)
--R
--R
--R (34) 5

```



```

--S 41 of 44
coerce a
--R
--R
--R (41) [18,19,13,14,15]
--R
--R                                          Type: OutputForm
--E 41

--S 42 of 44
hash a
--R
--R
--R (42) 36310821
--R
--R                                          Type: SingleInteger
--E 42

--S 43 of 44
latex a
--R
--R
--R (43) "\mbox{\bf Unimplemented}"
--R
--R                                          Type: String
--E 43

--S 44 of 44
)show ArrayStack
--R
--R ArrayStack S: SetCategory is a domain constructor
--R Abbreviation for ArrayStack is ASTACK
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASTACK
--R
--R----- Operations -----
--R arrayStack : List S -> %          bag : List S -> %
--R copy : % -> %                    depth : % -> NonNegativeInteger
--R empty : () -> %                  empty? : % -> Boolean
--R eq? : (%,% ) -> Boolean          extract! : % -> S
--R insert! : (S,% ) -> %            inspect : % -> S
--R map : ((S -> S),%) -> %          pop! : % -> S
--R push! : (S,% ) -> S              sample : () -> %
--R top : % -> S
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ==? : (%,% ) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,% ) -> NonNegativeInteger if $ has finiteAggregate and S has SETCAT
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (% ,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (% ,S,S) -> % if S has EVALAB S and S has SETCAT

```



```

--R eval : (% ,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (% ,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (% ,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R members : % -> List S if $ has finiteAggregate
--R more? : (% ,NonNegativeInteger) -> Boolean
--R parts : % -> List S if $ has finiteAggregate
--R size? : (% ,NonNegativeInteger) -> Boolean
--R ~=? : (% ,%) -> Boolean if S has SETCAT
--R
--E 44

)spool
)lisp (bye)

```

---

— ArrayStack.help —

```

=====
ArrayStack examples
=====

```

An ArrayStack object is represented as a list ordered by last-in, first-out. It operates like a pile of books, where the "next" book is the one on the top of the pile.

Here we create an array stack of integers from a list. Notice that the order in the list is the order in the stack.

```

a:ArrayStack INT:= arrayStack [1,2,3,4,5]
[1,2,3,4,5]

```

We can remove the top of the stack using pop!:

```

pop! a
1

```

Notice that the use of pop! is destructive (destructive operations in Axiom usually end with ! to indicate that the underlying data structure is changed).

```

a
[2,3,4,5]

```

The `extract!` operation is another name for the `pop!` operation and has the same effect. This operation treats the stack as a `BagAggregate`:

```
extract! a
2
```

and you can see that it also has destructively modified the stack:

```
a
[3,4,5]
```

Next we push a new element on top of the stack:

```
push!(9,a)
9
```

Again, the `push!` operation is destructive so the stack is changed:

```
a
[9,3,4,5]
```

Another name for `push!` is `insert!`, which treats the stack as a `BagAggregate`:

```
insert!(8,a)
[8,9,3,4,5]
```

and it modifies the stack:

```
a
[8,9,3,4,5]
```

The `inspect` function returns the top of the stack without modification, viewed as a `BagAggregate`:

```
inspect a
8
```

The `empty?` operation returns `true` only if there are no element on the stack, otherwise it returns `false`:

```
empty? a
false
```

The `top` operation returns the top of stack without modification, viewed as a `Stack`:

```
top a
8
```

The depth operation returns the number of elements on the stack:

```
depth a
5
```

which is the same as the # (length) operation:

```
#a
5
```

The less? predicate will compare the stack length to an integer:

```
less?(a,9)
true
```

The more? predicate will compare the stack length to an integer:

```
more?(a,9)
false
```

The size? operation will compare the stack length to an integer:

```
size?(a,#a)
true
```

and since the last computation must always be true we try:

```
size?(a,9)
false
```

The parts function will return the stack as a list of its elements:

```
parts a
[8,9,3,4,5]
```

If we have a BagAggregate of elements we can use it to construct a stack. Notice that the elements are pushed in reverse order:

```
bag([1,2,3,4,5])$ArrayStack(INT)
[5,4,3,2,1]
```

The empty function will construct an empty stack of a given type:

```
b:=empty()$(ArrayStack INT)
[]
```

and the empty? predicate allows us to find out if a stack is empty:

```
empty? b
true
```

The sample function returns a sample, empty stack:

```
sample()$ArrayStack(INT)
[]
```

We can copy a stack and it does not share storage so subsequent modifications of the original stack will not affect the copy:

```
c:=copy a
      [8,9,3,4,5]
```

The eq? function is only true if the lists are the same reference, so even though c is a copy of a, they are not the same:

```
eq?(a,c)
false
```

However, a clearly shares a reference with itself:

```
eq?(a,a)
true
```

But we can compare a and c for equality:

```
(a=c)@Boolean
true
```

and clearly a is equal to itself:

```
(a=a)@Boolean
true
```

and since a and c are equal, they are clearly NOT not-equal:

```
a~=c
false
```

We can use the any? function to see if a predicate is true for any element:

```
any?(x+>(x=4),a)
true
```

or false for every element:

```
any?(x+>(x=11),a)
false
```

We can use the every? function to check every element satisfies a predicate:

```
every?(x+-(x=11),a)
false
```

We can count the elements that are equal to an argument of this type:

```
count(4,a)
1
```

or we can count against a boolean function:

```
count(x+-(x>2),a)
5
```

You can also map a function over every element, returning a new stack:

```
map(x+>x+10,a)
[18,19,13,14,15]
```

Notice that the original stack is unchanged:

```
a
[8,9,3,4,5]
```

You can use `map!` to map a function over every element and change the original stack since `map!` is destructive:

```
map!(x+>x+10,a)
[18,19,13,14,15]
```

Notice that the original stack has been changed:

```
a
[18,19,13,14,15]
```

The `members` function can also get the element of the stack as a list:

```
members a
[18,19,13,14,15]
```

and using `member?` we can test if the stack holds a given element:

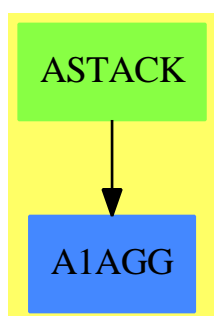
```
member?(14,a)
true
```

See Also:

- o `)show Stack`
- o `)show ArrayStack`
- o `)show Queue`
- o `)show Dequeue`
- o `)show Heap`

o )show BagAggregate

### 2.10.1 ArrayStack (ASTACK)



See

⇒ “Stack” (STACK) 20.28.1 on page 2521  
 ⇒ “Queue” (QUEUE) 18.5.1 on page 2143  
 ⇒ “Dequeue” (DEQUEUE) 5.5.1 on page 497  
 ⇒ “Heap” (HEAP) 9.2.1 on page 1100

#### Exports:

|         |            |          |        |         |
|---------|------------|----------|--------|---------|
| any?    | arrayStack | bag      | coerce | copy    |
| count   | depth      | empty    | empty? | eq?     |
| eval    | every?     | extract! | hash   | insert! |
| inspect | latex      | less?    | map    | map!    |
| member? | members    | more?    | parts  | pop!    |
| push!   | sample     | size?    | top    | #?      |
| ?~=?    | ?=?        |          |        |         |

— domain ASTACK ArrayStack —

```

)abbrev domain ASTACK ArrayStack
++ Author: Michael Monagan, Stephen Watt, Timothy Daly
++ Date Created: June 86 and July 87
++ Date Last Updated: Feb 92
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
  
```

```

++ Description:
++ A stack represented as a flexible array.
--% Dequeue and Heap data types

ArrayStack(S:SetCategory): StackAggregate(S) with
  arrayStack: List S -> %
  ++ arrayStack([x,y,...,z]) creates an array stack with first (top)
  ++ element x, second element y,...,and last element z.
  ++
  ++E c:ArrayStack INT:= arrayStack [1,2,3,4,5]

-- Inherited Signatures repeated for examples documentation

pop_! : % -> S
++
++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
++X pop! a
++X a
extract_! : % -> S
++
++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
++X extract! a
++X a
push_! : (S,%) -> S
++
++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
++X push!(9,a)
++X a
insert_! : (S,%) -> %
++
++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
++X insert!(8,a)
++X a
inspect : % -> S
++
++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
++X inspect a
top : % -> S
++
++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
++X top a
depth : % -> NonNegativeInteger
++
++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
++X depth a
less? : (%,NonNegativeInteger) -> Boolean
++
++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
++X less?(a,9)
more? : (%,NonNegativeInteger) -> Boolean

```

```

++
++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
++X more?(a,9)
size? : (% ,NonNegativeInteger) -> Boolean
++
++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
++X size?(a,5)
bag : List S -> %
++
++X bag([1,2,3,4,5])$ArrayStack(INT)
empty? : % -> Boolean
++
++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
++X empty? a
empty : () -> %
++
++X b:=empty()$(ArrayStack INT)
sample : () -> %
++
++X sample()$ArrayStack(INT)
copy : % -> %
++
++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
++X copy a
eq? : (% ,%) -> Boolean
++
++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
++X b:=copy a
++X eq?(a,b)
map : ((S -> S),%) -> %
++
++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
++X map(x+>x+10,a)
++X a
if $ has shallowlyMutable then
  map! : ((S -> S),%) -> %
  ++
  ++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
  ++X map!(x+>x+10,a)
  ++X a
if S has SetCategory then
  latex : % -> String
  ++
  ++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
  ++X latex a
hash : % -> SingleInteger
++
++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
++X hash a
coerce : % -> OutputForm

```



```

++
++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
++X coerce a
"=: (%,% ) -> Boolean
++
++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
++X b:ArrayStack INT:= arrayStack [1,2,3,4,5]
++X (a=b)@Boolean
"~=" : (%,% ) -> Boolean
++
++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
++X b:=copy a
++X (a~b)
if % has finiteAggregate then
every? : ((S -> Boolean),%) -> Boolean
++
++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
++X every?(x+-(x=4),a)
any? : ((S -> Boolean),%) -> Boolean
++
++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
++X any?(x+-(x=4),a)
count : ((S -> Boolean),%) -> NonNegativeInteger
++
++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
++X count(x+-(x>2),a)
_# : % -> NonNegativeInteger
++
++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
++X #a
parts : % -> List S
++
++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
++X parts a
members : % -> List S
++
++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
++X members a
if % has finiteAggregate and S has SetCategory then
member? : (S,%) -> Boolean
++
++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
++X member?(3,a)
count : (S,%) -> NonNegativeInteger
++
++X a:ArrayStack INT:= arrayStack [1,2,3,4,5]
++X count(4,a)

== add
Rep := IndexedFlexibleArray(S,0)

```

```

-- system operations
# s == _#(s)$Rep
s = t == s =$Rep t
copy s == copy(s)$Rep
coerce(d):OutputForm ==
  empty? d => empty()$(List S) ::OutputForm
  [(d.i::OutputForm) for i in 0..#d-1] ::OutputForm

-- stack operations
depth s == # s
empty? s == empty?(s)$Rep
extract_! s == pop_! s
insert_!(e,s) == (push_!(e,s);s)
push_!(e,s) == (concat(e,s); e)
pop_! s ==
  if empty? s then error "empty stack"
  r := s.0
  delete_!(s,0)
  r
top s == if empty? s then error "empty stack" else s.0
arrayStack l == construct(l)$Rep
empty() == new(0,0 pretend S)
parts s == [s.i for i in 0..#s-1]::List(S)
map(f,s) == construct [f(s.i) for i in 0..#s-1]
map!(f,s) == ( for i in 0..#s-1 repeat qsetelt!(s,i,f(s.i)) ; s )
inspect(s) ==
  if empty? s then error "empty stack"
  qelt(s,0)

```

---

— ASTACK.dotabb —

```

"ASTACK" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASTACK"]
"A1AGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=A1AGG"]
"ASTACK" -> "A1AGG"

```

---

## 2.11 domain ASP1 Asp1

— Asp1.input —

```

)set break resume

```

```

)sys rm -f Asp1.output
)spool Asp1.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp1
--R Asp1 name: Symbol is a domain constructor
--R Abbreviation for Asp1 is ASP1
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP1
--R
--R----- Operations -----
--R coerce : FortranCode -> %           coerce : List FortranCode -> %
--R coerce : % -> OutputForm           outputAsFortran : % -> Void
--R retract : Polynomial Integer -> %   retract : Polynomial Float -> %
--R retract : Expression Integer -> %   retract : Expression Float -> %
--R coerce : FortranExpression([construct,QUOTEX],[construct],MachineFloat) -> %
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Fraction Polynomial Integer -> %
--R retract : Fraction Polynomial Float -> %
--R retractIfCan : Fraction Polynomial Integer -> Union(%, "failed")
--R retractIfCan : Fraction Polynomial Float -> Union(%, "failed")
--R retractIfCan : Polynomial Integer -> Union(%, "failed")
--R retractIfCan : Polynomial Float -> Union(%, "failed")
--R retractIfCan : Expression Integer -> Union(%, "failed")
--R retractIfCan : Expression Float -> Union(%, "failed")
--R
--E 1

)spool
)lisp (bye)

```

---

— Asp1.help —

```

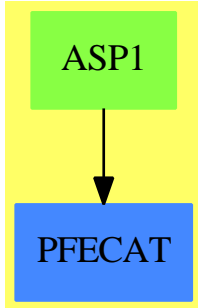
=====
Asp1 examples
=====

```

See Also:

- o )show Asp1

## 2.11.1 Asp1 (ASP1)

**Exports:**

coerce outputAsFortran retract retractIfCan

— domain ASP1 Asp1 —

```

)abbrev domain ASP1 Asp1
++ Author: Mike Dewar, Grant Keady, Godfrey Nolan
++ Date Created: Mar 1993
++ Date Last Updated: 18 March 1994, 6 October 1994
++ Related Constructors: FortranFunctionCategory, FortranProgramCategory.
++ Description:
++ \spadtype{Asp1} produces Fortran for Type 1 ASPs, needed for various
++ NAG routines. Type 1 ASPs take a univariate expression (in the symbol x)
++ and turn it into a Fortran Function like the following:
++
++ \tab{5}DOUBLE PRECISION FUNCTION F(X)\br
++ \tab{5}DOUBLE PRECISION X\br
++ \tab{5}F=DSIN(X)\br
++ \tab{5}RETURN\br
++ \tab{5}END
  
```

```

Asp1(name): Exports == Implementation where
  name : Symbol
  
```

```

FEXPR ==> FortranExpression
FST    ==> FortranScalarType
FT     ==> FortranType
SYMTAB ==> SymbolTable
RSFC   ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
FRAC   ==> Fraction
POLY   ==> Polynomial
EXPR   ==> Expression
INT    ==> Integer
FLOAT  ==> Float
  
```

```

Exports ==> FortranFunctionCategory with
  coerce : FEXPR(['X'],[],MachineFloat) -> $
    ++coerce(f) takes an object from the appropriate instantiation of
    ++\spadtype{FortranExpression} and turns it into an ASP.

Implementation ==> add

-- Build Symbol Table for Rep
syms : SYMTAB := empty()$SYMTAB
declare!(X,fortranReal()$FT,syms)$SYMTAB
real : FST := "real":FST

Rep := FortranProgram(name,[real]$Union(fst:FST,void:"void"),[X],syms)

retract(u:FRAC POLY INT):$ == (retract(u)$FEXPR(['X'],[],MachineFloat)):$
retractIfCan(u:FRAC POLY INT):Union($,"failed") ==
  foo : Union(FEXPR(['X'],[],MachineFloat),"failed")
  foo := retractIfCan(u)$FEXPR(['X'],[],MachineFloat)
  foo case "failed" => "failed"
  foo:$FEXPR(['X'],[],MachineFloat):$

retract(u:FRAC POLY FLOAT):$ == (retract(u)$FEXPR(['X'],[],MachineFloat)):$
retractIfCan(u:FRAC POLY FLOAT):Union($,"failed") ==
  foo : Union(FEXPR(['X'],[],MachineFloat),"failed")
  foo := retractIfCan(u)$FEXPR(['X'],[],MachineFloat)
  foo case "failed" => "failed"
  foo:$FEXPR(['X'],[],MachineFloat):$

retract(u:EXPR FLOAT):$ == (retract(u)$FEXPR(['X'],[],MachineFloat)):$
retractIfCan(u:EXPR FLOAT):Union($,"failed") ==
  foo : Union(FEXPR(['X'],[],MachineFloat),"failed")
  foo := retractIfCan(u)$FEXPR(['X'],[],MachineFloat)
  foo case "failed" => "failed"
  foo:$FEXPR(['X'],[],MachineFloat):$

retract(u:EXPR INT):$ == (retract(u)$FEXPR(['X'],[],MachineFloat)):$
retractIfCan(u:EXPR INT):Union($,"failed") ==
  foo : Union(FEXPR(['X'],[],MachineFloat),"failed")
  foo := retractIfCan(u)$FEXPR(['X'],[],MachineFloat)
  foo case "failed" => "failed"
  foo:$FEXPR(['X'],[],MachineFloat):$

retract(u:POLY FLOAT):$ == (retract(u)$FEXPR(['X'],[],MachineFloat)):$
retractIfCan(u:POLY FLOAT):Union($,"failed") ==
  foo : Union(FEXPR(['X'],[],MachineFloat),"failed")
  foo := retractIfCan(u)$FEXPR(['X'],[],MachineFloat)
  foo case "failed" => "failed"
  foo:$FEXPR(['X'],[],MachineFloat):$

retract(u:POLY INT):$ == (retract(u)$FEXPR(['X'],[],MachineFloat)):$

```

```

retractIfCan(u:POLY INT):Union($,"failed") ==
  foo : Union(FEXPR(['X'],[],MachineFloat),"failed")
  foo := retractIfCan(u)$FEXPR(['X'],[],MachineFloat)
  foo case "failed" => "failed"
  foo::FEXPR(['X'],[],MachineFloat)::$

coerce(u:FEXPR(['X'],[],MachineFloat)):$ ==
  coerce((u::Expression(MachineFloat))$FEXPR(['X'],[],MachineFloat))$Rep

coerce(c:List FortranCode):$ == coerce(c)$Rep

coerce(r:RSFC):$ == coerce(r)$Rep

coerce(c:FortranCode):$ == coerce(c)$Rep

coerce(u:$):OutputForm == coerce(u)$Rep

outputAsFortran(u):Void ==
  p := checkPrecision()$NAGLinkSupportPackage
  outputAsFortran(u)$Rep
  p => restorePrecision()$NAGLinkSupportPackage

```

---

— ASP1.dotabb —

```

"ASP1" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP1"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"ASP1" -> "PFECAT"

```

---

## 2.12 domain ASP10 Asp10

— Asp10.input —

```

)set break resume
)sys rm -f Asp10.output
)spool Asp10.output
)set message test on
)set message auto off
)clear all

```

--S 1 of 1

```

)show Asp10
--R Asp10 name: Symbol  is a domain constructor
--R Abbreviation for Asp10 is ASP10
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP10
--R
--R----- Operations -----
--R coerce : FortranCode -> %          coerce : List FortranCode -> %
--R coerce : % -> OutputForm          outputAsFortran : % -> Void
--R coerce : Vector FortranExpression([construct,QUOTEJINT,QUOTEX,QUOTEELAM],[construct],Mac
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Vector Fraction Polynomial Integer -> %
--R retract : Vector Fraction Polynomial Float -> %
--R retract : Vector Polynomial Integer -> %
--R retract : Vector Polynomial Float -> %
--R retract : Vector Expression Integer -> %
--R retract : Vector Expression Float -> %
--R retractIfCan : Vector Fraction Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Vector Fraction Polynomial Float -> Union(%,"failed")
--R retractIfCan : Vector Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Vector Polynomial Float -> Union(%,"failed")
--R retractIfCan : Vector Expression Integer -> Union(%,"failed")
--R retractIfCan : Vector Expression Float -> Union(%,"failed")
--R
--E 1

```

```

)spool
)lisp (bye)

```

---

### — Asp10.help —

```

=====
Asp10 examples
=====

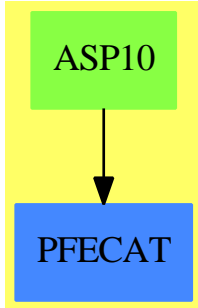
```

```

See Also:
o )show Asp10

```

## 2.12.1 Asp10 (ASP10)

**Exports:**

coerce outputAsFortran retract retractIfCan

— domain ASP10 Asp10 —

```

)abbrev domain ASP10 Asp10
++ Author: Mike Dewar and Godfrey Nolan
++ Date Created: Mar 1993
++ Date Last Updated: 18 March 1994
++               6 October 1994
++ Related Constructors: FortranVectorFunctionCategory, FortranProgramCategory
++ Description:
++ \spadtype{ASP10} produces Fortran for Type 10 ASPs, needed for NAG routine
++ d02kef. This ASP computes the values of a set of functions, for example:
++
++ \tab{5}SUBROUTINE COEFFN(P,Q,DQDL,X,ELAM,JINT)\br
++ \tab{5}DOUBLE PRECISION ELAM,P,Q,X,DQDL\br
++ \tab{5}INTEGER JINT\br
++ \tab{5}P=1.0D0\br
++ \tab{5}Q=(-1.0D0*X**3)+ELAM*X*X-2.0D0)/(X*X)\br
++ \tab{5}DQDL=1.0D0\br
++ \tab{5}RETURN\br
++ \tab{5}END
  
```

```

Asp10(name): Exports == Implementation where
  name : Symbol
  
```

```

FST ==> FortranScalarType
FT ==> FortranType
SYMTAB ==> SymbolTable
EXF ==> Expression Float
RSFC ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
FEXPR ==> FortranExpression(['JINT','X','ELAM'],[],MFLOAT)
MFLOAT ==> MachineFloat
FRAC ==> Fraction
  
```



```

POLY    ==> Polynomial
EXPR    ==> Expression
INT      ==> Integer
FLOAT    ==> Float
VEC      ==> Vector
VF2      ==> VectorFunctions2

Exports ==> FortranVectorFunctionCategory with
  coerce : Vector FEXPR -> %
  ++coerce(f) takes objects from the appropriate instantiation of
  ++\spadtype{FortranExpression} and turns them into an ASP.

Implementation ==> add

real : FST := "real"::FST
syms : SYMTAB := empty()$SYMTAB
declare!(P,fortranReal()$FT,syms)$SYMTAB
declare!(Q,fortranReal()$FT,syms)$SYMTAB
declare!(DQDL,fortranReal()$FT,syms)$SYMTAB
declare!(X,fortranReal()$FT,syms)$SYMTAB
declare!(ELAM,fortranReal()$FT,syms)$SYMTAB
declare!(JINT,fortranInteger()$FT,syms)$SYMTAB
Rep := FortranProgram(name,["void"]$Union(fst:FST,void:"void"),
  [P,Q,DQDL,X,ELAM,JINT],syms)

retract(u:VEC FRAC POLY INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY INT,FEXPR)
  v::$

retractIfCan(u:VEC FRAC POLY INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC FRAC POLY FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY FLOAT,FEXPR)
  v::$

retractIfCan(u:VEC FRAC POLY FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC EXPR INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(EXPR INT,FEXPR)
  v::$

retractIfCan(u:VEC EXPR INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR INT,FEXPR)
  v case "failed" => "failed"

```

```

(v::VEC FEXPR):: $

retract(u:VEC EXPR FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(EXPR FLOAT,FEXPR)
  v::$

retractIfCan(u:VEC EXPR FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR):: $

retract(u:VEC POLY INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(POLY INT,FEXPR)
  v::$

retractIfCan(u:VEC POLY INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR):: $

retract(u:VEC POLY FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(POLY FLOAT,FEXPR)
  v::$

retractIfCan(u:VEC POLY FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR):: $

coerce(c:FortranCode):% == coerce(c)$Rep

coerce(r:RSFC):% == coerce(r)$Rep

coerce(c:List FortranCode):% == coerce(c)$Rep

-- To help the poor old compiler!
localAssign(s:Symbol,u:Expression MFLOAT):FortranCode ==
  assign(s,u)$FortranCode

coerce(u:Vector FEXPR):% ==
  import Vector FEXPR
  not (#u = 3) => error "Incorrect Dimension For Vector"
  ([localAssign(P,elt(u,1)::Expression MFLOAT),_
    localAssign(Q,elt(u,2)::Expression MFLOAT),_
    localAssign(DQDL,elt(u,3)::Expression MFLOAT),_
    returns()$FortranCode ]$List(FortranCode))::Rep

coerce(u:%):OutputForm == coerce(u)$Rep

outputAsFortran(u):Void ==

```

```
p := checkPrecision()$NAGLinkSupportPackage
outputAsFortran(u)$Rep
p => restorePrecision()$NAGLinkSupportPackage
```

---

— ASP10.dotabb —

```
"ASP10" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP10"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"ASP10" -> "PFECAT"
```

---

## 2.13 domain ASP12 Asp12

— Asp12.input —

```
)set break resume
)sys rm -f Asp12.output
)spool Asp12.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp12
--R Asp12 name: Symbol is a domain constructor
--R Abbreviation for Asp12 is ASP12
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP12
--R
--R----- Operations -----
--R coerce : % -> OutputForm          outputAsFortran : () -> Void
--R outputAsFortran : % -> Void
--R
--E 1

)spool
)lisp (bye)
```

---

— Asp12.help —

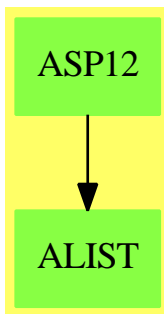
```
=====
Asp12 examples
=====
```

See Also:

o )show Asp12

—————

### 2.13.1 Asp12 (ASP12)



#### Exports:

coerce    outputAsFortran

— domain ASP12 Asp12 —

```
)abbrev domain ASP12 Asp12
++ Author: Mike Dewar and Godfrey Nolan
++ Date Created: Oct 1993
++ Date Last Updated: 18 March 1994
++          21 June 1994 Changed print to printStatement
++ Related Constructors:
++ Description:
++ \spadtype{Asp12} produces Fortran for Type 12 ASPs, needed for NAG routine
++ d02kef etc., for example:
++
++ \tab{5}SUBROUTINE MONIT (MAXIT,IFLAG,ELAM,FINFO)\br
++ \tab{5}DOUBLE PRECISION ELAM,FINFO(15)\br
++ \tab{5}INTEGER MAXIT,IFLAG\br
++ \tab{5}IF(MAXIT.EQ.-1)THEN\br
++ \tab{7}PRINT*,"Output from Monit"\br
++ \tab{5}ENDIF\br
++ \tab{5}PRINT*,MAXIT,IFLAG,ELAM,(FINFO(I),I=1,4)\br
++ \tab{5}RETURN\br
```

```

++ \tab{5}END\

Asp12(name): Exports == Implementation where
  name : Symbol

  O      ==> OutputForm
  S      ==> Symbol
  FST    ==> FortranScalarType
  FT     ==> FortranType
  FC     ==> FortranCode
  SYMTAB ==> SymbolTable
  EXI    ==> Expression Integer
  RSFC   ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
  U      ==> Union(I: Expression Integer,F: Expression Float,
                  CF: Expression Complex Float,switch:Switch)
  UFST   ==> Union(fst:FST,void:"void")

Exports ==> FortranProgramCategory with
  outputAsFortran:() -> Void
  ++outputAsFortran() generates the default code for \spadtype{ASP12}.

Implementation ==> add

import FC
import Switch

real : FST := "real":FST
syms : SYMTAB := empty()$SYMTAB
declare!(MAXIT,fortranInteger()$FT,syms)$SYMTAB
declare!(IFLAG,fortranInteger()$FT,syms)$SYMTAB
declare!(ELAM,fortranReal()$FT,syms)$SYMTAB
fType : FT := construct([real]$UFST,["15":Symbol],false)$FT
declare!(FINFO,fType,syms)$SYMTAB
Rep := FortranProgram(name,["void"]$UFST,[MAXIT,IFLAG,ELAM,FINFO],syms)

-- eqn : O := (I::O)=(1@Integer::EXI::O)
code:=[cond(EQ([MAXIT@S::EXI]$U,[-1::EXI]$U),
  printStatement(["_Output from Monit_":O])),
  printStatement([MAXIT::O,IFLAG::O,ELAM::O,subscript("(FINFO":S,[I::O]):O,"I=1"
returns()$List(FortranCode))::Rep

coerce(u:%):OutputForm == coerce(u)$Rep

outputAsFortran(u:%):Void == outputAsFortran(u)$Rep
outputAsFortran():Void == outputAsFortran(code)$Rep

```

---

— ASP12.dotabb —

```
"ASP12" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP12"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"ASP12" -> "ALIST"
```

—

## 2.14 domain ASP19 Asp19

— Asp19.input —

```
)set break resume
)sys rm -f Asp19.output
)spool Asp19.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp19
--R Asp19 name: Symbol is a domain constructor
--R Abbreviation for Asp19 is ASP19
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP19
--R
--R----- Operations -----
--R coerce : FortranCode -> %               coerce : List FortranCode -> %
--R coerce : % -> OutputForm               outputAsFortran : % -> Void
--R coerce : Vector FortranExpression([construct],[construct,QUOTEXC],MachineFloat) -> %
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Vector Fraction Polynomial Integer -> %
--R retract : Vector Fraction Polynomial Float -> %
--R retract : Vector Polynomial Integer -> %
--R retract : Vector Polynomial Float -> %
--R retract : Vector Expression Integer -> %
--R retract : Vector Expression Float -> %
--R retractIfCan : Vector Fraction Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Vector Fraction Polynomial Float -> Union(%,"failed")
--R retractIfCan : Vector Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Vector Polynomial Float -> Union(%,"failed")
--R retractIfCan : Vector Expression Integer -> Union(%,"failed")
--R retractIfCan : Vector Expression Float -> Union(%,"failed")
--R
--E 1
```

```
)spool
)lisp (bye)
```

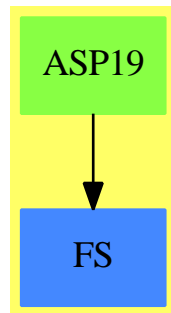
---

— Asp19.help —

```
=====
Asp19 examples
=====
```

```
See Also:
o )show Asp19
```

### 2.14.1 Asp19 (ASP19)



#### Exports:

```
coerce outputAsFortran retract retractIfCan
```

---

— domain ASP19 Asp19 —

```
)abbrev domain ASP19 Asp19
++ Author: Mike Dewar, Godfrey Nolan, Grant Keady
++ Date Created: Mar 1993
++ Date Last Updated: 18 March 1994
++                               6 October 1994
++ Related Constructors: FortranVectorFunctionCategory, FortranProgramCategory
++ Description:
++\spadtype{Asp19} produces Fortran for Type 19 ASPs, evaluating a set of
++functions and their jacobian at a given point, for example:
++
```

```

++\tab{5}SUBROUTINE LSFUN2(M,N,XC,FVECC,FJACC,LJC)\br
++\tab{5}DOUBLE PRECISION FVECC(M),FJACC(LJC,N),XC(N)\br
++\tab{5}INTEGER M,N,LJC\br
++\tab{5}INTEGER I,J\br
++\tab{5}DO 25003 I=1,LJC\br
++\tab{7}DO 25004 J=1,N\br
++\tab{9}FJACC(I,J)=0.0D0\br
++25004 CONTINUE\br
++25003 CONTINUE\br
++\tab{5}FVECC(1)=(XC(1)-0.14D0)*XC(3)+(15.0D0*XC(1)-2.1D0)*XC(2)+1.0D0)/(\br
++\tab{4}&XC(3)+15.0D0*XC(2))\br
++\tab{5}FVECC(2)=(XC(1)-0.18D0)*XC(3)+(7.0D0*XC(1)-1.26D0)*XC(2)+1.0D0)/(\br
++\tab{4}&XC(3)+7.0D0*XC(2))\br
++\tab{5}FVECC(3)=(XC(1)-0.22D0)*XC(3)+(4.333333333333333D0*XC(1)-0.953333\br
++\tab{4}&3333333333D0)*XC(2)+1.0D0)/(XC(3)+4.333333333333333D0*XC(2))\br
++\tab{5}FVECC(4)=(XC(1)-0.25D0)*XC(3)+(3.0D0*XC(1)-0.75D0)*XC(2)+1.0D0)/(\br
++\tab{4}&XC(3)+3.0D0*XC(2))\br
++\tab{5}FVECC(5)=(XC(1)-0.29D0)*XC(3)+(2.2D0*XC(1)-0.637999999999999D0)*\br
++\tab{4}&XC(2)+1.0D0)/(XC(3)+2.2D0*XC(2))\br
++\tab{5}FVECC(6)=(XC(1)-0.32D0)*XC(3)+(1.666666666666667D0*XC(1)-0.533333\br
++\tab{4}&3333333333D0)*XC(2)+1.0D0)/(XC(3)+1.666666666666667D0*XC(2))\br
++\tab{5}FVECC(7)=(XC(1)-0.35D0)*XC(3)+(1.285714285714286D0*XC(1)-0.45D0)*\br
++\tab{4}&XC(2)+1.0D0)/(XC(3)+1.285714285714286D0*XC(2))\br
++\tab{5}FVECC(8)=(XC(1)-0.39D0)*XC(3)+(XC(1)-0.39D0)*XC(2)+1.0D0)/(XC(3)+\br
++\tab{4}&XC(2))\br
++\tab{5}FVECC(9)=(XC(1)-0.37D0)*XC(3)+(XC(1)-0.37D0)*XC(2)+1.285714285714\br
++\tab{4}&286D0)/(XC(3)+XC(2))\br
++\tab{5}FVECC(10)=(XC(1)-0.58D0)*XC(3)+(XC(1)-0.58D0)*XC(2)+1.66666666666\br
++\tab{4}&6667D0)/(XC(3)+XC(2))\br
++\tab{5}FVECC(11)=(XC(1)-0.73D0)*XC(3)+(XC(1)-0.73D0)*XC(2)+2.2D0)/(XC(3)\br
++\tab{4}&+XC(2))\br
++\tab{5}FVECC(12)=(XC(1)-0.96D0)*XC(3)+(XC(1)-0.96D0)*XC(2)+3.0D0)/(XC(3)\br
++\tab{4}&+XC(2))\br
++\tab{5}FVECC(13)=(XC(1)-1.34D0)*XC(3)+(XC(1)-1.34D0)*XC(2)+4.33333333333\br
++\tab{4}&3333D0)/(XC(3)+XC(2))\br
++\tab{5}FVECC(14)=(XC(1)-2.1D0)*XC(3)+(XC(1)-2.1D0)*XC(2)+7.0D0)/(XC(3)+X\br
++\tab{4}&C(2))\br
++\tab{5}FVECC(15)=(XC(1)-4.39D0)*XC(3)+(XC(1)-4.39D0)*XC(2)+15.0D0)/(XC(3)\br
++\tab{4}&+XC(2))\br
++\tab{5}FJACC(1,1)=1.0D0\br
++\tab{5}FJACC(1,2)=-15.0D0/(XC(3)**2+30.0D0*XC(2)*XC(3)+225.0D0*XC(2)**2)\br
++\tab{5}FJACC(1,3)=-1.0D0/(XC(3)**2+30.0D0*XC(2)*XC(3)+225.0D0*XC(2)**2)\br
++\tab{5}FJACC(2,1)=1.0D0\br
++\tab{5}FJACC(2,2)=-7.0D0/(XC(3)**2+14.0D0*XC(2)*XC(3)+49.0D0*XC(2)**2)\br
++\tab{5}FJACC(2,3)=-1.0D0/(XC(3)**2+14.0D0*XC(2)*XC(3)+49.0D0*XC(2)**2)\br
++\tab{5}FJACC(3,1)=1.0D0\br
++\tab{5}FJACC(3,2)=((-0.1110223024625157D-15*XC(3))-4.333333333333333D0)/(\br
++\tab{4}&XC(3)**2+8.666666666666666D0*XC(2)*XC(3)+18.77777777777778D0*XC(2)\br
++\tab{4}&**2)\br
++\tab{5}FJACC(3,3)=(0.1110223024625157D-15*XC(2)-1.0D0)/(XC(3)**2+8.666666\br

```



```

++\tab{4}&66666666D0*XC(2)*XC(3)+18.77777777777778D0*XC(2)**2)\br
++\tab{5}FJACC(4,1)=1.0D0\br
++\tab{5}FJACC(4,2)=-3.0D0/(XC(3)**2+6.0D0*XC(2)*XC(3)+9.0D0*XC(2)**2)\br
++\tab{5}FJACC(4,3)=-1.0D0/(XC(3)**2+6.0D0*XC(2)*XC(3)+9.0D0*XC(2)**2)\br
++\tab{5}FJACC(5,1)=1.0D0\br
++\tab{5}FJACC(5,2)=((-0.1110223024625157D-15*XC(3))-2.2D0)/(XC(3)**2+4.399\br
++\tab{4}&999999999999999D0*XC(2)*XC(3)+4.839999999999998D0*XC(2)**2)\br
++\tab{5}FJACC(5,3)=(0.1110223024625157D-15*XC(2)-1.0D0)/(XC(3)**2+4.399999\br
++\tab{4}&999999999999999D0*XC(2)*XC(3)+4.839999999999998D0*XC(2)**2)\br
++\tab{5}FJACC(6,1)=1.0D0\br
++\tab{5}FJACC(6,2)=((-0.2220446049250313D-15*XC(3))-1.66666666666667D0)/(\br
++\tab{4}&XC(3)**2+3.333333333333333D0*XC(2)*XC(3)+2.777777777777777D0*XC(2)\br
++\tab{4}&**2)\br
++\tab{5}FJACC(6,3)=(0.2220446049250313D-15*XC(2)-1.0D0)/(XC(3)**2+3.333333\br
++\tab{4}&333333333333333D0*XC(2)*XC(3)+2.777777777777777D0*XC(2)**2)\br
++\tab{5}FJACC(7,1)=1.0D0\br
++\tab{5}FJACC(7,2)=((-0.5551115123125783D-16*XC(3))-1.285714285714286D0)/(\br
++\tab{4}&XC(3)**2+2.571428571428571D0*XC(2)*XC(3)+1.653061224489796D0*XC(2)\br
++\tab{4}&**2)\br
++\tab{5}FJACC(7,3)=(0.5551115123125783D-16*XC(2)-1.0D0)/(XC(3)**2+2.571428\br
++\tab{4}&571428571D0*XC(2)*XC(3)+1.653061224489796D0*XC(2)**2)\br
++\tab{5}FJACC(8,1)=1.0D0\br
++\tab{5}FJACC(8,2)=-1.0D0/(XC(3)**2+2.0D0*XC(2)*XC(3)+XC(2)**2)\br
++\tab{5}FJACC(8,3)=-1.0D0/(XC(3)**2+2.0D0*XC(2)*XC(3)+XC(2)**2)\br
++\tab{5}FJACC(9,1)=1.0D0\br
++\tab{5}FJACC(9,2)=-1.285714285714286D0/(XC(3)**2+2.0D0*XC(2)*XC(3)+XC(2)*\br
++\tab{4}&**2)\br
++\tab{5}FJACC(9,3)=-1.285714285714286D0/(XC(3)**2+2.0D0*XC(2)*XC(3)+XC(2)*\br
++\tab{4}&**2)\br
++\tab{5}FJACC(10,1)=1.0D0\br
++\tab{5}FJACC(10,2)=-1.666666666666667D0/(XC(3)**2+2.0D0*XC(2)*XC(3)+XC(2)\br
++\tab{4}&**2)\br
++\tab{5}FJACC(10,3)=-1.666666666666667D0/(XC(3)**2+2.0D0*XC(2)*XC(3)+XC(2)\br
++\tab{4}&**2)\br
++\tab{5}FJACC(11,1)=1.0D0\br
++\tab{5}FJACC(11,2)=-2.2D0/(XC(3)**2+2.0D0*XC(2)*XC(3)+XC(2)**2)\br
++\tab{5}FJACC(11,3)=-2.2D0/(XC(3)**2+2.0D0*XC(2)*XC(3)+XC(2)**2)\br
++\tab{5}FJACC(12,1)=1.0D0\br
++\tab{5}FJACC(12,2)=-3.0D0/(XC(3)**2+2.0D0*XC(2)*XC(3)+XC(2)**2)\br
++\tab{5}FJACC(12,3)=-3.0D0/(XC(3)**2+2.0D0*XC(2)*XC(3)+XC(2)**2)\br
++\tab{5}FJACC(13,1)=1.0D0\br
++\tab{5}FJACC(13,2)=-4.333333333333333D0/(XC(3)**2+2.0D0*XC(2)*XC(3)+XC(2)\br
++\tab{4}&**2)\br
++\tab{5}FJACC(13,3)=-4.333333333333333D0/(XC(3)**2+2.0D0*XC(2)*XC(3)+XC(2)\br
++\tab{4}&**2)\br
++\tab{5}FJACC(14,1)=1.0D0\br
++\tab{5}FJACC(14,2)=-7.0D0/(XC(3)**2+2.0D0*XC(2)*XC(3)+XC(2)**2)\br
++\tab{5}FJACC(14,3)=-7.0D0/(XC(3)**2+2.0D0*XC(2)*XC(3)+XC(2)**2)\br
++\tab{5}FJACC(15,1)=1.0D0\br
++\tab{5}FJACC(15,2)=-15.0D0/(XC(3)**2+2.0D0*XC(2)*XC(3)+XC(2)**2)\br

```

```

++\tab{5}FJACC(15,3)=-15.0D0/(XC(3)**2+2.0D0*XC(2)*XC(3)+XC(2)**2)\br
++\tab{5}RETURN\br
++\tab{5}END

```

```

Asp19(name): Exports == Implementation where
  name : Symbol

```

```

FST  ==> FortranScalarType
FT   ==> FortranType
FC   ==> FortranCode
SYMTAB ==> SymbolTable
RSFC ==> Record(localSymbols:SymbolTable,code:List(FC))
FSTU ==> Union(fst:FST,void:"void")
FRAC ==> Fraction
POLY ==> Polynomial
EXPR ==> Expression
INT  ==> Integer
FLOAT ==> Float
MFLOAT ==> MachineFloat
VEC  ==> Vector
VF2  ==> VectorFunctions2
MF2  ==> MatrixCategoryFunctions2(FEXPR,VEC FEXPR,VEC FEXPR,Matrix FEXPR,EXPR MFLOAT,VEC EXPR MFLOAT)
FEXPR ==> FortranExpression([],['XC'],MFLOAT)
S    ==> Symbol

```

```

Exports ==> FortranVectorFunctionCategory with
  coerce : VEC FEXPR -> $
    ++coerce(f) takes objects from the appropriate instantiation of
    ++\spadtype{FortranExpression} and turns them into an ASP.

```

```

Implementation ==> add

```

```

real : FSTU := ["real":FST]$FSTU
syms : SYMTAB := empty()$SYMTAB
declare!(M,fortranInteger()$FT,syms)$SYMTAB
declare!(N,fortranInteger()$FT,syms)$SYMTAB
declare!(LJC,fortranInteger()$FT,syms)$SYMTAB
xcType : FT := construct(real,[N],false)$FT
declare!(XC,xcType,syms)$SYMTAB
fveccType : FT := construct(real,[M],false)$FT
declare!(FVECC,fveccType,syms)$SYMTAB
fjaccType : FT := construct(real,[LJC,N],false)$FT
declare!(FJACC,fjaccType,syms)$SYMTAB
Rep := FortranProgram(name,["void"]$FSTU,[M,N,XC,FVECC,FJACC,LJC],syms)

coerce(c:List FC):$ == coerce(c)$Rep

coerce(r:RSFC):$ == coerce(r)$Rep

coerce(c:FC):$ == coerce(c)$Rep

```

```

-- Take a symbol, pull of the script and turn it into an integer!!
o2int(u:S):Integer ==
  o : OutputForm := first elt(scripts(u)$S,sub)
  o pretend Integer

-- To help the poor old compiler!
fexpr2expr(u:FEXPR):EXPR MFLOAT == coerce(u)$FEXPR

localAssign1(s:S,j:Matrix FEXPR):FC ==
  j' : Matrix EXPR MFLOAT := map(fexpr2expr,j)$MF2
  assign(s,j')$FC

localAssign2(s:S,j:VEC FEXPR):FC ==
  j' : VEC EXPR MFLOAT := map(fexpr2expr,j)$VF2(FEXPR,EXPR MFLOAT)
  assign(s,j')$FC

coerce(u:VEC FEXPR):$ ==
  -- First zero the Jacobian matrix in case we miss some derivatives which
  -- are zero.
  import POLY INT
  seg1 : Segment (POLY INT) := segment(1::(POLY INT),LJC@S::(POLY INT))
  seg2 : Segment (POLY INT) := segment(1::(POLY INT),N@S::(POLY INT))
  s1 : SegmentBinding POLY INT := equation(I@S,seg1)
  s2 : SegmentBinding POLY INT := equation(J@S,seg2)
  as : FC := assign(FJACC,[I@S::(POLY INT),J@S::(POLY INT)],0.0::EXPR FLOAT)
  clear : FC := forLoop(s1,forLoop(s2,as))
  j:Integer
  x:S := XC::S
  pu:List(S) := []
  -- Work out which variables appear in the expressions
  for e in entries(u) repeat
    pu := setUnion(pu,variables(e)$FEXPR)
  scriptList : List Integer := map(o2int,pu)$ListFunctions2(S,Integer)
  -- This should be the maximum XC_n which occurs (there may be others
  -- which don't):
  n:Integer := reduce(max,scriptList)$List(Integer)
  p:List(S) := []
  for j in 1..n repeat p:= cons(subscript(x,[j::OutputForm])$S,p)
  p:= reverse(p)
  jac:Matrix(FEXPR) := _
  jacobian(u,p)$MultiVariableCalculusFunctions(S,FEXPR,VEC FEXPR,List(S))
  c1:FC := localAssign2(FVECC,u)
  c2:FC := localAssign1(FJACC,jac)
  [clear,c1,c2,returns()$List(FC)::$

coerce(u:$):OutputForm == coerce(u)$Rep

outputAsFortran(u):Void ==
  p := checkPrecision()$NAGLinkSupportPackage

```

```

outputAsFortran(u)$Rep
p => restorePrecision()$NAGLinkSupportPackage

retract(u:VEC FRAC POLY INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY INT,FEXPR)
  v::$

retractIfCan(u:VEC FRAC POLY INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC FRAC POLY FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY FLOAT,FEXPR)
  v::$

retractIfCan(u:VEC FRAC POLY FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC EXPR INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(EXPR INT,FEXPR)
  v::$

retractIfCan(u:VEC EXPR INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC EXPR FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(EXPR FLOAT,FEXPR)
  v::$

retractIfCan(u:VEC EXPR FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC POLY INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(POLY INT,FEXPR)
  v::$

retractIfCan(u:VEC POLY INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC POLY FLOAT):$ ==

```

```

v : VEC FEXPR := map(retract,u)$VF2(POLY FLOAT,FEXPR)
v::$

retractIfCan(u:VEC POLY FLOAT):Union($,"failed") ==
v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY FLOAT,FEXPR)
v case "failed" => "failed"
(v::VEC FEXPR):: $

```

---

— ASP19.dotabb —

```

"ASP19" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP19"]
"FS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FS"]
"ASP19" -> "FS"

```

---

## 2.15 domain ASP20 Asp20

---

— Asp20.input —

```

)set break resume
)sys rm -f Asp20.output
)spool Asp20.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp20
--R Asp20 name: Symbol is a domain constructor
--R Abbreviation for Asp20 is ASP20
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP20
--R
--R----- Operations -----
--R coerce : FortranCode -> %               coerce : List FortranCode -> %
--R coerce : % -> OutputForm               outputAsFortran : % -> Void
--R coerce : Matrix FortranExpression([construct],[construct,QUOTEX,QUOTEHESS],MachineFloat)
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Matrix Fraction Polynomial Integer -> %
--R retract : Matrix Fraction Polynomial Float -> %
--R retract : Matrix Polynomial Integer -> %

```

```

--R retract : Matrix Polynomial Float -> %
--R retract : Matrix Expression Integer -> %
--R retract : Matrix Expression Float -> %
--R retractIfCan : Matrix Fraction Polynomial Integer -> Union(%, "failed")
--R retractIfCan : Matrix Fraction Polynomial Float -> Union(%, "failed")
--R retractIfCan : Matrix Polynomial Integer -> Union(%, "failed")
--R retractIfCan : Matrix Polynomial Float -> Union(%, "failed")
--R retractIfCan : Matrix Expression Integer -> Union(%, "failed")
--R retractIfCan : Matrix Expression Float -> Union(%, "failed")
--R
--E 1

)spool
)lisp (bye)

```

---

— Asp20.help —

=====

Asp20 examples

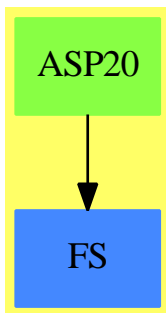
=====

See Also:

- o )show Asp20

---

### 2.15.1 Asp20 (ASP20)



#### Exports:

coerce outputAsFortran retract retractIfCan

— domain ASP20 Asp20 —

```

)abbrev domain ASP20 Asp20
++ Author: Mike Dewar and Godfrey Nolan and Grant Keady
++ Date Created: Dec 1993
++ Date Last Updated: 21 March 1994
++                      6 October 1994
++ Related Constructors: FortranVectorFunctionCategory, FortranProgramCategory
++ Description:
++ \spadtype{Asp20} produces Fortran for Type 20 ASPs, for example:
++
++ \tab{5}SUBROUTINE QPHESS(N,NROWH,NCOLH,JTHCOL,HESS,X,HX)\br
++ \tab{5}DOUBLE PRECISION HX(N),X(N),HESS(NROWH,NCOLH)\br
++ \tab{5}INTEGER JTHCOL,N,NROWH,NCOLH\br
++ \tab{5}HX(1)=2.0D0*X(1)\br
++ \tab{5}HX(2)=2.0D0*X(2)\br
++ \tab{5}HX(3)=2.0D0*X(4)+2.0D0*X(3)\br
++ \tab{5}HX(4)=2.0D0*X(4)+2.0D0*X(3)\br
++ \tab{5}HX(5)=2.0D0*X(5)\br
++ \tab{5}HX(6)=(-2.0D0*X(7))+(-2.0D0*X(6))\br
++ \tab{5}HX(7)=(-2.0D0*X(7))+(-2.0D0*X(6))\br
++ \tab{5}RETURN\br
++ \tab{5}END

```

```

Asp20(name): Exports == Implementation where
name : Symbol

```

```

FST    ==> FortranScalarType
FT     ==> FortranType
SYMTAB ==> SymbolTable
PI     ==> PositiveInteger
UFST   ==> Union(fst:FST,void:"void")
RSFC   ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
FRAC   ==> Fraction
POLY   ==> Polynomial
EXPR   ==> Expression
INT    ==> Integer
FLOAT  ==> Float
VEC    ==> Vector
MAT    ==> Matrix
VF2    ==> VectorFunctions2
MFLOAT ==> MachineFloat
FEXPR  ==> FortranExpression([],['X','HESS'],MFLOAT)
O      ==> OutputForm
M2     ==> MatrixCategoryFunctions2
MF2a   ==> M2(FRAC POLY INT,VEC FRAC POLY INT,VEC FRAC POLY INT,
             MAT FRAC POLY INT,FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)
MF2b   ==> M2(FRAC POLY FLOAT,VEC FRAC POLY FLOAT,VEC FRAC POLY FLOAT,
             MAT FRAC POLY FLOAT, FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)
MF2c   ==> M2(POLY INT,VEC POLY INT,VEC POLY INT,MAT POLY INT,
             FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)
MF2d   ==> M2(POLY FLOAT,VEC POLY FLOAT,VEC POLY FLOAT,

```

```

      MAT POLY FLOAT, FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)
MF2e ==> M2(EXPR INT,VEC EXPR INT,VEC EXPR INT,MAT EXPR INT,
      FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)
MF2f ==> M2(EXPR FLOAT,VEC EXPR FLOAT,VEC EXPR FLOAT,
      MAT EXPR FLOAT, FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)

Exports ==> FortranMatrixFunctionCategory with
  coerce: MAT FEXPR -> $
    ++coerce(f) takes objects from the appropriate instantiation of
    ++\spadtype{FortranExpression} and turns them into an ASP.

Implementation ==> add

  real : UFST := ["real"::FST]$UFST
  syms : SYMTAB := empty()
  declare!(N,fortranInteger(),syms)$SYMTAB
  declare!(NROWH,fortranInteger(),syms)$SYMTAB
  declare!(NCOLH,fortranInteger(),syms)$SYMTAB
  declare!(JTHCOL,fortranInteger(),syms)$SYMTAB
  hessType : FT := construct(real,[NROWH,NCOLH],false)$FT
  declare!(HESS,hessType,syms)$SYMTAB
  xType : FT := construct(real,[N],false)$FT
  declare!(X,xType,syms)$SYMTAB
  declare!(HX,xType,syms)$SYMTAB
  Rep := FortranProgram(name,["void"]$UFST,
      [N,NROWH,NCOLH,JTHCOL,HESS,X,HX],syms)

  coerce(c:List FortranCode):$ == coerce(c)$Rep

  coerce(r:RSFC):$ == coerce(r)$Rep

  coerce(c:FortranCode):$ == coerce(c)$Rep

  -- To help the poor old compiler!
  fexpr2expr(u:FEXPR):EXPR MFLOAT == coerce(u)$FEXPR

  localAssign(s:Symbol,j:VEC FEXPR):FortranCode ==
    j' : VEC EXPR MFLOAT := map(fexpr2expr,j)$VF2(FEXPR,EXPR MFLOAT)
    assign(s,j')$FortranCode

  coerce(u:MAT FEXPR):$ ==
    j:Integer
    x:Symbol := X::Symbol
    n := nrows(u)::PI
    p:VEC FEXPR := [retract(subscript(x,[j::0])$Symbol)$FEXPR for j in 1..n]
    prod:VEC FEXPR := u*p
    ([localAssign(HX,prod),returns()$FortranCode]$List(FortranCode)):$

  retract(u:MAT FRAC POLY INT):$ ==

```



```

v : MAT FEXPR := map(retract,u)$MF2a
v::$

retractIfCan(u:MAT FRAC POLY INT):Union($,"failed") ==
v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2a
v case "failed" => "failed"
(v::MAT FEXPR):: $

retract(u:MAT FRAC POLY FLOAT):$ ==
v : MAT FEXPR := map(retract,u)$MF2b
v::$

retractIfCan(u:MAT FRAC POLY FLOAT):Union($,"failed") ==
v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2b
v case "failed" => "failed"
(v::MAT FEXPR):: $

retract(u:MAT EXPR INT):$ ==
v : MAT FEXPR := map(retract,u)$MF2e
v::$

retractIfCan(u:MAT EXPR INT):Union($,"failed") ==
v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2e
v case "failed" => "failed"
(v::MAT FEXPR):: $

retract(u:MAT EXPR FLOAT):$ ==
v : MAT FEXPR := map(retract,u)$MF2f
v::$

retractIfCan(u:MAT EXPR FLOAT):Union($,"failed") ==
v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2f
v case "failed" => "failed"
(v::MAT FEXPR):: $

retract(u:MAT POLY INT):$ ==
v : MAT FEXPR := map(retract,u)$MF2c
v::$

retractIfCan(u:MAT POLY INT):Union($,"failed") ==
v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2c
v case "failed" => "failed"
(v::MAT FEXPR):: $

retract(u:MAT POLY FLOAT):$ ==
v : MAT FEXPR := map(retract,u)$MF2d
v::$

retractIfCan(u:MAT POLY FLOAT):Union($,"failed") ==
v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2d

```

```

v case "failed" => "failed"
(v::MAT FEXPR):: $

coerce(u:$):0 == coerce(u)$Rep

outputAsFortran(u):Void ==
  p := checkPrecision()$NAGLinkSupportPackage
  outputAsFortran(u)$Rep
  p => restorePrecision()$NAGLinkSupportPackage

-----

— ASP20.dotabb —

"ASP20" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP20"]
"FS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FS"]
"ASP20" -> "FS"

-----

```

## 2.16 domain ASP24 Asp24

```

— Asp24.input —

)set break resume
)sys rm -f Asp24.output
)spool Asp24.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp24
--R Asp24 name: Symbol is a domain constructor
--R Abbreviation for Asp24 is ASP24
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP24
--R
--R----- Operations -----
--R coerce : FortranCode -> %          coerce : List FortranCode -> %
--R coerce : % -> OutputForm          outputAsFortran : % -> Void
--R retract : Polynomial Integer -> %  retract : Polynomial Float -> %
--R retract : Expression Integer -> %   retract : Expression Float -> %
--R coerce : FortranExpression([construct],[construct,QUOTEXC],MachineFloat) -> %

```

```

--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Fraction Polynomial Integer -> %
--R retract : Fraction Polynomial Float -> %
--R retractIfCan : Fraction Polynomial Integer -> Union(%, "failed")
--R retractIfCan : Fraction Polynomial Float -> Union(%, "failed")
--R retractIfCan : Polynomial Integer -> Union(%, "failed")
--R retractIfCan : Polynomial Float -> Union(%, "failed")
--R retractIfCan : Expression Integer -> Union(%, "failed")
--R retractIfCan : Expression Float -> Union(%, "failed")
--R
--E 1

)spool
)lisp (bye)

```

---

— Asp24.help —

=====

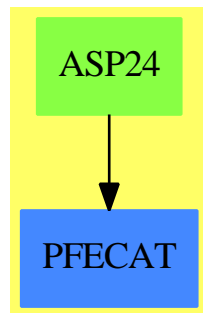
Asp24 examples

=====

See Also:

- o )show Asp24

### 2.16.1 Asp24 (ASP24)



#### Exports:

coerce outputAsFortran retract retractIfCan

— domain ASP24 Asp24 —

```

)abbrev domain ASP24 Asp24
++ Author: Mike Dewar, Grant Keady and Godfrey Nolan
++ Date Created: Mar 1993
++ Date Last Updated: 21 March 1994
++                               6 October 1994
++ Related Constructors: FortranScalarFunctionCategory, FortranProgramCategory
++ Description:
++\spadtype{Asp24} produces Fortran for Type 24 ASPs which evaluate a
++multivariate function at a point (needed for NAG routine e04jaf),
++for example:
++
++\tab{5}SUBROUTINE FUNCT1(N,XC,FC)\br
++\tab{5}DOUBLE PRECISION FC,XC(N)\br
++\tab{5}INTEGER N\br
++\tab{5}FC=10.0D0*XC(4)**4+(-40.0D0*XC(1)*XC(4)**3)+(60.0D0*XC(1)**2+5\br
++\tab{4}&.0D0)*XC(4)**2+((-10.0D0*XC(3))+(-40.0D0*XC(1)**3))*XC(4)+16.0D0*X\br
++\tab{4}&C(3)**4+(-32.0D0*XC(2)*XC(3)**3)+(24.0D0*XC(2)**2+5.0D0)*XC(3)**2+\br
++\tab{4}&(-8.0D0*XC(2)**3*XC(3))+XC(2)**4+100.0D0*XC(2)**2+20.0D0*XC(1)*XC(\br
++\tab{4}&2)+10.0D0*XC(1)**4+XC(1)**2\br
++\tab{5}RETURN\br
++\tab{5}END\br

Asp24(name): Exports == Implementation where
  name : Symbol

FST ==> FortranScalarType
FT ==> FortranType
SYMTAB ==> SymbolTable
RSFC ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
FSTU ==> Union(fst:FST,void:"void")
FEXPR ==> FortranExpression([],['XC'],MachineFloat)
FRAC ==> Fraction
POLY ==> Polynomial
EXPR ==> Expression
INT ==> Integer
FLOAT ==> Float

Exports ==> FortranFunctionCategory with
  coerce : FEXPR -> $
  ++ coerce(f) takes an object from the appropriate instantiation of
  ++ \spadtype{FortranExpression} and turns it into an ASP.

Implementation ==> add

real : FSTU := ["real":FST]$FSTU
syms : SYMTAB := empty()
declare!(N,fortranInteger(),syms)$SYMTAB
xcType : FT := construct(real,[N:Symbol],false)$FT

```

```

declare!(XC,xcType,syms)$SYMTAB
declare!(FC,fortranReal(),syms)$SYMTAB
Rep := FortranProgram(name,["void"]$FSTU,[N,XC,FC],syms)

coerce(c:List FortranCode):$ == coerce(c)$Rep

coerce(r:RSFC):$ == coerce(r)$Rep

coerce(c:FortranCode):$ == coerce(c)$Rep

coerce(u:FEXPR):$ ==
  coerce(assign(FC,u::Expression(MachineFloat))$FortranCode)$Rep

retract(u:FRAC POLY INT):$ == (retract(u)@FEXPR)::$
retractIfCan(u:FRAC POLY INT):Union($,"failed") ==
  foo : Union(FEXPR,"failed")
  foo := retractIfCan(u)$FEXPR
  foo case "failed" => "failed"
  (foo::FEXPR)::$

retract(u:FRAC POLY FLOAT):$ == (retract(u)@FEXPR)::$
retractIfCan(u:FRAC POLY FLOAT):Union($,"failed") ==
  foo : Union(FEXPR,"failed")
  foo := retractIfCan(u)$FEXPR
  foo case "failed" => "failed"
  (foo::FEXPR)::$

retract(u:EXPR FLOAT):$ == (retract(u)@FEXPR)::$
retractIfCan(u:EXPR FLOAT):Union($,"failed") ==
  foo : Union(FEXPR,"failed")
  foo := retractIfCan(u)$FEXPR
  foo case "failed" => "failed"
  (foo::FEXPR)::$

retract(u:EXPR INT):$ == (retract(u)@FEXPR)::$
retractIfCan(u:EXPR INT):Union($,"failed") ==
  foo : Union(FEXPR,"failed")
  foo := retractIfCan(u)$FEXPR
  foo case "failed" => "failed"
  (foo::FEXPR)::$

retract(u:POLY FLOAT):$ == (retract(u)@FEXPR)::$
retractIfCan(u:POLY FLOAT):Union($,"failed") ==
  foo : Union(FEXPR,"failed")
  foo := retractIfCan(u)$FEXPR
  foo case "failed" => "failed"
  (foo::FEXPR)::$

retract(u:POLY INT):$ == (retract(u)@FEXPR)::$
retractIfCan(u:POLY INT):Union($,"failed") ==

```

```

foo : Union(FEXPR,"failed")
foo := retractIfCan(u)$FEXPR
foo case "failed" => "failed"
(foo::FEXPR)::$

coerce(u:$):OutputForm == coerce(u)$Rep

outputAsFortran(u):Void ==
  p := checkPrecision()$NAGLinkSupportPackage
  outputAsFortran(u)$Rep
  p => restorePrecision()$NAGLinkSupportPackage

```

---

— ASP24.dotabb —

```

"ASP24" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP24"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"ASP24" -> "PFECAT"

```

---

## 2.17 domain ASP27 Asp27

---

— Asp27.input —

```

)set break resume
)sys rm -f Asp27.output
)spool Asp27.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp27
--R Asp27 name: Symbol is a domain constructor
--R Abbreviation for Asp27 is ASP27
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP27
--R
--R----- Operations -----
--R coerce : FortranCode -> %           coerce : List FortranCode -> %
--R coerce : Matrix MachineFloat -> %   coerce : % -> OutputForm
--R outputAsFortran : % -> Void

```

```
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R
--E 1
```

```
)spool
)lisp (bye)
```

---

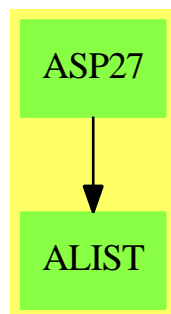
— Asp27.help —

```
=====
Asp27 examples
=====
```

```
See Also:
o )show Asp27
```

---

### 2.17.1 Asp27 (ASP27)



#### Exports:

```
coerce  outputAsFortran
```

— domain ASP27 Asp27 —

```
)abbrev domain ASP27 Asp27
++ Author: Mike Dewar and Godfrey Nolan
++ Date Created: Nov 1993
++ Date Last Updated: 27 April 1994
++                               6 October 1994
++ Related Constructors: FortranScalarFunctionCategory, FortranProgramCategory
```

```

++ Description:
++\spadtype{Asp27} produces Fortran for Type 27 ASPs, needed for NAG routine
++f02fjf ,for example:
++
++\tab{5}FUNCTION DOT(IFLAG,N,Z,W,RWORK,LRWORK,IWORK,LIWORK)\br
++\tab{5}DOUBLE PRECISION W(N),Z(N),RWORK(LRWORK)\br
++\tab{5}INTEGER N,LIWORK,IFLAG,LRWORK,IWORK(LIWORK)\br
++\tab{5}DOT=(W(16)+(-0.5D0*W(15)))*Z(16)+((-0.5D0*W(16))+W(15)+(-0.5D0*W(1\br
++\tab{4}&4)))*Z(15)+((-0.5D0*W(15))+W(14)+(-0.5D0*W(13)))*Z(14)+((-0.5D0*W(\br
++\tab{4}&14))+W(13)+(-0.5D0*W(12)))*Z(13)+((-0.5D0*W(13))+W(12)+(-0.5D0*W(1\br
++\tab{4}&11)))*Z(12)+((-0.5D0*W(12))+W(11)+(-0.5D0*W(10)))*Z(11)+((-0.5D0*W(\br
++\tab{4}&11))+W(10)+(-0.5D0*W(9)))*Z(10)+((-0.5D0*W(10))+W(9)+(-0.5D0*W(8))\br
++\tab{4}&8)*Z(9)+((-0.5D0*W(9))+W(8)+(-0.5D0*W(7)))*Z(8)+((-0.5D0*W(8))+W(7)\br
++\tab{4}&5)*Z(7)+((-0.5D0*W(7))+W(6)+(-0.5D0*W(5)))*Z(6)+((-0.\br
++\tab{4}&5D0*W(6))+W(5)+(-0.5D0*W(4)))*Z(5)+((-0.5D0*W(5))+W(4)+(-0.5D0*W(3\br
++\tab{4}&3)))*Z(4)+((-0.5D0*W(4))+W(3)+(-0.5D0*W(2)))*Z(3)+((-0.5D0*W(3))+W(\br
++\tab{4}&2)+(-0.5D0*W(1)))*Z(2)+((-0.5D0*W(2))+W(1))*Z(1)\br
++\tab{5}RETURN\br
++\tab{5}END

```

```

Asp27(name): Exports == Implementation where
  name : Symbol

```

```

O      ==> OutputForm
FST    ==> FortranScalarType
FT     ==> FortranType
SYMTAB ==> SymbolTable
UFST   ==> Union(fst:FST,void:"void")
FC     ==> FortranCode
PI     ==> PositiveInteger
RSFC   ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
EXPR   ==> Expression
MAT    ==> Matrix
MFLOAT ==> MachineFloat

```

```
Exports == FortranMatrixCategory
```

```
Implementation == add
```

```

real : UFST := ["real":FST]$UFST
integer : UFST := ["integer":FST]$UFST
syms : SYMTAB := empty()$SYMTAB
declare!(IFLAG,fortranInteger(),syms)$SYMTAB
declare!(N,fortranInteger(),syms)$SYMTAB
declare!(LRWORK,fortranInteger(),syms)$SYMTAB
declare!(LIWORK,fortranInteger(),syms)$SYMTAB
zType : FT := construct(real,[N],false)$FT

```



```

declare!(Z,zType,syms)$SYMTAB
declare!(W,zType,syms)$SYMTAB
rType : FT := construct(real,[LRWORK],false)$FT
declare!(RWORK,rType,syms)$SYMTAB
iType : FT := construct(integer,[LIWORK],false)$FT
declare!(IWORK,iType,syms)$SYMTAB
Rep := FortranProgram(name,real,
                      [IFLAG,N,Z,W,RWORK,LRWORK,IWORK,LIWORK],syms)

-- To help the poor old compiler!
localCoerce(u:Symbol):EXPR(MFLOAT) == coerce(u)$EXPR(MFLOAT)

coerce (u:MAT MFLOAT):$ ==
  Ws: Symbol := W
  Zs: Symbol := Z
  code : List FC
  l:EXPR MFLOAT := "+"/_
    [("+"/[localCoerce(elt(Ws,[j::0])$Symbol) * u(j,i)_
                                     for j in 1..nrows(u)::PI])_
    *localCoerce(elt(Zs,[i::0])$Symbol) for i in 1..ncols(u)::PI]
  c := assign(name,l)$FC
  code := [c,returns()]$List(FC)
  code:::$

coerce(c:List FortranCode):$ == coerce(c)$Rep

coerce(r:RSFC):$ == coerce(r)$Rep

coerce(c:FortranCode):$ == coerce(c)$Rep

coerce(u:$):OutputForm == coerce(u)$Rep

outputAsFortran(u):Void ==
  p := checkPrecision()$NAGLinkSupportPackage
  outputAsFortran(u)$Rep
  p => restorePrecision()$NAGLinkSupportPackage

```

---

— ASP27.dotabb —

```

"ASP27" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP27"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"ASP27" -> "ALIST"

```

---

## 2.18 domain ASP28 Asp28

— Asp28.input —

```
)set break resume
)sys rm -f Asp28.output
)spool Asp28.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp28
--R Asp28 name: Symbol is a domain constructor
--R Abbreviation for Asp28 is ASP28
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP28
--R
--R----- Operations -----
--R coerce : FortranCode -> %          coerce : List FortranCode -> %
--R coerce : Matrix MachineFloat -> %  coerce : % -> OutputForm
--R outputAsFortran : % -> Void
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R
--E 1

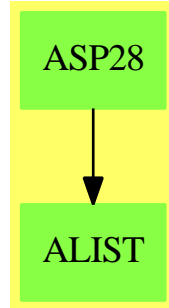
)spool
)lisp (bye)
```

— Asp28.help —

```
=====
Asp28 examples
=====
```

See Also:  
o )show Asp28

### 2.18.1 Asp28 (ASP28)



#### Exports:

coerce outputAsFortran

— domain ASP28 Asp28 —

```

)abbrev domain ASP28 Asp28
++ Author: Mike Dewar
++ Date Created: 21 March 1994
++ Date Last Updated: 28 April 1994
++               6 October 1994
++ Related Constructors: FortranVectorFunctionCategory, FortranProgramCategory
++ Description:
++\spadtype{Asp28} produces Fortran for Type 28 ASPs, used in NAG routine
++f02fjf, for example:
++
++\tab{5}SUBROUTINE IMAGE(IFLAG,N,Z,W,RWORK,LRWORK,IWORK,LIWORK)\br
++\tab{5}DOUBLE PRECISION Z(N),W(N),IWORK(LRWORK),RWORK(LRWORK)\br
++\tab{5}INTEGER N,LIWORK,IFLAG,LRWORK\br
++\tab{5}W(1)=0.01707454969713436D0*Z(16)+0.001747395874954051D0*Z(15)+0.00\br
++\tab{4}&2106973900813502D0*Z(14)+0.002957434991769087D0*Z(13)+(-0.00700554\br
++\tab{4}&0882865317D0*Z(12))+(-0.01219194009813166D0*Z(11))+0.0037230647365\br
++\tab{4}&3087D0*Z(10)+0.04932374658377151D0*Z(9)+(-0.03586220812223305D0*Z(\br
++\tab{4}&8))+(-0.04723268012114625D0*Z(7))+(-0.02434652144032987D0*Z(6))+0.\br
++\tab{4}&2264766947290192D0*Z(5))+(-0.1385343580686922D0*Z(4))+(-0.116530050\br
++\tab{4}&8238904D0*Z(3))+(-0.2803531651057233D0*Z(2))+1.019463911841327D0*Z\br
++\tab{4}&(1)\br
++\tab{5}W(2)=0.0227345011107737D0*Z(16)+0.008812321197398072D0*Z(15)+0.010\br
++\tab{4}&94012210519586D0*Z(14)+(-0.01764072463999744D0*Z(13))+(-0.01357136\br
++\tab{4}&72105995D0*Z(12))+0.00157466157362272D0*Z(11)+0.05258889186338282D\br
++\tab{4}&0*Z(10)+(-0.01981532388243379D0*Z(9))+(-0.06095390688679697D0*Z(8)\br
++\tab{4}&)+(-0.04153119955569051D0*Z(7))+0.2176561076571465D0*Z(6)+(-0.0532\br
++\tab{4}&5555586632358D0*Z(5))+(-0.1688977368984641D0*Z(4))+(-0.32440166056\br
++\tab{4}&67343D0*Z(3))+0.9128222941872173D0*Z(2)+(-0.2419652703415429D0*Z(1\br
++\tab{4}&))\br
++\tab{5}W(3)=0.03371198197190302D0*Z(16)+0.02021603150122265D0*Z(15)+(-0.0\br

```

```

++\tab{4}&06607305534689702D0*Z(14))+(-0.03032392238968179D0*Z(13))+0.002033\br
++\tab{4}&305231024948D0*Z(12))+0.05375944956767728D0*Z(11))+(-0.0163213312502\br
++\tab{4}&9967D0*Z(10))+(-0.05483186562035512D0*Z(9))+(-0.04901428822579872D\br
++\tab{4}&0*Z(8))+0.2091097927887612D0*Z(7))+(-0.05760560341383113D0*Z(6))+(-\br
++\tab{4}&0.1236679206156403D0*Z(5))+(-0.3523683853026259D0*Z(4))+0.88929961\br
++\tab{4}&32269974D0*Z(3))+(-0.2995429545781457D0*Z(2))+(-0.02986582812574917\br
++\tab{4}&D0*Z(1))\br
++\tab{5}W(4)=0.05141563713660119D0*Z(16))+0.005239165960779299D0*Z(15))+(-0.\br
++\tab{4}&01623427735779699D0*Z(14))+(-0.01965809746040371D0*Z(13))+0.054688\br
++\tab{4}&97337339577D0*Z(12))+(-0.014224695935687D0*Z(11))+(-0.0505181779315\br
++\tab{4}&6355D0*Z(10))+(-0.04353074206076491D0*Z(9))+0.2012230497530726D0*Z\br
++\tab{4}&(8))+(-0.06630874514535952D0*Z(7))+(-0.1280829963720053D0*Z(6))+(-0\br
++\tab{4}&.305169742604165D0*Z(5))+0.8600427128450191D0*Z(4))+(-0.32415033802\br
++\tab{4}&68184D0*Z(3))+(-0.09033531980693314D0*Z(2))+0.09089205517109111D0*\br
++\tab{4}&Z(1))\br
++\tab{5}W(5)=0.04556369767776375D0*Z(16))+(-0.001822737697581869D0*Z(15))+(\br
++\tab{4}&-0.002512226501941856D0*Z(14))+0.02947046460707379D0*Z(13))+(-0.014\br
++\tab{4}&45079632086177D0*Z(12))+(-0.05034242196614937D0*Z(11))+(-0.0376966\br
++\tab{4}&3291725935D0*Z(10))+0.2171103102175198D0*Z(9))+(-0.0824949256021352\br
++\tab{4}&4D0*Z(8))+(-0.1473995209288945D0*Z(7))+(-0.315042193418466D0*Z(6))\br
++\tab{4}&+0.9591623347824002D0*Z(5))+(-0.3852396953763045D0*Z(4))+(-0.141718\br
++\tab{4}&5427288274D0*Z(3))+(-0.03423495461011043D0*Z(2))+0.319820917706851\br
++\tab{4}&6D0*Z(1))\br
++\tab{5}W(6)=0.04015147277405744D0*Z(16))+0.01328585741341559D0*Z(15))+0.048\br
++\tab{4}&26082005465965D0*Z(14))+(-0.04319641116207706D0*Z(13))+(-0.04931323\br
++\tab{4}&319055762D0*Z(12))+(-0.03526886317505474D0*Z(11))+0.22295383396730\br
++\tab{4}&01D0*Z(10))+(-0.07375317649315155D0*Z(9))+(-0.1589391311991561D0*Z(\br
++\tab{4}&8))+(-0.328001910890377D0*Z(7))+0.952576555482747D0*Z(6))+(-0.31583\br
++\tab{4}&09975786731D0*Z(5))+(-0.1846882042225383D0*Z(4))+(-0.0703762046700\br
++\tab{4}&4427D0*Z(3))+0.2311852964327382D0*Z(2))+0.04254083491825025D0*Z(1)\br
++\tab{5}W(7)=0.06069778964023718D0*Z(16))+0.06681263884671322D0*Z(15))+(-0.0\br
++\tab{4}&2113506688615768D0*Z(14))+(-0.083996867458326D0*Z(13))+(-0.0329843\br
++\tab{4}&8523869648D0*Z(12))+0.2276878326327734D0*Z(11))+(-0.067356038933017\br
++\tab{4}&95D0*Z(10))+(-0.1559813965382218D0*Z(9))+(-0.3363262957694705D0*Z(\br
++\tab{4}&8))+0.9442791158560948D0*Z(7))+(-0.3199955249404657D0*Z(6))+(-0.136\br
++\tab{4}&2463839920727D0*Z(5))+(-0.1006185171570586D0*Z(4))+0.2057504515015\br
++\tab{4}&423D0*Z(3))+(-0.02065879269286707D0*Z(2))+0.03160990266745513D0*Z(1\br
++\tab{4}&)\br
++\tab{5}W(8)=0.126386868896738D0*Z(16))+0.002563370039476418D0*Z(15))+(-0.05\br
++\tab{4}&581757739455641D0*Z(14))+(-0.07777893205900685D0*Z(13))+0.23117338\br
++\tab{4}&45834199D0*Z(12))+(-0.06031581134427592D0*Z(11))+(-0.14805474755869\br
++\tab{4}&52D0*Z(10))+(-0.3364014128402243D0*Z(9))+0.9364014128402244D0*Z(8)\br
++\tab{4}&+(-0.3269452524413048D0*Z(7))+(-0.1396841886557241D0*Z(6))+(-0.056\br
++\tab{4}&1733845834199D0*Z(5))+0.1777789320590069D0*Z(4))+(-0.04418242260544\br
++\tab{4}&359D0*Z(3))+(-0.02756337003947642D0*Z(2))+0.07361313110326199D0*Z(\br
++\tab{4}&1)\br
++\tab{5}W(9)=0.07361313110326199D0*Z(16))+(-0.02756337003947642D0*Z(15))+(-\br
++\tab{4}&0.04418242260544359D0*Z(14))+0.1777789320590069D0*Z(13))+(-0.056173\br
++\tab{4}&3845834199D0*Z(12))+(-0.1396841886557241D0*Z(11))+(-0.326945252441\br
++\tab{4}&3048D0*Z(10))+0.9364014128402244D0*Z(9))+(-0.3364014128402243D0*Z(8)\br

```

```

++\tab{4}&))+(-0.1480547475586952D0*Z(7))+(-0.06031581134427592D0*Z(6))+0.23\br
++\tab{4}&11733845834199D0*Z(5))+(-0.07777893205900685D0*Z(4))+(-0.0558175773\br
++\tab{4}&9455641D0*Z(3))+0.002563370039476418D0*Z(2))+0.126386868896738D0*Z(\br
++\tab{4}&1)\br
++\tab{5}W(10)=0.03160990266745513D0*Z(16))+(-0.02065879269286707D0*Z(15))+0\br
++\tab{4}&.2057504515015423D0*Z(14))+(-0.1006185171570586D0*Z(13))+(-0.136246\br
++\tab{4}&3839920727D0*Z(12))+(-0.3199955249404657D0*Z(11))+0.94427911585609\br
++\tab{4}&48D0*Z(10))+(-0.3363262957694705D0*Z(9))+(-0.1559813965382218D0*Z(8)\br
++\tab{4}&))+(-0.06735603893301795D0*Z(7))+0.2276878326327734D0*Z(6))+(-0.032\br
++\tab{4}&98438523869648D0*Z(5))+(-0.083996867458326D0*Z(4))+(-0.02113506688\br
++\tab{4}&615768D0*Z(3))+0.06681263884671322D0*Z(2))+0.06069778964023718D0*Z(\br
++\tab{4}&1)\br
++\tab{5}W(11)=0.04254083491825025D0*Z(16))+0.2311852964327382D0*Z(15))+(-0.0\br
++\tab{4}&7037620467004427D0*Z(14))+(-0.1846882042225383D0*Z(13))+(-0.315830\br
++\tab{4}&9975786731D0*Z(12))+0.952576555482747D0*Z(11))+(-0.328001910890377D\br
++\tab{4}&0*Z(10))+(-0.1589391311991561D0*Z(9))+(-0.07375317649315155D0*Z(8)\br
++\tab{4}&)+0.2229538339673001D0*Z(7))+(-0.03526886317505474D0*Z(6))+(-0.0493\br
++\tab{4}&1323319055762D0*Z(5))+(-0.04319641116207706D0*Z(4))+0.048260820054\br
++\tab{4}&65965D0*Z(3))+0.01328585741341559D0*Z(2))+0.04015147277405744D0*Z(1)\br
++\tab{5}W(12)=0.3198209177068516D0*Z(16))+(-0.03423495461011043D0*Z(15))+(-\br
++\tab{4}&0.1417185427288274D0*Z(14))+(-0.3852396953763045D0*Z(13))+0.959162\br
++\tab{4}&3347824002D0*Z(12))+(-0.315042193418466D0*Z(11))+(-0.14739952092889\br
++\tab{4}&45D0*Z(10))+(-0.08249492560213524D0*Z(9))+0.2171103102175198D0*Z(8\br
++\tab{4}&)+(-0.03769663291725935D0*Z(7))+(-0.05034242196614937D0*Z(6))+(-0.\br
++\tab{4}&01445079632086177D0*Z(5))+0.02947046460707379D0*Z(4))+(-0.002512226\br
++\tab{4}&501941856D0*Z(3))+(-0.001822737697581869D0*Z(2))+0.045563697677763\br
++\tab{4}&75D0*Z(1)\br
++\tab{5}W(13)=0.09089205517109111D0*Z(16))+(-0.09033531980693314D0*Z(15))+(\br
++\tab{4}&-0.3241503380268184D0*Z(14))+0.8600427128450191D0*Z(13))+(-0.305169\br
++\tab{4}&742604165D0*Z(12))+(-0.1280829963720053D0*Z(11))+(-0.0663087451453\br
++\tab{4}&5952D0*Z(10))+0.2012230497530726D0*Z(9))+(-0.04353074206076491D0*Z(\br
++\tab{4}&8))+(-0.05051817793156355D0*Z(7))+(-0.014224695935687D0*Z(6))+0.05\br
++\tab{4}&468897337339577D0*Z(5))+(-0.01965809746040371D0*Z(4))+(-0.016234277\br
++\tab{4}&35779699D0*Z(3))+0.005239165960779299D0*Z(2))+0.05141563713660119D0\br
++\tab{4}&*Z(1)\br
++\tab{5}W(14)=(-0.02986582812574917D0*Z(16))+(-0.2995429545781457D0*Z(15))\br
++\tab{4}&+0.8892996132269974D0*Z(14))+(-0.3523683853026259D0*Z(13))+(-0.1236\br
++\tab{4}&679206156403D0*Z(12))+(-0.05760560341383113D0*Z(11))+0.20910979278\br
++\tab{4}&87612D0*Z(10))+(-0.04901428822579872D0*Z(9))+(-0.05483186562035512D\br
++\tab{4}&0*Z(8))+(-0.01632133125029967D0*Z(7))+0.05375944956767728D0*Z(6))+0\br
++\tab{4}&.002033305231024948D0*Z(5))+(-0.03032392238968179D0*Z(4))+(-0.00660\br
++\tab{4}&7305534689702D0*Z(3))+0.02021603150122265D0*Z(2))+0.033711981971903\br
++\tab{4}&02D0*Z(1)\br
++\tab{5}W(15)=(-0.2419652703415429D0*Z(16))+0.9128222941872173D0*Z(15))+(-0\br
++\tab{4}&.3244016605667343D0*Z(14))+(-0.1688977368984641D0*Z(13))+(-0.05325\br
++\tab{4}&555586632358D0*Z(12))+0.2176561076571465D0*Z(11))+(-0.0415311995556\br
++\tab{4}&9051D0*Z(10))+(-0.06095390688679697D0*Z(9))+(-0.01981532388243379D\br
++\tab{4}&0*Z(8))+0.05258889186338282D0*Z(7))+0.00157466157362272D0*Z(6))+(-0.\br
++\tab{4}&0135713672105995D0*Z(5))+(-0.01764072463999744D0*Z(4))+0.010940122\br
++\tab{4}&10519586D0*Z(3))+0.008812321197398072D0*Z(2))+0.0227345011107737D0*Z\br

```

```

++\tab{4}&(1)\br
++\tab{5}W(16)=1.019463911841327D0*Z(16)+(-0.2803531651057233D0*Z(15))+(-0.\br
++\tab{4}&1165300508238904D0*Z(14))+(-0.1385343580686922D0*Z(13))+0.22647669\br
++\tab{4}&47290192D0*Z(12))+(-0.02434652144032987D0*Z(11))+(-0.04723268012114\br
++\tab{4}&625D0*Z(10))+(-0.03586220812223305D0*Z(9))+0.04932374658377151D0*Z\br
++\tab{4}&(8)+0.00372306473653087D0*Z(7)+(-0.01219194009813166D0*Z(6))+(-0.0\br
++\tab{4}&07005540882865317D0*Z(5))+0.002957434991769087D0*Z(4)+0.0021069739\br
++\tab{4}&00813502D0*Z(3)+0.001747395874954051D0*Z(2)+0.01707454969713436D0*\br
++\tab{4}&Z(1)\br
++\tab{5}RETURN\br
++\tab{5}END\br

```

```

Asp28(name): Exports == Implementation where
  name : Symbol

```

```

FST    ==> FortranScalarType
FT     ==> FortranType
SYMTAB ==> SymbolTable
FC     ==> FortranCode
PI     ==> PositiveInteger
RSFC   ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
EXPR   ==> Expression
MFLOAT ==> MachineFloat
VEC     ==> Vector
UFST   ==> Union(fst:FST,void:"void")
MAT     ==> Matrix

```

```

Exports == FortranMatrixCategory

```

```

Implementation == add

```

```

real : UFST := ["real":FST]$UFST
syms : SYMTAB := empty()
declare!(IFLAG,fortranInteger(),syms)$SYMTAB
declare!(N,fortranInteger(),syms)$SYMTAB
declare!(LRWORK,fortranInteger(),syms)$SYMTAB
declare!(LIWORK,fortranInteger(),syms)$SYMTAB
xType : FT := construct(real,[N],false)$FT
declare!(Z,xType,syms)$SYMTAB
declare!(W,xType,syms)$SYMTAB
rType : FT := construct(real,[LRWORK],false)$FT
declare!(RWORK,rType,syms)$SYMTAB
iType : FT := construct(real,[LIWORK],false)$FT
declare!(IWORK,rType,syms)$SYMTAB
Rep := FortranProgram(name,["void"]$UFST,
                        [IFLAG,N,Z,W,RWORK,LRWORK,IWORK,LIWORK],syms)

```

```

-- To help the poor old compiler!
localCoerce(u:Symbol):EXPR(MFLOAT) == coerce(u)$EXPR(MFLOAT)

```

```

coerce (u:MAT MFLOAT):$ ==
  Zs: Symbol := Z
  code : List FC
  r: List Expr MFLOAT
  r := ["+"/[u(j,i)*localCoerce(elt(Zs,[i::OutputForm]))$Symbol)_
        for i in 1..ncols(u)$MAT(MFLOAT)::PI]_
        for j in 1..nrows(u)$MAT(MFLOAT)::PI]
  code := [assign(W@Symbol,vector(r)$VEC(EXPR MFLOAT)),returns()]$List(FC)
  code:::

coerce(c:FortranCode):$ == coerce(c)$Rep

coerce(r:RSFC):$ == coerce(r)$Rep

coerce(c:List FortranCode):$ == coerce(c)$Rep

coerce(u:$):OutputForm == coerce(u)$Rep

outputAsFortran(u):Void ==
  p := checkPrecision()$NAGLinkSupportPackage
  outputAsFortran(u)$Rep
  p => restorePrecision()$NAGLinkSupportPackage

```

---

— ASP28.dotabb —

```

"ASP28" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP28"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"ASP28" -> "ALIST"

```

## 2.19 domain ASP29 Asp29

---

— Asp29.input —

```

)set break resume
)sys rm -f Asp29.output
)spool Asp29.output
)set message test on
)set message auto off
)clear all

```

```

--S 1 of 1
)show Asp29
--R Asp29 name: Symbol is a domain constructor
--R Abbreviation for Asp29 is ASP29
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP29
--R
--R----- Operations -----
--R coerce : % -> OutputForm          outputAsFortran : () -> Void
--R outputAsFortran : % -> Void
--R
--E 1

)spool
)lisp (bye)

```

\_\_\_\_\_

— Asp29.help —

```

=====
Asp29 examples
=====

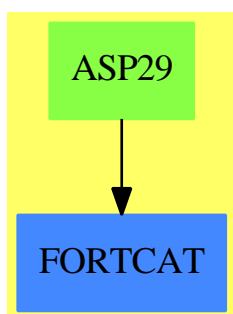
```

See Also:

- o )show Asp29

\_\_\_\_\_

### 2.19.1 Asp29 (ASP29)



#### Exports:

coerce outputAsFortran



## — domain ASP29 Asp29 —

```

)abbrev domain ASP29 Asp29
++ Author: Mike Dewar and Godfrey Nolan
++ Date Created: Nov 1993
++ Date Last Updated: 18 March 1994
++ Related Constructors: FortranScalarFunctionCategory, FortranProgramCategory
++ Description:
++\spadtype{Asp29} produces Fortran for Type 29 ASPs, needed for NAG routine
++f02fjf, for example:
++
++\tab{5}SUBROUTINE MONIT(ISTATE,NEXTIT,NEVALS,NEVECS,K,F,D)\br
++\tab{5}DOUBLE PRECISION D(K),F(K)\br
++\tab{5}INTEGER K,NEXTIT,NEVALS,NVECS,ISTATE\br
++\tab{5}CALL F02FJZ(ISTATE,NEXTIT,NEVALS,NEVECS,K,F,D)\br
++\tab{5}RETURN\br
++\tab{5}END\br

Asp29(name): Exports == Implementation where
  name : Symbol

FST ==> FortranScalarType
FT ==> FortranType
FSTU ==> Union(fst:FST,void:"void")
SYMTAB ==> SymbolTable
FC ==> FortranCode
PI ==> PositiveInteger
EXF ==> Expression Float
EXI ==> Expression Integer
VEF ==> Vector Expression Float
VEI ==> Vector Expression Integer
MEI ==> Matrix Expression Integer
MEF ==> Matrix Expression Float
UEXPR ==> Union(I: Expression Integer,F: Expression Float,_
               CF: Expression Complex Float)
RSFC ==> Record(localSymbols:SymbolTable,code:List(FortranCode))

Exports == FortranProgramCategory with
  outputAsFortran:() -> Void
  ++outputAsFortran() generates the default code for \spadtype{ASP29}.

Implementation == add

import FST
import FT
import FC
import SYMTAB

```

```

real : FSTU := ["real"::FST]$FSTU
integer : FSTU := ["integer"::FST]$FSTU
syms : SYMTAB := empty()
declare!(ISTATE,fortranInteger(),syms)
declare!(NEXTIT,fortranInteger(),syms)
declare!(NEVALS,fortranInteger(),syms)
declare!(NEVECS,fortranInteger(),syms)
declare!(K,fortranInteger(),syms)
kType : FT := construct(real,[K],false)$FT
declare!(F,kType,syms)
declare!(D,kType,syms)
Rep := FortranProgram(name,["void"]$FSTU,
                        [ISTATE,NEXTIT,NEVALS,NEVECS,K,F,D],syms)

outputAsFortran():Void ==
  callOne := call("F02FJZ(ISTATE,NEXTIT,NEVALS,NEVECS,K,F,D)")
  code : List FC := [callOne,returns()$List(FC)
  outputAsFortran(coerce(code)$Rep)$Rep

```

---

— ASP29.dotabb —

```

"ASP29" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP29"]
"FORTCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FORTCAT"]
"ASP29" -> "FORTCAT"

```

## 2.20 domain ASP30 Asp30

---

— Asp30.input —

```

)set break resume
)sys rm -f Asp30.output
)spool Asp30.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp30
--R Asp30 name: Symbol is a domain constructor

```

```

--R Abbreviation for Asp30 is ASP30
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP30
--R
--R----- Operations -----
--R coerce : FortranCode -> %          coerce : List FortranCode -> %
--R coerce : Matrix MachineFloat -> %  coerce : % -> OutputForm
--R outputAsFortran : % -> Void
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R
--E 1

)spool
)lisp (bye)

```

---

— Asp30.help —

=====

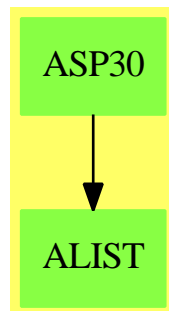
Asp30 examples

=====

See Also:

- o )show Asp30

### 2.20.1 Asp30 (ASP30)



**Exports:**

coerce outputAsFortran

— domain ASP30 Asp30 —

```

)abbrev domain ASP30 Asp30
++ Author: Mike Dewar and Godfrey Nolan
++ Date Created: Nov 1993
++ Date Last Updated: 28 March 1994
++                               6 October 1994
++ Related Constructors: FortranScalarFunctionCategory, FortranProgramCategory
++ Description:
++\spadtype{Asp30} produces Fortran for Type 30 ASPs, needed for NAG routine
++f04qaf, for example:
++
++\tab{5}SUBROUTINE APROD(MODE,M,N,X,Y,RWORK,LRWORK,IWORK,LIWORK)\br
++\tab{5}DOUBLE PRECISION X(N),Y(M),RWORK(LRWORK)\br
++\tab{5}INTEGER M,N,LIWORK,IFAIL,LRWORK,IWORK(LIWORK),MODE\br
++\tab{5}DOUBLE PRECISION A(5,5)\br
++\tab{5}EXTERNAL F06PAF\br
++\tab{5}A(1,1)=1.0D0\br
++\tab{5}A(1,2)=0.0D0\br
++\tab{5}A(1,3)=0.0D0\br
++\tab{5}A(1,4)=-1.0D0\br
++\tab{5}A(1,5)=0.0D0\br
++\tab{5}A(2,1)=0.0D0\br
++\tab{5}A(2,2)=1.0D0\br
++\tab{5}A(2,3)=0.0D0\br
++\tab{5}A(2,4)=0.0D0\br
++\tab{5}A(2,5)=-1.0D0\br
++\tab{5}A(3,1)=0.0D0\br
++\tab{5}A(3,2)=0.0D0\br
++\tab{5}A(3,3)=1.0D0\br
++\tab{5}A(3,4)=-1.0D0\br
++\tab{5}A(3,5)=0.0D0\br
++\tab{5}A(4,1)=-1.0D0\br
++\tab{5}A(4,2)=0.0D0\br
++\tab{5}A(4,3)=-1.0D0\br
++\tab{5}A(4,4)=4.0D0\br
++\tab{5}A(4,5)=-1.0D0\br
++\tab{5}A(5,1)=0.0D0\br
++\tab{5}A(5,2)=-1.0D0\br
++\tab{5}A(5,3)=0.0D0\br
++\tab{5}A(5,4)=-1.0D0\br
++\tab{5}A(5,5)=4.0D0\br
++\tab{5}IF(MODE.EQ.1)THEN\br
++\tab{7}CALL F06PAF('N',M,N,1.0D0,A,M,X,1,1.0D0,Y,1)\br
++\tab{5}ELSEIF(MODE.EQ.2)THEN\br
++\tab{7}CALL F06PAF('T',M,N,1.0D0,A,M,Y,1,1.0D0,X,1)\br
++\tab{5}ENDIF\br
++\tab{5}RETURN\br
++\tab{5}END

```

```

Asp30(name): Exports == Implementation where
  name : Symbol

```

```

FST    ==> FortranScalarType
FT     ==> FortranType
SYMTAB ==> SymbolTable
FC     ==> FortranCode
PI     ==> PositiveInteger
RSFC   ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
UFST   ==> Union(fst:FST,void:"void")
MAT     ==> Matrix
MFLOAT ==> MachineFloat
EXI     ==> Expression Integer
UEXPR  ==> Union(I:Expression Integer,F:Expression Float,_
                  CF:Expression Complex Float,switch:Switch)
S       ==> Symbol

Exports == FortranMatrixCategory

Implementation == add

import FC
import FT
import Switch

real : UFST := ["real"::FST]$UFST
integer : UFST := ["integer"::FST]$UFST
syms : SYMTAB := empty()$SYMTAB
declare!(MODE,fortranInteger()$FT,syms)$SYMTAB
declare!(M,fortranInteger()$FT,syms)$SYMTAB
declare!(N,fortranInteger()$FT,syms)$SYMTAB
declare!(LRWORK,fortranInteger()$FT,syms)$SYMTAB
declare!(LIWORK,fortranInteger()$FT,syms)$SYMTAB
xType : FT := construct(real,[N],false)$FT
declare!(X,xType,syms)$SYMTAB
yType : FT := construct(real,[M],false)$FT
declare!(Y,yType,syms)$SYMTAB
rType : FT := construct(real,[LRWORK],false)$FT
declare!(RWORK,rType,syms)$SYMTAB
iType : FT := construct(integer,[LIWORK],false)$FT
declare!(IWORK,iType,syms)$SYMTAB
declare!(IFAIL,fortranInteger()$FT,syms)$SYMTAB
Rep := FortranProgram(name,["void"]$UFST,
                      [MODE,M,N,X,Y,RWORK,LRWORK,IWORK,LIWORK],syms)

coerce(a:MAT MFLOAT):$ ==
  locals : SYMTAB := empty()
  numRows := nrows(a) :: Polynomial Integer
  numCols := ncols(a) :: Polynomial Integer
  declare!(A,[real,[numRows,numCols],false]$FT,locals)
  declare!(F06PAF@S,construct(["void"]$UFST,[],@List(S),true)$FT,locals)
  ptA:UEXPR := [("MODE"::S)::EXI]

```

```

ptB:UEXPR := [1::EXI]
ptC:UEXPR := [2::EXI]
sw1 : Switch := EQ(ptA,ptB)$Switch
sw2 : Switch := EQ(ptA,ptC)$Switch
callOne := call("F06PAF('N',M,N,1.0D0,A,M,X,1,1.0D0,Y,1)")
callTwo := call("F06PAF('T',M,N,1.0D0,A,M,Y,1,1.0D0,X,1)")
c : FC := cond(sw1,callOne,cond(sw2,callTwo))
code : List FC := [assign(A,a),c,returns()]
([locals,code]$RSFC)::$

coerce(c:List FortranCode):$ == coerce(c)$Rep

coerce(r:RSFC):$ == coerce(r)$Rep

coerce(c:FortranCode):$ == coerce(c)$Rep

coerce(u:$):OutputForm == coerce(u)$Rep

outputAsFortran(u):Void ==
  p := checkPrecision()$NAGLinkSupportPackage
  outputAsFortran(u)$Rep
  p => restorePrecision()$NAGLinkSupportPackage

```

---

— ASP30.dotabb —

```

"ASP30" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP30"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"ASP30" -> "ALIST"

```

---

## 2.21 domain ASP31 Asp31

— Asp31.input —

```

)set break resume
)sys rm -f Asp31.output
)spool Asp31.output
)set message test on
)set message auto off
)clear all

```

```

--S 1 of 1
)show Asp31
--R Asp31 name: Symbol is a domain constructor
--R Abbreviation for Asp31 is ASP31
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP31
--R
--R----- Operations -----
--R coerce : FortranCode -> %               coerce : List FortranCode -> %
--R coerce : % -> OutputForm               outputAsFortran : % -> Void
--R coerce : Vector FortranExpression([construct,QUOTEX],[construct,QUOTEY],MachineFloat) ->
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Vector Fraction Polynomial Integer -> %
--R retract : Vector Fraction Polynomial Float -> %
--R retract : Vector Polynomial Integer -> %
--R retract : Vector Polynomial Float -> %
--R retract : Vector Expression Integer -> %
--R retract : Vector Expression Float -> %
--R retractIfCan : Vector Fraction Polynomial Integer -> Union(%, "failed")
--R retractIfCan : Vector Fraction Polynomial Float -> Union(%, "failed")
--R retractIfCan : Vector Polynomial Integer -> Union(%, "failed")
--R retractIfCan : Vector Polynomial Float -> Union(%, "failed")
--R retractIfCan : Vector Expression Integer -> Union(%, "failed")
--R retractIfCan : Vector Expression Float -> Union(%, "failed")
--R
--E 1

)spool
)lisp (bye)

```

---

— Asp31.help —

```

=====
Asp31 examples
=====

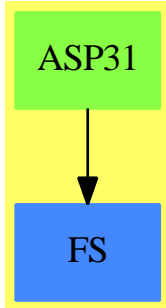
```

```

See Also:
o )show Asp31

```

## 2.21.1 Asp31 (ASP31)

**Exports:**

coerce outputAsFortran retract retractIfCan

— domain ASP31 Asp31 —

```

)abbrev domain ASP31 Asp31
++ Author: Mike Dewar, Grant Keady and Godfrey Nolan
++ Date Created: Mar 1993
++ Date Last Updated: 22 March 1994
++               6 October 1994
++ Related Constructors: FortranMatrixFunctionCategory, FortranProgramCategory
++ Description:
++ \spadtype{Asp31} produces Fortran for Type 31 ASPs, needed for NAG routine
++ d02ejf, for example:
++
++ \tab{5}SUBROUTINE PEDERV(X,Y,PW)\br
++ \tab{5}DOUBLE PRECISION X,Y(*)\br
++ \tab{5}DOUBLE PRECISION PW(3,3)\br
++ \tab{5}PW(1,1)=-0.03999999999999999D0\br
++ \tab{5}PW(1,2)=10000.0D0*Y(3)\br
++ \tab{5}PW(1,3)=10000.0D0*Y(2)\br
++ \tab{5}PW(2,1)=0.03999999999999999D0\br
++ \tab{5}PW(2,2)=(-10000.0D0*Y(3))+(-60000000.0D0*Y(2))\br
++ \tab{5}PW(2,3)=-10000.0D0*Y(2)\br
++ \tab{5}PW(3,1)=0.0D0\br
++ \tab{5}PW(3,2)=60000000.0D0*Y(2)\br
++ \tab{5}PW(3,3)=0.0D0\br
++ \tab{5}RETURN\br
++ \tab{5}END
  
```

```

Asp31(name): Exports == Implementation where
  name : Symbol
  
```

```

0      ==> OutputForm
FST    ==> FortranScalarType
  
```



```

UFST  ==> Union(fst:FST,void:"void")
MFLOAT ==> MachineFloat
FEXPR ==> FortranExpression(['X'],['Y'],MFLOAT)
FT     ==> FortranType
FC     ==> FortranCode
SYMTAB ==> SymbolTable
RSFC   ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
FRAC   ==> Fraction
POLY   ==> Polynomial
EXPR   ==> Expression
INT    ==> Integer
FLOAT  ==> Float
VEC    ==> Vector
MAT    ==> Matrix
VF2    ==> VectorFunctions2
MF2    ==> MatrixCategoryFunctions2(FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR,
                                   EXPR MFLOAT,VEC EXPR MFLOAT,VEC EXPR MFLOAT,MAT EXPR MFLOAT)

```

```

Exports ==> FortranVectorFunctionCategory with
  coerce : VEC FEXPR -> $
    ++coerce(f) takes objects from the appropriate instantiation of
    ++\spadtype{FortranExpression} and turns them into an ASP.

```

```

Implementation ==> add

```

```

real : UFST := ["real"::FST]$UFST
syms : SYMTAB := empty()
declare!(X,fortranReal(),syms)$SYMTAB
yType : FT := construct(real,["*":Symbol],false)$FT
declare!(Y,yType,syms)$SYMTAB
Rep := FortranProgram(name,["void"]$UFST,[X,Y,PW],syms)

-- To help the poor old compiler!
fexpr2expr(u:FEXPR):EXPR MFLOAT == coerce(u)$FEXPR

localAssign(s:Symbol,j:MAT FEXPR):FC ==
  j' : MAT EXPR MFLOAT := map(fexpr2expr,j)$MF2
  assign(s,j')$FC

makeXList(n:Integer):List(Symbol) ==
  j:Integer
  y:Symbol := Y::Symbol
  p:List(Symbol) := []
  for j in 1 .. n repeat p:= cons(subscript(y,[j::OutputForm])$Symbol,p)
  p:= reverse(p)

coerce(u:VEC FEXPR):$ ==

```

```

dimension := #u::Polynomial Integer
locals : SYMTAB := empty()
declare!(PW,[real,[dimension,dimension],false]$FT,locals)$SYMTAB
n:Integer := maxIndex(u)$VEC(FEXPR)
p:List(Symbol) := makeXList(n)
jac: MAT FEXPR := jacobian(u,p)$MultiVariableCalculusFunctions(
    Symbol,FEXPR ,VEC FEXPR,List(Symbol))
code : List FC := [localAssign(PW,jac),returns()$FC]$List(FC)
([locals,code]$RSFC)::$

retract(u:VEC FRAC POLY INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY INT,FEXPR)
  v::$

retractIfCan(u:VEC FRAC POLY INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC FRAC POLY FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY FLOAT,FEXPR)
  v::$

retractIfCan(u:VEC FRAC POLY FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC EXPR INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(EXPR INT,FEXPR)
  v::$

retractIfCan(u:VEC EXPR INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC EXPR FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(EXPR FLOAT,FEXPR)
  v::$

retractIfCan(u:VEC EXPR FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC POLY INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(POLY INT,FEXPR)
  v::$

```

```

retractIfCan(u:VEC POLY INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR):: $

retract(u:VEC POLY FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(POLY FLOAT,FEXPR)
  v:: $

retractIfCan(u:VEC POLY FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR):: $

coerce(c:List FC):$ == coerce(c)$Rep

coerce(r:RSFC):$ == coerce(r)$Rep

coerce(c:FC):$ == coerce(c)$Rep

coerce(u:$):0 == coerce(u)$Rep

outputAsFortran(u):Void ==
  p := checkPrecision()$NAGLinkSupportPackage
  outputAsFortran(u)$Rep
  p => restorePrecision()$NAGLinkSupportPackage

```

---

— ASP31.dotabb —

```

"ASP31" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP31"]
"FS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FS"]
"ASP31" -> "FS"

```

---

## 2.22 domain ASP33 Asp33

— Asp33.input —

```

)set break resume
)sys rm -f Asp33.output
)spool Asp33.output

```

```

)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp33
--R Asp33 name: Symbol is a domain constructor
--R Abbreviation for Asp33 is ASP33
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP33
--R
--R----- Operations -----
--R coerce : % -> OutputForm          outputAsFortran : () -> Void
--R outputAsFortran : % -> Void
--R
--E 1

)spool
)lisp (bye)

```

\_\_\_\_\_

### — Asp33.help —

```

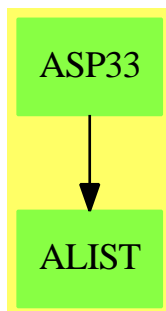
=====
Asp33 examples
=====

```

See Also:  
o )show Asp33

\_\_\_\_\_

#### 2.22.1 Asp33 (ASP33)



**Exports:**

coerce outputAsFortran

— domain ASP33 Asp33 —

```
)abbrev domain ASP33 Asp33
++ Author: Mike Dewar and Godfrey Nolan
++ Date Created: Nov 1993
++ Date Last Updated: 30 March 1994
++ Related Constructors: FortranScalarFunctionCategory, FortranProgramCategory.
++ Description:
++\spadtype{Asp33} produces Fortran for Type 33 ASPs, needed for NAG routine
++d02kef. The code is a dummy ASP:
++
++\tab{5}SUBROUTINE REPORT(X,V,JINT)\br
++\tab{5}DOUBLE PRECISION V(3),X\br
++\tab{5}INTEGER JINT\br
++\tab{5}RETURN\br
++\tab{5}END
```

```
Asp33(name): Exports == Implementation where
  name : Symbol
```

```
FST    ==> FortranScalarType
UFST   ==> Union(fst:FST,void:"void")
FT     ==> FortranType
SYMTAB ==> SymbolTable
FC     ==> FortranCode
RSFC   ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
```

```
Exports ==> FortranProgramCategory with
  outputAsFortran(): -> Void
  ++outputAsFortran() generates the default code for \spadtype{ASP33}.
```

```
Implementation ==> add
```

```
real : UFST := ["real"::FST]$UFST
syms : SYMTAB := empty()
declare!(JINT,fortranInteger(),syms)$SYMTAB
declare!(X,fortranReal(),syms)$SYMTAB
vType : FT := construct(real,["3"::Symbol],false)$FT
declare!(V,vType,syms)$SYMTAB
Rep := FortranProgram(name,["void"]$UFST,[X,V,JINT],syms)

outputAsFortran():Void ==
  outputAsFortran( (returns())$FortranCode)::Rep )$Rep

outputAsFortran(u):Void == outputAsFortran(u)$Rep
```

```
coerce(u:$):OutputForm == coerce(u)$Rep
```

---

— ASP33.dotabb —

```
"ASP33" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP33"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"ASP33" -> "ALIST"
```

---

## 2.23 domain ASP34 Asp34

— Asp34.input —

```
)set break resume
)sys rm -f Asp34.output
)spool Asp34.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp34
--R Asp34 name: Symbol is a domain constructor
--R Abbreviation for Asp34 is ASP34
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP34
--R
--R----- Operations -----
--R coerce : FortranCode -> %           coerce : List FortranCode -> %
--R coerce : Matrix MachineFloat -> %   coerce : % -> OutputForm
--R outputAsFortran : % -> Void
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R
--E 1

)spool
)lisp (bye)
```

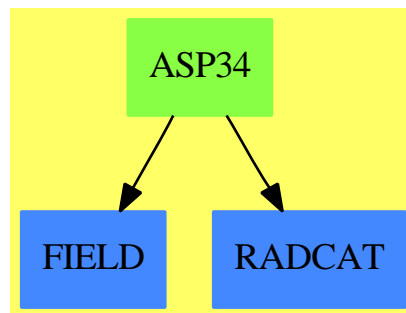
---

## — Asp34.help —

```
=====
Asp34 examples
=====
```

```
See Also:
o )show Asp34
```

## 2.23.1 Asp34 (ASP34)

**Exports:**

```
coerce  outputAsFortran
```

## — domain ASP34 Asp34 —

```
)abbrev domain ASP34 Asp34
++ Author: Mike Dewar and Godfrey Nolan
++ Date Created: Nov 1993
++ Date Last Updated: 14 June 1994 (Themos Tsikas)
++                      6 October 1994
++ Related Constructors: FortranScalarFunctionCategory, FortranProgramCategory
++ Description:
++\spadtype{Asp34} produces Fortran for Type 34 ASPs, needed for NAG routine
++f04mbf, for example:
++
++\tab{5}SUBROUTINE MSOLVE(IFLAG,N,X,Y,RWORK,LRWORK,IWORK,LIWORK)\br
++\tab{5}DOUBLE PRECISION RWORK(LRWORK),X(N),Y(N)\br
++\tab{5}INTEGER I,J,N,LIWORK,IFLAG,LRWORK,IWORK(LIWORK)\br
++\tab{5}DOUBLE PRECISION W1(3),W2(3),MS(3,3)\br
++\tab{5}IFLAG=-1\br
```

```

++\tab{5}MS(1,1)=2.0D0\br
++\tab{5}MS(1,2)=1.0D0\br
++\tab{5}MS(1,3)=0.0D0\br
++\tab{5}MS(2,1)=1.0D0\br
++\tab{5}MS(2,2)=2.0D0\br
++\tab{5}MS(2,3)=1.0D0\br
++\tab{5}MS(3,1)=0.0D0\br
++\tab{5}MS(3,2)=1.0D0\br
++\tab{5}MS(3,3)=2.0D0\br
++\tab{5}CALL F04ASF(MS,N,X,N,Y,W1,W2,IFLAG)\br
++\tab{5}IFLAG=-IFLAG\br
++\tab{5}RETURN\br
++\tab{5}END

```

```

Asp34(name): Exports == Implementation where
name : Symbol

```

```

FST  ==> FortranScalarType
FT   ==> FortranType
UFST ==> Union(fst:FST,void:"void")
SYMTAB ==> SymbolTable
FC    ==> FortranCode
PI    ==> PositiveInteger
EXI   ==> Expression Integer
RSFC  ==> Record(localSymbols:SymbolTable,code:List(FortranCode))

```

```
Exports == FortranMatrixCategory
```

```
Implementation == add
```

```

real : UFST := ["real":FST]$UFST
integer : UFST := ["integer":FST]$UFST
syms : SYMTAB := empty()$SYMTAB
declare!(IFLAG,fortranInteger(),syms)$SYMTAB
declare!(N,fortranInteger(),syms)$SYMTAB
xType : FT := construct(real,[N],false)$FT
declare!(X,xType,syms)$SYMTAB
declare!(Y,xType,syms)$SYMTAB
declare!(LRWORK,fortranInteger(),syms)$SYMTAB
declare!(LIWORK,fortranInteger(),syms)$SYMTAB
rType : FT := construct(real,[LRWORK],false)$FT
declare!(RWORK,rType,syms)$SYMTAB
iType : FT := construct(integer,[LIWORK],false)$FT
declare!(IWORK,iType,syms)$SYMTAB
Rep := FortranProgram(name,["void"]$UFST,
                        [IFLAG,N,X,Y,RWORK,LRWORK,IWORK,LIWORK],syms)

```

```

-- To help the poor old compiler
localAssign(s:Symbol,u:EXI):FC == assign(s,u)$FC

```



```

coerce(u:Matrix MachineFloat):$ ==
  dimension := nrows(u)::Polynomial Integer
  locals : SYMTAB := empty()$SYMTAB
  declare!(I,fortranInteger(),syms)$SYMTAB
  declare!(J,fortranInteger(),syms)$SYMTAB
  declare!(W1,[real,[dimension],false]$FT,locals)$SYMTAB
  declare!(W2,[real,[dimension],false]$FT,locals)$SYMTAB
  declare!(MS,[real,[dimension,dimension],false]$FT,locals)$SYMTAB
  assign1 : FC := localAssign(IFLAG@Symbol,(-1)@EXI)
  call : FC := call("F04ASF(MS,N,X,N,Y,W1,W2,IFLAG)")$FC
  assign2 : FC := localAssign(IFLAG::Symbol,-(IFLAG@Symbol::EXI))
  assign3 : FC := assign(MS,u)$FC
  code : List FC := [assign1,assign3,call,assign2,returns()]$List(FC)
  ([locals,code]$RSFC)::$

coerce(c:List FortranCode):$ == coerce(c)$Rep

coerce(r:RSFC):$ == coerce(r)$Rep

coerce(c:FortranCode):$ == coerce(c)$Rep

coerce(u:$):OutputForm == coerce(u)$Rep

outputAsFortran(u):Void ==
  p := checkPrecision()$NAGLinkSupportPackage
  outputAsFortran(u)$Rep
  p => restorePrecision()$NAGLinkSupportPackage

```

---

— ASP34.dotabb —

```

"ASP34" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP34"]
"FIELD" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FIELD"]
"RADCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=RADCAT"]
"ASP34" -> "FIELD"
"ASP34" -> "RADCAT"

```

## 2.24 domain ASP35 Asp35

---

— Asp35.input —

```

)set break resume

```

```

)sys rm -f Asp35.output
)spool Asp35.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp35
--R Asp35 name: Symbol is a domain constructor
--R Abbreviation for Asp35 is ASP35
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP35
--R
--R----- Operations -----
--R coerce : FortranCode -> %               coerce : List FortranCode -> %
--R coerce : % -> OutputForm               outputAsFortran : % -> Void
--R coerce : Vector FortranExpression([construct],[construct,QUOTEX],MachineFloat) -> %
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Vector Fraction Polynomial Integer -> %
--R retract : Vector Fraction Polynomial Float -> %
--R retract : Vector Polynomial Integer -> %
--R retract : Vector Polynomial Float -> %
--R retract : Vector Expression Integer -> %
--R retract : Vector Expression Float -> %
--R retractIfCan : Vector Fraction Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Vector Fraction Polynomial Float -> Union(%,"failed")
--R retractIfCan : Vector Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Vector Polynomial Float -> Union(%,"failed")
--R retractIfCan : Vector Expression Integer -> Union(%,"failed")
--R retractIfCan : Vector Expression Float -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)

```

---

— Asp35.help —

```

=====
Asp35 examples
=====

```

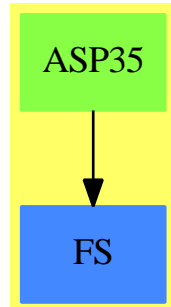
```

See Also:
o )show Asp35

```

---

### 2.24.1 Asp35 (ASP35)



#### Exports:

coerce outputAsFortran retract retractIfCan

— domain **ASP35** **Asp35** —

```

)abbrev domain ASP35 Asp35
++ Author: Mike Dewar, Godfrey Nolan, Grant Keady
++ Date Created: Mar 1993
++ Date Last Updated: 22 March 1994
++                               6 October 1994
++ Related Constructors: FortranVectorFunctionCategory, FortranProgramCategory
++ Description:
++\spadtype{Asp35} produces Fortran for Type 35 ASPs, needed for NAG routines
++c05pbf, c05pcf, for example:
++
++\tab{5}SUBROUTINE FCN(N,X,FVEC,FJAC,LDFJAC,IFLAG)\br
++\tab{5}DOUBLE PRECISION X(N),FVEC(N),FJAC(LDFJAC,N)\br
++\tab{5}INTEGER LDFJAC,N,IFLAG\br
++\tab{5}IF(IFLAG.EQ.1)THEN\br
++\tab{7}FVEC(1)=(-1.0D0*X(2))+X(1)\br
++\tab{7}FVEC(2)=(-1.0D0*X(3))+2.0D0*X(2)\br
++\tab{7}FVEC(3)=3.0D0*X(3)\br
++\tab{5}ELSEIF(IFLAG.EQ.2)THEN\br
++\tab{7}FJAC(1,1)=1.0D0\br
++\tab{7}FJAC(1,2)=-1.0D0\br
++\tab{7}FJAC(1,3)=0.0D0\br
++\tab{7}FJAC(2,1)=0.0D0\br
++\tab{7}FJAC(2,2)=2.0D0\br
++\tab{7}FJAC(2,3)=-1.0D0\br
++\tab{7}FJAC(3,1)=0.0D0\br
++\tab{7}FJAC(3,2)=0.0D0\br
++\tab{7}FJAC(3,3)=3.0D0\br
++\tab{5}ENDIF\br
++\tab{5}END
  
```

```

Asp35(name): Exports == Implementation where
  name : Symbol

  FST  ==> FortranScalarType
  FT   ==> FortranType
  UFST ==> Union(fst:FST,void:"void")
  SYMTAB ==> SymbolTable
  FC    ==> FortranCode
  PI    ==> PositiveInteger
  RSFC  ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
  FRAC  ==> Fraction
  POLY  ==> Polynomial
  EXPR  ==> Expression
  INT   ==> Integer
  FLOAT ==> Float
  VEC   ==> Vector
  MAT   ==> Matrix
  VF2   ==> VectorFunctions2
  MFLOAT ==> MachineFloat
  FEXPR ==> FortranExpression([],['X'],MFLOAT)
  MF2   ==> MatrixCategoryFunctions2(FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR,
    EXPR MFLOAT,VEC EXPR MFLOAT,VEC EXPR MFLOAT,MAT EXPR MFLOAT)
  SWU   ==> Union(I:Expression Integer,F:Expression Float,
    CF:Expression Complex Float,switch:Switch)

Exports ==> FortranVectorFunctionCategory with
  coerce : VEC FEXPR -> $
    ++coerce(f) takes objects from the appropriate instantiation of
    ++\spadtype{FortranExpression} and turns them into an ASP.

Implementation ==> add

  real : UFST := ["real":FST]$UFST
  syms : SYMTAB := empty()$SYMTAB
  declare!(N,fortranInteger(),syms)$SYMTAB
  xType : FT := construct(real,[N],false)$FT
  declare!(X,xType,syms)$SYMTAB
  declare!(FVEC,xType,syms)$SYMTAB
  declare!(LDFJAC,fortranInteger(),syms)$SYMTAB
  jType : FT := construct(real,[LDFJAC,N],false)$FT
  declare!(FJAC,jType,syms)$SYMTAB
  declare!(IFLAG,fortranInteger(),syms)$SYMTAB
  Rep := FortranProgram(name,["void"]$UFST,[N,X,FVEC,FJAC,LDFJAC,IFLAG],syms)

  coerce(u:$):OutputForm == coerce(u)$Rep

  makeXList(n:Integer):List(Symbol) ==
    x:Symbol := X::Symbol
    [subscript(x,[j::OutputForm])$Symbol for j in 1..n]

```

```

fexpr2expr(u:FEXPR):EXPR MFLOAT == coerce(u)$FEXPR

localAssign1(s:Symbol,j:MAT FEXPR):FC ==
  j' : MAT EXPR MFLOAT := map(fexpr2expr,j)$MF2
  assign(s,j')$FC

localAssign2(s:Symbol,j:VEC FEXPR):FC ==
  j' : VEC EXPR MFLOAT := map(fexpr2expr,j)$VF2(FEXPR,EXPR MFLOAT)
  assign(s,j')$FC

coerce(u:VEC FEXPR):$ ==
  n:Integer := maxIndex(u)
  p:List(Symbol) := makeXList(n)
  jac: MAT FEXPR := jacobian(u,p)$MultiVariableCalculusFunctions(
    Symbol,FEXPR,VEC FEXPR,List(Symbol))
  assf:FC := localAssign2(FVEC,u)
  assj:FC := localAssign1(FJAC,jac)
  iflag:SWU := [IFLAG@Symbol::EXPR(INT)]$SWU
  sw1:Switch := EQ(iflag,[1::EXPR(INT)]$SWU)
  sw2:Switch := EQ(iflag,[2::EXPR(INT)]$SWU)
  cond(sw1,assf,cond(sw2,assj)$FC)$FC:::$

coerce(c:List FC):$ == coerce(c)$Rep

coerce(r:RSFC):$ == coerce(r)$Rep

coerce(c:FC):$ == coerce(c)$Rep

outputAsFortran(u):Void ==
  p := checkPrecision()$NAGLinkSupportPackage
  outputAsFortran(u)$Rep
  p => restorePrecision()$NAGLinkSupportPackage

retract(u:VEC FRAC POLY INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY INT,FEXPR)
  v::$

retractIfCan(u:VEC FRAC POLY INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR):::$

retract(u:VEC FRAC POLY FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY FLOAT,FEXPR)
  v::$

retractIfCan(u:VEC FRAC POLY FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR):::$

```

```

retract(u:VEC EXPR INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(EXPR INT,FEXPR)
  v::$

retractIfCan(u:VEC EXPR INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC EXPR FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(EXPR FLOAT,FEXPR)
  v::$

retractIfCan(u:VEC EXPR FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC POLY INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(POLY INT,FEXPR)
  v::$

retractIfCan(u:VEC POLY INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC POLY FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(POLY FLOAT,FEXPR)
  v::$

retractIfCan(u:VEC POLY FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

```

---

— ASP35.dotabb —

```

"ASP35" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP35"]
"FS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FS"]
"ASP35" -> "FS"

```

---

## 2.25 domain ASP4 Asp4

— Asp4.input —

```
)set break resume
)sys rm -f Asp4.output
)spool Asp4.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp4
--R Asp4 name: Symbol is a domain constructor
--R Abbreviation for Asp4 is ASP4
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP4
--R
--R----- Operations -----
--R coerce : FortranCode -> %          coerce : List FortranCode -> %
--R coerce : % -> OutputForm          outputAsFortran : % -> Void
--R retract : Polynomial Integer -> %   retract : Polynomial Float -> %
--R retract : Expression Integer -> %   retract : Expression Float -> %
--R coerce : FortranExpression([construct],[construct,QUOTEX],MachineFloat) -> %
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Fraction Polynomial Integer -> %
--R retract : Fraction Polynomial Float -> %
--R retractIfCan : Fraction Polynomial Integer -> Union(%, "failed")
--R retractIfCan : Fraction Polynomial Float -> Union(%, "failed")
--R retractIfCan : Polynomial Integer -> Union(%, "failed")
--R retractIfCan : Polynomial Float -> Union(%, "failed")
--R retractIfCan : Expression Integer -> Union(%, "failed")
--R retractIfCan : Expression Float -> Union(%, "failed")
--R
--E 1

)spool
)lisp (bye)
```

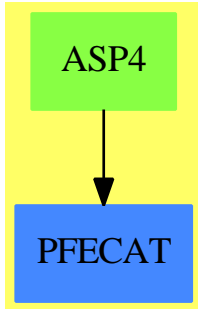
— Asp4.help —

```
=====
Asp4 examples
=====
```

See Also:

o )show Asp4

### 2.25.1 Asp4 (ASP4)



#### Exports:

coerce outputAsFortran retract retractIfCan

— domain ASP4 Asp4 —

```

)abbrev domain ASP4 Asp4
++ Author: Mike Dewar, Grant Keady and Godfrey Nolan
++ Date Created: Mar 1993
++ Date Last Updated: 18 March 1994
++                      6 October 1994
++ Related Constructors: FortranScalarFunctionCategory, FortranProgramCategory
++ Description:
++\spadtype{Asp4} produces Fortran for Type 4 ASPs, which take an expression
++in X(1) .. X(NDIM) and produce a real function of the form:
++
++\tab{5}DOUBLE PRECISION FUNCTION FUNCTN(NDIM,X)\br
++\tab{5}DOUBLE PRECISION X(NDIM)\br
++\tab{5}INTEGER NDIM\br
++\tab{5}FUNCTN=(4.0D0*X(1)*X(3)**2*DEXP(2.0D0*X(1)*X(3)))/(X(4)**2+(2.0D0*\br
++\tab{4}&X(2)+2.0D0)*X(4)+X(2)**2+2.0D0*X(2)+1.0D0)\br
++\tab{5}RETURN\br
++\tab{5}END
  
```

```

Asp4(name): Exports == Implementation where
  name : Symbol
  
```

```

FEXPR ==> FortranExpression([],['X'],MachineFloat)
FST    ==> FortranScalarType
  
```



```

FT      ==> FortranType
SYMTAB ==> SymbolTable
RSFC    ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
FSTU    ==> Union(fst:FST,void:"void")
FRAC    ==> Fraction
POLY    ==> Polynomial
EXPR    ==> Expression
INT      ==> Integer
FLOAT   ==> Float

Exports ==> FortranFunctionCategory with
  coerce : FEXPR -> $
    ++coerce(f) takes an object from the appropriate instantiation of
    ++\spadtype{FortranExpression} and turns it into an ASP.

Implementation ==> add

real : FSTU := ["real"::FST]$FSTU
syms : SYMTAB := empty()$SYMTAB
declare!(NDIM,fortranInteger(),syms)$SYMTAB
xType : FT := construct(real,[NDIM],false)$FT
declare!(X,xType,syms)$SYMTAB
Rep := FortranProgram(name,real,[NDIM,X],syms)

retract(u:FRAC POLY INT):$ == (retract(u)$FEXPR)::FSTU
retractIfCan(u:FRAC POLY INT):Union($,"failed") ==
  foo : Union(FEXPR,"failed")
  foo := retractIfCan(u)$FEXPR
  foo case "failed" => "failed"
  foo::FEXPR::FSTU

retract(u:FRAC POLY FLOAT):$ == (retract(u)$FEXPR)::FSTU
retractIfCan(u:FRAC POLY FLOAT):Union($,"failed") ==
  foo : Union(FEXPR,"failed")
  foo := retractIfCan(u)$FEXPR
  foo case "failed" => "failed"
  foo::FEXPR::FSTU

retract(u:EXPR FLOAT):$ == (retract(u)$FEXPR)::FSTU
retractIfCan(u:EXPR FLOAT):Union($,"failed") ==
  foo : Union(FEXPR,"failed")
  foo := retractIfCan(u)$FEXPR
  foo case "failed" => "failed"
  foo::FEXPR::FSTU

retract(u:EXPR INT):$ == (retract(u)$FEXPR)::FSTU
retractIfCan(u:EXPR INT):Union($,"failed") ==
  foo : Union(FEXPR,"failed")
  foo := retractIfCan(u)$FEXPR
  foo case "failed" => "failed"
  foo::FEXPR::FSTU

```

```

foo::FEXPR::$

retract(u:POLY FLOAT):$ == (retract(u)@FEXPR)::$
retractIfCan(u:POLY FLOAT):Union($,"failed") ==
  foo : Union(FEXPR,"failed")
  foo := retractIfCan(u)$FEXPR
  foo case "failed" => "failed"
foo::FEXPR::$

retract(u:POLY INT):$ == (retract(u)@FEXPR)::$
retractIfCan(u:POLY INT):Union($,"failed") ==
  foo : Union(FEXPR,"failed")
  foo := retractIfCan(u)$FEXPR
  foo case "failed" => "failed"
foo::FEXPR::$

coerce(u:FEXPR):$ ==
  coerce((u::Expression(MachineFloat))$FEXPR)$Rep

coerce(c:List FortranCode):$ == coerce(c)$Rep

coerce(r:RSFC):$ == coerce(r)$Rep

coerce(c:FortranCode):$ == coerce(c)$Rep

coerce(u:$):OutputForm == coerce(u)$Rep

outputAsFortran(u):Void ==
  p := checkPrecision()$NAGLinkSupportPackage
  outputAsFortran(u)$Rep
  p => restorePrecision()$NAGLinkSupportPackage

```

---

— ASP4.dotabb —

```

"ASP4" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP4"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"ASP4" -> "PFECAT"

```

---

## 2.26 domain ASP41 Asp41

— Asp41.input —

```

)set break resume
)sys rm -f Asp41.output
)spool Asp41.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp41
--R Asp41(nameOne: Symbol,nameTwo: Symbol,nameThree: Symbol) is a domain constructor
--R Abbreviation for Asp41 is ASP41
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP41
--R
--R----- Operations -----
--R coerce : FortranCode -> %               coerce : List FortranCode -> %
--R coerce : % -> OutputForm               outputAsFortran : % -> Void
--R coerce : Vector FortranExpression([construct,QUOTEX,QUOTEEPS],[construct,QUOTEY],Machine)
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Vector Fraction Polynomial Integer -> %
--R retract : Vector Fraction Polynomial Float -> %
--R retract : Vector Polynomial Integer -> %
--R retract : Vector Polynomial Float -> %
--R retract : Vector Expression Integer -> %
--R retract : Vector Expression Float -> %
--R retractIfCan : Vector Fraction Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Vector Fraction Polynomial Float -> Union(%,"failed")
--R retractIfCan : Vector Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Vector Polynomial Float -> Union(%,"failed")
--R retractIfCan : Vector Expression Integer -> Union(%,"failed")
--R retractIfCan : Vector Expression Float -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)

```

---

— Asp41.help —

=====

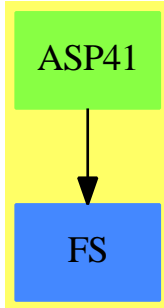
Asp41 examples

=====

See Also:

- o )show Asp41

## 2.26.1 Asp41 (ASP41)

**Exports:**

coerce outputAsFortran retract retractIfCan

— domain ASP41 Asp41 —

```

)abbrev domain ASP41 Asp41
++ Author: Mike Dewar, Godfrey Nolan
++ Date Created:
++ Date Last Updated: 29 March 1994
++                  6 October 1994
++ Related Constructors: FortranFunctionCategory, FortranProgramCategory.
++ Description:
++\spadtype{Asp41} produces Fortran for Type 41 ASPs, needed for NAG
++routines d02raf and d02saf in particular. These ASPs are in fact
++three Fortran routines which return a vector of functions, and their
++derivatives wrt Y(i) and also a continuation parameter EPS, for example:
++
++\tab{5}SUBROUTINE FCN(X,EPS,Y,F,N)\br
++\tab{5}DOUBLE PRECISION EPS,F(N),X,Y(N)\br
++\tab{5}INTEGER N\br
++\tab{5}F(1)=Y(2)\br
++\tab{5}F(2)=Y(3)\br
++\tab{5}F(3)=(-1.0D0*Y(1)*Y(3))+2.0D0*EPS*Y(2)**2+(-2.0D0*EPS)\br
++\tab{5}RETURN\br
++\tab{5}END\br
++\tab{5}SUBROUTINE JACOB(X,EPS,Y,F,N)\br
++\tab{5}DOUBLE PRECISION EPS,F(N,N),X,Y(N)\br
++\tab{5}INTEGER N\br
++\tab{5}F(1,1)=0.0D0\br
++\tab{5}F(1,2)=1.0D0\br
++\tab{5}F(1,3)=0.0D0\br
++\tab{5}F(2,1)=0.0D0\br
++\tab{5}F(2,2)=0.0D0\br
++\tab{5}F(2,3)=1.0D0\br
++\tab{5}F(3,1)=-1.0D0*Y(3)\br
  
```

```

++\tab{5}F(3,2)=4.0D0*EPS*Y(2)\br
++\tab{5}F(3,3)=-1.0D0*Y(1)\br
++\tab{5}RETURN\br
++\tab{5}END\br
++\tab{5}SUBROUTINE JACEPS(X,EPS,Y,F,N)\br
++\tab{5}DOUBLE PRECISION EPS,F(N),X,Y(N)\br
++\tab{5}INTEGER N\br
++\tab{5}F(1)=0.0D0\br
++\tab{5}F(2)=0.0D0\br
++\tab{5}F(3)=2.0D0*Y(2)**2-2.0D0\br
++\tab{5}RETURN\br
++\tab{5}END

```

Asp41(nameOne,nameTwo,nameThree): Exports == Implementation where

```

nameOne : Symbol
nameTwo : Symbol
nameThree : Symbol

```

```

D      ==> differentiate
FST    ==> FortranScalarType
UFST   ==> Union(fst:FST,void:"void")
FT     ==> FortranType
FC     ==> FortranCode
SYMTAB ==> SymbolTable
RSFC   ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
FRAC   ==> Fraction
POLY   ==> Polynomial
EXPR   ==> Expression
INT    ==> Integer
FLOAT  ==> Float
VEC    ==> Vector
VF2    ==> VectorFunctions2
MFLOAT ==> MachineFloat
FEXPR  ==> FortranExpression(['X','EPS'],['Y'],MFLOAT)
S      ==> Symbol
MF2    ==> MatrixCategoryFunctions2(FEXPR,VEC FEXPR,VEC FEXPR,Matrix FEXPR,
                                   EXPR MFLOAT,VEC EXPR MFLOAT,VEC EXPR MFLOAT,Matrix EXPR MFLOAT)

```

Exports ==> FortranVectorFunctionCategory with

```

coerce : VEC FEXPR -> $
++coerce(f) takes objects from the appropriate instantiation of
++\spadtype{FortranExpression} and turns them into an ASP.

```

Implementation ==> add

```

real : UFST := ["real"::FST]$UFST

```

```

symOne : SYMTAB := empty()$SYMTAB
declare!(N,fortranInteger(),symOne)$SYMTAB
declare!(X,fortranReal(),symOne)$SYMTAB
declare!(EPS,fortranReal(),symOne)$SYMTAB

```

```

yType : FT := construct(real,[N],false)$FT
declare!(Y,yType,symOne)$SYMTAB
declare!(F,yType,symOne)$SYMTAB

symTwo : SYMTAB := empty()$SYMTAB
declare!(N,fortranInteger(),symTwo)$SYMTAB
declare!(X,fortranReal(),symTwo)$SYMTAB
declare!(EPS,fortranReal(),symTwo)$SYMTAB
declare!(Y,yType,symTwo)$SYMTAB
fType : FT := construct(real,[N,N],false)$FT
declare!(F,fType,symTwo)$SYMTAB

symThree : SYMTAB := empty()$SYMTAB
declare!(N,fortranInteger(),symThree)$SYMTAB
declare!(X,fortranReal(),symThree)$SYMTAB
declare!(EPS,fortranReal(),symThree)$SYMTAB
declare!(Y,yType,symThree)$SYMTAB
declare!(F,yType,symThree)$SYMTAB

R1:=FortranProgram(nameOne,["void"]$UFST,[X,EPS,Y,F,N],symOne)
R2:=FortranProgram(nameTwo,["void"]$UFST,[X,EPS,Y,F,N],symTwo)
R3:=FortranProgram(nameThree,["void"]$UFST,[X,EPS,Y,F,N],symThree)
Rep := Record(f:R1,fJacob:R2,eJacob:R3)
Fsym:Symbol:=coerce "F"

fexpr2expr(u:FEXPR):EXPR MFLOAT == coerce(u)$FEXPR

localAssign1(s:S,j:Matrix FEXPR):FC ==
  j' : Matrix EXPR MFLOAT := map(fexpr2expr,j)$MF2
  assign(s,j')$FC

localAssign2(s:S,j:VEC FEXPR):FC ==
  j' : VEC EXPR MFLOAT := map(fexpr2expr,j)$VF2(FEXPR,EXPR MFLOAT)
  assign(s,j')$FC

makeCodeOne(u:VEC FEXPR):FortranCode ==
  -- simple assign
  localAssign2(Fsym,u)

makeCodeThree(u:VEC FEXPR):FortranCode ==
  -- compute jacobian wrt to eps
  jacEps:VEC FEXPR := [D(v,EPS) for v in entries(u)]$VEC(FEXPR)
  makeCodeOne(jacEps)

makeYList(n:Integer):List(Symbol) ==
  j:Integer
  y:Symbol := Y::Symbol
  p:List(Symbol) := []
  [subscript(y,[j::OutputForm])$Symbol for j in 1..n]

```

```

makeCodeTwo(u:VEC FEXPR):FortranCode ==
  -- compute jacobian wrt to f
  n:Integer := maxIndex(u)$VEC(FEXPR)
  p:List(Symbol) := makeYList(n)
  jac:Matrix(FEXPR) := _
  jacobian(u,p)$MultiVariableCalculusFunctions(S,FEXPR,VEC FEXPR,List(S))
  localAssign1(Fsym,jac)

coerce(u:VEC FEXPR):$ ==
  aF:FortranCode := makeCodeOne(u)
  bF:FortranCode := makeCodeTwo(u)
  cF:FortranCode := makeCodeThree(u)
  -- add returns() to complete subroutines
  aLF:List(FortranCode) := [aF,returns()$FortranCode]$List(FortranCode)
  bLF:List(FortranCode) := [bF,returns()$FortranCode]$List(FortranCode)
  cLF:List(FortranCode) := [cF,returns()$FortranCode]$List(FortranCode)
  [coerce(aLF)$R1,coerce(bLF)$R2,coerce(cLF)$R3]

coerce(u:$):OutputForm ==
  bracket commaSeparate
  [nameOne::OutputForm,nameTwo::OutputForm,nameThree::OutputForm]

outputAsFortran(u:$):Void ==
  p := checkPrecision()$NAGLinkSupportPackage
  outputAsFortran elt(u,f)$Rep
  outputAsFortran elt(u,fJacob)$Rep
  outputAsFortran elt(u,eJacob)$Rep
  p => restorePrecision()$NAGLinkSupportPackage

retract(u:VEC FRAC POLY INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY INT,FEXPR)
  v::$

retractIfCan(u:VEC FRAC POLY INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR):: $

retract(u:VEC FRAC POLY FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY FLOAT,FEXPR)
  v::$

retractIfCan(u:VEC FRAC POLY FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR):: $

retract(u:VEC EXPR INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(EXPR INT,FEXPR)
  v::$

```

```

retractIfCan(u:VEC EXPR INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC EXPR FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(EXPR FLOAT,FEXPR)
  v::$

retractIfCan(u:VEC EXPR FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC POLY INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(POLY INT,FEXPR)
  v::$

retractIfCan(u:VEC POLY INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC POLY FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(POLY FLOAT,FEXPR)
  v::$

retractIfCan(u:VEC POLY FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

```

---

— ASP41.dotabb —

```

"ASP41" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP41"]
"FS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FS"]
"ASP41" -> "FS"

```

---

## 2.27 domain ASP42 Asp42



## — Asp42.input —

```

)set break resume
)sys rm -f Asp42.output
)spool Asp42.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp42
--R Asp42(nameOne: Symbol,nameTwo: Symbol,nameThree: Symbol) is a domain constructor
--R Abbreviation for Asp42 is ASP42
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP42
--R
--R----- Operations -----
--R coerce : FortranCode -> %          coerce : List FortranCode -> %
--R coerce : % -> OutputForm          outputAsFortran : % -> Void
--R coerce : Vector FortranExpression([construct,QUOTEEPS],[construct,QUOTEYA,QUOTEYB],Machi
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Vector Fraction Polynomial Integer -> %
--R retract : Vector Fraction Polynomial Float -> %
--R retract : Vector Polynomial Integer -> %
--R retract : Vector Polynomial Float -> %
--R retract : Vector Expression Integer -> %
--R retract : Vector Expression Float -> %
--R retractIfCan : Vector Fraction Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Vector Fraction Polynomial Float -> Union(%,"failed")
--R retractIfCan : Vector Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Vector Polynomial Float -> Union(%,"failed")
--R retractIfCan : Vector Expression Integer -> Union(%,"failed")
--R retractIfCan : Vector Expression Float -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)

```

## — Asp42.help —

```

=====
Asp42 examples
=====

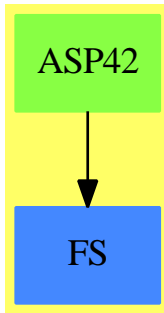
```

```

See Also:
o )show Asp42

```

### 2.27.1 Asp42 (ASP42)



#### Exports:

coerce outputAsFortran retract retractIfCan

— domain ASP42 Asp42 —

```

)abbrev domain ASP42 Asp42
++ Author: Mike Dewar, Godfrey Nolan
++ Date Created:
++ Date Last Updated: 29 March 1994
++               6 October 1994
++ Related Constructors: FortranFunctionCategory, FortranProgramCategory.
++ Description:
++\spadtype{Asp42} produces Fortran for Type 42 ASPs, needed for NAG
++routines d02raf and d02saf
++in particular. These ASPs are in fact
++three Fortran routines which return a vector of functions, and their
++derivatives wrt Y(i) and also a continuation parameter EPS, for example:
++
++\tab{5}SUBROUTINE G(EPS,YA,YB,BC,N)\br
++\tab{5}DOUBLE PRECISION EPS,YA(N),YB(N),BC(N)\br
++\tab{5}INTEGER N\br
++\tab{5}BC(1)=YA(1)\br
++\tab{5}BC(2)=YA(2)\br
++\tab{5}BC(3)=YB(2)-1.0D0\br
++\tab{5}RETURN\br
++\tab{5}END\br
++\tab{5}SUBROUTINE JACOBG(EPS,YA,YB,AJ,BJ,N)\br
++\tab{5}DOUBLE PRECISION EPS,YA(N),AJ(N,N),BJ(N,N),YB(N)\br
++\tab{5}INTEGER N\br
  
```

```

++\tab{5}AJ(1,1)=1.0D0\br
++\tab{5}AJ(1,2)=0.0D0\br
++\tab{5}AJ(1,3)=0.0D0\br
++\tab{5}AJ(2,1)=0.0D0\br
++\tab{5}AJ(2,2)=1.0D0\br
++\tab{5}AJ(2,3)=0.0D0\br
++\tab{5}AJ(3,1)=0.0D0\br
++\tab{5}AJ(3,2)=0.0D0\br
++\tab{5}AJ(3,3)=0.0D0\br
++\tab{5}BJ(1,1)=0.0D0\br
++\tab{5}BJ(1,2)=0.0D0\br
++\tab{5}BJ(1,3)=0.0D0\br
++\tab{5}BJ(2,1)=0.0D0\br
++\tab{5}BJ(2,2)=0.0D0\br
++\tab{5}BJ(2,3)=0.0D0\br
++\tab{5}BJ(3,1)=0.0D0\br
++\tab{5}BJ(3,2)=1.0D0\br
++\tab{5}BJ(3,3)=0.0D0\br
++\tab{5}RETURN\br
++\tab{5}END\br
++\tab{5}SUBROUTINE JACGEP(EPS,YA,YB,BCEP,N)\br
++\tab{5}DOUBLE PRECISION EPS,YA(N),YB(N),BCEP(N)\br
++\tab{5}INTEGER N\br
++\tab{5}BCEP(1)=0.0D0\br
++\tab{5}BCEP(2)=0.0D0\br
++\tab{5}BCEP(3)=0.0D0\br
++\tab{5}RETURN\br
++\tab{5}END

```

```

Asp42(nameOne,nameTwo,nameThree): Exports == Implementation where
  nameOne : Symbol
  nameTwo : Symbol
  nameThree : Symbol

D      ==> differentiate
FST    ==> FortranScalarType
FT     ==> FortranType
FP     ==> FortranProgram
FC     ==> FortranCode
PI     ==> PositiveInteger
NNI    ==> NonNegativeInteger
SYMTAB ==> SymbolTable
RSFC   ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
UFST   ==> Union(fst:FST,void:"void")
FRAC   ==> Fraction
POLY   ==> Polynomial
EXPR   ==> Expression
INT    ==> Integer
FLOAT  ==> Float
VEC    ==> Vector

```

```

VF2    ==> VectorFunctions2
MFLOAT ==> MachineFloat
FEXPR  ==> FortranExpression(['EPS'], ['YA', 'YB'], MFLOAT)
S       ==> Symbol
MF2     ==> MatrixCategoryFunctions2(FEXPR, VEC FEXPR, VEC FEXPR, Matrix FEXPR,
                                     EXPR MFLOAT, VEC EXPR MFLOAT, VEC EXPR MFLOAT, Matrix EXPR MFLOAT)

Exports ==> FortranVectorFunctionCategory with
  coerce : VEC FEXPR -> $
    ++coerce(f) takes objects from the appropriate instantiation of
    ++\spadtype{FortranExpression} and turns them into an ASP.

Implementation ==> add
  real : UFST := ["real"::FST]$UFST

  symOne : SYMTAB := empty()$SYMTAB
  declare!(EPS,fortranReal(),symOne)$SYMTAB
  declare!(N,fortranInteger(),symOne)$SYMTAB
  yType : FT := construct(real,[N],false)$FT
  declare!(YA,yType,symOne)$SYMTAB
  declare!(YB,yType,symOne)$SYMTAB
  declare!(BC,yType,symOne)$SYMTAB

  symTwo : SYMTAB := empty()$SYMTAB
  declare!(EPS,fortranReal(),symTwo)$SYMTAB
  declare!(N,fortranInteger(),symTwo)$SYMTAB
  declare!(YA,yType,symTwo)$SYMTAB
  declare!(YB,yType,symTwo)$SYMTAB
  ajType : FT := construct(real,[N,N],false)$FT
  declare!(AJ,ajType,symTwo)$SYMTAB
  declare!(BJ,ajType,symTwo)$SYMTAB

  symThree : SYMTAB := empty()$SYMTAB
  declare!(EPS,fortranReal(),symThree)$SYMTAB
  declare!(N,fortranInteger(),symThree)$SYMTAB
  declare!(YA,yType,symThree)$SYMTAB
  declare!(YB,yType,symThree)$SYMTAB
  declare!(BCEP,yType,symThree)$SYMTAB

  rt := ["void"]$UFST
  R1:=FortranProgram(nameOne,rt,[EPS,YA,YB,BC,N],symOne)
  R2:=FortranProgram(nameTwo,rt,[EPS,YA,YB,AJ,BJ,N],symTwo)
  R3:=FortranProgram(nameThree,rt,[EPS,YA,YB,BCEP,N],symThree)
  Rep := Record(g:R1,gJacob:R2,geJacob:R3)
  BCsym:Symbol:=coerce "BC"
  AJsym:Symbol:=coerce "AJ"
  BJsym:Symbol:=coerce "BJ"
  BCEPsym:Symbol:=coerce "BCEP"

  makeList(n:Integer,s:Symbol):List(Symbol) ==

```

```

j:Integer
p:List(Symbol) := []
for j in 1 .. n repeat p:= cons(subscript(s,[j::OutputForm])$Symbol,p)
reverse(p)

fexpr2expr(u:FEXPR):EXPR MFLOAT == coerce(u)$FEXPR

localAssign1(s:S,j:Matrix FEXPR):FC ==
  j' : Matrix EXPR MFLOAT := map(fexpr2expr,j)$MF2
  assign(s,j')$FC

localAssign2(s:S,j:VEC FEXPR):FC ==
  j' : VEC EXPR MFLOAT := map(fexpr2expr,j)$VF2(FEXPR,EXPR MFLOAT)
  assign(s,j')$FC

makeCodeOne(u:VEC FEXPR):FortranCode ==
  -- simple assign
  localAssign2(BCsym,u)

makeCodeTwo(u:VEC FEXPR):List(FortranCode) ==
  -- compute jacobian wrt to ya
  n:Integer := maxIndex(u)
  p:List(Symbol) := makeList(n,YA::Symbol)
  jacYA:Matrix(FEXPR) := _
    jacobian(u,p)$MultiVariableCalculusFunctions(S,FEXPR,VEC FEXPR,List(S))
  -- compute jacobian wrt to yb
  p:List(Symbol) := makeList(n,YB::Symbol)
  jacYB: Matrix(FEXPR) := _
    jacobian(u,p)$MultiVariableCalculusFunctions(S,FEXPR,VEC FEXPR,List(S))
  -- assign jacobians to AJ & BJ
  [localAssign1(AJsym,jacYA),localAssign1(BJsym,jacYB),returns()$FC]$List(FC)

makeCodeThree(u:VEC FEXPR):FortranCode ==
  -- compute jacobian wrt to eps
  jacEps:VEC FEXPR := [D(v,EPS) for v in entries u]$VEC(FEXPR)
  localAssign2(BCEPsym,jacEps)

coerce(u:VEC FEXPR):$ ==
  aF:FortranCode := makeCodeOne(u)
  bF:List(FortranCode) := makeCodeTwo(u)
  cF:FortranCode := makeCodeThree(u)
  -- add returns() to complete subroutines
  aLF:List(FortranCode) := [aF,returns()$FC]$List(FortranCode)
  cLF:List(FortranCode) := [cF,returns()$FC]$List(FortranCode)
  [coerce(aLF)$R1,coerce(bF)$R2,coerce(cLF)$R3]

coerce(u:$) : OutputForm ==
  bracket commaSeparate
    [nameOne::OutputForm,nameTwo::OutputForm,nameThree::OutputForm]

```

```

outputAsFortran(u:$):Void ==
  p := checkPrecision()$NAGLinkSupportPackage
  outputAsFortran elt(u,g)$Rep
  outputAsFortran elt(u,gJacob)$Rep
  outputAsFortran elt(u,geJacob)$Rep
  p => restorePrecision()$NAGLinkSupportPackage

retract(u:VEC FRAC POLY INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY INT,FEXPR)
  v::$

retractIfCan(u:VEC FRAC POLY INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC FRAC POLY FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY FLOAT,FEXPR)
  v::$

retractIfCan(u:VEC FRAC POLY FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC EXPR INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(EXPR INT,FEXPR)
  v::$

retractIfCan(u:VEC EXPR INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC EXPR FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(EXPR FLOAT,FEXPR)
  v::$

retractIfCan(u:VEC EXPR FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC POLY INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(POLY INT,FEXPR)
  v::$

retractIfCan(u:VEC POLY INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY INT,FEXPR)
  v case "failed" => "failed"

```

```

(v::VEC FEXPR)::$(

retract(u:VEC POLY FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(POLY FLOAT,FEXPR)
  v::$(

retractIfCan(u:VEC POLY FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$(

-----

— ASP42.dotabb —

"ASP42" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP42"]
"FS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FS"]
"ASP42" -> "FS"

-----

```

## 2.28 domain ASP49 Asp49

```

— Asp49.input —

)set break resume
)sys rm -f Asp49.output
)spool Asp49.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp49
--R Asp49 name: Symbol is a domain constructor
--R Abbreviation for Asp49 is ASP49
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP49
--R
--R----- Operations -----
--R coerce : FortranCode -> %               coerce : List FortranCode -> %
--R coerce : % -> OutputForm               outputAsFortran : % -> Void
--R retract : Polynomial Integer -> %       retract : Polynomial Float -> %
--R retract : Expression Integer -> %       retract : Expression Float -> %

```

```

--R coerce : FortranExpression([construct],[construct,QUOTEX],MachineFloat) -> %
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Fraction Polynomial Integer -> %
--R retract : Fraction Polynomial Float -> %
--R retractIfCan : Fraction Polynomial Integer -> Union(%, "failed")
--R retractIfCan : Fraction Polynomial Float -> Union(%, "failed")
--R retractIfCan : Polynomial Integer -> Union(%, "failed")
--R retractIfCan : Polynomial Float -> Union(%, "failed")
--R retractIfCan : Expression Integer -> Union(%, "failed")
--R retractIfCan : Expression Float -> Union(%, "failed")
--R
--E 1

)spool
)lisp (bye)

```

—————

— Asp49.help —

=====

Asp49 examples

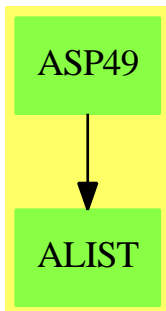
=====

See Also:

- o )show Asp49

—————

### 2.28.1 Asp49 (ASP49)



#### Exports:

coerce   outputAsFortran   retract   retractIfCan



## — domain ASP49 Asp49 —

```

)abbrev domain ASP49 Asp49
++ Author: Mike Dewar, Grant Keady and Godfrey Nolan
++ Date Created: Mar 1993
++ Date Last Updated: 23 March 1994
++                               6 October 1994
++ Related Constructors: FortranScalarFunctionCategory, FortranProgramCategory
++ Description:
++\spadtype{Asp49} produces Fortran for Type 49 ASPs, needed for NAG routines
++e04dgm, e04ucf, for example:
++
++\tab{5}SUBROUTINE  OBJFUN(MODE,N,X,OBJF,OBJGRD,NSTATE,IUSER,USER)\br
++\tab{5}DOUBLE  PRECISION  X(N),OBJF,OBJGRD(N),USER(*)\br
++\tab{5}INTEGER  N,IUSER(*),MODE,NSTATE\br
++\tab{5}OBJF=X(4)*X(9)+((-1.0D0*X(5))+X(3))*X(8)+((-1.0D0*X(3))+X(1))*X(7)\br
++\tab{4}&+(-1.0D0*X(2)*X(6))\br
++\tab{5}OBJGRD(1)=X(7)\br
++\tab{5}OBJGRD(2)=-1.0D0*X(6)\br
++\tab{5}OBJGRD(3)=X(8)+(-1.0D0*X(7))\br
++\tab{5}OBJGRD(4)=X(9)\br
++\tab{5}OBJGRD(5)=-1.0D0*X(8)\br
++\tab{5}OBJGRD(6)=-1.0D0*X(2)\br
++\tab{5}OBJGRD(7)=(-1.0D0*X(3))+X(1)\br
++\tab{5}OBJGRD(8)=(-1.0D0*X(5))+X(3)\br
++\tab{5}OBJGRD(9)=X(4)\br
++\tab{5}RETURN\br
++\tab{5}END

```

```

Asp49(name): Exports == Implementation where
  name : Symbol

```

```

FST    ==> FortranScalarType
UFST   ==> Union(fst:FST,void:"void")
FT     ==> FortranType
FC     ==> FortranCode
SYMTAB ==> SymbolTable
RSFC   ==> Record(localSymbols:SymbolTable,code:List(FC))
MFLOAT ==> MachineFloat
FEXPR  ==> FortranExpression([],['X'],MFLOAT)
FRAC   ==> Fraction
POLY   ==> Polynomial
EXPR   ==> Expression
INT     ==> Integer
FLOAT  ==> Float
VEC     ==> Vector
VF2    ==> VectorFunctions2
S       ==> Symbol

```

```

Exports ==> FortranFunctionCategory with
  coerce : FEXPR -> $
    ++coerce(f) takes an object from the appropriate instantiation of
    ++\spadtype{FortranExpression} and turns it into an ASP.

Implementation ==> add

real : UFST := ["real"::FST]$UFST
integer : UFST := ["integer"::FST]$UFST
syms : SYMTAB := empty()$SYMTAB
declare!(MODE,fortranInteger(),syms)$SYMTAB
declare!(N,fortranInteger(),syms)$SYMTAB
xType : FT := construct(real,[N::S],false)$FT
declare!(X,xType,syms)$SYMTAB
declare!(OBJF,fortranReal(),syms)$SYMTAB
declare!(OBJGRD,xType,syms)$SYMTAB
declare!(NSTATE,fortranInteger(),syms)$SYMTAB
iuType : FT := construct(integer,["*":S],false)$FT
declare!(IUSER,iuType,syms)$SYMTAB
uType : FT := construct(real,["*":S],false)$FT
declare!(USER,uType,syms)$SYMTAB
Rep := FortranProgram(name,["void"]$UFST,
                        [MODE,N,X,OBJF,OBJGRD,NSTATE,IUSER,USER],syms)

fexpr2expr(u:FEXPR):EXPR MFLOAT == coerce(u)$FEXPR

localAssign(s:S,j:VEC FEXPR):FC ==
  j' : VEC EXPR MFLOAT := map(fexpr2expr,j)$VF2(FEXPR,EXPR MFLOAT)
  assign(s,j')$FC

coerce(u:FEXPR):$ ==
  vars:List(S) := variables(u)
  grd:VEC FEXPR := gradient(u,vars)$MultiVariableCalculusFunctions(_
                           S,FEXPR,VEC FEXPR,List(S))
  code : List(FC) := [assign(OBJF@S,fexpr2expr u)$FC,_
                     localAssign(OBJGRD@S,grd),_
                     returns()$FC]
  code:::$

coerce(u:$):OutputForm == coerce(u)$Rep

coerce(c:List FC):$ == coerce(c)$Rep

coerce(r:RSFC):$ == coerce(r)$Rep

coerce(c:FC):$ == coerce(c)$Rep

outputAsFortran(u):Void ==
  p := checkPrecision()$NAGLinkSupportPackage
  outputAsFortran(u)$Rep

```

```

p => restorePrecision()$NAGLinkSupportPackage

retract(u:FRAC POLY INT):$ == (retract(u)@FEXPR)::$
retractIfCan(u:FRAC POLY INT):Union($,"failed") ==
  foo : Union(FEXPR,"failed")
  foo := retractIfCan(u)$FEXPR
  foo case "failed" => "failed"
  (foo::FEXPR)::$

retract(u:FRAC POLY FLOAT):$ == (retract(u)@FEXPR)::$
retractIfCan(u:FRAC POLY FLOAT):Union($,"failed") ==
  foo : Union(FEXPR,"failed")
  foo := retractIfCan(u)$FEXPR
  foo case "failed" => "failed"
  (foo::FEXPR)::$

retract(u:EXPR FLOAT):$ == (retract(u)@FEXPR)::$
retractIfCan(u:EXPR FLOAT):Union($,"failed") ==
  foo : Union(FEXPR,"failed")
  foo := retractIfCan(u)$FEXPR
  foo case "failed" => "failed"
  (foo::FEXPR)::$

retract(u:EXPR INT):$ == (retract(u)@FEXPR)::$
retractIfCan(u:EXPR INT):Union($,"failed") ==
  foo : Union(FEXPR,"failed")
  foo := retractIfCan(u)$FEXPR
  foo case "failed" => "failed"
  (foo::FEXPR)::$

retract(u:POLY FLOAT):$ == (retract(u)@FEXPR)::$
retractIfCan(u:POLY FLOAT):Union($,"failed") ==
  foo : Union(FEXPR,"failed")
  foo := retractIfCan(u)$FEXPR
  foo case "failed" => "failed"
  (foo::FEXPR)::$

retract(u:POLY INT):$ == (retract(u)@FEXPR)::$
retractIfCan(u:POLY INT):Union($,"failed") ==
  foo : Union(FEXPR,"failed")
  foo := retractIfCan(u)$FEXPR
  foo case "failed" => "failed"
  (foo::FEXPR)::$

```

---

```
"ASP49" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP49"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"ASP49" -> "ALIST"
```

## 2.29 domain ASP50 Asp50

— Asp50.input —

```
)set break resume
)sys rm -f Asp50.output
)spool Asp50.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp50
--R Asp50 name: Symbol is a domain constructor
--R Abbreviation for Asp50 is ASP50
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP50
--R
--R----- Operations -----
--R coerce : FortranCode -> %          coerce : List FortranCode -> %
--R coerce : % -> OutputForm          outputAsFortran : % -> Void
--R coerce : Vector FortranExpression([construct],[construct,QUOTEXC],MachineFloat) -> %
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Vector Fraction Polynomial Integer -> %
--R retract : Vector Fraction Polynomial Float -> %
--R retract : Vector Polynomial Integer -> %
--R retract : Vector Polynomial Float -> %
--R retract : Vector Expression Integer -> %
--R retract : Vector Expression Float -> %
--R retractIfCan : Vector Fraction Polynomial Integer -> Union(%, "failed")
--R retractIfCan : Vector Fraction Polynomial Float -> Union(%, "failed")
--R retractIfCan : Vector Polynomial Integer -> Union(%, "failed")
--R retractIfCan : Vector Polynomial Float -> Union(%, "failed")
--R retractIfCan : Vector Expression Integer -> Union(%, "failed")
--R retractIfCan : Vector Expression Float -> Union(%, "failed")
--R
--E 1

)spool
)lisp (bye)
```

---

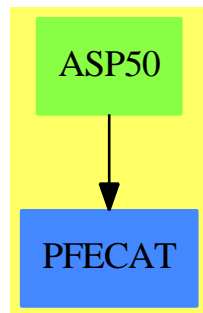
— Asp50.help —

```
=====
Asp50 examples
=====
```

```
See Also:
o )show Asp50
```

---

### 2.29.1 Asp50 (ASP50)



#### Exports:

```
coerce outputAsFortran retract retractIfCan
```

— domain ASP50 Asp50 —

```
)abbrev domain ASP50 Asp50
++ Author: Mike Dewar, Grant Keady and Godfrey Nolan
++ Date Created: Mar 1993
++ Date Last Updated: 23 March 1994
++                               6 October 1994
++ Related Constructors: FortranVectorFunctionCategory, FortranProgramCategory
++ Description:
++\spadtype{Asp50} produces Fortran for Type 50 ASPs, needed for NAG routine
++e04fdf, for example:
++
++\tab{5}SUBROUTINE LSFUN1(M,N,XC,FVECC)\br
++\tab{5}DOUBLE PRECISION FVECC(M),XC(N)\br
++\tab{5}INTEGER I,M,N\br
++\tab{5}FVECC(1)=((XC(1)-2.4D0)*XC(3)+(15.0D0*XC(1)-36.0D0)*XC(2)+1.0D0)/(\br
```

```

++\tab{4}&XC(3)+15.0D0*XC(2))\br
++\tab{5}FVECC(2)=((XC(1)-2.8D0)*XC(3)+(7.0D0*XC(1)-19.6D0)*XC(2)+1.0D0)/(X\br
++\tab{4}&C(3)+7.0D0*XC(2))\br
++\tab{5}FVECC(3)=((XC(1)-3.2D0)*XC(3)+(4.33333333333333D0*XC(1)-13.866666\br
++\tab{4}&66666667D0)*XC(2)+1.0D0)/(XC(3)+4.33333333333333D0*XC(2))\br
++\tab{5}FVECC(4)=((XC(1)-3.5D0)*XC(3)+(3.0D0*XC(1)-10.5D0)*XC(2)+1.0D0)/(X\br
++\tab{4}&C(3)+3.0D0*XC(2))\br
++\tab{5}FVECC(5)=((XC(1)-3.9D0)*XC(3)+(2.2D0*XC(1)-8.57999999999999D0)*XC\br
++\tab{4}&(2)+1.0D0)/(XC(3)+2.2D0*XC(2))\br
++\tab{5}FVECC(6)=((XC(1)-4.19999999999999D0)*XC(3)+(1.66666666666667D0*X\br
++\tab{4}&C(1)-7.0D0)*XC(2)+1.0D0)/(XC(3)+1.66666666666667D0*XC(2))\br
++\tab{5}FVECC(7)=((XC(1)-4.5D0)*XC(3)+(1.285714285714286D0*XC(1)-5.7857142\br
++\tab{4}&85714286D0)*XC(2)+1.0D0)/(XC(3)+1.285714285714286D0*XC(2))\br
++\tab{5}FVECC(8)=((XC(1)-4.89999999999999D0)*XC(3)+(XC(1)-4.899999999999\br
++\tab{4}&99D0)*XC(2)+1.0D0)/(XC(3)+XC(2))\br
++\tab{5}FVECC(9)=((XC(1)-4.69999999999999D0)*XC(3)+(XC(1)-4.699999999999\br
++\tab{4}&99D0)*XC(2)+1.285714285714286D0)/(XC(3)+XC(2))\br
++\tab{5}FVECC(10)=((XC(1)-6.8D0)*XC(3)+(XC(1)-6.8D0)*XC(2)+1.666666666666\br
++\tab{4}&67D0)/(XC(3)+XC(2))\br
++\tab{5}FVECC(11)=((XC(1)-8.29999999999999D0)*XC(3)+(XC(1)-8.299999999999\br
++\tab{4}&999D0)*XC(2)+2.2D0)/(XC(3)+XC(2))\br
++\tab{5}FVECC(12)=((XC(1)-10.6D0)*XC(3)+(XC(1)-10.6D0)*XC(2)+3.0D0)/(XC(3)\br
++\tab{4}&+XC(2))\br
++\tab{5}FVECC(13)=((XC(1)-1.34D0)*XC(3)+(XC(1)-1.34D0)*XC(2)+4.3333333333\br
++\tab{4}&3333D0)/(XC(3)+XC(2))\br
++\tab{5}FVECC(14)=((XC(1)-2.1D0)*XC(3)+(XC(1)-2.1D0)*XC(2)+7.0D0)/(XC(3)+X\br
++\tab{4}&C(2))\br
++\tab{5}FVECC(15)=((XC(1)-4.39D0)*XC(3)+(XC(1)-4.39D0)*XC(2)+15.0D0)/(XC(3\br
++\tab{4}&)+XC(2))\br
++\tab{5}END

```

```

Asp50(name): Exports == Implementation where
  name : Symbol

```

```

FST    ==> FortranScalarType
FT     ==> FortranType
UFST   ==> Union(fst:FST,void:"void")
SYMTAB ==> SymbolTable
RSFC   ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
FRAC   ==> Fraction
POLY   ==> Polynomial
EXPR   ==> Expression
INT     ==> Integer
FLOAT  ==> Float
VEC     ==> Vector
VF2     ==> VectorFunctions2
FEXPR  ==> FortranExpression([],['XC'],MFLOAT)
MFLOAT ==> MachineFloat

```

```

Exports ==> FortranVectorFunctionCategory with

```

```

coerce : VEC FEXPR -> $
  ++coerce(f) takes objects from the appropriate instantiation of
  ++\spadtype{FortranExpression} and turns them into an ASP.

Implementation ==> add

real : UFST := ["real"::FST]$UFST
syms : SYMTAB := empty()$SYMTAB
declare!(M,fortranInteger(),syms)$SYMTAB
declare!(N,fortranInteger(),syms)$SYMTAB
xcType : FT := construct(real,[N],false)$FT
declare!(XC,xcType,syms)$SYMTAB
fveccType : FT := construct(real,[M],false)$FT
declare!(FVECC,fveccType,syms)$SYMTAB
declare!(I,fortranInteger(),syms)$SYMTAB
tType : FT := construct(real,[M,N],false)$FT
-- declare!(TC,tType,syms)$SYMTAB
-- declare!(Y,fveccType,syms)$SYMTAB
Rep := FortranProgram(name,["void"]$UFST, [M,N,XC,FVECC],syms)

fexpr2expr(u:FEXPR):EXPR MFLOAT == coerce(u)$FEXPR

coerce(u:VEC FEXPR):$ ==
  u' : VEC EXPR MFLOAT := map(fexpr2expr,u)$VF2(FEXPR,EXPR MFLOAT)
  assign(FVECC,u')$FortranCode::$

retract(u:VEC FRAC POLY INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY INT,FEXPR)
  v::$

retractIfCan(u:VEC FRAC POLY INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR):: $

retract(u:VEC FRAC POLY FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY FLOAT,FEXPR)
  v::$

retractIfCan(u:VEC FRAC POLY FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR):: $

retract(u:VEC EXPR INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(EXPR INT,FEXPR)
  v::$

retractIfCan(u:VEC EXPR INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR INT,FEXPR)

```

```

    v case "failed" => "failed"
    (v::VEC FEXPR):: $

retract(u:VEC EXPR FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(EXPR FLOAT,FEXPR)
  v:: $

retractIfCan(u:VEC EXPR FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR):: $

retract(u:VEC POLY INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(POLY INT,FEXPR)
  v:: $

retractIfCan(u:VEC POLY INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR):: $

retract(u:VEC POLY FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(POLY FLOAT,FEXPR)
  v:: $

retractIfCan(u:VEC POLY FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR):: $

coerce(c:List FortranCode):$ == coerce(c)$Rep

coerce(r:RSFC):$ == coerce(r)$Rep

coerce(c:FortranCode):$ == coerce(c)$Rep

coerce(u:$):OutputForm == coerce(u)$Rep

outputAsFortran(u):Void ==
  p := checkPrecision()$NAGLinkSupportPackage
  outputAsFortran(u)$Rep
  p => restorePrecision()$NAGLinkSupportPackage

```

---

— ASP50.dotabb —

"ASP50" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP50"]



```
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"ASP50" -> "PFECAT"
```

## 2.30 domain ASP55 Asp55

— Asp55.input —

```
)set break resume
)sys rm -f Asp55.output
)spool Asp55.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp55
--R Asp55 name: Symbol is a domain constructor
--R Abbreviation for Asp55 is ASP55
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP55
--R
--R----- Operations -----
--R coerce : FortranCode -> %               coerce : List FortranCode -> %
--R coerce : % -> OutputForm               outputAsFortran : % -> Void
--R coerce : Vector FortranExpression([construct],[construct,QUOTEX],MachineFloat) -> %
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Vector Fraction Polynomial Integer -> %
--R retract : Vector Fraction Polynomial Float -> %
--R retract : Vector Polynomial Integer -> %
--R retract : Vector Polynomial Float -> %
--R retract : Vector Expression Integer -> %
--R retract : Vector Expression Float -> %
--R retractIfCan : Vector Fraction Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Vector Fraction Polynomial Float -> Union(%,"failed")
--R retractIfCan : Vector Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Vector Polynomial Float -> Union(%,"failed")
--R retractIfCan : Vector Expression Integer -> Union(%,"failed")
--R retractIfCan : Vector Expression Float -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

```

-----

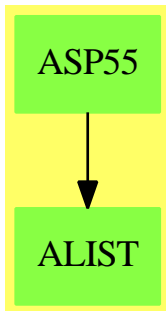
— Asp55.help —

=====
Asp55 examples
=====

See Also:
o )show Asp55

```

### 2.30.1 Asp55 (ASP55)



#### Exports:

coerce outputAsFortran retract retractIfCan

— domain ASP55 Asp55 —

```

)abbrev domain ASP55 Asp55
++ Author: Mike Dewar, Grant Keady and Godfrey Nolan
++ Date Created: June 1993
++ Date Last Updated: 23 March 1994
++                               6 October 1994
++ Related Constructors: FortranScalarFunctionCategory, FortranProgramCategory
++ Description:
++\spadtype{Asp55} produces Fortran for Type 55 ASPs, needed for NAG routines
++e04dgm and e04ucf, for example:
++
++\tab{5}SUBROUTINE CONFUN(MODE,NCNLN,N,NROWJ,NEEDC,X,C,CJAC,NSTATE,IUSER\br
++\tab{4}&,USER)\br
++\tab{5}DOUBLE PRECISION C(NCNLN),X(N),CJAC(NROWJ,N),USER(*)\br
++\tab{5}INTEGER N,IUSER(*),NEEDC(NCNLN),NROWJ,MODE,NCNLN,NSTATE\br

```

```

++\tab{5}IF(NEEDEC(1).GT.0)THEN\br
++\tab{7}C(1)=X(6)**2+X(1)**2\br
++\tab{7}CJAC(1,1)=2.0D0*X(1)\br
++\tab{7}CJAC(1,2)=0.0D0\br
++\tab{7}CJAC(1,3)=0.0D0\br
++\tab{7}CJAC(1,4)=0.0D0\br
++\tab{7}CJAC(1,5)=0.0D0\br
++\tab{7}CJAC(1,6)=2.0D0*X(6)\br
++\tab{5}ENDIF\br
++\tab{5}IF(NEEDEC(2).GT.0)THEN\br
++\tab{7}C(2)=X(2)**2+(-2.0D0*X(1)*X(2))+X(1)**2\br
++\tab{7}CJAC(2,1)=(-2.0D0*X(2))+2.0D0*X(1)\br
++\tab{7}CJAC(2,2)=2.0D0*X(2)+(-2.0D0*X(1))\br
++\tab{7}CJAC(2,3)=0.0D0\br
++\tab{7}CJAC(2,4)=0.0D0\br
++\tab{7}CJAC(2,5)=0.0D0\br
++\tab{7}CJAC(2,6)=0.0D0\br
++\tab{5}ENDIF\br
++\tab{5}IF(NEEDEC(3).GT.0)THEN\br
++\tab{7}C(3)=X(3)**2+(-2.0D0*X(1)*X(3))+X(2)**2+X(1)**2\br
++\tab{7}CJAC(3,1)=(-2.0D0*X(3))+2.0D0*X(1)\br
++\tab{7}CJAC(3,2)=2.0D0*X(2)\br
++\tab{7}CJAC(3,3)=2.0D0*X(3)+(-2.0D0*X(1))\br
++\tab{7}CJAC(3,4)=0.0D0\br
++\tab{7}CJAC(3,5)=0.0D0\br
++\tab{7}CJAC(3,6)=0.0D0\br
++\tab{5}ENDIF\br
++\tab{5}RETURN\br
++\tab{5}END

```

```

Asp55(name): Exports == Implementation where
    name : Symbol

```

```

FST    ==> FortranScalarType
FT     ==> FortranType
FSTU   ==> Union(fst:FST,void:"void")
SYMTAB ==> SymbolTable
FC     ==> FortranCode
RSFC   ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
FRAC   ==> Fraction
POLY   ==> Polynomial
EXPR   ==> Expression
INT    ==> Integer
S      ==> Symbol
FLOAT  ==> Float
VEC    ==> Vector
VF2    ==> VectorFunctions2
MAT    ==> Matrix
MFLOAT ==> MachineFloat
FEXPR  ==> FortranExpression([],['X'],MFLOAT)

```

```

MF2    ==> MatrixCategoryFunctions2(FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR,
      EXPR MFLOAT,VEC EXPR MFLOAT,VEC EXPR MFLOAT,MAT EXPR MFLOAT)
SWU    ==> Union(I:Expression Integer,F:Expression Float,
      CF:Expression Complex Float,switch:Switch)

Exports ==> FortranVectorFunctionCategory with
  coerce : VEC FEXPR -> $
  ++coerce(f) takes objects from the appropriate instantiation of
  ++\spadtype{FortranExpression} and turns them into an ASP.

Implementation ==> add

real : FSTU := ["real"::FST]$FSTU
integer : FSTU := ["integer"::FST]$FSTU
syms : SYMTAB := empty()$SYMTAB
declare!(MODE,fortranInteger(),syms)$SYMTAB
declare!(NCNLN,fortranInteger(),syms)$SYMTAB
declare!(N,fortranInteger(),syms)$SYMTAB
declare!(NROWJ,fortranInteger(),syms)$SYMTAB
needcType : FT := construct(integer,[NCNLN::Symbol],false)$FT
declare!(NEEDC,needcType,syms)$SYMTAB
xType : FT := construct(real,[N::Symbol],false)$FT
declare!(X,xType,syms)$SYMTAB
cType : FT := construct(real,[NCNLN::Symbol],false)$FT
declare!(C,cType,syms)$SYMTAB
cjacType : FT := construct(real,[NROWJ::Symbol,N::Symbol],false)$FT
declare!(CJAC,cjacType,syms)$SYMTAB
declare!(NSTATE,fortranInteger(),syms)$SYMTAB
iuType : FT := construct(integer,["* "::Symbol],false)$FT
declare!(IUSER,iuType,syms)$SYMTAB
uType : FT := construct(real,["* "::Symbol],false)$FT
declare!(USER,uType,syms)$SYMTAB
Rep := FortranProgram(name,["void"]$FSTU,
      [MODE,NCNLN,N,NROWJ,NEEDC,X,C,CJAC,NSTATE,IUSER,USER],syms)

-- Take a symbol, pull of the script and turn it into an integer!!
o2int(u:S):Integer ==
  o : OutputForm := first elt(scripts(u)$S,sub)
  o pretend Integer

localAssign(s:Symbol,dim:List POLY INT,u:FEXPR):FC ==
  assign(s,dim,(u::EXPR MFLOAT)$FEXPR)$FC

makeCond(index:INT,fun:FEXPR,jac:VEC FEXPR):FC ==
  needc : EXPR INT := (subscript(NEEDC,[index::OutputForm])$S)::EXPR(INT)
  sw : Switch := GT([needc]$SWU,[0::EXPR(INT)]$SWU)$Switch
  ass : List FC := [localAssign(CJAC,[index::POLY INT,i::POLY INT],jac.i)_
      for i in 1..maxIndex(jac)]
  cond(sw,block([localAssign(C,[index::POLY INT],fun),:ass])$FC)$FC

```

```

coerce(u:VEC FEXPR):$ ==
  ncnln:Integer := maxIndex(u)
  x:S := X::S
  pu:List(S) := []
  -- Work out which variables appear in the expressions
  for e in entries(u) repeat
    pu := setUnion(pu,variables(e)$FEXPR)
  scriptList : List Integer := map(o2int,pu)$ListFunctions2(S,Integer)
  -- This should be the maximum X_n which occurs (there may be others
  -- which don't):
  n:Integer := reduce(max,scriptList)$List(Integer)
  p:List(S) := []
  for j in 1..n repeat p:= cons(subscript(x,[j::OutputForm])$S,p)
  p:= reverse(p)
  jac:MAT FEXPR := _
    jacobian(u,p)$MultiVariableCalculusFunctions(S,FEXPR,VEC FEXPR,List(S))
  code : List FC := [makeCond(j,u.j,row(jac,j)) for j in 1..ncnln]
  [:code,returns()$FC]:::$

coerce(c:List FC):$ == coerce(c)$Rep

coerce(r:RSFC):$ == coerce(r)$Rep

coerce(c:FC):$ == coerce(c)$Rep

coerce(u:$):OutputForm == coerce(u)$Rep

outputAsFortran(u):Void ==
  p := checkPrecision()$NAGLinkSupportPackage
  outputAsFortran(u)$Rep
  p => restorePrecision()$NAGLinkSupportPackage

retract(u:VEC FRAC POLY INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY INT,FEXPR)
  v::$

retractIfCan(u:VEC FRAC POLY INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR):::$

retract(u:VEC FRAC POLY FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY FLOAT,FEXPR)
  v::$

retractIfCan(u:VEC FRAC POLY FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR):::$

```

```

retract(u:VEC EXPR INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(EXPR INT,FEXPR)
  v::$

retractIfCan(u:VEC EXPR INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR):: $

retract(u:VEC EXPR FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(EXPR FLOAT,FEXPR)
  v::$

retractIfCan(u:VEC EXPR FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR):: $

retract(u:VEC POLY INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(POLY INT,FEXPR)
  v::$

retractIfCan(u:VEC POLY INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR):: $

retract(u:VEC POLY FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(POLY FLOAT,FEXPR)
  v::$

retractIfCan(u:VEC POLY FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR):: $

```

---

— ASP55.dotabb —

```

"ASP55" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP55"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"ASP55" -> "ALIST"

```

---

## 2.31 domain ASP6 Asp6

— Asp6.input —

```
)set break resume
)sys rm -f Asp6.output
)spool Asp6.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp6
--R Asp6 name: Symbol is a domain constructor
--R Abbreviation for Asp6 is ASP6
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP6
--R
--R----- Operations -----
--R coerce : FortranCode -> %               coerce : List FortranCode -> %
--R coerce : % -> OutputForm               outputAsFortran : % -> Void
--R coerce : Vector FortranExpression([construct],[construct,QUOTEX],MachineFloat) -> %
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Vector Fraction Polynomial Integer -> %
--R retract : Vector Fraction Polynomial Float -> %
--R retract : Vector Polynomial Integer -> %
--R retract : Vector Polynomial Float -> %
--R retract : Vector Expression Integer -> %
--R retract : Vector Expression Float -> %
--R retractIfCan : Vector Fraction Polynomial Integer -> Union(%, "failed")
--R retractIfCan : Vector Fraction Polynomial Float -> Union(%, "failed")
--R retractIfCan : Vector Polynomial Integer -> Union(%, "failed")
--R retractIfCan : Vector Polynomial Float -> Union(%, "failed")
--R retractIfCan : Vector Expression Integer -> Union(%, "failed")
--R retractIfCan : Vector Expression Float -> Union(%, "failed")
--R
--E 1

)spool
)lisp (bye)
```

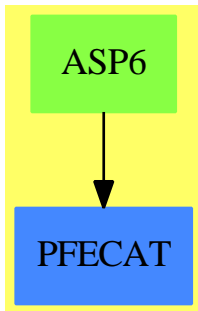
— Asp6.help —

```
=====
Asp6 examples
=====
```

See Also:

- o `)show Asp6`

### 2.31.1 Asp6 (ASP6)



#### Exports:

`coerce` `outputAsFortran` `retract` `retractIfCan`

— domain **ASP6** **Asp6** —

```

)abbrev domain ASP6 Asp6
++ Author: Mike Dewar and Godfrey Nolan and Grant Keady
++ Date Created: Mar 1993
++ Date Last Updated: 18 March 1994
++               6 October 1994
++ Related Constructors: FortranVectorFunctionCategory, FortranProgramCategory
++ Description:
++ \spadtype{Asp6} produces Fortran for Type 6 ASPs, needed for NAG routines
++ c05nbf, c05ncf. These represent vectors of functions of X(i) and look like:
++
++ \tab{5}SUBROUTINE FCN(N,X,FVEC,IFLAG)
++ \tab{5}DOUBLE PRECISION X(N),FVEC(N)
++ \tab{5}INTEGER N,IFLAG
++ \tab{5}FVEC(1)=(-2.0D0*X(2))+(-2.0D0*X(1)**2)+3.0D0*X(1)+1.0D0
++ \tab{5}FVEC(2)=(-2.0D0*X(3))+(-2.0D0*X(2)**2)+3.0D0*X(2)+(-1.0D0*X(1))+1.
++ \tab{4}&OD0
++ \tab{5}FVEC(3)=(-2.0D0*X(4))+(-2.0D0*X(3)**2)+3.0D0*X(3)+(-1.0D0*X(2))+1.
++ \tab{4}&OD0
++ \tab{5}FVEC(4)=(-2.0D0*X(5))+(-2.0D0*X(4)**2)+3.0D0*X(4)+(-1.0D0*X(3))+1.
++ \tab{4}&OD0
++ \tab{5}FVEC(5)=(-2.0D0*X(6))+(-2.0D0*X(5)**2)+3.0D0*X(5)+(-1.0D0*X(4))+1.
  
```



```

++ \tab{4}&OD0
++ \tab{5}FVEC(6)=(-2.0D0*X(7))+(-2.0D0*X(6)**2)+3.0D0*X(6)+(-1.0D0*X(5))+1.
++ \tab{4}&OD0
++ \tab{5}FVEC(7)=(-2.0D0*X(8))+(-2.0D0*X(7)**2)+3.0D0*X(7)+(-1.0D0*X(6))+1.
++ \tab{4}&OD0
++ \tab{5}FVEC(8)=(-2.0D0*X(9))+(-2.0D0*X(8)**2)+3.0D0*X(8)+(-1.0D0*X(7))+1.
++ \tab{4}&OD0
++ \tab{5}FVEC(9)=(-2.0D0*X(9)**2)+3.0D0*X(9)+(-1.0D0*X(8))+1.0D0
++ \tab{5}RETURN
++ \tab{5}END

```

```

Asp6(name): Exports == Implementation where
  name : Symbol

```

```

FEXPR ==> FortranExpression([],['X'],MFLOAT)
MFLOAT ==> MachineFloat
FST ==> FortranScalarType
FT ==> FortranType
SYMTAB ==> SymbolTable
RSFC ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
UFST ==> Union(fst:FST,void:"void")
FRAC ==> Fraction
POLY ==> Polynomial
EXPR ==> Expression
INT ==> Integer
FLOAT ==> Float
VEC ==> Vector
VF2 ==> VectorFunctions2

```

```

Exports == FortranVectorFunctionCategory with
  coerce: Vector FEXPR -> %
  ++coerce(f) takes objects from the appropriate instantiation of
  ++\spadtype{FortranExpression} and turns them into an ASP.

```

```

Implementation == add

```

```

real : UFST := ["real":FST]$UFST
syms : SYMTAB := empty()$SYMTAB
declare!(N,fortranInteger()$FT,syms)$SYMTAB
xType : FT := construct(real,[N],false)$FT
declare!(X,xType,syms)$SYMTAB
declare!(FVEC,xType,syms)$SYMTAB
declare!(IFLAG,fortranInteger()$FT,syms)$SYMTAB
Rep := FortranProgram(name,["void"]$Union(fst:FST,void:"void"),
  [N,X,FVEC,IFLAG],syms)

retract(u:VEC FRAC POLY INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY INT,FEXPR)
  v::$

```

```

retractIfCan(u:VEC FRAC POLY INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC FRAC POLY FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VectorFunctions2(FRAC POLY FLOAT,FEXPR)
  v::$

retractIfCan(u:VEC FRAC POLY FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=_
    map(retractIfCan,u)$VF2(FRAC POLY FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC EXPR INT):$ ==
  v : VEC FEXPR := map(retract,u)$VectorFunctions2(EXPR INT,FEXPR)
  v::$

retractIfCan(u:VEC EXPR INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC EXPR FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VectorFunctions2(EXPR FLOAT,FEXPR)
  v::$

retractIfCan(u:VEC EXPR FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC POLY INT):$ ==
  v : VEC FEXPR := map(retract,u)$VectorFunctions2(POLY INT,FEXPR)
  v::$

retractIfCan(u:VEC POLY INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC POLY FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VectorFunctions2(POLY FLOAT,FEXPR)
  v::$

retractIfCan(u:VEC POLY FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

```

```

fexpr2expr(u:FEXPR):EXPR MFLOAT ==
  (u::EXPR MFLOAT)$FEXPR

coerce(u:VEC FEXPR):% ==
  v : VEC EXPR MFLOAT
  v := map(fexpr2expr,u)$VF2(FEXPR,EXPR MFLOAT)
  ([assign(FVEC,v)$FortranCode,returns()$FortranCode]$List(FortranCode))::$

coerce(c:List FortranCode):% == coerce(c)$Rep

coerce(r:RSFC):% == coerce(r)$Rep

coerce(c:FortranCode):% == coerce(c)$Rep

coerce(u:%):OutputForm == coerce(u)$Rep

outputAsFortran(u):Void ==
  p := checkPrecision()$NAGLinkSupportPackage
  outputAsFortran(u)$Rep
  p => restorePrecision()$NAGLinkSupportPackage

```

---

— ASP6.dotabb —

```

"ASP6" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP6"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"ASP6" -> "PFECAT"

```

---

## 2.32 domain ASP7 Asp7

— Asp7.input —

```

)set break resume
)sys rm -f Asp7.output
)spool Asp7.output
)set message test on
)set message auto off
)clear all

```

--S 1 of 1

```

)show Asp7
--R Asp7 name: Symbol is a domain constructor
--R Abbreviation for Asp7 is ASP7
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP7
--R
--R----- Operations -----
--R coerce : FortranCode -> %          coerce : List FortranCode -> %
--R coerce : % -> OutputForm          outputAsFortran : % -> Void
--R coerce : Vector FortranExpression([construct,QUOTEX],[construct,QUOTEY],MachineFloat) -> %
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Vector Fraction Polynomial Integer -> %
--R retract : Vector Fraction Polynomial Float -> %
--R retract : Vector Polynomial Integer -> %
--R retract : Vector Polynomial Float -> %
--R retract : Vector Expression Integer -> %
--R retract : Vector Expression Float -> %
--R retractIfCan : Vector Fraction Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Vector Fraction Polynomial Float -> Union(%,"failed")
--R retractIfCan : Vector Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Vector Polynomial Float -> Union(%,"failed")
--R retractIfCan : Vector Expression Integer -> Union(%,"failed")
--R retractIfCan : Vector Expression Float -> Union(%,"failed")
--R
--E 1

```

```

)spool
)lisp (bye)

```

---

— Asp7.help —

```

=====
Asp7 examples
=====

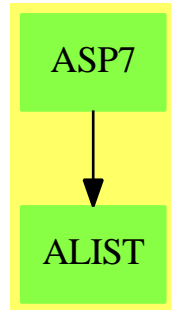
```

See Also:

- o )show Asp7

---

### 2.32.1 Asp7 (ASP7)



#### Exports:

coerce outputAsFortran retract retractIfCan

— domain ASP7 Asp7 —

```

)abbrev domain ASP7 Asp7
++ Author: Mike Dewar and Godfrey Nolan and Grant Keady
++ Date Created: Mar 1993
++ Date Last Updated: 18 March 1994
++               6 October 1994
++ Related Constructors: FortranVectorFunctionCategory, FortranProgramCategory
++ Description:
++ \spadtype{Asp7} produces Fortran for Type 7 ASPs, needed for NAG routines
++ d02bbf, d02gaf. These represent a vector of functions of the scalar X and
++ the array Z, and look like:
++
++ \tab{5}SUBROUTINE FCN(X,Z,F)\br
++ \tab{5}DOUBLE PRECISION F(*),X,Z(*)\br
++ \tab{5}F(1)=DTAN(Z(3))\br
++ \tab{5}F(2)=((-0.03199999999999999D0*DCOS(Z(3))*DTAN(Z(3)))+(-0.02D0*Z(2)\br
++ \tab{4}& **2))/(Z(2)*DCOS(Z(3)))\br
++ \tab{5}F(3)=-0.03199999999999999D0/(X*Z(2)**2)\br
++ \tab{5}RETURN\br
++ \tab{5}END
  
```

Asp7(name): Exports == Implementation where  
 name : Symbol

```

FST    ==> FortranScalarType
FT     ==> FortranType
SYMTAB ==> SymbolTable
RSFC   ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
MFLOAT ==> MachineFloat
FEXPR  ==> FortranExpression(['X'],['Y'],MFLOAT)
UFST   ==> Union(fst:FST,void:"void")
  
```

```

FRAC    ==> Fraction
POLY    ==> Polynomial
EXPR    ==> Expression
INT      ==> Integer
FLOAT   ==> Float
VEC      ==> Vector
VF2      ==> VectorFunctions2

Exports ==> FortranVectorFunctionCategory with
  coerce : Vector FEXPR -> %
    ++coerce(f) takes objects from the appropriate instantiation of
    ++\spadtype{FortranExpression} and turns them into an ASP.

Implementation ==> add

real : UFST := ["real"::FST]$UFST
syms : SYMTAB := empty()$SYMTAB
declare!(X,fortranReal(),syms)$SYMTAB
yType : FT := construct(real,["* "::Symbol],false)$FT
declare!(Y,yType,syms)$SYMTAB
declare!(F,yType,syms)$SYMTAB
Rep := FortranProgram(name,["void"]$UFST,[X,Y,F],syms)

retract(u:VEC FRAC POLY INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY INT,FEXPR)
  v::$

retractIfCan(u:VEC FRAC POLY INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR):: $

retract(u:VEC FRAC POLY FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY FLOAT,FEXPR)
  v::$

retractIfCan(u:VEC FRAC POLY FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR):: $

retract(u:VEC EXPR INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(EXPR INT,FEXPR)
  v::$

retractIfCan(u:VEC EXPR INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR):: $

```

```

retract(u:VEC EXPR FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(EXPR FLOAT,FEXPR)
  v::$

retractIfCan(u:VEC EXPR FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC POLY INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(POLY INT,FEXPR)
  v::$

retractIfCan(u:VEC POLY INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC POLY FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(POLY FLOAT,FEXPR)
  v::$

retractIfCan(u:VEC POLY FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

fexpr2expr(u:FEXPR):EXPR MFLOAT ==
  (u::EXPR MFLOAT)$FEXPR

coerce(u:Vector FEXPR):% ==
  v : Vector EXPR MFLOAT
  v:=map(fexpr2expr,u)$VF2(FEXPR,EXPR MFLOAT)
  ([assign(F,v)$FortranCode,returns()$FortranCode]$List(FortranCode))::%

coerce(c:List FortranCode):% == coerce(c)$Rep

coerce(r:RSFC):% == coerce(r)$Rep

coerce(c:FortranCode):% == coerce(c)$Rep

coerce(u:%):OutputForm == coerce(u)$Rep

outputAsFortran(u):Void ==
  p := checkPrecision()$NAGLinkSupportPackage
  outputAsFortran(u)$Rep
  p => restorePrecision()$NAGLinkSupportPackage

```

---

— ASP7.dotabb —

```
"ASP7" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP7"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"ASP7" -> "ALIST"
```

—

## 2.33 domain ASP73 Asp73

— Asp73.input —

```
)set break resume
)sys rm -f Asp73.output
)spool Asp73.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp73
--R Asp73 name: Symbol is a domain constructor
--R Abbreviation for Asp73 is ASP73
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP73
--R
--R----- Operations -----
--R coerce : FortranCode -> %               coerce : List FortranCode -> %
--R coerce : % -> OutputForm               outputAsFortran : % -> Void
--R coerce : Vector FortranExpression([construct,QUOTEX,QUOTEY],[construct],MachineFloat) -> %
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Vector Fraction Polynomial Integer -> %
--R retract : Vector Fraction Polynomial Float -> %
--R retract : Vector Polynomial Integer -> %
--R retract : Vector Polynomial Float -> %
--R retract : Vector Expression Integer -> %
--R retract : Vector Expression Float -> %
--R retractIfCan : Vector Fraction Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Vector Fraction Polynomial Float -> Union(%,"failed")
--R retractIfCan : Vector Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Vector Polynomial Float -> Union(%,"failed")
--R retractIfCan : Vector Expression Integer -> Union(%,"failed")
--R retractIfCan : Vector Expression Float -> Union(%,"failed")
--R
--E 1
```



```
)spool
)lisp (bye)
```

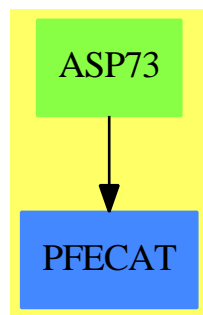
---

— Asp73.help —

```
=====
Asp73 examples
=====
```

```
See Also:
o )show Asp73
```

### 2.33.1 Asp73 (ASP73)



#### Exports:

```
coerce outputAsFortran retract retractIfCan
```

---

— domain ASP73 Asp73 —

```
)abbrev domain ASP73 Asp73
++ Author: Mike Dewar, Grant Keady and Godfrey Nolan
++ Date Created: Mar 1993
++ Date Last Updated: 30 March 1994, 6 October 1994
++ Related Constructors: FortranVectorFunctionCategory, FortranProgramCategory
++ Description:
++ \spadtype{Asp73} produces Fortran for Type 73 ASPs, needed for NAG routine
++ d03eef, for example:
++
```

```

++ \tab{5}SUBROUTINE PDEF(X,Y,ALPHA,BETA,GAMMA,DELTA,EPSOLN,PHI,PSI)\br
++ \tab{5}DOUBLE PRECISION ALPHA,EPSOLN,PHI,X,Y,BETA,DELTA,GAMMA,PSI\br
++ \tab{5}ALPHA=DSIN(X)\br
++ \tab{5}BETA=Y\br
++ \tab{5}GAMMA=X*Y\br
++ \tab{5}DELTA=DCOS(X)*DSIN(Y)\br
++ \tab{5}EPSOLN=Y+X\br
++ \tab{5}PHI=X\br
++ \tab{5}PSI=Y\br
++ \tab{5}RETURN\br
++ \tab{5}END

```

```

Asp73(name): Exports == Implementation where
  name : Symbol

```

```

FST  ==> FortranScalarType
FSTU ==> Union(fst:FST,void:"void")
FEXPR ==> FortranExpression(['X','Y'],[],MachineFloat)
FT    ==> FortranType
SYMTAB ==> SymbolTable
RSFC  ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
FRAC  ==> Fraction
POLY  ==> Polynomial
EXPR  ==> Expression
INT    ==> Integer
FLOAT ==> Float
VEC    ==> Vector
VF2    ==> VectorFunctions2

```

```

Exports ==> FortranVectorFunctionCategory with
  coerce : VEC FEXPR -> $
    ++coerce(f) takes objects from the appropriate instantiation of
    ++\spadtype{FortranExpression} and turns them into an ASP.

```

```

Implementation ==> add

```

```

syms : SYMTAB := empty()$SYMTAB
declare!(X,fortranReal(),syms) $SYMTAB
declare!(Y,fortranReal(),syms) $SYMTAB
declare!(ALPHA,fortranReal(),syms)$SYMTAB
declare!(BETA,fortranReal(),syms) $SYMTAB
declare!(GAMMA,fortranReal(),syms) $SYMTAB
declare!(DELTA,fortranReal(),syms) $SYMTAB
declare!(EPSOLN,fortranReal(),syms) $SYMTAB
declare!(PHI,fortranReal(),syms) $SYMTAB
declare!(PSI,fortranReal(),syms) $SYMTAB
Rep := FortranProgram(name,["void"]$FSTU,
                        [X,Y,ALPHA,BETA,GAMMA,DELTA,EPSOLN,PHI,PSI],syms)

```

```

-- To help the poor compiler!

```

```

localAssign(u:Symbol,v:FEXPR):FortranCode ==
  assign(u,(v::EXPR MachineFloat)$FEXPR)$FortranCode

coerce(u:VEC FEXPR):$ ==
  maxIndex(u) ^= 7 => error "Vector is not of dimension 7"
  [localAssign(ALPHA@Symbol,elt(u,1)),_
   localAssign(BETA@Symbol,elt(u,2)),_
   localAssign(GAMMA@Symbol,elt(u,3)),_
   localAssign(DELTA@Symbol,elt(u,4)),_
   localAssign(EPSOLN@Symbol,elt(u,5)),_
   localAssign(PHI@Symbol,elt(u,6)),_
   localAssign(PSI@Symbol,elt(u,7)),_
   returns()$FortranCode]$List(FortranCode)::$

coerce(c:FortranCode):$ == coerce(c)$Rep

coerce(r:RSFC):$ == coerce(r)$Rep

coerce(c:List FortranCode):$ == coerce(c)$Rep

coerce(u:$):OutputForm == coerce(u)$Rep

outputAsFortran(u):Void ==
  p := checkPrecision()$NAGLinkSupportPackage
  outputAsFortran(u)$Rep
  p => restorePrecision()$NAGLinkSupportPackage

retract(u:VEC FRAC POLY INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY INT,FEXPR)
  v::$

retractIfCan(u:VEC FRAC POLY INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC FRAC POLY FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY FLOAT,FEXPR)
  v::$

retractIfCan(u:VEC FRAC POLY FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC EXPR INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(EXPR INT,FEXPR)
  v::$

retractIfCan(u:VEC EXPR INT):Union($,"failed") ==

```

```

v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR INT,FEXPR)
v case "failed" => "failed"
(v::VEC FEXPR):: $

retract(u:VEC EXPR FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(EXPR FLOAT,FEXPR)
  v:: $

retractIfCan(u:VEC EXPR FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR):: $

retract(u:VEC POLY INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(POLY INT,FEXPR)
  v:: $

retractIfCan(u:VEC POLY INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR):: $

retract(u:VEC POLY FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(POLY FLOAT,FEXPR)
  v:: $

retractIfCan(u:VEC POLY FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR):: $

```

---

— ASP73.dotabb —

```

"ASP73" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP73"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"ASP73" -> "PFECAT"

```

---

## 2.34 domain ASP74 Asp74

— Asp74.input —

```

)set break resume
)sys rm -f Asp74.output
)spool Asp74.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp74
--R Asp74 name: Symbol is a domain constructor
--R Abbreviation for Asp74 is ASP74
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP74
--R
--R----- Operations -----
--R coerce : FortranCode -> %               coerce : List FortranCode -> %
--R coerce : % -> OutputForm               outputAsFortran : % -> Void
--R coerce : Matrix FortranExpression([construct,QUOTEX,QUOTEY],[construct],MachineFloat) ->
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Matrix Fraction Polynomial Integer -> %
--R retract : Matrix Fraction Polynomial Float -> %
--R retract : Matrix Polynomial Integer -> %
--R retract : Matrix Polynomial Float -> %
--R retract : Matrix Expression Integer -> %
--R retract : Matrix Expression Float -> %
--R retractIfCan : Matrix Fraction Polynomial Integer -> Union(%, "failed")
--R retractIfCan : Matrix Fraction Polynomial Float -> Union(%, "failed")
--R retractIfCan : Matrix Polynomial Integer -> Union(%, "failed")
--R retractIfCan : Matrix Polynomial Float -> Union(%, "failed")
--R retractIfCan : Matrix Expression Integer -> Union(%, "failed")
--R retractIfCan : Matrix Expression Float -> Union(%, "failed")
--R
--E 1

)spool
)lisp (bye)

```

---

— Asp74.help —

```

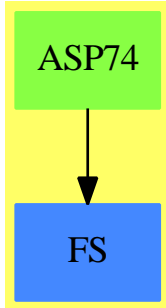
=====
Asp74 examples
=====

```

See Also:

- o )show Asp74

## 2.34.1 Asp74 (ASP74)

**Exports:**

coerce outputAsFortran retract retractIfCan

— domain ASP74 Asp74 —

```

)abbrev domain ASP74 Asp74
++ Author: Mike Dewar and Godfrey Nolan
++ Date Created: Oct 1993
++ Date Last Updated: 30 March 1994
++               6 October 1994
++ Related Constructors: FortranScalarFunctionCategory, FortranProgramCategory.
++ Description:
++ \spadtype{Asp74} produces Fortran for Type 74 ASPs, needed for NAG routine
++ d03eef, for example:
++
++ \tab{5} SUBROUTINE BNDY(X,Y,A,B,C,IBND)\br
++ \tab{5} DOUBLE PRECISION A,B,C,X,Y\br
++ \tab{5} INTEGER IBND\br
++ \tab{5} IF (IBND.EQ.0) THEN\br
++ \tab{7} A=0.0D0\br
++ \tab{7} B=1.0D0\br
++ \tab{7} C=-1.0D0*DSIN(X)\br
++ \tab{5} ELSEIF (IBND.EQ.1) THEN\br
++ \tab{7} A=1.0D0\br
++ \tab{7} B=0.0D0\br
++ \tab{7} C=DSIN(X)*DSIN(Y)\br
++ \tab{5} ELSEIF (IBND.EQ.2) THEN\br
++ \tab{7} A=1.0D0\br
++ \tab{7} B=0.0D0\br
++ \tab{7} C=DSIN(X)*DSIN(Y)\br
++ \tab{5} ELSEIF (IBND.EQ.3) THEN\br
++ \tab{7} A=0.0D0\br
++ \tab{7} B=1.0D0\br
++ \tab{7} C=-1.0D0*DSIN(Y)\br
++ \tab{5} ENDIF\br
  
```

```

++ \tab{5} END

Asp74(name): Exports == Implementation where
  name : Symbol

  FST    ==> FortranScalarType
  FSTU   ==> Union(fst:FST,void:"void")
  FT     ==> FortranType
  SYMTAB ==> SymbolTable
  FC     ==> FortranCode
  PI     ==> PositiveInteger
  RSFC   ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
  FRAC   ==> Fraction
  POLY   ==> Polynomial
  EXPR   ==> Expression
  INT    ==> Integer
  FLOAT  ==> Float
  MFLOAT ==> MachineFloat
  FEXPR  ==> FortranExpression(['X','Y'],[],MFLOAT)
  U      ==> Union(I: Expression Integer,F: Expression Float,_
                  CF: Expression Complex Float,switch:Switch)
  VEC    ==> Vector
  MAT    ==> Matrix
  M2     ==> MatrixCategoryFunctions2
  MF2a   ==> M2(FRAC POLY INT,VEC FRAC POLY INT,VEC FRAC POLY INT,
               MAT FRAC POLY INT, FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)
  MF2b   ==> M2(FRAC POLY FLOAT,VEC FRAC POLY FLOAT,VEC FRAC POLY FLOAT,
               MAT FRAC POLY FLOAT, FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)
  MF2c   ==> M2(POLY INT,VEC POLY INT,VEC POLY INT,MAT POLY INT,
               FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)
  MF2d   ==> M2(POLY FLOAT,VEC POLY FLOAT,VEC POLY FLOAT,
               MAT POLY FLOAT, FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)
  MF2e   ==> M2(EXPR INT,VEC EXPR INT,VEC EXPR INT,MAT EXPR INT,
               FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)
  MF2f   ==> M2(EXPR FLOAT,VEC EXPR FLOAT,VEC EXPR FLOAT,
               MAT EXPR FLOAT, FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)

Exports ==> FortranMatrixFunctionCategory with
  coerce : MAT FEXPR -> $
  ++coerce(f) takes objects from the appropriate instantiation of
  ++\spadtype{FortranExpression} and turns them into an ASP.

Implementation ==> add

  syms : SYMTAB := empty()$SYMTAB
  declare!(X,fortranReal(),syms)$SYMTAB
  declare!(Y,fortranReal(),syms)$SYMTAB
  declare!(A,fortranReal(),syms)$SYMTAB
  declare!(B,fortranReal(),syms)$SYMTAB
  declare!(C,fortranReal(),syms)$SYMTAB

```

```

declare!(IBND,fortranInteger(),syms)$SYMTAB
Rep := FortranProgram(name,["void"]$FSTU,[X,Y,A,B,C,IBND],syms)

-- To help the poor compiler!
localAssign(u:Symbol,v:FEXPR):FC == assign(u,(v::EXPR MFLOAT)$FEXPR)$FC

coerce(u:MAT FEXPR):$ ==
  (nrows(u) ^= 4 or ncols(u) ^= 3) => error "Not a 4X3 matrix"
  flag:U := [IBND@Symbol::EXPR INT]$U
  pt0:U := [0::EXPR INT]$U
  pt1:U := [1::EXPR INT]$U
  pt2:U := [2::EXPR INT]$U
  pt3:U := [3::EXPR INT]$U
  sw1: Switch := EQ(flag,pt0)$Switch
  sw2: Switch := EQ(flag,pt1)$Switch
  sw3: Switch := EQ(flag,pt2)$Switch
  sw4: Switch := EQ(flag,pt3)$Switch
  a11 : FC := localAssign(A,u(1,1))
  a12 : FC := localAssign(B,u(1,2))
  a13 : FC := localAssign(C,u(1,3))
  a21 : FC := localAssign(A,u(2,1))
  a22 : FC := localAssign(B,u(2,2))
  a23 : FC := localAssign(C,u(2,3))
  a31 : FC := localAssign(A,u(3,1))
  a32 : FC := localAssign(B,u(3,2))
  a33 : FC := localAssign(C,u(3,3))
  a41 : FC := localAssign(A,u(4,1))
  a42 : FC := localAssign(B,u(4,2))
  a43 : FC := localAssign(C,u(4,3))
  c : FC := cond(sw1,block([a11,a12,a13])$FC,
    cond(sw2,block([a21,a22,a23])$FC,
      cond(sw3,block([a31,a32,a33])$FC,
        cond(sw4,block([a41,a42,a43])$FC)$FC)$FC)$FC)$FC
  c::$

coerce(u:$):OutputForm == coerce(u)$Rep

coerce(c:FortranCode):$ == coerce(c)$Rep

coerce(r:RSFC):$ == coerce(r)$Rep

coerce(c:List FortranCode):$ == coerce(c)$Rep

outputAsFortran(u):Void ==
  p := checkPrecision()$NAGLinkSupportPackage
  outputAsFortran(u)$Rep
  p => restorePrecision()$NAGLinkSupportPackage

retract(u:MAT FRAC POLY INT):$ ==
  v : MAT FEXPR := map(retract,u)$MF2a

```



```

v::$

retractIfCan(u:MAT FRAC POLY INT):Union($,"failed") ==
  v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2a
  v case "failed" => "failed"
  (v::MAT FEXPR)::$

retract(u:MAT FRAC POLY FLOAT):$ ==
  v : MAT FEXPR := map(retract,u)$MF2b
  v::$

retractIfCan(u:MAT FRAC POLY FLOAT):Union($,"failed") ==
  v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2b
  v case "failed" => "failed"
  (v::MAT FEXPR)::$

retract(u:MAT EXPR INT):$ ==
  v : MAT FEXPR := map(retract,u)$MF2e
  v::$

retractIfCan(u:MAT EXPR INT):Union($,"failed") ==
  v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2e
  v case "failed" => "failed"
  (v::MAT FEXPR)::$

retract(u:MAT EXPR FLOAT):$ ==
  v : MAT FEXPR := map(retract,u)$MF2f
  v::$

retractIfCan(u:MAT EXPR FLOAT):Union($,"failed") ==
  v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2f
  v case "failed" => "failed"
  (v::MAT FEXPR)::$

retract(u:MAT POLY INT):$ ==
  v : MAT FEXPR := map(retract,u)$MF2c
  v::$

retractIfCan(u:MAT POLY INT):Union($,"failed") ==
  v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2c
  v case "failed" => "failed"
  (v::MAT FEXPR)::$

retract(u:MAT POLY FLOAT):$ ==
  v : MAT FEXPR := map(retract,u)$MF2d
  v::$

retractIfCan(u:MAT POLY FLOAT):Union($,"failed") ==
  v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2d
  v case "failed" => "failed"

```

```
(v:MAT FEXPR)::
```

---

— ASP74.dotabb —

```
"ASP74" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP74"]
"FS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FS"]
"ASP74" -> "FS"
```

---

## 2.35 domain ASP77 Asp77

— Asp77.input —

```
)set break resume
)sys rm -f Asp77.output
)spool Asp77.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp77
--R Asp77 name: Symbol is a domain constructor
--R Abbreviation for Asp77 is ASP77
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP77
--R
--R----- Operations -----
--R coerce : FortranCode -> %               coerce : List FortranCode -> %
--R coerce : % -> OutputForm               outputAsFortran : % -> Void
--R coerce : Matrix FortranExpression([construct,QUOTEX],[construct],MachineFloat) -> %
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Matrix Fraction Polynomial Integer -> %
--R retract : Matrix Fraction Polynomial Float -> %
--R retract : Matrix Polynomial Integer -> %
--R retract : Matrix Polynomial Float -> %
--R retract : Matrix Expression Integer -> %
--R retract : Matrix Expression Float -> %
--R retractIfCan : Matrix Fraction Polynomial Integer -> Union(%,"failed")
--R retractIfCan : Matrix Fraction Polynomial Float -> Union(%,"failed")
--R retractIfCan : Matrix Polynomial Integer -> Union(%,"failed")
```

```

--R retractIfCan : Matrix Polynomial Float -> Union(%,"failed")
--R retractIfCan : Matrix Expression Integer -> Union(%,"failed")
--R retractIfCan : Matrix Expression Float -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)

```

---

— Asp77.help —

```

=====
Asp77 examples
=====

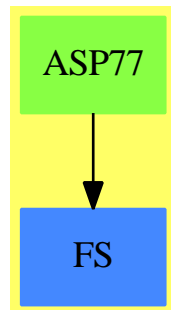
```

```

See Also:
o )show Asp77

```

### 2.35.1 Asp77 (ASP77)



#### Exports:

```

coerce  outputAsFortran  retract  retractIfCan

```

---

— domain ASP77 Asp77 —

```

)abbrev domain ASP77 Asp77
++ Author: Mike Dewar, Grant Keady and Godfrey Nolan
++ Date Created: Mar 1993
++ Date Last Updated: 30 March 1994

```

```

++                               6 October 1994
++ Related Constructors: FortranMatrixFunctionCategory, FortranProgramCategory
++ Description:
++ \spadtype{Asp77} produces Fortran for Type 77 ASPs, needed for NAG routine
++ d02gbf, for example:
++
++ \tab{5}SUBROUTINE FCNF(X,F)\br
++ \tab{5}DOUBLE PRECISION X\br
++ \tab{5}DOUBLE PRECISION F(2,2)\br
++ \tab{5}F(1,1)=0.0D0\br
++ \tab{5}F(1,2)=1.0D0\br
++ \tab{5}F(2,1)=0.0D0\br
++ \tab{5}F(2,2)=-10.0D0\br
++ \tab{5}RETURN\br
++ \tab{5}END

```

```

Asp77(name): Exports == Implementation where
name : Symbol

```

```

FST    ==> FortranScalarType
FSTU   ==> Union(fst:FST,void:"void")
FT     ==> FortranType
FC     ==> FortranCode
SYMTAB ==> SymbolTable
RSFC   ==> Record(localSymbols:SymbolTable,code:List(FC))
FRAC   ==> Fraction
POLY   ==> Polynomial
EXPR   ==> Expression
INT    ==> Integer
FLOAT  ==> Float
MFLOAT ==> MachineFloat
FEXPR  ==> FortranExpression(['X'],[],MFLOAT)
VEC    ==> Vector
MAT    ==> Matrix
M2     ==> MatrixCategoryFunctions2
MF2    ==> M2(FEXPR,VEC FEXPR,VEC FEXPR,Matrix FEXPR,EXPR MFLOAT,
             VEC EXPR MFLOAT,VEC EXPR MFLOAT,Matrix EXPR MFLOAT)
MF2a   ==> M2(FRAC POLY INT,VEC FRAC POLY INT,VEC FRAC POLY INT,
             MAT FRAC POLY INT, FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)
MF2b   ==> M2(FRAC POLY FLOAT,VEC FRAC POLY FLOAT,VEC FRAC POLY FLOAT,
             MAT FRAC POLY FLOAT, FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)
MF2c   ==> M2(POLY INT,VEC POLY INT,VEC POLY INT,MAT POLY INT,
             FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)
MF2d   ==> M2(POLY FLOAT,VEC POLY FLOAT,VEC POLY FLOAT,
             MAT POLY FLOAT, FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)
MF2e   ==> M2(EXPR INT,VEC EXPR INT,VEC EXPR INT,MAT EXPR INT,
             FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)
MF2f   ==> M2(EXPR FLOAT,VEC EXPR FLOAT,VEC EXPR FLOAT,
             MAT EXPR FLOAT, FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)

```

```

Exports ==> FortranMatrixFunctionCategory with
  coerce : MAT FEXPR -> $
    ++coerce(f) takes objects from the appropriate instantiation of
    ++\spadtype{FortranExpression} and turns them into an ASP.

Implementation ==> add

real : FSTU := ["real":FST]$FSTU
syms : SYMTAB := empty()$SYMTAB
declare!(X,fortranReal(),syms)$SYMTAB
Rep := FortranProgram(name,["void"]$FSTU,[X,F],syms)

fexpr2expr(u:FEXPR):EXPR MFLOAT == coerce(u)$FEXPR

localAssign(s:Symbol,j:MAT FEXPR):FortranCode ==
  j' : MAT EXPR MFLOAT := map(fexpr2expr,j)$MF2
  assign(s,j')$FortranCode

coerce(u:MAT FEXPR):$ ==
  dimension := nrows(u)::POLY(INT)
  locals : SYMTAB := empty()
  declare!(F,[real,[dimension,dimension]$List(POLY(INT)),false]$FT,locals)
  code : List FC := [localAssign(F,u),returns()$FC]
  ([locals,code]$RSFC)::$

coerce(c:List FC):$ == coerce(c)$Rep

coerce(r:RSFC):$ == coerce(r)$Rep

coerce(c:FC):$ == coerce(c)$Rep

coerce(u:$):OutputForm == coerce(u)$Rep

outputAsFortran(u):Void ==
  p := checkPrecision()$NAGLinkSupportPackage
  outputAsFortran(u)$Rep
  p => restorePrecision()$NAGLinkSupportPackage

retract(u:MAT FRAC POLY INT):$ ==
  v : MAT FEXPR := map(retract,u)$MF2a
  v::$

retractIfCan(u:MAT FRAC POLY INT):Union($,"failed") ==
  v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2a
  v case "failed" => "failed"
  (v::MAT FEXPR)::$

retract(u:MAT FRAC POLY FLOAT):$ ==
  v : MAT FEXPR := map(retract,u)$MF2b

```

```

v::$

retractIfCan(u:MAT FRAC POLY FLOAT):Union($,"failed") ==
  v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2b
  v case "failed" => "failed"
  (v::MAT FEXPR):: $

retract(u:MAT EXPR INT):$ ==
  v : MAT FEXPR := map(retract,u)$MF2e
  v::$

retractIfCan(u:MAT EXPR INT):Union($,"failed") ==
  v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2e
  v case "failed" => "failed"
  (v::MAT FEXPR):: $

retract(u:MAT EXPR FLOAT):$ ==
  v : MAT FEXPR := map(retract,u)$MF2f
  v::$

retractIfCan(u:MAT EXPR FLOAT):Union($,"failed") ==
  v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2f
  v case "failed" => "failed"
  (v::MAT FEXPR):: $

retract(u:MAT POLY INT):$ ==
  v : MAT FEXPR := map(retract,u)$MF2c
  v::$

retractIfCan(u:MAT POLY INT):Union($,"failed") ==
  v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2c
  v case "failed" => "failed"
  (v::MAT FEXPR):: $

retract(u:MAT POLY FLOAT):$ ==
  v : MAT FEXPR := map(retract,u)$MF2d
  v::$

retractIfCan(u:MAT POLY FLOAT):Union($,"failed") ==
  v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2d
  v case "failed" => "failed"
  (v::MAT FEXPR):: $

```

---

— ASP77.dotabb —

"ASP77" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP77"]

```
"FS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FS"]
"ASP77" -> "FS"
```

## 2.36 domain ASP78 Asp78

— Asp78.input —

```
)set break resume
)sys rm -f Asp78.output
)spool Asp78.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp78
--R Asp78 name: Symbol is a domain constructor
--R Abbreviation for Asp78 is ASP78
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP78
--R
--R----- Operations -----
--R coerce : FortranCode -> %               coerce : List FortranCode -> %
--R coerce : % -> OutputForm               outputAsFortran : % -> Void
--R coerce : Vector FortranExpression([construct,QUOTEX],[construct],MachineFloat) -> %
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Vector Fraction Polynomial Integer -> %
--R retract : Vector Fraction Polynomial Float -> %
--R retract : Vector Polynomial Integer -> %
--R retract : Vector Polynomial Float -> %
--R retract : Vector Expression Integer -> %
--R retract : Vector Expression Float -> %
--R retractIfCan : Vector Fraction Polynomial Integer -> Union(%, "failed")
--R retractIfCan : Vector Fraction Polynomial Float -> Union(%, "failed")
--R retractIfCan : Vector Polynomial Integer -> Union(%, "failed")
--R retractIfCan : Vector Polynomial Float -> Union(%, "failed")
--R retractIfCan : Vector Expression Integer -> Union(%, "failed")
--R retractIfCan : Vector Expression Float -> Union(%, "failed")
--R
--E 1

)spool
)lisp (bye)
```

```

-----

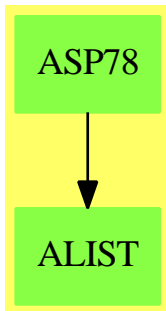
— Asp78.help —

=====
Asp78 examples
=====

See Also:
o )show Asp78

```

### 2.36.1 Asp78 (ASP78)



#### Exports:

```
coerce  outputAsFortran  retract  retractIfCan
```

— domain **ASP78** **Asp78** —

```

)abbrev domain ASP78 Asp78
++ Author: Mike Dewar, Grant Keady and Godfrey Nolan
++ Date Created: Mar 1993
++ Date Last Updated: 30 March 1994
++                               6 October 1994
++ Related Constructors: FortranVectorFunctionCategory, FortranProgramCategory
++ Description:
++ \spadtype{Asp78} produces Fortran for Type 78 ASPs, needed for NAG routine
++ d02gbf, for example:
++
++ \tab{5}SUBROUTINE FCNG(X,G)\br
++ \tab{5}DOUBLE PRECISION G(*),X\br
++ \tab{5}G(1)=0.0D0\br
++ \tab{5}G(2)=0.0D0\br

```



```

++ \tab{5}END

Asp78(name): Exports == Implementation where
  name : Symbol

  FST    ==> FortranScalarType
  FSTU   ==> Union(fst:FST,void:"void")
  FT     ==> FortranType
  FC     ==> FortranCode
  SYMTAB ==> SymbolTable
  RSFC   ==> Record(localSymbols:SymbolTable,code:List(FC))
  FRAC   ==> Fraction
  POLY   ==> Polynomial
  EXPR   ==> Expression
  INT    ==> Integer
  FLOAT  ==> Float
  VEC    ==> Vector
  VF2    ==> VectorFunctions2
  MFLOAT ==> MachineFloat
  FEXPR  ==> FortranExpression(['X'],[],MFLOAT)

Exports ==> FortranVectorFunctionCategory with
  coerce : VEC FEXPR -> $
  ++coerce(f) takes objects from the appropriate instantiation of
  ++\spadtype{FortranExpression} and turns them into an ASP.

Implementation ==> add

  real : FSTU := ["real":FST]$FSTU
  syms : SYMTAB := empty()$SYMTAB
  declare!(X,fortranReal(),syms)$SYMTAB
  gType : FT := construct(real,["*":Symbol],false)$FT
  declare!(G,gType,syms)$SYMTAB
  Rep := FortranProgram(name,["void"]$FSTU,[X,G],syms)

  fexpr2expr(u:FEXPR):EXPR MFLOAT == coerce(u)$FEXPR

  coerce(u:VEC FEXPR):$ ==
    u' : VEC EXPR MFLOAT := map(fexpr2expr,u)$VF2(FEXPR,EXPR MFLOAT)
    (assign(G,u')$FC):: $

  coerce(u:$):OutputForm == coerce(u)$Rep

  outputAsFortran(u):Void ==
    p := checkPrecision()$NAGLinkSupportPackage
    outputAsFortran(u)$Rep
    p => restorePrecision()$NAGLinkSupportPackage

  coerce(c:List FC):$ == coerce(c)$Rep

```

```

coerce(r:RSFC):$ == coerce(r)$Rep

coerce(c:FC):$ == coerce(c)$Rep

retract(u:VEC FRAC POLY INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY INT,FEXPR)
  v::$

retractIfCan(u:VEC FRAC POLY INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC FRAC POLY FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(FRAC POLY FLOAT,FEXPR)
  v::$

retractIfCan(u:VEC FRAC POLY FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(FRAC POLY FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC EXPR INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(EXPR INT,FEXPR)
  v::$

retractIfCan(u:VEC EXPR INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC EXPR FLOAT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(EXPR FLOAT,FEXPR)
  v::$

retractIfCan(u:VEC EXPR FLOAT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(EXPR FLOAT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC POLY INT):$ ==
  v : VEC FEXPR := map(retract,u)$VF2(POLY INT,FEXPR)
  v::$

retractIfCan(u:VEC POLY INT):Union($,"failed") ==
  v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY INT,FEXPR)
  v case "failed" => "failed"
  (v::VEC FEXPR)::$

retract(u:VEC POLY FLOAT):$ ==

```

```

v : VEC FEXPR := map(retract,u)$VF2(POLY FLOAT,FEXPR)
v::$

retractIfCan(u:VEC POLY FLOAT):Union($,"failed") ==
v:Union(VEC FEXPR,"failed"):=map(retractIfCan,u)$VF2(POLY FLOAT,FEXPR)
v case "failed" => "failed"
(v::VEC FEXPR):: $

```

---

— ASP78.dotabb —

```

"ASP78" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP78"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"ASP78" -> "ALIST"

```

---

## 2.37 domain ASP8 Asp8

---

— Asp8.input —

```

)set break resume
)sys rm -f Asp8.output
)spool Asp8.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp8
--R Asp8 name: Symbol is a domain constructor
--R Abbreviation for Asp8 is ASP8
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP8
--R
--R----- Operations -----
--R coerce : FortranCode -> %          coerce : List FortranCode -> %
--R coerce : Vector MachineFloat -> %  coerce : % -> OutputForm
--R outputAsFortran : % -> Void
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R
--E 1

```

```
)spool
)lisp (bye)
```

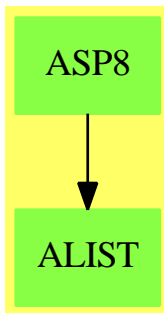
---

— Asp8.help —

```
=====
Asp8 examples
=====
```

```
See Also:
o )show Asp8
```

### 2.37.1 Asp8 (ASP8)



**Exports:**  
 coerce   outputAsFortran

---

— domain ASP8 Asp8 —

```
)abbrev domain ASP8 Asp8
++ Author: Godfrey Nolan and Mike Dewar
++ Date Created: 11 February 1994
++ Date Last Updated: 18 March 1994
++           31 May 1994 to use alternative interface. MCD
++           30 June 1994 to handle the end condition correctly. MCD
++           6 October 1994
++ Related Constructors: FortranVectorFunctionCategory, FortranProgramCategory
++ Description:
++ \spadtype{Asp8} produces Fortran for Type 8 ASPs, needed for NAG routine
```

++ d02bbf. This ASP prints intermediate values of the computed solution of  
++ an ODE and might look like:

```
++
++ \tab{5}SUBROUTINE OUTPUT(XSOL,Y,COUNT,M,N,RESULT,FORWRD)\br
++ \tab{5}DOUBLE PRECISION Y(N),RESULT(M,N),XSOL\br
++ \tab{5}INTEGER M,N,COUNT\br
++ \tab{5}LOGICAL FORWRD\br
++ \tab{5}DOUBLE PRECISION X02ALF,POINTS(8)\br
++ \tab{5}EXTERNAL X02ALF\br
++ \tab{5}INTEGER I\br
++ \tab{5}POINTS(1)=1.0D0\br
++ \tab{5}POINTS(2)=2.0D0\br
++ \tab{5}POINTS(3)=3.0D0\br
++ \tab{5}POINTS(4)=4.0D0\br
++ \tab{5}POINTS(5)=5.0D0\br
++ \tab{5}POINTS(6)=6.0D0\br
++ \tab{5}POINTS(7)=7.0D0\br
++ \tab{5}POINTS(8)=8.0D0\br
++ \tab{5}COUNT=COUNT+1\br
++ \tab{5}DO 25001 I=1,N\br
++ \tab{7} RESULT(COUNT,I)=Y(I)\br
++ 25001 CONTINUE\br
++ \tab{5}IF(COUNT.EQ.M)THEN\br
++ \tab{7}IF(FORWRD)THEN\br
++ \tab{9}XSOL=X02ALF()\br
++ \tab{7}ELSE\br
++ \tab{9}XSOL=-X02ALF()\br
++ \tab{7}ENDIF\br
++ \tab{5}ELSE\br
++ \tab{7} XSOL=POINTS(COUNT)\br
++ \tab{5}ENDIF\br
++ \tab{5}END
```

Asp8(name): Exports == Implementation where

name : Symbol

```
O      ==> OutputForm
S      ==> Symbol
FST    ==> FortranScalarType
UFST   ==> Union(fst:FST,void:"void")
FT     ==> FortranType
FC     ==> FortranCode
SYMTAB ==> SymbolTable
RSFC   ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
EX     ==> Expression Integer
MFLOAT ==> MachineFloat
EXPR   ==> Expression
PI     ==> Polynomial Integer
EXU    ==> Union(I: EXPR Integer,F: EXPR Float,CF: EXPR Complex Float,
                switch: Switch)
```

Exports ==> FortranVectorCategory

Implementation ==> add

```

real : UFST := ["real"::FST]$UFST
syms : SYMTAB := empty()$SYMTAB
declare!([COUNT,M,N],fortranInteger(),syms)$SYMTAB
declare!(XSOL,fortranReal(),syms)$SYMTAB
yType : FT := construct(real,[N],false)$FT
declare!(Y,yType,syms)$SYMTAB
declare!(FORWRD,fortranLogical(),syms)$SYMTAB
declare!(RESULT,construct(real,[M,N],false)$FT,syms)$SYMTAB
Rep := _
  FortranProgram(name,["void"]$UFST,[XSOL,Y,COUNT,M,N,RESULT,FORWRD],syms)

coerce(c:List FC):% == coerce(c)$Rep

coerce(r:RSFC):% == coerce(r)$Rep

coerce(c:FC):% == coerce(c)$Rep

coerce(u:%):0 == coerce(u)$Rep

outputAsFortran(u:%):Void ==
  p := checkPrecision()$NAGLinkSupportPackage
  outputAsFortran(u)$Rep
  p => restorePrecision()$NAGLinkSupportPackage

f2ex(u:MFLOAT):EXPR MFLOAT == (u::EXPR MFLOAT)$EXPR(MFLOAT)

coerce(points:Vector MFLOAT):% ==
  import PI
  import EXPR Integer
  -- Create some extra declarations
  locals : SYMTAB := empty()$SYMTAB
  nPol : PI := "N"::S::PI
  iPol : PI := "I"::S::PI
  countPol : PI := "COUNT"::S::PI
  pointsDim : PI := max(#points,1)::PI
  declare!(POINTS,[real,[pointsDim],false]$FT,locals)$SYMTAB
  declare!(XO2ALF,[real,[],true]$FT,locals)$SYMTAB
  -- Now build up the code fragments
  index : SegmentBinding PI := equation(I@S,1::PI..nPol)$SegmentBinding(PI)
  ySym : EX := (subscript("Y"::S,[I::0])$S)::EX
  loop := forLoop(index,assign(RESULT,[countPol,iPol],ySym)$FC)$FC
  v:Vector EXPR MFLOAT
  v := map(f2ex,points)$VectorFunctions2(MFLOAT,EXPR MFLOAT)
  assign1 : FC := assign(POINTS,v)$FC

```

```

countExp: EX := COUNT@S::EX
newValue: EX := 1 + countExp
assign2 : FC := assign(COUNT,newValue)$FC
newSymbol : S := subscript(POINTS,[COUNT]@List(0))$S
assign3 : FC := assign(XSOL, newSymbol::EX )$FC
fphuge : EX := kernel(operator X02ALF,empty())$List(EX))
assign4 : FC := assign(XSOL, fphuge)$FC
assign5 : FC := assign(XSOL, -fphuge)$FC
innerCond : FC := cond("FORWRD"::Symbol::Switch,assign4,assign5)
mExp : EX := M@S::EX
endCase : FC := cond(EQ([countExp]$EXU,[mExp]$EXU)$Switch,innerCond,assign3)
code := [assign1, assign2, loop, endCase]$List(FC)
([locals,code]$RSFC)::%

```

---

— ASP8.dotabb —

```

"ASP8" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP8"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"ASP8" -> "ALIST"

```

---

## 2.38 domain ASP80 Asp80

— Asp80.input —

```

)set break resume
)sys rm -f Asp80.output
)spool Asp80.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp80
--R Asp80 name: Symbol is a domain constructor
--R Abbreviation for Asp80 is ASP80
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP80
--R
--R----- Operations -----
--R coerce : FortranCode -> %          coerce : List FortranCode -> %

```

```

--R coerce : % -> OutputForm          outputAsFortran : % -> Void
--R coerce : Matrix FortranExpression([construct,QUOTEXL,QUOTEXR,QUOTEELAM],[construct],MachineFloat) -> %
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Matrix Fraction Polynomial Integer -> %
--R retract : Matrix Fraction Polynomial Float -> %
--R retract : Matrix Polynomial Integer -> %
--R retract : Matrix Polynomial Float -> %
--R retract : Matrix Expression Integer -> %
--R retract : Matrix Expression Float -> %
--R retractIfCan : Matrix Fraction Polynomial Integer -> Union(%, "failed")
--R retractIfCan : Matrix Fraction Polynomial Float -> Union(%, "failed")
--R retractIfCan : Matrix Polynomial Integer -> Union(%, "failed")
--R retractIfCan : Matrix Polynomial Float -> Union(%, "failed")
--R retractIfCan : Matrix Expression Integer -> Union(%, "failed")
--R retractIfCan : Matrix Expression Float -> Union(%, "failed")
--R
--E 1

```

```

)spool
)lisp (bye)

```

\_\_\_\_\_

— Asp80.help —

```

=====
Asp80 examples
=====

```

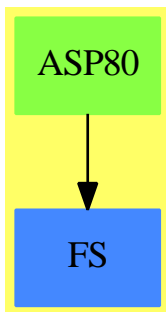
```

See Also:
o )show Asp80

```

\_\_\_\_\_

### 2.38.1 Asp80 (ASP80)





**Exports:**

```
coerce  outputAsFortran  retract  retractIfCan
```

## — domain ASP80 Asp80 —

```
)abbrev domain ASP80 Asp80
++ Author: Mike Dewar and Godfrey Nolan
++ Date Created: Oct 1993
++ Date Last Updated: 30 March 1994
++           6 October 1994
++ Related Constructors: FortranMatrixFunctionCategory, FortranProgramCategory
++ Description:
++ \spadtype{Asp80} produces Fortran for Type 80 ASPs, needed for NAG routine
++ d02kef, for example:
++
++ \tab{5}SUBROUTINE BDYVAL(XL,XR,ELAM,YL,YR)\br
++ \tab{5}DOUBLE PRECISION ELAM,XL,YL(3),XR,YR(3)\br
++ \tab{5}YL(1)=XL\br
++ \tab{5}YL(2)=2.0D0\br
++ \tab{5}YR(1)=1.0D0\br
++ \tab{5}YR(2)=-1.0D0*DSQRT(XR+(-1.0D0*ELAM))\br
++ \tab{5}RETURN\br
++ \tab{5}END
```

```
Asp80(name): Exports == Implementation where
  name : Symbol
```

```
FST    ==> FortranScalarType
FSTU   ==> Union(fst:FST,void:"void")
FT     ==> FortranType
FC     ==> FortranCode
SYMTAB ==> SymbolTable
RSFC   ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
FRAC   ==> Fraction
POLY   ==> Polynomial
EXPR   ==> Expression
INT     ==> Integer
FLOAT  ==> Float
MFLOAT ==> MachineFloat
FEXPR  ==> FortranExpression(['XL','XR','ELAM'],[],MFLOAT)
VEC     ==> Vector
MAT     ==> Matrix
VF2     ==> VectorFunctions2
M2      ==> MatrixCategoryFunctions2
MF2a    ==> M2(FRAC POLY INT,VEC FRAC POLY INT,VEC FRAC POLY INT,
              MAT FRAC POLY INT, FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)
MF2b    ==> M2(FRAC POLY FLOAT,VEC FRAC POLY FLOAT,VEC FRAC POLY FLOAT,
              MAT FRAC POLY FLOAT, FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)
```

```

MF2c ==> M2(POLY INT,VEC POLY INT,VEC POLY INT,MAT POLY INT,
            FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)
MF2d ==> M2(POLY FLOAT,VEC POLY FLOAT,VEC POLY FLOAT,
            MAT POLY FLOAT, FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)
MF2e ==> M2(EXPR INT,VEC EXPR INT,VEC EXPR INT,MAT EXPR INT,
            FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)
MF2f ==> M2(EXPR FLOAT,VEC EXPR FLOAT,VEC EXPR FLOAT,
            MAT EXPR FLOAT, FEXPR,VEC FEXPR,VEC FEXPR,MAT FEXPR)

Exports ==> FortranMatrixFunctionCategory with
  coerce : MAT FEXPR -> $
    ++coerce(f) takes objects from the appropriate instantiation of
    ++\spadtype{FortranExpression} and turns them into an ASP.

Implementation ==> add

real : FSTU := ["real"::FST]$FSTU
syms : SYMTAB := empty()$SYMTAB
declare!(XL,fortranReal(),syms)$SYMTAB
declare!(XR,fortranReal(),syms)$SYMTAB
declare!(ELAM,fortranReal(),syms)$SYMTAB
yType : FT := construct(real,["3"::Symbol],false)$FT
declare!(YL,yType,syms)$SYMTAB
declare!(YR,yType,syms)$SYMTAB
Rep := FortranProgram(name,["void"]$FSTU, [XL,XR,ELAM,YL,YR],syms)

fexpr2expr(u:FEXPR):EXPR MFLOAT == coerce(u)$FEXPR

vecAssign(s:Symbol,u:VEC FEXPR):FC ==
  u' : VEC EXPR MFLOAT := map(fexpr2expr,u)$VF2(FEXPR,EXPR MFLOAT)
  assign(s,u')$FC

coerce(u:MAT FEXPR):$ ==
  [vecAssign(YL,row(u,1)),vecAssign(YR,row(u,2)),returns()$FC]$List(FC):: $

coerce(c:List FortranCode):$ == coerce(c)$Rep

coerce(r:RSFC):$ == coerce(r)$Rep

coerce(c:FortranCode):$ == coerce(c)$Rep

coerce(u:$):OutputForm == coerce(u)$Rep

outputAsFortran(u):Void ==
  p := checkPrecision()$NAGLinkSupportPackage
  outputAsFortran(u)$Rep
  p => restorePrecision()$NAGLinkSupportPackage

retract(u:MAT FRAC POLY INT):$ ==
  v : MAT FEXPR := map(retract,u)$MF2a

```

```

v::$

retractIfCan(u:MAT FRAC POLY INT):Union($,"failed") ==
  v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2a
  v case "failed" => "failed"
  (v::MAT FEXPR)::$

retract(u:MAT FRAC POLY FLOAT):$ ==
  v : MAT FEXPR := map(retract,u)$MF2b
  v::$

retractIfCan(u:MAT FRAC POLY FLOAT):Union($,"failed") ==
  v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2b
  v case "failed" => "failed"
  (v::MAT FEXPR)::$

retract(u:MAT EXPR INT):$ ==
  v : MAT FEXPR := map(retract,u)$MF2e
  v::$

retractIfCan(u:MAT EXPR INT):Union($,"failed") ==
  v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2e
  v case "failed" => "failed"
  (v::MAT FEXPR)::$

retract(u:MAT EXPR FLOAT):$ ==
  v : MAT FEXPR := map(retract,u)$MF2f
  v::$

retractIfCan(u:MAT EXPR FLOAT):Union($,"failed") ==
  v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2f
  v case "failed" => "failed"
  (v::MAT FEXPR)::$

retract(u:MAT POLY INT):$ ==
  v : MAT FEXPR := map(retract,u)$MF2c
  v::$

retractIfCan(u:MAT POLY INT):Union($,"failed") ==
  v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2c
  v case "failed" => "failed"
  (v::MAT FEXPR)::$

retract(u:MAT POLY FLOAT):$ ==
  v : MAT FEXPR := map(retract,u)$MF2d
  v::$

retractIfCan(u:MAT POLY FLOAT):Union($,"failed") ==
  v:Union(MAT FEXPR,"failed"):=map(retractIfCan,u)$MF2d
  v case "failed" => "failed"

```

```
(v::MAT FEXPR)::
```

---

— ASP80.dotabb —

```
"ASP80" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP80"]
"FS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FS"]
"ASP80" -> "FS"
```

---

## 2.39 domain ASP9 Asp9

— Asp9.input —

```
)set break resume
)sys rm -f Asp9.output
)spool Asp9.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Asp9
--R Asp9 name: Symbol is a domain constructor
--R Abbreviation for Asp9 is ASP9
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ASP9
--R
--R----- Operations -----
--R coerce : FortranCode -> %          coerce : List FortranCode -> %
--R coerce : % -> OutputForm          outputAsFortran : % -> Void
--R retract : Polynomial Integer -> %   retract : Polynomial Float -> %
--R retract : Expression Integer -> %   retract : Expression Float -> %
--R coerce : FortranExpression([construct,QUOTEX],[construct,QUOTEY],MachineFloat) -> %
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R retract : Fraction Polynomial Integer -> %
--R retract : Fraction Polynomial Float -> %
--R retractIfCan : Fraction Polynomial Integer -> Union(%, "failed")
--R retractIfCan : Fraction Polynomial Float -> Union(%, "failed")
--R retractIfCan : Polynomial Integer -> Union(%, "failed")
--R retractIfCan : Polynomial Float -> Union(%, "failed")
--R retractIfCan : Expression Integer -> Union(%, "failed")
```

```
--R retractIfCan : Expression Float -> Union(%,"failed")
--R
--E 1
```

```
)spool
)lisp (bye)
```

---

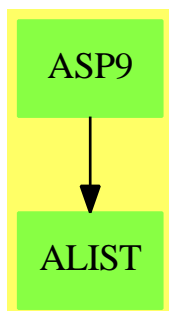
### — Asp9.help —

```
=====
Asp9 examples
=====
```

```
See Also:
o )show Asp9
```

---

### 2.39.1 Asp9 (ASP9)



#### Exports:

```
coerce outputAsFortran retract retractIfCan
```

### — domain ASP9 Asp9 —

```
)abbrev domain ASP9 Asp9
++ Author: Mike Dewar, Grant Keady and Godfrey Nolan
++ Date Created: Mar 1993
++ Date Last Updated: 18 March 1994
++               12 July 1994 added COMMON blocks for d02cjf, d02ejf
++               6 October 1994
```

```

++ Related Constructors: FortranVectorFunctionCategory, FortranProgramCategory
++ Description:
++ \spadtype{Asp9} produces Fortran for Type 9 ASPs, needed for NAG routines
++ d02bhf, d02cjf, d02ejf.
++ These ASPs represent a function of a scalar X and a vector Y, for example:
++
++ \tab{5}DOUBLE PRECISION FUNCTION G(X,Y)\br
++ \tab{5}DOUBLE PRECISION X,Y(*)\br
++ \tab{5}G=X+Y(1)\br
++ \tab{5}RETURN\br
++ \tab{5}END
++
++ If the user provides a constant value for G, then extra information is added
++ via COMMON blocks used by certain routines. This specifies that the value
++ returned by G in this case is to be ignored.

```

```

Asp9(name): Exports == Implementation where
  name : Symbol

```

```

FEXPR ==> FortranExpression(['X'],['Y'],MFLOAT)
MFLOAT ==> MachineFloat
FC ==> FortranCode
FST ==> FortranScalarType
FT ==> FortranType
SYMTAB ==> SymbolTable
RSFC ==> Record(localSymbols:SymbolTable,code:List(FortranCode))
UFST ==> Union(fst:FST,void:"void")
FRAC ==> Fraction
POLY ==> Polynomial
EXPR ==> Expression
INT ==> Integer
FLOAT ==> Float

```

```

Exports ==> FortranFunctionCategory with
  coerce : FEXPR -> %
    ++coerce(f) takes an object from the appropriate instantiation of
    ++\spadtype{FortranExpression} and turns it into an ASP.

```

```

Implementation ==> add

```

```

real : FST := "real"::FST
syms : SYMTAB := empty()$SYMTAB
declare!(X,fortranReal()$FT,syms)$SYMTAB
yType : FT := construct([real]$UFST,["*":Symbol],false)$FT
declare!(Y,yType,syms)$SYMTAB
Rep := FortranProgram(name,[real]$UFST,[X,Y],syms)

retract(u:FRAC POLY INT):$ == (retract(u)@FEXPR)::$
retractIfCan(u:FRAC POLY INT):Union($,"failed") ==
  foo : Union(FEXPR,"failed")

```

```

foo := retractIfCan(u)$FEXPR
foo case "failed" => "failed"
(foo::FEXPR)::$

retract(u:FRAC POLY FLOAT):$ == (retract(u)@FEXPR)::$
retractIfCan(u:FRAC POLY FLOAT):Union($,"failed") ==
  foo : Union(FEXPR,"failed")
  foo := retractIfCan(u)$FEXPR
  foo case "failed" => "failed"
  (foo::FEXPR)::$

retract(u:EXPR FLOAT):$ == (retract(u)@FEXPR)::$
retractIfCan(u:EXPR FLOAT):Union($,"failed") ==
  foo : Union(FEXPR,"failed")
  foo := retractIfCan(u)$FEXPR
  foo case "failed" => "failed"
  (foo::FEXPR)::$

retract(u:EXPR INT):$ == (retract(u)@FEXPR)::$
retractIfCan(u:EXPR INT):Union($,"failed") ==
  foo : Union(FEXPR,"failed")
  foo := retractIfCan(u)$FEXPR
  foo case "failed" => "failed"
  (foo::FEXPR)::$

retract(u:POLY FLOAT):$ == (retract(u)@FEXPR)::$
retractIfCan(u:POLY FLOAT):Union($,"failed") ==
  foo : Union(FEXPR,"failed")
  foo := retractIfCan(u)$FEXPR
  foo case "failed" => "failed"
  (foo::FEXPR)::$

retract(u:POLY INT):$ == (retract(u)@FEXPR)::$
retractIfCan(u:POLY INT):Union($,"failed") ==
  foo : Union(FEXPR,"failed")
  foo := retractIfCan(u)$FEXPR
  foo case "failed" => "failed"
  (foo::FEXPR)::$

coerce(u:FEXPR):% ==
  expr : Expression MachineFloat := (u::Expression(MachineFloat))$FEXPR
  (retractIfCan(u)@Union(MFLOAT,"failed"))$FEXPR case "failed" =>
    coerce(expr)$Rep
  locals : SYMTAB := empty()
  charType : FT := construct(["character":FST]$UFST,[6::POLY(INT)],false)$FT
  declare!([CHDUM1,CHDUM2,GOPT1,CHDUM,GOPT2],charType,locals)$SYMTAB
  common1 := common(CD02EJ,[CHDUM1,CHDUM2,GOPT1])$FC
  common2 := common(AD02CJ,[CHDUM,GOPT2])$FC
  assign1 := assign(GOPT1,"NOGOPT")$FC
  assign2 := assign(GOPT2,"NOGOPT")$FC

```

```

result := assign(name,expr)$FC
code : List FC := [common1,common2,assign1,assign2,result]
([locals,code]$RSFC)::Rep

coerce(c>List FortranCode):% == coerce(c)$Rep

coerce(r:RSFC):% == coerce(r)$Rep

coerce(c:FortranCode):% == coerce(c)$Rep

coerce(u:%):OutputForm == coerce(u)$Rep

outputAsFortran(u):Void ==
p := checkPrecision()$NAGLinkSupportPackage
outputAsFortran(u)$Rep
p => restorePrecision()$NAGLinkSupportPackage

-----

— ASP9.dotabb —

"ASP9" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ASP9"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"ASP9" -> "ALIST"

-----

```

## 2.40 domain JORDAN AssociatedJordanAlgebra

```

— AssociatedJordanAlgebra.input —

)set break resume
)sys rm -f AssociatedJordanAlgebra.output
)spool AssociatedJordanAlgebra.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show AssociatedJordanAlgebra
--R AssociatedJordanAlgebra(R: CommutativeRing,A: NonAssociativeAlgebra R) is a domain constructor
--R Abbreviation for AssociatedJordanAlgebra is JORDAN
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for JORDAN

```



```

--R
--R----- Operations -----
--R ?? : (R,%) -> %                ?? : (% ,R) -> %
--R ?? : (% ,%) -> %                ?? : (Integer,%) -> %
--R ?? : (PositiveInteger,%) -> %    ??? : (% ,PositiveInteger) -> %
--R +? : (% ,%) -> %                ?-? : (% ,%) -> %
--R -? : % -> %                     ?? : (% ,%) -> Boolean
--R 0 : () -> %                     antiCommutator : (% ,%) -> %
--R associator : (% ,% ,%) -> %      coerce : A -> %
--R coerce : % -> A                  coerce : % -> OutputForm
--R commutator : (% ,%) -> %          hash : % -> SingleInteger
--R latex : % -> String               sample : () -> %
--R zero? : % -> Boolean              ~=? : (% ,%) -> Boolean
--R ?? : (NonNegativeInteger,%) -> %
--R alternative? : () -> Boolean if A has FINAALG R
--R antiAssociative? : () -> Boolean if A has FINAALG R
--R antiCommutative? : () -> Boolean if A has FINAALG R
--R apply : (Matrix R,%) -> % if A has FRNAALG R
--R associative? : () -> Boolean if A has FINAALG R
--R associatorDependence : () -> List Vector R if A has FINAALG R and R has INTDOM or A has INTDOM
--R basis : () -> Vector % if A has FRNAALG R
--R commutative? : () -> Boolean if A has FINAALG R
--R conditionsForIdempotents : Vector % -> List Polynomial R if A has FINAALG R
--R conditionsForIdempotents : () -> List Polynomial R if A has FRNAALG R
--R convert : % -> Vector R if A has FRNAALG R
--R convert : Vector R -> % if A has FRNAALG R
--R coordinates : (% ,Vector %) -> Vector R if A has FINAALG R
--R coordinates : (Vector % ,Vector %) -> Matrix R if A has FINAALG R
--R coordinates : % -> Vector R if A has FRNAALG R
--R coordinates : Vector % -> Matrix R if A has FRNAALG R
--R ?.? : (% ,Integer) -> R if A has FRNAALG R
--R flexible? : () -> Boolean if A has FINAALG R
--R jacobiIdentity? : () -> Boolean if A has FINAALG R
--R jordanAdmissible? : () -> Boolean if A has FINAALG R
--R jordanAlgebra? : () -> Boolean if A has FINAALG R
--R leftAlternative? : () -> Boolean if A has FINAALG R
--R leftCharacteristicPolynomial : % -> SparseUnivariatePolynomial R if A has FINAALG R
--R leftDiscriminant : Vector % -> R if A has FINAALG R
--R leftDiscriminant : () -> R if A has FRNAALG R
--R leftMinimalPolynomial : % -> SparseUnivariatePolynomial R if A has FINAALG R and R has INTDOM
--R leftNorm : % -> R if A has FINAALG R
--R leftPower : (% ,PositiveInteger) -> %
--R leftRankPolynomial : () -> SparseUnivariatePolynomial Polynomial R if A has FRNAALG R and R has INTDOM
--R leftRecip : % -> Union(% ,"failed") if A has FINAALG R and R has INTDOM or A has FRNAALG R
--R leftRegularRepresentation : (% ,Vector %) -> Matrix R if A has FINAALG R
--R leftRegularRepresentation : % -> Matrix R if A has FRNAALG R
--R leftTrace : % -> R if A has FINAALG R
--R leftTraceMatrix : Vector % -> Matrix R if A has FINAALG R
--R leftTraceMatrix : () -> Matrix R if A has FRNAALG R
--R leftUnit : () -> Union(% ,"failed") if A has FINAALG R and R has INTDOM or A has FRNAALG R

```

```

--R leftUnits : () -> Union(Record(particular: %,basis: List %),"failed") if A has FINAALG R and R has INTDOM
--R lieAdmissible? : () -> Boolean if A has FINAALG R
--R lieAlgebra? : () -> Boolean if A has FINAALG R
--R noncommutativeJordanAlgebra? : () -> Boolean if A has FINAALG R
--R plenaryPower : (%,PositiveInteger) -> %
--R powerAssociative? : () -> Boolean if A has FINAALG R
--R rank : () -> PositiveInteger if A has FINAALG R
--R recip : % -> Union(%,"failed") if A has FINAALG R and R has INTDOM or A has FRNAALG R and R has INTDOM
--R represents : (Vector R,Vector %) -> % if A has FINAALG R
--R represents : Vector R -> % if A has FRNAALG R
--R rightAlternative? : () -> Boolean if A has FINAALG R
--R rightCharacteristicPolynomial : % -> SparseUnivariatePolynomial R if A has FINAALG R
--R rightDiscriminant : Vector % -> R if A has FINAALG R
--R rightDiscriminant : () -> R if A has FRNAALG R
--R rightMinimalPolynomial : % -> SparseUnivariatePolynomial R if A has FINAALG R and R has INTDOM or A has FRNAALG R
--R rightNorm : % -> R if A has FINAALG R
--R rightPower : (%,PositiveInteger) -> %
--R rightRankPolynomial : () -> SparseUnivariatePolynomial Polynomial R if A has FRNAALG R and R has FINAALG R
--R rightRecip : % -> Union(%,"failed") if A has FINAALG R and R has INTDOM or A has FRNAALG R and R has INTDOM
--R rightRegularRepresentation : (%,Vector %) -> Matrix R if A has FINAALG R
--R rightRegularRepresentation : % -> Matrix R if A has FRNAALG R
--R rightTrace : % -> R if A has FINAALG R
--R rightTraceMatrix : Vector % -> Matrix R if A has FINAALG R
--R rightTraceMatrix : () -> Matrix R if A has FRNAALG R
--R rightUnit : () -> Union(%,"failed") if A has FINAALG R and R has INTDOM or A has FRNAALG R and R has INTDOM
--R rightUnits : () -> Union(Record(particular: %,basis: List %),"failed") if A has FINAALG R and R has INTDOM
--R someBasis : () -> Vector % if A has FINAALG R
--R structuralConstants : Vector % -> Vector Matrix R if A has FINAALG R
--R structuralConstants : () -> Vector Matrix R if A has FRNAALG R
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unit : () -> Union(%,"failed") if A has FINAALG R and R has INTDOM or A has FRNAALG R and R has INTDOM
--R
--E 1

)spool
)lisp (bye)

```

---

— AssociatedJordanAlgebra.help —

```

=====
AssociatedJordanAlgebra examples
=====

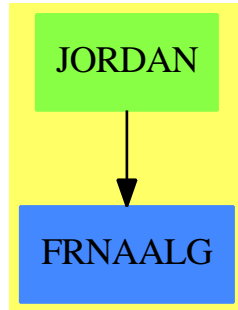
```

```

See Also:
o )show AssociatedJordanAlgebra

```

### 2.40.1 AssociatedJordanAlgebra (JORDAN)



See

⇒ “AssociatedLieAlgebra” (LIE) 2.41.1 on page 211

⇒ “LieSquareMatrix” (LSQM) 13.5.1 on page 1419

**Exports:**

|                               |                              |
|-------------------------------|------------------------------|
| 0                             | alternative?                 |
| antiAssociative?              | antiCommutative?             |
| antiCommutator                | apply                        |
| associative?                  | associator                   |
| associatorDependence          | basis                        |
| coerce                        | commutative?                 |
| commutator                    | conditionsForIdempotents     |
| convert                       | coordinates                  |
| flexible?                     | hash                         |
| jacobiIdentity?               | jordanAdmissible?            |
| jordanAlgebra?                | latex                        |
| leftAlternative?              | leftCharacteristicPolynomial |
| leftDiscriminant              | leftMinimalPolynomial        |
| leftNorm                      | leftPower                    |
| leftRankPolynomial            | leftRecip                    |
| leftRegularRepresentation     | leftTrace                    |
| leftTraceMatrix               | leftUnit                     |
| leftUnits                     | lieAdmissible?               |
| lieAlgebra?                   | noncommutativeJordanAlgebra? |
| plenaryPower                  | powerAssociative?            |
| rank                          | recip                        |
| represents                    | rightAlternative?            |
| rightCharacteristicPolynomial | rightDiscriminant            |
| rightMinimalPolynomial        | rightNorm                    |
| rightPower                    | rightRankPolynomial          |
| rightRecip                    | rightRegularRepresentation   |
| rightTrace                    | rightTraceMatrix             |
| rightUnit                     | rightUnits                   |
| sample                        | someBasis                    |
| structuralConstants           | subtractIfCan                |
| unit                          | zero?                        |
| ?*?                           | ?**?                         |
| ?+?                           | ?-?                          |
| -?                            | ?=?                          |
| ?~=?                          | ?..?                         |

— domain JORDAN AssociatedJordanAlgebra —

```
)abbrev domain JORDAN AssociatedJordanAlgebra
++ Author: J. Grabmeier
++ Date Created: 14 June 1991
++ Date Last Updated: 14 June 1991
++ Basic Operations: *,**,+, -
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords: associated Jordan algebra
```

```

++ References:
++ Description:
++ AssociatedJordanAlgebra takes an algebra \spad{A} and uses \spadfun{*$A}
++ to define the new multiplications \spad{a*b := (a *$A b + b *$A a)/2}
++ (anticommutator).
++ The usual notation \spad{{a,b}_+} cannot be used due to
++ restrictions in the current language.
++ This domain only gives a Jordan algebra if the
++ Jordan-identity \spad{(a*b)*c + (b*c)*a + (c*a)*b = 0} holds
++ for all \spad{a}, \spad{b}, \spad{c} in \spad{A}.
++ This relation can be checked by
++ \spadfun{jordanAdmissible?()}$A}.
++
++ If the underlying algebra is of type
++ \spadtype{FramedNonAssociativeAlgebra(R)} (i.e. a non
++ associative algebra over R which is a free R-module of finite
++ rank, together with a fixed R-module basis), then the same
++ is true for the associated Jordan algebra.
++ Moreover, if the underlying algebra is of type
++ \spadtype{FiniteRankNonAssociativeAlgebra(R)} (i.e. a non
++ associative algebra over R which is a free R-module of finite
++ rank), then the same true for the associated Jordan algebra.

AssociatedJordanAlgebra(R:CommutativeRing,A:NonAssociativeAlgebra R):
  public == private where
  public ==> Join (NonAssociativeAlgebra R, CoercibleTo A) with
    coerce : A -> %
      ++ coerce(a) coerces the element \spad{a} of the algebra \spad{A}
      ++ to an element of the Jordan algebra
      ++ \spadtype{AssociatedJordanAlgebra}(R,A).
  if A has FramedNonAssociativeAlgebra(R) then _
    FramedNonAssociativeAlgebra(R)
  if A has FiniteRankNonAssociativeAlgebra(R) then _
    FiniteRankNonAssociativeAlgebra(R)

private ==> A add
  Rep := A
  two : R := (1$R + 1$R)
  oneHalf : R := (recip two) :: R
  (a:%) * (b:%) ==
    zero? two => error
    "constructor must no be called with Ring of characteristic 2"
    ((a::Rep) * $Rep (b::Rep) + $Rep (b::Rep) * $Rep (a::Rep)) * oneHalf
  -- (a::Rep) * $Rep (b::Rep) + $Rep (b::Rep) * $Rep (a::Rep)
  coerce(a:%):A == a :: Rep
  coerce(a:A):% == a :: %
  (a:%) ** (n:PositiveInteger) == a

```

---

— JORDAN.dotabb —

```
"JORDAN" [color="#88FF44",href="bookvol10.3.pdf#nameddest=JORDAN"]
"FRNAALG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FRNAALG"]
"JORDAN" -> "FRNAALG"
```

## 2.41 domain LIE AssociatedLieAlgebra

— AssociatedLieAlgebra.input —

```
)set break resume
)sys rm -f AssociatedLieAlgebra.output
)spool AssociatedLieAlgebra.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show AssociatedLieAlgebra
--R AssociatedLieAlgebra(R: CommutativeRing,A: NonAssociativeAlgebra R) is a domain constructor
--R Abbreviation for AssociatedLieAlgebra is LIE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for LIE
--R
--R----- Operations -----
--R ?? : (R,%) -> %           ?? : (%,R) -> %
--R ?? : (%,%) -> %           ?? : (Integer,%) -> %
--R ?? : (PositiveInteger,%) -> %   ??? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %           ?-? : (%,%) -> %
--R -? : % -> %               ?=? : (%,%) -> Boolean
--R 0 : () -> %               antiCommutator : (%,%) -> %
--R associator : (%,%,%) -> %     coerce : A -> %
--R coerce : % -> A             coerce : % -> OutputForm
--R commutator : (%,%) -> %       hash : % -> SingleInteger
--R latex : % -> String          sample : () -> %
--R zero? : % -> Boolean         ?~=? : (%,%) -> Boolean
--R ?? : (NonNegativeInteger,%) -> %
--R alternative? : () -> Boolean if A has FINAALG R
--R antiAssociative? : () -> Boolean if A has FINAALG R
--R antiCommutative? : () -> Boolean if A has FINAALG R
--R apply : (Matrix R,%) -> % if A has FRNAALG R
--R associative? : () -> Boolean if A has FINAALG R
--R associatorDependence : () -> List Vector R if A has FINAALG R and R has INTDOM or A has FRNAALG R and
```

```

--R basis : () -> Vector % if A has FRNAALG R
--R commutative? : () -> Boolean if A has FINAALG R
--R conditionsForIdempotents : Vector % -> List Polynomial R if A has FINAALG R
--R conditionsForIdempotents : () -> List Polynomial R if A has FRNAALG R
--R convert : % -> Vector R if A has FRNAALG R
--R convert : Vector R -> % if A has FRNAALG R
--R coordinates : (%,Vector %) -> Vector R if A has FINAALG R
--R coordinates : (Vector %,Vector %) -> Matrix R if A has FINAALG R
--R coordinates : % -> Vector R if A has FRNAALG R
--R coordinates : Vector % -> Matrix R if A has FRNAALG R
--R ?.? : (%,Integer) -> R if A has FRNAALG R
--R flexible? : () -> Boolean if A has FINAALG R
--R jacobIdentity? : () -> Boolean if A has FINAALG R
--R jordanAdmissible? : () -> Boolean if A has FINAALG R
--R jordanAlgebra? : () -> Boolean if A has FINAALG R
--R leftAlternative? : () -> Boolean if A has FINAALG R
--R leftCharacteristicPolynomial : % -> SparseUnivariatePolynomial R if A has FINAALG R
--R leftDiscriminant : Vector % -> R if A has FINAALG R
--R leftDiscriminant : () -> R if A has FRNAALG R
--R leftMinimalPolynomial : % -> SparseUnivariatePolynomial R if A has FINAALG R and R has INTDOM
--R leftNorm : % -> R if A has FINAALG R
--R leftPower : (%,PositiveInteger) -> %
--R leftRankPolynomial : () -> SparseUnivariatePolynomial Polynomial R if A has FRNAALG R and R has INTDOM
--R leftRecip : % -> Union(%, "failed") if A has FINAALG R and R has INTDOM or A has FRNAALG R and R has INTDOM
--R leftRegularRepresentation : (%,Vector %) -> Matrix R if A has FINAALG R
--R leftRegularRepresentation : % -> Matrix R if A has FRNAALG R
--R leftTrace : % -> R if A has FINAALG R
--R leftTraceMatrix : Vector % -> Matrix R if A has FINAALG R
--R leftTraceMatrix : () -> Matrix R if A has FRNAALG R
--R leftUnit : () -> Union(%, "failed") if A has FINAALG R and R has INTDOM or A has FRNAALG R and R has INTDOM
--R leftUnits : () -> Union(Record(particular: %,basis: List %), "failed") if A has FINAALG R and R has INTDOM
--R lieAdmissible? : () -> Boolean if A has FINAALG R
--R lieAlgebra? : () -> Boolean if A has FINAALG R
--R noncommutativeJordanAlgebra? : () -> Boolean if A has FINAALG R
--R plenaryPower : (%,PositiveInteger) -> %
--R powerAssociative? : () -> Boolean if A has FINAALG R
--R rank : () -> PositiveInteger if A has FINAALG R
--R recip : % -> Union(%, "failed") if A has FINAALG R and R has INTDOM or A has FRNAALG R and R has INTDOM
--R represents : (Vector R,Vector %) -> % if A has FINAALG R
--R represents : Vector R -> % if A has FRNAALG R
--R rightAlternative? : () -> Boolean if A has FINAALG R
--R rightCharacteristicPolynomial : % -> SparseUnivariatePolynomial R if A has FINAALG R
--R rightDiscriminant : Vector % -> R if A has FINAALG R
--R rightDiscriminant : () -> R if A has FRNAALG R
--R rightMinimalPolynomial : % -> SparseUnivariatePolynomial R if A has FINAALG R and R has INTDOM
--R rightNorm : % -> R if A has FINAALG R
--R rightPower : (%,PositiveInteger) -> %
--R rightRankPolynomial : () -> SparseUnivariatePolynomial Polynomial R if A has FRNAALG R and R has INTDOM
--R rightRecip : % -> Union(%, "failed") if A has FINAALG R and R has INTDOM or A has FRNAALG R and R has INTDOM
--R rightRegularRepresentation : (%,Vector %) -> Matrix R if A has FINAALG R

```

```

--R rightRegularRepresentation : % -> Matrix R if A has FRNAALG R
--R rightTrace : % -> R if A has FINAALG R
--R rightTraceMatrix : Vector % -> Matrix R if A has FINAALG R
--R rightTraceMatrix : () -> Matrix R if A has FRNAALG R
--R rightUnit : () -> Union(%, "failed") if A has FINAALG R and R has INTDOM or A has FRNAALG R and R has
--R rightUnits : () -> Union(Record(particular: %, basis: List %), "failed") if A has FINAALG R and R has
--R someBasis : () -> Vector % if A has FINAALG R
--R structuralConstants : Vector % -> Vector Matrix R if A has FINAALG R
--R structuralConstants : () -> Vector Matrix R if A has FRNAALG R
--R subtractIfCan : (%, %) -> Union(%, "failed")
--R unit : () -> Union(%, "failed") if A has FINAALG R and R has INTDOM or A has FRNAALG R and R has INTD
--R
--E 1

)spool
)lisp (bye)

```

---

— AssociatedLieAlgebra.help —

```

=====
AssociatedLieAlgebra examples
=====

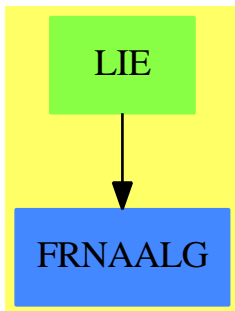
```

```

See Also:
o )show AssociatedLieAlgebra

```

### 2.41.1 AssociatedLieAlgebra (LIE)



See

⇒ “AssociatedJordanAlgebra” (JORDAN) 2.40.1 on page 206  
 ⇒ “LieSquareMatrix” (LSQM) 13.5.1 on page 1419



**Exports:**

|                            |                               |
|----------------------------|-------------------------------|
| 0                          | alternative?                  |
| antiAssociative?           | antiCommutative?              |
| antiCommutator             | apply                         |
| associative?               | associator                    |
| associatorDependence       | basis                         |
| coerce                     | commutative?                  |
| commutator                 | conditionsForIdempotents      |
| convert                    | coordinates                   |
| flexible?                  | hash                          |
| jacobiIdentity?            | jordanAdmissible?             |
| jordanAlgebra?             | latex                         |
| leftAlternative?           | leftCharacteristicPolynomial  |
| leftDiscriminant           | leftMinimalPolynomial         |
| leftNorm                   | leftPower                     |
| leftRankPolynomial         | leftRecip                     |
| leftRegularRepresentation  | leftTrace                     |
| leftTraceMatrix            | leftUnit                      |
| leftUnits                  | lieAdmissible?                |
| lieAlgebra?                | noncommutativeJordanAlgebra?  |
| plenaryPower               | powerAssociative?             |
| rank                       | recip                         |
| represents                 | represents                    |
| rightAlternative?          | rightCharacteristicPolynomial |
| rightDiscriminant          | rightMinimalPolynomial        |
| rightNorm                  | rightPower                    |
| rightRankPolynomial        | rightRecip                    |
| rightRegularRepresentation | rightTrace                    |
| rightTraceMatrix           | rightUnit                     |
| rightUnits                 | sample                        |
| someBasis                  | structuralConstants           |
| structuralConstants        | subtractIfCan                 |
| unit                       | zero?                         |
| ?*?                        | ?**?                          |
| ?+?                        | ?-?                           |
| -?                         | ?=?                           |
| ?~=?                       | ?..?                          |

— domain LIE AssociatedLieAlgebra —

```

)abbrev domain LIE AssociatedLieAlgebra
++ Author: J. Grabmeier
++ Date Created: 07 March 1991
++ Date Last Updated: 14 June 1991
++ Basic Operations: *,**,+, -
++ Related Constructors:
++ Also See:

```

```

++ AMS Classifications:
++ Keywords: associated Liealgebra
++ References:
++ Description:
++ AssociatedLieAlgebra takes an algebra \spad{A}
++ and uses \spadfun{*$A} to define the
++ Lie bracket \spad{a*b := (a *$A b - b *$A a)} (commutator). Note that
++ the notation \spad{[a,b]} cannot be used due to
++ restrictions of the current compiler.
++ This domain only gives a Lie algebra if the
++ Jacobi-identity \spad{(a*b)*c + (b*c)*a + (c*a)*b = 0} holds
++ for all \spad{a}, \spad{b}, \spad{c} in \spad{A}.
++ This relation can be checked by
++ \spad{lieAdmissible?()$A}.
++
++ If the underlying algebra is of type
++ \spadtype{FramedNonAssociativeAlgebra(R)} (i.e. a non
++ associative algebra over R which is a free \spad{R}-module of finite
++ rank, together with a fixed \spad{R}-module basis), then the same
++ is true for the associated Lie algebra.
++ Also, if the underlying algebra is of type
++ \spadtype{FiniteRankNonAssociativeAlgebra(R)} (i.e. a non
++ associative algebra over R which is a free R-module of finite
++ rank), then the same is true for the associated Lie algebra.

AssociatedLieAlgebra(R:CommutativeRing,A:NonAssociativeAlgebra R):
  public == private where
  public ==> Join (NonAssociativeAlgebra R, CoercibleTo A) with
    coerce : A -> %
      ++ coerce(a) coerces the element \spad{a} of the algebra \spad{A}
      ++ to an element of the Lie
      ++ algebra \spadtype{AssociatedLieAlgebra}(R,A).
  if A has FramedNonAssociativeAlgebra(R) then
    FramedNonAssociativeAlgebra(R)
  if A has FiniteRankNonAssociativeAlgebra(R) then
    FiniteRankNonAssociativeAlgebra(R)

private ==> A add
  Rep := A
  (a:%) * (b:%) == (a::Rep) * $Rep (b::Rep) -$Rep (b::Rep) * $Rep (a::Rep)
  coerce(a:%):A == a :: Rep
  coerce(a:A):% == a :: %
  (a:%) ** (n:PositiveInteger) ==
    n = 1 => a
    0

```

---

— LIE.dotabb —

```
"LIE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=LIE"]
"FRNAALG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FRNAALG"]
"LIE" -> "FRNAALG"
```

—————

## 2.42 domain ALIST AssociationList

— AssociationList.input —

```
)set break resume
)sys rm -f AssociationList.output
)spool AssociationList.output
)set message test on
)set message auto off
)clear all
--S 1 of 10
Data := Record(monthsOld : Integer, gender : String)
--R
--R
--R (1) Record(monthsOld: Integer,gender: String)
--R
--R                                          Type: Domain
--E 1

--S 2 of 10
al : AssociationList(String,Data)
--R
--R
--R                                          Type: Void
--E 2

--S 3 of 10
al := table()
--R
--R
--R (3) table()
--R      Type: AssociationList(String,Record(monthsOld: Integer,gender: String))
--E 3

--S 4 of 10
al."bob" := [407,"male"]$Data
--R
--R
--R (4) [monthsOld= 407,gender= "male"]
--R
--R                                          Type: Record(monthsOld: Integer,gender: String)
```

```

--E 4

--S 5 of 10
al."judith" := [366,"female"]$Data
--R
--R
--R (5) [monthsOld= 366,gender= "female"]
--R                                     Type: Record(monthsOld: Integer,gender: String)
--E 5

--S 6 of 10
al."katie" := [24,"female"]$Data
--R
--R
--R (6) [monthsOld= 24,gender= "female"]
--R                                     Type: Record(monthsOld: Integer,gender: String)
--E 6

--S 7 of 10
al."smokie" := [200,"female"]$Data
--R
--R
--R (7) [monthsOld= 200,gender= "female"]
--R                                     Type: Record(monthsOld: Integer,gender: String)
--E 7

--S 8 of 10
al
--R
--R
--R (8)
--R table
--R      "smokie"= [monthsOld= 200,gender= "female"]
--R      ,
--R      "katie"= [monthsOld= 24,gender= "female"]
--R      ,
--R      "judith"= [monthsOld= 366,gender= "female"]
--R      ,
--R      "bob"= [monthsOld= 407,gender= "male"]
--R      Type: AssociationList(String,Record(monthsOld: Integer,gender: String))
--E 8

--S 9 of 10
al."katie" := [23,"female"]$Data
--R
--R
--R (9) [monthsOld= 23,gender= "female"]
--R                                     Type: Record(monthsOld: Integer,gender: String)
--E 9

```

```

--S 10 of 10
delete!(al,1)
--R
--R
--R (10)
--R table
--R      "katie"= [monthsOld= 23,gender= "female"]
--R ,
--R      "judith"= [monthsOld= 366,gender= "female"]
--R ,
--R      "bob"= [monthsOld= 407,gender= "male"]
--R      Type: AssociationList(String,Record(monthsOld: Integer,gender: String))
--E 10
)spool
)lisp (bye)

```

---

— AssociationList.help —

=====  
AssociationList examples  
=====

The AssociationList constructor provides a general structure for associative storage. This type provides association lists in which data objects can be saved according to keys of any type. For a given association list, specific types must be chosen for the keys and entries. You can think of the representation of an association list as a list of records with key and entry fields.

Association lists are a form of table and so most of the operations available for Table are also available for AssociationList. They can also be viewed as lists and can be manipulated accordingly.

This is a Record type with age and gender fields.

```

Data := Record(monthsOld : Integer, gender : String)
      Record(monthsOld: Integer,gender: String)
      Type: Domain

```

In this expression, al is declared to be an association list whose keys are strings and whose entries are the above records.

```

al : AssociationList(String,Data)
      Type: Void

```

The table operation is used to create an empty association list.

```
al := table()
table()
Type: AssociationList(String,Record(monthsOld: Integer,gender: String))
```

You can use assignment syntax to add things to the association list.

```
al."bob" := [407,"male"]$Data
[monthsOld=407, gender= "male"]
Type: Record(monthsOld: Integer,gender: String)

al."judith" := [366,"female"]$Data
[monthsOld=366, gender= "female"]
Type: Record(monthsOld: Integer,gender: String)

al."katie" := [24,"female"]$Data
[monthsOld=24, gender= "female"]
Type: Record(monthsOld: Integer,gender: String)
```

Perhaps we should have included a species field.

```
al."smokie" := [200,"female"]$Data
[monthsOld=200, gender= "female"]
Type: Record(monthsOld: Integer,gender: String)
```

Now look at what is in the association list. Note that the last-added (key, entry) pair is at the beginning of the list.

```
al
table("smokie" = [monthsOld=200, gender= "female"],
      "katie" = [monthsOld=24, gender= "female"],
      "judith" = [monthsOld=366, gender= "female"],
      "bob" = [monthsOld=407, gender= "male"])
Type: AssociationList(String,Record(monthsOld: Integer,gender: String))
```

You can reset the entry for an existing key.

```
al."katie" := [23,"female"]$Data
[monthsOld=23, gender= "female"]
Type: Record(monthsOld: Integer,gender: String)
```

Use delete! to destructively remove an element of the association list. Use delete to return a copy of the association list with the element deleted. The second argument is the index of the element to delete.

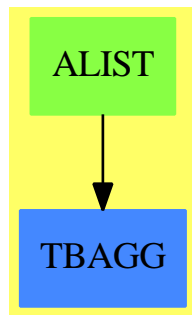
```
delete!(al,1)
table("katie" = [monthsOld=23, gender= "female"],
      "judith" = [monthsOld=366, gender= "female"],
      "bob" = [monthsOld=407, gender= "male"])
Type: AssociationList(String,Record(monthsOld: Integer,gender: String))
```

See Also:

- o )help Table
- o )help List
- o )show AssociationList

---

### 2.42.1 AssociationList (ALIST)



See

⇒ “IndexedList” (ILIST) 10.11.1 on page 1196

⇒ “List” (LIST) 13.9.1 on page 1468

**Exports:**

|                   |                  |                   |            |             |
|-------------------|------------------|-------------------|------------|-------------|
| any?              | assoc            | bag               | child?     | children    |
| coerce            | concat           | concat!           | construct  | convert     |
| copy              | copyInto!        | count             | cycleEntry | cycleLength |
| cycleSplit!       | cycleTail        | cyclic?           | delete     | delete!     |
| dictionary        | distance         | elt               | empty      | empty?      |
| entries           | entry?           | eq?               | eval       | every?      |
| explicitlyFinite? | extract!         | fill!             | find       | first       |
| hash              | index?           | indices           | insert     | insert!     |
| inspect           | key?             | keys              | last       | latex       |
| leaf?             | leaves           | less?             | list       | map         |
| map!              | max              | maxIndex          | member?    | members     |
| merge             | merge!           | min               | minIndex   | more?       |
| new               | node?            | nodes             | parts      | position    |
| possiblyInfinite? | qelt             | qsetelt!          | reduce     | remove      |
| remove!           | removeDuplicates | removeDuplicates! | rest       | reverse     |
| reverse!          | sample           | search            | second     | select      |
| select!           | setchildren!     | setelt            | setfirst!  | setlast!    |
| setrest!          | setvalue!        | size?             | sort       | sort!       |
| sorted?           | split!           | swap!             | table      | tail        |
| third             | value            | #?                | ?<?        | ?<=?        |
| ?=?               | ?>?              | ?>=?              | ?~=?       | ?..rest     |
| ?.value           | ?.first          | ?.last            | ?.?        |             |

— domain ALIST AssociationList —

```
)abbrev domain ALIST AssociationList
++ Author: Mark Botch
++ Date Created:
++ Change History:
++ Basic Operations: empty, empty?, keys, \#, concat, first, rest,
++   setrest!, search, setelt, remove!
++ Related Constructors:
++ Also See: List
++ AMS Classification:
++ Keywords: list, association list
++ Description:
++ \spadtype{AssociationList} implements association lists. These
++ may be viewed as lists of pairs where the first part is a key
++ and the second is the stored value. For example, the key might
++ be a string with a persons employee identification number and
++ the value might be a record with personnel data.
```

```
AssociationList(Key:SetCategory, Entry:SetCategory):
  AssociationListAggregate(Key, Entry) == add
    Pair ==> Record(key:Key, entry:Entry)
    Rep := Reference List Pair

  dictionary() == ref empty()
```



```

empty()                == dictionary()
empty? t               == empty? deref t
entries(t:%):List(Pair) == deref t
parts(t:%):List(Pair)  == deref t
keys t                 == [k.key for k in deref t]
# t                    == # deref t
first(t:%):Pair        == first deref t
rest t                 == ref rest deref t
concat(p:Pair, t:%)     == ref concat(p, deref t)
setrest_!(a:%, b:%)     == ref setrest_!(deref a, deref b)
setfirst_!(a:%, p:Pair) == setfirst_!(deref a, p)
minIndex(a:%):Integer  == minIndex(deref a)
maxIndex(a:%):Integer  == maxIndex(deref a)

search(k, t) ==
  for r in deref t repeat
    k = r.key => return(r.entry)
  "failed"

latex(a : %) : String ==
  l : List Pair := entries a
  s : String := "\left["
  while not empty?(l) repeat
    r : Pair := first l
    l       := rest l
    s := concat(s, concat(latex r.key, concat(" = ", latex r.entry)$String)$String)$String
    if not empty?(l) then s := concat(s, ", ")$String
  concat(s, " \right]")$String

-- assoc(k, l) ==
-- (r := find(#1.key=k, l)) case "failed" => "failed"
-- r

assoc(k, t) ==
  for r in deref t repeat
    k = r.key => return r
  "failed"

setelt(t:%, k:Key, e:Entry) ==
  (r := assoc(k, t)) case Pair => (r::Pair).entry := e
  setref(t, concat([k, e], deref t))
  e

remove_!(k:Key, t:%) ==
  empty?(l := deref t) => "failed"
  k = first(l).key =>
    setref(t, rest l)
    first(l).entry
  prev := l
  curr := rest l

```

```

while not empty? curr and first(curr).key ^= k repeat
  prev := curr
  curr := rest curr
empty? curr => "failed"
setrest_!(prev, rest curr)
first(curr).entry

```

---

— ALIST.dotabb —

```

"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"TBAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=TBAGG"]
"ALIST" -> "TBAGG"

```

---

## 2.43 domain ATTRIBUT AttributeButtons

— AttributeButtons.input —

```

)set break resume
)sys rm -f AttributeButtons.output
)spool AttributeButtons.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show AttributeButtons
--R AttributeButtons is a domain constructor
--R Abbreviation for AttributeButtons is ATTRIBUT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ATTRIBUT
--R
--R----- Operations -----
--R ==? : (%,% ) -> Boolean          coerce : % -> OutputForm
--R decrease : String -> Float      hash : % -> SingleInteger
--R increase : String -> Float      latex : % -> String
--R resetAttributeButtons : () -> Void  ?~=? : (%,% ) -> Boolean
--R decrease : (String,String) -> Float
--R getButtonValue : (String,String) -> Float
--R increase : (String,String) -> Float
--R setAttributeButtonStep : Float -> Float

```

```

--R setButtonValue : (String,String,Float) -> Float
--R setButtonValue : (String,Float) -> Float
--R
--E 1

)spool
)lisp (bye)

```

---

— AttributeButtons.help —

```

=====
AttributeButtons examples
=====

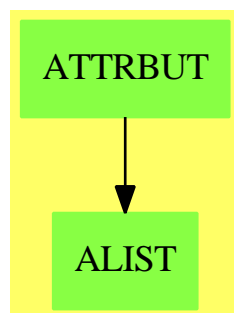
```

```

See Also:
o )show AttributeButtons

```

### 2.43.1 AttributeButtons (ATTRBUT)



#### Exports:

|                |          |                       |                        |
|----------------|----------|-----------------------|------------------------|
| coerce         | decrease | getButtonValue        | hash                   |
| increase       | latex    | resetAttributeButtons | setAttributeButtonStep |
| setButtonValue | ?~=?     | ?=?                   |                        |

— domain ATTRBUT AttributeButtons —

```

)abbrev domain ATTRBUT AttributeButtons
++ Author: Brian Dupee
++ Date Created: April 1996
++ Date Last Updated: December 1997

```

```

++ Basic Operations: increase, decrease, getButtonValue, setButtonValue
++ Related Constructors: Table(String,Float)
++ Description:
++ \axiomType{AttributeButtons} implements a database and associated
++ adjustment mechanisms for a set of attributes.
++
++ For ODEs these attributes are "stiffness", "stability" (i.e. how much
++ affect the cosine or sine component of the solution has on the stability of
++ the result), "accuracy" and "expense" (i.e. how expensive is the evaluation
++ of the ODE). All these have bearing on the cost of calculating the
++ solution given that reducing the step-length to achieve greater accuracy
++ requires considerable number of evaluations and calculations.
++
++ The effect of each of these attributes can be altered by increasing or
++ decreasing the button value.
++
++ For Integration there is a button for increasing and decreasing the preset
++ number of function evaluations for each method. This is automatically used
++ by ANNA when a method fails due to insufficient workspace or where the
++ limit of function evaluations has been reached before the required
++ accuracy is achieved.

```

```

AttributeButtons(): E == I where
  F      ==> Float
  ST     ==> String
  LST    ==> List String
  Rec    ==> Record(key:Symbol,entry:Any)
  RList  ==> List(Record(key:Symbol,entry:Any))
  IFL    ==> List(Record(iffail:Integer,instruction:ST))
  Entry  ==> Record(chapter:ST, type:ST, domainName: ST,
                   defaultMin:F, measure:F, failList:IFL, explList:LST)

```

```

E ==> SetCategory with

```

```

increase:(ST,ST) -> F
  ++ \axiom{increase(routineName,attributeName)} increases the value
  ++ for the effect of the attribute \axiom{attributeName} with routine
  ++ \axiom{routineName}.
  ++
  ++ \axiom{attributeName} should be one of the values
  ++ "stiffness", "stability", "accuracy", "expense" or
  ++ "functionEvaluations".
increase:(ST) -> F
  ++ \axiom{increase(attributeName)} increases the value for the
  ++ effect of the attribute \axiom{attributeName} with all routines.
  ++
  ++ \axiom{attributeName} should be one of the values
  ++ "stiffness", "stability", "accuracy", "expense" or
  ++ "functionEvaluations".

```

```

decrease:(ST,ST) -> F
  ++ \axiom{decrease(routineName,attributeName)} decreases the value
  ++ for the effect of the attribute \axiom{attributeName} with routine
  ++ \axiom{routineName}.
  ++
  ++ \axiom{attributeName} should be one of the values
  ++ "stiffness", "stability", "accuracy", "expense" or
  ++ "functionEvaluations".
decrease:(ST) -> F
  ++ \axiom{decrease(attributeName)} decreases the value for the
  ++ effect of the attribute \axiom{attributeName} with all routines.
  ++
  ++ \axiom{attributeName} should be one of the values
  ++ "stiffness", "stability", "accuracy", "expense" or
  ++ "functionEvaluations".
getButtonValue:(ST,ST) -> F
  ++ \axiom{getButtonValue(routineName,attributeName)} returns the
  ++ current value for the effect of the attribute \axiom{attributeName}
  ++ with routine \axiom{routineName}.
  ++
  ++ \axiom{attributeName} should be one of the values
  ++ "stiffness", "stability", "accuracy", "expense" or
  ++ "functionEvaluations".
resetAttributeButtons:() -> Void
  ++ \axiom{resetAttributeButtons()} resets the Attribute buttons to a
  ++ neutral level.
setAttributeButtonStep:(F) -> F
  ++ \axiom{setAttributeButtonStep(n)} sets the value of the steps for
  ++ increasing and decreasing the button values. \axiom{n} must be
  ++ greater than 0 and less than 1. The preset value is 0.5.
setButtonValue:(ST,F) -> F
  ++ \axiom{setButtonValue(attributeName,n)} sets the
  ++ value of all buttons of attribute \spad{attributeName}
  ++ to \spad{n}. \spad{n} must be in the range [0..1].
  ++
  ++ \axiom{attributeName} should be one of the values
  ++ "stiffness", "stability", "accuracy", "expense" or
  ++ "functionEvaluations".
setButtonValue:(ST,ST,F) -> F
  ++ \axiom{setButtonValue(attributeName,routineName,n)} sets the
  ++ value of the button of attribute \spad{attributeName} to routine
  ++ \spad{routineName} to \spad{n}. \spad{n} must be in the range [0..1].
  ++
  ++ \axiom{attributeName} should be one of the values
  ++ "stiffness", "stability", "accuracy", "expense" or
  ++ "functionEvaluations".
finiteAggregate

```

I ==> add

```

Rep := StringTable(F)
import Rep

buttons:() -> $

buttons():$ ==
  eList := empty()$List(Record(key:ST,entry:F))
  l1:List String := ["stability","stiffness","accuracy","expense"]
  l2:List String := ["functionEvaluations"]
  ro1 := selectODEIVPRoutines(r := routines()$RoutinesTable)$RoutinesTable
  ro2 := selectIntegrationRoutines(r)$RoutinesTable
  k1:List String := [string(i)$Symbol for i in keys(ro1)$RoutinesTable]
  k2:List String := [string(i)$Symbol for i in keys(ro2)$RoutinesTable]
  for i in k1 repeat
    for j in l1 repeat
      e:Record(key:ST,entry:F) := [i j,0.5]
      eList := cons(e,eList)$List(Record(key:ST,entry:F))
  for i in k2 repeat
    for j in l2 repeat
      e:Record(key:ST,entry:F) := [i j,0.5]
      eList := cons(e,eList)$List(Record(key:ST,entry:F))
  construct(eList)$Rep

attributeButtons:$ := buttons()

attributeStep:F := 0.5

setAttributeButtonStep(n:F):F ==
  positive?(n)$F and (n<1$F) => attributeStep:F := n
  error("setAttributeButtonStep","New value must be in (0..1)")$ErrorFunctions

resetAttributeButtons():Void ==
  attributeButtons := buttons()
  void()$Void

setButtonValue(routineName:ST,attributeName:ST,n:F):F ==
  f := search(routineName attributeName,attributeButtons)$Rep
  f case Float =>
    n>=0$F and n<=1$F =>
      setelt(attributeButtons,routineName attributeName,n)$Rep
      error("setAttributeButtonStep","New value must be in [0..1]")$ErrorFunctions
  error("setButtonValue","attribute name " attributeName
    " not found for routine " routineName)$ErrorFunctions

setButtonValue(attributeName:ST,n:F):F ==
  ro1 := selectODEIVPRoutines(r := routines()$RoutinesTable)$RoutinesTable
  ro2 := selectIntegrationRoutines(r)$RoutinesTable
  l1:List String := ["stability","stiffness","accuracy","expense"]
  l2:List String := ["functionEvaluations"]
  if attributeName="functionEvaluations" then

```

```

        for i in keys(ro2)$RoutinesTable repeat
            setButtonValue(string(i)$Symbol,attributeName,n)
        else
            for i in keys(ro1)$RoutinesTable repeat
                setButtonValue(string(i)$Symbol,attributeName,n)
            n

increase(routineName:ST,attributeName:ST):F ==
f := search(routineName attributeName,attributeButtons)$Rep
f case Float =>
    newValue:F := (1$F-attributeStep)*f+attributeStep
    setButtonValue(routineName,attributeName,newValue)
error("increase","attribute name " attributeName
      " not found for routine " routineName)$ErrorFunctions

increase(attributeName:ST):F ==
ro1 := selectODEIVPRoutines(r := routines())$RoutinesTable$RoutinesTable
ro2 := selectIntegrationRoutines(r)$RoutinesTable
l1:List String := ["stability","stiffness","accuracy","expense"]
l2:List String := ["functionEvaluations"]
if attributeName="functionEvaluations" then
    for i in keys(ro2)$RoutinesTable repeat
        increase(string(i)$Symbol,attributeName)
    else
        for i in keys(ro1)$RoutinesTable repeat
            increase(string(i)$Symbol,attributeName)
        getButtonValue(string(i)$Symbol,attributeName)

decrease(routineName:ST,attributeName:ST):F ==
f := search(routineName attributeName,attributeButtons)$Rep
f case Float =>
    newValue:F := (1$F-attributeStep)*f
    setButtonValue(routineName,attributeName,newValue)
error("increase","attribute name " attributeName
      " not found for routine " routineName)$ErrorFunctions

decrease(attributeName:ST):F ==
ro1 := selectODEIVPRoutines(r := routines())$RoutinesTable$RoutinesTable
ro2 := selectIntegrationRoutines(r)$RoutinesTable
l1:List String := ["stability","stiffness","accuracy","expense"]
l2:List String := ["functionEvaluations"]
if attributeName="functionEvaluations" then
    for i in keys(ro2)$RoutinesTable repeat
        decrease(string(i)$Symbol,attributeName)
    else
        for i in keys(ro1)$RoutinesTable repeat
            decrease(string(i)$Symbol,attributeName)
        getButtonValue(string(i)$Symbol,attributeName)

```

```

getButtonValue(routineName:ST,attributeName:ST):F ==
  f := search(routineName attributeName,attributeButtons)$Rep
  f case Float => f
  error("getButtonValue","attribute name " attributeName
        " not found for routine " routineName)$ErrorFunctions

```

---

— ATTRBUT.dotabb —

```

"ATTRBUT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ATTRBUT"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"ATTRBUT" -> "ALIST"

```

---

## 2.44 domain AUTOMOR Automorphism

— Automorphism.input —

```

)set break resume
)sys rm -f Automorphism.output
)spool Automorphism.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Automorphism
--R Automorphism R: Ring is a domain constructor
--R Abbreviation for Automorphism is AUTOMOR
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for AUTOMOR
--R
--R----- Operations -----
--R ?? : (% ,%) -> %
--R ??? : (% ,PositiveInteger) -> %
--R ?? : (% ,%) -> Boolean
--R ?? : (% ,Integer) -> %
--R coerce : % -> OutputForm
--R conjugate : (% ,%) -> %
--R hash : % -> SingleInteger
--R latex : % -> String
--R one? : % -> Boolean
--R ??? : (% ,Integer) -> %
--R ?? : (% ,%) -> %
--R 1 : () -> %
--R ?? : (% ,PositiveInteger) -> %
--R commutator : (% ,%) -> %
--R ?.? : (% ,R) -> R
--R inv : % -> %
--R morphism : (R -> R) -> %
--R recip : % -> Union(% ,"failed")

```



```

--R sample : () -> %
--R ***? : (% , NonNegativeInteger) -> %
--R ?? : (% , NonNegativeInteger) -> %
--R morphism : ((R , Integer) -> R) -> %
--R morphism : ((R -> R) , (R -> R)) -> %
--R
--E 1

)spool
)lisp (bye)

```

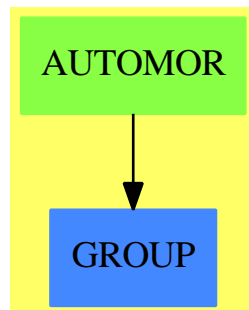
---

— Automorphism.help —

Automorphism examples

See Also:  
o )show Automorphism

### 2.44.1 Automorphism (AUTOMOR)



See

⇒ “SparseUnivariateSkewPolynomial” (ORESUP) 20.21.1 on page 2450

⇒ “UnivariateSkewPolynomial” (OREUP) 22.8.1 on page 2829

**Exports:**

|        |        |            |           |       |
|--------|--------|------------|-----------|-------|
| 1      | coerce | commutator | conjugate | hash  |
| inv    | latex  | morphism   | one?      | recip |
| sample | ?~=?   | ***?       | ?^?       | ?.?   |
| ?*?    | ?/?    | ?=?        |           |       |

## — domain AUTOMOR Automorphism —

```

)abbrev domain AUTOMOR Automorphism
++ Author: Manuel Bronstein
++ Date Created: 31 January 1994
++ Date Last Updated: 31 January 1994
++ References:
++ Description:
++ Automorphism R is the multiplicative group of automorphisms of R.
-- In fact, non-invertible endomorphism are allowed as partial functions.
-- This domain is noncanonical in that  $f*f^{-1}$  will be the identity
-- function but won't be equal to 1.

Automorphism(R:Ring): Join(Group, Eltable(R, R)) with
  morphism: (R -> R) -> %
    ++ morphism(f) returns the non-invertible morphism given by f.
  morphism: (R -> R, R -> R) -> %
    ++ morphism(f, g) returns the invertible morphism given by f, where
    ++ g is the inverse of f..
  morphism: ((R, Integer) -> R) -> %
    ++ morphism(f) returns the morphism given by  $\text{spad}\{f^n(x) = f(x,n)\}$ .
== add
err:   R -> R
ident: (R, Integer) -> R
iter:  (R -> R, NonNegativeInteger, R) -> R
iterat: (R -> R, R -> R, Integer, R) -> R
apply: (% , R, Integer) -> R

Rep := ((R, Integer) -> R)

1 == ident
err r == error "Morphism is not invertible"
ident(r, n) == r
f = g == EQ(f, g)$Lisp
elt(f, r) == apply(f, r, 1)
inv f == (r1:R, i2:Integer):R +-> apply(f, r1, - i2)
f ** n == (r1:R, i2:Integer):R +-> apply(f, r1, n * i2)
coerce(f:%):OutputForm == message("R -> R")
morphism(f:(R, Integer) -> R):% == f
morphism(f:R -> R):% == morphism(f, err)
morphism(f, g) == (r1:R, i2:Integer):R +-> iterat(f, g, i2, r1)
apply(f, r, n) == (g := f pretend ((R, Integer) -> R); g(r, n))

iterat(f, g, n, r) ==
  n < 0 => iter(g, (-n)::NonNegativeInteger, r)
  iter(f, n::NonNegativeInteger, r)

iter(f, n, r) ==
  for i in 1..n repeat r := f r

```

```

r

f * g ==
f = g => f**2
(r1:R, i2:Integer):R +->
  iterat((u1:R):R +-> f g u1,
        (v1:R):R +-> (inv g)(inv f) v1,
        i2, r1)

```

---

— AUTOMOR.dotabb —

```

"AUTOMOR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=AUTOMOR"]
"GROUP"   [color="#4488FF",href="bookvol10.2.pdf#nameddest=GROUP"]
"AUTOMOR" -> "GROUP"

```

---

## Chapter 3

# Chapter B

### 3.1 domain BBTREE BalancedBinaryTree

— BalancedBinaryTree.input —

```
)set break resume
)sys rm -f BalancedBinaryTree.output
)spool BalancedBinaryTree.output
)set message test on
)set message auto off
)clear all
--S 1 of 7
lm := [3,5,7,11]
--E 1

--S 2 of 7
t := balancedBinaryTree(#lm, 0)
--E 2

--S 3 of 7
setleaves!(t,lm)
--E 3

--S 4 of 7
mapUp!(t,_)
--E 4

--S 5 of 7
t
--E 5

--S 6 of 7
```

```
mapDown!(t,12,_rem)
--E 6

--S 7 of 7
leaves %
--E 7

)spool
)lisp (bye)
```

---

— **BalancedBinaryTree.help** —

```
=====
BalancedBinaryTree examples
=====
```

BalancedBinaryTrees(S) is the domain of balanced binary trees with elements of type S at the nodes. A binary tree is either empty or else consists of a node having a value and two branches, each branch a binary tree. A balanced binary tree is one that is balanced with respect its leaves. One with  $2^k$  leaves is perfectly "balanced": the tree has minimum depth, and the left and right branch of every interior node is identical in shape.

Balanced binary trees are useful in algebraic computation for so-called "divide-and-conquer" algorithms. Conceptually, the data for a problem is initially placed at the root of the tree. The original data is then split into two subproblems, one for each subtree. And so on. Eventually, the problem is solved at the leaves of the tree. A solution to the original problem is obtained by some mechanism that can reassemble the pieces. In fact, an implementation of the Chinese Remainder Algorithm using balanced binary trees was first proposed by David Y. Y. Yun at the IBM T. J. Watson Research Center in Yorktown Heights, New York, in 1978. It served as the prototype for polymorphic algorithms in Axiom.

In what follows, rather than perform a series of computations with a single expression, the expression is reduced modulo a number of integer primes, a computation is done with modular arithmetic for each prime, and the Chinese Remainder Algorithm is used to obtain the answer to the original problem. We illustrate this principle with the computation of  $12^2 = 144$ .

A list of moduli:

```
lm := [3,5,7,11]
      [3,5,7,11]
```

Type: PositiveInteger

The expression `modTree(n, lm)` creates a balanced binary tree with leaf values  $n \bmod m$  for each modulus  $m$  in `lm`.

```
modTree(12,lm)
[0, 2, 5, 1]
```

Type: List Integer

Operation `modTree` does this using operations on balanced binary trees. We trace its steps. Create a balanced binary tree `t` of zeros with four leaves.

```
t := balancedBinaryTree(#lm, 0)
[[0, 0, 0], 0, [0, 0, 0]]
```

Type: BalancedBinaryTree NonNegativeInteger

The leaves of the tree are set to the individual moduli.

```
setleaves!(t,lm)
[[3, 0, 5], 0, [7, 0, 11]]
```

Type: BalancedBinaryTree NonNegativeInteger

`mapUp!` to do a bottom-up traversal of `t`, setting each interior node to the product of the values at the nodes of its children.

```
mapUp!(t,_)
1155
```

Type: PositiveInteger

The value at the node of every subtree is the product of the moduli of the leaves of the subtree.

```
t
[[3, 15, 5], 1155, [7, 77, 11]]
```

Type: BalancedBinaryTree NonNegativeInteger

Operation `mapDown!(t,a,fn)` replaces the value  $v$  at each node of `t` by `fn(a,v)`.

```
mapDown!(t,12,_rem)
[[0, 12, 2], 12, [5, 12, 1]]
```

Type: BalancedBinaryTree NonNegativeInteger

The operation `leaves` returns the leaves of the resulting tree. In this case, it returns the list of  $12 \bmod m$  for each modulus  $m$ .

```
leaves %
[0, 2, 5, 1]
```

Type: List NonNegativeInteger

Compute the square of the images of 12 modulo each m.

```
squares := [x**2 rem m for x in % for m in lm]
[0, 4, 4, 1]
Type: List NonNegativeInteger
```

Call the Chinese Remainder Algorithm to get the answer for  $12^2$ .

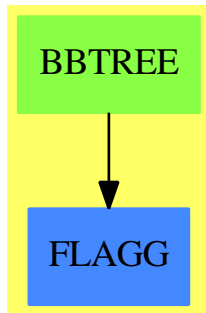
```
chineseRemainder(%,lm)
144
Type: PositiveInteger
```

See Also:

o )show BalancedBinaryTree

—————

### 3.1.1 BalancedBinaryTree (BBTREE)



See

- ⇒ “Tree” (TREE) 21.10.1 on page 2699
- ⇒ “BinaryTree” (BTREE) 3.11.1 on page 292
- ⇒ “BinarySearchTree” (BSTREE) 3.9.1 on page 285
- ⇒ “BinaryTournament” (BTourn) 3.10.1 on page 289
- ⇒ “PendantTree” (PENDTREE) 17.13.1 on page 1904

**Exports:**

|            |                    |           |              |          |
|------------|--------------------|-----------|--------------|----------|
| any?       | balancedBinaryTree | child?    | children     | coerce   |
| copy       | count              | cyclic?   | distance     | empty    |
| empty?     | eq?                | eval      | every?       | hash     |
| latex      | leaf?              | leaves    | left         | less?    |
| map        | map!               | mapDown!  | mapUp!       | member?  |
| members    | more?              | node?     | node         | nodes    |
| parts      | right              | sample    | setchildren! | setelt   |
| setleaves! | setleft!           | setright! | setvalue!    | size?    |
| value      | #?                 | ?=?       | ?~=?         | ?..right |
| ?..left    | ?..value           |           |              |          |

— domain BBTREE BalancedBinaryTree —

```

)abbrev domain BBTREE BalancedBinaryTree
++ Description:
++ \spadtype{BalancedBinaryTree(S)} is the domain of balanced
++ binary trees (bbtree). A balanced binary tree of \spad{2**k} leaves,
++ for some \spad{k > 0}, is symmetric, that is, the left and right
++ subtree of each interior node have identical shape.
++ In general, the left and right subtree of a given node can differ
++ by at most leaf node.

BalancedBinaryTree(S: SetCategory): Exports == Implementation where
  Exports == BinaryTreeCategory(S) with
    finiteAggregate
    shallowlyMutable
-- BUG: applies wrong fnct for balancedBinaryTree(0,[1,2,3,4])
--   balancedBinaryTree: (S, List S) -> %
--     ++ balancedBinaryTree(s, ls) creates a balanced binary tree with
--     ++ s at the interior nodes and elements of ls at the
--     ++ leaves.
  balancedBinaryTree: (NonNegativeInteger, S) -> %
    ++ balancedBinaryTree(n, s) creates a balanced binary tree with
    ++ n nodes each with value s.
    ++
    ++X balancedBinaryTree(4, 0)

setleaves_!: (% , List S) -> %
  ++ setleaves!(t, ls) sets the leaves of t in left-to-right order
  ++ to the elements of ls.
  ++
  ++X t1:=balancedBinaryTree(4, 0)
  ++X setleaves!(t1,[1,2,3,4])

mapUp_!: (% , (S,S) -> S) -> S
  ++ mapUp!(t,f) traverses balanced binary tree t in an "endorder"
  ++ (left then right then node) fashion returning t with the value
  ++ at each successive interior node of t replaced by
  ++ f(l,r) where l and r are the values at the immediate

```



```

++ left and right nodes.
++
++X T1:=BalancedBinaryTree Integer
++X t2:=balancedBinaryTree(4, 0)$T1
++X setleaves!(t2,[1,2,3,4]::List(Integer))
++X adder(a:Integer,b:Integer):Integer == a+b
++X mapUp!(t2,adder)
++X t2

mapUp_!: (% , % , (S,S,S,S) -> S) -> %
++ mapUp!(t,t1,f) traverses balanced binary tree t in an "endorder"
++ (left then right then node) fashion returning t with the value
++ at each successive interior node of t replaced by
++ f(l,r,l1,r1) where l and r are the values at the immediate
++ left and right nodes. Values l1 and r1 are values at the
++ corresponding nodes of a balanced binary tree t1, of identical
++ shape at t.
++
++X T1:=BalancedBinaryTree Integer
++X t2:=balancedBinaryTree(4, 0)$T1
++X setleaves!(t2,[1,2,3,4]::List(Integer))
++X adder4(i:INT,j:INT,k:INT,l:INT):INT == i+j+k+l
++X mapUp!(t2,t2,adder4)
++X t2

mapDown_!: (% ,S, (S,S,S) -> S) -> %
++ mapDown!(t,p,f) returns t after traversing t in "preorder"
++ (node then left then right) fashion replacing the successive
++ interior nodes as follows. The root value x is
++ replaced by q := f(p,x). The mapDown!(l,q,f) and
++ mapDown!(r,q,f) are evaluated for the left and right subtrees
++ l and r of t.
++
++X T1:=BalancedBinaryTree Integer
++X t2:=balancedBinaryTree(4, 0)$T1
++X setleaves!(t2,[1,2,3,4]::List(Integer))
++X adder(i:Integer,j:Integer):Integer == i+j
++X mapDown!(t2,4::INT,adder)
++X t2

mapDown_!: (% ,S, (S,S,S) -> List S) -> %
++ mapDown!(t,p,f) returns t after traversing t in "preorder"
++ (node then left then right) fashion replacing the successive
++ interior nodes as follows. Let l and r denote the left and
++ right subtrees of t. The root value x of t is replaced by p.
++ Then f(value l, value r, p), where l and r denote the left
++ and right subtrees of t, is evaluated producing two values
++ pl and pr. Then \spad{mapDown!(l,pl,f)} and \spad{mapDown!(l,pr,f)}
++ are evaluated.
++

```

```

++X T1:=BalancedBinaryTree Integer
++X t2:=balancedBinaryTree(4, 0)$T1
++X setleaves!(t2,[1,2,3,4]::List(Integer))
++X adder3(i:Integer,j:Integer,k:Integer):List Integer == [i+j,j+k]
++X mapDown!(t2,4::INT,adder3)
++X t2

Implementation == BinaryTree(S) add
Rep := BinaryTree(S)
leaf? x ==
  empty? x => false
  empty? left x and empty? right x
-- balancedBinaryTree(x: S, u: List S) ==
--   n := #u
--   n = 0 => empty()
--   setleaves_!(balancedBinaryTree(n, x), u)
setleaves_!(t, u) ==
  n := #u
  n = 0 =>
    empty? t => t
    error "the tree and list must have the same number of elements"
  n = 1 =>
    setvalue_!(t,first u)
    t
  m := n quo 2
  acc := empty()$(List S)
  for i in 1..m repeat
    acc := [first u,:acc]
    u := rest u
  setleaves_!(left t, reverse_! acc)
  setleaves_!(right t, u)
  t
balancedBinaryTree(n: NonNegativeInteger, val: S) ==
  n = 0 => empty()
  n = 1 => node(empty(),val,empty())
  m := n quo 2
  node(balancedBinaryTree(m, val), val,
    balancedBinaryTree((n - m) pretend NonNegativeInteger, val))
mapUp_!(x,fn) ==
  empty? x => error "mapUp! called on a null tree"
  leaf? x => x.value
  x.value := fn(mapUp_!(x.left,fn),mapUp_!(x.right,fn))
mapUp_!(x,y,fn) ==
  empty? x => error "mapUp! is called on a null tree"
  leaf? x =>
    leaf? y => x
    error "balanced binary trees are incompatible"
  leaf? y => error "balanced binary trees are incompatible"
  mapUp_!(x.left,y.left,fn)
  mapUp_!(x.right,y.right,fn)

```

```

    x.value := fn(x.left.value,x.right.value,y.left.value,y.right.value)
  x
mapDown_!(x: %, p: S, fn: (S,S) -> S ) ==
  empty? x => x
  x.value := fn(p, x.value)
  mapDown_!(x.left, x.value, fn)
  mapDown_!(x.right, x.value, fn)
  x
mapDown_!(x: %, p: S, fn: (S,S,S) -> List S) ==
  empty? x => x
  x.value := p
  leaf? x => x
  u := fn(x.left.value, x.right.value, p)
  mapDown_!(x.left, u.1, fn)
  mapDown_!(x.right, u.2, fn)
  x

```

---

— BBTREE.dotabb —

```

"BBTREE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=BBTREE"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"BBTREE" -> "FLAGG"

```

---

## 3.2 domain BPADIC BalancedPAdicInteger

— BalancedPAdicInteger.input —

```

)set break resume
)sys rm -f BalancedPAdicInteger.output
)spool BalancedPAdicInteger.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show BalancedPAdicInteger
--R BalancedPAdicInteger p: Integer is a domain constructor
--R Abbreviation for BalancedPAdicInteger is BPADIC
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for BPADIC

```

```

--R
--R----- Operations -----
--R ?? : (%,% ) -> %           ?? : (Integer,% ) -> %
--R ?? : (PositiveInteger,% ) -> %   ??? : (% ,PositiveInteger) -> %
--R ?+? : (%,% ) -> %           ?-? : (%,% ) -> %
--R -? : % -> %                 ?=? : (%,% ) -> Boolean
--R 1 : () -> %                 0 : () -> %
--R ?? : (% ,PositiveInteger) -> %   associates? : (%,% ) -> Boolean
--R coerce : % -> %             coerce : Integer -> %
--R coerce : % -> OutputForm      complete : % -> %
--R digits : % -> Stream Integer  extend : (% ,Integer) -> %
--R gcd : List % -> %            gcd : (%,% ) -> %
--R hash : % -> SingleInteger     latex : % -> String
--R lcm : List % -> %            lcm : (%,% ) -> %
--R moduloP : % -> Integer        modulus : () -> Integer
--R one? : % -> Boolean           order : % -> NonNegativeInteger
--R ?quo? : (%,% ) -> %          quotientByP : % -> %
--R recip : % -> Union(% ,"failed") ?rem? : (%,% ) -> %
--R sample : () -> %             sizeLess? : (%,% ) -> Boolean
--R sqrt : (% ,Integer) -> %      unit? : % -> Boolean
--R unitCanonical : % -> %        zero? : % -> Boolean
--R ~=? : (%,% ) -> Boolean
--R *? : (NonNegativeInteger,% ) -> %
--R **? : (% ,NonNegativeInteger) -> %
--R ^? : (% ,NonNegativeInteger) -> %
--R approximate : (% ,Integer) -> Integer
--R characteristic : () -> NonNegativeInteger
--R divide : (%,% ) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List % ,%) -> Union(List % ,"failed")
--R exquo : (%,% ) -> Union(% ,"failed")
--R extendedEuclidean : (% ,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,% ) -> Record(coef1: %,coef2: %,generator: %)
--R gcdPolynomial : (SparseUnivariatePolynomial % ,SparseUnivariatePolynomial %) -> SparseUnivariatePolynomial
--R multiEuclidean : (List % ,%) -> Union(List % ,"failed")
--R principalIdeal : List % -> Record(coef: List % ,generator: %)
--R root : (SparseUnivariatePolynomial Integer,Integer) -> %
--R subtractIfCan : (%,% ) -> Union(% ,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)

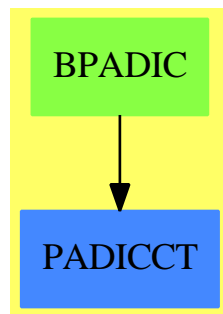
```

```
=====
BalancedPAdicInteger examples
=====
```

See Also:

```
o )show BalancedPAdicInteger
```

### 3.2.1 BalancedPAdicInteger (BPADIC)



See

- ⇒ “InnerPAdicInteger” (IPADIC) 10.24.1 on page 1258
- ⇒ “PAdicInteger” (PADIC) 17.1.1 on page 1841
- ⇒ “PAdicRationalConstructor” (PADICRC) 17.3.1 on page 1850
- ⇒ “PAdicRational” (PADICRAT) 17.2.1 on page 1845
- ⇒ “BalancedPAdicRational” (BPADICRT) 3.3.1 on page 244

#### Exports:

|                |                   |                    |               |
|----------------|-------------------|--------------------|---------------|
| 0              | 1                 | approximate        | associates?   |
| characteristic | coerce            | complete           | digits        |
| divide         | euclideanSize     | expressIdealMember | exquo         |
| extend         | extendedEuclidean | gcd                | gcdPolynomial |
| hash           | latex             | lcm                | moduloP       |
| modulus        | multiEuclidean    | one?               | order         |
| principalIdeal | quotientByP       | recip              | root          |
| sample         | sizeLess?         | sqrt               | subtractIfCan |
| unit?          | unitCanonical     | unitNormal         | zero?         |
| ?~=?           | ?*?               | ?**?               | ?^?           |
| ?+?            | ?-?               | -?                 | ?=?           |
| ?quo?          | ?rem?             |                    |               |

— domain BPADIC BalancedPAdicInteger —

```
)abbrev domain BPADIC BalancedPAdicInteger
```

```

++ Author: Clifton J. Williamson
++ Date Created: 15 May 1990
++ Date Last Updated: 15 May 1990
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: p-adic, complementation
++ Examples:
++ References:
++ Description:
++ Stream-based implementation of  $\mathbb{Z}_p$ : p-adic numbers are represented as
++  $\sum_{i=0}^{\infty} a[i] \cdot p^i$ , where the  $a[i]$  lie in  $-(p-1)/2, \dots, (p-1)/2$ .

BalancedPAdicInteger(p:Integer) == InnerPAdicInteger(p,false$Boolean)

```

---

— BPADIC.dotabb —

```

"BPADIC" [color="#88FF44",href="bookvol10.3.pdf#nameddest=BPADIC"]
"PADICCT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PADICCT"]
"BPADIC" -> "PADICCT"

```

---

### 3.3 domain BPADICRT BalancedPAdicRational

— BalancedPAdicRational.input —

```

)set break resume
)sys rm -f BalancedPAdicRational.output
)spool BalancedPAdicRational.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show BalancedPAdicRational
--R BalancedPAdicRational p: Integer is a domain constructor
--R Abbreviation for BalancedPAdicRational is BPADICRT
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for BPADICRT
--R

```

```

--R----- Operations -----
--R ?? : (Fraction Integer,%) -> %      ?? : (%,Fraction Integer) -> %
--R ?? : (%,%) -> %                    ?? : (Integer,%) -> %
--R ?? : (PositiveInteger,%) -> %      ***? : (%,Integer) -> %
--R ***? : (%,PositiveInteger) -> %    ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                  -? : % -> %
--R ?/? : (%,%) -> %                  ?? : (%,%) -> Boolean
--R 1 : () -> %                       0 : () -> %
--R ?? : (%,Integer) -> %             ?? : (%,PositiveInteger) -> %
--R associates? : (%,%) -> Boolean    coerce : Fraction Integer -> %
--R coerce : % -> %                   coerce : Integer -> %
--R coerce : % -> OutputForm          denominator : % -> %
--R factor : % -> Factored %          gcd : List % -> %
--R gcd : (%,%) -> %                  hash : % -> SingleInteger
--R inv : % -> %                       latex : % -> String
--R lcm : List % -> %                  lcm : (%,%) -> %
--R numerator : % -> %                 one? : % -> Boolean
--R prime? : % -> Boolean              ?quo? : (%,%) -> %
--R recip : % -> Union(%, "failed")    ?rem? : (%,%) -> %
--R removeZeroes : (Integer,%) -> %    removeZeroes : % -> %
--R sample : () -> %                  sizeLess? : (%,%) -> Boolean
--R squareFree : % -> Factored %       squareFreePart : % -> %
--R unit? : % -> Boolean               unitCanonical : % -> %
--R zero? : % -> Boolean               ?~=? : (%,%) -> Boolean
--R ?? : (%,BalancedPAdicInteger p) -> %
--R ?? : (BalancedPAdicInteger p,%) -> %
--R ?? : (NonNegativeInteger,%) -> %
--R ***? : (%,NonNegativeInteger) -> %
--R ?/? : (BalancedPAdicInteger p,BalancedPAdicInteger p) -> %
--R ?<? : (%,%) -> Boolean if BalancedPAdicInteger p has ORDSET
--R ?<=? : (%,%) -> Boolean if BalancedPAdicInteger p has ORDSET
--R ??? : (%,%) -> Boolean if BalancedPAdicInteger p has ORDSET
--R ?>=? : (%,%) -> Boolean if BalancedPAdicInteger p has ORDSET
--R D : (%,(BalancedPAdicInteger p -> BalancedPAdicInteger p)) -> %
--R D : (%,(BalancedPAdicInteger p -> BalancedPAdicInteger p),NonNegativeInteger) -> %
--R D : (%,List Symbol,List NonNegativeInteger) -> % if BalancedPAdicInteger p has PDRING SYMBOL
--R D : (%,Symbol,NonNegativeInteger) -> % if BalancedPAdicInteger p has PDRING SYMBOL
--R D : (%,List Symbol) -> % if BalancedPAdicInteger p has PDRING SYMBOL
--R D : (%,Symbol) -> % if BalancedPAdicInteger p has PDRING SYMBOL
--R D : (%,NonNegativeInteger) -> % if BalancedPAdicInteger p has DIFRING
--R D : % -> % if BalancedPAdicInteger p has DIFRING
--R ?? : (%,NonNegativeInteger) -> %
--R abs : % -> % if BalancedPAdicInteger p has OINTDOM
--R approximate : (%,Integer) -> Fraction Integer
--R ceiling : % -> BalancedPAdicInteger p if BalancedPAdicInteger p has INS
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%, "failed") if $ has CHARNZ and BalancedPAdicInteger p has PFECA
--R coerce : Symbol -> % if BalancedPAdicInteger p has RETRACT SYMBOL
--R coerce : BalancedPAdicInteger p -> %
--R conditionP : Matrix % -> Union(Vector %, "failed") if $ has CHARNZ and BalancedPAdicInteger

```

```

--R continuedFraction : % -> ContinuedFraction Fraction Integer
--R convert : % -> DoubleFloat if BalancedPAdicInteger p has REAL
--R convert : % -> Float if BalancedPAdicInteger p has REAL
--R convert : % -> InputForm if BalancedPAdicInteger p has KONVERT INFORM
--R convert : % -> Pattern Float if BalancedPAdicInteger p has KONVERT PATTERN FLOAT
--R convert : % -> Pattern Integer if BalancedPAdicInteger p has KONVERT PATTERN INT
--R denom : % -> BalancedPAdicInteger p
--R differentiate : (%,(BalancedPAdicInteger p -> BalancedPAdicInteger p)) -> %
--R differentiate : (%,(BalancedPAdicInteger p -> BalancedPAdicInteger p),NonNegativeInteger) -> %
--R differentiate : (%,(List Symbol,List NonNegativeInteger) -> % if BalancedPAdicInteger p has PDRING SYMBOL
--R differentiate : (%,(Symbol,NonNegativeInteger) -> % if BalancedPAdicInteger p has PDRING SYMBOL
--R differentiate : (%,(List Symbol) -> % if BalancedPAdicInteger p has PDRING SYMBOL
--R differentiate : (%,(Symbol) -> % if BalancedPAdicInteger p has PDRING SYMBOL
--R differentiate : (%,(NonNegativeInteger) -> % if BalancedPAdicInteger p has DIFRING
--R differentiate : % -> % if BalancedPAdicInteger p has DIFRING
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R ?.? : (%,(BalancedPAdicInteger p) -> % if BalancedPAdicInteger p has ELTAB(BPADIC p,BPADIC p)
--R euclideanSize : % -> NonNegativeInteger
--R eval : (%,(Symbol,BalancedPAdicInteger p) -> % if BalancedPAdicInteger p has IEVALAB(SYMBOL,BPADIC p)
--R eval : (%,(List Symbol,List BalancedPAdicInteger p) -> % if BalancedPAdicInteger p has IEVALAB(SYMBOL
--R eval : (%,(List Equation BalancedPAdicInteger p) -> % if BalancedPAdicInteger p has EVALAB BPADIC p
--R eval : (%,(Equation BalancedPAdicInteger p) -> % if BalancedPAdicInteger p has EVALAB BPADIC p
--R eval : (%,(BalancedPAdicInteger p,BalancedPAdicInteger p) -> % if BalancedPAdicInteger p has EVALAB B
--R eval : (%,(List BalancedPAdicInteger p,List BalancedPAdicInteger p) -> % if BalancedPAdicInteger p ha
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % if Balanced
--R factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % i
--R floor : % -> BalancedPAdicInteger p if BalancedPAdicInteger p has INS
--R fractionPart : % -> % if BalancedPAdicInteger p has EUCDOM
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolyn
--R init : () -> % if BalancedPAdicInteger p has STEP
--R map : ((BalancedPAdicInteger p -> BalancedPAdicInteger p),%) -> %
--R max : (%,%) -> % if BalancedPAdicInteger p has ORDSET
--R min : (%,%) -> % if BalancedPAdicInteger p has ORDSET
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R negative? : % -> Boolean if BalancedPAdicInteger p has OINTDOM
--R nextItem : % -> Union(%,"failed") if BalancedPAdicInteger p has STEP
--R numer : % -> BalancedPAdicInteger p
--R patternMatch : (%,(Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchResult(Float,%) if Balan
--R patternMatch : (%,(Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMatchResult(Integer,%) if
--R positive? : % -> Boolean if BalancedPAdicInteger p has OINTDOM
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R random : () -> % if BalancedPAdicInteger p has INS
--R reducedSystem : Matrix % -> Matrix BalancedPAdicInteger p
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix BalancedPAdicInteger p,vec: Vector Balance
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) if BalancedPA
--R reducedSystem : Matrix % -> Matrix Integer if BalancedPAdicInteger p has LINEXP INT

```



```

--R retract : % -> Integer if BalancedPAdicInteger p has RETRACT INT
--R retract : % -> Fraction Integer if BalancedPAdicInteger p has RETRACT INT
--R retract : % -> Symbol if BalancedPAdicInteger p has RETRACT SYMBOL
--R retract : % -> BalancedPAdicInteger p
--R retractIfCan : % -> Union(Integer,"failed") if BalancedPAdicInteger p has RETRACT INT
--R retractIfCan : % -> Union(Fraction Integer,"failed") if BalancedPAdicInteger p has RETRACT INT
--R retractIfCan : % -> Union(Symbol,"failed") if BalancedPAdicInteger p has RETRACT SYMBOL
--R retractIfCan : % -> Union(BalancedPAdicInteger p,"failed")
--R sign : % -> Integer if BalancedPAdicInteger p has OINTDOM
--R solveLinearPolynomialEquation : (List SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> List SparseUnivariatePolynomial %
--R squareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial %
--R subtractIfCan : (%,%) -> Union(%, "failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R wholePart : % -> BalancedPAdicInteger p if BalancedPAdicInteger p has EUCDOM
--R
--E 1

)spool
)lisp (bye)

```

---

— **BalancedPAdicRational.help** —

```

=====
BalancedPAdicRational examples
=====

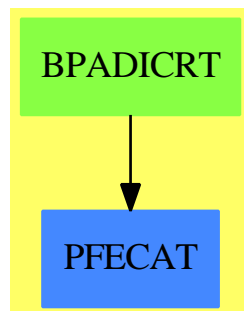
```

```

See Also:
o )show BalancedPAdicRational

```

### 3.3.1 BalancedPAdicRational (BPADICRT)



See

- ⇒ “InnerPAdicInteger” (IPADIC) 10.24.1 on page 1258
- ⇒ “PAdicInteger” (PADIC) 17.1.1 on page 1841
- ⇒ “BalancedPAdicInteger” (BPADIC) 3.2.1 on page 240
- ⇒ “PAdicRationalConstructor” (PADICRC) 17.3.1 on page 1850
- ⇒ “PAdicRational” (PADICRAT) 17.2.1 on page 1845

**Exports:**

|                               |                            |                    |
|-------------------------------|----------------------------|--------------------|
| 0                             | 1                          | abs                |
| approximate                   | associates?                | ceiling            |
| characteristic                | charthRoot                 | coerce             |
| conditionP                    | continuedFraction          | convert            |
| D                             | denom                      | denominator        |
| differentiate                 | divide                     | ?.?                |
| euclideanSize                 | eval                       | expressIdealMember |
| exquo                         | extendedEuclidean          | factor             |
| factorPolynomial              | factorSquareFreePolynomial | floor              |
| fractionPart                  | gcd                        | gcdPolynomial      |
| hash                          | init                       | inv                |
| latex                         | lcm                        | map                |
| max                           | min                        | multiEuclidean     |
| negative?                     | nextItem                   | numer              |
| numerator                     | one?                       | patternMatch       |
| positive?                     | prime?                     | principalIdeal     |
| random                        | recip                      | reducedSystem      |
| removeZeroes                  | retract                    | retractIfCan       |
| sample                        | sign                       | sizeLess?          |
| solveLinearPolynomialEquation | squareFree                 | squareFreePart     |
| squareFreePolynomial          | subtractIfCan              | unit?              |
| unitCanonical                 | unitNormal                 | wholePart          |
| zero?                         | ?*?                        | ?**?               |
| ?+?                           | ?-?                        | -?                 |
| ?/?                           | ?=?                        | ?^?                |
| ?~=?                          | ?<?                        | ?<=?               |
| ?>?                           | ?>=?                       | ?quo?              |
| ?rem?                         |                            |                    |

— domain BPADICRT BalancedPAdicRational —

```

)abbrev domain BPADICRT BalancedPAdicRational
++ Author: Clifton J. Williamson
++ Date Created: 15 May 1990
++ Date Last Updated: 15 May 1990
++ Keywords: p-adic, complementation
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:

```

```

++ Keywords: p-adic, completion
++ Examples:
++ References:
++ Description:
++ Stream-based implementation of  $\mathbb{Q}_p$ : numbers are represented as
++  $\sum(i = k.., a[i] * p^i)$ , where the  $a[i]$  lie in  $-(p-1)/2, \dots, (p-1)/2$ .

```

```

BalancedPAdicRational(p:Integer) ==
  PAdicRationalConstructor(p,BalancedPAdicInteger p)

```

---

— BPADICRT.dotabb —

```

"BPADICRT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=BPADICRT"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"BPADICRT" -> "PFECAT"

```

---

### 3.4 domain BFUNCT BasicFunctions

— BasicFunctions.input —

```

)set break resume
)sys rm -f BasicFunctions.output
)spool BasicFunctions.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show BasicFunctions
--R BasicFunctions is a domain constructor
--R Abbreviation for BasicFunctions is BFUNCT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for BFUNCT
--R
--R----- Operations -----
--R ?=? : (%,% ) -> Boolean          bfKeys : () -> List Symbol
--R coerce : % -> OutputForm        hash : % -> SingleInteger
--R latex : % -> String             ?~=? : (%,% ) -> Boolean
--R bfEntry : Symbol -> Record(zeros: Stream DoubleFloat,ones: Stream DoubleFloat,singularit:
--R

```

```
--E 1
```

```
)spool
)lisp (bye)
```

```
_____
```

```
— BasicFunctions.help —
```

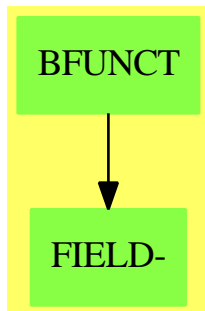
```
=====
BasicFunctions examples
=====
```

```
See Also:
```

```
o )show BasicFunctions
```

```
_____
```

### 3.4.1 BasicFunctions (BFUNCT)



#### Exports:

```
bfEntry bfKeys coerce hash latex
?~=?    ?=?
```

```
— domain BFUNCT BasicFunctions —
```

```
)abbrev domain BFUNCT BasicFunctions
++ Author: Brian Dupee
++ Date Created: August 1994
++ Date Last Updated: April 1996
++ Basic Operations: bfKeys, bfEntry
++ Description:
++ A Domain which implements a table containing details of
++ points at which particular functions have evaluation problems.
```

```

BasicFunctions(): E == I where
  DF ==> DoubleFloat
  SDF ==> Stream DoubleFloat
  RS ==> Record(zeros: SDF, ones: SDF, singularities: SDF)

E ==> SetCategory with
  bfKeys:() -> List Symbol
    ++ bfKeys() returns the names of each function in the
    ++ \axiomType{BasicFunctions} table
  bfEntry:Symbol -> RS
    ++ bfEntry(k) returns the entry in the \axiomType{BasicFunctions} table
    ++ corresponding to \spad{k}
  finiteAggregate

I ==> add

Rep := Table(Symbol,RS)
import Rep, SDF

f(x:DF):DF ==
  positive?(x) => -x
  -x+1

bf():$ ==
  import RS
  dpi := pi()$DF
  ndpi:SDF := map(x1+>->x1*dpi,(z := generate(f,0))) -- [n pi for n in Z]
  n1dpi:SDF := map(x1+>->-(2*(x1)-1)*dpi/2,z) -- [(n+1) pi /2]
  n2dpi:SDF := map(x1+>->2*x1*dpi,z) -- [2 n pi for n in Z]
  n3dpi:SDF := map(x1+>->-(4*(x1)-1)*dpi/4,z)
  n4dpi:SDF := map(x1+>->-(4*(x1)-1)*dpi/2,z)
  sinEntry:RS := [ndpi, n4dpi, empty()$SDF]
  cosEntry:RS := [n1dpi, n2dpi, esdf := empty()$SDF]
  tanEntry:RS := [ndpi, n3dpi, n1dpi]
  asinEntry:RS := [construct([0$DF])$SDF,
    construct([float(8414709848078965,-16,10)$DF]), esdf]
  acosEntry:RS := [construct([1$DF])$SDF,
    construct([float(54030230586813977,-17,10)$DF]), esdf]
  atanEntry:RS := [construct([0$DF])$SDF,
    construct([float(15574077246549023,-16,10)$DF]), esdf]
  secEntry:RS := [esdf, n2dpi, n1dpi]
  cscEntry:RS := [esdf, n4dpi, ndpi]
  cotEntry:RS := [n1dpi, n3dpi, ndpi]
  logEntry:RS := [construct([1$DF])$SDF,esdf, construct([0$DF])$SDF]
  entryList:List(Record(key:Symbol,entry:RS)) :=
    [[sin@Symbol, sinEntry], [cos@Symbol, cosEntry],
     [tan@Symbol, tanEntry], [sec@Symbol, secEntry],
     [csc@Symbol, cscEntry], [cot@Symbol, cotEntry],
     [asin@Symbol, asinEntry], [acos@Symbol, acosEntry],

```

```

[atan@Symbol, atanEntry], [log@Symbol, logEntry]]
construct(entryList)$Rep

bfKeys():List Symbol == keys(bf())$Rep

bfEntry(k:Symbol):RS == qelt(bf(),k)$Rep

-----

— BFUNCT.dotabb —

"BFUNCT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=BFUNCT"]
"FIELD-" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FIELD"]
"BFUNCT" -> "FIELD-"

```

### 3.5 domain BOP BasicOperator

```

— BasicOperator.input —

)set break resume
)sys rm -f BasicOperator.output
)spool BasicOperator.output
)set message test on
)set message auto off
)clear all
--S 1 of 18
y := operator 'y
--R
--R
--R (1) y
--R
--R                                          Type: BasicOperator
--E 1

--S 2 of 18
deq := D(y x, x, 2) + D(y x, x) + y x = 0
--R
--R
--R      , ,
--R (2) y '(x) + y '(x) + y(x)= 0
--R
--R                                          Type: Equation Expression Integer
--E 2

```







```

--S 15 of 18
properties y
--R
--R
--R (15) table("use"= NONE)
--R
--R                                         Type: AssociationList(String,None)
--E 15

--S 16 of 18
property(y, "use") :: None pretend String
--R
--R
--R (16) "unknown function"
--R
--R                                         Type: String
--E 16

--S 17 of 18
deleteProperty!(y, "use")
--R
--R
--R (17) y
--R
--R                                         Type: BasicOperator
--E 17

--S 18 of 18
properties y
--R
--R
--R (18) table()
--R
--R                                         Type: AssociationList(String,None)
--E 18
)spool
)lisp (bye)

```

---

— BasicOperator.help —

```

=====
BasicOperator examples
=====

```

A basic operator is an object that can be symbolically applied to a list of arguments from a set, the result being a kernel over that set or an expression.

You create an object of type BasicOperator by using the operator operation. This first form of this operation has one argument and it

must be a symbol. The symbol should be quoted in case the name has been used as an identifier to which a value has been assigned.

A frequent application of BasicOperator is the creation of an operator to represent the unknown function when solving a differential equation.

Let  $y$  be the unknown function in terms of  $x$ .

```
y := operator 'y
y
Type: BasicOperator
```

This is how you enter the equation  $y'' + y' + y = 0$ .

```
deq := D(y x, x, 2) + D(y x, x) + y x = 0
y''(x) + y'(x) + y(x) = 0
Type: Equation Expression Integer
```

To solve the above equation, enter this.

```
solve(deq, y, x)
[particular= 0, basis= [cos(-----)%ex/3, %ex/3 sin(-----)]]
2 2
Type: Union(Record(particular: Expression Integer,
basis: List Expression Integer),...)
```

Use the single argument form of BasicOperator (as above) when you intend to use the operator to create functional expressions with an arbitrary number of arguments

Nary means an arbitrary number of arguments can be used in the functional expressions.

```
nary? y
true
Type: Boolean
```

```
unary? y
false
Type: Boolean
```

Use the two-argument form when you want to restrict the number of arguments in the functional expressions created with the operator.

This operator can only be used to create functional expressions with one argument.

```
opOne := operator('opOne, 1)
opOne
Type: BasicOperator
```

```
nary? opOne
false
Type: Boolean
```

```
unary? opOne
true
Type: Boolean
```

Use `arity` to learn the number of arguments that can be used. It returns "false" if the operator is nary.

```
arity opOne
1
Type: Union(NonNegativeInteger,...)
```

Use `name` to learn the name of an operator.

```
name opOne
opOne
Type: Symbol
```

Use `is?` to learn if an operator has a particular name.

```
is?(opOne, 'z2)
false
Type: Boolean
```

You can also use a string as the name to be tested against.

```
is?(opOne, "opOne")
true
Type: Boolean
```

You can attached named properties to an operator. These are rarely used at the top-level of the Axiom interactive environment but are used with Axiom library source code.

By default, an operator has no properties.

```
properties y
table()
Type: AssociationList(String,None)
```

The interface for setting and getting properties is somewhat awkward because the property values are stored as values of type `None`.

Attach a property by using `setProperty`.

```
setProperty(y, "use", "unknown function" :: None )
y
                                     Type: BasicOperator

properties y
  table("use"=NONE)
                                     Type: AssociationList(String,None)
```

We know the property value has type `String`.

```
property(y, "use") :: None pretend String
  "unknown function"
                                     Type: String
```

Use `deleteProperty!` to destructively remove a property.

```
deleteProperty!(y, "use")
y
                                     Type: BasicOperator

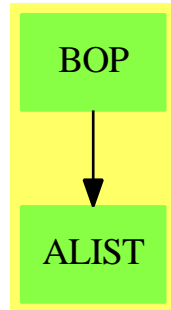
properties y
  table()
                                     Type: AssociationList(String,None)
```

See Also

- o `)help Expression`
- o `)help Kernel`
- o `)show BasicOperator`

---

### 3.5.1 BasicOperator (BOP)



#### Exports:

|                 |             |          |            |          |
|-----------------|-------------|----------|------------|----------|
| arity           | assert      | coerce   | comparison | copy     |
| deleteProperty! | display     | display  | display    | equality |
| has?            | hash        | input    | input      | is?      |
| latex           | max         | min      | name       | nary?    |
| nullary?        | operator    | operator | properties | property |
| setProperty     | setProperty | unary?   | weight     | weight   |
| ?~=?            | ?<?         | ?<=?     | ?=?        | ?>?      |
| ?>=?            |             |          |            |          |

— domain BOP BasicOperator —

```

)abbrev domain BOP BasicOperator
++ Author: Manuel Bronstein
++ Date Created: 22 March 1988
++ Date Last Updated: 11 October 1993
++ Keywords: operator, kernel.
++ Description:
++ Basic system operators.
++ A basic operator is an object that can be applied to a list of
++ arguments from a set, the result being a kernel over that set.

```

```

BasicOperator(): Exports == Implementation where

```

```

  O ==> OutputForm
  P ==> AssociationList(String, None)
  L ==> List Record(key:String, entry:None)
  SEX ==> InputForm
-- some internal properties
LESS? ==> "%less?"
EQUAL? ==> "%equal?"
WEIGHT ==> "%weight"
DISPLAY ==> "%display"
SEXPR ==> "%input"

```

```

Exports ==> OrderedSet with

```

```

name      : $ -> Symbol
  ++ name(op) returns the name of op.
properties: $ -> P
  ++ properties(op) returns the list of all the properties
  ++ currently attached to op.
copy      : $ -> $
  ++ copy(op) returns a copy of op.
operator  : Symbol -> $
  ++ operator(f) makes f into an operator with arbitrary arity.
operator  : (Symbol, NonNegativeInteger) -> $
  ++ operator(f, n) makes f into an n-ary operator.
arity     : $ -> Union(NonNegativeInteger, "failed")
  ++ arity(op) returns n if op is n-ary, and
  ++ "failed" if op has arbitrary arity.
nullary?  : $ -> Boolean
  ++ nullary?(op) tests if op is nullary.
unary?    : $ -> Boolean
  ++ unary?(op) tests if op is unary.
nary?     : $ -> Boolean
  ++ nary?(op) tests if op has arbitrary arity.
weight    : $ -> NonNegativeInteger
  ++ weight(op) returns the weight attached to op.
weight    : ($, NonNegativeInteger) -> $
  ++ weight(op, n) attaches the weight n to op.
equality   : ($, ($, $) -> Boolean) -> $
  ++ equality(op, foo?) attaches foo? as the "%equal?" property
  ++ to op. If op1 and op2 have the same name, and one of them
  ++ has an "%equal?" property f, then \spad{f(op1, op2)} is called to
  ++ decide whether op1 and op2 should be considered equal.
comparison : ($, ($, $) -> Boolean) -> $
  ++ comparison(op, foo?) attaches foo? as the "%less?" property
  ++ to op. If op1 and op2 have the same name, and one of them
  ++ has a "%less?" property f, then \spad{f(op1, op2)} is called to
  ++ decide whether \spad{op1 < op2}.
display    : $ -> Union(List 0 -> 0, "failed")
  ++ display(op) returns the "%display" property of op if
  ++ it has one attached, and "failed" otherwise.
display    : ($, List 0 -> 0) -> $
  ++ display(op, foo) attaches foo as the "%display" property
  ++ of op. If op has a "%display" property f, then \spad{op(a1,...,an)}
  ++ gets converted to OutputForm as \spad{f(a1,...,an)}.
display    : ($, 0 -> 0) -> $
  ++ display(op, foo) attaches foo as the "%display" property
  ++ of op. If op has a "%display" property f, then \spad{op(a)}
  ++ gets converted to OutputForm as \spad{f(a)}.
  ++ Argument op must be unary.
input      : ($, List SEX -> SEX) -> $
  ++ input(op, foo) attaches foo as the "%input" property
  ++ of op. If op has a "%input" property f, then \spad{op(a1,...,an)}
  ++ gets converted to InputForm as \spad{f(a1,...,an)}.

```

```

input      : $ -> Union(List SEX -> SEX, "failed")
++ input(op) returns the "%input" property of op if
++ it has one attached, "failed" otherwise.
is?        : ($, Symbol) -> Boolean
++ is?(op, s) tests if the name of op is s.
has?       : ($, String) -> Boolean
++ has?(op, s) tests if property s is attached to op.
assert     : ($, String) -> $
++ assert(op, s) attaches property s to op.
++ Argument op is modified "in place", i.e. no copy is made.
deleteProperty_!: ($, String) -> $
++ deleteProperty!(op, s) unattaches property s from op.
++ Argument op is modified "in place", i.e. no copy is made.
property   : ($, String) -> Union(None, "failed")
++ property(op, s) returns the value of property s if
++ it is attached to op, and "failed" otherwise.
setProperty : ($, String, None) -> $
++ setProperty(op, s, v) attaches property s to op,
++ and sets its value to v.
++ Argument op is modified "in place", i.e. no copy is made.
setProperties : ($, P) -> $
++ setProperties(op, l) sets the property list of op to l.
++ Argument op is modified "in place", i.e. no copy is made.

Implementation ==> add
-- if nargs < 0 then the operator has variable arity.
Rep := Record(opname:Symbol, nargs:SingleInteger, props:P)

oper: (Symbol, SingleInteger, P) -> $

is?(op, s)          == name(op) = s
name op             == op.opname
properties op        == op.props
setProperties(op, l) == (op.props := l; op)
operator s           == oper(s, -1::SingleInteger, table())
operator(s, n)       == oper(s, n::Integer::SingleInteger, table())
property(op, name)   == search(name, op.props)
assert(op, s)        == setProperty(op, s, NIL$Lisp)
has?(op, name)       == key?(name, op.props)
oper(se, n, prop)    == [se, n, prop]
weight(op, n)        == setProperty(op, WEIGHT, n pretend None)
nullary? op          == zero?(op.nargs)
-- unary? op         == one?(op.nargs)
unary? op            == ((op.nargs) = 1)
nary? op             == negative?(op.nargs)
equality(op, func)   == setProperty(op, EQUAL?, func pretend None)
comparison(op, func) == setProperty(op, LESS?, func pretend None)
display(op:$, f:0 -> 0) == display(op, (x1:List(0)):0 +-> f first x1)
deleteProperty_!(op, name) == (remove_!(name, properties op); op)
setProperty(op, name, valu) == (op.props.name := valu; op)

```

```

coerce(op:$):OutputForm      == name(op)::OutputForm
input(op:$, f:List SEX -> SEX) == setProperty(op, SEXPR, f pretend None)
display(op:$, f:List 0 -> 0)  == setProperty(op, DISPLAY, f pretend None)

display op ==
  (u := property(op, DISPLAY)) case "failed" => "failed"
  (u::None) pretend (List 0 -> 0)

input op ==
  (u := property(op, SEXPR)) case "failed" => "failed"
  (u::None) pretend (List SEX -> SEX)

arity op ==
  negative?(n := op.narg) => "failed"
  convert(n)@Integer :: NonNegativeInteger

copy op ==
  oper(name op, op.narg,
    table([[r.key, r.entry] for r in entries(properties op)@L]$L))

-- property EQUAL? contains a function f: (BOP, BOP) -> Boolean
-- such that f(o1, o2) is true iff o1 = o2
op1 = op2 ==
  (EQ$Lisp)(op1, op2) => true
  name(op1) ^= name(op2) => false
  op1.narg ^= op2.narg => false
  brace(keys properties op1)^=$Set(String) _
    brace(keys properties op2) => false
  (func := property(op1, EQUAL?)) case None =>
    ((func::None) pretend (($, $) -> Boolean)) (op1, op2)
  true

-- property WEIGHT allows one to change the ordering around
-- by default, every operator has weight 1
weight op ==
  (w := property(op, WEIGHT)) case "failed" => 1
  (w::None) pretend NonNegativeInteger

-- property LESS? contains a function f: (BOP, BOP) -> Boolean
-- such that f(o1, o2) is true iff o1 < o2
op1 < op2 ==
  (w1 := weight op1) ^= (w2 := weight op2) => w1 < w2
  op1.narg ^= op2.narg => op1.narg < op2.narg
  name(op1) ^= name(op2) => name(op1) < name(op2)
  n1 := #(k1 := brace(keys(properties op1))$Set(String))
  n2 := #(k2 := brace(keys(properties op2))$Set(String))
  n1 ^= n2 => n1 < n2
  not zero?(n1 := #(d1 := difference(k1, k2))) =>
    n1 ^= (n2 := #(d2 := difference(k2, k1))) => n1 < n2
    inspect(d1) < inspect(d2)

```



```

(func := property(op1, LESS?)) case None =>
  ((func::None) pretend (($, $) -> Boolean)) (op1, op2)
(func := property(op1, EQUAL?)) case None =>
  not(((func::None) pretend (($, $) -> Boolean)) (op1, op2))
false

```

---

— BOP.dotabb —

```

"BOP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=BOP"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"BOP" -> "ALIST"

```

---

### 3.6 domain BSD BasicStochasticDifferential

---

— BasicStochasticDifferential.input —

```

)set break resume
)sys rm -f BasicStochasticDifferential.output
)spool BasicStochasticDifferential.output
)set message test on
)set message auto off
)clear all

--S 1 of 30
print copyBSD()
--R
--R  []
--R
--R
--R                                          Type: Void
--E 1

--S 2 of 30
print copyIto()
--R
--R  table()
--R
--R                                          Type: Void
--E 2

--S 3 of 30
dX:=introduce!(X,dX) -- dX is a new stochastic differential
--R

```

```

--R
--R (3) dX
--R                                     Type: Union(BasicStochasticDifferential,...)
--E 3

--S 4 of 30
print copyBSD()
--R
--R [dX]
--R                                     Type: Void
--E 4

--S 5 of 30
print copyIto()
--R
--R table(X= dX)
--R                                     Type: Void
--E 5

--S 6 of 30
getSmgl(dX)
--R
--R
--R (6) X
--R                                     Type: Union(Symbol,...)
--E 6

--S 7 of 30
print dX
--R
--R dX
--R                                     Type: Void
--E 7

--S 8 of 30
print domainOf(dX)
--R
--R BasicStochasticDifferential()
--R                                     Type: Void
--E 8

--S 9 of 30
print d X
--R
--R dX
--R                                     Type: Void
--E 9

--S 10 of 30
print ((d$BSD) X)

```







```

--E 26

--S 27 of 30
[introduce!(A[i],dA[i]) for i in 1..2]
--R
--R
--R (23) [dA ,dA ]
--R      1 2
--R                                     Type: List Union(BasicStochasticDifferential,"failed")
--E 27

--S 28 of 30
print copyBSD()
--R
--R [dA ,dA ,dX,dt]
--R   1 2
--R                                     Type: Void
--E 28

--S 29 of 30
print copyIto()
--R
--R table(A = dA ,A = dA ,t= dt,X= dX)
--R          2 2 1 1
--R                                     Type: Void
--E 29

--S 30 of 30
[d A[i] for i in 1..2]
--R
--R
--R (26) [dA ,dA ]
--R      1 2
--R                                     Type: List Union(BasicStochasticDifferential,Integer)
--E 30

)spool
)lisp (bye)

```

---

— BasicStochasticDifferential.help —

```

=====
BasicStochasticDifferential examples
=====

```

Based on Symbol: a domain of symbols representing basic stochastic

differentials, used in `StochasticDifferential(R)` in the underlying sparse multivariate polynomial representation.

We create new BSD only by coercion from `Symbol` using a special function `introduce!` first of all to add to a private set `SDset`. We allow a separate function `convertIfCan` which will check whether the argument has previously been declared as a BSD.

The `copyBSD()` returns `setBSD` as a list of BSD.

Initially the BSD table of symbols is empty:

```
print copyBSD()
[]
```

as is the Ito table. The `copyIto()` returns the table relating semimartingales to basic stochastic differentials.

```
print copyIto()
table()
```

We introduce a new stochastic differential:

```
dX:=introduce!(X,dX) -- dX is a new stochastic differential
dX
```

and now it is in the BSD table of symbols:

```
print copyBSD()
[dX]
```

and the Ito table:

```
print copyIto()
table(X= dX)
```

The `getSmgl(bsd)` returns the semimartingale `S` related to the basic stochastic differential `bsd` by `introduce!`:

```
getSmgl(dX)
X
```

Note that the `dX` symbol is of type `BasicStochasticDifferential`:

```
print domainOf(dX)
BasicStochasticDifferential()
```

The `d` function, `d(X)` returns `dX` if `tableIto(X)=dX` otherwise 0:

```
print d X
dX
```

and we can specify the domain:

```
print ((d$BSD) X)
dX
```

which clearly is of domain `BasicStochasticDifferential`

```
print domainOf((d$BSD) X)
```

```
BasicStochasticDifferential()
```

The `introduce!(X,dX)` returns `dX` as BSD if it isn't already in BSD

```
introduce!(t,dt) -- dt is a new stochastic differential
dt
```

Now we can see that `dt` is in the BSD table:

```
print copyBSD()
[dX,dt]
```

and the Ito table:

```
print copyIto()
table(t= dt,X= dX)
```

You cannot repeat an `introduce!` for an existing entry:

```
introduce!(t,dt)
"failed"
```

Regular symbols can be added, especially with subscripts:

```
[introduce!(A[i],dA[i]) for i in 1..2]
[dA ,dA ]
 1    2
```

And we can see them in the BSD table:

```
print copyBSD()
[dA ,dA ,dX,dt]
 1    2
```

and the Ito table:

```
print copyIto()
table(A = dA ,A = dA ,t= dt,X= dX)
      2      2 1      1
```

The `d` function can extract these symbols:

```
[d A[i] for i in 1..2]
[dA ,dA ]
 1    2
```

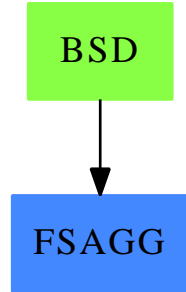
See Also:

```
o )show BasicStochasticDifferential
```

---



### 3.6.1 BasicStochasticDifferential (BSD)



See

⇒ “StochasticDifferential” (SD) 20.29.1 on page 2530

#### Exports:

|            |         |              |         |
|------------|---------|--------------|---------|
| coerce     | convert | convertIfCan | copyBSD |
| copyIto    | d       | getSmgl      | hash    |
| introduce! | latex   | max          | min     |
| ?~=?       | ?<?     | ?<=?         | ?=?     |
| ?>?        | ?>=?    |              |         |

— domain BSD BasicStochasticDifferential —

```

)abbrev domain BSD BasicStochasticDifferential
++ Author: Wilfrid S. Kendall
++ Date Last Updated: July 26, 1999
++ Related Domains: StochasticDifferential(R)
++ Keywords: stochastic differential, semimartingale.
++ References: Ito (1975), Kendall (1991a,b; 1993a,b; 1999a,b).
++ Description:
++ Based on Symbol: a domain of symbols representing basic stochastic
++ differentials, used in StochasticDifferential(R) in the underlying
++ sparse multivariate polynomial representation.
++
++ We create new BSD only by coercion from Symbol using a special
++ function introduce! first of all to add to a private set SDset.
++ We allow a separate function convertIfCan which will check whether the
++ argument has previously been declared as a BSD.
  
```

```

BasicStochasticDifferential(): Category == Implementation where
  INT ==> Integer
  OF ==> OutputForm
  Category ==> OrderedSet with
    ConvertibleTo(Symbol)
  
```

```

convertIfCan: Symbol -> Union(%, "failed")
++ convertIfCan(ds) transforms \axiom{dX} into a \axiom{BSD}
  
```

```

++ if possible (if \axiom{introduce(X,dX)} has
++ been invoked previously).

convert: Symbol -> %
++ convert(dX) transforms \axiom{dX} into a \axiom{BSD}
++ if possible and otherwise produces an error.

introduce!: (Symbol,Symbol) -> Union(%, "failed")
++ introduce!(X,dX) returns \axiom{dX} as \axiom{BSD} if it
++ isn't already in \axiom{BSD}
++
++X introduce!(t,dt) -- dt is a new stochastic differential
++X copyBSD()

d: Symbol -> Union(%,INT)
++ d(X) returns \axiom{dX} if \axiom{tableIto(X)=dX}
++ and otherwise returns \axiom{0}

copyBSD(): -> List %
++ copyBSD() returns \axiom{setBSD} as a list of \axiom{BSD}.
++
++X introduce!(t,dt) -- dt is a new stochastic differential
++X copyBSD()

copyIto(): -> Table(Symbol,%)
++ copyIto() returns the table relating semimartingales
++ to basic stochastic differentials.
++
++X introduce!(t,dt) -- dt is a new stochastic differential
++X copyIto()

getSmgl: % -> Union(Symbol,"failed")
++ getSmgl(bsd) returns the semimartingale \axiom{S} related
++ to the basic stochastic differential \axiom{bsd} by
++ \axiom{introduce!}
++
++X introduce!(t,dt) -- dt is a new stochastic differential
++X getSmgl(dt::BSD)

Implementation ==> Symbol add

Rep := Symbol

setBSD := empty()$Set(Symbol)
tableIto:Table(Symbol,%) := table()
tableBSD:Table(%,Symbol) := table()

convertIfCan(ds:Symbol):Union(%, "failed") ==
not(member?(ds,setBSD)) => "failed"
ds::%

```

```

convert(ds:Symbol):% ==
  (du:=convertIfCan(ds))
  case "failed" =>
    print(hconcat(ds::Symbol::OF,
      message(" is not a stochastic differential")$OF))
    error "above causes failure in convert$BSD"
  du

introduce!(X,dX) ==
  member?(dX,setBSD) => "failed"
  insert!(dX,setBSD)
  tableBSD(dX::%) := X
  tableIto(X) := dX::%

d(X) ==
  search(X,tableIto) case "failed" => 0::INT
  tableIto(X)

copyBSD() == [ds::% for ds in members(setBSD)]
copyIto() == tableIto
getSmgl(ds::%):Union(Symbol,"failed") == tableBSD(ds)

```

---

— BSD.dotabb —

```

"BSD" [color="#88FF44",href="bookvol10.3.pdf#nameddest=BSD"]
"FSAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FSAGG"]
"BSD" -> "FSAGG"

```

---

### 3.7 domain BINARY BinaryExpansion

— BinaryExpansion.input —

```

)set break resume
)sys rm -f BinaryExpansion.output
)spool BinaryExpansion.output
)set message test on
)set message auto off
)clear all
--S 1 of 7

```

```

r := binary(22/7)
--R
--R
--R      ---
--R  (1)  11.001
--R
--R                                          Type: BinaryExpansion
--E 1

--S 2 of 7
r + binary(6/7)
--R
--R
--R  (2)  100
--R
--R                                          Type: BinaryExpansion
--E 2

--S 3 of 7
[binary(1/i) for i in 102..106]
--R
--R
--R  (3)
--R      -----
--R  [0.000000101, 0.000000100111110001000101100101111001110010010101001,
--R      -----
--R      0.000000100111011, 0.000000100111,
--R      -----
--R      0.00000010011010100100001110011111011001010110111100011]
--R
--R                                          Type: List BinaryExpansion
--E 3

--S 4 of 7
binary(1/1007)
--R
--R
--R  (4)
--R  0.
--R  OVERBAR
--R      0000000001000001000101001001011110000011111100001011111001011000111110
--R      1000100111001001100110001100100101011110110100110000000011000011001
--R      1110111000110100010111101001000111101100001010111011100111010111001
--R      100101001011100000001110001111001000000100100100110111001010100111010
--R      001101110110101110001001000001100101101100000010110010111110001010000
--R      010101011010110000011011011101001010111111010111010100110010000101
--R      0011011000100110001000100001000011000111010011110001
--R
--R                                          Type: BinaryExpansion
--E 4

--S 5 of 7
p := binary(1/4)*x**2 + binary(2/3)*x + binary(4/9)
--R

```

```

--R
--R      2      --      -----
--R      (5)  0.01x  + 0.10x + 0.011100
--R
--R                                          Type: Polynomial BinaryExpansion
--E 5

--S 6 of 7
q := D(p, x)
--R
--R
--R      --
--R      (6)  0.1x + 0.10
--R
--R                                          Type: Polynomial BinaryExpansion
--E 6

--S 7 of 7
g := gcd(p, q)
--R
--R
--R      --
--R      (7)  x + 1.01
--R
--R                                          Type: Polynomial BinaryExpansion
--E 7
)spool
)lisp (bye)

```

---

— BinaryExpansion.help —

```

=====
BinaryExpansion examples
=====
All rational numbers have repeating binary expansions.  Operations to
access the individual bits of a binary expansion can be obtained by
converting the value to RadixExpansion(2).  More examples of
expansions are available with

```

The expansion (of type BinaryExpansion) of a rational number is returned by the binary operation.

```

r := binary(22/7)
--
11.001
Type: BinaryExpansion

```

Arithmetic is exact.

```

r + binary(6/7)

```

100

Type: BinaryExpansion

The period of the expansion can be short or long.

```
[binary(1/i) for i in 102..106]
```

```
-----
[0.00000101,
-----
0.000000100111110001000101100101111001110010010101001,
-----
0.000000100111011, 0.000000100111,
-----
0.00000010011010100100001110011111011001010110111100011]
```

Type: List BinaryExpansion

or very long.

```
binary(1/1007)
```

```
-----
0.00000000010000010001010010010111100000111111000010111110010110001111101
-----
000100111001001100110001100100101010111101101001100000000110000110011110
-----
1110001101000101111010010001111011000010101110111001110101110011001010
-----
010111000000011100011110010000001001001001101110010101001110100011011101
-----
101011100010010000011001011011000000101100101111100010100000101010101101
-----
011000001101101110100101011111101011101010011001000001010011011000100110
-----
001000100001000011000111010011110001
-----
```

Type: BinaryExpansion

These numbers are bona fide algebraic objects.

```
p := binary(1/4)*x**2 + binary(2/3)*x + binary(4/9)
```

```
0.01 x^2 + 0.10 x + 0.011100
```

Type: Polynomial BinaryExpansion

```
q := D(p, x)
```

```
0.1 x + 0.10
```

Type: Polynomial BinaryExpansion

```
g := gcd(p, q)
```

```
x+1.01
```

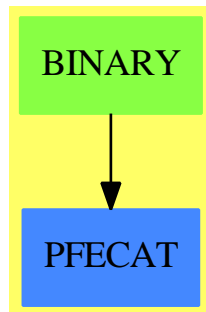
Type: Polynomial BinaryExpansion

See Also:

- o )help DecimalExpansion
- o )help HexadecimalExpansion
- o )show BinaryExpansion

—————▶

### 3.7.1 BinaryExpansion (BINARY)



See

- ⇒ “RadixExpansion” (RADIX) 19.2.1 on page 2165
- ⇒ “DecimalExpansion” (DECIMAL) 5.3.1 on page 451
- ⇒ “HexadecimalExpansion” (HEXADEC) 9.3.1 on page 1108

**Exports:**

|                               |                  |                            |
|-------------------------------|------------------|----------------------------|
| 0                             | 1                | abs                        |
| associates?                   | binary           | ceiling                    |
| characteristic                | charthRoot       | coerce                     |
| conditionP                    | convert          | D                          |
| denom                         | denominator      | differentiate              |
| divide                        | euclideanSize    | eval                       |
| expressIdealMember            | exquo            | extendedEuclidean          |
| factor                        | factorPolynomial | factorSquareFreePolynomial |
| floor                         | fractionPart     | gcd                        |
| gcdPolynomial                 | hash             | init                       |
| inv                           | latex            | lcm                        |
| map                           | max              | min                        |
| multiEuclidean                | negative?        | nextItem                   |
| numer                         | numerator        | one?                       |
| patternMatch                  | positive?        | prime?                     |
| principalIdeal                | random           | recip                      |
| reducedSystem                 | retract          | retractIfCan               |
| sample                        | sign             | sizeLess?                  |
| solveLinearPolynomialEquation | squareFree       | squareFreePart             |
| squareFreePolynomial          | subtractIfCan    | unit?                      |
| unitCanonical                 | unitNormal       | wholePart                  |
| zero?                         | ?*?              | ?**?                       |
| ?+?                           | ?-?              | -?                         |
| ?/?                           | ?=?              | ?^?                        |
| ?~=?                          | ?<?              | ?<=?                       |
| ?>?                           | ?>=?             | ?..?                       |
| ?quo?                         | ?rem?            |                            |

— domain BINARY BinaryExpansion —

```
)abbrev domain BINARY BinaryExpansion
++ Author: Clifton J. Williamson
++ Date Created: April 26, 1990
++ Date Last Updated: May 15, 1991
++ Basic Operations:
++ Related Domains: RadixExpansion
++ Also See:
++ AMS Classifications:
++ Keywords: radix, base, binary
++ Examples:
++ References:
++ Description:
++ This domain allows rational numbers to be presented as repeating
++ binary expansions.
```

```
BinaryExpansion(): Exports == Implementation where
  Exports ==> QuotientFieldCategory(Integer) with
    coerce: % -> Fraction Integer
```



```

++ coerce(b) converts a binary expansion to a rational number.
coerce: % -> RadixExpansion(2)
++ coerce(b) converts a binary expansion to a radix expansion with base 2.
fractionPart: % -> Fraction Integer
++ fractionPart(b) returns the fractional part of a binary expansion.
binary: Fraction Integer -> %
++ binary(r) converts a rational number to a binary expansion.
++
++X binary(22/7)

Implementation ==> RadixExpansion(2) add
binary r == r :: %
coerce(x:%): RadixExpansion(2) == x pretend RadixExpansion(2)

```

---

— BINARY.dotabb —

```

"BINARY" [color="#88FF44",href="bookvol10.3.pdf#nameddest=BINARY"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"BINARY" -> "PFECAT"

```

---

### 3.8 domain BINFILE BinaryFile

— BinaryFile.input —

```

)set break resume
)sys rm -f BinaryFile.output
)spool BinaryFile.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show BinaryFile
--R BinaryFile is a domain constructor
--R Abbreviation for BinaryFile is BINFILE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for BINFILE
--R
--R----- Operations -----
--R ?=? : (%,% ) -> Boolean          close! : % -> %

```

```

--R coerce : % -> OutputForm          flush : % -> Void
--R hash : % -> SingleInteger          iomode : % -> String
--R latex : % -> String                name : % -> FileName
--R open : (FileName,String) -> %      open : FileName -> %
--R position : % -> SingleInteger       read! : % -> SingleInteger
--R reopen! : (% ,String) -> %         ?~=? : (% ,%) -> Boolean
--R position! : (% ,SingleInteger) -> SingleInteger
--R readIfCan! : % -> Union(SingleInteger,"failed")
--R write! : (% ,SingleInteger) -> SingleInteger
--R
--E 1

)spool
)lisp (bye)

```

---

— BinaryFile.help —

```

=====
BinaryFile examples
=====

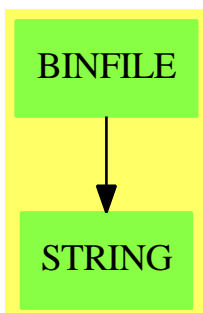
```

```

See Also:
o )show BinaryFile

```

### 3.8.1 BinaryFile (BINFILE)



See

- ⇒ “File” (FILE) 7.2.1 on page 770
- ⇒ “TextFile” (TEXTFILE) 21.5.1 on page 2651
- ⇒ “KeyedAccessFile” (KAFfile) 12.2.1 on page 1377
- ⇒ “Library” (LIB) 13.2.1 on page 1392

**Exports:**

|            |         |          |           |       |
|------------|---------|----------|-----------|-------|
| close!     | coerce  | hash     | iomode    | latex |
| name       | open    | position | position! | read! |
| readIfCan! | reopen! | write!   | ?=?       | ?~=?  |

— domain BINFILE BinaryFile —

```

)abbrev domain BINFILE BinaryFile
++ Author: Barry M. Trager
++ Date Created: 1993
++ Date Last Updated:
++ Basic Operations: writeByte! readByte! readByteIfCan!
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This domain provides an implementation of binary files. Data is
++ accessed one byte at a time as a small integer.

BinaryFile: Cat == Def where

  Cat == FileCategory(FileName, SingleInteger) with
    readIfCan_!: % -> Union(SingleInteger, "failed")
      ++ readIfCan!(f) returns a value from the file f, if possible.
      ++ If f is not open for reading, or if f is at the end of file
      ++ then \spad{"failed"} is the result.

--      "#": % -> SingleInteger
--      ++ #(f) returns the length of the file f in bytes.

    position: % -> SingleInteger
      ++ position(f) returns the current byte-position in the file f.

    position_!: (%, SingleInteger) -> SingleInteger
      ++ position!(f, i) sets the current byte-position to i.

  Def == File(SingleInteger) add
    FileState ==> SExpression

    Rep := Record(fileName:  FileName,      _
                  fileState: FileState,    _
                  fileIOMode: String)

--    direc : Symbol := INTERN("DIRECTION","KEYWORD")$Lisp
--    input  : Symbol := INTERN("INPUT","KEYWORD")$Lisp
--    output : Symbol := INTERN("OUTPUT","KEYWORD")$Lisp
--    eltype : Symbol := INTERN("ELEMENT-TYPE","KEYWORD")$Lisp
--    bytesize : SExpression := LIST(QUOTE(UNSIGNED$Lisp)$Lisp,8)$Lisp

```

```

defstream(fn: FileName, mode: String): FileState ==
  mode = "input" =>
    not readable? fn => error ["File is not readable", fn]
    BINARY__OPEN__INPUT(fn::String)$Lisp
--    OPEN(fn::String, direc, input, eltype, bytesize)$Lisp
  mode = "output" =>
    not writable? fn => error ["File is not writable", fn]
    BINARY__OPEN__OUTPUT(fn::String)$Lisp
--    OPEN(fn::String, direc, output, eltype, bytesize)$Lisp
    error ["IO mode must be input or output", mode]

open(fname, mode) ==
  fstream := defstream(fname, mode)
  [fname, fstream, mode]

reopen_!(f, mode) ==
  fname := f.fileName
  f.fileState := defstream(fname, mode)
  f.fileIOmode := mode
  f

close_! f ==
  f.fileIOmode = "output" =>
    BINARY__CLOSE__OUTPUT()$Lisp
    f
  f.fileIOmode = "input" =>
    BINARY__CLOSE__INPUT()$Lisp
    f
  error "file must be in read or write state"

read! f ==
  f.fileIOmode ^= "input" => error "File not in read state"
  BINARY__SELECT__INPUT(f.fileState)$Lisp
  BINARY__READBYTE()$Lisp
--  READ_-BYTE(f.fileState)$Lisp
readIfCan_! f ==
  f.fileIOmode ^= "input" => error "File not in read state"
  BINARY__SELECT__INPUT(f.fileState)$Lisp
  n:SingleInteger:=BINARY__READBYTE()$Lisp
  n = -1 => "failed"
  n::Union(SingleInteger,"failed")
--  READ_-BYTE(f.fileState,NIL$Lisp,
--  "failed)::Union(SingleInteger,"failed"))$Lisp
write_!(f, x) ==
  f.fileIOmode ^= "output" => error "File not in write state"
  x < 0 or x>255 => error "integer cannot be represented as a byte"
  BINARY__PRINBYTE(x)$Lisp
--  WRITE_-BYTE(x, f.fileState)$Lisp

```

```

x

--      # f == FILE_-LENGTH(f.fileState)$Lisp
position f ==
    f.fileIOMode ^= "input" => error "file must be in read state"
    FILE_-POSITION(f.fileState)$Lisp
position_!(f,i) ==
    f.fileIOMode ^= "input" => error "file must be in read state"
    (FILE_-POSITION(f.fileState,i)$Lisp ; i)

```

---

— BINFILE.dotabb —

```

"BINFILE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=BINFILE"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"BINFILE" -> "STRING"

```

---

### 3.9 domain BSTREE BinarySearchTree

— BinarySearchTree.input —

```

)set break resume
)sys rm -f BinarySearchTree.output
)spool BinarySearchTree.output
)set message test on
)set message auto off
)clear all
--S 1 of 12
lv := [8,3,5,4,6,2,1,5,7]
--R
--R
--R (1) [8,3,5,4,6,2,1,5,7]
--R
--R                                          Type: List PositiveInteger
--E 1

--S 2 of 12
t := binarySearchTree lv
--R
--R
--R (2) [[[1,2,.],3,[4,5,[5,6,7]]],8,.]
--R
--R                                          Type: BinarySearchTree PositiveInteger

```

```

--E 2

--S 3 of 12
emptybst := empty()$BSTREE(INT)
--R
--R
--R (3) []
--R
--R                                          Type: BinarySearchTree Integer
--E 3

--S 4 of 12
t1 := insert!(8,emptybst)
--R
--R
--R (4) 8
--R
--R                                          Type: BinarySearchTree Integer
--E 4

--S 5 of 12
insert!(3,t1)
--R
--R
--R (5) [3,8,.]
--R
--R                                          Type: BinarySearchTree Integer
--E 5

--S 6 of 12
leaves t
--R
--R
--R (6) [1,4,5,7]
--R
--R                                          Type: List PositiveInteger
--E 6

--S 7 of 12
split(3,t)
--R
--R
--R (7) [less= [1,2,.],greater= [[.,3,[4,5,[5,6,7]]],8,.]]
--RType: Record(less: BinarySearchTree PositiveInteger,greater: BinarySearchTree PositiveInteger)
--E 7

--S 8 of 12
insertRoot: (INT,BSTREE INT) -> BSTREE INT
--R
--R
--R                                          Type: Void
--E 8

--S 9 of 12
insertRoot(x, t) ==

```

```

      a := split(x, t)
      node(a.less, x, a.greater)
--R
--R
--R                                          Type: Void
--E 9

--S 10 of 12
buildFromRoot ls == reduce(insertRoot,ls,emptybst)
--R
--R
--R                                          Type: Void
--E 10

--S 11 of 12
rt := buildFromRoot reverse lv
--R
--R   Compiling function buildFromRoot with type List PositiveInteger ->
--R       BinarySearchTree Integer
--R   Compiling function insertRoot with type (Integer,BinarySearchTree
--R       Integer) -> BinarySearchTree Integer
--R
--R   (11)  [[[1,2,.],3,[4,5,[5,6,7]]],8,.]
--R
--R                                          Type: BinarySearchTree Integer
--E 11

--S 12 of 12
(t = rt)@Boolean
--R
--R
--R   (12)  true
--R
--R                                          Type: Boolean
--E 12
)spool
)lisp (bye)

```

---

— BinarySearchTree.help —

=====

BinarySearchTree examples

=====

BinarySearchTree(R) is the domain of binary trees with elements of type R, ordered across the nodes of the tree. A non-empty binary search tree has a value of type R, and right and left binary search subtrees. If a subtree is empty, it is displayed as a period (".").

Define a list of values to be placed across the tree. The resulting tree has 8 at the root; all other elements are in the left subtree.

```
lv := [8,3,5,4,6,2,1,5,7]
      [8, 3, 5, 4, 6, 2, 1, 5, 7]
      Type: List PositiveInteger
```

A convenient way to create a binary search tree is to apply the operation `binarySearchTree` to a list of elements.

```
t := binarySearchTree lv
    [[[1, 2, .], 3, [4, 5, [5, 6, 7]]], 8, .]
    Type: BinarySearchTree PositiveInteger
```

Another approach is to first create an empty binary search tree of integers.

```
emptybst := empty()$BSTREE(INT)
[]
      Type: BinarySearchTree Integer
```

Insert the value 8. This establishes 8 as the root of the binary search tree. Values inserted later that are less than 8 get stored in the left subtree, others in the right subtree.

```
t1 := insert!(8,emptybst)
8
      Type: BinarySearchTree Integer
```

Insert the value 3. This number becomes the root of the left subtree of `t1`. For optimal retrieval, it is thus important to insert the middle elements first.

```
insert!(3,t1)
[3, 8, .]
      Type: BinarySearchTree Integer
```

We go back to the original tree `t`. The leaves of the binary search tree are those which have empty left and right subtrees.

```
leaves t
[1, 4, 5, 7]
      Type: List PositiveInteger
```

The operation `split(k,t)` returns a record containing the two subtrees: one with all elements "less" than `k`, another with elements "greater" than `k`.

```
split(3,t)
[less=[1, 2, .], greater=[[., 3, [4, 5, [5, 6, 7]]], 8, .]]
      Type: Record(less: BinarySearchTree PositiveInteger,greater:
      BinarySearchTree PositiveInteger)
```



Define `insertRoot` to insert new elements by creating a new node.

```
insertRoot: (INT,BSTREE INT) -> BSTREE INT
Type: Void
```

The new node puts the inserted value between its "less" tree and "greater" tree.

```
insertRoot(x, t) ==
  a := split(x, t)
  node(a.less, x, a.greater)
Type: Void
```

Function `buildFromRoot` builds a binary search tree from a list of elements `ls` and the empty tree `emptybst`.

```
buildFromRoot ls == reduce(insertRoot,ls,emptybst)
Type: Void
```

Apply this to the reverse of the list `lv`.

```
rt := buildFromRoot reverse lv
[[[1, 2, . ], 3, [4, 5, [5, 6, 7]]], 8, .]
Type: BinarySearchTree Integer
```

Have Axiom check that these are equal.

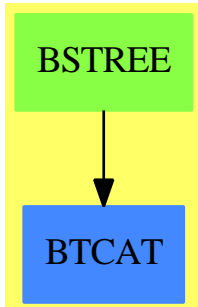
```
(t = rt)@Boolean
true
Type: Boolean
```

See Also:

o `)show BinarySearchTree`

---

## 3.9.1 BinarySearchTree (BSTREE)



See

- ⇒ “Tree” (TREE) 21.10.1 on page 2699
- ⇒ “BinaryTree” (BTREE) 3.11.1 on page 292
- ⇒ “BinaryTournament” (BTOURN) 3.10.1 on page 289
- ⇒ “BalancedBinaryTree” (BBTREE) 3.1.1 on page 234
- ⇒ “PendantTree” (PENDTREE) 17.13.1 on page 1904

**Exports:**

|             |                  |              |           |          |
|-------------|------------------|--------------|-----------|----------|
| any?        | binarySearchTree | child?       | children  | coerce   |
| copy        | count            | count        | cyclic?   | distance |
| empty       | empty?           | eq?          | eval      | eval     |
| eval        | eval             | every?       | hash      | insert!  |
| insertRoot! | latex            | leaf?        | leaves    | left     |
| less?       | map              | map!         | member?   | members  |
| more?       | node             | node?        | nodes     | parts    |
| right       | sample           | setchildren! | setelt    | setelt   |
| setelt      | setleft!         | setright!    | setvalue! | size?    |
| split       | value            | #?           | ?=?       | ?~=?     |
| ?right      | ?left            | ?.value      |           |          |

— domain **BSTREE BinarySearchTree** —

```

)abbrev domain BSTREE BinarySearchTree
++ Author: Mark Botch
++ Description:
++ BinarySearchTree(S) is the domain of
++ a binary trees where elements are ordered across the tree.
++ A binary search tree is either empty or has
++ a value which is an S, and a
++ right and left which are both BinaryTree(S)
++ Elements are ordered across the tree.

BinarySearchTree(S: OrderedSet): Exports == Implementation where
  Exports == BinaryTreeCategory(S) with
    shallowlyMutable

```

```

finiteAggregate
binarySearchTree: List S -> %
  ++ binarySearchTree(1) is not documented
  ++
  ++X binarySearchTree [1,2,3,4]

insert_!: (S,%) -> %
  ++ insert!(x,b) inserts element x as leaves into binary search tree b.
  ++
  ++X t1:=binarySearchTree [1,2,3,4]
  ++X insert!(5,t1)

insertRoot_!: (S,%) -> %
  ++ insertRoot!(x,b) inserts element x as a root of binary search tree b.
  ++
  ++X t1:=binarySearchTree [1,2,3,4]
  ++X insertRoot!(5,t1)

split:      (S,%) -> Record(less: %, greater: %)
  ++ split(x,b) splits binary tree b into two trees, one with elements
  ++ greater than x, the other with elements less than x.
  ++
  ++X t1:=binarySearchTree [1,2,3,4]
  ++X split(3,t1)

Implementation == BinaryTree(S) add
Rep := BinaryTree(S)
binarySearchTree(u:List S) ==
  null u => empty()
  tree := binaryTree(first u)
  for x in rest u repeat insert_!(x,tree)
  tree
insert_!(x,t) ==
  empty? t => binaryTree(x)
  x >= value t =>
    setright_!(t,insert_!(x,right t))
    t
  setleft_!(t,insert_!(x,left t))
  t
split(x,t) ==
  empty? t => [empty(),empty()]
  x > value t =>
    a := split(x,right t)
    [node(left t, value t, a.less), a.greater]
    a := split(x,left t)
    [a.less, node(a.greater, value t, right t)]
insertRoot_!(x,t) ==
  a := split(x,t)
  node(a.less, x, a.greater)

```

---

— BSTREE.dotabb —

```
"BSTREE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=BSTREE"]
"BTCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=BTCAT"]
"BSTREE" -> "BTCAT"
```

---

### 3.10 domain BTOURN BinaryTournament

A BinaryTournament(S) is the domain of binary trees where elements are ordered down the tree. A binary search tree is either empty or is a node containing a value of type S, and a right and a left which are both BinaryTree(S)

— BinaryTournament.input —

```
)set break resume
)sys rm -f BinaryTournament.output
)spool BinaryTournament.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show BinaryTournament
--R BinaryTournament S: OrderedSet is a domain constructor
--R Abbreviation for BinaryTournament is BTOURN
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for BTOURN
--R
--R----- Operations -----
--R binaryTournament : List S -> %      children : % -> List %
--R copy : % -> %                      cyclic? : % -> Boolean
--R distance : (%,% ) -> Integer        ?.right : (% ,right) -> %
--R ?.left : (% ,left) -> %             ?.value : (% ,value) -> S
--R empty : () -> %                    empty? : % -> Boolean
--R eq? : (%,% ) -> Boolean             insert! : (S,% ) -> %
--R leaf? : % -> Boolean                leaves : % -> List S
--R left : % -> %                      map : ((S -> S),%) -> %
--R node : (% ,S,% ) -> %              nodes : % -> List %
--R right : % -> %                    sample : () -> %
--R value : % -> S
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?? : (% ,%) -> Boolean if S has SETCAT
```

```

--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R child? : (%,%) -> Boolean if S has SETCAT
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if $ has finiteAggregate and S has SETCAT
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R members : % -> List S if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R node? : (%,%) -> Boolean if S has SETCAT
--R parts : % -> List S if $ has finiteAggregate
--R setchildren! : (%,List %) -> % if $ has shallowlyMutable
--R setelt : (%,right,%) -> % if $ has shallowlyMutable
--R setelt : (%,left,%) -> % if $ has shallowlyMutable
--R setelt : (%,value,S) -> S if $ has shallowlyMutable
--R setleft! : (%,%) -> % if $ has shallowlyMutable
--R setright! : (%,%) -> % if $ has shallowlyMutable
--R setvalue! : (%,S) -> S if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R ~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)

```

---

— BinaryTournament.help —

```

=====
BinaryTournament examples
=====

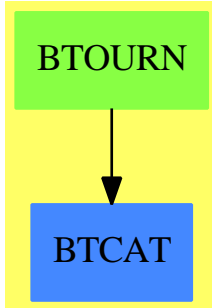
```

```

See Also:
o )show BinaryTournament

```

## 3.10.1 BinaryTournament (BTOURN)



See

- ⇒ “Tree” (TREE) 21.10.1 on page 2699
- ⇒ “BinaryTree” (BTREE) 3.11.1 on page 292
- ⇒ “BinarySearchTree” (BSTREE) 3.9.1 on page 285
- ⇒ “BalancedBinaryTree” (BBTREE) 3.1.1 on page 234
- ⇒ “PendantTree” (PENDTREE) 17.13.1 on page 1904

**Exports:**

|           |                  |              |          |          |
|-----------|------------------|--------------|----------|----------|
| any?      | binaryTournament | child?       | children | coerce   |
| copy      | count            | cyclic?      | distance | empty    |
| empty?    | eq?              | eval         | every?   | hash     |
| insert!   | latex            | leaf?        | leaves   | left     |
| less?     | map              | map!         | member?  | members  |
| more?     | node             | node?        | nodes    | parts    |
| right     | sample           | setchildren! | setelt   | setleft! |
| setright! | setvalue!        | size?        | value    | #?       |
| ?=?       | ?~=?             | ?..right     | ?..left  | ?..value |

— domain BTOURN BinaryTournament —

```

)abbrev domain BTOURN BinaryTournament
++ Author: Mark Botch
++ Description:
++ BinaryTournament creates a binary tournament with the
++ elements of ls as values at the nodes.

BinaryTournament(S: OrderedSet): Exports == Implementation where
  Exports == BinaryTreeCategory(S) with
    shallowlyMutable
  binaryTournament: List S -> %
    ++ binaryTournament(ls) creates a binary tournament with the
    ++ elements of ls as values at the nodes.
    ++
    ++X binaryTournament [1,2,3,4]
  
```

```

insert_!: (S,%) -> %
  ++ insert!(x,b) inserts element x as leaves into binary tournament b.
  ++
  ++X t1:=binaryTournament [1,2,3,4]
  ++X insert!(5,t1)
  ++X t1

Implementation == BinaryTree(S) add
Rep := BinaryTree(S)
binaryTournament(u:List S) ==
  null u => empty()
  tree := binaryTree(first u)
  for x in rest u repeat insert_!(x,tree)
  tree
insert_!(x,t) ==
  empty? t => binaryTree(x)
  x > value t =>
    setleft_!(t,copy t)
    setvalue_!(t,x)
    setright_!(t,empty())
  setright_!(t,insert_!(x,right t))
  t

```

---

— BTOURN.dotabb —

```

"BTourn" [color="#88FF44",href="bookvol10.3.pdf#nameddest=BTourn"]
"BTCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=BTCAT"]
"BTourn" -> "BTCAT"

```

---

### 3.11 domain BTREE BinaryTree

— BinaryTree.input —

```

)set break resume
)sys rm -f BinaryTree.output
)spool BinaryTree.output
)set message test on
)set message auto off
)clear all

```

```

--S 1 of 1
)show BinaryTree
--R BinaryTree S: SetCategory is a domain constructor
--R Abbreviation for BinaryTree is BTREE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for BTREE
--R
--R----- Operations -----
--R binaryTree : (%,S,%) -> %          binaryTree : S -> %
--R children : % -> List %             copy : % -> %
--R cyclic? : % -> Boolean             distance : (%,%) -> Integer
--R ?.right : (%,right) -> %          ?.left : (%,left) -> %
--R ?.value : (%,value) -> S          empty : () -> %
--R empty? : % -> Boolean             eq? : (%,%) -> Boolean
--R leaf? : % -> Boolean             leaves : % -> List S
--R left : % -> %                    map : ((S -> S),%) -> %
--R node : (%,S,%) -> %             nodes : % -> List %
--R right : % -> %                  sample : () -> %
--R value : % -> S
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ==? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R child? : (%,%) -> Boolean if S has SETCAT
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if $ has finiteAggregate and S has SETCAT
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R members : % -> List S if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R node? : (%,%) -> Boolean if S has SETCAT
--R parts : % -> List S if $ has finiteAggregate
--R setchildren! : (%,List %) -> % if $ has shallowlyMutable
--R setelt : (%,right,%) -> % if $ has shallowlyMutable
--R setelt : (%,left,%) -> % if $ has shallowlyMutable
--R setelt : (%,value,S) -> S if $ has shallowlyMutable
--R setleft! : (%,%) -> % if $ has shallowlyMutable
--R setright! : (%,%) -> % if $ has shallowlyMutable
--R setvalue! : (%,S) -> S if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R

```



```
--E 1
```

```
)spool
)lisp (bye)
```

---

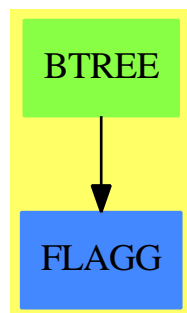
— BinaryTree.help —

```
=====
BinaryTree examples
=====
```

```
See Also:
o )show BinaryTree
```

---

### 3.11.1 BinaryTree (BTREE)



**See**

- ⇒ “Tree” (TREE) 21.10.1 on page 2699
- ⇒ “BinarySearchTree” (BSTREE) 3.9.1 on page 285
- ⇒ “BinaryTournament” (BTourn) 3.10.1 on page 289
- ⇒ “BalancedBinaryTree” (BBTREE) 3.1.1 on page 234
- ⇒ “PendantTree” (PENDTREE) 17.13.1 on page 1904

**Exports:**

|           |              |         |          |           |
|-----------|--------------|---------|----------|-----------|
| any?      | binaryTree   | child?  | children | coerce    |
| copy      | count        | cyclic? | distance | empty     |
| empty?    | eq?          | eval    | every?   | hash      |
| latex     | leaf?        | leaves  | left     | less?     |
| map       | map!         | member? | members  | more?     |
| node      | node?        | nodes   | parts    | right     |
| sample    | setchildren! | setelt  | setleft! | setright! |
| setvalue! | size?        | value   | #?       | ?=?       |
| ?~=?      | ?..right     | ?..left | ?..value |           |

— domain BTREE BinaryTree —

```

)abbrev domain BTREE BinaryTree
++ Author: Mark Botch
++ Description:
++ \spadtype{BinaryTree(S)} is the domain of all
++ binary trees. A binary tree over \spad{S} is either empty or has
++ a \spadfun{value} which is an S and a \spadfun{right}
++ and \spadfun{left} which are both binary trees.

```

```

BinaryTree(S: SetCategory): Exports == Implementation where
Exports == BinaryTreeCategory(S) with

```

```

binaryTree: S -> %
++ binaryTree(v) is an non-empty binary tree
++ with value v, and left and right empty.
++
++X t1:=binaryTree([1,2,3])

binaryTree: (%S,%) -> %
++ binaryTree(l,v,r) creates a binary tree with
++ value v with left subtree l and right subtree r.
++
++X t1:=binaryTree([1,2,3])
++X t2:=binaryTree([4,5,6])
++X binaryTree(t1,[7,8,9],t2)

```

```

Implementation == add
Rep := List Tree S
t1 = t2 == (t1::Rep) =$Rep (t2::Rep)
empty() == [] pretend %
empty() == [] pretend %
node(l,v,r) == cons(tree(v,l:Rep),r:Rep)
binaryTree(l,v,r) == node(l,v,r)
binaryTree(v:S) == node(empty(),v,empty())
empty? t == empty?(t)$Rep
leaf? t == empty? t or empty? left t and empty? right t
right t ==
  empty? t => error "binaryTree:no right"

```

```

    rest t
  left t ==
    empty? t => error "binaryTree:no left"
    children first t
  value t==
    empty? t => error "binaryTree:no value"
    value first t
  setvalue_! (t,nd)==
    empty? t => error "binaryTree:no value to set"
    setvalue_!(first(t:Rep),nd)
    nd
  setleft_!(t1,t2) ==
    empty? t1 => error "binaryTree:no left to set"
    setchildren_!(first(t1:Rep),t2:Rep)
    t1
  setright_!(t1,t2) ==
    empty? t1 => error "binaryTree:no right to set"
    setrest_!(t1>List Tree S,t2)

```

---

— BTREE.dotabb —

```

"BTREE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=BTREE"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"BTREE" -> "FLAGG"

```

## 3.12 domain BITS Bits

---

— Bits.input —

```

)set break resume
)sys rm -f Bits.output
)spool Bits.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Bits
--R Bits is a domain constructor
--R Abbreviation for Bits is BITS

```

```

--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for BITS
--R
--R----- Operations -----
--R ?/\? : (%,%) -> %                ?<? : (%,%) -> Boolean
--R ?<=? : (%,%) -> Boolean          ?=? : (%,%) -> Boolean
--R ?>? : (%,%) -> Boolean          ?>=? : (%,%) -> Boolean
--R ?\/? : (%,%) -> %                ^? : % -> %
--R ?and? : (%,%) -> %              coerce : % -> OutputForm
--R concat : (%,Boolean) -> %        concat : (Boolean,%) -> %
--R concat : (%,%) -> %              concat : List % -> %
--R construct : List Boolean -> %    copy : % -> %
--R delete : (%,Integer) -> %        ?.? : (%,Integer) -> Boolean
--R empty : () -> %                  empty? : % -> Boolean
--R entries : % -> List Boolean      eq? : (%,%) -> Boolean
--R hash : % -> SingleInteger        index? : (Integer,%) -> Boolean
--R indices : % -> List Integer      insert : (%,%,Integer) -> %
--R latex : % -> String              max : (%,%) -> %
--R min : (%,%) -> %                 nand : (%,%) -> %
--R nor : (%,%) -> %                 not? : % -> %
--R ?or? : (%,%) -> %                 qelt : (%,Integer) -> Boolean
--R reverse : % -> %                 sample : () -> %
--R xor : (%,%) -> %                 ~? : % -> %
--R ?~=? : (%,%) -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R any? : ((Boolean -> Boolean),%) -> Boolean if $ has finiteAggregate
--R bits : (NonNegativeInteger,Boolean) -> %
--R convert : % -> InputForm if Boolean has KONVERT INFORM
--R copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
--R count : ((Boolean -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R count : (Boolean,%) -> NonNegativeInteger if $ has finiteAggregate and Boolean has SETCAT
--R delete : (%,UniversalSegment Integer) -> %
--R elt : (%,Integer,Boolean) -> Boolean
--R ?.? : (%,UniversalSegment Integer) -> %
--R entry? : (Boolean,%) -> Boolean if $ has finiteAggregate and Boolean has SETCAT
--R eval : (%,List Equation Boolean) -> % if Boolean has EVALAB BOOLEAN and Boolean has SETCAT
--R eval : (%,Equation Boolean) -> % if Boolean has EVALAB BOOLEAN and Boolean has SETCAT
--R eval : (%,Boolean,Boolean) -> % if Boolean has EVALAB BOOLEAN and Boolean has SETCAT
--R eval : (%,List Boolean,List Boolean) -> % if Boolean has EVALAB BOOLEAN and Boolean has SETCAT
--R every? : ((Boolean -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,Boolean) -> % if $ has shallowlyMutable
--R find : ((Boolean -> Boolean),%) -> Union(Boolean,"failed")
--R first : % -> Boolean if Integer has ORDSET
--R insert : (Boolean,%,Integer) -> %
--R less? : (%,NonNegativeInteger) -> Boolean
--R map : ((Boolean -> Boolean),%) -> %
--R map : (((Boolean,Boolean) -> Boolean),%,%) -> %
--R map! : ((Boolean -> Boolean),%) -> % if $ has shallowlyMutable
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (Boolean,%) -> Boolean if $ has finiteAggregate and Boolean has SETCAT

```

```

--R members : % -> List Boolean if $ has finiteAggregate
--R merge : (((Boolean,Boolean) -> Boolean),%,%) -> %
--R merge : (%,%) -> % if Boolean has ORDSET
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R new : (NonNegativeInteger,Boolean) -> %
--R parts : % -> List Boolean if $ has finiteAggregate
--R position : ((Boolean -> Boolean),%) -> Integer
--R position : (Boolean,%) -> Integer if Boolean has SETCAT
--R position : (Boolean,%,Integer) -> Integer if Boolean has SETCAT
--R qsetelt! : (%,Integer,Boolean) -> Boolean if $ has shallowlyMutable
--R reduce : (((Boolean,Boolean) -> Boolean),%,Boolean,Boolean) -> Boolean if $ has finiteAg
--R reduce : (((Boolean,Boolean) -> Boolean),%,Boolean) -> Boolean if $ has finiteAggregate
--R reduce : (((Boolean,Boolean) -> Boolean),%) -> Boolean if $ has finiteAggregate
--R remove : (Boolean,%) -> % if $ has finiteAggregate and Boolean has SETCAT
--R remove : ((Boolean -> Boolean),%) -> % if $ has finiteAggregate
--R removeDuplicates : % -> % if $ has finiteAggregate and Boolean has SETCAT
--R reverse! : % -> % if $ has shallowlyMutable
--R select : ((Boolean -> Boolean),%) -> % if $ has finiteAggregate
--R setelt : (%,Integer,Boolean) -> Boolean if $ has shallowlyMutable
--R setelt : (%,UniversalSegment Integer,Boolean) -> Boolean if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : (((Boolean,Boolean) -> Boolean),%) -> %
--R sort : % -> % if Boolean has ORDSET
--R sort! : (((Boolean,Boolean) -> Boolean),%) -> % if $ has shallowlyMutable
--R sort! : % -> % if $ has shallowlyMutable and Boolean has ORDSET
--R sorted? : (((Boolean,Boolean) -> Boolean),%) -> Boolean
--R sorted? : % -> Boolean if Boolean has ORDSET
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R
--E 1

)spool
)lisp (bye)

```

---

— Bits.help —

```

=====
Bits examples
=====

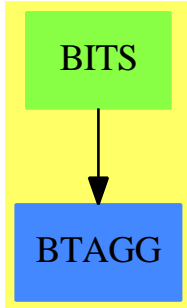
```

```

See Also:
o )show Bits

```

## 3.12.1 Bits (BITS)



See

- ⇒ “Reference” (REF) 19.5.1 on page 2209
- ⇒ “Boolean” (BOOLEAN) 3.15.1 on page 304
- ⇒ “IndexedBits” (IBITS) 10.3.1 on page 1165

**Exports:**

|          |         |                  |         |           |
|----------|---------|------------------|---------|-----------|
| any?     | bits    | coerce           | concat  | construct |
| convert  | copy    | copyInto!        | count   | delete    |
| elt      | empty   | empty?           | entries | entry?    |
| eq?      | eval    | every?           | fill!   | find      |
| first    | hash    | index?           | indices | insert    |
| latex    | less?   | map              | map!    | max       |
| maxIndex | member? | members          | merge   | min       |
| minIndex | more?   | nand             | new     | nor       |
| not?     | parts   | position         | qelt    | qsetelt!  |
| reduce   | remove  | removeDuplicates | reverse | reverse!  |
| sample   | setelt  | size?            | sort    | sort!     |
| sorted?  | swap!   | xor              | #?      | ?.?       |
| ?/\?     | ?<?     | ?<=?             | ?=?     | ?>?       |
| ?>=?     | ?\\/?   | ^?               | ?and?   | ?or?      |
| ?.?      | ~?      | ?~=?             |         |           |

— domain BITS Bits —

```
)abbrev domain BITS Bits
++ Author: Stephen M. Watt
++ Date Created:
++ Change History:
++ Basic Operations: And, Not, Or
++ Related Constructors:
++ Keywords: bits
++ Description:
++ \spadtype{Bits} provides logical functions for Indexed Bits.
```

Bits(): Exports == Implementation where

```

Exports == BitAggregate() with
  bits: (NonNegativeInteger, Boolean) -> %
    ++ bits(n,b) creates bits with n values of b
Implementation == IndexedBits(1) add
  bits(n,b) == new(n,b)

```

---

— BITS.dotabb —

```

"BITS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=BITS"]
"BTAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=BTAGG"]
"BITS" -> "BTAGG"

```

---

### 3.13 domain BLHN BlowUpWithHamburgerNoether

— BlowUpWithHamburgerNoether.input —

```

)set break resume
)sys rm -f BlowUpWithHamburgerNoether.output
)spool BlowUpWithHamburgerNoether.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show BlowUpWithHamburgerNoether
--R BlowUpWithHamburgerNoether is a domain constructor
--R Abbreviation for BlowUpWithHamburgerNoether is BLHN
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for BLHN
--R
--R----- Operations -----
--R ?=? : (% ,%) -> Boolean          chartCoord : % -> Integer
--R coerce : List Integer -> %      coerce : % -> OutputForm
--R excepCoord : % -> Integer        hash : % -> SingleInteger
--R infClsPt? : % -> Boolean          latex : % -> String
--R quotValuation : % -> Integer      ramifMult : % -> Integer
--R transCoord : % -> Integer         ?~=? : (% ,%) -> Boolean
--R createHN : (Integer,Integer,Integer,Integer,Integer,Boolean,Union(left,center,right,vert.
--R type : % -> Union(left,center,right,vertical,horizontal)
--R

```

```
--E 1
```

```
)spool
)lisp (bye)
```

```
-----
```

```
— BlowUpWithHamburgerNoether.help —
```

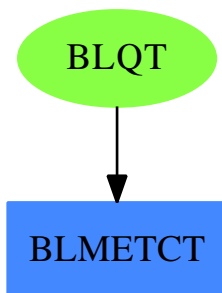
```
=====
BlowUpWithHamburgerNoether examples
=====
```

See Also:

```
o )show BlowUpWithHamburgerNoether
```

```
-----
```

### 3.13.1 BlowUpWithHamburgerNoether (BLHN)



#### Exports:

```
?=?      ?~=?      chartCoord  coerce  createHN
excepCoord hash      infClsPt?   latex  quotValuation
ramifMult  transCoord type
```

```
— domain BLHN BlowUpWithHamburgerNoether —
```

```
)abbrev domain BLHN BlowUpWithHamburgerNoether
++ Authors: Gaetan Hache
++ Date Created: june 1996
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
```



```

++ This domain is part of the PAFF package
BlowUpWithHamburgerNoether: Exports == Implementation where
  MetRec ==> Record(_
    ex:Integer, tr:Integer, ch:Integer , quotVal:Integer, _
    ramif:Integer, infClsPt:Boolean, _
    type:Union("left","center","right","vertical","horizontal") )

Exports ==> BlowUpMethodCategory with HamburgerNoether

Implementation == add
  Rep := MetRec

  infClsPt_? a == a.infClsPt

  createHN( a,b,c,d,e,f,g)==[a,b,c,d,e,f,g]$Rep

  excepCoord a == a.ex

  chartCoord a == a.ch

  transCoord a == a.tr

  ramifMult a == a.ramif

  quotValuation a == a.quotVal

  type a == a.type

  coerce(c:%):OutputForm== ( (c :: Rep ) :: MetRec) :: OutputForm

  -----

  — BLHN.dotabb —

  "BLHN" [color="#88FF44",href="bookvol10.3.pdf#nameddest=BLHN",
    shape=ellipse]
  "BLMETCT" [color="#4488FF",href="bookvol10.3.pdf#nameddest=BLMETCT"]
  "BLHN" -> "BLMETCT"

  -----

```

### 3.14 domain BLQT BlowUpWithQuadTrans

```

  — BlowUpWithQuadTrans.input —

)set break resume

```

```

)sys rm -f BlowUpWithQuadTrans.output
)spool BlowUpWithQuadTrans.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show BlowUpWithQuadTrans
--R BlowUpWithQuadTrans is a domain constructor
--R Abbreviation for BlowUpWithQuadTrans is BLQT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for BLQT
--R
--R----- Operations -----
--R ?? : (% ,%) -> Boolean          chartCoord : % -> Integer
--R coerce : List Integer -> %      coerce : % -> OutputForm
--R excepCoord : % -> Integer        hash : % -> SingleInteger
--R infClsPt? : % -> Boolean          latex : % -> String
--R quotValuation : % -> Integer      ramifMult : % -> Integer
--R transCoord : % -> Integer          ?~=? : (% ,%) -> Boolean
--R createHN : (Integer,Integer,Integer,Integer,Integer,Boolean,Union(left,center,right,vertical,horizontal))
--R type : % -> Union(left,center,right,vertical,horizontal)
--R
--E 1

)spool
)lisp (bye)

```

---

— BlowUpWithQuadTrans.help —

```

=====
BlowUpWithQuadTrans examples
=====

```

See Also:

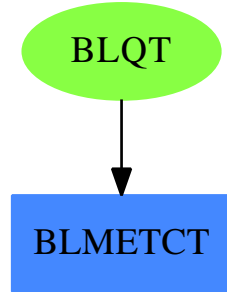
```

o )show BlowUpWithQuadTrans

```

---

## 3.14.1 BlowUpWithQuadTrans (BLQT)

**Exports:**

|                         |                         |                         |                     |                            |
|-------------------------|-------------------------|-------------------------|---------------------|----------------------------|
| <code>?=?</code>        | <code>?~=?</code>       | <code>chartCoord</code> | <code>coerce</code> | <code>createHN</code>      |
| <code>excepCoord</code> | <code>hash</code>       | <code>infClsPt?</code>  | <code>latex</code>  | <code>quotValuation</code> |
| <code>ramifMult</code>  | <code>transCoord</code> | <code>type</code>       |                     |                            |

— domain BLQT BlowUpWithQuadTrans —

```

)abbrev domain BLQT BlowUpWithQuadTrans
++ Authors: Gaetan Hache
++ Date Created: june 1996
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
++ This domain is part of the PAFF package
BlowUpWithQuadTrans: Exports == Implementation where

MetRec ==> Record( ex:Integer, tr: Integer, ch: Integer , ramif: Integer )
outRec ==> Record( exCoord:Integer, affNeigh: Integer )
Exports ==> BlowUpMethodCategory with

  QuadraticTransform

Implementation == add
  Rep := MetRec

  coerce(la:List(Integer)):% == [la.1, la.2,la.3, 1 ]$Rep

  ramifMult a == One$Integer

  excepCoord a == a.ex

  chartCoord a == a.ch

  transCoord a == a.tr

  ramifMult a == a.ramif

```

```

quotValuation a == One$Integer

coerce(c: %): OutputForm ==
  oo: outRec := [ excepCoord(c) , chartCoord(c) ]$outRec
  oo :: OutputForm

```

---

— BLQT.dotabb —

```

"BLQT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=BLQT",
        shape=ellipse]
"BLMETCT" [color="#4488FF",href="bookvol10.3.pdf#nameddest=BLMETCT"]
"BLQT" -> "BLMETCT"

```

---

## 3.15 domain BOOLEAN Boolean

— Boolean.input —

```

)set break resume
)sys rm -f Boolean.output
)spool Boolean.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Boolean
--R Boolean is a domain constructor
--R Abbreviation for Boolean is BOOLEAN
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for BOOLEAN
--R
--R----- Operations -----
--R ?/\? : (%,% ) -> %
--R ?<=? : (%,% ) -> Boolean
--R ?>? : (%,% ) -> Boolean
--R ?\/? : (%,% ) -> %
--R ?and? : (%,% ) -> %
--R convert : % -> InputForm
--R hash : % -> SingleInteger
--R ?<? : (%,% ) -> Boolean
--R ?=? : (%,% ) -> Boolean
--R ?>=? : (%,% ) -> Boolean
--R ^? : % -> %
--R coerce : % -> OutputForm
--R false : () -> %
--R implies : (%,% ) -> %

```

```

--R index : PositiveInteger -> %
--R lookup : % -> PositiveInteger
--R min : (%,% ) -> %
--R nor : (%,% ) -> %
--R ?or? : (%,% ) -> %
--R size : () -> NonNegativeInteger
--R true : () -> %
--R ~? : % -> %
--R
--R
--E 1

)spool
)lisp (bye)

```

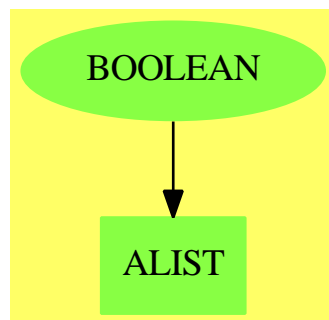
---

— Boolean.help —

=====  
 Boolean examples  
 =====

See Also:  
 o )show Boolean

### 3.15.1 Boolean (BOOLEAN)



See

- ⇒ “Reference” (REF) 19.5.1 on page 2209
- ⇒ “IndexedBits” (IBITS) 10.3.1 on page 1165
- ⇒ “Bits” (BITS) 3.12.1 on page 297

**Exports:**

|        |         |        |        |         |
|--------|---------|--------|--------|---------|
| coerce | convert | false  | hash   | implies |
| index  | latex   | lookup | max    | min     |
| nand   | nor     | not?   | random | size    |
| test   | true    | xor    | ~?     | ?~=?    |
| ?/\?   | ?<?     | ?<=?   | ?=?    | ?>?     |
| ?>=?   | ?\/?    | ^?     | ?and?  | ?or?    |

— domain BOOLEAN Boolean —

```

)abbrev domain BOOLEAN Boolean
++ Author: Stephen M. Watt
++ Date Created:
++ Change History:
++ Basic Operations: true, false, not, and, or, xor, nand, nor, implies, ^
++ Related Constructors:
++ Keywords: boolean
++ Description:
++ \spadtype{Boolean} is the elementary logic with 2 values:
++ true and false

Boolean(): Join(OrderedSet, Finite, Logic, ConvertibleTo InputForm) with
  true  : constant -> %
    ++ true is a logical constant.
  false : constant -> %
    ++ false is a logical constant.
  _^     : % -> %
    ++ ^ n returns the negation of n.
  _not   : % -> %
    ++ not n returns the negation of n.
  _and   : (% , %) -> %
    ++ a and b returns the logical and of Boolean \spad{a} and b.
  _or    : (% , %) -> %
    ++ a or b returns the logical inclusive or
    ++ of Boolean \spad{a} and b.
  xor    : (% , %) -> %
    ++ xor(a,b) returns the logical exclusive or
    ++ of Boolean \spad{a} and b.
  nand   : (% , %) -> %
    ++ nand(a,b) returns the logical negation of \spad{a} and b.
  nor    : (% , %) -> %
    ++ nor(a,b) returns the logical negation of \spad{a} or b.
  implies: (% , %) -> %
    ++ implies(a,b) returns the logical implication
    ++ of Boolean \spad{a} and b.
  test: % -> Boolean
    ++ test(b) returns b and is provided for compatibility with the
    ++ new compiler.

== add

```

```

nt: % -> %

test a      == a pretend Boolean

nt b        == (b pretend Boolean => false; true)
true        == EQ(2,2)$Lisp  --well, 1 is rather special
false       == NIL$Lisp
sample()    == true
not b       == (test b => false; true)
_~ b        == (test b => false; true)
_~ b        == (test b => false; true)
_and(a, b)  == (test a => b; false)
_/_(a, b)   == (test a => b; false)
_or(a, b)   == (test a => true; b)
_\\_(a, b)  == (test a => true; b)
xor(a, b)   == (test a => nt b; b)
nor(a, b)   == (test a => false; nt b)
nand(a, b)  == (test a => nt b; true)
a = b       == BooleanEquality(a, b)$Lisp
implies(a, b) == (test a => b; true)
a < b       == (test b => not(test a);false)

size()      == 2
index i     ==
  even?(i::Integer) => false
  true
lookup a    ==
  a pretend Boolean => 1
  2
random()    ==
  even?(random()$Integer) => false
  true

convert(x:%):InputForm ==
  x pretend Boolean => convert("true"::Symbol)
  convert("false"::Symbol)

coerce(x:%):OutputForm ==
  x pretend Boolean => message "true"
  message "false"

```

---

— BOOLEAN.dotabb —

"BOOLEAN" [color="#88FF44",href="bookvol10.3.pdf#nameddest=BOOLEAN",  
 shape=ellipse]  
 "ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]

"BOOLEAN" -> "ALIST"

---





## Chapter 4

# Chapter C

### 4.1 domain CARD CardinalNumber

— CardinalNumber.input —

```
)set break resume
)sys rm -f CardinalNumber.output
)spool CardinalNumber.output
)set message test on
)set message auto off
)clear all
--S 1 of 20
c0 := 0 :: CardinalNumber
--R
--R
--R (1) 0
--R
--R                                          Type: CardinalNumber
--E 1

--S 2 of 20
c1 := 1 :: CardinalNumber
--R
--R
--R (2) 1
--R
--R                                          Type: CardinalNumber
--E 2

--S 3 of 20
c2 := 2 :: CardinalNumber
--R
--R
--R (3) 2
```

[illegible]

```

--S 10 of 20
countable? A0
--R
--R
--R (10) true
--R
--R                                          Type: Boolean
--E 10

--S 11 of 20
countable? A1
--R
--R
--R (11) false
--R
--R                                          Type: Boolean
--E 11

--S 12 of 20
[c2 + c2, c2 + A1]
--R
--R
--R (12) [4,Aleph(1)]
--R
--R                                          Type: List CardinalNumber
--E 12

--S 13 of 20
[c0*c2, c1*c2, c2*c2, c0*A1, c1*A1, c2*A1, A0*A1]
--R
--R
--R (13) [0,2,4,0,Aleph(1),Aleph(1),Aleph(1)]
--R
--R                                          Type: List CardinalNumber
--E 13

--S 14 of 20
[c2**c0, c2**c1, c2**c2, A1**c0, A1**c1, A1**c2]
--R
--R
--R (14) [1,2,4,1,Aleph(1),Aleph(1)]
--R
--R                                          Type: List CardinalNumber
--E 14

--S 15 of 20
[c2-c1, c2-c2, c2-c3, A1-c2, A1-A0, A1-A1]
--R
--R
--R (15) [1,0,"failed",Aleph(1),Aleph(1),"failed"]
--R                                          Type: List Union(CardinalNumber,"failed")
--E 15

--S 16 of 20

```

```

generalizedContinuumHypothesisAssumed true
--R
--R
--R (16) true
--R
--R                                          Type: Boolean
--E 16

--S 17 of 20
[c0**A0, c1**A0, c2**A0, A0**A0, A0**A1, A1**A0, A1**A1]
--R
--R
--R (17) [0,1,Aleph(1),Aleph(1),Aleph(2),Aleph(1),Aleph(2)]
--R
--R                                          Type: List CardinalNumber
--E 17

--S 18 of 20
a := Aleph 0
--R
--R
--R (18) Aleph(0)
--R
--R                                          Type: CardinalNumber
--E 18

--S 19 of 20
c := 2**a
--R
--R
--R (19) Aleph(1)
--R
--R                                          Type: CardinalNumber
--E 19

--S 20 of 20
f := 2**c
--R
--R
--R (20) Aleph(2)
--R
--R                                          Type: CardinalNumber
--E 20
)spool
)lisp (bye)

```

---

— CardinalNumber.help —

```

=====
CardinalNumber examples
=====

```

The CardinalNumber domain can be used for values indicating the cardinality of sets, both finite and infinite. For example, the dimension operation in the category VectorSpace returns a cardinal number.

The non-negative integers have a natural construction as cardinals

$$0 = \#\{\}, 1 = \{0\}, 2 = \{0, 1\}, \dots, n = \{i \mid 0 \leq i < n\}.$$

The fact that 0 acts as a zero for the multiplication of cardinals is equivalent to the axiom of choice.

Cardinal numbers can be created by conversion from non-negative integers.

```
c0 := 0 :: CardinalNumber
0
                                Type: CardinalNumber

c1 := 1 :: CardinalNumber
1
                                Type: CardinalNumber

c2 := 2 :: CardinalNumber
2
                                Type: CardinalNumber

c3 := 3 :: CardinalNumber
3
                                Type: CardinalNumber
```

They can also be obtained as the named cardinal Aleph(n).

```
A0 := Aleph 0
Aleph(0)
                                Type: CardinalNumber

A1 := Aleph 1
Aleph(1)
                                Type: CardinalNumber
```

The finite? operation tests whether a value is a finite cardinal, that is, a non-negative integer.

```
finite? c2
true
                                Type: Boolean

finite? A0
false
                                Type: Boolean
```

Similarly, the `countable?` operation determines whether a value is a countable cardinal, that is, finite or `Aleph(0)`.

```
countable? c2
true
                                Type: Boolean

countable? A0
true
                                Type: Boolean

countable? A1
false
                                Type: Boolean
```

Arithmetic operations are defined on cardinal numbers as follows:  
If  $x = \#X$  and  $y = \#Y$  then

```
x+y = #(X+Y) cardinality of the disjoint union
x-y = #(X-Y) cardinality of the relative complement
x*y = #(X*Y) cardinality of the Cartesian product
x**y = #(X**Y) cardinality of the set of maps from Y to X
```

Here are some arithmetic examples.

```
[c2 + c2, c2 + A1]
[4, Aleph(1)]
                                Type: List CardinalNumber

[c0*c2, c1*c2, c2*c2, c0*A1, c1*A1, c2*A1, A0*A1]
[0, 2, 4, 0, Aleph(1), Aleph(1), Aleph(1)]
                                Type: List CardinalNumber

[c2**c0, c2**c1, c2**c2, A1**c0, A1**c1, A1**c2]
[1, 2, 4, 1, Aleph(1), Aleph(1)]
                                Type: List CardinalNumber
```

Subtraction is a partial operation: it is not defined when subtracting a larger cardinal from a smaller one, nor when subtracting two equal infinite cardinals.

```
[c2-c1, c2-c2, c2-c3, A1-c2, A1-A0, A1-A1]
[1, 0, "failed", Aleph(1), Aleph(1), "failed"]
                                Type: List Union(CardinalNumber,"failed")
```

The generalized continuum hypothesis asserts that

```
2**Aleph i = Aleph(i+1)
```

and is independent of the axioms of set theory.

(reference: Goedel, The consistency of the continuum hypothesis, Ann. Math. Studies, Princeton Univ. Press, 1940.)

The CardinalNumber domain provides an operation to assert whether the hypothesis is to be assumed.

```
generalizedContinuumHypothesisAssumed true
true
                                Type: Boolean
```

When the generalized continuum hypothesis is assumed, exponentiation to a transfinite power is allowed.

```
[c0**A0, c1**A0, c2**A0, A0**A0, A0**A1, A1**A0, A1**A1]
[0, 1, Aleph(1), Aleph(1), Aleph(2), Aleph(1), Aleph(2)]
                                Type: List CardinalNumber
```

Three commonly encountered cardinal numbers are

```
a = #Z countable infinity
c = #R the continuum
f = #{g| g: [0,1] -> R}
```

In this domain, these values are obtained under the generalized continuum hypothesis in this way.

```
a := Aleph 0
Aleph(0)
                                Type: CardinalNumber

c := 2**a
Aleph(1)
                                Type: CardinalNumber

f := 2**c
Aleph(2)
                                Type: CardinalNumber
```

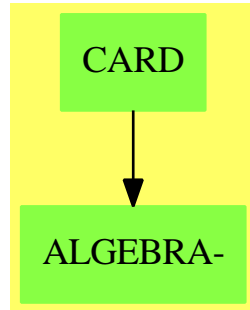
See Also:

```
o )show CardinalNumber
```

---



### 4.1.1 CardinalNumber (CARD)



#### Exports:

|  |              |
|--|--------------|
| 0                                      | 1            |
| Aleph                                  | coerce       |
| countable?                             | finite?      |
| generalizedContinuumHypothesisAssumed  | hash         |
| generalizedContinuumHypothesisAssumed? | latex        |
| max                                    | min          |
| one?                                   | recip        |
| retract                                | retractIfCan |
| sample                                 | zero?        |
| ?^?                                    | ?~=?         |
| ?*?                                    | ?**?         |
| ?-?                                    | ?+?          |
| ?<?                                    | ?<=?         |
| ?=?                                    | ?>?          |
| ?>=?                                   |              |

— domain CARD CardinalNumber —

```

)abbrev domain CARD CardinalNumber
++ Author: S.M. Watt
++ Date Created: June 1986
++ Date Last Updated: May 1990
++ Basic Operations: Aleph, +, -, *, **
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: cardinal number, transfinite arithmetic
++ Examples:
++ References:
++   Goedel, "The consistency of the continuum hypothesis",
++   Ann. Math. Studies, Princeton Univ. Press, 1940
++ Description:
++ Members of the domain CardinalNumber are values indicating the

```

```

++ cardinality of sets, both finite and infinite. Arithmetic operations
++ are defined on cardinal numbers as follows.
++
++ If \spad{x = #X} and \spad{y = #Y} then\br
++ \tab{5}\spad{x+y = #(X+Y)} \tab{5}disjoint union\br
++ \tab{5}\spad{x-y = #(X-Y)} \tab{5}relative complement\br
++ \tab{5}\spad{x*y = #(X*Y)} \tab{5}cartesian product\br
++ \tab{5}\spad{x**y = #(X**Y)} \tab{4}\spad{X**Y = g| g:Y->X}
++
++ The non-negative integers have a natural construction as cardinals\br
++ \spad{0 = #\{\}} , \spad{1 = \{0\}} ,
++ \spad{2 = \{0, 1\}} , ... , \spad{n = \{i| 0 <= i < n\}} .
++
++ That \spad{0} acts as a zero for the multiplication of cardinals is
++ equivalent to the axiom of choice.
++
++ The generalized continuum hypothesis asserts \br
++ \spad{2**Aleph i = Aleph(i+1)}
++ and is independent of the axioms of set theory [Goedel 1940].
++
++ Three commonly encountered cardinal numbers are\br
++ \tab{5}\spad{a = #Z} \tab{5}countable infinity\br
++ \tab{5}\spad{c = #R} \tab{5}the continuum\br
++ \tab{5}\spad{f = # g | g:[0,1]->R}
++
++ In this domain, these values are obtained using\br
++ \tab{5}\spad{a := Aleph 0}, \spad{c := 2**a}, \spad{f := 2**c}.

CardinalNumber: Join(OrderedSet, AbelianMonoid, Monoid,
                    RetractableTo NonNegativeInteger) with
    commutative "*"
        ++ a domain D has \spad{commutative("*")} if it has an operation
        ++ \spad{"*": (D,D) -> D} which is commutative.

    "-": (%,% ) -> Union(%, "failed")
        ++ \spad{x - y} returns an element z such that
        ++ \spad{z+y=x} or "failed" if no such element exists.
        ++
        ++X c2:=2::CardinalNumber
        ++X c2-c2
        ++X A1:=Aleph 1
        ++X A1-c2

    "**": (% , %) -> %
        ++ \spad{x**y} returns \spad{#(X**Y)} where \spad{X**Y} is defined
        ++ as \spad{\{g| g:Y->X\}}.
        ++
        ++X c2:=2::CardinalNumber
        ++X c2**c2
        ++X A1:=Aleph 1

```

```

++X A1**c2
++X generalizedContinuumHypothesisAssumed true
++X A1**A1

Aleph: NonNegativeInteger -> %
++ Aleph(n) provides the named (infinite) cardinal number.
++
++X A0:=Aleph 0

finite?: % -> Boolean
++ finite?(\spad{a}) determines whether
++ \spad{a} is a finite cardinal, i.e. an integer.
++
++X c2:=2::CardinalNumber
++X finite? c2
++X A0:=Aleph 0
++X finite? A0

countable?: % -> Boolean
++ countable?(\spad{a}) determines
++ whether \spad{a} is a countable cardinal,
++ i.e. an integer or \spad{Aleph 0}.
++
++X c2:=2::CardinalNumber
++X countable? c2
++X A0:=Aleph 0
++X countable? A0
++X A1:=Aleph 1
++X countable? A1

generalizedContinuumHypothesisAssumed?: () -> Boolean
++ generalizedContinuumHypothesisAssumed?()
++ tests if the hypothesis is currently assumed.
++
++X generalizedContinuumHypothesisAssumed?

generalizedContinuumHypothesisAssumed: Boolean -> Boolean
++ generalizedContinuumHypothesisAssumed(bool)
++ is used to dictate whether the hypothesis is to be assumed.
++
++X generalizedContinuumHypothesisAssumed true
++X a:=Aleph 0
++X c:=2**a
++X f:=2**c

== add
NNI ==> NonNegativeInteger
FINord ==> -1
DUMMYval ==> -1

Rep := Record(order: Integer, ival: Integer)

```

```

GCHypothesis: Reference(Boolean) := ref false

-- Creation
0          == [FINord, 0]
1          == [FINord, 1]
coerce(n:NonNegativeInteger):% == [FINord, n]
Aleph n    == [n, DUMMYval]

-- Output
ALEPExpr := "Aleph":OutputForm

coerce(x: %): OutputForm ==
  x.order = FINord => (x.ival)::OutputForm
  prefix(ALEPExpr, [(x.order)::OutputForm])

-- Manipulation
x = y ==
  x.order ^= y.order => false
  finite? x          => x.ival = y.ival
  true              -- equal transfinite
x < y ==
  x.order < y.order => true
  x.order > y.order => false
  finite? x          => x.ival < y.ival
  false              -- equal transfinite
x:% + y:% ==
  finite? x and finite? y => [FINord, x.ival+y.ival]
  max(x, y)
x - y ==
  x < y          => "failed"
  finite? x => [FINord, x.ival-y.ival]
  x > y          => x
  "failed" -- equal transfinite
x:% * y:% ==
  finite? x and finite? y => [FINord, x.ival*y.ival]
  x = 0 or y = 0          => 0
  max(x, y)
n:NonNegativeInteger * x:% ==
  finite? x => [FINord, n*x.ival]
  n = 0     => 0
  x
x**y ==
  y = 0 =>
    x ^= 0 => 1
    error "0**0 not defined for cardinal numbers."
  finite? y =>
    not finite? x => x
    [FINord,x.ival**(y.ival):NNI]
  x = 0 => 0

```

```

x = 1 => 1
GCHypothesis() => [max(x.order-1, y.order) + 1, DUMMYval]
error "Transfinite exponentiation only implemented under GCH"

finite? x == x.order = FINord
countable? x == x.order < 1

retract(x:):NonNegativeInteger ==
  finite? x => (x.ival):NNI
  error "Not finite"

retractIfCan(x:):Union(NonNegativeInteger, "failed") ==
  finite? x => (x.ival):NNI
  "failed"

-- State manipulation
generalizedContinuumHypothesisAssumed?() == GCHypothesis()
generalizedContinuumHypothesisAssumed b == (GCHypothesis() := b)



---



— CARD.dotabb —

"CARD" [color="#88FF44",href="bookvol10.3.pdf#nameddest=CARD"]
"ALGEBRA-" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALGEBRA"]
"CARD" -> "ALGEBRA-"

```

---

## 4.2 domain CARTEN CartesianTensor

```

— CartesianTensor.input —

)set break resume
)sys rm -f CartesianTensor.output
)spool CartesianTensor.output
)set message test on
)set message auto off
)clear all
--S 1 of 48
CT := CARTEN(i0 := 1, 2, Integer)
--R
--R
--R (1) CartesianTensor(1,2,Integer)

```

```

--R                                                    Type: Domain
--E 1

--S 2 of 48
t0: CT := 8
--R
--R
--R      (2)  8
--R
--R                                                    Type: CartesianTensor(1,2,Integer)
--E 2

--S 3 of 48
rank t0
--R
--R
--R      (3)  0
--R
--R                                                    Type: NonNegativeInteger
--E 3

--S 4 of 48
v: DirectProduct(2, Integer) := directProduct [3,4]
--R
--R
--R      (4)  [3,4]
--R
--R                                                    Type: DirectProduct(2,Integer)
--E 4

--S 5 of 48
Tv: CT := v
--R
--R
--R      (5)  [3,4]
--R
--R                                                    Type: CartesianTensor(1,2,Integer)
--E 5

--S 6 of 48
m: SquareMatrix(2, Integer) := matrix [ [1,2],[4,5] ]
--R
--R
--R      (6)  +1  2+
--R          |    |
--R          +4  5+
--R
--R                                                    Type: SquareMatrix(2,Integer)
--E 6

--S 7 of 48
Tm: CT := m
--R
--R
--R      +1  2+

```

```

--R      (7)  |      |
--R      +4  5+
--R
--R                                          Type: CartesianTensor(1,2,Integer)
--E 7

--S 8 of 48
n: SquareMatrix(2, Integer) := matrix [ [2,3],[0,1] ]
--R
--R
--R      +2  3+
--R      (8)  |      |
--R      +0  1+
--R
--R                                          Type: SquareMatrix(2,Integer)
--E 8

--S 9 of 48
Tn: CT := n
--R
--R
--R      +2  3+
--R      (9)  |      |
--R      +0  1+
--R
--R                                          Type: CartesianTensor(1,2,Integer)
--E 9

--S 10 of 48
t1: CT := [2, 3]
--R
--R
--R      (10)  [2,3]
--R
--R                                          Type: CartesianTensor(1,2,Integer)
--E 10

--S 11 of 48
rank t1
--R
--R
--R      (11)  1
--R
--R                                          Type: PositiveInteger
--E 11

--S 12 of 48
t2: CT := [t1, t1]
--R
--R
--R      +2  3+
--R      (12)  |      |
--R      +2  3+
--R
--R                                          Type: CartesianTensor(1,2,Integer)
--E 12

```

```

--S 13 of 48
t3: CT := [t2, t2]
--R
--R
--R      +2  3+ +2  3+
--R  (13) [|   |, |   |]
--R      +2  3+ +2  3+
--R
--R                                          Type: CartesianTensor(1,2,Integer)
--E 13

--S 14 of 48
tt: CT := [t3, t3]; tt := [tt, tt]
--R
--R
--R      ++2  3+  +2  3++ ++2  3+  +2  3++
--R      ||   |   |   ||  ||   |   |   ||
--R      |+2  3+  +2  3+| |+2  3+  +2  3+|
--R  (14) [|           |, |           |]
--R      |+2  3+  +2  3+| |+2  3+  +2  3+|
--R      ||   |   |   ||  ||   |   |   ||
--R      ++2  3+  +2  3++ ++2  3+  +2  3++
--R
--R                                          Type: CartesianTensor(1,2,Integer)
--E 14

--S 15 of 48
rank tt
--R
--R
--R  (15)  5
--R
--R                                          Type: PositiveInteger
--E 15

--S 16 of 48
Tmn := product(Tm, Tn)
--R
--R
--R      ++2  3+  +4  6+ +
--R      ||   |   |   ||  |
--R      |+0  1+  +0  2+ |
--R  (16) |           |
--R      |+8  12+ +10  15+|
--R      ||   |   |   ||
--R      ++0  4+  +0  5  ++
--R
--R                                          Type: CartesianTensor(1,2,Integer)
--E 16

--S 17 of 48
Tmv := contract(Tm,2,Tv,1)
--R

```



```

--R
--R (17) [11,32]
--R
--R                                         Type: CartesianTensor(1,2,Integer)
--E 17

--S 18 of 48
Tm*Tv
--R
--R
--R (18) [11,32]
--R
--R                                         Type: CartesianTensor(1,2,Integer)
--E 18

--S 19 of 48
Tmv = m * v
--R
--R
--R (19) [11,32]= [11,32]
--R
--R                                         Type: Equation CartesianTensor(1,2,Integer)
--E 19

--S 20 of 48
t0()
--R
--R
--R (20) 8
--R
--R                                         Type: PositiveInteger
--E 20

--S 21 of 48
t1(1+1)
--R
--R
--R (21) 3
--R
--R                                         Type: PositiveInteger
--E 21

--S 22 of 48
t2(2,1)
--R
--R
--R (22) 2
--R
--R                                         Type: PositiveInteger
--E 22

--S 23 of 48
t3(2,1,2)
--R
--R
--R (23) 3

```

[illegible]

```

--S 30 of 48
cTmn := contract(Tmn,1,2)
--R
--R
--R      +12 18+
--R (30) |   |
--R      +0 6 +
--R
--R                                          Type: CartesianTensor(1,2,Integer)
--E 30

--S 31 of 48
trace(m) * n
--R
--R
--R      +12 18+
--R (31) |   |
--R      +0 6 +
--R
--R                                          Type: SquareMatrix(2,Integer)
--E 31

--S 32 of 48
contract(Tmn,1,2) = trace(m) * n
--R
--R
--R      +12 18+ +12 18+
--R (32) |   | = |   |
--R      +0 6 + +0 6 +
--R
--R                                          Type: Equation CartesianTensor(1,2,Integer)
--E 32

--S 33 of 48
contract(Tmn,1,3) = transpose(m) * n
--R
--R
--R      +2 7 + +2 7 +
--R (33) |   | = |   |
--R      +4 11+ +4 11+
--R
--R                                          Type: Equation CartesianTensor(1,2,Integer)
--E 33

--S 34 of 48
contract(Tmn,1,4) = transpose(m) * transpose(n)
--R
--R
--R      +14 4+ +14 4+
--R (34) |   | = |   |
--R      +19 5+ +19 5+
--R
--R                                          Type: Equation CartesianTensor(1,2,Integer)
--E 34

```

```

--S 35 of 48
contract(Tmn,2,3) = m * n
--R
--R
--R      +2  5 +  +2  5 +
--R  (35) |      |= |      |
--R      +8 17+ +8 17+
--R
--R                                          Type: Equation CartesianTensor(1,2,Integer)
--E 35

```

```

--S 36 of 48
contract(Tmn,2,4) = m * transpose(n)
--R
--R
--R      +8  2+ +8  2+
--R  (36) |      |= |      |
--R      +23 5+ +23 5+
--R
--R                                          Type: Equation CartesianTensor(1,2,Integer)
--E 36

```

```

--S 37 of 48
contract(Tmn,3,4) = trace(n) * m
--R
--R
--R      +3  6 +  +3  6 +
--R  (37) |      |= |      |
--R      +12 15+ +12 15+
--R
--R                                          Type: Equation CartesianTensor(1,2,Integer)
--E 37

```

```

--S 38 of 48
tTmn := transpose(Tmn,1,3)
--R
--R
--R      ++2  3 +  +4  6 ++
--R      ||      |  |      ||
--R      |+8 12+ +10 15+|
--R  (38) |      |
--R      |+0 1+  +0 2+ |
--R      ||      |  |  ||
--R      ++0 4+  +0 5+ +
--R
--R                                          Type: CartesianTensor(1,2,Integer)
--E 38

```

```

--S 39 of 48
transpose Tmn
--R
--R
--R      ++2  8+  +4 10++

```

```

--R      ||      |      |      ||
--R      |+0  0+      +0  0 +|
--R  (39) |      |      |
--R      |+3  12+  +6  15+|
--R      ||      |      |      ||
--R      ++1  4 +  +2  5 ++
--R
--E 39

```

Type: CartesianTensor(1,2,Integer)

```

--S 40 of 48
transpose Tm = transpose m
--R
--R
--R      +1  4+  +1  4+
--R  (40) |      |= |      |
--R      +2  5+  +2  5+
--R
--E 40

```

Type: Equation CartesianTensor(1,2,Integer)

```

--S 41 of 48
rTmn := reindex(Tmn, [1,4,2,3])
--R
--R
--R      ++2  0+      +3  1+ +
--R      ||      |      |      ||
--R      |+4  0+      +6  2+ |
--R  (41) |      |
--R      |+8  0+      +12  4+|
--R      ||      |      |      ||
--R      ++10  0+      +15  5++
--R
--E 41

```

Type: CartesianTensor(1,2,Integer)

```

--S 42 of 48
tt := transpose(Tm)*Tn - Tn*transpose(Tm)
--R
--R
--R      +- 6  - 16+
--R  (42) |      |
--R      + 2      6  +
--R
--E 42

```

Type: CartesianTensor(1,2,Integer)

```

--S 43 of 48
Tv*(tt+Tn)
--R
--R
--R  (43) [- 4,- 11]
--R
--E 43

```

Type: CartesianTensor(1,2,Integer)

```

--S 44 of 48
reindex(product(Tn,Tn),[4,3,2,1])+3*Tn*product(Tm,Tm)
--R
--R
--R      ++46  84 +  +57  114++
--R      ||      |  |      ||
--R      |+174 212+ +228 285+|
--R (44) |      |      |
--R      | +18 24+  +17 30+ |
--R      | |      |      | |
--R      + +57 63+  +63 76+ +
--R
--R                                          Type: CartesianTensor(1,2,Integer)
--E 44

--S 45 of 48
delta: CT := kroneckerDelta()
--R
--R
--R      +1  0+
--R (45) |      |
--R      +0  1+
--R
--R                                          Type: CartesianTensor(1,2,Integer)
--E 45

--S 46 of 48
contract(Tmn, 2, delta, 1) = reindex(Tmn, [1,3,4,2])
--R
--R
--R      + +2  4+  +0  0++ + +2  4+  +0  0++
--R      | |      |  |      || | |      |  |      ||
--R      | +3  6+  +1  2+| | +3  6+  +1  2+|
--R (46) |      |      | = |      |
--R      |+8   10+ +0  0+| |+8   10+ +0  0+|
--R      ||      |  |      || ||      |  |      ||
--R      ++12 15+ +4  5++ ++12 15+ +4  5++
--R
--R                                          Type: Equation CartesianTensor(1,2,Integer)
--E 46

--S 47 of 48
epsilon:CT := leviCivitaSymbol()
--R
--R
--R      + 0  1+
--R (47) |      |
--R      +- 1  0+
--R
--R                                          Type: CartesianTensor(1,2,Integer)
--E 47

--S 48 of 48

```

```

contract(epsilon*Tm*epsilon, 1,2) = 2 * determinant m
--R
--R
--R (48) - 6= - 6
--R
--R                                         Type: Equation CartesianTensor(1,2,Integer)
--E 48
)spool
)lisp (bye)

```

---

— CartesianTensor.help —

=====  
 CartesianTensor examples  
 =====

CartesianTensor(i0,dim,R) provides Cartesian tensors with components belonging to a commutative ring R. Tensors can be described as a generalization of vectors and matrices. This gives a concise tensor algebra for multilinear objects supported by the CartesianTensor domain. You can form the inner or outer product of any two tensors and you can add or subtract tensors with the same number of components. Additionally, various forms of traces and transpositions are useful.

The CartesianTensor constructor allows you to specify the minimum index for subscripting. In what follows we discuss in detail how to manipulate tensors.

Here we construct the domain of Cartesian tensors of dimension 2 over the integers, with indices starting at 1.

```

CT := CARTEN(i0 := 1, 2, Integer)
      CartesianTensor(1,2,Integer)
                        Type: Domain

```

=====  
 Forming tensors  
 =====

Scalars can be converted to tensors of rank zero.

```

t0: CT := 8
      8
                        Type: CartesianTensor(1,2,Integer)

rank t0
      0
                        Type: NonNegativeInteger

```

Vectors (mathematical direct products, rather than one dimensional array structures) can be converted to tensors of rank one.

```
v: DirectProduct(2, Integer) := directProduct [3,4]
[3, 4]
Type: DirectProduct(2,Integer)

Tv: CT := v
[3, 4]
Type: CartesianTensor(1,2,Integer)
```

Matrices can be converted to tensors of rank two.

```
m: SquareMatrix(2, Integer) := matrix [ [1,2],[4,5] ]
+1 2+
|   |
+4 5+
Type: SquareMatrix(2,Integer)

Tm: CT := m
+1 2+
|   |
+4 5+
Type: CartesianTensor(1,2,Integer)

n: SquareMatrix(2, Integer) := matrix [ [2,3],[0,1] ]
+2 3+
|   |
+0 1+
Type: SquareMatrix(2,Integer)

Tn: CT := n
+2 3+
|   |
+0 1+
Type: CartesianTensor(1,2,Integer)
```

In general, a tensor of rank  $k$  can be formed by making a list of rank  $k-1$  tensors or, alternatively, a  $k$ -deep nested list of lists.

```
t1: CT := [2, 3]
[2, 3]
Type: CartesianTensor(1,2,Integer)

rank t1
1
Type: PositiveInteger

t2: CT := [t1, t1]
```



```

+2  3+
|    |
+2  3+

```

Type: CartesianTensor(1,2,Integer)

```
t3: CT := [t2, t2]
```

```

+2  3+ +2  3+
[|    |, |    |]
+2  3+ +2  3+

```

Type: CartesianTensor(1,2,Integer)

```

tt: CT := [t3, t3]; tt := [tt, tt]
++2  3+  +2  3++ ++2  3+  +2  3++
||    |  |    || ||    |  |    ||
|+2  3+  +2  3+| |+2  3+  +2  3+|
[|          |, |          |]
|+2  3+  +2  3+| |+2  3+  +2  3+|
||    |  |    || ||    |  |    ||
++2  3+  +2  3++ ++2  3+  +2  3++

```

Type: CartesianTensor(1,2,Integer)

```

rank tt
5

```

Type: PositiveInteger

#### Multiplication

Given two tensors of rank  $k_1$  and  $k_2$ , the outer product forms a new tensor of rank  $k_1+k_2$ . Here

$$T_{mn}(i,j,k,l) = T_m(i,j)T_n(k,l)$$

```

Tmn := product(Tm, Tn)
++2  3+    +4  6+ +
||    |    |    | |
|+0  1+    +0  2+ |
|          |
|+8  12+  +10  15+|
||    |    |    ||
++0  4 +  +0  5 ++

```

Type: CartesianTensor(1,2,Integer)

The inner product (contract) forms a tensor of rank  $k_1+k_2-2$ . This product generalizes the vector dot product and matrix-vector product by summing component products along two indices.

Here we sum along the second index of  $T_m$  and the first index of  $T_v$ . Here

```

Tmv = sum {j=1..dim} Tm(i,j) Tv(j)

Tmv := contract(Tm,2,Tv,1)
[11,32]
Type: CartesianTensor(1,2,Integer)

```

The multiplication operator `*` is scalar multiplication or an inner product depending on the ranks of the arguments.

If either argument is rank zero it is treated as scalar multiplication. Otherwise, `a*b` is the inner product summing the last index of `a` with the first index of `b`.

```

Tm*Tv
[11,32]
Type: CartesianTensor(1,2,Integer)

```

This definition is consistent with the inner product on matrices and vectors.

```

Tmv = m * v
[11,32] = [11,32]
Type: Equation CartesianTensor(1,2,Integer)

```

```

=====
Selecting Components
=====

```

For tensors of low rank (that is, four or less), components can be selected by applying the tensor to its indices.

```

t0()
8
Type: PositiveInteger

t1(1+1)
3
Type: PositiveInteger

t2(2,1)
2
Type: PositiveInteger

t3(2,1,2)
3
Type: PositiveInteger

Tmn(2,1,2,1)
0

```

Type: NonNegativeInteger

A general indexing mechanism is provided for a list of indices.

```
t0[]
  8
```

Type: PositiveInteger

```
t1[2]
  3
```

Type: PositiveInteger

```
t2[2,1]
  2
```

Type: PositiveInteger

The general mechanism works for tensors of arbitrary rank, but is somewhat less efficient since the intermediate index list must be created.

```
t3[2,1,2]
  3
```

Type: PositiveInteger

```
Tmn[2,1,2,1]
  0
```

Type: NonNegativeInteger

```
=====
Contraction
=====
```

A "contraction" between two tensors is an inner product, as we have seen above. You can also contract a pair of indices of a single tensor. This corresponds to a "trace" in linear algebra. The expression `contract(t,k1,k2)` forms a new tensor by summing the diagonal given by indices in position `k1` and `k2`.

This is the tensor given by

```
xTmn = sum{k=1..dim} Tmn(k,k,i,j)
```

```
cTmn := contract(Tmn,1,2)
```

```
+12 18+
|    |
+0  6 +
```

Type: CartesianTensor(1,2,Integer)

Since `Tmn` is the outer product of matrix `m` and matrix `n`, the above is equivalent to this.

```
trace(m) * n
```

```

+12 18+
|    |
+0  6 +

```

Type: SquareMatrix(2,Integer)

In this and the next few examples, we show all possible contractions of Tmn and their matrix algebra equivalents.

```
contract(Tmn,1,2) = trace(m) * n
```

```

+12 18+ +12 18+
|      |= |      |
+0  6 + +0  6 +

```

Type: Equation CartesianTensor(1,2,Integer)

```
contract(Tmn,1,3) = transpose(m) * n
```

```

+2  7 + +2  7 +
|      |= |      |
+4 11+ +4 11+

```

Type: Equation CartesianTensor(1,2,Integer)

```
contract(Tmn,1,4) = transpose(m) * transpose(n)
```

```

+14 4+ +14 4+
|      |= |      |
+19 5+ +19 5+

```

Type: Equation CartesianTensor(1,2,Integer)

```
contract(Tmn,2,3) = m * n
```

```

+2  5 + +2  5 +
|      |= |      |
+8 17+ +8 17+

```

Type: Equation CartesianTensor(1,2,Integer)

```
contract(Tmn,2,4) = m * transpose(n)
```

```

+8  2+ +8  2+
|      |= |      |
+23 5+ +23 5+

```

Type: Equation CartesianTensor(1,2,Integer)

```
contract(Tmn,3,4) = trace(n) * m
```

```

+3  6 + +3  6 +
|      |= |      |
+12 15+ +12 15+

```

Type: Equation CartesianTensor(1,2,Integer)

```

=====
Transpositions
=====

```

You can exchange any desired pair of indices using the transpose operation.

Here the indices in positions one and three are exchanged, that is,  
 $tTmn(i,j,k,l) = Tmn(k,j,i,l)$

```
tTmn := transpose(Tmn,1,3)
++2  3 +  +4  6 ++
||    |  |    ||
|+8  12+ +10 15+|
|      |
|+0  1+   +0  2+ |
||    |   |    ||
++0  4+   +0  5+ +
Type: CartesianTensor(1,2,Integer)
```

If no indices are specified, the first and last index are exchanged.

```
transpose Tmn
++2  8+   +4  10++
||    |   |    ||
|+0  0+   +0  0 +|
|      |
|+3  12+  +6  15+|
||    |   |    ||
++1  4 +  +2  5 ++
Type: CartesianTensor(1,2,Integer)
```

This is consistent with the matrix transpose.

```
transpose Tm = transpose m
+1  4+  +1  4+
|    |= |    |
+2  5+  +2  5+
Type: Equation CartesianTensor(1,2,Integer)
```

If a more complicated reordering of the indices is required, then the `reindex` operation can be used. This operation allows the indices to be arbitrarily permuted.

```
rTmn(i,j,k,l) = Tmn(i,l,j,k)

rTmn := reindex(Tmn, [1,4,2,3])
++2  0+   +3  1+ +
||    |   |    ||
|+4  0+   +6  2+ |
|      |
|+8  0+  +12  4+|
||    |   |    ||
++10 0+  +15  5++
Type: CartesianTensor(1,2,Integer)
```

```
=====
Arithmetic
=====
```

Tensors of equal rank can be added or subtracted so arithmetic expressions can be used to produce new tensors.

```
tt := transpose(Tm)*Tn - Tn*transpose(Tm)
+- 6  - 16+
|      |
+ 2      6  +
Type: CartesianTensor(1,2,Integer)
```

```
Tv*(tt+Tn)
[- 4,- 11]
Type: CartesianTensor(1,2,Integer)
```

```
reindex(product(Tn,Tn),[4,3,2,1])+3*Tn*product(Tm,Tm)
++46  84 +  +57  114++
||      |  |      ||
|+174 212+ +228 285+|
|      |      |
| +18 24+  +17 30+ |
| |      |  |      |
+ +57 63+  +63 76+ +
Type: CartesianTensor(1,2,Integer)
```

```
=====
Specific Tensors
=====
```

Two specific tensors have properties which depend only on the dimension.

The Kronecker delta satisfies

```

+-          +-
|  1  if i = j |
delta(i,j) = |      |
|  0  if i != j |
+-          +-

```

```
delta: CT := kroneckerDelta()
+1  0+
|      |
+0  1+
Type: CartesianTensor(1,2,Integer)
```

This can be used to reindex via contraction.

```
contract(Tmn, 2, delta, 1) = reindex(Tmn, [1,3,4,2])
+ +2 4+ +0 0++ + +2 4+ +0 0++
| | | | | | | |
| +3 6+ +1 2+| | +3 6+ +1 2+|
| | | | | | | |
|+8 10+ +0 0+| |+8 10+ +0 0+|
|| | | || || | |
++12 15+ +4 5++ ++12 15+ +4 5++
Type: Equation CartesianTensor(1,2,Integer)
```

The Levi Civita symbol determines the sign of a permutation of indices.

```
epsilon:CT := leviCivitaSymbol()
+ 0 1+
| |
+- 1 0+
Type: CartesianTensor(1,2,Integer)
```

Here we have:

```
epsilon(i1,...,idim)
= +1 if i1,...,idim is an even permutation of i0,...,i0+dim-1
= -1 if i1,...,idim is an odd permutation of i0,...,i0+dim-1
= 0 if i1,...,idim is not a permutation of i0,...,i0+dim-1
```

This property can be used to form determinants.

```
contract(epsilon*Tm*epsilon, 1,2) = 2 * determinant m
- 6= - 6
Type: Equation CartesianTensor(1,2,Integer)
```

#### Properties of the CartesianTensor domain

GradedModule(R,E) denotes "E-graded R-module", that is, a collection of R-modules indexed by an abelian monoid E. An element  $g$  of  $G[s]$  for some specific  $s$  in  $E$  is said to be an element of  $G$  with degree  $s$ . Sums are defined in each module  $G[s]$  so two elements of  $G$  can be added if they have the same degree. Morphisms can be defined and composed by degree to give the mathematical category of graded modules.

GradedAlgebra(R,E) denotes "E-graded R-algebra". A graded algebra is a graded module together with a degree preserving R-bilinear map, called the product.

```
degree(product(a,b)) = degree(a) + degree(b)
```

```

product(r*a,b)      = product(a,r*b) = r*product(a,b)
product(a1+a2,b)    = product(a1,b) + product(a2,b)
product(a,b1+b2)    = product(a,b1) + product(a,b2)
product(a,product(b,c)) = product(product(a,b),c)

```

The domain `CartesianTensor(i0, dim, R)` belongs to the category `GradedAlgebra(R, NonNegativeInteger)`. The non-negative integer degree is the tensor rank and the graded algebra product is the tensor outer product. The graded module addition captures the notion that only tensors of equal rank can be added.

If  $V$  is a vector space of dimension  $\dim$  over  $R$ , then the tensor module  $T[k](V)$  is defined as

```

T[0](V) = R
T[k](V) = T[k-1](V) * V

```

where  $*$  denotes the  $R$ -module tensor product. `CartesianTensor(i0,dim,R)` is the graded algebra in which the degree  $k$  module is  $T[k](V)$ .

```

=====
Tensor Calculus
=====

```

It should be noted here that often tensors are used in the context of tensor-valued manifold maps. This leads to the notion of covariant and contravariant bases with tensor component functions transforming in specific ways under a change of coordinates on the manifold. This is no more directly supported by the `CartesianTensor` domain than it is by the `Vector` domain. However, it is possible to have the components implicitly represent component maps by choosing a polynomial or expression type for the components. In this case, it is up to the user to satisfy any constraints which arise on the basis of this interpretation.

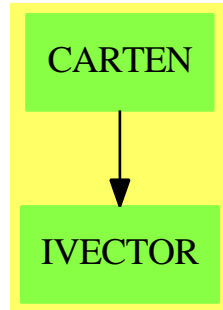
See Also

o `)show CartesianTensor`

---



### 4.2.1 CartesianTensor (CARTEN)



#### Exports:

|              |        |                |          |                  |
|--------------|--------|----------------|----------|------------------|
| 0            | 1      | coerce         | contract | degree           |
| elt          | hash   | kroneckerDelta | latex    | leviCivitaSymbol |
| product      | rank   | ravel          | reindex  | retract          |
| retractIfCan | sample | transpose      | unravel  | ?~=?             |
| ?.?          | ?*?    | ?+?            | ?-?      | -?               |
| ?=?          |        |                |          |                  |

— domain CARTEN CartesianTensor —

```

)abbrev domain CARTEN CartesianTensor
++ Author: Stephen M. Watt
++ Date Created: December 1986
++ Date Last Updated: May 15, 1991
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: tensor, graded algebra
++ Examples:
++ References:
++ Description:
++ CartesianTensor(minix,dim,R) provides Cartesian tensors with
++ components belonging to a commutative ring R. These tensors
++ can have any number of indices. Each index takes values from
++ \spad{minix} to \spad{minix + dim - 1}.
  
```

```

CartesianTensor(minix, dim, R): Exports == Implementation where
  NNI ==> NonNegativeInteger
  I   ==> Integer
  DP  ==> DirectProduct
  SM  ==> SquareMatrix

  minix: Integer
  dim: NNI
  
```

```

R: CommutativeRing

Exports ==> Join(GradedAlgebra(R, NNI), GradedModule(I, NNI)) with

coerce: DP(dim, R) -> %
  ++ coerce(v) views a vector as a rank 1 tensor.
  ++
  ++X v:DirectProduct(2,Integer):=directProduct [3,4]
  ++X tv:CartesianTensor(1,2,Integer):=v

coerce: SM(dim, R) -> %
  ++ coerce(m) views a matrix as a rank 2 tensor.
  ++
  ++X v:SquareMatrix(2,Integer):=[[1,2],[3,4]]
  ++X tv:CartesianTensor(1,2,Integer):=v

coerce: List R -> %
  ++ coerce([r_1,...,r_dim]) allows tensors to be constructed
  ++ using lists.
  ++
  ++X v:=[2,3]
  ++X tv:CartesianTensor(1,2,Integer):=v

coerce: List % -> %
  ++ coerce([t_1,...,t_dim]) allows tensors to be constructed
  ++ using lists.
  ++
  ++X v:=[2,3]
  ++X tv:CartesianTensor(1,2,Integer):=v
  ++X tm:CartesianTensor(1,2,Integer):=[tv,tv]

rank: % -> NNI
  ++ rank(t) returns the tensorial rank of t (that is, the
  ++ number of indices). This is the same as the graded module
  ++ degree.
  ++
  ++X CT:=CARTEN(1,2,Integer)
  ++X t0:CT:=8
  ++X rank t0

elt: (%) -> R
  ++ elt(t) gives the component of a rank 0 tensor.
  ++
  ++X tv:CartesianTensor(1,2,Integer):=8
  ++X elt(tv)
  ++X tv[]

elt: (% , I) -> R
  ++ elt(t,i) gives a component of a rank 1 tensor.
  ++

```

```

++X v:=[2,3]
++X tv:CartesianTensor(1,2,Integer):=v
++X elt(tv,2)
++X tv[2]

elt: (% , I, I) -> R
++ elt(t,i,j) gives a component of a rank 2 tensor.
++
++X v:=[2,3]
++X tv:CartesianTensor(1,2,Integer):=v
++X tm:CartesianTensor(1,2,Integer):=[tv,tv]
++X elt(tm,2,2)
++X tm[2,2]

elt: (% , I, I, I) -> R
++ elt(t,i,j,k) gives a component of a rank 3 tensor.
++
++X v:=[2,3]
++X tv:CartesianTensor(1,2,Integer):=v
++X tm:CartesianTensor(1,2,Integer):=[tv,tv]
++X tn:CartesianTensor(1,2,Integer):=[tm,tm]
++X elt(tn,2,2,2)
++X tn[2,2,2]

elt: (% , I, I, I, I) -> R
++ elt(t,i,j,k,l) gives a component of a rank 4 tensor.
++
++X v:=[2,3]
++X tv:CartesianTensor(1,2,Integer):=v
++X tm:CartesianTensor(1,2,Integer):=[tv,tv]
++X tn:CartesianTensor(1,2,Integer):=[tm,tm]
++X tp:CartesianTensor(1,2,Integer):=[tn,tn]
++X elt(tp,2,2,2,2)
++X tp[2,2,2,2]

elt: (% , List I) -> R
++ elt(t,[i1,...,iN]) gives a component of a rank \spad{N} tensor.
++
++X v:=[2,3]
++X tv:CartesianTensor(1,2,Integer):=v
++X tm:CartesianTensor(1,2,Integer):=[tv,tv]
++X tn:CartesianTensor(1,2,Integer):=[tm,tm]
++X tp:CartesianTensor(1,2,Integer):=[tn,tn]
++X tq:CartesianTensor(1,2,Integer):=[tp,tp]
++X elt(tq,[2,2,2,2,2])

-- This specializes the documentation from GradedAlgebra.
product: (% ,%) -> %
++ product(s,t) is the outer product of the tensors s and t.
++ For example, if \spad{r = product(s,t)} for rank 2 tensors

```

```

++ s and t, then \spad{r} is a rank 4 tensor given by
++   \spad{r(i,j,k,l) = s(i,j)*t(k,l)}.
++
++X m:SquareMatrix(2,Integer):=matrix [[1,2],[4,5]]
++X Tm:CartesianTensor(1,2,Integer):=m
++X n:SquareMatrix(2,Integer):=matrix [[2,3],[0,1]]
++X Tn:CartesianTensor(1,2,Integer):=n
++X Tmn:=product(Tm,Tn)

" * ": (% , %) -> %
++ s*t is the inner product of the tensors s and t which contracts
++ the last index of s with the first index of t, that is,
++   \spad{t*s = contract(t,rank t, s, 1)}
++   \spad{t*s = sum(k=1..N, t[i1,...,iN,k]*s[k,j1,...,jM])}
++ This is compatible with the use of \spad{M*v} to denote
++ the matrix-vector inner product.
++
++X m:SquareMatrix(2,Integer):=matrix [[1,2],[4,5]]
++X Tm:CartesianTensor(1,2,Integer):=m
++X v:DirectProduct(2,Integer):=directProduct [3,4]
++X Tv:CartesianTensor(1,2,Integer):=v
++X Tm*Tv

contract: (% , Integer, % , Integer) -> %
++ contract(t,i,s,j) is the inner product of tensors s and t
++ which sums along the \spad{k1}-th index of
++ t and the \spad{k2}-th index of s.
++ For example, if \spad{r = contract(s,2,t,1)} for rank 3 tensors
++ rank 3 tensors \spad{s} and \spad{t}, then \spad{r} is
++ the rank 4 \spad{(= 3 + 3 - 2)} tensor given by
++   \spad{r(i,j,k,l) = sum(h=1..dim,s(i,h,j)*t(h,k,l))}.
++
++X m:SquareMatrix(2,Integer):=matrix [[1,2],[4,5]]
++X Tm:CartesianTensor(1,2,Integer):=m
++X v:DirectProduct(2,Integer):=directProduct [3,4]
++X Tv:CartesianTensor(1,2,Integer):=v
++X Tmv:=contract(Tm,2,Tv,1)

contract: (% , Integer, Integer) -> %
++ contract(t,i,j) is the contraction of tensor t which
++ sums along the \spad{i}-th and \spad{j}-th indices.
++ For example, if
++ \spad{r = contract(t,1,3)} for a rank 4 tensor t, then
++ \spad{r} is the rank 2 \spad{(= 4 - 2)} tensor given by
++   \spad{r(i,j) = sum(h=1..dim,t(h,i,h,j))}.
++
++X m:SquareMatrix(2,Integer):=matrix [[1,2],[4,5]]
++X Tm:CartesianTensor(1,2,Integer):=m
++X v:DirectProduct(2,Integer):=directProduct [3,4]
++X Tv:CartesianTensor(1,2,Integer):=v

```

```

++X Tmv:=contract(Tm,2,1)

transpose: % -> %
++ transpose(t) exchanges the first and last indices of t.
++ For example, if \spad{r = transpose(t)} for a rank 4
++ tensor t, then \spad{r} is the rank 4 tensor given by
++   \spad{r(i,j,k,l) = t(l,j,k,i)}.
++
++X m:SquareMatrix(2,Integer):=matrix [[1,2],[4,5]]
++X Tm:CartesianTensor(1,2,Integer):=m
++X transpose(Tm)

transpose: (% , Integer, Integer) -> %
++ transpose(t,i,j) exchanges the \spad{i}-th and \spad{j}-th
++ indices of t. For example, if \spad{r = transpose(t,2,3)}
++ for a rank 4 tensor t, then \spad{r} is the rank 4 tensor
++ given by
++   \spad{r(i,j,k,l) = t(i,k,j,l)}.
++
++X m:SquareMatrix(2,Integer):=matrix [[1,2],[4,5]]
++X tm:CartesianTensor(1,2,Integer):=m
++X tn:CartesianTensor(1,2,Integer):=[tm,tm]
++X transpose(tn,1,2)

reindex: (% , List Integer) -> %
++ reindex(t,[i1,...,idim]) permutes the indices of t.
++ For example, if \spad{r = reindex(t, [4,1,2,3])}
++ for a rank 4 tensor t,
++ then \spad{r} is the rank for tensor given by
++   \spad{r(i,j,k,l) = t(l,i,j,k)}.
++
++X n:SquareMatrix(2,Integer):=matrix [[2,3],[0,1]]
++X tn:CartesianTensor(1,2,Integer):=n
++X p:=product(tn,tn)
++X reindex(p,[4,3,2,1])

kroneckerDelta: () -> %
++ kroneckerDelta() is the rank 2 tensor defined by
++   \spad{kroneckerDelta()(i,j)}
++   \spad{= 1 if i = j}
++   \spad{= 0 if i != j}
++
++X delta:CartesianTensor(1,2,Integer):=kroneckerDelta()

leviCivitaSymbol: () -> %
++ leviCivitaSymbol() is the rank \spad{dim} tensor defined by
++ \spad{leviCivitaSymbol()(i1,...,idim) = +1/0/-1}
++ if \spad{i1,...,idim} is an even/is nota /is an odd permutation
++ of \spad{minix,...,minix+dim-1}.
++

```

```

++X lcs:CartesianTensor(1,2,Integer):=leviCivitaSymbol()

ravel:      % -> List R
++ ravel(t) produces a list of components from a tensor such that
++   \spad{unravel(ravel(t)) = t}.
++
++X n:SquareMatrix(2,Integer):=matrix [[2,3],[0,1]]
++X tn:CartesianTensor(1,2,Integer):=n
++X ravel tn

unravel:    List R -> %
++ unravel(t) produces a tensor from a list of
++ components such that
++   \spad{unravel(ravel(t)) = t}.

sample:     () -> %
++ sample() returns an object of type %.

Implementation ==> add

PERM ==> Vector Integer -- 1-based entries from 1..n
INDEX ==> Vector Integer -- 1-based entries from minix..minix+dim-1

get ==> elt$Rep
set_! ==> setelt$Rep

-- Use row-major order:
--   x[h,i,j] <-> x[(h-minix)*dim**2+(i-minix)*dim+(j-minix)]

Rep := IndexedVector(R,0)

n:      Integer
r,s:    R
x,y,z:  %

---- Local stuff
dim2: NNI := dim**2
dim3: NNI := dim**3
dim4: NNI := dim**4

sample()==kroneckerDelta()$%
int2index(n: Integer, indv: INDEX): INDEX ==
  n < 0 => error "Index error (too small)"
  rnk := #indv
  for i in 1..rnk repeat
    qr := divide(n, dim)
    n := qr.quotient
    indv.((rnk-i+1) pretend NNI) := qr.remainder + minix
  n ^= 0 => error "Index error (too big)"

```

```

indv

index2int(indv: INDEX): Integer ==
  n: I := 0
  for i in 1..#indv repeat
    ix := indv.i - minix
    ix < 0 or ix > dim-1 => error "Index error (out of range)"
    n := dim*n + ix
  n

lengthRankOrElse(v: Integer): NNI ==
  v = 1    => 0
  v = dim  => 1
  v = dim2 => 2
  v = dim3 => 3
  v = dim4 => 4
  rx := 0
  while v ^= 0 repeat
    qr := divide(v, dim)
    v := qr.quotient
    if v ^= 0 then
      qr.remainder ^= 0 => error "Rank is not a whole number"
      rx := rx + 1
  rx

-- l must be a list of the numbers 1..#l
mkPerm(n: NNI, l: List Integer): PERM ==
  #l ^= n =>
    error "The list is not a permutation."
  p: PERM := new(n, 0)
  seen: Vector Boolean := new(n, false)
  for i in 1..n for e in l repeat
    e < 1 or e > n => error "The list is not a permutation."
    p.i := e
    seen.e := true
  for e in 1..n repeat
    not seen.e => error "The list is not a permutation."
  p

-- permute s according to p into result t.
permute_!(t: INDEX, s: INDEX, p: PERM): INDEX ==
  for i in 1..#p repeat t.i := s.(p.i)
  t

-- permsign!(v) = 1, 0, or -1 according as
-- v is an even, is not, or is an odd permutation of minix..minix+#v-1.
permsign_(v: INDEX): Integer ==
  -- sum minix..minix+#v-1.
  maxix := minix+#v-1
  psum := (((maxix+1)*maxix - minix*(minix-1)) exquo 2)::Integer

```

```

-- +/v ^= psum => 0
n := 0
for i in 1..#v repeat n := n + v.i
n ^= psum => 0
-- Bubble sort! This is pretty grotesque.
totTrans: Integer := 0
nTrans: Integer := 1
while nTrans ^= 0 repeat
  nTrans := 0
  for i in 1..#v-1 for j in 2..#v repeat
    if v.i > v.j then
      nTrans := nTrans + 1
      e := v.i; v.i := v.j; v.j := e
  totTrans := totTrans + nTrans
for i in 1..dim repeat
  if v.i ^= minix+i-1 then return 0
odd? totTrans => -1
1

---- Exported functions
ravel x ==
  [get(x,i) for i in 0..#x-1]

unravel l ==
  -- lengthRankOrElse #l gives sytnax error
  nz: NNI := # l
  lengthRankOrElse nz
  z := new(nz, 0)
  for i in 0..nz-1 for r in l repeat set_!(z, i, r)
  z

kroneckerDelta() ==
  z := new(dim2, 0)
  for i in 1..dim for zi in 0.. by (dim+1) repeat set_!(z, zi, 1)
  z

leviCivitaSymbol() ==
  nz := dim**dim
  z := new(nz, 0)
  indv: INDEX := new(dim, 0)
  for i in 0..nz-1 repeat
    set_!(z, i, permsign_!(int2index(i, indv))::R)
  z

-- from GradedModule
degree x ==
  rank x

rank x ==
  n := #x

```



```

lengthRankOrElse n

elt(x) ==
  #x ^= 1  => error "Index error (the rank is not 0)"
  get(x,0)
elt(x, i: I) ==
  #x ^= dim => error "Index error (the rank is not 1)"
  get(x,(i-minix))
elt(x, i: I, j: I) ==
  #x ^= dim2 => error "Index error (the rank is not 2)"
  get(x,(dim*(i-minix) + (j-minix)))
elt(x, i: I, j: I, k: I) ==
  #x ^= dim3 => error "Index error (the rank is not 3)"
  get(x,(dim2*(i-minix) + dim*(j-minix) + (k-minix)))
elt(x, i: I, j: I, k: I, l: I) ==
  #x ^= dim4 => error "Index error (the rank is not 4)"
  get(x,(dim3*(i-minix)+dim2*(j-minix)+dim*(k-minix)+(l-minix)))

elt(x, i: List I) ==
  #i ^= rank x => error "Index error (wrong rank)"
  n: I := 0
  for ii in i repeat
    ix := ii - minix
    ix<0 or ix>dim-1 => error "Index error (out of range)"
    n := dim*n + ix
  get(x,n)

coerce(lr: List R): % ==
  #lr ^= dim => error "Incorrect number of components"
  z := new(dim, 0)
  for r in lr for i in 0..dim-1 repeat set_!(z, i, r)
  z
coerce(lx: List %): % ==
  #lx ^= dim => error "Incorrect number of slices"
  rx := rank first lx
  for x in lx repeat
    rank x ^= rx => error "Inhomogeneous slice ranks"
  nx := # first lx
  z := new(dim * nx, 0)
  for x in lx for offz in 0.. by nx repeat
    for i in 0..nx-1 repeat set_!(z, offz + i, get(x,i))
  z

retractIfCan(x:%):Union(R,"failed") ==
  zero? rank(x) => x()
  "failed"
Outf ==> OutputForm

mkOutf(x:%, i0:I, rnk:NNI): Outf ==
  odd? rnk =>

```

```

    rnk1 := (rnk-1) pretend NNI
    nskip := dim**rnk1
    [mkOutf(x, i0+nskip*i, rnk1) for i in 0..dim-1]::Outf
  rnk = 0 =>
    get(x,i0)::Outf
  rnk1 := (rnk-2) pretend NNI
  nskip := dim**rnk1
  matrix [[mkOutf(x, i0+nskip*(dim*i + j), rnk1)
    for j in 0..dim-1] for i in 0..dim-1]
coerce(x): Outf ==
  mkOutf(x, 0, rank x)

0 == 0$R::Rep
1 == 1$R::Rep

--coerce(n: I): % == new(1, n::R)
coerce(r: R): % == new(1,r)

coerce(v: DP(dim,R)): % ==
  z := new(dim, 0)
  for i in 0..dim-1 for j in minIndex v .. maxIndex v repeat
    set_!(z, i, v.j)
  z
coerce(m: SM(dim,R)): % ==
  z := new(dim**2, 0)
  offz := 0
  for i in 0..dim-1 repeat
    for j in 0..dim-1 repeat
      set_!(z, offz + j, m(i+1,j+1))
    offz := offz + dim
  z

x = y ==
  #x ^= #y => false
  for i in 0..#x-1 repeat
    if get(x,i) ^= get(y,i) then return false
  true
x + y ==
  #x ^= #y => error "Rank mismatch"
  -- z := [xi + yi for xi in x for yi in y]
  z := new(#x, 0)
  for i in 0..#x-1 repeat set_!(z, i, get(x,i) + get(y,i))
  z
x - y ==
  #x ^= #y => error "Rank mismatch"
  -- [xi - yi for xi in x for yi in y]
  z := new(#x, 0)
  for i in 0..#x-1 repeat set_!(z, i, get(x,i) - get(y,i))
  z
- x ==

```

```

-- [-xi for xi in x]
z := new(#x, 0)
for i in 0..#x-1 repeat set_!(z, i, -get(x,i))
z
n * x ==
-- [n * xi for xi in x]
z := new(#x, 0)
for i in 0..#x-1 repeat set_!(z, i, n * get(x,i))
z
x * n ==
-- [n * xi for xi in x]
z := new(#x, 0)
for i in 0..#x-1 repeat set_!(z, i, n* get(x,i)) -- Commutative!!
z
r * x ==
-- [r * xi for xi in x]
z := new(#x, 0)
for i in 0..#x-1 repeat set_!(z, i, r * get(x,i))
z
x * r ==
-- [xi*r for xi in x]
z := new(#x, 0)
for i in 0..#x-1 repeat set_!(z, i, r* get(x,i)) -- Commutative!!
z
product(x, y) ==
nx := #x; ny := #y
z := new(nx * ny, 0)
for i in 0..nx-1 for ioff in 0.. by ny repeat
  for j in 0..ny-1 repeat
    set_!(z, ioff + j, get(x,i) * get(y,j))
z
x * y ==
rx := rank x
ry := rank y
rx = 0 => get(x,0) * y
ry = 0 => x * get(y,0)
contract(x, rx, y, 1)

contract(x, i, j) ==
rx := rank x
i < 1 or i > rx or j < 1 or j > rx or i = j =>
  error "Improper index for contraction"
if i > j then (i,j) := (j,i)

rl:= (rx- j) pretend NNI; nl:= dim**rl; zol:= 1;      xol:= zol
rm:= (j-i-1) pretend NNI; nm:= dim**rm; zom:= nl;    xom:= zom*dim
rh:= (i - 1) pretend NNI; nh:= dim**rh; zoh:= nl*nm
xoh:= zoh*dim**2
xok := nl*(1 + nm*dim)
z   := new(nl*nm*nh, 0)

```

```

for h in 1..nh _
for xh in 0.. by xoh for zh in 0.. by zoh repeat
  for m in 1..nm _
  for xm in xh.. by xom for zm in zh.. by zom repeat
    for l in 1..nl _
    for xl in xm.. by xol for zl in zm.. by zol repeat
      set_!(z, zl, 0)
      for k in 1..dim for xk in xl.. by xok repeat
        set_!(z, zl, get(z,zl) + get(x,xk))
      z
contract(x, i, y, j) ==
  rx := rank x
  ry := rank y

  i < 1 or i > rx or j < 1 or j > ry =>
    error "Improper index for contraction"

  rly:= (ry-j) pretend NNI; nly:= dim**rly; oly:= 1;    zoly:= 1
  rhy:= (j -1) pretend NNI; nhy:= dim**rhy
  ohy:= nly*dim; zohy:= zoly*nly
  rlx:= (rx-i) pretend NNI; nlx:= dim**rlx
  olx:= 1;      zolx:= zohy*nhy
  rhx:= (i -1) pretend NNI; nhx:= dim**rhx
  ohx:= nlx*dim; zohx:= zolx*nlx

  z := new(nlx*nhx*nly*nhy, 0)

  for dxh in 1..nhx _
  for xh in 0.. by ohx for zhx in 0.. by zohx repeat
    for dxl in 1..nlx _
    for xl in xh.. by olx for zlx in zhx.. by zolx repeat
      for dyh in 1..nhy _
      for yh in 0.. by ohy for zhy in zlx.. by zohy repeat
        for dyl in 1..nly _
        for yl in yh.. by oly for zly in zhy.. by zoly repeat
          set_!(z, zly, 0)
          for k in 1..dim _
          for xk in xl.. by nlx for yk in yl.. by nly repeat
            set_!(z, zly, get(z,zly)+get(x,xk)*get(y,yk))
          z
transpose x ==
  transpose(x, 1, rank x)
transpose(x, i, j) ==
  rx := rank x
  i < 1 or i > rx or j < 1 or j > rx or i = j =>
    error "Improper indicies for transposition"
  if i > j then (i,j) := (j,i)

```

```

rl:= (rx- j) pretend NNI; nl:= dim**rl; zol:= 1;      zoi := zol*nl
rm:= (j-i-1) pretend NNI; nm:= dim**rm; zom:= nl*dim; zoj := zom*nm
rh:= (i - 1) pretend NNI; nh:= dim**rh; zoh:= nl*nm*dim**2
z  := new(#x, 0)
for h in 1..nh for zh in 0.. by zoh repeat _
for m in 1..nm for zm in zh.. by zom repeat _
for l in 1..nl for zl in zm.. by zol repeat _
  for p in 1..dim _
    for zp in zl.. by zoi for xp in zl.. by zoj repeat
      for q in 1..dim _
        for zq in zp.. by zoj for xq in xp.. by zoi repeat
          set_!(z, zq, get(x,xq))
z

reindex(x, 1) ==
  nx := #x
  z: % := new(nx, 0)

rx := rank x
p  := mkPerm(rx, 1)
xiv: INDEX := new(rx, 0)
ziv: INDEX := new(rx, 0)

-- Use permutation
for i in 0..#x-1 repeat
  pi := index2int(permute_!(ziv, int2index(i,xiv),p))
  set_!(z, pi, get(x,i))
z

-----

— CARTEN.dotabb —

"CARTEN" [color="#88FF44",href="bookvol10.3.pdf#nameddest=CARTEN"]
"IVECTOR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IVECTOR"]
"CARTEN" -> "IVECTOR"

-----

```

### 4.3 domain CHAR Character

— Character.input —

```
)set break resume
```

```

)sys rm -f Character.output
)spool Character.output
)set message test on
)set message auto off
)clear all
--S 1 of 13
chars := [char "a", char "A", char "X", char "8", char "+"]
--R
--R
--R (1) [a,A,X,8,+]
--R
--R                                          Type: List Character
--E 1

--S 2 of 13
space()
--R
--R
--R (2)
--R
--R                                          Type: Character
--E 2

--S 3 of 13
quote()
--R
--R
--R (3) "
--R
--R                                          Type: Character
--E 3

--S 4 of 13
escape()
--R
--R
--R (4) _
--R
--R                                          Type: Character
--E 4

--S 5 of 13
[ord c for c in chars]
--R
--R
--R (5) [97,65,88,56,43]
--R
--R                                          Type: List Integer
--E 5

--S 6 of 13
[upperCase c for c in chars]
--R
--R
--R (6) [A,A,X,8,+]

```



```

--S 13 of 13
[alphanumeric? c for c in chars]
--R
--R
--R (13) [true,true,true,true,false]
--R
--R                                          Type: List Boolean
--E 13
)spool
)lisp (bye)

```

---

— Character.help —

```

=====
Character examples
=====

```

The members of the domain Character are values representing letters, numerals and other text elements.

Characters can be obtained using String notation.

```

chars := [char "a", char "A", char "X", char "8", char "+"]
[a,A,X,8,+]

```

Type: List Character

Certain characters are available by name. This is the blank character.

```

space()

```

Type: Character

This is the quote that is used in strings.

```

quote()
"

```

Type: Character

This is the escape character that allows quotes and other characters within strings.

```

escape()
-

```

Type: Character

Characters are represented as integers in a machine-dependent way. The integer value can be obtained using the ord operation. It is



always true that `char(ord c) = c` and `ord(char i) = i`, provided that `i` is in the range `0..size()-1`.

```
[ord c for c in chars]
[97,65,88,56,43]
Type: List Integer
```

The `lowerCase` operation converts an upper case letter to the corresponding lower case letter. If the argument is not an upper case letter, then it is returned unchanged.

```
[upperCase c for c in chars]
[A,A,X,8,+]
Type: List Character
```

The `upperCase` operation converts lower case letters to upper case.

```
[lowerCase c for c in chars]
[a,a,x,8,+]
Type: List Character
```

A number of tests are available to determine whether characters belong to certain families.

```
[alphabetic? c for c in chars]
[true,true,true,false,false]
Type: List Boolean
```

```
[upperCase? c for c in chars]
[false,true,true,false,false]
Type: List Boolean
```

```
[lowerCase? c for c in chars]
[true,false,false,false,false]
Type: List Boolean
```

```
[digit? c for c in chars]
[false,false,false,true,false]
Type: List Boolean
```

```
[hexDigit? c for c in chars]
[true,true,false,true,false]
Type: List Boolean
```

```
[alphanumeric? c for c in chars]
[true,true,true,true,false]
Type: List Boolean
```

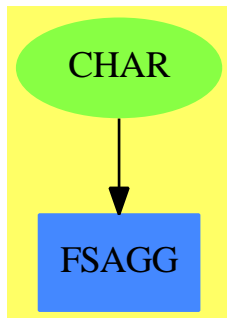
See Also:

- o `)help CharacterClass`

```
o )help String
o )show Character
```

---

### 4.3.1 Character (CHAR)



See

⇒ “CharacterClass” (CCLASS) 4.4.1 on page 365  
 ⇒ “IndexedString” (ISTRING) 10.14.1 on page 1214  
 ⇒ “String” (STRING) 20.31.1 on page 2565

#### Exports:

|               |               |            |        |        |
|---------------|---------------|------------|--------|--------|
| alphanumeric? | alphanumeric? | char       | coerce | digit? |
| escape        | hash          | hexDigit?  | index  | latex  |
| lookup        | lowerCase     | lowerCase? | max    | min    |
| ord           | quote         | random     | size   | space  |
| upperCase     | upperCase?    | ?~=?       | ?<?    | ?<=?   |
| ?=?           | ?>?           | ?>=?       |        |        |

— domain CHAR Character —

```
)abbrev domain CHAR Character
++ Author: Stephen M. Watt
++ Date Created: July 1986
++ Date Last Updated: June 20, 1991
++ Basic Operations: char
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: character, string
++ Examples:
++ References:
++ Description:
```

++ This domain provides the basic character data type.

```

Character: OrderedFinite() with
ord: % -> Integer
  ++ ord(c) provides an integral code corresponding to the
  ++ character c. It is always true that \spad{char ord c = c}.
  ++
  ++X chars := [char "a", char "A", char "X", char "8", char "+"]
  ++X [ord c for c in chars]
char: Integer -> %
  ++ char(i) provides a character corresponding to the integer
  ++ code i. It is always true that \spad{ord char i = i}.
  ++
  ++X [char c for c in [97,65,88,56,43]]
char: String -> %
  ++ char(s) provides a character from a string s of length one.
  ++
  ++X [char c for c in ["a","A","X","8","+"]]
space: () -> %
  ++ space() provides the blank character.
  ++
  ++X space()
quote: () -> %
  ++ quote() provides the string quote character, \spad{"}.
  ++
  ++X quote()
escape: () -> %
  ++ escape() provides the escape character, \spad{\\}, which
  ++ is used to allow quotes and other characters within
  ++ strings.
  ++
  ++X escape()
upperCase: % -> %
  ++ upperCase(c) converts a lower case letter to the corresponding
  ++ upper case letter. If c is not a lower case letter, then
  ++ it is returned unchanged.
  ++
  ++X chars := [char "a", char "A", char "X", char "8", char "+"]
  ++X [upperCase c for c in chars]
lowerCase: % -> %
  ++ lowerCase(c) converts an upper case letter to the corresponding
  ++ lower case letter. If c is not an upper case letter, then
  ++ it is returned unchanged.
  ++
  ++X chars := [char "a", char "A", char "X", char "8", char "+"]
  ++X [lowerCase c for c in chars]
digit?: % -> Boolean
  ++ digit?(c) tests if c is a digit character,
  ++ i.e. one of 0..9.
  ++

```

```

    ++X chars := [char "a", char "A", char "X", char "8", char "+"]
    ++X [digit? c for c in chars]
hexDigit?: % -> Boolean
    ++ hexDigit?(c) tests if c is a hexadecimal numeral,
    ++ i.e. one of 0..9, a..f or A..F.
    ++
    ++X chars := [char "a", char "A", char "X", char "8", char "+"]
    ++X [hexDigit? c for c in chars]
alphabetic?: % -> Boolean
    ++ alphabetic?(c) tests if c is a letter,
    ++ i.e. one of a..z or A..Z.
    ++
    ++X chars := [char "a", char "A", char "X", char "8", char "+"]
    ++X [alphabetic? c for c in chars]
upperCase?: % -> Boolean
    ++ upperCase?(c) tests if c is an upper case letter,
    ++ i.e. one of A..Z.
    ++
    ++X chars := [char "a", char "A", char "X", char "8", char "+"]
    ++X [upperCase? c for c in chars]
lowerCase?: % -> Boolean
    ++ lowerCase?(c) tests if c is an lower case letter,
    ++ i.e. one of a..z.
    ++
    ++X chars := [char "a", char "A", char "X", char "8", char "+"]
    ++X [lowerCase? c for c in chars]
alphanumeric?: % -> Boolean
    ++ alphanumeric?(c) tests if c is either a letter or number,
    ++ i.e. one of 0..9, a..z or A..Z.
    ++
    ++X chars := [char "a", char "A", char "X", char "8", char "+"]
    ++X [alphanumeric? c for c in chars]

== add

Rep := SingleInteger -- 0..255

CC ==> CharacterClass()
import CC

OutChars:PrimitiveArray(OutputForm) :=
    construct [CODE_-CHAR(i)$Lisp for i in 0..255]

minChar := minIndex OutChars

a = b                == a =$Rep b
a < b                == a <$Rep b
size()               == 256
index n              == char((n - 1)::Integer)
lookup c             == (1 + ord c)::PositiveInteger

```

```

char(n:Integer)      == n::%
ord c                == convert(c)$Rep
random()             == char(random()$Integer rem size())
space                == QENUM("  ", 0$Lisp)$Lisp
quote                == QENUM("_" , 0$Lisp)$Lisp
escape               == QENUM("__ ", 0$Lisp)$Lisp
coerce(c:%):OutputForm == OutChars(minChar + ord c)
digit? c             == member?(c pretend Character, digit())
hexDigit? c          == member?(c pretend Character, hexDigit())
upperCase? c         == member?(c pretend Character, upperCase())
lowerCase? c         == member?(c pretend Character, lowerCase())
alphabetic? c        == member?(c pretend Character, alphabetic())
alphanumeric? c      == member?(c pretend Character, alphanumeric())

latex c ==
  concat("\mbox{'", concat(new(1,c pretend Character)$String, "'}")_
    $String)$String

char(s:String) ==
  (#s) = 1 => s(minIndex s) pretend %
  error "String is not a single character"

upperCase c ==
  QENUM(PNAME(UPCASE(CODE_-CHAR(ord c)$Lisp)$Lisp)$Lisp,0$Lisp)$Lisp

lowerCase c ==
  QENUM(PNAME(DOWNCASE(CODE_-CHAR(ord c)$Lisp)$Lisp)$Lisp,0$Lisp)$Lisp

```

---

— CHAR.dotabb —

```

"CHAR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=CHAR",shape=ellipse]
"FSAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FSAGG"]
"CHAR" -> "FSAGG"

```

---

## 4.4 domain CCLASS CharacterClass

---

— CharacterClass.input —

```

)set break resume
)sys rm -f CharacterClass.output

```

```

)spool CharacterClass.output
)set message test on
)set message auto off
)clear all
--S 1 of 16
cl1:=charClass[char "a",char "e",char "i",char "o",char "u",char "y"]
--R
--R
--R (1) "aeiouy"
--R
--R Type: CharacterClass
--E 1

--S 2 of 16
cl2 := charClass "bcdfghjklmnpqrstvwxyz"
--R
--R
--R (2) "bcdfghjklmnpqrstvwxyz"
--R
--R Type: CharacterClass
--E 2

--S 3 of 16
digit()
--R
--R
--R (3) "0123456789"
--R
--R Type: CharacterClass
--E 3

--S 4 of 16
hexDigit()
--R
--R
--R (4) "0123456789ABCDEFabcdef"
--R
--R Type: CharacterClass
--E 4

--S 5 of 16
upperCase()
--R
--R
--R (5) "ABCDEFGHIJKLMNOPQRSTUVWXYZ"
--R
--R Type: CharacterClass
--E 5

--S 6 of 16
lowerCase()
--R
--R
--R (6) "abcdefghijklmnopqrstuvwxyz"
--R
--R Type: CharacterClass

```

[illegible]

```

--S 13 of 16
difference(c11,c12)
--R
--R
--R (13) "aeiou"
--R
--R                                         Type: CharacterClass
--E 13

--S 14 of 16
intersect(complement(c11),c12)
--R
--R
--R (14) "bcdfghjklmnpqrstvwxyz"
--R
--R                                         Type: CharacterClass
--E 14

--S 15 of 16
insert!(char "a", c12)
--R
--R
--R (15) "abcfghjklmnpqrstvwxyz"
--R
--R                                         Type: CharacterClass
--E 15

--S 16 of 16
remove!(char "b", c12)
--R
--R
--R (16) "acdfghjklmnpqrstvwxyz"
--R
--R                                         Type: CharacterClass
--E 16
)spool
)lisp (bye)

```

---

— CharacterClass.help —

=====

CharacterClass examples

=====

The CharacterClass domain allows classes of characters to be defined and manipulated efficiently.

Character classes can be created by giving either a string or a list of characters.

```
c11:=charClass[char "a",char "e",char "i",char "o",char "u",char "y"]
```



```
"aeiouy"
                                Type: CharacterClass
```

```
c12 := charClass "bcdfghjklmnpqrstvwxyz"
"bcdfghjklmnpqrstvwxyz"
                                Type: CharacterClass
```

A number of character classes are predefined for convenience.

```
digit()
"0123456789"
                                Type: CharacterClass
```

```
hexDigit()
"0123456789ABCDEFabcdef"
                                Type: CharacterClass
```

```
upperCase()
"ABCDEFGHIJKLMNOPQRSTUVWXYZ"
                                Type: CharacterClass
```

```
lowerCase()
"abcdefghijklmnopqrstuvwxyz"
                                Type: CharacterClass
```

```
alphabetic()
"ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz"
                                Type: CharacterClass
```

```
alphanumeric()
"0123456789ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz"
                                Type: CharacterClass
```

You can quickly test whether a character belongs to a class.

```
member?(char "a", c11)
true
                                Type: Boolean
```

```
member?(char "a", c12)
false
                                Type: Boolean
```

Classes have the usual set operations because the `CharacterClass` domain belongs to the category `FiniteSetAggregate(Character)`.

```
intersect(c11, c12)
"y"
                                Type: CharacterClass
```

```
union(c11,c12)
"abcdefghijklmnopqrstuvwxyz"
                                Type: CharacterClass
```

```
difference(c11,c12)
"aeiou"
                                Type: CharacterClass
```

```
intersect(complement(c11),c12)
"bcdfghjklmnpqrstvwxyz"
                                Type: CharacterClass
```

You can modify character classes by adding or removing characters.

```
insert!(char "a", c12)
"abcdefghijklmnopqrstuvwxyz"
                                Type: CharacterClass
```

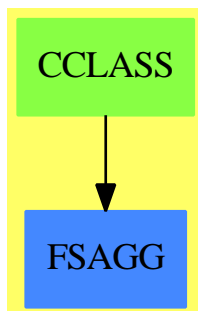
```
remove!(char "b", c12)
"acdfghjklmnpqrstvwxyz"
                                Type: CharacterClass
```

See Also:

- o )help Character
- o )help String
- o )show CharacterClass

---

#### 4.4.1 CharacterClass (CCLASS)



See

⇒ “Character” (CHAR) 4.3.1 on page 357

⇒ “IndexedString” (ISTRING) 10.14.1 on page 1214

⇒ “String” (STRING) 20.31.1 on page 2565

**Exports:**

|            |             |                     |           |                  |
|------------|-------------|---------------------|-----------|------------------|
| any?       | alphabetic  | alphanumeric        | bag       | brace            |
| brace      | cardinality | charClass           | coerce    | complement       |
| construct  | convert     | copy                | count     | count            |
| dictionary | difference  | digit               | empty     | empty?           |
| eq?        | eval        | eval                | eval      | eval             |
| every?     | extract!    | find                | hash      | hexDigit         |
| index      | insert!     | inspect             | intersect | latex            |
| less?      | lookup      | lowerCase           | map       | map!             |
| max        | member?     | members             | min       | more?            |
| parts      | random      | reduce              | reduce    | reduce           |
| remove     | remove      | remove!             | remove!   | removeDuplicates |
| sample     | select      | select!             | set       | size             |
| size?      | subset?     | symmetricDifference | union     | universe         |
| upperCase  | #?          | ?<?                 | ?=?       | ?~=?             |

— domain CCLASS CharacterClass —

```
)abbrev domain CCLASS CharacterClass
++ Author: Stephen M. Watt
++ Date Created: July 1986
++ Date Last Updated: June 20, 1991
++ Basic Operations: charClass
++ Related Domains: Character, Bits
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ This domain allows classes of characters to be defined and manipulated
++ efficiently.
```

```
CharacterClass: Join(SetCategory, ConvertibleTo String,
  FiniteSetAggregate Character, ConvertibleTo List Character) with
  charClass: String -> %
    ++ charClass(s) creates a character class which contains
    ++ exactly the characters given in the string s.
  charClass: List Character -> %
    ++ charClass(l) creates a character class which contains
    ++ exactly the characters given in the list l.
  digit: constant -> %
    ++ digit() returns the class of all characters
    ++ for which digit? is true.
  hexDigit: constant -> %
    ++ hexDigit() returns the class of all characters for which
    ++ hexDigit? is true.
```

```

upperCase: constant -> %
  ++ upperCase() returns the class of all characters for which
  ++ upperCase? is true.
lowerCase: constant -> %
  ++ lowerCase() returns the class of all characters for which
  ++ lowerCase? is true.
alphabetic : constant -> %
  ++ alphabetic() returns the class of all characters for which
  ++ alphabetic? is true.
alphanumeric: constant -> %
  ++ alphanumeric() returns the class of all characters for which
  ++ alphanumeric? is true.

== add
Rep := IndexedBits(0)
N    := size()$Character

a, b: %

digit()      == charClass "0123456789"
hexDigit()   == charClass "0123456789abcdefABCDEF"
upperCase()  == charClass "ABCDEFGHIJKLMNOPQRSTUVWXYZ"
lowerCase()  == charClass "abcdefghijklmnopqrstuvwxyz"
alphabetic() == union(upperCase(), lowerCase())
alphanumeric() == union(alphabetic(), digit())

a = b        == a = $Rep b

member?(c, a) == a(ord c)
union(a,b)    == Or(a, b)
intersect (a,b) == And(a, b)
difference(a,b) == And(a, Not b)
complement a  == Not a

convert(cl):String ==
  construct(convert(cl)@List(Character))
convert(cl:%):List(Character) ==
  [char(i) for i in 0..N-1 | cl.i]

charClass(s: String) ==
  cl := new(N, false)
  for i in minIndex(s)..maxIndex(s) repeat cl(ord s.i) := true
  cl

charClass(l: List Character) ==
  cl := new(N, false)
  for c in l repeat cl(ord c) := true
  cl

coerce(cl):OutputForm == (convert(cl)@String)::OutputForm

```

```

-- Stuff to make a legal SetAggregate view
# a == (n := 0; for i in 0..N-1 | a.i repeat n := n+1; n)
empty():% == charClass []
brace():% == charClass []

insert_!(c, a) == (a(ord c) := true; a)
remove_!(c, a) == (a(ord c) := false; a)

inspect(a) ==
  for i in 0..N-1 | a.i repeat
    return char i
  error "Cannot take a character from an empty class."
extract_!(a) ==
  for i in 0..N-1 | a.i repeat
    a.i := false
    return char i
  error "Cannot take a character from an empty class."

map(f, a) ==
  b := new(N, false)
  for i in 0..N-1 | a.i repeat b(ord f char i) := true
  b

temp: % := new(N, false)$Rep
map_!(f, a) ==
  fill_!(temp, false)
  for i in 0..N-1 | a.i repeat temp(ord f char i) := true
  copyInto_!(a, temp, 0)

parts a ==
  [char i for i in 0..N-1 | a.i]

```

---

— CCLASS.dotabb —

```

"CCLASS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=CCLASS"]
"FSAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FSAGG"]
"CCLASS" -> "FSAGG"

```

---

## 4.5 domain CLIF CliffordAlgebra[?, ?]

### 4.5.1 Vector (linear) spaces

This information is originally from Paul Leopardi's presentation on the *Introduction to Clifford Algebras* and is included here as an outline with his permission. Further details are based on the book by Doran and Lasenby called *Geometric Algebra for Physicists*.

Consider the various kinds of products that can occur between vectors. There are scalar and vector products from 3D geometry. There are the complex and quaterion products. There is also the *outer* or *exterior* product.

Vector addition commutes:

$$a + b = b + a$$

Vector addition is associative:

$$a + (b + c) = (a + b) + c$$

The identity vector exists:

$$a + 0 = a$$

Every vector has an inverse:

$$a + (-a) = 0$$

If we consider vectors to be directed line segments, thus establishing a geometric meaning for a vector, then each of these properties has a geometric meaning.

A multiplication operator exists between scalars and vectors with the properties:

$$\lambda(a + b) = \lambda a + \lambda b$$

$$(\lambda + \mu)a = \lambda a + \mu a$$

$$(\lambda\mu)a = \lambda(\mu a)$$

If  $1\lambda = \lambda$  for all scalars  $\lambda$  then  $1a = a$  for all vectors  $a$

These properties completely define a vector (linear) space. The  $+$  operation for scalar arithmetic is not the same as the  $+$  operation for vectors.

**Definition: Isomorphic** The vector space  $A$  is isomorphic to the vector space  $B$  if there exists a one-to-one correspondence between their elements which preserves sums and there is a one-to-one correspondence between the scalars which preserves sums and products.

**Definition: Subspace** Vector space  $B$  is a subspace of vector space  $A$  if all of the elements of  $B$  are contained in  $A$  and they share the same scalars.

**Definition: Linear Combination** Given vectors  $a_1, \dots, a_n$  the vector  $b$  is a linear combination of the vectors if we can find scalars  $\lambda_i$  such that

$$b = \lambda_1 a_1 + \dots + \lambda_n a_n = \sum_{k=1}^n \lambda_k a_k$$

**Definition: Linearly Independent** If there exists scalars  $\lambda_i$  such that

$$\lambda_1 a_1 + \dots + \lambda_n a_n = 0$$

and at least one of the  $\lambda_i$  is not zero then the vectors  $a_1, \dots, a_n$  are linearly dependent. If no such scalars exist then the vectors are linearly independent.

**Definition: Span** If every vector can be written as a linear combination of a fixed set of vectors  $a_1, \dots, a_n$  then this set of vectors is said to span the vector space.

**Definition: Basis** If a set of vectors  $a_1, \dots, a_n$  is linearly independent and spans a vector space  $A$  then the vectors form a basis for  $A$ .

**Definition: Dimension** The dimension of a vector space is the number of basis elements, which is unique since all bases of a vector space have the same number of elements.

### 4.5.2 Quadratic Forms[?]

For vector space  $\mathbb{V}$  over field  $\mathbb{F}$ , characteristic  $\neq 2$ :

Map  $f : \mathbb{V} \rightarrow \mathbb{F}$ , with

$$f(\lambda x) = \lambda^2 f(x), \forall \lambda \in \mathbb{F}, x \in \mathbb{V}$$

$f(x) = b(x, x)$ , where

$$b : \mathbb{V} \times \mathbb{V} \rightarrow \mathbb{F}, \text{ given by}$$

$$b(x, y) := \frac{1}{2}(f(x+y) - f(x) - f(y))$$

is a symmetric bilinear form

### 4.5.3 Quadratic spaces, Clifford Maps[?, ?]

A quadratic space is the pair  $(\mathbb{V}, f)$ , where  $f$  is a quadratic form on  $\mathbb{V}$

A Clifford map is a vector space homomorphism

$$\rho : \mathbb{V} \rightarrow \mathbb{A}$$

where  $\mathbb{A}$  is an associated algebra, and

$$(\rho v)^2 = f(v), \quad \forall v \in \mathbb{V}$$

### 4.5.4 Universal Clifford algebras[?]

The *universal Clifford algebra*  $Cl(f)$  for the quadratic space  $(\mathbb{V}, f)$  is the algebra generated by the image of the Clifford map  $\phi_f$  such that  $Cl(f)$  is the universal initial object such that  $\forall$  suitable algebra  $\mathbb{A}$  with Clifford map  $\phi_{\mathbb{A}} \exists$  a homomorphism

$$P_{\mathbb{A}} : Cl(f) \rightarrow \mathbb{A}$$

$$\rho_{\mathbb{A}} = P_{\mathbb{A}} \circ \rho_f$$

### 4.5.5 Real Clifford algebras $\mathbb{R}_{p,q}$ [?]

The real quadratic space  $\mathbb{R}^{p,q}$  is  $\mathbb{R}^{p+q}$  with

$$\phi(x) := - \sum_{k=-q}^{-1} x_k^2 + \sum_{k=1}^p x_k^2$$

For each  $p, q \in \mathbb{N}$ , the real universal Clifford algebra for  $\mathbb{R}^{p,q}$  is called  $\mathbb{R}_{p,q}$

$\mathbb{R}_{p,q}$  is isomorphic to some matrix algebra over one of:  $\mathbb{R}, \mathbb{R} \oplus \mathbb{R}, \mathbb{C}, \mathbb{H}, \mathbb{H} \oplus \mathbb{H}$

For example,  $\mathbb{R}_{1,1} \cong \mathbb{R}(2)$

### 4.5.6 Notation for integer sets

For  $S \subseteq \mathbb{Z}$ , define

$$\sum_{k \in S} f_k := \sum_{k=\min S, k \in S}^{\max S} f_k$$

$$\prod_{k \in S} f_k := \prod_{k=\min S, k \in S}^{\max S} f_k$$

$\mathbb{P}(S) :=$  the *power set* of  $S$

For  $m \leq n \in \mathbb{Z}$ , define

$$\zeta(m, n) := \{m, m+1, \dots, n-1, n\} \setminus \{0\}$$

### 4.5.7 Frames for Clifford algebras[?, ?, ?]

A *frame* is an ordered basis  $(\gamma_{-q}, \dots, \gamma_p)$  for  $\mathbb{R}^{p,q}$  which puts a quadratic form into the canonical form  $\phi$

For  $p, q \in \mathbb{N}$ , embed the frame for  $\mathbb{R}^{p,q}$  into  $\mathbb{R}_{p,q}$  via the maps

$$\gamma : \zeta(-q, p) \rightarrow \mathbb{R}^{p,q}$$

$$\rho : \mathbb{R}^{p,q} \rightarrow \mathbb{R}_{p,q}$$

$$(\rho\gamma k)^2 = \phi\gamma k = \text{sgn } k$$

### 4.5.8 Real frame groups[?, ?]

For  $p, q \in \mathbb{N}$ , define the real *frame group*  $\mathbb{G}_{p,q}$  via the map

$$g : \zeta(-q, p) \rightarrow \mathbb{G}_{p,q}$$



with generators and relations

$$\begin{aligned} \langle \mu, g_k | \mu g_k &= g_k \mu, \quad \mu^2 = 1, \\ (g_k)^2 &= \begin{cases} \mu, & \text{if } k < 0 \\ 1 & \text{if } k > 0 \end{cases} \\ g_k g_m &= \mu g_m g_k \quad \forall k \neq m \rangle \end{aligned}$$

#### 4.5.9 Canonical products[?, ?, ?]

The real frame group  $\mathbb{G}_{p,q}$  has order  $2^{p+q+1}$

Each member  $w$  can be expressed as the canonically ordered product

$$\begin{aligned} w &= \mu^a \prod_{k \in T} g_k \\ &= \mu^a \prod_{k=-q, k \neq 0}^p g_k^{b_k} \end{aligned}$$

where  $T \subseteq \zeta(-q, p)$ ,  $a, b_k \in \{0, 1\}$

#### 4.5.10 Clifford algebra of frame group[?, ?, ?, ?]

For  $p, q \in \mathbb{N}$  embed  $\mathbb{G}_{p,q}$  into  $\mathbb{R}_{p,q}$  via the map

$$\begin{aligned} \alpha \mathbb{G}_{p,q} &\rightarrow \mathbb{R}_{p,q} \\ \alpha 1 &:= 1, \quad \alpha \mu := -1 \\ \alpha g_k &:= \rho \gamma_k, \quad \alpha(gh) := (\alpha g)(\alpha h) \end{aligned}$$

Define *basis elements* via the map

$$e : \mathbb{P}\zeta(-q, p) \rightarrow \mathbb{R}_{p,q}, \quad e_T := \alpha \prod_{k \in T} g_k$$

Each  $a \in \mathbb{R}_{p,q}$  can be expressed as

$$a = \sum_{T \subseteq \zeta(-q, p)} a_T e_T$$

## 4.5.11 Neutral matrix representations[?, ?, ?]

The representation map  $P_m$  and representation matrix  $R_m$  make the following diagram

$$\begin{array}{ccc}
 & \text{coord} & \\
 \mathbb{R}_{m,m} & \longrightarrow & \mathbb{R}^{4^m} \\
 \downarrow P_m & & \downarrow R_m \\
 \text{commute: } & & \\
 V & & V \\
 \mathbb{R}(2^m) & \xrightarrow{\text{reshape}} & \mathbb{R}^{4^m} \\
 & \text{--- CliffordAlgebra.input ---} &
 \end{array}$$

```

)set break resume
)sys rm -f CliffordAlgebra.output
)spool CliffordAlgebra.output
)set message test on
)set message auto off
)clear all
--S 1 of 36
K := Fraction Polynomial Integer
--R
--R
--R (1) Fraction Polynomial Integer
--R
--R                                          Type: Domain
--E 1

--S 2 of 36
m := matrix [ [-1] ]
--R
--R
--R (2) [- 1]
--R
--R                                          Type: Matrix Integer
--E 2

--S 3 of 36
C := CliffordAlgebra(1, K, quadraticForm m)
--R
--R
--R (3) CliffordAlgebra(1,Fraction Polynomial Integer,MATRIX)
--R
--R                                          Type: Domain
--E 3

--S 4 of 36
i: C := e(1)
--R
--R

```

```

--R (4) e
--R 1
--R Type: CliffordAlgebra(1,Fraction Polynomial Integer,MATRIX)
--E 4

--S 5 of 36
x := a + b * i
--R
--R
--R (5) a + b e
--R 1
--R Type: CliffordAlgebra(1,Fraction Polynomial Integer,MATRIX)
--E 5

--S 6 of 36
y := c + d * i
--R
--R
--R (6) c + d e
--R 1
--R Type: CliffordAlgebra(1,Fraction Polynomial Integer,MATRIX)
--E 6

--S 7 of 36
x * y
--R
--R
--R (7) - b d + a c + (a d + b c)e
--R 1
--R Type: CliffordAlgebra(1,Fraction Polynomial Integer,MATRIX)
--E 7
)clear all

--S 8 of 36
K := Fraction Polynomial Integer
--R
--R
--R (1) Fraction Polynomial Integer
--R
--R Type: Domain
--E 8

--S 9 of 36
m := matrix [ [-1,0],[0,-1] ]
--R
--R
--R +- 1 0 +
--R (2) | |
--R + 0 - 1+
--R
--R Type: Matrix Integer
--E 9

```

```

--S 10 of 36
H := CliffordAlgebra(2, K, quadraticForm m)
--R
--R
--R (3) CliffordAlgebra(2,Fraction Polynomial Integer,MATRIX)
--R
--R                                         Type: Domain
--E 10

--S 11 of 36
i: H := e(1)
--R
--R
--R (4) e
--R      1
--R
--R                                         Type: CliffordAlgebra(2,Fraction Polynomial Integer,MATRIX)
--E 11

--S 12 of 36
j: H := e(2)
--R
--R
--R (5) e
--R      2
--R
--R                                         Type: CliffordAlgebra(2,Fraction Polynomial Integer,MATRIX)
--E 12

--S 13 of 36
k: H := i * j
--R
--R
--R (6) e e
--R      1 2
--R
--R                                         Type: CliffordAlgebra(2,Fraction Polynomial Integer,MATRIX)
--E 13

--S 14 of 36
x := a + b * i + c * j + d * k
--R
--R
--R (7) a + b e + c e + d e e
--R      1      2      1 2
--R
--R                                         Type: CliffordAlgebra(2,Fraction Polynomial Integer,MATRIX)
--E 14

--S 15 of 36
y := e + f * i + g * j + h * k
--R
--R
--R (8) e + f e + g e + h e e

```

```

--R          1      2      1 2
--R          Type: CliffordAlgebra(2,Fraction Polynomial Integer,MATRIX)
--E 15

--S 16 of 36
x + y
--R
--R
--R
--R      (9)  e + a + (f + b)e + (g + c)e + (h + d)e e
--R          1      2      1 2
--R          Type: CliffordAlgebra(2,Fraction Polynomial Integer,MATRIX)
--E 16

--S 17 of 36
x * y
--R
--R
--R
--R      (10)
--R      - d h - c g - b f + a e + (c h - d g + a f + b e)e
--R          1
--R      +
--R      (- b h + a g + d f + c e)e + (a h + b g - c f + d e)e e
--R          2      1 2
--R          Type: CliffordAlgebra(2,Fraction Polynomial Integer,MATRIX)
--E 17

--S 18 of 36
y * x
--R
--R
--R
--R      (11)
--R      - d h - c g - b f + a e + (- c h + d g + a f + b e)e
--R          1
--R      +
--R      (b h + a g - d f + c e)e + (a h - b g + c f + d e)e e
--R          2      1 2
--R          Type: CliffordAlgebra(2,Fraction Polynomial Integer,MATRIX)
--E 18
)clear all

--S 19 of 36
K := Fraction Polynomial Integer
--R
--R
--R      (1)  Fraction Polynomial Integer
--R
--R          Type: Domain
--E 19

--S 20 of 36
Ext := CliffordAlgebra(3, K, quadraticForm 0)

```

```

--R
--R
--R (2) CliffordAlgebra(3,Fraction Polynomial Integer,MATRIX)
--R                                         Type: Domain
--E 20

--S 21 of 36
i: Ext := e(1)
--R
--R
--R (3) e
--R      1
--R                                         Type: CliffordAlgebra(3,Fraction Polynomial Integer,MATRIX)
--E 21

--S 22 of 36
j: Ext := e(2)
--R
--R
--R (4) e
--R      2
--R                                         Type: CliffordAlgebra(3,Fraction Polynomial Integer,MATRIX)
--E 22

--S 23 of 36
k: Ext := e(3)
--R
--R
--R (5) e
--R      3
--R                                         Type: CliffordAlgebra(3,Fraction Polynomial Integer,MATRIX)
--E 23

--S 24 of 36
x := x1*i + x2*j + x3*k
--R
--R
--R (6) x1 e + x2 e + x3 e
--R      1   2   3
--R                                         Type: CliffordAlgebra(3,Fraction Polynomial Integer,MATRIX)
--E 24

--S 25 of 36
y := y1*i + y2*j + y3*k
--R
--R
--R (7) y1 e + y2 e + y3 e
--R      1   2   3
--R                                         Type: CliffordAlgebra(3,Fraction Polynomial Integer,MATRIX)
--E 25

```

```

--S 26 of 36
x + y
--R
--R
--R (8)  $(y_1 + x_1)e_1 + (y_2 + x_2)e_2 + (y_3 + x_3)e_3$ 
--R
--R Type: CliffordAlgebra(3,Fraction Polynomial Integer,MATRIX)
--E 26

--S 27 of 36
x * y + y * x
--R
--R
--R (9) 0
--R
--R Type: CliffordAlgebra(3,Fraction Polynomial Integer,MATRIX)
--E 27

--S 28 of 36
dual2 a == coefficient(a,[2,3]) * i + coefficient(a,[3,1]) * j + coefficient(a,[1,2]) * k
--R
--R
--R Type: Void
--E 28

--S 29 of 36
dual2(x*y)
--R
--R Compiling function dual2 with type CliffordAlgebra(3,Fraction
--R Polynomial Integer,MATRIX) -> CliffordAlgebra(3,Fraction
--R Polynomial Integer,MATRIX)
--R
--R (11)  $(x_2 y_3 - x_3 y_2)e_1 + (-x_1 y_3 + x_3 y_1)e_2 + (x_1 y_2 - x_2 y_1)e_3$ 
--R
--R Type: CliffordAlgebra(3,Fraction Polynomial Integer,MATRIX)
--E 29
)clear all

--S 30 of 36
K := Fraction Integer
--R
--R
--R (1) Fraction Integer
--R
--R Type: Domain
--E 30

--S 31 of 36
g := matrix [ [1,0,0,0], [0,-1,0,0], [0,0,-1,0], [0,0,0,-1] ]
--R
--R
--R +1 0 0 0 +

```

```

--R      |
--R      |0  - 1  0  0 |
--R  (2) |
--R      |0  0  - 1  0 |
--R      |
--R      +0  0  0  - 1+
--R
--R                                          Type: Matrix Integer
--E 31

```

```

--S 32 of 36
D := CliffordAlgebra(4,K, quadraticForm g)
--R
--R
--R  (3) CliffordAlgebra(4,Fraction Integer,MATRIX)
--R
--R                                          Type: Domain
--E 32

```

```

--S 33 of 36
gam := [e(i)$D for i in 1..4]
--R
--R
--R  (4) [e ,e ,e ,e ]
--R      1 2 3 4
--R
--R                                          Type: List CliffordAlgebra(4,Fraction Integer,MATRIX)
--E 33

```

```

--S 34 of 36
m := 1; n:= 2; r := 3; s := 4;
--R
--R
--R
--R                                          Type: PositiveInteger
--E 34

```

```

--S 35 of 36
lhs := reduce(+, [reduce(+, [ g(l,t)*gam(l)*gam(m)*gam(n)*gam(r)*gam(s)*gam(t) for l in 1..4]) for t in
--R
--R
--R  (6) - 4e e e e
--R      1 2 3 4
--R
--R                                          Type: CliffordAlgebra(4,Fraction Integer,MATRIX)
--E 35

```

```

--S 36 of 36
rhs := 2*(gam s * gam m*gam n*gam r + gam r*gam n*gam m*gam s)
--R
--R
--R  (7) - 4e e e e
--R      1 2 3 4
--R
--R                                          Type: CliffordAlgebra(4,Fraction Integer,MATRIX)
--E 36

```



```
)spool
)lisp (bye)
```

---

— CliffordAlgebra.help —

```
=====
CliffordAlgebra examples
=====
```

CliffordAlgebra(n,K,Q) defines a vector space of dimension  $2^n$  over the field K with a given quadratic form Q. If  $\{e_1..e_n\}$  is a basis for  $K^n$  then

```
{ 1,
  e(i)          1 <= i <= n,
  e(i1)*e(i2)   1 <= i1 < i2 <=n,
  ...,
  e(1)*e(2)*...*e(n) }
```

is a basis for the Clifford algebra. The algebra is defined by the relations

```
e(i)*e(i) = Q(e(i))
e(i)*e(j) = -e(j)*e(i), for i ^= j
```

Examples of Clifford Algebras are gaussians (complex numbers), quaternions, exterior algebras and spin algebras.

```
=====
The Complex Numbers as a Clifford Algebra
=====
```

This is the field over which we will work, rational functions with integer coefficients.

```
K := Fraction Polynomial Integer
Fraction Polynomial Integer
Type: Domain
```

We use this matrix for the quadratic form.

```
m := matrix [ [-1] ]
[- 1]
Type: Matrix Integer
```

We get complex arithmetic by using this domain.

```

C := CliffordAlgebra(1, K, quadraticForm m)
      CliffordAlgebra(1, Fraction Polynomial Integer, MATRIX)
                        Type: Domain

```

Here is i, the usual square root of -1.

```

i: C := e(1)
  e
  1
      Type: CliffordAlgebra(1, Fraction Polynomial Integer, MATRIX)

```

Here are some examples of the arithmetic.

```

x := a + b * i
  a + b e
      1
      Type: CliffordAlgebra(1, Fraction Polynomial Integer, MATRIX)

```

```

y := c + d * i
  c + d e
      1
      Type: CliffordAlgebra(1, Fraction Polynomial Integer, MATRIX)

```

```

x * y
- b d + a c + (a d + b c) e
                        1
      Type: CliffordAlgebra(1, Fraction Polynomial Integer, MATRIX)

```

```

=====
The Quaternion Numbers as a Clifford Algebra
=====

```

This is the field over which we will work, rational functions with integer coefficients.

```

K := Fraction Polynomial Integer
      Fraction Polynomial Integer
                        Type: Domain

```

We use this matrix for the quadratic form.

```

m := matrix [ [-1,0],[0,-1] ]
+- 1    0 +
|      |
+ 0    - 1+
      Type: Matrix Integer

```

The resulting domain is the quaternions.

```

H := CliffordAlgebra(2, K, quadraticForm m)

```

```
CliffordAlgebra(2,Fraction Polynomial Integer,MATRIX)
Type: Domain
```

We use Hamilton's notation for  $i, j, k$ .

```
i: H := e(1)
e
1
Type: CliffordAlgebra(2,Fraction Polynomial Integer,MATRIX)
```

```
j: H := e(2)
e
2
Type: CliffordAlgebra(2,Fraction Polynomial Integer,MATRIX)
```

```
k: H := i * j
e e
1 2
Type: CliffordAlgebra(2,Fraction Polynomial Integer,MATRIX)
```

```
x := a + b * i + c * j + d * k
a + b e + c e + d e e
1 2 1 2
Type: CliffordAlgebra(2,Fraction Polynomial Integer,MATRIX)
```

```
y := e + f * i + g * j + h * k
e + f e + g e + h e e
1 2 1 2
Type: CliffordAlgebra(2,Fraction Polynomial Integer,MATRIX)
```

```
x + y
e + a + (f + b)e + (g + c)e + (h + d)e e
1 2 1 2
Type: CliffordAlgebra(2,Fraction Polynomial Integer,MATRIX)
```

```
x * y
- d h - c g - b f + a e + (c h - d g + a f + b e)e
1
+
(- b h + a g + d f + c e)e + (a h + b g - c f + d e)e e
2 1 2
Type: CliffordAlgebra(2,Fraction Polynomial Integer,MATRIX)
```

```
y * x
- d h - c g - b f + a e + (- c h + d g + a f + b e)e
1
+
(b h + a g - d f + c e)e + (a h - b g + c f + d e)e e
2 1 2
Type: CliffordAlgebra(2,Fraction Polynomial Integer,MATRIX)
```

```
=====
The Exterior Algebra on a Three Space
=====
```

This is the field over which we will work, rational functions with integer coefficients.

```
K := Fraction Polynomial Integer
      Fraction Polynomial Integer
      Type: Domain
```

If we chose the three by three zero quadratic form, we obtain the exterior algebra on  $e(1), e(2), e(3)$ .

```
Ext := CliffordAlgebra(3, K, quadraticForm 0)
      CliffordAlgebra(3, Fraction Polynomial Integer, MATRIX)
      Type: Domain
```

This is a three dimensional vector algebra. We define  $i, j, k$  as the unit vectors.

```
i: Ext := e(1)
e
1
      Type: CliffordAlgebra(3, Fraction Polynomial Integer, MATRIX)
```

```
j: Ext := e(2)
e
2
      Type: CliffordAlgebra(3, Fraction Polynomial Integer, MATRIX)
```

```
k: Ext := e(3)
e
3
      Type: CliffordAlgebra(3, Fraction Polynomial Integer, MATRIX)
```

Now it is possible to do arithmetic.

```
x := x1*i + x2*j + x3*k
x1 e  + x2 e  + x3 e
  1    2    3
      Type: CliffordAlgebra(3, Fraction Polynomial Integer, MATRIX)
```

```
y := y1*i + y2*j + y3*k
y1 e  + y2 e  + y3 e
  1    2    3
      Type: CliffordAlgebra(3, Fraction Polynomial Integer, MATRIX)
```

```
x + y
```

```

(y1 + x1)e1 + (y2 + x2)e2 + (y3 + x3)e3
Type: CliffordAlgebra(3,Fraction Polynomial Integer,MATRIX)

```

```

x * y + y * x
0
Type: CliffordAlgebra(3,Fraction Polynomial Integer,MATRIX)

```

On an  $n$  space, a grade  $p$  form has a dual  $n-p$  form. In particular, in three space the dual of a grade two element identifies

```

e1*e2 -> e3, e2*e3 -> e1, e3*e1 -> e2.

```

```

dual2 a == coefficient(a,[2,3]) * i + coefficient(a,[3,1]) * j + coefficient(a,[1,2]) * k
Type: Void

```

The vector cross product is then given by this.

```

dual2(x*y)
(x2 y3 - x3 y2)e1 + (- x1 y3 + x3 y1)e2 + (x1 y2 - x2 y1)e3
Type: CliffordAlgebra(3,Fraction Polynomial Integer,MATRIX)

```

```

=====
The Dirac Spin Algebra
=====

```

In this section we will work over the field of rational numbers.

```

K := Fraction Integer
Fraction Integer
Type: Domain

```

We define the quadratic form to be the Minkowski space-time metric.

```

g := matrix [ [1,0,0,0], [0,-1,0,0], [0,0,-1,0], [0,0,0,-1] ]
+1  0  0  0 +
|
|0  - 1  0  0 |
|
|0  0  - 1  0 |
|
+0  0  0  - 1+
Type: Matrix Integer

```

We obtain the Dirac spin algebra used in Relativistic Quantum Field Theory.

```

D := CliffordAlgebra(4,K, quadraticForm g)
CliffordAlgebra(4,Fraction Integer,MATRIX)
Type: Domain

```

The usual notation for the basis is gamma with a superscript. For Axiom input we will use gam(i):

```
gam := [e(i)$D for i in 1..4]
      [e ,e ,e ,e ]
        1 2 3 4
      Type: List CliffordAlgebra(4,Fraction Integer,MATRIX)
```

There are various contraction identities of the form

$$g(l,t)*gam(l)*gam(m)*gam(n)*gam(r)*gam(s)*gam(t) = \\ 2*(gam(s)gam(m)gam(n)gam(r) + gam(r)*gam(n)*gam(m)*gam(s))$$

where a sum over l and t is implied.

Verify this identity for particular values of m,n,r,s.

```
m := 1; n:= 2; r := 3; s := 4;
      Type: PositiveInteger
```

```
lhs := reduce(+, [reduce(+, [ g(l,t)*gam(l)*gam(m)*gam(n)*gam(r)*gam(s)*gam(t) for l in 1..4]) for t in 1..4]
      - 4e e e e
        1 2 3 4
      Type: CliffordAlgebra(4,Fraction Integer,MATRIX)
```

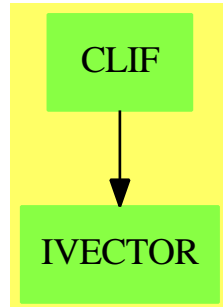
```
rhs := 2*(gam s * gam m*gam n*gam r + gam r*gam n*gam m*gam s)
      - 4e e e e
        1 2 3 4
      Type: CliffordAlgebra(4,Fraction Integer,MATRIX)
```

See Also:

- o )help Complex
- o )help Quaternion
- o )show CliffordAlgebra
- o \$AXIOM/doc/src/algebra/clifford.spad

---

## 4.5.12 CliffordAlgebra (CLIF)



See

⇒ “QuadraticForm” (QFORM) 18.1.1 on page 2114

**Exports:**

|           |       |                |               |          |
|-----------|-------|----------------|---------------|----------|
| 0         | 1     | characteristic | coefficient   | coerce   |
| dimension | e     | hash           | latex         | monomial |
| one?      | recip | sample         | subtractIfCan | zero?    |
| ?~=?      | ?*?   | ?**?           | ?^?           | ?+?      |
| ?-?       | -?    | ?/?            | ?=?           |          |

— domain CLIF CliffordAlgebra —

```

)abbrev domain CLIF CliffordAlgebra
++ Author: Stephen M. Watt
++ Date Created: August 1988
++ Date Last Updated: May 17, 1991
++ Basic Operations: wholeRadix, fractRadix, wholeRagits, fractRagits
++ Related Domains: QuadraticForm, Quaternion, Complex
++ Also See:
++ AMS Classifications:
++ Keywords: clifford algebra, grassman algebra, spin algebra
++ Examples:
++ References:
++
++ Description:
++ CliffordAlgebra(n, K, Q) defines a vector space of dimension \spad{2**n}
++ over K, given a quadratic form Q on \spad{K**n}.
++
++ If \spad{e[i]}, \spad{1<=i<=n} is a basis for \spad{K**n} then
++ 1, \spad{e[i]} (\spad{1<=i<=n}), \spad{e[i1]*e[i2]}
++ (\spad{1<=i1<i2<=n}), ..., \spad{e[1]*e[2]*...*e[n]}
++ is a basis for the Clifford Algebra.
++
++ The algebra is defined by the relations\br
++ \tab{5}\spad{e[i]*e[j]} = -e[j]*e[i] (\spad{i \~~= j}),\br
++ \tab{5}\spad{e[i]*e[i]} = Q(e[i])
  
```

```

++
++ Examples of Clifford Algebras are: gaussians, quaternions, exterior
++ algebras and spin algebras.

CliffordAlgebra(n, K, Q): T == Impl where
  n: PositiveInteger
  K: Field
  Q: QuadraticForm(n, K)

  PI ==> PositiveInteger
  NNI==> NonNegativeInteger

  T ==> Join(Ring, Algebra(K), VectorSpace(K)) with
    e: PI -> %
      ++ e(n) produces the appropriate unit element.
    monomial: (K, List PI) -> %
      ++ monomial(c,[i1,i2,...,iN]) produces the value given by
      ++ \spad{c*e(i1)*e(i2)*...*e(iN)}.
    coefficient: (% , List PI) -> K
      ++ coefficient(x,[i1,i2,...,iN]) extracts the coefficient of
      ++ \spad{e(i1)*e(i2)*...*e(iN)} in x.
    recip: % -> Union(%, "failed")
      ++ recip(x) computes the multiplicative inverse of x or "failed"
      ++ if x is not invertible.

  Impl ==> add
    Qeelist := [Q unitVector(i::PositiveInteger) for i in 1..n]
    dim      := 2**n

    Rep      := PrIMITIVEArray K

    New      ==> new(dim, 0$K)$Rep

    x, y, z: %
    c: K
    m: Integer

    characteristic() == characteristic()$K
    dimension()      == dim::CardinalNumber

    x = y ==
      for i in 0..dim-1 repeat
        if x.i ^= y.i then return false
      true

    x + y == (z := New; for i in 0..dim-1 repeat z.i := x.i + y.i; z)
    x - y == (z := New; for i in 0..dim-1 repeat z.i := x.i - y.i; z)
    - x    == (z := New; for i in 0..dim-1 repeat z.i := - x.i; z)
    m * x == (z := New; for i in 0..dim-1 repeat z.i := m*x.i; z)
    c * x == (z := New; for i in 0..dim-1 repeat z.i := c*x.i; z)

```



```

0          == New
1          == (z := New; z.0 := 1; z)
coerce(m): % == (z := New; z.0 := m::K; z)
coerce(c): % == (z := New; z.0 := c; z)

e b ==
  b::NNI > n => error "No such basis element"
  iz := 2**((b-1)::NNI)
  z := New; z.iz := 1; z

-- The ei*ej products could instead be precomputed in
-- a (2**n)**2 multiplication table.
addMonomProd(c1: K, b1: NNI, c2: K, b2: NNI, z: %): % ==
  c := c1 * c2
  bz := b2
  for i in 0..n-1 | bit?(b1,i) repeat
    -- Apply rule ei*ej = -ej*ei for i^=j
    k := 0
    for j in i+1..n-1 | bit?(b1, j) repeat k := k+1
    for j in 0..i-1 | bit?(bz, j) repeat k := k+1
    if odd? k then c := -c
    -- Apply rule ei**2 = Q(ei)
    if bit?(bz,i) then
      c := c * Qeelist.(i+1)
      bz:= (bz - 2**i)::NNI
    else
      bz:= bz + 2**i
  z.bz := z.bz + c
  z

x * y ==
  z := New
  for ix in 0..dim-1 repeat
    if x.ix ^= 0 then for iy in 0..dim-1 repeat
      if y.iy ^= 0 then addMonomProd(x.ix,ix,y.iy,iy,z)
  z

canonMonom(c: K, lb: List PI): Record(coef: K, basel: NNI) ==
  -- 0. Check input
  for b in lb repeat b > n => error "No such basis element"

  -- 1. Apply identity ei*ej = -ej*ei, i^=j.
  -- The Rep assumes n is small so bubble sort is ok.
  -- Using bubble sort keeps the exchange info obvious.
  wasordered := false
  exchanges := 0
  while not wasordered repeat
    wasordered := true
    for i in 1..#lb-1 repeat

```

```

        if lb.i > lb.(i+1) then
            t := lb.i; lb.i := lb.(i+1); lb.(i+1) := t
            exchanges := exchanges + 1
            wasordered := false
        if odd? exchanges then c := -c

-- 2. Prepare the basis element
-- Apply identity ei*ei = Q(ei).
bz := 0
for b in lb repeat
    bn := (b-1)::NNI
    if bit?(bz, bn) then
        c := c * Qeelist bn
        bz := (bz - 2**bn)::NNI
    else
        bz := bz + 2**bn
[c, bz::NNI]

monomial(c, lb) ==
    r := canonMonom(c, lb)
    z := New
    z r.basel := r.coef
    z
coefficient(z, lb) ==
    r := canonMonom(1, lb)
    r.coef = 0 => error "Cannot take coef of 0"
    z r.basel/r.coef

Ex ==> OutputForm

coerceMonom(c: K, b: NNI): Ex ==
    b = 0 => c::Ex
    ml := [sub("e"::Ex, i::Ex) for i in 1..n | bit?(b,i-1)]
    be := reduce("?", ml)
    c = 1 => be
    c::Ex * be

coerce(x): Ex ==
    t1 := [coerceMonom(x.i,i) for i in 0..dim-1 | x.i^=0]
    null t1 => "0"::Ex
    reduce("+", t1)

localPowerSets(j::NNI): List(List(PI)) ==
    l: List List PI := list []
    j = 0 => l
    Sm := localPowerSets((j-1)::NNI)
    Sn: List List PI := []
    for x in Sm repeat Sn := cons(cons(j pretend PI, x), Sn)
    append(Sn, Sm)

```

```

powerSets(j:NNI):List List PI == map(reverse, localPowerSets j)

Pn:List List PI := powerSets(n)

recip(x: %): Union(%, "failed") ==
  one:% := 1
  -- tmp:c := x*yC - 1$C
  rhsEqs : List K := []
  lhsEqs: List List K := []
  lhsEqi: List K
  for pi in Pn repeat
    rhsEqs := cons(coefficient(one, pi), rhsEqs)

    lhsEqi := []
    for pj in Pn repeat
      lhsEqi := cons(coefficient(x*monomial(1,pj),pi),lhsEqi)
    lhsEqs := cons(reverse(lhsEqi),lhsEqs)
  ans := particularSolution(matrix(lhsEqs),vector(rhsEqs)_
    )$LinearSystemMatrixPackage(K, Vector K, Vector K, Matrix K)
  ans case "failed" => "failed"
  ansP := parts(ans)
  ansC:% := 0
  for pj in Pn repeat
    cj:= first ansP
    ansP := rest ansP
    ansC := ansC + cj*monomial(1,pj)
  ansC

```

---

— CLIF.dotabb —

```

"CLIF" [color="#88FF44",href="bookvol10.3.pdf#nameddest=CLIF"]
"IVECTOR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IVECTOR"]
"CLIF" -> "IVECTOR"

```

---

## 4.6 domain COLOR Color

— Color.input —

```

)set break resume
)sys rm -f Color.output

```

```

)spool Color.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Color
--R Color is a domain constructor
--R Abbreviation for Color is COLOR
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for COLOR
--R
--R----- Operations -----
--R ?? : (DoubleFloat,%) -> %      ?? : (PositiveInteger,%) -> %
--R ?+? : (%,%) -> %              ?=? : (%,%) -> Boolean
--R blue : () -> %                coerce : % -> OutputForm
--R color : Integer -> %           green : () -> %
--R hash : % -> SingleInteger      hue : % -> Integer
--R latex : % -> String            red : () -> %
--R yellow : () -> %              ?~=? : (%,%) -> Boolean
--R numberOfHues : () -> PositiveInteger
--R
--E 1

)spool
)lisp (bye)

```

---

— Color.help —

=====

Color examples

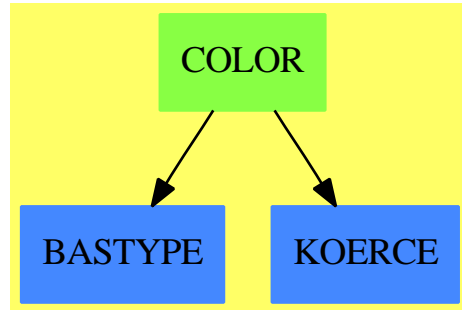
=====

See Also:

- o )show Color

---

### 4.6.1 Color (COLOR)



See

⇒ “Palette” (PALETTE) 17.4.1 on page 1856

#### Exports:

|      |        |              |       |        |
|------|--------|--------------|-------|--------|
| blue | coerce | color        | green | hash   |
| hue  | latex  | numberOfHues | red   | yellow |
| ?~=? | ?*?    | ?+?          | ?=?   |        |

— domain **COLOR** Color —

```

)abbrev domain COLOR Color
++ Author: Jim Wen
++ Date Created: 10 May 1989
++ Date Last Updated: 19 Mar 1991 by Jon Steinbach
++ Basic Operations: red, yellow, green, blue, hue, numberOfHues, color, +, *, =
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ Color() specifies a domain of 27 colors provided in the
++ \Language{} system (the colors mix additively).

```

Color(): Exports == Implementation where

```

I      ==> Integer
PI     ==> PositiveInteger
SF     ==> DoubleFloat

```

Exports ==> AbelianSemiGroup with

```

"*"    : (PI, %) -> %
++ s * c, returns the color c, whose weighted shade has been scaled by s.
"*"    : (SF, %) -> %
++ s * c, returns the color c, whose weighted shade has been scaled by s.
"+"    : (%, %) -> %
++ c1 + c2 additively mixes the two colors c1 and c2.

```

```

red    : ()      -> %
  ++ red() returns the position of the red hue from total hues.
yellow : ()      -> %
  ++ yellow() returns the position of the yellow hue from total hues.
green  : ()      -> %
  ++ green() returns the position of the green hue from total hues.
blue   : ()      -> %
  ++ blue() returns the position of the blue hue from total hues.
hue    : %       -> I
  ++ hue(c) returns the hue index of the indicated color c.
numberOfHues : () -> PI
  ++ numberOfHues() returns the number of total hues, set in totalHues.
color   : Integer -> %
  ++ color(i) returns a color of the indicated hue i.

Implementation ==> add
totalHues ==> 27 --see (header.h file) for the current number

Rep := Record(hue:I, weight:SF)

f:SF * c:% ==
  -- s * c returns the color c, whose weighted shade has been scaled by s
  zero? f => c
  -- 0 is the identity function...or maybe an error is better?
  [c.hue, f * c.weight]

x + y ==
  x.hue = y.hue => [x.hue, x.weight + y.weight]
  if y.weight > x.weight then -- let x be color with bigger weight
    c := x
    x := y
    y := c
  diff := x.hue - y.hue
  if (xHueSmaller:=(diff < 0)) then diff := -diff
  if (moreThanHalf:=(diff > totalHues quo 2)) then diff := totalHues-diff
  offset : I := wholePart(round (diff::SF/(2::SF)**(x.weight/y.weight)) )
  if (xHueSmaller and ^moreThanHalf) or (^xHueSmaller and moreThanHalf) then
    ans := x.hue + offset
  else
    ans := x.hue - offset
  if (ans < 0) then ans := totalHues + ans
  else if (ans > totalHues) then ans := ans - totalHues
  [ans,1]

x = y == (x.hue = y.hue) and (x.weight = y.weight)
red() == [1,1]
yellow() == [11::I,1]
green() == [14::I,1]
blue() == [22::I,1]

```

```

sample() == red()
hue c     == c.hue
i:PositiveInteger * c:% == i::SF * c
numberOfHues() == totalHues

color i ==
  if (i<0) or (i>totalHues) then
    error concat("Color should be in the range 1..",totalHues::String)
  [i::I, 1]

coerce(c:%):OutputForm ==
  hconcat ["Hue: " ::OutputForm, (c.hue)::OutputForm,
           " Weight: " ::OutputForm, (c.weight)::OutputForm]

```

---

— COLOR.dotabb —

```

"COLOR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=COLOR"]
"BASTYPE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=BASTYPE"]
"KOERCE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=KOERCE"]
"COLOR" -> "BASTYPE"
"COLOR" -> "KOERCE"

```

---

## 4.7 domain COMM Commutator

— Commutator.input —

```

)set break resume
)sys rm -f Commutator.output
)spool Commutator.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Commutator
--R Commutator is a domain constructor
--R Abbreviation for Commutator is COMM
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for COMM
--R

```

```

--R----- Operations -----
--R ==? : (%,% ) -> Boolean          coerce : % -> OutputForm
--R hash : % -> SingleInteger        latex : % -> String
--R mkcomm : (%,% ) -> %             mkcomm : Integer -> %
--R ==~? : (%,% ) -> Boolean
--R
--E 1

```

```

)spool
)lisp (bye)

```

— Commutator.help —

```

=====
Commutator examples
=====

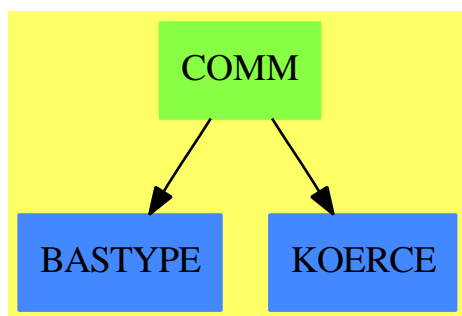
```

```

See Also:
o )show Commutator

```

#### 4.7.1 Commutator (COMM)



See

⇒ “OrdSetInts” (OSI) 16.20.1 on page 1825  
 ⇒ “FreeNilpotentLie” (FNLA) 7.33.1 on page 993

**Exports:**

coerce hash latex mkcomm ==? ==~?

— domain COMM Commutator —



```

)abbrev domain COMM Commutator
++ Author : Larry Lambe
++ Date created: 30 June 1988.
++ Updated   : 10 March 1991
++ Description:
++ A type for basic commutators

Commutator: Export == Implement where
  I  ==> Integer
  OSI ==> OrdSetInts
  0  ==> OutputForm

Export == SetCategory with
  mkcomm : I -> %
    ++ mkcomm(i) is not documented
  mkcomm : (%,% ) -> %
    ++ mkcomm(i,j) is not documented

Implement == add
  P  := Record(left:%,right:%)
  Rep := Union(OSI,P)
  x,y: %
  i : I

  x = y ==
    (x case OSI) and (y case OSI) => x::OSI = y::OSI
    (x case P) and (y case P) =>
      xx:P := x::P
      yy:P := y::P
      (xx.right = yy.right) and (xx.left = yy.left)
    false

  mkcomm(i) == i::OSI
  mkcomm(x,y) == construct(x,y)$P

  coerce(x: %): 0 ==
    x case OSI => x::OSI::0
    xx := x::P
    bracket([xx.left::0,xx.right::0])$0

```

— COMM.dotabb —

```

"COMM" [color="#88FF44",href="bookvol10.3.pdf#nameddest=COMM"]
"BASTYPE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=BASTYPE"]
"KOERCE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=KOERCE"]
"COMM" -> "BASTYPE"

```

"COMM" -> "KOERCE"

## 4.8 domain COMPLEX Complex

— Complex.input —

```
)set break resume
)sys rm -f Complex.output
)spool Complex.output
)set message test on
)set message auto off
)clear all
--S 1 of 16
a := complex(4/3,5/2)
--R
--R
--R      4 5
--R  (1) - + - %i
--R      3 2
--R
--R                                          Type: Complex Fraction Integer
--E 1

--S 2 of 16
b := complex(4/3,-5/2)
--R
--R
--R      4 5
--R  (2) - - - %i
--R      3 2
--R
--R                                          Type: Complex Fraction Integer
--E 2

--S 3 of 16
a + b
--R
--R
--R      8
--R  (3) -
--R      3
--R
--R                                          Type: Complex Fraction Integer
--E 3

--S 4 of 16
a - b
```

```

--R
--R
--R (4) 5%i
--R
--R                                          Type: Complex Fraction Integer
--E 4

--S 5 of 16
a * b
--R
--R
--R
--R      289
--R (5) ---
--R      36
--R
--R                                          Type: Complex Fraction Integer
--E 5

--S 6 of 16
a / b
--R
--R
--R      161  240
--R (6) - --- + --- %i
--R      289  289
--R
--R                                          Type: Complex Fraction Integer
--E 6

--S 7 of 16
% :: Fraction Complex Integer
--R
--R
--R      - 15 + 8%i
--R (7) -----
--R      15 + 8%i
--R
--R                                          Type: Fraction Complex Integer
--E 7

--S 8 of 16
3.4 + 6.7 * %i
--R
--R
--R (8) 3.4 + 6.7 %i
--R
--R                                          Type: Complex Float
--E 8

--S 9 of 16
conjugate a
--R
--R
--R      4  5
--R (9) - - - %i

```

```

--R      3  2
--R
--R      Type: Complex Fraction Integer
--E 9

--S 10 of 16
norm a
--R
--R
--R      289
--R      (10) ---
--R      36
--R
--R      Type: Fraction Integer
--E 10

--S 11 of 16
real a
--R
--R
--R      4
--R      (11) -
--R      3
--R
--R      Type: Fraction Integer
--E 11

--S 12 of 16
imag a
--R
--R
--R      5
--R      (12) -
--R      2
--R
--R      Type: Fraction Integer
--E 12

--S 13 of 16
gcd(13 - 13*%i, 31 + 27*%i)
--R
--R
--R      (13) 5 + %i
--R
--R      Type: Complex Integer
--E 13

--S 14 of 16
lcm(13 - 13*%i, 31 + 27*%i)
--R
--R
--R      (14) 143 - 39%i
--R
--R      Type: Complex Integer
--E 14

```

```

--S 15 of 16
factor(13 - 13*i)
--R
--R
--R      (15)  - (1 + %i)(2 + 3%i)(3 + 2%i)
--R
--R                                          Type: Factored Complex Integer
--E 15

--S 16 of 16
factor complex(2,0)
--R
--R
--R      (16)  - %i (1 + %i)2
--R
--R                                          Type: Factored Complex Integer
--E 16
)spool
)lisp (bye)

```

---

— Complex.help —

=====  
Complex examples  
=====

The Complex constructor implements complex objects over a commutative ring R. Typically, the ring R is Integer, Fraction Integer, Float or DoubleFloat. R can also be a symbolic type, like Polynomial Integer.

Complex objects are created by the complex operation.

```

a := complex(4/3,5/2)
4   5
- + - %i
3   2
                                Type: Complex Fraction Integer

b := complex(4/3,-5/2)
4   5
- - - %i
3   2
                                Type: Complex Fraction Integer

```

The standard arithmetic operations are available.

```

a + b
8

```

```
-
3
Type: Complex Fraction Integer
```

```
a - b
5%i
Type: Complex Fraction Integer
```

```
a * b
289
---
36
Type: Complex Fraction Integer
```

If  $R$  is a field, you can also divide the complex objects.

```
a / b
161 240
- --- + --- %i
289 289
Type: Complex Fraction Integer
```

We can view the last object as a fraction of complex integers.

```
% :: Fraction Complex Integer
- 15 + 8%i
-----
15 + 8%i
Type: Fraction Complex Integer
```

The predefined macro `%i` is defined to be `complex(0,1)`.

```
3.4 + 6.7 * %i
3.4 + 6.7 %i
Type: Complex Float
```

You can also compute the conjugate and norm of a complex number.

```
conjugate a
4 5
- - - %i
3 2
Type: Complex Fraction Integer
```

```
norm a
289
---
36
Type: Fraction Integer
```

The `real` and `imag` operations are provided to extract the real and imaginary parts, respectively.

```
real a
4
-
3
Type: Fraction Integer
```

```
imag a
5
-
2
Type: Fraction Integer
```

The domain `Complex Integer` is also called the Gaussian integers. If `R` is the integers (or, more generally, a `EuclideanDomain`), you can compute greatest common divisors.

```
gcd(13 - 13*i, 31 + 27*i)
5 + i
Type: Complex Integer
```

You can also compute least common multiples.

```
lcm(13 - 13*i, 31 + 27*i)
143 - 39*i
Type: Complex Integer
```

You can factor Gaussian integers.

```
factor(13 - 13*i)
- (1 + i)(2 + 3i)(3 + 2i)
Type: Factored Complex Integer

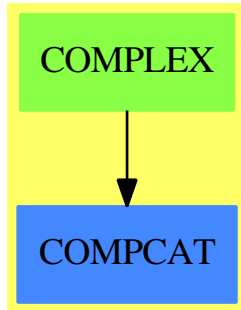
factor complex(2,0)
2
- i (1 + i)
Type: Factored Complex Integer
```

See Also

o `)show Complex`

---

4.8.1 Complex (COMPLEX)





**Exports:**

|                               |                            |                          |
|-------------------------------|----------------------------|--------------------------|
| 0                             | 1                          | abs                      |
| acos                          | acosh                      | acot                     |
| acoth                         | acsc                       | acsch                    |
| argument                      | asec                       | asech                    |
| asin                          | asinh                      | associates?              |
| atan                          | atanh                      | basis                    |
| characteristic                | characteristicPolynomial   | charthRoot               |
| coerce                        | complex                    | conditionP               |
| conjugate                     | convert                    | coordinates              |
| cos                           | cosh                       | cot                      |
| coth                          | createPrimitiveElement     | csc                      |
| csch                          | D                          | definingPolynomial       |
| derivationCoordinates         | differentiate              | discreteLog              |
| discriminant                  | divide                     | euclideanSize            |
| eval                          | exp                        | expressIdealMember       |
| exquo                         | extendedEuclidean          | factor                   |
| factorPolynomial              | factorSquareFreePolynomial | factorsOfCyclicGroupSize |
| gcd                           | gcdPolynomial              | generator                |
| hash                          | imag                       | imaginary                |
| index                         | init                       | inv                      |
| latex                         | lcm                        | lift                     |
| log                           | lookup                     | map                      |
| max                           | min                        | minimalPolynomial        |
| multiEuclidean                | nextItem                   | norm                     |
| nthRoot                       | OMwrite                    | one?                     |
| order                         | patternMatch               | pi                       |
| polarCoordinates              | prime?                     | primeFrobenius           |
| primitive?                    | primitiveElement           | principalIdeal           |
| random                        | rank                       | rational                 |
| rational?                     | rationalIfCan              | real                     |
| recip                         | reduce                     | reducedSystem            |
| regularRepresentation         | representationType         | represents               |
| retract                       | retractIfCan               | sample                   |
| sec                           | sech                       | sin                      |
| sinh                          | size                       | sizeLess?                |
| solveLinearPolynomialEquation | sqrt                       | squareFree               |
| squareFreePart                | squareFreePolynomial       | subtractIfCan            |
| tableForDiscreteLogarithm     | tan                        | tanh                     |
| trace                         | traceMatrix                | unit?                    |
| unitCanonical                 | unitNormal                 | zero?                    |
| ?*?                           | ?**?                       | ?+?                      |
| ?-?                           | -?                         | ?=?                      |
| ?^?                           | ?~=?                       | ?/?                      |
| ?<?                           | ?<=?                       | ?>?                      |
| ?>=?                          | ??                         | ?quo?                    |
| ?rem?                         |                            |                          |

## — domain COMPLEX Complex —

```

)abbrev domain COMPLEX Complex
++ Author: Mark Botch
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ \spadtype{Complex(R)} creates the domain of elements of the form
++ \spad{a + b * i} where \spad{a} and b come from the ring R,
++ and i is a new element such that \spad{i**2 = -1}.

Complex(R:CommutativeRing): ComplexCategory(R) with
  if R has OpenMath then OpenMath
== add
  Rep := Record(real:R, imag:R)

  if R has OpenMath then
    writeOMComplex(dev: OpenMathDevice, x: %): Void ==
      MputApp(dev)
      MputSymbol(dev, "complex1", "complex__cartesian")
      Mwrite(dev, real x)
      Mwrite(dev, imag x)
      MputEndApp(dev)

    OMwrite(x: %): String ==
      s: String := ""
      sp := OM_-STRINGTOSTRINGPTR(s)$Lisp
      dev: OpenMathDevice := OMopenString(sp pretend String, OMencodingXML)
      MputObject(dev)
      writeOMComplex(dev, x)
      MputEndObject(dev)
      Mclose(dev)
      s := OM_-STRINGPTRTOSTRING(sp)$Lisp pretend String
      s

    OMwrite(x: %, wholeObj: Boolean): String ==
      s: String := ""
      sp := OM_-STRINGTOSTRINGPTR(s)$Lisp
      dev: OpenMathDevice := OMopenString(sp pretend String, OMencodingXML)
      if wholeObj then
        MputObject(dev)
      writeOMComplex(dev, x)
      if wholeObj then

```

```

    OMputEndObject(dev)
  OMclose(dev)
  s := OM_STRINGPTRTOSTRING(sp)$Lisp pretend String
  s

OMwrite(dev: OpenMathDevice, x: %): Void ==
  OMputObject(dev)
  writeOMComplex(dev, x)
  OMputEndObject(dev)

OMwrite(dev: OpenMathDevice, x: %, wholeObj: Boolean): Void ==
  if wholeObj then
    OMputObject(dev)
    writeOMComplex(dev, x)
  if wholeObj then
    OMputEndObject(dev)

0 == [0, 0]
1 == [1, 0]
zero? x == zero?(x.real) and zero?(x.imag)
-- one? x == one?(x.real) and zero?(x.imag)
one? x == ((x.real) = 1) and zero?(x.imag)
coerce(r:R):% == [r, 0]
complex(r, i) == [r, i]
real x == x.real
imag x == x.imag
x + y == [x.real + y.real, x.imag + y.imag]
-- by re-defining this here, we save 5 fn calls
x:% * y:% ==
  [x.real * y.real - x.imag * y.imag,
   x.imag * y.real + y.imag * x.real] -- here we save nine!

if R has IntegralDomain then
  _exquo(x:%, y:%) == -- to correct bad defaulting problem
  zero? y.imag => x exquo y.real
  x * conjugate(y) exquo norm(y)

```

---

— COMPLEX.dotabb —

```

"COMPLEX" [color="#88FF44",href="bookvol10.3.pdf#nameddest=COMPLEX"]
"COMPCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=COMPCAT"]
"COMPLEX" -> "COMPCAT"

```

---

## 4.9 domain CDFMAT ComplexDoubleFloatMatrix

— ComplexDoubleFloatMatrix.input —

```
)set break resume
)sys rm -f ComplexDoubleFloatMatrix.output
)spool ComplexDoubleFloatMatrix.output
)set message test on
)set message auto off
)clear all

--S 1 of 6
)show ComplexDoubleFloatMatrix
--R ComplexDoubleFloatMatrix is a domain constructor
--R Abbreviation for ComplexDoubleFloatMatrix is CDFMAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for CDFMAT
--R
--R----- Operations -----
--R ?? : (Integer,%) -> %           ?? : (%,%) -> %
--R ?+? : (%,%) -> %               -? : % -> %
--R ?-? : (%,%) -> %               antisymmetric? : % -> Boolean
--R copy : % -> %                  diagonal? : % -> Boolean
--R diagonalMatrix : List % -> %   empty : () -> %
--R empty? : % -> Boolean           eq? : (%,%) -> Boolean
--R horizConcat : (%,%) -> %       maxColIndex : % -> Integer
--R maxRowIndex : % -> Integer      minColIndex : % -> Integer
--R minRowIndex : % -> Integer      ncols : % -> NonNegativeInteger
--R nrows : % -> NonNegativeInteger qnew : (Integer,Integer) -> %
--R sample : () -> %               square? : % -> Boolean
--R squareTop : % -> %             symmetric? : % -> Boolean
--R transpose : % -> %             vertConcat : (%,%) -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?? : (ComplexDoubleFloatVector,%) -> ComplexDoubleFloatVector
--R ?? : (%,ComplexDoubleFloatVector) -> ComplexDoubleFloatVector
--R ?? : (%,Complex DoubleFloat) -> %
--R ?? : (Complex DoubleFloat,%) -> %
--R ??? : (%,Integer) -> % if Complex DoubleFloat has FIELD
--R ??? : (%,NonNegativeInteger) -> %
--R ?/? : (%,Complex DoubleFloat) -> % if Complex DoubleFloat has FIELD
--R ?=? : (%,%) -> Boolean if Complex DoubleFloat has SETCAT
--R any? : ((Complex DoubleFloat -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : ComplexDoubleFloatVector -> %
--R coerce : % -> OutputForm if Complex DoubleFloat has SETCAT
--R column : (%,Integer) -> ComplexDoubleFloatVector
--R columnSpace : % -> List ComplexDoubleFloatVector if Complex DoubleFloat has EUCDOM
--R count : (Complex DoubleFloat,%) -> NonNegativeInteger if $ has finiteAggregate and Complex DoubleFlo
--R count : ((Complex DoubleFloat -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
```

```

--R determinant : % -> Complex DoubleFloat if Complex DoubleFloat has commutative *
--R diagonalMatrix : List Complex DoubleFloat -> %
--R elt : (% ,List Integer,List Integer) -> %
--R elt : (% ,Integer,Integer,Complex DoubleFloat) -> Complex DoubleFloat
--R elt : (% ,Integer,Integer) -> Complex DoubleFloat
--R eval : (% ,List Complex DoubleFloat,List Complex DoubleFloat) -> % if Complex DoubleFloat
--R eval : (% ,Complex DoubleFloat,Complex DoubleFloat) -> % if Complex DoubleFloat has EVALAB
--R eval : (% ,Equation Complex DoubleFloat) -> % if Complex DoubleFloat has EVALAB COMPLEX D
--R eval : (% ,List Equation Complex DoubleFloat) -> % if Complex DoubleFloat has EVALAB COMPI
--R every? : ((Complex DoubleFloat -> Boolean),%) -> Boolean if $ has finiteAggregate
--R exquo : (% ,Complex DoubleFloat) -> Union(%,"failed") if Complex DoubleFloat has INTDOM
--R fill! : (% ,Complex DoubleFloat) -> %
--R hash : % -> SingleInteger if Complex DoubleFloat has SETCAT
--R inverse : % -> Union(%,"failed") if Complex DoubleFloat has FIELD
--R latex : % -> String if Complex DoubleFloat has SETCAT
--R less? : (% ,NonNegativeInteger) -> Boolean
--R listOfLists : % -> List List Complex DoubleFloat
--R map : (((Complex DoubleFloat,Complex DoubleFloat) -> Complex DoubleFloat),%,%,Complex DoubleFlo
--R map : (((Complex DoubleFloat,Complex DoubleFloat) -> Complex DoubleFloat),%,%) -> %
--R map : ((Complex DoubleFloat -> Complex DoubleFloat),%) -> %
--R map! : ((Complex DoubleFloat -> Complex DoubleFloat),%) -> %
--R matrix : List List Complex DoubleFloat -> %
--R member? : (Complex DoubleFloat,%) -> Boolean if $ has finiteAggregate and Complex DoubleFlo
--R members : % -> List Complex DoubleFloat if $ has finiteAggregate
--R minordet : % -> Complex DoubleFloat if Complex DoubleFloat has commutative *
--R more? : (% ,NonNegativeInteger) -> Boolean
--R new : (NonNegativeInteger,NonNegativeInteger,Complex DoubleFloat) -> %
--R nullSpace : % -> List ComplexDoubleFloatVector if Complex DoubleFloat has INTDOM
--R nullity : % -> NonNegativeInteger if Complex DoubleFloat has INTDOM
--R parts : % -> List Complex DoubleFloat
--R pfaffian : % -> Complex DoubleFloat if Complex DoubleFloat has COMRING
--R qelt : (% ,Integer,Integer) -> Complex DoubleFloat
--R qsetelt! : (% ,Integer,Integer,Complex DoubleFloat) -> Complex DoubleFloat
--R rank : % -> NonNegativeInteger if Complex DoubleFloat has INTDOM
--R row : (% ,Integer) -> ComplexDoubleFloatVector
--R rowEchelon : % -> % if Complex DoubleFloat has EUCDOM
--R scalarMatrix : (NonNegativeInteger,Complex DoubleFloat) -> %
--R setColumn! : (% ,Integer,ComplexDoubleFloatVector) -> %
--R setRow! : (% ,Integer,ComplexDoubleFloatVector) -> %
--R setelt : (% ,List Integer,List Integer,%) -> %
--R setelt : (% ,Integer,Integer,Complex DoubleFloat) -> Complex DoubleFloat
--R setsubMatrix! : (% ,Integer,Integer,%) -> %
--R size? : (% ,NonNegativeInteger) -> Boolean
--R subMatrix : (% ,Integer,Integer,Integer,Integer) -> %
--R swapColumns! : (% ,Integer,Integer) -> %
--R swapRows! : (% ,Integer,Integer) -> %
--R transpose : ComplexDoubleFloatVector -> %
--R zero : (NonNegativeInteger,NonNegativeInteger) -> %
--R ~=? : (% ,%) -> Boolean if Complex DoubleFloat has SETCAT
--R

```

```

--E 1

--S 2 of 6
a:CDFMAT:=qnew(2,3)
--R
--R      +0.  0.  0.+
--R  (1) |      |
--R      +0.  0.  0.+
--R
--R                                          Type: ComplexDoubleFloatMatrix
--E 2

--S 3 of 6
qsetelt!(a,1,1,1.0+2*%i)
--R
--R  (2)  1. + 2. %i
--R
--R                                          Type: Complex DoubleFloat
--E 3

--S 4 of 6
a
--R
--R      +0.      0.      0.+
--R  (3) |      |
--R      +0.  1. + 2. %i  0.+
--R
--R                                          Type: ComplexDoubleFloatMatrix
--E 4

--S 5 of 6
qsetelt!(a,0,0,2.0+4*%i)
--R
--R  (4)  2. + 4. %i
--R
--R                                          Type: Complex DoubleFloat
--E 5

--S 6 of 6
a
--R
--R      +2. + 4. %i      0.      0.+
--R  (5) |      |
--R      +  0.      1. + 2. %i  0.+
--R
--R                                          Type: ComplexDoubleFloatMatrix
--E 6

)spool
)lisp (bye)

```

---

## — ComplexDoubleFloatMatrix.help —

```
=====
ComplexDoubleFloatMatrix examples
=====
```

This domain creates a lisp simple array of machine doublefloats.  
It provides one new function called qnew which takes an integer  
that gives the array length.

NOTE: Unlike normal Axiom arrays the ComplexDoubleFloatMatrix arrays  
are 0-based so the first element is 0. Axiom arrays normally  
start at 1.

```
a:CDFMAT:=qnew(2,3)
```

```
  +0.  0.  0.+
  |      |
  +0.  0.  0.+
```

```
qsetelt!(a,1,1,1.0+2*%i)
      1. + 2. %i
```

```
a
```

```
  +0.      0.      0.+
  |      |      |
  +0.  1. + 2. %i  0.+
```

```
qsetelt!(a,0,0,2.0+4*%i)
      2. + 4. %i
```

```
a
```

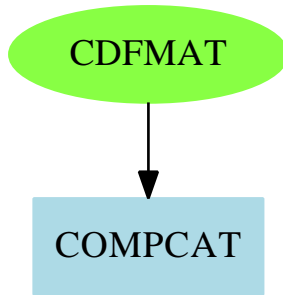
```
  +2. + 4. %i      0.      0.+
  |      |      |
  +  0.      1. + 2. %i  0.+
```

See Also:

- o )help Float
- o )help DoubleFloat
- o )show ComplexDoubleFloatMatrix

---

## 4.9.1 ComplexDoubleFloatMatrix (CDFMAT)

**Exports:**

|             |                |                |              |
|-------------|----------------|----------------|--------------|
| #?          | -?             | ?**?           | ?*?          |
| ?+?         | ?-?            | ?/?            | ?=?          |
| ?~=?        | antisymmetric? | any?           | coerce       |
| column      | columnSpace    | copy           | count        |
| determinant | diagonal?      | diagonalMatrix | elt          |
| empty       | empty?         | eq?            | eval         |
| every?      | exquo          | fill!          | hash         |
| horizConcat | inverse        | latex          | less?        |
| listOfLists | map            | map!           | matrix       |
| maxColIndex | maxRowIndex    | member?        | members      |
| minColIndex | minRowIndex    | minordet       | more?        |
| ncols       | new            | nrows          | nullSpace    |
| nullity     | parts          | pfaffian       | qelt         |
| qnew        | qsetelt!       | rank           | row          |
| rowEchelon  | sample         | scalarMatrix   | setColumn!   |
| setRow!     | setelt         | setsubMatrix!  | size?        |
| square?     | squareTop      | subMatrix      | swapColumns! |
| swapRows!   | symmetric?     | transpose      | vertConcat   |
| zero        |                |                |              |

— domain CDFMAT ComplexDoubleFloatMatrix —

```

)abbrev domain CDFMAT ComplexDoubleFloatMatrix
++ Author: Waldek Hebisch
++ Description: This is a low-level domain which implements matrices
++ (two dimensional arrays) of complex double precision floating point
++ numbers. Indexing is 0 based, there is no bound checking (unless
++ provided by lower level).

```

```

ComplexDoubleFloatMatrix : MatrixCategory(Complex DoubleFloat,
                                           ComplexDoubleFloatVector,
                                           ComplexDoubleFloatVector) with
  qnew : (Integer, Integer) -> %

```



```

++ qnew(n, m) creates a new uninitialized n by m matrix.
++
++X t1:CDFMAT:=qnew(3,4)

== add

NNI ==> Integer
Qelt2 ==> CDAREF2$Lisp
Qsetelt2 ==> CDSETAREF2$Lisp
Qnrows ==> CDANROWS$Lisp
Qncols ==> CDANCOLS$Lisp
Qnew ==> MAKE_-CDOUBLE_-MATRIX$Lisp

minRowIndex x == 0
minColIndex x == 0
nrows x == Qnrows(x)
ncols x == Qncols(x)
maxRowIndex x == Qnrows(x) - 1
maxColIndex x == Qncols(x) - 1

qelt(m, i, j) == Qelt2(m, i, j)
qsetelt_!(m, i, j, r) == Qsetelt2(m, i, j, r)

empty() == Qnew(0$Integer, 0$Integer)
qnew(rows, cols) == Qnew(rows, cols)
new(rows, cols, a) ==
  res := Qnew(rows, cols)
  for i in 0..(rows - 1) repeat
    for j in 0..(cols - 1) repeat
      Qsetelt2(res, i, j, a)
  res

```

---

— CDFMAT.dotabb —

```

"CDFMAT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=CDFMAT",
          shape=ellipse]
"COMPCAT" [color=lightblue,href="bookvol10.2.pdf#nameddest=COMPCAT"];
"CDFMAT" -> "COMPCAT"

```

---

## 4.10 domain CDFVEC ComplexDoubleFloatVector

— ComplexDoubleFloatVector.input —

```

)set break resume
)sys rm -f ComplexDoubleFloatVector.output
)spool ComplexDoubleFloatVector.output
)set message test on
)set message auto off
)clear all

--S 1 of 6
)show ComplexDoubleFloatVector
--R ComplexDoubleFloatVector is a domain constructor
--R Abbreviation for ComplexDoubleFloatVector is CDFVEC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for CDFVEC
--R
--R----- Operations -----
--R concat : List % -> %                concat : (%,%) -> %
--R copy : % -> %                      delete : (%,Integer) -> %
--R empty : () -> %                   empty? : % -> Boolean
--R eq? : (%,%) -> Boolean             index? : (Integer,%) -> Boolean
--R indices : % -> List Integer        insert : (%,%,Integer) -> %
--R qnew : Integer -> %               reverse : % -> %
--R sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?? : (%,Complex DoubleFloat) -> % if Complex DoubleFloat has MONOID
--R ?? : (Complex DoubleFloat,%) -> % if Complex DoubleFloat has MONOID
--R ?? : (Integer,%) -> % if Complex DoubleFloat has ABELGRP
--R ?+? : (%,%) -> % if Complex DoubleFloat has ABELSG
--R ?-? : (%,%) -> % if Complex DoubleFloat has ABELGRP
--R -? : % -> % if Complex DoubleFloat has ABELGRP
--R ?<? : (%,%) -> Boolean if Complex DoubleFloat has ORDSET
--R ?<=? : (%,%) -> Boolean if Complex DoubleFloat has ORDSET
--R ==? : (%,%) -> Boolean if Complex DoubleFloat has SETCAT
--R ??? : (%,%) -> Boolean if Complex DoubleFloat has ORDSET
--R ?>=? : (%,%) -> Boolean if Complex DoubleFloat has ORDSET
--R any? : ((Complex DoubleFloat -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if Complex DoubleFloat has SETCAT
--R concat : (Complex DoubleFloat,%) -> %
--R concat : (%,Complex DoubleFloat) -> %
--R construct : List Complex DoubleFloat -> %
--R convert : % -> InputForm if Complex DoubleFloat has KONVERT INFORM
--R copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
--R count : (Complex DoubleFloat,%) -> NonNegativeInteger if $ has finiteAggregate and Complex DoubleFlo
--R count : ((Complex DoubleFloat -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R cross : (%,%) -> % if Complex DoubleFloat has RING

```

```

--R delete : (% , UniversalSegment Integer) -> %
--R dot : (% , %) -> Complex DoubleFloat if Complex DoubleFloat has RING
--R ?.? : (% , UniversalSegment Integer) -> %
--R ?.? : (% , Integer) -> Complex DoubleFloat
--R elt : (% , Integer, Complex DoubleFloat) -> Complex DoubleFloat
--R entries : % -> List Complex DoubleFloat
--R entry? : (Complex DoubleFloat, %) -> Boolean if $ has finiteAggregate and Complex DoubleFloat
--R eval : (% , List Complex DoubleFloat, List Complex DoubleFloat) -> % if Complex DoubleFloat
--R eval : (% , Complex DoubleFloat, Complex DoubleFloat) -> % if Complex DoubleFloat has EVALAB
--R eval : (% , Equation Complex DoubleFloat) -> % if Complex DoubleFloat has EVALAB COMPLEX D
--R eval : (% , List Equation Complex DoubleFloat) -> % if Complex DoubleFloat has EVALAB COMPI
--R every? : ((Complex DoubleFloat -> Boolean), %) -> Boolean if $ has finiteAggregate
--R fill! : (% , Complex DoubleFloat) -> % if $ has shallowlyMutable
--R find : ((Complex DoubleFloat -> Boolean), %) -> Union(Complex DoubleFloat, "failed")
--R first : % -> Complex DoubleFloat if Integer has ORDSET
--R hash : % -> SingleInteger if Complex DoubleFloat has SETCAT
--R insert : (Complex DoubleFloat, % , Integer) -> %
--R latex : % -> String if Complex DoubleFloat has SETCAT
--R length : % -> Complex DoubleFloat if Complex DoubleFloat has RADCAT and Complex DoubleFlo
--R less? : (% , NonNegativeInteger) -> Boolean
--R magnitude : % -> Complex DoubleFloat if Complex DoubleFloat has RADCAT and Complex Double
--R map : (((Complex DoubleFloat, Complex DoubleFloat) -> Complex DoubleFloat), %, %) -> %
--R map : ((Complex DoubleFloat -> Complex DoubleFloat), %) -> %
--R map! : ((Complex DoubleFloat -> Complex DoubleFloat), %) -> % if $ has shallowlyMutable
--R max : (% , %) -> % if Complex DoubleFloat has ORDSET
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (Complex DoubleFloat, %) -> Boolean if $ has finiteAggregate and Complex DoubleFlo
--R members : % -> List Complex DoubleFloat if $ has finiteAggregate
--R merge : (% , %) -> % if Complex DoubleFloat has ORDSET
--R merge : (((Complex DoubleFloat, Complex DoubleFloat) -> Boolean), %, %) -> %
--R min : (% , %) -> % if Complex DoubleFloat has ORDSET
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (% , NonNegativeInteger) -> Boolean
--R new : (NonNegativeInteger, Complex DoubleFloat) -> %
--R outerProduct : (% , %) -> Matrix Complex DoubleFloat if Complex DoubleFloat has RING
--R parts : % -> List Complex DoubleFloat if $ has finiteAggregate
--R position : (Complex DoubleFloat, % , Integer) -> Integer if Complex DoubleFloat has SETCAT
--R position : (Complex DoubleFloat, %) -> Integer if Complex DoubleFloat has SETCAT
--R position : ((Complex DoubleFloat -> Boolean), %) -> Integer
--R qelt : (% , Integer) -> Complex DoubleFloat
--R qsetelt! : (% , Integer, Complex DoubleFloat) -> Complex DoubleFloat if $ has shallowlyMutabl
--R reduce : (((Complex DoubleFloat, Complex DoubleFloat) -> Complex DoubleFloat), %) -> Compl
--R reduce : (((Complex DoubleFloat, Complex DoubleFloat) -> Complex DoubleFloat), %, Complex D
--R reduce : (((Complex DoubleFloat, Complex DoubleFloat) -> Complex DoubleFloat), %, Complex D
--R remove : ((Complex DoubleFloat -> Boolean), %) -> % if $ has finiteAggregate
--R remove : (Complex DoubleFloat, %) -> % if $ has finiteAggregate and Complex DoubleFloat ha
--R removeDuplicates : % -> % if $ has finiteAggregate and Complex DoubleFloat has SETCAT
--R reverse! : % -> % if $ has shallowlyMutable
--R select : ((Complex DoubleFloat -> Boolean), %) -> % if $ has finiteAggregate
--R setelt : (% , UniversalSegment Integer, Complex DoubleFloat) -> Complex DoubleFloat if $ has

```

```

--R setelt : (%,Integer,Complex DoubleFloat) -> Complex DoubleFloat if $ has shallowlyMutable
--R size? : (% ,NonNegativeInteger) -> Boolean
--R sort : % -> % if Complex DoubleFloat has ORDSET
--R sort : (((Complex DoubleFloat,Complex DoubleFloat) -> Boolean),%) -> %
--R sort! : % -> % if $ has shallowlyMutable and Complex DoubleFloat has ORDSET
--R sort! : (((Complex DoubleFloat,Complex DoubleFloat) -> Boolean),%) -> % if $ has shallowlyMutable
--R sorted? : % -> Boolean if Complex DoubleFloat has ORDSET
--R sorted? : (((Complex DoubleFloat,Complex DoubleFloat) -> Boolean),%) -> Boolean
--R swap! : (% ,Integer,Integer) -> Void if $ has shallowlyMutable
--R vector : List Complex DoubleFloat -> %
--R zero : NonNegativeInteger -> % if Complex DoubleFloat has ABELMON
--R ~=?: (% ,%) -> Boolean if Complex DoubleFloat has SETCAT
--R
--E 1

--S 2 of 6
t1:CDFVEC:=qnew(5)
--R
--R (1) [0.,0.,0.,0.,0.]
--R
--R Type: ComplexDoubleFloatVector
--E 2

-- NOTE: CDFVEC arrays are 0-based, normal Axiom arrays are 1-based
--S 3 of 6
t1.1:=1.0+2*%i
--R
--R (2) 1. + 2. %i
--R
--R Type: Complex DoubleFloat
--E 3

--S 4 of 6
t1
--R
--R (3) [0.,1. + 2. %i,0.,0.,0.]
--R
--R Type: ComplexDoubleFloatVector
--E 4

--S 5 of 6
t1.0:=3.0+4.0*%i
--R
--R (4) 3. + 4. %i
--R
--R Type: Complex DoubleFloat
--E 5

--S 6 of 6
t1
--R
--R (5) [3. + 4. %i,1. + 2. %i,0.,0.,0.]
--R
--R Type: ComplexDoubleFloatVector
--E 6

```

```
)spool
)lisp (bye)
```

---

— ComplexDoubleFloatVector.help —

```
=====
ComplexDoubleFloatVector examples
=====
This domain creates a lisp simple array of machine complex doublefloats.
It provides one new function called qnew which takes an integer
that gives the array length.

NOTE: Unlike normal Axiom arrays the ComplexDoubleFloatVector arrays
are 0-based so the first element is 0. Axiom arrays normally
start at 1.

t1:CDFVEC:=qnew(5)
      [0.,0.,0.,0.,0.]

t1.1:=1.0+2*%i
      1. + 2. %i

t1
      [0.,1. + 2. %i,0.,0.,0.]

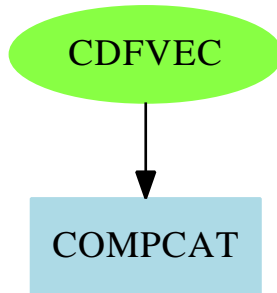
t1.0:=3.0+4.0*%i
      3. + 4. %i

t1
      [3. + 4. %i,1. + 2. %i,0.,0.,0.]

See Also:
o )help Float
o )help DoubleFloat
o )show ComplexDoubleFloatVector
```

---

## 4.10.1 ComplexDoubleFloatVector (CDFVEC)

**Exports:**

|         |                  |           |           |
|---------|------------------|-----------|-----------|
| #?      | -?               | ?*?       | ?+?       |
| ?-?     | ?..?             | ?<=?      | ?<?       |
| ?=?     | ?>=?             | ?>?       | ?~=?      |
| any?    | coerce           | concat    | construct |
| convert | copy             | copyInto! | count     |
| cross   | delete           | dot       | elt       |
| empty   | empty?           | entries   | entry?    |
| eq?     | eval             | every?    | fill!     |
| find    | first            | hash      | index?    |
| indices | insert           | latex     | length    |
| less?   | magnitude        | map       | map!      |
| max     | maxIndex         | member?   | members   |
| merge   | min              | minIndex  | more?     |
| new     | outerProduct     | parts     | position  |
| qelt    | qnew             | qsetelt!  | reduce    |
| remove  | removeDuplicates | reverse   | reverse!  |
| sample  | select           | setelt    | size?     |
| sort    | sort!            | sorted?   | swap!     |
| vector  | zero             |           |           |

— domain CDFVEC ComplexDoubleFloatVector —

```

)abbrev domain CDFVEC ComplexDoubleFloatVector
++ Author: Waldek Hebisch
++ Description: This is a low-level domain which implements vectors
++ (one dimensional arrays) of complex double precision floating point
++ numbers. Indexing is 0 based, there is no bound checking (unless
++ provided by lower level).
ComplexDoubleFloatVector : VectorCategory Complex DoubleFloat with
  qnew : Integer -> %
    ++ qnew(n) creates a new uninitialized vector of length n.
    ++
    ++X t1:CDFVEC:=qnew 7
  
```

```

vector: List Complex DoubleFloat -> %
  ++ vector(l) converts the list l to a vector.
  ++
  ++X t1:List(Complex(DoubleFloat)):= [1+2*%i, 3+4*%i, -5-6*%i]
  ++X t2:CDFVEC:=vector(t1)
== add

Qelt1 ==> CDELT$Lisp
Qsetelt1 ==> CDSELT$Lisp

qelt(x, i) == Qelt1(x, i)
qsetelt_!(x, i, s) == Qsetelt1(x, i, s)
Qsize ==> CDLEN$Lisp
Qnew ==> MAKE_-CDOUBLE_-VECTOR$Lisp

#x == Qsize x
minIndex x == 0
empty() == Qnew(0$Lisp)
qnew(n) == Qnew(n)
new(n, x) ==
  res := Qnew(n)
  fill_!(res, x)
qelt(x, i) == Qelt1(x, i)
elt(x:%, i:Integer) == Qelt1(x, i)
qsetelt_!(x, i, s) == Qsetelt1(x, i, s)
setelt(x : %, i : Integer, s : Complex DoubleFloat) ==
  Qsetelt1(x, i, s)
fill_!(x, s) ==
  for i in 0..((Qsize(x)) - 1) repeat Qsetelt1(x, i, s)
x

```

— CDFVEC.dotabb —

```

"CDFVEC" [color="#88FF44",href="bookvol10.3.pdf#nameddest=CDFVEC",
  shape=ellipse]
"COMPCAT" [color=lightblue,href="bookvol10.2.pdf#nameddest=COMPCAT"];
"CDFVEC" -> "COMPCAT"

```

## 4.11 domain CONTFRAC ContinuedFraction

— ContinuedFraction.input —

```

)set break resume
)sys rm -f ContinuedFraction.output
)spool ContinuedFraction.output
)set message test on
)set message auto off
)clear all
--S 1 of 22
c := continuedFraction(314159/100000)
--R
--R
--R      1 |      1 |      1 |      1 |      1 |      1 |      1 |
--R  (1)  3 + +---+ + +---+ + +---+ + +---+ + +---+ + +---+ + +---+
--R      | 7      | 15      | 1      | 25      | 1      | 7      | 4
--R
--R                                          Type: ContinuedFraction Integer
--E 1

--S 2 of 22
partialQuotients c
--R
--R
--R  (2)  [3,7,15,1,25,1,7,4]
--R
--R                                          Type: Stream Integer
--E 2

--S 3 of 22
convergents c
--R
--R
--R      22 333 355 9208 9563 76149 314159
--R  (3)  [3,--,---,---,---,---,---,---]
--R      7 106 113 2931 3044 24239 100000
--R
--R                                          Type: Stream Fraction Integer
--E 3

--S 4 of 22
approximants c
--R
--R
--R      22 333 355 9208 9563 76149 314159
--R  (4)  [3,--,---,---,---,---,---,---]
--R      7 106 113 2931 3044 24239 100000
--R
--R                                          Type: Stream Fraction Integer
--E 4

--S 5 of 22
pq := partialQuotients(1/c)
--R
--R
--R  (5)  [0,3,7,15,1,25,1,7,4]

```





```

--S 10 of 22
ccf := convergents cf
--R
--R
--R      6 61  860 15541 342762  8927353 268163352  9126481321
--R (10) [0,1,-,--,---,-----,-----,-----,-----,-----,...]
--R      7 71 1001 18089 398959 10391023 312129649 10622799089
--R                                          Type: Stream Fraction Integer
--E 10

--S 11 of 22
eConvergents := [2*e + 1 for e in ccf]
--R
--R
--R      19 193 2721 49171 1084483 28245729 848456353 28875761731
--R (11) [1,3,-,--,---,-----,-----,-----,-----,-----,...]
--R      7 71 1001 18089 398959 10391023 312129649 10622799089
--R                                          Type: Stream Fraction Integer
--E 11

--S 12 of 22
eConvergents :: Stream Float
--R
--R
--R (12)
--R [1.0, 3.0, 2.7142857142 857142857, 2.7183098591 549295775,
--R 2.7182817182 817182817, 2.7182818287 356957267, 2.7182818284 585634113,
--R 2.7182818284 590458514, 2.7182818284 590452348, 2.7182818284 590452354,
--R ...]
--R                                          Type: Stream Float
--E 12

--S 13 of 22
exp 1.0
--R
--R
--R (13) 2.7182818284 590452354
--R                                          Type: Float
--E 13

--S 14 of 22
cf := continuedFraction(1,[(2*i+1)**2 for i in 0..],repeating [2])
--R
--R
--R (14)
--R      1 |      9 |      25 |      49 |      81 |      121 |      169 |      225 |
--R 1 + +---+ + +---+ + +---+ + +---+ + +---+ + +---+ + +---+ + +---+
--R   | 2   | 2   | 2   | 2   | 2   | 2   | 2   | 2   |
--R +

```



```

--R
--R
--R                                          Type: Void
--E 19

--S 20 of 22
r := ((x - 1) * (x - 2)) / ((x-3) * (x-4))
--R
--R
--R          2
--R      x  - 3x + 2
--R  (20)  -----
--R          2
--R      x  - 7x + 12
--R                                          Type: Fraction UnivariatePolynomial(x,Fraction Integer)
--E 20

--S 21 of 22
continuedFraction r
--R
--R
--R          1      |      1      |
--R  (21)  1 + +-----+ + +-----+
--R          | 1      9      | 16      40
--R          | - x - -      | -- x - --
--R          | 4      8      | 3      3
--R          Type: ContinuedFraction UnivariatePolynomial(x,Fraction Integer)
--E 21

--S 22 of 22
[i*i for i in convergents(z) :: Stream Float]
--R
--R
--R  (22)
--R  [9.0, 11.1111111111 11111111, 10.9944598337 9501385, 11.0002777777 77777778,
--R   10.9999860763 98799786, 11.0000006979 29731039, 10.9999999650 15834446,
--R   11.0000000017 53603304, 10.9999999999 12099531, 11.0000000000 04406066,
--R   ...]
--R                                          Type: Stream Float
--E 22
)spool
)lisp (bye)

```

— ContinuedFraction.help —

```

=====
ContinuedFraction examples
=====

```

Continued fractions have been a fascinating and useful tool in mathematics for well over three hundred years. Axiom implements continued fractions for fractions of any Euclidean domain. In practice, this usually means rational numbers. In this section we demonstrate some of the operations available for manipulating both finite and infinite continued fractions.

The ContinuedFraction domain is a field and therefore you can add, subtract, multiply and divide the fractions.

The continuedFraction operation converts its fractional argument to a continued fraction.

```
c := continuedFraction(314159/100000)
      1 |      1 |      1 |      1 |      1 |      1 |      1 |
3 + +---+ + +---+ + +---+ + +---+ + +---+ + +---+ + +---+
   | 7   | 15   | 1   | 25   | 1   | 7   | 4
                                     Type: ContinuedFraction Integer
```

This display is a compact form of the bulkier

```
3 +
  -----
7 +
  -----
15 +
  -----
1 +
  -----
25 +
  -----
1 +
  -----
7 +
  -----
4
```

You can write any rational number in a similar form. The fraction will be finite and you can always take the "numerators" to be 1. That is, any rational number can be written as a simple, finite continued fraction of the form

```
a(1) +
  -----
a(2) +
  -----
a(3) +
  .
  .
  .
```

$$\frac{1}{a(n-1) + \frac{1}{a(n)}}$$

The  $a(i)$  are called partial quotients and the operation `partialQuotients` creates a stream of them.

```
partialQuotients c
[3,7,15,1,25,1,7,4]
Type: Stream Integer
```

By considering more and more of the fraction, you get the convergents. For example, the first convergent is  $a(1)$ , the second is  $a(1) + 1/a(2)$  and so on.

```
convergents c
22 333 355 9208 9563 76149 314159
[3,--,---,---,----,----,-----,-----]
7 106 113 2931 3044 24239 100000
Type: Stream Fraction Integer
```

Since this is a finite continued fraction, the last convergent is the original rational number, in reduced form. The result of `approximants` is always an infinite stream, though it may just repeat the "last" value.

```
approximants c
22 333 355 9208 9563 76149 314159
[3,--,---,---,----,----,-----,-----]
7 106 113 2931 3044 24239 100000
Type: Stream Fraction Integer
```

Inverting `c` only changes the partial quotients of its fraction by inserting a 0 at the beginning of the list.

```
pq := partialQuotients(1/c)
[0,3,7,15,1,25,1,7,4]
Type: Stream Integer
```

Do this to recover the original continued fraction from this list of partial quotients. The three-argument form of the `continuedFraction` operation takes an element which is the whole part of the fraction, a stream of elements which are the numerators of the fraction, and a stream of elements which are the denominators of the fraction.

```
continuedFraction(first pq, repeating [1], rest pq)
```

```

      1 |      1 |      1 |      1 |      1 |      1 |      1 |      1 |
+---+ + +---+ + +---+ + +---+ + +---+ + +---+ + +---+ + +---+
| 3      | 7      | 15     | 1      | 25     | 1      | 7      | 4
Type: ContinuedFraction Integer

```

The streams need not be finite for continuedFraction. Can you guess which irrational number has the following continued fraction? See the end of this section for the answer.

```

z:=continuedFraction(3,repeating [1],repeating [3,6])
      1 |      1 |      1 |      1 |      1 |      1 |      1 |      1 |
3 + +---+ + +---+ + +---+ + +---+ + +---+ + +---+ + +---+ + +---+
   | 3      | 6      | 3      | 6      | 3      | 6      | 3      | 6      | 3
+
  1 |
+---+ + ...
| 6
Type: ContinuedFraction Integer

```

In 1737 Euler discovered the infinite continued fraction expansion

$$\begin{array}{r}
 e - 1 \\
 \hline
 2 \quad 1 + \cfrac{1}{6 + \cfrac{1}{10 + \cfrac{1}{14 + \dots}}}
 \end{array}$$

We use this expansion to compute rational and floating point approximations of  $e$ . For this and other interesting expansions, see C. D. Olds, Continued Fractions, New Mathematical Library, (New York: Random House, 1963), pp. 134--139.}

By looking at the above expansion, we see that the whole part is 0 and the numerators are all equal to 1. This constructs the stream of denominators.

```

dens:Stream Integer := cons(1,generate((x+>x+4),6))
[1,6,10,14,18,22,26,30,34,38,...]
Type: Stream Integer

```

Therefore this is the continued fraction expansion for  $(e - 1) / 2$ .

```

cf := continuedFraction(0,repeating [1],dens)
      1 |      1 |      1 |      1 |      1 |      1 |      1 |      1 |
+---+ + +---+ + +---+ + +---+ + +---+ + +---+ + +---+ + +---+

```

```

      | 1      | 6      | 10      | 14      | 18      | 22      | 26      | 30
+
      1 |      1 |
+-----+ + +-----+ + ...
      | 34      | 38

```

Type: ContinuedFraction Integer

These are the rational number convergents.

```

ccf := convergents cf
      6 61 860 15541 342762 8927353 268163352 9126481321
[0,1,-,--,---,-----,-----,-----,-----,...]
      7 71 1001 18089 398959 10391023 312129649 10622799089

```

Type: Stream Fraction Integer

You can get rational convergents for e by multiplying by 2 and adding 1.

```

eConvergents := [2*e + 1 for e in ccf]
      19 193 2721 49171 1084483 28245729 848456353 28875761731
[1,3,--,---,-----,-----,-----,-----,-----,...]
      7 71 1001 18089 398959 10391023 312129649 10622799089

```

Type: Stream Fraction Integer

You can also compute the floating point approximations to these convergents.

```

eConvergents :: Stream Float
[1.0, 3.0, 2.7142857142 857142857, 2.7183098591 549295775,
 2.7182817182 817182817, 2.7182818287 356957267, 2.7182818284 585634113,
 2.7182818284 590458514, 2.7182818284 590452348, 2.7182818284 590452354,
 ...]

```

Type: Stream Float

Compare this to the value of e computed by the exp operation in Float.

```

exp 1.0
2.7182818284 590452354

```

Type: Float

In about 1658, Lord Brouncker established the following expansion for  $4 / \pi$ ,

$$\begin{array}{r}
 1 + \cfrac{1}{\cfrac{2 + \cfrac{9}{2 + \cfrac{25}{2 + \cfrac{49}{2 + \cfrac{81}{\dots}}}}}}
 \end{array}$$



$$\frac{\text{-----}}{2 + \dots}$$

Let's use this expansion to compute rational and floating point approximations for pi.

```
cf := continuedFraction(1,[(2*i+1)**2 for i in 0..],repeating [2])
      1 |    9 |    25 |    49 |    81 |    121 |    169 |    225 |
      1 + +---+ + +---+ + +---+ + +---+ + +---+ + +---+ + +---+
        | 2    | 2    | 2    | 2    | 2    | 2    | 2    | 2
      +
      289 |    361 |
      +-----+ + +-----+ + ...
        | 2      | 2
Type: ContinuedFraction Integer
```

```
ccf := convergents cf
      3 15 105 315 3465 45045 45045 765765 14549535
[1,-,--,---,---,---,---,---,---,---,---,---,---,...]
      2 13 76 263 2578 36979 33976 622637 11064338
Type: Stream Fraction Integer
```

```
piConvergents := [4/p for p in ccf]
      8 52 304 1052 10312 147916 135904 2490548 44257352
[4,-,--,---,---,---,---,---,---,---,---,---,---,...]
      3 15 105 315 3465 45045 45045 765765 14549535
Type: Stream Fraction Integer
```

As you can see, the values are converging to  
pi = 3.14159265358979323846..., but not very quickly.

```
piConvergents :: Stream Float
[4.0, 2.66666666666 666666667, 3.46666666666 666666667,
 2.8952380952 380952381, 3.3396825396 825396825, 2.9760461760 461760462,
 3.2837384837 384837385, 3.0170718170 718170718, 3.2523659347 188758953,
 3.0418396189 294022111, ...]
Type: Stream Float
```

You need not restrict yourself to continued fractions of integers.  
Here is an expansion for a quotient of Gaussian integers.

```
continuedFraction((- 122 + 597*i)/(4 - 4*i))
      1 |    1 |
      - 90 + 59*i + +-----+ + +-----+
        | 1 - 2*i    | - 1 + 2*i
Type: ContinuedFraction Complex Integer
```

This is an expansion for a quotient of polynomials in one variable  
with rational number coefficients.

```

r : Fraction UnivariatePolynomial(x,Fraction Integer)
      Type: Void

r := ((x - 1) * (x - 2)) / ((x-3) * (x-4))
      2
      x  - 3x + 2
      -----
      2
      x  - 7x + 12
      Type: Fraction UnivariatePolynomial(x,Fraction Integer)

continuedFraction r
      1 | 1 |
1 + +-----+ + +-----+
   | 1   9   | 16   40
   | - x - - | -- x - --
   | 4   8   | 3    3
      Type: ContinuedFraction UnivariatePolynomial(x,Fraction Integer)

```

To conclude this section, we give you evidence that

$$\begin{aligned}
 z = 3 + & \cfrac{1}{\cfrac{3 + \cfrac{1}{\cfrac{6 + \cfrac{1}{\cfrac{3 + \cfrac{1}{\cfrac{6 + \cfrac{1}{3 + \dots}}}}}}}}}{\cfrac{1}{\cfrac{3 + \cfrac{1}{\cfrac{6 + \cfrac{1}{\cfrac{3 + \cfrac{1}{\cfrac{6 + \cfrac{1}{3 + \dots}}}}}}}}}}
 \end{aligned}$$

is the expansion of `sqrt(11)`.

```

[i*i for i in convergents(z) :: Stream Float]
[9.0, 11.111111111 11111111, 10.9944598337 9501385, 11.0002777777 77777778,
 10.9999860763 98799786, 11.0000006979 29731039, 10.9999999650 15834446,
 11.0000000017 53603304, 10.9999999999 12099531, 11.0000000000 04406066,
 ...]
      Type: Stream Float

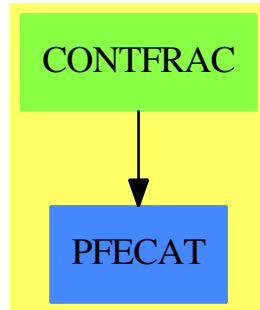
```

See Also:

- o )help Stream
- o )show ContinuedFraction

---

## 4.11.1 ContinuedFraction (CONTFRAC)

**Exports:**

|                   |                    |                          |
|-------------------|--------------------|--------------------------|
| 0                 | 1                  | approximants             |
| associates?       | characteristic     | coerce                   |
| complete          | continuedFraction  | convergents              |
| denominators      | divide             | extend                   |
| euclideanSize     | expressIdealMember | exquo                    |
| extendedEuclidean | factor             | gcd                      |
| gcdPolynomial     | hash               | inv                      |
| latex             | lcm                | multiEuclidean           |
| numerators        | one?               | partialDenominators      |
| partialNumerators | partialQuotients   | prime?                   |
| principalIdeal    | recip              | reducedContinuedFraction |
| reducedForm       | sample             | sizeLess?                |
| squareFree        | squareFreePart     | subtractIfCan            |
| unit?             | unitCanonical      | unitNormal               |
| wholePart         | zero?              | ?*?                      |
| ?**?              | ?+?                | ?-?                      |
| -?                | ?/?                | ?=?                      |
| ?^?               | ?~=?               | ?quo?                    |
| ?rem?             |                    |                          |

— domain CONTFRAC ContinuedFraction —

```

)abbrev domain CONTFRAC ContinuedFraction
++ Author: Stephen M. Watt
++ Date Created: January 1987
++ Change History:
++   11 April   1990
++   7 October 1991 -- SMW: Treat whole part specially. Added comments.
++ Basic Operations:
++   (Field), (Algebra),
++   approximants, complete, continuedFraction, convergents, denominators,
++   extend, numerators, partialDenominators, partialNumerators,
++   partialQuotients, reducedContinuedFraction, reducedForm, wholePart
  
```

```

++ Related Constructors:
++ Also See: Fraction
++ AMS Classifications: 11A55 11J70 11K50 11Y65 30B70 40A15
++ Keywords: continued fraction, convergent
++ References:
++ Description:
++ \spadtype{ContinuedFraction} implements general
++ continued fractions. This version is not restricted to simple,
++ finite fractions and uses the \spadtype{Stream} as a
++ representation. The arithmetic functions assume that the
++ approximants alternate below/above the convergence point.
++ This is enforced by ensuring the partial numerators and partial
++ denominators are greater than 0 in the Euclidean domain view of \spad{R}
++ (i.e. \spad{sizeLess?(0, x)}).

```

ContinuedFraction(R): Exports == Implementation where

```

R :      EuclideanDomain
Q  ==> Fraction R
MT  ==> MoebiusTransform Q
OUT ==> OutputForm

```

Exports ==> Join(Algebra R, Algebra Q, Field) with

```

continuedFraction:      Q -> %
++ continuedFraction(r) converts the fraction \spadvar{r} with
++ components of type \spad{R} to a continued fraction over
++ \spad{R}.

```

```

continuedFraction:      (R, Stream R, Stream R) -> %
++ continuedFraction(b0,a,b) constructs a continued fraction in
++ the following way: if \spad{a = [a1,a2,...]} and \spad{b =
++ [b1,b2,...]} then the result is the continued fraction
++ \spad{b0 + a1/(b1 + a2/(b2 + ...))}.

```

```

reducedContinuedFraction: (R, Stream R) -> %
++ reducedContinuedFraction(b0,b) constructs a continued
++ fraction in the following way: if \spad{b = [b1,b2,...]}
++ then the result is the continued fraction \spad{b0 + 1/(b1 +
++ 1/(b2 + ...))}. That is, the result is the same as
++ \spad{continuedFraction(b0,[1,1,1,...],[b1,b2,b3,...])}.

```

```

partialNumerators:      % -> Stream R
++ partialNumerators(x) extracts the numerators in \spadvar{x}.
++ That is, if \spad{x = continuedFraction(b0, [a1,a2,a3,...],
++ [b1,b2,b3,...])}, then \spad{partialNumerators(x) =
++ [a1,a2,a3,...]}.

```

```

partialDenominators: % -> Stream R
++ partialDenominators(x) extracts the denominators in
++ \spadvar{x}. That is, if \spad{x = continuedFraction(b0,
++ [a1,a2,a3,...], [b1,b2,b3,...])}, then

```

```

++ \spad{partialDenominators(x) = [b1,b2,b3,...]}.

partialQuotients:    % -> Stream R
++ partialQuotients(x) extracts the partial quotients in
++ \spadvar{x}. That is, if \spad{x = continuedFraction(b0,
++ [a1,a2,a3,...], [b1,b2,b3,...])}, then
++ \spad{partialQuotients(x) = [b0,b1,b2,b3,...]}.

wholePart:           % -> R
++ wholePart(x) extracts the whole part of \spadvar{x}. That
++ is, if \spad{x = continuedFraction(b0, [a1,a2,a3,...],
++ [b1,b2,b3,...])}, then \spad{wholePart(x) = b0}.

reducedForm:         % -> %
++ reducedForm(x) puts the continued fraction \spadvar{x} in
++ reduced form, i.e. the function returns an equivalent
++ continued fraction of the form
++ \spad{continuedFraction(b0,[1,1,1,...],[b1,b2,b3,...])}.

approximants:        % -> Stream Q
++ approximants(x) returns the stream of approximants of the
++ continued fraction \spadvar{x}. If the continued fraction is
++ finite, then the stream will be infinite and periodic with
++ period 1.

convergents:         % -> Stream Q
++ convergents(x) returns the stream of the convergents of the
++ continued fraction \spadvar{x}. If the continued fraction is
++ finite, then the stream will be finite.

numerators:          % -> Stream R
++ numerators(x) returns the stream of numerators of the
++ approximants of the continued fraction \spadvar{x}. If the
++ continued fraction is finite, then the stream will be finite.

denominators:        % -> Stream R
++ denominators(x) returns the stream of denominators of the
++ approximants of the continued fraction \spadvar{x}. If the
++ continued fraction is finite, then the stream will be finite.

extend:              (%,Integer) -> %
++ extend(x,n) causes the first \spadvar{n} entries in the
++ continued fraction \spadvar{x} to be computed. Normally
++ entries are only computed as needed.

complete:            % -> %
++ complete(x) causes all entries in \spadvar{x} to be computed.
++ Normally entries are only computed as needed. If \spadvar{x}
++ is an infinite continued fraction, a user-initiated interrupt is
++ necessary to stop the computation.

```

```

Implementation ==> add

-- isOrdered ==> R is Integer
isOrdered ==> R has OrderedRing and R has multiplicativeValuation
canReduce? ==> isOrdered or R has additiveValuation

Rec ==> Record(num: R, den: R)
Str ==> Stream Rec
Rep := Record(value: Record(whole: R, fract: Str), reduced?: Boolean)

import Str

genFromSequence:      Stream Q -> %
genReducedForm:        (Q, Stream Q, MT) -> Stream Rec
genFractionA:          (Stream R, Stream R) -> Stream Rec
genFractionB:          (Stream R, Stream R) -> Stream Rec
genNumDen:             (R, R, Stream Rec) -> Stream R

genApproximants:       (R, R, R, R, Stream Rec) -> Stream Q
genConvergents:        (R, R, R, R, Stream Rec) -> Stream Q
iGenApproximants:      (R, R, R, R, Stream Rec) -> Stream Q
iGenConvergents:       (R, R, R, R, Stream Rec) -> Stream Q

reducedForm c ==
  c.reduced? => c
  explicitlyFinite? c.value.fract =>
    continuedFraction last complete convergents c
  canReduce? => genFromSequence approximants c
  error "Reduced form not defined for this continued fraction."

eucWhole(a: Q): R == numer a quo denom a

eucWhole0(a: Q): R ==
  isOrdered =>
    n := numer a
    d := denom a
    q := n quo d
    r := n - q*d
    if r < 0 then q := q - 1
    q
  eucWhole a

x = y ==
  x := reducedForm x
  y := reducedForm y

x.value.whole ^= y.value.whole => false

x1 := x.value.fract; y1 := y.value.fract

```

```

while not empty? x1 and not empty? y1 repeat
  frst.x1.den ^= frst.y1.den => return false
  x1 := rst x1; y1 := rst y1
empty? x1 and empty? y1

continuedFraction q == q :: %

if isOrdered then
  continuedFraction(wh,nums,dens) == [[wh,genFractionA(nums,dens)],false]

  genFractionA(nums,dens) ==
    empty? nums or empty? dens => empty()
    n := frst nums
    d := frst dens
    n < 0 => error "Numerators must be greater than 0."
    d < 0 => error "Denominators must be greater than 0."
    concat([n,d]$Rec, delay genFractionA(rst nums,rst dens))
else
  continuedFraction(wh,nums,dens) == [[wh,genFractionB(nums,dens)],false]

  genFractionB(nums,dens) ==
    empty? nums or empty? dens => empty()
    n := frst nums
    d := frst dens
    concat([n,d]$Rec, delay genFractionB(rst nums,rst dens))

reducedContinuedFraction(wh,dens) ==
  continuedFraction(wh, repeating [1], dens)

coerce(n:Integer):% == [[n::R,empty()], true]
coerce(r:R):% == [[r, empty()], true]

coerce(a: Q): % ==
  wh := eucWhole0 a
  fr := a - wh::Q
  zero? fr => [[wh, empty()], true]

l : List Rec := empty()
n := numer fr
d := denom fr
while not zero? d repeat
  qr := divide(n,d)
  l := concat([1,qr.quotient],l)
  n := d
  d := qr.remainder
[[wh, construct rest reverse! l], true]

characteristic() == characteristic()$Q

```

```

genFromSequence apps ==
  lo := first apps; apps := rst apps
  hi := first apps; apps := rst apps
  while eucWhole0 lo ^= eucWhole0 hi repeat
    lo := first apps; apps := rst apps
    hi := first apps; apps := rst apps
  wh := eucWhole0 lo
  [[wh, genReducedForm(wh::Q, apps, moebius(1,0,0,1))], canReduce?]

genReducedForm(wh0, apps, mt) ==
  lo: Q := first apps - wh0; apps := rst apps
  hi: Q := first apps - wh0; apps := rst apps
  lo = hi and zero? eval(mt, lo) => empty()
  mt := recip mt
  wlo := eucWhole eval(mt, lo)
  whi := eucWhole eval(mt, hi)
  while wlo ^= whi repeat
    wlo := eucWhole eval(mt, first apps - wh0); apps := rst apps
    whi := eucWhole eval(mt, first apps - wh0); apps := rst apps
  concat([1,wlo], delay genReducedForm(wh0, apps, shift(mt, -wlo::Q)))

wholePart c ==
  c.value.whole
partialNumerators c ==
  map(x1+>x1.num, c.value.fract)$StreamFunctions2(Rec,R)
partialDenominators c ==
  map(x1+>x1.den, c.value.fract)$StreamFunctions2(Rec,R)
partialQuotients c ==
  concat(c.value.whole, partialDenominators c)

approximants c ==
  empty? c.value.fract => repeating [c.value.whole::Q]
  genApproximants(1,0,c.value.whole,1,c.value.fract)
convergents c ==
  empty? c.value.fract => concat(c.value.whole::Q, empty())
  genConvergents (1,0,c.value.whole,1,c.value.fract)
numerators c ==
  empty? c.value.fract => concat(c.value.whole, empty())
  genNumDen(1,c.value.whole,c.value.fract)
denominators c ==
  genNumDen(0,1,c.value.fract)

extend(x,n) == (extend(x.value.fract,n); x)
complete(x) == (complete(x.value.fract); x)

iGenApproximants(pm2,qm2,pm1,qm1,fr) == delay
  nd := frst fr
  pm := nd.num*pm2 + nd.den*pm1
  qm := nd.num*qm2 + nd.den*qm1

```



```

genApproximants(pm1,qm1,pm,qm,rst fr)

genApproximants(pm2,qm2,pm1,qm1,fr) ==
  empty? fr => repeating [pm1/qm1]
  concat(pm1/qm1,iGenApproximants(pm2,qm2,pm1,qm1,fr))

iGenConvergents(pm2,qm2,pm1,qm1,fr) == delay
  nd := frst fr
  pm := nd.num*pm2 + nd.den*pm1
  qm := nd.num*qm2 + nd.den*qm1
  genConvergents(pm1,qm1,pm,qm,rst fr)

genConvergents(pm2,qm2,pm1,qm1,fr) ==
  empty? fr => concat(pm1/qm1, empty())
  concat(pm1/qm1,iGenConvergents(pm2,qm2,pm1,qm1,fr))

genNumDen(m2,m1,fr) ==
  empty? fr => concat(m1,empty())
  concat(m1,delay genNumDen(m1,m2*frst(fr).num + m1*frst(fr).den,rst fr))

gen ==> genFromSequence
apx ==> approximants

c, d: %
a: R
q: Q
n: Integer

0 == (0$R) :: %
1 == (1$R) :: %

c + d == genFromSequence map((x,y) +-> x + y, apx c, apx d)
c - d == genFromSequence map((x,y) +-> x - y, apx c, rest apx d)
- c == genFromSequence map(x +-> - x, rest apx c)
c * d == genFromSequence map((x,y) +-> x * y, apx c, apx d)
a * d == genFromSequence map(x +-> a * x, apx d)
q * d == genFromSequence map(x +-> q * x, apx d)
n * d == genFromSequence map(x +-> n * x, apx d)
c / d == genFromSequence map((x,y) +-> x / y, apx c, rest apx d)
recip c == (c = 0 => "failed";
  genFromSequence map(x +-> 1/x, rest apx c))

showAll?: () -> Boolean
showAll?() ==
  NULL(_$streamsShowAll$Lisp)$Lisp => false
  true

zagRec(t:Rec):OUT == zag(t.num :: OUT,t.den :: OUT)

coerce(c:%): OUT ==

```

```

wh := c.value.whole
fr := c.value.fract
empty? fr => wh :: OUT
count : NonNegativeInteger := _$streamCount$Lisp
l : List OUT := empty()
for n in 1..count while not empty? fr repeat
  l := concat(zagRec frst fr,l)
  fr := rst fr
if showAll?() then
  for n in (count + 1).. while explicitEntries? fr repeat
    l := concat(zagRec frst fr,l)
    fr := rst fr
if not explicitlyEmpty? fr then l := concat("..." :: OUT,l)
l := reverse_! l
e := reduce("+",l)
zero? wh => e
(wh :: OUT) + e

```

---

— CONTFRAC.dotabb —

```

"CONTRFAC" [color="#88FF44",href="bookvol10.3.pdf#nameddest=CONTRFAC"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"CONTRFAC" -> "PFECAT"

```

---



## Chapter 5

# Chapter D

### 5.1 domain DBASE Database

— Database.input —

```
)set break resume
)sys rm -f Database.output
)spool Database.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Database
--R Database S where
--R   S: OrderedSet with
--R      ?? : (%,Symbol) -> String
--R      display : % -> Void
--R      fullDisplay : % -> Void  is a domain constructor
--R Abbreviation for Database is DBASE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for DBASE
--R
--R----- Operations -----
--R ?+? : (%,%) -> %              ?-? : (%,%) -> %
--R ?=? : (%,%) -> Boolean        coerce : List S -> %
--R coerce : % -> OutputForm      display : % -> Void
--R ?? : (%,QueryEquation) -> %   fullDisplay : % -> Void
--R hash : % -> SingleInteger      latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R ?? : (%,Symbol) -> DataList String
--R fullDisplay : (%,PositiveInteger,PositiveInteger) -> Void
```

```
--R
--E 1

)spool
)lisp (bye)
```

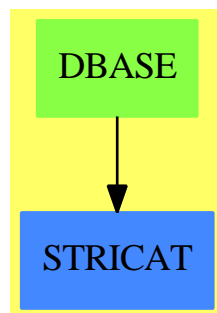
---

— Database.help —

```
=====
Database examples
=====
```

```
See Also:
o )show Database
```

### 5.1.1 Database (DBASE)



See

⇒ “DataList” (DLIST) 5.2.1 on page 445  
 ⇒ “IndexCard” (ICARD) 10.2.1 on page 1159  
 ⇒ “QueryEquation” (QEQUAT) 18.4.1 on page 2129

**Exports:**

```
coerce  display  fullDisplay  hash  latex
?+?    ?-?      ?=?         ?~=?  ??
```

---

— domain DBASE Database —

```
)abbrev domain DBASE Database
++ Author: Mark Botch
++ Description:
```

```

++ This domain implements a simple view of a database whose fields are
++ indexed by symbols

Database(S): Exports == Implementation where
  S: OrderedSet with
    elt: (%,Symbol) -> String
      ++ elt(x,s) returns an element of x indexed by s
    display: % -> Void
      ++ display(x) displays x in some form
    fullDisplay: % -> Void
      ++ fullDisplay(x) displays x in detail
  Exports == SetCategory with
    elt: (%,QueryEquation) -> %
      ++ elt(db,q) returns all elements of \axiom{db} which satisfy \axiom{q}.
    elt: (%,Symbol) -> DataList String
      ++ elt(db,s) returns the \axiom{s} field of each element of \axiom{db}.
    _+: (%,%) -> %
      ++ db1+db2 returns the merge of databases db1 and db2
    _-: (%,%) -> %
      ++ db1-db2 returns the difference of databases db1 and db2 i.e. consisting
      ++ of elements in db1 but not in db2
    coerce: List S -> %
      ++ coerce(l) makes a database out of a list
    display: % -> Void
      ++ display(db) prints a summary line for each entry in \axiom{db}.
    fullDisplay: % -> Void
      ++ fullDisplay(db) prints full details of each entry in \axiom{db}.
    fullDisplay: (%,PositiveInteger,PositiveInteger) -> Void
      ++ fullDisplay(db,start,end ) prints full details of entries in the range
      ++ \axiom{start..end} in \axiom{db}.
  Implementation == List S add
    s: Symbol
    Rep := List S
    coerce(u: List S):% == u@%
    elt(data: %,s: Symbol) == [x.s for x in data] :: DataList(String)
    elt(data: %,eq: QueryEquation) ==
      field := variable eq
      val := value eq
      [x for x in data | stringMatches?(val,x.field)$Lisp]
    x+y==removeDuplicates_! merge(x,y)
    x-y==mergeDifference(copy(x::Rep),y::Rep)$MergeThing(S)
    coerce(data): OutputForm == (#data):: OutputForm
    display(data) == for x in data repeat display x
    fullDisplay(data) == for x in data repeat fullDisplay x
    fullDisplay(data,n,m) == for x in data for i in 1..m repeat
      if i >= n then fullDisplay x

```

---

— DBASE.dotabb —

```
"DBASE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=DBASE"]
"STRICAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=STRICAT"]
"DBASE" -> "STRICAT"
```

— —

## 5.2 domain DLIST DataList

— DataList.input —

```
)set break resume
)sys rm -f DataList.output
)spool DataList.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show DataList
--R DataList S: OrderedSet is a domain constructor
--R Abbreviation for DataList is DLIST
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for DLIST
--R
--R----- Operations -----
--R children : % -> List %
--R coerce : List S -> %
--R concat : List % -> %
--R concat : (%,% ) -> %
--R concat! : (%,% ) -> %
--R copy : % -> %
--R cycleTail : % -> %
--R datalist : List S -> %
--R delete! : (%,Integer) -> %
--R ?.sort : (% ,sort) -> %
--R elt : (% ,Integer,S) -> S
--R ?.last : (% ,last) -> S
--R ?.first : (% ,first) -> S
--R empty : () -> %
--R entries : % -> List S
--R explicitlyFinite? : % -> Boolean
--R index? : (Integer,% ) -> Boolean
--R insert : (S,% ,Integer) -> %
--R coerce : % -> List S
--R concat : (% ,S) -> %
--R concat : (S,% ) -> %
--R concat! : (% ,S) -> %
--R construct : List S -> %
--R cycleEntry : % -> %
--R cyclic? : % -> Boolean
--R delete : (% ,Integer) -> %
--R distance : (% ,%) -> Integer
--R ?.unique : (% ,unique) -> %
--R ?.? : (% ,Integer) -> S
--R ?.rest : (% ,rest) -> %
--R ?.value : (% ,value) -> S
--R empty? : % -> Boolean
--R eq? : (% ,%) -> Boolean
--R first : % -> S
--R indices : % -> List Integer
--R insert : (% ,% ,Integer) -> %
```

```

--R insert! : (S,%,Integer) -> %          insert! : (%,%,Integer) -> %
--R last : % -> S                          leaf? : % -> Boolean
--R leaves : % -> List S                    list : S -> %
--R map : ((S,S) -> S),%,%) -> %           map : ((S -> S),%) -> %
--R new : (NonNegativeInteger,S) -> %       nodes : % -> List %
--R possiblyInfinite? : % -> Boolean        qelt : (%,Integer) -> S
--R rest : % -> %                          reverse : % -> %
--R sample : () -> %                       second : % -> S
--R tail : % -> %                          third : % -> S
--R value : % -> S
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?<? : (%,% ) -> Boolean if S has ORDSET
--R ?<=? : (%,% ) -> Boolean if S has ORDSET
--R ?=? : (%,% ) -> Boolean if S has SETCAT
--R ??? : (%,% ) -> Boolean if S has ORDSET
--R ?>? : (%,% ) -> Boolean if S has ORDSET
--R ?>=? : (%,% ) -> Boolean if S has ORDSET
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R child? : (%,% ) -> Boolean if S has SETCAT
--R coerce : % -> OutputForm if S has SETCAT
--R convert : % -> InputForm if S has KONVERT INFORM
--R copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
--R count : (S,% ) -> NonNegativeInteger if $ has finiteAggregate and S has SETCAT
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R cycleLength : % -> NonNegativeInteger
--R cycleSplit! : % -> % if $ has shallowlyMutable
--R delete : (% ,UniversalSegment Integer) -> %
--R delete! : (% ,UniversalSegment Integer) -> %
--R ?.count : (% ,count) -> NonNegativeInteger
--R ?.? : (% ,UniversalSegment Integer) -> %
--R entry? : (S,% ) -> Boolean if $ has finiteAggregate and S has SETCAT
--R eval : (% ,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (% ,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (% ,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (% ,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (% ,S) -> % if $ has shallowlyMutable
--R find : ((S -> Boolean),%) -> Union(S,"failed")
--R first : (% ,NonNegativeInteger) -> %
--R hash : % -> SingleInteger if S has SETCAT
--R last : (% ,NonNegativeInteger) -> %
--R latex : % -> String if S has SETCAT
--R less? : (% ,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R max : (%,% ) -> % if S has ORDSET
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (S,% ) -> Boolean if $ has finiteAggregate and S has SETCAT
--R members : % -> List S if $ has finiteAggregate
--R merge : (((S,S) -> Boolean),%,%) -> %
--R merge : (%,% ) -> % if S has ORDSET
--R merge! : (((S,S) -> Boolean),%,%) -> %

```



```

--R merge! : (%,% ) -> % if S has ORDSET
--R min : (%,% ) -> % if S has ORDSET
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (% ,NonNegativeInteger) -> Boolean
--R node? : (%,% ) -> Boolean if S has SETCAT
--R parts : % -> List S if $ has finiteAggregate
--R position : ((S -> Boolean),%) -> Integer
--R position : (S,% ) -> Integer if S has SETCAT
--R position : (S,% ,Integer) -> Integer if S has SETCAT
--R qsetelt! : (% ,Integer,S) -> S if $ has shallowlyMutable
--R reduce : (((S,S) -> S),% ,S,S) -> S if $ has finiteAggregate and S has SETCAT
--R reduce : (((S,S) -> S),% ,S) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
--R remove : (S,% ) -> % if $ has finiteAggregate and S has SETCAT
--R remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove! : ((S -> Boolean),%) -> %
--R remove! : (S,% ) -> % if S has SETCAT
--R removeDuplicates : % -> % if $ has finiteAggregate and S has SETCAT
--R removeDuplicates! : % -> % if S has SETCAT
--R rest : (% ,NonNegativeInteger) -> %
--R reverse! : % -> % if $ has shallowlyMutable
--R select : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R select! : ((S -> Boolean),%) -> %
--R setchildren! : (% ,List %) -> % if $ has shallowlyMutable
--R setelt : (% ,Integer,S) -> S if $ has shallowlyMutable
--R setelt : (% ,UniversalSegment Integer,S) -> S if $ has shallowlyMutable
--R setelt : (% ,last,S) -> S if $ has shallowlyMutable
--R setelt : (% ,rest,% ) -> % if $ has shallowlyMutable
--R setelt : (% ,first,S) -> S if $ has shallowlyMutable
--R setelt : (% ,value,S) -> S if $ has shallowlyMutable
--R setfirst! : (% ,S) -> S if $ has shallowlyMutable
--R setlast! : (% ,S) -> S if $ has shallowlyMutable
--R setrest! : (% ,%) -> % if $ has shallowlyMutable
--R setvalue! : (% ,S) -> S if $ has shallowlyMutable
--R size? : (% ,NonNegativeInteger) -> Boolean
--R sort : (((S,S) -> Boolean),%) -> %
--R sort : % -> % if S has ORDSET
--R sort! : (((S,S) -> Boolean),%) -> % if $ has shallowlyMutable
--R sort! : % -> % if $ has shallowlyMutable and S has ORDSET
--R sorted? : (((S,S) -> Boolean),%) -> Boolean
--R sorted? : % -> Boolean if S has ORDSET
--R split! : (% ,Integer) -> % if $ has shallowlyMutable
--R swap! : (% ,Integer,Integer) -> Void if $ has shallowlyMutable
--R ~=? : (% ,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)

```

---

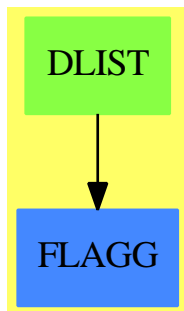
— DataList.help —

```
=====
DataList examples
=====
```

See Also:

- o )show DataList

### 5.2.1 DataList (DLIST)



See

- ⇒ “IndexCard” (ICARD) 10.2.1 on page 1159
- ⇒ “Database” (DBASE) 5.1.1 on page 440
- ⇒ “QueryEquation” (QEQUAT) 18.4.1 on page 2129

**Exports:**

|                   |                  |                   |                   |
|-------------------|------------------|-------------------|-------------------|
| any?              | child?           | children          | coerce            |
| concat            | concat!          | construct         | convert           |
| copy              | copyInto!        | count             | cycleEntry        |
| cycleLength       | cycleSplit!      | cycleTail         | cyclic?           |
| datalist          | delete           | delete!           | distance          |
| elt               | empty            | empty?            | entries           |
| entry?            | eq?              | eval              | every?            |
| explicitlyFinite? | fill!            | find              | first             |
| hash              | index?           | indices           | insert            |
| insert!           | last             | latex             | leaf?             |
| leaves            | less?            | list              | map               |
| map!              | max              | maxIndex          | member?           |
| members           | merge            | merge!            | min               |
| minIndex          | more?            | new               | node?             |
| nodes             | parts            | position          | possiblyInfinite? |
| qelt              | qsetelt!         | reduce            | remove            |
| remove!           | removeDuplicates | removeDuplicates! | rest              |
| reverse           | reverse!         | sample            | second            |
| select            | select!          | setchildren!      | setelt            |
| setfirst!         | setlast!         | setrest!          | setvalue!         |
| size?             | sort             | sort!             | sorted?           |
| split!            | swap!            | tail              | third             |
| value             | #?               | ?<?               | ?<=?              |
| ?=?               | ?>?              | ?>=?              | ??                |
| ?~=?              | ?.count          | ?.sort            | ?.unique          |
| ?.last            | ?.rest           | ?.first           | ?.value           |

— domain DLIST DataList —

```

)abbrev domain DLIST DataList
++ Author: Mark Botch
++ Description:
++ This domain provides some nice functions on lists

DataList(S:OrderedSet) : Exports == Implementation where
  Exports == ListAggregate(S) with
    coerce: List S -> %
      ++ coerce(l) creates a datalist from l
    coerce: % -> List S
      ++ coerce(x) returns the list of elements in x
    datalist: List S -> %
      ++ datalist(l) creates a datalist from l
    elt: (%, "unique") -> %
      ++ \axiom{l.unique} returns \axiom{l} with duplicates removed.
      ++ Note: \axiom{l.unique} = removeDuplicates(l)}.
    elt: (%, "sort") -> %
      ++ \axiom{l.sort} returns \axiom{l} with elements sorted.

```

```

++ Note: \axiom{l.sort = sort(l)}
elt: (%,"count") -> NonNegativeInteger
++ \axiom{l."count"} returns the number of elements in \axiom{l}.
Implementation == List(S) add
elt(x,"unique") == removeDuplicates(x)
elt(x,"sort") == sort(x)
elt(x,"count") == #x
coerce(x:List S) == x pretend %
coerce(x:%):List S == x pretend (List S)
coerce(x:%): OutputForm == (x :: List S) :: OutputForm
datalist(x:List S) == x::%

```

---

— DLIST.dotabb —

```

"DLIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=DLIST"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"DLIST" -> "FLAGG"

```

---

### 5.3 domain DECIMAL DecimalExpansion

---

— DecimalExpansion.input —

```

)set break resume
)sys rm -f DecimalExpansion.output
)spool DecimalExpansion.output
)set message test on
)set message auto off
)clear all
--S 1 of 7
r := decimal(22/7)
--R
--R
--R
--R (1) 3.142857
--R
--R                                          Type: DecimalExpansion
--E 1

--S 2 of 7
r + decimal(6/7)
--R

```

```

--R
--R (2) 4
--R
--R                                         Type: DecimalExpansion
--E 2

--S 3 of 7
[decimal(1/i) for i in 350..354]
--R
--R
--R (3)
--R
--R [0.00285714, 0.002849, 0.0028409, 0.00283286118980169971671388101983,
--R
--R 0.00282485875706214689265536723163841807909604519774011299435]
--R
--R                                         Type: List DecimalExpansion
--E 3

--S 4 of 7
decimal(1/2049)
--R
--R
--R (4)
--R 0.
--R
--R OVERBAR
--R 00048804294777940458760370912640312347486578818936066373840897999023914
--R 104441190824792581747193753050268423621278672523182040019521717911176
--R 183504148365056124938994631527574426549536359199609565641776476329917
--R 032698877501220107369448511469009272816007808687164470473401659346022
--R 449975597852611029770619814543679843826256710590531966813079551
--R
--R                                         Type: DecimalExpansion
--E 4

--S 5 of 7
p := decimal(1/4)*x**2 + decimal(2/3)*x + decimal(4/9)
--R
--R
--R
--R      2      -      -
--R (5) 0.25x  + 0.6x + 0.4
--R
--R                                         Type: Polynomial DecimalExpansion
--E 5

--S 6 of 7
q := differentiate(p, x)
--R
--R
--R
--R      -
--R (6) 0.5x + 0.6
--R
--R                                         Type: Polynomial DecimalExpansion
--E 6

```

```

--S 7 of 7
g := gcd(p, q)
--R
--R
--R
--R      (7)  x + 1.3
--R
--R                                          Type: Polynomial DecimalExpansion
--E 7
)spool
)lisp (bye)

```

— DecimalExpansion.help —

=====

DecimalExpansion examples

=====

All rationals have repeating decimal expansions. Operations to access the individual digits of a decimal expansion can be obtained by converting the value to RadixExpansion(10).

The operation decimal is used to create this expansion of type DecimalExpansion.

```

r := decimal(22/7)
-----
3.142857
Type: DecimalExpansion

```

Arithmetic is exact.

```

r + decimal(6/7)
4
Type: DecimalExpansion

```

The period of the expansion can be short or long ...

```

[decimal(1/i) for i in 350..354]
-----
[0.00285714, 0.002849, 0.0028409, 0.00283286118980169971671388101983,
-----
0.00282485875706214689265536723163841807909604519774011299435]
Type: List DecimalExpansion

```

or very long.

```

decimal(1/2049)

```

```

-----
0.00048804294777940458760370912640312347486578818936066373840897999023914
-----
104441190824792581747193753050268423621278672523182040019521717911176
-----
183504148365056124938994631527574426549536359199609565641776476329917
-----
032698877501220107369448511469009272816007808687164470473401659346022
-----
449975597852611029770619814543679843826256710590531966813079551
Type: DecimalExpansion

```

These numbers are bona fide algebraic objects.

```

p := decimal(1/4)*x**2 + decimal(2/3)*x + decimal(4/9)
      2      -      -
0.25x  + 0.6x + 0.4
Type: Polynomial DecimalExpansion

q := differentiate(p, x)
      -
0.5x + 0.6
Type: Polynomial DecimalExpansion

g := gcd(p, q)
      -
x + 1.3
Type: Polynomial DecimalExpansion

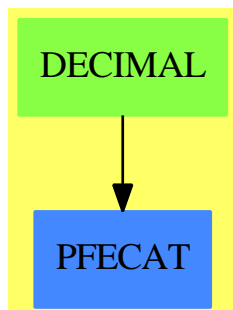
```

See Also:

- o )help RadixExpansion
- o )help BinaryExpansion
- o )help HexadecimalExpansion
- o )show DecimalExpansion

---

## 5.3.1 DecimalExpansion (DECIMAL)



See

⇒ “RadixExpansion” (RADIX) 19.2.1 on page 2165

⇒ “BinaryExpansion” (BINARY) 3.7.1 on page 274

⇒ “HexadecimalExpansion” (HEXADEC) 9.3.1 on page 1108

**Exports:**

|                               |                  |                            |
|-------------------------------|------------------|----------------------------|
| 0                             | 1                | abs                        |
| associates?                   | ceiling          | characteristic             |
| charthRoot                    | coerce           | conditionP                 |
| convert                       | D                | decimal                    |
| denom                         | denominator      | differentiate              |
| divide                        | euclideanSize    | eval                       |
| expressIdealMember            | exquo            | extendedEuclidean          |
| factor                        | factorPolynomial | factorSquareFreePolynomial |
| floor                         | fractionPart     | gcd                        |
| gcdPolynomial                 | hash             | init                       |
| inv                           | latex            | lcm                        |
| map                           | max              | min                        |
| multiEuclidean                | negative?        | nextItem                   |
| numer                         | numerator        | one?                       |
| patternMatch                  | positive?        | prime?                     |
| principalIdeal                | random           | recip                      |
| reducedSystem                 | retract          | retractIfCan               |
| sample                        | sign             | sizeLess?                  |
| solveLinearPolynomialEquation | squareFree       | squareFreePart             |
| squareFreePolynomial          | subtractIfCan    | unit?                      |
| unitCanonical                 | unitNormal       | wholePart                  |
| zero?                         | ?*?              | ?**?                       |
| ?+?                           | ?-?              | -?                         |
| ?/?                           | ?=?              | ?^?                        |
| ?~=?                          | ?<?              | ?<=?                       |
| ?>?                           | ?>=?             | ?..?                       |
| ?quo?                         | ?rem?            |                            |



## — domain DECIMAL DecimalExpansion —

```

)abbrev domain DECIMAL DecimalExpansion
++ Author: Stephen M. Watt
++ Date Created: October, 1986
++ Date Last Updated: May 15, 1991
++ Basic Operations:
++ Related Domains: RadixExpansion
++ Also See:
++ AMS Classifications:
++ Keywords: radix, base, repeating decimal
++ Examples:
++ References:
++ Description:
++ This domain allows rational numbers to be presented as repeating
++ decimal expansions.

DecimalExpansion(): Exports == Implementation where
  Exports ==> QuotientFieldCategory(Integer) with
    coerce: % -> Fraction Integer
      ++ coerce(d) converts a decimal expansion to a rational number.
    coerce: % -> RadixExpansion(10)
      ++ coerce(d) converts a decimal expansion to a radix expansion
      ++ with base 10.
    fractionPart: % -> Fraction Integer
      ++ fractionPart(d) returns the fractional part of a decimal expansion.
    decimal: Fraction Integer -> %
      ++ decimal(r) converts a rational number to a decimal expansion.

Implementation ==> RadixExpansion(10) add
  decimal r == r :: %
  coerce(x:%): RadixExpansion(10) == x pretend RadixExpansion(10)

```

## — DECIMAL.dotabb —

```

"DECIMAL" [color="#88FF44",href="bookvol10.3.pdf#nameddest=DECIMAL"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"DECIMAL" -> "PFECAT"

```

## 5.4 Denavit-Hartenberg Matrices

### 5.4.1 Homogeneous Transformations

The study of robot manipulation is concerned with the relationship between objects, and between objects and manipulators. In this chapter we will develop the representation necessary to describe these relationships. Similar problems of representation have already been solved in the field of computer graphics, where the relationship between objects must also be described. Homogeneous transformations are used in this field and in computer vision [Duda] [Roberts63] [Roberts65]. These transformations were employed by Denavit to describe linkages [Denavit] and are now used to describe manipulators [Pieper] [Paul72] [Paul77b].

We will first establish notation for vectors and planes and then introduce transformations on them. These transformations consist primarily of translation and rotation. We will then show that these transformations can also be considered as coordinate frames in which to represent objects, including the manipulator. The inverse transformation will then be introduced. A later section describes the general rotation transformation representing a rotation about a vector. An algorithm is then described to find the equivalent axis and angle of rotations represented by any given transformation. A brief section on stretching and scaling transforms is included together with a section on the perspective transformation. The chapter concludes with a section on transformation equations.

### 5.4.2 Notation

In describing the relationship between objects we will make use of point vectors, planes, and coordinate frames. Point vectors are denoted by lower case, bold face characters. Planes are denoted by script characters, and coordinate frames by upper case, bold face characters. For example:

|                   |   |
|-------------------|---|
| vectors           | $\mathbf{v}, \mathbf{x1}, \mathbf{x}$   |
| planes            | $\mathcal{P}, \mathcal{Q}$              |
| coordinate frames | $\mathbf{I}, \mathbf{A}, \mathbf{CONV}$ |

We will use point vectors, planes, and coordinate frames as variables which have associated values. For example, a point vector has as value its three Cartesian coordinate components.

If we wish to describe a point in space, which we will call  $p$ , with respect to a coordinate frame  $\mathbf{E}$ , we will use a vector which we will call  $\mathbf{v}$ . We will write this as

$${}^E\mathbf{v}$$

The leading superscript describes the defining coordinate frame.

We might also wish to describe this same point,  $p$ , with respect to a different coordinate frame, for example  $\mathbf{H}$ , using a vector  $\mathbf{w}$  as

$${}^H\mathbf{w}$$

$\mathbf{v}$  and  $\mathbf{w}$  are two vectors which probably have different component values and  $\mathbf{v} \neq \mathbf{w}$  even though both vectors describe the same point  $p$ . The case might also exist of a vector  $\mathbf{a}$  describing a point 3 inches above any frame

$${}^{F^1}\mathbf{a} \quad {}^{F^2}\mathbf{a}$$

In this case the vectors are identical but describe different points. Frequently, the defining frame will be obvious from the text and the superscripts will be left off. In many cases the name of the vector will be the same as the name of the object described, for example, the tip of a pin might be described by a vector **tip** with respect to a frame **BASE** as

$${}^{BASE}\mathbf{tip}$$

If it were obvious from the text that we were describing the vector with respect to **BASE** then we might simply write

$$\mathbf{tip}$$

If we also wish to describe this point with respect to another coordinate frame say, **HAND**, then we must use another vector to describe this relationship, for example

$${}^{HAND}\mathbf{tv}$$

${}^{HAND}\mathbf{tv}$  and **tip** both describe the same feature but have different values. In order to refer to individual components of coordinate frames, point vectors, or planes, we add subscripts to indicate the particular component. For example, the vector  ${}^{HAND}\mathbf{tv}$  has components  ${}^{HAND}\mathbf{tv}_x$ ,  ${}^{HAND}\mathbf{tv}_y$ ,  ${}^{HAND}\mathbf{tv}_z$ .

### 5.4.3 Vectors

The homogeneous coordinate representation of objects in  $n$ -space is an  $(n + 1)$ -space entity such that a particular perspective projection recreates the  $n$ -space. This can also be viewed as the addition of an extra coordinate to each vector, a scale factor, such that the vector has the same meaning if each component, including the scale factor, is multiplied by a constant.

A point vector

$$\mathbf{v} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k} \tag{1.1}$$

where  $\mathbf{i}$ ,  $\mathbf{j}$ , and  $\mathbf{k}$  are unit vectors along the  $x$ ,  $y$ , and  $z$  coordinate axes, respectively, is represented in homogeneous coordinates as a column matrix

$$\mathbf{v} = \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \\ \mathbf{z} \\ \mathbf{w} \end{bmatrix} \tag{1.2}$$

where

$$\begin{aligned}\mathbf{a} &= \mathbf{x}/\mathbf{w} \\ \mathbf{b} &= \mathbf{y}/\mathbf{w} \\ \mathbf{c} &= \mathbf{z}/\mathbf{w}\end{aligned}\tag{1.3}$$

Thus the vector  $3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$  can be represented as  $[3, 4, 5, 1]^T$  or as  $[6, 8, 10, 2]^T$  or again as  $[-30, -40, -50, -10]^T$ , etc. The superscript  $T$  indicates the transpose of the row vector into a column vector. The vector at the origin, the null vector, is represented as  $[0, 0, 0, n]^T$  where  $n$  is any non-zero scale factor. The vector  $[0, 0, 0, 0]^T$  is undefined. Vectors of the form  $[a, b, c, 0]^T$  represent vectors at infinity and are used to represent directions; the addition of any other finite vector does not change their value in any way.

We will also make use of the vector dot and cross products. Given two vectors

$$\begin{aligned}\mathbf{a} &= a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k} \\ \mathbf{b} &= b_x\mathbf{i} + b_y\mathbf{j} + b_z\mathbf{k}\end{aligned}\tag{1.4}$$

we define the vector dot product, indicated by “ $\cdot$ ” as

$$\mathbf{a} \cdot \mathbf{b} = a_x b_x + a_y b_y + a_z b_z\tag{1.5}$$

The dot product of two vectors is a scalar. The cross product, indicated by an “ $\times$ ”, is another vector perpendicular to the plane formed by the vectors of the product and is defined by

$$\mathbf{a} \times \mathbf{b} = (a_y b_z - a_z b_y)\mathbf{i} + (a_z b_x - a_x b_z)\mathbf{j} + (a_x b_y - a_y b_x)\mathbf{k}\tag{1.6}$$

This definition is easily remembered as the expansion of the determinant

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}\tag{1.7}$$

#### 5.4.4 Planes

A plane is represented as a row matrix

$$\mathcal{P} = [a, b, c, d]\tag{1.8}$$

such that if a point  $\mathbf{v}$  lies in a plane  $\mathcal{P}$  the matrix product

$$\mathcal{P} \mathbf{v} = 0\tag{1.9}$$

or in expanded form

$$xa + yb + zc + wd = 0 \quad (1.10)$$

If we define a constant

$$m = +\sqrt{a^2 + b^2 + c^2} \quad (1.11)$$

and divide Equation 1.10 by  $wm$  we obtain

$$\frac{x}{w} \frac{a}{m} + \frac{y}{w} \frac{b}{m} + \frac{z}{w} \frac{c}{m} = -\frac{d}{m} \quad (1.12)$$

The left hand side of Equation 1.12 is the vector dot product of two vectors  $(x/w)\mathbf{i} + (y/w)\mathbf{j} + (z/w)\mathbf{k}$  and  $(a/m)\mathbf{i} + (b/m)\mathbf{j} + (c/m)\mathbf{k}$  and represents the directed distance of the point  $(x/w)\mathbf{i} + (y/w)\mathbf{j} + (z/w)\mathbf{k}$  along the vector  $(a/m)\mathbf{i} + (b/m)\mathbf{j} + (c/m)\mathbf{k}$ . The vector  $(a/m)\mathbf{i} + (b/m)\mathbf{j} + (c/m)\mathbf{k}$  can be interpreted as the outward pointing normal of a plane situated a distance  $-d/m$  from the origin in the direction of the normal. Thus a plane  $\mathcal{P}$  parallel to the  $x,y$  plane, one unit along the  $z$  axis, is represented as

$$\mathcal{P} = [0, 0, 1, -1] \quad (1.13)$$

$$\text{or as } \mathcal{P} = [0, 0, 2, -2] \quad (1.14)$$

$$\text{or as } \mathcal{P} = [0, 0, -100, 100] \quad (1.15)$$

A point  $\mathbf{v} = [10, 20, 1, 1]$  should lie in this plane

$$[0, 0, -100, 100] \begin{bmatrix} 10 \\ 20 \\ 1 \\ 1 \end{bmatrix} = 0 \quad (1.16)$$

or

$$[0, 0, 1, -1] \begin{bmatrix} -5 \\ -10 \\ -.5 \\ -.5 \end{bmatrix} = 0 \quad (1.17)$$

The point  $\mathbf{v} = [0, 0, 2, 1]$  lies above the plane

$$[0, 0, 2, -2] \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix} = 2 \quad (1.18)$$

and  $\mathcal{P} \mathbf{v}$  is indeed positive, indicating that the point is outside the plane in the direction of the outward pointing normal. A point  $\mathbf{v} = [0, 0, 0, 1]$  lies below the plane

$$[0, 0, 1, -1] \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = -1 \quad (1.19)$$

The plane  $[0, 0, 0, 0]$  is undefined.

### 5.4.5 Transformations

A transformation of the space  $\mathbf{H}$  is a 4x4 matrix and can represent translation, rotation, stretching, and perspective transformations. Given a point  $\mathbf{u}$ , its transformation  $\mathbf{v}$  is represented by the matrix product

$$\mathbf{v} = \mathbf{H}\mathbf{u} \quad (1.20)$$

The corresponding plane transformation  $\mathcal{P}$  to  $\mathcal{Q}$  is

$$\mathcal{Q} = \mathcal{P} \mathbf{H}^{-1} \quad (1.21)$$

as we require that the condition

$$\mathcal{Q} \mathbf{v} = \mathcal{P} \mathbf{u} \quad (1.22)$$

is invariant under all transformations. To verify this we substitute from Equations 1.20 and 1.21 into the left hand side of 1.22 and we obtain on the right hand side  $\mathbf{H}^{-1}\mathbf{H}$  which is the identity matrix  $\mathbf{I}$

$$\mathcal{P} \mathbf{H}^{-1} \mathbf{H} \mathbf{u} = \mathcal{P} \mathbf{u} \quad (1.23)$$

### 5.4.6 Translation Transformation

The transformation  $\mathbf{H}$  corresponding to a translation by a vector  $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$  is

$$\mathbf{H} = \text{Trans}(\mathbf{a}, \mathbf{b}, \mathbf{c}) = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1.24)$$

Given a vector  $\mathbf{u} = [x, y, z, w]^T$  the transformed vector  $\mathbf{v}$  is given by

$$\mathbf{H} = \mathbf{Trans}(\mathbf{a}, \mathbf{b}, \mathbf{c}) = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \quad (1.25)$$

$$\mathbf{v} = \begin{bmatrix} x + aw \\ y + bw \\ z + cw \\ w \end{bmatrix} = \begin{bmatrix} x/w + a \\ y/w + b \\ z/w + c \\ 1 \end{bmatrix} \quad (1.26)$$

The translation may also be interpreted as the addition of the two vectors  $(x/w)\mathbf{i} + (y/w)\mathbf{j} + (z/w)\mathbf{k}$  and  $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ .

Every element of a transformation matrix may be multiplied by a non-zero constant without changing the transformation, in the same manner as points and planes. Consider the vector  $2\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$  translated by, or added to  $4\mathbf{i} - 3\mathbf{j} + 7\mathbf{k}$

$$\begin{bmatrix} 6 \\ 0 \\ 9 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 2 \\ 1 \end{bmatrix} \quad (1.27)$$

If we multiply the transformation matrix elements by, say, -5, and the vector elements by 2, we obtain

$$\begin{bmatrix} -60 \\ 0 \\ -90 \\ -10 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 0 & -20 \\ 0 & -5 & 0 & 15 \\ 0 & 0 & -5 & -35 \\ 0 & 0 & 0 & -5 \end{bmatrix} \begin{bmatrix} 4 \\ 6 \\ 4 \\ 2 \end{bmatrix} \quad (1.28)$$

which corresponds to the vector  $[6, 0, 9, 1]^T$  as before. The point  $[2, 3, 2, 1]$  lies in the plane  $[1, 0, 0, -2]$

$$[1, 0, 0, -2] \begin{bmatrix} 2 \\ 3 \\ 2 \\ 1 \end{bmatrix} = 0 \quad (1.29)$$

The transformed point is, as we have already found,  $[6, 0, 9, 1]^T$ . We will now compute the transformed plane. The inverse of the transform is

$$\begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and the transformed plane

$$[1 \ 0 \ 0 \ -6] = [1 \ 0 \ 0 \ -2] \begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1.30)$$

Once again the transformed point lies in the transformed plane

$$[1 \ 0 \ 0 \ -6] \begin{bmatrix} 6 \\ 0 \\ 9 \\ 1 \end{bmatrix} = 0 \quad (1.31)$$

The general translation operation can be represented in Axiom as

— **translate** —

```

translate(x,y,z) ==
matrix(
  [[1,0,0,x],_
   [0,1,0,y],_
   [0,0,1,z],_
   [0,0,0,1]])

```

---

#### 5.4.7 Rotation Transformations

The transformations corresponding to rotations about the  $x$ ,  $y$ , and  $z$  axes by an angle  $\theta$  are

$$\mathbf{Rot}(\mathbf{x}, \theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta & 0 \\ 0 & \sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1.32)$$

Rotations can be described in Axiom as functions that return matrices. We can define a function for each of the rotation matrices that correspond to the rotations about each axis. Note that the sine and cosine functions in Axiom expect their argument to be in radians rather than degrees. This conversion is

$$radians = \frac{degrees * \pi}{180}$$

The Axiom code for **Rot**(**x**, **degree**) is

— **rotatex** —



```

rotatex(degree) ==
angle := degree * pi() / 180::R
cosAngle := cos(angle)
sinAngle := sin(angle)
matrix(_
[[1, 0, 0, 0], _
[0, cosAngle, -sinAngle, 0], _
[0, sinAngle, cosAngle, 0], _
[0, 0, 0, 1]])

```

---

$$\mathbf{Rot}(y, \theta) = \begin{bmatrix} \cos \theta & 0 & \sin \theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \theta & 0 & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1.33)$$

The Axiom code for  $\mathbf{Rot}(y, \text{degree})$  is

— rotatey —

```

rotatey(degree) ==
angle := degree * pi() / 180::R
cosAngle := cos(angle)
sinAngle := sin(angle)
matrix(_
[[cosAngle, 0, sinAngle, 0], _
[0, 1, 0, 0], _
[-sinAngle, 0, cosAngle, 0], _
[0, 0, 0, 1]])

```

---

$$\mathbf{Rot}(z, \theta) = \begin{bmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1.34)$$

And the Axiom code for  $\mathbf{Rot}(z, \text{degree})$  is

— rotatez —

```

rotatez(degree) ==
angle := degree * pi() / 180::R
cosAngle := cos(angle)
sinAngle := sin(angle)
matrix(_

```

```
[[cosAngle, -sinAngle, 0, 0], _
 [sinAngle,  cosAngle, 0, 0], _
 [ 0,          0,      1, 0], _
 [ 0,          0,      0, 1]])
```

---

Let us interpret these rotations by means of an example. Given a point  $\mathbf{u} = 7\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$  what is the effect of rotating it  $90^\circ$  about the  $\mathbf{z}$  axis to  $\mathbf{v}$ ? The transform is obtained from Equation 1.34 with  $\sin \theta = 1$  and  $\cos \theta = 0$ .

$$\begin{bmatrix} -3 \\ 7 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 7 \\ 3 \\ 2 \\ 1 \end{bmatrix} \quad (1.35)$$

Let us now rotate  $\mathbf{v}$   $90^\circ$  about the  $y$  axis to  $\mathbf{w}$ . The transform is obtained from Equation 1.33 and we have

$$\begin{bmatrix} 2 \\ 7 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -3 \\ 7 \\ 2 \\ 1 \end{bmatrix} \quad (1.36)$$

If we combine these two rotations we have

$$\mathbf{v} = \mathbf{Rot}(\mathbf{z}, 90)\mathbf{u} \quad (1.37)$$

$$\text{and } \mathbf{w} = \mathbf{Rot}(\mathbf{y}, 90)\mathbf{v} \quad (1.38)$$

Substituting for  $\mathbf{v}$  from Equation 1.37 into Equation 1.38 we obtain

$$\mathbf{w} = \mathbf{Rot}(\mathbf{y}, 90) \mathbf{Rot}(\mathbf{z}, 90) \mathbf{u} \quad (1.39)$$

$$\mathbf{Rot}(\mathbf{y}, 90) \mathbf{Rot}(\mathbf{z}, 90) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1.40)$$

$$\mathbf{Rot}(\mathbf{y}, 90) \mathbf{Rot}(\mathbf{z}, 90) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1.41)$$

thus

$$\mathbf{w} = \begin{bmatrix} 2 \\ 7 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 7 \\ 3 \\ 2 \\ 1 \end{bmatrix} \quad (1.42)$$

as we obtained before.

If we reverse the order of rotations and first rotate  $90^\circ$  about the  $y$  axis and then  $90^\circ$  about the  $z$  axis, we obtain a different position

$$\mathbf{Rot}(z, 90)\mathbf{Rot}(y, 90) = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1.43)$$

and the point  $\mathbf{u}$  transforms into  $\mathbf{w}$  as

$$\begin{bmatrix} -3 \\ 2 \\ -7 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 7 \\ 3 \\ 2 \\ 1 \end{bmatrix} \quad (1.44)$$

We should expect this, as matrix multiplication is noncommutative.

$$\mathbf{AB} \neq \mathbf{BA} \quad (1.45)$$

We will now combine the original rotation with a translation  $4\mathbf{i} - 3\mathbf{j} + 7\mathbf{k}$ . We obtain the translation from Equation 1.27 and the rotation from Equation 1.41. The matrix expression is

$$\mathbf{Trans}(4, -3, 7)\mathbf{Rot}(y, 90)\mathbf{Rot}(z, 90) = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 4 \\ 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1.46)$$

and our point  $\mathbf{w} = 7\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$  transforms into  $\mathbf{x}$  as

$$\begin{bmatrix} 6 \\ 4 \\ 10 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 4 \\ 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 7 \\ 3 \\ 2 \\ 1 \end{bmatrix} \quad (1.47)$$

### 5.4.8 Coordinate Frames

We can interpret the elements of the homogeneous transformation as four vectors describing a second coordinate frame. The vector  $[0, 0, 0, 1]^T$  lies at the origin of the second coordinate frame. Its transformation corresponds to the right hand column of the transformation matrix. Consider the transform in Equation 1.47

$$\begin{bmatrix} 4 \\ -3 \\ 7 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 4 \\ 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad (1.48)$$

The transform of the null vector is  $[4, -3, 7, 1]^T$ , the right hand column. If we transform vectors corresponding to unit vectors along the  $x$ ,  $y$ , and  $z$  axes, we obtain  $[4, -2, 7, 1]^T$ ,  $[4, -3, 8, 1]^T$ , and  $[5, -3, 7, 1]^T$ , respectively. Those four vectors form a coordinate frame.

The direction of these unit vectors is formed by subtracting the vector representing the origin of this coordinate frame and extending the vectors to infinity by reducing their scale factors to zero. The direction of the  $x$ ,  $y$ , and  $z$  axes of this frame are  $[0, 1, 0, 0]^T$ ,  $[0, 0, 1, 0]^T$ , and  $[1, 0, 0, 0]^T$ , respectively. These direction vectors correspond to the first three columns of the transformation matrix. The transformation matrix thus describes the three axis directions and the position of the origin of a coordinate frame rotated and translated away from the reference coordinate frame. When a vector is transformed, as in Equation 1.47, the original vector can be considered as a vector described in the coordinate frame. The transformed vector is the same vector described with respect to the reference coordinate frame.

### 5.4.9 Relative Transformations

The rotations and translations we have been describing have all been made with respect to the fixed reference coordinate frame. Thus, in the example given,

$$\mathbf{Trans}(4, -3, 7)\mathbf{Rot}(y, 90)\mathbf{Rot}(z, 90) = \begin{bmatrix} 0 & 0 & 1 & 4 \\ 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1.49)$$

the frame is first rotated around the reference  $z$  axis by  $90^\circ$ , then rotated  $90^\circ$  around the reference  $y$  axis, and finally translated by  $4\mathbf{i} - 3\mathbf{j} + 7\mathbf{k}$ . We may also interpret the operation in the reverse order, from left to right, as follows: the object is first translated by  $4\mathbf{i} - 3\mathbf{j} + 7\mathbf{k}$ ; it is then rotated  $90^\circ$  around the current frames axes, which in this case are the same as the reference axes; it is then rotated  $90^\circ$  about the newly rotated (current) frames axes.

In general, if we postmultiply a transform representing a frame by a second transformation describing a rotation and/or translation, we make that translation and/or rotation with respect to the frame axes described by the first transformation. If we premultiply the frame transformation by a transformation representing a translation and/or rotation, then that translation and/or rotation is made with respect to the base reference coordinate frame.

Thus, given a frame  $\mathbf{C}$  and a transformation  $\mathbf{T}$ , corresponding to a rotation of  $90^\circ$  about the  $z$  axis, and a translation of 10 units in the  $x$  direction, we obtain a new position  $\mathbf{X}$  when the change is made in the base coordinates  $\mathbf{X} = \mathbf{T}\mathbf{C}$

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 20 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 10 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 20 \\ 0 & 0 & -1 & 10 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1.50)$$

and a new position  $\mathbf{Y}$  when the change is made relative to the frame axes as  $\mathbf{Y} = \mathbf{C}\mathbf{T}$

$$\begin{bmatrix} 0 & -1 & 0 & 30 \\ 0 & 0 & -1 & 10 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 20 \\ 0 & 0 & -1 & 10 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 10 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1.51)$$

#### 5.4.10 Objects

Transformations are used to describe the position and orientation of objects. An object is described by six points with respect to a coordinate frame fixed in the object.

If we rotate the object  $90^\circ$  about the  $z$  axis and then  $90^\circ$  about the  $y$  axis, followed by a translation of four units in the  $x$  direction, we can describe the transformation as

$$\mathbf{Trans}(4, 0, 0)\mathbf{Rot}(y, 90)\mathbf{Rot}(z, 90) = \begin{bmatrix} 0 & 0 & 1 & 4 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1.52)$$

The transformation matrix represents the operation of rotation and translation on a coordinate frame originally aligned with the reference coordinate frame. We may transform the six points of the object as

$$\begin{bmatrix} 4 & 4 & 6 & 6 & 4 & 4 \\ 1 & -1 & -1 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 4 & 4 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 4 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & -1 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 4 & 4 \\ 0 & 0 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \quad (1.53)$$

It can be seen that the object described bears the same fixed relationship to its coordinate frame, whose position and orientation are described by the transformation. Given an object described by a reference coordinate frame, and a transformation representing the position and orientation of the object's axes, the object can be simply reconstructed, without the necessity of transforming all the points, by noting the direction and orientation of key features with respect to the describing frame's coordinate axes. By drawing the transformed coordinate frame, the object can be related to the new axis directions.

### 5.4.11 Inverse Transformations

We are now in a position to develop the inverse transformation as the transform which carries the transformed coordinate frame back to the original frame. This is simply the description of the reference coordinate frame with respect to the transformed frame. Suppose the direction of the reference frame  $x$  axis is  $[0, 0, 1, 0]^T$  with respect to the transformed frame. The  $y$  and  $z$  axes are  $[1, 0, 0, 0]^T$  and  $[0, 1, 0, 0]^T$ , respectively. The location of the origin is  $[0, 0, -4, 1]^T$  with respect to the transformed frame and thus the inverse transformation is

$$\mathbf{T}^{-1} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & -4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1.54)$$

That this is indeed the transform inverse is easily verified by multiplying it by the transform  $\mathbf{T}$  to obtain the identity transform

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & -4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 4 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1.55)$$

In general, given a transform with elements

$$\mathbf{T} = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1.56)$$

then the inverse is

$$\mathbf{T}^{-1} = \begin{bmatrix} n_x & n_y & n_z & -\mathbf{p} \cdot \mathbf{n} \\ o_x & o_y & o_z & -\mathbf{p} \cdot \mathbf{o} \\ a_x & a_y & a_z & -\mathbf{p} \cdot \mathbf{a} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1.57)$$

where  $\mathbf{p}$ ,  $\mathbf{n}$ ,  $\mathbf{o}$ , and  $\mathbf{a}$  are the four column vectors and “ $\cdot$ ” represents the vector dot product. This result is easily verified by postmultiplying Equation 1.56 by Equation 1.57.

### 5.4.12 General Rotation Transformation

We state the rotation transformations for rotations about the  $x$ ,  $y$ , and  $z$  axes (Equations 1.32, 1.33 and 1.34). These transformations have a simple geometric interpretation. For example, in the case of a rotation about the  $z$  axis, the column representing the  $z$  axis will remain constant, while the column elements representing the  $x$  and  $y$  axes will vary.

We will now develop the transformation matrix representing a rotation around an arbitrary vector  $\mathbf{k}$  located at the origin. In order to do this we will imagine that  $\mathbf{k}$  is the  $z$  axis unit vector of a coordinate frame  $\mathbf{C}$

$$\mathbf{C} = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1.58)$$

$$\mathbf{k} = a_x \mathbf{i} + a_y \mathbf{j} + a_z \mathbf{k} \quad (1.59)$$

Rotating around the vector  $\mathbf{k}$  is then equivalent to rotating around the  $z$  axis of the frame  $\mathbf{C}$ .

$$\mathbf{Rot}(\mathbf{k}, \theta) = \mathbf{Rot}(\mathbf{C}\mathbf{z}, \theta) \quad (1.60)$$

If we are given a frame  $\mathbf{T}$  described with respect to the reference coordinate frame, we can find a frame  $\mathbf{X}$  which describes the same frame with respect to frame  $\mathbf{C}$  as

$$\mathbf{T} = \mathbf{C}\mathbf{X} \quad (1.61)$$

where  $\mathbf{X}$  describes the position of  $\mathbf{T}$  with respect to frame  $\mathbf{C}$ . Solving for  $\mathbf{X}$  we obtain

$$\mathbf{X} = \mathbf{C}^{-1}\mathbf{T} \quad (1.62)$$

Rotation  $\mathbf{T}$  around  $\mathbf{k}$  is equivalent to rotating  $\mathbf{X}$  around the  $z$  axis of frame  $\mathbf{C}$

$$\mathbf{Rot}(\mathbf{k}, \theta)\mathbf{T} = \mathbf{C}\mathbf{Rot}(\mathbf{z}, \theta)\mathbf{X} \quad (1.63)$$

$$\mathbf{Rot}(\mathbf{k}, \theta)\mathbf{T} = \mathbf{C}\mathbf{Rot}(\mathbf{z}, \theta)\mathbf{C}^{-1}\mathbf{T}. \quad (1.64)$$

Thus

$$\mathbf{Rot}(\mathbf{k}, \theta) = \mathbf{C}\mathbf{Rot}(\mathbf{z}, \theta)\mathbf{C}^{-1} \quad (1.65)$$

However, we have only  $\mathbf{k}$ , the  $z$  axis of the frame  $\mathbf{C}$ . By expanding equation 1.65 we will discover that  $\mathbf{C}\mathbf{Rot}(\mathbf{z}, \theta)\mathbf{C}^{-1}$  is a function of  $\mathbf{k}$  only.

Multiplying  $\mathbf{Rot}(\mathbf{z}, \theta)$  on the right by  $\mathbf{C}^{-1}$  we obtain

$$\mathbf{Rot}(\mathbf{z}, \theta)\mathbf{C}^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n_x & n_y & n_z & 0 \\ o_x & o_x & o_z & 0 \\ a_x & a_y & a_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} n_x \cos \theta - o_x \sin \theta & n_y \cos \theta - o_y \sin \theta & n_z \cos \theta - o_z \sin \theta & 0 \\ n_x \sin \theta + o_x \cos \theta & n_y \sin \theta + o_y \cos \theta & n_z \sin \theta + o_z \cos \theta & 0 \\ a_x & a_y & a_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1.66)$$

premultiplying by

$$\mathbf{C} = \begin{bmatrix} n_x & o_x & a_x & 0 \\ n_y & o_y & a_y & 0 \\ n_z & o_z & a_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1.67)$$

we obtain  $\mathbf{C} \mathbf{Rot}(\mathbf{z}, \theta) \mathbf{C}^{-1}$

$$\begin{bmatrix} n_x n_x \cos \theta - n_x o_x \sin \theta + n_x o_x \sin \theta + o_x o_x \cos \theta + a_x a_x & n_y n_x \cos \theta - n_y o_x \sin \theta + n_x o_y \sin \theta + o_x o_y \cos \theta + a_y a_x & n_z n_x \cos \theta - n_z o_x \sin \theta + n_x o_z \sin \theta + o_x o_z \cos \theta + a_z a_x & 0 \\ n_x n_y \cos \theta - n_x o_y \sin \theta + n_y o_x \sin \theta + o_y o_x \cos \theta + a_x a_y & n_y n_y \cos \theta - n_y o_y \sin \theta + n_y o_y \sin \theta + o_y o_y \cos \theta + a_y a_y & n_z n_y \cos \theta - n_z o_y \sin \theta + n_y o_z \sin \theta + o_y o_z \cos \theta + a_z a_y & 0 \\ n_x n_z \cos \theta - n_x o_z \sin \theta + n_z o_x \sin \theta + o_z o_x \cos \theta + a_x a_z & n_y n_z \cos \theta - n_y o_z \sin \theta + n_z o_y \sin \theta + o_z o_y \cos \theta + a_y a_z & n_z n_z \cos \theta - n_z o_z \sin \theta + n_z o_z \sin \theta + o_z o_z \cos \theta + a_z a_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1.68)$$

Simplifying, using the following relationships:

the dot product of any row or column of  $\mathbf{C}$  with any other row or column is zero, as the vectors are orthogonal;

the dot product of any row or column of  $\mathbf{C}$  with itself is  $\mathbf{1}$  as the vectors are of unit magnitude;

the  $z$  unit vector is the vector cross product of the  $x$  and  $y$  vectors or

$$\mathbf{a} = \mathbf{n} \times \mathbf{o} \quad (1.69)$$

which has components

$$a_x = n_y o_z - n_z o_y$$

$$a_y = n_z o_x - n_x o_z$$

$$a_z = n_x o_y - n_y o_x$$



the versine, abbreviated **vers**  $\theta$ , is defined as **vers**  $\theta = (1 - \cos \theta)$ ,  $k_x = a_x$ ,  $k_y = a_y$  and  $k_z = a_z$ . We obtain **Rot**(**k**,  $\theta$ ) =

$$\begin{bmatrix} k_x k_x \text{vers} \theta + \cos \theta & k_y k_x \text{vers} \theta - k_z \sin \theta & k_z k_x \text{vers} \theta + k_y \sin \theta & 0 \\ k_x k_y \text{vers} \theta + k_z \sin \theta & k_y k_y \text{vers} \theta + \cos \theta & k_z k_y \text{vers} \theta - k_x \sin \theta & 0 \\ k_x k_z \text{vers} \theta - k_y \sin \theta & k_y k_z \text{vers} \theta + k_x \sin \theta & k_z k_z \text{vers} \theta + \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1.70)$$

This is an important result and should be thoroughly understood before proceeding further. From this general rotation transformation we can obtain each of the elementary rotation transforms. For example **Rot**(**x**,  $\theta$ ) is **Rot**(**k**,  $\theta$ ) where  $k_x = 1$ ,  $k_y = 0$ , and  $k_z = 0$ . Substituting these values of **k** into Equation 1.70 we obtain

$$\mathbf{Rot}(\mathbf{x}, \theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta & 0 \\ 0 & \sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1.71)$$

as before.

#### 5.4.13 Equivalent Angle and Axis of Rotation

Given any arbitrary rotational transformation, we can use Equation 1.70 to obtain an axis about which an equivalent rotation  $\theta$  is made as follows. Given a rotational transformation **R**

$$\mathbf{R} = \begin{bmatrix} n_x & o_x & a_x & 0 \\ n_y & o_y & a_y & 0 \\ n_z & o_z & a_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1.72)$$

we may equate **R** to **Rot**(**k**,  $\theta$ )

$$\begin{bmatrix} n_x & o_x & a_x & 0 \\ n_y & o_y & a_y & 0 \\ n_z & o_z & a_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} k_x k_x \text{vers} \theta + \cos \theta & k_y k_x \text{vers} \theta - k_z \sin \theta & k_z k_x \text{vers} \theta + k_y \sin \theta & 0 \\ k_x k_y \text{vers} \theta + k_z \sin \theta & k_y k_y \text{vers} \theta + \cos \theta & k_z k_y \text{vers} \theta - k_x \sin \theta & 0 \\ k_x k_z \text{vers} \theta - k_y \sin \theta & k_y k_z \text{vers} \theta + k_x \sin \theta & k_z k_z \text{vers} \theta + \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1.73)$$

Summing the diagonal terms of Equation 1.73 we obtain

$$n_x + o_y + a_z + 1 = k_x^2 \text{vers} \theta + \cos \theta + k_y^2 \text{vers} \theta + \cos \theta + k_z^2 \text{vers} \theta + \cos \theta + 1 \quad (1.74)$$

$$\begin{aligned} n_x + o_y + a_z &= (k_x^2 + k_y^2 + k_z^2) \cos \theta + 3 \cos \theta \\ &= 1 + 2 \cos \theta \end{aligned} \quad (1.75)$$

and the cosine of the angle of rotation is

$$\cos \theta = \frac{1}{2}(n_x + o_y + a_z - 1) \quad (1.76)$$

Differencing pairs of off-diagonal terms in Equation 1.73 we obtain

$$o_z - a_y = 2k_x \sin \theta \quad (1.77)$$

$$a_x - n_z = 2k_y \sin \theta \quad (1.78)$$

$$n_y - o_x = 2k_z \sin \theta \quad (1.79)$$

Squaring and adding Equations 1.77-1.79 we obtain an expression for  $\sin \theta$

$$(o_z - a_y)^2 + (a_x - n_z)^2 + (n_y - o_x)^2 = 4 \sin^2 \theta \quad (1.80)$$

and the sine of the angle of rotation is

$$\sin \theta = \pm \frac{1}{2} \sqrt{(o_z - a_y)^2 + (a_x - n_z)^2 + (n_y - o_x)^2} \quad (1.81)$$

We may define the rotation to be positive about the vector  $\mathbf{k}$  such that  $0 \leq \theta \leq 180^\circ$ . In this case the  $+$  sign is appropriate in Equation 1.81 and thus the angle of rotation  $\theta$  is uniquely defined as

$$\tan \theta = \frac{\sqrt{(o_z - a_y)^2 + (a_x - n_z)^2 + (n_y - o_x)^2}}{(n_x + o_y + a_z - 1)} \quad (1.82)$$

The components of  $\mathbf{k}$  may be obtained from Equations 1.77-1.79 as

$$k_x = \frac{o_z - a_y}{2 \sin \theta} \quad (1.83)$$

$$k_y = \frac{a_x - n_z}{2 \sin \theta} \quad (1.84)$$

$$k_z = \frac{n_y - o_x}{2 \sin \theta} \quad (1.85)$$

When the angle of rotation is very small, the axis of rotation is physically not well defined due to the small magnitude of both numerator and denominator in Equations 1.83-1.85. If the resulting angle is small, the vector  $\mathbf{k}$  should be renormalized to ensure that  $|\mathbf{k}| = 1$ . When the angle of rotation approaches  $180^\circ$  the vector  $\mathbf{k}$  is once again poorly defined by Equation 1.83-1.85 as the magnitude of the sine is again decreasing. The axis of rotation is, however, physically well defined in this case. When  $\theta < 150^\circ$ , the denominator of Equations

1.83-1.85 is less than 1. As the angle increases to  $180^\circ$  the rapidly decreasing magnitude of both numerator and denominator leads to considerable inaccuracies in the determination of  $\mathbf{k}$ . At  $\theta = 180^\circ$ , Equations 1.83-1.85 are of the form  $0/0$ , yielding no information at all about a physically well defined vector  $\mathbf{k}$ . If the angle of rotation is greater than  $90^\circ$ , then we must follow a different approach in determining  $\mathbf{k}$ . Equating the diagonal elements of Equation 1.73 we obtain

$$k_x^2 \text{vers}\theta + \cos\theta = n_x \quad (1.86)$$

$$k_y^2 \text{vers}\theta + \cos\theta = o_y \quad (1.87)$$

$$k_z^2 \text{vers}\theta + \cos\theta = a_z \quad (1.88)$$

Substituting for  $\cos\theta$  and  $\text{vers}\theta$  from Equation 1.76 and solving for the elements of  $\mathbf{k}$  we obtain further

$$k_x = \pm \sqrt{\frac{n_x - \cos\theta}{1 - \cos\theta}} \quad (1.89)$$

$$k_y = \pm \sqrt{\frac{o_y - \cos\theta}{1 - \cos\theta}} \quad (1.90)$$

$$k_z = \pm \sqrt{\frac{a_z - \cos\theta}{1 - \cos\theta}} \quad (1.91)$$

The largest component of  $\mathbf{k}$  defined by Equations 1.89-1.91 corresponds to the most positive component of  $n_x$ ,  $o_y$ , and  $a_z$ . For this largest element, the sign of the radical can be obtained from Equations 1.77-1.79. As the sine of the angle of rotation  $\theta$  must be positive, then the sign of the component of  $\mathbf{k}$  defined by Equations 1.77-1.79 must be the same as the sign of the left hand side of these equations. Thus we may combine Equations 1.89-1.91 with the information contained in Equations 1.77-1.79 as follows

$$k_x = \text{sgn}(o_z - a_y) \sqrt{\frac{(n_x - \cos\theta)}{1 - \cos\theta}} \quad (1.92)$$

$$k_y = \text{sgn}(a_x - n_z) \sqrt{\frac{(o_y - \cos\theta)}{1 - \cos\theta}} \quad (1.93)$$

$$k_z = \text{sgn}(n_y - o_x) \sqrt{\frac{(a_z - \cos\theta)}{1 - \cos\theta}} \quad (1.94)$$

where  $\text{sgn}(e) = +1$  if  $e \geq 0$  and  $\text{sgn}(e) = -1$  if  $e \leq 0$ .

Only the largest element of  $\mathbf{k}$  is determined from Equations 1.92-1.94, corresponding to the most positive element of  $n_x$ ,  $o_y$ , and  $a_z$ . The remaining elements are more accurately determined by the following equations formed by summing pairs of off-diagonal elements of Equation 1.73

$$n_y + o_x = 2k_x k_y \text{vers}\theta \quad (1.95)$$

$$o_z + a_y = 2k_y k_z \text{vers}\theta \quad (1.96)$$

$$n_z + a_x = 2k_z k_x \text{vers}\theta \quad (1.97)$$

If  $k_x$  is largest then

$$k_y = \frac{n_y + o_x}{2k_x \text{vers}\theta} \quad \text{from Equation 1.95} \quad (1.98)$$

$$k_z = \frac{a_x + n_z}{2k_x \text{vers}\theta} \quad \text{from Equation 1.97} \quad (1.99)$$

If  $k_y$  is largest then

$$k_x = \frac{n_y + o_x}{2k_y \text{vers}\theta} \quad \text{from Equation 1.95} \quad (1.100)$$

$$k_z = \frac{o_z + a_y}{2k_y \text{vers}\theta} \quad \text{from Equation 1.96} \quad (1.101)$$

If  $k_z$  is largest then

$$k_x = \frac{a_x + n_z}{2k_z \text{vers}\theta} \quad \text{from Equation 1.97} \quad (1.102)$$

$$k_y = \frac{o_z + a_y}{2k_z \text{vers}\theta} \quad \text{from Equation 1.96} \quad (1.103)$$

#### 5.4.14 Example 1.1

Determine the equivalent axis and angle of rotation for the matrix given in Equations 1.41

$$\mathbf{Rot}(\mathbf{y}, 90)\mathbf{Rot}(\mathbf{z}, 90) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1.104)$$

We first determine  $\cos \theta$  from Equation 1.76

$$\cos \theta = \frac{1}{2}(0 + 0 + 0 - 1) = -\frac{1}{2} \quad (1.105)$$

and  $\sin \theta$  from Equation 1.81

$$\sin\theta = \frac{1}{2}\sqrt{(1-0)^2 + (1-0)^2 + (1-0)^2} = \frac{\sqrt{3}}{2} \quad (1.106)$$

Thus

$$\theta = \tan^{-1} \left( \frac{\frac{\sqrt{3}}{2}}{\frac{-1}{2}} \right) = 120^\circ \quad (1.107)$$

As  $\theta > 90$ , we determine the largest component of  $\mathbf{k}$  corresponding to the largest element on the diagonal. As all diagonal elements are equal in this example we may pick any one. We will pick  $k_x$  given by Equation 1.92

$$k_x = +\sqrt{(0 + \frac{1}{2}) / (1 + \frac{1}{2})} = \frac{1}{\sqrt{3}} \quad (1.108)$$

As we have determined  $k_x$  we may now determine  $k_y$  and  $k_z$  from Equations 1.98 and 1.99, respectively

$$k_y = \frac{1+0}{\sqrt{3}} = \frac{1}{\sqrt{3}} \quad (1.109)$$

$$k_z = \frac{1+0}{\sqrt{3}} = \frac{1}{\sqrt{3}} \quad (1.110)$$

In summary, then

$$\mathbf{Rot}(\mathbf{y}, 90)\mathbf{Rot}(\mathbf{z}, 90) = \mathbf{Rot}(\mathbf{k}, 120) \quad (1.111)$$

where

$$\mathbf{k} = \frac{1}{\sqrt{3}}\mathbf{i} + \frac{1}{\sqrt{3}}\mathbf{j} + \frac{1}{\sqrt{3}}\mathbf{k} \quad (1.112)$$

Any combination of rotations is always equivalent to a single rotation about some axis  $\mathbf{k}$  by an angle  $\theta$ , an important result that we will make use of later.

### 5.4.15 Stretching and Scaling

A transform  $\mathbf{T}$

$$\mathbf{T} = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1.113)$$

will stretch objects uniformly along the  $x$  axis by a factor  $a$ , along the  $y$  axis by a factor  $b$ , and along the  $z$  axis by a factor  $c$ . Consider any point on an object  $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ ; its transform is

$$\begin{bmatrix} ax \\ by \\ cz \\ 1 \end{bmatrix} = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \quad (1.114)$$

indicating stretching as stated. Thus a cube could be transformed into a rectangular parallelepiped by such a transform.

The Axiom code to perform this scale change is:

```

— scale —

scale(scalex, scaley, scalez) ==
matrix(
  [[scalex, 0, 0, 0],
   [0, scaley, 0, 0],
   [0, 0, scalez, 0],
   [0, 0, 0, 1]])

```

The transform  $\mathbf{S}$  where

$$\mathbf{S} = \begin{bmatrix} s & 0 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & s & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1.115)$$

will scale any object by the factor  $s$ .

### 5.4.16 Perspective Transformations

Consider the image formed of an object by a simple lens.

The axis of the lens is along the  $y$  axis for convenience. An object point  $x, y, z$  is imaged at  $x', y', z'$  if the lens has a focal length  $f$  ( $f$  is considered positive).  $y'$  represents the image distance and varies with object distance  $y$ . If we plot points on a plane perpendicular to the  $y$  axis located at  $y'$  (the film plane in a camera), then a perspective image is formed.

We will first obtain values of  $x'$ ,  $y'$ , and  $z'$ , then introduce a perspective transformation and show that the same values are obtained.

Based on the fact that a ray passing through the center of the lens is undeviated we may write

$$\frac{z}{y} = \frac{z'}{y'} \quad (1.116)$$

$$\text{and } \frac{x}{y} = \frac{x'}{y'} \quad (1.117)$$

Based on the additional fact that a ray parallel to the lens axis passes through the focal point  $f$ , we may write

$$\frac{z}{f} = \frac{z'}{y' + f} \quad (1.118)$$

$$\text{and } \frac{x}{f} = \frac{x'}{y' + f} \quad (1.119)$$

Notice that  $x'$ ,  $y'$ , and  $z'$  are negative and that  $f$  is positive. Eliminating  $y'$  between Equations 1.116 and 1.118 we obtain

$$\frac{z}{f} = \frac{z'}{\left(\frac{z'y}{z} + f\right)} \quad (1.120)$$

and solving for  $z'$  we obtain the result

$$z' = \frac{z}{\left(1 - \frac{y}{f}\right)} \quad (1.121)$$

Working with Equations 1.117 and 1.119 we can similarly obtain

$$x' = \frac{x}{\left(1 - \frac{y}{f}\right)} \quad (1.122)$$

In order to obtain the image distance  $y'$  we rewrite Equations 1.116 and 1.118 as

$$\frac{z}{z'} = \frac{y}{y'} \quad (1.123)$$

and

$$\frac{z}{z'} = \frac{f}{y' + f} \quad (1.124)$$

thus

$$\frac{y}{y'} = \frac{f}{y' + f} \quad (1.125)$$

and solving for  $y'$  we obtain the result

$$y' = \frac{y}{(1 - \frac{y}{f})} \quad (1.126)$$

The homogeneous transformation  $\mathbf{P}$  which produces the same result is

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -\frac{1}{f} & 0 & 1 \end{bmatrix} \quad (1.127)$$

as any point  $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$  transforms as

$$\begin{bmatrix} x \\ y \\ z \\ 1 - \frac{y}{f} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -\frac{1}{f} & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \quad (1.128)$$

The image point  $x', y', z'$ , obtained by dividing through by the weight factor  $(1 - \frac{y}{f})$ , is

$$\frac{x}{(1 - \frac{y}{f})}\mathbf{i} + \frac{y}{(1 - \frac{y}{f})}\mathbf{j} + \frac{z}{(1 - \frac{y}{f})}\mathbf{k} \quad (1.129)$$

This is the same result that we obtained above.

A transform similar to  $\mathbf{P}$  but with  $-\frac{1}{f}$  at the bottom of the first column produces a perspective transformation along the  $x$  axis. If the  $-\frac{1}{f}$  term is in the third column then the projection is along the  $z$  axis.

#### 5.4.17 Transform Equations

We will frequently be required to deal with transform equations in which a coordinate frame is described in two or more ways. A manipulator is positioned with respect to base coordinates by a transform  $\mathbf{Z}$ . The end of the manipulator is described by a transform  ${}^Z\mathbf{T}_6$ , and the end effector is described by  ${}^{T_6}\mathbf{E}$ . An object is positioned with respect to base coordinates by a transform  $\mathbf{B}$ , and finally the manipulator end effector is positioned with respect to the object by  ${}^B\mathbf{G}$ . We have two descriptions of the position of the end effector, one with respect to the object and one with respect to the manipulator. As both positions are the same, we may equate the two descriptions

$$\mathbf{Z}{}^Z\mathbf{T}_6{}^{T_6}\mathbf{E} = \mathbf{B}{}^B\mathbf{G} \quad (1.130)$$

If we wish to solve Equation 1.130 for the manipulator transform  $\mathbf{T}_6$  we must premultiply Equation 1.130 by  $\mathbf{Z}^{-1}$  and postmultiply by  $\mathbf{E}^{-1}$  to obtain



$$\mathbf{T}_6 = \mathbf{Z}^{-1} \mathbf{B} \mathbf{G} \mathbf{E}^{-1} \quad (1.131)$$

As a further example, consider that the position of the object  $\mathbf{B}$  is unknown, but that the manipulator is moved such that the end effector is positioned over the object correctly. We may then solve for  $\mathbf{B}$  from Equation 1.130 by postmultiplying by  $\mathbf{G}^{-1}$ .

$$\mathbf{B} = \mathbf{Z} \mathbf{T}_6 \mathbf{E} \mathbf{G}^{-1} \quad (1.133)$$

#### 5.4.18 Summary

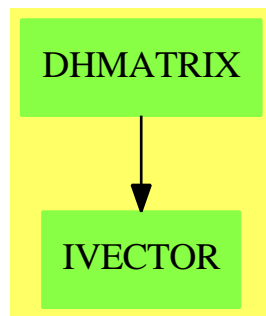
Homogeneous transformations may be readily used to describe the positions and orientations of coordinate frames in space. If a coordinate frame is embedded in an object then the position and orientation of the object are also readily described.

The description of object A in terms of object B by means of a homogeneous transformation may be inverted to obtain the description of object B in terms of object A. This is not a property of a simple vector description of the relative displacement of one object with respect to another.

Transformations may be interpreted as a product of rotation and translation transformations. If they are interpreted from left to right, then the rotations and translations are in terms of the currently defined coordinate frame. If they are interpreted from right to left, then the rotations and translations are described with respect to the reference coordinate frame.

Homogeneous transformations describe coordinate frames in terms of rectangular components, which are the sines and cosines of angles. This description may be related to rotations in which case the description is in terms of a vector and angle of rotation.

#### 5.4.19 DenavitHartenbergMatrix (DHMATRIX)



**Exports:**

|                |             |             |                |           |
|----------------|-------------|-------------|----------------|-----------|
| antisymmetric? | any?        | coerce      | column         | copy      |
| count          | determinant | diagonal?   | diagonalMatrix | elt       |
| empty          | empty?      | eq?         | eval           | every?    |
| exquo          | fill!       | hash        | horizConcat    | identity  |
| inverse        | latex       | less?       | listOfLists    | map       |
| map!           | matrix      | maxColIndex | maxRowIndex    | member?   |
| members        | minColIndex | minordet    | minRowIndex    | more?     |
| ncols          | new         | nrows       | nullSpace      | nullity   |
| parts          | qelt        | qsetelt!    | rank           | rotatex   |
| rotatey        | rotatez     | row         | rowEchelon     | sample    |
| scalarMatrix   | scale       | setColumn!  | setRow!        | setelt    |
| setsubMatrix!  | size?       | square?     | squareTop      | subMatrix |
| swapColumns!   | swapRows!   | symmetric?  | translate      | transpose |
| vertConcat     | zero        | #?          | ***?           | ?/?       |
| ?=?            | ?~=?        | ?*?         | ?+?            | -?        |
| ?-?            |             |             |                |           |

— domain DHMATRIX DenavitHartenbergMatrix —

```
)abbrev domain DHMATRIX DenavitHartenbergMatrix

++ Author: Timothy Daly
++ Date Created: June 26, 1991
++ Date Last Updated: 26 June 1991
++ Description:
++ 4x4 Matrices for coordinate transformations\br
++ This package contains functions to create 4x4 matrices
++ useful for rotating and transforming coordinate systems.
++ These matrices are useful for graphics and robotics.
++ (Reference: Robot Manipulators Richard Paul MIT Press 1981)
++
++ A Denavit-Hartenberg Matrix is a 4x4 Matrix of the form:\br
++ \tab{5}\spad{nx ox ax px}\br
++ \tab{5}\spad{ny oy ay py}\br
++ \tab{5}\spad{nz oz az pz}\br
++ \tab{5}\spad{0 0 0 1}\br
++ (n, o, and a are the direction cosines)

DenavitHartenbergMatrix(R): Exports == Implementation where
  R : Join(Field, TranscendentalFunctionCategory)

-- for the implementation of dhmatrix
  minrow ==> 1
  mincolumn ==> 1
--
  nx ==> x(1,1)::R
  ny ==> x(2,1)::R
```

```

nz ==> x(3,1)::R
ox ==> x(1,2)::R
oy ==> x(2,2)::R
oz ==> x(3,2)::R
ax ==> x(1,3)::R
ay ==> x(2,3)::R
az ==> x(3,3)::R
px ==> x(1,4)::R
py ==> x(2,4)::R
pz ==> x(3,4)::R
row ==> Vector(R)
col ==> Vector(R)
radians ==> pi()/180

Exports ==> MatrixCategory(R,row,col) with
"*.": (% , Point R) -> Point R
  ++ t*p applies the dhmatrix t to point p
identity: () -> %
  ++ identity() create the identity dhmatrix
rotatex: R -> %
  ++ rotatex(r) returns a dhmatrix for rotation about axis x for r degrees
rotatey: R -> %
  ++ rotatey(r) returns a dhmatrix for rotation about axis y for r degrees
rotatez: R -> %
  ++ rotatez(r) returns a dhmatrix for rotation about axis z for r degrees
scale: (R,R,R) -> %
  ++ scale(sx,sy,sz) returns a dhmatrix for scaling in the x, y and z
  ++ directions
translate: (R,R,R) -> %
  ++ translate(x,y,z) returns a dhmatrix for translation by x, y, and z

Implementation ==> Matrix(R) add

identity() == matrix([[1,0,0,0],[0,1,0,0],[0,0,1,0],[0,0,0,1]])

-- inverse(x) == (inverse(x pretend (Matrix R))$Matrix(R)) pretend %
-- dhinverse(x) == matrix( _
--   [[nx,ny,nz,-(px*nx+py*ny+pz*nz)],_
--   [ox,oy,oz,-(px*ox+py*oy+pz*oz)],_
--   [ax,ay,az,-(px*ax+py*ay+pz*az)],_
--   [ 0, 0, 0, 1]])

d * p ==
  v := p pretend Vector R
  v := concat(v, 1$R)
  v := d * v
  point ([v.1, v.2, v.3]$List(R))

\getchunk{rotatex}

```



---

## Type: PositiveInteger

```

--S 3 of 63
a
--R
--R
--R (3) [2,3,4,5]
--R
--R                                         Type: Dequeue Integer
--E 3

--S 4 of 63
extract! a
--R
--R
--R (4) 2
--R
--R                                         Type: PositiveInteger
--E 4

--S 5 of 63
a
--R
--R
--R (5) [3,4,5]
--R
--R                                         Type: Dequeue Integer
--E 5

--S 6 of 63
enqueue!(9,a)
--R
--R
--R (6) 9
--R
--R                                         Type: PositiveInteger
--E 6

--S 7 of 63
a
--R
--R
--R (7) [3,4,5,9]
--R
--R                                         Type: Dequeue Integer
--E 7

--S 8 of 63
insert!(8,a)
--R
--R
--R (8) [3,4,5,9,8]
--R
--R                                         Type: Dequeue Integer
--E 8

--S 9 of 63

```

```

a
--R
--R
--R (9) [3,4,5,9,8]
--R
--R                                         Type: Dequeue Integer
--E 9

--S 10 of 63
front a
--R
--R
--R (10) 3
--R
--R                                         Type: PositiveInteger
--E 10

--S 11 of 63
back a
--R
--R
--R (11) 8
--R
--R                                         Type: PositiveInteger
--E 11

--S 12 of 63
bottom! a
--R
--R
--R (12) 8
--R
--R                                         Type: PositiveInteger
--E 12

--S 13 of 63
a
--R
--R
--R (13) [3,4,5,9]
--R
--R                                         Type: Dequeue Integer
--E 13

--S 14 of 63
depth a
--R
--R
--R (14) 4
--R
--R                                         Type: PositiveInteger
--E 14

--S 15 of 63
height a
--R

```

```

--R
--R (15)  4
--R
--R                                         Type: PositiveInteger
--E 15

--S 16 of 63
insertBottom!(6,a)
--R
--R
--R (16)  6
--R
--R                                         Type: PositiveInteger
--E 16

--S 17 of 63
a
--R
--R
--R (17)  [3,4,5,9,6]
--R
--R                                         Type: Dequeue Integer
--E 17

--S 18 of 63
extractBottom! a
--R
--R
--R (18)  6
--R
--R                                         Type: PositiveInteger
--E 18

--S 19 of 63
a
--R
--R
--R (19)  [3,4,5,9]
--R
--R                                         Type: Dequeue Integer
--E 19

--S 20 of 63
insertTop!(7,a)
--R
--R
--R (20)  7
--R
--R                                         Type: PositiveInteger
--E 20

--S 21 of 63
a
--R
--R
--R (21)  [7,3,4,5,9]

```

[illegible]



```
--S 28 of 63
reverse! a
--R
--R
--R (28) [9,5,4]
--R
--E 28
```

Type: Dequeue Integer

```
--S 29 of 63
rotate! a
--R
--R
--R (29) [5,4,9]
--R
--E 29
```

Type: Dequeue Integer

```
--S 30 of 63
inspect a
--R
--R
--R (30) 5
--R
--E 30
```

Type: PositiveInteger

```
--S 31 of 63
empty? a
--R
--R
--R (31) false
--R
--E 31
```

Type: Boolean

```
--S 32 of 63
#a
--R
--R
--R (32) 3
--R
--E 32
```

Type: PositiveInteger

```
--S 33 of 63
length a
--R
--R
--R (33) 3
--R
--E 33
```

Type: PositiveInteger

```
--S 34 of 63
```

```

less?(a,9)
--R
--R
--R (34) true
--R
--R                                          Type: Boolean
--E 34

--S 35 of 63
more?(a,9)
--R
--R
--R (35) false
--R
--R                                          Type: Boolean
--E 35

--S 36 of 63
size?(a,#a)
--R
--R
--R (36) true
--R
--R                                          Type: Boolean
--E 36

--S 37 of 63
size?(a,9)
--R
--R
--R (37) false
--R
--R                                          Type: Boolean
--E 37

--S 38 of 63
parts a
--R
--R
--R (38) [5,4,9]
--R
--R                                          Type: List Integer
--E 38

--S 39 of 63
bag([1,2,3,4,5])$Dequeue(INT)
--R
--R
--R (39) [1,2,3,4,5]
--R
--R                                          Type: Dequeue Integer
--E 39

--S 40 of 63
b:=empty()$(Dequeue INT)
--R

```

```

--R
--R (40) []
--R
--R                                         Type: Dequeue Integer
--E 40

--S 41 of 63
empty? b
--R
--R
--R (41) true
--R
--R                                         Type: Boolean
--E 41

--S 42 of 63
sample()$Dequeue(INT)
--R
--R
--R (42) []
--R
--R                                         Type: Dequeue Integer
--E 42

--S 43 of 63
c:=copy a
--R
--R
--R (43) [5,4,9]
--R
--R                                         Type: Dequeue Integer
--E 43

--S 44 of 63
eq?(a,c)
--R
--R
--R (44) false
--R
--R                                         Type: Boolean
--E 44

--S 45 of 63
eq?(a,a)
--R
--R
--R (45) true
--R
--R                                         Type: Boolean
--E 45

--S 46 of 63
(a=c)@Boolean
--R
--R
--R (46) true

```

```

--R
--E 46
Type: Boolean

--S 47 of 63
(a=a)@Boolean
--R
--R
--R (47) true
--R
--R
--E 47
Type: Boolean

--S 48 of 63
a~=c
--R
--R
--R (48) false
--R
--R
--E 48
Type: Boolean

--S 49 of 63
any?(x+>(x=4),a)
--R
--R
--R (49) true
--R
--R
--E 49
Type: Boolean

--S 50 of 63
any?(x+>(x=11),a)
--R
--R
--R (50) false
--R
--R
--E 50
Type: Boolean

--S 51 of 63
every?(x+>(x=11),a)
--R
--R
--R (51) false
--R
--R
--E 51
Type: Boolean

--S 52 of 63
count(4,a)
--R
--R
--R (52) 1
--R
--R
--E 52
Type: PositiveInteger

```

```
--S 53 of 63
count(x+-->(x>2),a)
--R
--R
--R (53)  3
--R
--E 53
```

Type: PositiveInteger

```
--S 54 of 63
map(x+-->x+10,a)
--R
--R
--R (54)  [15,14,19]
--R
--E 54
```

Type: Dequeue Integer

```
--S 55 of 63
a
--R
--R
--R (55)  [5,4,9]
--R
--E 55
```

Type: Dequeue Integer

```
--S 56 of 63
map!(x+-->x+10,a)
--R
--R
--R (56)  [15,14,19]
--R
--E 56
```

Type: Dequeue Integer

```
--S 57 of 63
a
--R
--R
--R (57)  [15,14,19]
--R
--E 57
```

Type: Dequeue Integer

```
--S 58 of 63
members a
--R
--R
--R (58)  [15,14,19]
--R
--E 58
```

Type: List Integer

```
--S 59 of 63
```

```

member?(14,a)
--R
--R
--R (59) true
--R
--R                                          Type: Boolean
--E 59

--S 60 of 63
coerce a
--R
--R
--R (60) [15,14,19]
--R
--R                                          Type: OutputForm
--E 60

--S 61 of 63
hash a
--R
--R
--R (61) 4999531
--R
--R                                          Type: SingleInteger
--E 61

--S 62 of 63
latex a
--R
--R
--R (62) "\mbox{\bf Unimplemented}"
--R
--R                                          Type: String
--E 62

--S 63 of 63
)show Dequeue
--R
--R Dequeue S: SetCategory is a domain constructor
--R Abbreviation for Dequeue is DEQUEUE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for DEQUEUE
--R
--R----- Operations -----
--R back : % -> S                bag : List S -> %
--R bottom! : % -> S            copy : % -> %
--R depth : % -> NonNegativeInteger dequeue : List S -> %
--R dequeue : () -> %          dequeue! : % -> S
--R empty : () -> %            empty? : % -> Boolean
--R enqueue! : (S,%) -> S      eq? : (%,%) -> Boolean
--R extract! : % -> S          extractBottom! : % -> S
--R extractTop! : % -> S       front : % -> S
--R height : % -> NonNegativeInteger insert! : (S,%) -> %
--R insertBottom! : (S,%) -> S insertTop! : (S,%) -> S

```

```

--R inspect : % -> S                                length : % -> NonNegativeInteger
--R map : ((S -> S),%) -> %                          pop! : % -> S
--R push! : (S,%) -> S                                reverse! : % -> %
--R rotate! : % -> %                                  sample : () -> %
--R top : % -> S                                       top! : % -> S
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ==? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if $ has finiteAggregate and S has SETCAT
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R members : % -> List S if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List S if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R ~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 63

)spool
)lisp (bye)

```

---

— Dequeue.help —

=====

Dequeue examples

=====

A Dequeue is a double-ended queue so elements can be added to either end.

Here we create an dequeue of integers from a list. Notice that the order in the list is the order in the dequeue.

```

a:Dequeue INT:= dequeue [1,2,3,4,5]
[1,2,3,4,5]

```

We can remove the top of the dequeue using dequeue!:

```
dequeue! a
1
```

Notice that the use of dequeue! is destructive (destructive operations in Axiom usually end with ! to indicate that the underlying data structure is changed).

```
a
[2,3,4,5]
```

The extract! operation is another name for the dequeue! operation and has the same effect. This operation treats the dequeue as a BagAggregate:

```
extract! a
2
```

and you can see that it also has destructively modified the dequeue:

```
a
[3,4,5]
```

Next we use enqueue! to add a new element to the end of the dequeue:

```
enqueue!(9,a)
9
```

Again, the enqueue! operation is destructive so the dequeue is changed:

```
a
[3,4,5,9]
```

Another name for enqueue! is insert!, which treats the dequeue as a BagAggregate:

```
insert!(8,a)
[3,4,5,9,8]
```

and it modifies the dequeue:

```
a
[3,4,5,9,8]
```

The front operation returns the item at the front of the dequeue:

```
front a
3
```



The back operation returns the item at the back of the dequeue:

```
back a
8
```

The bottom! operation returns the item at the back of the dequeue:

```
bottom! a
8
```

and it modifies the dequeue:

```
a
[3,4,5,9]
```

The depth function returns the number of elements in the dequeue:

```
depth a
4
```

The height function returns the number of elements in the dequeue:

```
height a
4
```

The insertBottom! function adds the element at the end:

```
insertBottom!(6,a)
6
```

and it modifies the dequeue:

```
a
[3,4,5,9,6]
```

The extractBottom! function removes the element at the end:

```
extractBottom! a
6
```

and it modifies the dequeue:

```
a
[3,4,5,9]
```

The insertTop! function adds the element at the top:

```
insertTop!(7,a)
7
```

and it modifies the dequeue:

```
a
  [7,3,4,5,9]
```

The extractTop! function adds the element at the top:

```
extractTop! a
  7
```

and it modifies the dequeue:

```
a
  [3,4,5,9]
```

The top function returns the top element:

```
top a
  3
```

and it does not modifies the dequeue:

```
a
  [3,4,5,9]
```

The top! function returns the top element:

```
top! a
  3
```

and it modifies the dequeue:

```
a
  [4,5,9]
```

The reverse! operation destructively reverses the elements of the dequeue:

```
reverse! a
  [9,5,4]
```

The rotate! operation moves the top element to the bottom:

```
rotate! a
  [5,4,9]
```

The inspect function returns the top of the dequeue without modification, viewed as a BagAggregate:

```
inspect a
```

5

The `empty?` operation returns `true` only if there are no element on the dequeue, otherwise it returns `false`:

```
empty? a
false
```

The `# (length)` operation:

```
#a
3
```

The `length` operation does the same thing:

```
length a
3
```

The `less?` predicate will compare the dequeue length to an integer:

```
less?(a,9)
true
```

The `more?` predicate will compare the dequeue length to an integer:

```
more?(a,9)
false
```

The `size?` operation will compare the dequeue length to an integer:

```
size?(a,#a)
true
```

and since the last computation must always be `true` we try:

```
size?(a,9)
false
```

The `parts` function will return the dequeue as a list of its elements:

```
parts a
[5,4,9]
```

If we have a `BagAggregate` of elements we can use it to construct a dequeue:

```
bag([1,2,3,4,5])$Dequeue(INT)
[1,2,3,4,5]
```

The `empty` function will construct an empty dequeue of a given type:

```
b:=empty()$(Dequeue INT)
[]
```

and the `empty?` predicate allows us to find out if a dequeue is empty:

```
empty? b
true
```

The `sample` function returns a sample, empty dequeue:

```
sample()$Dequeue(INT)
[]
```

We can copy a dequeue and it does not share storage so subsequent modifications of the original dequeue will not affect the copy:

```
c:=copy a
[5,4,9]
```

The `eq?` function is only true if the lists are the same reference, so even though `c` is a copy of `a`, they are not the same:

```
eq?(a,c)
false
```

However, `a` clearly shares a reference with itself:

```
eq?(a,a)
true
```

But we can compare `a` and `c` for equality:

```
(a=c)@Boolean
true
```

and clearly `a` is equal to itself:

```
(a=a)@Boolean
true
```

and since `a` and `c` are equal, they are clearly NOT not-equal:

```
a~c
false
```

We can use the `any?` function to see if a predicate is true for any element:

```
any?(x+>(x=4),a)
true
```

or false for every element:

```
any?(x+-->(x=11),a)
false
```

We can use the `every?` function to check every element satisfies a predicate:

```
every?(x+-->(x=11),a)
false
```

We can count the elements that are equal to an argument of this type:

```
count(4,a)
1
```

or we can count against a boolean function:

```
count(x+-->(x>2),a)
3
```

You can also map a function over every element, returning a new dequeue:

```
map(x+-->x+10,a)
[15,14,19]
```

Notice that the original dequeue is unchanged:

```
a
[5,4,9]
```

You can use `map!` to map a function over every element and change the original dequeue since `map!` is destructive:

```
map!(x+-->x+10,a)
[15,14,19]
```

Notice that the original dequeue has been changed:

```
a
[15,14,19]
```

The `member` function can also get the element of the dequeue as a list:

```
members a
[15,14,19]
```

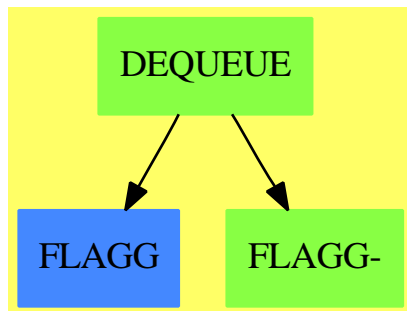
and using `member?` we can test if the dequeue holds a given element:

```
member?(14,a)
true
```

See Also:

- o )show Stack
- o )show ArrayStack
- o )show Queue
- o )show Dequeue
- o )show Heap
- o )show BagAggregate

### 5.5.1 Dequeue (DEQUEUE)



See

- ⇒ “Stack” (STACK) 20.28.1 on page 2521
- ⇒ “ArrayStack” (ASTACK) 2.10.1 on page 65
- ⇒ “Queue” (QUEUE) 18.5.1 on page 2143
- ⇒ “Heap” (HEAP) 9.2.1 on page 1100

#### Exports:

|         |          |                |               |            |
|---------|----------|----------------|---------------|------------|
| any?    | back     | bag            | bottom!       | coerce     |
| copy    | count    | depth          | dequeue       | dequeue!   |
| empty   | empty?   | enqueue!       | eq?           | eval       |
| every?  | extract! | extractBottom! | extractTop!   | front      |
| height  | hash     | insert!        | insertBottom! | insertTop! |
| inspect | latex    | length         | less?         | map        |
| map!    | member?  | members        | more?         | parts      |
| pop!    | push!    | reverse!       | rotate!       | sample     |
| size?   | top      | top!           | #?            | ?=?        |
| ?~=?    |          |                |               |            |

— domain DEQUEUE Dequeue —

```

)abbrev domain DEQUEUE Dequeue
++ Author: Michael Monagan and Stephen Watt
++ Date Created: June 86 and July 87
++ Date Last Updated: Feb 92
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ Linked list implementation of a Dequeue

Dequeue(S:SetCategory): DequeueAggregate S with
  dequeue: List S -> %
    ++ dequeue([x,y,...,z]) creates a dequeue with first (top or front)
    ++ element x, second element y,...,and last (bottom or back) element z.
    ++
    ++E g:Dequeue INT:= dequeue [1,2,3,4,5]

-- Inherited Signatures repeated for examples documentation

dequeue_! : % -> S
++
++X a:Dequeue INT:= dequeue [1,2,3,4,5]
++X dequeue! a
++X a
extract_! : % -> S
++
++X a:Dequeue INT:= dequeue [1,2,3,4,5]
++X extract! a
++X a
enqueue_! : (S,%) -> S
++
++X a:Dequeue INT:= dequeue [1,2,3,4,5]
++X enqueue! (9,a)
++X a
insert_! : (S,%) -> %
++
++X a:Dequeue INT:= dequeue [1,2,3,4,5]
++X insert! (8,a)
++X a
inspect : % -> S
++
++X a:Dequeue INT:= dequeue [1,2,3,4,5]
++X inspect a
front : % -> S
++
++X a:Dequeue INT:= dequeue [1,2,3,4,5]

```

```

++X front a
back : % -> S
++
++X a:Dequeue INT:= dequeue [1,2,3,4,5]
++X back a
rotate_! : % -> %
++
++X a:Dequeue INT:= dequeue [1,2,3,4,5]
++X rotate! a
length : % -> NonNegativeInteger
++
++X a:Dequeue INT:= dequeue [1,2,3,4,5]
++X length a
less? : (% , NonNegativeInteger) -> Boolean
++
++X a:Dequeue INT:= dequeue [1,2,3,4,5]
++X less?(a,9)
more? : (% , NonNegativeInteger) -> Boolean
++
++X a:Dequeue INT:= dequeue [1,2,3,4,5]
++X more?(a,9)
size? : (% , NonNegativeInteger) -> Boolean
++
++X a:Dequeue INT:= dequeue [1,2,3,4,5]
++X size?(a,5)
bag : List S -> %
++
++X bag([1,2,3,4,5])$Dequeue(INT)
empty? : % -> Boolean
++
++X a:Dequeue INT:= dequeue [1,2,3,4,5]
++X empty? a
empty : () -> %
++
++X b:=empty()$(Dequeue INT)
sample : () -> %
++
++X sample()$Dequeue(INT)
copy : % -> %
++
++X a:Dequeue INT:= dequeue [1,2,3,4,5]
++X copy a
eq? : (% , %) -> Boolean
++
++X a:Dequeue INT:= dequeue [1,2,3,4,5]
++X b:=copy a
++X eq?(a,b)
map : ((S -> S) , %) -> %
++
++X a:Dequeue INT:= dequeue [1,2,3,4,5]

```



```

++X map(x+>x+10,a)
++X a
depth : % -> NonNegativeInteger
++
++X a:Dequeue INT:= dequeue [1,2,3,4,5]
++X depth a
dequeue : () -> %
++
++X a:Dequeue INT:= dequeue ()
height : % -> NonNegativeInteger
++
++X a:Dequeue INT:= dequeue [1,2,3,4,5]
++X height a
top : % -> S
++
++X a:Dequeue INT:= dequeue [1,2,3,4,5]
++X top a
bottom_! : % -> S
++
++X a:Dequeue INT:= dequeue [1,2,3,4,5]
++X bottom! a
++X a
extractBottom_! : % -> S
++
++X a:Dequeue INT:= dequeue [1,2,3,4,5]
++X extractBottom! a
++X a
extractTop_! : % -> S
++
++X a:Dequeue INT:= dequeue [1,2,3,4,5]
++X extractTop! a
++X a
insertBottom_! : (S,%) -> S
++
++X a:Dequeue INT:= dequeue [1,2,3,4,5]
++X insertBottom! a
++X a
insertTop_! : (S,%) -> S
++
++X a:Dequeue INT:= dequeue [1,2,3,4,5]
++X insertTop! a
++X a
pop_! : % -> S
++
++X a:Dequeue INT:= dequeue [1,2,3,4,5]
++X pop! a
++X a
push_! : (S,%) -> S
++
++X a:Dequeue INT:= dequeue [1,2,3,4,5]

```

```

++X push! a
++X a
reverse_! : % -> %
++
++X a:Dequeue INT:= dequeue [1,2,3,4,5]
++X reverse! a
++X a
top_! : % -> S
++
++X a:Dequeue INT:= dequeue [1,2,3,4,5]
++X top! a
++X a
if $ has shallowlyMutable then
  map! : ((S -> S),%) -> %
  ++
  ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
  ++X map!(x+>x+10,a)
  ++X a
if S has SetCategory then
  latex : % -> String
  ++
  ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
  ++X latex a
  hash : % -> SingleInteger
  ++
  ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
  ++X hash a
  coerce : % -> OutputForm
  ++
  ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
  ++X coerce a
  "=" : (%,% ) -> Boolean
  ++
  ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
  ++X b:Dequeue INT:= dequeue [1,2,3,4,5]
  ++X (a=b)@Boolean
  "~=" : (%,% ) -> Boolean
  ++
  ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
  ++X b:=copy a
  ++X (a~=b)
if % has finiteAggregate then
  every? : ((S -> Boolean),%) -> Boolean
  ++
  ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
  ++X every?(x+>(x=4),a)
  any? : ((S -> Boolean),%) -> Boolean
  ++
  ++X a:Dequeue INT:= dequeue [1,2,3,4,5]
  ++X any?(x+>(x=4),a)

```

```

count : ((S -> Boolean),%) -> NonNegativeInteger
++
++X a:Dequeue INT:= dequeue [1,2,3,4,5]
++X count(x+>(x>2),a)
_# : % -> NonNegativeInteger
++
++X a:Dequeue INT:= dequeue [1,2,3,4,5]
++X #a
parts : % -> List S
++
++X a:Dequeue INT:= dequeue [1,2,3,4,5]
++X parts a
members : % -> List S
++
++X a:Dequeue INT:= dequeue [1,2,3,4,5]
++X members a
if % has finiteAggregate and S has SetCategory then
member? : (S,%) -> Boolean
++
++X a:Dequeue INT:= dequeue [1,2,3,4,5]
++X member?(3,a)
count : (S,%) -> NonNegativeInteger
++
++X a:Dequeue INT:= dequeue [1,2,3,4,5]
++X count(4,a)

== Queue S add
Rep := Reference List S
bottom! d == extractBottom! d
dequeue d == ref copy d
extractBottom! d ==
  if empty? d then error "empty dequeue"
  p := deref d
  n := maxIndex p
  n = 1 =>
    r := first p
    setref(d,[])
    r
  q := rest(p,(n-2)::NonNegativeInteger)
  r := first rest q
  q.rest := []
  r
top! d == extractTop! d
extractTop! d ==
  if empty? d then error "empty dequeue"
  e := top d
  setref(d,rest deref d)
  e
height d == # deref d
depth d == # deref d

```

```

insertTop!(e,d) == (setref(d,cons(e,deref d)); e)
lastTail==> LAST$Lisp
insertBottom!(e,d) ==
  if empty? d then setref(d, list e)
  else lastTail.(deref d).rest := list e
  e
top d == if empty? d then error "empty dequeue" else first deref d
reverse! d == (setref(d,reverse deref d); d)

```

---

— DEQUEUE.dotabb —

```

"DEQUEUE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=DEQUEUE"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"FLAGG-" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FLAGG"]
"DEQUEUE" -> "FLAGG-"
"DEQUEUE" -> "FLAGG"

```

---

## 5.6 domain DERHAM DeRhamComplex

— DeRhamComplex.input —

```

)set break resume
)sys rm -f DeRhamComplex.output
)spool DeRhamComplex.output
)set message test on
)set message auto off
)clear all
--S 1 of 34
coefRing := Integer
--R
--R
--R (1) Integer
--R
--R                                          Type: Domain
--E 1

--S 2 of 34
lv : List Symbol := [x,y,z]
--R
--R
--R (2) [x,y,z]

```

```

--R
--E 2
Type: List Symbol

--S 3 of 34
der := DERHAM(coefRing,lv)
--R
--R
--R (3) DeRhamComplex(Integer,[x,y,z])
--R
--E 3
Type: Domain

--S 4 of 34
R := Expression coefRing
--R
--R
--R (4) Expression Integer
--R
--E 4
Type: Domain

--S 5 of 34
f : R := x**2*y*z-5*x**3*y**2*z**5
--R
--R
--R      3 2 5      2
--R (5) - 5x y z + x y z
--R
--E 5
Type: Expression Integer

--S 6 of 34
g : R := z**2*y*cos(z)-7*sin(x**3*y**2)*z**2
--R
--R
--R      2      3 2      2
--R (6) - 7z sin(x y ) + y z cos(z)
--R
--E 6
Type: Expression Integer

--S 7 of 34
h : R :=x*y*z-2*x**3*y*z**2
--R
--R
--R      3      2
--R (7) - 2x y z + x y z
--R
--E 7
Type: Expression Integer

--S 8 of 34
dx : der := generator(1)
--R
--R

```

```

--R (8) dx
--R                                         Type: DeRhamComplex(Integer,[x,y,z])
--E 8

--S 9 of 34
dy : der := generator(2)
--R
--R
--R (9) dy
--R                                         Type: DeRhamComplex(Integer,[x,y,z])
--E 9

--S 10 of 34
dz : der := generator(3)
--R
--R
--R (10) dz
--R                                         Type: DeRhamComplex(Integer,[x,y,z])
--E 10

--S 11 of 34
[dx,dy,dz] := [generator(i)$der for i in 1..3]
--R
--R
--R (11) [dx,dy,dz]
--R                                         Type: List DeRhamComplex(Integer,[x,y,z])
--E 11

--S 12 of 34
alpha : der := f*dx + g*dy + h*dz
--R
--R
--R (12)
--R      3 2      2      3 2      2
--R      (- 2x y z + x y z)dz + (- 7z sin(x y ) + y z cos(z))dy
--R      +
--R      3 2 5      2
--R      (- 5x y z + x y z)dx
--R                                         Type: DeRhamComplex(Integer,[x,y,z])
--E 12

--S 13 of 34
beta : der := cos(tan(x*y*z)+x*y*z)*dx + x*dy
--R
--R
--R (13) x dy + cos(tan(x y z) + x y z)dx
--R                                         Type: DeRhamComplex(Integer,[x,y,z])
--E 13

--S 14 of 34

```

Type: BasicOperator

```

--E 18

--S 19 of 34
b : BOP := operator('b)
--R
--R
--R (19) b
--R
--R                                          Type: BasicOperator
--E 19

--S 20 of 34
c : BOP := operator('c)
--R
--R
--R (20) c
--R
--R                                          Type: BasicOperator
--E 20

--S 21 of 34
sigma := a(x,y,z) * dx + b(x,y,z) * dy + c(x,y,z) * dz
--R
--R
--R (21) c(x,y,z)dz + b(x,y,z)dy + a(x,y,z)dx
--R
--R                                          Type: DeRhamComplex(Integer,[x,y,z])
--E 21

--S 22 of 34
theta := a(x,y,z) * dx * dy + b(x,y,z) * dx * dz + c(x,y,z) * dy * dz
--R
--R
--R (22) c(x,y,z)dy dz + b(x,y,z)dx dz + a(x,y,z)dx dy
--R
--R                                          Type: DeRhamComplex(Integer,[x,y,z])
--E 22

--S 23 of 34
totalDifferential(a(x,y,z))$der
--R
--R
--R (23) a (x,y,z)dz + a (x,y,z)dy + a (x,y,z)dx
--R      ,3          ,2          ,1
--R
--R                                          Type: DeRhamComplex(Integer,[x,y,z])
--E 23

--S 24 of 34
exteriorDifferential sigma
--R
--R
--R (24)
--R      (c (x,y,z) - b (x,y,z))dy dz + (c (x,y,z) - a (x,y,z))dx dz
--R      ,2          ,3          ,1          ,3

```



```

--R      +
--R      (b (x,y,z) - a (x,y,z))dx dy
--R      ,1      ,2
--R
--R                                          Type: DeRhamComplex(Integer,[x,y,z])
--E 24

--S 25 of 34
exteriorDifferential theta
--R
--R
--R      (25) (c (x,y,z) - b (x,y,z) + a (x,y,z))dx dy dz
--R      ,1      ,2      ,3
--R
--R                                          Type: DeRhamComplex(Integer,[x,y,z])
--E 25

--S 26 of 34
one : der := 1
--R
--R
--R      (26) 1
--R
--R                                          Type: DeRhamComplex(Integer,[x,y,z])
--E 26

--S 27 of 34
g1 : der := a([x,t,y,u,v,z,e]) * one
--R
--R
--R      (27) a(x,t,y,u,v,z,e)
--R
--R                                          Type: DeRhamComplex(Integer,[x,y,z])
--E 27

--S 28 of 34
h1 : der := a([x,y,x,t,x,z,y,r,u,x]) * one
--R
--R
--R      (28) a(x,y,x,t,x,z,y,r,u,x)
--R
--R                                          Type: DeRhamComplex(Integer,[x,y,z])
--E 28

--S 29 of 34
exteriorDifferential g1
--R
--R
--R      (29) a (x,t,y,u,v,z,e)dz + a (x,t,y,u,v,z,e)dy + a (x,t,y,u,v,z,e)dx
--R      ,6      ,3      ,1
--R
--R                                          Type: DeRhamComplex(Integer,[x,y,z])
--E 29

--S 30 of 34
exteriorDifferential h1

```

```

--R
--R
--R (30)
--R      a (x,y,x,t,x,z,y,r,u,x)dz
--R      ,6
--R +
--R      (a (x,y,x,t,x,z,y,r,u,x) + a (x,y,x,t,x,z,y,r,u,x))dy
--R      ,7      ,2
--R +
--R      a (x,y,x,t,x,z,y,r,u,x) + a (x,y,x,t,x,z,y,r,u,x)
--R      ,10      ,5
--R +
--R      a (x,y,x,t,x,z,y,r,u,x) + a (x,y,x,t,x,z,y,r,u,x)
--R      ,3      ,1
--R *
--R      dx
--R
--R                                          Type: DeRhamComplex(Integer,[x,y,z])
--E 30

--S 31 of 34
coefficient(gamma, dx*dy)
--R
--R
--R      2      3 2      2      4 2 5      3
--R (31) (7z sin(x y ) - y z cos(z))cos(tan(x y z) + x y z) - 5x y z + x y z
--R                                          Type: Expression Integer
--E 31

--S 32 of 34
coefficient(gamma, one)
--R
--R
--R (32) 0
--R
--R                                          Type: Expression Integer
--E 32

--S 33 of 34
coefficient(g1,one)
--R
--R
--R (33) a(x,t,y,u,v,z,e)
--R
--R                                          Type: Expression Integer
--E 33

--S 34 of 34
gamma := alpha * beta
--R
--R
--R (34)
--R      4      2      2      3      2

```

```

--R      (2x y z - x y z)dy dz + (2x y z - x y z)cos(tan(x y z) + x y z)dx dz
--R      +
--R      2      3 2      2      4 2 5      3
--R      ((7z sin(x y ) - y z cos(z))cos(tan(x y z) + x y z) - 5x y z + x y z)dx dy
--R                                          Type: DeRhamComplex(Integer,[x,y,z])
--E 34
)spool
)lisp (bye)

```

---

— DeRhamComplex.help —

=====

DeRhamComplex examples

=====

The domain constructor DeRhamComplex creates the class of differential forms of arbitrary degree over a coefficient ring. The De Rham complex constructor takes two arguments: a ring, coefRing, and a list of coordinate variables.

This is the ring of coefficients.

```

coefRing := Integer
Integer
Type: Domain

```

These are the coordinate variables.

```

lv : List Symbol := [x,y,z]
[x,y,z]
Type: List Symbol

```

This is the De Rham complex of Euclidean three-space using coordinates x, y and z.

```

der := DERHAM(coefRing,lv)
DeRhamComplex(Integer,[x,y,z])
Type: Domain

```

This complex allows us to describe differential forms having expressions of integers as coefficients. These coefficients can involve any number of variables, for example,  $f(x,t,r,y,u,z)$ . As we've chosen to work with ordinary Euclidean three-space, expressions involving these forms are treated as functions of x, y and z with the additional arguments t, r and u regarded as symbolic constants.

Here are some examples of coefficients.

```

R := Expression coefRing
Expression Integer
Type: Domain

f : R := x**2*y*z-5*x**3*y**2*z**5
      3 2 5      2
      - 5x y z + x y z
Type: Expression Integer

g : R := z**2*y*cos(z)-7*sin(x**3*y**2)*z**2
      2      3 2      2
      - 7z sin(x y ) + y z cos(z)
Type: Expression Integer

h : R := x*y*z-2*x**3*y*z**2
      3      2
      - 2x y z + x y z
Type: Expression Integer

```

We now define the multiplicative basis elements for the exterior algebra over R.

```

dx : der := generator(1)
dx
Type: DeRhamComplex(Integer,[x,y,z])

dy : der := generator(2)
dy
Type: DeRhamComplex(Integer,[x,y,z])

dz : der := generator(3)
dz
Type: DeRhamComplex(Integer,[x,y,z])

```

This is an alternative way to give the above assignments.

```

[dx,dy,dz] := [generator(i)$der for i in 1..3]
[dx,dy,dz]
Type: List DeRhamComplex(Integer,[x,y,z])

```

Now we define some one-forms.

```

alpha : der := f*dx + g*dy + h*dz
      3 2      2      3 2      2
      (- 2x y z + x y z)dz + (- 7z sin(x y ) + y z cos(z))dy
+
      3 2 5      2
      (- 5x y z + x y z)dx

```

Type: DeRhamComplex(Integer,[x,y,z])

```
beta : der := cos(tan(x*y*z)+x*y*z)*dx + x*dy
      x dy + cos(tan(x y z) + x y z)dx
      Type: DeRhamComplex(Integer,[x,y,z])
```

A well-known theorem states that the composition of exteriorDifferential with itself is the zero map for continuous forms. Let's verify this theorem for alpha.

```
exteriorDifferential alpha
      2          3 2          3 2
      (y z sin(z) + 14z sin(x y ) - 2y z cos(z) - 2x z + x z)dy dz
+
      3 2 4      2 2      2
      (25x y z - 6x y z + y z - x y)dx dz
+
      2 2 2      3 2      3 5      2
      (- 21x y z cos(x y ) + 10x y z - x z)dx dy
      Type: DeRhamComplex(Integer,[x,y,z])
```

We see a lengthy output of the last expression, but nevertheless, the composition is zero.

```
exteriorDifferential %
0
      Type: DeRhamComplex(Integer,[x,y,z])
```

Now we check that exteriorDifferential is a "graded derivation" D, that is, D satisfies:

```
D(a*b) = D(a)*b + (-1)**degree(a)*a*D(b)

gamma := alpha * beta
      4 2 2          3 2
      (2x y z - x y z)dy dz + (2x y z - x y z)cos(tan(x y z) + x y z)dx dz
+
      2 3 2      2          4 2 5      3
      ((7z sin(x y ) - y z cos(z))cos(tan(x y z) + x y z) - 5x y z + x y z)dx dy
      Type: DeRhamComplex(Integer,[x,y,z])
```

We try this for the one-forms alpha and beta.

```
exteriorDifferential(gamma) - (exteriorDifferential(alpha)*beta - alpha * exteriorDifferential(beta))
0
      Type: DeRhamComplex(Integer,[x,y,z])
```

Now we define some "basic operators"

```
a : BOP := operator('a)
```

```

a
                                     Type: BasicOperator

b : BOP := operator('b)
b
                                     Type: BasicOperator

c : BOP := operator('c)
c
                                     Type: BasicOperator

```

We also define some indeterminate one- and two-forms using these operators.

```

sigma := a(x,y,z) * dx + b(x,y,z) * dy + c(x,y,z) * dz
        c(x,y,z)dz + b(x,y,z)dy + a(x,y,z)dx
                                     Type: DeRhamComplex(Integer,[x,y,z])

theta  := a(x,y,z) * dx * dy + b(x,y,z) * dx * dz + c(x,y,z) * dy * dz
        c(x,y,z)dy dz + b(x,y,z)dx dz + a(x,y,z)dx dy
                                     Type: DeRhamComplex(Integer,[x,y,z])

```

This allows us to get formal definitions for the "gradient" ...

```

totalDifferential(a(x,y,z))$der
(23)  a (x,y,z)dz + a (x,y,z)dy + a (x,y,z)dx
      ,3          ,2          ,1
                                     Type: DeRhamComplex(Integer,[x,y,z])
the "curl" ...

```

```

exteriorDifferential sigma
(c (x,y,z) - b (x,y,z))dy dz + (c (x,y,z) - a (x,y,z))dx dz
 ,2          ,3          ,1          ,3
+
(b (x,y,z) - a (x,y,z))dx dy
 ,1          ,2
                                     Type: DeRhamComplex(Integer,[x,y,z])

```

and the "divergence."

```

exteriorDifferential theta
(c (x,y,z) - b (x,y,z) + a (x,y,z))dx dy dz
 ,1          ,2          ,3
                                     Type: DeRhamComplex(Integer,[x,y,z])

```

Note that the De Rham complex is an algebra with unity. This element 1 is the basis for elements for zero-forms, that is, functions in our space.

```

one : der := 1

```

1

Type: DeRhamComplex(Integer,[x,y,z])

To convert a function to a function lying in the De Rham complex, multiply the function by "one."

```
g1 : der := a([x,t,y,u,v,z,e]) * one
      a(x,t,y,u,v,z,e)
```

Type: DeRhamComplex(Integer,[x,y,z])

A current limitation of Axiom forces you to write functions with more than four arguments using square brackets in this way.

```
h1 : der := a([x,y,x,t,x,z,y,r,u,x]) * one
      a(x,y,x,t,x,z,y,r,u,x)
```

Type: DeRhamComplex(Integer,[x,y,z])

Now note how the system keeps track of where your coordinate functions are located in expressions.

```
exteriorDifferential g1
      a (x,t,y,u,v,z,e)dz + a (x,t,y,u,v,z,e)dy + a (x,t,y,u,v,z,e)dx
      ,6 ,3 ,1
      Type: DeRhamComplex(Integer,[x,y,z])
```

```
exteriorDifferential h1
      a (x,y,x,t,x,z,y,r,u,x)dz
      ,6
+
      (a (x,y,x,t,x,z,y,r,u,x) + a (x,y,x,t,x,z,y,r,u,x))dy
      ,7 ,2
+
      a (x,y,x,t,x,z,y,r,u,x) + a (x,y,x,t,x,z,y,r,u,x)
      ,10 ,5
+
      a (x,y,x,t,x,z,y,r,u,x) + a (x,y,x,t,x,z,y,r,u,x)
      ,3 ,1
*
      dx
      Type: DeRhamComplex(Integer,[x,y,z])
```

In this example of Euclidean three-space, the basis for the De Rham complex consists of the eight forms: 1, dx, dy, dz, dx\*dy, dx\*dz, dy\*dz, and dx\*dy\*dz.

```
coefficient(gamma, dx*dy)
      2 3 2 2 4 2 5 3
      (7z sin(x y ) - y z cos(z))cos(tan(x y z) + x y z) - 5x y z + x y z
      Type: Expression Integer
```

```
coefficient(gamma, one)
```

```

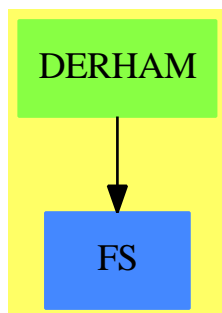
0
                                     Type: Expression Integer

coefficient(g1,one)
  a(x,t,y,u,v,z,e)
                                     Type: Expression Integer

See Also:
o )help Operator
o )show DeRhamComplex

```

### 5.6.1 DeRhamComplex (DERHAM)



See

⇒ “ExtAlgBasis” (EAB) 6.8.1 on page 711  
 ⇒ “AntiSymm” (ANTISYM) 2.8.1 on page 40

#### Exports:

|                    |               |                      |                  |
|--------------------|---------------|----------------------|------------------|
| 0                  | 1             | characteristic       | coefficient      |
| coerce             | degree        | exteriorDifferential | generator        |
| hash               | homogeneous?  | latex                | leadingBasisTerm |
| leadingCoefficient | map           | one?                 | recip            |
| reductum           | retract       | retractable?         | retractIfCan     |
| sample             | subtractIfCan | totalDifferential    | zero?            |
| ?~=?               | ?*?           | ?**?                 | ?^?              |
| ?+?                | ?-?           | -?                   | ?=?              |

— domain DERHAM DeRhamComplex —

```

)abbrev domain DERHAM DeRhamComplex
++ Author: Larry A. Lambe
++ Date   : 01/26/91.

```



```

++ Revised : 12/01/91.
++
++ based on code from '89 (AntiSymmetric)
++
++ Needs: LeftAlgebra, ExtAlgBasis, FreeMod(Ring,OrderedSet)
++
++ Description:
++ The deRham complex of Euclidean space, that is, the
++ class of differential forms of arbitrary degree over a coefficient ring.
++ See Flanders, Harley, Differential Forms, With Applications to the Physical
++ Sciences, New York, Academic Press, 1963.

DeRhamComplex(CoefRing,listIndVar>List Symbol): Export == Implement where
  CoefRing : Join(Ring, OrderedSet)
  ASY      ==> AntiSymm(R,listIndVar)
  DIFRING  ==> DifferentialRing
  LALG     ==> LeftAlgebra
  FMR      ==> FreeMod(R,EAB)
  I        ==> Integer
  L        ==> List
  EAB      ==> ExtAlgBasis -- these are exponents of basis elements in order
  NNI      ==> NonNegativeInteger
  O        ==> OutputForm
  R        ==> Expression(CoefRing)

Export == Join(LALG(R), RetractableTo(R)) with
  leadingCoefficient : %          -> R
    ++ leadingCoefficient(df) returns the leading
    ++ coefficient of differential form df.
  leadingBasisTerm   : %          -> %
    ++ leadingBasisTerm(df) returns the leading
    ++ basis term of differential form df.
  reductum           : %          -> %
    ++ reductum(df), where df is a differential form,
    ++ returns df minus the leading
    ++ term of df if df has two or more terms, and
    ++ 0 otherwise.
  coefficient         : (%,% )    -> R
    ++ coefficient(df,u), where df is a differential form,
    ++ returns the coefficient of df containing the basis term u
    ++ if such a term exists, and 0 otherwise.
  generator           : NNI        -> %
    ++ generator(n) returns the nth basis term for a differential form.
  homogeneous?       : %          -> Boolean
    ++ homogeneous?(df) tests if all of the terms of
    ++ differential form df have the same degree.
  retractable?       : %          -> Boolean
    ++ retractable?(df) tests if differential form df is a 0-form,
    ++ i.e., if degree(df) = 0.
  degree             : %          -> I

```

```

    ++ degree(df) returns the homogeneous degree of differential form df.
map      : (R -> R, %) -> %
    ++ map(f,df) replaces each coefficient x of differential
    ++ form df by \spad{f(x)}.
totalDifferential : R -> %
    ++ totalDifferential(x) returns the total differential
    ++ (gradient) form for element x.
exteriorDifferential : % -> %
    ++ exteriorDifferential(df) returns the exterior
    ++ derivative (gradient, curl, divergence, ...) of
    ++ the differential form df.

Implement == ASY add
Rep := ASY

dim := #listIndVar

totalDifferential(f) ==
  divs:=[differentiate(f,listIndVar.i)*generator(i)$ASY for i in 1..dim]
  reduce("+",divs)

termDiff : (R, %) -> %
termDiff(r,e) ==
  totalDifferential(r) * e

exteriorDifferential(x) ==
  x = 0 => 0
  termDiff(leadingCoefficient(x)$Rep,leadingBasisTerm x) + exteriorDifferential(reductum x)

lv := [concat("d",string(liv))$String::Symbol for liv in listIndVar]

displayList:EAB -> 0
displayList(x):0 ==
  le: L I := exponents(x)$EAB
  -- reduce(*,[lv.i)::0 for i in 1..dim | le.i = 1])$L(0)
  -- reduce(*,[lv.i)::0 for i in 1..dim | one?(le.i)])$L(0)
  reduce(*,[lv.i)::0 for i in 1..dim | ((le.i) = 1)])$L(0)

makeTerm:(R,EAB) -> 0
makeTerm(r,x) ==
  -- we know that r ^= 0
  x = Nul(dim)$EAB => r::0
  -- one? r => displayList(x)
  (r = 1) => displayList(x)
  -- r = 1 => displayList(x)
  r::0 * displayList(x)

terms : % -> List Record(k: EAB, c: R)
terms(a) ==
  -- it is the case that there are at least two terms in a

```

```

a pretend List Record(k: EAB, c: R)

coerce(a):0 ==
  a          = 0$Rep => 0$I::0
  ta := terms a
-- reductum(a) = 0$Rep => makeTerm(leadingCoefficient a, a.first.k)
  null ta.rest => makeTerm(ta.first.c, ta.first.k)
  reduce(_+, [makeTerm(t.c, t.k) for t in ta])$L(0)

```

---

— DERHAM.dotabb —

```

"DERHAM" [color="#88FF44",href="bookvol10.3.pdf#nameddest=DERHAM"]
"FS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FS"]
"DERHAM" -> "FS"

```

---

## 5.7 domain DSTREE DesingTree

— DesingTree.input —

```

)set break resume
)sys rm -f DesingTree.output
)spool DesingTree.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show DesingTree
--R DesingTree S: SetCategory is a domain constructor
--R Abbreviation for DesingTree is DSTREE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for DSTREE
--R
--R----- Operations -----
--R children : % -> List %          copy : % -> %
--R cyclic? : % -> Boolean          distance : (% , %) -> Integer
--R ?.value : (% , value) -> S      empty : () -> %
--R empty? : % -> Boolean           encode : % -> String
--R eq? : (% , %) -> Boolean        fullOut : % -> OutputForm
--R fullOutput : () -> Boolean      fullOutput : Boolean -> Boolean

```

```

--R leaf? : % -> Boolean                leaves : % -> List S
--R map : ((S -> S),%) -> %              nodes : % -> List %
--R sample : () -> %                     tree : List S -> %
--R tree : S -> %                        tree : (S,List %) -> %
--R value : % -> S
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R child? : (%,%) -> Boolean if S has SETCAT
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if $ has finiteAggregate and S has SETCAT
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R members : % -> List S if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R node? : (%,%) -> Boolean if S has SETCAT
--R parts : % -> List S if $ has finiteAggregate
--R setchildren! : (%,List %) -> % if $ has shallowlyMutable
--R setelt : (%,value,S) -> S if $ has shallowlyMutable
--R setvalue! : (%,S) -> S if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R ~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)

```

---

— DesingTree.help —

```

=====
DesingTree examples
=====

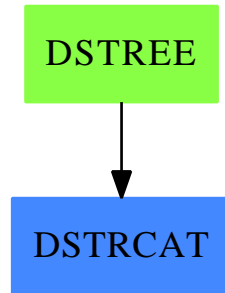
```

See Also:

- o )show DesingTree

---

### 5.7.1 DesingTree (DSTREE)



#### Exports:

|         |           |         |         |              |
|---------|-----------|---------|---------|--------------|
| #?      | ?.value   | ?=?     | ?~=?    | any?         |
| child?  | children  | coerce  | copy    | count        |
| cyclic? | distance  | empty   | empty?  | encode       |
| eq?     | eval      | every?  | fullOut | fullOutput   |
| hash    | latex     | leaf?   | leaves  | less?        |
| map     | map!      | member? | members | more?        |
| node?   | nodes     | parts   | sample  | setchildren! |
| setelt  | setvalue! | size?   | tree    | value        |

— domain DSTREE DesingTree —

```

)abbrev domain DSTREE DesingTree
++ Authors: Gaetan Hache
++ Date Created: jan 1998
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
++ This category is part of the PAFF package
DesingTree(S: SetCategory): T==C where

T == DesingTreeCategory(S) with

  encode: % -> String
    ++ encode(t) returns a string indicating the "shape" of the tree

  fullOut: % -> OutputForm
    ++ fullOut(tr) yields a full output of tr (see function fullOutput).

  fullOutput: Boolean -> Boolean
    ++ fullOutput(b) sets a flag such that when true,
    ++ a coerce to OutputForm yields the full output of
    ++ tr, otherwise encode(tr) is output (see encode function).
  
```

```

    ++ The default is false.

fullOutput: () -> Boolean
    ++ fullOutput returns the value of the flag set by fullOutput(b).

C == add
Rep ==> Record(value: S, args: List %)

fullOut(t:%): OutputForm ==
    empty? children t => (value t) :: OutputForm
    prefix((value t)::OutputForm, [fullOut(tr) for tr in children t])

fullOutputFlag:Boolean:=false()

fullOutput(f)== fullOutputFlag:=f

fullOutput == fullOutputFlag

leaves(t)==
    empty?(chdr:=children(t)) => list(value(t))
    concat([leaves(subt) for subt in chdr])

t1=t2 == value t1 = value t2 and children t1 = children t2

coerce(t:%):OutputForm==
    ^fullOutput() => encode(t) :: OutputForm
    fullOut(t)

tree(s,ls) == ([s,ls]:Rep):%

tree(s:S) == ([s,[]]:Rep):%

tree(ls:List(S))==
    empty?(ls) =>
        error "Cannot create a tree with an empty list"
    f:=first(ls)
    empty?(rs:=rest(ls)) =>
        tree(f)
    tree(f,[tree(rs)])

value t == (t:Rep).value

children t == ((t:Rep).args):List %

setchildren_!(t,ls) == ((t:Rep).args:=ls;t pretend %)

setvalue_!(t,s) == ((t:Rep).value:=s;s)

encode(t)==
    empty?(chtr:=children(t)) => empty()$String

```

```
concat([concat(["U",encode(arb),"."]) for arb in chtr])
```

---

— DSTREE.dotabb —

```
"DSTREE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=DSTREE"];
"DSTRCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=DSTRCAT"]
"DSTREE" -> "DSTRCAT"
```

---

## 5.8 domain DSMP DifferentialSparseMultivariatePolynomial

— DifferentialSparseMultivariatePolynomial.input —

```
)set break resume
)sys rm -f DifferentialSparseMultivariatePolynomial.output
)spool DifferentialSparseMultivariatePolynomial.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show DifferentialSparseMultivariatePolynomial
--R DifferentialSparseMultivariatePolynomial(R: Ring,S: OrderedSet,V: DifferentialVariableCa
--R Abbreviation for DifferentialSparseMultivariatePolynomial is DSMP
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for DSMP
--R
--R----- Operations -----
--R ?? : (%,R) -> %                ?? : (R,%) -> %
--R ?? : (%,%) -> %                ?? : (Integer,%) -> %
--R ?? : (PositiveInteger,%) -> %  *** : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %              ?-? : (%,%) -> %
--R -? : % -> %                    ?? : (%,%) -> Boolean
--R D : (%,(R -> R)) -> %          D : % -> % if R has DIFRING
--R D : (%,List V) -> %            D : (%,V) -> %
--R 1 : () -> %                    0 : () -> %
--R ?? : (%,PositiveInteger) -> %  coefficients : % -> List R
--R coerce : S -> %                 coerce : V -> %
--R coerce : R -> %                 coerce : Integer -> %
```

```

--R coerce : % -> OutputForm
--R differentiate : (% , List V) -> %
--R eval : (% , List V , List %) -> %
--R eval : (% , List V , List R) -> %
--R eval : (% , List % , List %) -> %
--R eval : (% , Equation %) -> %
--R ground : % -> R
--R hash : % -> SingleInteger
--R isobaric? : % -> Boolean
--R leader : % -> V
--R leadingMonomial : % -> %
--R monomial? : % -> Boolean
--R one? : % -> Boolean
--R primitiveMonomials : % -> List %
--R reductum : % -> %
--R retract : % -> V
--R sample : () -> %
--R variables : % -> List V
--R zero? : % -> Boolean
--R ?? : (Fraction Integer , %) -> % if R has ALGEBRA FRAC INT
--R ?? : (% , Fraction Integer) -> % if R has ALGEBRA FRAC INT
--R ?? : (NonNegativeInteger , %) -> %
--R ??? : (% , NonNegativeInteger) -> %
--R ?/? : (% , R) -> % if R has FIELD
--R ?<? : (% , %) -> Boolean if R has ORDSET
--R ?<=? : (% , %) -> Boolean if R has ORDSET
--R ?>? : (% , %) -> Boolean if R has ORDSET
--R ?>=? : (% , %) -> Boolean if R has ORDSET
--R D : (% , (R -> R) , NonNegativeInteger) -> %
--R D : (% , List Symbol , List NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (% , Symbol , NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (% , List Symbol) -> % if R has PDRING SYMBOL
--R D : (% , Symbol) -> % if R has PDRING SYMBOL
--R D : (% , NonNegativeInteger) -> % if R has DIFRING
--R D : (% , List V , List NonNegativeInteger) -> %
--R D : (% , V , NonNegativeInteger) -> %
--R ?^? : (% , NonNegativeInteger) -> %
--R associates? : (% , %) -> Boolean if R has INTDOM
--R binomThmExpt : (% , % , NonNegativeInteger) -> % if R has COMRING
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(% , "failed") if $ has CHARNZ and R has PFECAT or R has CHARNZ
--R coefficient : (% , List V , List NonNegativeInteger) -> %
--R coefficient : (% , V , NonNegativeInteger) -> %
--R coefficient : (% , IndexedExponents V) -> R
--R coerce : % -> % if R has INTDOM
--R coerce : Fraction Integer -> % if R has ALGEBRA FRAC INT or R has RETRACT FRAC INT
--R coerce : SparseMultivariatePolynomial(R , S) -> %
--R conditionP : Matrix % -> Union(Vector % , "failed") if $ has CHARNZ and R has PFECAT
--R content : (% , V) -> % if R has GCDDOM
--R content : % -> R if R has GCDDOM
degree : % -> IndexedExponents V
differentiate : (% , V) -> %
eval : (% , V , %) -> %
eval : (% , V , R) -> %
eval : (% , % , %) -> %
eval : (% , List Equation %) -> %
ground? : % -> Boolean
initial : % -> %
latex : % -> String
leadingCoefficient : % -> R
map : ((R -> R) , %) -> %
monomials : % -> List %
order : % -> NonNegativeInteger
recip : % -> Union(% , "failed")
retract : % -> S
retract : % -> R
separant : % -> %
weight : % -> NonNegativeInteger
?~=? : (% , %) -> Boolean

```



```

--R convert : % -> InputForm if R has KONVERT INFORM and V has KONVERT INFORM
--R convert : % -> Pattern Integer if R has KONVERT PATTERN INT and V has KONVERT PATTERN INT
--R convert : % -> Pattern Float if R has KONVERT PATTERN FLOAT and V has KONVERT PATTERN FLOAT
--R degree : (% , S) -> NonNegativeInteger
--R degree : (% , List V) -> List NonNegativeInteger
--R degree : (% , V) -> NonNegativeInteger
--R differentialVariables : % -> List S
--R differentiate : (% , (R -> R)) -> %
--R differentiate : (% , (R -> R), NonNegativeInteger) -> %
--R differentiate : (% , List Symbol, List NonNegativeInteger) -> % if R has PDRING SYMBOL
--R differentiate : (% , Symbol, NonNegativeInteger) -> % if R has PDRING SYMBOL
--R differentiate : (% , List Symbol) -> % if R has PDRING SYMBOL
--R differentiate : (% , Symbol) -> % if R has PDRING SYMBOL
--R differentiate : (% , NonNegativeInteger) -> % if R has DIFRING
--R differentiate : % -> % if R has DIFRING
--R differentiate : (% , List V, List NonNegativeInteger) -> %
--R differentiate : (% , V, NonNegativeInteger) -> %
--R discriminant : (% , V) -> % if R has COMRING
--R eval : (% , List S, List R) -> % if R has DIFRING
--R eval : (% , S, R) -> % if R has DIFRING
--R eval : (% , List S, List %) -> % if R has DIFRING
--R eval : (% , S, %) -> % if R has DIFRING
--R exquo : (% , %) -> Union(% , "failed") if R has INTDOM
--R exquo : (% , R) -> Union(% , "failed") if R has INTDOM
--R factor : % -> Factored % if R has PFECAT
--R factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial %
--R factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial %
--R gcd : (% , %) -> % if R has GCDDOM
--R gcd : List % -> % if R has GCDDOM
--R gcdPolynomial : (SparseUnivariatePolynomial % , SparseUnivariatePolynomial %) -> SparseUnivariatePolynomial %
--R isExpt : % -> Union(Record(var: V, exponent: NonNegativeInteger) , "failed")
--R isPlus : % -> Union(List % , "failed")
--R isTimes : % -> Union(List % , "failed")
--R lcm : (% , %) -> % if R has GCDDOM
--R lcm : List % -> % if R has GCDDOM
--R mainVariable : % -> Union(V , "failed")
--R makeVariable : % -> (NonNegativeInteger -> %) if R has DIFRING
--R makeVariable : S -> (NonNegativeInteger -> %)
--R mapExponents : ((IndexedExponents V -> IndexedExponents V) , %) -> %
--R max : (% , %) -> % if R has ORDSET
--R min : (% , %) -> % if R has ORDSET
--R minimumDegree : (% , List V) -> List NonNegativeInteger
--R minimumDegree : (% , V) -> NonNegativeInteger
--R minimumDegree : % -> IndexedExponents V
--R monicDivide : (% , %, V) -> Record(quotient: % , remainder: %)
--R monomial : (% , List V, List NonNegativeInteger) -> %
--R monomial : (% , V, NonNegativeInteger) -> %
--R monomial : (R , IndexedExponents V) -> %
--R multivariate : (SparseUnivariatePolynomial % , V) -> %
--R multivariate : (SparseUnivariatePolynomial R , V) -> %

```

```

--R numberOfMonomials : % -> NonNegativeInteger
--R order : (% ,S) -> NonNegativeInteger
--R patternMatch : (% ,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMatchResult(Integer,%) if
--R patternMatch : (% ,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchResult(Float,%) if R has
--R pomopo! : (% ,R,IndexedExponents V,% ) -> %
--R prime? : % -> Boolean if R has PFECAT
--R primitivePart : (% ,V) -> % if R has GCDDOM
--R primitivePart : % -> % if R has GCDDOM
--R reducedSystem : Matrix % -> Matrix R
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) if R has LINE
--R reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT
--R resultant : (% ,%,V) -> % if R has COMRING
--R retract : % -> SparseMultivariatePolynomial(R,S)
--R retract : % -> Integer if R has RETRACT INT
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(SparseMultivariatePolynomial(R,S),"failed")
--R retractIfCan : % -> Union(S,"failed")
--R retractIfCan : % -> Union(V,"failed")
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(R,"failed")
--R solveLinearPolynomialEquation : (List SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) ->
--R squareFree : % -> Factored % if R has GCDDOM
--R squareFreePart : % -> % if R has GCDDOM
--R squareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % if R ha
--R subtractIfCan : (% ,%) -> Union(%,"failed")
--R totalDegree : (% ,List V) -> NonNegativeInteger
--R totalDegree : % -> NonNegativeInteger
--R unit? : % -> Boolean if R has INTDOM
--R unitCanonical : % -> % if R has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has INTDOM
--R univariate : % -> SparseUnivariatePolynomial R
--R univariate : (% ,V) -> SparseUnivariatePolynomial %
--R weight : (% ,S) -> NonNegativeInteger
--R weights : (% ,S) -> List NonNegativeInteger
--R weights : % -> List NonNegativeInteger
--R
--E 1

)spool
)lisp (bye)

```

---

— DifferentialSparseMultivariatePolynomial.help —

=====

DifferentialSparseMultivariatePolynomial examples

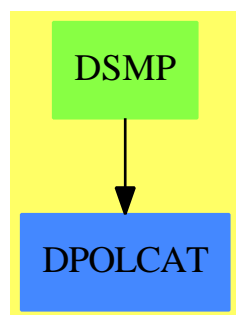
=====

See Also:

o )show DifferentialSparseMultivariatePolynomial

\_\_\_\_\_

### 5.8.1 DifferentialSparseMultivariatePolynomial (DSMP)



See

- ⇒ “OrderlyDifferentialVariable” (ODVAR) 16.17.1 on page 1816
- ⇒ “SequentialDifferentialVariable” (SDVAR) 20.7.1 on page 2348
- ⇒ “OrderlyDifferentialPolynomial” (ODPOL) 16.16.1 on page 1813
- ⇒ “SequentialDifferentialPolynomial” (SDPOL) 20.6.1 on page 2345

**Exports:**

|                            |                               |                       |
|----------------------------|-------------------------------|-----------------------|
| 0                          | 1                             | associates?           |
| binomThmExpt               | characteristic                | charthRoot            |
| coefficient                | coefficients                  | coerce                |
| conditionP                 | content                       | convert               |
| D                          | degree                        | differentialVariables |
| differentiate              | discriminant                  | eval                  |
| exquo                      | factor                        | factorPolynomial      |
| factorSquareFreePolynomial | gcd                           | gcdPolynomial         |
| ground                     | ground?                       | hash                  |
| initial                    | isExpt                        | isobaric?             |
| isPlus                     | isTimes                       | latex                 |
| lcm                        | leader                        | leadingCoefficient    |
| leadingMonomial            | makeVariable                  | map                   |
| mapExponents               | max                           | min                   |
| minimumDegree              | monicDivide                   | monomial              |
| monomials                  | monomial?                     | multivariate          |
| numberOfMonomials          | one?                          | order                 |
| patternMatch               | pomopo!                       | prime?                |
| primitiveMonomials         | primitivePart                 | recip                 |
| reducedSystem              | reductum                      | resultant             |
| retract                    | retractIfCan                  | sample                |
| separant                   | solveLinearPolynomialEquation | squareFree            |
| squareFreePart             | squareFreePolynomial          | subtractIfCan         |
| totalDegree                | unit?                         | unitCanonical         |
| unitNormal                 | univariate                    | variables             |
| weight                     | weights                       | zero?                 |
| ?*?                        | ?**?                          | ?+?                   |
| ?-?                        | -?                            | ?=?                   |
| ?^?                        | ?~=?                          | ?/?                   |
| ?<?                        | ?<=?                          | ?>?                   |
| ?>=?                       |                               |                       |

— domain DSMP DifferentialSparseMultivariatePolynomial —

```
)abbrev domain DSMP DifferentialSparseMultivariatePolynomial
++ Author: William Sit
++ Date Created: 19 July 1990
++ Date Last Updated: 13 September 1991
++ Basic Operations:DifferentialPolynomialCategory
++ Related Constructors:
++ See Also:
++ AMS Classifications:12H05
++ Keywords: differential indeterminates, ranking, differential polynomials,
++          order, weight, leader, separant, initial, isobaric
++ References:Kolchin, E.R. "Differential Algebra and Algebraic Groups"
++          (Academic Press, 1973).
++ Description:
```

```

++ \spadtype{DifferentialSparseMultivariatePolynomial} implements
++ an ordinary differential polynomial ring by combining a
++ domain belonging to the category \spadtype{DifferentialVariableCategory}
++ with the domain \spadtype{SparseMultivariatePolynomial}.

```

```

DifferentialSparseMultivariatePolynomial(R, S, V):
  Exports == Implementation where
  R: Ring
  S: OrderedSet
  V: DifferentialVariableCategory S
  E ==> IndexedExponents(V)
  PC ==> PolynomialCategory(R, IndexedExponents(V), V)
  PCL ==> PolynomialCategoryLifting
  P ==> SparseMultivariatePolynomial(R, V)
  SUP ==> SparseUnivariatePolynomial
  SMP ==> SparseMultivariatePolynomial(R, S)

  Exports ==> Join(DifferentialPolynomialCategory(R, S, V, E),
    RetractableTo SMP)

  Implementation ==> P add
    retractIfCan(p:$):Union(SMP, "failed") ==
      zero? order p =>
        map(x+>retract(x)@S :: SMP, y+>y::SMP, p)$PCL(
          IndexedExponents V, V, R, $, SMP)
        "failed"

    coerce(p:SMP):$ ==
      map(x+>x::V::$, y+>y::$, p)$PCL(IndexedExponents S, S, R, SMP, $)

```

---

— DSMP.dotabb —

```

"DSMP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=DSMP"]
"DPOLCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=DPOLCAT"]
"DSMP" -> "DPOLCAT"

```

---

## 5.9 domain DIRPROD DirectProduct

---

— DirectProduct.input —

```

)set break resume

```

```

)sys rm -f DirectProduct.output
)spool DirectProduct.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show DirectProduct
--R DirectProduct(dim: NonNegativeInteger,R: Type) is a domain constructor
--R Abbreviation for DirectProduct is DIRPROD
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for DIRPROD
--R
--R----- Operations -----
--R -? : % -> % if R has RING          1 : () -> % if R has MONOID
--R 0 : () -> % if R has CABMON        coerce : % -> Vector R
--R copy : % -> %                     directProduct : Vector R -> %
--R ?.? : (%,Integer) -> R            elt : (%,Integer,R) -> R
--R empty : () -> %                  empty? : % -> Boolean
--R entries : % -> List R             eq? : (%,% ) -> Boolean
--R index? : (Integer,% ) -> Boolean  indices : % -> List Integer
--R map : ((R -> R),%) -> %          qelt : (%,Integer) -> R
--R sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?? : (PositiveInteger,% ) -> % if R has ABELSG
--R ?? : (NonNegativeInteger,% ) -> % if R has CABMON
--R ?? : (R,% ) -> % if R has RING
--R ?? : (% ,R) -> % if R has RING
--R ?? : (% ,%) -> % if R has MONOID
--R ?? : (Integer,% ) -> % if R has RING
--R ??? : (% ,PositiveInteger) -> % if R has MONOID
--R ??? : (% ,NonNegativeInteger) -> % if R has MONOID
--R ?+? : (% ,%) -> % if R has ABELSG
--R ?-? : (% ,%) -> % if R has RING
--R ?/? : (% ,R) -> % if R has FIELD
--R ?<? : (% ,%) -> Boolean if R has OAMONS or R has ORDRING
--R ?<=? : (% ,%) -> Boolean if R has OAMONS or R has ORDRING
--R ?=? : (% ,%) -> Boolean if R has SETCAT
--R ?>? : (% ,%) -> Boolean if R has OAMONS or R has ORDRING
--R ?>=? : (% ,%) -> Boolean if R has OAMONS or R has ORDRING
--R D : (% ,(R -> R)) -> % if R has RING
--R D : (% ,(R -> R),NonNegativeInteger) -> % if R has RING
--R D : (% ,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL and R has RING
--R D : (% ,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL and R has RING
--R D : (% ,List Symbol) -> % if R has PDRING SYMBOL and R has RING
--R D : (% ,Symbol) -> % if R has PDRING SYMBOL and R has RING
--R D : (% ,NonNegativeInteger) -> % if R has DIFRING and R has RING
--R D : % -> % if R has DIFRING and R has RING
--R ?? : (% ,PositiveInteger) -> % if R has MONOID
--R ?? : (% ,NonNegativeInteger) -> % if R has MONOID

```

```

--R abs : % -> % if R has ORDRING
--R any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R characteristic : () -> NonNegativeInteger if R has RING
--R coerce : R -> % if R has SETCAT
--R coerce : Fraction Integer -> % if R has RETRACT FRAC INT and R has SETCAT
--R coerce : Integer -> % if R has RETRACT INT and R has SETCAT or R has RING
--R coerce : % -> OutputForm if R has SETCAT
--R count : (R,%) -> NonNegativeInteger if $ has finiteAggregate and R has SETCAT
--R count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R differentiate : (%,(R -> R)) -> % if R has RING
--R differentiate : (%,(R -> R),NonNegativeInteger) -> % if R has RING
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL and R has RING
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL and R has RING
--R differentiate : (%,List Symbol) -> % if R has PDRING SYMBOL and R has RING
--R differentiate : (%,Symbol) -> % if R has PDRING SYMBOL and R has RING
--R differentiate : (%,NonNegativeInteger) -> % if R has DIFRING and R has RING
--R differentiate : % -> % if R has DIFRING and R has RING
--R dimension : () -> CardinalNumber if R has FIELD
--R dot : (%,%) -> R if R has RING
--R entry? : (R,%) -> Boolean if $ has finiteAggregate and R has SETCAT
--R eval : (%,List R,List R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,R,R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,Equation R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,List Equation R) -> % if R has EVALAB R and R has SETCAT
--R every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,R) -> % if $ has shallowlyMutable
--R first : % -> R if Integer has ORDSET
--R hash : % -> SingleInteger if R has SETCAT
--R index : PositiveInteger -> % if R has FINITE
--R latex : % -> String if R has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R lookup : % -> PositiveInteger if R has FINITE
--R map! : ((R -> R),%) -> % if $ has shallowlyMutable
--R max : (%,%) -> % if R has OAMONS or R has ORDRING
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (R,%) -> Boolean if $ has finiteAggregate and R has SETCAT
--R members : % -> List R if $ has finiteAggregate
--R min : (%,%) -> % if R has OAMONS or R has ORDRING
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R negative? : % -> Boolean if R has ORDRING
--R one? : % -> Boolean if R has MONOID
--R parts : % -> List R if $ has finiteAggregate
--R positive? : % -> Boolean if R has ORDRING
--R qsetelt! : (%,Integer,R) -> R if $ has shallowlyMutable
--R random : () -> % if R has FINITE
--R recip : % -> Union(%, "failed") if R has MONOID
--R reducedSystem : Matrix % -> Matrix R if R has RING
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R) if R has RING
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) if R has RING

```

```

--R reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT and R has RING
--R retract : % -> R if R has SETCAT
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT and R has SETCAT
--R retract : % -> Integer if R has RETRACT INT and R has SETCAT
--R retractIfCan : % -> Union(R,"failed") if R has SETCAT
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT and R has SETCAT
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT and R has SETCAT
--R setelt : (% ,Integer,R) -> R if $ has shallowlyMutable
--R sign : % -> Integer if R has ORDRING
--R size : () -> NonNegativeInteger if R has FINITE
--R size? : (% ,NonNegativeInteger) -> Boolean
--R subtractIfCan : (% ,%) -> Union(% ,"failed") if R has CABMON
--R sup : (% ,%) -> % if R has OAMONS
--R swap! : (% ,Integer,Integer) -> Void if $ has shallowlyMutable
--R unitVector : PositiveInteger -> % if R has RING
--R zero? : % -> Boolean if R has CABMON
--R ?~=? : (% ,%) -> Boolean if R has SETCAT
--R
--E 1

)spool
)lisp (bye)

```

---

— DirectProduct.help —

```

=====
DirectProduct examples
=====

```

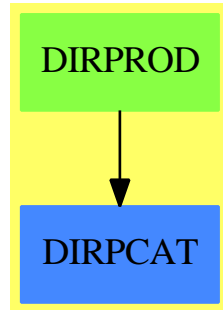
```

See Also:
o )show DirectProduct

```



### 5.9.1 DirectProduct (DIRPROD)



#### Exports:

|               |               |               |            |                |
|---------------|---------------|---------------|------------|----------------|
| 0             | 1             | abs           | any?       | characteristic |
| coerce        | copy          | count         | D          | differentiate  |
| dimension     | directProduct | dot           | elt        | empty          |
| empty?        | entries       | entry?        | eq?        | eval           |
| every?        | fill!         | first         | hash       | index          |
| index?        | indices       | latex         | less?      | lookup         |
| map           | map!          | max           | maxIndex   | member?        |
| members       | min           | minIndex      | more?      | negative?      |
| one?          | parts         | positive?     | qelt       | qsetelt!       |
| random        | recip         | reducedSystem | retract    | retractIfCan   |
| sample        | setelt        | sign          | size       | size?          |
| subtractIfCan | sup           | swap!         | unitVector | zero?          |
| #?            | ?*?           | ?**?          | ?+?        | ?-?            |
| ?/?           | ?<?           | ?<=?          | ?=?        | ?>?            |
| ?>=?          | ?^?           | ?~=?          | -?         | ?.?            |

— domain DIRPROD DirectProduct —

```

)abbrev domain DIRPROD DirectProduct
++ Author: Mark Botch
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors: Vector, IndexedVector
++ Also See: OrderedDirectProduct
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This type represents the finite direct or cartesian product of an
++ underlying component type. This contrasts with simple vectors in that
++ the members can be viewed as having constant length. Thus many
++ categorical properties can be lifted from the underlying component type.
  
```

```

++ Component extraction operations are provided but no updating operations.
++ Thus new direct product elements can either be created by converting
++ vector elements using the \spadfun{directProduct} function
++ or by taking appropriate linear combinations of basis vectors provided
++ by the \spad{unitVector} operation.

```

```

DirectProduct(dim:NonNegativeInteger, R:Type):
  DirectProductCategory(dim, R) == Vector R add

  Rep := Vector R

  coerce(z:%):Vector(R)      == copy(z)$Rep pretend Vector(R)
  coerce(r:R):%              == new(dim, r)$Rep

  parts x == VEC2LIST(x)$Lisp

  directProduct z ==
    size?(z, dim) => copy(z)$Rep
    error "Not of the correct length"

  if R has SetCategory then
    same?: % -> Boolean
    same? z == every?(x +-> x = z(minIndex z), z)

    x = y == _and/[qelt(x,i)$Rep = qelt(y,i)$Rep for i in 1..dim]

    retract(z:%):R ==
      same? z => z(minIndex z)
      error "Not retractable"

    retractIfCan(z:%):Union(R, "failed") ==
      same? z => z(minIndex z)
      "failed"

  if R has AbelianSemiGroup then
    u:% + v:% == map(_+ , u, v)$Rep

  if R has AbelianMonoid then
    0 == zero(dim)$Vector(R) pretend %

  if R has Monoid then
    1 == new(dim, 1)$Vector(R) pretend %
    u:% * r:R      == map(x +-> x * r, u)
    r:R * u:%      == map(x +-> r * x, u)
    x:% * y:% == [x.i * y.i for i in 1..dim]$Vector(R) pretend %

  if R has CancellationAbelianMonoid then
    subtractIfCan(u:%, v:%):Union(%, "failed") ==

```

```

w := new(dim,0)$Vector(R)
for i in 1..dim repeat
  (c:=subtractIfCan(qelt(u, i)$Rep, qelt(v,i)$Rep)) case "failed" =>
    return "failed"
  qsetelt_!(w, i, c::R)$Rep
w pretend %

if R has Ring then

u:% * v:% == map(_* , u, v)$Rep

recip z ==
w := new(dim,0)$Vector(R)
for i in minIndex w .. maxIndex w repeat
  (u := recip qelt(z, i)) case "failed" => return "failed"
  qsetelt_!(w, i, u::R)
w pretend %

unitVector i ==
v:= new(dim,0)$Vector(R)
v.i := 1
v pretend %

if R has OrderedSet then
x < y ==
for i in 1..dim repeat
  qelt(x,i) < qelt(y,i) => return true
  qelt(x,i) > qelt(y,i) => return false
false

if R has OrderedAbelianMonoidSup then sup(x, y) == map(sup, x, y)

-->bo $noSubsumption := false

```

---

— DIRPROD.dotabb —

```

"DIRPROD" [color="#88FF44",href="bookvol10.3.pdf#nameddest=DIRPROD"]
"DIRPCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=DIRPCAT"]
"DIRPROD" -> "DIRPCAT"

```

---

## 5.10 domain DPMM DirectProductMatrixModule

— DirectProductMatrixModule.input —

```
)set break resume
)sys rm -f DirectProductMatrixModule.output
)spool DirectProductMatrixModule.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show DirectProductMatrixModule
--R DirectProductMatrixModule(n: PositiveInteger,R: Ring,M: SquareMatrixCategory(n,R,DirectProduct(n,R),
--R Abbreviation for DirectProductMatrixModule is DPMM
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for DPMM
--R
--R----- Operations -----
--R ?? : (M,%) -> %                ?? : (PositiveInteger,%) -> %
--R ?? : (Integer,%) -> %          ?? : (R,%) -> %
--R +? : (%,%) -> %                -? : % -> %
--R -? : (%,%) -> %                ?? : (%,%) -> Boolean
--R 0 : () -> %                    coerce : % -> OutputForm
--R coerce : % -> Vector S         copy : % -> %
--R directProduct : Vector S -> %  ?.? : (%,Integer) -> S
--R elt : (%,Integer,S) -> S       empty : () -> %
--R empty? : % -> Boolean          entries : % -> List S
--R eq? : (%,%) -> Boolean         hash : % -> SingleInteger
--R index? : (Integer,%) -> Boolean indices : % -> List Integer
--R latex : % -> String            map : ((S -> S),%) -> %
--R qelt : (%,Integer) -> S        sample : () -> %
--R zero? : % -> Boolean           ~=? : (%,%) -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?? : (%,%) -> % if S has DIFRING and S has RING or S has LINEXP INT and S has RING or S has MONOID
--R ?? : (S,%) -> % if S has RING
--R ?? : (%,S) -> % if S has RING
--R ?? : (NonNegativeInteger,%) -> %
--R ?? : (%,PositiveInteger) -> % if S has DIFRING and S has RING or S has LINEXP INT and S has RING or S has MONOID
--R ?? : (%,NonNegativeInteger) -> % if S has DIFRING and S has RING or S has LINEXP INT and S has RING or S has MONOID
--R ?/? : (%,S) -> % if S has FIELD
--R ?<? : (%,%) -> Boolean if S has OAMONS or S has ORDRING
--R ?<=? : (%,%) -> Boolean if S has OAMONS or S has ORDRING
--R ?>? : (%,%) -> Boolean if S has OAMONS or S has ORDRING
--R ?>=? : (%,%) -> Boolean if S has OAMONS or S has ORDRING
--R D : % -> % if S has DIFRING and S has RING
--R D : (%,NonNegativeInteger) -> % if S has DIFRING and S has RING
--R D : (%,Symbol) -> % if S has PDRING SYMBOL and S has RING
```

```

--R D : (%,List Symbol) -> % if S has PDRING SYMBOL and S has RING
--R D : (%,Symbol,NonNegativeInteger) -> % if S has PDRING SYMBOL and S has RING
--R D : (%,List Symbol,List NonNegativeInteger) -> % if S has PDRING SYMBOL and S has RING
--R D : (%,(S -> S)) -> % if S has RING
--R D : (%,(S -> S),NonNegativeInteger) -> % if S has RING
--R 1 : () -> % if S has DIFRING and S has RING or S has LINEXP INT and S has RING or S has RING
--R ?? : (%,PositiveInteger) -> % if S has DIFRING and S has RING or S has LINEXP INT and S has RING
--R ?? : (%,NonNegativeInteger) -> % if S has DIFRING and S has RING or S has LINEXP INT and S has RING
--R abs : % -> % if S has ORDRING
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R characteristic : () -> NonNegativeInteger if S has RING
--R coerce : Fraction Integer -> % if S has RETRACT FRAC INT and S has SETCAT
--R coerce : Integer -> % if S has RETRACT INT and S has SETCAT or S has RING
--R coerce : S -> % if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if $ has finiteAggregate and S has SETCAT
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R differentiate : % -> % if S has DIFRING and S has RING
--R differentiate : (%,NonNegativeInteger) -> % if S has DIFRING and S has RING
--R differentiate : (%,Symbol) -> % if S has PDRING SYMBOL and S has RING
--R differentiate : (%,List Symbol) -> % if S has PDRING SYMBOL and S has RING
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if S has PDRING SYMBOL and S has RING
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if S has PDRING SYMBOL and S has RING
--R differentiate : (%,(S -> S)) -> % if S has RING
--R differentiate : (%,(S -> S),NonNegativeInteger) -> % if S has RING
--R dimension : () -> CardinalNumber if S has FIELD
--R dot : (%,%) -> S if S has RING
--R entry? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,S) -> % if $ has shallowlyMutable
--R first : % -> S if Integer has ORDSET
--R index : PositiveInteger -> % if S has FINITE
--R less? : (%,NonNegativeInteger) -> Boolean
--R lookup : % -> PositiveInteger if S has FINITE
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R max : (%,%) -> % if S has OAMONS or S has ORDRING
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R members : % -> List S if $ has finiteAggregate
--R min : (%,%) -> % if S has OAMONS or S has ORDRING
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R negative? : % -> Boolean if S has ORDRING
--R one? : % -> Boolean if S has DIFRING and S has RING or S has LINEXP INT and S has RING or S has RING
--R parts : % -> List S if $ has finiteAggregate
--R positive? : % -> Boolean if S has ORDRING
--R qsetelt! : (%,Integer,S) -> S if $ has shallowlyMutable

```

```

--R random : () -> % if S has FINITE
--R recip : % -> Union(%, "failed") if S has DIFRING and S has RING or S has LINEXP INT and S has RING or
--R reducedSystem : Matrix % -> Matrix Integer if S has LINEXP INT and S has RING
--R reducedSystem : (Matrix %, Vector %) -> Record(mat: Matrix Integer, vec: Vector Integer) if S has LINE
--R reducedSystem : Matrix % -> Matrix S if S has RING
--R reducedSystem : (Matrix %, Vector %) -> Record(mat: Matrix S, vec: Vector S) if S has RING
--R retract : % -> Fraction Integer if S has RETRACT FRAC INT and S has SETCAT
--R retract : % -> Integer if S has RETRACT INT and S has SETCAT
--R retract : % -> S if S has SETCAT
--R retractIfCan : % -> Union(Fraction Integer, "failed") if S has RETRACT FRAC INT and S has SETCAT
--R retractIfCan : % -> Union(Integer, "failed") if S has RETRACT INT and S has SETCAT
--R retractIfCan : % -> Union(S, "failed") if S has SETCAT
--R setelt : (%, Integer, S) -> S if $ has shallowlyMutable
--R sign : % -> Integer if S has ORDRING
--R size : () -> NonNegativeInteger if S has FINITE
--R size? : (%, NonNegativeInteger) -> Boolean
--R subtractIfCan : (%, %) -> Union(%, "failed")
--R sup : (%, %) -> % if S has OAMONS
--R swap! : (%, Integer, Integer) -> Void if $ has shallowlyMutable
--R unitVector : PositiveInteger -> % if S has RING
--R
--E 1

)spool
)lisp (bye)

```

---

— DirectProductMatrixModule.help —

```

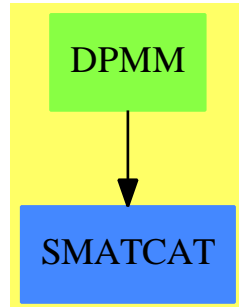
=====
DirectProductMatrixModule examples
=====

```

See Also:

- o )show DirectProductMatrixModule

### 5.10.1 DirectProductMatrixModule (DPMM)



See

⇒ “OppositeMonogenicLinearOperator” (OMLO) 16.11.1 on page 1768

⇒ “OrdinaryDifferentialRing” (ODR) 16.18.1 on page 1820

⇒ “DirectProductModule” (DPMO) 5.11.1 on page 542

#### Exports:

|               |                |               |           |               |
|---------------|----------------|---------------|-----------|---------------|
| 0             | 1              | coerce        | copy      | directProduct |
| elt           | empty          | empty?        | entries   | eq?           |
| hash          | index?         | indices       | latex     | map           |
| qelt          | sample         | zero?         | D         | abs           |
| any?          | characteristic | coerce        | count     | differentiate |
| dimension     | dot            | entry?        | eval      | every?        |
| fill!         | first          | index         | less?     | lookup        |
| map!          | max            | maxIndex      | member?   | members       |
| min           | minIndex       | more?         | negative? | one?          |
| parts         | positive?      | qsetelt!      | random    | recip         |
| reducedSystem | retract        | retractIfCan  | setelt    | sign          |
| size          | size?          | subtractIfCan | sup       | swap!         |
| unitVector    | #?             | ?*?           | ?**?      | ?/?           |
| ?<?           | ?<=?           | ?>?           | ?>=?      | ?^?           |
| ?^?           | -?             | ?-?           | ?=?       | ?..?          |
| ?~=?          |                |               |           |               |

— domain DPMM DirectProductMatrixModule —

```

)abbrev domain DPMM DirectProductMatrixModule
++ Author: Stephen M. Watt
++ Date Created: 1986
++ Date Last Updated: June 4, 1991
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: equation
++ Examples:
  
```

```

++ References:
++ Description:
++ This constructor provides a direct product type with a
++ left matrix-module view.

DirectProductMatrixModule(n, R, M, S): DPcategory == DPcapsule where
  n: PositiveInteger
  R: Ring
  RowCol ==> DirectProduct(n,R)
  M: SquareMatrixCategory(n,R,RowCol,RowCol)
  S: LeftModule(R)

DPcategory == Join(DirectProductCategory(n,S), LeftModule(R), LeftModule(M))

DPcapsule == DirectProduct(n, S) add
  Rep := Vector(S)
  r:R * x:$ == [r*x.i for i in 1..n]
  m:M * x:$ == [ +/[m(i,j)*x.j for j in 1..n] for i in 1..n]

-----

— DPMM.dotabb —

"DPMM" [color="#88FF44",href="bookvol10.3.pdf#nameddest=DPMM"]
"SMATCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=SMATCAT"]
"DPMM" -> "SMATCAT"

-----

```

## 5.11 domain DPMO DirectProductModule

```

— DirectProductModule.input —

)set break resume
)sys rm -f DirectProductModule.output
)spool DirectProductModule.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show DirectProductModule
--R DirectProductModule(n: NonNegativeInteger,R: Ring,S: LeftModule R) is a domain constructor
--R Abbreviation for DirectProductModule is DPMO

```



```

--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for DPMO
--R
--R----- Operations -----
--R ?? : (PositiveInteger,%) -> %      ?? : (Integer,%) -> %
--R ?? : (R,%) -> %                    +? : (%,%) -> %
--R -? : % -> %                        ?-? : (%,%) -> %
--R ?? : (%,%) -> Boolean              0 : () -> %
--R coerce : % -> OutputForm           coerce : % -> Vector S
--R copy : % -> %                      directProduct : Vector S -> %
--R ?.? : (%,Integer) -> S             elt : (%,Integer,S) -> S
--R empty : () -> %                   empty? : % -> Boolean
--R entries : % -> List S              eq? : (%,%) -> Boolean
--R hash : % -> SingleInteger           index? : (Integer,%) -> Boolean
--R indices : % -> List Integer         latex : % -> String
--R map : ((S -> S),%) -> %            qelt : (%,Integer) -> S
--R sample : () -> %                  zero? : % -> Boolean
--R ~=? : (%,%) -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?? : (%,%) -> % if S has DIFRING and S has RING or S has LINEXP INT and S has RING or S
--R ?? : (S,%) -> % if S has RING
--R ?? : (%,S) -> % if S has RING
--R ?? : (NonNegativeInteger,%) -> %
--R ??? : (%,PositiveInteger) -> % if S has DIFRING and S has RING or S has LINEXP INT and S
--R ??? : (%,NonNegativeInteger) -> % if S has DIFRING and S has RING or S has LINEXP INT and S
--R ?/? : (%,S) -> % if S has FIELD
--R ?<? : (%,%) -> Boolean if S has OAMONS or S has ORDRING
--R ?<=? : (%,%) -> Boolean if S has OAMONS or S has ORDRING
--R ?>? : (%,%) -> Boolean if S has OAMONS or S has ORDRING
--R ?>=? : (%,%) -> Boolean if S has OAMONS or S has ORDRING
--R D : % -> % if S has DIFRING and S has RING
--R D : (%,NonNegativeInteger) -> % if S has DIFRING and S has RING
--R D : (%,Symbol) -> % if S has PDRING SYMBOL and S has RING
--R D : (%,List Symbol) -> % if S has PDRING SYMBOL and S has RING
--R D : (%,Symbol,NonNegativeInteger) -> % if S has PDRING SYMBOL and S has RING
--R D : (%,List Symbol,List NonNegativeInteger) -> % if S has PDRING SYMBOL and S has RING
--R D : (%,(S -> S)) -> % if S has RING
--R D : (%,(S -> S),NonNegativeInteger) -> % if S has RING
--R 1 : () -> % if S has DIFRING and S has RING or S has LINEXP INT and S has RING or S has L
--R ?? : (%,PositiveInteger) -> % if S has DIFRING and S has RING or S has LINEXP INT and S
--R ?? : (%,NonNegativeInteger) -> % if S has DIFRING and S has RING or S has LINEXP INT and S
--R abs : % -> % if S has ORDRING
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R characteristic : () -> NonNegativeInteger if S has RING
--R coerce : Fraction Integer -> % if S has RETRACT FRAC INT and S has SETCAT
--R coerce : Integer -> % if S has RETRACT INT and S has SETCAT or S has RING
--R coerce : S -> % if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if $ has finiteAggregate and S has SETCAT
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R differentiate : % -> % if S has DIFRING and S has RING

```

```

--R differentiate : (%,NonNegativeInteger) -> % if S has DIFRING and S has RING
--R differentiate : (%,Symbol) -> % if S has PDRING SYMBOL and S has RING
--R differentiate : (%,List Symbol) -> % if S has PDRING SYMBOL and S has RING
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if S has PDRING SYMBOL and S has RING
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if S has PDRING SYMBOL and S has RING
--R differentiate : (%,(S -> S)) -> % if S has RING
--R differentiate : (%,(S -> S),NonNegativeInteger) -> % if S has RING
--R dimension : () -> CardinalNumber if S has FIELD
--R dot : (%,%) -> S if S has RING
--R entry? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,S) -> % if $ has shallowlyMutable
--R first : % -> S if Integer has ORDSET
--R index : PositiveInteger -> % if S has FINITE
--R less? : (%,NonNegativeInteger) -> Boolean
--R lookup : % -> PositiveInteger if S has FINITE
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R max : (%,%) -> % if S has OAMONS or S has ORDRING
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R members : % -> List S if $ has finiteAggregate
--R min : (%,%) -> % if S has OAMONS or S has ORDRING
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R negative? : % -> Boolean if S has ORDRING
--R one? : % -> Boolean if S has DIFRING and S has RING or S has LINEXP INT and S has RING or S has MONO
--R parts : % -> List S if $ has finiteAggregate
--R positive? : % -> Boolean if S has ORDRING
--R qsetelt! : (%,Integer,S) -> S if $ has shallowlyMutable
--R random : () -> % if S has FINITE
--R recip : % -> Union(%, "failed") if S has DIFRING and S has RING or S has LINEXP INT and S has RING or
--R reducedSystem : Matrix % -> Matrix Integer if S has LINEXP INT and S has RING
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) if S has LINEXP INT
--R reducedSystem : Matrix % -> Matrix S if S has RING
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix S,vec: Vector S) if S has RING
--R retract : % -> Fraction Integer if S has RETRACT FRAC INT and S has SETCAT
--R retract : % -> Integer if S has RETRACT INT and S has SETCAT
--R retract : % -> S if S has SETCAT
--R retractIfCan : % -> Union(Fraction Integer, "failed") if S has RETRACT FRAC INT and S has SETCAT
--R retractIfCan : % -> Union(Integer, "failed") if S has RETRACT INT and S has SETCAT
--R retractIfCan : % -> Union(S, "failed") if S has SETCAT
--R setelt : (%,Integer,S) -> S if $ has shallowlyMutable
--R sign : % -> Integer if S has ORDRING
--R size : () -> NonNegativeInteger if S has FINITE
--R size? : (%,NonNegativeInteger) -> Boolean
--R subtractIfCan : (%,%) -> Union(%, "failed")

```

```

--R sup : (%,% ) -> % if S has OAMONS
--R swap! : (% ,Integer,Integer) -> Void if $ has shallowlyMutable
--R unitVector : PositiveInteger -> % if S has RING
--R
--E 1

)spool
)lisp (bye)

```

---

— DirectProductModule.help —

```

=====
DirectProductModule examples
=====

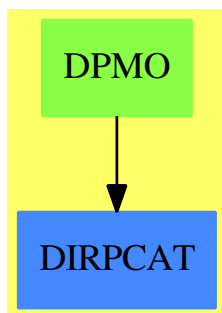
```

```

See Also:
o )show DirectProductModule

```

### 5.11.1 DirectProductModule (DPMO)



See

- ⇒ “OppositeMonogenicLinearOperator” (OMLO) 16.11.1 on page 1768
- ⇒ “OrdinaryDifferentialRing” (ODR) 16.18.1 on page 1820
- ⇒ “DirectProductMatrixModule” (DPMM) 5.10.1 on page 538

**Exports:**

|               |               |               |            |                |
|---------------|---------------|---------------|------------|----------------|
| 0             | 1             | abs           | any?       | characteristic |
| coerce        | copy          | count         | D          | differentiate  |
| dimension     | directProduct | dot           | elt        | empty          |
| empty?        | entries       | entry?        | eval       | every?         |
| eq?           | fill!         | first         | hash       | index          |
| index?        | indices       | latex         | less?      | lookup         |
| map           | map!          | max           | maxIndex   | member?        |
| members       | min           | minIndex      | more?      | negative?      |
| one?          | parts         | positive?     | qelt       | qsetelt!       |
| random        | recip         | reducedSystem | retract    | retractIfCan   |
| sample        | setelt        | sign          | size       | size?          |
| subtractIfCan | sup           | swap!         | unitVector | zero?          |
| #?            | ?*?           | ?**?          | ?/?        | ?<?            |
| ?<=?          | ?>?           | ?>=?          | ?^?        | ?+?            |
| -?            | ?-?           | ?=?           | ?.?        | ?~=?           |

— domain DPMO DirectProductModule —

```

)abbrev domain DPMO DirectProductModule
++ Author: Stephen M. Watt
++ Date Created: 1986
++ Date Last Updated: June 4, 1991
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: equation
++ Examples:
++ References:
++ Description:
++ This constructor provides a direct product of R-modules
++ with an R-module view.

DirectProductModule(n, R, S): DPcategory == DPcapsule where
  n: NonNegativeInteger
  R: Ring
  S: LeftModule(R)

DPcategory == Join(DirectProductCategory(n,S), LeftModule(R))
-- with if S has Algebra(R) then Algebra(R)
-- <above line leads to matchMmCond: unknown form of condition>

DPcapsule == DirectProduct(n,S) add
  Rep := Vector(S)
  r:R * x:$ == [r * x.i for i in 1..n]

```

— DPMO.dotabb —

```
"DPMO" [color="#88FF44",href="bookvol10.3.pdf#nameddest=DPMO"]
"DIRPCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=DIRPCAT"]
"DPMO" -> "DIRPCAT"
```

—————

## 5.12 domain DIRRING DirichletRing

The Dirichlet Ring is the ring of arithmetical functions

$$f : \mathbb{N}_+ \rightarrow R$$

(see [http://en.wikipedia.org/wiki/Arithmetic\\_function](http://en.wikipedia.org/wiki/Arithmetic_function)) together with the Dirichlet convolution (see [http://en.wikipedia.org/wiki/Dirichlet\\_convolution](http://en.wikipedia.org/wiki/Dirichlet_convolution)) as multiplication and component-wise addition. Since we can consider the values an arithmetic functions assumes as the coefficients of a Dirichlet generating series, we call  $R$  the coefficient ring of a function.

In general we only assume that the coefficient ring  $R$  is a ring. If  $R$  happens to be commutative, then so is the Dirichlet ring, and in this case it is even an algebra.

Apart from the operations inherited from those categories, we only provide some convenient coercion functions.

— DirichletRing.input —

```
)set break resume
)sys rm -f DirichletRing.output
)spool DirichletRing.output
)set message test on
)set message auto off
)clear all

--S 1 of 21
t1:DIRRING INT := (n:PI):INT +-> moebiusMu n
--R
--R
--R (1) [1,- 1,- 1,0,- 1,1,- 1,0,0,1,...]
--R
--R                                          Type: DirichletRing Integer
--E 1

--S 2 of 21
[t1.i for i in 1..4]
--R
--R
```

```
--R      (2) [1,- 1,- 1,0]
--R
--E 2
Type: List Integer

--S 3 of 21
t2:DIRRING INT := [moebiusMu n for n in 1..]
--R
--R
--R      (3) [1,- 1,- 1,0,- 1,1,- 1,0,0,1,...]
--R
--E 3
Type: DirichletRing Integer

--S 4 of 21
[t2.i for i in 1..4]
--R
--R
--R      (4) [1,- 1,- 1,0]
--R
--E 4
Type: List Integer

--S 5 of 21
DIRRING INT has CommutativeRing
--R
--R
--R      (5) true
--R
--E 5
Type: Boolean

--S 6 of 21
mu:DIRRING FRAC INT := (n:PI):FRAC INT --> moebiusMu n
--R
--R
--R      (6) [1,- 1,- 1,0,- 1,1,- 1,0,0,1,...]
--R
--E 6
Type: DirichletRing Fraction Integer

--S 7 of 21
phi:DIRRING FRAC INT := (n:PI):FRAC INT --> eulerPhi n
--R
--R
--R      (7) [1,1,2,2,4,2,6,4,6,4,...]
--R
--E 7
Type: DirichletRing Fraction Integer

--S 8 of 21
t3:=[(recip mu * phi).n for n in 1..10]
--R
--R
--R      (8) [1,2,3,4,5,6,7,8,9,10]
--R
--E 8
Type: List Fraction Integer
```

--E 8

--S 9 of 21

t4:=[(phi \* recip mu).n for n in 1..10]

--R

--R

--R (9) [1,2,3,4,5,6,7,8,9,10]

--R

Type: List Fraction Integer

--E 9

--S 10 of 21

reduce(\_and,[(x = y)@Boolean for x in t3 for y in t4])

--R

--R

--R (10) true

--R

Type: Boolean

--E 10

--S 11 of 21

DIRRING FRAC INT has Algebra FRAC INT

--R

--R

--R (11) true

--R

Type: Boolean

--E 11

--S 12 of 21

t5:=[(1/2 \* phi).n for n in 1..10]

--R

--R

--R 1 1

--R (12) [-,-,1,1,2,1,3,2,3,2]

--R 2 2

--R

Type: List Fraction Integer

--E 12

--S 13 of 21

t6:=[eulerPhi n/2 for n in 1..10]

--R

--R

--R 1 1

--R (13) [-,-,1,1,2,1,3,2,3,2]

--R 2 2

--R

Type: List Fraction Integer

--E 13

--S 14 of 21

reduce(\_and,[(x = y)@Boolean for x in t5 for y in t6])

--R

--R





```

--E 20

--S 21 of 21
)show DirichletRing
--R DirichletRing Coef: Ring is a domain constructor
--R Abbreviation for DirichletRing is DIRRING
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for DIRRING
--R
--R----- Operations -----
--R ?? : (%,%) -> %                ?? : (Integer,%) -> %
--R ?? : (PositiveInteger,%) -> %  ??? : (%,PositiveInteger) -> %
--R ?? : (%,%) -> %                ?-? : (%,%) -> %
--R -? : % -> %                    ?? : (%,%) -> Boolean
--R 1 : () -> %                    0 : () -> %
--R ?? : (%,PositiveInteger) -> %  coerce : % -> Stream Coef
--R coerce : Stream Coef -> %       coerce : Integer -> %
--R coerce : % -> OutputForm        ?.? : (%,PositiveInteger) -> Coef
--R hash : % -> SingleInteger       latex : % -> String
--R one? : % -> Boolean              recip : % -> Union(%, "failed")
--R sample : () -> %                zero? : % -> Boolean
--R zeta : () -> %                  ?~=? : (%,%) -> Boolean
--R ?? : (%,Coef) -> % if Coef has COMRING
--R ?? : (Coef,%) -> % if Coef has COMRING
--R ?? : (NonNegativeInteger,%) -> %
--R ??? : (%,NonNegativeInteger) -> %
--R ?? : (%,NonNegativeInteger) -> %
--R additive? : (%,PositiveInteger) -> Boolean
--R associates? : (%,%) -> Boolean if Coef has COMRING
--R characteristic : () -> NonNegativeInteger
--R coerce : % -> % if Coef has COMRING
--R coerce : Coef -> % if Coef has COMRING
--R coerce : % -> (PositiveInteger -> Coef)
--R coerce : (PositiveInteger -> Coef) -> %
--R exquo : (%,%) -> Union(%, "failed") if Coef has COMRING
--R multiplicative? : (%,PositiveInteger) -> Boolean
--R subtractIfCan : (%,%) -> Union(%, "failed")
--R unit? : % -> Boolean if Coef has COMRING
--R unitCanonical : % -> % if Coef has COMRING
--R unitNormal : % -> Record(unit: %, canonical: %, associate: %) if Coef has COMRING
--R
--E 21

)spool
)lisp (bye)

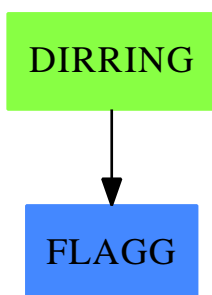
```

```
=====
DirichletRing examples
=====
```

```
See Also:
o )show DirichletRing
```

—————

### 5.12.1 DirichletRing (DIRRING)



#### Exports:

|            |           |               |                 |               |
|------------|-----------|---------------|-----------------|---------------|
| 0          | 1         | ?*?           | ?**?            | ?+?           |
| ?-?        | -?        | ?=?           | ?^?             | ?..?          |
| ? =?       | additive? | associates?   | characteristic  | coerce        |
| exquo      | hash      | latex         | multiplicative? | one?          |
| recip      | sample    | subtractIfCan | unit?           | unitCanonical |
| unitNormal | zero?     | zeta          |                 |               |

— domain DIRRING DirichletRing —

```
)abbrev domain DIRRING DirichletRing
++ Author: Martin Rubey
++ Description: DirichletRing is the ring of arithmetical functions
++ with Dirichlet convolution as multiplication
DirichletRing(Coef: Ring):
    Exports == Implementation where

    PI ==> PositiveInteger
    FUN ==> PI -> Coef

    Exports ==> Join(Ring, Eltable(PI, Coef)) with

    if Coef has CommutativeRing then
        IntegralDomain
```

```

if Coef has CommutativeRing then
  Algebra Coef

coerce: FUN -> %
coerce: % -> FUN
coerce: Stream Coef -> %
coerce: % -> Stream Coef

zeta: constant -> %
  ++ zeta() returns the function which is constantly one

multiplicative?: (% , PI) -> Boolean
  ++ multiplicative?(a, n) returns true if the first
  ++ n coefficients of a are multiplicative

additive?: (% , PI) -> Boolean
  ++ additive?(a, n) returns true if the first
  ++ n coefficients of a are additive

Implementation ==> add

Rep := Record(function: FUN)

per(f: Rep): % == f pretend %
rep(a: %): Rep == a pretend Rep

elt(a: % , n: PI): Coef ==
  f: FUN := (rep a).function
  f n

coerce(a: %): FUN == (rep a).function

coerce(f: FUN): % == per [f]

indices: Stream Integer
  := integers(1)$StreamTaylorSeriesOperations(Integer)

coerce(a: %): Stream Coef ==
  f: FUN := (rep a).function
  map((n: Integer): Coef +-> f(n::PI), indices)
  $StreamFunctions2(Integer, Coef)

coerce(f: Stream Coef): % ==
  ((n: PI): Coef +-> f.(n::Integer))::%

coerce(f: %): OutputForm == f::Stream Coef::OutputForm

1: % ==
  ((n: PI): Coef +-> (if one? n then 1$Coef else 0$Coef))::%

```

```

0: % ==
  ((n: PI): Coef +-> 0$Coef)::%

zeta: % ==
  ((n: PI): Coef +-> 1$Coef)::%

(f: %) + (g: %) ==
  ((n: PI): Coef +-> f(n)+g(n))::%

- (f: %) ==
  ((n: PI): Coef +-> -f(n))::%

(a: Integer) * (f: %) ==
  ((n: PI): Coef +-> a*f(n))::%

(a: Coef) * (f: %) ==
  ((n: PI): Coef +-> a*f(n))::%

import IntegerNumberTheoryFunctions

(f: %) * (g: %) ==
  conv := (n: PI): Coef +-> _
    reduce((a: Coef, b: Coef): Coef +-> a + b, _
      [f(d::PI) * g((n quo d)::PI) for d in divisors(n::Integer)], 0)
    $ListFunctions2(Coef, Coef)
  conv::%

unit?(a: %): Boolean == not (recip(a(1$PI))$Coef case "failed")

qrecip: (% , Coef, PI) -> Coef
qrecip(f: %, flinv: Coef, n: PI): Coef ==
  if one? n then flinv
  else
    -flinv * reduce(_+, [f(d::PI) * qrecip(f, flinv, (n quo d)::PI) _
      for d in rest divisors(n)], 0) _
    $ListFunctions2(Coef, Coef)

recip f ==
  if (flinv := recip(f(1$PI))$Coef) case "failed" then "failed"
  else
    mp := (n: PI): Coef +-> qrecip(f, flinv, n)

    mp::%::Union(%, "failed")

multiplicative?(a, n) ==
  for i in 2..n repeat
    fl := factors(factor i)$Factored(Integer)
    rl := [a.(((f.factor)::PI)**((f.exponent)::PI)) for f in fl]
    if a.(i::PI) ~= reduce((r:Coef, s:Coef):Coef +-> r*s, rl)

```

```

      then
        output(i::OutputForm)$OutputPackage
        output(r1::OutputForm)$OutputPackage
        return false
      true

additive?(a, n) ==
  for i in 2..n repeat
    fl := factors(factor i)$Factored(Integer)
    rl := [a.(((f.factor)::PI)**((f.exponent)::PI)) for f in fl]
    if a.(i::PI) ~= reduce((r:Coef, s:Coef):Coef +-> r+s, rl)
    then
      output(i::OutputForm)$OutputPackage
      output(rl::OutputForm)$OutputPackage
      return false
    true

```

---

— DIRRING.dotabb —

```

"DIRRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=DIRRING"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"DIRRING" -> "FLAGG"

```

---

### 5.13 domain DMP DistributedMultivariatePolynomial

— DistributedMultivariatePolynomial.input —

```

)set break resume
)sys rm -f DistributedMultivariatePolynomial.output
)spool DistributedMultivariatePolynomial.output
)set message test on
)set message auto off
)clear all
--S 1 of 10
(d1,d2,d3) : DMP([z,y,x],FRAC INT)
--R
--R
--E 1

```

Type: Void

```

--S 2 of 10

```

```

d1 := -4*z + 4*y**2*x + 16*x**2 + 1
--R
--R
--R      2      2
--R      (2)  - 4z + 4y x + 16x + 1
--R      Type: DistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 2

--S 3 of 10
d2 := 2*z*y**2 + 4*x + 1
--R
--R
--R      2
--R      (3)  2z y + 4x + 1
--R      Type: DistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 3

--S 4 of 10
d3 := 2*z*x**2 - 2*y**2 - x
--R
--R
--R      2      2
--R      (4)  2z x - 2y - x
--R      Type: DistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 4

--S 5 of 10
groebner [d1,d2,d3]
--R
--R
--R      (5)
--R      1568 6 1264 5 6 4 182 3 2047 2 103 2857
--R      [z - ---- x - ---- x + --- x + --- x - ---- x - ---- x - ----,
--R      2745 305 305 549 610 2745 10980
--R      2 112 6 84 5 1264 4 13 3 84 2 1772 2
--R      y + ---- x - --- x - ---- x - --- x + --- x + ---- x + ----,
--R      2745 305 305 549 305 2745 2745
--R      7 29 6 17 4 11 3 1 2 15 1
--R      x + -- x - -- x - -- x + -- x + -- x + -]
--R      4 16 8 32 16 4
--R      Type: List DistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 5

--S 6 of 10
(n1,n2,n3) : HDMP([z,y,x],FRAC INT)
--R
--R
--R      Type: Void
--E 6

--S 7 of 10

```

```

n1 := d1
--R
--R
--R      2      2
--R (7)  4y x + 16x - 4z + 1
--R Type: HomogeneousDistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 7

--S 8 of 10
n2 := d2
--R
--R
--R      2
--R (8)  2z y + 4x + 1
--R Type: HomogeneousDistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 8

--S 9 of 10
n3 := d3
--R
--R
--R      2      2
--R (9)  2z x - 2y - x
--R Type: HomogeneousDistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 9

--S 10 of 10
groebner [n1,n2,n3]
--R
--R
--R (10)
--R      4      3      3      2      1      1      4      29      3      1      2      7      9      1
--R [y + 2x - - x + - z - -, x + - x - - y - - z x - - x - -,
--R      2      2      2      8      4      8      4      16      4
--R      2      1      2      2      1      2      2      1
--R      z y + 2x + -, y x + 4x - z + -, z x - y - - x,
--R      2      2      2      4      2
--R      z - 4y + 2x - - z - - x]
--R
--RType: List HomogeneousDistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 10
)spool
)lisp (bye)

```

```
=====
MultivariatePolynomial
DistributedMultivariatePolynomial
HomogeneousDistributedMultivariatePolynomial
GeneralDistributedMultivariatePolynomial
=====
```

DistributedMultivariatePolynomial which is abbreviated as DMP and HomogeneousDistributedMultivariatePolynomial, which is abbreviated as HDMP, are very similar to MultivariatePolynomial except that they are represented and displayed in a non-recursive manner.

```
(d1,d2,d3) : DMP([z,y,x],FRAC INT)
              Type: Void
```

The constructor DMP orders its monomials lexicographically while HDMP orders them by total order refined by reverse lexicographic order.

```
d1 := -4*z + 4*y**2*x + 16*x**2 + 1
      2      2
    - 4z + 4y x + 16x + 1
      Type: DistributedMultivariatePolynomial([z,y,x],Fraction Integer)

d2 := 2*z*y**2 + 4*x + 1
      2
    2z y + 4x + 1
      Type: DistributedMultivariatePolynomial([z,y,x],Fraction Integer)

d3 := 2*z*x**2 - 2*y**2 - x
      2      2
    2z x - 2y - x
      Type: DistributedMultivariatePolynomial([z,y,x],Fraction Integer)
```

These constructors are mostly used in Groebner basis calculations.

```
groebner [d1,d2,d3]
      1568 6 1264 5 6 4 182 3 2047 2 103 2857
[z - ---- x - ---- x + --- x + --- x - ---- x - ---- x - ----,
 2745 305 305 549 610 2745 10980
 2 112 6 84 5 1264 4 13 3 84 2 1772 2
y + ---- x - --- x - ---- x - --- x + --- x + ---- x + ----,
 2745 305 305 549 305 2745 2745
 7 29 6 17 4 11 3 1 2 15 1
x + -- x - -- x - -- x + -- x + -- x + -]
 4 16 8 32 16 4
      Type: List DistributedMultivariatePolynomial([z,y,x],Fraction Integer)

(n1,n2,n3) : HDMP([z,y,x],FRAC INT)
              Type: Void
```



```

n1 := d1
      2      2
    4y x + 16x - 4z + 1
Type: HomogeneousDistributedMultivariatePolynomial([z,y,x],Fraction Integer)

n2 := d2
      2
    2z y + 4x + 1
Type: HomogeneousDistributedMultivariatePolynomial([z,y,x],Fraction Integer)

n3 := d3
      2      2
    2z x - 2y - x
Type: HomogeneousDistributedMultivariatePolynomial([z,y,x],Fraction Integer)

```

Note that we get a different Groebner basis when we use the HDMP polynomials, as expected.

```

groebner [n1,n2,n3]
      4      3      3      2      1      1      4      29      3      1      2      7      9      1
[y + 2x - - x + - z - -, x + - x - - y - - z x - - x - -,
      2      2      8      4      8      4      16      4
      2      1      2      2      1      2      2      1
z y + 2x + -, y x + 4x - z + -, z x - y - - x,
      2      4      2
      2      2      2      1      3
z - 4y + 2x - - z - - x]
      4      2
Type: List HomogeneousDistributedMultivariatePolynomial([z,y,x],
Fraction Integer)

```

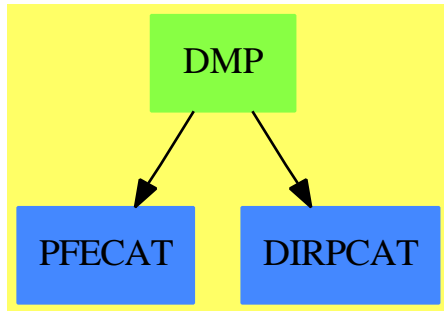
GeneralDistributedMultivariatePolynomial is somewhat more flexible in the sense that as well as accepting a list of variables to specify the variable ordering, it also takes a predicate on exponent vectors to specify the term ordering. With this polynomial type the user can experiment with the effect of using completely arbitrary term orderings. This flexibility is mostly important for algorithms such as Groebner basis calculations which can be very sensitive to term ordering.

See Also:

- o )help Polynomial
- o )help UnivariatePolynomial
- o )help MultivariatePolynomial
- o )help HomogeneousDistributedMultivariatePolynomial
- o )help GeneralDistributedMultivariatePolynomial
- o )show DistributedMultivariatePolynomial

---

### 5.13.1 DistributedMultivariatePolynomial (DMP)



See

⇒ “GeneralDistributedMultivariatePolynomial” (GDMP) 8.1.1 on page 1018

⇒ “HomogeneousDistributedMultivariatePolynomial” (HDMP) 9.10.1 on page 1145

**Exports:**

|                    |                               |                |
|--------------------|-------------------------------|----------------|
| 0                  | 1                             | associates?    |
| binomThmExpt       | characteristic                | charthRoot     |
| coefficient        | coefficients                  | coerce         |
| conditionP         | convert                       | D              |
| degree             | differentiate                 | discriminant   |
| eval               | exquo                         | factor         |
| factorPolynomial   | factorSquareFreePolynomial    | gcd            |
| gcdPolynomial      | ground                        | ground?        |
| hash               | isExpt                        | isPlus         |
| isTimes            | latex                         | lcm            |
| leadingCoefficient | leadingMonomial               | mainVariable   |
| map                | mapExponents                  | max            |
| min                | minimumDegree                 | monicDivide    |
| monomial           | monomial?                     | monomials      |
| multivariate       | numberOfMonomials             | one?           |
| patternMatch       | popopo!                       | prime?         |
| primitiveMonomials | primitivePart                 | recip          |
| reducedSystem      | reductum                      | resultant      |
| retract            | retractIfCan                  | reorder        |
| retract            | solveLinearPolynomialEquation | sample         |
| squareFree         | squareFreePolynomial          | squareFreePart |
| subtractIfCan      | totalDegree                   | unit?          |
| unitCanonical      | unitNormal                    | univariate     |
| variables          | zero?                         | ?*             |
| ?**?               | ?+?                           | ?-?            |
| -?                 | ?=?                           | ?^?            |
| ?~=?               | ?/?                           | ?<?            |
| ?<=?               | ?>?                           | ?>=?           |

— domain DMP DistributedMultivariatePolynomial —

```

)abbrev domain DMP DistributedMultivariatePolynomial
++ Author: Barry Trager
++ Date Created:
++ Date Last Updated:
++ Basic Functions: Ring, degree, eval, coefficient, monomial, differentiate,
++ resultant, gcd, leadingCoefficient
++ Related Constructors: GeneralDistributedMultivariatePolynomial,
++ HomogeneousDistributedMultivariatePolynomial
++ Also See: Polynomial
++ AMS Classifications:
++ Keywords: polynomial, multivariate, distributed
++ References:
++ Description:
++ This type supports distributed multivariate polynomials
++ whose variables are from a user specified list of symbols.
++ The coefficient ring may be non commutative,
++ but the variables are assumed to commute.

```

```

++ The term ordering is lexicographic specified by the variable
++ list parameter with the most significant variable first in the list.

DistributedMultivariatePolynomial(vl,R): public == private where
  vl : List Symbol
  R   : Ring
  E   ==> DirectProduct(#vl,NonNegativeInteger)
  OV  ==> OrderedVariableList(vl)
  public == PolynomialCategory(R,E,OV) with
    reorder: (% ,List Integer) -> %
    ++ reorder(p, perm) applies the permutation perm to the variables
    ++ in a polynomial and returns the new correctly ordered polynomial

private ==
  GeneralDistributedMultivariatePolynomial(vl,R,E)

```

---

— DMP.dotabb —

```

"DMP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=DMP"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"DIRPCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=DIRPCAT"]
"DMP" -> "PFECAT"
"DMP" -> "DIRPCAT"

```

---

## 5.14 domain DIV Divisor

— Divisor.input —

```

)set break resume
)sys rm -f Divisor.output
)spool Divisor.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Divisor
--R Divisor S: SetCategoryWithDegree is a domain constructor
--R Abbreviation for Divisor is DIV
--R This constructor is exposed in this frame.

```

```

--R Issue )edit bookvol10.3.pamphlet to see algebra source code for DIV
--R
--R----- Operations -----
--R ??? : (Integer,S) -> %          ??? : (Integer,%) -> %
--R ??? : (%,Integer) -> %          ??? : (Integer,%) -> %
--R ??? : (PositiveInteger,%) -> %  ?+? : (S,%) -> %
--R ?+? : (%,%) -> %                ?-? : (%,%) -> %
--R -? : % -> %                      ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean           0 : () -> %
--R coefficient : (S,%) -> Integer   coerce : S -> %
--R coerce : % -> OutputForm          collect : % -> %
--R concat : (%,%) -> %              degree : % -> Integer
--R divOfPole : % -> %                divOfZero : % -> %
--R effective? : % -> Boolean          hash : % -> SingleInteger
--R incr : % -> %                      latex : % -> String
--R mapGen : ((S -> S),%) -> %         nthCoef : (%,Integer) -> Integer
--R nthFactor : (%,Integer) -> S       reductum : % -> %
--R retract : % -> S                  sample : () -> %
--R size : % -> NonNegativeInteger     split : % -> List %
--R supp : % -> List S                 suppOfPole : % -> List S
--R suppOfZero : % -> List S           zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ??? : (NonNegativeInteger,%) -> %
--R head : % -> Record(gen: S,exp: Integer)
--R highCommonTerms : (%,%) -> % if Integer has OAMON
--R mapCoef : ((Integer -> Integer),%) -> %
--R retractIfCan : % -> Union(S,"failed")
--R subtractIfCan : (%,%) -> Union(%, "failed")
--R terms : % -> List Record(gen: S,exp: Integer)
--R
--E 1

)spool
)lisp (bye)

```

---

— Divisor.help —

```

=====
Divisor examples
=====

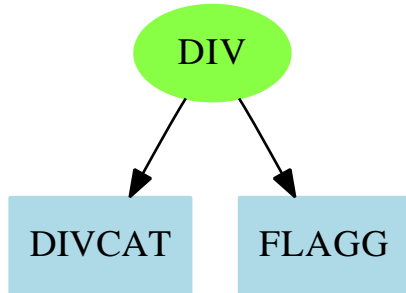
```

```

See Also:
o )show Divisor

```

## 5.14.1 Divisor (DIV)

**Exports:**

|             |            |                 |               |
|-------------|------------|-----------------|---------------|
| 0           | -?         | ?*?             | ?+?           |
| ?-?         | ?<=?       | ?=?             | ?~=?          |
| coefficient | coerce     | collect         | concat        |
| degree      | divOfPole  | divOfZero       | effective?    |
| hash        | head       | highCommonTerms | incr          |
| latex       | mapCoef    | mapGen          | nthCoef       |
| nthFactor   | reductum   | retract         | retractIfCan  |
| sample      | size       | split           | subtractIfCan |
| supp        | suppOfPole | suppOfZero      | terms         |
| zero?       |            |                 |               |

— domain DIV Divisor —

```

)abbrev domain DIV Divisor
++ Author: Gaetan Hache
++ Date Created: 17 nov 1992
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
++ The following is part of the PAFF package
Divisor(S:SetCategoryWithDegree):Exports == Implementation where

```

```

PT      ==> Record(gen:S,exp: Integer)
INT      ==> Integer
BOOLEAN ==> Boolean
LIST     ==> List

```

```

Exports == DivisorCategory(S) with

```

```

  head: % -> PT

```

```

  reductum: % -> %

```

```

Implementation == List PT add

Rep := List PT

incr(d)==
  [ [ pt.gen , pt.exp + 1 ] for pt in d ]

inOut: PT -> OutputForm

inOut(pp)==
  one?(pp.exp) => pp.gen :: OutputForm
  bl:OutputForm:= " " ::OutputForm
  (pp.exp :: OutputForm) * hconcat(bl,pp.gen :: OutputForm)

coerce(d:%):OutputForm==
  zero?(d) => ("0"::OutputForm)
  ll:List OutputForm:=[inOut df for df in d]
  reduce("+",ll)

reductum(d)==
  zero?(d) => d
  dl:Rep:= d pretend Rep
  dlr := rest dl
  empty?(dlr) => 0
  dlr

head(d)==
  zero?(d) => error "Cannot take head of zero"
  dl:Rep:= d pretend Rep
  first dl

coerce(s:S) == [[s,1]$PT]::%

split(a) ==
  zero?(a) => []
  [[r]::% for r in a]

coefficient(s,a)==
  r:INT:=0
  for pt in terms(a) repeat
    if pt.gen=s then
      r:=pt.exp
  r

terms(a)==a::Rep

0==empty()$Rep

supp(a)==

```

```

aa:=terms(collect(a))
[p.gen for p in aa | ^zero?(p.exp)]

suppOfZero(a)==
aa:=terms(collect(a))
[p.gen for p in aa | (p.exp) > 0 ]

suppOfPole(a)==
aa:=terms(collect(a))
[p.gen for p in aa | p.exp < 0 ]

divOfZero(a)==
aa:=terms(collect(a))
[p for p in aa | (p.exp) > 0 ]::%

divOfPole(a)==
aa:=terms(collect(a))
[p for p in aa | p.exp < 0 ]::%

zero?(a)==
((collect(a)::Rep)=empty()$Rep)::BOOLEAN

collect(d)==
a:=d::Rep
empty?(a) => 0
t:Rep:=empty()
ff:PT:=first(a)
one?(#(a)) =>
  if zero?(ff.exp) then
    t::%
  else
    a::%
inList?:Boolean:=false()
newC:INT
restred:=terms(collect((rest(a)::%)))
zero?(ff.exp) =>
  restred::%
for bb in restred repeat
  b:=bb::PT
  if b.gen=ff.gen then
    newC:=b.exp+ff.exp
    if ^zero?(newC) then
      t:=concat(t,[b.gen,newC]$PT)
      inList?:=true()
    else
      t:=concat(t,b)
if ^inList? then
  t:=cons(ff,t)
t::%

```



```

a:% + b:% ==
  collect(concat(a pretend Rep,b pretend Rep))

a:% - b:% ==
  a + (-1)*b

-a:% == (-1)*a

n:INT * a:% ==
  zero?(n) => 0
  t:Rep:=empty()
  for p in a pretend Rep repeat
    t:=concat(t,[ p.gen, n*p.exp]$PT)
  t::%

a:% <= b:% ==
  bma:= b - a
  effective? bma => true
  false

effective?(a)== empty?(suppOfPole(a))

degree(d:%):Integer==
  reduce("+",[p.exp * degree(p.gen)) for p in d @ Rep],0$INT)

```

---

— DIV.dotabb —

```

"DIV" [color="#88FF44",href="bookvol10.3.pdf#nameddest=DIV",shape=ellipse]
"DIVCAT" [color=lightblue,href="bookvol10.2.pdf#nameddest=DIVCAT"];
"FLAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=FLAGG"];
"DIV" -> "DIVCAT"
"DIV" -> "FLAGG"

```

---

## 5.15 domain DFLOAT DoubleFloat

Greg Vanuxem has added some functionality to allow the user to modify the printed format of floating point numbers. The format of the numbers follows the common lisp format specification for floats. First we include Greg's email to show the use of this feature:

```

PS: For those who use the Doublefloat domain
    there is an another (undocumented) patch that adds a
    lisp format to the DoubleFloat output routine. Copy

```

int/algebra/DFLOAT.spad to your working directory,  
patch it, compile it and ")lib" it when necessary.

```
(1) -> )boot $useBFasDefault:=false

(SPADLET |$useBFasDefault| NIL)
Value = NIL
(1) -> a:= matrix [ [0.5978,0.2356], [0.4512,0.2355] ]

      +      0.5978      0.2356      +
(1)  |
      +0.4511999999999999 0.2354999999999999+
                                Type: Matrix DoubleFloat
(2) -> )lib DFLOAT
      DoubleFloat is now explicitly exposed in frame initial
      DoubleFloat will be automatically loaded when needed
from /home/greg/Axiom/DFLOAT.nrlib/code
(2) -> doubleFloatFormat("~",4,,F")

(2)  "~G"
                                Type: String
(3) -> a

      +0.5978  0.2356+
(3)  |
      +0.4512  0.2355+
                                Type: Matrix DoubleFloat
```

So it is clear that he has added a new function called `doubleFloatFormat` which takes a string argument that specifies the common lisp format control string ("`~",4,,F`"). For reference we quote from the common lisp manual [?]. On page 582 we find:

A format directive consists of a tilde (~), optional prefix parameters separated by commas, optional colon (:), and at-sign (@) modifiers, and a single character indicating what kind of directive this is. The alphabetic case of the directive character is ignored. The prefix parameters are generally integers, notated as optionally signed decimal numbers.

X3J13 voted in June 1987 (80) to specify that if both colon and at-sign modifiers are present, they may appear in either order; thus `~:@R` and `~@:R` mean the same thing. However, it is traditional to put the colon first, and all examples in the book put colon before at-signs.

On page 588 we find:

`~F`

*Fixed-format floating-point.* The next *arg* is printed as a floating point number.

The full form is `~w,d,k,overflowchar,padcharF`. The parameter *w* is the width of the field to be printed; *d* is the number of digits to print after the decimal point; *k* is a scale factor that defaults to zero.

Exactly *w* characters will be output. First, leading copies of the character *padchar* (which defaults to a space) are printed, if necessary, to pad the field on the left. If the *arg* is negative, then a minus sign is printed; if the *arg* is not negative, then a plus sign is printed if and only if the `@` modifier was specified. Then a sequence of digits, containing a single embedded decimal point, is printed; this represents the magnitude of the value of *arg* times  $10^k$ , rounded to *d* fractional digits. (When rounding up and rounding down would produce printed values equidistant from the scaled value of *arg*, then the implementation is free to use either one. For example, printing the argument 6.375 using the format `~4.2F` may correctly produce either 6.37 or 6.38.) Leading zeros are not permitted, except that a single zero digit is output before the decimal point if the printed value is less than 1, and this single zero digit is not output after all if  $w = d + 1$ .

If it is impossible to print the value in the required format in the field of width *w*, then one of two actions is taken. If the parameter *overflowchar* is specified, then *w* copies of that parameter are printed instead of the scaled value of *arg*. If the *overflowchar* parameter is omitted, then the scaled value is printed using more than *w* characters, as many more as may be needed.

If the *w* parameter is omitted, then the field is of variable width. In effect, a value is chosen for *w* in such a way that no leading pad characters need to be printed and exactly *d* characters will follow the decimal point. For example, the directive `~,2F` will print exactly two digits after the decimal point and as many as necessary before the decimal point.

If the parameter *d* is omitted, then there is no constraint on the number of digits to appear after the decimal point. A value is chosen for *d* in such a way that as many digits as possible may be printed subject to the width constraint imposed by the parameter *w* and the constraint that no trailing zero digits may appear in the fraction, except that if the fraction to be printed is zero, then a single zero digit should appear after the decimal point if permitted by the width constraint.

If both *w* and *d* are omitted, then the effect is to print the value using ordinary free-format output; `prin1` uses this format for any number whose magnitude is either zero or between  $10^{-3}$  (inclusive) and  $10^7$  (exclusive).

If *w* is omitted, then if the magnitude of *arg* is so large (or, if *d* is also omitted, so small) that more than 100 digits would have to be printed, then an implementation is free, at its discretion, to print the number using exponential notation instead, as if by the directive `~E` (with all parameters of `~E` defaulted, not taking their valued from the `~F` directive).

If *arg* is a rational number, then it is coerced to be a **single-float** and then printed. (Alternatively, an implementation is permitted to process a rational number by any other method that has essentially the same behavior but avoids

If *arg* is a complex number or some non-numeric object, then it is printed using the format directive `~wD`, thereby printing it in decimal radix and a minimum field width of *w*. (If it is desired to print each of the real part and imaginary part of a complex number using a `~F` directive, then this must be done explicitly with two `~F` directives and code to extract the two parts of the complex number.)

— DoubleFloat.input —

[illegible]

```

--S 4 of 13
eApprox : DoubleFloat := 2.71828
--R
--R
--R (4) 2.71828
--R
--R                                          Type: DoubleFloat
--E 4

--S 5 of 13
avg : List DoubleFloat -> DoubleFloat
--R
--R
--R                                          Type: Void
--E 5

--S 6 of 13
avg l ==
  empty? l => 0 :: DoubleFloat
  reduce(+,l) / #l
--R
--R
--R                                          Type: Void
--E 6

--S 7 of 13
avg []
--R
--R
--R Compiling function avg with type List DoubleFloat -> DoubleFloat
--R
--R (7) 0.
--R
--R                                          Type: DoubleFloat
--E 7

--S 8 of 13
avg [3.4,9.7,-6.8]
--R
--R
--R (8) 2.0999999999999996
--R
--R                                          Type: DoubleFloat
--E 8

--S 9 of 13
cos(3.1415926)$DoubleFloat
--R
--R
--R (9) - 0.99999999999999856
--R
--R                                          Type: DoubleFloat
--E 9

--S 10 of 13
cos(3.1415926 :: DoubleFloat)

```

```

--R
--R
--R (10) - 0.99999999999999856
--R
--R                                          Type: DoubleFloat
--E 10

--S 11 of 13
a:DFLOAT := -1.0/3.0
--R
--R
--R (11) - 0.3333333333333337
--R
--R                                          Type: DoubleFloat
--E 11

--S 12 of 13
integerDecode a
--R
--R
--R (12) [6004799503160662,- 54,- 1]
--R
--R                                          Type: List Integer
--E 12

--S 13 of 13
machineFraction a
--R
--R
--R          3002399751580331
--R (13) -  -----
--R          9007199254740992
--R
--R                                          Type: Fraction Integer
--E 13

)spool
)lisp (bye)

```

---

— DoubleFloat.help —

```

=====
DoubleFloat examples
=====

```

Axiom provides two kinds of floating point numbers. The domain Float (abbreviation FLOAT) implements a model of arbitrary precision floating point numbers. The domain DoubleFloat (abbreviation DFLOAT) is intended to make available hardware floating point arithmetic in Axiom. The actual model of floating point DoubleFloat that provides

is system-dependent. For example, on the IBM system 370 Axiom uses IBM double precision which has fourteen hexadecimal digits of precision or roughly sixteen decimal digits. Arbitrary precision floats allow the user to specify the precision at which arithmetic operations are computed. Although this is an attractive facility, it comes at a cost. Arbitrary-precision floating-point arithmetic typically takes twenty to two hundred times more time than hardware floating point.

The usual arithmetic and elementary functions are available for DoubleFloat. By default, floating point numbers that you enter into Axiom are of type Float.

```
2.71828
2.71828
                                Type: Float
```

You must therefore tell Axiom that you want to use DoubleFloat values and operations. The following are some conservative guidelines for getting Axiom to use DoubleFloat.

To get a value of type DoubleFloat, use a target with @, ...

```
2.71828@DoubleFloat
2.71828
                                Type: DoubleFloat
```

a conversion, ...

```
2.71828 :: DoubleFloat
2.71828
                                Type: DoubleFloat
```

or an assignment to a declared variable. It is more efficient if you use a target rather than an explicit or implicit conversion.

```
eApprox : DoubleFloat := 2.71828
2.71828
                                Type: DoubleFloat
```

You also need to declare functions that work with DoubleFloat.

```
avg : List DoubleFloat -> DoubleFloat
                                Type: Void

avg l ==
  empty? l => 0 :: DoubleFloat
  reduce(_+,l) / #l
                                Type: Void
```

```
avg []
0.
                                Type: DoubleFloat
```

```
avg [3.4,9.7,-6.8]
2.1000000000000001
                                Type: DoubleFloat
```

Use package-calling for operations from DoubleFloat unless the arguments themselves are already of type DoubleFloat.

```
cos(3.1415926)$DoubleFloat
-0.99999999999999856
                                Type: DoubleFloat
```

```
cos(3.1415926 :: DoubleFloat)
-0.99999999999999856
                                Type: DoubleFloat
```

By far, the most common usage of DoubleFloat is for functions to be graphed.

You can get the exact machine-specific bits of a DoubleFloat in two ways. The first is to use the `integerDecode` function to break the DoubleFloat into components.

```
a := -1.0/3.0
-0.3333333333333331

integerDecode a

[6004799503160661,- 54,- 1]
```

This is the mantissa, exact to 54 bits, the power of 2, and the sign. Thus it is:

```
6004799503160661 * 2^-54
```

You can get the same information as a fraction with

```
machineFraction a

6004799503160661
- ----
18014398509481984
```

where the denominator 18014398509481984 is  $2^{54}$

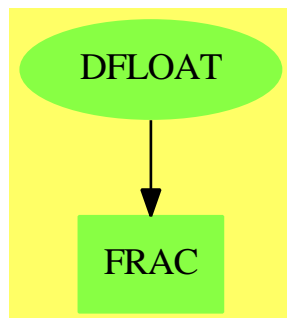


See Also:

- o `)help Float`
- o `)show DoubleFloat`

---

### 5.15.1 DoubleFloat (DFLOAT)



**Exports:**

|                    |                   |                |                       |
|--------------------|-------------------|----------------|-----------------------|
| 0                  | 1                 | abs            | acos                  |
| acosh              | acot              | acoth          | acsc                  |
| acsch              | airyAi            | airyBi         | asec                  |
| asech              | asin              | asinh          | associates?           |
| atan               | atanh             | base           | besselI               |
| besselJ            | besselK           | besselY        | Beta                  |
| bits               | ceiling           | characteristic | coerce                |
| convert            | cos               | cosh           | cot                   |
| coth               | csc               | csch           | D                     |
| decreasePrecision  | differentiate     | digamma        | digits                |
| divide             | doubleFloatFormat | euclideanSize  | exp                   |
| expressIdealMember | exp1              | exponent       | exquo                 |
| extendedEuclidean  | factor            | float          | floor                 |
| fractionPart       | Gamma             | gcd            | gcdPolynomial         |
| hash               | increasePrecision | integerDecode  | inv                   |
| latex              | lcm               | log            | log10                 |
| log2               | machineFraction   | mantissa       | max                   |
| min                | multiEuclidean    | negative?      | norm                  |
| nthRoot            | OMwrite           | one?           | order                 |
| patternMatch       | pi                | polygamma      | positive?             |
| precision          | prime?            | principalIdeal | rationalApproximation |
| recip              | retract           | retractIfCan   | round                 |
| sample             | sec               | sech           | sign                  |
| sin                | sinh              | sizeLess?      | sqrt                  |
| squareFree         | squareFreePart    | subtractIfCan  | tan                   |
| tanh               | truncate          | unit?          | unitCanonical         |
| unitNormal         | wholePart         | zero?          | ?*?                   |
| ?**?               | ?+?               | ?-?            | -?                    |
| ?/?                | ?<?               | ?<=?           | ?=?                   |
| ?>?                | ?>=?              | ?^?            | ?quo?                 |
| ?rem?              | ?~=?              |                |                       |

— domain DFLOAT DoubleFloat —

```

)abbrev domain DFLOAT DoubleFloat
++ Author: Michael Monagan
++ Date Created:
++   January 1988
++ Change History:
++ Basic Operations: exp1, hash, log2, log10, rationalApproximation, / , **
++ Related Constructors:
++ Keywords: small float
++ Description:
++ \spadtype{DoubleFloat} is intended to make accessible
++ hardware floating point arithmetic in Axiom, either native double
++ precision, or IEEE. On most machines, there will be hardware support for

```

```

++ the arithmetic operations: ++ +, *, / and possibly also the
++ sqrt operation.
++ The operations exp, log, sin, cos, atan are normally coded in
++ software based on minimax polynomial/rational approximations.
++
++ Some general comments about the accuracy of the operations:
++ the operations +, *, / and sqrt are expected to be fully accurate.
++ The operations exp, log, sin, cos and atan are not expected to be
++ fully accurate. In particular, sin and cos
++ will lose all precision for large arguments.
++
++ The Float domain provides an alternative to the DoubleFloat domain.
++ It provides an arbitrary precision model of floating point arithmetic.
++ This means that accuracy problems like those above are eliminated
++ by increasing the working precision where necessary. \spadtype{Float}
++ provides some special functions such as erf, the error function
++ in addition to the elementary functions. The disadvantage of Float is that
++ it is much more expensive than small floats when the latter can be used.

DoubleFloat(): Join(FloatingPointSystem, DifferentialRing, OpenMath,
  TranscendentalFunctionCategory, SpecialFunctionCategory, _
  ConvertibleTo InputForm) with
  _/ : (%, Integer) -> %
    ++ x / i computes the division from x by an integer i.
  _*_ : (%,%) -> %
    ++ x ** y returns the yth power of x (equal to \spad{exp(y log x)}).
  exp1 : () -> %
    ++ exp1() returns the natural log base \spad{2.718281828...}.
  hash : % -> Integer
    ++ hash(x) returns the hash key for x
  log2 : % -> %
    ++ log2(x) computes the logarithm with base 2 for x.
  log10 : % -> %
    ++ log10(x) computes the logarithm with base 10 for x.
  atan : (%,%) -> %
    ++ atan(x,y) computes the arc tangent from x with phase y.
  Gamma : % -> %
    ++ Gamma(x) is the Euler Gamma function.
  Beta : (%,%) -> %
    ++ Beta(x,y) is \spad{Gamma(x) * Gamma(y)/Gamma(x+y)}.
  doubleFloatFormat : String -> String
    ++ change the output format for doublefloats using lisp format strings
  rationalApproximation: (%, NonNegativeInteger) -> Fraction Integer
    ++ rationalApproximation(f, n) computes a rational approximation
    ++ r to f with relative error \spad{< 10**(-n)}.
  rationalApproximation: (%, NonNegativeInteger, NonNegativeInteger) -> _
    Fraction Integer
    ++ rationalApproximation(f, n, b) computes a rational
    ++ approximation r to f with relative error \spad{< b**(-n)}
    ++ (that is, \spad{|(r-f)/f| < b**(-n)}).

```

```

machineFraction : % -> Fraction Integer
  ++ machineFraction(x) returns a bit-exact fraction of the machine
  ++ floating point number using the common lisp integer-decode-float
  ++ function. See Steele, ISBN 0-13-152414-3 p354
  ++ This function can be used to print results which do not depend
  ++ on binary-to-decimal conversions
  ++
  ++X a:DFLOAT:=-1.0/3.0
  ++X machineFraction a
integerDecode : % -> List Integer
  ++ integerDecode(x) returns the multiple values of the common
  ++ lisp integer-decode-float function.
  ++ See Steele, ISBN 0-13-152414-3 p354. This function can be used
  ++ to ensure that the results are bit-exact and do not depend on
  ++ the binary-to-decimal conversions.
  ++
  ++X a:DFLOAT:=-1.0/3.0
  ++X integerDecode a

== add
format: String := "~G"
MER ==> Record(MANTISSA:Integer,EXPONENT:Integer)

manexp: % -> MER

doubleFloatFormat(s:String): String ==
  ss: String := format
  format := s
  ss

OMwrite(x: %): String ==
  s: String := ""
  sp := OM_-STRINGTOSTRINGPTR(s)$Lisp
  dev: OpenMathDevice := OMopenString(sp pretend String, OMencodingXML)
  OMputObject(dev)
  OMputFloat(dev, convert x)
  OMputEndObject(dev)
  OMclose(dev)
  s := OM_-STRINGPTRTOSTRING(sp)$Lisp pretend String
  s

OMwrite(x: %, wholeObj: Boolean): String ==
  s: String := ""
  sp := OM_-STRINGTOSTRINGPTR(s)$Lisp
  dev: OpenMathDevice := OMopenString(sp pretend String, OMencodingXML)
  if wholeObj then
    OMputObject(dev)
  OMputFloat(dev, convert x)
  if wholeObj then
    OMputEndObject(dev)

```

```

OMclose(dev)
s := OM_STRINGPTRTOSTRING(sp)$Lisp pretend String
s

OMwrite(dev: OpenMathDevice, x: %): Void ==
  OMputObject(dev)
  OMputFloat(dev, convert x)
  OMputEndObject(dev)

OMwrite(dev: OpenMathDevice, x: %, wholeObj: Boolean): Void ==
  if wholeObj then
    OMputObject(dev)
    OMputFloat(dev, convert x)
  if wholeObj then
    OMputEndObject(dev)

checkComplex(x:%):% == C_-TO_-R(x)$Lisp
-- In AKCL we used to have to make the arguments to ASIN ACOS ACOSH ATANH
-- complex to get the correct behaviour.
--makeComplex(x: %):% == COMPLEX(x, 0$%)$Lisp

machineFraction(df:%):Fraction(Integer) ==
  numer:Integer:=INTEGER_-DECODE_-FLOAT_-NUMERATOR(df)$Lisp
  denom:Integer:=INTEGER_-DECODE_-FLOAT_-DENOMINATOR(df)$Lisp
  sign:Integer:=INTEGER_-DECODE_-FLOAT_-SIGN(df)$Lisp
  sign*numер/denom

integerDecode(df:%):List(Integer) ==
  numer:Integer:=INTEGER_-DECODE_-FLOAT_-NUMERATOR(df)$Lisp
  exp:Integer:=INTEGER_-DECODE_-FLOAT_-EXPONENT(df)$Lisp
  sign:Integer:=INTEGER_-DECODE_-FLOAT_-SIGN(df)$Lisp
  [numer,exp,sign]

base()          == FLOAT_-RADIX(0$%)$Lisp
mantissa x      == manexp(x).MANTISSA
exponent x      == manexp(x).EXPONENT
precision()     == FLOAT_-DIGITS(0$%)$Lisp
bits()          ==
  base() = 2 => precision()
  base() = 16 => 4*precision()
  wholePart(precision()*log2(base():%)):PositiveInteger
max()           == MOST_-POSITIVE_-DOUBLE_-FLOAT$Lisp
min()           == MOST_-NEGATIVE_-DOUBLE_-FLOAT$Lisp
order(a) == precision() + exponent a - 1
0               == FLOAT(0$Lisp,MOST_-POSITIVE_-DOUBLE_-FLOAT$Lisp)$Lisp
1               == FLOAT(1$Lisp,MOST_-POSITIVE_-DOUBLE_-FLOAT$Lisp)$Lisp
-- rational approximation to e accurate to 23 digits
exp1() == FLOAT(534625820200,MOST_-POSITIVE_-DOUBLE_-FLOAT$Lisp)$Lisp / _
        FLOAT(196677847971,MOST_-POSITIVE_-DOUBLE_-FLOAT$Lisp)$Lisp
pi()           == FLOAT(PI$Lisp,MOST_-POSITIVE_-DOUBLE_-FLOAT$Lisp)$Lisp

```

```

coerce(x:%):OutputForm ==
  x >= 0 => message(FORMAT(NIL$Lisp,format,x)$Lisp pretend String)
  - (message(FORMAT(NIL$Lisp,format,-x)$Lisp pretend String))
convert(x:%):InputForm == convert(x pretend DoubleFloat)$InputForm
x < y          == DFLESSTHAN(x,y)$Lisp
- x            == DFUNARYMINUS(x)$Lisp
x + y          == DFADD(x,y)$Lisp
x:% - y:%      == DFSUBTRACT(x,y)$Lisp
x:% * y:%      == DFMULTIPLY(x,y)$Lisp
i:Integer * x:% == DFINTEGERMULTIPLY(i,x)$Lisp
max(x,y)       == DFMAX(x,y)$Lisp
min(x,y)       == DFMIN(x,y)$Lisp
x = y          == DFEQL(x,y)$Lisp
x:% / i:Integer == DFINTEGERDIVIDE(x,i)$Lisp
sqrt x         == checkComplex DFSQRT(x)$Lisp
log10 x        == checkComplex DFLOG(x,10)$Lisp
x:% ** i:Integer == DFINTEGEREXPT(x,i)$Lisp
x:% ** y:%      == checkComplex DFEXPT(x,y)$Lisp
coerce(i:Integer):% == FLOAT(i,MOST_-POSITIVE_-DOUBLE_-FLOAT$Lisp)$Lisp
exp x          == DFEXP(x)$Lisp
log x          == checkComplex DFLOG(x)$Lisp
log2 x         == checkComplex DFLOG(x,2)$Lisp
sin x          == DFSIN(x)$Lisp
cos x          == DFCOS(x)$Lisp
tan x          == DFTAN(x)$Lisp
cot x          == COT(x)$Lisp
sec x          == SEC(x)$Lisp
csc x          == CSC(x)$Lisp
asin x         == checkComplex DFASIN(x)$Lisp -- can be complex
acos x         == checkComplex DFACOS(x)$Lisp -- can be complex
atan x         == DFATAN(x)$Lisp
acsc x         == checkComplex ACSC(x)$Lisp
acot x         == ACOT(x)$Lisp
asec x         == checkComplex ASEC(x)$Lisp
sinh x         == SINH(x)$Lisp
cosh x         == COSH(x)$Lisp
tanh x         == TANH(x)$Lisp
csch x         == CSCH(x)$Lisp
coth x         == COTH(x)$Lisp
sech x         == SECH(x)$Lisp
asinh x        == DFASINH(x)$Lisp
acosh x        == checkComplex DFACOSH(x)$Lisp -- can be complex
atanh x        == checkComplex DFATANH(x)$Lisp -- can be complex
acsch x        == ACSCH(x)$Lisp
acoth x        == checkComplex ACOTH(x)$Lisp
asech x        == checkComplex ASECH(x)$Lisp
x:% / y:%      == DFDIVIDE(x,y)$Lisp
negative? x    == DFMINUSP(x)$Lisp
zero? x        == ZEROP(x)$Lisp
hash x         == HASHEQ(x)$Lisp

```

```

recip(x)          == (zero? x => "failed"; 1 / x)
differentiate x   == 0

SFSFUN           ==> DoubleFloatSpecialFunctions()
sfx              ==> x pretend DoubleFloat
sfy              ==> y pretend DoubleFloat
airyAi x         == airyAi(sfx)$SFSFUN pretend %
airyBi x         == airyBi(sfx)$SFSFUN pretend %
besselI(x,y)     == besselI(sfx,sfy)$SFSFUN pretend %
besselJ(x,y)     == besselJ(sfx,sfy)$SFSFUN pretend %
besselK(x,y)     == besselK(sfx,sfy)$SFSFUN pretend %
bessely(x,y)     == bessely(sfx,sfy)$SFSFUN pretend %
Beta(x,y)        == Beta(sfx,sfy)$SFSFUN pretend %
digamma x        == digamma(sfx)$SFSFUN pretend %
Gamma x          == Gamma(sfx)$SFSFUN pretend %
-- not implemented in SFSFUN
-- Gamma(x,y)     == Gamma(sfx,sfy)$SFSFUN pretend %
polygamma(x,y)   ==
  if (n := retractIfCan(x:%):Union(Integer, "failed")) case Integer _
    and n >= 0
  then polygamma(n::Integer::NonNegativeInteger,sfy)$SFSFUN pretend %
  else error "polygamma: first argument should be a nonnegative integer"

wholePart x       == TRUNCATE(x)$Lisp
float(ma,ex,b)    == ma*(b::%)**ex
convert(x:%):DoubleFloat == x pretend DoubleFloat
convert(x:%):Float == convert(x pretend DoubleFloat)$Float
rationalApproximation(x, d) == rationalApproximation(x, d, 10)

atan(x,y) ==
  x = 0 =>
    y > 0 => pi()/2
    y < 0 => -pi()/2
    0
  -- Only count on first quadrant being on principal branch.
  theta := atan abs(y/x)
  if x < 0 then theta := pi() - theta
  if y < 0 then theta := - theta
  theta

retract(x:%):Fraction(Integer) ==
  rationalApproximation(x, (precision() - 1)::NonNegativeInteger, base())

retractIfCan(x:%):Union(Fraction Integer, "failed") ==
  rationalApproximation(x, (precision() - 1)::NonNegativeInteger, base())

retract(x:%):Integer ==
  x = ((n := wholePart x)::%) => n
  error "Not an integer"

```

```

retractIfCan(x:%):Union(Integer, "failed") ==
  x = ((n := wholePart x)::%) => n
  "failed"

sign(x) == retract FLOAT_-SIGN(x,1)$Lisp

abs x    == FLOAT_-SIGN(1,x)$Lisp

manexp(x) ==
  zero? x => [0,0]
  s := sign x; x := abs x
  if x > max()%% then return [s*mantissa(max())+1,exponent max()]
  me:Record(man:%,exp:Integer) := MANEXP(x)$Lisp
  two53:= base()**precision()
  [s*wholePart(two53 * me.man ),me.exp-precision()]

-- rationalApproximation(y,d,b) ==
--   this is the quotient remainder algorithm (requires wholePart operation)
--   x := y
--   if b < 2 then error "base must be > 1"
--   tol := (b::%)**d
--   p0,p1,q0,q1 : Integer
--   p0 := 0; p1 := 1; q0 := 1; q1 := 0
--   repeat
--     a := wholePart x
--     x := fractionPart x
--     p2 := p0+a*p1
--     q2 := q0+a*q1
--     if x = 0 or tol*abs(q2*y-(p2::%)) < abs(q2*y) then
--       return (p2/q2)
--     (p0,p1) := (p1,p2)
--     (q0,q1) := (q1,q2)
--     x := 1/x

rationalApproximation(f,d,b) ==
  -- this algorithm expresses f as n / d where d = BASE ** k
  -- then all arithmetic operations are done over the integers
  (nu, ex) := manexp f
  BASE := base()
  ex >= 0 => (nu * BASE ** (ex::NonNegativeInteger))::Fraction(Integer)
  de :Integer := BASE**((-ex)::NonNegativeInteger)
  b < 2 => error "base must be > 1"
  tol := b**d
  s := nu; t := de
  p0:Integer := 0; p1:Integer := 1; q0:Integer := 1; q1:Integer := 0
  repeat
    (q,r) := divide(s, t)
    p2 := q*p1+p0
    q2 := q*q1+q0
    r = 0 or tol*abs(nu*q2-de*p2) < de*abs(p2) => return(p2/q2)

```



```

      (p0,p1) := (p1,p2)
      (q0,q1) := (q1,q2)
      (s,t) := (t,r)

x:% ** r:Fraction Integer ==
zero? x =>
  zero? r => error "0**0 is undefined"
  negative? r => error "division by 0"
  0
--      zero? r or one? x => 1
zero? r or (x = 1) => 1
--      one? r => x
      (r = 1) => x
      n := numer r
      d := denom r
      negative? x =>
        odd? d =>
          odd? n => return -((-x)**r)
          return ((-x)**r)
        error "negative root"
      d = 2 => sqrt(x) ** n
      x ** (n:% / d:%)

```

---

— DFLOAT.dotabb —

```

"DFLOAT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=DFLOAT",
          shape=ellipse]
"FRAC" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FRAC"]
"DFLOAT" -> "FRAC"

```

---

## 5.16 domain DFMAT DoubleFloatMatrix

— DoubleFloatMatrix.input —

```

)set break resume
)sys rm -f DoubleFloatMatrix.output
)spool DoubleFloatMatrix.output
)set message test on
)set message auto off
)clear all

```

```

--S 1 of 6
)show DoubleFloatMatrix
--R DoubleFloatMatrix is a domain constructor
--R Abbreviation for DoubleFloatMatrix is DFMAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for DFMAT
--R
--R----- Operations -----
--R ?? : (Integer,%) -> %           ?? : (%,DoubleFloat) -> %
--R ?? : (DoubleFloat,%) -> %      ?? : (%,%) -> %
--R +? : (%,%) -> %                -? : % -> %
--R ?-? : (%,%) -> %               antisymmetric? : % -> Boolean
--R coerce : DoubleFloatVector -> % copy : % -> %
--R diagonal? : % -> Boolean        diagonalMatrix : List % -> %
--R empty : () -> %                empty? : % -> Boolean
--R eq? : (%,%) -> Boolean          fill! : (%,DoubleFloat) -> %
--R horizConcat : (%,%) -> %       maxColIndex : % -> Integer
--R maxRowIndex : % -> Integer      minColIndex : % -> Integer
--R minRowIndex : % -> Integer      ncols : % -> NonNegativeInteger
--R nrows : % -> NonNegativeInteger parts : % -> List DoubleFloat
--R qnew : (Integer,Integer) -> %   sample : () -> %
--R square? : % -> Boolean          squareTop : % -> %
--R symmetric? : % -> Boolean       transpose : % -> %
--R vertConcat : (%,%) -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?? : (DoubleFloatVector,%) -> DoubleFloatVector
--R ?? : (%,DoubleFloatVector) -> DoubleFloatVector
--R ***? : (%,Integer) -> % if DoubleFloat has FIELD
--R ***? : (%,NonNegativeInteger) -> %
--R ?/? : (%,DoubleFloat) -> % if DoubleFloat has FIELD
--R ==? : (%,%) -> Boolean if DoubleFloat has SETCAT
--R any? : ((DoubleFloat -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if DoubleFloat has SETCAT
--R column : (%,Integer) -> DoubleFloatVector
--R columnSpace : % -> List DoubleFloatVector if DoubleFloat has EUCLDOM
--R count : (DoubleFloat,%) -> NonNegativeInteger if $ has finiteAggregate and DoubleFloat has SETCAT
--R count : ((DoubleFloat -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R determinant : % -> DoubleFloat if DoubleFloat has commutative *
--R diagonalMatrix : List DoubleFloat -> %
--R elt : (%,List Integer,List Integer) -> %
--R elt : (%,Integer,Integer,DoubleFloat) -> DoubleFloat
--R elt : (%,Integer,Integer) -> DoubleFloat
--R eval : (%,List DoubleFloat,List DoubleFloat) -> % if DoubleFloat has EVALAB DFLOAT and DoubleFloat has SETCAT
--R eval : (%,DoubleFloat,DoubleFloat) -> % if DoubleFloat has EVALAB DFLOAT and DoubleFloat has SETCAT
--R eval : (%,Equation DoubleFloat) -> % if DoubleFloat has EVALAB DFLOAT and DoubleFloat has SETCAT
--R eval : (%,List Equation DoubleFloat) -> % if DoubleFloat has EVALAB DFLOAT and DoubleFloat has SETCAT
--R every? : ((DoubleFloat -> Boolean),%) -> Boolean if $ has finiteAggregate
--R exquo : (%,DoubleFloat) -> Union(%, "failed") if DoubleFloat has INTDOM
--R hash : % -> SingleInteger if DoubleFloat has SETCAT

```

```

--R inverse : % -> Union(%, "failed") if DoubleFloat has FIELD
--R latex : % -> String if DoubleFloat has SETCAT
--R less? : (%, NonNegativeInteger) -> Boolean
--R listOfLists : % -> List List DoubleFloat
--R map : (((DoubleFloat, DoubleFloat) -> DoubleFloat), %, %, DoubleFloat) -> %
--R map : (((DoubleFloat, DoubleFloat) -> DoubleFloat), %, %) -> %
--R map : ((DoubleFloat -> DoubleFloat), %) -> %
--R map! : ((DoubleFloat -> DoubleFloat), %) -> %
--R matrix : List List DoubleFloat -> %
--R member? : (DoubleFloat, %) -> Boolean if $ has finiteAggregate and DoubleFloat has SETCAT
--R members : % -> List DoubleFloat if $ has finiteAggregate
--R minordet : % -> DoubleFloat if DoubleFloat has commutative *
--R more? : (%, NonNegativeInteger) -> Boolean
--R new : (NonNegativeInteger, NonNegativeInteger, DoubleFloat) -> %
--R nullSpace : % -> List DoubleFloatVector if DoubleFloat has INTDOM
--R nullity : % -> NonNegativeInteger if DoubleFloat has INTDOM
--R pfaffian : % -> DoubleFloat if DoubleFloat has COMRING
--R qelt : (%, Integer, Integer) -> DoubleFloat
--R qsetelt! : (%, Integer, Integer, DoubleFloat) -> DoubleFloat
--R rank : % -> NonNegativeInteger if DoubleFloat has INTDOM
--R row : (%, Integer) -> DoubleFloatVector
--R rowEchelon : % -> % if DoubleFloat has EUCDOM
--R scalarMatrix : (NonNegativeInteger, DoubleFloat) -> %
--R setColumn! : (%, Integer, DoubleFloatVector) -> %
--R setRow! : (%, Integer, DoubleFloatVector) -> %
--R setelt : (%, List Integer, List Integer, %) -> %
--R setelt : (%, Integer, Integer, DoubleFloat) -> DoubleFloat
--R setsubMatrix! : (%, Integer, Integer, %) -> %
--R size? : (%, NonNegativeInteger) -> Boolean
--R subMatrix : (%, Integer, Integer, Integer, Integer) -> %
--R swapColumns! : (%, Integer, Integer) -> %
--R swapRows! : (%, Integer, Integer) -> %
--R transpose : DoubleFloatVector -> %
--R zero : (NonNegativeInteger, NonNegativeInteger) -> %
--R ?~=? : (%, %) -> Boolean if DoubleFloat has SETCAT
--R
--E 1

--S 2 of 6
a:DFMAT:=qnew(2,3)
--R
--R      +0.  0.  0.+
--R  (1)  |      |
--R      +0.  0.  0.+
--R
--R                                          Type: DoubleFloatMatrix
--E 2

--S 3 of 6
qsetelt!(a,1,1,1.0)
--R

```

```

--R (2) 1.
--R
--R                                          Type: DoubleFloat
--E 3

--S 4 of 6
a
--R
--R      +0.  0.  0.+
--R (3)  |      |
--R      +0.  1.  0.+
--R
--R                                          Type: DoubleFloatMatrix
--E 4

--S 5 of 6
qsetelt!(a,0,0,2.0)
--R
--R (4) 2.
--R
--R                                          Type: DoubleFloat
--E 5

--S 6 of 6
a
--R
--R      +2.  0.  0.+
--R (5)  |      |
--R      +0.  1.  0.+
--R
--R                                          Type: DoubleFloatMatrix
--E 6

)spool
)lisp (bye)

```

---

— DoubleFloatMatrix.help —

```

=====
DoubleFloatMatrix examples
=====

```

This domain creates a lisp simple array of machine doublefloats.  
It provides one new function called qnew which takes an integer  
that gives the array length.

NOTE: Unlike normal Axiom arrays the DoubleFloatMatrix arrays  
are 0-based so the first element is 0. Axiom arrays normally  
start at 1.

```
a:DFMAT:=qnew(2,3)
```

```

+0.  0.  0.+
|      |
+0.  0.  0.+

```

```

qsetelt!(a,1,1,1.0)
1.

```

```

a
+0.  0.  0.+
|      |
+0.  1.  0.+

```

```

qsetelt!(a,0,0,2.0)
2.

```

```

a
+2.  0.  0.+
|      |
+0.  1.  0.+

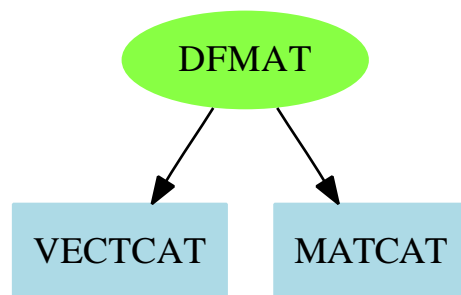
```

See Also:

- o )help Float
- o )help DoubleFloat
- o )show DoubleFloatMatrix

---

### 5.16.1 DoubleFloatMatrix (DFMAT)



**Exports:**

|                |                |              |             |
|----------------|----------------|--------------|-------------|
| #?             | -?             | ***?         | ?*?         |
| ?+?            | ?-?            | ?/?          | ?=?         |
| ?~=?           | antisymmetric? | any?         | coerce      |
| coerce         | column         | columnSpace  | copy        |
| count          | count          | determinant  | diagonal?   |
| diagonalMatrix | diagonalMatrix | elt          | elt         |
| elt            | empty          | empty?       | eq?         |
| eval           | eval           | eval         | eval        |
| every?         | exquo          | fill!        | hash        |
| horizConcat    | inverse        | latex        | less?       |
| listOfLists    | map            | map          | map         |
| map!           | matrix         | maxColIndex  | maxRowIndex |
| member?        | members        | minColIndex  | minRowIndex |
| minordet       | more?          | ncols        | new         |
| nrows          | nullSpace      | nullity      | parts       |
| pfaffian       | qelt           | qnew         | qsetelt!    |
| rank           | row            | rowEchelon   | sample      |
| scalarMatrix   | setColumn!     | setRow!      | setelt      |
| setelt         | setsubMatrix!  | size?        | square?     |
| squareTop      | subMatrix      | swapColumns! | swapRows!   |
| symmetric?     | transpose      | transpose    | vertConcat  |
| zero           |                |              |             |

— domain DFMAT DoubleFloatMatrix —

```

)abbrev domain DFMAT DoubleFloatMatrix
++ Author: Waldek Hebisch
++ Description: This is a low-level domain which implements matrices
++ (two dimensional arrays) of double precision floating point
++ numbers. Indexing is 0 based, there is no bound checking (unless
++ provided by lower level).
DoubleFloatMatrix : MatrixCategory(DoubleFloat,
                                   DoubleFloatVector,
                                   DoubleFloatVector) with
  qnew : (Integer, Integer) -> %
    ++ qnew(n, m) creates a new uninitialized n by m matrix.
    ++
    ++X t1:DFMAT:=qnew(3,4)

== add

Qelt2 ==> DAREF2$Lisp
Qsetelt2 ==> DSETAREF2$Lisp
Qnrows ==> DANROWS$Lisp
Qncols ==> DANCOLS$Lisp
Qnew ==> MAKE_-DOUBLE_-MATRIX$Lisp
Qnew1 ==> MAKE_-DOUBLE_-MATRIX1$Lisp

```

```

minRowIndex x == 0
minColIndex x == 0
nrows x == Qnrows(x)
ncols x == Qncols(x)
maxRowIndex x == Qnrows(x) - 1
maxColIndex x == Qncols(x) - 1

qelt(m, i, j) == Qelt2(m, i, j)
qsetelt_!(m, i, j, r) == Qsetelt2(m, i, j, r)

empty() == Qnew(0$Integer, 0$Integer)
qnew(rows, cols) == Qnew(rows, cols)
new(rows, cols, a) == Qnew1(rows, cols, a)

```

---

— DFMAT.dotabb —

```

"DFMAT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=DFMAT",
        shape=ellipse]
"VECTCAT" [color=lightblue,href="bookvol10.2.pdf#nameddest=VECTCAT"];
"MATCAT" [color=lightblue,href="bookvol10.2.pdf#nameddest=MATCAT"];
"DFMAT" -> "VECTCAT"
"DFMAT" -> "MATCAT"

```

---

## 5.17 domain DFVEC DoubleFloatVector

— DoubleFloatVector.input —

```

)set break resume
)sys rm -f DoubleFloatVector.output
)spool DoubleFloatVector.output
)set message test on
)set message auto off
)clear all

--S 1 of 6
)show DoubleFloatVector
--R DoubleFloatVector is a domain constructor
--R Abbreviation for DoubleFloatVector is DFVEC
--R This constructor is exposed in this frame.

```

```

--R Issue )edit bookvol10.3.pamphlet to see algebra source code for DFVEC
--R
--R----- Operations -----
--R concat : List % -> %                concat : (%,% ) -> %
--R concat : (DoubleFloat,% ) -> %      concat : (% ,DoubleFloat) -> %
--R construct : List DoubleFloat -> %    copy : % -> %
--R delete : (% ,Integer) -> %           ?.? : (% ,Integer) -> DoubleFloat
--R empty : () -> %                     empty? : % -> Boolean
--R entries : % -> List DoubleFloat      eq? : (% ,%) -> Boolean
--R index? : (Integer,% ) -> Boolean      indices : % -> List Integer
--R insert : (% ,% ,Integer) -> %         qelt : (% ,Integer) -> DoubleFloat
--R qnew : Integer -> %                  reverse : % -> %
--R sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?? : (% ,DoubleFloat) -> % if DoubleFloat has MONOID
--R ?? : (DoubleFloat,% ) -> % if DoubleFloat has MONOID
--R ?? : (Integer,% ) -> % if DoubleFloat has ABELGRP
--R ?+? : (% ,%) -> % if DoubleFloat has ABELSG
--R ?-? : (% ,%) -> % if DoubleFloat has ABELGRP
--R -? : % -> % if DoubleFloat has ABELGRP
--R ?<? : (% ,%) -> Boolean if DoubleFloat has ORDSET
--R ?<=? : (% ,%) -> Boolean if DoubleFloat has ORDSET
--R ?=? : (% ,%) -> Boolean if DoubleFloat has SETCAT
--R ?>? : (% ,%) -> Boolean if DoubleFloat has ORDSET
--R ?>=? : (% ,%) -> Boolean if DoubleFloat has ORDSET
--R any? : ((DoubleFloat -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if DoubleFloat has SETCAT
--R convert : % -> InputForm if DoubleFloat has KONVERT INFORM
--R copyInto! : (% ,% ,Integer) -> % if $ has shallowlyMutable
--R count : (DoubleFloat,% ) -> NonNegativeInteger if $ has finiteAggregate and DoubleFloat has SETCAT
--R count : ((DoubleFloat -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R cross : (% ,%) -> % if DoubleFloat has RING
--R delete : (% ,UniversalSegment Integer) -> %
--R dot : (% ,%) -> DoubleFloat if DoubleFloat has RING
--R ?.? : (% ,UniversalSegment Integer) -> %
--R elt : (% ,Integer,DoubleFloat) -> DoubleFloat
--R entry? : (DoubleFloat,% ) -> Boolean if $ has finiteAggregate and DoubleFloat has SETCAT
--R eval : (% ,List DoubleFloat,List DoubleFloat) -> % if DoubleFloat has EVALAB DFLOAT and DoubleFloat has SETCAT
--R eval : (% ,DoubleFloat,DoubleFloat) -> % if DoubleFloat has EVALAB DFLOAT and DoubleFloat has SETCAT
--R eval : (% ,Equation DoubleFloat) -> % if DoubleFloat has EVALAB DFLOAT and DoubleFloat has SETCAT
--R eval : (% ,List Equation DoubleFloat) -> % if DoubleFloat has EVALAB DFLOAT and DoubleFloat has SETCAT
--R every? : ((DoubleFloat -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (% ,DoubleFloat) -> % if $ has shallowlyMutable
--R find : ((DoubleFloat -> Boolean),%) -> Union(DoubleFloat,"failed")
--R first : % -> DoubleFloat if Integer has ORDSET
--R hash : % -> SingleInteger if DoubleFloat has SETCAT
--R insert : (DoubleFloat,% ,Integer) -> %
--R latex : % -> String if DoubleFloat has SETCAT
--R length : % -> DoubleFloat if DoubleFloat has RADCAT and DoubleFloat has RING
--R less? : (% ,NonNegativeInteger) -> Boolean

```



[illegible]

```

-- NOTE: DFVEC arrays are 0-based, normal Axiom arrays are 1-based
--S 3 of 6
t1.1:=1.0

(3)  1.
                                         Type: DoubleFloat
--E 3

--S 4 of 6
t1

(4)  [0.,1.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.]
                                         Type: DoubleFloatVector
--E 4

--S 5 of 6
t1.0:=2.0

(5)  2.
                                         Type: DoubleFloat
--E 5

--S 6 of 6
t1

(6)  [2.,1.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.]
                                         Type: DoubleFloatVector
--E 6

)spool
)lisp (bye)

```

---

— DoubleFloatVector.help —

```

=====
DoubleFloatVector examples
=====

```

This domain creates a lisp simple array of machine doublefloats.  
It provides one new function called qnew which takes an integer  
that gives the array length.

NOTE: Unlike normal Axiom arrays the DoubleFloatVector arrays  
are 0-based so the first element is 0. Axiom arrays normally  
start at 1.

```

a:DFVEC:=qnew 17

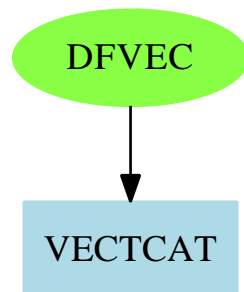
```

```
[0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.]  
  
a.1:=1.0  
1.  
  
a  
[0.,1.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.]  
  
a.0:=2.0  
2.  
  
a  
[2.,1.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.]
```

See Also:  
o )help Float  
o )help DoubleFloat  
o )show DoubleFloatVector

---

### 5.17.1 DoubleFloatVector (DFVEC)



**Exports:**

|           |                  |         |              |        |
|-----------|------------------|---------|--------------|--------|
| #?        | -?               | ?*?     | ?+?          | ?-?    |
| ?.?       | ?<=?             | ?<?     | ?=?          | ?>=?   |
| ?>?       | ?~=?             | any?    | coerce       | concat |
| construct | convert          | copy    | copyInto!    | count  |
| cross     | delete           | dot     | elt          | empty  |
| empty?    | entries          | entry?  | eq?          | eval   |
| every?    | fill!            | find    | first        | hash   |
| index?    | indices          | insert  | latex        | length |
| less?     | magnitude        | map     | map!         | max    |
| maxIndex  | member?          | members | merge        | min    |
| minIndex  | more?            | new     | outerProduct | parts  |
| position  | qelt             | qnew    | qsetelt!     | reduce |
| remove    | removeDuplicates | reverse | reverse!     | sample |
| select    | setelt           | size?   | sort         | sort!  |
| sorted?   | swap!            | zero    |              |        |

— domain DFVEC DoubleFloatVector —

```
)abbrev domain DFVEC DoubleFloatVector
++ Author: Waldek Hebisch
++ Description: This is a low-level domain which implements vectors
++ (one dimensional arrays) of double precision floating point
++ numbers. Indexing is 0 based, there is no bound checking (unless
++ provided by lower level).
DoubleFloatVector : VectorCategory DoubleFloat with
  qnew : Integer -> %
  ++ qnew(n) creates a new uninitialized vector of length n.
  ++
  ++X t1:DFVEC:=qnew(7)
== add

Qelt1 ==> DELT$Lisp
Qsetelt1 ==> DSETELT$Lisp

qelt(x, i) == Qelt1(x, i)
qsetelt!(x, i, s) == Qsetelt1(x, i, s)
Qsize ==> DLEN$Lisp
Qnew ==> MAKE_-DOUBLE_-VECTOR$Lisp
Qnew1 ==> MAKE_-DOUBLE_-VECTOR1$Lisp

#x == Qsize x
minIndex x == 0
empty() == Qnew(0$Lisp)
qnew(n) == Qnew(n)
new(n, x) == Qnew1(n, x)
qelt(x, i) == Qelt1(x, i)
elt(x:%, i:Integer) == Qelt1(x, i)
qsetelt!(x, i, s) == Qsetelt1(x, i, s)
```

```

setelt(x : %, i : Integer, s : DoubleFloat) == Qsetelt1(x, i, s)
fill_!(x, s) ==
  for i in 0..((Qsize(x)) - 1) repeat Qsetelt1(x, i, s)
x

```

---

— DFVEC.dotabb —

```

"DFVEC" [color="#88FF44",href="bookvol10.3.pdf#nameddest=DFVEC",
        shape=ellipse]
"VECTCAT" [color=lightblue,href="bookvol10.2.pdf#nameddest=VECTCAT"];
"DFVEC" -> "VECTCAT"

```

---

## 5.18 domain DROPT DrawOption

---

— DrawOption.input —

```

)set break resume
)sys rm -f DrawOption.output
)spool DrawOption.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show DrawOption
--R DrawOption is a domain constructor
--R Abbreviation for DrawOption is DROPT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for DROPT
--R
--R----- Operations -----
--R ?? : (%,%) -> Boolean          adaptive : Boolean -> %
--R clip : List Segment Float -> % clip : Boolean -> %
--R coerce : % -> OutputForm      curveColor : Palette -> %
--R curveColor : Float -> %       hash : % -> SingleInteger
--R latex : % -> String           pointColor : Palette -> %
--R pointColor : Float -> %       range : List Segment Float -> %
--R ranges : List Segment Float -> % style : String -> %
--R title : String -> %          toScale : Boolean -> %
--R tubePoints : PositiveInteger -> % tubeRadius : Float -> %

```

```

--R unit : List Float -> %          var1Steps : PositiveInteger -> %
--R var2Steps : PositiveInteger -> %    ?~=? : (%,% ) -> Boolean
--R colorFunction : ((DoubleFloat,DoubleFloat,DoubleFloat) -> DoubleFloat) -> %
--R colorFunction : ((DoubleFloat,DoubleFloat) -> DoubleFloat) -> %
--R colorFunction : (DoubleFloat -> DoubleFloat) -> %
--R coord : (Point DoubleFloat -> Point DoubleFloat) -> %
--R coordinates : (Point DoubleFloat -> Point DoubleFloat) -> %
--R option : (List %,Symbol) -> Union(Any,"failed")
--R option? : (List %,Symbol) -> Boolean
--R range : List Segment Fraction Integer -> %
--R space : ThreeSpace DoubleFloat -> %
--R viewpoint : Record(theta: DoubleFloat,phi: DoubleFloat,scale: DoubleFloat,scaleX: DoubleFloat,scaleY: DoubleFloat)
--R
--E 1

```

```

)spool
)lisp (bye)

```

\_\_\_\_\_

— DrawOption.help —

```

=====
DrawOption examples
=====

```

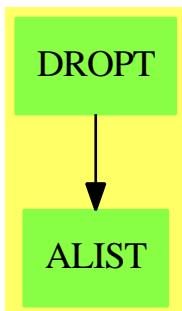
```

See Also:
o )show DrawOption

```

\_\_\_\_\_

### 5.18.1 DrawOption (DROPT)



**Exports:**

|             |            |           |               |            |
|-------------|------------|-----------|---------------|------------|
| adaptive    | clip       | coerce    | colorFunction | coord      |
| coordinates | curveColor | hash      | latex         | option     |
| option?     | pointColor | range     | ranges        | space      |
| style       | title      | toScale   | tubePoints    | tubeRadius |
| unit        | var1Steps  | var2Steps | viewpoint     | ?=?        |
| ?~=?        |            |           |               |            |

## — domain DROPT DrawOption —

```

)abbrev domain DROPT DrawOption
++ Author: Stephen Watt
++ Date Created: 1 March 1990
++ Date Last Updated: 31 Oct 1990, Jim Wen
++ Basic Operations: adaptive, clip, title, style, toScale, coordinates,
++ pointColor, curveColor, colorFunction, tubeRadius, range, ranges,
++ var1Steps, var2Steps, tubePoints, unit
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ DrawOption allows the user to specify defaults for the
++ creation and rendering of plots.

DrawOption(): Exports == Implementation where
  RANGE ==> List Segment Float
  UNIT  ==> List Float
  PAL   ==> Palette
  POINT ==> Point(DoubleFloat)
  SEG   ==> Segment Float
  SF    ==> DoubleFloat
  SPACE3 ==> ThreeSpace(DoubleFloat)
  VIEWPT ==> Record(theta:SF, phi:SF, scale:SF, scaleX:SF, scaleY:SF, scaleZ:SF, deltaX:SF, d

Exports ==> SetCategory with
  adaptive : Boolean -> %
    ++ adaptive(b) turns adaptive 2D plotting on if b is true, or off if b is
    ++ false. This option is expressed in the form \spad{adaptive == b}.
  clip : Boolean -> %
    ++ clip(b) turns 2D clipping on if b is true, or off if b is false.
    ++ This option is expressed in the form \spad{clip == b}.
  viewpoint : VIEWPT -> %
    ++ viewpoint(vp) creates a viewpoint data structure corresponding to the
    ++ list of values. The values are interpreted as [theta, phi, scale,
    ++ scaleX, scaleY, scaleZ, deltaX, deltaY]. This option is expressed
    ++ in the form \spad{viewpoint == ls}.
  title : String -> %

```

```

++ title(s) specifies a title for a plot by the indicated string s.
++ This option is expressed in the form \spad{title == s}.
style : String -> %
++ style(s) specifies the drawing style in which the graph will be plotted
++ by the indicated string s. This option is expressed in the
++ form \spad{style == s}.
toScale : Boolean -> %
++ toScale(b) specifies whether or not a plot is to be drawn to scale;
++ if b is true it is drawn to scale, if b is false it is not. This option
++ is expressed in the form \spad{toScale == b}.
clip : List SEG -> %
++ clip([l]) provides ranges for user-defined clipping as specified
++ in the list l. This option is expressed in the form \spad{clip == [l]}.
coordinates : (POINT -> POINT) -> %
++ coordinates(p) specifies a change of coordinate systems of point p.
++ This option is expressed in the form \spad{coordinates == p}.
pointColor : Float -> %
++ pointColor(v) specifies a color, v, for 2D graph points. This option
++ is expressed in the form \spad{pointColor == v}.
pointColor : PAL -> %
++ pointColor(p) specifies a color index for 2D graph points from the
++ spadcolors palette p. This option is expressed in the
++ form \spad{pointColor == p}.
curveColor : Float -> %
++ curveColor(v) specifies a color, v, for 2D graph curves.
++ This option is expressed in the form \spad{curveColor == v}.
curveColor : PAL -> %
++ curveColor(p) specifies a color index for 2D graph curves from the
++ spadcolors palette p.
++ This option is expressed in the form \spad{curveColor == p}.
colorFunction : (SF -> SF) -> %
++ colorFunction(f(z)) specifies the color based upon the z-component of
++ three dimensional plots. This option is expressed in the
++ form \spad{colorFunction == f(z)}.
colorFunction : ((SF,SF) -> SF) -> %
++ colorFunction(f(u,v)) specifies the color for three dimensional plots
++ as a function based upon the two parametric variables. This option
++ is expressed in the form \spad{colorFunction == f(u,v)}.
colorFunction : ((SF,SF,SF) -> SF) -> %
++ colorFunction(f(x,y,z)) specifies the color for three dimensional
++ plots as a function of x, y, and z coordinates. This option is
++ expressed in the form \spad{colorFunction == f(x,y,z)}.
tubeRadius : Float -> %
++ tubeRadius(r) specifies a radius, r, for a tube plot around a 3D curve;
++ is expressed in the form \spad{tubeRadius == 4}.
range : List SEG -> %
++ range([l]) provides a user-specified range l.
++ This option is expressed in the form \spad{range == [l]}.
range : List Segment Integer -> %
++ range([i]) provides a user-specified range i.

```



```

    ++ This option is expressed in the form \spad{range == [i]}.
ranges : RANGE -> %
    ++ ranges(l) provides a list of user-specified ranges l.
    ++ This option is expressed in the form \spad{ranges == l}.
space : SPACE3 -> %
    ++ space specifies the space into which we will draw. If none is given
    ++ then a new space is created.
var1Steps : PositiveInteger -> %
    ++ var1Steps(n) indicates the number of subdivisions, n, of the first
    ++ range variable. This option is expressed in the
    ++ form \spad{var1Steps == n}.
var2Steps : PositiveInteger -> %
    ++ var2Steps(n) indicates the number of subdivisions, n, of the second
    ++ range variable. This option is expressed in the
    ++ form \spad{var2Steps == n}.
tubePoints : PositiveInteger -> %
    ++ tubePoints(n) specifies the number of points, n, defining the circle
    ++ which creates the tube around a 3D curve, the default is 6.
    ++ This option is expressed in the form \spad{tubePoints == n}.
coord : (POINT->POINT) -> %
    ++ coord(p) specifies a change of coordinates of point p.
    ++ This option is expressed in the form \spad{coord == p}.
unit : UNIT -> %
    ++ unit(lf) will mark off the units according to the indicated list lf.
    ++ This option is expressed in the form \spad{unit == [f1,f2]}.
option : (List %, Symbol) -> Union(Any, "failed")
    ++ option() is not to be used at the top level;
    ++ option determines internally which drawing options are indicated in
    ++ a draw command.
option?: (List %, Symbol) -> Boolean
    ++ option?() is not to be used at the top level;
    ++ option? internally returns true for drawing options which are
    ++ indicated in a draw command, or false for those which are not.
Implementation ==> add
import AnyFunctions1(String)
import AnyFunctions1(Segment Float)
import AnyFunctions1(VIEWPT)
import AnyFunctions1(List Segment Float)
import AnyFunctions1(List Segment Fraction Integer)
import AnyFunctions1(List Integer)
import AnyFunctions1(PositiveInteger)
import AnyFunctions1(Boolean)
import AnyFunctions1(RANGE)
import AnyFunctions1(UNIT)
import AnyFunctions1(Float)
import AnyFunctions1(POINT -> POINT)
import AnyFunctions1(SF -> SF)
import AnyFunctions1((SF,SF) -> SF)
import AnyFunctions1((SF,SF,SF) -> SF)
import AnyFunctions1(POINT)

```

```

import AnyFunctions1(PAL)
import AnyFunctions1(SPACE3)

Rep := Record(keyword:Symbol, value:Any)

length:List SEG -> NonNegativeInteger
-- these lists will become tuples in a later version
length tup == # tup

lengthR:List Segment Fraction Integer -> NonNegativeInteger
-- these lists will become tuples in a later version
lengthR tup == # tup

lengthI:List Integer -> NonNegativeInteger
-- these lists will become tuples in a later version
lengthI tup == # tup

viewpoint vp ==
  ["viewpoint":Symbol, vp:Any]

title s == ["title":Symbol, s:Any]
style s == ["style":Symbol, s:Any]
toScale b == ["toScale":Symbol, b:Any]
clip(b:Boolean) == ["clipBoolean":Symbol, b:Any]
adaptive b == ["adaptive":Symbol, b:Any]

pointColor(x:Float) == ["pointColorFloat":Symbol, x:Any]
pointColor(c:PAL) == ["pointColorPalette":Symbol, c:Any]
curveColor(x:Float) == ["curveColorFloat":Symbol, x:Any]
curveColor(c:PAL) == ["curveColorPalette":Symbol, c:Any]
colorFunction(f:SF -> SF) == ["colorFunction1":Symbol, f:Any]
colorFunction(f:(SF,SF) -> SF) == ["colorFunction2":Symbol, f:Any]
colorFunction(f:(SF,SF,SF) -> SF) == ["colorFunction3":Symbol, f:Any]
clip(tup:List SEG) ==
  length tup > 3 =>
    error "clip: at most 3 segments may be specified"
  ["clipSegment":Symbol, tup:Any]
coordinates f == ["coordinates":Symbol, f:Any]
tubeRadius x == ["tubeRadius":Symbol, x:Any]
range(tup:List Segment Float) ==
  ((n := length tup) > 3) =>
    error "range: at most 3 segments may be specified"
  n < 2 =>
    error "range: at least 2 segments may be specified"
  ["rangeFloat":Symbol, tup:Any]
range(tup:List Segment Fraction Integer) ==
  ((n := lengthR tup) > 3) =>
    error "range: at most 3 segments may be specified"
  n < 2 =>
    error "range: at least 2 segments may be specified"

```

```

["rangeRat"::Symbol, tup::Any]

ranges s          == ["ranges"::Symbol, s::Any]
space s           == ["space"::Symbol, s::Any]
var1Steps s       == ["var1Steps"::Symbol, s::Any]
var2Steps s       == ["var2Steps"::Symbol, s::Any]
tubePoints s      == ["tubePoints"::Symbol, s::Any]
coord s           == ["coord"::Symbol, s::Any]
unit s            == ["unit"::Symbol, s::Any]
coerce(x:%):OutputForm == x.keyword::OutputForm = x.value::OutputForm
x:% = y:%         == x.keyword = y.keyword and x.value = y.value

option?(l, s) ==
  for x in l repeat
    x.keyword = s => return true
  false

option(l, s) ==
  for x in l repeat
    x.keyword = s => return(x.value)
  "failed"

```

---

— DROPT.dotabb —

```

"DROPT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=DROPT"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"DROPT" -> "ALIST"

```

---

## 5.19 domain D01AJFA d01ajfAnnaType

— d01ajfAnnaType.input —

```

)set break resume
)sys rm -f d01ajfAnnaType.output
)spool d01ajfAnnaType.output
)set message test on
)set message auto off
)clear all

```

--S 1 of 1

```

)show d01ajfAnnaType
--R d01ajfAnnaType is a domain constructor
--R Abbreviation for d01ajfAnnaType is D01AJFA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for D01AJFA
--R
--R----- Operations -----
--R ?=? : (% ,%) -> Boolean          coerce : % -> OutputForm
--R hash : % -> SingleInteger       latex : % -> String
--R ?~=? : (% ,%) -> Boolean
--R measure : (RoutinesTable,Record(fn: Expression DoubleFloat,range: List Segment OrderedCompletion Dou
--R measure : (RoutinesTable,Record(var: Symbol,fn: Expression DoubleFloat,range: Segment OrderedCompleti
--R numericalIntegration : (Record(fn: Expression DoubleFloat,range: List Segment OrderedCompletion Dou
--R numericalIntegration : (Record(var: Symbol,fn: Expression DoubleFloat,range: Segment OrderedCompleti
--R
--E 1

)spool
)lisp (bye)

```

\_\_\_\_\_

— d01ajfAnnaType.help —

```

=====
d01ajfAnnaType examples
=====

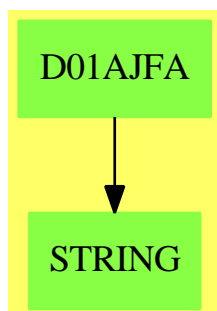
```

See Also:

- o )show d01ajfAnnaType

\_\_\_\_\_

### 5.19.1 d01ajfAnnaType (D01AJFA)



**Exports:**

coerce hash latex measure numericalIntegration ?~=? ?=?

— domain D01AJFA d01ajfAnnaType —

```
)abbrev domain D01AJFA d01ajfAnnaType
++ Author: Brian Dupee
++ Date Created: March 1994
++ Date Last Updated: December 1997
++ Basic Operations: measure, numericalIntegration
++ Related Constructors: Result, RoutinesTable
++ Description:
++ \axiomType{d01ajfAnnaType} is a domain of
++ \axiomType{NumericalIntegrationCategory}
++ for the NAG routine D01AJF, a general numerical integration routine which
++ can handle some singularities in the input function. The function
++ \axiomFun{measure} measures the usefulness of the routine D01AJF
++ for the given problem. The function \axiomFun{numericalIntegration}
++ performs the integration by using \axiomType{NagIntegrationPackage}.

d01ajfAnnaType(): NumericalIntegrationCategory == Result add
  EF2 ==> ExpressionFunctions2
  EDF ==> Expression DoubleFloat
  LDF ==> List DoubleFloat
  SDF ==> Stream DoubleFloat
  DF ==> DoubleFloat
  FI ==> Fraction Integer
  EFI ==> Expression Fraction Integer
  SOCDF ==> Segment OrderedCompletion DoubleFloat
  NIA ==> Record(var:Symbol,fn:EDF,range:SOCDF,abserr:DF,relerr:DF)
  MDNIA ==> Record(fn:EDF,range:List SOCDF,abserr:DF,relerr:DF)
  INT ==> Integer
  BOP ==> BasicOperator
  S ==> Symbol
  ST ==> String
  LST ==> List String
  RT ==> RoutinesTable
  Rep:=Result
  import Rep, NagIntegrationPackage, d01AgentsPackage

measure(R:RT,args:NIA) ==
  ext:Result := empty()$Result
  pp:SDF := singularitiesOf(args)
  not (empty?(pp)$SDF) =>
    [0.1,"d01ajf: There is a possible problem at the following point(s): "
      commaSeparate(sdf2lst(pp)) ,ext]
  [getMeasure(R,d01ajf :: S)$RT,
    "The general routine d01ajf is our default",ext]
```

```

numericalIntegration(args:NIA,hints:Result) ==
  ArgsFn := map(x+>convert(x)$DF,args.fn)$EF2(DF,Float)
  b:Float := getButtonValue("d01ajf","functionEvaluations")$AttributeButtons
  fEvals:INT := wholePart exp(1.1513*(1.0/(2.0*(1.0-b))))
  iw:INT := 75*fEvals
  f : Union(fn:FileName,fp:Asp1(F)) := [retract(ArgsFn)$Asp1(F)]
  d01ajf(getlo(args.range),gethi(args.range),args.abserr,_
    args.relerr,4*iw,iw,-1,f)

```

---

— D01AJFA.dotabb —

```

"D01AJFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=D01AJFA"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"D01AJFA" -> "STRING"

```

---

## 5.20 domain D01AKFA d01akfAnnaType

— d01akfAnnaType.input —

```

)set break resume
)sys rm -f d01akfAnnaType.output
)spool d01akfAnnaType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show d01akfAnnaType
--R d01akfAnnaType is a domain constructor
--R Abbreviation for d01akfAnnaType is D01AKFA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for D01AKFA
--R
--R----- Operations -----
--R ?=? : (%,% ) -> Boolean          coerce : % -> OutputForm
--R hash : % -> SingleInteger        latex : % -> String
--R ?~=? : (%,% ) -> Boolean
--R measure : (RoutinesTable,Record(fn: Expression DoubleFloat,range: List Segment OrderedCompletion Dou
--R measure : (RoutinesTable,Record(var: Symbol,fn: Expression DoubleFloat,range: Segment OrderedComple
--R numericalIntegration : (Record(fn: Expression DoubleFloat,range: List Segment OrderedCompletion Dou

```

```
--R numericalIntegration : (Record(var: Symbol,fn: Expression DoubleFloat,range: Segment Ord
--R
--E 1
```

```
)spool
)lisp (bye)
```

---

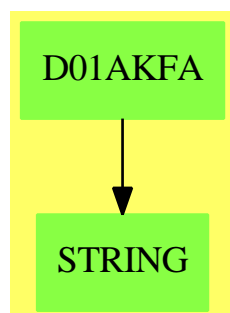
### — d01akfAnnaType.help —

```
=====
d01akfAnnaType examples
=====
```

```
See Also:
o )show d01akfAnnaType
```

---

#### 5.20.1 d01akfAnnaType (D01AKFA)



#### Exports:

```
coerce hash latex measure numericalIntegration ?=? ?~=?
```

### — domain D01AKFA d01akfAnnaType —

```
)abbrev domain D01AKFA d01akfAnnaType
++ Author: Brian Dupee
++ Date Created: March 1994
++ Date Last Updated: December 1997
++ Basic Operations: measure, numericalIntegration
++ Related Constructors: Result, RoutinesTable
```

```

++ Description:
++ \axiomType{d01akfAnnaType} is a domain of
++ \axiomType{NumericalIntegrationCategory}
++ for the NAG routine D01AKF, a numerical integration routine which is
++ is suitable for oscillating, non-singular functions. The function
++ \axiomFun{measure} measures the usefulness of the routine D01AKF
++ for the given problem. The function \axiomFun{numericalIntegration}
++ performs the integration by using \axiomType{NagIntegrationPackage}.

d01akfAnnaType(): NumericalIntegrationCategory == Result add
  EF2 ==> ExpressionFunctions2
  EDF ==> Expression DoubleFloat
  LDF ==> List DoubleFloat
  SDF ==> Stream DoubleFloat
  DF ==> DoubleFloat
  FI ==> Fraction Integer
  EFI ==> Expression Fraction Integer
  SOCDF ==> Segment OrderedCompletion DoubleFloat
  NIA ==> Record(var:Symbol,fn:EDF,range:SOCDF,abserr:DF,relerr:DF)
  MDNIA ==> Record(fn:EDF,range:List SOCDF,abserr:DF,relerr:DF)
  INT ==> Integer
  BOP ==> BasicOperator
  S ==> Symbol
  ST ==> String
  LST ==> List String
  RT ==> RoutinesTable
  Rep:=Result
  import Rep, d01AgentsPackage, NagIntegrationPackage

measure(R:RT,args:NIA) ==
  ext:Result := empty()$Result
  pp:SDF := singularitiesOf(args)
  not (empty?(pp)$SDF) =>
    [0.0,"d01akf: There is a possible problem at the following point(s): "
      commaSeparate(sdf2lst(pp)) ,ext]
  o:Float := functionIsOscillatory(args)
  one := 1.0
  m:Float := (getMeasure(R,d01akf@S)$RT)*(one-one/(one+sqrt(o)))**2
  m > 0.8 => [m,"d01akf: The expression shows much oscillation",ext]
  m > 0.6 => [m,"d01akf: The expression shows some oscillation",ext]
  m > 0.5 => [m,"d01akf: The expression shows little oscillation",ext]
  [m,"d01akf: The expression shows little or no oscillation",ext]

numericalIntegration(args:NIA,hints:Result) ==
  ArgsFn := map(x+>convert(x)$DF,args.fn)$EF2(DF,Float)
  b:Float := getButtonValue("d01akf","functionEvaluations")$AttributeButtons
  fEvals:INT := wholePart exp(1.1513*(1.0/(2.0*(1.0-b))))
  iw:INT := 75*fEvals
  f : Union(fn:FileName,fp:Asp1(F)) := [retract(ArgsFn)$Asp1(F)]
  d01akf(getlo(args.range),gethi(args.range),args.abserr,_

```



```
args.relerr,4*iw,iw,-1,f)
```

---

— D01AKFA.dotabb —

```
"D01AKFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=D01AKFA"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"D01AKFA" -> "STRING"
```

---

## 5.21 domain D01ALFA d01alfAnnaType

— d01alfAnnaType.input —

```
)set break resume
)sys rm -f d01alfAnnaType.output
)spool d01alfAnnaType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show d01alfAnnaType
--R d01alfAnnaType is a domain constructor
--R Abbreviation for d01alfAnnaType is D01ALFA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for D01ALFA
--R
--R----- Operations -----
--R ==? : (%,% ) -> Boolean          coerce : % -> OutputForm
--R hash : % -> SingleInteger       latex : % -> String
--R ~=? : (%,% ) -> Boolean
--R measure : (RoutinesTable,Record(fn: Expression DoubleFloat,range: List Segment OrderedComp
--R measure : (RoutinesTable,Record(var: Symbol,fn: Expression DoubleFloat,range: Segment Ord
--R numericalIntegration : (Record(fn: Expression DoubleFloat,range: List Segment OrderedComp
--R numericalIntegration : (Record(var: Symbol,fn: Expression DoubleFloat,range: Segment Ord
--R
--E 1

)spool
)lisp (bye)
```

---

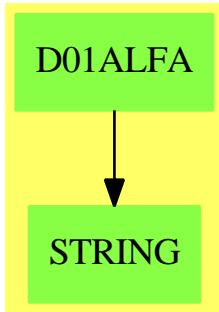
— d01alfAnnaType.help —

```
=====
d01alfAnnaType examples
=====
```

```
See Also:
o )show d01alfAnnaType
```

---

### 5.21.1 d01alfAnnaType (D01ALFA)



#### Exports:

```
coerce hash latex measure numericalIntegration ?~=? ?=?
```

— domain D01ALFA d01alfAnnaType —

```
)abbrev domain D01ALFA d01alfAnnaType
++ Author: Brian Dupee
++ Date Created: March 1994
++ Date Last Updated: December 1997
++ Basic Operations: measure, numericalIntegration
++ Related Constructors: Result, RoutinesTable
++ Description:
++ \axiomType{d01alfAnnaType} is a domain of
++ \axiomType{NumericalIntegrationCategory}
++ for the NAG routine D01ALF, a general numerical integration routine which
++ can handle a list of singularities. The
++ function \axiomFun{measure} measures the usefulness of the routine D01ALF
++ for the given problem. The function \axiomFun{numericalIntegration}
++ performs the integration by using \axiomType{NagIntegrationPackage}.
```

```

d01alfAnnaType(): NumericalIntegrationCategory == Result add
  EF2 ==> ExpressionFunctions2
  EDF ==> Expression DoubleFloat
  LDF ==> List DoubleFloat
  SDF ==> Stream DoubleFloat
  DF ==> DoubleFloat
  FI ==> Fraction Integer
  EFI ==> Expression Fraction Integer
  SOCDF ==> Segment OrderedCompletion DoubleFloat
  NIA ==> Record(var:Symbol,fn:EDF,range:SOCDF,abserr:DF,relerr:DF)
  MDNIA ==> Record(fn:EDF,range:List SOCDF,abserr:DF,relerr:DF)
  INT ==> Integer
  BOP ==> BasicOperator
  S ==> Symbol
  ST ==> String
  LST ==> List String
  RT ==> RoutinesTable
  Rep:=Result
  import Rep, d01AgentsPackage, NagIntegrationPackage

measure(R:RT,args:NIA) ==
  ext:Result := empty()$Result
  streamOfZeros:SDF := singularitiesOf(args)
  listOfZeros:LST := removeDuplicates!(sdf2lst(streamOfZeros))
  numberOfZeros:INT := # listOfZeros
  (numberOfZeros > 15)$Boolean =>
    [0.0,"d01alf: The list of singularities is too long", ext]
  positive?(numberOfZeros) =>
    l:LDF := entries(complete(streamOfZeros)$SDF)$SDF
    lany:Any := coerce(l)$AnyFunctions1(LDF)
    ex:Record(key:S,entry:Any) := [d01alfextra@S,lany]
    ext := insert!(ex,ext)$Result
    st:ST := "Recommended is d01alf with the singularities "
              commaSeparate(listOfZeros)

  m :=
--    one?(numberOfZeros) => 0.4
    (numberOfZeros = 1) => 0.4
    getMeasure(R,d01alf@S)$RT
  [m, st, ext]
  [0.0, "d01alf: A list of suitable singularities has not been found", ext]

numericalIntegration(args:NIA,hints:Result) ==
  la:Any := coerce(search((d01alfextra@S),hints)$Result)$Any
  listOfZeros:LDF := retract(la)$AnyFunctions1(LDF)
  l:= removeDuplicates(listOfZeros)$LDF
  n:Integer := (#(l))$List(DF)
  M:Matrix DF := matrix([l])$(Matrix DF)
  b:Float := getButtonValue("d01alf","functionEvaluations")$AttributeButtons
  fEvals:INT := wholePart exp(1.1513*(1.0/(2.0*(1.0-b))))

```

```

iw:INT := 75*fEvals
ArgsFn := map(x+>convert(x)$DF,args.fn)$EF2(DF,Float)
f : Union(fn:FileName,fp:Asp1(F)) := [retract(ArgsFn)$Asp1(F)]
d01alf(getlo(args.range),gethi(args.range),n,M,
      args.abserr,args.relerr,2*n*iw,n*iw,-1,f)

```

---

— D01ALFA.dotabb —

```

"D01ALFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=D01ALFA"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"D01ALFA" -> "STRING"

```

---

## 5.22 domain D01AMFA d01amfAnnaType

---

— d01amfAnnaType.input —

```

)set break resume
)sys rm -f d01amfAnnaType.output
)spool d01amfAnnaType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show d01amfAnnaType
--R d01amfAnnaType is a domain constructor
--R Abbreviation for d01amfAnnaType is D01AMFA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for D01AMFA
--R
--R----- Operations -----
--R ?? : (% ,%) -> Boolean          coerce : % -> OutputForm
--R hash : % -> SingleInteger      latex : % -> String
--R ?~? : (% ,%) -> Boolean
--R measure : (RoutinesTable,Record(fn: Expression DoubleFloat,range: List Segment OrderedCompletion Dou
--R measure : (RoutinesTable,Record(var: Symbol,fn: Expression DoubleFloat,range: Segment OrderedCompleti
--R numericalIntegration : (Record(fn: Expression DoubleFloat,range: List Segment OrderedCompletion Dou
--R numericalIntegration : (Record(var: Symbol,fn: Expression DoubleFloat,range: Segment OrderedCompleti
--R
--E 1

```

```
)spool
)lisp (bye)
```

---

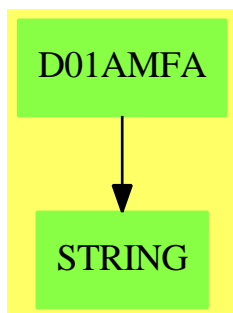
— d01amfAnnaType.help —

```
=====
d01amfAnnaType examples
=====

See Also:
o )show d01amfAnnaType
```

---

### 5.22.1 d01amfAnnaType (D01AMFA)



#### Exports:

coerce hash latex measure numericalIntegration ?=? ?~=?

— domain D01AMFA d01amfAnnaType —

```
)abbrev domain D01AMFA d01amfAnnaType
++ Author: Brian Dupee
++ Date Created: March 1994
++ Date Last Updated: December 1997
++ Basic Operations: measure, numericalIntegration
++ Related Constructors: Result, RoutinesTable
++ Description:
++ \axiomType{d01amfAnnaType} is a domain of
++ \axiomType{NumericalIntegrationCategory}
```

```

++ for the NAG routine D01AMF, a general numerical integration routine which
++ can handle infinite or semi-infinite range of the input function. The
++ function \axiomFun{measure} measures the usefulness of the routine D01AMF
++ for the given problem. The function \axiomFun{numericalIntegration}
++ performs the integration by using \axiomType{NagIntegrationPackage}.

d01amfAnnaType(): NumericalIntegrationCategory == Result add
  EF2 ==> ExpressionFunctions2
  EDF ==> Expression DoubleFloat
  LDF ==> List DoubleFloat
  SDF ==> Stream DoubleFloat
  DF ==> DoubleFloat
  FI ==> Fraction Integer
  EFI ==> Expression Fraction Integer
  SOCDF ==> Segment OrderedCompletion DoubleFloat
  NIA ==> Record(var:Symbol,fn:EDF,range:SOCDF,abserr:DF,relerr:DF)
  MDNIA ==> Record(fn:EDF,range:List SOCDF,abserr:DF,relerr:DF)
  INT ==> Integer
  BOP ==> BasicOperator
  S ==> Symbol
  ST ==> String
  LST ==> List String
  RT ==> RoutinesTable
  Rep:=Result
  import Rep, d01AgentsPackage, NagIntegrationPackage

measure(R:RT,args:NIA) ==
  ext:Result := empty()$Result
  Range:=rangeIsFinite(args)
  pp:SDF := singularitiesOf(args)
  not (empty?(pp)$SDF) =>
    [0.0,"d01amf: There is a possible problem at the following point(s): "
      commaSeparate(sdf2lst(pp)), ext]
  [getMeasure(R,d01amf@S)$RT, "d01amf is a reasonable choice if the "
    "integral is infinite or semi-infinite and d01transform cannot "
    "do better than using general routines",ext]

numericalIntegration(args:NIA,hints:Result) ==
  r:INT
  bound:DF
  ArgsFn := map(x+>convert(x)$DF,args.fn)$EF2(DF,Float)
  b:Float := getButtonValue("d01amf","functionEvaluations")$AttributeButtons
  fEvals:INT := wholePart exp(1.1513*(1.0/(2.0*(1.0-b))))
  iw:INT := 150*fEvals
  f : Union(fn:FileName,fp:Asp1(F)) := [retract(ArgsFn)$Asp1(F)]
  Range:=rangeIsFinite(args)
  if (Range case upperInfinite) then
    bound := getlo(args.range)
    r := 1
  else if (Range case lowerInfinite) then

```

```

    bound := gethi(args.range)
    r := -1
else
    bound := 0$DF
    r := 2
d01amf(bound,r,args.abserr,args.relerr,4*iw,iw,-1,f)

```

---

— D01AMFA.dotabb —

```

"D01AMFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=D01AMFA"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"D01AMFA" -> "STRING"

```

---

## 5.23 domain D01ANFA d01anfAnnaType

---

— d01anfAnnaType.input —

```

)set break resume
)sys rm -f d01anfAnnaType.output
)spool d01anfAnnaType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show d01anfAnnaType
--R d01anfAnnaType is a domain constructor
--R Abbreviation for d01anfAnnaType is D01ANFA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for D01ANFA
--R
--R----- Operations -----
--R ?? : (%,% ) -> Boolean               coerce : % -> OutputForm
--R hash : % -> SingleInteger           latex : % -> String
--R ??=? : (%,% ) -> Boolean
--R measure : (RoutinesTable,Record(fn: Expression DoubleFloat,range: List Segment OrderedComp
--R measure : (RoutinesTable,Record(var: Symbol,fn: Expression DoubleFloat,range: Segment Or
--R numericalIntegration : (Record(fn: Expression DoubleFloat,range: List Segment OrderedComp
--R numericalIntegration : (Record(var: Symbol,fn: Expression DoubleFloat,range: Segment Or
--R

```

```
--E 1
```

```
)spool
)lisp (bye)
```

```
_____
```

```
— d01anfAnnaType.help —
```

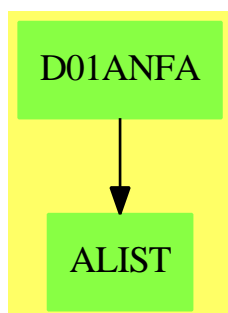
```
=====
d01anfAnnaType examples
=====
```

See Also:

o )show d01anfAnnaType

```
_____
```

### 5.23.1 d01anfAnnaType (D01ANFA)



#### Exports:

coerce hash latex measure numericalIntegration ?~=? ?=?

```
— domain D01ANFA d01anfAnnaType —
```

```
)abbrev domain D01ANFA d01anfAnnaType
++ Author: Brian Dupee
++ Date Created: March 1994
++ Date Last Updated: December 1997
++ Basic Operations: measure, numericalIntegration
++ Related Constructors: Result, RoutinesTable
++ Description:
++ \axiomType{d01anfAnnaType} is a domain of
```



```

++ \axiomType{NumericalIntegrationCategory}
++ for the NAG routine D01ANF, a numerical integration routine which can
++ handle weight functions of the form cos(\omega x) or sin(\omega x). The
++ function \axiomFun{measure} measures the usefulness of the routine D01ANF
++ for the given problem. The function \axiomFun{numericalIntegration}
++ performs the integration by using \axiomType{NagIntegrationPackage}.

d01anfAnnaType(): NumericalIntegrationCategory == Result add
  EF2 ==> ExpressionFunctions2
  EDF ==> Expression DoubleFloat
  LDF ==> List DoubleFloat
  SDF ==> Stream DoubleFloat
  DF ==> DoubleFloat
  FI ==> Fraction Integer
  EFI ==> Expression Fraction Integer
  SOCDF ==> Segment OrderedCompletion DoubleFloat
  NIA ==> Record(var:Symbol,fn:EDF,range:SOCDF,abserr:DF,relerr:DF)
  MDNIA ==> Record(fn:EDF,range:List SOCDF,abserr:DF,relerr:DF)
  INT ==> Integer
  BOP ==> BasicOperator
  S ==> Symbol
  ST ==> String
  LST ==> List String
  RT ==> RoutinesTable
  Rep:=Result
  import Rep, d01WeightsPackage, d01AgentsPackage, NagIntegrationPackage

measure(R:RT,args:NIA) ==
  ext:Result := empty()$Result
  weight:Union(Record(op:BOP,w:DF),"failed") :=
    exprHasWeightCosWXorSinWX(args)
  weight case "failed" =>
    [0.0,"d01anf: A suitable weight has not been found", ext]
  weight case Record(op:BOP,w:DF) =>
    wany := coerce(weight)$AnyFunctions1(Record(op:BOP,w:DF))
    ex:Record(key:S,entry:Any) := [d01anfextra@S,wany]
    ext := insert!(ex,ext)$Result
    ws:ST := string(name(weight.op)$BOP)$S "(" df2st(weight.w)
              string(args.var)$S ")"
    [getMeasure(R,d01anf@S)$RT,
     "d01anf: The expression has a suitable weight:- " ws, ext]

numericalIntegration(args:NIA,hints:Result) ==
  a:INT
  r:Any := coerce(search((d01anfextra@S),hints)$Result)$Any
  rec:Record(op:BOP,w:DF) := retract(r)$AnyFunctions1(Record(op:BOP,w:DF))
  Var := args.var :: EDF
  o:BOP := rec.op
  den:EDF := o((rec.w*Var)$EDF)
  Argsfn:EDF := args.fn/den

```

```

if (name(o) = cos@S)@Boolean then a := 1
else a := 2
b:Float := getButtonValue("d01anf","functionEvaluations")$AttributeButtons
fEvals:INT := wholePart exp(1.1513*(1.0/(2.0*(1.0-b))))
iw:INT := 75*fEvals
ArgsFn := map(x+>convert(x)$DF,Argsfn)$EF2(DF,Float)
f : Union(fn:FileName,fp:Asp1(G)) := [retract(ArgsFn)$Asp1(G)]
d01anf(getlo(args.range),gethi(args.range),rec.w,a,
      args.abterr,args.relerr,4*iw,iw,-1,f)

```

---

— D01ANFA.dotabb —

```

"D01ANFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=D01ANFA"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"D01ANFA" -> "ALIST"

```

---

## 5.24 domain D01APFA d01apfAnnaType

— d01apfAnnaType.input —

```

)set break resume
)sys rm -f d01apfAnnaType.output
)spool d01apfAnnaType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show d01apfAnnaType
--R d01apfAnnaType is a domain constructor
--R Abbreviation for d01apfAnnaType is D01APFA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for D01APFA
--R
--R----- Operations -----
--R ?=? : (%,% ) -> Boolean          coerce : % -> OutputForm
--R hash : % -> SingleInteger        latex : % -> String
--R ?~=? : (%,% ) -> Boolean
--R measure : (RoutinesTable,Record(fn: Expression DoubleFloat,range: List Segment OrderedCompletion Dou
--R measure : (RoutinesTable,Record(var: Symbol,fn: Expression DoubleFloat,range: Segment OrderedCompleto

```

```

--R numericalIntegration : (Record(fn: Expression DoubleFloat,range: List Segment OrderedComp
--R numericalIntegration : (Record(var: Symbol,fn: Expression DoubleFloat,range: Segment Ord
--R
--E 1

)spool
)lisp (bye)

```

---

— d01apfAnnaType.help —

```

=====
d01apfAnnaType examples
=====

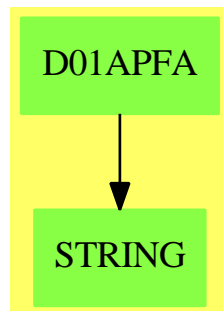
```

```

See Also:
o )show d01apfAnnaType

```

### 5.24.1 d01apfAnnaType (D01APFA)



#### Exports:

```

coerce hash latex measure numericalIntegration ?=? ?~=?

```

---

— domain D01APFA d01apfAnnaType —

```

)abbrev domain D01APFA d01apfAnnaType
++ Author: Brian Dupee
++ Date Created: March 1994
++ Date Last Updated: December 1997
++ Basic Operations: measure, numericalIntegration

```

```

++ Related Constructors: Result, RoutinesTable
++ Description:
++ \axiomType{d01apfAnnaType} is a domain of
++ \axiomType{NumericalIntegrationCategory}
++ for the NAG routine D01APF, a general numerical integration routine which
++ can handle end point singularities of the algebraico-logarithmic form
++  $w(x) = (x-a)^c * (b-x)^d$ . The
++ function \axiomFun{measure} measures the usefulness of the routine D01APF
++ for the given problem. The function \axiomFun{numericalIntegration}
++ performs the integration by using \axiomType{NagIntegrationPackage}.

```

```

d01apfAnnaType(): NumericalIntegrationCategory == Result add
  EF2 ==> ExpressionFunctions2
  EDF ==> Expression DoubleFloat
  LDF ==> List DoubleFloat
  SDF ==> Stream DoubleFloat
  DF ==> DoubleFloat
  FI ==> Fraction Integer
  EFI ==> Expression Fraction Integer
  SOCDF ==> Segment OrderedCompletion DoubleFloat
  NIA ==> Record(var:Symbol,fn:EDF,range:SOCDF,abserr:DF,relerr:DF)
  MDNIA ==> Record(fn:EDF,range:List SOCDF,abserr:DF,relerr:DF)
  INT ==> Integer
  BOP ==> BasicOperator
  S ==> Symbol
  ST ==> String
  LST ==> List String
  RT ==> RoutinesTable
  Rep:=Result
  import Rep, NagIntegrationPackage, d01AgentsPackage, d01WeightsPackage

  measure(R:RT,args:NIA) ==
    ext:Result := empty()$Result
    d := (c := 0$DF)
    if ((a := exprHasAlgebraicWeight(args)) case LDF) then
      if (a.1 > -1) then c := a.1
      if (a.2 > -1) then d := a.2
    l:INT := exprHasLogarithmicWeights(args)
    -- (zero? c) and (zero? d) and (one? l) =>
    (zero? c) and (zero? d) and (l = 1) =>
      [0.0,"d01apf: A suitable singularity has not been found", ext]
    out:LDF := [c,d,l :: DF]
    outany:Any := coerce(out)$AnyFunctions1(LDF)
    ex:Record(key:S,entry:Any) := [d01apfextra@S,outany]
    ext := insert!(ex,ext)$Result
    st:ST := "Recommended is d01apf with c = " df2st(c) ", d = "
              df2st(d) " and l = " string(l)$ST
    [getMeasure(R,d01apf@S)$RT, st, ext]

  numericalIntegration(args:NIA,hints:Result) ==

```

```

Var:EDF := coerce(args.var)$EDF
la:Any := coerce(search((d01apfextra@S),hints)$Result)@Any
list:LDF := retract(la)$AnyFunctions1(LDF)
Fac1:EDF := (Var - (getlo(args.range) :: EDF))$EDF
Fac2:EDF := ((gethi(args.range) :: EDF) - Var)$EDF
c := first(list)$LDF
d := second(list)$LDF
l := (retract(third(list)$LDF)@INT)$DF
thebiz:EDF := (Fac1**(c :: EDF))*(Fac2**(d :: EDF))
if l > 1 then
  if l = 2 then
    thebiz := thebiz*log(Fac1)
  else if l = 3 then
    thebiz := thebiz*log(Fac2)
  else
    thebiz := thebiz*log(Fac1)*log(Fac2)
Fn := (args.fn/thebiz)$EDF
ArgsFn := map(x+>convert(x)$DF,Fn)$EF2(DF,Float)
b:Float := getButtonValue("d01apf","functionEvaluations")$AttributeButtons
fEvals:INT := wholePart exp(1.1513*(1.0/(2.0*(1.0-b))))
iw:INT := 75*fEvals
f : Union(fn:FileName,fp:Asp1(G)) := [retract(ArgsFn)$Asp1(G)]
d01apf(getlo(args.range),gethi(args.range),c,d,l,_
      args.abserr,args.relerr,4*iw,iw,-1,f)

```

---

— D01APFA.dotabb —

```

"D01APFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=D01APFA"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"D01APFA" -> "STRING"

```

## 5.25 domain D01AQFA d01aqfAnnaType

---

— d01aqfAnnaType.input —

```

)set break resume
)sys rm -f d01aqfAnnaType.output
)spool d01aqfAnnaType.output
)set message test on

```

```

)set message auto off
)clear all

--S 1 of 1
)show d01aqfAnnaType
--R d01aqfAnnaType is a domain constructor
--R Abbreviation for d01aqfAnnaType is D01AQFA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for D01AQFA
--R
--R----- Operations -----
--R ?=? : (% ,%) -> Boolean          coerce : % -> OutputForm
--R hash : % -> SingleInteger        latex : % -> String
--R ?~=? : (% ,%) -> Boolean
--R measure : (RoutinesTable,Record(fn: Expression DoubleFloat,range: List Segment OrderedCompletion Dou
--R measure : (RoutinesTable,Record(var: Symbol,fn: Expression DoubleFloat,range: Segment OrderedCompleti
--R numericalIntegration : (Record(fn: Expression DoubleFloat,range: List Segment OrderedCompletion Dou
--R numericalIntegration : (Record(var: Symbol,fn: Expression DoubleFloat,range: Segment OrderedCompleti
--R
--E 1

)spool
)lisp (bye)

```

---

— d01aqfAnnaType.help —

```

=====
d01aqfAnnaType examples
=====

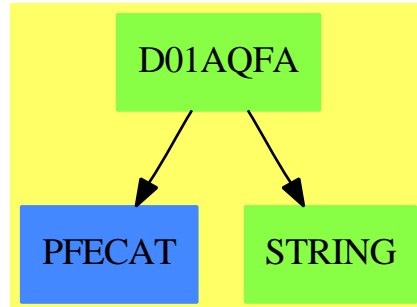
```

See Also:

o )show d01aqfAnnaType

---

## 5.25.1 d01aqfAnnaType (D01AQFA)

**Exports:**

coerce hash latex measure numericalIntegration ?=? ?~=?

— domain D01AQFA d01aqfAnnaType —

```

)abbrev domain D01AQFA d01aqfAnnaType
++ Author: Brian Dupee
++ Date Created: March 1994
++ Date Last Updated: December 1997
++ Basic Operations: measure, numericalIntegration
++ Related Constructors: Result, RoutinesTable
++ Description:
++ \axiomType{d01aqfAnnaType} is a domain of
++ \axiomType{NumericalIntegrationCategory}
++ for the NAG routine D01AQF, a general numerical integration routine which
++ can solve an integral of the form
++ /home/bjd/Axiom/anna/hypertext/bitmaps/d01aqf.xbm
++ The function \axiomFun{measure} measures the usefulness of the routine
++ D01AQF for the given problem. The function \axiomFun{numericalIntegration}
++ performs the integration by using \axiomType{NagIntegrationPackage}.

```

```

d01aqfAnnaType(): NumericalIntegrationCategory == Result add
  EF2 ==> ExpressionFunctions2
  EDF ==> Expression DoubleFloat
  LDF ==> List DoubleFloat
  SDF ==> Stream DoubleFloat
  DF ==> DoubleFloat
  FI ==> Fraction Integer
  EFI ==> Expression Fraction Integer
  SOCDF ==> Segment OrderedCompletion DoubleFloat
  NIA ==> Record(var:Symbol,fn:EDF,range:SOCDF,abserr:DF,relerr:DF)
  MDNIA ==> Record(fn:EDF,range:List SOCDF,abserr:DF,relerr:DF)
  INT ==> Integer
  BOP ==> BasicOperator
  S ==> Symbol

```

```

ST ==> String
LST ==> List String
RT ==> RoutinesTable
Rep:=Result
import Rep, d01AgentsPackage, NagIntegrationPackage

measure(R:RT,args:NIA) ==
  ext:Result := empty()$Result
  Den := denominator(args.fn)
--  one? Den =>
  (Den = 1) =>
    [0.0,"d01aqf: A suitable weight function has not been found", ext]
  listOfZeros:LDF := problemPoints(args.fn,args.var,args.range)
  numberOfZeros := (#(listOfZeros))$LDF
  zero?(numberOfZeros) =>
    [0.0,"d01aqf: A suitable weight function has not been found", ext]
  numberOfZeros = 1 =>
    s:SDF := singularitiesOf(args)
    more?(s,1)$SDF =>
      [0.0,"d01aqf: Too many singularities have been found", ext]
    cFloat:Float := (convert(first(listOfZeros)$LDF)$Float)$SDF
    cString:ST := (convert(cFloat)$ST)$Float
    lany:Any := coerce(listOfZeros)$AnyFunctions1(LDF)
    ex:Record(key:S,entry:Any) := [d01aqfextra@S,lany]
    ext := insert!(ex,ext)$Result
    [getMeasure(R,d01aqf@S)$RT, "Recommended is d01aqf with the "
      "hilbertian weight function of 1/(x-c) where c = " cString, ext]
    [0.0,"d01aqf: More than one factor has been found and so does not "
      "have a suitable weight function",ext]

numericalIntegration(args:NIA,hints:Result) ==
  Args := copy args
  ca:Any := coerce(search((d01aqfextra@S),hints)$Result)$Any
  c:DF := first(retract(ca)$AnyFunctions1(LDF))$LDF
  ci:FI := df2fi(c)$ExpertSystemToolsPackage
  Var:EFI := Args.var :: EFI
  Gx:EFI := (Var-(ci::EFI))*(edf2efi(Args.fn)$ExpertSystemToolsPackage)
  ArgsFn := map(x+>convert(x)$FI,Gx)$EF2(FI,Float)
  b:Float := getButtonValue("d01aqf","functionEvaluations")$AttributeButtons
  fEvals:INT := wholePart exp(1.1513*(1.0/(2.0*(1.0-b))))
  iw:INT := 75*fEvals
  f : Union(fn:FileName,fp:Asp1(G)) := [retract(ArgsFn)$Asp1(G)]
  d01aqf(getlo(Args.range),gethi(Args.range),c,_
    Args.abserr,Args.relerr,4*iw,iw,-1,f)

```



```

"D01AQFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=D01AQFA"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"D01AQFA" -> "STRING"
"D01AQFA" -> "PFECAT"

```

---

## 5.26 domain D01ASFA d01asfAnnaType

— d01asfAnnaType.input —

```

)set break resume
)sys rm -f d01asfAnnaType.output
)spool d01asfAnnaType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show d01asfAnnaType
--R d01asfAnnaType is a domain constructor
--R Abbreviation for d01asfAnnaType is D01ASFA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for D01ASFA
--R
--R----- Operations -----
--R ?=? : (%,% ) -> Boolean          coerce : % -> OutputForm
--R hash : % -> SingleInteger        latex : % -> String
--R ?~=? : (%,% ) -> Boolean
--R measure : (RoutinesTable,Record(fn: Expression DoubleFloat,range: List Segment OrderedComp
--R measure : (RoutinesTable,Record(var: Symbol,fn: Expression DoubleFloat,range: Segment Or
--R numericalIntegration : (Record(fn: Expression DoubleFloat,range: List Segment OrderedComp
--R numericalIntegration : (Record(var: Symbol,fn: Expression DoubleFloat,range: Segment Or
--R
--E 1

)spool
)lisp (bye)

```

---

— d01asfAnnaType.help —

=====

d01asfAnnaType examples

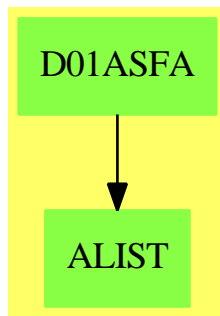
=====

See Also:

o )show d01asfAnnaType

\_\_\_\_\_

### 5.26.1 d01asfAnnaType (D01ASFA)



**Exports:**

coerce hash latex measure numericalIntegration ?? ?~=?

— domain D01ASFA d01asfAnnaType —

```

)abbrev domain D01ASFA d01asfAnnaType
++ Author: Brian Dupee
++ Date Created: March 1994
++ Date Last Updated: December 1997
++ Basic Operations: measure, numericalIntegration
++ Related Constructors: Result, RoutinesTable
++ Description:
++ \axiomType{d01asfAnnaType} is a domain of
++ \axiomType{NumericalIntegrationCategory}
++ for the NAG routine D01ASF, a numerical integration routine which can
++ handle weight functions of the form cos(\omega x) or sin(\omega x) on an
++ semi-infinite range. The
++ function \axiomFun{measure} measures the usefulness of the routine D01ASF
++ for the given problem. The function \axiomFun{numericalIntegration}
++ performs the integration by using \axiomType{NagIntegrationPackage}.

d01asfAnnaType(): NumericalIntegrationCategory == Result add
  EF2 ==> ExpressionFunctions2
  EDF ==> Expression DoubleFloat
  
```

```

LDF ==> List DoubleFloat
SDF ==> Stream DoubleFloat
DF ==> DoubleFloat
FI ==> Fraction Integer
EFI ==> Expression Fraction Integer
SOCDF ==> Segment OrderedCompletion DoubleFloat
NIA ==> Record(var:Symbol,fn:EDF,range:SOCDF,abserr:DF,relerr:DF)
MDNIA ==> Record(fn:EDF,range:List SOCDF,abserr:DF,relerr:DF)
INT ==> Integer
BOP ==> BasicOperator
S ==> Symbol
ST ==> String
LST ==> List String
RT ==> RoutinesTable
Rep:=Result
import Rep, d01WeightsPackage, d01AgentsPackage, NagIntegrationPackage

measure(R:RT,args:NIA) ==
  ext:Result := empty()$Result
  Range := rangeIsFinite(args)
  not(Range case upperInfinite) =>
    [0.0,"d01asf is not a suitable routine for infinite integrals",ext]
  weight: Union(Record(op:BOP,w:DF),"failed") :=
    exprHasWeightCosWXorSinWX(args)
  weight case "failed" =>
    [0.0,"d01asf: A suitable weight has not been found", ext]
  weight case Record(op:BOP,w:DF) =>
    wany := coerce(weight)$AnyFunctions1(Record(op:BOP,w:DF))
    ex:Record(key:S,entry:Any) := [d01asfextra@S,wany]
    ext := insert!(ex,ext)$Result
    ws:ST := string(name(weight.op)$BOP)$S "(" df2st(weight.w)
              string(args.var)$S ")"
    [getMeasure(R,d01asf@S)$RT,
     "d01asf: A suitable weight has been found:- " ws, ext]

numericalIntegration(args:NIA,hints:Result) ==
  i:INT
  r:Any := coerce(search((d01asfextra@S),hints)$Result)$Any
  rec:Record(op:BOP,w:DF) := retract(r)$AnyFunctions1(Record(op:BOP,w:DF))
  Var := args.var :: EDF
  o:BOP := rec.op
  den:EDF := o((rec.w*Var)$EDF)
  Argsfn:EDF := args.fn/den
  if (name(o) = cos@S)$Boolean then i := 1
  else i := 2
  b:Float := getButtonValue("d01asf","functionEvaluations")$AttributeButtons
  fEvals:INT := wholePart exp(1.1513*(1.0/(2.0*(1.0-b))))
  iw:INT := 75*fEvals
  ArgsFn := map(x +-> convert(x)$DF,Argsfn)$EF2(DF,Float)
  f : Union(fn:FileName,fp:Asp1(G)) := [retract(ArgsFn)$Asp1(G)]

```

```

err :=
  positive?(args.abserr) => args.abserr
  args.relerr
d01asf(getlo(args.range),rec.w,i,err,50,4*iw,2*iw,-1,f)

```

---

— D01ASFA.dotabb —

```

"D01ASFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=D01ASFA"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"D01ASFA" -> "ALIST"

```

---

## 5.27 domain D01FCFA d01fcfAnnaType

— d01fcfAnnaType.input —

```

)set break resume
)sys rm -f d01fcfAnnaType.output
)spool d01fcfAnnaType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show d01fcfAnnaType
--R d01fcfAnnaType is a domain constructor
--R Abbreviation for d01fcfAnnaType is D01FCFA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for D01FCFA
--R
--R----- Operations -----
--R ==? : (% ,%) -> Boolean          coerce : % -> OutputForm
--R hash : % -> SingleInteger       latex : % -> String
--R ?~=? : (% ,%) -> Boolean
--R measure : (RoutinesTable,Record(fn: Expression DoubleFloat,range: List Segment OrderedCompletion Dou
--R measure : (RoutinesTable,Record(var: Symbol,fn: Expression DoubleFloat,range: Segment OrderedCompleti
--R numericalIntegration : (Record(fn: Expression DoubleFloat,range: List Segment OrderedCompletion Dou
--R numericalIntegration : (Record(var: Symbol,fn: Expression DoubleFloat,range: Segment OrderedCompleti
--R
--E 1

```

```
)spool
)lisp (bye)
```

---

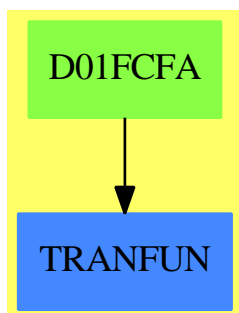
— d01fcfAnnaType.help —

```
=====
d01fcfAnnaType examples
=====
```

```
See Also:
o )show d01fcfAnnaType
```

---

### 5.27.1 d01fcfAnnaType (D01FCFA)



#### Exports:

```
coerce hash latex measure numericalIntegration ?=? ?~=?
```

— domain D01FCFA d01fcfAnnaType —

```
)abbrev domain D01FCFA d01fcfAnnaType
++ Author: Brian Dupee
++ Date Created: March 1994
++ Date Last Updated: December 1997
++ Basic Operations: measure, numericalIntegration
++ Related Constructors: Result, RoutinesTable
++ Description:
++ \axiomType{d01fcfAnnaType} is a domain of
++ \axiomType{NumericalIntegrationCategory}
++ for the NAG routine D01FCF, a numerical integration routine which can
```

```

++ handle multi-dimensional quadrature over a finite region. The
++ function \axiomFun{measure} measures the usefulness of the routine D01GBF
++ for the given problem. The function \axiomFun{numericalIntegration}
++ performs the integration by using \axiomType{NagIntegrationPackage}.

d01fcfAnnaType(): NumericalIntegrationCategory == Result add
  EF2 ==> ExpressionFunctions2
  EDF ==> Expression DoubleFloat
  LDF ==> List DoubleFloat
  SDF ==> Stream DoubleFloat
  DF ==> DoubleFloat
  FI ==> Fraction Integer
  EFI ==> Expression Fraction Integer
  SOCDF ==> Segment OrderedCompletion DoubleFloat
  NIA ==> Record(var:Symbol,fn:EDF,range:SOCDF,abserr:DF,relerr:DF)
  MDNIA ==> Record(fn:EDF,range:List SOCDF,abserr:DF,relerr:DF)
  INT ==> Integer
  BOP ==> BasicOperator
  S ==> Symbol
  ST ==> String
  LST ==> List String
  RT ==> RoutinesTable
  Rep:=Result
  import Rep, d01AgentsPackage, NagIntegrationPackage

measure(R:RT,args:MDNIA) ==
  ext:Result := empty()$Result
  segs := args.range
  vars := variables(args.fn)$EDF
  for i in 1..# vars repeat
    nia:NIA := [vars.i,args.fn,segs.i,args.abserr,args.relerr]
    not rangeIsFinite(nia) case finite => return
    [0.0,"d01fcf is not a suitable routine for infinite integrals",ext]
  [getMeasure(R,d01fcf@S)$RT, "Recommended is d01fcf", ext]

numericalIntegration(args:MDNIA,hints:Result) ==
  import Integer
  segs := args.range
  dim := # segs
  err := args.relerr
  low:Matrix DF := matrix([[getlo(segs.i) for i in 1..dim]])$(Matrix DF)
  high:Matrix DF := matrix([[gethi(segs.i) for i in 1..dim]])$(Matrix DF)
  b:Float := getButtonValue("d01fcf","functionEvaluations")$AttributeButtons
  a:Float:= exp(1.1513*(1.0/(2.0*(1.0-b))))
  alpha:INT := 2**dim+2*dim**2+2*dim+1
  d:Float := max(1.e-3,nthRoot(convert(err)@Float,4))$Float
  minpts:INT := (fEvals := wholePart(a))*wholePart(alpha::Float/d)
  maxpts:INT := 5*minpts
  lenwrk:INT := (dim+2)*(1+(33*fEvals))
  ArgsFn := map(x+>convert(x)$DF,args.fn)$EF2(DF,Float)

```

```
f : Union(fn:FileName,fp:Asp4(FUNCTN)) := [retract(ArgsFn)$Asp4(FUNCTN)]
out:Result := d01fcf(dim,low,high,maxpts,err,lenwrk,minpts,-1,f)
changeName(finval@Symbol,result@Symbol,out)
```

---

— D01FCFA.dotabb —

```
"D01FCFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=D01FCFA"]
"TRANFUN" [color="#4488FF",href="bookvol10.2.pdf#nameddest=TRANFUN"]
"D01FCFA" -> "TRANFUN"
```

---

## 5.28 domain D01GBFA d01gbfAnnaType

— d01gbfAnnaType.input —

```
)set break resume
)sys rm -f d01gbfAnnaType.output
)spool d01gbfAnnaType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show d01gbfAnnaType
--R d01gbfAnnaType is a domain constructor
--R Abbreviation for d01gbfAnnaType is D01GBFA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for D01GBFA
--R
--R----- Operations -----
--R ?=? : (%,% ) -> Boolean          coerce : % -> OutputForm
--R hash : % -> SingleInteger        latex : % -> String
--R ?~=? : (%,% ) -> Boolean
--R measure : (RoutinesTable,Record(fn: Expression DoubleFloat,range: List Segment OrderedCom
--R measure : (RoutinesTable,Record(var: Symbol,fn: Expression DoubleFloat,range: Segment Or
--R numericalIntegration : (Record(fn: Expression DoubleFloat,range: List Segment OrderedComp
--R numericalIntegration : (Record(var: Symbol,fn: Expression DoubleFloat,range: Segment Ord
--R
--E 1

)spool
)lisp (bye)
```

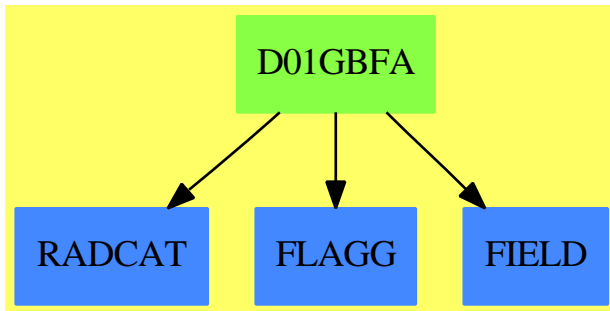
— d01gbfAnnaType.help —

```
=====
d01gbfAnnaType examples
=====
```

See Also:

```
o )show d01gbfAnnaType
```

### 5.28.1 d01gbfAnnaType (D01GBFA)



**Exports:**

```
coerce hash latex measure numericalIntegration ?? ?~=?
```

— domain D01GBFA d01gbfAnnaType —

```
)abbrev domain D01GBFA d01gbfAnnaType
++ Author: Brian Dupee
++ Date Created: March 1994
++ Date Last Updated: December 1997
++ Basic Operations: measure, numericalIntegration
++ Related Constructors: Result, RoutinesTable
++ Description:
++ \axiomType{d01gbfAnnaType} is a domain of
++ \axiomType{NumericalIntegrationCategory}
++ for the NAG routine D01GBF, a numerical integration routine which can
++ handle multi-dimensional quadrature over a finite region. The
++ function \axiomFun{measure} measures the usefulness of the routine D01GBF
++ for the given problem. The function \axiomFun{numericalIntegration}
++ performs the integration by using \axiomType{NagIntegrationPackage}.
```



```

d01gbfAnnaType(): NumericalIntegrationCategory == Result add
  EF2 ==> ExpressionFunctions2
  EDF ==> Expression DoubleFloat
  LDF ==> List DoubleFloat
  SDF ==> Stream DoubleFloat
  DF ==> DoubleFloat
  FI ==> Fraction Integer
  EFI ==> Expression Fraction Integer
  SOCDF ==> Segment OrderedCompletion DoubleFloat
  NIA ==> Record(var:Symbol,fn:EDF,range:SOCDF,abserr:DF,relerr:DF)
  MDNIA ==> Record(fn:EDF,range:List SOCDF,abserr:DF,relerr:DF)
  INT ==> Integer
  BOP ==> BasicOperator
  S ==> Symbol
  ST ==> String
  LST ==> List String
  RT ==> RoutinesTable
  Rep:=Result
  import Rep, d01AgentsPackage, NagIntegrationPackage

measure(R:RT,args:MDNIA) ==
  ext:Result := empty()$Result
  (rel := args.relerr) < 0.01 :: DF =>
    [0.1, "d01gbf: The relative error requirement is too small",ext]
  segs := args.range
  vars := variables(args.fn)$EDF
  for i in 1..# vars repeat
    nia:NIA := [vars.i,args.fn,segs.i,args.abserr,rel]
    not rangeIsFinite(nia) case finite => return
    [0.0,"d01gbf is not a suitable routine for infinite integrals",ext]
  [getMeasure(R,d01gbf@S)$RT, "Recommended is d01gbf", ext]

numericalIntegration(args:MDNIA,hints:Result) ==
  import Integer
  segs := args.range
  dim:INT := # segs
  low:Matrix DF := matrix([[getlo(segs.i) for i in 1..dim]])$(Matrix DF)
  high:Matrix DF := matrix([[gethi(segs.i) for i in 1..dim]])$(Matrix DF)
  b:Float := getButtonValue("d01gbf","functionEvaluations")$AttributeButtons
  a:Float:= exp(1.1513*(1.0/(2.0*(1.0-b))))
  maxcls:INT := 1500*(dim+1)*(fEvals:INT := wholePart(a))
  mincls:INT := 300*fEvals
  c:Float := nthRoot((maxcls::Float)/4.0,dim)$Float
  lenwrk:INT := 3*dim*(d:INT := wholePart(c))+10*dim
  wrkstr:Matrix DF := matrix([[0$DF for i in 1..lenwrk]])$(Matrix DF)
  ArgsFn := map(x+>convert(x)$DF,args.fn)$EF2(DF,Float)
  f : Union(fn:FileName,fp:Asp4(FUNCTN)) := [retract(ArgsFn)$Asp4(FUNCTN)]
  out:Result := _
    d01gbf(dim,low,high,maxcls,args.relerr,lenwrk,mincls,wrkstr,-1,f)

```

```
changeName(finest@Symbol,result@Symbol,out)
```

---

— D01GBFA.dotabb —

```
"D01GBFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=D01GBFA"]
"RADCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=RADCAT"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"FIELD" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FIELD"]
"D01GBFA" -> "FIELD"
"D01GBFA" -> "RADCAT"
"D01GBFA" -> "FLAGG"
```

---

## 5.29 domain D01TRNS d01TransformFunctionType

— d01TransformFunctionType.input —

```
)set break resume
)sys rm -f d01TransformFunctionType.output
)spool d01TransformFunctionType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show d01TransformFunctionType
--R d01TransformFunctionType is a domain constructor
--R Abbreviation for d01TransformFunctionType is D01TRNS
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for D01TRNS
--R
--R----- Operations -----
--R ?=? : (% ,%) -> Boolean          coerce : % -> OutputForm
--R hash : % -> SingleInteger       latex : % -> String
--R ?~=? : (% ,%) -> Boolean
--R measure : (RoutinesTable,Record(fn: Expression DoubleFloat,range: List Segment OrderedCompletion Dou
--R measure : (RoutinesTable,Record(var: Symbol,fn: Expression DoubleFloat,range: Segment OrderedComple
--R numericalIntegration : (Record(fn: Expression DoubleFloat,range: List Segment OrderedCompletion Dou
--R numericalIntegration : (Record(var: Symbol,fn: Expression DoubleFloat,range: Segment OrderedCompleti
--R
--E 1
```

```
)spool
)lisp (bye)
```

---

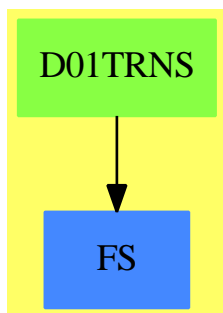
— d01TransformFunctionType.help —

```
=====
d01TransformFunctionType examples
=====
```

```
See Also:
o )show d01TransformFunctionType
```

---

### 5.29.1 d01TransformFunctionType (D01TRNS)



#### Exports:

```
coerce hash latex measure numericalIntegration ?=? ?~=?
```

— domain D01TRNS d01TransformFunctionType —

```
)abbrev domain D01TRNS d01TransformFunctionType
++ Author: Brian Dupee
++ Date Created: April 1994
++ Date Last Updated: December 1997
++ Basic Operations: measure, numericalIntegration
++ Related Constructors: Result, RoutinesTable
++ Description:
++ Since an infinite integral cannot be evaluated numerically
++ it is necessary to transform the integral onto finite ranges.
```

```

++ \axiomType{d01TransformFunctionType} uses the mapping \spad{x -> 1/x}
++ and contains the functions \axiomFun{measure} and
++ \axiomFun{numericalIntegration}.

EDF ==> Expression DoubleFloat
EEDF ==> Equation Expression DoubleFloat
FI ==> Fraction Integer
EFI ==> Expression Fraction Integer
EEFI ==> Equation Expression Fraction Integer
EF2 ==> ExpressionFunctions2
DF ==> DoubleFloat
F ==> Float
SOCDF ==> Segment OrderedCompletion DoubleFloat
OCDF ==> OrderedCompletion DoubleFloat
NIA ==> Record(var:Symbol,fn:EDF,range:SOCDF,abserr:DF,relerr:DF)
INT ==> Integer
PI ==> PositiveInteger
HINT ==> Record(str:String,fn:EDF,range:SOCDF,ext:Result)
S ==> Symbol
ST ==> String
LST ==> List String
Measure ==> Record(measure:F,explanations:ST,extra:Result)
MS ==> Record(measure:F,name:ST,explanations:LST,extra:Result)

d01TransformFunctionType():NumericalIntegrationCategory == Result add
  Rep:=Result
  import d01AgentsPackage,Rep

  rec2any(re:Record(str:ST,fn:EDF,range:SOCDF)):Any ==
    coerce(re)$AnyFunctions1(Record(str:ST,fn:EDF,range:SOCDF))

  changeName(ans:Result,name:ST):Result ==
    sy:S := coerce(name "Answer")$S
    anyAns:Any := coerce(ans)$AnyFunctions1(Result)
    construct([[sy,anyAns]])$Result

  getIntegral(args:NIA,hint:HINT) : Result ==
    Args := copy args
    Args.fn := hint.fn
    Args.range := hint.range
    integrate(Args::NumericalIntegrationProblem)$AnnaNumericalIntegrationPackage

  transformFunction(args:NIA) : NIA ==
    Args := copy args
    Var := Args.var :: EFI -- coerce Symbol to EFI
    NewVar:EFI := inv(Var)$EFI -- invert it
    VarEqn:EEFI:=equation(Var,NewVar)$EEFI -- turn it into an equation
    Afn:EFI := edf2efi(Args.fn)$ExpertSystemToolsPackage
    Afn := subst(Afn,VarEqn)$EFI -- substitute into function
    Var2:EFI := Var**2

```

```

Afn:= simplify(Afn/Var2)$TranscendentalManipulations(FI,EFI)
Args.fn:= map(x+>convert(x)$FI,Afn)$EF2(FI,DF)
Args

doit(seg:SOCDF,args:NIA):MS ==
  Args := copy args
  Args.range := seg
  measure(Args::NumericalIntegrationProblem)$AnnaNumericalIntegrationPackage

transform(c:Boolean,args:NIA):Measure ==
  if c then
    l := coerce( recip(lo(args.range)))@OCDF
    Seg:SOCDF := segment(0$OCDF,l)
  else
    h := coerce( recip(hi(args.range)))@OCDF
    Seg:SOCDF := segment(h,0$OCDF)
  Args := transformFunction(args)
  m:MS := doit(Seg,Args)
  out1:ST :=
    "The recommendation is to transform the function and use " m.name
  out2:List(HINT) := [[m.name,Args.fn,Seg,m.extra]]
  out2Any:Any := coerce(out2)$AnyFunctions1(List(HINT))
  ex:Record(key:S,entry:Any) := [d01transformextra@S,out2Any]
  extr:Result := construct([ex])$Result
  [m.measure,out1,extr]

split(c:PI,args:NIA):Measure ==
  Args := copy args
  Args.relerr := Args.relerr/2
  Args.abserr := Args.abserr/2
  if (c = 1)@Boolean then
    seg1:SOCDF := segment(-1$OCDF,1$OCDF)
  else if (c = 2)@Boolean then
    seg1 := segment(lo(Args.range),1$OCDF)
  else
    seg1 := segment(-1$OCDF,hi(Args.range))
  m1:MS := doit(seg1,Args)
  Args := transformFunction Args
  if (c = 2)@Boolean then
    seg2:SOCDF := segment(0$OCDF,1$OCDF)
  else if (c = 3)@Boolean then
    seg2 := segment(-1$OCDF,0$OCDF)
  else seg2 := seg1
  m2:MS := doit(seg2,Args)
  m1m:F := m1.measure
  m2m:F := m2.measure
  m:F := m1m*m2m/((m1m*m2m)+(1.0-m1m)*(1.0-m2m))
  out1:ST := "The recommendation is to transform the function and use "
    m1.name " and " m2.name
  out2:List(HINT) :=

```

```

      [[m1.name,args.fn,seg1,m1.extra],[m2.name,Args.fn,seg2,m2.extra]]
out2Any:Any := coerce(out2)$AnyFunctions1(List(HINT))
ex:Record(key:S,entry:Any) := [d01transformextra@S,out2Any]
extr:Result := construct([ex])$Result
[m,out1,extr]

measure(R:RoutinesTable,args:NIA) ==
  Range:=rangeIsFinite(args)
  Range case bothInfinite => split(1,args)
  Range case upperInfinite =>
    positive?(lo(args.range))$OCDF =>
      transform(true,args)
      split(2,args)
  Range case lowerInfinite =>
    negative?(hi(args.range))$OCDF =>
      transform(false,args)
      split(3,args)

numericalIntegration(args:NIA,hints:Result) ==
  mainResult:DF := mainAbserr:DF := 0$DF
  ans:Result := empty()$Result
  hla:Any := coerce(search((d01transformextra@S),hints)$Result)@Any
  hintList := retract(hla)$AnyFunctions1(List(HINT))
  methodName:ST := empty()$ST
  repeat
    if (empty?(hintList)$(List(HINT)))
      then leave
    item := first(hintList)$List(HINT)
    a:Result := getIntegral(args,item)
    anyRes := coerce(search((result@S),a)$Result)@Any
    midResult := retract(anyRes)$AnyFunctions1(DF)
    anyErr := coerce(search((abserr pretend S),a)$Result)@Any
    midAbserr := retract(anyErr)$AnyFunctions1(DF)
    mainResult := mainResult+midResult
    mainAbserr := mainAbserr+midAbserr
    if (methodName = item.str)@Boolean then
      methodName := concat([item.str,"1"])$ST
    else
      methodName := item.str
    ans := concat(ans,changeName(a,methodName))$ExpertSystemToolsPackage
    hintList := rest(hintList)$(List(HINT))
  anyResult := coerce(mainResult)$AnyFunctions1(DF)
  anyAbserr := coerce(mainAbserr)$AnyFunctions1(DF)
  recResult:Record(key:S,entry:Any):=[result@S,anyResult]
  recAbserr:Record(key:S,entry:Any):=[abserr pretend S,anyAbserr]
  insert!(recAbserr,insert!(recResult,ans))$Result

```

---

— D01TRNS.dotabb —

```
"D01TRNS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=D01TRNS"]
"FS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FS"]
"D01TRNS" -> "FS"
```

—————

### 5.30 domain D02BBFA d02bbfAnnaType

— d02bbfAnnaType.input —

```
)set break resume
)sys rm -f d02bbfAnnaType.output
)spool d02bbfAnnaType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show d02bbfAnnaType
--R d02bbfAnnaType is a domain constructor
--R Abbreviation for d02bbfAnnaType is D02BBFA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for D02BBFA
--R
--R----- Operations -----
--R ?=? : (%,% ) -> Boolean          coerce : % -> OutputForm
--R hash : % -> SingleInteger        latex : % -> String
--R ?~=? : (%,% ) -> Boolean
--R ODESolve : Record(xinit: DoubleFloat,xend: DoubleFloat,fn: Vector Expression DoubleFloat
--R measure : (RoutinesTable,Record(xinit: DoubleFloat,xend: DoubleFloat,fn: Vector Expression
--R
--E 1

)spool
)lisp (bye)
```

—————

— d02bbfAnnaType.help —

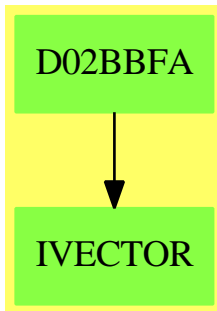
```
=====
d02bbfAnnaType examples
```

=====

See Also:

o )show d02bbfAnnaType

### 5.30.1 d02bbfAnnaType (D02BBFA)



**Exports:**

coerce hash latex measure ODESolve ==? ?~=?

— domain D02BBFA d02bbfAnnaType —

```

)abbrev domain D02BBFA d02bbfAnnaType
++ Author: Brian Dupee
++ Date Created: February 1995
++ Date Last Updated: January 1996
++ Basic Operations:
++ Description:
++ \axiomType{d02bbfAnnaType} is a domain of
++ \axiomType{OrdinaryDifferentialEquationsInitialValueProblemSolverCategory}
++ for the NAG routine D02BBF, a ODE routine which uses an
++ Runge-Kutta method to solve a system of differential
++ equations. The function \axiomFun{measure} measures the
++ usefulness of the routine D02BBF for the given problem. The
++ function \axiomFun{ODESolve} performs the integration by using
++ \axiomType{NagOrdinaryDifferentialEquationsPackage}.

d02bbfAnnaType():OrdinaryDifferentialEquationsSolverCategory == Result add
-- Runge Kutta

EDF ==> Expression DoubleFloat
LDF ==> List DoubleFloat
  
```



```

MDF ==> Matrix DoubleFloat
DF ==> DoubleFloat
F ==> Float
FI ==> Fraction Integer
EFI ==> Expression Fraction Integer
SOCDF ==> Segment OrderedCompletion DoubleFloat
VEDF ==> Vector Expression DoubleFloat
VEF ==> Vector Expression Float
EF ==> Expression Float
VDF ==> Vector DoubleFloat
VMF ==> Vector MachineFloat
MF ==> MachineFloat
ODEA ==> Record(xinit:DF,xend:DF,fn:VEDF,yinit:LDF,intvals:LDF,_
               g:EDF,abserr:DF,relerr:DF)
RSS ==> Record(stiffnessFactor:F,stabilityFactor:F)
INT ==> Integer
EF2 ==> ExpressionFunctions2

import d02AgentsPackage, NagOrdinaryDifferentialEquationsPackage
import AttributeButtons

accuracyCF(ode:ODEA):F ==
  b := getButtonValue("d02bbf","accuracy")$AttributeButtons
  accuracyIntensityValue := combineFeatureCompatibility(b,accuracyIF(ode))
  accuracyIntensityValue > 0.999 => 0$F
  0.8*exp(-((10*accuracyIntensityValue)**3)$F/266)$F

stiffnessCF(stiffnessIntensityValue:F):F ==
  b := getButtonValue("d02bbf","stiffness")$AttributeButtons
  0.5*exp(-(2*combineFeatureCompatibility(b,stiffnessIntensityValue)**2)$F

stabilityCF(stabilityIntensityValue:F):F ==
  b := getButtonValue("d02bbf","stability")$AttributeButtons
  0.5 * cos(combineFeatureCompatibility(b,stabilityIntensityValue))$F

expenseOfEvaluationCF(ode:ODEA):F ==
  b := getButtonValue("d02bbf","expense")$AttributeButtons
  expenseOfEvaluationIntensityValue :=
    combineFeatureCompatibility(b,expenseOfEvaluationIF(ode))
  0.35+0.2*exp(-(2.0*expenseOfEvaluationIntensityValue)**3)$F

measure(R:RoutinesTable,args:ODEA) ==
  m := getMeasure(R,d02bbf :: Symbol)$RoutinesTable
  ssf := stiffnessAndStabilityOfODEIF args
  m := combineFeatureCompatibility(m,[accuracyCF(args),
    stiffnessCF(ssf.stiffnessFactor),
    expenseOfEvaluationCF(args),
    stabilityCF(ssf.stabilityFactor)])
  [m,"Runge-Kutta Merson method"]

```

```

ODESolve(ode:ODEA) ==
  i:LDF := ode.intvals
  M := inc(# i)$INT
  irelab := 0$INT
  if positive?(a := ode.abserr) then
    inc(irelab)$INT
  if positive?(r := ode.relerr) then
    inc(irelab)$INT
  if positive?(a+r) then
    tol:DF := a + r
  else
    tol := float(1,-4,10)$DF
  asp7:Union(fn:FileName,fp:Asp7(FCN)) :=
    [retract(vedf2vef(ode.fn)$ExpertSystemToolsPackage)$Asp7(FCN)]
  asp8:Union(fn:FileName,fp:Asp8(OUTPUT)) :=
    [coerce(ldf2vmf(i)$ExpertSystemToolsPackage)$Asp8(OUTPUT)]
  d02bbf(ode.xend,M,# ode.fn,irelab,ode.xinit,matrix([ode.yinit])$MDF,
    tol,-1,asp7,asp8)

```

---

— D02BBFA.dotabb —

```

"D02BBFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=D02BBFA"]
"IVECTOR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IVECTOR"]
"D02BBFA" -> "IVECTOR"

```

## 5.31 domain D02BHFA d02bhfAnnaType

---

— d02bhfAnnaType.input —

```

)set break resume
)sys rm -f d02bhfAnnaType.output
)spool d02bhfAnnaType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show d02bhfAnnaType
--R d02bhfAnnaType is a domain constructor
--R Abbreviation for d02bhfAnnaType is D02BHFA

```

```
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for D02BHFA
--R
--R----- Operations -----
--R ==? : (%,% )-> Boolean          coerce : % -> OutputForm
--R hash : % -> SingleInteger      latex : % -> String
--R ==~? : (%,% )-> Boolean
--R ODESolve : Record(xinit: DoubleFloat,xend: DoubleFloat,fn: Vector Expression DoubleFloat
--R measure : (RoutinesTable,Record(xinit: DoubleFloat,xend: DoubleFloat,fn: Vector Expressi
--R
--E 1

)spool
)lisp (bye)
```

— d02bhfAnnaType.help —

=====

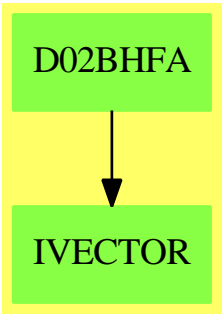
d02bhfAnnaType examples

=====

See Also:

- o )show d02bhfAnnaType

5.31.1 d02bhfAnnaType (D02BHFA)



**Exports:**

coerce hash latex measure ODESolve ==? ==~?

— domain D02BHFA d02bhfAnnaType —

```

)abbrev domain D02BHFA d02bhfAnnaType
++ Author: Brian Dupee
++ Date Created: February 1995
++ Date Last Updated: January 1996
++ Basic Operations:
++ Description:
++ \axiomType{d02bhfAnnaType} is a domain of
++ \axiomType{OrdinaryDifferentialEquationsInitialValueProblemSolverCategory}
++ for the NAG routine D02BHF, a ODE routine which uses an
++ Runge-Kutta method to solve a system of differential
++ equations. The function \axiomFun{measure} measures the
++ usefulness of the routine D02BHF for the given problem. The
++ function \axiomFun{ODESolve} performs the integration by using
++ \axiomType{NagOrdinaryDifferentialEquationsPackage}.

d02bhfAnnaType():OrdinaryDifferentialEquationsSolverCategory == Result add
-- Runge Kutta
EDF ==> Expression DoubleFloat
LDF ==> List DoubleFloat
MDF ==> Matrix DoubleFloat
DF ==> DoubleFloat
F ==> Float
FI ==> Fraction Integer
EFI ==> Expression Fraction Integer
SOCDF ==> Segment OrderedCompletion DoubleFloat
VEDF ==> Vector Expression DoubleFloat
VEF ==> Vector Expression Float
EF ==> Expression Float
VDF ==> Vector DoubleFloat
VMF ==> Vector MachineFloat
MF ==> MachineFloat
ODEA ==> Record(xinit:DF,xend:DF,fn:VEDF,yinit:LDF,intvals:LDF,
               g:EDF,abserr:DF,relerr:DF)
RSS ==> Record(stiffnessFactor:F,stabilityFactor:F)
INT ==> Integer
EF2 ==> ExpressionFunctions2

import d02AgentsPackage, NagOrdinaryDifferentialEquationsPackage
import AttributeButtons

accuracyCF(ode:ODEA):F ==
  b := getButtonValue("d02bhf","accuracy")$AttributeButtons
  accuracyIntensityValue := combineFeatureCompatibility(b,accuracyIF(ode))
  accuracyIntensityValue > 0.999 => 0$F
  0.8*exp(-(10*accuracyIntensityValue)**3)$F/266)$F

stiffnessCF(stiffnessIntensityValue:F):F ==
  b := getButtonValue("d02bhf","stiffness")$AttributeButtons
  0.5*exp(-(2*combineFeatureCompatibility(b,stiffnessIntensityValue)**2)$F

```

```

stabilityCF(stabilityIntensityValue:F):F ==
  b := getButtonValue("d02bhf","stability")$AttributeButtons
  0.5 * cos(combineFeatureCompatibility(b,stabilityIntensityValue))$F

expenseOfEvaluationCF(ode:ODEA):F ==
  b := getButtonValue("d02bhf","expense")$AttributeButtons
  expenseOfEvaluationIntensityValue :=
    combineFeatureCompatibility(b,expenseOfEvaluationIF(ode))
  0.35+0.2*exp(-(2.0*expenseOfEvaluationIntensityValue)**3)$F

measure(R:RoutinesTable,args:ODEA) ==
  m := getMeasure(R,d02bhf :: Symbol)$RoutinesTable
  ssf := stiffnessAndStabilityOfODEIF args
  m := combineFeatureCompatibility(m,[accuracyCF(args),
    stiffnessCF(ssf.stiffnessFactor),
    expenseOfEvaluationCF(args),
    stabilityCF(ssf.stabilityFactor)])
  [m,"Runge-Kutta Merson method"]

ODESolve(ode:ODEA) ==
  irelab := 0$INT
  if positive?(a := ode.abserr) then
    inc(irelab)$INT
  if positive?(r := ode.relerr) then
    inc(irelab)$INT
  if positive?(a+r) then
    tol := max(a,r)$DF
  else
    tol:DF := float(1,-4,10)$DF
  asp7:Union(fn:FileName,fp:Asp7(FCN)) :=
    [retract(e:VEF := vedf2vef(ode.fn)$ExpertSystemToolsPackage)$Asp7(FCN)]
  asp9:Union(fn:FileName,fp:Asp9(G)) :=
    [retract(edf2ef(ode.g)$ExpertSystemToolsPackage)$Asp9(G)]
  d02bhf(ode.xend,# e,irelab,0$DF,ode.xinit,matrix([ode.yinit])$MDF,
    tol,-1,asp9,asp7)

```

---

— D02BHFA.dotabb —

```

"D02BHFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=D02BHFA"]
"IVECTOR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IVECTOR"]
"D02BHFA" -> "IVECTOR"

```

---

## 5.32 domain D02CJFA d02cjfAnnaType

— d02cjfAnnaType.input —

```
)set break resume
)sys rm -f d02cjfAnnaType.output
)spool d02cjfAnnaType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show d02cjfAnnaType
--R d02cjfAnnaType is a domain constructor
--R Abbreviation for d02cjfAnnaType is D02CJFA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for D02CJFA
--R
--R----- Operations -----
--R ==? : (%,% ) -> Boolean          coerce : % -> OutputForm
--R hash : % -> SingleInteger        latex : % -> String
--R ==~? : (%,% ) -> Boolean
--R ODESolve : Record(xinit: DoubleFloat,xend: DoubleFloat,fn: Vector Expression DoubleFloat,yinit: List
--R measure : (RoutinesTable,Record(xinit: DoubleFloat,xend: DoubleFloat,fn: Vector Expression DoubleFlo
--R
--E 1

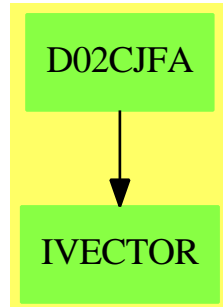
)spool
)lisp (bye)
```

— d02cjfAnnaType.help —

```
=====
d02cjfAnnaType examples
=====
```

See Also:  
o )show d02cjfAnnaType

## 5.32.1 d02cjfAnnaType (D02CJFA)

**Exports:**

coerce hash latex measure ODESolve ?? ?~=?

— domain D02CJFA d02cjfAnnaType —

```

)abbrev domain D02CJFA d02cjfAnnaType
++ Author: Brian Dupee
++ Date Created: February 1995
++ Date Last Updated: January 1996
++ Basic Operations:
++ Description:
++ \axiomType{d02cjfAnnaType} is a domain of
++ \axiomType{OrdinaryDifferentialEquationsInitialValueProblemSolverCategory}
++ for the NAG routine D02CJF, a ODE routine which uses an
++ Adams-Moulton-Bashworth method to solve a system of differential
++ equations. The function \axiomFun{measure} measures the
++ usefulness of the routine D02CJF for the given problem. The
++ function \axiomFun{ODESolve} performs the integration by using
++ \axiomType{NagOrdinaryDifferentialEquationsPackage}.

d02cjfAnnaType():OrdinaryDifferentialEquationsSolverCategory == Result add
-- Adams
EDF ==> Expression DoubleFloat
LDF ==> List DoubleFloat
MDF ==> Matrix DoubleFloat
DF ==> DoubleFloat
F ==> Float
FI ==> Fraction Integer
EFI ==> Expression Fraction Integer
SOCDF ==> Segment OrderedCompletion DoubleFloat
VEDF ==> Vector Expression DoubleFloat
VEF ==> Vector Expression Float
EF ==> Expression Float
VDF ==> Vector DoubleFloat
VMF ==> Vector MachineFloat
  
```

```

MF ==> MachineFloat
ODEA ==> Record(xinit:DF,xend:DF,fn:VEDF,yinit:LDF,intvals:LDF,_
               g:EDF,abserr:DF,relerr:DF)
RSS ==> Record(stiffnessFactor:F,stabilityFactor:F)
INT ==> Integer
EF2 ==> ExpressionFunctions2

import d02AgentsPackage, NagOrdinaryDifferentialEquationsPackage

accuracyCF(ode:ODEA):F ==
  b := getButtonValue("d02cjf","accuracy")$AttributeButtons
  accuracyIntensityValue := combineFeatureCompatibility(b,accuracyIF(ode))
  accuracyIntensityValue > 0.9999 => 0$F
  0.6*(cos(accuracyIntensityValue*(pi()$F)/2)$F)**0.755

stiffnessCF(ode:ODEA):F ==
  b := getButtonValue("d02cjf","stiffness")$AttributeButtons
  ssf := stiffnessAndStabilityOfODEIF ode
  stiffnessIntensityValue :=
    combineFeatureCompatibility(b,ssf.stiffnessFactor)
  0.5*exp(-(1.1*stiffnessIntensityValue)**3)$F

measure(R:RoutinesTable,args:ODEA) ==
  m := getMeasure(R,d02cjf :: Symbol)$RoutinesTable
  m := combineFeatureCompatibility(m,[accuracyCF(args), stiffnessCF(args)])
  [m,"Adams method"]

ODESolve(ode:ODEA) ==
  i:LDF := ode.intvals
  if empty?(i) then
    i := [ode.xend]
  M := inc(# i)$INT
  if positive?((a := ode.abserr)*(r := ode.relerr))$DF then
    ire:String := "D"
  else
    if positive?(a) then
      ire:String := "A"
    else
      ire:String := "R"
  tol := max(a,r)$DF
  asp7:Union(fn:FileName,fp:Asp7(FCN)) :=
    [retract(e:VEF := vedf2vef(ode.fn)$ExpertSystemToolsPackage)$Asp7(FCN)]
  asp8:Union(fn:FileName,fp:Asp8(OUTPUT)) :=
    [coerce(ldf2vmf(i)$ExpertSystemToolsPackage)$Asp8(OUTPUT)]
  asp9:Union(fn:FileName,fp:Asp9(G)) :=
    [retract(edf2ef(ode.g)$ExpertSystemToolsPackage)$Asp9(G)]
  d02cjf(ode.xend,M,# e,tol,ire,ode.xinit,matrix([ode.yinit])$MDF,
        -1,asp9,asp7,asp8)

```



---

— D02CJFA.dotabb —

```
"D02CJFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=D02CJFA"]
"IVECTOR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IVECTOR"]
"D02CJFA" -> "IVECTOR"
```

---

### 5.33 domain D02EJFA d02ejfAnnaType

— d02ejfAnnaType.input —

```
)set break resume
)sys rm -f d02ejfAnnaType.output
)spool d02ejfAnnaType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show d02ejfAnnaType
--R d02ejfAnnaType is a domain constructor
--R Abbreviation for d02ejfAnnaType is D02EJFA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for D02EJFA
--R
--R----- Operations -----
--R ?? : (%,% )-> Boolean          coerce : % -> OutputForm
--R hash : % -> SingleInteger      latex : % -> String
--R ?~=? : (%,% )-> Boolean
--R ODEsolve : Record(xinit: DoubleFloat,xend: DoubleFloat,fn: Vector Expression DoubleFloat
--R measure : (RoutinesTable,Record(xinit: DoubleFloat,xend: DoubleFloat,fn: Vector Expression
--R
--E 1

)spool
)lisp (bye)
```

---

— d02ejfAnnaType.help —

=====

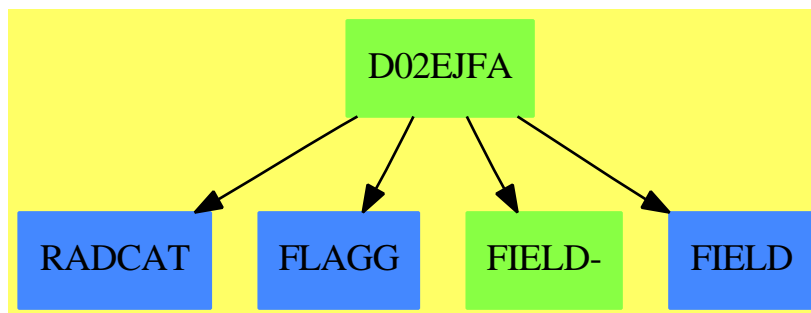
d02ejfAnnaType examples

=====

See Also:

o )show d02ejfAnnaType

### 5.33.1 d02ejfAnnaType (D02EJFA)



Exports:

coerce hash latex measure ODESolve ==? ?~=?

— domain D02EJFA d02ejfAnnaType —

```

)abbrev domain D02EJFA d02ejfAnnaType
++ Author: Brian Dupee
++ Date Created: February 1995
++ Date Last Updated: January 1996
++ Basic Operations:
++ Description:
++ \axiomType{d02ejfAnnaType} is a domain of
++ \axiomType{OrdinaryDifferentialEquationsInitialValueProblemSolverCategory}
++ for the NAG routine D02EJF, a ODE routine which uses a backward
++ differentiation formulae method to handle a stiff system
++ of differential equations. The function \axiomFun{measure} measures
++ the usefulness of the routine D02EJF for the given problem. The
++ function \axiomFun{ODESolve} performs the integration by using
++ \axiomType{NagOrdinaryDifferentialEquationsPackage}.

d02ejfAnnaType():OrdinaryDifferentialEquationsSolverCategory == Result add
-- BDF "Stiff"
EDF ==> Expression DoubleFloat
LDF ==> List DoubleFloat
  
```

```

MDF ==> Matrix DoubleFloat
DF ==> DoubleFloat
F ==> Float
FI ==> Fraction Integer
EFI ==> Expression Fraction Integer
SOCDF ==> Segment OrderedCompletion DoubleFloat
VEDF ==> Vector Expression DoubleFloat
VEF ==> Vector Expression Float
EF ==> Expression Float
VDF ==> Vector DoubleFloat
VMF ==> Vector MachineFloat
MF ==> MachineFloat
ODEA ==> Record(xinit:DF,xend:DF,fn:VEDF,yinit:LDF,intvals:LDF,_
               g:EDF,abserr:DF,relerr:DF)
RSS ==> Record(stiffnessFactor:F,stabilityFactor:F)
INT ==> Integer
EF2 ==> ExpressionFunctions2

import d02AgentsPackage, NagOrdinaryDifferentialEquationsPackage

accuracyCF(ode:ODEA):F ==
  b := getButtonValue("d02ejf","accuracy")$AttributeButtons
  accuracyIntensityValue := combineFeatureCompatibility(b,accuracyIF(ode))
  accuracyIntensityValue > 0.999 => 0$F
  0.5*exp(-((10*accuracyIntensityValue)**3)$F/250)$F

intermediateResultsCF(ode:ODEA):F ==
  intermediateResultsIntensityValue := intermediateResultsIF(ode)
  i := 0.5 * exp(-(intermediateResultsIntensityValue/1.649)**3)$F
  a := accuracyCF(ode)
  i+(0.5-i)*(0.5-a)

stabilityCF(ode:ODEA):F ==
  b := getButtonValue("d02ejf","stability")$AttributeButtons
  ssf := stiffnessAndStabilityOfODEIF ode
  stabilityIntensityValue :=
    combineFeatureCompatibility(b,ssf.stabilityFactor)
  0.68 - 0.5 * exp(-(stabilityIntensityValue)**3)$F

expenseOfEvaluationCF(ode:ODEA):F ==
  b := getButtonValue("d02ejf","expense")$AttributeButtons
  expenseOfEvaluationIntensityValue :=
    combineFeatureCompatibility(b,expenseOfEvaluationIF(ode))
  0.5 * exp(-(1.7*expenseOfEvaluationIntensityValue)**3)$F

systemSizeCF(args:ODEA):F ==
  (1$F - systemSizeIF(args))/2.0

measure(R:RoutinesTable,args:ODEA) ==
  arg := copy args

```

```

m := getMeasure(R,d02ejf :: Symbol)$RoutinesTable
m := combineFeatureCompatibility(m,[intermediateResultsCF(arg),
    accuracyCF(arg),
    systemSizeCF(arg),
    expenseOfEvaluationCF(arg),
    stabilityCF(arg)])
[m,"BDF method for Stiff Systems"]

ODESolve(ode:ODEA) ==
  i:LDF := ode.intvals
  m := inc(# i)$INT
  if positive?((a := ode.abserr)*(r := ode.relerr))$DF then
    ire:String := "D"
  else
    if positive?(a) then
      ire:String := "A"
    else
      ire:String := "R"
  if positive?(a+r)$DF then
    tol := max(a,r)$DF
  else
    tol := float(1,-4,10)$DF
asp7:Union(fn:FileName,fp:Asp7(FCN)) :=
  [retract(e:VEF := vedf2vef(ode.fn)$ExpertSystemToolsPackage)$Asp7(FCN)]
asp31:Union(fn:FileName,fp:Asp31(PEDERV)) :=
  [retract(e)$Asp31(PEDERV)]
asp8:Union(fn:FileName,fp:Asp8(OUTPUT)) :=
  [coerce(ldf2vmf(i)$ExpertSystemToolsPackage)$Asp8(OUTPUT)]
asp9:Union(fn:FileName,fp:Asp9(G)) :=
  [retract(edf2ef(ode.g)$ExpertSystemToolsPackage)$Asp9(G)]
n:INT := # ode.yinit
iw:INT := (12+n)*n+50
ans := d02ejf(ode.xend,m,n,ire,iw,ode.xinit,matrix([ode.yinit])$MDF,
    tol,-1,asp9,asp7,asp31,asp8)

```

— D02EJFA.dotabb —

```

"D02EJFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=D02EJFA"]
"RADCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=RADCAT"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"FIELD-" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FIELD"]
"FIELD" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FIELD"]
"D02EJFA" -> "FLAGG"
"D02EJFA" -> "FIELD-"
"D02EJFA" -> "FIELD"
"D02EJFA" -> "RADCAT"

```

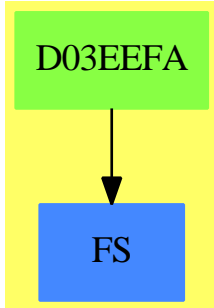
— d03eefAnnaType.input —

— d03eefAnnaType.help —

```
=====
d03eefAnnaType examples
=====

See Also:
o )show d03eefAnnaType
```

## 5.34.1 d03eefAnnaType (D03EEFA)

**Exports:**

coerce hash latex measure PDESolve ?? ?~=?

— domain D03EEFA d03eefAnnaType —

```

)abbrev domain D03EEFA d03eefAnnaType
++ Author: Brian Dupee
++ Date Created: June 1996
++ Date Last Updated: June 1996
++ Basic Operations:
++ Description:
++ \axiomType{d03eefAnnaType} is a domain of
++ \axiomType{PartialDifferentialEquationsSolverCategory}
++ for the NAG routines D03EEF/D03EDF.

d03eefAnnaType():PartialDifferentialEquationsSolverCategory == Result add
-- 2D Elliptic PDE
LEDF ==> List Expression DoubleFloat
EDF ==> Expression DoubleFloat
LDF ==> List DoubleFloat
MDF ==> Matrix DoubleFloat
DF ==> DoubleFloat
F ==> Float
FI ==> Fraction Integer
VEF ==> Vector Expression Float
EF ==> Expression Float
MEF ==> Matrix Expression Float
NNI ==> NonNegativeInteger
INT ==> Integer
PDEC ==> Record(start:DF, finish:DF, grid:NNI, boundaryType:INT,
                dStart:MDF, dFinish:MDF)
PDEB ==> Record(pde:LEDF, constraints:List PDEC,
                f:List LEDF, st:String, tol:DF)

import d03AgentsPackage, NagPartialDifferentialEquationsPackage

```

```

import ExpertSystemToolsPackage

measure(R:RoutinesTable,args:PDEB) ==
  (# (args.constraints) > 2)@Boolean =>
    [0$F,"d03eef/d03edf is unsuitable for PDEs of order more than 2"]
  elliptic?(args) =>
    m := getMeasure(R,d03eef :: Symbol)$RoutinesTable
    [m,"d03eef/d03edf is suitable"]
    [0$F,"d03eef/d03edf is unsuitable for hyperbolic or parabolic PDEs"]

PDESolve(args:PDEB) ==
  xcon := first(args.constraints)
  ycon := second(args.constraints)
  nx := xcon.grid
  ny := ycon.grid
  p := args.pde
  x1 := xcon.start
  x2 := xcon.finish
  y1 := ycon.start
  y2 := ycon.finish
  lda := ((4*(nx+1)*(ny+1)+2) quo 3)$INT
  scheme:String :=
    central?((x2-x1)/2,(y2-y1)/2,args.pde) => "C"
    "U"
  asp73:Union(fn:FileName,fp:Asp73(PDEF)) :=
    [retract(vector([edf2ef u for u in p])$VEF)$Asp73(PDEF)]
  asp74:Union(fn:FileName,fp:Asp74(BNDY)) :=
    [retract(matrix([[edf2ef v for v in w] for w in args.f])$MEF)$Asp74(BNDY)]
  fde := d03eef(x1,x2,y1,y2,nx,ny,lda,scheme,-1,asp73,asp74)
  ub := new(1,nx*ny,0$DF)$MDF
  A := search(a::Symbol,fde)$Result
  A case "failed" => empty()$Result
  AA := A::Any
  fdea := retract(AA)$AnyFunctions1(MDF)
  r := search(rhs::Symbol,fde)$Result
  r case "failed" => empty()$Result
  rh := r::Any
  fderhs := retract(rh)$AnyFunctions1(MDF)
  d03edf(nx,ny,lda,15,args.tol,0,fdea,fderhs,ub,-1)

```

---

— D03EEFA.dotabb —

```

"D03EEFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=D03EEFA"]
"FS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FS"]
"D03EEFA" -> "FS"

```

## 5.35 domain D03FAFA d03fafAnnaType

— d03fafAnnaType.input —

```
)set break resume
)sys rm -f d03fafAnnaType.output
)spool d03fafAnnaType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show d03fafAnnaType
--R d03fafAnnaType is a domain constructor
--R Abbreviation for d03fafAnnaType is D03FAFA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for D03FAFA
--R
--R----- Operations -----
--R ?=? : (%,% ) -> Boolean          coerce : % -> OutputForm
--R hash : % -> SingleInteger        latex : % -> String
--R ?~=? : (%,% ) -> Boolean
--R PDESolve : Record(pde: List Expression DoubleFloat,constraints: List Record(start: DoubleFloat,finis
--R measure : (RoutinesTable,Record(pde: List Expression DoubleFloat,constraints: List Record(start: Dou
--R
--E 1

)spool
)lisp (bye)
```

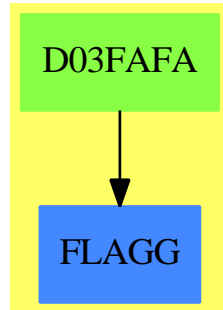
— d03fafAnnaType.help —

```
=====
d03fafAnnaType examples
=====
```

See Also:  
o )show d03fafAnnaType



### 5.35.1 d03fafAnnaType (D03FAFA)



#### Exports:

coerce hash latex measure PDESolve  $\text{?=?}$   $\text{?}^{\sim}\text{=?}$

— domain D03FAFA d03fafAnnaType —

```

)abbrev domain D03FAFA d03fafAnnaType
++ Author: Brian Dupee
++ Date Created: July 1996
++ Date Last Updated: July 1996
++ Basic Operations:
++ Description:
++ \axiomType{d03fafAnnaType} is a domain of
++ \axiomType{PartialDifferentialEquationsSolverCategory}
++ for the NAG routine D03FAF.

d03fafAnnaType():PartialDifferentialEquationsSolverCategory == Result add
-- 3D Helmholtz PDE
LEDF ==> List Expression DoubleFloat
EDF ==> Expression DoubleFloat
LDF ==> List DoubleFloat
MDF ==> Matrix DoubleFloat
DF ==> DoubleFloat
F ==> Float
FI ==> Fraction Integer
VEF ==> Vector Expression Float
EF ==> Expression Float
MEF ==> Matrix Expression Float
NNI ==> NonNegativeInteger
INT ==> Integer
PDEC ==> Record(start:DF, finish:DF, grid:NNI, boundaryType:INT,
               dStart:MDF, dFinish:MDF)
PDEB ==> Record(pde:LEDF, constraints:List PDEC,
               f:List LEDF, st:String, tol:DF)

import d03AgentsPackage, NagPartialDifferentialEquationsPackage

```

```
import ExpertSystemToolsPackage

measure(R:RoutinesTable,args:PDEB) ==
  (# (args.constraints) < 3)@Boolean =>
    [0$F,"d03faf is unsuitable for PDEs of order other than 3"]
    [0$F,"d03faf isn't finished"]
```

---

— D03FAFAs.dotabb —

```
"D03FAFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=D03FAFA"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"D03FAFA" -> "FLAGG"
```

---



## Chapter 6

# Chapter E

### 6.1 domain EQ Equation

— Equation.input —

```
)set break resume
)sys rm -f Equation.output
)spool Equation.output
)set message test on
)set message auto off
)clear all
--S 1 of 12
eq1 := 3*x + 4*y = 5
--R
--R
--R (1)  $4y + 3x = 5$ 
--R
--R                                          Type: Equation Polynomial Integer
--E 1

--S 2 of 12
eq2 := 2*x + 2*y = 3
--R
--R
--R (2)  $2y + 2x = 3$ 
--R
--R                                          Type: Equation Polynomial Integer
--E 2

--S 3 of 12
lhs eq1
--R
--R
--R (3)  $4y + 3x$ 
```

```

--R                                         Type: Polynomial Integer
--E 3

--S 4 of 12
rhs eq1
--R
--R
--R      (4)  5
--R
--R                                         Type: Polynomial Integer
--E 4

--S 5 of 12
eq1 + eq2
--R
--R
--R      (5)  6y + 5x = 8
--R
--R                                         Type: Equation Polynomial Integer
--E 5

--S 6 of 12
eq1 * eq2
--R
--R
--R      (6)  8y2 + 14x y + 6x2 = 15
--R
--R                                         Type: Equation Polynomial Integer
--E 6

--S 7 of 12
2*eq2 - eq1
--R
--R
--R      (7)  x = 1
--R
--R                                         Type: Equation Polynomial Integer
--E 7

--S 8 of 12
eq1**2
--R
--R
--R      (8)  16y2 + 24x y + 9x2 = 25
--R
--R                                         Type: Equation Polynomial Integer
--E 8

--S 9 of 12
if x+1 = y then "equal" else "unequal"
--R
--R
--R      (9)  "unequal"

```

Type: String

Type: Equation Polynomial Integer

Type: String

Type: Boolean

## Equation examples

Equations are created using the equals symbol, =.

Type: Equation Polynomial Integer

Type: Equation Polynomial Integer

The left- and right-hand sides of an equation are accessible using the operations `lhs` and `rhs`.

```
lhs eq1
4y + 3x
Type: Polynomial Integer
```

```
rhs eq1
5
Type: Polynomial Integer
```

Arithmetic operations are supported and operate on both sides of the equation.

```
eq1 + eq2
6y + 5x = 8
Type: Equation Polynomial Integer
```

```
eq1 * eq2
      2      2
8y  + 14x y + 6x = 15
Type: Equation Polynomial Integer
```

```
2*eq2 - eq1
x = 1
Type: Equation Polynomial Integer
```

Equations may be created for any type so the arithmetic operations will be defined only when they make sense. For example, exponentiation is not defined for equations involving non-square matrices.

```
eq1**2
      2      2
16y  + 24x y + 9x = 25
Type: Equation Polynomial Integer
```

Note that an equals symbol is also used to test for equality of values in certain contexts. For example, `x+1` and `y` are unequal as polynomials.

```
if x+1 = y then "equal" else "unequal"
"unequal"
Type: String
```

```
eqpol := x+1 = y
x + 1 = y
Type: Equation Polynomial Integer
```

If an equation is used where a Boolean value is required, then it is evaluated using the equality test from the operand type.

```
if eqpol then "equal" else "unequal"
"unequal"
```

Type: String

If one wants a Boolean value rather than an equation, all one has to do is ask!

```
eqpol::Boolean
false
```

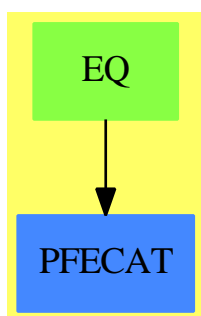
Type: Boolean

See Also:

o )show Equation

---

### 6.1.1 Equation (EQ)



#### Exports:

|           |                |                |           |            |
|-----------|----------------|----------------|-----------|------------|
| 0         | 1              | characteristic | coerce    | commutator |
| conjugate | D              | differentiate  | dimension | equation   |
| eval      | factorAndSplit | hash           | inv       | latex      |
| leftOne   | leftZero       | lhs            | map       | one?       |
| recip     | rhs            | rightOne       | rightZero | sample     |
| subst     | subtractIfCan  | swap           | zero?     | ?^=?       |
| -?        | ?=?            | ?*?            | ?**?      | ?+?        |
| ?-?       | ?/?            | ?=?            | ?^?       |            |

— domain EQ Equation —

```
)abbrev domain EQ Equation
--FOR THE BENEFIT OF LIBAXO GENERATION
```



```

++ Author: Stephen M. Watt, enhancements by Johannes Grabmeier
++ Date Created: April 1985
++ Date Last Updated: June 3, 1991; September 2, 1992
++ Basic Operations: =
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: equation
++ Examples:
++ References:
++ Description:
++ Equations as mathematical objects. All properties of the basis domain,
++ e.g. being an abelian group are carried over the equation domain, by
++ performing the structural operations on the left and on the
++ right hand side.
-- The interpreter translates "=" to "equation". Otherwise, it will
-- find a modemap for "=" in the domain of the arguments.

Equation(S: Type): public == private where
  Ex ==> OutputForm
  public ==> Type with
    "=": (S, S) -> $
      ++ a=b creates an equation.
    equation: (S, S) -> $
      ++ equation(a,b) creates an equation.
    swap: $ -> $
      ++ swap(eq) interchanges left and right hand side of equation eq.
    lhs: $ -> S
      ++ lhs(eqn) returns the left hand side of equation eqn.
    rhs: $ -> S
      ++ rhs(eqn) returns the right hand side of equation eqn.
    map: (S -> S, $) -> $
      ++ map(f,eqn) constructs a new equation by applying f to both
      ++ sides of eqn.
    if S has InnerEvalable(Symbol,S) then
      InnerEvalable(Symbol,S)
    if S has SetCategory then
      SetCategory
      CoercibleTo Boolean
      if S has Evalable(S) then
        eval: ($, $) -> $
          ++ eval(eqn, x=f) replaces x by f in equation eqn.
        eval: ($, List $) -> $
          ++ eval(eqn, [x1=v1, ... xn=vn]) replaces xi by vi in equation eqn.
    if S has AbelianSemiGroup then
      AbelianSemiGroup
      "+": (S, $) -> $
        ++ x+eqn produces a new equation by adding x to both sides of
        ++ equation eqn.
      "+": ($, S) -> $

```

```

        ++ eqn+x produces a new equation by adding x to both sides of
        ++ equation eqn.
if S has AbelianGroup then
  AbelianGroup
  leftZero : $ -> $
  ++ leftZero(eq) subtracts the left hand side.
  rightZero : $ -> $
  ++ rightZero(eq) subtracts the right hand side.
  "-": (S, $) -> $
  ++ x-eqn produces a new equation by subtracting both sides of
  ++ equation eqn from x.
  "-": ($, S) -> $
  ++ eqn-x produces a new equation by subtracting x from both sides of
  ++ equation eqn.
if S has SemiGroup then
  SemiGroup
  "*": (S, $) -> $
  ++ x*eqn produces a new equation by multiplying both sides of
  ++ equation eqn by x.
  "*": ($, S) -> $
  ++ eqn*x produces a new equation by multiplying both sides of
  ++ equation eqn by x.
if S has Monoid then
  Monoid
  leftOne : $ -> Union($,"failed")
  ++ leftOne(eq) divides by the left hand side, if possible.
  rightOne : $ -> Union($,"failed")
  ++ rightOne(eq) divides by the right hand side, if possible.
if S has Group then
  Group
  leftOne : $ -> Union($,"failed")
  ++ leftOne(eq) divides by the left hand side.
  rightOne : $ -> Union($,"failed")
  ++ rightOne(eq) divides by the right hand side.
if S has Ring then
  Ring
  BiModule(S,S)
if S has CommutativeRing then
  Module(S)
  --Algebra(S)
if S has IntegralDomain then
  factorAndSplit : $ -> List $
  ++ factorAndSplit(eq) make the right hand side 0 and
  ++ factors the new left hand side. Each factor is equated
  ++ to 0 and put into the resulting list without repetitions.
if S has PartialDifferentialRing(Symbol) then
  PartialDifferentialRing(Symbol)
if S has Field then
  VectorSpace(S)
  "/": ($, $) -> $

```

```

    ++ e1/e2 produces a new equation by dividing the left and right
    ++ hand sides of equations e1 and e2.
  inv: $ -> $
    ++ inv(x) returns the multiplicative inverse of x.
  if S has ExpressionSpace then
    subst: ($, $) -> $
      ++ subst(eq1,eq2) substitutes eq2 into both sides of eq1
      ++ the lhs of eq2 should be a kernel

private ==> add
  Rep := Record(lhs: S, rhs: S)
  eq1,eq2: $
  s : S
  if S has IntegralDomain then
    factorAndSplit eq ==
      (S has factor : S -> Factored S) =>
        eq0 := rightZero eq
        [equation(rcf.factor,0) for rcf in factors factor lhs eq0]
        [eq]
  l:S = r:S == [l, r]
  equation(l, r) == [l, r] -- hack! See comment above.
  lhs eqn == eqn.lhs
  rhs eqn == eqn.rhs
  swap eqn == [rhs eqn, lhs eqn]
  map(fn, eqn) == equation(fn(eqn.lhs), fn(eqn.rhs))

  if S has InnerEvalable(Symbol,S) then
    s:Symbol
    ls:List Symbol
    x:S
    lx:List S
    eval(eqn,s,x) == eval(eqn.lhs,s,x) = eval(eqn.rhs,s,x)
    eval(eqn,ls,lx) == eval(eqn.lhs,ls,lx) = eval(eqn.rhs,ls,lx)
  if S has Evalable(S) then
    eval(eqn1:$, eqn2:$):$ ==
      eval(eqn1.lhs, eqn2 pretend Equation S) =
        eval(eqn1.rhs, eqn2 pretend Equation S)
    eval(eqn1:$, leqn2:List $):$ ==
      eval(eqn1.lhs, leqn2 pretend List Equation S) =
        eval(eqn1.rhs, leqn2 pretend List Equation S)
  if S has SetCategory then
    eq1 = eq2 == (eq1.lhs = eq2.lhs)@Boolean and
      (eq1.rhs = eq2.rhs)@Boolean
    coerce(eqn:$):Ex == eqn.lhs::Ex = eqn.rhs::Ex
    coerce(eqn:$):Boolean == eqn.lhs = eqn.rhs
  if S has AbelianSemiGroup then
    eq1 + eq2 == eq1.lhs + eq2.lhs = eq1.rhs + eq2.rhs
    s + eq2 == [s,s] + eq2
    eq1 + s == eq1 + [s,s]
  if S has AbelianGroup then

```

```

- eq == (- lhs eq) = (-rhs eq)
s - eq2 == [s,s] - eq2
eq1 - s == eq1 - [s,s]
leftZero eq == 0 = rhs eq - lhs eq
rightZero eq == lhs eq - rhs eq = 0
0 == equation(0$S,0$S)
eq1 - eq2 == eq1.lhs - eq2.lhs = eq1.rhs - eq2.rhs
if S has SemiGroup then
  eq1:$ * eq2:$ == eq1.lhs * eq2.lhs = eq1.rhs * eq2.rhs
  1:S * eqn:$ == 1 * eqn.lhs = 1 * eqn.rhs
  1:S * eqn:$ == 1 * eqn.lhs = 1 * eqn.rhs
  eqn:$ * 1:S == eqn.lhs * 1 = eqn.rhs * 1
  -- We have to be a bit careful here: raising to a +ve integer is OK
  -- (since it's the equivalent of repeated multiplication)
  -- but other powers may cause contradictions
  -- Watch what else you add here! JHD 2/Aug 1990
if S has Monoid then
  1 == equation(1$S,1$S)
  recip eq ==
    (lh := recip lhs eq) case "failed" => "failed"
    (rh := recip rhs eq) case "failed" => "failed"
    [lh :: S, rh :: S]
  leftOne eq ==
    (re := recip lhs eq) case "failed" => "failed"
    1 = rhs eq * re
  rightOne eq ==
    (re := recip rhs eq) case "failed" => "failed"
    lhs eq * re = 1
if S has Group then
  inv eq == [inv lhs eq, inv rhs eq]
  leftOne eq == 1 = rhs eq * inv rhs eq
  rightOne eq == lhs eq * inv lhs eq = 1
if S has Ring then
  characteristic() == characteristic()$S
  i:Integer * eq:$ == (i::S) * eq
if S has IntegralDomain then
  factorAndSplit eq ==
    (S has factor : S -> Factored S) =>
      eq0 := rightZero eq
      [equation(rcf.factor,0) for rcf in factors factor lhs eq0]
    (S has Polynomial Integer) =>
      eq0 := rightZero eq
      MF ==> MultivariateFactorize(Symbol, IndexedExponents Symbol, _
        Integer, Polynomial Integer)
      p : Polynomial Integer := (lhs eq0) pretend Polynomial Integer
      [equation((rcf.factor) pretend S,0) for rcf in factors factor(p)$MF]
      [eq]
if S has PartialDifferentialRing(Symbol) then
  differentiate(eq:$, sym:Symbol):$ ==
    [differentiate(lhs eq, sym), differentiate(rhs eq, sym)]

```

---

— EqTable.input —

```
Type: EqTable(List Integer,Integer)
```

```

--S 3 of 6
l2 := [1,2,3]
--R
--R
--R (3) [1,2,3]
--R
--R                                         Type: List PositiveInteger
--E 3

--S 4 of 6
e.l1 := 111
--R
--R
--R (4) 111
--R
--R                                         Type: PositiveInteger
--E 4

--S 5 of 6
e.l2 := 222
--R
--R
--R (5) 222
--R
--R                                         Type: PositiveInteger
--E 5

--S 6 of 6
e.l1
--R
--R
--R (6) 111
--R
--R                                         Type: PositiveInteger
--E 6
)spool
)lisp (bye)

```

---

— EqTable.help —

=====

EqTable examples

=====

The EqTable domain provides tables where the keys are compared using eq?. Keys are considered equal only if they are the same instance of a structure. This is useful if the keys are themselves updatable structures. Otherwise, all operations are the same as for type Table.

The operation table is here used to create a table where the keys are

lists of integers.

```
e: EqTable(List Integer, Integer) := table()
table()
Type: EqTable(List Integer,Integer)
```

These two lists are equal according to =, but not according to eq?.

```
l1 := [1,2,3]
[1,2,3]
Type: List PositiveInteger
```

```
l2 := [1,2,3]
[1,2,3]
Type: List PositiveInteger
```

Because the two lists are not eq?, separate values can be stored under each.

```
e.l1 := 111
111
Type: PositiveInteger
```

```
e.l2 := 222
222
Type: PositiveInteger
```

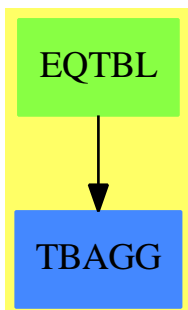
```
e.l1
111
Type: PositiveInteger
```

See Also:

- o )help Table
- o )show EqTable

---

## 6.2.1 EqTable (EQTBL)



See

- ⇒ “HashTable” (HASHTBL) 9.1.1 on page 1085
- ⇒ “InnerTable” (INTABL) 10.27.1 on page 1299
- ⇒ “Table” (TABLE) 21.1.1 on page 2621
- ⇒ “StringTable” (STRTBL) 20.32.1 on page 2569
- ⇒ “GeneralSparseTable” (GSTBL) 8.5.1 on page 1044
- ⇒ “SparseTable” (STBL) 20.16.1 on page 2409

**Exports:**

|        |          |                  |           |          |
|--------|----------|------------------|-----------|----------|
| any?   | bag      | coerce           | construct | convert  |
| copy   | count    | dictionary       | elt       | empty    |
| empty? | entries  | entry?           | eq?       | eval     |
| every? | extract! | fill!            | find      | first    |
| hash   | index?   | indices          | insert!   | inspect  |
| key?   | keys     | latex            | less?     | map      |
| map!   | maxIndex | member?          | members   | minIndex |
| more?  | parts    | qelt             | qsetelt!  | reduce   |
| remove | remove!  | removeDuplicates | sample    | search   |
| select | select!  | setelt           | size?     | swap!    |
| table  | table    | #?               | ?=?       | ?~=?     |
| ??     |          |                  |           |          |

— domain EQTBL EqTable —

```

)abbrev domain EQTBL EqTable
++ Author: Stephen M. Watt
++ Date Created:
++ Date Last Updated: June 21, 1991
++ Basic Operations:
++ Related Domains: HashTable, Table, StringTable
++ Also See:
++ AMS Classifications:
++ Keywords: equation
++ Examples:
++ References:
  
```



```

++ Description:
++ This domain provides tables where the keys are compared using
++ \spadfun{eq?}. Thus keys are considered equal only if they
++ are the same instance of a structure.

EqTable(Key: SetCategory, Entry: SetCategory) ==
    HashTable(Key, Entry, "EQ")

```

---

— EQTBL.dotabb —

```

"EQTBL" [color="#88FF44",href="bookvol10.3.pdf#nameddest=EQTBL"]
"TBAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=TBAGG"]
"EQTBL" -> "TBAGG"

```

---

### 6.3 domain EMR EuclideanModularRing

— EuclideanModularRing.input —

```

)set break resume
)sys rm -f EuclideanModularRing.output
)spool EuclideanModularRing.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show EuclideanModularRing
--R EuclideanModularRing(S: CommutativeRing,R: UnivariatePolynomialCategory S,Mod: AbelianMonoid)
--R Abbreviation for EuclideanModularRing is EMR
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for EMR
--R
--R----- Operations -----
--R ??? : (%,% ) -> %
--R ??? : (PositiveInteger,% ) -> %
--R ??+? : (%,% ) -> %
--R -? : % -> %
--R 1 : () -> %
--R ??^? : (% ,PositiveInteger) -> %
--R coerce : % -> R
--R ??? : (Integer,% ) -> %
--R ??? : (% ,PositiveInteger) -> %
--R ?-? : (%,% ) -> %
--R ?? : (%,% ) -> Boolean
--R 0 : () -> %
--R associates? : (%,% ) -> Boolean
--R coerce : % -> %

```

```

--R coerce : Integer -> %
--R ?.? : (% , R) -> R
--R gcd : (% , %) -> %
--R inv : % -> %
--R lcm : List % -> %
--R modulus : % -> Mod
--R ?quo? : (% , %) -> %
--R reduce : (R , Mod) -> %
--R sample : () -> %
--R unit? : % -> Boolean
--R zero? : % -> Boolean
--R ?*? : (NonNegativeInteger , %) -> %
--R ?**? : (% , NonNegativeInteger) -> %
--R ?^? : (% , NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R divide : (% , %) -> Record(quotient: % , remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R exQuo : (% , %) -> Union(% , "failed")
--R expressIdealMember : (List % , %) -> Union(List % , "failed")
--R exquo : (% , %) -> Union(% , "failed")
--R extendedEuclidean : (% , % , %) -> Union(Record(coef1: % , coef2: % ) , "failed")
--R extendedEuclidean : (% , %) -> Record(coef1: % , coef2: % , generator: %)
--R gcdPolynomial : (SparseUnivariatePolynomial % , SparseUnivariatePolynomial %) -> SparseUnivariatePolynomial %
--R multiEuclidean : (List % , %) -> Union(List % , "failed")
--R principalIdeal : List % -> Record(coef: List % , generator: %)
--R subtractIfCan : (% , %) -> Union(% , "failed")
--R unitNormal : % -> Record(unit: % , canonical: % , associate: %)
--R
--E 1

)spool
)lisp (bye)

```

---

— EuclideanModularRing.help —

```

=====
EuclideanModularRing examples
=====

```

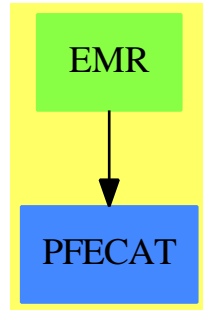
See Also:

```

o )show EuclideanModularRing

```

### 6.3.1 EuclideanModularRing (EMR)



See

⇒ “ModularRing” (MODRING) 14.10.1 on page 1604

⇒ “ModularField” (MODFIELD) 14.9.1 on page 1602

#### Exports:

|                   |               |               |                    |           |
|-------------------|---------------|---------------|--------------------|-----------|
| 0                 | 1             | associates?   | characteristic     | coerce    |
| divide            | euclideanSize | exQuo         | expressIdealMember | exquo     |
| extendedEuclidean | gcd           | gcdPolynomial | hash               | inv       |
| latex             | lcm           | modulus       | multiEuclidean     | one?      |
| principalIdeal    | recip         | reduce        | sample             | sizeLess? |
| subtractIfCan     | unit?         | unitCanonical | unitNormal         | zero?     |
| ?~=?              | ?*?           | ?**?          | ?^?                | ?+?       |
| ?-?               | ~?            | ?=?           | ?..?               | ?quo?     |
| ?rem?             |               |               |                    |           |

— domain EMR EuclideanModularRing —

```

)abbrev domain EMR EuclideanModularRing
++ Author: Mark Botch
++ Description:
++ These domains are used for the factorization and gcds
++ of univariate polynomials over the integers in order to work modulo
++ different primes.
++ See \spadtype{ModularRing}, \spadtype{ModularField}

EuclideanModularRing(S,R,Mod,reduction:(R,Mod) -> R,
                    merge:(Mod,Mod) -> Union(Mod,"failed"),
                    exactQuo : (R,R,Mod) -> Union(R,"failed")) : C == T

where
  S    : CommutativeRing
  R    : UnivariatePolynomialCategory S
  Mod  : AbelianMonoid

C == EuclideanDomain with
  modulus : % -> Mod

```

```

    ++ modulus(x) is not documented
coerce : %      -> R
    ++ coerce(x) is not documented
reduce : (R,Mod) -> %
    ++ reduce(r,m) is not documented
exQuo  : (%,% ) -> Union(%, "failed")
    ++ exQuo(x,y) is not documented
recip  : %      -> Union(%, "failed")
    ++ recip(x) is not documented
inv    : %      -> %
    ++ inv(x) is not documented
elt    : (% , R) -> R
    ++ elt(x,r) or x.r is not documented

T == ModularRing(R,Mod,reduction,merge,exactQuo) add

--representation
Rep:= Record(val:R,module:Mod)
--declarations
x,y,z: %

divide(x,y) ==
  t:=merge(x.module,y.module)
  t case "failed" => error "incompatible moduli"
  xm:=t::Mod
  yv:=y.val
  invlcy:R
--   if one? leadingCoefficient yv then invlcy:=1
  if (leadingCoefficient yv = 1) then invlcy:=1
  else
    invlcy:=(inv reduce((leadingCoefficient yv)::R,xm)).val
    yv:=reduction(invlcy*yv,xm)
  r:=monicDivide(x.val,yv)
  [reduce(invlcy*r.quotient,xm),reduce(r.remainder,xm)]

if R has fmecg:(R,NonNegativeInteger,S,R)->R
  then x rem y ==
    t:=merge(x.module,y.module)
    t case "failed" => error "incompatible moduli"
    xm:=t::Mod
    yv:=y.val
    invlcy:R
--   if not one? leadingCoefficient yv then
  if not (leadingCoefficient yv = 1) then
    invlcy:=(inv reduce((leadingCoefficient yv)::R,xm)).val
    yv:=reduction(invlcy*yv,xm)
  dy:=degree yv
  xv:=x.val
  while (d:=degree xv - dy)>=0 repeat
    xv:=reduction(fmecg(xv,d::NonNegativeInteger,

```

```

                                leadingCoefficient xv,yv),xm)
      xv = 0 => return [xv,xm]$Rep
    [xv,xm]$Rep
  else x rem y ==
    t:=merge(x.modulo,y.modulo)
    t case "failed" => error "incompatible moduli"
    xm:=t::Mod
    yv:=y.val
    invlcy:R
--    if not one? leadingCoefficient yv then
    if not (leadingCoefficient yv = 1) then
      invlcy:=(inv reduce((leadingCoefficient yv)::R,xm)).val
      yv:=reduction(invlcy*yv,xm)
      r:=monicDivide(x.val,yv)
      reduce(r.remainder,xm)

  euclideanSize x == degree x.val

  unitCanonical x ==
    zero? x => x
    degree(x.val) = 0 => 1
--    one? leadingCoefficient(x.val) => x
    (leadingCoefficient(x.val) = 1) => x
    invlcx:=%:=inv reduce((leadingCoefficient(x.val))::R,x.modulo)
    invlcx * x

  unitNormal x ==
--    zero?(x) or one?(leadingCoefficient(x.val)) => [1, x, 1]
    zero?(x) or ((leadingCoefficient(x.val)) = 1) => [1, x, 1]
    lcx := reduce((leadingCoefficient(x.val))::R,x.modulo)
    invlcx:=inv lcx
    degree(x.val) = 0 => [lcx, 1, invlcx]
    [lcx, invlcx * x, invlcx]

  elt(x : %,s : R) : R == reduction(elt(x.val,s),x.modulo)

```

---

— EMR.dotabb —

```

"EMR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=EMR"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"EMR" -> "PFECAT"

```

---

## 6.4 domain EXIT Exit

— Exit.input —

```

)set break resume
)sys rm -f Exit.output
)spool Exit.output
)set message test on
)set message auto off
)clear all
--S 1
n := 0
--R
--R
--R (1)  0
--R
--R                                          Type: NonNegativeInteger
--E 1

--S 2
gasp(): Exit ==
  free n
  n := n + 1
  error "Oh no!"
--R
--R Function declaration gasp : () -> Exit has been added to workspace.
--R
--R                                          Type: Void
--E 2

--S 3
half(k) ==
  if odd? k then gasp()
  else k quo 2
--R
--R
--R                                          Type: Void
--E 3

--S 4
half 4
--R
--R Compiling function gasp with type () -> Exit
--R Compiling function half with type PositiveInteger -> Integer
--R
--R (4)  2
--R
--R                                          Type: PositiveInteger
--E 4

--S 5
half 3

```

```

--R
--R
--RDaly Bug
--R   Error signalled from user code in function gasp:
--R       Oh no!
--E 5

--S 6
n
--R
--R
--R   (5)  1
--R
--R                                          Type: NonNegativeInteger
--E 6
)spool
)lisp (bye)

```

---

— Exit.help —

```

=====
Exit examples
=====

```

A function that does not return directly to its caller has Exit as its return type. The operation error is an example of one which does not return to its caller. Instead, it causes a return to top-level.

```
n := 0
```

The function gasp is given return type Exit since it is guaranteed never to return a value to its caller.

```

gasp(): Exit ==
  free n
  n := n + 1
  error "Oh no!"

```

The return type of half is determined by resolving the types of the two branches of the if.

```

half(k) ==
  if odd? k then gasp()
  else k quo 2

```

Because gasp has the return type Exit, the type of if in half is resolved to be Integer.

```
half 4
```

```
half 3
```

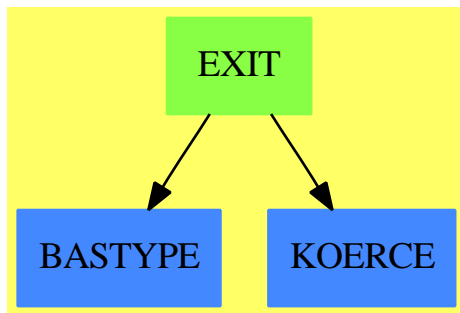
```
n
```

See Also:

o )show Exit

---

### 6.4.1 Exit (EXIT)



**Exports:**

```
coerce hash latex ?=? ?~=?
```

— domain EXIT Exit —

```

)abbrev domain EXIT Exit
++ Author: Stephen M. Watt
++ Date Created: 1986
++ Date Last Updated: May 30, 1991
++ Basic Operations:
++ Related Domains: ErrorFunctions, ResolveLatticeCompletion, Void
++ Also See:
++ AMS Classifications:
++ Keywords: exit, throw, error, non-local return
++ Examples:
++ References:
++ Description:
++ A function which does not return directly to its caller should
++ have Exit as its return type.
++
  
```



```

++ Note that It is convenient to have a formal \spad{coerce} into each type
++ from type Exit. This allows, for example, errors to be raised in
++ one half of a type-balanced \spad{if}.

```

```

Exit: SetCategory == add
      coerce(n:%) == error "Cannot use an Exit value."
      n1 = n2      == error "Cannot use an Exit value."

```

---

— EXIT.dotabb —

```

"EXIT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=EXIT"]
"BASTYPE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=BASTYPE"]
"KOERCE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=KOERCE"]
"EXIT" -> "BASTYPE"
"EXIT" -> "KOERCE"

```

---

## 6.5 domain EXPEXPAN ExponentialExpansion

— ExponentialExpansion.input —

```

)set break resume
)sys rm -f ExponentialExpansion.output
)spool ExponentialExpansion.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ExponentialExpansion
--R ExponentialExpansion(R: Join(OrderedSet,RetractableTo Integer,LinearlyExplicitRingOver I
--R Abbreviation for ExponentialExpansion is EXPEXPAN
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for EXPEXPAN
--R
--R----- Operations -----
--R ?? : (Fraction Integer,%) -> %      ?? : (%,Fraction Integer) -> %
--R ?? : (%,%) -> %                    ?? : (Integer,%) -> %
--R ?? : (PositiveInteger,%) -> %      ??? : (%,Integer) -> %
--R ??? : (%,PositiveInteger) -> %    ?? : (%,%) -> %
--R ?? : (%,%) -> %                    -? : % -> %

```

```

--R ?/? : (% , %) -> %
--R 1 : () -> %
--R ?? : (% , Integer) -> %
--R associates? : (% , %) -> Boolean
--R coerce : % -> %
--R coerce : % -> OutputForm
--R factor : % -> Factored %
--R gcd : (% , %) -> %
--R inv : % -> %
--R lcm : List % -> %
--R numerator : % -> %
--R prime? : % -> Boolean
--R recip : % -> Union(% , "failed")
--R sample : () -> %
--R squareFree : % -> Factored %
--R unit? : % -> Boolean
--R zero? : % -> Boolean
--R ?? : (% , UnivariatePuisseuxSeriesWithExponentialSingularity(R, FE, var, cen)) -> %
--R ?? : (UnivariatePuisseuxSeriesWithExponentialSingularity(R, FE, var, cen), %) -> %
--R ?? : (NonNegativeInteger, %) -> %
--R ??? : (% , NonNegativeInteger) -> %
--R ?/? : (UnivariatePuisseuxSeriesWithExponentialSingularity(R, FE, var, cen), UnivariatePuisseuxSeriesWithExponentialSingularity(R, FE, var, cen)) -> %
--R ?<? : (% , %) -> Boolean if UnivariatePuisseuxSeriesWithExponentialSingularity(R, FE, var, cen) has ORDSE
--R ?<=? : (% , %) -> Boolean if UnivariatePuisseuxSeriesWithExponentialSingularity(R, FE, var, cen) has ORDSE
--R ?>? : (% , %) -> Boolean if UnivariatePuisseuxSeriesWithExponentialSingularity(R, FE, var, cen) has ORDSE
--R ?>=? : (% , %) -> Boolean if UnivariatePuisseuxSeriesWithExponentialSingularity(R, FE, var, cen) has ORDSE
--R D : (% , (UnivariatePuisseuxSeriesWithExponentialSingularity(R, FE, var, cen) -> UnivariatePuisseuxSeriesWithExponentialSingularity(R, FE, var, cen))) -> %
--R D : (% , (UnivariatePuisseuxSeriesWithExponentialSingularity(R, FE, var, cen) -> UnivariatePuisseuxSeriesWithExponentialSingularity(R, FE, var, cen))) -> %
--R D : (% , List Symbol, List NonNegativeInteger) -> % if UnivariatePuisseuxSeriesWithExponentialSingularity(R, FE, var, cen) has ORDSE
--R D : (% , Symbol, NonNegativeInteger) -> % if UnivariatePuisseuxSeriesWithExponentialSingularity(R, FE, var, cen) has ORDSE
--R D : (% , List Symbol) -> % if UnivariatePuisseuxSeriesWithExponentialSingularity(R, FE, var, cen) has PDRING SY
--R D : (% , Symbol) -> % if UnivariatePuisseuxSeriesWithExponentialSingularity(R, FE, var, cen) has PDRING SY
--R D : (% , NonNegativeInteger) -> % if UnivariatePuisseuxSeriesWithExponentialSingularity(R, FE, var, cen) has DIFRING
--R D : % -> % if UnivariatePuisseuxSeriesWithExponentialSingularity(R, FE, var, cen) has DIFRING
--R ?? : (% , NonNegativeInteger) -> %
--R abs : % -> % if UnivariatePuisseuxSeriesWithExponentialSingularity(R, FE, var, cen) has OINTDOM
--R ceiling : % -> UnivariatePuisseuxSeriesWithExponentialSingularity(R, FE, var, cen) if UnivariatePuisseuxSeriesWithExponentialSingularity(R, FE, var, cen) has OINTDOM
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(% , "failed") if $ has CHARNZ and UnivariatePuisseuxSeriesWithExponentialSingularity(R, FE, var, cen) has CHARNZ
--R coerce : UnivariatePuisseuxSeries(FE, var, cen) -> %
--R coerce : Symbol -> % if UnivariatePuisseuxSeriesWithExponentialSingularity(R, FE, var, cen) has RETRACT
--R coerce : UnivariatePuisseuxSeriesWithExponentialSingularity(R, FE, var, cen) -> %
--R conditionP : Matrix % -> Union(Vector % , "failed") if $ has CHARNZ and UnivariatePuisseuxSeriesWithExponentialSingularity(R, FE, var, cen) has CHARNZ
--R convert : % -> DoubleFloat if UnivariatePuisseuxSeriesWithExponentialSingularity(R, FE, var, cen) has REAL
--R convert : % -> Float if UnivariatePuisseuxSeriesWithExponentialSingularity(R, FE, var, cen) has REAL
--R convert : % -> InputForm if UnivariatePuisseuxSeriesWithExponentialSingularity(R, FE, var, cen) has KONV
--R convert : % -> Pattern Float if UnivariatePuisseuxSeriesWithExponentialSingularity(R, FE, var, cen) has KONV
--R convert : % -> Pattern Integer if UnivariatePuisseuxSeriesWithExponentialSingularity(R, FE, var, cen) has KONV
--R denom : % -> UnivariatePuisseuxSeriesWithExponentialSingularity(R, FE, var, cen)
--R differentiate : (% , (UnivariatePuisseuxSeriesWithExponentialSingularity(R, FE, var, cen) -> UnivariatePuisseuxSeriesWithExponentialSingularity(R, FE, var, cen))) -> %

```

```

--R differentiate : (%,(UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen) -> UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen)) -> % if UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen)
--R differentiate : (%,(List Symbol,List NonNegativeInteger) -> % if UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen)
--R differentiate : (%,(Symbol,NonNegativeInteger) -> % if UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen)
--R differentiate : (%,(List Symbol) -> % if UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen)
--R differentiate : (%,(Symbol) -> % if UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen)
--R differentiate : (%,(NonNegativeInteger) -> % if UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen)
--R differentiate : % -> % if UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen)
--R divide : (%,(%) -> Record(quotient: %,remainder: %)
--R ?.? : (%,(UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen)) -> % if UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen)
--R euclideanSize : % -> NonNegativeInteger
--R eval : (%,(Symbol,UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen)) -> % if UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen)
--R eval : (%,(List Symbol,List UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen)) -> % if UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen)
--R eval : (%,(List Equation UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen)) -> % if UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen)
--R eval : (%,(Equation UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen)) -> % if UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen)
--R eval : (%,(UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen),UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen)) -> % if UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen)
--R eval : (%,(List UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen),List UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen)) -> % if UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen)
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,(%) -> Union(%,"failed")
--R extendedEuclidean : (%,(%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,(%) -> Record(coef1: %,coef2: %,generator: %)
--R factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial %
--R factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial %
--R floor : % -> UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen) if UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen)
--R fractionPart : % -> % if UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen)
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolynomial %
--R init : () -> % if UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen) has ST
--R limitPlus : % -> Union(OrderedCompletion FE,"failed")
--R map : ((UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen) -> UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen)) -> UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen)
--R max : (%,(%) -> % if UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen) has ST
--R min : (%,(%) -> % if UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen) has ST
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R negative? : % -> Boolean if UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen)
--R nextItem : % -> Union(%,"failed") if UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen)
--R numer : % -> UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen)
--R patternMatch : (%,(Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchResult(Float,%))
--R patternMatch : (%,(Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMatchResult(Integer,%))
--R positive? : % -> Boolean if UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen)
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R random : () -> % if UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen) has ST
--R reducedSystem : Matrix % -> Matrix UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen)
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen),vec: Vector %)
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) if UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen)
--R reducedSystem : Matrix % -> Matrix Integer if UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen)
--R retract : % -> UnivariatePuisseuxSeries(FE,var,cen)
--R retract : % -> Integer if UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen)
--R retract : % -> Fraction Integer if UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen)
--R retract : % -> Symbol if UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen)
--R retract : % -> UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen)
--R retractIfCan : % -> Union(UnivariatePuisseuxSeries(FE,var,cen),"failed")
--R retractIfCan : % -> Union(Integer,"failed") if UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen)

```

```

--R retractIfCan : % -> Union(Fraction Integer,"failed") if UnivariatePuisseuxSeriesWithExponentialSingul
--R retractIfCan : % -> Union(Symbol,"failed") if UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE
--R retractIfCan : % -> Union(UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen),"failed")
--R sign : % -> Integer if UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen) has OINTDOM
--R solveLinearPolynomialEquation : (List SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) ->
--R squareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % if Univ
--R subtractIfCan : (%,%) -> Union(%, "failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R wholePart : % -> UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen) if UnivariatePuisseu
--R
--E 1

)spool
)lisp (bye)

```

---

### — ExponentialExpansion.help —

```

=====
ExponentialExpansion examples
=====

```

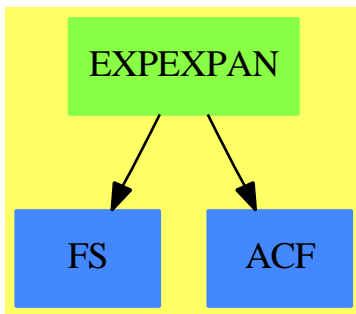
```

See Also:
o )show ExponentialExpansion

```

---

## 6.5.1 ExponentialExpansion (EXPEXPAN)



See

- ⇒ “ExponentialOfUnivariatePuisseuxSeries” (EXPUPXS) 6.7.1 on page 707
- ⇒ “UnivariatePuisseuxSeriesWithExponentialSingularity” (UPXSSING) 22.7.1 on page 2809

**Exports:**

|                               |                    |                |
|-------------------------------|--------------------|----------------|
| 0                             | 1                  | associates?    |
| abs                           | ceiling            | characteristic |
| charthRoot                    | coerce             | conditionP     |
| convert                       | D                  | denom          |
| denominator                   | differentiate      | divide         |
| euclideanSize                 | expressIdealMember | eval           |
| exquo                         | extendedEuclidean  | factor         |
| factorSquareFreePolynomial    | factorPolynomial   | floor          |
| fractionPart                  | gcd                | gcdPolynomial  |
| hash                          | init               | inv            |
| latex                         | lcm                | limitPlus      |
| map                           | max                | min            |
| multiEuclidean                | negative?          | nextItem       |
| numer                         | numerator          | one?           |
| patternMatch                  | positive?          | prime?         |
| principalIdeal                | random             | recip          |
| reducedSystem                 | retract            | retractIfCan   |
| sample                        | sign               | sizeLess?      |
| solveLinearPolynomialEquation | squareFree         | squareFreePart |
| squareFreePolynomial          | subtractIfCan      | unit?          |
| unitCanonical                 | unitNormal         | wholePart      |
| zero?                         | ?*?                | ?**?           |
| ?+?                           | ?-?                | -?             |
| ?/?                           | ?=?                | ?^?            |
| ?~=?                          | ?<?                | ?<=?           |
| ?>?                           | ?>=?               | ?..?           |
| ?quo?                         | ?rem?              |                |

## — domain EXPEXPAN ExponentialExpansion —

```

)abbrev domain EXPEXPAN ExponentialExpansion
++ Author: Clifton J. Williamson
++ Date Created: 13 August 1992
++ Date Last Updated: 27 August 1992
++ Basic Operations:
++ Related Domains: UnivariatePuisseuxSeries(FE,var,cen),
++                  ExponentialOfUnivariatePuisseuxSeries(FE,var,cen)
++ Also See:
++ AMS Classifications:
++ Keywords: limit, functional expression, power series
++ Examples:
++ References:
++ Description:
++ UnivariatePuisseuxSeriesWithExponentialSingularity is a domain used to
++ represent essential singularities of functions. Objects in this domain
++ are quotients of sums, where each term in the sum is a univariate Puisseux
++ series times the exponential of a univariate Puisseux series.

```

```

ExponentialExpansion(R,FE,var,cen): Exports == Implementation where
  R      : Join(OrderedSet,RetractableTo Integer,
               LinearlyExplicitRingOver Integer,GcdDomain)
  FE     : Join(AlgebraicallyClosedField,TranscendentalFunctionCategory,
               FunctionSpace R)
  var    : Symbol
  cen    : FE
  RN      ==> Fraction Integer
  UPXS    ==> UnivariatePuisseuxSeries(FE,var,cen)
  EXPUPXS ==> ExponentialOfUnivariatePuisseuxSeries(FE,var,cen)
  UPXSING ==> UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen)
  OFE     ==> OrderedCompletion FE
  Result  ==> Union(OFE,"failed")
  PxRec   ==> Record(k: Fraction Integer,c:FE)
  Term    ==> Record(%coef:UPXS,%expon:EXPUPXS,%expTerms:List PxRec)
  TypedTerm ==> Record(%term:Term,%type:String)
  SIGNEF  ==> ElementaryFunctionSign(R,FE)

Exports ==> Join(QuotientFieldCategory UPXSING,RetractableTo UPXS) with
  limitPlus : % -> Union(OFE,"failed")
  ++ limitPlus(f(var)) returns \spad{limit(var -> a+,f(var))}.
  coerce: UPXS -> %
  ++ coerce(f) converts a \spadtype{UnivariatePuisseuxSeries} to
  ++ an \spadtype{ExponentialExpansion}.

Implementation ==> Fraction(UPXSING) add
  coeff : Term -> UPXS
  exponent : Term -> EXPUPXS
  upxssingIfCan : % -> Union(UPXSING,"failed")
  seriesQuotientLimit: (UPXS,UPXS) -> Union(OFE,"failed")
  seriesQuotientInfinity: (UPXS,UPXS) -> Union(OFE,"failed")

Rep := Fraction UPXSING

ZEROCOUNT : RN := 1000/1

coeff term == term.%coef
exponent term == term.%expon

--!! why is this necessary?
--!! code can run forever in retractIfCan if original assignment
--!! for 'ff' is used
upxssingIfCan f ==
--   one? denom f => numer f
   (denom f = 1) => numer f
   "failed"

retractIfCan(f:%):Union(UPXS,"failed") ==
  --ff := (retractIfCan$Rep)(f)@Union(UPXSING,"failed")

```

```

--ff case "failed" => "failed"
(ff := upxssingIfCan f) case "failed" => "failed"
(fff := retractIfCan(ff::UPXSSING)@Union(UPXS,"failed")) case "failed" =>
  "failed"
fff :: UPXS

f:UPXSSING / g:UPXSSING ==
  (rec := recip g) case "failed" => f /$Rep g
  f * (rec :: UPXSSING) :: %

f:% / g:% ==
  (rec := recip numer g) case "failed" => f /$Rep g
  (rec :: UPXSSING) * (denom g) * f

coerce(f:UPXS) == f :: UPXSSING :: %

seriesQuotientLimit(num,den) ==
  -- limit of the quotient of two series
  series := num / den
  (ord := order(series,1)) > 0 => 0
  coef := coefficient(series,ord)
  member?(var,variables coef) => "failed"
  ord = 0 => coef :: OFE
  (sig := sign(coef)$SIGNEF) case "failed" => return "failed"
  (sig :: Integer) = 1 => plusInfinity()
  minusInfinity()

seriesQuotientInfinity(num,den) ==
  -- infinite limit: plus or minus?
  -- look at leading coefficients of series to tell
  (numOrd := order(num,ZEROCOUNT)) = ZEROCOUNT => "failed"
  (denOrd := order(den,ZEROCOUNT)) = ZEROCOUNT => "failed"
  cc := coefficient(num,numOrd)/coefficient(den,denOrd)
  member?(var,variables cc) => "failed"
  (sig := sign(cc)$SIGNEF) case "failed" => return "failed"
  (sig :: Integer) = 1 => plusInfinity()
  minusInfinity()

limitPlus f ==
  zero? f => 0
  (den := denom f) = 1 => limitPlus numer f
  (numTerm := dominantTerm(num := numer f)) case "failed" => "failed"
  numType := (numTerm := numTerm :: TypedTerm).%type
  (denomTerm := dominantTerm den) case "failed" => "failed"
  denType := (denTerm := denomTerm :: TypedTerm).%type
  numExpon := exponent numTerm.%term; denExpon := exponent denTerm.%term
  numCoef := coeff numTerm.%term; denCoef := coeff denTerm.%term
  -- numerator tends to zero exponentially
  (numType = "zero") =>
    -- denominator tends to zero exponentially

```

```

(denType = "zero") =>
  (exponDiff := numExpon - denExpon) = 0 =>
    seriesQuotientLimit(numCoef,denCoef)
  expCoef := coefficient(exponDiff,order exponDiff)
  (sig := sign(expCoef)$SIGNEF) case "failed" => return "failed"
  (sig :: Integer) = -1 => 0
  seriesQuotientInfinity(numCoef,denCoef)
0 -- otherwise limit is zero
-- numerator is a Puiseux series
(numType = "series") =>
  -- denominator tends to zero exponentially
  (denType = "zero") =>
    seriesQuotientInfinity(numCoef,denCoef)
  -- denominator is a series
  (denType = "series") => seriesQuotientLimit(numCoef,denCoef)
0
-- remaining case: numerator tends to infinity exponentially
-- denominator tends to infinity exponentially
(denType = "infinity") =>
  (exponDiff := numExpon - denExpon) = 0 =>
    seriesQuotientLimit(numCoef,denCoef)
  expCoef := coefficient(exponDiff,order exponDiff)
  (sig := sign(expCoef)$SIGNEF) case "failed" => return "failed"
  (sig :: Integer) = -1 => 0
  seriesQuotientInfinity(numCoef,denCoef)
-- denominator tends to zero exponentially or is a series
seriesQuotientInfinity(numCoef,denCoef)

```

---

— EXPEXPAN.dotabb —

```

"EXPEXPAN" [color="#88FF44",href="bookvol10.3.pdf#nameddest=EXPEXPAN"]
"FS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FS"]
"ACF" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ACF"]
"EXPEXPAN" -> "ACF"
"EXPEXPAN" -> "FS"

```

---

## 6.6 domain EXPR Expression

— Expression.input —

```
)set break resume
```



```

)sys rm -f Expression.output
)spool Expression.output
)set message test on
)set message auto off
)clear all
--S 1 of 23
sin(x) + 3*cos(x)**2
--R
--R
--R
--R      2
--R      (1) sin(x) + 3cos(x)
--R
--R                                          Type: Expression Integer
--E 1

--S 2 of 23
tan(x) - 3.45*x
--R
--R
--R      (2) tan(x) - 3.45 x
--R
--R                                          Type: Expression Float
--E 2

--S 3 of 23
(tan sqrt 7 - sin sqrt 11)**2 / (4 - cos(x - y))
--R
--R
--R      +-+ 2      +---+      +-+      +---+ 2
--R      - tan(\|7 ) + 2sin(\|11 )tan(\|7 ) - sin(\|11 )
--R      (3) -----
--R                                  cos(y - x) - 4
--R
--R                                          Type: Expression Integer
--E 3

--S 4 of 23
log(exp x)@Expression(Integer)
--R
--R
--R      (4) x
--R
--R                                          Type: Expression Integer
--E 4

--S 5 of 23
log(exp x)@Expression(Complex Integer)
--R
--R
--R      x
--R      (5) log(%e )
--R
--R                                          Type: Expression Complex Integer
--E 5

```

```

--S 6 of 23
sqrt 3 + sqrt(2 + sqrt(-5))
--R
--R
--R      +-----+
--R      | +---+      +-+
--R (6)  \|\|- 5  + 2  + \3
--R
--R                                          Type: AlgebraicNumber
--E 6

--S 7 of 23
% :: Expression Integer
--R
--R
--R      +-----+
--R      | +---+      +-+
--R (7)  \|\|- 5  + 2  + \3
--R
--R                                          Type: Expression Integer
--E 7

--S 8 of 23
height mainKernel sin(x + 4)
--R
--R
--R (8)  2
--R
--R                                          Type: PositiveInteger
--E 8

--S 9 of 23
e := (sin(x) - 4)**2 / ( 1 - 2*y*sqrt(- y) )
--R
--R
--R      2
--R      - sin(x)  + 8sin(x) - 16
--R (9)  -----
--R      +---+
--R      2y\|- y  - 1
--R
--R                                          Type: Expression Integer
--E 9

--S 10 of 23
numer e
--R
--R
--R      2
--R (10) - sin(x)  + 8sin(x) - 16
--R      Type: SparseMultivariatePolynomial(Integer,Kernel Expression Integer)
--E 10

--S 11 of 23

```

```

denom e
--R
--R
--R      +----+
--R      (11) 2y\|- y - 1
--R      Type: SparseMultivariatePolynomial(Integer,Kernel Expression Integer)
--E 11

--S 12 of 23
D(e, x)
--R
--R
--R      +----+
--R      (4y cos(x)sin(x) - 16y cos(x))\|- y - 2cos(x)sin(x) + 8cos(x)
--R      (12) -----
--R      +----+      3
--R      4y\|- y + 4y - 1
--R
--R      Type: Expression Integer
--E 12

--S 13 of 23
D(e, [x, y], [1, 2])
--R
--R
--R      (13)
--R      7      4      7      4      +----+
--R      ((- 2304y + 960y )cos(x)sin(x) + (9216y - 3840y )cos(x))\|- y
--R      +
--R      9      6      3
--R      (- 960y + 2160y - 180y - 3)cos(x)sin(x)
--R      +
--R      9      6      3
--R      (3840y - 8640y + 720y + 12)cos(x)
--R      /
--R      12      9      6      3      +----+      11      8      5
--R      (256y - 1792y + 1120y - 112y + 1)\|- y - 1024y + 1792y - 448y
--R      +
--R      2
--R      16y
--R
--R      Type: Expression Integer
--E 13

--S 14 of 23
complexNumeric(cos(2 - 3%i))
--R
--R
--R      (14) - 4.1896256909 688072301 + 9.1092278937 55336598 %i
--R
--R      Type: Complex Float
--E 14

```

[illegible]

--E 20

--S 21 of 23

cos(%pi / 4)

--R

--R

--R           +++

--R           \|2

--R   (21) ----

--R           2

--R

Type: Expression Integer

--E 21

--S 22 of 23

tan(x)\*\*6 + 3\*tan(x)\*\*4 + 3\*tan(x)\*\*2 + 1

--R

--R

--R           6           4           2

--R   (22) tan(x) + 3tan(x) + 3tan(x) + 1

--R

Type: Expression Integer

--E 22

--S 23 of 23

simplify %

--R

--R

--R           1

--R   (23) -----

--R           6

--R       cos(x)

--R

Type: Expression Integer

--E 23

)spool

)lisp (bye)

-----

### — Expression.help —

=====  
Expression examples  
=====

Expression is a constructor that creates domains whose objects  
can have very general symbolic forms. Here are some examples:

This is an object of type Expression Integer.

sin(x) + 3\*cos(x)\*\*2

This is an object of type Expression Float.

```
tan(x) - 3.45*x
```

This object contains symbolic function applications, sums, products, square roots, and a quotient.

```
(tan sqrt 7 - sin sqrt 11)**2 / (4 - cos(x - y))
```

As you can see, Expression actually takes an argument domain. The coefficients of the terms within the expression belong to the argument domain. Integer and Float, along with Complex Integer and Complex Float are the most common coefficient domains.

The choice of whether to use a Complex coefficient domain or not is important since Axiom can perform some simplifications on real-valued objects

```
log(exp x)@Expression(Integer)
```

... which are not valid on complex ones.

```
log(exp x)@Expression(Complex Integer)
```

Many potential coefficient domains, such as AlgebraicNumber, are not usually used because Expression can subsume them.

```
sqrt 3 + sqrt(2 + sqrt(-5))
```

```
% :: Expression Integer
```

Note that we sometimes talk about "an object of type Expression." This is not really correct because we should say, for example, "an object of type Expression Integer" or "an object of type Expression Float." By a similar abuse of language, when we refer to an "expression" in this section we will mean an object of type Expression R for some domain R.

The Axiom documentation contains many examples of the use of Expression. For the rest of this section, we'll give you some pointers to those examples plus give you some idea of how to manipulate expressions.

It is important for you to know that Expression creates domains that have category Field. Thus you can invert any non-zero expression and you shouldn't expect an operation like factor to give you much information. You can imagine expressions as being represented as quotients of "multivariate" polynomials where the "variables" are kernels. A kernel can either be a symbol such as x or a symbolic

function application like `sin(x + 4)`. The second example is actually a nested kernel since the argument to `sin` contains the kernel `x`.

```
height mainKernel sin(x + 4)
```

Actually, the argument to `sin` is an expression, and so the structure of `Expression` is recursive. See `Kernel` which demonstrates how to extract the kernels in an expression.

Use the HyperDoc Browse facility to see what operations are applicable to expression. At the time of this writing, there were 262 operations with 147 distinct name in `Expression Integer`. For example, `numer` and `denom` extract the numerator and denominator of an expression.

```
e := (sin(x) - 4)**2 / ( 1 - 2*y*sqrt(- y) )

numer e

denom e
```

Use `D` to compute partial derivatives.

```
D(e, x)

D(e, [x, y], [1, 2])
```

When an expression involves no ‘symbol kernels’ (for example, `x`), it may be possible to numerically evaluate the expression.

If you suspect the evaluation will create a complex number, use `complexNumeric`.

```
complexNumeric(cos(2 - 3*i))
```

If you know it will be real, use `numeric`.

```
numeric(tan 3.8)
```

The `numeric` operation will display an error message if the evaluation yields a value with a non-zero imaginary part. Both of these operations have an optional second argument `n` which specifies that the accuracy of the approximation be up to `n` decimal places.

When an expression involves no ‘symbolic application’ kernels, it may be possible to convert it a polynomial or rational function in the variables that are present.

```
e2 := cos(x**2 - y + 3)

e3 := asin(e2) - %pi/2
```

```
e3 :: Polynomial Integer
```

This also works for the polynomial types where specific variables and their ordering are given.

```
e3 :: DMP([x, y], Integer)
```

Finally, a certain amount of simplification takes place as expressions are constructed.

```
sin %pi
```

```
cos(%pi / 4)
```

For simplifications that involve multiple terms of the expression, use `simplify`.

```
tan(x)**6 + 3*tan(x)**4 + 3*tan(x)**2 + 1
```

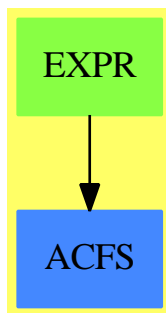
```
simplify %
```

See Also:

- o `)show Kernel`
- o `)show Expression`

---

### 6.6.1 Expression (EXPR)



See

⇒ “Pi” (HACKPI) 17.16.1 on page 1937



**Exports:**

|                    |                      |                   |                |
|--------------------|----------------------|-------------------|----------------|
| 0                  | 1                    | abs               | acos           |
| acosh              | acot                 | acoth             | acsc           |
| acsch              | airyAi               | airyBi            | applyQuote     |
| asec               | asech                | asin              | asinh          |
| associates?        | atan                 | atanh             | belong?        |
| besselI            | besselJ              | besselK           | besselY        |
| Beta               | binomial             | box               | characteristic |
| charthRoot         | Ci                   | coerce            | commutator     |
| conjugate          | convert              | cos               | cosh           |
| cot                | coth                 | csc               | csch           |
| D                  | definingPolynomial   | denom             | denominator    |
| differentiate      | digamma              | dilog             | distribute     |
| divide             | Ei                   | elt               | erf            |
| euclideanSize      | eval                 | even?             | exp            |
| expressIdealMember | exquo                | extendedEuclidean | factor         |
| factorPolynomial   | factorial            | factorials        | freeOf?        |
| Gamma              | gcd                  | gcdPolynomial     | ground         |
| ground?            | hash                 | height            | integral       |
| inv                | is?                  | isExpt            | isMult         |
| isPlus             | isPower              | isTimes           | kernel         |
| kernels            | latex                | lcm               | li             |
| log                | mainKernel           | map               | max            |
| min                | minPoly              | multiEuclidean    | nthRoot        |
| number?            | numer                | numerator         | odd?           |
| one?               | operator             | operators         | paren          |
| patternMatch       | permutation          | pi                | polygamma      |
| prime?             | principalIdeal       | product           | recip          |
| reduce             | reducedSystem        | retract           | retractIfCan   |
| rootOf             | rootsOf              | sample            | sec            |
| sech               | Si                   | simplifyPower     | sin            |
| sinh               | sizeLess?            | sqrt              | squareFree     |
| squareFreePart     | squareFreePolynomial | subst             | subtractIfCan  |
| summation          | tan                  | tanh              | tower          |
| unit?              | unitCanonical        | unitNormal        | univariate     |
| variables          | zero?                | zeroOf            | zerosOf        |
| ?<?                | ?<=?                 | ?=?               | ?>?            |
| ?>=?               | ?~=?                 | ?*?               | ?**?           |
| ?+?                | -?                   | ?-?               | ?/?            |
| ?^?                | ?quo?                | ?rem?             |                |

— domain EXPR Expression —

```
)abbrev domain EXPR Expression
++ Author: Manuel Bronstein
++ Date Created: 19 July 1988
```

```

++ Date Last Updated: October 1993 (P.Gianni), February 1995 (MB)
++ Keywords: operator, kernel, function.
++ Description:
++ Top-level mathematical expressions involving symbolic functions.

```

```

Expression(R:OrderedSet): Exports == Implementation where

```

```

Q  ==> Fraction Integer
K  ==> Kernel %
MP ==> SparseMultivariatePolynomial(R, K)
AF ==> AlgebraicFunction(R, %)
EF ==> ElementaryFunction(R, %)
CF ==> CombinatorialFunction(R, %)
LF ==> LiouvillianFunction(R, %)
AN ==> AlgebraicNumber
KAN ==> Kernel AN
FSF ==> FunctionalSpecialFunction(R, %)
ESD ==> ExpressionSpace_&(&())
FSD ==> FunctionSpace_&(&(), R)
SYMBOL ==> "%symbol"
ALGOP  ==> "%alg"
POWER  ==> "%power"::Symbol
SUP    ==> SparseUnivariatePolynomial

```

```

Exports ==> FunctionSpace R with
  if R has IntegralDomain then
    AlgebraicallyClosedFunctionSpace R
    TranscendentalFunctionCategory
    CombinatorialOpsCategory
    LiouvillianFunctionCategory
    SpecialFunctionCategory
    reduce: % -> %
      ++ reduce(f) simplifies all the unreduced algebraic quantities
      ++ present in f by applying their defining relations.
    number?: % -> Boolean
      ++ number?(f) tests if f is rational
    simplifyPower: (%,Integer) -> %
      ++ simplifyPower?(f,n) is not documented
  if R has GcdDomain then
    factorPolynomial : SUP % -> Factored SUP %
      ++ factorPolynomial(p) is not documented
    squareFreePolynomial: SUP % -> Factored SUP %
      ++ squareFreePolynomial(p) is not documented
  if R has RetractableTo Integer then RetractableTo AN

```

```

Implementation ==> add
  import KernelFunctions2(R, %)

  retNotUnit      : % -> R
  retNotUnitIfCan: % -> Union(R, "failed")

```

```

belong? op == true

retNotUnit x ==
  (u := constantIfCan(k := retract(x)@K)) case R => u::R
  error "Not retractable"

retNotUnitIfCan x ==
  (r := retractIfCan(x)@Union(K,"failed")) case "failed" => "failed"
  constantIfCan(r::K)

if R has IntegralDomain then
  reduc : (% , List Kernel %) -> %
  commonk : (% , %) -> List K
  commonk0 : (List K, List K) -> List K
  topnat : % -> %
  algkernels: List K -> List K
  evl : (MP, K, SparseUnivariatePolynomial %) -> Fraction MP
  evl0 : (MP, K) -> SparseUnivariatePolynomial Fraction MP

Rep := Fraction MP
0 == 0$Rep
1 == 1$Rep
-- one? x == one?(x)$Rep
one? x == (x = 1)$Rep
zero? x == zero?(x)$Rep
- x:% == -$Rep x
n:Integer * x:% == n *$Rep x
coerce(n:Integer) == coerce(n)$Rep@Rep::%
x:% * y:% == reduc(x *$Rep y, commonk(x, y))
x:% + y:% == reduc(x +$Rep y, commonk(x, y))
(x:% - y:%):% == reduc(x -$Rep y, commonk(x, y))
x:% / y:% == reduc(x /$Rep y, commonk(x, y))

number?(x:%):Boolean ==
  if R has RetractableTo(Integer) then
    ground?(x) or ((retractIfCan(x)@Union(Q,"failed")) case Q)
  else
    ground?(x)

simplifyPower(x:%,n:Integer):% ==
  k : List K := kernels x
  is?(x,POWER) =>
    -- Look for a power of a number in case we can do a simplification
    args : List % := argument first k
    not(#args = 2) => error "Too many arguments to **"
    number?(args.1) =>
      reduc((args.1) **$Rep n, algkernels kernels (args.1))**(args.2)
      (first args)**(n*second(args))
    reduc(x **$Rep n, algkernels k)

```

```

x:% ** n:NonNegativeInteger ==
  n = 0 => 1$%
  n = 1 => x
  simplifyPower(numerator x,n pretend Integer) /
    simplifyPower(denominator x,n pretend Integer)

x:% ** n:Integer ==
  n = 0 => 1$%
  n = 1 => x
  n = -1 => 1/x
  simplifyPower(numerator x,n) /
    simplifyPower(denominator x,n)

x:% ** n:PositiveInteger ==
  n = 1 => x
  simplifyPower(numerator x,n pretend Integer) /
    simplifyPower(denominator x,n pretend Integer)

x:% < y:%      == x <$Rep y
x:% = y:%      == x =$Rep y
numer x        == numer(x)$Rep
denom x        == denom(x)$Rep
coerce(p:MP):% == coerce(p)$Rep
reduce x       == reduc(x, algkernels kernels x)
commonk(x, y)  == commonk0(algkernels kernels x, algkernels kernels y)
algkernels l   == select_!(x +-> has?(operator x, ALGOP), l)
toprat f == ratDenom(f,algkernels kernels f)$AlgebraicManipulations(R, %)

x:MP / y:MP ==
  reduc(x /$Rep y,commonk0(algkernels variables x,algkernels variables y))

-- since we use the reduction from FRAC SMP which asssumes that the
-- variables are independent, we must remove algebraic from the denominators
reducedSystem(m:Matrix %):Matrix(R) ==
  mm:Matrix(MP) := reducedSystem(map(toprat, m))$Rep
  reducedSystem(mm)$MP

-- since we use the reduction from FRAC SMP which asssumes that the
-- variables are independent, we must remove algebraic from the denominators
reducedSystem(m:Matrix %, v:Vector %):
Record(mat:Matrix R, vec:Vector R) ==
  r:Record(mat:Matrix MP, vec:Vector MP) :=
    reducedSystem(map(toprat, m), map(toprat, v))$Rep
  reducedSystem(r.mat, r.vec)$MP

-- The result MUST be left sorted deepest first  MB 3/90
commonk0(x, y) ==
  ans := empty()$List(K)
  for k in reverse_! x repeat if member?(k, y) then ans := concat(k, ans)
  ans

```

```

rootOf(x: SparseUnivariatePolynomial %, v: Symbol) == rootOf(x,v)$AF
pi() == pi()$EF
exp x == exp(x)$EF
log x == log(x)$EF
sin x == sin(x)$EF
cos x == cos(x)$EF
tan x == tan(x)$EF
cot x == cot(x)$EF
sec x == sec(x)$EF
csc x == csc(x)$EF
asin x == asin(x)$EF
acos x == acos(x)$EF
atan x == atan(x)$EF
acot x == acot(x)$EF
asec x == asec(x)$EF
acsc x == acsc(x)$EF
sinh x == sinh(x)$EF
cosh x == cosh(x)$EF
tanh x == tanh(x)$EF
coth x == coth(x)$EF
sech x == sech(x)$EF
csch x == csch(x)$EF
asinh x == asinh(x)$EF
acosh x == acosh(x)$EF
atanh x == atanh(x)$EF
acoth x == acoth(x)$EF
asech x == asech(x)$EF
acsch x == acsch(x)$EF

abs x == abs(x)$FSF
Gamma x == Gamma(x)$FSF
Gamma(a, x) == Gamma(a, x)$FSF
Beta(x,y) == Beta(x,y)$FSF
digamma x == digamma(x)$FSF
polygamma(k,x) == polygamma(k,x)$FSF
besselJ(v,x) == besselJ(v,x)$FSF
besselY(v,x) == besselY(v,x)$FSF
besselI(v,x) == besselI(v,x)$FSF
besselK(v,x) == besselK(v,x)$FSF
airyAi x == airyAi(x)$FSF
airyBi x == airyBi(x)$FSF

x:% ** y:% == x **$CF y
factorial x == factorial(x)$CF
binomial(n, m) == binomial(n, m)$CF
permutation(n, m) == permutation(n, m)$CF
factorials x == factorials(x)$CF
factorials(x, n) == factorials(x, n)$CF
summation(x:%, n: Symbol) == summation(x, n)$CF

```

```

summation(x:%, s:SegmentBinding %) == summation(x, s)$CF
product(x:%, n:Symbol) == product(x, n)$CF
product(x:%, s:SegmentBinding %) == product(x, s)$CF

erf x == erf(x)$LF
Ei x == Ei(x)$LF
Si x == Si(x)$LF
Ci x == Ci(x)$LF
li x == li(x)$LF
dilog x == dilog(x)$LF
fresnelS x == fresnelS(x)$LF
fresnelC x == fresnelC(x)$LF
integral(x:%, n:Symbol) == integral(x, n)$LF
integral(x:%, s:SegmentBinding %) == integral(x, s)$LF

operator op ==
  belong?(op)$AF => operator(op)$AF
  belong?(op)$EF => operator(op)$EF
  belong?(op)$CF => operator(op)$CF
  belong?(op)$LF => operator(op)$LF
  belong?(op)$FSF => operator(op)$FSF
  belong?(op)$FSD => operator(op)$FSD
  belong?(op)$ESD => operator(op)$ESD
  nullary? op and has?(op, SYMBOL) => operator(kernel(name op)$K)
  (n := arity op) case "failed" => operator name op
  operator(name op, n::NonNegativeInteger)

reduc(x, l) ==
  for k in l repeat
    p := minPoly k
    x := evl(numer x, k, p) /$Rep evl(denom x, k, p)
  x

evl0(p, k) ==
  numer univariate(p::Fraction(MP),
    k)$PolynomialCategoryQuotientFunctions(IndexedExponents K,
      K,R,MP,Fraction MP)

-- uses some operations from Rep instead of % in order not to
-- reduce recursively during those operations.
evl(p, k, m) ==
  degree(p, k) < degree m => p::Fraction(MP)
  (((evl0(p, k) pretend SparseUnivariatePolynomial($)) rem m)
    pretend SparseUnivariatePolynomial Fraction MP) (k::MP::Fraction(MP))

if R has GcdDomain then
  noalg?: SUP % -> Boolean

noalg? p ==
  while p ^= 0 repeat

```

```

    not empty? algkernels kernels leadingCoefficient p => return false
    p := reductum p
    true

gcdPolynomial(p:SUP %, q:SUP %) ==
  noalg? p and noalg? q => gcdPolynomial(p, q)$Rep
  gcdPolynomial(p, q)$GcdDomain_&(&%)

factorPolynomial(x:SUP %) : Factored SUP % ==
  uf:= factor(x pretend SUP(Rep))$SupFractionFactorizer(
                                     IndexedExponents K,K,R,MP)
  uf pretend Factored SUP %

squareFreePolynomial(x:SUP %) : Factored SUP % ==
  uf:= squareFree(x pretend SUP(Rep))$SupFractionFactorizer(
                                     IndexedExponents K,K,R,MP)
  uf pretend Factored SUP %

if R is AN then
  -- this is to force the coercion R -> EXPR R to be used
  -- instead of the coercion AN -> EXPR R which loops.
  -- simpler looking code will fail! MB 10/91
  coerce(x:AN):% == (monomial(x, 0$IndexedExponents(K))$MP)::%

if (R has RetractableTo Integer) then
  x:% ** r:Q == x **$AF r
  minPoly k == minPoly(k)$AF
  definingPolynomial x == definingPolynomial(x)$AF
  retract(x:~):Q == retract(x)$Rep
  retractIfCan(x:~):Union(Q, "failed") == retractIfCan(x)$Rep

if not(R is AN) then
  k2expr : KAN -> %
  smp2expr: SparseMultivariatePolynomial(Integer, KAN) -> %
  R2AN : R -> Union(AN, "failed")
  k2an : K -> Union(AN, "failed")
  smp2an : MP -> Union(AN, "failed")

  coerce(x:AN):% == smp2expr( Numer x ) / smp2expr( Denom x )
  k2expr k == map(x+>x:~%, k)$ExpressionSpaceFunctions2(AN, %)

  smp2expr p ==
    map(k2expr,x+>x:~%,p)_
    $PolynomialCategoryLifting(IndexedExponents KAN,
                               KAN, Integer, SparseMultivariatePolynomial(Integer, KAN), %)

  retractIfCan(x:~):Union(AN, "failed") ==
    ((n:= smp2an Numer x) case AN) and ((d:= smp2an Denom x) case AN)
    => (n::AN) / (d::AN)

```

```

"failed"

R2AN r ==
  (u := retractIfCan(r::%)@Union(Q, "failed")) case Q => u::Q::AN
  "failed"

k2an k ==
  not(belong?(op := operator k)$AN) => "failed"
  arg:List(AN) := empty()
  for x in argument k repeat
    if (a := retractIfCan(x)@Union(AN, "failed")) case "failed" then
      return "failed"
    else arg := concat(a::AN, arg)
  (operator(op)$AN) reverse_!(arg)

smp2an p ==
  (x1 := mainVariable p) case "failed" => R2AN leadingCoefficient p
  up := univariate(p, k := x1::K)
  (t := k2an k) case "failed" => "failed"
  ans:AN := 0
  while not ground? up repeat
    (c:=smp2an leadingCoefficient up) case "failed" => return "failed"
    ans := ans + (c::AN) * (t::AN) ** (degree up)
    up := reductum up
  (c := smp2an leadingCoefficient up) case "failed" => "failed"
  ans + c::AN

if R has ConvertibleTo InputForm then
  convert(x::%):InputForm == convert(x)$Rep
  import MakeUnaryCompiledFunction(%, %, %)
  eval(f::%, op: BasicOperator, g::%, x:Symbol)::% ==
    eval(f,[op],[g],x)
  eval(f::%, ls:List BasicOperator, lg:List %, x:Symbol) ==
    -- handle subscripted symbols by renaming -> eval -> renaming back
    llsym:List List Symbol:=[variables g for g in lg]
    lsym:List Symbol:= removeDuplicates concat llsym
    lsd:List Symbol:=select (scripted?,lsym)
    empty? lsd=> eval(f,ls,[compiledFunction(g, x) for g in lg])
    ns:List Symbol:=[new()$Symbol for i in lsd]
    lforwardSubs:List Equation % := [(i::%)= (j::%) for i in lsd for j in ns]
    lbackwardSubs:List Equation % := [(j::%)= (i::%) for i in lsd for j in ns]
    nlg:List % :=[subst(g,lforwardSubs) for g in lg]
    res:=eval(f, ls, [compiledFunction(g, x) for g in nlg])
    subst(res,lbackwardSubs)

if R has PatternMatchable Integer then
  patternMatch(x::%, p:Pattern Integer,
    l:PatternMatchResult(Integer, %)) ==
    patternMatch(x, p, l)$PatternMatchFunctionSpace(Integer, R, %)

if R has PatternMatchable Float then

```



```

patternMatch(x:%, p:Pattern Float,
  l:PatternMatchResult(Float, %)) ==
  patternMatch(x, p, l)$PatternMatchFunctionSpace(Float, R, %)

else -- R is not an integral domain
operator op ==
  belong?(op)$FSD => operator(op)$FSD
  belong?(op)$ESD => operator(op)$ESD
  nullary? op and has?(op, SYMBOL) => operator(kernel(name op)$K)
  (n := arity op) case "failed" => operator name op
  operator(name op, n::NonNegativeInteger)

if R has Ring then
  Rep := MP
  0 == 0$Rep
  1 == 1$Rep
  - x:% == -$Rep x
  n:Integer *x:% == n*$Rep x
  x:% * y:% == x*$Rep y
  x:% + y:% == x+$Rep y
  x:% = y:% == x=$Rep y
  x:% < y:% == x <$Rep y
  numer x == x@Rep
  coerce(p:MP):% == p

  reducedSystem(m:Matrix %):Matrix(R) ==
    reducedSystem(m)$Rep

  reducedSystem(m:Matrix %, v:Vector %):
  Record(mat:Matrix R, vec:Vector R) ==
    reducedSystem(m, v)$Rep

if R has ConvertibleTo InputForm then
  convert(x:%):InputForm == convert(x)$Rep

if R has PatternMatchable Integer then
  kintmatch: (K,Pattern Integer,PatternMatchResult(Integer,Rep))
    -> PatternMatchResult(Integer, Rep)

  kintmatch(k, p, l) ==
    patternMatch(k, p, l pretend PatternMatchResult(Integer, %)
    )$PatternMatchKernel(Integer, %)
    pretend PatternMatchResult(Integer, Rep)

  patternMatch(x:%, p:Pattern Integer,
    l:PatternMatchResult(Integer, %)) ==
    patternMatch(x@Rep, p,
      l pretend PatternMatchResult(Integer, Rep),
      kintmatch
    )$PatternMatchPolynomialCategory(Integer,

```

```

IndexedExponents K, K, R, Rep)
  pretend PatternMatchResult(Integer, %)

if R has PatternMatchable Float then
  kfltmach: (K, Pattern Float, PatternMatchResult(Float, Rep))
    -> PatternMatchResult(Float, Rep)

  kfltmach(k, p, l) ==
    patternMatch(k, p, l pretend PatternMatchResult(Float, %)
    )$PatternMatchKernel(Float, %)
    pretend PatternMatchResult(Float, Rep)

  patternMatch(x:%, p:Pattern Float,
    l:PatternMatchResult(Float, %)) ==
    patternMatch(x@Rep, p,
      l pretend PatternMatchResult(Float, Rep),
      kfltmach
    )$PatternMatchPolynomialCategory(Float,
      IndexedExponents K, K, R, Rep)
      pretend PatternMatchResult(Float, %)

else -- R is not even a ring
  if R has AbelianMonoid then
    import ListToMap(K, %)

    kereval      : (K, List K, List %) -> %
    subeval      : (K, List K, List %) -> %

    Rep := FreeAbelianGroup K

    0 == 0$Rep
    x:% + y:% == x +$Rep y
    x:% = y:% == x =$Rep y
    x:% < y:% == x <$Rep y
    coerce(k:K):% == coerce(k)$Rep
    kernels x == [f.gen for f in terms x]
    coerce(x:R):% == (zero? x => 0; constantKernel(x):%)
    retract(x:%):R == (zero? x => 0; retNotUnit x)
    coerce(x:%):OutputForm == coerce(x)$Rep
    kereval(k, lk, lv) ==
      match(lk, lv, k, (x2:K):% +-> map(x1 +-> eval(x1, lk, lv), x2))

    subeval(k, lk, lv) ==
      match(lk, lv, k,
        (x:K):% +->
          kernel(operator x, [subst(a, lk, lv) for a in argument x]))

    isPlus x ==
      empty?(l := terms x) or empty? rest l => "failed"
      [t.exp *$Rep t.gen for t in l]$List(%)

```

```

isMult x ==
  empty?(l := terms x) or not empty? rest l => "failed"
  t := first l
  [t.exp, t.gen]

eval(x:%, lk:List K, lv:List %) ==
  _+/[t.exp * kereval(t.gen, lk, lv) for t in terms x]

subst(x:%, lk:List K, lv:List %) ==
  _+/[t.exp * subeval(t.gen, lk, lv) for t in terms x]

retractIfCan(x:%):Union(R, "failed") ==
  zero? x => 0
  retNotUnitIfCan x

if R has AbelianGroup then -(x:%) == -$Rep x

--      else      -- R is not an AbelianMonoid
--      if R has SemiGroup then
--      Rep := FreeGroup K
--      1      == 1$Rep
--      x:% * y:%      == x *$Rep y
--      x:% = y:%      == x =$Rep y
--      coerce(k:K):%  == k::$Rep
--      kernels x      == [f.gen for f in factors x]
--      coerce(x:R):%  == (one? x => 1; constantKernel x)
--      retract(x:%):R == (one? x => 1; retNotUnit x)
--      coerce(x:%):OutputForm == coerce(x)$Rep

--      retractIfCan(x:%):Union(R, "failed") ==
--      one? x => 1
--      retNotUnitIfCan x

--      if R has Group then inv(x:%):% == inv(x)$Rep

else -- R is nothing
  import ListToMap(K, %)

  Rep := K

  x:% < y:%      == x <$Rep y
  x:% = y:%      == x =$Rep y
  coerce(k:K):%  == k
  kernels x      == [x pretend K]
  coerce(x:R):%  == constantKernel x
  retract(x:%):R == retNotUnit x
  retractIfCan(x:%):Union(R, "failed") == retNotUnitIfCan x
  coerce(x:%):OutputForm == coerce(x)$Rep

```

```

eval(x:%, lk:List K, lv:List %) ==
  match(lk, lv, x pretend K,
    (x1:K):% +-> map(x2 +-> eval(x2, lk, lv), x1))

subst(x, lk, lv) ==
  match(lk, lv, x pretend K,
    (x1:K):% +->
      kernel(operator x1, [subst(a, lk, lv) for a in argument x1]))

if R has ConvertibleTo InputForm then
  convert(x:%):InputForm == convert(x)$Rep

--
if R has PatternMatchable Integer then
--
  convert(x:%):Pattern(Integer) == convert(x)$Rep
--
  patternMatch(x:%, p:Pattern Integer,
--
    l:PatternMatchResult(Integer, %)) ==
--
    patternMatch(x pretend K,p,l)$PatternMatchKernel(Integer, %)
--
if R has PatternMatchable Float then
--
  convert(x:%):Pattern(Float) == convert(x)$Rep
--
  patternMatch(x:%, p:Pattern Float,
--
    l:PatternMatchResult(Float, %)) ==
--
    patternMatch(x pretend K, p, l)$PatternMatchKernel(Float, %)

```

---

— EXPR.dotabb —

```

"EXPR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=EXPR"]
"ACFS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ACFS"]
"EXPR" -> "ACFS"

```

---

## 6.7 domain EXPUPXS ExponentialOfUnivariatePuisseuxSeries

— ExponentialOfUnivariatePuisseuxSeries.input —

```

)set break resume
)sys rm -f ExponentialOfUnivariatePuisseuxSeries.output
)spool ExponentialOfUnivariatePuisseuxSeries.output
)set message test on

```

```

)set message auto off
)clear all

--S 1 of 1
)show ExponentialOfUnivariatePuisseuxSeries
--R ExponentialOfUnivariatePuisseuxSeries(FE: Join(Field,OrderedSet),var: Symbol,cen: FE) is
--R Abbreviation for ExponentialOfUnivariatePuisseuxSeries is EXPUPXS
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for EXPUPXS
--R
--R----- Operations -----
--R ?? : (FE,%) -> %           ?? : (%,FE) -> %
--R ?? : (%,%) -> %           ?? : (Integer,%) -> %
--R ?? : (PositiveInteger,%) -> %   ??? : (%,PositiveInteger) -> %
--R ?? : (%,%) -> %           ?-? : (%,%) -> %
--R -? : % -> %               ?<? : (%,%) -> Boolean
--R ?<=? : (%,%) -> Boolean      ?=? : (%,%) -> Boolean
--R ?>? : (%,%) -> Boolean      ?>=? : (%,%) -> Boolean
--R 1 : () -> %               0 : () -> %
--R ?? : (%,PositiveInteger) -> %   center : % -> FE
--R coerce : Integer -> %           coerce : % -> OutputForm
--R complete : % -> %             degree : % -> Fraction Integer
--R ?.? : (%,Fraction Integer) -> FE   hash : % -> SingleInteger
--R inv : % -> % if FE has FIELD      latex : % -> String
--R leadingCoefficient : % -> FE      leadingMonomial : % -> %
--R map : ((FE -> FE),%) -> %         max : (%,%) -> %
--R min : (%,%) -> %               monomial? : % -> Boolean
--R one? : % -> Boolean            order : % -> Fraction Integer
--R pole? : % -> Boolean           recip : % -> Union(%, "failed")
--R reductum : % -> %             sample : () -> %
--R variable : % -> Symbol         zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?? : (%,Fraction Integer) -> % if FE has ALGEBRA FRAC INT
--R ?? : (Fraction Integer,%) -> % if FE has ALGEBRA FRAC INT
--R ?? : (NonNegativeInteger,%) -> %
--R ??? : (%,Fraction Integer) -> % if FE has ALGEBRA FRAC INT
--R ??? : (%,%) -> % if FE has ALGEBRA FRAC INT
--R ??? : (%,Integer) -> % if FE has FIELD
--R ??? : (%,NonNegativeInteger) -> %
--R ?/? : (%,%) -> % if FE has FIELD
--R ?/? : (%,FE) -> % if FE has FIELD
--R D : % -> % if FE has *: (Fraction Integer,FE) -> FE
--R D : (%,NonNegativeInteger) -> % if FE has *: (Fraction Integer,FE) -> FE
--R D : (%,Symbol) -> % if FE has *: (Fraction Integer,FE) -> FE and FE has PDRING SYMBOL
--R D : (%,List Symbol) -> % if FE has *: (Fraction Integer,FE) -> FE and FE has PDRING SYMBOL
--R D : (%,Symbol,NonNegativeInteger) -> % if FE has *: (Fraction Integer,FE) -> FE and FE has PDRING SYMBOL
--R D : (%,List Symbol,List NonNegativeInteger) -> % if FE has *: (Fraction Integer,FE) -> FE and FE has PDRING SYMBOL
--R ?? : (%,Integer) -> % if FE has FIELD
--R ?? : (%,NonNegativeInteger) -> %
--R acos : % -> % if FE has ALGEBRA FRAC INT

```

```

--R acosh : % -> % if FE has ALGEBRA FRAC INT
--R acot : % -> % if FE has ALGEBRA FRAC INT
--R acoth : % -> % if FE has ALGEBRA FRAC INT
--R acsc : % -> % if FE has ALGEBRA FRAC INT
--R acsch : % -> % if FE has ALGEBRA FRAC INT
--R approximate : (% , Fraction Integer) -> FE if FE has **: (FE, Fraction Integer) -> FE and FE has coercion
--R asec : % -> % if FE has ALGEBRA FRAC INT
--R asech : % -> % if FE has ALGEBRA FRAC INT
--R asin : % -> % if FE has ALGEBRA FRAC INT
--R asinh : % -> % if FE has ALGEBRA FRAC INT
--R associates? : (% , %) -> Boolean if FE has INTDOM
--R atan : % -> % if FE has ALGEBRA FRAC INT
--R atanh : % -> % if FE has ALGEBRA FRAC INT
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%, "failed") if FE has CHARNZ
--R coefficient : (% , Fraction Integer) -> FE
--R coerce : % -> % if FE has INTDOM
--R coerce : Fraction Integer -> % if FE has ALGEBRA FRAC INT
--R coerce : FE -> % if FE has COMRING
--R cos : % -> % if FE has ALGEBRA FRAC INT
--R cosh : % -> % if FE has ALGEBRA FRAC INT
--R cot : % -> % if FE has ALGEBRA FRAC INT
--R coth : % -> % if FE has ALGEBRA FRAC INT
--R csc : % -> % if FE has ALGEBRA FRAC INT
--R csch : % -> % if FE has ALGEBRA FRAC INT
--R differentiate : % -> % if FE has *: (Fraction Integer, FE) -> FE
--R differentiate : (% , NonNegativeInteger) -> % if FE has *: (Fraction Integer, FE) -> FE
--R differentiate : (% , Symbol) -> % if FE has *: (Fraction Integer, FE) -> FE and FE has PDRING SYMBOL
--R differentiate : (% , List Symbol) -> % if FE has *: (Fraction Integer, FE) -> FE and FE has PDRING SYMBOL
--R differentiate : (% , Symbol, NonNegativeInteger) -> % if FE has *: (Fraction Integer, FE) -> FE and FE has PDRING SYMBOL
--R differentiate : (% , List Symbol, List NonNegativeInteger) -> % if FE has *: (Fraction Integer, FE) -> FE
--R divide : (% , %) -> Record(quotient: %, remainder: %) if FE has FIELD
--R ?? : (% , %) -> % if Fraction Integer has SGROUP
--R euclideanSize : % -> NonNegativeInteger if FE has FIELD
--R eval : (% , FE) -> Stream FE if FE has **: (FE, Fraction Integer) -> FE
--R exp : % -> % if FE has ALGEBRA FRAC INT
--R exponent : % -> UnivariatePuisseuxSeries(FE, var, cen)
--R exponential : UnivariatePuisseuxSeries(FE, var, cen) -> %
--R exponentialOrder : % -> Fraction Integer
--R expressIdealMember : (List %, %) -> Union(List %, "failed") if FE has FIELD
--R exquo : (% , %) -> Union(%, "failed") if FE has INTDOM
--R extend : (% , Fraction Integer) -> %
--R extendedEuclidean : (% , %) -> Record(coef1: %, coef2: %, generator: %) if FE has FIELD
--R extendedEuclidean : (% , %, %) -> Union(Record(coef1: %, coef2: %), "failed") if FE has FIELD
--R factor : % -> Factored % if FE has FIELD
--R gcd : (% , %) -> % if FE has FIELD
--R gcd : List % -> % if FE has FIELD
--R gcdPolynomial : (SparseUnivariatePolynomial %, SparseUnivariatePolynomial %) -> SparseUnivariatePolynomial
--R integrate : (% , Symbol) -> % if FE has integrate: (FE, Symbol) -> FE and FE has variables: FE -> List
--R integrate : % -> % if FE has ALGEBRA FRAC INT

```

```

--R lcm : (% , %) -> % if FE has FIELD
--R lcm : List % -> % if FE has FIELD
--R log : % -> % if FE has ALGEBRA FRAC INT
--R monomial : (% , List SingletonAsOrderedSet, List Fraction Integer) -> %
--R monomial : (% , SingletonAsOrderedSet, Fraction Integer) -> %
--R monomial : (FE, Fraction Integer) -> %
--R multiEuclidean : (List % , %) -> Union(List % , "failed") if FE has FIELD
--R multiplyExponents : (% , Fraction Integer) -> %
--R multiplyExponents : (% , PositiveInteger) -> %
--R nthRoot : (% , Integer) -> % if FE has ALGEBRA FRAC INT
--R order : (% , Fraction Integer) -> Fraction Integer
--R pi : () -> % if FE has ALGEBRA FRAC INT
--R prime? : % -> Boolean if FE has FIELD
--R principalIdeal : List % -> Record(coef: List % , generator: %) if FE has FIELD
--R ?quo? : (% , %) -> % if FE has FIELD
--R ?rem? : (% , %) -> % if FE has FIELD
--R sec : % -> % if FE has ALGEBRA FRAC INT
--R sech : % -> % if FE has ALGEBRA FRAC INT
--R series : (NonNegativeInteger, Stream Record(k: Fraction Integer, c: FE)) -> %
--R sin : % -> % if FE has ALGEBRA FRAC INT
--R sinh : % -> % if FE has ALGEBRA FRAC INT
--R sizeLess? : (% , %) -> Boolean if FE has FIELD
--R sqrt : % -> % if FE has ALGEBRA FRAC INT
--R squareFree : % -> Factored % if FE has FIELD
--R squareFreePart : % -> % if FE has FIELD
--R subtractIfCan : (% , %) -> Union(% , "failed")
--R tan : % -> % if FE has ALGEBRA FRAC INT
--R tanh : % -> % if FE has ALGEBRA FRAC INT
--R terms : % -> Stream Record(k: Fraction Integer, c: FE)
--R truncate : (% , Fraction Integer, Fraction Integer) -> %
--R truncate : (% , Fraction Integer) -> %
--R unit? : % -> Boolean if FE has INTDOM
--R unitCanonical : % -> % if FE has INTDOM
--R unitNormal : % -> Record(unit: % , canonical: % , associate: %) if FE has INTDOM
--R variables : % -> List SingletonAsOrderedSet
--R
--E 1

)spool
)lisp (bye)

```

---

— ExponentialOfUnivariatePuisseuxSeries.help —

```

=====
ExponentialOfUnivariatePuisseuxSeries examples
=====

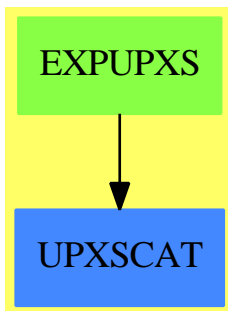
```

See Also:

o )show ExponentialOfUnivariatePuisseuxSeries

\_\_\_\_\_

### 6.7.1 ExponentialOfUnivariatePuisseuxSeries (EXPUPXS)



See

⇒ “UnivariatePuisseuxSeriesWithExponentialSingularity” (UPXSSING) 22.7.1 on page 2809  
 ⇒ “ExponentialExpansion” (EXPEXPAN) 6.5.1 on page 679



**Exports:**

|                   |                   |                    |                   |
|-------------------|-------------------|--------------------|-------------------|
| 0                 | 1                 | acos               | acosh             |
| acot              | acoth             | acsc               | acsch             |
| approximate       | asec              | asech              | asin              |
| asinh             | associates?       | atan               | atanh             |
| center            | characteristic    | charthRoot         | coefficient       |
| coerce            | complete          | cos                | cosh              |
| cot               | coth              | csc                | csch              |
| D                 | degree            | differentiate      | divide            |
| euclideanSize     | eval              | exp                | exponent          |
| exponential       | exponentialOrder  | expressIdealMember | exquo             |
| extend            | extendedEuclidean | factor             | gcd               |
| gcdPolynomial     | hash              | integrate          | inv               |
| latex             | lcm               | leadingCoefficient | leadingMonomial   |
| log               | map               | max                | min               |
| monomial          | monomial?         | multiEuclidean     | multiplyExponents |
| multiplyExponents | nthRoot           | one?               | order             |
| pi                | pole?             | prime?             | principalIdeal    |
| recip             | reductum          | sample             | sec               |
| sech              | series            | sin                | sinh              |
| sizeLess?         | sqrt              | squareFree         | squareFreePart    |
| subtractIfCan     | tan               | tanh               | terms             |
| truncate          | unit?             | unitCanonical      | unitNormal        |
| variable          | variables         | zero?              | ?*?               |
| ?**?              | ?+?               | ?-?                | -?                |
| ?<?               | ?<=?              | ?=?                | ?>?               |
| ?>=?              | ?^?               | ?.??               | ?~=?              |
| ?/?               | ?quo?             | ?rem?              |                   |

## — domain EXPUPXS ExponentialOfUnivariatePuisseuxSeries —

```

)abbrev domain EXPUPXS ExponentialOfUnivariatePuisseuxSeries
++ Author: Clifton J. Williamson
++ Date Created: 4 August 1992
++ Date Last Updated: 27 August 1992
++ Basic Operations:
++ Related Domains: UnivariatePuisseuxSeries(FE,var,cen)
++ Also See:
++ AMS Classifications:
++ Keywords: limit, functional expression, power series, essential singularity
++ Examples:
++ References:
++ Description:
++ ExponentialOfUnivariatePuisseuxSeries is a domain used to represent
++ essential singularities of functions. An object in this domain is a
++ function of the form \spad{exp(f(x))}, where \spad{f(x)} is a Puiseux
++ series with no terms of non-negative degree. Objects are ordered
++ according to order of singularity, with functions which tend more

```

```

++ rapidly to zero or infinity considered to be larger. Thus, if
++ \spad{order(f(x)) < order(g(x))}, i.e. the first non-zero term of
++ \spad{f(x)} has lower degree than the first non-zero term of \spad{g(x)},
++ then \spad{exp(f(x)) > exp(g(x))}. If \spad{order(f(x)) = order(g(x))},
++ then the ordering is essentially random. This domain is used
++ in computing limits involving functions with essential singularities.

```

```

ExponentialOfUnivariatePuisseuxSeries(FE,var,cen):_

```

```

    Exports == Implementation where

```

```

    FE : Join(Field,OrderedSet)

```

```

    var : Symbol

```

```

    cen : FE

```

```

    UPXS ==> UnivariatePuisseuxSeries(FE,var,cen)

```

```

Exports ==> Join(UnivariatePuisseuxSeriesCategory(FE),OrderedAbelianMonoid) _
    with

```

```

    exponential : UPXS -> %

```

```

    ++ exponential(f(x)) returns \spad{exp(f(x))}.

```

```

    ++ Note: the function does NOT check that \spad{f(x)} has no
    ++ non-negative terms.

```

```

    exponent : % -> UPXS

```

```

    ++ exponent(exp(f(x))) returns \spad{f(x)}

```

```

    exponentialOrder: % -> Fraction Integer

```

```

    ++ exponentialOrder(exp(c * x **(-n) + ...)) returns \spad{-n}.

```

```

    ++ exponentialOrder(0) returns \spad{0}.

```

```

Implementation ==> UPXS add

```

```

    Rep := UPXS

```

```

    exponential f == complete f

```

```

    exponent f == f pretend UPXS

```

```

    exponentialOrder f == order(exponent f,0)

```

```

    zero? f == empty? entries complete terms f

```

```

    f = g ==

```

```

    -- we redefine equality because we know that we are dealing with

```

```

    -- a FINITE series, so there is no danger in computing all terms

```

```

    (entries complete terms f) = (entries complete terms g)

```

```

    f < g ==

```

```

    zero? f => not zero? g

```

```

    zero? g => false

```

```

    (ordf := exponentialOrder f) > (ordg := exponentialOrder g) => true

```

```

    ordf < ordg => false

```

```

    (fCoef := coefficient(f,ordf)) = (gCoef := coefficient(g,ordg)) =>

```

```

    reductum(f) < reductum(g)

```

```

    fCoef < gCoef -- this is "random" if FE is EXPR INT

```

```

coerce(f: %): OutputForm ==
  ("%e" :: OutputForm) ** ((coerce$Rep)(complete f)@OutputForm)

-----

— EXPUPXS.dotabb —

"EXPUPXS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=EXPUPXS"]
"UPXSCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=UPXSCAT"]
"EXPUPXS" -> "UPXSCAT"

```

## 6.8 domain EAB ExtAlgBasis

```

— ExtAlgBasis.input —

)set break resume
)sys rm -f ExtAlgBasis.output
)spool ExtAlgBasis.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ExtAlgBasis
--R ExtAlgBasis is a domain constructor
--R Abbreviation for ExtAlgBasis is EAB
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for EAB
--R
--R----- Operations -----
--R ?<? : (%,% ) -> Boolean          ?<=? : (%,% ) -> Boolean
--R ?=? : (%,% ) -> Boolean          ?>? : (%,% ) -> Boolean
--R ?>=? : (%,% ) -> Boolean          Nul : NonNegativeInteger -> %
--R coerce : List Integer -> %       coerce : % -> OutputForm
--R degree : % -> NonNegativeInteger exponents : % -> List Integer
--R hash : % -> SingleInteger         latex : % -> String
--R max : (%,% ) -> %                 min : (%,% ) -> %
--R ?~=? : (%,% ) -> Boolean
--R
--E 1

)spool
)lisp (bye)

```

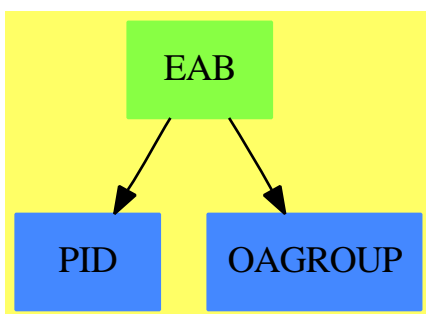
---

— ExtAlgBasis.help —

```
=====
ExtAlgBasis examples
=====
```

```
See Also:
o )show ExtAlgBasis
```

### 6.8.1 ExtAlgBasis (EAB)



See

⇒ “AntiSymm” (ANTISYM) 2.8.1 on page 40  
 ⇒ “DeRhamComplex” (DERHAM) 5.6.1 on page 515

#### Exports:

|        |        |           |      |       |
|--------|--------|-----------|------|-------|
| coerce | degree | exponents | hash | latex |
| max    | min    | Nul       | ?~=? | ?<?   |
| ?<=?   | ?=?    | ?>?       | ?>=? |       |

— domain EAB ExtAlgBasis —

```
)abbrev domain EAB ExtAlgBasis
++ Author: Larry Lambe
++ Date created: 03/14/89
++ Description:
++ A domain used in the construction of the exterior algebra on a set
++ X over a ring R. This domain represents the set of all ordered
++ subsets of the set X, assumed to be in correspondance with
++ {1,2,3, ...}. The ordered subsets are themselves ordered
++ lexicographically and are in bijective correspondance with an ordered
```

```

++ basis of the exterior algebra. In this domain we are dealing strictly
++ with the exponents of basis elements which can only be 0 or 1.
-- Thus we really have  $L(\{0,1\})$ .
++
++ The multiplicative identity element of the exterior algebra corresponds
++ to the empty subset of  $X$ . A coerce from List Integer to an
++ ordered basis element is provided to allow the convenient input of
++ expressions. Another exported function forgets the ordered structure
++ and simply returns the list corresponding to an ordered subset.

ExtAlgBasis(): Export == Implement where
  I ==> Integer
  L ==> List
  NNI ==> NonNegativeInteger

Export == OrderedSet with
  coerce : L I -> %
    ++ coerce(l) converts a list of 0's and 1's into a basis
    ++ element, where 1 (respectively 0) designates that the
    ++ variable of the corresponding index of l is (respectively, is not)
    ++ present.
    ++ Error: if an element of l is not 0 or 1.
  degree : % -> NNI
    ++ degree(x) gives the numbers of 1's in x, i.e., the number
    ++ of non-zero exponents in the basis element that x represents.
  exponents : % -> L I
    ++ exponents(x) converts a domain element into a list of zeros
    ++ and ones corresponding to the exponents in the basis element
    ++ that x represents.
-- subscripts : % -> L I
  -- subscripts(x) looks at the exponents in x and converts
  -- them to the proper subscripts
  Nul : NNI -> %
    ++ Nul() gives the basis element 1 for the algebra generated
    ++ by n generators.

Implement == add
  Rep := L I
  x,y : %

  x = y == x =$Rep y

  x < y ==
    null x ==> not null y
    null y ==> false
    first x = first y ==> rest x < rest y
    first x > first y

  coerce(li:(L I)) ==
    for x in li repeat

```

```

        if x ^= 1 and x ^= 0 then error "coerce: values can only be 0 and 1"
    li

    degree x          == (_+/x)::NNI

    exponents x       == copy(x @ Rep)

--   subscripts x     ==
--       cntr:I := 1
--       result: L I := []
--       for j in x repeat
--           if j = 1 then result := cons(cntr,result)
--           cntr:=cntr+1
--       reverse_! result

    Nul n             == [0 for i in 1..n]

    coerce x          == coerce(x @ Rep)$(L I)

_____

— EAB.dotabb —

"EAB" [color="#88FF44",href="bookvol10.3.pdf#nameddest=EAB"]
"PID" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PID"]
"OAGROUP" [color="#4488FF",href="bookvol10.2.pdf#nameddest=OAGROUP"]
"EAB" -> "PID"
"EAB" -> "OAGROUP"

_____

```

## 6.9 domain E04DGFA e04dgfAnnaType

```

_____

— e04dgfAnnaType.input —

)set break resume
)sys rm -f e04dgfAnnaType.output
)spool e04dgfAnnaType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show e04dgfAnnaType

```

```

--R e04dgfAnnaType is a domain constructor
--R Abbreviation for e04dgfAnnaType is E04DGFA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for E04DGFA
--R
--R----- Operations -----
--R ==? : (%,% ) -> Boolean          coerce : % -> OutputForm
--R hash : % -> SingleInteger       latex : % -> String
--R ==~? : (%,% ) -> Boolean
--R measure : (RoutinesTable,Record(lfn: List Expression DoubleFloat,init: List DoubleFloat))
--R measure : (RoutinesTable,Record(fn: Expression DoubleFloat,init: List DoubleFloat,lb: List DoubleFloat))
--R numericalOptimization : Record(fn: Expression DoubleFloat,init: List DoubleFloat,lb: List DoubleFloat)
--R numericalOptimization : Record(lfn: List Expression DoubleFloat,init: List DoubleFloat)
--R
--E 1

)spool
)lisp (bye)

```

---

— e04dgfAnnaType.help —

```

=====
e04dgfAnnaType examples
=====

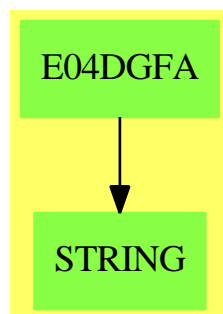
```

```

See Also:
o )show e04dgfAnnaType

```

### 6.9.1 e04dgfAnnaType (E04DGFA)



See

⇒ “E04FDFA” (e04fdfAnnaType) 6.10.1 on page 718  
 ⇒ “E04GCFA” (e04gcfAnnaType) 6.11.1 on page 721  
 ⇒ “E04JAFA” (e04jafAnnaType) 6.12.1 on page 726  
 ⇒ “E04MBFA” (e04mbfAnnaType) 6.13.1 on page 729  
 ⇒ “E04NAFA” (e04nafAnnaType) 6.14.1 on page 733  
 ⇒ “E04UCFA” (e04ucfAnnaType) 6.15.1 on page 736

**Exports:**

coerce hash latex measure numericalOptimization ?=? ?~=?

## — domain E04DGFA e04dgfAnnaType —

```

)abbrev domain E04DGFA e04dgfAnnaType
++ Author: Brian Dupee
++ Date Created: February 1996
++ Date Last Updated: February 1996
++ Basic Operations: measure, numericalOptimization
++ Related Constructors: Result, RoutinesTable
++ Description:
++ \axiomType{e04dgfAnnaType} is a domain of \axiomType{NumericalOptimization}
++ for the NAG routine E04DGF, a general optimization routine which
++ can handle some singularities in the input function. The function
++ \axiomFun{measure} measures the usefulness of the routine E04DGF
++ for the given problem. The function \axiomFun{numericalOptimization}
++ performs the optimization by using \axiomType{NagOptimisationPackage}.

e04dgfAnnaType(): NumericalOptimizationCategory == Result add
  DF ==> DoubleFloat
  EF ==> Expression Float
  EDF ==> Expression DoubleFloat
  PDF ==> Polynomial DoubleFloat
  VPDF ==> Vector Polynomial DoubleFloat
  LDF ==> List DoubleFloat
  LOCDF ==> List OrderedCompletion DoubleFloat
  MDF ==> Matrix DoubleFloat
  MPDF ==> Matrix Polynomial DoubleFloat
  MF ==> Matrix Float
  MEF ==> Matrix Expression Float
  LEDF ==> List Expression DoubleFloat
  VEF ==> Vector Expression Float
  NOA ==> Record(fn:EDF, init:LDF, lb:LOCDF, cf:LEDF, ub:LOCDF)
  LSA ==> Record(lfn:LEDF, init:LDF)
  EF2 ==> ExpressionFunctions2
  MI ==> Matrix Integer
  INT ==> Integer
  F ==> Float
  NNI ==> NonNegativeInteger
  S ==> Symbol
  LS ==> List Symbol

```



```

MVCF ==> MultiVariableCalculusFunctions
ESTOOLS2 ==> ExpertSystemToolsPackage2
SDF ==> Stream DoubleFloat
LSDF ==> List Stream DoubleFloat
SOCDF ==> Segment OrderedCompletion DoubleFloat
OCDF ==> OrderedCompletion DoubleFloat

Rep:=Result
import Rep, NagOptimisationPackage, ExpertSystemToolsPackage

measure(R:RoutinesTable,args:NOA) ==
  string:String := "e04dgm is "
  positive?((args.cf) + (args.lb) + (args.ub)) =>
    string := concat(string,"unsuitable for constrained problems. ")
    [0.0,string]
  string := concat(string,"recommended")
  [getMeasure(R,e04dgm@Symbol)$RoutinesTable, string]

numericalOptimization(args:NOA) ==
  argsFn:EDF := args.fn
  n:NMI := #(variables(argsFn)$EDF)
  fu:DF := float(4373903597,-24,10)$DF
  it:INT := max(50,5*n)
  lin:DF := float(9,-1,10)$DF
  ma:DF := float(1,20,10)$DF
  op:DF := float(326,-14,10)$DF
  x:MDF := mat(args.init,n)
  ArgsFn:Expression Float := edf2ef(argsFn)
  f:Union(fn:FileName,fp:Asp49(OBJFUN)) := [retract(ArgsFn)$Asp49(OBJFUN)]
  e04dgm(n,1$DF,fu,it,lin,true,ma,op,1,1,n,0,x,-1,f)

```

---

— E04DGFA.dotabb —

```

"E04DGFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=E04DGFA"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"E04DGFA" -> "STRING"

```

---

## 6.10 domain E04FDFA e04fdfAnnaType

— e04fdfAnnaType.input —

```

)set break resume
)sys rm -f e04fdfAnnaType.output
)spool e04fdfAnnaType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show e04fdfAnnaType
--R e04fdfAnnaType is a domain constructor
--R Abbreviation for e04fdfAnnaType is E04FDFA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for E04FDFA
--R
--R----- Operations -----
--R ?? : (%,%) -> Boolean          coerce : % -> OutputForm
--R hash : % -> SingleInteger      latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R measure : (RoutinesTable,Record(lfn: List Expression DoubleFloat,init: List DoubleFloat)) -> Record(
--R measure : (RoutinesTable,Record(fn: Expression DoubleFloat,init: List DoubleFloat,lb: List OrderedCo
--R numericalOptimization : Record(fn: Expression DoubleFloat,init: List DoubleFloat,lb: List OrderedCom
--R numericalOptimization : Record(lfn: List Expression DoubleFloat,init: List DoubleFloat) -> Result
--R
--E 1

)spool
)lisp (bye)

```

---

— e04fdfAnnaType.help —

```

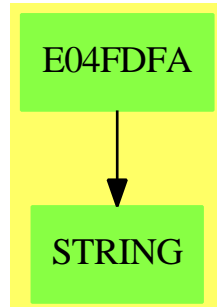
=====
e04fdfAnnaType examples
=====

```

See Also:

- o )show e04fdfAnnaType

### 6.10.1 e04fdfAnnaType (E04FDFA)



See

- ⇒ “E04DGFA” (e04dggAnnaType) 6.9.1 on page 714
- ⇒ “E04GCFA” (e04gcfAnnaType) 6.11.1 on page 721
- ⇒ “E04JAFA” (e04jafAnnaType) 6.12.1 on page 726
- ⇒ “E04MBFA” (e04mbfAnnaType) 6.13.1 on page 729
- ⇒ “E04NAFA” (e04nafAnnaType) 6.14.1 on page 733
- ⇒ “E04UCFA” (e04ucfAnnaType) 6.15.1 on page 736

#### Exports:

coerce hash latex measure numericalOptimization ?=? ?~=?

— domain E04FDFA e04fdfAnnaType —

```

)abbrev domain E04FDFA e04fdfAnnaType
++ Author: Brian Dupee
++ Date Created: February 1996
++ Date Last Updated: February 1996
++ Basic Operations: measure, numericalOptimization
++ Related Constructors: Result, RoutinesTable
++ Description:
++ \axiomType{e04fdfAnnaType} is a domain of \axiomType{NumericalOptimization}
++ for the NAG routine E04FDF, a general optimization routine which
++ can handle some singularities in the input function. The function
++ \axiomFun{measure} measures the usefulness of the routine E04FDF
++ for the given problem. The function \axiomFun{numericalOptimization}
++ performs the optimization by using \axiomType{NagOptimisationPackage}.
  
```

```

e04fdfAnnaType(): NumericalOptimizationCategory == Result add
  DF ==> DoubleFloat
  EF ==> Expression Float
  EDF ==> Expression DoubleFloat
  PDF ==> Polynomial DoubleFloat
  VPDF ==> Vector Polynomial DoubleFloat
  LDF ==> List DoubleFloat
  LOCDF ==> List OrderedCompletion DoubleFloat
  
```

```

MDF ==> Matrix DoubleFloat
MPDF ==> Matrix Polynomial DoubleFloat
MF ==> Matrix Float
MEF ==> Matrix Expression Float
LEDF ==> List Expression DoubleFloat
VEF ==> Vector Expression Float
NOA ==> Record(fn:EDF, init:LDF, lb:LOCDF, cf:LEDF, ub:LOCDF)
LSA ==> Record(lfn:LEDF, init:LDF)
EF2 ==> ExpressionFunctions2
MI ==> Matrix Integer
INT ==> Integer
F ==> Float
NNI ==> NonNegativeInteger
S ==> Symbol
LS ==> List Symbol
MVCF ==> MultiVariableCalculusFunctions
ESTOOLS2 ==> ExpertSystemToolsPackage2
SDF ==> Stream DoubleFloat
LSDF ==> List Stream DoubleFloat
SOCDF ==> Segment OrderedCompletion DoubleFloat
OCDF ==> OrderedCompletion DoubleFloat

Rep:=Result
import Rep, NagOptimisationPackage
import e04AgentsPackage,ExpertSystemToolsPackage

measure(R:RoutinesTable,args:NOA) ==
  argsFn := args.fn
  string:String := "e04fdf is "
  positive?((args.cf) + (args.lb) + (args.ub)) =>
    string := concat(string,"unsuitable for constrained problems. ")
    [0.0,string]
  n:NNI := #(variables(argsFn)$EDF)
  (n>1)@Boolean =>
    string := concat(string,"unsuitable for single instances of multivariate problems. ")
    [0.0,string]
  sumOfSquares(argsFn) case "failed" =>
    string := concat(string,"unsuitable.")
    [0.0,string]
  string := concat(string,"recommended since the function is a sum of squares.")
  [getMeasure(R,e04fdf@Symbol)$RoutinesTable, string]

measure(R:RoutinesTable,args:LSA) ==
  string:String := "e04fdf is recommended"
  [getMeasure(R,e04fdf@Symbol)$RoutinesTable, string]

numericalOptimization(args:NOA) ==
  argsFn := args.fn
  lw:INT := 14
  x := mat(args.init,1)

```

```

(a := sumOfSquares(argsFn)) case EDF =>
  ArgsFn := vector([edf2ef(a)])$VEF
  f : Union(fn:FileName,fp:Asp50(LSFUN1)) := [retract(ArgsFn)$Asp50(LSFUN1)]
  out:Result := e04fdf(1,1,1,lw,x,-1,f)
  changeNameToObjf(fsumsq@Symbol,out)
empty()$Result

numericalOptimization(args:LSA) ==
  argsFn := copy args.lfn
  m:INT := #(argsFn)
  n:NNI := #(variables(args))
  nn:INT := n
  lw:INT :=
--    one?(nn) => 9+5*m
    (nn = 1) => 9+5*m
    nn*(7+n+2*m+((nn-1) quo 2)$INT)+3*m
  x := mat(args.init,n)
  ArgsFn := vector([edf2ef(i)$ExpertSystemToolsPackage for i in argsFn])$VEF
  f : Union(fn:FileName,fp:Asp50(LSFUN1)) := [retract(ArgsFn)$Asp50(LSFUN1)]
  out:Result := e04fdf(m,n,1,lw,x,-1,f)
  changeNameToObjf(fsumsq@Symbol,out)

```

---

— E04FDFA.dotabb —

```

"E04FDFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=E04FDFA"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"E04FDFA" -> "STRING"

```

---

## 6.11 domain E04GCFA e04gcfAnnaType

— e04gcfAnnaType.input —

```

)set break resume
)sys rm -f e04gcfAnnaType.output
)spool e04gcfAnnaType.output
)set message test on
)set message auto off
)clear all

```

```
--S 1 of 1
```

```

)show e04gcfAnnaType
--R e04gcfAnnaType is a domain constructor
--R Abbreviation for e04gcfAnnaType is E04GCFA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for E04GCFA
--R
--R----- Operations -----
--R ?=? : (% ,%) -> Boolean          coerce : % -> OutputForm
--R hash : % -> SingleInteger        latex : % -> String
--R ?~=? : (% ,%) -> Boolean
--R measure : (RoutinesTable,Record(lfn: List Expression DoubleFloat,init: List DoubleFloat)) -> Record(
--R measure : (RoutinesTable,Record(fn: Expression DoubleFloat,init: List DoubleFloat,lb: List OrderedCo
--R numericalOptimization : Record(fn: Expression DoubleFloat,init: List DoubleFloat,lb: List OrderedCom
--R numericalOptimization : Record(lfn: List Expression DoubleFloat,init: List DoubleFloat) -> Result
--R
--E 1

)spool
)lisp (bye)

```

\_\_\_\_\_

— e04gcfAnnaType.help —

```

=====
e04gcfAnnaType examples
=====

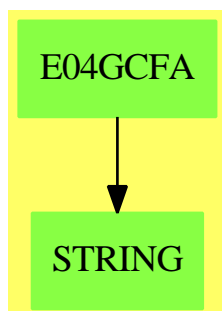
```

See Also:

- o )show e04gcfAnnaType

\_\_\_\_\_

### 6.11.1 e04gcfAnnaType (E04GCFA)



See

⇒ “E04DGFA” (e04dgfAnnaType) 6.9.1 on page 714  
 ⇒ “E04FDFA” (e04fdfAnnaType) 6.10.1 on page 718  
 ⇒ “E04JAFa” (e04jafAnnaType) 6.12.1 on page 726  
 ⇒ “E04MBFA” (e04mbfAnnaType) 6.13.1 on page 729  
 ⇒ “E04NAFA” (e04nafAnnaType) 6.14.1 on page 733  
 ⇒ “E04UCFA” (e04ucfAnnaType) 6.15.1 on page 736

**Exports:**

coerce hash latex measure numericalOptimization ?=? ?~=?

— domain E04GCFA e04gcfAnnaType —

```

)abbrev domain E04GCFA e04gcfAnnaType
++ Author: Brian Dupee
++ Date Created: February 1996
++ Date Last Updated: February 1996
++ Basic Operations: measure, numericalOptimization
++ Related Constructors: Result, RoutinesTable
++ Description:
++ \axiomType{e04gcfAnnaType} is a domain of \axiomType{NumericalOptimization}
++ for the NAG routine E04GCF, a general optimization routine which
++ can handle some singularities in the input function. The function
++ \axiomFun{measure} measures the usefulness of the routine E04GCF
++ for the given problem. The function \axiomFun{numericalOptimization}
++ performs the optimization by using \axiomType{NagOptimisationPackage}.

e04gcfAnnaType(): NumericalOptimizationCategory == Result add
  DF ==> DoubleFloat
  EF ==> Expression Float
  EDF ==> Expression DoubleFloat
  PDF ==> Polynomial DoubleFloat
  VPDF ==> Vector Polynomial DoubleFloat
  LDF ==> List DoubleFloat
  LOCDF ==> List OrderedCompletion DoubleFloat
  MDF ==> Matrix DoubleFloat
  MPDF ==> Matrix Polynomial DoubleFloat
  MF ==> Matrix Float
  MEF ==> Matrix Expression Float
  LEDF ==> List Expression DoubleFloat
  VEF ==> Vector Expression Float
  NOA ==> Record(fn:EDF, init:LDF, lb:LOCDF, cf:LEDF, ub:LOCDF)
  LSA ==> Record(lfn:LEDF, init:LDF)
  EF2 ==> ExpressionFunctions2
  MI ==> Matrix Integer
  INT ==> Integer
  F ==> Float
  NNI ==> NonNegativeInteger
  S ==> Symbol

```

```

LS ==> List Symbol
MVCF ==> MultiVariableCalculusFunctions
ESTOOLS2 ==> ExpertSystemToolsPackage2
SDF ==> Stream DoubleFloat
LSDF ==> List Stream DoubleFloat
SOCDF ==> Segment OrderedCompletion DoubleFloat
OCDF ==> OrderedCompletion DoubleFloat

Rep:=Result
import Rep, NagOptimisationPackage,ExpertSystemContinuityPackage
import e04AgentsPackage,ExpertSystemToolsPackage

measure(R:RoutinesTable,args:NOA) ==
  argsFn:EDF := args.fn
  string:String := "e04gcf is "
  positive? (#(args.cf) + #(args.lb) + #(args.ub)) =>
    string := concat(string,"unsuitable for constrained problems. ")
    [0.0,string]
  n:NNI := #(variables(argsFn)$EDF)
  (n>1)@Boolean =>
    string := concat(string,"unsuitable for single instances of multivariate problems. ")
    [0.0,string]
  a := coerce(float(10,0,10))$OCDF
  seg:SOCDF := -a..a
  sings := singularitiesOf(argsFn,variables(argsFn)$EDF,seg)
  s := #(sdf2lst(sings))
  positive? s =>
    string := concat(string,"not recommended for discontinuous functions.")
    [0.0,string]
  sumOfSquares(args.fn) case "failed" =>
    string := concat(string,"unsuitable.")
    [0.0,string]
  string := concat(string,"recommended since the function is a sum of squares.")
  [getMeasure(R,e04gcf@Symbol)$RoutinesTable, string]

measure(R:RoutinesTable,args:LSA) ==
  string:String := "e04gcf is "
  a := coerce(float(10,0,10))$OCDF
  seg:SOCDF := -a..a
  sings := concat([singularitiesOf(i,variables(args),seg) for i in args.lfn])$SDF
  s := #(sdf2lst(sings))
  positive? s =>
    string := concat(string,"not recommended for discontinuous functions.")
    [0.0,string]
  string := concat(string,"recommended.")
  m := getMeasure(R,e04gcf@Symbol)$RoutinesTable
  m := m-(1-exp(-(expenseOfEvaluation(args)**3))
  [m, string]

numericalOptimization(args:NOA) ==

```



```

argsFn:EDF := args.fn
lw:INT := 16
x := mat(args.init,1)
(a := sumOfSquares(argsFn)) case EDF =>
  ArgsFn := vector([edf2ef(a)$ExpertSystemToolsPackage])$VEF
  f : Union(fn:FileName,fp:Asp19(LSFUN2)) := [retract(ArgsFn)$Asp19(LSFUN2)]
  out:Result := e04gcf(1,1,1,lw,x,-1,f)
  changeNameToObjf(fsumsq@Symbol,out)
empty()$Result

numericalOptimization(args:LSA) ==
  argsFn := copy args.lfn
  m:NNI := #(argsFn)
  n:NNI := #(variables(args))
  lw:INT :=
--    one?(n) => 11+5*m
    (n = 1) => 11+5*m
    2*n*(4+n+m)+3*m
  x := mat(args.init,n)
  ArgsFn := vector([edf2ef(i)$ExpertSystemToolsPackage for i in argsFn])$VEF
  f : Union(fn:FileName,fp:Asp19(LSFUN2)) := [retract(ArgsFn)$Asp19(LSFUN2)]
  out:Result := e04gcf(m,n,1,lw,x,-1,f)
  changeNameToObjf(fsumsq@Symbol,out)

```

---

— E04GCFA.dotabb —

```

"E04GCFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=E04GCFA"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"E04GCFA" -> "STRING"

```

---

## 6.12 domain E04JAFA e04jafAnnaType

— e04jafAnnaType.input —

```

)set break resume
)sys rm -f e04jafAnnaType.output
)spool e04jafAnnaType.output
)set message test on
)set message auto off
)clear all

```

```

--S 1 of 1
)show e04jafAnnaType
--R e04jafAnnaType is a domain constructor
--R Abbreviation for e04jafAnnaType is E04JAFA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for E04JAFA
--R
--R----- Operations -----
--R ?=? : (%,% ) -> Boolean          coerce : % -> OutputForm
--R hash : % -> SingleInteger        latex : % -> String
--R ?~=? : (%,% ) -> Boolean
--R measure : (RoutinesTable,Record(lfn: List Expression DoubleFloat,init: List DoubleFloat)) -> Record(
--R measure : (RoutinesTable,Record(fn: Expression DoubleFloat,init: List DoubleFloat,lb: List OrderedCo
--R numericalOptimization : Record(fn: Expression DoubleFloat,init: List DoubleFloat,lb: List OrderedCom
--R numericalOptimization : Record(lfn: List Expression DoubleFloat,init: List DoubleFloat) -> Result
--R
--E 1

)spool
)lisp (bye)

```

---

— e04jafAnnaType.help —

```

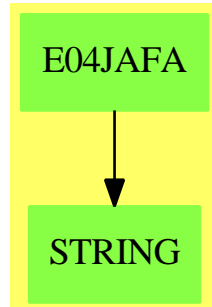
=====
e04jafAnnaType examples
=====

```

See Also:  
o )show e04jafAnnaType

---

### 6.12.1 e04jafAnnaType (E04JAFA)



See

- ⇒ “E04DGFA” (e04dgmAnnaType) 6.9.1 on page 714
- ⇒ “E04FDFA” (e04fdfAnnaType) 6.10.1 on page 718
- ⇒ “E04GCFA” (e04gcfAnnaType) 6.11.1 on page 721
- ⇒ “E04MBFA” (e04mbfAnnaType) 6.13.1 on page 729
- ⇒ “E04NAFA” (e04nafAnnaType) 6.14.1 on page 733
- ⇒ “E04UCFA” (e04ucfAnnaType) 6.15.1 on page 736

#### Exports:

coerce hash latex measure numericalOptimization ?=? ?~=?

— domain E04JAFA e04jafAnnaType —

```

)abbrev domain E04JAFA e04jafAnnaType
++ Author: Brian Dupee
++ Date Created: February 1996
++ Date Last Updated: February 1996
++ Basic Operations: measure, numericalOptimization
++ Related Constructors: Result, RoutinesTable
++ Description:
++ \axiomType{e04jafAnnaType} is a domain of \axiomType{NumericalOptimization}
++ for the NAG routine E04JAF, a general optimization routine which
++ can handle some singularities in the input function. The function
++ \axiomFun{measure} measures the usefulness of the routine E04JAF
++ for the given problem. The function \axiomFun{numericalOptimization}
++ performs the optimization by using \axiomType{NagOptimisationPackage}.
  
```

```

e04jafAnnaType(): NumericalOptimizationCategory == Result add
  DF ==> DoubleFloat
  EF ==> Expression Float
  EDF ==> Expression DoubleFloat
  PDF ==> Polynomial DoubleFloat
  VPDF ==> Vector Polynomial DoubleFloat
  LDF ==> List DoubleFloat
  LOCDF ==> List OrderedCompletion DoubleFloat
  
```

```

MDF ==> Matrix DoubleFloat
MPDF ==> Matrix Polynomial DoubleFloat
MF ==> Matrix Float
MEF ==> Matrix Expression Float
LEDF ==> List Expression DoubleFloat
VEF ==> Vector Expression Float
NOA ==> Record(fn:EDF, init:LDF, lb:LOCDF, cf:LEDF, ub:LOCDF)
LSA ==> Record(lfn:LEDF, init:LDF)
EF2 ==> ExpressionFunctions2
MI ==> Matrix Integer
INT ==> Integer
F ==> Float
NNI ==> NonNegativeInteger
S ==> Symbol
LS ==> List Symbol
MVCF ==> MultiVariableCalculusFunctions
ESTOOLS2 ==> ExpertSystemToolsPackage2
SDF ==> Stream DoubleFloat
LSDF ==> List Stream DoubleFloat
SOCDF ==> Segment OrderedCompletion DoubleFloat
OCDF ==> OrderedCompletion DoubleFloat

Rep:=Result
import Rep, NagOptimisationPackage
import e04AgentsPackage,ExpertSystemToolsPackage

bound(a:LOCDF,b:LOCDF):Integer ==
  empty?(concat(a,b)) => 1
--  one?((#(removeDuplicates(a))) and zero?(first(a))) => 2
  ((#(removeDuplicates(a)) = 1) and zero?(first(a))) => 2
--  one?((#(removeDuplicates(a))) and one?((#(removeDuplicates(b)))) => 3
  ((#(removeDuplicates(a)) = 1) and ((#(removeDuplicates(b)) = 1) => 3
  0

measure(R:RoutinesTable,args:NOA) ==
  string:String := "e04jaf is "
  if positive?((#(args.cf))) then
    if not simpleBounds?(args.cf) then
      string :=
        concat(string,"suitable for simple bounds only, not constraint functions.")
    (# string) < 20 =>
      if zero?((#(args.lb) + #(args.ub))) then
        string := concat(string, "usable if there are no constraints")
        [getMeasure(R,e04jaf@Symbol)$RoutinesTable*0.5,string]
      else
        string := concat(string,"recommended")
        [getMeasure(R,e04jaf@Symbol)$RoutinesTable, string]
    [0.0,string]

numericalOptimization(args:NOA) ==

```

```

argsFn:EDF := args.fn
n:NNI := #(variables(argsFn)$EDF)
ibound:INT := bound(args.lb,args.ub)
m:INT := n
lw:INT := max(13,12 * m + ((m * (m - 1)) quo 2)$INT)$INT
bl := mat(finiteBound(args.lb,float(1,6,10)$DF),n)
bu := mat(finiteBound(args.ub,float(1,6,10)$DF),n)
x := mat(args.init,n)
ArgsFn:EF := edf2ef(argsFn)
fr:Union(fn:FileName,fp:Asp24(FUNCT1)) := [retract(ArgsFn)$Asp24(FUNCT1)]
out:Result := e04jaf(n,ibound,n+2,lw,bl,bu,x,-1,fr)
changeNameToObjf(f@Symbol,out)

```

---

— E04JAFA.dotabb —

```

"E04JAFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=E04JAFA"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"E04JAFA" -> "STRING"

```

---

## 6.13 domain E04MBFA e04mbfAnnaType

— e04mbfAnnaType.input —

```

)set break resume
)sys rm -f e04mbfAnnaType.output
)spool e04mbfAnnaType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show e04mbfAnnaType
--R e04mbfAnnaType is a domain constructor
--R Abbreviation for e04mbfAnnaType is E04MBFA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for E04MBFA
--R
--R----- Operations -----
--R ?=? : (%,% ) -> Boolean          coerce : % -> OutputForm
--R hash : % -> SingleInteger       latex : % -> String

```

```

--R ?~=? : (%,% ) -> Boolean
--R measure : (RoutinesTable,Record(lfn: List Expression DoubleFloat,init: List DoubleFloat)) -> Record(
--R measure : (RoutinesTable,Record(fn: Expression DoubleFloat,init: List DoubleFloat,lb: List OrderedCo
--R numericalOptimization : Record(fn: Expression DoubleFloat,init: List DoubleFloat,lb: List OrderedCom
--R numericalOptimization : Record(lfn: List Expression DoubleFloat,init: List DoubleFloat) -> Result
--R
--E 1

```

```

)spool
)lisp (bye)

```

— e04mbfAnnaType.help —

```

=====
e04mbfAnnaType examples
=====

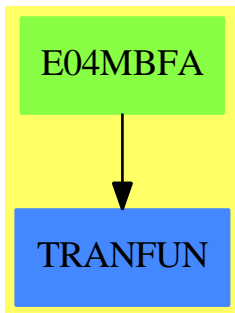
```

```

See Also:
o )show e04mbfAnnaType

```

### 6.13.1 e04mbfAnnaType (E04MBFA)



See

- ⇒ “E04DGFA” (e04dgfAnnaType) 6.9.1 on page 714
- ⇒ “E04FDFA” (e04fdfAnnaType) 6.10.1 on page 718
- ⇒ “E04GCFA” (e04gcfAnnaType) 6.11.1 on page 721
- ⇒ “E04JAFA” (e04jafAnnaType) 6.12.1 on page 726
- ⇒ “E04NAFA” (e04nafAnnaType) 6.14.1 on page 733
- ⇒ “E04UCFA” (e04ucfAnnaType) 6.15.1 on page 736

**Exports:**

coerce hash latex measure numericalOptimization ==? ?~=?

— domain E04MBFA e04mbfAnnaType —

```
)abbrev domain E04MBFA e04mbfAnnaType
++ Author: Brian Dupee
++ Date Created: February 1996
++ Date Last Updated: February 1996
++ Basic Operations: measure, numericalOptimization
++ Related Constructors: Result, RoutinesTable
++ Description:
++ \axiomType{e04mbfAnnaType} is a domain of \axiomType{NumericalOptimization}
++ for the NAG routine E04MBF, an optimization routine for Linear functions.
++ The function
++ \axiomFun{measure} measures the usefulness of the routine E04MBF
++ for the given problem. The function \axiomFun{numericalOptimization}
++ performs the optimization by using \axiomType{NagOptimisationPackage}.

e04mbfAnnaType(): NumericalOptimizationCategory == Result add
DF ==> DoubleFloat
EF ==> Expression Float
EDF ==> Expression DoubleFloat
PDF ==> Polynomial DoubleFloat
VPDF ==> Vector Polynomial DoubleFloat
LDF ==> List DoubleFloat
LOCDF ==> List OrderedCompletion DoubleFloat
MDF ==> Matrix DoubleFloat
MPDF ==> Matrix Polynomial DoubleFloat
MF ==> Matrix Float
MEF ==> Matrix Expression Float
LEDF ==> List Expression DoubleFloat
VEF ==> Vector Expression Float
NOA ==> Record(fn:EDF, init:LDF, lb:LOCDF, cf:LEDF, ub:LOCDF)
LSA ==> Record(lfn:LEDF, init:LDF)
EF2 ==> ExpressionFunctions2
MI ==> Matrix Integer
INT ==> Integer
F ==> Float
NNI ==> NonNegativeInteger
S ==> Symbol
LS ==> List Symbol
MVCF ==> MultiVariableCalculusFunctions
ESTOOLS2 ==> ExpertSystemToolsPackage2
SDF ==> Stream DoubleFloat
LSDF ==> List Stream DoubleFloat
SOCDF ==> Segment OrderedCompletion DoubleFloat
OCDF ==> OrderedCompletion DoubleFloat
```

```

Rep:=Result
import Rep, NagOptimisationPackage
import e04AgentsPackage,ExpertSystemToolsPackage

measure(R:RoutinesTable,args:NOA) ==
  (not linear?([args.fn])) or (not linear?(args.cf)) =>
    [0.0,"e04mbf is for a linear objective function and constraints only."]
  [getMeasure(R,e04mbf@Symbol)$RoutinesTable,"e04mbf is recommended" ]

numericalOptimization(args:NOA) ==
  argsFn:EDF := args.fn
  c := args.cf
  listVars:List LS := concat(variables(argsFn)$EDF,[variables(z)$EDF for z in c])
  n:NNI := #(v := removeDuplicates(concat(listVars)$LS)$LS)
  A:MDF := linearMatrix(args.cf,n)
  nclin:NNI := # linearPart(c)
  nrowa:NNI := max(1,nclin)
  bl:MDF := mat(finiteBound(args.lb,float(1,21,10)$DF),n)
  bu:MDF := mat(finiteBound(args.ub,float(1,21,10)$DF),n)
  cvec:MDF := mat(coefficients(retract(argsFn)@PDF)$PDF,n)
  x := mat(args.init,n)
  lwork:INT :=
    nclin < n => 2*nclin*(nclin+4)+2+6*n+nrowa
    2*(n+3)*n+4*nclin+nrowa
  out:Result := e04mbf(20,1,n,nclin,n+nclin,nrowa,A,bl,bu,cvec,true,2*n,lwork,x,-1)
  changeNameToObjf(objlp@Symbol,out)

```

---

— E04MBFA.dotabb —

```

"E04MBFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=E04MBFA"]
"TRANFUN" [color="#4488FF",href="bookvol10.2.pdf#nameddest=TRANFUN"]
"E04MBFA" -> "TRANFUN"

```

---

## 6.14 domain E04NAFA e04nafAnnaType

— e04nafAnnaType.input —

```

)set break resume
)sys rm -f e04nafAnnaType.output
)spool e04nafAnnaType.output

```



```

)set message test on
)set message auto off
)clear all

--S 1 of 1
)show e04nafAnnaType
--R e04nafAnnaType is a domain constructor
--R Abbreviation for e04nafAnnaType is EO4NAFA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for EO4NAFA
--R
--R----- Operations -----
--R ?=? : (% ,%) -> Boolean          coerce : % -> OutputForm
--R hash : % -> SingleInteger        latex : % -> String
--R ?~=? : (% ,%) -> Boolean
--R measure : (RoutinesTable,Record(lfn: List Expression DoubleFloat,init: List DoubleFloat))
--R measure : (RoutinesTable,Record(fn: Expression DoubleFloat,init: List DoubleFloat,lb: List DoubleFloat))
--R numericalOptimization : Record(fn: Expression DoubleFloat,init: List DoubleFloat,lb: List DoubleFloat)
--R numericalOptimization : Record(lfn: List Expression DoubleFloat,init: List DoubleFloat)
--R
--E 1

)spool
)lisp (bye)

```

---

— e04nafAnnaType.help —

```

=====
e04nafAnnaType examples
=====

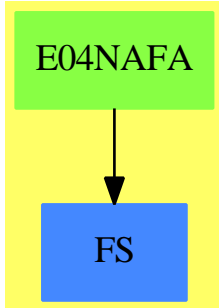
```

```

See Also:
o )show e04nafAnnaType

```

## 6.14.1 e04nafAnnaType (E04NAFA)



See

- ⇒ “E04DGFA” (e04dgfAnnaType) 6.9.1 on page 714
- ⇒ “E04FDFA” (e04fdfAnnaType) 6.10.1 on page 718
- ⇒ “E04GCFA” (e04gcfAnnaType) 6.11.1 on page 721
- ⇒ “E04JAFA” (e04jafAnnaType) 6.12.1 on page 726
- ⇒ “E04MBFA” (e04mbfAnnaType) 6.13.1 on page 729
- ⇒ “E04UCFA” (e04ucfAnnaType) 6.15.1 on page 736

**Exports:**

coerce hash latex measure numericalOptimization ?? ?~=?

— domain E04NAFA e04nafAnnaType —

```

)abbrev domain E04NAFA e04nafAnnaType
++ Author: Brian Dupee
++ Date Created: February 1996
++ Date Last Updated: February 1996
++ Basic Operations: measure, numericalOptimization
++ Related Constructors: Result, RoutinesTable
++ Description:
++ \axiomType{e04nafAnnaType} is a domain of \axiomType{NumericalOptimization}
++ for the NAG routine E04NAF, an optimization routine for Quadratic functions.
++ The function
++ \axiomFun{measure} measures the usefulness of the routine E04NAF
++ for the given problem. The function \axiomFun{numericalOptimization}
++ performs the optimization by using \axiomType{NagOptimisationPackage}.

```

```

e04nafAnnaType(): NumericalOptimizationCategory == Result add
  DF ==> DoubleFloat
  EF ==> Expression Float
  EDF ==> Expression DoubleFloat
  PDF ==> Polynomial DoubleFloat
  VPDF ==> Vector Polynomial DoubleFloat
  LDF ==> List DoubleFloat
  LOCDF ==> List OrderedCompletion DoubleFloat

```

```

MDF ==> Matrix DoubleFloat
MPDF ==> Matrix Polynomial DoubleFloat
MF ==> Matrix Float
MEF ==> Matrix Expression Float
LEDF ==> List Expression DoubleFloat
VEF ==> Vector Expression Float
NOA ==> Record(fn:EDF, init:LDF, lb:LOCDF, cf:LEDF, ub:LOCDF)
LSA ==> Record(lfn:LEDF, init:LDF)
EF2 ==> ExpressionFunctions2
MI ==> Matrix Integer
INT ==> Integer
F ==> Float
NNI ==> NonNegativeInteger
S ==> Symbol
LS ==> List Symbol
MVCF ==> MultiVariableCalculusFunctions
ESTOOLS2 ==> ExpertSystemToolsPackage2
SDF ==> Stream DoubleFloat
LSDF ==> List Stream DoubleFloat
SOCDF ==> Segment OrderedCompletion DoubleFloat
OCDF ==> OrderedCompletion DoubleFloat

Rep:=Result
import Rep, NagOptimisationPackage
import e04AgentsPackage,ExpertSystemToolsPackage

measure(R:RoutinesTable,args:NOA) ==
  string:String := "e04naf is "
  argsFn:EDF := args.fn
  if not (quadratic?(argsFn) and linear?(args.cf)) then
    string :=
      concat(string,"for a quadratic function with linear constraints only.")
  (# string) < 20 =>
    string := concat(string,"recommended")
    [getMeasure(R,e04naf@Symbol)$RoutinesTable, string]
  [0.0,string]

numericalOptimization(args:NOA) ==
  argsFn:EDF := args.fn
  c := args.cf
  listVars:List LS := concat(variables(argsFn)$EDF,[variables(z)$EDF for z in c])
  n:NNI := #(v := sort(removeDuplicates(concat(listVars)$LS)$LS)$LS)
  A:MDF := linearMatrix(c,n)
  nclin:NNI := # linearPart(c)
  nrowa:NNI := max(1,nclin)
  big:DF := float(1,10,10)$DF
  fea:MDF := new(1,n+nclin,float(1053,-11,10)$DF)$MDF
  bl:MDF := mat(finiteBound(args.lb,float(1,21,10)$DF),n)
  bu:MDF := mat(finiteBound(args.ub,float(1,21,10)$DF),n)
  alin:EDF := splitLinear(argsFn)

```

```

p:PDF := retract(alin)@PDF
pl:List PDF := [coefficient(p,i,1)$PDF for i in v]
cvec:MDF := mat([pdf2df j for j in pl],n)
h1:MPDF := hessian(p,v)$MVCF(S,PDF,VPDF,LS)
hess:MDF := map(pdf2df,h1)$ESTTOOLS2(PDF,DF)
h2:MEF := map(df2ef,hess)$ESTTOOLS2(DF,EF)
x := mat(args.init,n)
istate:MI := zero(1,n+nclin)$MI
lwork:INT := 2*n*(n+2*nclin)+nrowa
qphess:Union(fn:FileName,fp:Asp20(QPHESS)) := [retract(h2)$Asp20(QPHESS)]
out:Result := e04naf(20,1,n,nclin,n+nclin,nrowa,n,n,big,A,bl,bu,cvec,fea,
                    hess,true,false,true,2*n,lwork,x,istate,-1,qphess)
changeNameToObjf(obj@Symbol,out)

```

---

— E04NAFA.dotabb —

```

"E04NAFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=E04NAFA"]
"FS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FS"]
"E04NAFA" -> "FS"

```

---

## 6.15 domain E04UCFA e04ucfAnnaType

— e04ucfAnnaType.input —

```

)set break resume
)sys rm -f e04ucfAnnaType.output
)spool e04ucfAnnaType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show e04ucfAnnaType
--R e04ucfAnnaType is a domain constructor
--R Abbreviation for e04ucfAnnaType is E04UCFA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for E04UCFA
--R
--R----- Operations -----
--R ?=? : (% ,%) -> Boolean          coerce : % -> OutputForm

```

```

--R hash : % -> SingleInteger          latex : % -> String
--R ?~=? : (% ,%) -> Boolean
--R measure : (RoutinesTable,Record(lfn: List Expression DoubleFloat,init: List DoubleFloat))
--R measure : (RoutinesTable,Record(fn: Expression DoubleFloat,init: List DoubleFloat,lb: List DoubleFloat))
--R numericalOptimization : Record(fn: Expression DoubleFloat,init: List DoubleFloat,lb: List DoubleFloat)
--R numericalOptimization : Record(lfn: List Expression DoubleFloat,init: List DoubleFloat)
--R
--E 1

```

```

)spool
)lisp (bye)

```

\_\_\_\_\_

— e04ucfAnnaType.help —

```

=====
e04ucfAnnaType examples
=====

```

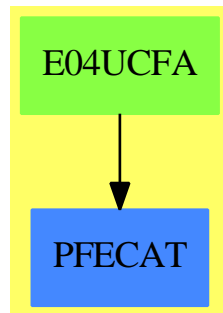
```

See Also:
o )show e04ucfAnnaType

```

\_\_\_\_\_

### 6.15.1 e04ucfAnnaType (E04UCFA)



See

- ⇒ “E04DGFA” (e04dgfAnnaType) 6.9.1 on page 714
- ⇒ “E04FDFA” (e04fdfAnnaType) 6.10.1 on page 718
- ⇒ “E04GCFA” (e04gcfAnnaType) 6.11.1 on page 721
- ⇒ “E04JAFA” (e04jafAnnaType) 6.12.1 on page 726
- ⇒ “E04MBFA” (e04mbfAnnaType) 6.13.1 on page 729
- ⇒ “E04NAFA” (e04nafAnnaType) 6.14.1 on page 733

**Exports:**

coerce hash latex measure numericalOptimization ?? ?~=?

## — domain E04UCFA e04ucfAnnaType —

```
)abbrev domain E04UCFA e04ucfAnnaType
++ Author: Brian Dupee
++ Date Created: February 1996
++ Date Last Updated: November 1997
++ Basic Operations: measure, numericalOptimization
++ Related Constructors: Result, RoutinesTable
++ Description:
++ \axiomType{e04ucfAnnaType} is a domain of \axiomType{NumericalOptimization}
++ for the NAG routine E04UCF, a general optimization routine which
++ can handle some singularities in the input function. The function
++ \axiomFun{measure} measures the usefulness of the routine E04UCF
++ for the given problem. The function \axiomFun{numericalOptimization}
++ performs the optimization by using \axiomType{NagOptimisationPackage}.
```

```
e04ucfAnnaType(): NumericalOptimizationCategory == Result add
  DF ==> DoubleFloat
  EF ==> Expression Float
  EDF ==> Expression DoubleFloat
  PDF ==> Polynomial DoubleFloat
  VPDF ==> Vector Polynomial DoubleFloat
  LDF ==> List DoubleFloat
  LOCDF ==> List OrderedCompletion DoubleFloat
  MDF ==> Matrix DoubleFloat
  MPDF ==> Matrix Polynomial DoubleFloat
  MF ==> Matrix Float
  MEF ==> Matrix Expression Float
  LEDF ==> List Expression DoubleFloat
  VEF ==> Vector Expression Float
  NOA ==> Record(fn:EDF, init:LDF, lb:LOCDF, cf:LEDF, ub:LOCDF)
  LSA ==> Record(lfn:LEDF, init:LDF)
  EF2 ==> ExpressionFunctions2
  MI ==> Matrix Integer
  INT ==> Integer
  F ==> Float
  NNI ==> NonNegativeInteger
  S ==> Symbol
  LS ==> List Symbol
  MVCF ==> MultiVariableCalculusFunctions
  ESTOOLS2 ==> ExpertSystemToolsPackage2
  SDF ==> Stream DoubleFloat
  LSDF ==> List Stream DoubleFloat
  SOCDF ==> Segment OrderedCompletion DoubleFloat
  OCDF ==> OrderedCompletion DoubleFloat
```

```

Rep:=Result
import Rep,NagOptimisationPackage
import e04AgentsPackage,ExpertSystemToolsPackage

measure(R:RoutinesTable,args:NOA) ==
  zero?((args.lb) + (args.ub)) =>
    [0.0,"e04ucf is not recommended if there are no bounds specified"]
  zero?((args.cf)) =>
    string:String := "e04ucf is usable but not always recommended if there are no constraints"
    [getMeasure(R,e04ucf@Symbol)$RoutinesTable*0.5,string]
    [getMeasure(R,e04ucf@Symbol)$RoutinesTable,"e04ucf is recommended"]

numericalOptimization(args:NOA) ==
  Args := sortConstraints(args)
  argsFn := Args.fn
  c := Args.cf
  listVars:List LS := concat(variables(argsFn)$EDF,[variables(z)$EDF for z in c])
  n:NNI := #(v := sort(removeDuplicates(concat(listVars)$LS)$LS)$LS)
  lin:NNI := #(linearPart(c))
  nlcf := nonLinearPart(c)
  nonlin:NNI := #(nlcf)
  if empty?(nlcf) then
    nlcf := new(n,coerce(first(v)$LS)$EDF)$LEDF
  nrowa:NNI := max(1,lin)
  nrowj:NNI := max(1,nonlin)
  A:MDF := linearMatrix(c,n)
  bl:MDF := mat(finiteBound(Args.lb,float(1,25,10)$DF),n)
  bu:MDF := mat(finiteBound(Args.ub,float(1,25,10)$DF),n)
  liwork:INT := 3*n+lin+2*nonlin
  lwork:INT :=
    zero?(lin+nonlin) => 20*n
    zero?(nonlin) => 2*n*(n+10)+11*lin
    2*n*(n+nonlin+10)+(11+n)*lin + 21*nonlin
  cra:DF := float(1,-2,10)$DF
  fea:DF := float(1053671201,-17,10)$DF
  fun:DF := float(4373903597,-24,10)$DF
  infb:DF := float(1,15,10)$DF
  lint:DF := float(9,-1,10)$DF
  maji:INT := max(50,3*(n+lin)+10*nonlin)
  mini:INT := max(50,3*(n+lin+nonlin))
  nonf:DF := float(105,-10,10)$DF
  opt:DF := float(326,-10,10)$DF
  ste:DF := float(2,0,10)$DF
  istate:MI := zero(1,n+lin+nonlin)$MI
  cjac:MDF :=
    positive?(nonlin) => zero(nrowj,n)$MDF
    zero(nrowj,1)$MDF
  clambda:MDF := zero(1,n+lin+nonlin)$MDF
  r:MDF := zero(n,n)$MDF
  x:MDF := mat(Args.init,n)

```

```

VectCF:VEF := vector([edf2ef e for e in nlcf])$VEF
ArgsFn:EF := edf2ef(argsFn)
fasp : Union(fn:FileName,fp:Asp49(OBJFUN)) := [retract(ArgsFn)$Asp49(OBJFUN)]
casp : Union(fn:FileName,fp:Asp55(CONFUN)) := [retract(VectCF)$Asp55(CONFUN)]
e04ucf(n,lin,nonlin,nrowa,nrowj,n,A,bl,bu,liwork,lwork,false,cra,3,fea,
      fun,true,infb,infb,fea,lint,true,maji,1,mini,0,-1,nonf,opt,ste,1,
      1,n,n,3,istate,cjac,clambda,r,x,-1,casp,fasp)

```

---

— E04UCFA.dotabb —

```

"E04UCFA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=E04UCFA"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"E04UCFA" -> "PFECAT"

```

---





## Chapter 7

# Chapter F

### 7.1 domain FR Factored

— Factored.input —

```
)set break resume
)sys rm -f Factored.output
)spool Factored.output
)set message test on
)set message auto off
)clear all
--S 1 of 38
g := factor(4312)
--R
--R
--R      3 2
--R  (1) 2 7 11
--R
--E 1
```

Type: Factored Integer

```
--S 2 of 38
unit(g)
--R
--R
--R  (2) 1
--R
--E 2
```

Type: PositiveInteger

```
--S 3 of 38
numberOfFactors(g)
--R
--R
```

```

--R (3) 3
--R                                         Type: PositiveInteger
--E 3

--S 4 of 38
[nthFactor(g,i) for i in 1..numberOfFactors(g)]
--R
--R
--R (4) [2,7,11]
--R                                         Type: List Integer
--E 4

--S 5 of 38
[nthExponent(g,i) for i in 1..numberOfFactors(g)]
--R
--R
--R (5) [3,2,1]
--R                                         Type: List Integer
--E 5

--S 6 of 38
[nthFlag(g,i) for i in 1..numberOfFactors(g)]
--R
--R
--R (6) ["prime","prime","prime"]
--R                                         Type: List Union("nil","sqfr","irred","prime")
--E 6

--S 7 of 38
factorList(g)
--R
--R
--R (7)
--R [[flg= "prime",fctr= 2,xpnt= 3], [flg= "prime",fctr= 7,xpnt= 2],
--R  [flg= "prime",fctr= 11,xpnt= 1]]
--RType: List Record(flg: Union("nil","sqfr","irred","prime"),fctr: Integer,xpnt: Integer)
--E 7

--S 8 of 38
factors(g)
--R
--R
--R (8)
--R [[factor= 2,exponent= 3],[factor= 7,exponent= 2],[factor= 11,exponent= 1]]
--R                                         Type: List Record(factor: Integer,exponent: Integer)
--E 8

--S 9 of 38
first(%).factor
--R

```

```

--R
--R (9)  2
--R
--R                                         Type: PositiveInteger
--E 9

--S 10 of 38
g := factor(4312)
--R
--R
--R          3 2
--R (10)  2 7 11
--R
--R                                         Type: Factored Integer
--E 10

--S 11 of 38
expand(g)
--R
--R
--R (11)  4312
--R
--R                                         Type: PositiveInteger
--E 11

--S 12 of 38
reduce(*,[t.factor for t in factors(g)])
--R
--R
--R (12)  154
--R
--R                                         Type: PositiveInteger
--E 12

--S 13 of 38
g := factor(4312)
--R
--R
--R          3 2
--R (13)  2 7 11
--R
--R                                         Type: Factored Integer
--E 13

--S 14 of 38
f := factor(246960)
--R
--R
--R          4 2 3
--R (14)  2 3 5 7
--R
--R                                         Type: Factored Integer
--E 14

--S 15 of 38
f * g

```

```
--R  
--R  
--R      7 2   5  
 $(15) \quad 2^3 5^7 11$   
--R  
Type: Factored Integer  
--E 15  
  
--S 16 of 38  
f**500  
--R  
--R  
--R      2000 1000 500 1500  
 $(16) \quad 2^2 3^3 5^5 7^7$   
--R  
Type: Factored Integer  
--E 16  
  
--S 17 of 38  
gcd(f,g)  
--R  
--R  
--R      3 2  
 $(17) \quad 2^7$   
--R  
Type: Factored Integer  
--E 17  
  
--S 18 of 38  
lcm(f,g)  
--R  
--R  
--R      4 2   3  
 $(18) \quad 2^3 5^7 11$   
--R  
Type: Factored Integer  
--E 18  
  
--S 19 of 38  
f + g  
--R  
--R  
--R      3 2  
 $(19) \quad 2^7 641$   
--R  
Type: Factored Integer  
--E 19  
  
--S 20 of 38  
f - g  
--R  
--R  
--R      3 2  
 $(20) \quad 2^7 619$   
--R  
Type: Factored Integer
```

[illegible]

```
--S 27 of 38
nilFactor(24,2)
--R
--R
--R      2
--R (27) 24
--R
--E 27
```

Type: Factored Integer

```
--S 28 of 38
nthFlag(%,1)
--R
--R
--R (28) "nil"
--R
--E 28
```

Type: Union("nil",...)

```
--S 29 of 38
sqfrFactor(30,2)
--R
--R
--R      2
--R (29) 30
--R
--E 29
```

Type: Factored Integer

```
--S 30 of 38
irreducibleFactor(13,10)
--R
--R
--R      10
--R (30) 13
--R
--E 30
```

Type: Factored Integer

```
--S 31 of 38
primeFactor(11,5)
--R
--R
--R      5
--R (31) 11
--R
--E 31
```

Type: Factored Integer

```
--S 32 of 38
h := factor(-720)
--R
--R
--R      4 2
--R (32) - 2 3 5
```

```

--R                                                    Type: Factored Integer
--E 32

--S 33 of 38
h - makeFR(unit(h),factorList(h))
--R
--R
--R (33)  0
--R
--R                                                    Type: Factored Integer
--E 33

--S 34 of 38
p := (4*x*x-12*x+9)*y*y + (4*x*x-12*x+9)*y + 28*x*x - 84*x + 63
--R
--R
--R (34)   $(4x^2 - 12x + 9)y^2 + (4x^2 - 12x + 9)y + 28x^2 - 84x + 63$ 
--R
--R                                                    Type: Polynomial Integer
--E 34

--S 35 of 38
fp := factor(p)
--R
--R
--R (35)   $(2x - 3)(y^2 + y + 7)$ 
--R
--R                                                    Type: Factored Polynomial Integer
--E 35

--S 36 of 38
D(p,x)
--R
--R
--R (36)   $(8x - 12)y^2 + (8x - 12)y + 56x - 84$ 
--R
--R                                                    Type: Polynomial Integer
--E 36

--S 37 of 38
D(fp,x)
--R
--R
--R (37)   $4(2x - 3)(y^2 + y + 7)$ 
--R
--R                                                    Type: Factored Polynomial Integer
--E 37

--S 38 of 38
numberOfFactors(%)
--R

```



```

--R
--R (38) 3
--R
--R                                         Type: PositiveInteger
--E 38
)spool
)lisp (bye)

```

---

— Factored.help —

=====

Factored examples

=====

Factored creates a domain whose objects are kept in factored form as long as possible. Thus certain operations like \* (multiplication) and gcd are relatively easy to do. Others, such as addition, require somewhat more work, and the result may not be completely factored unless the argument domain R provides a factor operation. Each object consists of a unit and a list of factors, where each factor consists of a member of R (the base), an exponent, and a flag indicating what is known about the base. A flag may be one of "nil", "sqfr", "irred" or "prime", which mean that nothing is known about the base, it is square-free, it is irreducible, or it is prime, respectively. The current restriction to factored objects of integral domains allows simplification to be performed without worrying about multiplication order.

=====

Decomposing Factored Objects

=====

In this section we will work with a factored integer.

```

g := factor(4312)
      3 2
      2 7 11

```

Type: Factored Integer

Let's begin by decomposing g into pieces. The only possible units for integers are 1 and -1.

```

unit(g)
1

```

Type: PositiveInteger

There are three factors.

```

numberOfFactors(g)
3
                                Type: PositiveInteger

```

We can make a list of the bases, ...

```

[nthFactor(g,i) for i in 1..numberOfFactors(g)]
[2,7,11]
                                Type: List Integer

```

and the exponents, ...

```

[nthExponent(g,i) for i in 1..numberOfFactors(g)]
[3,2,1]
                                Type: List Integer

```

and the flags. You can see that all the bases (factors) are prime.

```

[nthFlag(g,i) for i in 1..numberOfFactors(g)]
["prime","prime","prime"]
                                Type: List Union("nil","sqfr","irred","prime")

```

A useful operation for pulling apart a factored object into a list of records of the components is `factorList`.

```

factorList(g)
[[flg= "prime",fctr= 2,xpnt= 3], [flg= "prime",fctr= 7,xpnt= 2],
 [flg= "prime",fctr= 11,xpnt= 1]]
                                Type: List Record(flg: Union("nil","sqfr","irred","prime"),
                                fctr: Integer,xpnt: Integer)

```

If you don't care about the flags, use `factors`.

```

factors(g)
[[factor= 2,exponent= 3],[factor= 7,exponent= 2],[factor= 11,exponent= 1]]
                                Type: List Record(factor: Integer,exponent: Integer)

```

Neither of these operations returns the unit.

```

first(%).factor
2
                                Type: PositiveInteger

```

```

=====
Expanding Factored Objects
=====

```

Recall that we are working with this factored integer.

```

g := factor(4312)

```

```

3 2
2 7 11

```

Type: Factored Integer

To multiply out the factors with their multiplicities, use `expand`.

```

expand(g)
4312

```

Type: PositiveInteger

If you would like, say, the distinct factors multiplied together but with multiplicity one, you could do it this way.

```

reduce(*,[t.factor for t in factors(g)])
154

```

Type: PositiveInteger

#### =====

#### Arithmetic with Factored Objects

#### =====

We're still working with this factored integer.

```

g := factor(4312)
3 2
2 7 11

```

Type: Factored Integer

We'll also define this factored integer.

```

f := factor(246960)
4 2 3
2 3 5 7

```

Type: Factored Integer

Operations involving multiplication and division are particularly easy with factored objects.

```

f * g
7 2 5
2 3 5 7 11

```

Type: Factored Integer

```

f**500
2000 1000 500 1500
2 3 5 7

```

Type: Factored Integer

```

gcd(f,g)
3 2

```

```
2 7
```

Type: Factored Integer

```
lcm(f,g)
  4 2 3
  2 3 5 7 11
```

Type: Factored Integer

If we use addition and subtraction things can slow down because we may need to compute greatest common divisors.

```
f + g
  3 2
  2 7 641
```

Type: Factored Integer

```
f - g
  3 2
  2 7 619
```

Type: Factored Integer

Test for equality with 0 and 1 by using zero? and one?, respectively.

```
zero?(factor(0))
true
```

Type: Boolean

```
zero?(g)
false
```

Type: Boolean

```
one?(factor(1))
true
```

Type: Boolean

```
one?(f)
false
```

Type: Boolean

Another way to get the zero and one factored objects is to use package calling.

```
0$Factored(Integer)
0
```

Type: Factored Integer

```
1$Factored(Integer)
1
```

Type: Factored Integer

```
=====
Creating New Factored Objects
=====
```

The map operation is used to iterate across the unit and bases of a factored object.

The following four operations take a base and an exponent and create a factored object. They differ in handling the flag component.

```
nilFactor(24,2)
  2
 24
                                Type: Factored Integer
```

This factor has no associated information.

```
nthFlag(%,1)
"nil"
                                Type: Union("nil",...)
```

This factor is asserted to be square-free.

```
sqfrFactor(30,2)
  2
 30
                                Type: Factored Integer
```

This factor is asserted to be irreducible.

```
irreducibleFactor(13,10)
  10
 13
                                Type: Factored Integer
```

This factor is asserted to be prime.

```
primeFactor(11,5)
  5
 11
                                Type: Factored Integer
```

A partial inverse to factorList is makeFR.

```
h := factor(-720)
  4 2
- 2 3 5
                                Type: Factored Integer
```

The first argument is the unit and the second is a list of records as

returned by factorList.

```
h - makeFR(unit(h),factorList(h))
0
Type: Factored Integer
```

=====

Factored Objects with Variables

=====

Some of the operations available for polynomials are also available for factored polynomials.

```
p := (4*x*x-12*x+9)*y*y + (4*x*x-12*x+9)*y + 28*x*x - 84*x + 63
      2      2      2      2
(4x - 12x + 9)y + (4x - 12x + 9)y + 28x - 84x + 63
Type: Polynomial Integer
```

```
fp := factor(p)
      2 2
(2x - 3) (y + y + 7)
Type: Factored Polynomial Integer
```

You can differentiate with respect to a variable.

```
D(p,x)
      2
(8x - 12)y + (8x - 12)y + 56x - 84
Type: Polynomial Integer
```

```
D(fp,x)
      2
4(2x - 3)(y + y + 7)
Type: Factored Polynomial Integer
```

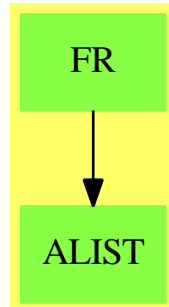
```
numberOfFactors(%)
3
Type: PositiveInteger
```

See Also:

- o )help FactoredFunctions2
- o )show Factored

---

### 7.1.1 Factored (FR)



#### Exports:

|               |                |               |                 |                   |
|---------------|----------------|---------------|-----------------|-------------------|
| 0             | 1              | associates?   | characteristic  | coerce            |
| convert       | D              | differentiate | eval            | expand            |
| exponent      | exquo          | factor        | factorList      | factors           |
| flagFactor    | gcd            | gcdPolynomial | hash            | irreducibleFactor |
| latex         | lcm            | makeFR        | map             | nilFactor         |
| nthExponent   | nthFactor      | nthFlag       | numberOfFactors | one?              |
| prime?        | primeFactor    | rational      | rational?       | rationalIfCan     |
| recip         | retract        | retractIfCan  | sample          | sqfrFactor        |
| squareFree    | squareFreePart | subtractIfCan | unit            | unit?             |
| unitCanonical | unitNormal     | unitNormalize | zero?           | ?*?               |
| ?**?          | ?+?            | ?-?           | -?              | ?=?               |
| ?^?           | ?~=?           | ?..?          |                 |                   |

— domain FR Factored —

```

)abbrev domain FR Factored
++ Author: Robert S. Sutor
++ Date Created: 1985
++ Change History:
++   21 Jan 1991  J Grabmeier    Corrected a bug in exquo.
++   16 Aug 1994  R S Sutor      Improved convert to InputForm
++ Basic Operations:
++   expand, exponent, factorList, factors, flagFactor, irreducibleFactor,
++   makeFR, map, nilFactor, nthFactor, nthFlag, numberOfFactors,
++   primeFactor, sqfrFactor, unit, unitNormalize,
++ Related Constructors: FactoredFunctionUtilities, FactoredFunctions2
++ Also See:
++ AMS Classifications: 11A51, 11Y05
++ Keywords: factorization, prime, square-free, irreducible, factor
++ References:
++ Description:
++ \spadtype{Factored} creates a domain whose objects are kept in
++ factored form as long as possible. Thus certain operations like
  
```

```

++ multiplication and gcd are relatively easy to do. Others, like
++ addition require somewhat more work, and unless the argument
++ domain provides a factor function, the result may not be
++ completely factored. Each object consists of a unit and a list of
++ factors, where a factor has a member of R (the "base"), and
++ exponent and a flag indicating what is known about the base. A
++ flag may be one of "nil", "sqfr", "irred" or "prime", which respectively mean
++ that nothing is known about the base, it is square-free, it is
++ irreducible, or it is prime. The current
++ restriction to integral domains allows simplification to be
++ performed without worrying about multiplication order.

```

```

Factored(R: IntegralDomain): Exports == Implementation where

```

```

  fUnion ==> Union("nil", "sqfr", "irred", "prime")
  FF      ==> Record(flag: fUnion, fctr: R, xpnt: Integer)
  SRFE    ==> Set(Record(factor:R, exponent:Integer))

```

```

Exports ==> Join(IntegralDomain, DifferentialExtension R, Algebra R,
  FullyEvaluableOver R, FullyRetractableTo R) with

```

```

expand: % -> R
++ expand(f) multiplies the unit and factors together, yielding an
++ "unfactored" object. Note: this is purposely not called
++ \spadfun{coerce} which would cause the interpreter to do this
++ automatically.
++
++X f:=nilFactor(y-x,3)
++X expand(f)

```

```

exponent: % -> Integer
++ exponent(u) returns the exponent of the first factor of
++ \spadvar{u}, or 0 if the factored form consists solely of a unit.
++
++X f:=nilFactor(y-x,3)
++X exponent(f)

```

```

makeFR : (R, List FF) -> %
++ makeFR(unit,listOfFactors) creates a factored object (for
++ use by factoring code).
++
++X f:=nilFactor(x-y,3)
++X g:=factorList f
++X makeFR(z,g)

```

```

factorList : % -> List FF
++ factorList(u) returns the list of factors with flags (for
++ use by factoring code).
++
++X f:=nilFactor(x-y,3)
++X factorList f

```



```

nilFactor: (R, Integer) -> %
++ nilFactor(base,exponent) creates a factored object with
++ a single factor with no information about the kind of
++ base (flag = "nil").
++
++X nilFactor(24,2)
++X nilFactor(x-y,3)

factors: % -> List Record(factor:R, exponent:Integer)
++ factors(u) returns a list of the factors in a form suitable
++ for iteration. That is, it returns a list where each element
++ is a record containing a base and exponent. The original
++ object is the product of all the factors and the unit (which
++ can be extracted by \axiom{unit(u)}).
++
++X f:=x*y^3-3*x^2*y^2+3*x^3*y-x^4
++X factors f
++X g:=makeFR(z,factorList f)
++X factors g

irreducibleFactor: (R, Integer) -> %
++ irreducibleFactor(base,exponent) creates a factored object with
++ a single factor whose base is asserted to be irreducible
++ (flag = "irred").
++
++X a:=irreducibleFactor(3,1)
++X nthFlag(a,1)

nthExponent: (% , Integer) -> Integer
++ nthExponent(u,n) returns the exponent of the nth factor of
++ \spadvar{u}. If \spadvar{n} is not a valid index for a factor
++ (for example, less than 1 or too big), 0 is returned.
++
++X a:=factor 9720000
++X nthExponent(a,2)

nthFactor: (% ,Integer) -> R
++ nthFactor(u,n) returns the base of the nth factor of
++ \spadvar{u}. If \spadvar{n} is not a valid index for a factor
++ (for example, less than 1 or too big), 1 is returned. If
++ \spadvar{u} consists only of a unit, the unit is returned.
++
++X a:=factor 9720000
++X nthFactor(a,2)

nthFlag: (% ,Integer) -> fUnion
++ nthFlag(u,n) returns the information flag of the nth factor of
++ \spadvar{u}. If \spadvar{n} is not a valid index for a factor
++ (for example, less than 1 or too big), "nil" is returned.
++

```

```

++X a:=factor 9720000
++X nthFlag(a,2)

numberOfFactors : % -> NonNegativeInteger
++ numberOfFactors(u) returns the number of factors in \spadvar{u}.
++
++X a:=factor 9720000
++X numberOfFactors a

primeFactor: (R,Integer) -> %
++ primeFactor(base,exponent) creates a factored object with
++ a single factor whose base is asserted to be prime
++ (flag = "prime").
++
++X a:=primeFactor(3,4)
++X nthFlag(a,1)

sqfrFactor: (R,Integer) -> %
++ sqfrFactor(base,exponent) creates a factored object with
++ a single factor whose base is asserted to be square-free
++ (flag = "sqfr").
++
++X a:=sqfrFactor(3,5)
++X nthFlag(a,1)

flagFactor: (R,Integer, fUnion) -> %
++ flagFactor(base,exponent,flag) creates a factored object with
++ a single factor whose base is asserted to be properly
++ described by the information flag.

unit: % -> R
++ unit(u) extracts the unit part of the factorization.
++
++X f:=x*y^3-3*x^2*y^2+3*x^3*y-x^4
++X unit f
++X g:=makeFR(z,factorList f)
++X unit g

unitNormalize: % -> %
++ unitNormalize(u) normalizes the unit part of the factorization.
++ For example, when working with factored integers, this operation will
++ ensure that the bases are all positive integers.

map: (R -> R, %) -> %
++ map(fn,u) maps the function \userfun{fn} across the factors of
++ \spadvar{u} and creates a new factored object. Note: this clears
++ the information flags (sets them to "nil") because the effect of
++ \userfun{fn} is clearly not known in general.
++
++X m(a:Factored Polynomial Integer):Factored Polynomial Integer == a^2

```

```

++X f:=x*y^3-3*x^2*y^2+3*x^3*y-x^4
++X map(m,f)
++X g:=makeFR(z,factorList f)
++X map(m,g)

-- the following operations are conditional on R

if R has GcdDomain then GcdDomain
if R has RealConstant then RealConstant
if R has UniqueFactorizationDomain then UniqueFactorizationDomain

if R has ConvertibleTo InputForm then ConvertibleTo InputForm

if R has IntegerNumberSystem then
  rational?      : % -> Boolean
  ++ rational?(u) tests if \spadvar{u} is actually a
  ++ rational number (see \spadtype{Fraction Integer}).
  rational       : % -> Fraction Integer
  ++ rational(u) assumes spadvar{u} is actually a rational number
  ++ and does the conversion to rational number
  ++ (see \spadtype{Fraction Integer}).
  rationalIfCan: % -> Union(Fraction Integer, "failed")
  ++ rationalIfCan(u) returns a rational number if u
  ++ really is one, and "failed" otherwise.

if R has Eltable(%, %) then Eltable(%, %)
if R has Evaluable(%) then Evaluable(%)
if R has InnerEvaluable(Symbol, %) then InnerEvaluable(Symbol, %)

Implementation ==> add

-- Representation:
-- Note: exponents are allowed to be integers so that some special cases
-- may be used in simplifications
Rep := Record(unt:R, fct:List FF)

if R has ConvertibleTo InputForm then
  convert(x:%):InputForm ==
    empty?(lf := reverse factorList x) => convert(unit x)@InputForm
    l := empty()$List(InputForm)
    for rec in lf repeat
--      one?(rec.fctr) => l
      ((rec.fctr) = 1) => l
      iFactor : InputForm := binary( convert("::" :: Symbol)@InputForm, [convert(rec.fctr)@InputForm]
      iExpon  : InputForm := convert(rec.xpnt)@InputForm
      iFun    : List InputForm :=
        rec.flg case "nil" =>
          [convert("nilFactor" :: Symbol)@InputForm, iFactor, iExpon]$List(InputForm)
        rec.flg case "sqfr" =>
          [convert("sqfrFactor" :: Symbol)@InputForm, iFactor, iExpon]$List(InputForm)

```

```

rec.flg case "prime" =>
  [convert("primeFactor" :: Symbol)@InputForm, iFactor, iExpon]$List(InputForm)
rec.flg case "irred" =>
  [convert("irreducibleFactor" :: Symbol)@InputForm, iFactor, iExpon]$List(InputForm)
  nil$List(InputForm)
  l := concat( iFun pretend InputForm, l )
-- one?(rec.xpnt) =>
--   l := concat(convert(rec.fctr)@InputForm, l)
--   l := concat(convert(rec.fctr)@InputForm ** rec.xpnt, l)
empty? l => convert(unit x)@InputForm
if unit x ^= 1 then l := concat(convert(unit x)@InputForm,l)
empty? rest l => first l
binary(convert(_::Symbol)@InputForm, l)@InputForm

orderedR? := R has OrderedSet

-- Private function signatures:
reciprocal      : % -> %
qexpand        : % -> R
negexp?        : % -> Boolean
SimplifyFactorization : List FF -> List FF
LispLessP      : (FF, FF) -> Boolean
mkFF           : (R, List FF) -> %
SimplifyFactorization1 : (FF, List FF) -> List FF
stricterFlag   : (fUnion, fUnion) -> fUnion

nilFactor(r, i)      == flagFactor(r, i, "nil")
sqfrFactor(r, i)     == flagFactor(r, i, "sqfr")
irreducibleFactor(r, i) == flagFactor(r, i, "irred")
primeFactor(r, i)    == flagFactor(r, i, "prime")
unit? u              == (empty? u.fct) and (not zero? u.unt)
factorList u         == u.fct
unit u               == u.unt
numberOfFactors u    == # u.fct
0                    == [1, [1, "nil", 0, 1]$FF]
zero? u              == # u.fct = 1 and
                      (first u.fct).flg case "nil" and
                      zero? (first u.fct).fctr and
--                      one? u.unt
                      (u.unt = 1)
1                    == [1, empty()]
one? u               == empty? u.fct and u.unt = 1
mkFF(r, x)           == [r, x]
coerce(j:Integer):%  == (j::R)::%
characteristic()     == characteristic()$R
i:Integer * u:%      == (i :: %) * u
r:R * u:%             == (r :: %) * u
factors u            == [[fe.fctr, fe.xpnt] for fe in factorList u]
expand u             == retract u
negexp? x            == "or"/[negative?(y.xpnt) for y in factorList x]

```

```

makeFR(u, l) ==
-- normalizing code to be installed when contents are handled better
-- current squareFree returns the content as a unit part.
--     if (not unit?(u)) then
--         l := cons(["nil", u, 1]$FF, l)
--         u := 1
--     unitNormalize mkFF(u, SimplifyFactorization l)

if R has IntegerNumberSystem then
    rational? x      == true
    rationalIfCan x == rational x

    rational x ==
        convert(unit x)@Integer *
        _*/[(convert(f.fctr)@Integer)::Fraction(Integer)
              ** f.xpnt for f in factorList x]

if R has Eltable(R, R) then
    elt(x:%, v:%) == x(expand v)

if R has Evaluable(R) then
    eval(x:%, l:List Equation %) ==
        eval(x, [expand lhs e = expand rhs e for e in l]$List(Equation R))

if R has InnerEvaluable(Symbol, R) then
    eval(x:%, ls:List Symbol, lv:List %) ==
        eval(x, ls, [expand v for v in lv]$List(R))

if R has RealConstant then
--! negcount and rest commented out since RealConstant doesn't support
--! positive? or negative?
-- negcount: % -> Integer
-- positive?(x:%):Boolean == not(zero? x) and even?(negcount x)
-- negative?(x:%):Boolean == not(zero? x) and odd?(negcount x)
-- negcount x ==
--     n := count(negative?(#1.fctr), factorList x)$List(FF)
--     negative? unit x => n + 1
--     n

    convert(x:%):Float ==
        convert(unit x)@Float *
        _*/[convert(f.fctr)@Float ** f.xpnt for f in factorList x]

    convert(x:%):DoubleFloat ==
        convert(unit x)@DoubleFloat *
        _*/[convert(f.fctr)@DoubleFloat ** f.xpnt for f in factorList x]

u:% * v:% ==
    zero? u or zero? v => 0

```

```

--      one? u => v
      (u = 1) => v
--      one? v => u
      (v = 1) => u
      mkFF(unit u * unit v,
        SimplifyFactorization concat(factorList u, copy factorList v))

u:% ** n:NonNegativeInteger ==
  mkFF(unit(u)**n, [[x.flg, x.fctr, n * x.xpnt] for x in factorList u])

SimplifyFactorization x ==
  empty? x => empty()
  x := sort_!(LispLessP, x)
  x := SimplifyFactorization1(first x, rest x)
  if orderedR? then x := sort_!(LispLessP, x)
  x

SimplifyFactorization1(f, x) ==
  empty? x =>
    zero?(f.xpnt) => empty()
    list f
  f1 := first x
  f.fctr = f1.fctr =>
    SimplifyFactorization1([stricterFlag(f.flg, f1.flg),
      f.fctr, f.xpnt + f1.xpnt], rest x)
  l := SimplifyFactorization1(first x, rest x)
  zero?(f.xpnt) => l
  concat(f, l)

coerce(x:%)::OutputForm ==
  empty?(lf := reverse factorList x) => (unit x)::OutputForm
  l := empty()$List(OutputForm)
  for rec in lf repeat
--      one?(rec.fctr) => l
      ((rec.fctr) = 1) => l
--      one?(rec.xpnt) =>
      ((rec.xpnt) = 1) =>
        l := concat(rec.fctr :: OutputForm, l)
        l := concat(rec.fctr::OutputForm ** rec.xpnt::OutputForm, l)
  empty? l => (unit x) :: OutputForm
  e :=
    empty? rest l => first l
    reduce(*, l)
  1 = unit x => e
  (unit x)::OutputForm * e

retract(u:%)::R ==
  negexp? u => error "Negative exponent in factored object"
  qexpand u

```

```

qexpand u ==
  unit u *
    _*/[y.fctr ** (y.xpnt::NonNegativeInteger) for y in factorList u]

retractIfCan(u:%):Union(R, "failed") ==
  negexp? u => "failed"
  qexpand u

LispLessP(y, y1) ==
  orderedR? => y.fctr < y1.fctr
  GGREATERP(y.fctr, y1.fctr)$Lisp => false
  true

stricterFlag(fl1, fl2) ==
  fl1 case "prime"   => fl1
  fl1 case "irred"   =>
    fl2 case "prime" => fl2
    fl1
  fl1 case "sqfr"    =>
    fl2 case "nil"   => fl1
    fl2
  fl2

if R has IntegerNumberSystem
then
  coerce(r:R):% ==
    factor(r)$IntegerFactorizationPackage(R) pretend %
else
  if R has UniqueFactorizationDomain
  then
    coerce(r:R):% ==
      zero? r => 0
      unit? r => mkFF(r, empty())
      unitNormalize(squareFree(r) pretend %)
  else
    coerce(r:R):% ==
--      one? r => 1
      (r = 1) => 1
      unitNormalize mkFF(1, [{"nil", r, 1}$FF])

u = v ==
  (unit u = unit v) and # u.fct = # v.fct and
  set(factors u)$SRFE == $SRFE set(factors v)$SRFE

- u ==
  zero? u => u
  mkFF(- unit u, factorList u)

recip u ==

```

```

not empty? factorList u => "failed"
(r := recip unit u) case "failed" => "failed"
mkFF(r::R, empty())

reciprocal u ==
  mkFF((recip unit u)::R,
    [[y.flg, y.fctr, - y.xpnt]$FF for y in factorList u])

exponent u == -- exponent of first factor
  empty?(fl := factorList u) or zero? u => 0
  first(fl).xpnt

nthExponent(u, i) ==
  l := factorList u
  zero? u or i < 1 or i > #l => 0
  (l.(minIndex(l) + i - 1)).xpnt

nthFactor(u, i) ==
  zero? u => 0
  zero? i => unit u
  l := factorList u
  negative? i or i > #l => 1
  (l.(minIndex(l) + i - 1)).fctr

nthFlag(u, i) ==
  l := factorList u
  zero? u or i < 1 or i > #l => "nil"
  (l.(minIndex(l) + i - 1)).flg

flagFactor(r, i, fl) ==
  zero? i => 1
  zero? r => 0
  unitNormalize mkFF(1, [[fl, r, i]$FF])

differentiate(u:%, deriv: R -> R) ==
  ans := deriv(unit u) * ((u exquo unit(u)::%):%)
  ans + (_+/[fact.xpnt * deriv(fact.fctr) *
    ((u exquo nilFactor(fact.fctr, 1)):%) for fact in factorList u])

map(fn, u) ==
  fn(unit u) * _*/[irreducibleFactor(fn(f.fctr),f.xpnt) for f in factorList u]

u exquo v ==
  empty?(x1 := factorList v) => unitNormal(retract v).associate * u
  empty? factorList u => "failed"
  v1 := u * reciprocal v
  goodQuotient:Boolean := true
  while (goodQuotient and (not empty? x1)) repeat
    if x1.first.xpnt < 0
      then goodQuotient := false

```



```

        else x1 := rest x1
    goodQuotient => v1
    "failed"

unitNormal u == -- does a bunch of work, but more canonical
    (ur := recip(un := unit u)) case "failed" => [1, u, 1]
    as := ur::R
    v1 := empty()$List(FF)
    for x in factorList u repeat
        ucar := unitNormal(x.fctr)
        e := abs(x.xpnt)::NonNegativeInteger
        if x.xpnt < 0
        then -- associate is recip of unit
            un := un * (ucar.associate ** e)
            as := as * (ucar.unit ** e)
        else
            un := un * (ucar.unit ** e)
            as := as * (ucar.associate ** e)
    --
    if not one?(ucar.canonical) then
    if not ((ucar.canonical) = 1) then
        v1 := concat([x.flg, ucar.canonical, x.xpnt], v1)
    [mkFF(un, empty()), mkFF(1, reverse_! v1), mkFF(as, empty())]

unitNormalize u ==
    uca := unitNormal u
    mkFF(unit(uca.unit)*unit(uca.canonical), factorList(uca.canonical))

if R has GcdDomain then
    u + v ==
        zero? u => v
        zero? v => u
        v1 := reciprocal(u1 := gcd(u, v))
        (expand(u * v1) + expand(v * v1)) * u1

gcd(u, v) ==
    --
    one? u or one? v => 1
    (u = 1) or (v = 1) => 1
    zero? u => v
    zero? v => u
    f1 := empty()$List(Integer) -- list of used factor indices in x
    f2 := f1 -- list of indices corresponding to a given factor
    f3 := empty()$List(List Integer) -- list of f2-like lists
    x := concat(factorList u, factorList v)
    for i in minIndex x .. maxIndex x repeat
        if not member?(i, f1) then
            f1 := concat(i, f1)
            f2 := [i]
        for j in i+1..maxIndex x repeat
            if x.i.fctr = x.j.fctr then
                f1 := concat(j, f1)

```

```

        f2 := concat(j, f2)
        f3 := concat(f2, f3)
x1 := empty()$List(FF)
while not empty? f3 repeat
  f1 := first f3
  if #f1 > 1 then
    i := first f1
    y := copy x.i
    f1 := rest f1
    while not empty? f1 repeat
      i := first f1
      if x.i.xpnt < y.xpnt then y.xpnt := x.i.xpnt
      f1 := rest f1
    x1 := concat(y, x1)
  f3 := rest f3
if orderedR? then x1 := sort_!(LispLessP, x1)
mkFF(1, x1)

else -- R not a GCD domain
  u + v ==
    zero? u => v
    zero? v => u
    irreducibleFactor(expand u + expand v, 1)

if R has UniqueFactorizationDomain then
  prime? u ==
    not(empty?(l := factorList u)) and (empty? rest l) and
--      one?(l.first.xpnt) and (l.first.flg case "prime")
      ((l.first.xpnt) = 1) and (l.first.flg case "prime")



---


— FR.dotabb —

"FR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FR"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"FR" -> "ALIST"



---



```

## 7.2 domain FILE File

— File.input —

```
)set break resume
```

```

)sys rm -f File.output
)spool File.output
)set message test on
)set message auto off
)clear all
--S 1 of 12
ifile:File List Integer:=open("jazz1","output")
--R
--R
--R (1) "jazz1"
--R
--R                                          Type: File List Integer
--E 1

--S 2 of 12
write!(ifile, [-1,2,3])
--R
--R
--R (2) [- 1,2,3]
--R
--R                                          Type: List Integer
--E 2

--S 3 of 12
write!(ifile, [10,-10,0,111])
--R
--R
--R (3) [10,- 10,0,111]
--R
--R                                          Type: List Integer
--E 3

--S 4 of 12
write!(ifile, [7])
--R
--R
--R (4) [7]
--R
--R                                          Type: List Integer
--E 4

--S 5 of 12
reopen!(ifile, "input")
--R
--R
--R (5) "jazz1"
--R
--R                                          Type: File List Integer
--E 5

--S 6 of 12
read! ifile
--R
--R
--R (6) [- 1,2,3]

```

[illegible]

---

```
=====
File examples
=====
```

Type: File List Integer

Type: List Integer

Type: List Integer

Type: List Integer

Type: File List Integer

```
read! ifile
```

```
[- 1,2,3]
```

Type: List Integer

```
read! ifile
[10,- 10,0,111]
```

Type: List Integer

The read operation can cause an error if one tries to read more data than is in the file. To guard against this possibility the readIfCan operation should be used.

```
readIfCan! ifile
[7]
```

Type: Union(List Integer,...)

```
readIfCan! ifile
"failed"
```

Type: Union("failed",...)

You can find the current mode of the file, and the file's name.

```
iomode ifile
"input"
```

Type: String

```
name ifile
"jazz1"
```

Type: FileName

When you are finished with a file, you should close it.

```
close! ifile
"jazz1"
```

Type: File List Integer

```
)system rm /tmp/jazz1
```

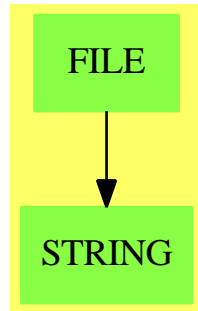
A limitation of the underlying LISP system is that not all values can be represented in a file. In particular, delayed values containing compiled functions cannot be saved.

See Also:

- o )help TextFile
- o )help KeyedAccessFile
- o )help Library
- o )help Filename
- o )show File

---

### 7.2.1 File (FILE)



See

- ⇒ “TextFile” (TEXTFILE) 21.5.1 on page 2651
- ⇒ “BinaryFile” (BINFILE) 3.8.1 on page 277
- ⇒ “KeyedAccessFile” (KAFfile) 12.2.1 on page 1377
- ⇒ “Library” (LIB) 13.2.1 on page 1392

#### Exports:

|         |        |       |            |        |
|---------|--------|-------|------------|--------|
| close!  | coerce | flush | hash       | iomode |
| latex   | name   | open  | readIfCan! | read!  |
| reopen! | write! | ?=?   | ?~=?       |        |

— domain **FILE** File —

```
)abbrev domain FILE File
++ Author: Stephen M. Watt, Victor Miller
++ Date Created: 1984
++ Date Last Updated: June 4, 1991
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ This domain provides a basic model of files to save arbitrary values.
++ The operations provide sequential access to the contents.

File(S:SetCategory): FileCategory(FileName, S) with
  readIfCan_!: % -> Union(S, "failed")
    ++ readIfCan!(f) returns a value from the file f, if possible.
    ++ If f is not open for reading, or if f is at the end of file
    ++ then \spad{"failed"} is the result.
```

```

== add
  FileState ==> SExpression
  IOMode     ==> String

Rep:=Record(fileName:  FileName,  _
             fileState: FileState, _
             fileIOMode: IOMode)

defstream(fn: FileName, mode: IOMode): FileState ==
  mode = "input" =>
    not readable? fn => error ["File is not readable", fn]
    MAKE_-INSTREAM(fn::String)$Lisp
  mode = "output" =>
    not writable? fn => error ["File is not writable", fn]
    MAKE_-OUTSTREAM(fn::String)$Lisp
  error ["IO mode must be input or output", mode]

f1 = f2 ==
  f1.fileName = f2.fileName
coerce(f: %): OutputForm ==
  f.fileName::OutputForm

open fname ==
  open(fname, "input")
open(fname, mode) ==
  fstream := defstream(fname, mode)
  [fname, fstream, mode]
reopen_!(f, mode) ==
  fname := f.fileName
  f.fileState := defstream(fname, mode)
  f.fileIOMode:= mode
  f
close_! f ==
  SHUT(f.fileState)$Lisp
  f.fileIOMode := "closed"
  f
name f ==
  f.fileName
iomode f ==
  f.fileIOMode
read_! f ==
  f.fileIOMode ^= "input" =>
    error "File not in read state"
  x := VMREAD(f.fileState)$Lisp
  PLACEP(x)$Lisp =>
    error "End of file"
  x
readIfCan_! f ==
  f.fileIOMode ^= "input" =>
    error "File not in read state"

```



```

x: S := VMREAD(f.fileState)$Lisp
PLACEP(x)$Lisp => "failed"
x
write_!(f, x) ==
  f.fileIOmode ^= "output" =>
    error "File not in write state"
  z := PRINT_-FULL(x, f.fileState)$Lisp
  TERPRI(f.fileState)$Lisp
x

flush f ==
  f.fileIOmode ^= "output" => error "File not in write state"
  FORCE_-OUTPUT(f.fileState)$Lisp

```

---

— FILE.dotabb —

```

"FILE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FILE"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"FILE" -> "STRING"

```

---

## 7.3 domain FNAME FileName

— FileName.input —

```

)set break resume
)sys rm -f FileName.output
)spool FileName.output
)set message test on
)set message auto off
)clear all
--S 1 of 18
fn: FileName
--R
--R
--E 1

```

Type: Void

```

--S 2 of 18
fn := "fname.input"
--R
--R

```

```

--R (2) "fname.input"
--R
--E 2
Type: FileName

--S 3 of 18
directory fn
--R
--R
--R (3) ""
--R
--E 3
Type: String

--S 4 of 18
name fn
--R
--R
--R (4) "fname"
--R
--E 4
Type: String

--S 5 of 18
extension fn
--R
--R
--R (5) "input"
--R
--E 5
Type: String

--S 6 of 18
fn := filename("/tmp", "fname", "input")
--R
--R
--R (6) "/tmp/fname.input"
--R
--E 6
Type: FileName

--S 7 of 18
objdir := "/tmp"
--R
--R
--R (7) "/tmp"
--R
--E 7
Type: String

--S 8 of 18
fn := filename(objdir, "table", "spad")
--R
--R
--R (8) "/tmp/table.spad"
--R
--E 8
Type: FileName

```

--E 8

--S 9 of 18

fn := filename("", "letter", "")

--R

--R

--R (9) "letter"

--R

Type: FileName

--E 9

--S 10 of 18

exists? "/etc/passwd"

--R

--R

--R (10) true

--R

Type: Boolean

--E 10

--S 11 of 18

readable? "/etc/passwd"

--R

--R

--R (11) true

--R

Type: Boolean

--E 11

--S 12 of 18

readable? "/etc/security/passwd"

--R

--R

--R (12) false

--R

Type: Boolean

--E 12

--S 13 of 18

readable? "/ect/passwd"

--R

--R

--R (13) false

--R

Type: Boolean

--E 13

--S 14 of 18

writable? "/etc/passwd"

--R

--R

--R (14) true

--R

Type: Boolean

--E 14

```

--S 15 of 18
writable? "/dev/null"
--R
--R
--R (15) true
--R
--R                                          Type: Boolean
--E 15

--S 16 of 18
writable? "/etc/DoesNotExist"
--R
--R
--R (16) true
--R
--R                                          Type: Boolean
--E 16

--S 17 of 18
writable? "/tmp/DoesNotExist"
--R
--R
--R (17) true
--R
--R                                          Type: Boolean
--E 17

--S 18 of 18
fn := new(objdir, "xxx", "yy")
--R
--R
--R (18)  "/tmp/xxx1419.yy"
--R
--R                                          Type: FileName
--E 18
)spool
)lisp (bye)

```

---

— **FileName.help** —

=====  
**FileName examples**  
=====

The FileName domain provides an interface to the computer's file system. Functions are provided to manipulate file names and to test properties of files.

The simplest way to use file names in the Axiom interpreter is to rely on conversion to and from strings. The syntax of these strings depends on the operating system.

```
fn: FileName
                                Type: Void
```

On Linux, this is a proper file syntax:

```
fn := "fname.input"
    "fname.input"
                                Type: FileName
```

Although it is very convenient to be able to use string notation for file names in the interpreter, it is desirable to have a portable way of creating and manipulating file names from within programs.

A measure of portability is obtained by considering a file name to consist of three parts: the directory, the name, and the extension.

```
directory fn
    ""
                                Type: String

name fn
    "fname"
                                Type: String

extension fn
    "input"
                                Type: String
```

The meaning of these three parts depends on the operating system. For example, on CMS the file "SPADPROF INPUT M" would have directory "M", name "SPADPROF" and extension "INPUT".

It is possible to create a filename from its parts.

```
fn := filename("/tmp", "fname", "input")
    "/tmp/fname.input"
                                Type: FileName
```

When writing programs, it is helpful to refer to directories via variables.

```
objdir := "/tmp"
        "/tmp"
                                Type: String

fn := filename(objdir, "table", "spad")
    "/tmp/table.spad"
                                Type: FileName
```

If the directory or the extension is given as an empty string, then a default is used. On AIX, the defaults are the current directory and no extension.

```
fn := filename("", "letter", "")
    "letter"
                                Type: FileName
```

Three tests provide information about names in the file system.

The exists? operation tests whether the named file exists.

```
exists? "/etc/passwd"
    true
                                Type: Boolean
```

The operation readable? tells whether the named file can be read. If the file does not exist, then it cannot be read.

```
readable? "/etc/passwd"
    true
                                Type: Boolean
```

```
readable? "/etc/security/passwd"
    false
                                Type: Boolean
```

```
readable? "/ect/passwd"
    false
                                Type: Boolean
```

Likewise, the operation writable? tells whether the named file can be written. If the file does not exist, the test is determined by the properties of the directory.

```
writable? "/etc/passwd"
    true
                                Type: Boolean
```

```
writable? "/dev/null"
    true
                                Type: Boolean
```

```
writable? "/etc/DoesNotExist"
    true
                                Type: Boolean
```

```
writable? "/tmp/DoesNotExist"
    true
                                Type: Boolean
```

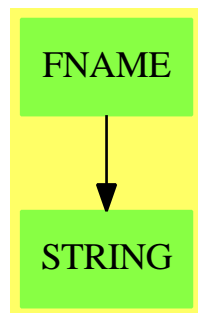
The new operation constructs the name of a new writable file. The argument sequence is the same as for filename, except that the name part is actually a prefix for a constructed unique name.

The resulting file is in the specified directory with the given extension, and the same defaults are used.

```
fn := new(objdir, "xxx", "yy")
    "/tmp/xxx1419.yy"
                                Type: FileName
```

See Also:  
o )show FileName

### 7.3.1 FileName (FNAME)



#### Exports:

|           |           |         |           |           |
|-----------|-----------|---------|-----------|-----------|
| coerce    | directory | exists? | extension | filename  |
| hash      | latex     | name    | new       | readable? |
| writable? | ?=?       | ?~=?    |           |           |

— domain FNAME FileName —

```
)abbrev domain FNAME FileName
++ Author: Stephen M. Watt
++ Date Created: 1985
++ Date Last Updated: June 20, 1991
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
```

```

++ Keywords:
++ Examples:
++ References:
++ Description:
++ This domain provides an interface to names in the file system.

```

```

FileName(): FileNameCategory == add

f1 = f2                                == EQUAL(f1, f2)$Lisp
coerce(f: %): OutputForm == f::String::OutputForm

coerce(f: %): String == NAMESTRING(f)$Lisp
coerce(s: String): % == PARSE_-NAMESTRING(s)$Lisp

filename(d,n,e) == fnameMake(d,n,e)$Lisp

directory(f:%): String == fnameDirectory(f)$Lisp
name(f:%): String == fnameName(f)$Lisp
extension(f:%): String == fnameType(f)$Lisp

exists? f == fnameExists?(f)$Lisp
readable? f == fnameReadable?(f)$Lisp
writable? f == fnameWritable?(f)$Lisp

new(d,pref,e) == fnameNew(d,pref,e)$Lisp

```

---

— FNAME.dotabb —

```

"FNAME" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FNAME"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"FNAME" -> "STRING"

```

---

## 7.4 domain FDIV FiniteDivisor

---

— FiniteDivisor.input —

```

)set break resume
)sys rm -f FiniteDivisor.output
)spool FiniteDivisor.output
)set message test on

```



```

)set message auto off
)clear all

--S 1 of 1
)show FiniteDivisor
--R FiniteDivisor(F: Field,UP: UnivariatePolynomialCategory F,UPUP: UnivariatePolynomialCategory F)
--R Abbreviation for FiniteDivisor is FDIV
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FDIV
--R
--R----- Operations -----
--R ?? : (Integer,%) -> %          ?? : (PositiveInteger,%) -> %
--R ?? : (%,%) -> %              ?-? : (%,%) -> %
--R -? : % -> %                  ?? : (%,%) -> Boolean
--R 0 : () -> %                  coerce : % -> OutputForm
--R divisor : (R,UP,UP,UP,F) -> % divisor : (F,F,Integer) -> %
--R divisor : (F,F) -> %         divisor : R -> %
--R finiteBasis : % -> Vector R   hash : % -> SingleInteger
--R lSpaceBasis : % -> Vector R   latex : % -> String
--R principal? : % -> Boolean     reduce : % -> %
--R sample : () -> %             zero? : % -> Boolean
--R ?? : (%,%) -> Boolean
--R ?? : (NonNegativeInteger,%) -> %
--R decompose : % -> Record(id: FractionalIdeal(UP,Fraction UP,UPUP,R),principalPart: R)
--R divisor : FractionalIdeal(UP,Fraction UP,UPUP,R) -> %
--R generator : % -> Union(R,"failed")
--R ideal : % -> FractionalIdeal(UP,Fraction UP,UPUP,R)
--R subtractIfCan : (%,%) -> Union(%, "failed")
--R
--E 1

)spool
)lisp (bye)

```

---

— FiniteDivisor.help —

```

=====
FiniteDivisor examples
=====

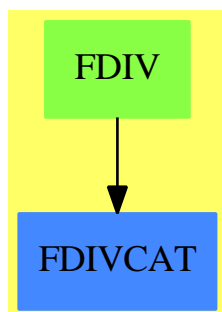
```

```

See Also:
o )show FiniteDivisor

```

## 7.4.1 FiniteDivisor (FDIV)



See

⇒ “FractionalIdeal” (FRIDEAL) 7.25.1 on page 961

⇒ “FramedModule” (FRMOD) 7.26.1 on page 967

⇒ “HyperellipticFiniteDivisor” (HELLFDIV) 9.11.1 on page 1149

**Exports:**

|            |        |           |               |             |
|------------|--------|-----------|---------------|-------------|
| 0          | coerce | decompose | divisor       | finiteBasis |
| generator  | hash   | ideal     | lSpaceBasis   | latex       |
| principal? | reduce | sample    | subtractIfCan | zero?       |
| ?~=?       | ?*?    | ?+?       | ?-?           | -?          |
| ?=?        |        |           |               |             |

— domain FDIV FiniteDivisor —

```
)abbrev domain FDIV FiniteDivisor
++ Author: Manuel Bronstein
++ Date Created: 1987
++ Date Last Updated: 29 July 1993
++ Description:
++ This domains implements finite rational divisors on a curve, that
++ is finite formal sums SUM(n * P) where the n's are integers and the
++ P's are finite rational points on the curve.
++ Keywords: divisor, algebraic, curve.
++ Examples: )r FDIV INPUT
```

```
FiniteDivisor(F, UP, UPUP, R): Exports == Implementation where
```

```
  F   : Field
  UP   : UnivariatePolynomialCategory F
  UPUP : UnivariatePolynomialCategory Fraction UP
  R    : FunctionFieldCategory(F, UP, UPUP)
```

```
N  ==> NonNegativeInteger
RF ==> Fraction UP
ID ==> FractionalIdeal(UP, RF, UPUP, R)
```

```
Exports ==> FiniteDivisorCategory(F, UP, UPUP, R) with
```

```

finiteBasis: % -> Vector R
  ++ finiteBasis(d) returns a basis for d as a module over K[x].
lSpaceBasis: % -> Vector R
  ++ lSpaceBasis(d) returns a basis for \spad{L(d) = {f | (f) >= -d}}
  ++ as a module over \spad{K[x]}.

Implementation ==> add
if hyperelliptic()$R case UP then
  Rep := HyperellipticFiniteDivisor(F, UP, UPUP, R)

  0 == 0$Rep
  coerce(d:$):OutputForm == coerce(d)$Rep
  d1 = d2 == d1 =$Rep d2
  n * d == n *$Rep d
  d1 + d2 == d1 +$Rep d2
  - d == -$Rep d
  ideal d == ideal(d)$Rep
  reduce d == reduce(d)$Rep
  generator d == generator(d)$Rep
  decompose d == decompose(d)$Rep
  divisor(i:ID) == divisor(i)$Rep
  divisor(f:R) == divisor(f)$Rep
  divisor(a, b) == divisor(a, b)$Rep
  divisor(a, b, n) == divisor(a, b, n)$Rep
  divisor(h, d, dp, g, r) == divisor(h, d, dp, g, r)$Rep
else
  Rep := Record(id:ID, fbasis:Vector(R))

  import CommonDenominator(UP, RF, Vector RF)
  import UnivariatePolynomialCommonDenominator(UP, RF, UPUP)

  makeDivisor : (UP, UPUP, UP) -> %
  intReduce : (R, UP) -> R

  ww := integralBasis()$R

  0 == [1, empty()]
  divisor(i:ID) == [i, empty()]
  divisor(f:R) == divisor ideal [f]
  coerce(d:$):OutputForm == ideal(d)::OutputForm
  ideal d == d.id
  decompose d == [ideal d, 1]
  d1 = d2 == basis(ideal d1) = basis(ideal d2)
  n * d == divisor(ideal(d) ** n)
  d1 + d2 == divisor(ideal d1 * ideal d2)
  - d == divisor inv ideal d
  divisor(h, d, dp, g, r) == makeDivisor(d, lift h - (r * dp)::RF::UPUP, g)

  intReduce(h, b) ==

```

```

v := integralCoordinates(h).num
integralRepresents(
    [qelt(v, i) rem b for i in minIndex v .. maxIndex v], 1)

divisor(a, b) ==
  x := monomial(1, 1)$UP
  not ground? gcd(d := x - a::UP, retract(discriminant())@UP) =>
    error "divisor: point is singular"
  makeDivisor(d, monomial(1, 1)$UPUP - b::UP::RF::UPUP, 1)

divisor(a, b, n) ==
  not ground? gcd(d := monomial(1, 1)$UP - a::UP,
    retract(discriminant())@UP)) and
    ((n exquo rank()) case "failed") =>
      error "divisor: point is singular"

m:N :=
  n < 0 => (-n)::N
  n::N
g := makeDivisor(d**m, (monomial(1, 1)$UPUP - b::UP::RF::UPUP)**m, 1)
n < 0 => -g
g

reduce d ==
  (i := minimize(j := ideal d)) = j => d
  #(n := numer i) ^= 2 => divisor i
  cd := splitDenominator lift n(1 + minIndex n)
  b := gcd(cd.den * retract(retract(n minIndex n)@RF)@UP,
    retract(norm reduce(cd.num))@UP)
  e := cd.den * denom i
  divisor ideal([(b / e)::R,
    reduce map((s:RF):RF+-->(retract(s)@UP rem b)/e, cd.num)]$Vector(R))

finiteBasis d ==
  if empty?(d.fbasis) then
    d.fbasis := normalizeAtInfinity
      basis module(ideal d)$FramedModule(UP, RF, UPUP, R, ww)
  d.fbasis

generator d ==
  bsis := finiteBasis d
  for i in minIndex bsis .. maxIndex bsis repeat
    integralAtInfinity? qelt(bsis, i) =>
      return primitivePart qelt(bsis, i)
  "failed"

lSpaceBasis d ==
  map_!(primitivePart, reduceBasisAtInfinity finiteBasis(-d))

-- b = center, hh = integral function, g = gcd(b, discriminant)
makeDivisor(b, hh, g) ==

```

```

b := gcd(b, retract(norm(h := reduce hh))@UP)
h := intReduce(h, b)
if not ground? gcd(g, b) then h := intReduce(h ** rank(), b)
divisor ideal [b::RF::R, h]$Vector(R)

```

---

— FDIV.dotabb —

```

"FDIV" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FDIV"]
"FDIVCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FDIVCAT"]
"FDIV" -> "FDIVCAT"

```

---

## 7.5 domain FF FiniteField

— FiniteField.input —

```

)set break resume
)sys rm -f FiniteField.output
)spool FiniteField.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FiniteField
--R FiniteField(p: PositiveInteger,n: PositiveInteger) is a domain constructor
--R Abbreviation for FiniteField is FF
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FF
--R
--R----- Operations -----
--R ??? : (PrimeField p,%) -> %          ??? : (%,PrimeField p) -> %
--R ??? : (Fraction Integer,%) -> %      ??? : (%,Fraction Integer) -> %
--R ??? : (%,%) -> %                    ??? : (Integer,%) -> %
--R ??? : (PositiveInteger,%) -> %        ??? : (%,Integer) -> %
--R ??? : (%,PositiveInteger) -> %        ??+ : (%,%) -> %
--R ??- : (%,%) -> %                    -? : % -> %
--R ??/? : (%,PrimeField p) -> %          ??/? : (%,%) -> %
--R ??=? : (%,%) -> Boolean                1 : () -> %
--R 0 : () -> %                            ??^ : (%,Integer) -> %
--R ??^ : (%,PositiveInteger) -> %        algebraic? : % -> Boolean

```

```

--R associates? : (%,%) -> Boolean
--R coerce : PrimeField p -> %
--R coerce : % -> %
--R coerce : % -> OutputForm
--R dimension : () -> CardinalNumber
--R gcd : List % -> %
--R hash : % -> SingleInteger
--R inv : % -> %
--R lcm : List % -> %
--R norm : % -> PrimeField p
--R prime? : % -> Boolean
--R recip : % -> Union(%, "failed")
--R retract : % -> PrimeField p
--R sizeLess? : (%,%) -> Boolean
--R squareFreePart : % -> %
--R transcendent? : % -> Boolean
--R unitCanonical : % -> %
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %
--R D : (%,NonNegativeInteger) -> % if PrimeField p has FINITE
--R D : % -> % if PrimeField p has FINITE
--R Frobenius : (%,NonNegativeInteger) -> % if PrimeField p has FINITE
--R Frobenius : % -> % if PrimeField p has FINITE
--R ?^? : (%,NonNegativeInteger) -> %
--R basis : PositiveInteger -> Vector %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%, "failed") if PrimeField p has CHARNZ or PrimeField p has FINITE
--R charthRoot : % -> % if PrimeField p has FINITE
--R conditionP : Matrix % -> Union(Vector %, "failed") if PrimeField p has FINITE
--R coordinates : Vector % -> Matrix PrimeField p
--R coordinates : % -> Vector PrimeField p
--R createNormalElement : () -> % if PrimeField p has FINITE
--R createPrimitiveElement : () -> % if PrimeField p has FINITE
--R definingPolynomial : () -> SparseUnivariatePolynomial PrimeField p
--R degree : % -> OnePointCompletion PositiveInteger
--R differentiate : (%,NonNegativeInteger) -> % if PrimeField p has FINITE
--R differentiate : % -> % if PrimeField p has FINITE
--R discreteLog : (%,%) -> Union(NonNegativeInteger, "failed") if PrimeField p has CHARNZ or PrimeField p has FINITE
--R discreteLog : % -> NonNegativeInteger if PrimeField p has FINITE
--R divide : (%,%) -> Record(quotient: %, remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %,%) -> Union(List %, "failed")
--R exquo : (%,%) -> Union(%, "failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %, coef2: %), "failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %, coef2: %, generator: %)
--R extensionDegree : () -> PositiveInteger
--R extensionDegree : () -> OnePointCompletion PositiveInteger
--R factorsOfCyclicGroupSize : () -> List Record(factor: Integer, exponent: Integer) if PrimeField p has FINITE
--R gcdPolynomial : (SparseUnivariatePolynomial %, SparseUnivariatePolynomial %) -> SparseUnivariatePolynomial PrimeField p
--R gcdPolynomial : (SparseUnivariatePolynomial %, SparseUnivariatePolynomial %) -> SparseUnivariatePolynomial PrimeField p
--R basis : () -> Vector %
--R coerce : Fraction Integer -> %
--R coerce : Integer -> %
--R degree : % -> PositiveInteger
--R factor : % -> Factored %
--R gcd : (%,%) -> %
--R inGroundField? : % -> Boolean
--R latex : % -> String
--R lcm : (%,%) -> %
--R one? : % -> Boolean
--R ?quo? : (%,%) -> %
--R ?rem? : (%,%) -> %
--R sample : () -> %
--R squareFree : % -> Factored %
--R trace : % -> PrimeField p
--R unit? : % -> Boolean
--R zero? : % -> Boolean

```

```

--R generator : () -> % if PrimeField p has FINITE
--R index : PositiveInteger -> % if PrimeField p has FINITE
--R init : () -> % if PrimeField p has FINITE
--R linearAssociatedExp : (%,SparseUnivariatePolynomial PrimeField p) -> % if PrimeField p has FINITE
--R linearAssociatedLog : (%,%) -> Union(SparseUnivariatePolynomial PrimeField p,"failed") if PrimeField p has FINITE
--R linearAssociatedLog : % -> SparseUnivariatePolynomial PrimeField p if PrimeField p has FINITE
--R linearAssociatedOrder : % -> SparseUnivariatePolynomial PrimeField p if PrimeField p has FINITE
--R lookup : % -> PositiveInteger if PrimeField p has FINITE
--R minimalPolynomial : (%,PositiveInteger) -> SparseUnivariatePolynomial % if PrimeField p has FINITE
--R minimalPolynomial : % -> SparseUnivariatePolynomial PrimeField p
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R nextItem : % -> Union(%,"failed") if PrimeField p has FINITE
--R norm : (%,PositiveInteger) -> % if PrimeField p has FINITE
--R normal? : % -> Boolean if PrimeField p has FINITE
--R normalElement : () -> % if PrimeField p has FINITE
--R order : % -> OnePointCompletion PositiveInteger if PrimeField p has CHARNZ or PrimeField p has FINITE
--R order : % -> PositiveInteger if PrimeField p has FINITE
--R primeFrobenius : % -> % if PrimeField p has CHARNZ or PrimeField p has FINITE
--R primeFrobenius : (%,NonNegativeInteger) -> % if PrimeField p has CHARNZ or PrimeField p has FINITE
--R primitive? : % -> Boolean if PrimeField p has FINITE
--R primitiveElement : () -> % if PrimeField p has FINITE
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R random : () -> % if PrimeField p has FINITE
--R representationType : () -> Union("prime",polynomial,normal,cyclic) if PrimeField p has FINITE
--R represents : Vector PrimeField p -> %
--R retractIfCan : % -> Union(PrimeField p,"failed")
--R size : () -> NonNegativeInteger if PrimeField p has FINITE
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger,NonNegativeInteger) if PrimeField p has FINITE
--R trace : (%,PositiveInteger) -> % if PrimeField p has FINITE
--R transcendenceDegree : () -> NonNegativeInteger
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)

```

---

— FiniteField.help —

```

=====
FiniteField examples
=====

```

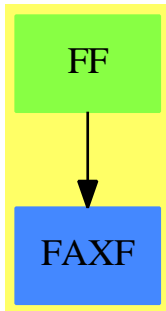
```

See Also:
o )show FiniteField

```

---

### 7.5.1 FiniteField (FF)



See

- ⇒ “FiniteFieldExtensionByPolynomial” (FFP) 7.10.1 on page 818
- ⇒ “FiniteFieldExtension” (FFX) 7.9.1 on page 813
- ⇒ “InnerFiniteField” (IFF) 10.21.1 on page 1247



**Exports:**

|                           |                          |                        |
|---------------------------|--------------------------|------------------------|
| 0                         | 1                        | algebraic?             |
| associates?               | basis                    | characteristic         |
| charthRoot                | coerce                   | conditionP             |
| coordinates               | createNormalElement      | createPrimitiveElement |
| D                         | definingPolynomial       | degree                 |
| differentiate             | dimension                | discreteLog            |
| divide                    | euclideanSize            | expressIdealMember     |
| exquo                     | extendedEuclidean        | extensionDegree        |
| factor                    | factorsOfCyclicGroupSize | Frobenius              |
| gcd                       | gcdPolynomial            | generator              |
| hash                      | index                    | inGroundField?         |
| init                      | inv                      | latex                  |
| lcm                       | linearAssociatedExp      | linearAssociatedLog    |
| linearAssociatedOrder     | lookup                   | minimalPolynomial      |
| multiEuclidean            | nextItem                 | norm                   |
| normal?                   | normalElement            | one?                   |
| order                     | prime?                   | primeFrobenius         |
| primitive?                | primitiveElement         | principalIdeal         |
| random                    | recip                    | representationType     |
| represents                | retract                  | retractIfCan           |
| sample                    | size                     | sizeLess?              |
| squareFree                | squareFreePart           | subtractIfCan          |
| tableForDiscreteLogarithm | trace                    | transcendenceDegree    |
| transcendent?             | unit?                    | unitCanonical          |
| unitNormal                | zero?                    | ?*                     |
| ?**?                      | ?+?                      | ?-?                    |
| -?                        | ?/?                      | ?=?                    |
| ?^?                       | ?~=?                     | ?quo?                  |
| ?rem?                     |                          |                        |

— domain FF FiniteField —

```

)abbrev domain FF FiniteField
++ Author: Mark Botch
++ Date Created: ???
++ Date Last Updated: 29 May 1990
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords: field, extension field, algebraic extension,
++ finite extension, finite field, Galois field
++ Reference:
++ R.Lidl, H.Niederreiter: Finite Field, Encyclopedia of Mathematics an
++ Its Applications, Vol. 20, Cambridge Univ. Press, 1983, ISBN 0 521 30240 4
++ J. Grabmeier, A. Scheerhorn: Finite Fields in AXIOM.

```

```

++ AXIOM Technical Report Series, ATR/5 NP2522.
++ Description:
++ FiniteField(p,n) implements finite fields with p**n elements.
++ This packages checks that p is prime.
++ For a non-checking version, see \spadtype{InnerFiniteField}.

FiniteField(p:PositiveInteger, n:PositiveInteger): _
  FiniteAlgebraicExtensionField(PrimeField p) ==_
  FiniteFieldExtensionByPolynomial(PrimeField p,_
    createIrreduciblePoly(n)$FiniteFieldPolynomialPackage(PrimeField p))
-- old code for generating irreducible polynomials:
-- now "better" order (sparse polys first)
-- generateIrredPoly(n)$IrredPolyOverFiniteField(GF))

```

---

— FF.dotabb —

```

"FF" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FF"]
"FAXF" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FAXF"]
"FF" -> "FAXF"

```

---

## 7.6 domain FFCG FiniteFieldCyclicGroup

— FiniteFieldCyclicGroup.input —

```

)set break resume
)sys rm -f FiniteFieldCyclicGroup.output
)spool FiniteFieldCyclicGroup.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FiniteFieldCyclicGroup
--R FiniteFieldCyclicGroup(p: PositiveInteger,extdeg: PositiveInteger) is a domain constructor
--R Abbreviation for FiniteFieldCyclicGroup is FFCG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FFCG
--R
--R----- Operations -----
--R ?? : (PrimeField p,%) -> %          ?? : (%,PrimeField p) -> %

```

```

--R ?? : (Fraction Integer,%) -> %
--R ?? : (%,%) -> %
--R ?? : (PositiveInteger,%) -> %
--R ??? : (%,PositiveInteger) -> %
--R ?-? : (%,%) -> %
--R ?/? : (%,PrimeField p) -> %
--R ?=? : (%,%) -> Boolean
--R 0 : () -> %
--R ?? : (%,PositiveInteger) -> %
--R associates? : (%,%) -> Boolean
--R coerce : PrimeField p -> %
--R coerce : % -> %
--R coerce : % -> OutputForm
--R dimension : () -> CardinalNumber
--R gcd : List % -> %
--R hash : % -> SingleInteger
--R inv : % -> %
--R lcm : List % -> %
--R norm : % -> PrimeField p
--R prime? : % -> Boolean
--R recip : % -> Union(%, "failed")
--R retract : % -> PrimeField p
--R sizeLess? : (%,%) -> Boolean
--R squareFreePart : % -> %
--R transcendent? : % -> Boolean
--R unitCanonical : % -> %
--R ?~=? : (%,%) -> Boolean
--R ?? : (NonNegativeInteger,%) -> %
--R ??? : (%,NonNegativeInteger) -> %
--R D : (%,NonNegativeInteger) -> % if PrimeField p has FINITE
--R D : % -> % if PrimeField p has FINITE
--R Frobenius : (%,NonNegativeInteger) -> % if PrimeField p has FINITE
--R Frobenius : % -> % if PrimeField p has FINITE
--R ?? : (%,NonNegativeInteger) -> %
--R basis : PositiveInteger -> Vector %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%, "failed") if PrimeField p has CHARNZ or PrimeField p has FINITE
--R charthRoot : % -> % if PrimeField p has FINITE
--R conditionP : Matrix % -> Union(Vector %, "failed") if PrimeField p has FINITE
--R coordinates : Vector % -> Matrix PrimeField p
--R coordinates : % -> Vector PrimeField p
--R createNormalElement : () -> % if PrimeField p has FINITE
--R createPrimitiveElement : () -> % if PrimeField p has FINITE
--R definingPolynomial : () -> SparseUnivariatePolynomial PrimeField p
--R degree : % -> OnePointCompletion PositiveInteger
--R differentiate : (%,NonNegativeInteger) -> % if PrimeField p has FINITE
--R differentiate : % -> % if PrimeField p has FINITE
--R discreteLog : (%,%) -> Union(NonNegativeInteger, "failed") if PrimeField p has CHARNZ or FINITE
--R discreteLog : % -> NonNegativeInteger if PrimeField p has FINITE
--R divide : (%,%) -> Record(quotient: %, remainder: %)

?? : (%,Fraction Integer) -> %
?? : (Integer,%) -> %
??? : (%,Integer) -> %
?+? : (%,%) -> %
-? : % -> %
?/? : (%,%) -> %
1 : () -> %
?? : (%,Integer) -> %
algebraic? : % -> Boolean
basis : () -> Vector %
coerce : Fraction Integer -> %
coerce : Integer -> %
degree : % -> PositiveInteger
factor : % -> Factored %
gcd : (%,%) -> %
inGroundField? : % -> Boolean
latex : % -> String
lcm : (%,%) -> %
one? : % -> Boolean
?quo? : (%,%) -> %
?rem? : (%,%) -> %
sample : () -> %
squareFree : % -> Factored %
trace : % -> PrimeField p
unit? : % -> Boolean
zero? : % -> Boolean

```

```

--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R extensionDegree : () -> PositiveInteger
--R extensionDegree : () -> OnePointCompletion PositiveInteger
--R factorsOfCyclicGroupSize : () -> List Record(factor: Integer,exponent: Integer) if PrimeField p has
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolyn
--R generator : () -> % if PrimeField p has FINITE
--R getZechTable : () -> PrimitiveArray SingleInteger
--R index : PositiveInteger -> % if PrimeField p has FINITE
--R init : () -> % if PrimeField p has FINITE
--R linearAssociatedExp : (%,SparseUnivariatePolynomial PrimeField p) -> % if PrimeField p has FINITE
--R linearAssociatedLog : (%,%) -> Union(SparseUnivariatePolynomial PrimeField p,"failed") if PrimeField
--R linearAssociatedLog : % -> SparseUnivariatePolynomial PrimeField p if PrimeField p has FINITE
--R linearAssociatedOrder : % -> SparseUnivariatePolynomial PrimeField p if PrimeField p has FINITE
--R lookup : % -> PositiveInteger if PrimeField p has FINITE
--R minimalPolynomial : (%,PositiveInteger) -> SparseUnivariatePolynomial % if PrimeField p has FINITE
--R minimalPolynomial : % -> SparseUnivariatePolynomial PrimeField p
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R nextItem : % -> Union(%,"failed") if PrimeField p has FINITE
--R norm : (%,PositiveInteger) -> % if PrimeField p has FINITE
--R normal? : % -> Boolean if PrimeField p has FINITE
--R normalElement : () -> % if PrimeField p has FINITE
--R order : % -> OnePointCompletion PositiveInteger if PrimeField p has CHARNZ or PrimeField p has FINIT
--R order : % -> PositiveInteger if PrimeField p has FINITE
--R primeFrobenius : % -> % if PrimeField p has CHARNZ or PrimeField p has FINITE
--R primeFrobenius : (%,NonNegativeInteger) -> % if PrimeField p has CHARNZ or PrimeField p has FINITE
--R primitive? : % -> Boolean if PrimeField p has FINITE
--R primitiveElement : () -> % if PrimeField p has FINITE
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R random : () -> % if PrimeField p has FINITE
--R representationType : () -> Union("prime",polynomial,normal,cyclic) if PrimeField p has FINITE
--R represents : Vector PrimeField p -> %
--R retractIfCan : % -> Union(PrimeField p,"failed")
--R size : () -> NonNegativeInteger if PrimeField p has FINITE
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger,NonNegativeInteger) if PrimeField p has
--R trace : (%,PositiveInteger) -> % if PrimeField p has FINITE
--R transcendenceDegree : () -> NonNegativeInteger
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)

```

---

## — FiniteFieldCyclicGroup.help —

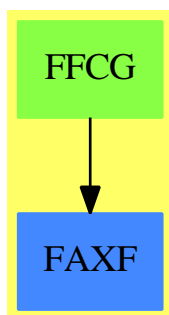
```
=====
FiniteFieldCyclicGroup examples
=====
```

See Also:

- o )show FiniteFieldCyclicGroup

---

## 7.6.1 FiniteFieldCyclicGroup (FFCG)



See

⇒ “FiniteFieldCyclicGroupExtensionByPolynomial” (FFCGP) 7.8.1 on page 802

⇒ “FiniteFieldCyclicGroupExtension” (FFCGX) 7.7.1 on page 797

**Exports:**

|                     |                           |                        |
|---------------------|---------------------------|------------------------|
| 0                   | 1                         | algebraic?             |
| associates?         | basis                     | characteristic         |
| charthRoot          | coerce                    | conditionP             |
| coordinates         | createNormalElement       | createPrimitiveElement |
| D                   | definingPolynomial        | degree                 |
| differentiate       | dimension                 | discreteLog            |
| divide              | euclideanSize             | expressIdealMember     |
| exquo               | extendedEuclidean         | extensionDegree        |
| factor              | factorsOfCyclicGroupSize  | Frobenius              |
| gcd                 | gcdPolynomial             | generator              |
| getZechTable        | hash                      | index                  |
| inGroundField?      | init                      | inv                    |
| latex               | lcm                       | linearAssociatedExp    |
| linearAssociatedLog | linearAssociatedOrder     | lookup                 |
| minimalPolynomial   | multiEuclidean            | nextItem               |
| norm                | normal?                   | normalElement          |
| one?                | order                     | prime?                 |
| primeFrobenius      | primitive?                | primitiveElement       |
| principalIdeal      | random                    | recip                  |
| representationType  | represents                | retract                |
| retractIfCan        | sample                    | size                   |
| sizeLess?           | squareFree                | squareFreePart         |
| subtractIfCan       | tableForDiscreteLogarithm | trace                  |
| transcendenceDegree | transcendent?             | unit?                  |
| unitCanonical       | unitNormal                | zero?                  |
| ?*?                 | ?**?                      | ?+?                    |
| ?-?                 | -?                        | ?/?                    |
| ?=?                 | ?^?                       | ?~=?                   |
| ?quo?               | ?rem?                     |                        |

— domain **FFCG** **FiniteFieldCyclicGroup** —

```

)abbrev domain FFCG FiniteFieldCyclicGroup
++ Authors: J.Grabmeier, A.Scheerhorn
++ Date Created: 04.04.1991
++ Date Last Updated:
++ Basic Operations:
++ Related Constructors: FiniteFieldCyclicGroupExtensionByPolynomial,
++ FiniteFieldPolynomialPackage
++ Also See: FiniteField, FiniteFieldNormalBasis
++ AMS Classifications:
++ Keywords: finite field, primitive elements, cyclic group
++ References:
++ R.Lidl, H.Niederreiter: Finite Field, Encycoldia of Mathematics and
++ Its Applications, Vol. 20, Cambridge Univ. Press, 1983, ISBN 0 521 30240 4
++ Description:

```

```

++ FiniteFieldCyclicGroup(p,n) implements a finite field extension of degree n
++ over the prime field with p elements. Its elements are represented by
++ powers of a primitive element, i.e. a generator of the multiplicative
++ (cyclic) group. As primitive element we choose the root of the extension
++ polynomial, which is created by createPrimitivePoly from
++ \spadtype{FiniteFieldPolynomialPackage}. The Zech logarithms are stored
++ in a table of size half of the field size, and use \spadtype{SingleInteger}
++ for representing field elements, hence, there are restrictions
++ on the size of the field.

FiniteFieldCyclicGroup(p,extdeg):_
  Exports == Implementation where
  p : PositiveInteger
  extdeg : PositiveInteger
  PI      ==> PositiveInteger
  FFPOLY  ==> FiniteFieldPolynomialPackage(PrimeField(p))
  SI      ==> SingleInteger
  Exports ==> FiniteAlgebraicExtensionField(PrimeField(p)) with
    getZechTable:() -> PrimitiveArray(SingleInteger)
    ++ getZechTable() returns the zech logarithm table of the field.
    ++ This table is used to perform additions in the field quickly.
  Implementation ==> FiniteFieldCyclicGroupExtensionByPolynomial(PrimeField(p),_
    createPrimitivePoly(extdeg)$FFPOLY)

```

---

— FFCG.dotabb —

```

"FFCG" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FFCG"]
"FAXF" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FAXF"]
"FFCG" -> "FAXF"

```

## 7.7 domain FFCGX FiniteFieldCyclicGroupExtension

---

— FiniteFieldCyclicGroupExtension.input —

```

)set break resume
)sys rm -f FiniteFieldCyclicGroupExtension.output
)spool FiniteFieldCyclicGroupExtension.output
)set message test on
)set message auto off
)clear all

```

```

--S 1 of 1
)show FiniteFieldCyclicGroupExtension
--R FiniteFieldCyclicGroupExtension(GF: FiniteFieldCategory,extdeg: PositiveInteger) is a domain constr
--R Abbreviation for FiniteFieldCyclicGroupExtension is FFCGX
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FFCGX
--R
--R----- Operations -----
--R ?? : (GF,%) -> %                ?? : (%,GF) -> %
--R ?? : (Fraction Integer,%) -> %  ?? : (%,Fraction Integer) -> %
--R ?? : (%,%) -> %                ?? : (Integer,%) -> %
--R ?? : (PositiveInteger,%) -> %    ??? : (%,Integer) -> %
--R ??? : (%,PositiveInteger) -> %  ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                -? : % -> %
--R ?/? : (%,GF) -> %               ?/? : (%,%) -> %
--R ?? : (%,%) -> Boolean           D : % -> % if GF has FINITE
--R 1 : () -> %                     0 : () -> %
--R ?? : (%,Integer) -> %           ?? : (%,PositiveInteger) -> %
--R algebraic? : % -> Boolean       associates? : (%,%) -> Boolean
--R basis : () -> Vector %          coerce : GF -> %
--R coerce : Fraction Integer -> %  coerce : % -> %
--R coerce : Integer -> %           coerce : % -> OutputForm
--R coordinates : % -> Vector GF     degree : % -> PositiveInteger
--R dimension : () -> CardinalNumber factor : % -> Factored %
--R gcd : List % -> %               gcd : (%,%) -> %
--R hash : % -> SingleInteger        inGroundField? : % -> Boolean
--R inv : % -> %                    latex : % -> String
--R lcm : List % -> %               lcm : (%,%) -> %
--R norm : % -> GF                  one? : % -> Boolean
--R prime? : % -> Boolean            ?quo? : (%,%) -> %
--R recip : % -> Union(%, "failed") ?rem? : (%,%) -> %
--R represents : Vector GF -> %     retract : % -> GF
--R sample : () -> %                sizeLess? : (%,%) -> Boolean
--R squareFree : % -> Factored %     squareFreePart : % -> %
--R trace : % -> GF                  transcendent? : % -> Boolean
--R unit? : % -> Boolean             unitCanonical : % -> %
--R zero? : % -> Boolean             ?~=? : (%,%) -> Boolean
--R ?? : (NonNegativeInteger,%) -> %
--R ??? : (%,NonNegativeInteger) -> %
--R D : (%,NonNegativeInteger) -> % if GF has FINITE
--R Frobenius : (%,NonNegativeInteger) -> % if GF has FINITE
--R Frobenius : % -> % if GF has FINITE
--R ?? : (%,NonNegativeInteger) -> %
--R basis : PositiveInteger -> Vector %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%, "failed") if GF has CHARNZ or GF has FINITE
--R charthRoot : % -> % if GF has FINITE
--R conditionP : Matrix % -> Union(Vector %, "failed") if GF has FINITE
--R coordinates : Vector % -> Matrix GF

```



```

--R createNormalElement : () -> % if GF has FINITE
--R createPrimitiveElement : () -> % if GF has FINITE
--R definingPolynomial : () -> SparseUnivariatePolynomial GF
--R degree : % -> OnePointCompletion PositiveInteger
--R differentiate : (% , NonNegativeInteger) -> % if GF has FINITE
--R differentiate : % -> % if GF has FINITE
--R discreteLog : (% , %) -> Union(NonNegativeInteger, "failed") if GF has CHARNZ or GF has FINITE
--R discreteLog : % -> NonNegativeInteger if GF has FINITE
--R divide : (% , %) -> Record(quotient: %, remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %, %) -> Union(List %, "failed")
--R exquo : (% , %) -> Union(%, "failed")
--R extendedEuclidean : (% , %, %) -> Union(Record(coef1: %, coef2: %), "failed")
--R extendedEuclidean : (% , %) -> Record(coef1: %, coef2: %, generator: %)
--R extensionDegree : () -> PositiveInteger
--R extensionDegree : () -> OnePointCompletion PositiveInteger
--R factorsOfCyclicGroupSize : () -> List Record(factor: Integer, exponent: Integer) if GF has FINITE
--R gcdPolynomial : (SparseUnivariatePolynomial %, SparseUnivariatePolynomial %) -> SparseUnivariatePolynomial GF
--R generator : () -> % if GF has FINITE
--R getZechTable : () -> PrimitiveArray SingleInteger
--R index : PositiveInteger -> % if GF has FINITE
--R init : () -> % if GF has FINITE
--R linearAssociatedExp : (% , SparseUnivariatePolynomial GF) -> % if GF has FINITE
--R linearAssociatedLog : (% , %) -> Union(SparseUnivariatePolynomial GF, "failed") if GF has FINITE
--R linearAssociatedLog : % -> SparseUnivariatePolynomial GF if GF has FINITE
--R linearAssociatedOrder : % -> SparseUnivariatePolynomial GF if GF has FINITE
--R lookup : % -> PositiveInteger if GF has FINITE
--R minimalPolynomial : (% , PositiveInteger) -> SparseUnivariatePolynomial % if GF has FINITE
--R minimalPolynomial : % -> SparseUnivariatePolynomial GF
--R multiEuclidean : (List %, %) -> Union(List %, "failed")
--R nextItem : % -> Union(%, "failed") if GF has FINITE
--R norm : (% , PositiveInteger) -> % if GF has FINITE
--R normal? : % -> Boolean if GF has FINITE
--R normalElement : () -> % if GF has FINITE
--R order : % -> OnePointCompletion PositiveInteger if GF has CHARNZ or GF has FINITE
--R order : % -> PositiveInteger if GF has FINITE
--R primeFrobenius : % -> % if GF has CHARNZ or GF has FINITE
--R primeFrobenius : (% , NonNegativeInteger) -> % if GF has CHARNZ or GF has FINITE
--R primitive? : % -> Boolean if GF has FINITE
--R primitiveElement : () -> % if GF has FINITE
--R principalIdeal : List % -> Record(coef: List %, generator: %)
--R random : () -> % if GF has FINITE
--R representationType : () -> Union("prime", polynomial, normal, cyclic) if GF has FINITE
--R retractIfCan : % -> Union(GF, "failed")
--R size : () -> NonNegativeInteger if GF has FINITE
--R subtractIfCan : (% , %) -> Union(%, "failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger, NonNegativeInteger) if GF has FINITE
--R trace : (% , PositiveInteger) -> % if GF has FINITE
--R transcendenceDegree : () -> NonNegativeInteger
--R unitNormal : % -> Record(unit: %, canonical: %, associate: %)

```

```
--R
--E 1

)spool
)lisp (bye)
```

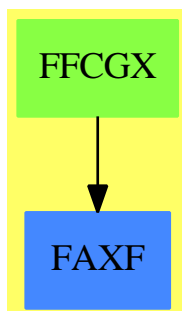
---

— FiniteFieldCyclicGroupExtension.help —

```
=====
FiniteFieldCyclicGroupExtension examples
=====
```

```
See Also:
o )show FiniteFieldCyclicGroupExtension
```

### 7.7.1 FiniteFieldCyclicGroupExtension (FFCGX)



See

⇒ “FiniteFieldCyclicGroupExtensionByPolynomial” (FFCGP) 7.8.1 on page 802  
 ⇒ “FiniteFieldCyclicGroup” (FFCG) 7.6.1 on page 792

**Exports:**

|                     |                           |                        |
|---------------------|---------------------------|------------------------|
| 0                   | 1                         | algebraic?             |
| associates?         | basis                     | characteristic         |
| charthRoot          | coerce                    | conditionP             |
| coordinates         | createNormalElement       | createPrimitiveElement |
| D                   | definingPolynomial        | degree                 |
| differentiate       | dimension                 | discreteLog            |
| divide              | euclideanSize             | expressIdealMember     |
| exquo               | extendedEuclidean         | extensionDegree        |
| factor              | factorsOfCyclicGroupSize  | Frobenius              |
| gcd                 | gcdPolynomial             | generator              |
| getZechTable        | hash                      | index                  |
| inGroundField?      | init                      | inv                    |
| latex               | lcm                       | linearAssociatedExp    |
| linearAssociatedLog | linearAssociatedOrder     | lookup                 |
| minimalPolynomial   | multiEuclidean            | nextItem               |
| norm                | normal?                   | normalElement          |
| one?                | order                     | prime?                 |
| primeFrobenius      | primitive?                | primitiveElement       |
| principalIdeal      | random                    | recip                  |
| representationType  | represents                | retract                |
| retractIfCan        | sample                    | size                   |
| sizeLess?           | squareFree                | squareFreePart         |
| subtractIfCan       | tableForDiscreteLogarithm | trace                  |
| transcendenceDegree | transcendent?             | unit?                  |
| unitCanonical       | unitNormal                | zero?                  |
| ?*?                 | ?**?                      | ?+?                    |
| ?-?                 | -?                        | ?/?                    |
| ?=?                 | ?^?                       | ?~=?                   |
| ?quo?               | ?rem?                     |                        |

— domain **FFCGX FiniteFieldCyclicGroupExtension** —

```

)abbrev domain FFCGX FiniteFieldCyclicGroupExtension
++ Authors: J.Grabmeier, A.Scheerhorn
++ Date Created: 04.04.1991
++ Date Last Updated:
++ Basic Operations:
++ Related Constructors: FiniteFieldCyclicGroupExtensionByPolynomial,
++ FiniteFieldPolynomialPackage
++ Also See: FiniteFieldExtension, FiniteFieldNormalBasisExtension
++ AMS Classifications:
++ Keywords: finite field, primitive elements, cyclic group
++ References:
++ R.Lidl, H.Niederreiter: Finite Field, Encycoldia of Mathematics and
++ Its Applications, Vol. 20, Cambridge Univ. Press, 1983, ISBN 0 521 30240 4
++ J. Grabmeier, A. Scheerhorn: Finite Fields in AXIOM.

```

```

++ AXIOM Technical Report Series, ATR/5 NP2522.
++ Description:
++ FiniteFieldCyclicGroupExtension(GF,n) implements a extension of degree n
++ over the ground field GF. Its elements are represented by powers of
++ a primitive element, i.e. a generator of the multiplicative (cyclic) group.
++ As primitive element we choose the root of the extension polynomial, which
++ is created by createPrimitivePoly from
++ \spadtype{FiniteFieldPolynomialPackage}. Zech logarithms are stored
++ in a table of size half of the field size, and use \spadtype{SingleInteger}
++ for representing field elements, hence, there are restrictions
++ on the size of the field.

```

```

FiniteFieldCyclicGroupExtension(GF,extdeg):_
  Exports == Implementation where
  GF      : FiniteFieldCategory
  extdeg  : PositiveInteger
  PI      ==> PositiveInteger
  FFPOLY  ==> FiniteFieldPolynomialPackage(GF)
  SI      ==> SingleInteger
  Exports ==> FiniteAlgebraicExtensionField(GF) with
    getZechTable:() -> PrimitiveArray(SingleInteger)
    ++ getZechTable() returns the zech logarithm table of the field.
    ++ This table is used to perform additions in the field quickly.
  Implementation ==> FiniteFieldCyclicGroupExtensionByPolynomial(GF,_
    createPrimitivePoly(extdeg)$FFPOLY)

```

---

— FFCGX.dotabb —

```

"FFCGX" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FFCGX"]
"FAXF" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FAXF"]
"FFCGX" -> "FAXF"

```

## 7.8 domain FFCGP FiniteFieldCyclicGroupExtension-ByPolynomial

---

— FiniteFieldCyclicGroupExtensionByPolynomial.input —

```

)set break resume
)sys rm -f FiniteFieldCyclicGroupExtensionByPolynomial.output

```

```

)spool FiniteFieldCyclicGroupExtensionByPolynomial.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FiniteFieldCyclicGroupExtensionByPolynomial
--R FiniteFieldCyclicGroupExtensionByPolynomial(GF: FiniteFieldCategory,defpol: SparseUnivar
--R Abbreviation for FiniteFieldCyclicGroupExtensionByPolynomial is FFCGP
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FFCGP
--R
--R----- Operations -----
--R ??? : (GF,%) -> %                ??? : (%,GF) -> %
--R ??? : (Fraction Integer,%) -> %    ??? : (%,Fraction Integer) -> %
--R ??? : (%,%) -> %                  ??? : (Integer,%) -> %
--R ??? : (PositiveInteger,%) -> %      ??? : (%,Integer) -> %
--R ??? : (%,PositiveInteger) -> %      ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                  -? : % -> %
--R ?/? : (%,GF) -> %                ?/? : (%,%) -> %
--R ?? : (%,%) -> Boolean              D : % -> % if GF has FINITE
--R 1 : () -> %                        0 : () -> %
--R ?? : (%,Integer) -> %              ?? : (%,PositiveInteger) -> %
--R algebraic? : % -> Boolean           associates? : (%,%) -> Boolean
--R basis : () -> Vector %              coerce : GF -> %
--R coerce : Fraction Integer -> %      coerce : % -> %
--R coerce : Integer -> %               coerce : % -> OutputForm
--R coordinates : % -> Vector GF         degree : % -> PositiveInteger
--R dimension : () -> CardinalNumber     factor : % -> Factored %
--R gcd : List % -> %                   gcd : (%,%) -> %
--R hash : % -> SingleInteger            inGroundField? : % -> Boolean
--R inv : % -> %                         latex : % -> String
--R lcm : List % -> %                   lcm : (%,%) -> %
--R norm : % -> GF                       one? : % -> Boolean
--R prime? : % -> Boolean                 ?quo? : (%,%) -> %
--R recip : % -> Union(%, "failed")      ?rem? : (%,%) -> %
--R represents : Vector GF -> %          retract : % -> GF
--R sample : () -> %                     sizeLess? : (%,%) -> Boolean
--R squareFree : % -> Factored %         squareFreePart : % -> %
--R trace : % -> GF                       transcendent? : % -> Boolean
--R unit? : % -> Boolean                  unitCanonical : % -> %
--R zero? : % -> Boolean                  ~=? : (%,%) -> Boolean
--R ??? : (NonNegativeInteger,%) -> %
--R ??? : (%,NonNegativeInteger) -> %
--R D : (%,NonNegativeInteger) -> % if GF has FINITE
--R Frobenius : (%,NonNegativeInteger) -> % if GF has FINITE
--R Frobenius : % -> % if GF has FINITE
--R ?? : (%,NonNegativeInteger) -> %
--R basis : PositiveInteger -> Vector %
--R characteristic : () -> NonNegativeInteger

```

## 7.8. DOMAIN FFCGP FINITEFIELD CYCLIC GROUPEXTENSION BY POLYNOMIAL 801

```

--R charthRoot : % -> Union(%, "failed") if GF has CHARNZ or GF has FINITE
--R charthRoot : % -> % if GF has FINITE
--R conditionP : Matrix % -> Union(Vector %, "failed") if GF has FINITE
--R coordinates : Vector % -> Matrix GF
--R createNormalElement : () -> % if GF has FINITE
--R createPrimitiveElement : () -> % if GF has FINITE
--R definingPolynomial : () -> SparseUnivariatePolynomial GF
--R degree : % -> OnePointCompletion PositiveInteger
--R differentiate : (%, NonNegativeInteger) -> % if GF has FINITE
--R differentiate : % -> % if GF has FINITE
--R discreteLog : (%, %) -> Union(NonNegativeInteger, "failed") if GF has CHARNZ or GF has FINITE
--R discreteLog : % -> NonNegativeInteger if GF has FINITE
--R divide : (%, %) -> Record(quotient: %, remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %, %) -> Union(List %, "failed")
--R exquo : (%, %) -> Union(%, "failed")
--R extendedEuclidean : (%, %, %) -> Union(Record(coef1: %, coef2: %), "failed")
--R extendedEuclidean : (%, %) -> Record(coef1: %, coef2: %, generator: %)
--R extensionDegree : () -> PositiveInteger
--R extensionDegree : () -> OnePointCompletion PositiveInteger
--R factorsOfCyclicGroupSize : () -> List Record(factor: Integer, exponent: Integer) if GF has FINITE
--R gcdPolynomial : (SparseUnivariatePolynomial %, SparseUnivariatePolynomial %) -> SparseUnivariatePolynomial
--R generator : () -> % if GF has FINITE
--R getZechTable : () -> PrimitiveArray SingleInteger
--R index : PositiveInteger -> % if GF has FINITE
--R init : () -> % if GF has FINITE
--R linearAssociatedExp : (%, SparseUnivariatePolynomial GF) -> % if GF has FINITE
--R linearAssociatedLog : (%, %) -> Union(SparseUnivariatePolynomial GF, "failed") if GF has FINITE
--R linearAssociatedLog : % -> SparseUnivariatePolynomial GF if GF has FINITE
--R linearAssociatedOrder : % -> SparseUnivariatePolynomial GF if GF has FINITE
--R lookup : % -> PositiveInteger if GF has FINITE
--R minimalPolynomial : (%, PositiveInteger) -> SparseUnivariatePolynomial % if GF has FINITE
--R minimalPolynomial : % -> SparseUnivariatePolynomial GF
--R multiEuclidean : (List %, %) -> Union(List %, "failed")
--R nextItem : % -> Union(%, "failed") if GF has FINITE
--R norm : (%, PositiveInteger) -> % if GF has FINITE
--R normal? : % -> Boolean if GF has FINITE
--R normalElement : () -> % if GF has FINITE
--R order : % -> OnePointCompletion PositiveInteger if GF has CHARNZ or GF has FINITE
--R order : % -> PositiveInteger if GF has FINITE
--R primeFrobenius : % -> % if GF has CHARNZ or GF has FINITE
--R primeFrobenius : (%, NonNegativeInteger) -> % if GF has CHARNZ or GF has FINITE
--R primitive? : % -> Boolean if GF has FINITE
--R primitiveElement : () -> % if GF has FINITE
--R principalIdeal : List % -> Record(coef: List %, generator: %)
--R random : () -> % if GF has FINITE
--R representationType : () -> Union("prime", polynomial, normal, cyclic) if GF has FINITE
--R retractIfCan : % -> Union(GF, "failed")
--R size : () -> NonNegativeInteger if GF has FINITE
--R subtractIfCan : (%, %) -> Union(%, "failed")

```

```

--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger,NonNegativeInteger) if GF has
--R trace : (%,PositiveInteger) -> % if GF has FINITE
--R transcendenceDegree : () -> NonNegativeInteger
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)

```

---

— FiniteFieldCyclicGroupExtensionByPolynomial.help —

```

=====
FiniteFieldCyclicGroupExtensionByPolynomial examples
=====

```

See Also:

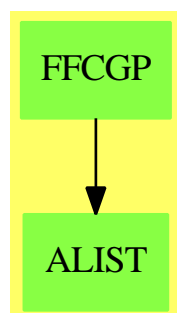
```

o )show FiniteFieldCyclicGroupExtensionByPolynomial

```

---

### 7.8.1 FiniteFieldCyclicGroupExtensionByPolynomial (FFCGP)



See

⇒ “FiniteFieldCyclicGroupExtension” (FFCGX) 7.7.1 on page 797

⇒ “FiniteFieldCyclicGroup” (FFCG) 7.6.1 on page 792

**Exports:**

|                     |                           |                        |
|---------------------|---------------------------|------------------------|
| 0                   | 1                         | algebraic?             |
| associates?         | basis                     | characteristic         |
| charthRoot          | coerce                    | conditionP             |
| coordinates         | createNormalElement       | createPrimitiveElement |
| D                   | definingPolynomial        | degree                 |
| differentiate       | dimension                 | discreteLog            |
| divide              | euclideanSize             | expressIdealMember     |
| exquo               | extendedEuclidean         | extensionDegree        |
| factor              | factorsOfCyclicGroupSize  | Frobenius              |
| gcd                 | gcdPolynomial             | generator              |
| getZechTable        | hash                      | index                  |
| inGroundField?      | init                      | inv                    |
| latex               | lcm                       | linearAssociatedExp    |
| linearAssociatedLog | linearAssociatedOrder     | lookup                 |
| minimalPolynomial   | multiEuclidean            | nextItem               |
| norm                | normal?                   | normalElement          |
| one?                | order                     | prime?                 |
| primeFrobenius      | primitive?                | primitiveElement       |
| principalIdeal      | random                    | recip                  |
| representationType  | represents                | retract                |
| retractIfCan        | sample                    | size                   |
| sizeLess?           | squareFree                | squareFreePart         |
| subtractIfCan       | tableForDiscreteLogarithm | trace                  |
| transcendenceDegree | transcendent?             | unit?                  |
| unitCanonical       | unitNormal                | zero?                  |
| ?*?                 | ?**?                      | ?+?                    |
| ?-?                 | -?                        | ?/?                    |
| ?=?                 | ?^?                       | ?~=?                   |
| ?quo?               | ?rem?                     |                        |

— domain FFCGP FiniteFieldCyclicGroupExtensionByPolynomial

```

)abbrev domain FFCGP FiniteFieldCyclicGroupExtensionByPolynomial
++ Authors: J.Grabmeier, A.Scheerhorn
++ Date Created: 26.03.1991
++ Date Last Updated: 31 March 1991
++ Basic Operations:
++ Related Constructors: FiniteFieldFunctions
++ Also See: FiniteFieldExtensionByPolynomial,
++ FiniteFieldNormalBasisExtensionByPolynomial
++ AMS Classifications:
++ Keywords: finite field, primitive elements, cyclic group
++ References:
++ R.Lidl, H.Niederreiter: Finite Field, Encycloedia of Mathematics and
++ Its Applications, Vol. 20, Cambridge Univ. Press, 1983, ISBN 0 521 30240 4

```



```

++ J. Grabmeier, A. Scheerhorn: Finite Fields in AXIOM.
++ AXIOM Technical Report Series, ATR/5 NP2522.
++ Description:
++ FiniteFieldCyclicGroupExtensionByPolynomial(GF,defpol) implements a
++ finite extension field of the ground field GF. Its elements are
++ represented by powers of a primitive element, i.e. a generator of the
++ multiplicative (cyclic) group. As primitive
++ element we choose the root of the extension polynomial defpol,
++ which MUST be primitive (user responsibility). Zech logarithms are stored
++ in a table of size half of the field size, and use \spadtype{SingleInteger}
++ for representing field elements, hence, there are restrictions
++ on the size of the field.

```

```

FiniteFieldCyclicGroupExtensionByPolynomial(GF,defpol):-
  Exports == Implementation where
  GF      : FiniteFieldCategory          -- the ground field
  defpol: SparseUnivariatePolynomial GF -- the extension polynomial
  -- the root of defpol is used as the primitive element

```

```

PI      ==> PositiveInteger
NNI     ==> NonNegativeInteger
I       ==> Integer
SI      ==> SingleInteger
SUP     ==> SparseUnivariatePolynomial
SAE     ==> SimpleAlgebraicExtension(GF,SUP GF,defpol)
V       ==> Vector GF
FFP     ==> FiniteFieldExtensionByPolynomial(GF,defpol)
FFF     ==> FiniteFieldFunctions(GF)
OUT     ==> OutputForm
ARR     ==> PrimitiveArray(SI)
TBL     ==> Table(PI,NNI)

```

```

Exports ==> FiniteAlgebraicExtensionField(GF) with

```

```

  getZechTable() -> ARR
  ++ getZechTable() returns the zech logarithm table of the field
  ++ it is used to perform additions in the field quickly.
Implementation ==> add

```

```

-- global variables =====

```

```

Rep:= SI
-- elements are represented by small integers in the range
-- (-1)..(size()-2). The (-1) representing the field element zero,
-- the other small integers representing the corresponding power
-- of the primitive element, the root of the defining polynomial

-- it would be very nice if we could use the representation
-- Rep:= Union("zero", IntegerMod(size())$GF ** degree(defpol) -1)),

```

## 7.8. DOMAIN FFCGP FINITEFIELD CYCLIC GROUP EXTENSION BY POLYNOMIAL 805

```

-- why doesn't the compiler like this ?

extdeg:NNI :=degree(defpol)$(SUP GF)::NNI
-- the extension degree

sizeFF:NNI:=(size())$GF ** extdeg) pretend NNI
-- the size of the field

if sizeFF > 2**20 then
  error "field too large for this representation"

sizeCG:SI:=(sizeFF - 1) pretend SI
-- the order of the cyclic group

sizeFG:SI:=(sizeCG quo (size())$GF-1)) pretend SI
-- the order of the factor group

zechlog:ARR:=new(((sizeFF+1) quo 2)::NNI,-1::SI)$ARR
-- the table for the zech logarithm

alpha :=new()$Symbol :: OutputForm
-- get a new symbol for the output representation of
-- the elements

primEltGF:GF:=
  odd?(extdeg)$I => -$GF coefficient(defpol,0)$(SUP GF)
  coefficient(defpol,0)$(SUP GF)
-- the corresponding primitive element of the groundfield
-- equals the trace of the primitive element w.r.t. the groundfield

facOfGroupSize := nil()$(List Record(factor:Integer,exponent:Integer))
-- the factorization of sizeCG

initzech?:Boolean:=true
-- gets false after initialization of the zech logarithm array

initelt?:Boolean:=true
-- gets false after initialization of the normal element

normalElt:SI:=0
-- the global variable containing a normal element

-- functions =====

-- for completeness we have to give a dummy implementation for
-- 'tableForDiscreteLogarithm', although this function is not
-- necessary in the cyclic group representation case

tableForDiscreteLogarithm(fac) == table()$TBL

```

```

getZechTable() == zechlog
initializeZech:() -> Void
initializeElt: () -> Void

order(x:$):PI ==
  zero?(x) =>
    error"order: order of zero undefined"
  (sizeCG quo gcd(sizeCG,x pretend NNI)):PI

primitive?(x:$) ==
--   zero?(x) or one?(x) => false
  zero?(x) or (x = 1) => false
  gcd(x::Rep,sizeCG)$Rep = 1$Rep => true
  false

coordinates(x:$) ==
  x=0 => new(extdeg,0)$(Vector GF)
  primElement:SAE:=convert(monomial(1,1)$(SUP GF))$SAE
-- the primitive element in the corresponding algebraic extension
  coordinates(primElement **$SAE (x pretend SI))$SAE

x:$ + y:$ ==
  if initzech? then initializeZech()
  zero? x => y
  zero? y => x
  d:Rep:=positiveRemainder(y -$Rep x,sizeCG)$Rep
  (d pretend SI) <= shift(sizeCG,-$SI (1$SI)) =>
    zechlog.(d pretend SI) =$SI -1::$SI => 0
    addmod(x,zechlog.(d pretend SI) pretend Rep,sizeCG)$Rep
--d:Rep:=positiveRemainder(x -$Rep y,sizeCG)$Rep
  d:Rep:=(sizeCG -$SI d)::Rep
  addmod(y,zechlog.(d pretend SI) pretend Rep,sizeCG)$Rep
--positiveRemainder(x +$Rep zechlog.(d pretend SI) -$Rep d,sizeCG)$Rep

initializeZech() ==
  zechlog:=createZechTable(defpol)$FFF
  -- set initialization flag
  initzech? := false
  void()$Void

basis(n:PI) ==
  extensionDegree() rem n ^= 0 =>
    error("argument must divide extension degree")
  m:=sizeCG quo (size()$GF**n-1)
  [index((1+i*m) ::PI) for i in 0..(n-1)]::Vector $

n:I * x:$ == ((n::GF)::)$ * x

```

## 7.8. DOMAIN FFCGP FINITEFIELD CYCLIC GROUPEXTENSION BY POLYNOMIAL 807

```

minimalPolynomial(a) ==
  f: SUP $ := monomial(1,1)$(SUP $) - monomial(a,0)$(SUP $)
  u: $ := Frobenius(a)
  while not(u = a) repeat
    f := f * (monomial(1,1)$(SUP $) - monomial(u,0)$(SUP $))
    u := Frobenius(u)
  p: SUP GF := 0$(SUP GF)
  while not zero?(f)$(SUP $) repeat
    g: GF := retract(leadingCoefficient(f)$(SUP $))
    p := p + monomial(g, _
                     degree(f)$(SUP $))$(SUP GF)
    f := reductum(f)$(SUP $)
  p

factorsOfCyclicGroupSize() ==
  if empty? facOfGroupSize then initializeElt()
  facOfGroupSize

representationType() == "cyclic"

definingPolynomial() == defpol

random() ==
  positiveRemainder(random()$Rep, sizeFF pretend Rep)$Rep - $Rep 1$Rep

represents(v) ==
  u: FFP := represents(v)$FFP
  u = $FFP 0$FFP => 0
  discreteLog(u)$FFP pretend Rep

coerce(e: GF): $ ==
  zero?(e)$GF => 0
  log: I := discreteLog(primEltGF, e)$GF::NNI *$I sizeFG
  -- version before 10.20.92: log pretend Rep
  -- 1$GF is coerced to sizeCG pretend Rep by old version
  -- now 1$GF is coerced to 0$Rep which is correct.
  positiveRemainder(log, sizeCG) pretend Rep

retractIfCan(x: $) ==
  zero? x => 0$GF
  u := (x::Rep) exquo$Rep (sizeFG pretend Rep)
  u = "failed" => "failed"
  primEltGF **$GF ((u::)$ pretend SI)

retract(x: $) ==
  a := retractIfCan(x)

```

```

a="failed" => error "element not in groundfield"
a :: GF

basis() == [index(i :: PI) for i in 1..extdeg]::Vector $

inGroundField?(x) ==
  zero? x=> true
  positiveRemainder(x::Rep,sizeFG pretend Rep)$Rep =$Rep 0$Rep => true
  false

discreteLog(b:$,x:$) ==
  zero? x => "failed"
  e:= extendedEuclidean(b,sizeCG,x)$Rep
  e = "failed" => "failed"
  e1:Record(coef1:$,coef2:$) := e :: Record(coef1:$,coef2:$)
  positiveRemainder(e1.coef1,sizeCG)$Rep pretend NNI

- x:$ ==
  zero? x => 0
  characteristic() =$I 2 => x
  addmod(x,shift(sizeCG,-1)$SI pretend Rep,sizeCG)

generator() == 1$SI
createPrimitiveElement() == 1$SI
primitiveElement() == 1$SI

discreteLog(x:$) ==
  zero? x => error "discrete logarithm error"
  x pretend NNI

normalElement() ==
  if initelt? then initializeElt()
  normalElt::$

initializeElt() ==
  facOfGroupSize := factors(factor(sizeCG)$Integer)
  normalElt:=createNormalElement() pretend SI
  initelt?:=false
  void()$Void

extensionDegree() == extdeg pretend PI

characteristic() == characteristic()$GF

lookup(x:$) ==
  x =$Rep (-$Rep 1$Rep) => sizeFF pretend PI
  (x +$Rep 1$Rep) pretend PI

index(a:PI) ==

```

## 7.8. DOMAIN FFCGP FINITEFIELD CYCLIC GROUPEXTENSION BY POLYNOMIAL 809

```

    positiveRemainder(a,sizeFF)$I pretend Rep -$Rep 1$Rep

0 == (-$Rep 1$Rep)

1 == 0$Rep

-- to get a "exponent like" output form
coerce(x:$):OUT ==
  x =$Rep (-$Rep 1$Rep) => "0":OUT
  x =$Rep 0$Rep => "1":OUT
  y:I:=lookup(x)-1
  alpha **$OUT (y:OUT)

x:$ = y:$ == x =$Rep y

x:$ * y:$ ==
  x = 0 => 0
  y = 0 => 0
  addmod(x,y,sizeCG)$Rep

a:GF * x:$ == coerce(a)@$ * x
x:$/a:GF == x/coerce(a)@$

-- x:$ / a:GF ==
-- a = 0$GF => error "division by zero"
-- x * inv(coerce(a))

inv(x:$) ==
  zero?(x) => error "inv: not invertible"
-- one?(x) => 1
  (x = 1) => 1
  sizeCG -$Rep x

x:$ ** n:PI == x ** n::I

x:$ ** n:NNI == x ** n::I

x:$ ** n:I ==
  m:Rep:=positiveRemainder(n,sizeCG)$I pretend Rep
  m =$Rep 0$Rep => 1
  x = 0 => 0
  mulmod(m,x,sizeCG::Rep)$Rep

```

---

— FFCGP.dotabb —

"FFCGP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FFCGP"]

```
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"FFCGP" -> "ALIST"
```

## 7.9 domain FFX FiniteFieldExtension

— FiniteFieldExtension.input —

```
)set break resume
)sys rm -f FiniteFieldExtension.output
)spool FiniteFieldExtension.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FiniteFieldExtension
--R FiniteFieldExtension(GF: FiniteFieldCategory,n: PositiveInteger) is a domain constructor
--R Abbreviation for FiniteFieldExtension is FFX
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FFX
--R
--R----- Operations -----
--R ??? : (GF,%) -> %          ??? : (%,GF) -> %
--R ??? : (Fraction Integer,%) -> %    ??? : (%,Fraction Integer) -> %
--R ??? : (%,%) -> %            ??? : (Integer,%) -> %
--R ??? : (PositiveInteger,%) -> %    ??? : (%,Integer) -> %
--R ??? : (%,PositiveInteger) -> %    ?+? : (%,%) -> %
--R ?-? : (%,%) -> %            -? : % -> %
--R ?/? : (%,GF) -> %          ?/? : (%,%) -> %
--R ?=? : (%,%) -> Boolean      D : % -> % if GF has FINITE
--R 1 : () -> %                0 : () -> %
--R ?^? : (%,Integer) -> %      ?^? : (%,PositiveInteger) -> %
--R algebraic? : % -> Boolean   associates? : (%,%) -> Boolean
--R basis : () -> Vector %      coerce : GF -> %
--R coerce : Fraction Integer -> %    coerce : % -> %
--R coerce : Integer -> %         coerce : % -> OutputForm
--R coordinates : % -> Vector GF    degree : % -> PositiveInteger
--R dimension : () -> CardinalNumber factor : % -> Factored %
--R gcd : List % -> %           gcd : (%,%) -> %
--R hash : % -> SingleInteger     inGroundField? : % -> Boolean
--R inv : % -> %                latex : % -> String
--R lcm : List % -> %           lcm : (%,%) -> %
--R norm : % -> GF              one? : % -> Boolean
--R prime? : % -> Boolean        ?quo? : (%,%) -> %
```

```

--R recip : % -> Union(%, "failed")
--R represents : Vector GF -> %
--R sample : () -> %
--R squareFree : % -> Factored %
--R trace : % -> GF
--R unit? : % -> Boolean
--R zero? : % -> Boolean
--R ?? : (NonNegativeInteger, %) -> %
--R ??? : (%, NonNegativeInteger) -> %
--R D : (%, NonNegativeInteger) -> % if GF has FINITE
--R Frobenius : (%, NonNegativeInteger) -> % if GF has FINITE
--R Frobenius : % -> % if GF has FINITE
--R ?? : (%, NonNegativeInteger) -> %
--R basis : PositiveInteger -> Vector %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%, "failed") if GF has CHARNZ or GF has FINITE
--R charthRoot : % -> % if GF has FINITE
--R conditionP : Matrix % -> Union(Vector %, "failed") if GF has FINITE
--R coordinates : Vector % -> Matrix GF
--R createNormalElement : () -> % if GF has FINITE
--R createPrimitiveElement : () -> % if GF has FINITE
--R definingPolynomial : () -> SparseUnivariatePolynomial GF
--R degree : % -> OnePointCompletion PositiveInteger
--R differentiate : (%, NonNegativeInteger) -> % if GF has FINITE
--R differentiate : % -> % if GF has FINITE
--R discreteLog : (%, %) -> Union(NonNegativeInteger, "failed") if GF has CHARNZ or GF has FINITE
--R discreteLog : % -> NonNegativeInteger if GF has FINITE
--R divide : (%, %) -> Record(quotient: %, remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %, %) -> Union(List %, "failed")
--R exquo : (%, %) -> Union(%, "failed")
--R extendedEuclidean : (%, %, %) -> Union(Record(coef1: %, coef2: %), "failed")
--R extendedEuclidean : (%, %) -> Record(coef1: %, coef2: %, generator: %)
--R extensionDegree : () -> PositiveInteger
--R extensionDegree : () -> OnePointCompletion PositiveInteger
--R factorsOfCyclicGroupSize : () -> List Record(factor: Integer, exponent: Integer) if GF has FINITE
--R gcdPolynomial : (SparseUnivariatePolynomial %, SparseUnivariatePolynomial %) -> SparseUnivariatePolynomial GF
--R generator : () -> % if GF has FINITE
--R index : PositiveInteger -> % if GF has FINITE
--R init : () -> % if GF has FINITE
--R linearAssociatedExp : (%, SparseUnivariatePolynomial GF) -> % if GF has FINITE
--R linearAssociatedLog : (%, %) -> Union(SparseUnivariatePolynomial GF, "failed") if GF has FINITE
--R linearAssociatedLog : % -> SparseUnivariatePolynomial GF if GF has FINITE
--R linearAssociatedOrder : % -> SparseUnivariatePolynomial GF if GF has FINITE
--R lookup : % -> PositiveInteger if GF has FINITE
--R minimalPolynomial : (%, PositiveInteger) -> SparseUnivariatePolynomial % if GF has FINITE
--R minimalPolynomial : % -> SparseUnivariatePolynomial GF
--R multiEuclidean : (List %, %) -> Union(List %, "failed")
--R nextItem : % -> Union(%, "failed") if GF has FINITE
--R norm : (%, PositiveInteger) -> % if GF has FINITE
--R rem? : (%, %) -> %
--R retract : % -> GF
--R sizeLess? : (%, %) -> Boolean
--R squareFreePart : % -> %
--R transcendent? : % -> Boolean
--R unitCanonical : % -> %
--R ~=? : (%, %) -> Boolean

```



```

--R normal? : % -> Boolean if GF has FINITE
--R normalElement : () -> % if GF has FINITE
--R order : % -> OnePointCompletion PositiveInteger if GF has CHARNZ or GF has FINITE
--R order : % -> PositiveInteger if GF has FINITE
--R primeFrobenius : % -> % if GF has CHARNZ or GF has FINITE
--R primeFrobenius : (% , NonNegativeInteger) -> % if GF has CHARNZ or GF has FINITE
--R primitive? : % -> Boolean if GF has FINITE
--R primitiveElement : () -> % if GF has FINITE
--R principalIdeal : List % -> Record(coef: List %, generator: %)
--R random : () -> % if GF has FINITE
--R representationType : () -> Union("prime", polynomial, normal, cyclic) if GF has FINITE
--R retractIfCan : % -> Union(GF, "failed")
--R size : () -> NonNegativeInteger if GF has FINITE
--R subtractIfCan : (% , %) -> Union(% , "failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger, NonNegativeInteger) if GF has FINITE
--R trace : (% , PositiveInteger) -> % if GF has FINITE
--R transcendenceDegree : () -> NonNegativeInteger
--R unitNormal : % -> Record(unit: %, canonical: %, associate: %)
--R
--E 1

)spool
)lisp (bye)

```

---

— FiniteFieldExtension.help —

```

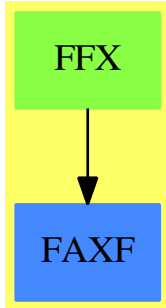
=====
FiniteFieldExtension examples
=====

```

```

See Also:
o )show FiniteFieldExtension

```

**7.9.1 FiniteFieldExtension (FFX)**

**See**

- ⇒ “FiniteFieldExtensionByPolynomial” (FFP) 7.10.1 on page 818
- ⇒ “InnerFiniteField” (IFF) 10.21.1 on page 1247
- ⇒ “FiniteField” (FF) 7.5.1 on page 787

**Exports:**

|                           |                          |                        |
|---------------------------|--------------------------|------------------------|
| 0                         | 1                        | algebraic?             |
| associates?               | basis                    | characteristic         |
| charthRoot                | coerce                   | conditionP             |
| coordinates               | createNormalElement      | createPrimitiveElement |
| D                         | definingPolynomial       | degree                 |
| differentiate             | dimension                | discreteLog            |
| divide                    | euclideanSize            | expressIdealMember     |
| exquo                     | extendedEuclidean        | extensionDegree        |
| factor                    | factorsOfCyclicGroupSize | Frobenius              |
| gcd                       | gcdPolynomial            | generator              |
| hash                      | index                    | inGroundField?         |
| init                      | inv                      | latex                  |
| lcm                       | linearAssociatedExp      | linearAssociatedLog    |
| linearAssociatedOrder     | lookup                   | minimalPolynomial      |
| multiEuclidean            | nextItem                 | norm                   |
| normal?                   | normalElement            | one?                   |
| order                     | prime?                   | primeFrobenius         |
| primitive?                | primitiveElement         | principalIdeal         |
| random                    | recip                    | representationType     |
| represents                | retract                  | retractIfCan           |
| sample                    | size                     | sizeLess?              |
| squareFree                | squareFreePart           | subtractIfCan          |
| tableForDiscreteLogarithm | trace                    | transcendenceDegree    |
| transcendent?             | unit?                    | unitCanonical          |
| unitNormal                | zero?                    | ??                     |
| ***?                      | ?+?                      | ?-?                    |
| -?                        | ?/?                      | ?=?                    |
| ?^?                       | ?~=?                     | ?quo?                  |
| ?rem?                     |                          |                        |

— domain FFX FiniteFieldExtension —

```

)abbrev domain FFX FiniteFieldExtension
++ Authors: R.Sutor, J. Grabmeier, A. Scheerhorn
++ Date Created:
++ Date Last Updated: 31 March 1991
++ Basic Operations:
++ Related Constructors: FiniteFieldExtensionByPolynomial,
++ FiniteFieldPolynomialPackage
++ Also See: FiniteFieldCyclicGroupExtension,
++ FiniteFieldNormalBasisExtension
++ AMS Classifications:
++ Keywords: field, extension field, algebraic extension,
++ finite extension, finite field, Galois field
++ Reference:
++ R.Lidl, H.Niederreiter: Finite Field, Encyclopedia of Mathematics an

```

```

++ Its Applications, Vol. 20, Cambridge Univ. Press, 1983, ISBN 0 521 30240 4
++ J. Grabmeier, A. Scheerhorn: Finite Fields in AXIOM.
++ AXIOM Technical Report Series, ATR/5 NP2522.
++ Description:
++ FiniteFieldExtensionByPolynomial(GF, n) implements an extension
++ of the finite field GF of degree n generated by the extension
++ polynomial constructed by createIrreduciblePoly from
++ \spadtype{FiniteFieldPolynomialPackage}.

FiniteFieldExtension(GF, n): Exports == Implementation where
  GF: FiniteFieldCategory
  n : PositiveInteger
Exports ==> FiniteAlgebraicExtensionField(GF)
-- MonogenicAlgebra(GF, SUP) with -- have to check this
Implementation ==> FiniteFieldExtensionByPolynomial(GF,
  createIrreduciblePoly(n)$FiniteFieldPolynomialPackage(GF))
-- old code for generating irreducible polynomials:
-- now "better" order (sparse polys first)
-- generateIrredPoly(n)$IrredPolyOverFiniteField(GF))

```

---

— FFX.dotabb —

```

"FFX" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FFX"]
"FAXF" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FAXF"]
"FFX" -> "FAXF"

```

---

## 7.10 domain FFP FiniteFieldExtensionByPolynomial

— FiniteFieldExtensionByPolynomial.input —

```

)set break resume
)sys rm -f FiniteFieldExtensionByPolynomial.output
)spool FiniteFieldExtensionByPolynomial.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FiniteFieldExtensionByPolynomial
--R FiniteFieldExtensionByPolynomial(GF: FiniteFieldCategory,defpol: SparseUnivariatePolynomial GF) is

```

```

--R Abbreviation for FiniteFieldExtensionByPolynomial is FFP
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FFP
--R
--R----- Operations -----
--R ?? : (GF,%) -> %           ?? : (%,GF) -> %
--R ?? : (Fraction Integer,%) -> %   ?? : (%,Fraction Integer) -> %
--R ?? : (%,%) -> %             ?? : (Integer,%) -> %
--R ?? : (PositiveInteger,%) -> %   ??? : (%,Integer) -> %
--R ??? : (%,PositiveInteger) -> %  ?+? : (%,%) -> %
--R ?-? : (%,%) -> %             -? : % -> %
--R ?/? : (%,GF) -> %           ?/? : (%,%) -> %
--R ?=? : (%,%) -> Boolean       D : % -> % if GF has FINITE
--R 1 : () -> %                 0 : () -> %
--R ?? : (%,Integer) -> %       ?? : (%,PositiveInteger) -> %
--R algebraic? : % -> Boolean   associates? : (%,%) -> Boolean
--R basis : () -> Vector %     coerce : GF -> %
--R coerce : Fraction Integer -> %   coerce : % -> %
--R coerce : Integer -> %       coerce : % -> OutputForm
--R coordinates : % -> Vector GF   degree : % -> PositiveInteger
--R dimension : () -> CardinalNumber   factor : % -> Factored %
--R gcd : List % -> %           gcd : (%,%) -> %
--R hash : % -> SingleInteger     inGroundField? : % -> Boolean
--R inv : % -> %                 latex : % -> String
--R lcm : List % -> %           lcm : (%,%) -> %
--R norm : % -> GF              one? : % -> Boolean
--R prime? : % -> Boolean        ?quo? : (%,%) -> %
--R recip : % -> Union(%, "failed")   ?rem? : (%,%) -> %
--R represents : Vector GF -> %     retract : % -> GF
--R sample : () -> %             sizeLess? : (%,%) -> Boolean
--R squareFree : % -> Factored %   squareFreePart : % -> %
--R trace : % -> GF              transcendent? : % -> Boolean
--R unit? : % -> Boolean          unitCanonical : % -> %
--R zero? : % -> Boolean         ~=? : (%,%) -> Boolean
--R ?? : (NonNegativeInteger,%) -> %
--R ??? : (%,NonNegativeInteger) -> %
--R D : (%,NonNegativeInteger) -> % if GF has FINITE
--R Frobenius : (%,NonNegativeInteger) -> % if GF has FINITE
--R Frobenius : % -> % if GF has FINITE
--R ?? : (%,NonNegativeInteger) -> %
--R basis : PositiveInteger -> Vector %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%, "failed") if GF has CHARNZ or GF has FINITE
--R charthRoot : % -> % if GF has FINITE
--R conditionP : Matrix % -> Union(Vector %, "failed") if GF has FINITE
--R coordinates : Vector % -> Matrix GF
--R createNormalElement : () -> % if GF has FINITE
--R createPrimitiveElement : () -> % if GF has FINITE
--R definingPolynomial : () -> SparseUnivariatePolynomial GF
--R degree : % -> OnePointCompletion PositiveInteger

```

```

--R differentiate : (% , NonNegativeInteger) -> % if GF has FINITE
--R differentiate : % -> % if GF has FINITE
--R discreteLog : (% , %) -> Union(NonNegativeInteger, "failed") if GF has CHARNZ or GF has FINITE
--R discreteLog : % -> NonNegativeInteger if GF has FINITE
--R divide : (% , %) -> Record(quotient: %, remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List % , %) -> Union(List % , "failed")
--R exquo : (% , %) -> Union(% , "failed")
--R extendedEuclidean : (% , % , %) -> Union(Record(coef1: %, coef2: %), "failed")
--R extendedEuclidean : (% , %) -> Record(coef1: %, coef2: %, generator: %)
--R extensionDegree : () -> PositiveInteger
--R extensionDegree : () -> OnePointCompletion PositiveInteger
--R factorsOfCyclicGroupSize : () -> List Record(factor: Integer, exponent: Integer) if GF has FINITE
--R gcdPolynomial : (SparseUnivariatePolynomial % , SparseUnivariatePolynomial %) -> SparseUnivariatePolynomial %
--R generator : () -> % if GF has FINITE
--R index : PositiveInteger -> % if GF has FINITE
--R init : () -> % if GF has FINITE
--R linearAssociatedExp : (% , SparseUnivariatePolynomial GF) -> % if GF has FINITE
--R linearAssociatedLog : (% , %) -> Union(SparseUnivariatePolynomial GF, "failed") if GF has FINITE
--R linearAssociatedLog : % -> SparseUnivariatePolynomial GF if GF has FINITE
--R linearAssociatedOrder : % -> SparseUnivariatePolynomial GF if GF has FINITE
--R lookup : % -> PositiveInteger if GF has FINITE
--R minimalPolynomial : (% , PositiveInteger) -> SparseUnivariatePolynomial % if GF has FINITE
--R minimalPolynomial : % -> SparseUnivariatePolynomial GF
--R multiEuclidean : (List % , %) -> Union(List % , "failed")
--R nextItem : % -> Union(% , "failed") if GF has FINITE
--R norm : (% , PositiveInteger) -> % if GF has FINITE
--R normal? : % -> Boolean if GF has FINITE
--R normalElement : () -> % if GF has FINITE
--R order : % -> OnePointCompletion PositiveInteger if GF has CHARNZ or GF has FINITE
--R order : % -> PositiveInteger if GF has FINITE
--R primeFrobenius : % -> % if GF has CHARNZ or GF has FINITE
--R primeFrobenius : (% , NonNegativeInteger) -> % if GF has CHARNZ or GF has FINITE
--R primitive? : % -> Boolean if GF has FINITE
--R primitiveElement : () -> % if GF has FINITE
--R principalIdeal : List % -> Record(coef: List % , generator: %)
--R random : () -> % if GF has FINITE
--R representationType : () -> Union("prime", polynomial, normal, cyclic) if GF has FINITE
--R retractIfCan : % -> Union(GF, "failed")
--R size : () -> NonNegativeInteger if GF has FINITE
--R subtractIfCan : (% , %) -> Union(% , "failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger, NonNegativeInteger) if GF has FINITE
--R trace : (% , PositiveInteger) -> % if GF has FINITE
--R transcendenceDegree : () -> NonNegativeInteger
--R unitNormal : % -> Record(unit: % , canonical: % , associate: %)
--R
--E 1

)spool
)lisp (bye)

```

---

— **FiniteFieldExtensionByPolynomial.help** —

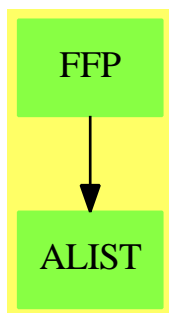
```
=====
FiniteFieldExtensionByPolynomial examples
=====
```

See Also:

o )show FiniteFieldExtensionByPolynomial

---

### 7.10.1 FiniteFieldExtensionByPolynomial (FFP)



See

⇒ “FiniteFieldExtension” (FFX) 7.9.1 on page 813

⇒ “InnerFiniteField” (IFF) 10.21.1 on page 1247

⇒ “FiniteField” (FF) 7.5.1 on page 787

**Exports:**

|                           |                          |                        |
|---------------------------|--------------------------|------------------------|
| 0                         | 1                        | algebraic?             |
| associates?               | basis                    | characteristic         |
| charthRoot                | coerce                   | conditionP             |
| coordinates               | createNormalElement      | createPrimitiveElement |
| D                         | definingPolynomial       | degree                 |
| differentiate             | dimension                | discreteLog            |
| divide                    | euclideanSize            | expressIdealMember     |
| exquo                     | extendedEuclidean        | extensionDegree        |
| factor                    | factorsOfCyclicGroupSize | Frobenius              |
| gcd                       | gcdPolynomial            | generator              |
| hash                      | index                    | inGroundField?         |
| init                      | inv                      | latex                  |
| lcm                       | linearAssociatedExp      | linearAssociatedLog    |
| linearAssociatedOrder     | lookup                   | minimalPolynomial      |
| multiEuclidean            | nextItem                 | norm                   |
| normal?                   | normalElement            | one?                   |
| order                     | prime?                   | primeFrobenius         |
| primitive?                | primitiveElement         | principalIdeal         |
| random                    | recip                    | representationType     |
| represents                | retract                  | retractIfCan           |
| sample                    | size                     | sizeLess?              |
| squareFree                | squareFreePart           | subtractIfCan          |
| tableForDiscreteLogarithm | trace                    | transcendenceDegree    |
| transcendent?             | unit?                    | unitCanonical          |
| unitNormal                | zero?                    | ?*?                    |
| ?**?                      | ?+?                      | ?-?                    |
| -?                        | ?/?                      | ?/?                    |
| ?=?                       | ?^?                      | ?~=?                   |
| ?quo?                     | ?rem?                    |                        |

— domain **FFP** FiniteFieldExtensionByPolynomial —

```
)abbrev domain FFP FiniteFieldExtensionByPolynomial
++ Authors: R.Sutor, J. Grabmeier, O. Gschnitzer, A. Scheerhorn
++ Date Created:
++ Date Last Updated: 31 March 1991
++ Basic Operations:
++ Related Constructors:
++ Also See: FiniteFieldCyclicGroupExtensionByPolynomial,
++ FiniteFieldNormalBasisExtensionByPolynomial
++ AMS Classifications:
++ Keywords: field, extension field, algebraic extension,
++ finite extension, finite field, Galois field
++ Reference:
++ R.Lidl, H.Niederreiter: Finite Field, Encyclopedia of Mathematics an
++ Its Applications, Vol. 20, Cambridge Univ. Press, 1983, ISBN 0 521 30240 4
```



```

++ J. Grabmeier, A. Scheerhorn: Finite Fields in AXIOM.
++ AXIOM Technical Report Series, ATR/5 NP2522.
++ Description:
++ FiniteFieldExtensionByPolynomial(GF, defpol) implements the extension
++ of the finite field GF generated by the extension polynomial
++ defpol which MUST be irreducible.
++ Note: the user has the responsibility to ensure that
++ defpol is irreducible.

FiniteFieldExtensionByPolynomial(GF:FiniteFieldCategory,_
  defpol: SparseUnivariatePolynomial GF): Exports == Implementation where
-- GF      : FiniteFieldCategory
-- defpol  : SparseUnivariatePolynomial GF

PI  ==> PositiveInteger
NNI ==> NonNegativeInteger
SUP ==> SparseUnivariatePolynomial
I   ==> Integer
R   ==> Record(key:PI,entry:NNI)
TBL ==> Table(PI,NNI)
SAE ==> SimpleAlgebraicExtension(GF,SUP GF,defpol)
OUT ==> OutputForm

Exports ==> FiniteAlgebraicExtensionField(GF)

Implementation ==> add

-- global variables =====

Rep:=SAE

extdeg:PI      := degree(defpol)$(SUP GF) pretend PI
-- the extension degree

alpha          := new()$Symbol :: OutputForm
-- a new symbol for the output form of field elements

sizeCG:Integer := size()$GF**extdeg - 1
-- the order of the multiplicative group

facOfGroupSize := nil()$(List Record(factor:Integer,exponent:Integer))
-- the factorization of sizeCG

normalElt:PI:=1
-- for the lookup of the normal Element computed by
-- createNormalElement

primitiveElt:PI:=1
-- for the lookup of the primitive Element computed by
-- createPrimitiveElement()

```

```

initlog?:Boolean:=true
-- gets false after initialization of the discrete logarithm table

initelt?:Boolean:=true
-- gets false after initialization of the primitive and the
-- normal element

discLogTable:Table(PI,TBL):=table()$Table(PI,TBL)
-- tables indexed by the factors of sizeCG,
-- discLogTable(factor) is a table with keys
-- primitiveElement() ** (i * (sizeCG quo factor)) and entries i for
-- i in 0..n-1, n computed in initialize() in order to use
-- the minimal size limit 'limit' optimal.

-- functions =====

-- createNormalElement() ==
--   a:=primitiveElement()
--   nElt:=generator()
--   for i in 1.. repeat
--     normal? nElt => return nElt
--   nElt:=nElt*a
--   nElt

generator() == reduce(monomial(1,1)$SUP(GF))$Rep
norm x == resultant(defpol, lift x)

initializeElt: () -> Void
initializeLog: () -> Void
basis(n:PI) ==
  (extdeg rem n) ^ 0 => error "argument must divide extension degree"
  a:=$:=norm(primitiveElement(),n)
  vector [a**i for i in 0..n-1]

degree(x) ==
  y:=$:=1
  m:=zero(extdeg,extdeg+1)$Matrix GF
  for i in 1..extdeg+1 repeat
    setColumn_!(m,i,coordinates(y))$Matrix GF
    y:=y*x
  rank(m)::PI

minimalPolynomial(x:$) ==
  y:=$:=1
  m:=zero(extdeg,extdeg+1)$Matrix GF
  for i in 1..extdeg+1 repeat
    setColumn_!(m,i,coordinates(y))$Matrix GF
    y:=y*x

```

```

v:=first nullSpace(m)$(Matrix GF)
+/[monomial(v.(i+1),i)$(SUP GF) for i in 0..extdeg]

normal?(x) ==
  l:List List GF:=[entries coordinates x]
  a:=x
  for i in 2..extdeg repeat
    a:=Frobenius(a)
    l:=concat(l,entries coordinates a)$(List List GF)
    ((rank matrix(l)$(Matrix GF)) = extdeg::NNI) => true
  false

a:GF * x:$ == a *$Rep x
n:I * x:$ == n *$Rep x
-x == -$Rep x
random() == random()$Rep
coordinates(x:$) == coordinates(x)$Rep
represents(v) == represents(v)$Rep
coerce(x:GF):$ == coerce(x)$Rep
definingPolynomial() == defpol
retract(x) == retract(x)$Rep
retractIfCan(x) == retractIfCan(x)$Rep
index(x) == index(x)$Rep
lookup(x) == lookup(x)$Rep
x:$ / y:$ == x /$Rep y
x:$ / a:GF == x /coerce(a)
--   x:$ / a:GF ==
--   a = 0$GF => error "division by zero"
--   x * inv(coerce(a))
x:$ * y:$ == x *$Rep y
x:$ + y:$ == x +$Rep y
x:$ - y:$ == x -$Rep y
x:$ = y:$ == x =$Rep y
basis() == basis()$Rep
0 == 0$Rep
1 == 1$Rep

factorsOfCyclicGroupSize() ==
  if empty? facOfGroupSize then initializeElt()
  facOfGroupSize

representationType() == "polynomial"

tableForDiscreteLogarithm(fac) ==
  if initlog? then initializeLog()
  tbl:=search(fac::PI,discLogTable)$Table(PI,TBL)
  tbl case "failed" =>
    error "tableForDiscreteLogarithm: argument must be prime divisor_

```

```

of the order of the multiplicative group"
    tbl pretend TBL

primitiveElement() ==
    if initelt? then initializeElt()
    index(primitiveElt)

normalElement() ==
    if initelt? then initializeElt()
    index(normalElt)

initializeElt() ==
    facOfGroupSize:=factors(factor(sizeCG)$Integer)
    -- get a primitive element
    pE:=createPrimitiveElement()
    primitiveElt:=lookup(pE)
    -- create a normal element
    nElt:=generator()
    while not normal? nElt repeat
        nElt:=nElt*pE
    normalElt:=lookup(nElt)
    -- set elements initialization flag
    initelt? := false
    void()$Void

initializeLog() ==
    if initelt? then initializeElt()
-- set up tables for discrete logarithm
    limit:Integer:=30
    -- the minimum size for the discrete logarithm table
    for f in facOfGroupSize repeat
        fac:=f.factor
        base:=$:=primitiveElement() ** (sizeCG quo fac)
        l:Integer:=length(fac)$Integer
        n:Integer:=0
        if odd?(l)$Integer then n:=shift(fac,-(l quo 2))
            else n:=shift(1,(l quo 2))
        if n < limit then
            d:=(fac-1) quo limit + 1
            n:=(fac-1) quo d + 1
            tbl:TBL:=table()$TBL
            a:=$:=1
            for i in (0::NNI)..(n-1)::NNI repeat
                insert_!([lookup(a),i::NNI]$R,tbl)$TBL
            a:=a*base
            insert_!([fac::PI,copy(tbl)$TBL]_
                $Record(key:PI,entry:TBL),discLogTable)$Table(PI,TBL)
    -- set logarithm initialization flag
    initlog? := false
    -- tell user about initialization

```

```

--print("discrete logarithm tables initialized":OUT)
void()$Void

coerce(e:$):OutputForm == outputForm(lift(e),alpha)

extensionDegree() == extdeg

size() == (sizeCG + 1) pretend NNI

-- sizeOfGroundField() == size()$GF

inGroundField?(x) ==
  retractIfCan(x) = "failed" => false
  true

characteristic() == characteristic()$GF

-----

— FFP.dotabb —

"FFP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FFP"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"FFP" -> "ALIST"

```

## 7.11 domain FFNB FiniteFieldNormalBasis

```

— FiniteFieldNormalBasis.input —

)set break resume
)sys rm -f FiniteFieldNormalBasis.output
)spool FiniteFieldNormalBasis.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FiniteFieldNormalBasis
--R FiniteFieldNormalBasis(p: PositiveInteger,extdeg: PositiveInteger) is a domain constructor
--R Abbreviation for FiniteFieldNormalBasis is FFNB
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FFNB

```

```

--R
--R----- Operations -----
--R ?? : (PrimeField p,%) -> %           ?? : (%,PrimeField p) -> %
--R ?? : (Fraction Integer,%) -> %       ?? : (%,Fraction Integer) -> %
--R ?? : (%,%) -> %                     ?? : (Integer,%) -> %
--R ?? : (PositiveInteger,%) -> %        ??? : (%,Integer) -> %
--R ??? : (%,PositiveInteger) -> %      ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                   -? : % -> %
--R ?/? : (%,PrimeField p) -> %         ?/? : (%,%) -> %
--R ?? : (%,%) -> Boolean               1 : () -> %
--R 0 : () -> %                         ?? : (%,Integer) -> %
--R ?? : (%,PositiveInteger) -> %       algebraic? : % -> Boolean
--R associates? : (%,%) -> Boolean      basis : () -> Vector %
--R coerce : PrimeField p -> %          coerce : Fraction Integer -> %
--R coerce : % -> %                     coerce : Integer -> %
--R coerce : % -> OutputForm            degree : % -> PositiveInteger
--R dimension : () -> CardinalNumber     factor : % -> Factored %
--R gcd : List % -> %                   gcd : (%,%) -> %
--R hash : % -> SingleInteger            inGroundField? : % -> Boolean
--R inv : % -> %                        latex : % -> String
--R lcm : List % -> %                   lcm : (%,%) -> %
--R norm : % -> PrimeField p             one? : % -> Boolean
--R prime? : % -> Boolean                ?quo? : (%,%) -> %
--R recip : % -> Union(%, "failed")      ?rem? : (%,%) -> %
--R retract : % -> PrimeField p          sample : () -> %
--R sizeLess? : (%,%) -> Boolean         squareFree : % -> Factored %
--R squareFreePart : % -> %              trace : % -> PrimeField p
--R transcendent? : % -> Boolean          unit? : % -> Boolean
--R unitCanonical : % -> %               zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?? : (NonNegativeInteger,%) -> %
--R ??? : (%,NonNegativeInteger) -> %
--R D : (%,NonNegativeInteger) -> % if PrimeField p has FINITE
--R D : % -> % if PrimeField p has FINITE
--R Frobenius : (%,NonNegativeInteger) -> % if PrimeField p has FINITE
--R Frobenius : % -> % if PrimeField p has FINITE
--R ?? : (%,NonNegativeInteger) -> %
--R basis : PositiveInteger -> Vector %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%, "failed") if PrimeField p has CHARNZ or PrimeField p has FINITE
--R charthRoot : % -> % if PrimeField p has FINITE
--R conditionP : Matrix % -> Union(Vector %, "failed") if PrimeField p has FINITE
--R coordinates : Vector % -> Matrix PrimeField p
--R coordinates : % -> Vector PrimeField p
--R createNormalElement : () -> % if PrimeField p has FINITE
--R createPrimitiveElement : () -> % if PrimeField p has FINITE
--R definingPolynomial : () -> SparseUnivariatePolynomial PrimeField p
--R degree : % -> OnePointCompletion PositiveInteger
--R differentiate : (%,NonNegativeInteger) -> % if PrimeField p has FINITE
--R differentiate : % -> % if PrimeField p has FINITE

```

```

--R discreteLog : (%,% ) -> Union(NonNegativeInteger,"failed") if PrimeField p has CHARNZ or
--R discreteLog : % -> NonNegativeInteger if PrimeField p has FINITE
--R divide : (%,% ) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %,% ) -> Union(List %,"failed")
--R exquo : (%,% ) -> Union(%,"failed")
--R extendedEuclidean : (%,%,% ) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,% ) -> Record(coef1: %,coef2: %,generator: %)
--R extensionDegree : () -> PositiveInteger
--R extensionDegree : () -> OnePointCompletion PositiveInteger
--R factorsOfCyclicGroupSize : () -> List Record(factor: Integer,exponent: Integer) if PrimeField p has CHARNZ or PrimeField p has FINITE
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolynomial % if PrimeField p has FINITE
--R generator : () -> % if PrimeField p has FINITE
--R getMultiplicationMatrix : () -> Matrix PrimeField p
--R getMultiplicationTable : () -> Vector List Record(value: PrimeField p,index: SingleInteger) if PrimeField p has FINITE
--R index : PositiveInteger -> % if PrimeField p has FINITE
--R init : () -> % if PrimeField p has FINITE
--R linearAssociatedExp : (% ,SparseUnivariatePolynomial PrimeField p) -> % if PrimeField p has FINITE
--R linearAssociatedLog : (%,% ) -> Union(SparseUnivariatePolynomial PrimeField p,"failed") if PrimeField p has CHARNZ or PrimeField p has FINITE
--R linearAssociatedLog : % -> SparseUnivariatePolynomial PrimeField p if PrimeField p has FINITE
--R linearAssociatedOrder : % -> SparseUnivariatePolynomial PrimeField p if PrimeField p has FINITE
--R lookup : % -> PositiveInteger if PrimeField p has FINITE
--R minimalPolynomial : (% ,PositiveInteger) -> SparseUnivariatePolynomial % if PrimeField p has FINITE
--R minimalPolynomial : % -> SparseUnivariatePolynomial PrimeField p if PrimeField p has FINITE
--R multiEuclidean : (List %,% ) -> Union(List %,"failed")
--R nextItem : % -> Union(%,"failed") if PrimeField p has FINITE
--R norm : (% ,PositiveInteger) -> % if PrimeField p has FINITE
--R normal? : % -> Boolean if PrimeField p has FINITE
--R normalElement : () -> % if PrimeField p has FINITE
--R order : % -> OnePointCompletion PositiveInteger if PrimeField p has CHARNZ or PrimeField p has FINITE
--R order : % -> PositiveInteger if PrimeField p has FINITE
--R primeFrobenius : % -> % if PrimeField p has CHARNZ or PrimeField p has FINITE
--R primeFrobenius : (% ,NonNegativeInteger) -> % if PrimeField p has CHARNZ or PrimeField p has FINITE
--R primitive? : % -> Boolean if PrimeField p has FINITE
--R primitiveElement : () -> % if PrimeField p has FINITE
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R random : () -> % if PrimeField p has FINITE
--R representationType : () -> Union("prime",polynomial,normal,cyclic) if PrimeField p has FINITE
--R represents : Vector PrimeField p -> %
--R retractIfCan : % -> Union(PrimeField p,"failed")
--R size : () -> NonNegativeInteger if PrimeField p has FINITE
--R sizeMultiplication : () -> NonNegativeInteger
--R subtractIfCan : (%,% ) -> Union(%,"failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger,NonNegativeInteger) if PrimeField p has FINITE
--R trace : (% ,PositiveInteger) -> % if PrimeField p has FINITE
--R transcendenceDegree : () -> NonNegativeInteger
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

```

```
)spool
)lisp (bye)
```

---

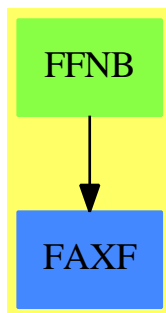
— **FiniteFieldNormalBasis.help** —

```
=====
FiniteFieldNormalBasis examples
=====
```

```
See Also:
o )show FiniteFieldNormalBasis
```

---

### 7.11.1 FiniteFieldNormalBasis (FFNB)



See

⇒ “FiniteFieldNormalBasisExtensionByPolynomial” (FFNBP) 7.13.1 on page 838  
 ⇒ “FiniteFieldNormalBasisExtension” (FFNBX) 7.12.1 on page 832



**Exports:**

|                           |                          |                        |
|---------------------------|--------------------------|------------------------|
| 0                         | 1                        | algebraic?             |
| associates?               | basis                    | characteristic         |
| charthRoot                | coerce                   | conditionP             |
| coordinates               | createNormalElement      | createPrimitiveElement |
| D                         | definingPolynomial       | degree                 |
| differentiate             | dimension                | discreteLog            |
| divide                    | euclideanSize            | expressIdealMember     |
| exquo                     | extendedEuclidean        | extensionDegree        |
| factor                    | factorsOfCyclicGroupSize | Frobenius              |
| gcd                       | gcdPolynomial            | generator              |
| getMultiplicationMatrix   | getMultiplicationTable   | hash                   |
| index                     | inGroundField?           | init                   |
| inv                       | latex                    | lcm                    |
| linearAssociatedExp       | linearAssociatedLog      | linearAssociatedOrder  |
| lookup                    | minimalPolynomial        | multiEuclidean         |
| nextItem                  | norm                     | normal?                |
| normalElement             | one?                     | order                  |
| prime?                    | primeFrobenius           | primitive?             |
| primitiveElement          | principalIdeal           | random                 |
| recip                     | representationType       | represents             |
| retract                   | retractIfCan             | sample                 |
| size                      | sizeLess?                | sizeMultiplication     |
| squareFree                | squareFreePart           | subtractIfCan          |
| tableForDiscreteLogarithm | trace                    | transcendenceDegree    |
| transcendent?             | unit?                    | unitCanonical          |
| unitNormal                | zero?                    | ?*                     |
| ***?                      | ?+?                      | ?-?                    |
| -?                        | ?/?                      | ?=?                    |
| ?^?                       | ?~=?                     | ?quo?                  |
| ?rem?                     |                          |                        |

— domain FFNB FiniteFieldNormalBasis —

```

)abbrev domain FFNB FiniteFieldNormalBasis
++ Authors: J.Grabmeier, A.Scheerhorn
++ Date Created: 26.03.1991
++ Date Last Updated:
++ Basic Operations:
++ Related Constructors: FiniteFieldNormalBasisExtensionByPolynomial,
++ FiniteFieldPolynomialPackage
++ Also See: FiniteField, FiniteFieldCyclicGroup
++ AMS Classifications:
++ Keywords: finite field, normal basis
++ References:
++ R.Lidl, H.Niederreiter: Finite Field, Encyclopedia of Mathematics and
++ Its Applications, Vol. 20, Cambridge Univ. Press, 1983, ISBN 0 521 30240 4

```

```

++ J. Grabmeier, A. Scheerhorn: Finite Fields in AXIOM.
++ AXIOM Technical Report Series, ATR/5 NP2522.
++ Description:
++ FiniteFieldNormalBasis(p,n) implements a
++ finite extension field of degree n over the prime field with p elements.
++ The elements are represented by coordinate vectors with respect to
++ a normal basis,
++ i.e. a basis consisting of the conjugates (q-powers) of an element, in
++ this case called normal element.
++ This is chosen as a root of the extension polynomial
++ created by createNormalPoly

FiniteFieldNormalBasis(p,extdeg):_
  Exports == Implementation where
  p : PositiveInteger
  extdeg: PositiveInteger          -- the extension degree
  NNI    ==> NonNegativeInteger
  FFF    ==> FiniteFieldFunctions(PrimeField(p))
  TERM   ==> Record(value:PrimeField(p),index:SingleInteger)
  Exports ==> FiniteAlgebraicExtensionField(PrimeField(p)) with
    getMultiplicationTable: () -> Vector List TERM
      ++ getMultiplicationTable() returns the multiplication
      ++ table for the normal basis of the field.
      ++ This table is used to perform multiplications between field elements.
    getMultiplicationMatrix: () -> Matrix PrimeField(p)
      ++ getMultiplicationMatrix() returns the multiplication table in
      ++ form of a matrix.
    sizeMultiplication: () -> NNI
      ++ sizeMultiplication() returns the number of entries in the
      ++ multiplication table of the field. Note: The time of multiplication
      ++ of field elements depends on this size.

  Implementation ==> FiniteFieldNormalBasisExtensionByPolynomial(PrimeField(p),_
    createLowComplexityNormalBasis(extdeg)$FFF)

-----

-- FFNB.dotabb --

"FFNB" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FFNB"]
"FAXF" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FAXF"]
"FFNB" -> "FAXF"

-----

```

## 7.12 domain FFNBX FiniteFieldNormalBasisExtension

— FiniteFieldNormalBasisExtension.input —

```

)set break resume
)sys rm -f FiniteFieldNormalBasisExtension.output
)spool FiniteFieldNormalBasisExtension.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FiniteFieldNormalBasisExtension
--R FiniteFieldNormalBasisExtension(GF: FiniteFieldCategory,extdeg: PositiveInteger) is a d
--R Abbreviation for FiniteFieldNormalBasisExtension is FFNBX
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FFNBX
--R
--R----- Operations -----
--R ???: (GF,%) -> %                ??? : (%,GF) -> %
--R ???: (Fraction Integer,%) -> %  ??? : (%,Fraction Integer) -> %
--R ???: (%,%) -> %                ??? : (Integer,%) -> %
--R ???: (PositiveInteger,%) -> %   ??? : (%,Integer) -> %
--R ??? : (%,PositiveInteger) -> %  ?+?: (%,%) -> %
--R ?-?: (%,%) -> %                -?: % -> %
--R ?/? : (%,GF) -> %              ?/? : (%,%) -> %
--R ?=: (%,%) -> Boolean           D : % -> % if GF has FINITE
--R 1 : () -> %                    0 : () -> %
--R ???: (%,Integer) -> %          ???: (%,PositiveInteger) -> %
--R algebraic? : % -> Boolean      associates? : (%,%) -> Boolean
--R basis : () -> Vector %         coerce : GF -> %
--R coerce : Fraction Integer -> % coerce : % -> %
--R coerce : Integer -> %          coerce : % -> OutputForm
--R coordinates : % -> Vector GF   degree : % -> PositiveInteger
--R dimension : () -> CardinalNumber factor : % -> Factored %
--R gcd : List % -> %              gcd : (%,%) -> %
--R hash : % -> SingleInteger      inGroundField? : % -> Boolean
--R inv : % -> %                   latex : % -> String
--R lcm : List % -> %              lcm : (%,%) -> %
--R norm : % -> GF                 one? : % -> Boolean
--R prime? : % -> Boolean           ?quo? : (%,%) -> %
--R recip : % -> Union(%, "failed") ?rem? : (%,%) -> %
--R represents : Vector GF -> %    retract : % -> GF
--R sample : () -> %               sizeLess? : (%,%) -> Boolean
--R squareFree : % -> Factored %   squareFreePart : % -> %
--R trace : % -> GF                transcendent? : % -> Boolean
--R unit? : % -> Boolean           unitCanonical : % -> %
--R zero? : % -> Boolean           ~=?: (%,%) -> Boolean

```

```

--R ?? : (NonNegativeInteger,%) -> %
--R ***? : (%,NonNegativeInteger) -> %
--R D : (%,NonNegativeInteger) -> % if GF has FINITE
--R Frobenius : (%,NonNegativeInteger) -> % if GF has FINITE
--R Frobenius : % -> % if GF has FINITE
--R ?? : (%,NonNegativeInteger) -> %
--R basis : PositiveInteger -> Vector %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%, "failed") if GF has CHARNZ or GF has FINITE
--R charthRoot : % -> % if GF has FINITE
--R conditionP : Matrix % -> Union(Vector %, "failed") if GF has FINITE
--R coordinates : Vector % -> Matrix GF
--R createNormalElement : () -> % if GF has FINITE
--R createPrimitiveElement : () -> % if GF has FINITE
--R definingPolynomial : () -> SparseUnivariatePolynomial GF
--R degree : % -> OnePointCompletion PositiveInteger
--R differentiate : (%,NonNegativeInteger) -> % if GF has FINITE
--R differentiate : % -> % if GF has FINITE
--R discreteLog : (%,%) -> Union(NonNegativeInteger, "failed") if GF has CHARNZ or GF has FINITE
--R discreteLog : % -> NonNegativeInteger if GF has FINITE
--R divide : (%,%) -> Record(quotient: %, remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %, %) -> Union(List %, "failed")
--R exquo : (%,%) -> Union(%, "failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %, coef2: %), "failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %, coef2: %, generator: %)
--R extensionDegree : () -> PositiveInteger
--R extensionDegree : () -> OnePointCompletion PositiveInteger
--R factorsOfCyclicGroupSize : () -> List Record(factor: Integer, exponent: Integer) if GF has FINITE
--R gcdPolynomial : (SparseUnivariatePolynomial %, SparseUnivariatePolynomial %) -> SparseUnivariatePolynomial
--R generator : () -> % if GF has FINITE
--R getMultiplicationMatrix : () -> Matrix GF
--R getMultiplicationTable : () -> Vector List Record(value: GF, index: SingleInteger)
--R index : PositiveInteger -> % if GF has FINITE
--R init : () -> % if GF has FINITE
--R linearAssociatedExp : (%, SparseUnivariatePolynomial GF) -> % if GF has FINITE
--R linearAssociatedLog : (%,%) -> Union(SparseUnivariatePolynomial GF, "failed") if GF has FINITE
--R linearAssociatedLog : % -> SparseUnivariatePolynomial GF if GF has FINITE
--R linearAssociatedOrder : % -> SparseUnivariatePolynomial GF if GF has FINITE
--R lookup : % -> PositiveInteger if GF has FINITE
--R minimalPolynomial : (%, PositiveInteger) -> SparseUnivariatePolynomial % if GF has FINITE
--R minimalPolynomial : % -> SparseUnivariatePolynomial GF
--R multiEuclidean : (List %, %) -> Union(List %, "failed")
--R nextItem : % -> Union(%, "failed") if GF has FINITE
--R norm : (%, PositiveInteger) -> % if GF has FINITE
--R normal? : % -> Boolean if GF has FINITE
--R normalElement : () -> % if GF has FINITE
--R order : % -> OnePointCompletion PositiveInteger if GF has CHARNZ or GF has FINITE
--R order : % -> PositiveInteger if GF has FINITE
--R primeFrobenius : % -> % if GF has CHARNZ or GF has FINITE

```

```

--R primeFrobenius : (% , NonNegativeInteger) -> % if GF has CHARNZ or GF has FINITE
--R primitive? : % -> Boolean if GF has FINITE
--R primitiveElement : () -> % if GF has FINITE
--R principalIdeal : List % -> Record(coef: List %, generator: %)
--R random : () -> % if GF has FINITE
--R representationType : () -> Union("prime", polynomial, normal, cyclic) if GF has FINITE
--R retractIfCan : % -> Union(GF, "failed")
--R size : () -> NonNegativeInteger if GF has FINITE
--R sizeMultiplication : () -> NonNegativeInteger
--R subtractIfCan : (% , %) -> Union(% , "failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger, NonNegativeInteger) if GF has FINITE
--R trace : (% , PositiveInteger) -> % if GF has FINITE
--R transcendenceDegree : () -> NonNegativeInteger
--R unitNormal : % -> Record(unit: % , canonical: % , associate: %)
--R
--E 1

)spool
)lisp (bye)

```

---

— FiniteFieldNormalBasisExtension.help —

```

=====
FiniteFieldNormalBasisExtension examples
=====

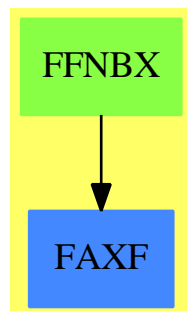
```

```

See Also:
o )show FiniteFieldNormalBasisExtension

```

### 7.12.1 FiniteFieldNormalBasisExtension (FFNBX)



See

⇒ “FiniteFieldNormalBasisExtensionByPolynomial” (FFNBP) 7.13.1 on page 838

⇒ “FiniteFieldNormalBasis” (FFNB) 7.11.1 on page 827

### Exports:

|                           |                          |                        |
|---------------------------|--------------------------|------------------------|
| 0                         | 1                        | algebraic?             |
| associates?               | basis                    | characteristic         |
| charthRoot                | coerce                   | conditionP             |
| coordinates               | createNormalElement      | createPrimitiveElement |
| D                         | definingPolynomial       | degree                 |
| differentiate             | dimension                | discreteLog            |
| divide                    | euclideanSize            | expressIdealMember     |
| exquo                     | extendedEuclidean        | extensionDegree        |
| factor                    | factorsOfCyclicGroupSize | Frobenius              |
| gcd                       | gcdPolynomial            | generator              |
| getMultiplicationMatrix   | getMultiplicationTable   | hash                   |
| index                     | inGroundField?           | init                   |
| inv                       | latex                    | lcm                    |
| linearAssociatedExp       | linearAssociatedLog      | linearAssociatedOrder  |
| lookup                    | minimalPolynomial        | multiEuclidean         |
| nextItem                  | norm                     | normal?                |
| normalElement             | one?                     | order                  |
| prime?                    | primeFrobenius           | primitive?             |
| primitiveElement          | principalIdeal           | random                 |
| recip                     | representationType       | represents             |
| retract                   | retractIfCan             | sample                 |
| size                      | sizeLess?                | sizeMultiplication     |
| squareFree                | squareFreePart           | subtractIfCan          |
| tableForDiscreteLogarithm | trace                    | transcendenceDegree    |
| transcendent?             | unit?                    | unitCanonical          |
| unitNormal                | zero?                    | ?*?                    |
| ?**?                      | ?+?                      | ?-?                    |
| -?                        | ?/?                      | ?=?                    |
| ?^?                       | ?~=?                     | ?quo?                  |
| ?rem?                     |                          |                        |

— domain FFNBX FiniteFieldNormalBasisExtension —

```
)abbrev domain FFNBX FiniteFieldNormalBasisExtension
++ Authors: J.Grabmeier, A.Scheerhorn
++ Date Created: 26.03.1991
++ Date Last Updated:
++ Basic Operations:
++ Related Constructors: FiniteFieldNormalBasisExtensionByPolynomial,
++ FiniteFieldPolynomialPackage
++ Also See: FiniteFieldExtension, FiniteFieldCyclicGroupExtension
++ AMS Classifications:
++ Keywords: finite field, normal basis
```

```

++ References:
++ R.Lidl, H.Niederreiter: Finite Field, Encyclopedia of Mathematics and
++ Its Applications, Vol. 20, Cambridge Univ. Press, 1983, ISBN 0 521 30240 4
++ J. Grabmeier, A. Scheerhorn: Finite Fields in AXIOM.
++ AXIOM Technical Report Series, ATR/5 NP2522.
++ Description:
++ FiniteFieldNormalBasisExtensionByPolynomial(GF,n) implements a
++ finite extension field of degree n over the ground field GF.
++ The elements are represented by coordinate vectors with respect
++ to a normal basis,
++ i.e. a basis consisting of the conjugates (q-powers) of an element, in
++ this case called normal element. This is chosen as a root of the extension
++ polynomial, created by createNormalPoly from
++ \spadtype{FiniteFieldPolynomialPackage}

FiniteFieldNormalBasisExtension(GF,extdeg):_
  Exports == Implementation where
  GF      : FiniteFieldCategory          -- the ground field
  extdeg: PositiveInteger                -- the extension degree
  NNI      ==> NonNegativeInteger
  FFF      ==> FiniteFieldFunctions(GF)
  TERM     ==> Record(value:GF,index:SingleInteger)
  Exports ==> FiniteAlgebraicExtensionField(GF) with
    getMultiplicationTable: () -> Vector List TERM
      ++ getMultiplicationTable() returns the multiplication
      ++ table for the normal basis of the field.
      ++ This table is used to perform multiplications between field elements.
    getMultiplicationMatrix: () -> Matrix GF
      ++ getMultiplicationMatrix() returns the multiplication table in
      ++ form of a matrix.
    sizeMultiplication:() -> NNI
      ++ sizeMultiplication() returns the number of entries in the
      ++ multiplication table of the field. Note: the time of multiplication
      ++ of field elements depends on this size.

  Implementation ==> FiniteFieldNormalBasisExtensionByPolynomial(GF,_
    createLowComplexityNormalBasis(extdeg)$FFF)

```

---

— FFNBX.dotabb —

```

"FFNBX" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FFNBX"]
"FAXF"  [color="#4488FF",href="bookvol10.2.pdf#nameddest=FAXF"]
"FFNBX" -> "FAXF"

```

---

## 7.13 domain FFNBP FiniteFieldNormalBasisExtension-ByPolynomial

— FiniteFieldNormalBasisExtensionByPolynomial.input —

```
)set break resume
)sys rm -f FiniteFieldNormalBasisExtensionByPolynomial.output
)spool FiniteFieldNormalBasisExtensionByPolynomial.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FiniteFieldNormalBasisExtensionByPolynomial
--R FiniteFieldNormalBasisExtensionByPolynomial(GF: FiniteFieldCategory, uni: Union(SparseUnivariatePolynom
--R Abbreviation for FiniteFieldNormalBasisExtensionByPolynomial is FFNBP
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FFNBP
--R
--R----- Operations -----
--R ?? : (GF,%) -> %          ?? : (%,GF) -> %
--R ?? : (Fraction Integer,%) -> %    ?? : (%,Fraction Integer) -> %
--R ?? : (%,%) -> %          ?? : (Integer,%) -> %
--R ?? : (PositiveInteger,%) -> %    ??? : (%,Integer) -> %
--R ??? : (%,PositiveInteger) -> %   ?+? : (%,%) -> %
--R ?-? : (%,%) -> %          -? : % -> %
--R ?/? : (%,GF) -> %         ?/? : (%,%) -> %
--R ?=? : (%,%) -> Boolean      D : % -> % if GF has FINITE
--R 1 : () -> %                0 : () -> %
--R ??^? : (%,Integer) -> %     ??^? : (%,PositiveInteger) -> %
--R algebraic? : % -> Boolean   associates? : (%,%) -> Boolean
--R basis : () -> Vector %     coerce : GF -> %
--R coerce : Fraction Integer -> %   coerce : % -> %
--R coerce : Integer -> %        coerce : % -> OutputForm
--R coordinates : % -> Vector GF    degree : % -> PositiveInteger
--R dimension : () -> CardinalNumber factor : % -> Factored %
--R gcd : List % -> %           gcd : (%,%) -> %
--R hash : % -> SingleInteger    inGroundField? : % -> Boolean
--R inv : % -> %                latex : % -> String
--R lcm : List % -> %           lcm : (%,%) -> %
--R norm : % -> GF              one? : % -> Boolean
--R prime? : % -> Boolean        ?quo? : (%,%) -> %
--R recip : % -> Union(%, "failed") ?rem? : (%,%) -> %
--R represents : Vector GF -> %   retract : % -> GF
--R sample : () -> %            sizeLess? : (%,%) -> Boolean
--R squareFree : % -> Factored %  squareFreePart : % -> %
--R trace : % -> GF             transcendent? : % -> Boolean
--R unit? : % -> Boolean         unitCanonical : % -> %
```



```

--R zero? : % -> Boolean                                ?~=? : (%,% ) -> Boolean
--R ?? : (NonNegativeInteger,% ) -> %
--R ??? : (% ,NonNegativeInteger) -> %
--R D : (% ,NonNegativeInteger) -> % if GF has FINITE
--R Frobenius : (% ,NonNegativeInteger) -> % if GF has FINITE
--R Frobenius : % -> % if GF has FINITE
--R ?^? : (% ,NonNegativeInteger) -> %
--R basis : PositiveInteger -> Vector %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(% ,"failed") if GF has CHARNZ or GF has FINITE
--R charthRoot : % -> % if GF has FINITE
--R conditionP : Matrix % -> Union(Vector % ,"failed") if GF has FINITE
--R coordinates : Vector % -> Matrix GF
--R createNormalElement : () -> % if GF has FINITE
--R createPrimitiveElement : () -> % if GF has FINITE
--R definingPolynomial : () -> SparseUnivariatePolynomial GF
--R degree : % -> OnePointCompletion PositiveInteger
--R differentiate : (% ,NonNegativeInteger) -> % if GF has FINITE
--R differentiate : % -> % if GF has FINITE
--R discreteLog : (% ,%) -> Union(NonNegativeInteger,"failed") if GF has CHARNZ or GF has FINITE
--R discreteLog : % -> NonNegativeInteger if GF has FINITE
--R divide : (% ,%) -> Record(quotient: % ,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List % ,%) -> Union(List % ,"failed")
--R exquo : (% ,%) -> Union(% ,"failed")
--R extendedEuclidean : (% ,%,%) -> Union(Record(coef1: % ,coef2: % ),"failed")
--R extendedEuclidean : (% ,%) -> Record(coef1: % ,coef2: % ,generator: %)
--R extensionDegree : () -> PositiveInteger
--R extensionDegree : () -> OnePointCompletion PositiveInteger
--R factorsOfCyclicGroupSize : () -> List Record(factor: Integer,exponent: Integer) if GF has FINITE
--R gcdPolynomial : (SparseUnivariatePolynomial % ,SparseUnivariatePolynomial % ) -> SparseUnivariatePolynomial GF
--R generator : () -> % if GF has FINITE
--R getMultiplicationMatrix : () -> Matrix GF
--R getMultiplicationTable : () -> Vector List Record(value: GF,index: SingleInteger)
--R index : PositiveInteger -> % if GF has FINITE
--R init : () -> % if GF has FINITE
--R linearAssociatedExp : (% ,SparseUnivariatePolynomial GF) -> % if GF has FINITE
--R linearAssociatedLog : (% ,%) -> Union(SparseUnivariatePolynomial GF,"failed") if GF has FINITE
--R linearAssociatedLog : % -> SparseUnivariatePolynomial GF if GF has FINITE
--R linearAssociatedOrder : % -> SparseUnivariatePolynomial GF if GF has FINITE
--R lookup : % -> PositiveInteger if GF has FINITE
--R minimalPolynomial : (% ,PositiveInteger) -> SparseUnivariatePolynomial % if GF has FINITE
--R minimalPolynomial : % -> SparseUnivariatePolynomial GF
--R multiEuclidean : (List % ,%) -> Union(List % ,"failed")
--R nextItem : % -> Union(% ,"failed") if GF has FINITE
--R norm : (% ,PositiveInteger) -> % if GF has FINITE
--R normal? : % -> Boolean if GF has FINITE
--R normalElement : () -> % if GF has FINITE
--R order : % -> OnePointCompletion PositiveInteger if GF has CHARNZ or GF has FINITE
--R order : % -> PositiveInteger if GF has FINITE

```

### 7.13. DOMAIN FFBP FINITEFIELDNORMALBASISEXTENSIONBYPOLYNOMIAL837

```

--R primeFrobenius : % -> % if GF has CHARNZ or GF has FINITE
--R primeFrobenius : (% , NonNegativeInteger) -> % if GF has CHARNZ or GF has FINITE
--R primitive? : % -> Boolean if GF has FINITE
--R primitiveElement : () -> % if GF has FINITE
--R principalIdeal : List % -> Record(coef: List %, generator: %)
--R random : () -> % if GF has FINITE
--R representationType : () -> Union("prime", polynomial, normal, cyclic) if GF has FINITE
--R retractIfCan : % -> Union(GF, "failed")
--R size : () -> NonNegativeInteger if GF has FINITE
--R sizeMultiplication : () -> NonNegativeInteger
--R subtractIfCan : (% , %) -> Union(% , "failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger, NonNegativeInteger) if GF has FINITE
--R trace : (% , PositiveInteger) -> % if GF has FINITE
--R transcendenceDegree : () -> NonNegativeInteger
--R unitNormal : % -> Record(unit: %, canonical: %, associate: %)
--R
--E 1

```

```

)spool
)lisp (bye)

```

---

#### — FiniteFieldNormalBasisExtensionByPolynomial.help —

```

=====
FiniteFieldNormalBasisExtensionByPolynomial examples
=====

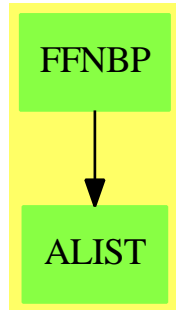
```

```

See Also:
o )show FiniteFieldNormalBasisExtensionByPolynomial

```

---

**7.13.1 FiniteFieldNormalBasisExtensionByPolynomial (FFNBP)**

**See**

⇒ “FiniteFieldNormalBasisExtension” (FFNBX) 7.12.1 on page 832

⇒ “FiniteFieldNormalBasis” (FFNB) 7.11.1 on page 827

**Exports:**

|                           |                          |                        |
|---------------------------|--------------------------|------------------------|
| 0                         | 1                        | algebraic?             |
| associates?               | basis                    | characteristic         |
| charthRoot                | coerce                   | conditionP             |
| coordinates               | createNormalElement      | createPrimitiveElement |
| D                         | definingPolynomial       | degree                 |
| differentiate             | dimension                | discreteLog            |
| divide                    | euclideanSize            | expressIdealMember     |
| exquo                     | extendedEuclidean        | extensionDegree        |
| factor                    | factorsOfCyclicGroupSize | Frobenius              |
| gcd                       | gcdPolynomial            | generator              |
| getMultiplicationMatrix   | getMultiplicationTable   | hash                   |
| index                     | inGroundField?           | init                   |
| inv                       | latex                    | lcm                    |
| linearAssociatedExp       | linearAssociatedLog      | linearAssociatedOrder  |
| lookup                    | minimalPolynomial        | multiEuclidean         |
| nextItem                  | norm                     | normal?                |
| normalElement             | one?                     | order                  |
| prime?                    | primeFrobenius           | primitive?             |
| primitiveElement          | principalIdeal           | recip                  |
| random                    | representationType       | represents             |
| retract                   | retractIfCan             | sample                 |
| size                      | sizeLess?                | sizeMultiplication     |
| squareFree                | squareFreePart           | subtractIfCan          |
| tableForDiscreteLogarithm | trace                    | transcendenceDegree    |
| transcendent?             | unit?                    | unitCanonical          |
| unitNormal                | zero?                    | ?*                     |
| ***?                      | ?+?                      | ?-?                    |
| -?                        | ?/?                      | ?=?                    |
| ?^?                       | ?~=?                     | ?quo?                  |
| ?rem?                     |                          |                        |

— domain **FFNBP** FiniteFieldNormalBasisExtensionByPolynomial

```
)abbrev domain FFNBP FiniteFieldNormalBasisExtensionByPolynomial
++ Authors: J.Grabmeier, A.Scheerhorn
++ Date Created: 26.03.1991
++ Date Last Updated: 08 May 1991
++ Basic Operations:
++ Related Constructors: InnerNormalBasisFieldFunctions, FiniteFieldFunctions,
++ Also See: FiniteFieldExtensionByPolynomial,
++ FiniteFieldCyclicGroupExtensionByPolynomial
++ AMS Classifications:
++ Keywords: finite field, normal basis
++ References:
++ R.Lidl, H.Niederreiter: Finite Field, Encyclopedia of Mathematics and
```

```

++ Its Applications, Vol. 20, Cambridge Univ. Press, 1983, ISBN 0 521 30240 4
++ J. Grabmeier, A. Scheerhorn: Finite Fields in AXIOM .
++ AXIOM Technical Report Series, ATR/5 NP2522.
++ Description:
++ FiniteFieldNormalBasisExtensionByPolynomial(GF,uni) implements a
++ finite extension of the ground field GF. The elements are
++ represented by coordinate vectors with respect to. a normal basis,
++ i.e. a basis
++ consisting of the conjugates (q-powers) of an element, in this case
++ called normal element, where q is the size of GF.
++ The normal element is chosen as a root of the extension
++ polynomial, which MUST be normal over GF (user responsibility)

FiniteFieldNormalBasisExtensionByPolynomial(GF,uni): Exports == _
Implementation where
GF      : FiniteFieldCategory          -- the ground field
uni     : Union(SparseUnivariatePolynomial GF, _
                Vector List Record(value:GF,index:SingleInteger))

PI      ==> PositiveInteger
NNI     ==> NonNegativeInteger
I       ==> Integer
SI      ==> SingleInteger
SUP     ==> SparseUnivariatePolynomial
V       ==> Vector GF
M       ==> Matrix GF
OUT     ==> OutputForm
TERM    ==> Record(value:GF,index:SI)
R       ==> Record(key:PI,entry:NNI)
TBL     ==> Table(PI,NNI)
FFF     ==> FiniteFieldFunctions(GF)
INBFF   ==> InnerNormalBasisFieldFunctions(GF)

Exports ==> FiniteAlgebraicExtensionField(GF) with

getMultiplicationTable: () -> Vector List TERM
++ getMultiplicationTable() returns the multiplication
++ table for the normal basis of the field.
++ This table is used to perform multiplications between field elements.
getMultiplicationMatrix:() -> M
++ getMultiplicationMatrix() returns the multiplication table in
++ form of a matrix.
sizeMultiplication:() -> NNI
++ sizeMultiplication() returns the number of entries in the
++ multiplication table of the field.
++ Note: the time of multiplication
++ of field elements depends on this size.
Implementation ==> add

-- global variables =====

```

### 7.13. DOMAIN FFBP FINITEFIELDNORMALBASISEXTENSIONBYPOLYNOMIAL841

```

Rep:= V      -- elements are represented by vectors over GF

alpha        :=new()$Symbol :: OutputForm
-- get a new Symbol for the output representation of the elements

initlog?:Boolean:=true
-- gets false after initialization of the logarithm table

initelt?:Boolean:=true
-- gets false after initialization of the primitive element

initmult?:Boolean:=true
-- gets false after initialization of the multiplication
-- table or the primitive element

extdeg:PI    :=1

defpol:SUP(GF):=0$SUP(GF)
-- the defining polynomial

multTable:Vector List TERM:=new(1,nil())$(List TERM))
-- global variable containing the multiplication table

if uni case (Vector List TERM) then
  multTable:=uni :: (Vector List TERM)
  extdeg:= (#multTable) pretend PI
  vv:V:=new(extdeg,0)$V
  vv.1:=1$GF
  setFieldInfo(multTable,1$GF)$INBFF
  defpol:=minimalPolynomial(vv)$INBFF
  initmult?:=false
else
  defpol:=uni :: SUP(GF)
  extdeg:=degree(defpol)$(SUP GF) pretend PI
  multTable:Vector List TERM:=new(extdeg,nil())$(List TERM))

basisOutput : List OUT :=
  qs:OUT:=(q::Symbol)::OUT
  append([alpha, alpha **$OUT qs],_
    [alpha **$OUT (qs **$OUT i::OUT) for i in 2..extdeg-1] )

facOfGroupSize :=nil()$(List Record(factor:Integer,exponent:Integer))
-- the factorization of the cyclic group size

traceAlpha:GF:=-$GF coefficient(defpol,(degree(defpol)-1)::NNI)
-- the inverse of the trace of the normalElt
-- is computed here. It defines the imbedding of

```

```

-- GF in the extension field

primitiveElt:PI:=1
-- for the lookup the primitive Element computed by createPrimitiveElement()

discLogTable:Table(PI,TBL):=table()$Table(PI,TBL)
-- tables indexed by the factors of sizeCG,
-- discLogTable(factor) is a table with keys
-- primitiveElement() ** (i * (sizeCG quo factor)) and entries i for
-- i in 0..n-1, n computed in initialize() in order to use
-- the minimal size limit 'limit' optimal.

-- functions =====

initializeLog: ()      -> Void
initializeElt: ()      -> Void
initializeMult: ()     -> Void

coerce(v:GF):$ == new(extdeg,v /$GF traceAlpha)$Rep
represents(v) == v::$

degree(a) ==
d:PI:=1
b:= qPot(a::Rep,1)$INBFF
while (b^=a) repeat
  b:= qPot(b::Rep,1)$INBFF
  d:=d+1
d

linearAssociatedExp(x,f) ==
xm:SUP(GF):=monomial(1$GF,extdeg)$(SUP GF) - 1$(SUP GF)
r:= (f * pol(x::Rep)$INBFF) rem xm
vectorise(r,extdeg)$(SUP GF)
linearAssociatedLog(x) == pol(x::Rep)$INBFF
linearAssociatedOrder(x) ==
xm:SUP(GF):=monomial(1$GF,extdeg)$(SUP GF) - 1$(SUP GF)
xm quo gcd(xm,pol(x::Rep)$INBFF)
linearAssociatedLog(b,x) ==
zero? x => 0
xm:SUP(GF):=monomial(1$GF,extdeg)$(SUP GF) - 1$(SUP GF)
e:= extendedEuclidean(pol(b::Rep)$INBFF,xm,pol(x::Rep)$INBFF)$(SUP GF)
e = "failed" => "failed"
e1:= e :: Record(coef1:(SUP GF),coef2:(SUP GF))
e1.coef1

getMultiplicationTable() ==
if initmult? then initializeMult()
multTable
getMultiplicationMatrix() ==

```

### 7.13. DOMAIN FFBP FINITEFIELDNORMALBASISEXTENSIONBYPOLYNOMIAL843

```

    if initmult? then initializeMult()
    createMultiplicationMatrix(multTable)$FFF
sizeMultiplication() ==
    if initmult? then initializeMult()
    sizeMultiplication(multTable)$FFF

trace(a:$) == retract trace(a,1)
norm(a:$) == retract norm(a,1)
generator() == normalElement(extdeg)$INBFF
basis(n:PI) ==
    (extdeg rem n) ^= 0 => error "argument must divide extension degree"
    [Frobenius(trace(normalElement,n),i) for i in 0..(n-1)]::(Vector $)

a:GF * x:$ == a *$Rep x

x:$/a:GF == x/coerce(a)
-- x:$ / a:GF ==
-- a = 0$GF => error "division by zero"
-- x * inv(coerce(a))

coordinates(x:$) == x::Rep

Frobenius(e) == qPot(e::Rep,1)$INBFF
Frobenius(e,n) == qPot(e::Rep,n)$INBFF

retractIfCan(x) ==
    inGroundField?(x) =>
        x.1 *$GF traceAlpha
        "failed"

retract(x) ==
    inGroundField?(x) =>
        x.1 *$GF traceAlpha
        error("element not in ground field")

-- to get a "normal basis like" output form
coerce(x:$):OUT ==
    l:List OUT:=nil()$(List OUT)
    n : PI := extdeg
-- one? n => (x.1) :: OUT
    (n = 1) => (x.1) :: OUT
    for i in 1..n for b in basisOutput repeat
        if not zero? x.i then
            mon : OUT :=
-- one? x.i => b
                (x.i = 1) => b
                ((x.i)::OUT) *$OUT b
            l:=cons(mon,l)$(List OUT)
    null(l)$(List OUT) => (0::OUT)

```



```

r:=reduce("+",1)$(List OUT)
r

initializeElt() ==
  facOfGroupSize := factors factor(size())$GF**extdeg-1)$I
  -- get a primitive element
  primitiveElt:=lookup(createPrimitiveElement())
  initelt?:=false
  void()$Void

initializeMult() ==
  multTable:=createMultiplicationTable(defpol)$FFF
  setFieldInfo(multTable,traceAlpha)$INBFF
  -- reset initialize flag
  initmult?:=false
  void()$Void

initializeLog() ==
  if initelt? then initializeElt()
  -- set up tables for discrete logarithm
  limit:Integer:=30
  -- the minimum size for the discrete logarithm table
  for f in facOfGroupSize repeat
    fac:=f.factor
    base:=index(primitiveElt)**((size())$GF**extdeg -$I 1$I) quo$I fac)
    l:Integer:=length(fac)$Integer
    n:Integer:=0
    if odd?(l)$I then n:=shift(fac,-$I (l quo$I 2))$I
      else n:=shift(1,l quo$I 2)$I
    if n <$I limit then
      d:=(fac -$I 1$I) quo$I limit +$I 1$I
      n:=(fac -$I 1$I) quo$I d +$I 1$I
      tbl:TBL:=table()$TBL
      a:=$:=1
      for i in (0::NNI)..(n-1)::NNI repeat
        insert_!([lookup(a),i::NNI]$R,tbl)$TBL
        a:=a*base
      insert_!([fac::PI,copy(tbl)$TBL]_
        $Record(key:PI,entry:TBL),discLogTable)$Table(PI,TBL)
    initlog?:=false
    -- tell user about initialization
    --print("discrete logarithm table initialized":OUT)
    void()$Void

tableForDiscreteLogarithm(fac) ==
  if initlog? then initializeLog()
  tbl:=search(fac::PI,discLogTable)$Table(PI,TBL)
  tbl case "failed" =>
    error "tableForDiscreteLogarithm: argument must be prime _
divisor of the order of the multiplicative group"

```

### 7.13. DOMAIN FFBP FINITEFIELDNORMALBASISEXTENSIONBYPOLYNOMIAL845

```

tbl :: TBL

primitiveElement() ==
  if initelt? then initializeElt()
  index(primitiveElt)

factorsOfCyclicGroupSize() ==
  if empty? facOfGroupSize then initializeElt()
  facOfGroupSize

extensionDegree() == extdeg

sizeOfGroundField() == size()$GF pretend NNI

definingPolynomial() == defpol

trace(a,d) ==
  v:=trace(a::Rep,d)$INBFF
  erg:=v
  for i in 2..(extdeg quo d) repeat
    erg:=concat(erg,v)$Rep
  erg

characteristic() == characteristic()$GF

random() == random(extdeg)$INBFF

x:$ * y:$ ==
  if initmult? then initializeMult()
  setFieldInfo(multTable,traceAlpha)$INBFF
  x::Rep *$INBFF y::Rep

1 == new(extdeg,inv(traceAlpha)$GF)$Rep

0 == zero(extdeg)$Rep

size() == size()$GF ** extdeg

index(n:PI) == index(extdeg,n)$INBFF

lookup(x:$) == lookup(x::Rep)$INBFF

basis() ==
  a:=basis(extdeg)$INBFF
  vector([e::$ for e in entries a])

x:$ ** e:I ==

```

```

    if initmult? then initializeMult()
    setFieldInfo(multTable,traceAlpha)$INBFF
    (x::Rep) **$INBFF e

normal?(x) == normal?(x::Rep)$INBFF

-(x:$) == -$Rep x
x:$ + y:$ == x +$Rep y
x:$ - y:$ == x -$Rep y
x:$ = y:$ == x =$Rep y
n:I * x:$ == x *$Rep (n::GF)

representationType() == "normal"

minimalPolynomial(a) ==
    if initmult? then initializeMult()
    setFieldInfo(multTable,traceAlpha)$INBFF
    minimalPolynomial(a::Rep)$INBFF

-- is x an element of the ground field GF ?
inGroundField?(x) ==
    erg:=true
    for i in 2..extdeg repeat
        not(x.i =$GF x.1) => erg:=false
    erg

x:$ / y:$ ==
    if initmult? then initializeMult()
    setFieldInfo(multTable,traceAlpha)$INBFF
    x::Rep /$INBFF y::Rep

inv(a) ==
    if initmult? then initializeMult()
    setFieldInfo(multTable,traceAlpha)$INBFF
    inv(a::Rep)$INBFF

norm(a,d) ==
    if initmult? then initializeMult()
    setFieldInfo(multTable,traceAlpha)$INBFF
    norm(a::Rep,d)$INBFF

normalElement() == normalElement(extdeg)$INBFF

```

---

— FFNBP.dotabb —

```
"FFNBP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FFNBP"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"FFNBP" -> "ALIST"
```

—

## 7.14 domain FARRAY FlexibleArray

— FlexibleArray.input —

```
)set break resume
)sys rm -f FlexibleArray.output
)spool FlexibleArray.output
)set message test on
)set message auto off
)clear all
--S 1 of 16
flexibleArray [i for i in 1..6]
--R
--R
--R (1) [1,2,3,4,5,6]
--R
--R                                          Type: FlexibleArray PositiveInteger
--E 1

--S 2 of 16
f : FARRAY INT := new(6,0)
--R
--R
--R (2) [0,0,0,0,0,0]
--R
--R                                          Type: FlexibleArray Integer
--E 2

--S 3 of 16
for i in 1..6 repeat f.i := i; f
--R
--R
--R (3) [1,2,3,4,5,6]
--R
--R                                          Type: FlexibleArray Integer
--E 3

--S 4 of 16
physicalLength f
--R
```

```

--R
--R (4) 6
--R
--R                                         Type: PositiveInteger
--E 4

--S 5 of 16
concat!(f,11)
--R
--R
--R (5) [1,2,3,4,5,6,11]
--R
--R                                         Type: FlexibleArray Integer
--E 5

--S 6 of 16
physicalLength f
--R
--R
--R (6) 10
--R
--R                                         Type: PositiveInteger
--E 6

--S 7 of 16
physicalLength!(f,15)
--R
--R
--R (7) [1,2,3,4,5,6,11]
--R
--R                                         Type: FlexibleArray Integer
--E 7

--S 8 of 16
concat!(f,f)
--R
--R
--R (8) [1,2,3,4,5,6,11,1,2,3,4,5,6,11]
--R
--R                                         Type: FlexibleArray Integer
--E 8

--S 9 of 16
insert!(22,f,1)
--R
--R
--R (9) [22,1,2,3,4,5,6,11,1,2,3,4,5,6,11]
--R
--R                                         Type: FlexibleArray Integer
--E 9

--S 10 of 16
g := f(10..)
--R
--R
--R (10) [2,3,4,5,6,11]

```



```
)spool
)lisp (bye)
```

---

— FlexibleArray.help —

```
=====
FlexibleArray
=====
```

The FlexibleArray domain constructor creates one-dimensional arrays of elements of the same type. Flexible arrays are an attempt to provide a data type that has the best features of both one-dimensional arrays (fast, random access to elements) and lists (flexibility). They are implemented by a fixed block of storage. When necessary for expansion, a new, larger block of storage is allocated and the elements from the old storage area are copied into the new block.

Flexible arrays have available most of the operations provided by OneDimensionalArray Vector. Since flexible arrays are also of category ExtensibleLinearAggregate they have operations concat!, delete!, insert!, merge!, remove!, removeDuplicates!, and select!. In addition, the operations physicalLength and physicalLength! provide user-control over expansion and contraction.

A convenient way to create a flexible array is to apply the operation flexibleArray to a list of values.

```
flexibleArray [i for i in 1..6]
[1,2,3,4,5,6]
Type: FlexibleArray PositiveInteger
```

Create a flexible array of six zeroes.

```
f : FARRAY INT := new(6,0)
[0,0,0,0,0,0]
Type: FlexibleArray Integer
```

For i=1..6 set the i-th element to i. Display f.

```
for i in 1..6 repeat f.i := i; f
[1,2,3,4,5,6]
Type: FlexibleArray Integer
```

Initially, the physical length is the same as the number of elements.

```
physicalLength f
```

6

Type: PositiveInteger

Add an element to the end of f.

```
concat!(f,11)
[1,2,3,4,5,6,11]
Type: FlexibleArray Integer
```

See that its physical length has grown.

```
physicalLength f
10
Type: PositiveInteger
```

Make f grow to have room for 15 elements.

```
physicalLength!(f,15)
[1,2,3,4,5,6,11]
Type: FlexibleArray Integer
```

Concatenate the elements of f to itself. The physical length allows room for three more values at the end.

```
concat!(f,f)
[1,2,3,4,5,6,11,1,2,3,4,5,6,11]
Type: FlexibleArray Integer
```

Use insert! to add an element to the front of a flexible array.

```
insert!(22,f,1)
[22,1,2,3,4,5,6,11,1,2,3,4,5,6,11]
Type: FlexibleArray Integer
```

Create a second flexible array from f consisting of the elements from index 10 forward.

```
g := f(10..)
[2,3,4,5,6,11]
Type: FlexibleArray Integer
```

Insert this array at the front of f.

```
insert!(g,f,1)
[2,3,4,5,6,11,22,1,2,3,4,5,6,11,1,2,3,4,5,6,11]
Type: FlexibleArray Integer
```

Merge the flexible array f into g after sorting each in place.

```
merge!(sort! f, sort! g)
```



```
[1,1,2,2,2,2,3,3,3,3,4,4,4,4,5,5,5,5,6,6,6,6,11,11,11,11,22]
Type: FlexibleArray Integer
```

Remove duplicates in place.

```
removeDuplicates! f
[1,2,3,4,5,6,11,22]
Type: FlexibleArray Integer
```

Remove all odd integers.

```
select!(i +-> even? i,f)
[2,4,6,22]
Type: FlexibleArray Integer
```

All these operations have shrunk the physical length of f.

```
physicalLength f
8
Type: PositiveInteger
```

To force Axiom not to shrink flexible arrays call the shrinkable operation with the argument false. You must package call this operation. The previous value is returned.

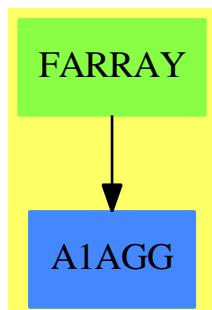
```
shrinkable(false)$FlexibleArray(Integer)
true
Type: Boolean
```

See Also:

- o )help OneDimensionalArray
- o )help Vector
- o )help ExtensibleLinearAggregate
- o )show FlexibleArray

---

## 7.14.1 FlexibleArray (FARRAY)



See

- ⇒ “PrimitiveArray” (PRIMARR) 17.30.1 on page 2069
- ⇒ “Tuple” (TUPLE) 21.12.1 on page 2711
- ⇒ “IndexedFlexibleArray” (IFARRAY) 10.10.1 on page 1187
- ⇒ “IndexedOneDimensionalArray” (IARRAY1) 10.13.1 on page 1208
- ⇒ “OneDimensionalArray” (ARRAY1) 16.3.1 on page 1736

**Exports:**

|          |                  |                   |                |                 |
|----------|------------------|-------------------|----------------|-----------------|
| any?     | coerce           | concat            | concat!        | construct       |
| convert  | copy             | copyInto!         | count          | delete          |
| delete!  | elt              | empty             | empty?         | entries         |
| entry?   | eq?              | eval              | every?         | fill!           |
| find     | first            | flexibleArray     | hash           | index?          |
| indices  | insert           | insert!           | latex          | less?           |
| map      | map!             | max               | maxIndex       | member?         |
| members  | merge            | merge!            | min            | minIndex        |
| more?    | new              | parts             | physicalLength | physicalLength! |
| position | qelt             | qsetelt!          | reduce         | remove          |
| remove!  | removeDuplicates | removeDuplicates! | reverse        | reverse!        |
| sample   | select           | select!           | setelt         | shrinkable      |
| size?    | sort             | sort!             | sorted?        | swap!           |
| #?       | ?<?              | ?<=?              | ?=?            | ?>?             |
| ?>=?     | ?~=?             | ?..?              |                |                 |

— domain FARRAY FlexibleArray —

```

)abbrev domain FARRAY FlexibleArray
++ Author: Mark Botch
++ Description:
++ A FlexibleArray is the notion of an array intended to allow for growth
++ at the end only. Hence the following efficient operations
++ \spad{append(x,a)} meaning append item x at the end of the array \spad{a}
++ \spad{delete(a,n)} meaning delete the last item from the array \spad{a}
++ Flexible arrays support the other operations inherited from
++ \spadtype{ExtensibleLinearAggregate}. However, these are not efficient.
  
```

```

++ Flexible arrays combine the \spad{0(1)} access time property of arrays
++ with growing and shrinking at the end in \spad{0(1)} (average) time.
++ This is done by using an ordinary array which may have zero or more
++ empty slots at the end. When the array becomes full it is copied
++ into a new larger (50% larger) array. Conversely, when the array
++ becomes less than 1/2 full, it is copied into a smaller array.
++ Flexible arrays provide for an efficient implementation of many
++ data structures in particular heaps, stacks and sets.

```

```

FlexibleArray(S: Type) == Implementation where
  ARRAYMININDEX ==> 1      -- if you want to change this, be my guest
  Implementation ==> IndexedFlexibleArray(S, ARRAYMININDEX)
-- Join(OneDimensionalArrayAggregate S, ExtensibleLinearAggregate S)

```

---

— FARRAY.dotabb —

```

"FARRAY" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FARRAY"]
"A1AGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=A1AGG"]
"FARRAY" -> "A1AGG"

```

---

## 7.15 domain FLOAT Float

As reported in bug number 4733 (rounding of negative numbers) errors were observed in operations such as

```

-> round(-3.9)
-> truncate(-3.9)

```

The problem is the unexpected behaviour of the shift with negative integer arguments.

```

-> shift(-7,-1)

```

returns -4 while the code here in float expects the value to be -3. shift uses the lisp function ASH 'arithmetic shift left' but the spad code expects an unsigned 'logical' shift. See

[http://www.lispworks.com/reference/HyperSpec/Body/f\\_ash.htm#ash](http://www.lispworks.com/reference/HyperSpec/Body/f_ash.htm#ash)

A new internal function shift2 is defined in terms of shift to compensate for the use of ASH and provide the required function.

It is currently unknown whether the unexpected behaviour of shift for negative arguments will cause bugs in other parts of Axiom.

— Float.input —

```

)set break resume
)sys rm -f Float.output
)spool Float.output
)set message test on
)set message auto off
)clear all
--S 1 of 64
1.234
--R
--R
--R (1) 1.234
--R
--R                                          Type: Float
--E 1

--S 2 of 64
1.234E2
--R
--R
--R (2) 123.4
--R
--R                                          Type: Float
--E 2

--S 3 of 64
sqrt(1.2 + 2.3 / 3.4 ** 4.5)
--R
--R
--R (3) 1.0996972790 671286226
--R
--R                                          Type: Float
--E 3

--S 4 of 64
i := 3 :: Float
--R
--R
--R (4) 3.0
--R
--R                                          Type: Float
--E 4

--S 5 of 64
i :: Integer
--R
--R
--R (5) 3
--R
--R                                          Type: Integer
--E 5

--S 6 of 64
i :: Fraction Integer
--R
--R

```

```

--R (6) 3
--R
--R                                          Type: Fraction Integer
--E 6

--S 7 of 64
r := 3/7 :: Float
--R
--R
--R (7) 0.4285714285 7142857143
--R
--R                                          Type: Float
--E 7

--S 8 of 64
r :: Fraction Integer
--R
--R
--R      3
--R (8) -
--R      7
--R
--R                                          Type: Fraction Integer
--E 8

--S 9 of 64
r :: Integer
--R
--R
--R      RDaly Bug
--R      Cannot convert from type Float to Integer for value
--R      0.4285714285 7142857143
--R
--R
--E 9

--S 10 of 64
truncate 3.6
--R
--R
--R (9) 3.0
--R
--R                                          Type: Float
--E 10

--S 11 of 64
round 3.6
--R
--R
--R (10) 4.0
--R
--R                                          Type: Float
--E 11

--S 12 of 64
truncate(-3.6)

```

```

--R
--R
--R (11) - 3.0
--R
--R                                          Type: Float
--E 12

--S 13 of 64
round(-3.6)
--R
--R
--R (12) - 4.0
--R
--R                                          Type: Float
--E 13

--S 14 of 64
fractionPart 3.6
--R
--R
--R (13) 0.6
--R
--R                                          Type: Float
--E 14

--S 15 of 64
digits 40
--R
--R
--R (14) 20
--R
--R                                          Type: PositiveInteger
--E 15

--S 16 of 64
sqrt 0.2
--R
--R
--R (15) 0.4472135954 9995793928 1834733746 2552470881
--R
--R                                          Type: Float
--E 16

--S 17 of 64
pi()$Float
--R
--R
--R (16) 3.1415926535 8979323846 2643383279 502884197
--R
--R                                          Type: Float
--E 17

--S 18 of 64
digits 500
--R
--R

```

```

--R (17) 40
--R
--R                                         Type: PositiveInteger
--E 18

--S 19 of 64
pi()$Float
--R
--R
--R (18)
--R 3.1415926535 8979323846 2643383279 5028841971 6939937510 5820974944 592307816
--R 4 0628620899 8628034825 3421170679 8214808651 3282306647 0938446095 505822317
--R 2 5359408128 4811174502 8410270193 8521105559 6446229489 5493038196 442881097
--R 5 6659334461 2847564823 3786783165 2712019091 4564856692 3460348610 454326648
--R 2 1339360726 0249141273 7245870066 0631558817 4881520920 9628292540 917153643
--R 6 7892590360 0113305305 4882046652 1384146951 9415116094 3305727036 575959195
--R 3 0921861173 8193261179 3105118548 0744623799 6274956735 1885752724 891227938
--R 1 830119491
--R
--R                                         Type: Float
--E 19

--S 20 of 64
digits 20
--R
--R
--R (19) 500
--R
--R                                         Type: PositiveInteger
--E 20

--S 21 of 64
outputSpacing 0; x := sqrt 0.2
--R
--R
--R (20) 0.44721359549995793928
--R
--R                                         Type: Float
--E 21

--S 22 of 64
outputSpacing 5; x
--R
--R
--R (21) 0.44721 35954 99957 93928
--R
--R                                         Type: Float
--E 22

--S 23 of 64
y := x/10**10
--R
--R
--R (22) 0.44721 35954 99957 93928 E -10
--R
--R                                         Type: Float

```

```

--E 23

--S 24 of 64
outputFloating(); x
--R
--R
--R (23) 0.44721 35954 99957 93928 E 0
--R
--R                                          Type: Float
--E 24

--S 25 of 64
outputFixed(); y
--R
--R
--R (24) 0.00000 00000 44721 35954 99957 93928
--R
--R                                          Type: Float
--E 25

--S 26 of 64
outputFloating 2; y
--R
--R
--R (25) 0.45 E -10
--R
--R                                          Type: Float
--E 26

--S 27 of 64
outputFixed 2; x
--R
--R
--R (26) 0.45
--R
--R                                          Type: Float
--E 27

--S 28 of 64
outputGeneral()
--R
--R
--R                                          Type: Void
--E 28

--S 29 of 64
a: Matrix Fraction Integer := matrix [ [1/(i+j+1) for j in 0..9] for i in 0..9]
--R
--R
--R
--R      +   1   1   1   1   1   1   1   1   1   1+
--R      |1   -   -   -   -   -   -   -   -   -   --|
--R      |   2   3   4   5   6   7   8   9   10|
--R      |
--R      |1   1   1   1   1   1   1   1   1   1|
--R      |-   -   -   -   -   -   -   -   -   -- --|

```



```

--R      |2  3  4  5  6  7  8  9  10 11|
--R      |
--R      |1  1  1  1  1  1  1  1  1  1|
--R      |- - - - - - - -- -- --|
--R      |3  4  5  6  7  8  9  10 11 12|
--R      |
--R      |1  1  1  1  1  1  1  1  1  1|
--R      |- - - - - - - -- -- --|
--R      |4  5  6  7  8  9  10 11 12 13|
--R      |
--R      |1  1  1  1  1  1  1  1  1  1|
--R      |- - - - - - - -- -- --|
--R      |5  6  7  8  9  10 11 12 13 14|
--R      (28) |
--R      |1  1  1  1  1  1  1  1  1  1|
--R      |- - - - - - - -- -- --|
--R      |6  7  8  9  10 11 12 13 14 15|
--R      |
--R      |1  1  1  1  1  1  1  1  1  1|
--R      |- - - -- -- -- -- -- --|
--R      |7  8  9  10 11 12 13 14 15 16|
--R      |
--R      |1  1  1  1  1  1  1  1  1  1|
--R      |- - -- -- -- -- -- -- --|
--R      |8  9  10 11 12 13 14 15 16 17|
--R      |
--R      |1  1  1  1  1  1  1  1  1  1|
--R      |- -- -- -- -- -- -- -- --|
--R      |9  10 11 12 13 14 15 16 17 18|
--R      |
--R      | 1  1  1  1  1  1  1  1  1  1|
--R      |-- -- -- -- -- -- -- -- --|
--R      +10 11 12 13 14 15 16 17 18 19+
--R
--R                                          Type: Matrix Fraction Integer
--E 29

--S 30 of 64
d:= determinant a
--R
--R
--R
--R
--R      (29) -----
--R      46206893947914691316295628839036278726983680000000000
--R
--R                                          Type: Fraction Integer
--E 30

--S 31 of 64
d :: Float
--R
--R

```

```

--R (30) 0.21641 79226 43149 18691 E -52
--R
--R                                          Type: Float
--E 31

--S 32 of 64
b: Matrix DoubleFloat := matrix [ [1/(i+j+1$DoubleFloat) for j in 0..9] for i in 0..9]
--R
--R
--R (31)
--R [
--R [1., 0.5, 0.3333333333333331, 0.25, 0.20000000000000001,
--R 0.16666666666666666, 0.14285714285714285, 0.125, 0.1111111111111111,
--R 0.10000000000000001]
--R ,
--R [0.5, 0.3333333333333331, 0.25, 0.20000000000000001, 0.1666666666666666,
--R 0.14285714285714285, 0.125, 0.1111111111111111, 0.10000000000000001,
--R 9.0909090909090912E-2]
--R ,
--R [0.3333333333333331, 0.25, 0.20000000000000001, 0.1666666666666666,
--R 0.14285714285714285, 0.125, 0.1111111111111111, 0.10000000000000001,
--R 9.0909090909090912E-2, 8.333333333333329E-2]
--R ,
--R [0.25, 0.20000000000000001, 0.1666666666666666, 0.14285714285714285,
--R 0.125, 0.1111111111111111, 0.10000000000000001, 9.0909090909090912E-2,
--R 8.333333333333329E-2, 7.6923076923076927E-2]
--R ,
--R [0.20000000000000001, 0.1666666666666666, 0.14285714285714285, 0.125,
--R 0.1111111111111111, 0.10000000000000001, 9.0909090909090912E-2,
--R 8.333333333333329E-2, 7.6923076923076927E-2, 7.1428571428571425E-2]
--R ,
--R [0.1666666666666666, 0.14285714285714285, 0.125, 0.1111111111111111,
--R 0.10000000000000001, 9.0909090909090912E-2, 8.333333333333329E-2,
--R 7.6923076923076927E-2, 7.1428571428571425E-2, 6.666666666666666E-2]
--R ,
--R [0.14285714285714285, 0.125, 0.1111111111111111, 0.10000000000000001,
--R 9.0909090909090912E-2, 8.333333333333329E-2, 7.6923076923076927E-2,
--R 7.1428571428571425E-2, 6.666666666666666E-2, 6.25E-2]
--R ,
--R [0.125, 0.1111111111111111, 0.10000000000000001, 9.0909090909090912E-2,
--R 8.333333333333329E-2, 7.6923076923076927E-2, 7.1428571428571425E-2,
--R 6.666666666666666E-2, 6.25E-2, 5.8823529411764705E-2]
--R ,
--R

```

```

--R      [0.1111111111111111, 0.10000000000000001, 9.09090909090912E-2,
--R      8.333333333333329E-2, 7.6923076923076927E-2, 7.1428571428571425E-2,
--R      6.666666666666666E-2, 6.25E-2, 5.8823529411764705E-2,
--R      5.555555555555552E-2]
--R      ,
--R
--R      [0.10000000000000001, 9.09090909090912E-2, 8.333333333333329E-2,
--R      7.6923076923076927E-2, 7.1428571428571425E-2, 6.666666666666666E-2,
--R      6.25E-2, 5.8823529411764705E-2, 5.555555555555552E-2,
--R      5.2631578947368418E-2]
--R      ]
--R
--R                                          Type: Matrix DoubleFloat
--E 32

--S 33 of 64
determinant b
--R
--R
--R      (32)  2.1643677945721411E-53
--R
--R                                          Type: DoubleFloat
--E 33

--S 34 of 64
digits 40
--R
--R
--R      (33)  20
--R
--R                                          Type: PositiveInteger
--E 34

--S 35 of 64
c: Matrix Float := matrix [ [1/(i+j+1$Float) for j in 0..9] for i in 0..9]
--R
--R
--R      (34)
--R      [
--R      [1.0, 0.5, 0.33333 33333 33333 33333 33333 33333 33333 33333 33333, 0.25, 0.2,
--R      0.16666 66666 66666 66666 66666 66666 66666 66666 66667,
--R      0.14285 71428 57142 85714 28571 42857 14285 71429, 0.125,
--R      0.11111 11111 11111 11111 11111 11111 11111 11111 11111, 0.1]
--R      ,
--R
--R      [0.5, 0.33333 33333 33333 33333 33333 33333 33333 33333 33333, 0.25, 0.2,
--R      0.16666 66666 66666 66666 66666 66666 66666 66666 66667,
--R      0.14285 71428 57142 85714 28571 42857 14285 71429, 0.125,
--R      0.11111 11111 11111 11111 11111 11111 11111 11111 11111, 0.1,
--R      0.09090 90909 09090 90909 09090 90909 09090 90909 1]
--R      ,
--R
--R      [0.33333 33333 33333 33333 33333 33333 33333 33333 33333, 0.25, 0.2,

```

```

--R      0.16666 66666 66666 66666 66666 66666 66666 66667,
--R      0.14285 71428 57142 85714 28571 42857 14285 71429, 0.125,
--R      0.11111 11111 11111 11111 11111 11111 11111 11111, 0.1,
--R      0.09090 90909 09090 90909 09090 90909 09090 90909 1,
--R      0.08333 33333 33333 33333 33333 33333 33333 33333 4]
--R      ,
--R
--R      [0.25, 0.2, 0.16666 66666 66666 66666 66666 66666 66667,
--R      0.14285 71428 57142 85714 28571 42857 14285 71429, 0.125,
--R      0.11111 11111 11111 11111 11111 11111 11111 11111, 0.1,
--R      0.09090 90909 09090 90909 09090 90909 09090 90909 1,
--R      0.08333 33333 33333 33333 33333 33333 33333 33333 4,
--R      0.07692 30769 23076 92307 69230 76923 07692 30769 2]
--R      ,
--R
--R      [0.2, 0.16666 66666 66666 66666 66666 66666 66667,
--R      0.14285 71428 57142 85714 28571 42857 14285 71429, 0.125,
--R      0.11111 11111 11111 11111 11111 11111 11111 11111, 0.1,
--R      0.09090 90909 09090 90909 09090 90909 09090 90909 1,
--R      0.08333 33333 33333 33333 33333 33333 33333 33333 4,
--R      0.07692 30769 23076 92307 69230 76923 07692 30769 2,
--R      0.07142 85714 28571 42857 14285 71428 57142 85714 3]
--R      ,
--R
--R      [0.16666 66666 66666 66666 66666 66666 66667,
--R      0.14285 71428 57142 85714 28571 42857 14285 71429, 0.125,
--R      0.11111 11111 11111 11111 11111 11111 11111 11111, 0.1,
--R      0.09090 90909 09090 90909 09090 90909 09090 90909 1,
--R      0.08333 33333 33333 33333 33333 33333 33333 33333 4,
--R      0.07692 30769 23076 92307 69230 76923 07692 30769 2,
--R      0.07142 85714 28571 42857 14285 71428 57142 85714 3,
--R      0.06666 66666 66666 66666 66666 66666 66666 66666 7]
--R      ,
--R
--R      [0.14285 71428 57142 85714 28571 42857 14285 71429, 0.125,
--R      0.11111 11111 11111 11111 11111 11111 11111 11111, 0.1,
--R      0.09090 90909 09090 90909 09090 90909 09090 90909 1,
--R      0.08333 33333 33333 33333 33333 33333 33333 33333 4,
--R      0.07692 30769 23076 92307 69230 76923 07692 30769 2,
--R      0.07142 85714 28571 42857 14285 71428 57142 85714 3,
--R      0.06666 66666 66666 66666 66666 66666 66666 66666 7, 0.0625]
--R      ,
--R
--R      [0.125, 0.11111 11111 11111 11111 11111 11111 11111 11111, 0.1,
--R      0.09090 90909 09090 90909 09090 90909 09090 90909 1,
--R      0.08333 33333 33333 33333 33333 33333 33333 33333 4,
--R      0.07692 30769 23076 92307 69230 76923 07692 30769 2,
--R      0.07142 85714 28571 42857 14285 71428 57142 85714 3,
--R      0.06666 66666 66666 66666 66666 66666 66666 66666 7, 0.0625,
--R      0.05882 35294 11764 70588 23529 41176 47058 82352 9]

```

```

--R      ,
--R
--R      [0.11111 11111 11111 11111 11111 11111 11111 11111, 0.1,
--R      0.09090 90909 09090 90909 09090 90909 09090 90909 1,
--R      0.08333 33333 33333 33333 33333 33333 33333 33333 4,
--R      0.07692 30769 23076 92307 69230 76923 07692 30769 2,
--R      0.07142 85714 28571 42857 14285 71428 57142 85714 3,
--R      0.06666 66666 66666 66666 66666 66666 66666 66666 7, 0.0625,
--R      0.05882 35294 11764 70588 23529 41176 47058 82352 9,
--R      0.05555 55555 55555 55555 55555 55555 55555 55555 6]
--R      ,
--R
--R      [0.1, 0.09090 90909 09090 90909 09090 90909 09090 90909 1,
--R      0.08333 33333 33333 33333 33333 33333 33333 33333 4,
--R      0.07692 30769 23076 92307 69230 76923 07692 30769 2,
--R      0.07142 85714 28571 42857 14285 71428 57142 85714 3,
--R      0.06666 66666 66666 66666 66666 66666 66666 66666 7, 0.0625,
--R      0.05882 35294 11764 70588 23529 41176 47058 82352 9,
--R      0.05555 55555 55555 55555 55555 55555 55555 55555 6,
--R      0.05263 15789 47368 42105 26315 78947 36842 10526 3]
--R      ]
--R
--R                                          Type: Matrix Float
--E 35

--S 36 of 64
determinant c
--R
--R
--R      (35)  0.21641 79226 43149 18690 60594 98362 26174 36159 E -52
--R
--R                                          Type: Float
--E 36

--S 37 of 64
digits 20
--R
--R
--R      (36)  40
--R
--R                                          Type: PositiveInteger
--E 37

)clear all

--S 38 of 64
outputFixed()
--R
--R
--R                                          Type: Void
--E 38

--S 39 of 64
a:=3.0

```

```
--R
--R
--R (2) 3.0
--R                                         Type: Float
--E 39

--S 40 of 64
b:=3.1
--R
--R
--R (3) 3.1
--R                                         Type: Float
--E 40

--S 41 of 64
c:=numeric pi()
--R
--R
--R (4) 3.14159 26535 89793 2385
--R                                         Type: Float
--E 41

--S 42 of 64
d:=0.0
--R
--R
--R (5) 0.0
--R                                         Type: Float
--E 42

--S 43 of 64
outputFixed 2
--R
--R                                         Type: Void
--E 43

--S 44 of 64
a
--R
--R
--R (7) 3.00
--R                                         Type: Float
--E 44

--S 45 of 64
b
--R
--R
--R (8) 3.10
--R                                         Type: Float
```

--E 45

--S 46 of 64

c

--R

--R

--R (9) 3.14

--R

Type: Float

--E 46

--S 47 of 64

d

--R

--R

--R (10) 0.00

--R

Type: Float

--E 47

--S 48 of 64

outputFixed 0

--R

--R

Type: Void

--E 48

--S 49 of 64

a

--R

--R

--R (12) 3.0

--R

Type: Float

--E 49

--S 50 of 64

b

--R

--R

--R (13) 3.

--R

Type: Float

--E 50

--S 51 of 64

c

--R

--R

--R (14) 3.

--R

Type: Float

--E 51

--S 52 of 64

31.1

```
--R
--R
--R (15) 31.
--R                                          Type: Float
--E 52

--S 53 of 64
310.1
--R
--R
--R (16) 310.
--R                                          Type: Float
--E 53

--S 54 of 64
d
--R
--R
--R (17) 0.0
--R                                          Type: Float
--E 54

--S 55 of 64
outputFixed(0)
--R                                          Type: Void
--E 55

--S 56 of 64
1.1
--R
--R
--R (19) 1.
--R                                          Type: Float
--E 56

--S 57 of 64
3111.1
--R
--R
--R (20) 3111.
--R                                          Type: Float
--E 57

--S 58 of 64
1234567890.1
--R
--R
--R (21) 12345 67890.
--R                                          Type: Float
--E 58

--S 59 of 64
outputFixed(12)
```



```

--R                                                    Type: Void
--E 59

--S 60 of 64
1234567890.1
--R
--R   (23)  12345 67890.09999 99999 99
--R                                                    Type: Float
--E 60

--S 61 of 64
outputFixed(15)
--R                                                    Type: Void
--E 61

--S 62 of 64
1234567890.1
--R
--R   (25)  12345 67890.09999 99999 98545
--R                                                    Type: Float
--E 62

--S 63 of 64
outputFixed(2)
--R                                                    Type: Void
--E 63

--S 64 of 64
1234567890.1
--R
--R   (27)  12345 67890.10
--R                                                    Type: Float
--E 64

)spool
)lisp (bye)

```

---

— Float.help —

```

=====
Float
=====

```

Axiom provides two kinds of floating point numbers. The domain Float implements a model of arbitrary precision floating point numbers. The domain DoubleFloat is intended to make available hardware floating point arithmetic in Axiom. The actual model of floating point that

DoubleFloat provides is system-dependent. For example, on the IBM system 370 Axiom uses IBM double precision which has fourteen hexadecimal digits of precision or roughly sixteen decimal digits. Arbitrary precision floats allow the user to specify the precision at which arithmetic operations are computed. Although this is an attractive facility, it comes at a cost. Arbitrary-precision floating-point arithmetic typically takes twenty to two hundred times more time than hardware floating point.

```
=====
Introduction to Float
=====
```

Scientific notation is supported for input and output of floating point numbers. A floating point number is written as a string of digits containing a decimal point optionally followed by the letter "E", and then the exponent.

We begin by doing some calculations using arbitrary precision floats. The default precision is twenty decimal digits.

```
1.234
1.234
Type: Float
```

A decimal base for the exponent is assumed, so the number 1.234E2 denotes  $1.234 \times 10^2$ .

```
1.234E2
123.4
Type: Float
```

The normal arithmetic operations are available for floating point numbers.

```
sqrt(1.2 + 2.3 / 3.4 ** 4.5)
1.0996972790 671286226
Type: Float
```

```
=====
Conversion Functions
=====
```

You can use conversion to go back and forth between Integer, Fraction Integer and Float, as appropriate.

```
i := 3 :: Float
3.0
Type: Float
```

```
i :: Integer
3
```

Type: Integer

```
i :: Fraction Integer
3
```

Type: Fraction Integer

Since you are explicitly asking for a conversion, you must take responsibility for any loss of exactness.

```
r := 3/7 :: Float
0.4285714285 7142857143
Type: Float
```

```
r :: Fraction Integer
3
-
7
```

Type: Fraction Integer

This conversion cannot be performed: use truncate or round if that is what you intend.

```
r :: Integer
Cannot convert from type Float to Integer for value
0.4285714285 7142857143
```

The operations truncate and round truncate ...

```
truncate 3.6
3.0
Type: Float
```

and round to the nearest integral Float respectively.

```
round 3.6
4.0
Type: Float
```

```
truncate(-3.6)
- 3.0
Type: Float
```

```
round(-3.6)
- 4.0
Type: Float
```

The operation fractionPart computes the fractional part of x, that is,  $x - \text{truncate } x$ .

```
fractionPart 3.6
```

```
0.6
```

```
Type: Float
```

The operation digits allows the user to set the precision. It returns the previous value it was using.

```
digits 40
20
```

```
Type: PositiveInteger
```

```
sqrt 0.2
0.4472135954 9995793928 1834733746 2552470881
```

```
Type: Float
```

```
pi()$Float
3.1415926535 8979323846 2643383279 502884197
```

```
Type: Float
```

The precision is only limited by the computer memory available. Calculations at 500 or more digits of precision are not difficult.

```
digits 500
40
```

```
Type: PositiveInteger
```

```
pi()$Float
3.1415926535 8979323846 2643383279 5028841971 6939937510 5820974944 592307816
4 0628620899 8628034825 3421170679 8214808651 3282306647 0938446095 505822317
2 5359408128 4811174502 8410270193 8521105559 6446229489 5493038196 442881097
5 6659334461 2847564823 3786783165 2712019091 4564856692 3460348610 454326648
2 1339360726 0249141273 7245870066 0631558817 4881520920 9628292540 917153643
6 7892590360 0113305305 4882046652 1384146951 9415116094 3305727036 575959195
3 0921861173 8193261179 3105118548 0744623799 6274956735 1885752724 891227938
1 830119491
```

```
Type: Float
```

Reset digits to its default value.

```
digits 20
500
```

```
Type: PositiveInteger
```

Numbers of type Float are represented as a record of two integers, namely, the mantissa and the exponent where the base of the exponent is binary. That is, the floating point number (m,e) represents the number  $m \times 2^e$ . A consequence of using a binary base is that decimal numbers can not, in general, be represented exactly.

```
=====
```

## Output Functions

=====

A number of operations exist for specifying how numbers of type Float are to be displayed. By default, spaces are inserted every ten digits in the output for readability. Note that you cannot include spaces in the input form of a floating point number, though you can use underscores.

Output spacing can be modified with the `outputSpacing` operation. This inserts no spaces and then displays the value of `x`.

```
outputSpacing 0; x := sqrt 0.2
0.44721359549995793928
Type: Float
```

Issue this to have the spaces inserted every 5 digits.

```
outputSpacing 5; x
0.44721 35954 99957 93928
Type: Float
```

By default, the system displays floats in either fixed format or scientific format, depending on the magnitude of the number.

```
y := x/10**10
0.44721 35954 99957 93928 E -10
Type: Float
```

A particular format may be requested with the operations `outputFloating` and `outputFixed`.

```
outputFloating(); x
0.44721 35954 99957 93928 E 0
Type: Float
```

```
outputFixed(); y
0.00000 00000 44721 35954 99957 93928
Type: Float
```

Additionally, you can ask for `n` digits to be displayed after the decimal point.

```
outputFloating 2; y
0.45 E -10
Type: Float
```

```
outputFixed 2; x
0.45
Type: Float
```

This resets the output printing to the default behavior.

```
outputGeneral()
```

```
      Type: Void
```

```
=====
An Example: Determinant of a Hilbert Matrix
=====
```

Consider the problem of computing the determinant of a 10 by 10 Hilbert matrix. The  $(i,j)$ -th entry of a Hilbert matrix is given by  $1/(i+j+1)$ .

First do the computation using rational numbers to obtain the exact result.

```
a: Matrix Fraction Integer:=matrix[ [1/(i+j+1) for j in 0..9] for i in 0..9]
```

```

+   1   1   1   1   1   1   1   1   1   1+
|1  -   -   -   -   -   -   -   -   -  --|
|   2   3   4   5   6   7   8   9   10|
|
|1  1   1   1   1   1   1   1   1   1|
|-  -   -   -   -   -   -   -   -  --|
|2  3   4   5   6   7   8   9   10  11|
|
|1  1   1   1   1   1   1   1   1   1|
|-  -   -   -   -   -   -   -  --|
|3  4   5   6   7   8   9   10  11  12|
|
|1  1   1   1   1   1   1   1   1   1|
|-  -   -   -   -   -   -  --|
|4  5   6   7   8   9   10  11  12  13|
|
|1  1   1   1   1   1   1   1   1   1|
|-  -   -   -   -   -  --|
|5  6   7   8   9   10  11  12  13  14|
|
|1  1   1   1   1   1   1   1   1   1|
|-  -   -   -  --|
|6  7   8   9   10  11  12  13  14  15|
|
|1  1   1   1   1   1   1   1   1   1|
|-  -   -  --|
|7  8   9   10  11  12  13  14  15  16|
|
|1  1   1   1   1   1   1   1   1   1|
|-  -  --|
|8  9   10  11  12  13  14  15  16  17|
|
```

```

|1  1  1  1  1  1  1  1  1  1|
|-  --  --  --  --  --  --  --  --  --|
|9  10 11 12 13 14 15 16 17 18|
|
| 1  1  1  1  1  1  1  1  1  1|
|--  --  --  --  --  --  --  --  --  --|
+10 11 12 13 14 15 16 17 18 19+
      Type: Matrix Fraction Integer

```

This version of determinant uses Gaussian elimination.

```

d:= determinant a
                        1
-----
46206893947914691316295628839036278726983680000000000
      Type: Fraction Integer

d :: Float
0.21641 79226 43149 18691 E -52
      Type: Float

```

Now use hardware floats. Note that a semicolon (;) is used to prevent the display of the matrix.

```

b: Matrix DoubleFloat:=matrix[ [1/(i+j+1)$DoubleFloat) for j in 0..9] for i in 0..9];

      Type: Matrix DoubleFloat

```

The result given by hardware floats is correct only to four significant digits of precision. In the jargon of numerical analysis, the Hilbert matrix is said to be "ill-conditioned."

```

determinant b
2.1643677945721411E-53
      Type: DoubleFloat

```

Now repeat the computation at a higher precision using Float.

```

digits 40
20
      Type: PositiveInteger

c: Matrix Float := matrix [ [1/(i+j+1)$Float) for j in 0..9] for i in 0..9];
      Type: Matrix Float

determinant c
0.21641 79226 43149 18690 60594 98362 26174 36159 E -52
      Type: Float

```

Reset digits to its default value.

```
digits 20
      40
```

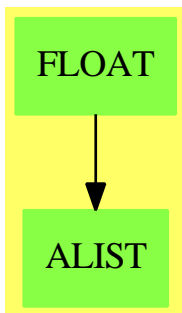
Type: PositiveInteger

See Also:

- o )help DoubleFloat
- o )show Float

---

### 7.15.1 Float (FLOAT)





**Exports:**

|               |                   |                |                       |
|---------------|-------------------|----------------|-----------------------|
| 0             | 1                 | abs            | acos                  |
| acosh         | acot              | acoth          | acsc                  |
| acsch         | asec              | asech          | asin                  |
| asinh         | associates?       | atan           | atanh                 |
| base          | bits              | ceiling        | characteristic        |
| coerce        | convert           | cos            | cosh                  |
| cot           | coth              | csc            | csch                  |
| D             | decreasePrecision | differentiate  | digits                |
| divide        | euclideanSize     | exp            | expressIdealMember    |
| exp1          | exponent          | exquo          | extendedEuclidean     |
| factor        | float             | floor          | fractionPart          |
| gcd           | gcdPolynomial     | hash           | increasePrecision     |
| inv           | latex             | lcm            | log                   |
| log10         | log2              | mantissa       | max                   |
| min           | multiEuclidean    | negative?      | norm                  |
| normalize     | nthRoot           | OMwrite        | one?                  |
| order         | outputFixed       | outputFloating | outputGeneral         |
| outputSpacing | patternMatch      | pi             | positive?             |
| precision     | prime?            | principalIdeal | rationalApproximation |
| recip         | relerror          | retract        | retractIfCan          |
| round         | sample            | sec            | sech                  |
| shift         | sign              | sin            | sinh                  |
| sizeLess?     | sqrt              | squareFree     | squareFreePart        |
| subtractIfCan | tan               | tanh           | truncate              |
| unit?         | unitCanonical     | unitNormal     | wholePart             |
| zero?         | ?*?               | ?**?           | ?+?                   |
| ?-?           | -?                | ?/?            | ?<?                   |
| ?<=?          | ?=?               | ?>?            | ?>=?                  |
| ?^?           | ?~=?              | ?quo?          | ?rem?                 |

— domain **Float** Float —)abbrev domain **Float** Float

B ==&gt; Boolean

I ==&gt; Integer

S ==&gt; String

PI ==&gt; PositiveInteger

RN ==&gt; Fraction Integer

SF ==&gt; DoubleFloat

N ==&gt; NonNegativeInteger

++ Author: Michael Monagan

++ Date Created:

++ December 1987

++ Change History:

```

++ 19 Jun 1990
++ Basic Operations: outputFloating, outputFixed, outputGeneral, outputSpacing,
++ atan, convert, exp1, log2, log10, normalize, rationalApproximation,
++ relderror, shift, / , **
++ Keywords: float, floating point, number
++ Description:
++ \spadtype{Float} implements arbitrary precision floating point arithmetic.
++ The number of significant digits of each operation can be set
++ to an arbitrary value (the default is 20 decimal digits).
++ The operation \spad{float(mantissa,exponent,base)} for integer
++ \spad{mantissa}, \spad{exponent} specifies the number
++ \spad{mantissa * base ** exponent}
++ The underlying representation for floats is binary
++ not decimal. The implications of this are described below.
++
++ The model adopted is that arithmetic operations are rounded to
++ to nearest unit in the last place, that is, accurate to within
++ \spad{2**(-bits)}. Also, the elementary functions and constants are
++ accurate to one unit in the last place.
++ A float is represented as a record of two integers, the mantissa
++ and the exponent. The base of the representation is binary, hence
++ a \spad{Record(m:mantissa,e:exponent)} represents the number
++ \spad{m * 2 ** e}.
++ Though it is not assumed that the underlying integers are represented
++ with a binary base, the code will be most efficient when this is the
++ the case (this is true in most implementations of Lisp).
++ The decision to choose the base to be binary has some unfortunate
++ consequences. First, decimal numbers like 0.3 cannot be represented
++ exactly. Second, there is a further loss of accuracy during
++ conversion to decimal for output. To compensate for this, if d
++ digits of precision are specified, \spad{1 + ceiling(log2 d)} bits are used.
++ Two numbers that are displayed identically may therefore be
++ not equal. On the other hand, a significant efficiency loss would
++ be incurred if we chose to use a decimal base when the underlying
++ integer base is binary.
++
++ Algorithms used:
++ For the elementary functions, the general approach is to apply
++ identities so that the taylor series can be used, and, so
++ that it will converge within \spad{O( sqrt n )} steps. For example,
++ using the identity \spad{exp(x) = exp(x/2)**2}, we can compute
++ \spad{exp(1/3)} to n digits of precision as follows. We have
++ \spad{exp(1/3) = exp(2 ** (-sqrt s) / 3) ** (2 ** sqrt s)}.
++ The taylor series will converge in less than sqrt n steps and the
++ exponentiation requires sqrt n multiplications for a total of
++ \spad{2 sqrt n} multiplications. Assuming integer multiplication costs
++ \spad{O( n**2 )} the overall running time is \spad{O( sqrt(n) n**2 )}.
++ This approach is the best known approach for precisions up to
++ about 10,000 digits at which point the methods of Brent
++ which are \spad{O( log(n) n**2 )} become competitive. Note also that

```

```

++ summing the terms of the taylor series for the elementary
++ functions is done using integer operations. This avoids the
++ overhead of floating point operations and results in efficient
++ code at low precisions. This implementation makes no attempt
++ to reuse storage, relying on the underlying system to do
++ \spadgloss{garbage collection}. I estimate that the efficiency of this
++ package at low precisions could be improved by a factor of 2
++ if in-place operations were available.
++
++ Running times: in the following, n is the number of bits of precision\br
++ \spad{*, \spad{/}, \spad{sqrt}, \spad{pi}, \spad{exp1}, \spad{log2},
++ \spad{log10}: \spad{ 0( n**2 )} \br
++ \spad{exp}, \spad{log}, \spad{sin}, \spad{atan}: \spad{0(sqrt(n) n**2)}\br
++ The other elementary functions are coded in terms of the ones above.

```

```
Float():
```

```

Join(FloatingPointSystem, DifferentialRing, ConvertibleTo String, OpenMath,
CoercibleTo DoubleFloat, TranscendentalFunctionCategory, ConvertibleTo InputForm) with
_/ : (%, I) -> %
++ x / i computes the division from x by an integer i.
_*. : (%, %) -> %
++ x ** y computes \spad{exp(y log x)} where \spad{x >= 0}.
normalize: % -> %
++ normalize(x) normalizes x at current precision.
relererror : (%, %) -> I
++ relererror(x,y) computes the absolute value of \spad{x - y} divided by
++ y, when \spad{y != 0}.
shift: (%, I) -> %
++ shift(x,n) adds n to the exponent of float x.
rationalApproximation: (%, N) -> RN
++ rationalApproximation(f, n) computes a rational approximation
++ r to f with relative error \spad{< 10**(-n)}.
rationalApproximation: (%, N, N) -> RN
++ rationalApproximation(f, n, b) computes a rational
++ approximation r to f with relative error \spad{< b**(-n)}, that is
++ \spad{|(r-f)/f| < b**(-n)}.
log2 : () -> %
++ log2() returns \spad{ln 2}, i.e. \spad{0.6931471805...}.
log10: () -> %
++ log10() returns \spad{ln 10}: \spad{2.3025809299...}.
exp1 : () -> %
++ exp1() returns exp 1: \spad{2.7182818284...}.
atan : (%,%) -> %
++ atan(x,y) computes the arc tangent from x with phase y.
log2 : % -> %
++ log2(x) computes the logarithm for x to base 2.
log10: % -> %
++ log10(x) computes the logarithm for x to base 10.
convert: SF -> %
++ convert(x) converts a \spadtype{DoubleFloat} x to a \spadtype{Float}.

```

```

outputFloating: () -> Void
  ++ outputFloating() sets the output mode to floating (scientific) notation, i.e.
  ++ \spad{mantissa * 10 exponent} is displayed as \spad{0.mantissa E exponent}.
outputFloating: N -> Void
  ++ outputFloating(n) sets the output mode to floating (scientific) notation
  ++ with n significant digits displayed after the decimal point.
outputFixed: () -> Void
  ++ outputFixed() sets the output mode to fixed point notation;
  ++ the output will contain a decimal point.
outputFixed: N -> Void
  ++ outputFixed(n) sets the output mode to fixed point notation,
  ++ with n digits displayed after the decimal point.
outputGeneral: () -> Void
  ++ outputGeneral() sets the output mode (default mode) to general
  ++ notation; numbers will be displayed in either fixed or floating
  ++ (scientific) notation depending on the magnitude.
outputGeneral: N -> Void
  ++ outputGeneral(n) sets the output mode to general notation
  ++ with n significant digits displayed.
outputSpacing: N -> Void
  ++ outputSpacing(n) inserts a space after n (default 10) digits on output;
  ++ outputSpacing(0) means no spaces are inserted.
arbitraryPrecision
arbitraryExponent
== add
BASE ==> 2
BITS:Reference(PI) := ref 68 -- 20 digits
LENGTH ==> INTEGER_-LENGTH$Lisp
ISQRT ==> approxSqrt$IntegerRoots(I)
Rep := Record( mantissa:I, exponent:I )
StoredConstant ==> Record( precision:PI, value:% )
UCA ==> Record( unit:%, coef:%, associate:% )
inc ==> increasePrecision
dec ==> decreasePrecision

-- local utility operations
shift2 : (I,I) -> I          -- WSP: fix bug in shift
times : (%,% ) -> %          -- multiply x and y with no rounding
itimes: (I,% ) -> %          -- multiply by a small integer
chop: (% ,PI) -> %           -- chop x at p bits of precision
dvide: (% ,%) -> %           -- divide x by y with no rounding
square: (% ,I) -> %          -- repeated squaring with chopping
power: (% ,I) -> %           -- x ** n with chopping
plus: (% ,%) -> %            -- addition with no rounding
sub: (% ,%) -> %             -- subtraction with no rounding
negate: % -> %               -- negation with no rounding
ceilog10base2: PI -> PI      -- rational approximation
floorln2: PI -> PI          -- rational approximation

atanSeries: % -> %           -- atan(x) by taylor series |x| < 1/2

```

```

atanInverse: I -> %           -- atan(1/n) for n an integer > 1
expInverse: I -> %           -- exp(1/n) for n an integer
expSeries: % -> %            -- exp(x) by taylor series |x| < 1/2
logSeries: % -> %            -- log(x) by taylor series 1/2 < x < 2
sinSeries: % -> %            -- sin(x) by taylor series |x| < 1/2
cosSeries: % -> %            -- cos(x) by taylor series |x| < 1/2
piRamanujan: () -> %         -- pi using Ramanujans series

writeOMFloat(dev: OpenMathDevice, x: %): Void ==
    MputApp(dev)
    MputSymbol(dev, "bigfloat1", "bigfloat")
    MputInteger(dev, mantissa x)
    MputInteger(dev, 2)
    MputInteger(dev, exponent x)
    MputEndApp(dev)

OMwrite(x: %): String ==
    s: String := ""
    sp := OM_STRINGTOSTRINGPTR(s)$Lisp
    dev: OpenMathDevice := OMopenString(sp pretend String, OMencodingXML)
    MputObject(dev)
    writeOMFloat(dev, x)
    MputEndObject(dev)
    OMclose(dev)
    s := OM_STRINGPTRTOSTRING(sp)$Lisp pretend String
    s

OMwrite(x: %, wholeObj: Boolean): String ==
    s: String := ""
    sp := OM_STRINGTOSTRINGPTR(s)$Lisp
    dev: OpenMathDevice := OMopenString(sp pretend String, OMencodingXML)
    if wholeObj then
        MputObject(dev)
        writeOMFloat(dev, x)
    if wholeObj then
        MputEndObject(dev)
    OMclose(dev)
    s := OM_STRINGPTRTOSTRING(sp)$Lisp pretend String
    s

OMwrite(dev: OpenMathDevice, x: %): Void ==
    MputObject(dev)
    writeOMFloat(dev, x)
    MputEndObject(dev)

OMwrite(dev: OpenMathDevice, x: %, wholeObj: Boolean): Void ==
    if wholeObj then
        MputObject(dev)
        writeOMFloat(dev, x)
    if wholeObj then

```

```

OMputEndObject(dev)

shift2(x,y) == sign(x)*shift(sign(x)*x,y)

asin x ==
  zero? x => 0
  negative? x => -asin(-x)
--  one? x => pi()/2
  (x = 1) => pi()/2
  x > 1 => error "asin: argument > 1 in magnitude"
  inc 5; r := atan(x/sqrt(sub(1,times(x,x)))); dec 5
  normalize r

acos x ==
  zero? x => pi()/2
  negative? x => (inc 3; r := pi()-acos(-x); dec 3; normalize r)
--  one? x => 0
  (x = 1) => 0
  x > 1 => error "acos: argument > 1 in magnitude"
  inc 5; r := atan(sqrt(sub(1,times(x,x)))/x); dec 5
  normalize r

atan(x,y) ==
  x = 0 =>
    y > 0 => pi()/2
    y < 0 => -pi()/2
    0
  -- Only count on first quadrant being on principal branch.
  theta := atan abs(y/x)
  if x < 0 then theta := pi() - theta
  if y < 0 then theta := - theta
  theta

atan x ==
  zero? x => 0
  negative? x => -atan(-x)
  if x > 1 then
    inc 4
    r := if zero? fractionPart x and x < [bits(),0] then atanInverse wholePart x
          else atan(1/x)
    r := pi/2 - r
    dec 4
    return normalize r
  -- make |x| < 0( 2**(-sqrt p) ) < 1/2 to speed series convergence
  -- by using the formula atan(x) = 2*atan(x/(1+sqrt(1+x**2)))
  k := ISQRT (bits()-100)::I quo 5
  k := max(0,2 + k + order x)
  inc(2*k)
  for i in 1..k repeat x := x/(1+sqrt(1+x*x))
  t := atanSeries x

```

```

dec(2*k)
t := shift(t,k)
normalize t

atanSeries x ==
-- atan(x) = x (1 - x**2/3 + x**4/5 - x**6/7 + ...) |x| < 1
p := bits() + LENGTH bits() + 2
s:I := d:I := shift(1,p)
y := times(x,x)
t := m := - shift2(y.mantissa,y.exponent+p)
for i in 3.. by 2 while t ^= 0 repeat
  s := s + t quo i
  t := (m * t) quo d
x * [s,-p]

atanInverse n ==
-- compute atan(1/n) for an integer n > 1
-- atan n = 1/n - 1/n**3/3 + 1/n**5/5 - ...
-- pi = 16 atan(1/5) - 4 atan(1/239)
n2 := -n*n
e:I := bits() + LENGTH bits() + LENGTH n + 1
s:I := shift(1,e) quo n
t:I := s quo n2
for k in 3.. by 2 while t ^= 0 repeat
  s := s + t quo k
  t := t quo n2
normalize [s,-e]

sin x ==
s := sign x; x := abs x; p := bits(); inc 4
if x > [6,0] then (inc p; x := 2*pi*fractionPart(x/pi/2); bits p)
if x > [3,0] then (inc p; s := -s; x := x - pi; bits p)
if x > [3,-1] then (inc p; x := pi - x; dec p)
-- make |x| < 0( 2**(-sqrt p) ) < 1/2 to speed series convergence
-- by using the formula sin(3*x/3) = 3 sin(x/3) - 4 sin(x/3)**3
-- the running time is O( sqrt p M(p) ) assuming |x| < 1
k := ISQRT (bits()-100)::I quo 4
k := max(0,2 + k + order x)
if k > 0 then (inc k; x := x / 3**k::N)
r := sinSeries x
for i in 1..k repeat r := itimes(3,r)-shift(r**3,2)
bits p
s * r

sinSeries x ==
-- sin(x) = x (1 - x**2/3! + x**4/5! - x**6/7! + ...) |x| < 1/2
p := bits() + LENGTH bits() + 2
y := times(x,x)
s:I := d:I := shift(1,p)
m:I := - shift2(y.mantissa,y.exponent+p)

```

```

t:I := m quo 6
for i in 4.. by 2 while t ^= 0 repeat
  s := s + t
  t := (m * t) quo (i*(i+1))
  t := t quo d
x * [s,-p]

cos x ==
s:I := 1; x := abs x; p := bits(); inc 4
if x > [6,0] then (inc p; x := 2*pi*fractionPart(x/pi/2); dec p)
if x > [3,0] then (inc p; s := -s; x := x-pi; dec p)
if x > [1,0] then
  -- take care of the accuracy problem near pi/2
  inc p; x := pi/2-x; bits p; x := normalize x
  return (s * sin x)
-- make |x| < 0( 2**(-sqrt p) ) < 1/2 to speed series convergence
-- by using the formula cos(2*x/2) = 2 cos(x/2)**2 - 1
-- the running time is O( sqrt p M(p) ) assuming |x| < 1
k := ISQRT (bits()-100)::I quo 3
k := max(0,2 + k + order x)
-- need to increase precision by more than k, otherwise recursion
-- causes loss of accuracy.
-- Michael Monagan suggests adding a factor of log(k)
if k > 0 then (inc(k+length(k)**2); x := shift(x,-k))
r := cosSeries x
for i in 1..k repeat r := shift(r*r,1)-1
bits p
s * r

cosSeries x ==
-- cos(x) = 1 - x**2/2! + x**4/4! - x**6/6! + ... |x| < 1/2
p := bits() + LENGTH bits() + 1
y := times(x,x)
s:I := d:I := shift(1,p)
m:I := - shift2(y.mantissa,y.exponent+p)
t:I := m quo 2
for i in 3.. by 2 while t ^= 0 repeat
  s := s + t
  t := (m * t) quo (i*(i+1))
  t := t quo d
normalize [s,-p]

tan x ==
s := sign x; x := abs x; p := bits(); inc 6
if x > [3,0] then (inc p; x := pi()*fractionPart(x/pi())); dec p)
if x > [3,-1] then (inc p; x := pi()-x; s := -s; dec p)
if x > 1 then (c := cos x; t := sqrt(1-c*c)/c)
else (c := sin x; t := c/sqrt(1-c*c))

```



```

bits p
s * t

P:StoredConstant := [1,[1,2]]
pi() ==
  -- We use Ramanujan's identity to compute pi.
  -- The running time is quadratic in the precision.
  -- This is about twice as fast as Machin's identity on Lisp/VM
  -- pi = 16 atan(1/5) - 4 atan(1/239)
  bits() <= P.precision => normalize P.value
  (P := [bits(), piRamanujan()]) value

piRamanujan() ==
  -- Ramanujans identity for 1/pi
  -- Reference: Shanks and Wrench, Math Comp, 1962
  -- "Calculation of pi to 100,000 Decimals".
  n := bits() + LENGTH bits() + 11
  t:I := shift(1,n) quo 882
  d:I := 4*882**2
  s:I := 0
  for i in 2.. by 2 for j in 1123.. by 21460 while t ^= 0 repeat
    s := s + j*t
    m := -(i-1)*(2*i-1)*(2*i-3)
    t := (m*t) quo (d*i**3)
  1 / [s,-n-2]

sinh x ==
  zero? x => 0
  lost:I := max(- order x,0)
  2*lost > bits() => x
  inc(5+lost); e := exp x; s := (e-1/e)/2; dec(5+lost)
  normalize s

cosh x ==
  (inc 5; e := exp x; c := (e+1/e)/2; dec 5; normalize c)

tanh x ==
  zero? x => 0
  lost:I := max(- order x,0)
  2*lost > bits() => x
  inc(6+lost); e := exp x; e := e*e; t := (e-1)/(e+1); dec(6+lost)
  normalize t

asinh x ==
  p := min(0,order x)
  if zero? x or 2*p < -bits() then return x
  inc(5-p); r := log(x+sqrt(1+x*x)); dec(5-p)
  normalize r

acosh x ==

```

```

    if x < 1 then error "invalid argument to acosh"
    inc 5; r := log(x+sqrt(sub(times(x,x),1))); dec 5
    normalize r

atanh x ==
    if x > 1 or x < -1 then error "invalid argument to atanh"
    p := min(0,order x)
    if zero? x or 2*p < -bits() then return x
    inc(5-p); r := log((x+1)/(1-x))/2; dec(5-p)
    normalize r

log x ==
    negative? x => error "negative log"
    zero? x => error "log 0 generated"
    p := bits(); inc 5
    -- apply log(x) = n log 2 + log(x/2**n) so that 1/2 < x < 2
    if (n := order x) < 0 then n := n+1
    l := if n = 0 then 0 else (x := shift(x,-n); n * log2)
    -- speed the series convergence by finding m and k such that
    -- | exp(m/2**k) x - 1 | < 1 / 2 ** 0(sqrt p)
    -- write log(exp(m/2**k) x) as m/2**k + log x
    k := ISQRT (p-100)::I quo 3
    if k > 1 then
        k := max(1,k+order(x-1))
        inc k
        ek := expInverse (2**k::N)
        dec(p quo 2); m := order square(x,k); inc(p quo 2)
        m := (6847196937 * m) quo 9878417065 -- m := m log 2
        x := x * ek ** (-m)
        l := l + [m,-k]
    l := l + logSeries x
    bits p
    normalize l

logSeries x ==
    -- log(x) = 2 y (1 + y**2/3 + y**4/5 ...) for y = (x-1) / (x+1)
    -- given 1/2 < x < 2 on input we have -1/3 < y < 1/3
    p := bits() + (g := LENGTH bits() + 3)
    inc g; y := (x-1)/(x+1); dec g
    s:I := d:I := shift(1,p)
    z := times(y,y)
    t := m := shift2(z.mantissa,z.exponent+p)
    for i in 3.. by 2 while t ^= 0 repeat
        s := s + t quo i
        t := m * t quo d
    y * [s,1-p]

L2:StoredConstant := [1,1]
log2() ==
    -- log x = 2 * sum( ((x-1)/(x+1))**(2*k+1)/(2*k+1), k=1.. )

```

```

-- log 2 = 2 * sum( 1/9**k / (2*k+1), k=0..n ) / 3
n := bits() :: N
n <= L2.precision => normalize L2.value
n := n + LENGTH n + 3 -- guard bits
s:I := shift(1,n+1) quo 3
t:I := s quo 9
for k in 3.. by 2 while t ^= 0 repeat
  s := s + t quo k
  t := t quo 9
L2 := [bits(),[s,-n]]
normalize L2.value

L10:StoredConstant := [1,[1,1]]
log10() ==
-- log x = 2 * sum( ((x-1)/(x+1))**(2*k+1)/(2*k+1), k=0.. )
-- log 5/4 = 2 * sum( 1/81**k / (2*k+1), k=0.. ) / 9
n := bits() :: N
n <= L10.precision => normalize L10.value
n := n + LENGTH n + 5 -- guard bits
s:I := shift(1,n+1) quo 9
t:I := s quo 81
for k in 3.. by 2 while t ^= 0 repeat
  s := s + t quo k
  t := t quo 81
-- We have log 10 = log 5 + log 2 and log 5/4 = log 5 - 2 log 2
inc 2; L10 := [bits(),[s,-n] + 3*log2]; dec 2
normalize L10.value

log2(x) == (inc 2; r := log(x)/log2; dec 2; normalize r)
log10(x) == (inc 2; r := log(x)/log10; dec 2; normalize r)

exp(x) ==
-- exp(n+x) = exp(1)**n exp(x) for n such that |x| < 1
p := bits(); inc 5; e1:% := 1
if (n := wholePart x) ^= 0 then
  inc LENGTH n; e1 := exp1 ** n; dec LENGTH n
x := fractionPart x
if zero? x then (bits p; return normalize e1)
-- make |x| < 0( 2**(-sqrt p) ) < 1/2 to speed series convergence
-- by repeated use of the formula exp(2*x/2) = exp(x/2)**2
-- results in an overall running time of O( sqrt p M(p) )
k := ISQRT (p-100)::I quo 3
k := max(0,2 + k + order x)
if k > 0 then (inc k; x := shift(x,-k))
e := expSeries x
if k > 0 then e := square(e,k)
bits p
e * e1

expSeries x ==

```

```

-- exp(x) = 1 + x + x**2/2 + ... + x**i/i!  valid for all x
p := bits() + LENGTH bits() + 1
s:I := d:I := shift(1,p)
t:I := n:I := shift2(x.mantissa,x.exponent+p)
for i in 2.. while t ^= 0 repeat
  s := s + t
  t := (n * t) quo i
  t := t quo d
normalize [s,-p]

expInverse k ==
-- computes exp(1/k) via continued fraction
p0:I := 2*k+1; p1:I := 6*k*p0+1
q0:I := 2*k-1; q1:I := 6*k*q0+1
for i in 10*k.. by 4*k while 2 * LENGTH p0 < bits() repeat
  (p0,p1) := (p1,i*p1+p0)
  (q0,q1) := (q1,i*q1+q0)
ddivide([p1,0],[q1,0])

E:StoredConstant := [1,[1,1]]
exp1() ==
  if bits() > E.precision then E := [bits(),expInverse 1]
  normalize E.value

sqrt x ==
  negative? x => error "negative sqrt"
  m := x.mantissa; e := x.exponent
  l := LENGTH m
  p := 2 * bits() - l + 2
  if odd?(e-l) then p := p - 1
  i := shift2(x.mantissa,p)
  -- ISQRT uses a variable precision newton iteration
  i := ISQRT i
  normalize [i,(e-p) quo 2]

bits() == BITS()
bits(n) == (t := bits(); BITS() := n; t)
precision() == bits()
precision(n) == bits(n)
increasePrecision n == (b := bits(); bits((b + n)::PI); b)
decreasePrecision n == (b := bits(); bits((b - n)::PI); b)
ceilllog10base2 n == ((13301 * n + 4003) quo 4004) :: PI
digits() == max(1,4004 * (bits()-1) quo 13301)::PI
digits(n) == (t := digits(); bits (1 + ceilllog10base2 n); t)

order(a) == LENGTH a.mantissa + a.exponent - 1
relerror(a,b) == order((a-b)/b)
0 == [0,0]
1 == [1,0]
base() == BASE

```

```

mantissa x == x.mantissa
exponent x == x.exponent
one? a == a = 1
zero? a == zero?(a.mantissa)
negative? a == negative?(a.mantissa)
positive? a == positive?(a.mantissa)

chop(x,p) ==
  e : I := LENGTH x.mantissa - p
  if e > 0 then x := [shift2(x.mantissa,-e),x.exponent+e]
  x
float(m,e) == normalize [m,e]
float(m,e,b) ==
  m = 0 => 0
  inc 4; r := m * [b,0] ** e; dec 4
  normalize r
normalize x ==
  m := x.mantissa
  m = 0 => 0
  e : I := LENGTH m - bits()
  if e > 0 then
    y := shift2(m,1-e)
    if odd? y then
      y := (if y>0 then y+1 else y-1) quo 2
      if LENGTH y > bits() then
        y := y quo 2
        e := e+1
    else y := y quo 2
    x := [y,x.exponent+e]
  x
shift(x:%,n:I) == [x.mantissa,x.exponent+n]

x = y ==
  order x = order y and sign x = sign y and zero? (x - y)
x < y ==
  y.mantissa = 0 => x.mantissa < 0
  x.mantissa = 0 => y.mantissa > 0
  negative? x and positive? y => true
  negative? y and positive? x => false
  order x < order y => positive? x
  order x > order y => negative? x
  negative? (x-y)

abs x == if negative? x then -x else normalize x
ceiling x ==
  if negative? x then return (-floor(-x))
  if zero? fractionPart x then x else truncate x + 1
wholePart x == shift2(x.mantissa,x.exponent)
floor x == if negative? x then -ceiling(-x) else truncate x
round x == (half := [sign x,-1]; truncate(x + half))

```

```

sign x == if x.mantissa < 0 then -1 else 1
truncate x ==
  if x.exponent >= 0 then return x
  normalize [shift2(x.mantissa,x.exponent),0]
recip(x) == if x=0 then "failed" else 1/x
differentiate x == 0

- x == normalize negate x
negate x == [-x.mantissa,x.exponent]
x + y == normalize plus(x,y)
x - y == normalize plus(x,negate y)
sub(x,y) == plus(x,negate y)
plus(x,y) ==
  mx := x.mantissa; my := y.mantissa
  mx = 0 => y
  my = 0 => x
  ex := x.exponent; ey := y.exponent
  ex = ey => [mx+my,ex]
  de := ex + LENGTH mx - ey - LENGTH my
  de > bits()+1 => x
  de < -(bits()+1) => y
  if ex < ey then (mx,my,ex,ey) := (my,mx,ey,ex)
  mw := my + shift2(mx,ex-ey)
  [mw,ey]

x:% * y:% == normalize times (x,y)
x:I * y:% ==
  if LENGTH x > bits() then normalize [x,0] * y
  else normalize [x * y.mantissa,y.exponent]
x:% / y:% == normalize dvide(x,y)
x:% / y:I ==
  if LENGTH y > bits() then x / normalize [y,0] else x / [y,0]
inv x == 1 / x

times(x:%,y:%) == [x.mantissa * y.mantissa, x.exponent + y.exponent]
itimes(n:I,y:%) == [n * y.mantissa,y.exponent]

dvide(x,y) ==
  ew := LENGTH y.mantissa - LENGTH x.mantissa + bits() + 1
  mw := shift2(x.mantissa,ew) quo y.mantissa
  ew := x.exponent - y.exponent - ew
  [mw,ew]

square(x,n) ==
  ma := x.mantissa; ex := x.exponent
  for k in 1..n repeat
    ma := ma * ma; ex := ex + ex
    l:I := bits():I - LENGTH ma
    ma := shift2(ma,l); ex := ex - l
  [ma,ex]

```

```

power(x,n) ==
  y:% := 1; z:% := x
  repeat
    if odd? n then y := chop( times(y,z), bits() )
    if (n := n quo 2) = 0 then return y
    z := chop( times(z,z), bits() )

x:% ** y:% ==
  x = 0 =>
    y = 0 => error "0**0 is undefined"
    y < 0 => error "division by 0"
    y > 0 => 0
  y = 0 => 1
  y = 1 => x
  x = 1 => 1
  p := abs order y + 5
  inc p; r := exp(y*log(x)); dec p
  normalize r

x:% ** r:RN ==
  x = 0 =>
    r = 0 => error "0**0 is undefined"
    r < 0 => error "division by 0"
    r > 0 => 0
  r = 0 => 1
  r = 1 => x
  x = 1 => 1
  n := numer r
  d := denom r
  negative? x =>
    odd? d =>
      odd? n => return -((-x)**r)
      return ((-x)**r)
    error "negative root"
  if d = 2 then
    inc LENGTH n; y := sqrt(x); y := y**n; dec LENGTH n
    return normalize y
  y := [n,0]/[d,0]
  x ** y

x:% ** n:I ==
  x = 0 =>
    n = 0 => error "0**0 is undefined"
    n < 0 => error "division by 0"
    n > 0 => 0
  n = 0 => 1
  n = 1 => x
  x = 1 => 1
  p := bits()

```

```

bits(p + LENGTH n + 2)
y := power(x,abs n)
if n < 0 then y := dvide(1,y)
bits p
normalize y

-- Utility routines for conversion to decimal
ceillength10: I -> I
chop10: (% ,I) -> %
convert10: (% ,I) -> %
floorlength10: I -> I
length10: I -> I
normalize10: (% ,I) -> %
quotient10: (% ,% ,I) -> %
power10: (% ,I ,I) -> %
times10: (% ,% ,I) -> %

convert10(x,d) ==
  m := x.mantissa; e := x.exponent
  --!! deal with bits here
  b := bits(); (q,r) := divide(abs e, b)
  b := 2**b::N; r := 2**r::N
  -- compute 2**e = b**q * r
  h := power10([b,0],q,d+5)
  h := chop10([r*h.mantissa,h.exponent],d+5)
  if e < 0 then h := quotient10([m,0],h,d)
  else times10([m,0],h,d)

ceillength10 n == 146 * LENGTH n quo 485 + 1
floorlength10 n == 643 * LENGTH n quo 2136
-- length10 n == DECIMAL_-LENGTH(n)$Lisp
length10 n ==
  ln := LENGTH(n:=abs n)
  upper := 76573 * ln quo 254370
  lower := 21306 * (ln-1) quo 70777
  upper = lower => upper + 1
  n := n quo (10**lower::N)
  while n >= 10 repeat
    n:= n quo 10
    lower := lower + 1
  lower + 1

chop10(x,p) ==
  e : I := floorlength10 x.mantissa - p
  if e > 0 then x := [x.mantissa quo 10**e::N,x.exponent+e]
  x
normalize10(x,p) ==
  ma := x.mantissa
  ex := x.exponent
  e : I := length10 ma - p

```



```

    if e > 0 then
        ma := ma quo 10**(e-1)::N
        ex := ex + e
        (ma,r) := divide(ma, 10)
        if r > 4 then
            ma := ma + 1
            if ma = 10**p::N then (ma := 1; ex := ex + p)
    [ma,ex]
times10(x,y,p) == normalize10(times(x,y),p)
quotient10(x,y,p) ==
    ew := floorLength10 y.mantissa - ceilLength10 x.mantissa + p + 2
    if ew < 0 then ew := 0
    mw := (x.mantissa * 10**ew::N) quo y.mantissa
    ew := x.exponent - y.exponent - ew
    normalize10([mw,ew],p)
power10(x,n,d) ==
    x = 0 => 0
    n = 0 => 1
    n = 1 => x
    x = 1 => 1
    p:I := d + LENGTH n + 1
    e:I := n
    y:% := 1
    z:% := x
    repeat
        if odd? e then y := chop10(times(y,z),p)
        if (e := e quo 2) = 0 then return y
        z := chop10(times(z,z),p)

-----
-- Output routines for Floats --
-----

zero ==> char("0")
separator ==> space()$Character

SPACING : Reference(N) := ref 10
OUTMODE : Reference(S) := ref "general"
OUTPREC : Reference(I) := ref(-1)

fixed : % -> S
floating : % -> S
general : % -> S

padFromLeft(s:S):S ==
    zero? SPACING() => s
    n:I := #s - 1
    t := new( (n + 1 + n quo SPACING()) :: N , separator )
    for i in 0..n for j in minIndex t .. repeat
        t.j := s.(i + minIndex s)
        if (i+1) rem SPACING() = 0 then j := j+1

```

```

t
padFromRight(s:S):S ==
  SPACING() = 0 => s
  n:I := #s - 1
  t := new( (n + 1 + n quo SPACING()) :: N , separator )
  for i in n..0 by -1 for j in maxIndex t .. by -1 repeat
    t.j := s.(i + minIndex s)
    if (n-i+1) rem SPACING() = 0 then j := j-1
  t

fixed f ==
  d := if OUTPREC() = -1 then digits::I else OUTPREC()
  dpos:N:= if (d > 0) then d::N else 1::N
  zero? f =>
    OUTPREC() = -1 => "0.0"
    concat("0",concat(".",padFromLeft new(dpos,zero)))
  zero? exponent f =>
    concat(padFromRight convert(mantissa f)@S,
      concat(".",padFromLeft new(dpos,zero)))
  negative? f => concat("-", fixed abs f)
  bl := LENGTH(f.mantissa) + f.exponent
  dd :=
    OUTPREC() = -1 => d
    bl > 0 => (146*bl) quo 485 + 1 + d
  d
  g := convert10(abs f,dd)
  m := g.mantissa
  e := g.exponent
  if OUTPREC() ^=- -1 then
    -- round g to OUTPREC digits after the decimal point
    l := length10 m
    if -e > OUTPREC() and -e < 2*digits::I then
      g := normalize10(g,l+e+OUTPREC())
      m := g.mantissa; e := g.exponent
  s := convert(m)@S; n := #s; o := e+n
  p := if OUTPREC() = -1 then n::I else OUTPREC()
  t:S
  if e >= 0 then
    s := concat(s, new(e::N, zero))
    t := ""
  else if o <= 0 then
    t := concat(new((-o)::N,zero), s)
    s := "0"
  else
    t := s(o + minIndex s .. n + minIndex s - 1)
    s := s(minIndex s .. o + minIndex s - 1)
  n := #t
  if OUTPREC() = -1 then
    t := rightTrim(t,zero)
    if t = "" then t := "0"

```

```

else if n > p then t := t(minIndex t .. p + minIndex t- 1)
      else t := concat(t, new((p-n)::N,zero))
concat(padFromRight s, concat(".", padFromLeft t))

floating f ==
zero? f => "0.0"
negative? f => concat("-", floating abs f)
t:S := if zero? SPACING() then "E" else " E "
zero? exponent f =>
  s := convert(mantissa f)@S
  concat ["0.", padFromLeft s, t, convert(#s)@S]
-- base conversion to decimal rounded to the requested precision
d := if OUTPREC() = -1 then digits::I else OUTPREC()
g := convert10(f,d); m := g.mantissa; e := g.exponent
-- I'm assuming that length10 m = # s given n > 0
s := convert(m)@S; n := #s; o := e+n
s := padFromLeft s
concat ["0.", s, t, convert(o)@S]

general(f) ==
zero? f => "0.0"
negative? f => concat("-", general abs f)
d := if OUTPREC() = -1 then digits::I else OUTPREC()
zero? exponent f =>
  d := d + 1
  s := convert(mantissa f)@S
  OUTPREC() ^= -1 and (e := #s) > d =>
    t:S := if zero? SPACING() then "E" else " E "
    concat ["0.", padFromLeft s, t, convert(e)@S]
  padFromRight concat(s, ".0")
-- base conversion to decimal rounded to the requested precision
g := convert10(f,d); m := g.mantissa; e := g.exponent
-- I'm assuming that length10 m = # s given n > 0
s := convert(m)@S; n := #s; o := n + e
-- Note: at least one digit is displayed after the decimal point
-- and trailing zeroes after the decimal point are dropped
if o > 0 and o <= max(n,d) then
  -- fixed format: add trailing zeroes before the decimal point
  if o > n then s := concat(s, new((o-n)::N,zero))
  t := rightTrim(s(o + minIndex s .. n + minIndex s - 1), zero)
  if t = "" then t := "0" else t := padFromLeft t
  s := padFromRight s(minIndex s .. o + minIndex s - 1)
  concat(s, concat(".", t))
else if o <= 0 and o >= -5 then
  -- fixed format: up to 5 leading zeroes after the decimal point
  concat("0.",padFromLeft concat(new((-o)::N,zero),rightTrim(s,zero)))
else
  -- print using E format written  0.mantissa E exponent
  t := padFromLeft rightTrim(s,zero)
  s := if zero? SPACING() then "E" else " E "

```

```

concat ["0.", t, s, convert(e+n)@S]

outputSpacing n == SPACING() := n
outputFixed() == (OUTMODE() := "fixed"; OUTPREC() := -1)
outputFixed n == (OUTMODE() := "fixed"; OUTPREC() := n::I)
outputGeneral() == (OUTMODE() := "general"; OUTPREC() := -1)
outputGeneral n == (OUTMODE() := "general"; OUTPREC() := n::I)
outputFloating() == (OUTMODE() := "floating"; OUTPREC() := -1)
outputFloating n == (OUTMODE() := "floating"; OUTPREC() := n::I)

convert(f):S ==
  b:Integer :=
    OUTPREC() = -1 and not zero? f =>
      bits(length(abs mantissa f)::PositiveInteger)
    0
  s :=
    OUTMODE() = "fixed" => fixed f
    OUTMODE() = "floating" => floating f
    OUTMODE() = "general" => general f
  empty()$String
  if b > 0 then bits(b::PositiveInteger)
  s = empty()$String => error "bad output mode"
  s

coerce(f):OutputForm ==
  f >= 0 => message(convert(f)@S)
  - (coerce(-f)@OutputForm)

convert(f):InputForm ==
  convert [convert("float"::Symbol), convert mantissa f,
    convert exponent f, convert base()]$List(InputForm)

-- Conversion routines
convert(x:%):Float == x pretend Float
convert(x:%):SF == makeSF(x.mantissa,x.exponent)$Lisp
coerce(x:%):SF == convert(x)@SF
convert(sf:SF):% == float(mantissa sf,exponent sf,base())$SF

retract(f:%):RN == rationalApproximation(f,(bits()-1)::N,BASE)

retractIfCan(f:%):Union(RN, "failed") ==
  rationalApproximation(f,(bits()-1)::N,BASE)

retract(f:%):I ==
  (f = (n := wholePart f)::%) => n
  error "Not an integer"

retractIfCan(f:%):Union(I, "failed") ==
  (f = (n := wholePart f)::%) => n
  "failed"

```

```

rationalApproximation(f,d) == rationalApproximation(f,d,10)

rationalApproximation(f,d,b) ==
  t: Integer
  nu := f.mantissa; ex := f.exponent
  if ex >= 0 then return ((nu*BASE**(ex::N))/1)
  de := BASE**((-ex)::N)
  if b < 2 then error "base must be > 1"
  tol := b**d
  s := nu; t := de
  p0,p1,q0,q1 : Integer
  p0 := 0; p1 := 1; q0 := 1; q1 := 0
  repeat
    (q,r) := divide(s, t)
    p2 := q*p1+p0
    q2 := q*q1+q0
    if r = 0 or tol*abs(nu*q2-de*p2) < de*abs(p2) then return (p2/q2)
    (p0,p1) := (p1,p2)
    (q0,q1) := (q1,q2)
    (s,t) := (t,r)

```

---

— **Float.dotabb** —

```

"FLOAT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FLOAT"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"FLOAT" -> "ALIST"

```

---

## 7.16 domain FC FortranCode

— **FortranCode.input** —

```

)set break resume
)sys rm -f FortranCode.output
)spool FortranCode.output
)set message test on
)set message auto off
)clear all

```

--S 1 of 1

```

)show FortranCode
--R FortranCode is a domain constructor
--R Abbreviation for FortranCode is FC
--R This constructor is exposed in this frame.
--R Issue )edit NIL to see algebra source code for FC
--R
--R----- Operations -----
--R ==? : (% ,%) -> Boolean          assign : (Symbol,String) -> %
--R block : List % -> %              call : String -> %
--R coerce : % -> OutputForm          comment : List String -> %
--R comment : String -> %             cond : (Switch,% ,%) -> %
--R cond : (Switch,% ,%) -> %         continue : SingleInteger -> %
--R getCode : % -> SExpression         goto : SingleInteger -> %
--R hash : % -> SingleInteger          latex : % -> String
--R printCode : % -> Void              returns : Expression Integer -> %
--R returns : Expression Float -> %    returns : () -> %
--R save : () -> %                    stop : () -> %
--R whileLoop : (Switch,% ,%) -> %     ?~=? : (% ,%) -> Boolean
--R assign : (Symbol,List Polynomial Integer,Expression Complex Float) -> %
--R assign : (Symbol,List Polynomial Integer,Expression Float) -> %
--R assign : (Symbol,List Polynomial Integer,Expression Integer) -> %
--R assign : (Symbol,Vector Expression Complex Float) -> %
--R assign : (Symbol,Vector Expression Float) -> %
--R assign : (Symbol,Vector Expression Integer) -> %
--R assign : (Symbol,Matrix Expression Complex Float) -> %
--R assign : (Symbol,Matrix Expression Float) -> %
--R assign : (Symbol,Matrix Expression Integer) -> %
--R assign : (Symbol,Expression Complex Float) -> %
--R assign : (Symbol,Expression Float) -> %
--R assign : (Symbol,Expression Integer) -> %
--R assign : (Symbol,List Polynomial Integer,Expression MachineComplex) -> %
--R assign : (Symbol,List Polynomial Integer,Expression MachineFloat) -> %
--R assign : (Symbol,List Polynomial Integer,Expression MachineInteger) -> %
--R assign : (Symbol,Vector Expression MachineComplex) -> %
--R assign : (Symbol,Vector Expression MachineFloat) -> %
--R assign : (Symbol,Vector Expression MachineInteger) -> %
--R assign : (Symbol,Matrix Expression MachineComplex) -> %
--R assign : (Symbol,Matrix Expression MachineFloat) -> %
--R assign : (Symbol,Matrix Expression MachineInteger) -> %
--R assign : (Symbol,Vector MachineComplex) -> %
--R assign : (Symbol,Vector MachineFloat) -> %
--R assign : (Symbol,Vector MachineInteger) -> %
--R assign : (Symbol,Matrix MachineComplex) -> %
--R assign : (Symbol,Matrix MachineFloat) -> %
--R assign : (Symbol,Matrix MachineInteger) -> %
--R assign : (Symbol,Expression MachineComplex) -> %
--R assign : (Symbol,Expression MachineFloat) -> %
--R assign : (Symbol,Expression MachineInteger) -> %
--R code : % -> Union(nullBranch: null,assignmentBranch: Record(var: Symbol,arrayIndex: List Polynomial
--R common : (Symbol,List Symbol) -> %

```

```

--R forLoop : (SegmentBinding Polynomial Integer,Polynomial Integer,%) -> %
--R forLoop : (SegmentBinding Polynomial Integer,%) -> %
--R operation : % -> Union(Null: null,Assignment: assignment,Conditional: conditional,Return
--R printStatement : List OutputForm -> %
--R repeatUntilLoop : (Switch,%) -> %
--R returns : Expression Complex Float -> %
--R returns : Expression MachineComplex -> %
--R returns : Expression MachineInteger -> %
--R returns : Expression MachineFloat -> %
--R setLabelValue : SingleInteger -> SingleInteger
--R
--E 1

)spool
)lisp (bye)

```

---

— FortranCode.help —

```

=====
FortranCode examples
=====

```

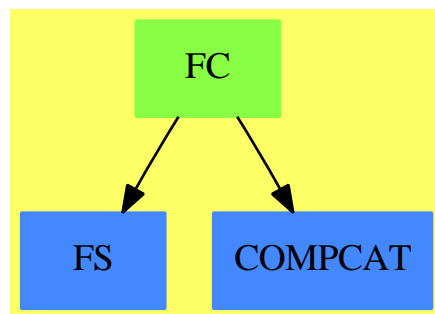
```

See Also:
o )show FortranCode

```

---

### 7.16.1 FortranCode (FC)



See

⇒ “Result” (RESULT) 19.9.1 on page 2260  
 ⇒ “FortranProgram” (FORTRAN) 7.18.1 on page 923

- ⇒ “ThreeDimensionalMatrix” (M3D) 21.7.1 on page 2661
- ⇒ “SimpleFortranProgram” (SFORT) 20.11.1 on page 2364
- ⇒ “Switch” (SWITCH) 20.36.1 on page 2588
- ⇒ “FortranTemplate” (FTEM) 7.20.1 on page 934
- ⇒ “FortranExpression” (FEXPR) 7.17.1 on page 914

**Exports:**

|               |                |                 |          |           |
|---------------|----------------|-----------------|----------|-----------|
| assign        | block          | call            | code     | coerce    |
| comment       | common         | cond            | continue | forLoop   |
| getCode       | goto           | hash            | latex    | operation |
| printCode     | printStatement | repeatUntilLoop | returns  | save      |
| setLabelValue | stop           | whileLoop       | ?=?      | ?=?       |

— domain FC FortranCode —

```

)abbrev domain FC FortranCode
++ Author: Mike Dewar
++ Date Created: April 1991
++ Date Last Updated: 22 March 1994
++
++      26 May 1994 Added common, MCD
++      21 June 1994 Changed print to printStatement, MCD
++      30 June 1994 Added stop, MCD
++      12 July 1994 Added assign for String, MCD
++      9 January 1995 Added fortran2Lines to getCall, MCD
++ Basic Operations:
++ Related Constructors: FortranProgram, Switch, FortranType
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This domain builds representations of program code segments for use with
++ the FortranProgram domain.

```

```

FortranCode(): public == private where
  L ==> List
  PI ==> PositiveInteger
  PIN ==> Polynomial Integer
  SEX ==> SExpression
  O ==> OutputForm
  OP ==> Union(Null:"null",
    Assignment:"assignment",
    Conditional:"conditional",
    Return:"return",
    Block:"block",
    Comment:"comment",
    Call:"call",
    For:"for",
    While:"while",
    Repeat:"repeat",

```



```

        Goto:"goto",
        Continue:"continue",
        ArrayAssignment:"arrayAssignment",
        Save:"save",
        Stop:"stop",
        Common:"common",
        Print:"print")
ARRAYASS ==> Record(var:Symbol, rand:0, ints2Floats?:Boolean)
EXPRESSION ==> Record(ints2Floats?:Boolean,expr:0)
ASS ==> Record(var:Symbol,
               arrayIndex:L PIN,
               rand:EXPRESSION
               )
COND ==> Record(switch: Switch(),
               thenClause: $,
               elseClause: $
               )
RETURN ==> Record(empty?:Boolean,value:EXPRESSION)
BLOCK ==> List $
COMMENT ==> List String
COMMON ==> Record(name:Symbol,contents:List Symbol)
CALL ==> String
FOR ==> Record(range:SegmentBinding PIN, span:PIN, body:$)
LABEL ==> SingleInteger
LOOP ==> Record(switch:Switch(),body:$)
PRINTLIST ==> List 0
OPREC ==> Union(nullBranch:"null", assignmentBranch:ASS,
               arrayAssignmentBranch:ARRAYASS,
               conditionalBranch:COND, returnBranch:RETURN,
               blockBranch:BLOCK, commentBranch:COMMENT, callBranch:CALL,
               forBranch:FOR, labelBranch:LABEL, loopBranch:LOOP,
               commonBranch:COMMON, printBranch:PRINTLIST)

public == SetCategory with
  coerce: $ -> 0
  ++ coerce(f) returns an object of type OutputForm.
  forLoop: (SegmentBinding PIN,$) -> $
  ++ forLoop(i=1..10,c) creates a representation of a FORTRAN DO loop with
  ++ \spad{i} ranging over the values 1 to 10.
  forLoop: (SegmentBinding PIN,PIN,$) -> $
  ++ forLoop(i=1..10,n,c) creates a representation of a FORTRAN DO loop with
  ++ \spad{i} ranging over the values 1 to 10 by n.
  whileLoop: (Switch,$) -> $
  ++ whileLoop(s,c) creates a while loop in FORTRAN.
  repeatUntilLoop: (Switch,$) -> $
  ++ repeatUntilLoop(s,c) creates a repeat ... until loop in FORTRAN.
  goto: SingleInteger -> $
  ++ goto(l) creates a representation of a FORTRAN GOTO statement
  continue: SingleInteger -> $
  ++ continue(l) creates a representation of a FORTRAN CONTINUE labelled

```

```

    ++ with l
comment: String -> $
    ++ comment(s) creates a representation of the String s as a single FORTRAN
    ++ comment.
comment: List String -> $
    ++ comment(s) creates a representation of the Strings s as a multi-line
    ++ FORTRAN comment.
call: String -> $
    ++ call(s) creates a representation of a FORTRAN CALL statement
returns: () -> $
    ++ returns() creates a representation of a FORTRAN RETURN statement.
returns: Expression MachineFloat -> $
    ++ returns(e) creates a representation of a FORTRAN RETURN statement
    ++ with a returned value.
returns: Expression MachineInteger -> $
    ++ returns(e) creates a representation of a FORTRAN RETURN statement
    ++ with a returned value.
returns: Expression MachineComplex -> $
    ++ returns(e) creates a representation of a FORTRAN RETURN statement
    ++ with a returned value.
returns: Expression Float -> $
    ++ returns(e) creates a representation of a FORTRAN RETURN statement
    ++ with a returned value.
returns: Expression Integer -> $
    ++ returns(e) creates a representation of a FORTRAN RETURN statement
    ++ with a returned value.
returns: Expression Complex Float -> $
    ++ returns(e) creates a representation of a FORTRAN RETURN statement
    ++ with a returned value.
cond: (Switch,$) -> $
    ++ cond(s,e) creates a representation of the FORTRAN expression
    ++ IF (s) THEN e.
cond: (Switch,$,$) -> $
    ++ cond(s,e,f) creates a representation of the FORTRAN expression
    ++ IF (s) THEN e ELSE f.
assign: (Symbol,String) -> $
    ++ assign(x,y) creates a representation of the FORTRAN expression
    ++ x=y.
assign: (Symbol,Expression MachineInteger) -> $
    ++ assign(x,y) creates a representation of the FORTRAN expression
    ++ x=y.
assign: (Symbol,Expression MachineFloat) -> $
    ++ assign(x,y) creates a representation of the FORTRAN expression
    ++ x=y.
assign: (Symbol,Expression MachineComplex) -> $
    ++ assign(x,y) creates a representation of the FORTRAN expression
    ++ x=y.
assign: (Symbol,Matrix MachineInteger) -> $
    ++ assign(x,y) creates a representation of the FORTRAN expression
    ++ x=y.

```

```

assign: (Symbol,Matrix MachineFloat) -> $
    ++ assign(x,y) creates a representation of the FORTRAN expression
    ++ x=y.
assign: (Symbol,Matrix MachineComplex) -> $
    ++ assign(x,y) creates a representation of the FORTRAN expression
    ++ x=y.
assign: (Symbol,Vector MachineInteger) -> $
    ++ assign(x,y) creates a representation of the FORTRAN expression
    ++ x=y.
assign: (Symbol,Vector MachineFloat) -> $
    ++ assign(x,y) creates a representation of the FORTRAN expression
    ++ x=y.
assign: (Symbol,Vector MachineComplex) -> $
    ++ assign(x,y) creates a representation of the FORTRAN expression
    ++ x=y.
assign: (Symbol,Matrix Expression MachineInteger) -> $
    ++ assign(x,y) creates a representation of the FORTRAN expression
    ++ x=y.
assign: (Symbol,Matrix Expression MachineFloat) -> $
    ++ assign(x,y) creates a representation of the FORTRAN expression
    ++ x=y.
assign: (Symbol,Matrix Expression MachineComplex) -> $
    ++ assign(x,y) creates a representation of the FORTRAN expression
    ++ x=y.
assign: (Symbol,Vector Expression MachineInteger) -> $
    ++ assign(x,y) creates a representation of the FORTRAN expression
    ++ x=y.
assign: (Symbol,Vector Expression MachineFloat) -> $
    ++ assign(x,y) creates a representation of the FORTRAN expression
    ++ x=y.
assign: (Symbol,Vector Expression MachineComplex) -> $
    ++ assign(x,y) creates a representation of the FORTRAN expression
    ++ x=y.
assign: (Symbol,L PIN,Expression MachineInteger) -> $
    ++ assign(x,l,y) creates a representation of the assignment of \spad{y}
    ++ to the \spad{l}'th element of array \spad{x} (\spad{l} is a list of
    ++ indices).
assign: (Symbol,L PIN,Expression MachineFloat) -> $
    ++ assign(x,l,y) creates a representation of the assignment of \spad{y}
    ++ to the \spad{l}'th element of array \spad{x} (\spad{l} is a list of
    ++ indices).
assign: (Symbol,L PIN,Expression MachineComplex) -> $
    ++ assign(x,l,y) creates a representation of the assignment of \spad{y}
    ++ to the \spad{l}'th element of array \spad{x} (\spad{l} is a list of
    ++ indices).
assign: (Symbol,Expression Integer) -> $
    ++ assign(x,y) creates a representation of the FORTRAN expression
    ++ x=y.
assign: (Symbol,Expression Float) -> $
    ++ assign(x,y) creates a representation of the FORTRAN expression

```

```

++ x=y.
assign: (Symbol,Expression Complex Float) -> $
++ assign(x,y) creates a representation of the FORTRAN expression
++ x=y.
assign: (Symbol,Matrix Expression Integer) -> $
++ assign(x,y) creates a representation of the FORTRAN expression
++ x=y.
assign: (Symbol,Matrix Expression Float) -> $
++ assign(x,y) creates a representation of the FORTRAN expression
++ x=y.
assign: (Symbol,Matrix Expression Complex Float) -> $
++ assign(x,y) creates a representation of the FORTRAN expression
++ x=y.
assign: (Symbol,Vector Expression Integer) -> $
++ assign(x,y) creates a representation of the FORTRAN expression
++ x=y.
assign: (Symbol,Vector Expression Float) -> $
++ assign(x,y) creates a representation of the FORTRAN expression
++ x=y.
assign: (Symbol,Vector Expression Complex Float) -> $
++ assign(x,y) creates a representation of the FORTRAN expression
++ x=y.
assign: (Symbol,L PIN,Expression Integer) -> $
++ assign(x,l,y) creates a representation of the assignment of \spad{y}
++ to the \spad{l}'th element of array \spad{x} (\spad{l} is a list of
++ indices).
assign: (Symbol,L PIN,Expression Float) -> $
++ assign(x,l,y) creates a representation of the assignment of \spad{y}
++ to the \spad{l}'th element of array \spad{x} (\spad{l} is a list of
++ indices).
assign: (Symbol,L PIN,Expression Complex Float) -> $
++ assign(x,l,y) creates a representation of the assignment of \spad{y}
++ to the \spad{l}'th element of array \spad{x} (\spad{l} is a list of
++ indices).
block: List($ ) -> $
++ block(l) creates a representation of the statements in l as a block.
stop: () -> $
++ stop() creates a representation of a STOP statement.
save: () -> $
++ save() creates a representation of a SAVE statement.
printStatement: List 0 -> $
++ printStatement(l) creates a representation of a PRINT statement.
common: (Symbol,List Symbol) -> $
++ common(name,contents) creates a representation a named common block.
operation: $ -> OP
++ operation(f) returns the name of the operation represented by \spad{f}.
code: $ -> OPREC
++ code(f) returns the internal representation of the object represented
++ by \spad{f}.
printCode: $ -> Void

```

```

    ++ printCode(f) prints out \spad{f} in FORTRAN notation.
getCode: $ -> SEX
    ++ getCode(f) returns a Lisp list of strings representing \spad{f}
    ++ in Fortran notation. This is used by the FortranProgram domain.
setLabelValue:SingleInteger -> SingleInteger
    ++ setLabelValue(i) resets the counter which produces labels to i

private == add
import Void
import ASS
import COND
import RETURN
import L PIN
import O
import SEX
import FortranType
import TheSymbolTable

Rep := Record(op: OP, data: OPREC)

-- We need to be able to generate unique labels
labelValue:SingleInteger := 25000::SingleInteger
setLabelValue(u:SingleInteger):SingleInteger == labelValue := u
newLabel():SingleInteger ==
    labelValue := labelValue + 1$SingleInteger
    labelValue

commaSep(l:List String):List(String) ==
    [(l.1),:[:["",u] for u in rest(l)]]

getReturn(rec:RETURN):SEX ==
    returnToken : SEX := convert("RETURN":Symbol::O)$SEX
    elt(rec,empty?)$RETURN =>
        getStatement(returnToken,NIL$Lisp)$Lisp
    rt : EXPRESSION := elt(rec,value)$RETURN
    rv : O := elt(rt,expr)$EXPRESSION
    getStatement([returnToken,convert(rv)$SEX]$Lisp,
        elt(rt,ints2Floats?)$EXPRESSION )$Lisp

getStop():SEX ==
    fortran2Lines(LIST("STOP")$Lisp)$Lisp

getSave():SEX ==
    fortran2Lines(LIST("SAVE")$Lisp)$Lisp

getCommon(u:COMMON):SEX ==
    fortran2Lines(APPEND(LIST("COMMON"," /",string (u.name),"/ ")$Lisp,_
        addCommas(u.contents)$Lisp)$Lisp)$Lisp

getPrint(l:PRINTLIST):SEX ==

```

```

ll : SEX := LIST("PRINT*")$Lisp
for i in l repeat
  ll := APPEND(ll,CONS(",",expression2Fortran(i)$Lisp)$Lisp)$Lisp
fortran2Lines(ll)$Lisp

getBlock(rec:BLOCK):SEX ==
  indentFortLevel(convert(1@Integer)$SEX)$Lisp
  expr : SEX := LIST()$Lisp
  for u in rec repeat
    expr := APPEND(expr,getCode(u))$Lisp
  indentFortLevel(convert(-1@Integer)$SEX)$Lisp
  expr

getBody(f:$):SEX ==
  operation(f) case Block => getCode f
  indentFortLevel(convert(1@Integer)$SEX)$Lisp
  expr := getCode f
  indentFortLevel(convert(-1@Integer)$SEX)$Lisp
  expr

getElseIf(f:$):SEX ==
  rec := code f
  expr :=
    fortFormatElseIf(elt(rec.conditionalBranch,switch)$COND::0)$Lisp
  expr :=
    APPEND(expr,getBody elt(rec.conditionalBranch,thenClause)$COND)$Lisp
  elseBranch := elt(rec.conditionalBranch,elseClause)$COND
  not(operation(elseBranch) case Null) =>
    operation(elseBranch) case Conditional =>
      APPEND(expr,getElseIf elseBranch)$Lisp
  expr := APPEND(expr, getStatement(ELSE::0,NIL$Lisp)$Lisp)$Lisp
  expr := APPEND(expr, getBody elseBranch)$Lisp
  expr

getContinue(label:SingleInteger):SEX ==
  lab : 0 := label::0
  if (width(lab) > 6) then error "Label too big"
  cnt : 0 := "CONTINUE":0
  --sp : 0 := hspace(6-width lab)
  sp : 0 := hspace(_$fortIndent$Lisp -width lab)
  LIST(STRCONC(STRINGIMAGE(lab)$Lisp,sp,cnt)$Lisp)$Lisp

getGoto(label:SingleInteger):SEX ==
  fortran2Lines(
    LIST(STRCONC("GOTO ",STRINGIMAGE(label::0)$Lisp)$Lisp)$Lisp)$Lisp

getRepeat(repRec:LOOP):SEX ==
  sw : Switch := NOT elt(repRec,switch)$LOOP
  lab := newLabel()
  bod := elt(repRec,body)$LOOP

```

```

APPEND(getContinue lab,getBody bod,
      fortFormatIfGoto(sw::0,lab)$Lisp)$Lisp

getWhile(whileRec:LOOP):SEX ==
  sw := NOT elt(whileRec,switch)$LOOP
  lab1 := newLabel()
  lab2 := newLabel()
  bod := elt(whileRec,body)$LOOP
  APPEND(fortFormatLabelledIfGoto(sw::0,lab1,lab2)$Lisp,
        getBody bod, getBody goto(lab1), getContinue lab2)$Lisp

getArrayAssign(rec:ARRAYASS):SEX ==
  getfortarrayexp((rec.var)::0,rec.rand,rec.ints2Floats?)$Lisp

getAssign(rec:ASS):SEX ==
  indices : L PIN := elt(rec,arrayIndex)$ASS
  if indices = []::(L PIN) then
    lhs := elt(rec,var)$ASS::0
  else
    lhs := cons(elt(rec,var)$ASS::PIN,indices)::0
    -- Must get the index brackets correct:
    lhs := (cdr car cdr convert(lhs)$SEX::SEX)::0 -- Yuck!
  elt(elt(rec,rnd)$ASS,ints2Floats?)$EXPRESSION =>
    assignment2Fortran1(lhs,elt(elt(rec,rnd)$ASS,expr)$EXPRESSION)$Lisp
  integerAssignment2Fortran1(lhs,elt(elt(rec,rnd)$ASS,expr)$EXPRESSION)$Lisp

getCond(rec:COND):SEX ==
  expr := APPEND(fortFormatIf(elt(rec,switch)$COND::0)$Lisp,
    getBody elt(rec,thenClause)$COND)$Lisp
  elseBranch := elt(rec,elseClause)$COND
  if not(operation(elseBranch) case Null) then
    operation(elseBranch) case Conditional =>
      expr := APPEND(expr,getElseIf elseBranch)$Lisp
      expr := APPEND(expr,getStateMENT(ELSE::0,NIL$Lisp)$Lisp,
        getBody elseBranch)$Lisp
  APPEND(expr,getStateMENT(ENDIF::0,NIL$Lisp)$Lisp)$Lisp

getComment(rec:COMMENT):SEX ==
  convert([convert(concat("C      ",c)$String)$SEX for c in rec])$SEX

getCall(rec:CALL):SEX ==
  expr := concat("CALL ",rec)$String
  #expr > 1320 => error "Fortran CALL too large"
  fortran2Lines(convert([convert(expr)$SEX ])$SEX)$Lisp

getFor(rec:FOR):SEX ==
  rng : SegmentBinding PIN := elt(rec,range)$FOR
  increment : PIN := elt(rec,span)$FOR
  lab : SingleInteger := newLabel()
  declare!(variable rng,fortranInteger())

```

```

expr := fortFormatDo(variable rng, (lo segment rng)::0, _
  (hi segment rng)::0, increment::0, lab)$Lisp
APPEND(expr, getBody elt(rec, body)$FOR, getContinue(lab))$Lisp

getCode(f:$):SEX ==
  opp:OP := operation f
  rec:OPREC:= code f
  opp case Assignment => getAssign(rec.assignmentBranch)
  opp case ArrayAssignment => getArrayAssign(rec.arrayAssignmentBranch)
  opp case Conditional => getCond(rec.conditionalBranch)
  opp case Return => getReturn(rec.returnBranch)
  opp case Block => getBlock(rec.blockBranch)
  opp case Comment => getComment(rec.commentBranch)
  opp case Call => getCall(rec.callBranch)
  opp case For => getFor(rec.forBranch)
  opp case Continue => getContinue(rec.labelBranch)
  opp case Goto => getGoto(rec.labelBranch)
  opp case Repeat => getRepeat(rec.loopBranch)
  opp case While => getWhile(rec.loopBranch)
  opp case Save => getSave()
  opp case Stop => getStop()
  opp case Print => getPrint(rec.printBranch)
  opp case Common => getCommon(rec.commonBranch)
  error "Unsupported program construct."
  convert(0)$SEX

printCode(f:$):Void ==
  displayLines1$Lisp getCode f
  void()$Void

code (f:$):OPREC ==
  elt(f, data)$Rep

operation (f:$):OP ==
  elt(f, op)$Rep

common(name:Symbol, contents:List Symbol):$ ==
  [["common"]$OP, [[name, contents]$COMMON]$OPREC]$Rep

stop():$ ==
  [["stop"]$OP, ["null"]$OPREC]$Rep

save():$ ==
  [["save"]$OP, ["null"]$OPREC]$Rep

printStatement(l:List 0):$ ==
  [["print"]$OP, [l]$OPREC]$Rep

comment(s:List String):$ ==
  [["comment"]$OP, [s]$OPREC]$Rep

```



```

comment(s:String):$ ==
  [["comment"]$OP, [list s]$OPREC]$Rep

forLoop(r:SegmentBinding PIN,body:$):$ ==
  [["for"]$OP, [[r,(incr segment r)::PIN,body]$FOR]$OPREC]$Rep

forLoop(r:SegmentBinding PIN,increment:PIN,body:$):$ ==
  [["for"]$OP, [[r,increment,body]$FOR]$OPREC]$Rep

goto(l:SingleInteger):$ ==
  [["goto"]$OP, [l]$OPREC]$Rep

continue(l:SingleInteger):$ ==
  [["continue"]$OP, [l]$OPREC]$Rep

whileLoop(sw:Switch,b:$):$ ==
  [["while"]$OP, [[sw,b]$LOOP]$OPREC]$Rep

repeatUntilLoop(sw:Switch,b:$):$ ==
  [["repeat"]$OP, [[sw,b]$LOOP]$OPREC]$Rep

returns():$ ==
  v := [false,0::0]$EXPRESSION
  [["return"]$OP, [[true,v]$RETURN]$OPREC]$Rep

returns(v:Expression MachineInteger):$ ==
  [["return"]$OP, [[false,[false,v::0]$EXPRESSION]$RETURN]$OPREC]$Rep

returns(v:Expression MachineFloat):$ ==
  [["return"]$OP, [[false,[false,v::0]$EXPRESSION]$RETURN]$OPREC]$Rep

returns(v:Expression MachineComplex):$ ==
  [["return"]$OP, [[false,[false,v::0]$EXPRESSION]$RETURN]$OPREC]$Rep

returns(v:Expression Integer):$ ==
  [["return"]$OP, [[false,[false,v::0]$EXPRESSION]$RETURN]$OPREC]$Rep

returns(v:Expression Float):$ ==
  [["return"]$OP, [[false,[false,v::0]$EXPRESSION]$RETURN]$OPREC]$Rep

returns(v:Expression Complex Float):$ ==
  [["return"]$OP, [[false,[false,v::0]$EXPRESSION]$RETURN]$OPREC]$Rep

block(l>List $):$ ==
  [["block"]$OP, [l]$OPREC]$Rep

cond(sw:Switch,thenC:$):$ ==
  [["conditional"]$OP,
    [[sw,thenC, [["null"]$OP, ["null"]$OPREC]$Rep]$COND]$OPREC]$Rep

```

```

cond(sw:Switch,thenC:$,elseC:$):$ ==
  [["conditional"]$OP,[[sw,thenC,elseC]$COND]$OPREC]$Rep

coerce(f : $):0 ==
  (f.op)::0

assign(v:Symbol,rhs:String):$ ==
  [["assignment"]$OP,[[v,nil()::L PIN,[false,rhs::0]$EXPRESSION]$ASS]$OPREC]$Rep

assign(v:Symbol,rhs:Matrix MachineInteger):$ ==
  [["arrayAssignment"]$OP,[[v,rhs::0,false]$ARRAYASS]$OPREC]$Rep

assign(v:Symbol,rhs:Matrix MachineFloat):$ ==
  [["arrayAssignment"]$OP,[[v,rhs::0,true]$ARRAYASS]$OPREC]$Rep

assign(v:Symbol,rhs:Matrix MachineComplex):$ ==
  [["arrayAssignment"]$OP,[[v,rhs::0,true]$ARRAYASS]$OPREC]$Rep

assign(v:Symbol,rhs:Vector MachineInteger):$ ==
  [["arrayAssignment"]$OP,[[v,rhs::0,false]$ARRAYASS]$OPREC]$Rep

assign(v:Symbol,rhs:Vector MachineFloat):$ ==
  [["arrayAssignment"]$OP,[[v,rhs::0,true]$ARRAYASS]$OPREC]$Rep

assign(v:Symbol,rhs:Vector MachineComplex):$ ==
  [["arrayAssignment"]$OP,[[v,rhs::0,true]$ARRAYASS]$OPREC]$Rep

assign(v:Symbol,rhs:Matrix Expression MachineInteger):$ ==
  [["arrayAssignment"]$OP,[[v,rhs::0,false]$ARRAYASS]$OPREC]$Rep

assign(v:Symbol,rhs:Matrix Expression MachineFloat):$ ==
  [["arrayAssignment"]$OP,[[v,rhs::0,true]$ARRAYASS]$OPREC]$Rep

assign(v:Symbol,rhs:Matrix Expression MachineComplex):$ ==
  [["arrayAssignment"]$OP,[[v,rhs::0,true]$ARRAYASS]$OPREC]$Rep

assign(v:Symbol,rhs:Vector Expression MachineInteger):$ ==
  [["arrayAssignment"]$OP,[[v,rhs::0,false]$ARRAYASS]$OPREC]$Rep

assign(v:Symbol,rhs:Vector Expression MachineFloat):$ ==
  [["arrayAssignment"]$OP,[[v,rhs::0,true]$ARRAYASS]$OPREC]$Rep

assign(v:Symbol,rhs:Vector Expression MachineComplex):$ ==
  [["arrayAssignment"]$OP,[[v,rhs::0,true]$ARRAYASS]$OPREC]$Rep

assign(v:Symbol,index:L PIN,rhs:Expression MachineInteger):$ ==
  [["assignment"]$OP,[[v,index,[false,rhs::0]$EXPRESSION]$ASS]$OPREC]$Rep

assign(v:Symbol,index:L PIN,rhs:Expression MachineFloat):$ ==

```

```

[[ "assignment" ] $OP, [[ v, index, [ true, rhs::0 ] $EXPRESSION ] $ASS ] $OPREC ] $Rep

assign(v:Symbol, index:L PIN, rhs:Expression MachineComplex):$ ==
[[ "assignment" ] $OP, [[ v, index, [ true, rhs::0 ] $EXPRESSION ] $ASS ] $OPREC ] $Rep

assign(v:Symbol, rhs:Expression MachineInteger):$ ==
[[ "assignment" ] $OP, [[ v, nil()::L PIN, [ false, rhs::0 ] $EXPRESSION ] $ASS ] $OPREC ] $Rep

assign(v:Symbol, rhs:Expression MachineFloat):$ ==
[[ "assignment" ] $OP, [[ v, nil()::L PIN, [ true, rhs::0 ] $EXPRESSION ] $ASS ] $OPREC ] $Rep

assign(v:Symbol, rhs:Expression MachineComplex):$ ==
[[ "assignment" ] $OP, [[ v, nil()::L PIN, [ true, rhs::0 ] $EXPRESSION ] $ASS ] $OPREC ] $Rep

assign(v:Symbol, rhs:Matrix Expression Integer):$ ==
[[ "arrayAssignment" ] $OP, [[ v, rhs::0, false ] $ARRAYASS ] $OPREC ] $Rep

assign(v:Symbol, rhs:Matrix Expression Float):$ ==
[[ "arrayAssignment" ] $OP, [[ v, rhs::0, true ] $ARRAYASS ] $OPREC ] $Rep

assign(v:Symbol, rhs:Matrix Expression Complex Float):$ ==
[[ "arrayAssignment" ] $OP, [[ v, rhs::0, true ] $ARRAYASS ] $OPREC ] $Rep

assign(v:Symbol, rhs:Vector Expression Integer):$ ==
[[ "arrayAssignment" ] $OP, [[ v, rhs::0, false ] $ARRAYASS ] $OPREC ] $Rep

assign(v:Symbol, rhs:Vector Expression Float):$ ==
[[ "arrayAssignment" ] $OP, [[ v, rhs::0, true ] $ARRAYASS ] $OPREC ] $Rep

assign(v:Symbol, rhs:Vector Expression Complex Float):$ ==
[[ "arrayAssignment" ] $OP, [[ v, rhs::0, true ] $ARRAYASS ] $OPREC ] $Rep

assign(v:Symbol, index:L PIN, rhs:Expression Integer):$ ==
[[ "assignment" ] $OP, [[ v, index, [ false, rhs::0 ] $EXPRESSION ] $ASS ] $OPREC ] $Rep

assign(v:Symbol, index:L PIN, rhs:Expression Float):$ ==
[[ "assignment" ] $OP, [[ v, index, [ true, rhs::0 ] $EXPRESSION ] $ASS ] $OPREC ] $Rep

assign(v:Symbol, index:L PIN, rhs:Expression Complex Float):$ ==
[[ "assignment" ] $OP, [[ v, index, [ true, rhs::0 ] $EXPRESSION ] $ASS ] $OPREC ] $Rep

assign(v:Symbol, rhs:Expression Integer):$ ==
[[ "assignment" ] $OP, [[ v, nil()::L PIN, [ false, rhs::0 ] $EXPRESSION ] $ASS ] $OPREC ] $Rep

assign(v:Symbol, rhs:Expression Float):$ ==
[[ "assignment" ] $OP, [[ v, nil()::L PIN, [ true, rhs::0 ] $EXPRESSION ] $ASS ] $OPREC ] $Rep

assign(v:Symbol, rhs:Expression Complex Float):$ ==
[[ "assignment" ] $OP, [[ v, nil()::L PIN, [ true, rhs::0 ] $EXPRESSION ] $ASS ] $OPREC ] $Rep

```

```
call(s:String):$ ==
  [["call"]$OP,[s]$OPREC]$Rep
```

---

— FC.dotabb —

```
"FC" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FC"]
"FS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FS"]
"COMPCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=COMPCAT"]
"FC" -> "COMPCAT"
"FC" -> "FS"
```

---

## 7.17 domain FEXPR FortranExpression

— FortranExpression.input —

```
)set break resume
)sys rm -f FortranExpression.output
)spool FortranExpression.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FortranExpression
--R FortranExpression(basicSymbols: List Symbol,subscriptedSymbols: List Symbol,R: FortranMachineTypeCat
--R Abbreviation for FortranExpression is FEXPR
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FEXPR
--R
--R----- Operations -----
--R ?? : (PositiveInteger,%) -> %      ?? : (Integer,%) -> %
--R ?? : (%,%) -> %                  ?? : (%,R) -> %
--R ?? : (R,%) -> %                  ??? : (%,PositiveInteger) -> %
--R ?? : (%,%) -> %                  -? : % -> %
--R ?? : (%,%) -> %                  ?<? : (%,%) -> Boolean
--R ?<=? : (%,%) -> Boolean          ?=? : (%,%) -> Boolean
--R ?>? : (%,%) -> Boolean          ?>=? : (%,%) -> Boolean
--R D : (%,Symbol) -> %              D : (%,List Symbol) -> %
--R 1 : () -> %                      0 : () -> %
--R ?? : (%,PositiveInteger) -> %    abs : % -> %
```

```

--R acos : % -> %
--R atan : % -> %
--R box : List % -> %
--R coerce : % -> Expression R
--R coerce : R -> %
--R coerce : % -> OutputForm
--R cosh : % -> %
--R distribute : (%,%) -> %
--R elt : (BasicOperator,%,%) -> %
--R eval : (%,List %,List %) -> %
--R eval : (%,Equation %) -> %
--R eval : (%,Kernel %,%) -> %
--R freeOf? : (%,Symbol) -> Boolean
--R hash : % -> SingleInteger
--R is? : (%,Symbol) -> Boolean
--R kernels : % -> List Kernel %
--R log : % -> %
--R map : ((% -> %),Kernel %) -> %
--R min : (%,%) -> %
--R paren : List % -> %
--R pi : () -> %
--R retract : Symbol -> %
--R retract : % -> R
--R sample : () -> %
--R sinh : % -> %
--R subst : (%,Equation %) -> %
--R tanh : % -> %
--R useNagFunctions : () -> Boolean
--R zero? : % -> Boolean
--R ?? : (NonNegativeInteger,%) -> %
--R ??? : (%,NonNegativeInteger) -> %
--R D : (%,Symbol,NonNegativeInteger) -> %
--R D : (%,List Symbol,List NonNegativeInteger) -> %
--R ?? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R definingPolynomial : % -> % if $ has RING
--R differentiate : (%,List Symbol) -> %
--R differentiate : (%,Symbol,NonNegativeInteger) -> %
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> %
--R elt : (BasicOperator,List %) -> %
--R elt : (BasicOperator,%,%,%,%) -> %
--R elt : (BasicOperator,%,%,%) -> %
--R eval : (%,BasicOperator,(% -> %)) -> %
--R eval : (%,BasicOperator,(List % -> %)) -> %
--R eval : (%,List BasicOperator,List (List % -> %)) -> %
--R eval : (%,List BasicOperator,List (% -> %)) -> %
--R eval : (%,Symbol,(% -> %)) -> %
--R eval : (%,Symbol,(List % -> %)) -> %
--R eval : (%,List Symbol,List (List % -> %)) -> %
--R eval : (%,List Symbol,List (% -> %)) -> %

asin : % -> %
belong? : BasicOperator -> Boolean
box : % -> %
coerce : Integer -> %
coerce : Kernel % -> %
cos : % -> %
differentiate : (%,Symbol) -> %
distribute : % -> %
elt : (BasicOperator,%) -> %
eval : (%,%,%) -> %
eval : (%,List Equation %) -> %
exp : % -> %
freeOf? : (%,%) -> Boolean
height : % -> NonNegativeInteger
kernel : (BasicOperator,%) -> %
latex : % -> String
log10 : % -> %
max : (%,%) -> %
one? : % -> Boolean
paren : % -> %
recip : % -> Union(%, "failed")
retract : Expression R -> %
retract : % -> Kernel %
sin : % -> %
sqrt : % -> %
tan : % -> %
tower : % -> List Kernel %
variables : % -> List Symbol
?~=? : (%,%) -> Boolean

```

```

--R eval : (% ,List Kernel % ,List %) -> %
--R even? : % -> Boolean if $ has RETRACT INT
--R is? : (% ,BasicOperator) -> Boolean
--R kernel : (BasicOperator ,List %) -> %
--R mainKernel : % -> Union(Kernel % , "failed")
--R minPoly : Kernel % -> SparseUnivariatePolynomial % if $ has RING
--R odd? : % -> Boolean if $ has RETRACT INT
--R operator : BasicOperator -> BasicOperator
--R operators : % -> List BasicOperator
--R retract : Polynomial Float -> % if R has RETRACT FLOAT
--R retract : Fraction Polynomial Float -> % if R has RETRACT FLOAT
--R retract : Expression Float -> % if R has RETRACT FLOAT
--R retract : Polynomial Integer -> % if R has RETRACT INT
--R retract : Fraction Polynomial Integer -> % if R has RETRACT INT
--R retract : Expression Integer -> % if R has RETRACT INT
--R retractIfCan : Polynomial Float -> Union(% , "failed") if R has RETRACT FLOAT
--R retractIfCan : Fraction Polynomial Float -> Union(% , "failed") if R has RETRACT FLOAT
--R retractIfCan : Expression Float -> Union(% , "failed") if R has RETRACT FLOAT
--R retractIfCan : Polynomial Integer -> Union(% , "failed") if R has RETRACT INT
--R retractIfCan : Fraction Polynomial Integer -> Union(% , "failed") if R has RETRACT INT
--R retractIfCan : Expression Integer -> Union(% , "failed") if R has RETRACT INT
--R retractIfCan : Symbol -> Union(% , "failed")
--R retractIfCan : Expression R -> Union(% , "failed")
--R retractIfCan : % -> Union(R , "failed")
--R retractIfCan : % -> Union(Kernel % , "failed")
--R subst : (% ,List Kernel % ,List %) -> %
--R subst : (% ,List Equation %) -> %
--R subtractIfCan : (% ,%) -> Union(% , "failed")
--R useNagFunctions : Boolean -> Boolean
--R
--E 1

)spool
)lisp (bye)

```

---

— FortranExpression.help —

```

=====
FortranExpression examples
=====

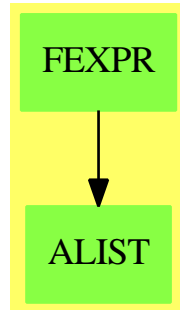
```

```

See Also:
o )show FortranExpression

```

### 7.17.1 FortranExpression (FEXPR)



See

- ⇒ “Result” (RESULT) 19.9.1 on page 2260
- ⇒ “FortranCode” (FC) 7.16.1 on page 898
- ⇒ “FortranProgram” (FORTRAN) 7.18.1 on page 923
- ⇒ “ThreeDimensionalMatrix” (M3D) 21.7.1 on page 2661
- ⇒ “SimpleFortranProgram” (SFORT) 20.11.1 on page 2364
- ⇒ “Switch” (SWITCH) 20.36.1 on page 2588
- ⇒ “FortranTemplate” (FTEM) 7.20.1 on page 934

#### Exports:

|            |          |                 |                    |               |
|------------|----------|-----------------|--------------------|---------------|
| 0          | 1        | abs             | acos               | asin          |
| atan       | belong?  | box             | characteristic     | coerce        |
| cos        | cosh     | D               | definingPolynomial | differentiate |
| distribute | elt      | eval            | even?              | exp           |
| freeOf?    | hash     | height          | is?                | kernel        |
| kernels    | latex    | log             | log10              | mainKernel    |
| map        | max      | min             | minPoly            | odd?          |
| one?       | operator | operators       | paren              | pi            |
| recip      | retract  | retractIfCan    | sample             | sin           |
| sinh       | sqrt     | subst           | subtractIfCan      | tan           |
| tanh       | tower    | useNagFunctions | variables          | zero?         |
| ?*?        | ?**?     | ?+?             | -?                 | ?-?           |
| ?<?        | ?<=?     | ?=?             | ?>?                | ?>=?          |
| ?^?        | ?~=?     |                 |                    |               |

— domain FEXPR FortranExpression —

```

)abbrev domain FEXPR FortranExpression
++ Author: Mike Dewar
++ Date Created: December 1993
++ Date Last Updated: 19 May 1994
++
++       7 July 1994 added %power to f77Functions
++      12 July 1994 added RetractableTo(R)
++ Basic Operations:
  
```

```

++ Related Domains:
++ Also See: FortranMachineTypeCategory, MachineInteger, MachineFloat,
++ MachineComplex
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ A domain of expressions involving functions which can be
++ translated into standard Fortran-77, with some extra extensions from
++ the NAG Fortran Library.

```

```

FortranExpression(basicSymbols,subscriptedSymbols,R):
    Exports==Implementation where

basicSymbols : List Symbol
subscriptedSymbols : List Symbol
R : FortranMachineTypeCategory

EXPR ==> Expression
EXF2 ==> ExpressionFunctions2
S ==> Symbol
L ==> List
BO ==> BasicOperator
FRAC ==> Fraction
POLY ==> Polynomial

Exports ==> Join(ExpressionSpace,Algebra(R),RetractableTo(R),
    PartialDifferentialRing(Symbol)) with
retract : EXPR R -> $
    ++ retract(e) takes e and transforms it into a
    ++ FortranExpression checking that it contains no non-Fortran
    ++ functions, and that it only contains the given basic symbols
    ++ and subscripted symbols which correspond to scalar and array
    ++ parameters respectively.
retractIfCan : EXPR R -> Union($,"failed")
    ++ retractIfCan(e) takes e and tries to transform it into a
    ++ FortranExpression checking that it contains no non-Fortran
    ++ functions, and that it only contains the given basic symbols
    ++ and subscripted symbols which correspond to scalar and array
    ++ parameters respectively.
retract : S -> $
    ++ retract(e) takes e and transforms it into a FortranExpression
    ++ checking that it is one of the given basic symbols
    ++ or subscripted symbols which correspond to scalar and array
    ++ parameters respectively.
retractIfCan : S -> Union($,"failed")
    ++ retractIfCan(e) takes e and tries to transform it into a
    ++ FortranExpression checking that it is one of the given basic symbols
    ++ or subscripted symbols which correspond to scalar and array
    ++ parameters respectively.

```



```

coerce : $ -> EXPR R
  ++ coerce(x) is not documented
if (R has RetractableTo(Integer)) then
  retract : EXPR Integer -> $
    ++ retract(e) takes e and transforms it into a
    ++ FortranExpression checking that it contains no non-Fortran
    ++ functions, and that it only contains the given basic symbols
    ++ and subscripted symbols which correspond to scalar and array
    ++ parameters respectively.
  retractIfCan : EXPR Integer -> Union($,"failed")
    ++ retractIfCan(e) takes e and tries to transform it into a
    ++ FortranExpression checking that it contains no non-Fortran
    ++ functions, and that it only contains the given basic symbols
    ++ and subscripted symbols which correspond to scalar and array
    ++ parameters respectively.
  retract : FRAC POLY Integer -> $
    ++ retract(e) takes e and transforms it into a
    ++ FortranExpression checking that it contains no non-Fortran
    ++ functions, and that it only contains the given basic symbols
    ++ and subscripted symbols which correspond to scalar and array
    ++ parameters respectively.
  retractIfCan : FRAC POLY Integer -> Union($,"failed")
    ++ retractIfCan(e) takes e and tries to transform it into a
    ++ FortranExpression checking that it contains no non-Fortran
    ++ functions, and that it only contains the given basic symbols
    ++ and subscripted symbols which correspond to scalar and array
    ++ parameters respectively.
  retract : POLY Integer -> $
    ++ retract(e) takes e and transforms it into a
    ++ FortranExpression checking that it contains no non-Fortran
    ++ functions, and that it only contains the given basic symbols
    ++ and subscripted symbols which correspond to scalar and array
    ++ parameters respectively.
  retractIfCan : POLY Integer -> Union($,"failed")
    ++ retractIfCan(e) takes e and tries to transform it into a
    ++ FortranExpression checking that it contains no non-Fortran
    ++ functions, and that it only contains the given basic symbols
    ++ and subscripted symbols which correspond to scalar and array
    ++ parameters respectively.
if (R has RetractableTo(Float)) then
  retract : EXPR Float -> $
    ++ retract(e) takes e and transforms it into a
    ++ FortranExpression checking that it contains no non-Fortran
    ++ functions, and that it only contains the given basic symbols
    ++ and subscripted symbols which correspond to scalar and array
    ++ parameters respectively.
  retractIfCan : EXPR Float -> Union($,"failed")
    ++ retractIfCan(e) takes e and tries to transform it into a
    ++ FortranExpression checking that it contains no non-Fortran
    ++ functions, and that it only contains the given basic symbols

```

```

    ++ and subscripted symbols which correspond to scalar and array
    ++ parameters respectively.
retract : FRAC POLY Float -> $
    ++ retract(e) takes e and transforms it into a
    ++ FortranExpression checking that it contains no non-Fortran
    ++ functions, and that it only contains the given basic symbols
    ++ and subscripted symbols which correspond to scalar and array
    ++ parameters respectively.
retractIfCan : FRAC POLY Float -> Union($,"failed")
    ++ retractIfCan(e) takes e and tries to transform it into a
    ++ FortranExpression checking that it contains no non-Fortran
    ++ functions, and that it only contains the given basic symbols
    ++ and subscripted symbols which correspond to scalar and array
    ++ parameters respectively.
retract : POLY Float -> $
    ++ retract(e) takes e and transforms it into a
    ++ FortranExpression checking that it contains no non-Fortran
    ++ functions, and that it only contains the given basic symbols
    ++ and subscripted symbols which correspond to scalar and array
    ++ parameters respectively.
retractIfCan : POLY Float -> Union($,"failed")
    ++ retractIfCan(e) takes e and tries to transform it into a
    ++ FortranExpression checking that it contains no non-Fortran
    ++ functions, and that it only contains the given basic symbols
    ++ and subscripted symbols which correspond to scalar and array
    ++ parameters respectively.
abs      : $ -> $
    ++ abs(x) represents the Fortran intrinsic function ABS
sqrt     : $ -> $
    ++ sqrt(x) represents the Fortran intrinsic function SQRT
exp      : $ -> $
    ++ exp(x) represents the Fortran intrinsic function EXP
log      : $ -> $
    ++ log(x) represents the Fortran intrinsic function LOG
log10    : $ -> $
    ++ log10(x) represents the Fortran intrinsic function LOG10
sin      : $ -> $
    ++ sin(x) represents the Fortran intrinsic function SIN
cos      : $ -> $
    ++ cos(x) represents the Fortran intrinsic function COS
tan      : $ -> $
    ++ tan(x) represents the Fortran intrinsic function TAN
asin     : $ -> $
    ++ asin(x) represents the Fortran intrinsic function ASIN
acos     : $ -> $
    ++ acos(x) represents the Fortran intrinsic function ACOS
atan     : $ -> $
    ++ atan(x) represents the Fortran intrinsic function ATAN
sinh     : $ -> $
    ++ sinh(x) represents the Fortran intrinsic function SINH

```

```

cosh  : $ -> $
  ++ cosh(x) represents the Fortran intrinsic function COSH
tanh  : $ -> $
  ++ tanh(x) represents the Fortran intrinsic function TANH
pi    : () -> $
  ++ pi(x) represents the NAG Library function X01AAF which returns
  ++ an approximation to the value of pi
variables : $ -> L S
  ++ variables(e) return a list of all the variables in \spad{e}.
useNagFunctions : () -> Boolean
  ++ useNagFunctions() indicates whether NAG functions are being used
  ++ for mathematical and machine constants.
useNagFunctions : Boolean -> Boolean
  ++ useNagFunctions(v) sets the flag which controls whether NAG functions
  ++ are being used for mathematical and machine constants. The previous
  ++ value is returned.

Implementation ==> EXPR R add

-- The standard FORTRAN-77 intrinsic functions, plus nthRoot which
-- can be translated into an arithmetic expression:
f77Functions : L S := [abs,sqrt,exp,log,log10,sin,cos,tan,asin,acos,
                      atan,sinh,cosh,tanh,nthRoot,%power]
nagFunctions : L S := [pi, X01AAF]
useNagFunctionsFlag : Boolean := true

-- Local functions to check for "unassigned" symbols etc.

mkEqn(s1:Symbol,s2:Symbol):Equation EXPR(R) ==
  equation(s2::EXPR(R),script(s1,scripts(s2))::EXPR(R))

fixUpSymbols(u:EXPR R):Union(EXPR R,"failed") ==
  -- If its a univariate expression then just fix it up:
  syms : L S := variables(u)
  -- one?(#basicSymbols) and zero?(#subscriptedSymbols) =>
  -- (#basicSymbols = 1) and zero?(#subscriptedSymbols) =>
  -- not one?(#syms) => "failed"
  -- not (#syms = 1) => "failed"
  subst(u,equation(first(syms)::EXPR(R),first(basicSymbols)::EXPR(R)))
  -- We have one variable but it is subscripted:
  -- zero?(#basicSymbols) and one?(#subscriptedSymbols) =>
  zero?(#basicSymbols) and (#subscriptedSymbols = 1) =>
  -- Make sure we don't have both X and X_i
  for s in syms repeat
    not scripted?(s) => return "failed"
  -- not one?((#(syms:=removeDuplicates! [name(s) for s in syms]))=> "failed"
  not ((#(syms:=removeDuplicates! [name(s) for s in syms])) = 1)=> "failed"
  sym : Symbol := first subscriptedSymbols
  subst(u,[mkEqn(sym,i) for i in variables(u)])
  "failed"

```

```

extraSymbols?(u:EXPR R):Boolean ==
  syms   : L S := [name(v) for v in variables(u)]
  extras : L S := setDifference(syms,
                                setUnion(basicSymbols,subscriptedSymbols))
  not empty? extras

checkSymbols(u:EXPR R):EXPR(R) ==
  syms   : L S := [name(v) for v in variables(u)]
  extras : L S := setDifference(syms,
                                setUnion(basicSymbols,subscriptedSymbols))
  not empty? extras =>
    m := fixUpSymbols(u)
    m case EXPR(R) => m::EXPR(R)
    error("Extra symbols detected:",[string(v) for v in extras]$L(String))
  u

notSymbol?(v:B0):Boolean ==
  s : S := name v
  member?(s,basicSymbols) or
  scripted?(s) and member?(name s,subscriptedSymbols) => false
  true

extraOperators?(u:EXPR R):Boolean ==
  ops   : L S := [name v for v in operators(u) | notSymbol?(v)]
  if useNagFunctionsFlag then
    fortranFunctions : L S := append(f77Functions,nagFunctions)
  else
    fortranFunctions : L S := f77Functions
  extras : L S := setDifference(ops,fortranFunctions)
  not empty? extras

checkOperators(u:EXPR R):Void ==
  ops   : L S := [name v for v in operators(u) | notSymbol?(v)]
  if useNagFunctionsFlag then
    fortranFunctions : L S := append(f77Functions,nagFunctions)
  else
    fortranFunctions : L S := f77Functions
  extras : L S := setDifference(ops,fortranFunctions)
  not empty? extras =>
    error("Non FORTRAN-77 functions detected:",[string(v) for v in extras])
  void()

checkForNagOperators(u:EXPR R):$ ==
  useNagFunctionsFlag =>
    import Pi
    import PiCoercions(R)
    piOp : BasicOperator := operator X01AAF
    piSub : Equation EXPR R :=
      equation(pi()$Pi::EXPR(R),kernel(piOp,0::EXPR(R))$EXPR(R))

```

```

    subst(u,piSub) pretend $
  u pretend $

-- Conditional retractions:

if R has RetractableTo(Integer) then

  retractIfCan(u:POLY Integer):Union($,"failed") ==
    retractIfCan((u:EXPR Integer)$EXPR(Integer))@Union($,"failed")

  retract(u:POLY Integer):$ ==
    retract((u:EXPR Integer)$EXPR(Integer))@$

  retractIfCan(u:FRAC POLY Integer):Union($,"failed") ==
    retractIfCan((u:EXPR Integer)$EXPR(Integer))@Union($,"failed")

  retract(u:FRAC POLY Integer):$ ==
    retract((u:EXPR Integer)$EXPR(Integer))@$

  int2R(u:Integer):R == u::R

  retractIfCan(u:EXPR Integer):Union($,"failed") ==
    retractIfCan(map(int2R,u)$EXF2(Integer,R))@Union($,"failed")

  retract(u:EXPR Integer):$ ==
    retract(map(int2R,u)$EXF2(Integer,R))@$

if R has RetractableTo(Float) then

  retractIfCan(u:POLY Float):Union($,"failed") ==
    retractIfCan((u:EXPR Float)$EXPR(Float))@Union($,"failed")

  retract(u:POLY Float):$ ==
    retract((u:EXPR Float)$EXPR(Float))@$

  retractIfCan(u:FRAC POLY Float):Union($,"failed") ==
    retractIfCan((u:EXPR Float)$EXPR(Float))@Union($,"failed")

  retract(u:FRAC POLY Float):$ ==
    retract((u:EXPR Float)$EXPR(Float))@$

  float2R(u:Float):R == (u::R)

  retractIfCan(u:EXPR Float):Union($,"failed") ==
    retractIfCan(map(float2R,u)$EXF2(Float,R))@Union($,"failed")

  retract(u:EXPR Float):$ ==
    retract(map(float2R,u)$EXF2(Float,R))@$

-- Exported Functions

```

```

useNagFunctions():Boolean == useNagFunctionsFlag
useNagFunctions(v:Boolean):Boolean ==
  old := useNagFunctionsFlag
  useNagFunctionsFlag := v
  old

log10(x:$):$ ==
  kernel(operator log10,x)

pi():$ == kernel(operator X01AAF,0)

coerce(u:$):EXPR R == u pretend EXPR(R)

retractIfCan(u:EXPR R):Union($,"failed") ==
  if (extraSymbols? u) then
    m := fixUpSymbols(u)
    m case "failed" => return "failed"
    u := m::EXPR(R)
  extraOperators? u => "failed"
  checkForNagOperators(u)

retract(u:EXPR R):$ ==
  u:=checkSymbols(u)
  checkOperators(u)
  checkForNagOperators(u)

retractIfCan(u:Symbol):Union($,"failed") ==
  not (member?(u,basicSymbols) or
    scripted?(u) and member?(name u,subscriptedSymbols)) => "failed"
  (((u::EXPR(R))$(EXPR R))pretend Rep)::$

retract(u:Symbol):$ ==
  res : Union($,"failed") := retractIfCan(u)
  res case "failed" => error("Illegal Symbol Detected:",u::String)
  res::$

```

---

— FEXPR.dotabb —

```

"FEXPR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FEXPR"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"FEXPR" -> "ALIST"

```

---

## 7.18 domain FORTRAN FortranProgram

— FortranProgram.input —

```
)set break resume
)sys rm -f FortranProgram.output
)spool FortranProgram.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FortranProgram
--R FortranProgram(name: Symbol,returnType: Union(fst: FortranScalarType,void: void),argument
--R Abbreviation for FortranProgram is FORTRAN
--R This constructor is exposed in this frame.
--R Issue )edit NIL to see algebra source code for FORTRAN
--R
--R----- Operations -----
--R coerce : Expression Float -> %          coerce : Expression Integer -> %
--R coerce : List FortranCode -> %          coerce : FortranCode -> %
--R coerce : % -> OutputForm                outputAsFortran : % -> Void
--R coerce : Equation Expression Complex Float -> %
--R coerce : Equation Expression Float -> %
--R coerce : Equation Expression Integer -> %
--R coerce : Expression Complex Float -> %
--R coerce : Equation Expression MachineComplex -> %
--R coerce : Equation Expression MachineFloat -> %
--R coerce : Equation Expression MachineInteger -> %
--R coerce : Expression MachineComplex -> %
--R coerce : Expression MachineFloat -> %
--R coerce : Expression MachineInteger -> %
--R coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
--R
--E 1

)spool
)lisp (bye)
```

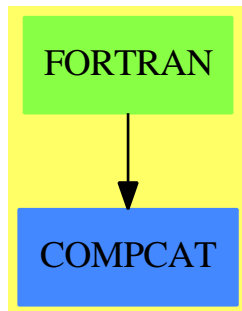
— FortranProgram.help —

```
=====
FortranProgram examples
=====
```

See Also:

```
o )show FortranProgram
```

### 7.18.1 FortranProgram (FORTRAN)



See

- ⇒ “Result” (RESULT) 19.9.1 on page 2260
- ⇒ “FortranCode” (FC) 7.16.1 on page 898
- ⇒ “ThreeDimensionalMatrix” (M3D) 21.7.1 on page 2661
- ⇒ “SimpleFortranProgram” (SFORT) 20.11.1 on page 2364
- ⇒ “Switch” (SWITCH) 20.36.1 on page 2588
- ⇒ “FortranTemplate” (FTEM) 7.20.1 on page 934
- ⇒ “FortranExpression” (FEXPR) 7.17.1 on page 914

#### Exports:

coerce outputAsFortran

— domain FORTRAN FortranProgram —

```

)abbrev domain FORTRAN FortranProgram
++ Author: Mike Dewar
++ Date Created: October 1992
++ Date Last Updated: 13 January 1994
++      23 January 1995 Added support for intrinsic functions
++ Basic Operations:
++ Related Constructors: FortranType, FortranCode, Switch
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ \axiomType{FortranProgram} allows the user to build and manipulate simple
++ models of FORTRAN subprograms. These can then be transformed into
++ actual FORTRAN notation.

```



```
FortranProgram(name,returnType,arguments,symbols): Exports == Implement where
```

```
  name      : Symbol
  returnType : Union(fst:FortranScalarType,void:"void")
  arguments  : List Symbol
  symbols    : SymbolTable
```

```
FC      ==> FortranCode
EXPR    ==> Expression
INT      ==> Integer
CMPX     ==> Complex
MINT     ==> MachineInteger
MFLOAT   ==> MachineFloat
MCMPLX   ==> MachineComplex
REP      ==> Record(localSymbols : SymbolTable, code : List FortranCode)
```

```
Exports ==> FortranProgramCategory with
```

```
  coerce : FortranCode -> $
    ++ coerce(fc) is not documented
  coerce : List FortranCode -> $
    ++ coerce(lfc) is not documented
  coerce : REP -> $
    ++ coerce(r) is not documented
  coerce : EXPR MINT -> $
    ++ coerce(e) is not documented
  coerce : EXPR MFLOAT -> $
    ++ coerce(e) is not documented
  coerce : EXPR MCMPLX -> $
    ++ coerce(e) is not documented
  coerce : Equation EXPR MINT -> $
    ++ coerce(eq) is not documented
  coerce : Equation EXPR MFLOAT -> $
    ++ coerce(eq) is not documented
  coerce : Equation EXPR MCMPLX -> $
    ++ coerce(eq) is not documented
  coerce : EXPR INT -> $
    ++ coerce(e) is not documented
  coerce : EXPR Float -> $
    ++ coerce(e) is not documented
  coerce : EXPR CMPX Float -> $
    ++ coerce(e) is not documented
  coerce : Equation EXPR INT -> $
    ++ coerce(eq) is not documented
  coerce : Equation EXPR Float -> $
    ++ coerce(eq) is not documented
  coerce : Equation EXPR CMPX Float -> $
    ++ coerce(eq) is not documented
```

```
Implement ==> add
```

```

Rep := REP

import SExpression
import TheSymbolTable
import FortranCode

makeRep(b:List FortranCode):$ ==
  construct(empty())$SymbolTable,b)$REP

codeFrom(u:$):List FortranCode ==
  elt(u::Rep,code)$REP

outputAsFortran(p:$):Void ==
  setLabelValue(25000::SingleInteger)$FC
  -- Do this first to catch any extra type declarations:
  tempName := "FPTEMP"::Symbol
  newSubProgram(tempName)
  initialiseIntrinsicList()$Lisp
  body : List SExpression := [getCode(l)$FortranCode for l in codeFrom(p)]
  intrinsics : SExpression := getIntrinsicList()$Lisp
  endSubProgram()
  fortFormatHead(returnType::OutputForm, name::OutputForm, _
    arguments::OutputForm)$Lisp
  printTypes(symbols)$SymbolTable
  printTypes((p::Rep).localSymbols)$SymbolTable
  printTypes(tempName)$TheSymbolTable
  fortFormatIntrinsics(intrinsics)$Lisp
  clearTheSymbolTable(tempName)
  for expr in body repeat displayLines1(expr)$Lisp
  dispStatement(END::OutputForm)$Lisp
  void()$Void

mkString(l:List Symbol):String ==
  unparse(convert(l::OutputForm)$InputForm)$InputForm

checkVariables(user:List Symbol,target:List Symbol):Void ==
  -- We don't worry about whether the user has subscripted the
  -- variables or not.
  setDifference(map(name$Symbol,user),target) ^= empty()$List(Symbol) =>
    s1 : String := mkString(user)
    s2 : String := mkString(target)
    error ["Incompatible variable lists:", s1, s2]
  void()$Void

coerce(u:EXPR MINT) : $ ==
  checkVariables(variables(u)$EXPR(MINT),arguments)
  l : List(FC) := [assign(name,u)$FC,returns()$FC]
  makeRep l

coerce(u:Equation EXPR MINT) : $ ==

```

```

retractIfCan(lhs u)@Union(Kernel(EXPR MINT),"failed") case "failed" =>
  error "left hand side is not a kernel"
vList : List Symbol := variables lhs u
#vList ^= #arguments =>
  error "Incorrect number of arguments"
veList : List EXPR MINT := [w::EXPR(MINT) for w in vList]
aeList : List EXPR MINT := [w::EXPR(MINT) for w in arguments]
eList : List Equation EXPR MINT :=
  [equation(w,v) for w in veList for v in aeList]
(subst(rhs u,eList)):$

coerce(u:EXPR MFLOAT) : $ ==
  checkVariables(variables(u)$EXPR(MFLOAT),arguments)
  l : List(FC) := [assign(name,u)$FC,returns()$FC]
  makeRep l

coerce(u:Equation EXPR MFLOAT) : $ ==
  retractIfCan(lhs u)@Union(Kernel(EXPR MFLOAT),"failed") case "failed" =>
    error "left hand side is not a kernel"
  vList : List Symbol := variables lhs u
  #vList ^= #arguments =>
    error "Incorrect number of arguments"
  veList : List EXPR MFLOAT := [w::EXPR(MFLOAT) for w in vList]
  aeList : List EXPR MFLOAT := [w::EXPR(MFLOAT) for w in arguments]
  eList : List Equation EXPR MFLOAT :=
    [equation(w,v) for w in veList for v in aeList]
  (subst(rhs u,eList)):$

coerce(u:EXPR MCMPLX) : $ ==
  checkVariables(variables(u)$EXPR(MCMPLX),arguments)
  l : List(FC) := [assign(name,u)$FC,returns()$FC]
  makeRep l

coerce(u:Equation EXPR MCMPLX) : $ ==
  retractIfCan(lhs u)@Union(Kernel(EXPR MCMPLX),"failed") case "failed"=>
    error "left hand side is not a kernel"
  vList : List Symbol := variables lhs u
  #vList ^= #arguments =>
    error "Incorrect number of arguments"
  veList : List EXPR MCMPLX := [w::EXPR(MCMPLX) for w in vList]
  aeList : List EXPR MCMPLX := [w::EXPR(MCMPLX) for w in arguments]
  eList : List Equation EXPR MCMPLX :=
    [equation(w,v) for w in veList for v in aeList]
  (subst(rhs u,eList)):$

coerce(u:REP):$ ==
  u@Rep

coerce(u:$):OutputForm ==

```

```

coerce(name)$Symbol

coerce(c:List FortranCode):$ ==
  makeRep c

coerce(c:FortranCode):$ ==
  makeRep [c]

coerce(u:EXPR INT) : $ ==
  checkVariables(variables(u)$EXPR(INT),arguments)
  l : List(FC) := [assign(name,u)$FC,returns()$FC]
  makeRep l

coerce(u:Equation EXPR INT) : $ ==
  retractIfCan(lhs u)@Union(Kernel(EXPR INT),"failed") case "failed" =>
    error "left hand side is not a kernel"
  vList : List Symbol := variables lhs u
  #vList ^= #arguments =>
    error "Incorrect number of arguments"
  veList : List EXPR INT := [w::EXPR(INT) for w in vList]
  aeList : List EXPR INT := [w::EXPR(INT) for w in arguments]
  eList : List Equation EXPR INT :=
    [equation(w,v) for w in veList for v in aeList]
  (subst(rhs u,eList))::$

coerce(u:EXPR Float) : $ ==
  checkVariables(variables(u)$EXPR(Float),arguments)
  l : List(FC) := [assign(name,u)$FC,returns()$FC]
  makeRep l

coerce(u:Equation EXPR Float) : $ ==
  retractIfCan(lhs u)@Union(Kernel(EXPR Float),"failed") case "failed" =>
    error "left hand side is not a kernel"
  vList : List Symbol := variables lhs u
  #vList ^= #arguments =>
    error "Incorrect number of arguments"
  veList : List EXPR Float := [w::EXPR(Float) for w in vList]
  aeList : List EXPR Float := [w::EXPR(Float) for w in arguments]
  eList : List Equation EXPR Float :=
    [equation(w,v) for w in veList for v in aeList]
  (subst(rhs u,eList))::$

coerce(u:EXPR Complex Float) : $ ==
  checkVariables(variables(u)$EXPR(Complex Float),arguments)
  l : List(FC) := [assign(name,u)$FC,returns()$FC]
  makeRep l

coerce(u:Equation EXPR CMPX Float) : $ ==
  retractIfCan(lhs u)@Union(Kernel EXPR CMPX Float,"failed") case "failed"=>
    error "left hand side is not a kernel"

```

```

vList : List Symbol := variables lhs u
#vList ^= #arguments =>
  error "Incorrect number of arguments"
veList : List EXPR CMPX Float := [w::EXPR(CMPX Float) for w in vList]
aeList : List EXPR CMPX Float := [w::EXPR(CMPX Float) for w in arguments]
eList : List Equation EXPR CMPX Float :=
  [equation(w,v) for w in veList for v in aeList]
(subst(rhs u,eList))::$

```

---

— FORTRAN.dotabb —

```

"FORTRAN" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FORTRAN"]
"COMPCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=COMPCAT"]
"FORTRAN" -> "COMPCAT"

```

## 7.19 domain FST FortranScalarType

---

— FortranScalarType.input —

```

)set break resume
)sys rm -f FortranScalarType.output
)spool FortranScalarType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FortranScalarType
--R FortranScalarType is a domain constructor
--R Abbreviation for FortranScalarType is FST
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FST
--R
--R----- Operations -----
--R ?=? : (% , %) -> Boolean          character? : % -> Boolean
--R coerce : % -> SExpression         coerce : % -> Symbol
--R coerce : Symbol -> %              coerce : String -> %
--R coerce : % -> OutputForm          complex? : % -> Boolean
--R double? : % -> Boolean            doubleComplex? : % -> Boolean
--R integer? : % -> Boolean           logical? : % -> Boolean

```

```
--R real? : % -> Boolean
--R
--E 1
```

```
)spool
)lisp (bye)
```

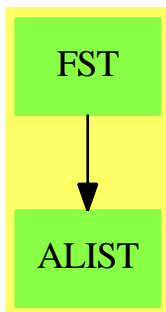
---

— FortranScalarType.help —

```
=====
FortranScalarType examples
=====
```

```
See Also:
o )show FortranScalarType
```

### 7.19.1 FortranScalarType (FST)



See

⇒ “FortranType” (FT) 7.21.1 on page 938  
 ⇒ “SymbolTable” (SYMTAB) 20.38.1 on page 2606  
 ⇒ “TheSymbolTable” (SYMS) 21.6.1 on page 2655

#### Exports:

character? coerce complex? double? doubleComplex? integer? logical? real? ?=?

---

— domain FST FortranScalarType —

```
)abbrev domain FST FortranScalarType
++ Author: Mike Dewar
```

```

++ Date Created:  October 1992
++ Date Last Updated:
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ Creates and manipulates objects which correspond to the
++ basic FORTRAN data types: REAL, INTEGER, COMPLEX, LOGICAL and CHARACTER

FortranScalarType() : exports == implementation where

exports == CoercibleTo OutputForm with
  coerce : String -> $
    ++ coerce(s) transforms the string s into an element of
    ++ FortranScalarType provided s is one of "real", "double precision",
    ++ "complex", "logical", "integer", "character", "REAL",
    ++ "COMPLEX", "LOGICAL", "INTEGER", "CHARACTER",
    ++ "DOUBLE PRECISION"
  coerce : Symbol -> $
    ++ coerce(s) transforms the symbol s into an element of
    ++ FortranScalarType provided s is one of real, complex, double precision,
    ++ logical, integer, character, REAL, COMPLEX, LOGICAL,
    ++ INTEGER, CHARACTER, DOUBLE PRECISION
  coerce : $ -> Symbol
    ++ coerce(x) returns the symbol associated with x
  coerce : $ -> SExpression
    ++ coerce(x) returns the s-expression associated with x
  real? : $ -> Boolean
    ++ real?(t) tests whether t is equivalent to the FORTRAN type REAL.
  double? : $ -> Boolean
    ++ double?(t) tests whether t is equivalent to the FORTRAN type
    ++ DOUBLE PRECISION
  integer? : $ -> Boolean
    ++ integer?(t) tests whether t is equivalent to the FORTRAN type INTEGER.
  complex? : $ -> Boolean
    ++ complex?(t) tests whether t is equivalent to the FORTRAN type COMPLEX.
  doubleComplex? : $ -> Boolean
    ++ doubleComplex?(t) tests whether t is equivalent to the (non-standard)
    ++ FORTRAN type DOUBLE COMPLEX.
  character? : $ -> Boolean
    ++ character?(t) tests whether t is equivalent to the FORTRAN type
    ++ CHARACTER.
  logical? : $ -> Boolean
    ++ logical?(t) tests whether t is equivalent to the FORTRAN type LOGICAL.
  "=" : ($,$) -> Boolean
    ++ x=y tests for equality

```

```

implementation == add

U == Union(RealThing:"real",
           IntegerThing:"integer",
           ComplexThing:"complex",
           CharacterThing:"character",
           LogicalThing:"logical",
           DoublePrecisionThing:"double precision",
           DoubleComplexThing:"double complex")
Rep := U

doubleSymbol : Symbol := "double precision"::Symbol
upperDoubleSymbol : Symbol := "DOUBLE PRECISION"::Symbol
doubleComplexSymbol : Symbol := "double complex"::Symbol
upperDoubleComplexSymbol : Symbol := "DOUBLE COMPLEX"::Symbol

u = v ==
  u case RealThing and v case RealThing => true
  u case IntegerThing and v case IntegerThing => true
  u case ComplexThing and v case ComplexThing => true
  u case LogicalThing and v case LogicalThing => true
  u case CharacterThing and v case CharacterThing => true
  u case DoublePrecisionThing and v case DoublePrecisionThing => true
  u case DoubleComplexThing and v case DoubleComplexThing => true
  false

coerce(t:$):OutputForm ==
  t case RealThing => coerce(REAL)$Symbol
  t case IntegerThing => coerce(INTEGER)$Symbol
  t case ComplexThing => coerce(COMPLEX)$Symbol
  t case CharacterThing => coerce(CHARACTER)$Symbol
  t case DoublePrecisionThing => coerce(upperDoubleSymbol)$Symbol
  t case DoubleComplexThing => coerce(upperDoubleComplexSymbol)$Symbol
  coerce(LOGICAL)$Symbol

coerce(t:$):SEExpression ==
  t case RealThing => convert(real::Symbol)@SEExpression
  t case IntegerThing => convert(integer::Symbol)@SEExpression
  t case ComplexThing => convert(complex::Symbol)@SEExpression
  t case CharacterThing => convert(character::Symbol)@SEExpression
  t case DoublePrecisionThing => convert(doubleSymbol)@SEExpression
  t case DoubleComplexThing => convert(doubleComplexSymbol)@SEExpression
  convert(logical::Symbol)@SEExpression

coerce(t:$):Symbol ==
  t case RealThing => real::Symbol
  t case IntegerThing => integer::Symbol
  t case ComplexThing => complex::Symbol
  t case CharacterThing => character::Symbol

```



```

t case DoublePrecisionThing => doubleSymbol
t case DoublePrecisionThing => doubleComplexSymbol
logical::Symbol

coerce(s:Symbol):$ ==
  s = real => ["real"]$Rep
  s = REAL => ["real"]$Rep
  s = integer => ["integer"]$Rep
  s = INTEGER => ["integer"]$Rep
  s = complex => ["complex"]$Rep
  s = COMPLEX => ["complex"]$Rep
  s = character => ["character"]$Rep
  s = CHARACTER => ["character"]$Rep
  s = logical => ["logical"]$Rep
  s = LOGICAL => ["logical"]$Rep
  s = doubleSymbol => ["double precision"]$Rep
  s = upperDoubleSymbol => ["double precision"]$Rep
  s = doubleComplexSymbol => ["double complex"]$Rep
  s = upperDoubleCComplexSymbol => ["double complex"]$Rep

coerce(s:String):$ ==
  s = "real" => ["real"]$Rep
  s = "integer" => ["integer"]$Rep
  s = "complex" => ["complex"]$Rep
  s = "character" => ["character"]$Rep
  s = "logical" => ["logical"]$Rep
  s = "double precision" => ["double precision"]$Rep
  s = "double complex" => ["double complex"]$Rep
  s = "REAL" => ["real"]$Rep
  s = "INTEGER" => ["integer"]$Rep
  s = "COMPLEX" => ["complex"]$Rep
  s = "CHARACTER" => ["character"]$Rep
  s = "LOGICAL" => ["logical"]$Rep
  s = "DOUBLE PRECISION" => ["double precision"]$Rep
  s = "DOUBLE COMPLEX" => ["double complex"]$Rep
  error concat([s," is invalid as a Fortran Type"])$String

real?(t:$):Boolean == t case RealThing

double?(t:$):Boolean == t case DoublePrecisionThing

logical?(t:$):Boolean == t case LogicalThing

integer?(t:$):Boolean == t case IntegerThing

character?(t:$):Boolean == t case CharacterThing

complex?(t:$):Boolean == t case ComplexThing

doubleComplex?(t:$):Boolean == t case DoubleComplexThing

```

---

— FST.dotabb —

```
"FST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FST"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"FST" -> "ALIST"
```

---

## 7.20 domain FTEM FortranTemplate

— FortranTemplate.input —

```
)set break resume
)sys rm -f FortranTemplate.output
)spool FortranTemplate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FortranTemplate
--R FortranTemplate is a domain constructor
--R Abbreviation for FortranTemplate is FTEM
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FTEM
--R
--R----- Operations -----
--R ?=? : (%,% ) -> Boolean          close! : % -> %
--R coerce : % -> OutputForm        flush : % -> Void
--R fortranCarriageReturn : () -> Void  fortranLiteral : String -> Void
--R hash : % -> SingleInteger        iomode : % -> String
--R latex : % -> String              name : % -> FileName
--R open : (FileName,String) -> %    open : FileName -> %
--R read! : % -> String              reopen! : (% ,String) -> %
--R write! : (% ,String) -> String    ?~=? : (% ,%) -> Boolean
--R fortranLiteralLine : String -> Void
--R processTemplate : FileName -> FileName
--R processTemplate : (FileName,FileName) -> FileName
--R
--E 1
```

```
)spool
)lisp (bye)
```

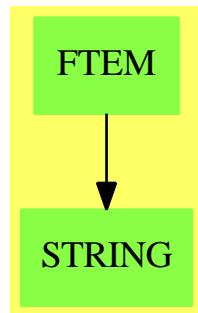
---

— FortranTemplate.help —

```
=====
FortranTemplate examples
=====
```

```
See Also:
o )show FortranTemplate
```

### 7.20.1 FortranTemplate (FTEM)



See

```
⇒ “Result” (RESULT) 19.9.1 on page 2260
⇒ “FortranCode” (FC) 7.16.1 on page 898
⇒ “FortranProgram” (FORTRAN) 7.18.1 on page 923
⇒ “ThreeDimensionalMatrix” (M3D) 21.7.1 on page 2661
⇒ “SimpleFortranProgram” (SFORT) 20.11.1 on page 2364
⇒ “Switch” (SWITCH) 20.36.1 on page 2588
⇒ “FortranExpression” (FEXPR) 7.17.1 on page 914
```

#### Exports:

|                 |        |                       |                |                    |
|-----------------|--------|-----------------------|----------------|--------------------|
| close!          | coerce | fortranCarriageReturn | fortranLiteral | fortranLiteralLine |
| hash            | iomode | latex                 | name           | open               |
| processTemplate | read!  | reopen!               | write!         | ?=?                |
| ?~=?            |        |                       |                |                    |

---

— domain FTEM FortranTemplate —

```

)abbrev domain FTEM FortranTemplate
++ Author: Mike Dewar
++ Date Created:  October 1992
++ Date Last Updated:
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ Code to manipulate Fortran templates

FortranTemplate() : specification == implementation where

specification == FileCategory(FileName, String) with

  processTemplate : (FileName, FileName) -> FileName
    ++ processTemplate(tp,fn) processes the template tp, writing the
    ++ result out to fn.
  processTemplate : (FileName) -> FileName
    ++ processTemplate(tp) processes the template tp, writing the
    ++ result to the current FORTRAN output stream.
  fortranLiterallLine : String -> Void
    ++ fortranLiterallLine(s) writes s to the current Fortran output stream,
    ++ followed by a carriage return
  fortranLiteral : String -> Void
    ++ fortranLiteral(s) writes s to the current Fortran output stream
  fortranCarriageReturn : () -> Void
    ++ fortranCarriageReturn() produces a carriage return on the current
    ++ Fortran output stream

implementation == TextFile add

import TemplateUtilities
import FortranOutputStackPackage

Rep := TextFile

fortranLiterallLine(s:String):Void ==
  PRINTEXP(s,_$fortranOutputStream$Lisp)$Lisp
  TERPRI(_$fortranOutputStream$Lisp)$Lisp

fortranLiteral(s:String):Void ==
  PRINTEXP(s,_$fortranOutputStream$Lisp)$Lisp

fortranCarriageReturn():Void ==
  TERPRI(_$fortranOutputStream$Lisp)$Lisp

```

```

writePassiveLine!(line:String):Void ==
-- We might want to be a bit clever here and look for new SubPrograms etc.
fortranLiteralLine line

processTemplate(tp:FileName, fn:FileName):FileName ==
  pushFortranOutputStack(fn)
  processTemplate(tp)
  popFortranOutputStack()
  fn

getLine(fp:TextFile):String ==
  line : String := stripCommentsAndBlanks readLine!(fp)
  while not empty?(line) and elt(line,maxIndex line) = char "__" repeat
    setelt(line,maxIndex line,char " ")
    line := concat(line, stripCommentsAndBlanks readLine!(fp))$String
  line

processTemplate(tp:FileName):FileName ==
  fp : TextFile := open(tp,"input")
  active : Boolean := true
  line : String
  endInput : Boolean := false
  while not (endInput or endOfFile? fp) repeat
    if active then
      line := getLine fp
      line = "endInput" => endInput := true
      if line = "beginVerbatim" then
        active := false
      else
        not empty? line => interpretString line
    else
      line := readLine!(fp)
      if line = "endVerbatim" then
        active := true
      else
        writePassiveLine! line
  close!(fp)
  if not active then
    error concat(["Missing 'endVerbatim' line in ",tp:String])$String
  string(_$fortranOutputFile$Lisp)::FileName

```

— FTEM.dotabb —

```

"FTEM" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FTEM"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"FTEM" -> "STRING"

```

## 7.21 domain FT FortranType

— FortranType.input —

```
)set break resume
)sys rm -f FortranType.output
)spool FortranType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FortranType
--R FortranType is a domain constructor
--R Abbreviation for FortranType is FT
--R This constructor is exposed in this frame.
--R Issue )edit NIL to see algebra source code for FT
--R
--R----- Operations -----
--R ?=? : (%,% ) -> Boolean          coerce : FortranScalarType -> %
--R coerce : % -> OutputForm          external? : % -> Boolean
--R fortranCharacter : () -> %         fortranComplex : () -> %
--R fortranDouble : () -> %           fortranDoubleComplex : () -> %
--R fortranInteger : () -> %          fortranLogical : () -> %
--R fortranReal : () -> %             hash : % -> SingleInteger
--R latex : % -> String               ?~=? : (%,% ) -> Boolean
--R construct : (Union(fst: FortranScalarType,void: void),List Polynomial Integer,Boolean) -> %
--R construct : (Union(fst: FortranScalarType,void: void),List Symbol,Boolean) -> %
--R dimensionsOf : % -> List Polynomial Integer
--R scalarTypeOf : % -> Union(fst: FortranScalarType,void: void)
--R
--E 1

)spool
)lisp (bye)
```

— FortranType.help —

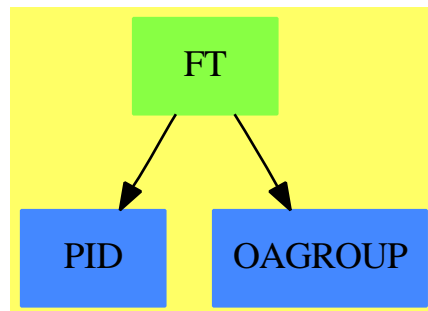
```
=====
FortranType examples
```

=====

See Also:

- o `)show FortranType`

### 7.21.1 FortranType (FT)



See

⇒ “FortranScalarType” (FST) 7.19.1 on page 929  
 ⇒ “SymbolTable” (SYMTAB) 20.38.1 on page 2606  
 ⇒ “TheSymbolTable” (SYMS) 21.6.1 on page 2655

#### Exports:

|                               |                             |                            |                                   |
|-------------------------------|-----------------------------|----------------------------|-----------------------------------|
| <code>coerce</code>           | <code>construct</code>      | <code>dimensionsOf</code>  | <code>external?</code>            |
| <code>fortranCharacter</code> | <code>fortranComplex</code> | <code>fortranDouble</code> | <code>fortranDoubleComplex</code> |
| <code>fortranInteger</code>   | <code>fortranLogical</code> | <code>fortranReal</code>   | <code>hash</code>                 |
| <code>latex</code>            | <code>scalarTypeOf</code>   | <code>?=?</code>           | <code>?~=?</code>                 |

— domain **FT** FortranType —

```

)abbrev domain FT FortranType
++ Author: Mike Dewar
++ Date Created: October 1992
++ Date Last Updated:
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ Creates and manipulates objects which correspond to FORTRAN
  
```

```

++ data types, including array dimensions.

FortranType() : exports == implementation where

FST    ==> FortranScalarType
FSTU   ==> Union(fst:FST,void:"void")

exports == SetCategory with
  coerce : $ -> OutputForm
    ++ coerce(x) provides a printable form for x
  coerce : FST -> $
    ++ coerce(t) creates an element from a scalar type
  scalarTypeOf : $ -> FSTU
    ++ scalarTypeOf(t) returns the FORTRAN data type of t
  dimensionsOf : $ -> List Polynomial Integer
    ++ dimensionsOf(t) returns the dimensions of t
  external? : $ -> Boolean
    ++ external?(u) returns true if u is declared to be EXTERNAL
  construct : (FSTU,List Symbol,Boolean) -> $
    ++ construct(type,dims) creates an element of FortranType
  construct : (FSTU,List Polynomial Integer,Boolean) -> $
    ++ construct(type,dims) creates an element of FortranType
  fortranReal : () -> $
    ++ fortranReal() returns REAL, an element of FortranType
  fortranDouble : () -> $
    ++ fortranDouble() returns DOUBLE PRECISION, an element of FortranType
  fortranInteger : () -> $
    ++ fortranInteger() returns INTEGER, an element of FortranType
  fortranLogical : () -> $
    ++ fortranLogical() returns LOGICAL, an element of FortranType
  fortranComplex : () -> $
    ++ fortranComplex() returns COMPLEX, an element of FortranType
  fortranDoubleComplex: () -> $
    ++ fortranDoubleComplex() returns DOUBLE COMPLEX, an element of
    ++ FortranType
  fortranCharacter : () -> $
    ++ fortranCharacter() returns CHARACTER, an element of FortranType

implementation == add

Dims == List Polynomial Integer
Rep := Record(type : FSTU, dimensions : Dims, external : Boolean)

coerce(a:$):OutputForm ==
  t : OutputForm
  if external?(a) then
    if scalarTypeOf(a) case void then
      t := "EXTERNAL":OutputForm
    else
      t := blankSeparate(["EXTERNAL":OutputForm,
```



```

                                coerce(scalarTypeOf a)$FSTU]$OutputForm
else
  t := coerce(scalarTypeOf a)$FSTU
  empty? dimensionsOf(a) => t
  sub(t,
    paren([u::OutputForm for u in dimensionsOf(a)])$OutputForm)$OutputForm

scalarTypeOf(u:$):FSTU ==
  u.type

dimensionsOf(u:$):Dims ==
  u.dimensions

external?(u:$):Boolean ==
  u.external

construct(t:FSTU, d:List Symbol, e:Boolean):$ ==
  e and not empty? d => error "EXTERNAL objects cannot have dimensions"
  not(e) and t case void => error "VOID objects must be EXTERNAL"
  construct(t,[l::Polynomial(Integer) for l in d],e)$Rep

construct(t:FSTU, d:List Polynomial Integer, e:Boolean):$ ==
  e and not empty? d => error "EXTERNAL objects cannot have dimensions"
  not(e) and t case void => error "VOID objects must be EXTERNAL"
  construct(t,d,e)$Rep

coerce(u:FST):$ ==
  construct([u]$FSTU,[],@List Polynomial Integer,false)

fortranReal():$ == ("real":FST):$

fortranDouble():$ == ("double precision":FST):$

fortranInteger():$ == ("integer":FST):$

fortranComplex():$ == ("complex":FST):$

fortranDoubleComplex():$ == ("double complex":FST):$

fortranCharacter():$ == ("character":FST):$

fortranLogical():$ == ("logical":FST):$

```

---

— FT.dotabb —

"FT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FT"]

```
"PID" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PID"]
"OAGROUP" [color="#4488FF",href="bookvol10.2.pdf#nameddest=OAGROUP"]
"FT" -> "PID"
"FT" -> "OAGROUP"
```

## 7.22 domain FCOMP FourierComponent

— FourierComponent.input —

```
)set break resume
)sys rm -f FourierComponent.output
)spool FourierComponent.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FourierComponent
--R FourierComponent E: OrderedSet is a domain constructor
--R Abbreviation for FourierComponent is FCOMP
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FCOMP
--R
--R----- Operations -----
--R ?<? : (% ,%) -> Boolean          ?<=? : (% ,%) -> Boolean
--R ?=? : (% ,%) -> Boolean          ?>? : (% ,%) -> Boolean
--R ?>=? : (% ,%) -> Boolean          argument : % -> E
--R coerce : % -> OutputForm          cos : E -> %
--R hash : % -> SingleInteger          latex : % -> String
--R max : (% ,%) -> %                  min : (% ,%) -> %
--R sin : E -> %                       sin? : % -> Boolean
--R ?~=? : (% ,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

— FourierComponent.help —

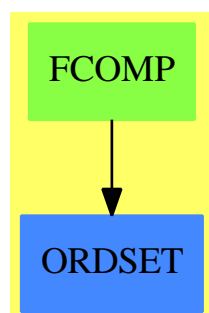
=====

FourierComponent examples

See Also:

o )show FourierComponent

### 7.22.1 FourierComponent (FCOMP)



See

⇒ “FourierSeries” (FSERIES) 7.23.1 on page 945

#### Exports:

|          |        |     |      |       |
|----------|--------|-----|------|-------|
| argument | coerce | cos | hash | latex |
| max      | min    | sin | sin? | ?~=?  |
| ?<?      | ?<=?   | ?=? | ?>?  | ?>=?  |

— domain FCOMP FourierComponent —

```

)abbrev domain FCOMP FourierComponent
++ Author: James Davenport
++ Date Created: 17 April 1992
++ Date Last Updated: 12 June 1992
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This domain creates kernels for use in Fourier series
  
```

```

FourierComponent(E:OrderedSet):
  OrderedSet with
    sin: E -> $
  
```

```

++ sin(x) makes a sin kernel for use in Fourier series
cos: E -> $
++ cos(x) makes a cos kernel for use in Fourier series
sin?: $ -> Boolean
++ sin?(x) returns true if term is a sin, otherwise false
argument: $ -> E
++ argument(x) returns the argument of a given sin/cos expressions
==
add
--representations
Rep:=Record(SinIfTrue:Boolean, arg:E)
e:E
x,y:$
sin e == [true,e]
cos e == [false,e]
sin? x == x.SinIfTrue
argument x == x.arg
coerce(x):OutputForm ==
  hconcat((if x.SinIfTrue then "sin" else "cos")::OutputForm,
    bracket((x.arg)::OutputForm))
x<y ==
  x.arg < y.arg => true
  y.arg < x.arg => false
  x.SinIfTrue => false
  y.SinIfTrue

```

---

— FCOMP.dotabb —

```

"FCOMP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FCOMP"]
"ORDSET" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ORDSET"]
"FCOMP" -> "ORDSET"

```

---

## 7.23 domain FSERIES FourierSeries

---

— FourierSeries.input —

```

)set break resume
)sys rm -f FourierSeries.output
)spool FourierSeries.output
)set message test on

```

```

)set message auto off
)clear all

--S 1 of 1
)show FourierSeries
--R FourierSeries(R: Join(CommutativeRing,Algebra Fraction Integer),E: Join(OrderedSet,AbelianGroup))
--R Abbreviation for FourierSeries is FSERIES
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FSERIES
--R
--R----- Operations -----
--R ?? : (R,%) -> %
--R ?? : (Integer,%) -> %
--R ?? : (PositiveInteger,%) -> %
--R ?? : (%,%) -> %
--R -? : % -> %
--R 1 : () -> %
--R ?? : (%,PositiveInteger) -> %
--R coerce : R -> %
--R coerce : % -> OutputForm
--R latex : % -> String
--R makeSin : (E,R) -> %
--R recip : % -> Union(%, "failed")
--R zero? : % -> Boolean
--R ?? : (NonNegativeInteger,%) -> %
--R ??? : (%,NonNegativeInteger) -> %
--R ?? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R subtractIfCan : (%,%) -> Union(%, "failed")
--R
--E 1

)spool
)lisp (bye)

```

---

— FourierSeries.help —

```

=====
FourierSeries examples
=====

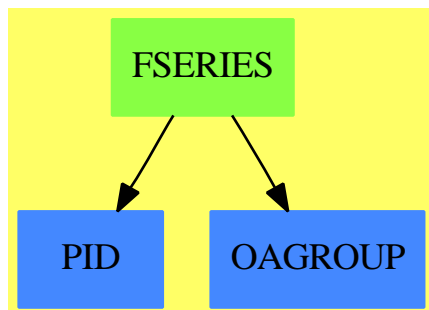
```

```

See Also:
o )show FourierSeries

```

## 7.23.1 FourierSeries (FSERIES)



See

⇒ “FourierComponent” (FCOMP) 7.22.1 on page 942

**Exports:**

|        |               |                |        |       |
|--------|---------------|----------------|--------|-------|
| 0      | 1             | characteristic | coerce | hash  |
| latex  | makeCos       | makeSin        | one?   | recip |
| sample | subtractIfCan | zero?          | ?~=?   | ?*?   |
| ?**?   | ?^?           | ?+?            | ?-?    | -?    |
| ?=?    |               |                |        |       |

— domain FSERIES FourierSeries —

```
)abbrev domain FSERIES FourierSeries
```

```
++ Author: James Davenport
```

```
++ Date Created: 17 April 1992
```

```
++ Date Last Updated:
```

```
++ Basic Functions:
```

```
++ Related Constructors:
```

```
++ Also See:
```

```
++ AMS Classifications:
```

```
++ Keywords:
```

```
++ References:
```

```
++ Description:
```

```
++ This domain converts terms into Fourier series
```

```
FourierSeries(R:Join(CommutativeRing,Algebra(Fraction Integer)),
  E:Join(OrderedSet,AbelianGroup)):
```

```
  Algebra(R) with
```

```
    if E has canonical and R has canonical then canonical
```

```
    coerce: R -> $
```

```
      ++ coerce(r) converts coefficients into Fourier Series
```

```
    coerce: FourierComponent(E) -> $
```

```
      ++ coerce(c) converts sin/cos terms into Fourier Series
```

```
    makeSin: (E,R) -> $
```

```
      ++ makeSin(e,r) makes a sin expression with given
```

```
      ++ argument and coefficient
```

```

makeCos: (E,R) -> $
  ++ makeCos(e,r) makes a sin expression with given
  ++argument and coefficient
== FreeModule(R,FourierComponent(E))
add
--representations
Term := Record(k:FourierComponent(E),c:R)
Rep  := List Term
multiply : (Term,Term) -> $
w,x1,x2:$
t1,t2:Term
n:NonNegativeInteger
z:Integer
e:FourierComponent(E)
a:E
r:R
1 == [[cos 0,1]]
coerce e ==
  sin? e and zero? argument e => 0
  if argument e < 0 then
    not sin? e => e:=cos(- argument e)
    return [[sin(- argument e),-1]]
  [[e,1]]
multiply(t1,t2) ==
  r:=(t1.c*t2.c)*(1/2)
  s1:=argument t1.k
  s2:=argument t2.k
  sum:=s1+s2
  diff:=s1-s2
  sin? t1.k =>
    sin? t2.k =>
      makeCos(diff,r) + makeCos(sum,-r)
      makeSin(sum,r) + makeSin(diff,r)
  sin? t2.k =>
    makeSin(sum,r) + makeSin(diff,r)
    makeCos(diff,r) + makeCos(sum,r)
x1*x2 ==
  null x1 => 0
  null x2 => 0
  +/[+/[multiply(t1,t2) for t2 in x2] for t1 in x1]
makeCos(a,r) ==
  a<0 => [[cos(-a),r]]
  [[cos a,r]]
makeSin(a,r) ==
  zero? a => []
  a<0 => [[sin(-a),-r]]
  [[sin a,r]]

```

---

— F SERIES.dotabb —

```
"F SERIES" [color="#88FF44",href="bookvol10.3.pdf#nameddest=F SERIES"]
"PID" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PID"]
"OAGROUP" [color="#4488FF",href="bookvol10.2.pdf#nameddest=OAGROUP"]
"F SERIES" -> "PID"
"F SERIES" -> "OAGROUP"
```

—————

## 7.24 domain FRAC Fraction

— Fraction.input —

```
)set break resume
)sys rm -f Fraction.output
)spool Fraction.output
)set message test on
)set message auto off
)clear all
--S 1 of 12
a := 11/12
--R
--R
--R      11
--R (1) --
--R      12
--R
--R                                          Type: Fraction Integer
--E 1

--S 2 of 12
b := 23/24
--R
--R
--R      23
--R (2) --
--R      24
--R
--R                                          Type: Fraction Integer
--E 2

--S 3 of 12
3 - a*b**2 + a + b/a
--R
--R
--R      313271
```



```

--R (3) -----
--R      76032
--R
--R                                          Type: Fraction Integer
--E 3

--S 4 of 12
numer(a)
--R
--R
--R (4)  11
--R
--R                                          Type: PositiveInteger
--E 4

--S 5 of 12
denom(b)
--R
--R
--R (5)  24
--R
--R                                          Type: PositiveInteger
--E 5

--S 6 of 12
r := (x**2 + 2*x + 1)/(x**2 - 2*x + 1)
--R
--R
--R      2
--R      x  + 2x + 1
--R (6) -----
--R      2
--R      x  - 2x + 1
--R
--R                                          Type: Fraction Polynomial Integer
--E 6

--S 7 of 12
factor(r)
--R
--R
--R      2
--R      x  + 2x + 1
--R (7) -----
--R      2
--R      x  - 2x + 1
--R
--R                                          Type: Factored Fraction Polynomial Integer
--E 7

--S 8 of 12
map(factor,r)
--R
--R
--R      2

```

```

--R      (x + 1)
--R (8)  -----
--R      2
--R      (x - 1)
--R
--R                                          Type: Fraction Factored Polynomial Integer
--E 8

```

```

--S 9 of 12
continuedFraction(7/12)
--R
--R
--R      1 |    1 |    1 |    1 |
--R (9)  +---+ + +---+ + +---+ + +---+
--R      | 1    | 1    | 2    | 2
--R
--R                                          Type: ContinuedFraction Integer
--E 9

```

```

--S 10 of 12
partialFraction(7,12)
--R
--R
--R      3  1
--R (10)  1 - -- + -
--R      2  3
--R      2
--R
--R                                          Type: PartialFraction Integer
--E 10

```

```

--S 11 of 12
g := 2/3 + 4/5*i
--R
--R
--R      2  4
--R (11)  - + - %i
--R      3  5
--R
--R                                          Type: Complex Fraction Integer
--E 11

```

```

--S 12 of 12
g :: FRAC COMPLEX INT
--R
--R
--R      10 + 12%i
--R (12)  -----
--R      15
--R
--R                                          Type: Fraction Complex Integer
--E 12

```

```

)spool
)lisp (bye)

```

---

— Fraction.help —

=====

Fraction examples

=====

The Fraction domain implements quotients. The elements must belong to a domain of category IntegralDomain: multiplication must be commutative and the product of two non-zero elements must not be zero. This allows you to make fractions of most things you would think of, but don't expect to create a fraction of two matrices! The abbreviation for Fraction is FRAC.

Use / to create a fraction.

```
a := 11/12
  11
  --
  12
                                Type: Fraction Integer
```

```
b := 23/24
  23
  --
  24
                                Type: Fraction Integer
```

The standard arithmetic operations are available.

```
3 - a*b**2 + a + b/a
313271
-----
76032
                                Type: Fraction Integer
```

Extract the numerator and denominator by using numer and denom, respectively.

```
numer(a)
  11
                                Type: PositiveInteger
```

```
denom(b)
  24
                                Type: PositiveInteger
```

Operations like max, min, negative?, positive? and zero?

are all available if they are provided for the numerators and denominators.

Don't expect a useful answer from factor, gcd or lcm if you apply them to fractions.

```
r := (x**2 + 2*x + 1)/(x**2 - 2*x + 1)
      2
      x  + 2x + 1
      -----
      2
      x  - 2x + 1
Type: Fraction Polynomial Integer
```

Since all non-zero fractions are invertible, these operations have trivial definitions.

```
factor(r)
      2
      x  + 2x + 1
      -----
      2
      x  - 2x + 1
Type: Factored Fraction Polynomial Integer
```

Use map to apply factor to the numerator and denominator, which is probably what you mean.

```
map(factor,r)
      2
      (x + 1)
      -----
      2
      (x - 1)
Type: Fraction Factored Polynomial Integer
```

Other forms of fractions are available. Use continuedFraction to create a continued fraction.

```
continuedFraction(7/12)
      1 |      1 |      1 |      1 |
      +---+ + +---+ + +---+ + +---+
      | 1      | 1      | 2      | 2
Type: ContinuedFraction Integer
```

Use partialFraction to create a partial fraction.

```
partialFraction(7,12)
      3      1
      1 - -- + -
```

$$\frac{2^2 + 3^2}{2}$$

Type: PartialFraction Integer

Use conversion to create alternative views of fractions with objects moved in and out of the numerator and denominator.

$$g := \frac{2}{3} + \frac{4}{5}i$$

Type: Complex Fraction Integer

$$g :: \frac{10 + 12i}{15}$$

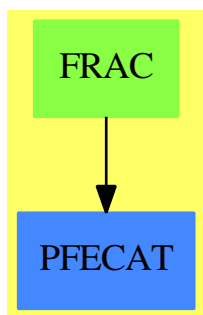
Type: Fraction Complex Integer

See Also:

- o )help ContinuedFraction
- o )help PartialFraction
- o )help Integer
- o )show Fraction

---

### 7.24.1 Fraction (FRAC)



See

⇒ “Localize” (LO) 13.13.1 on page 1486

⇒ “LocalAlgebra” (LA) 13.12.1 on page 1484

**Exports:**

|                               |                            |                    |
|-------------------------------|----------------------------|--------------------|
| 0                             | 1                          | abs                |
| associates?                   | characteristic             | charthRoot         |
| ceiling                       | coerce                     | conditionP         |
| convert                       | D                          | denom              |
| denominator                   | differentiate              | divide             |
| euclideanSize                 | eval                       | expressIdealMember |
| exquo                         | extendedEuclidean          | factor             |
| factorPolynomial              | factorSquareFreePolynomial | floor              |
| fractionPart                  | gcd                        | gcdPolynomial      |
| hash                          | init                       | inv                |
| latex                         | lcm                        | map                |
| max                           | min                        | multiEuclidean     |
| negative?                     | nextItem                   | numer              |
| numerator                     | OMwrite                    | one?               |
| patternMatch                  | positive?                  | prime?             |
| principalIdeal                | random                     | recip              |
| reducedSystem                 | retract                    | retractIfCan       |
| sample                        | sign                       | sizeLess?          |
| solveLinearPolynomialEquation | squareFree                 | squareFreePart     |
| squareFreePolynomial          | subtractIfCan              | unit?              |
| unitCanonical                 | unitNormal                 | wholePart          |
| zero?                         | ?*?                        | ?**?               |
| ?+?                           | ?-?                        | -?                 |
| ?/?                           | ?=?                        | ?^?                |
| ?~=?                          | ?<?                        | ?<=?               |
| ?>?                           | ?>=?                       | ?..?               |
| ?quo?                         | ?rem?                      |                    |

## — domain FRAC Fraction —

```

)abbrev domain FRAC Fraction
++ Author: Mark Botch
++ Date Created:
++ Date Last Updated: 12 February 1992
++ Basic Functions: Field, numer, denom
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords: fraction, localization
++ References:
++ Description:
++ Fraction takes an IntegralDomain S and produces
++ the domain of Fractions with numerators and denominators from S.
++ If S is also a GcdDomain, then gcd's between numerator and
++ denominator will be cancelled during all operations.

```

```

Fraction(S: IntegralDomain): QuotientFieldCategory S with

```

```

    if S has IntegerNumberSystem and S has OpenMath then OpenMath
    if S has canonical and S has GcdDomain and S has canonicalUnitNormal
    then canonical
        ++ \spad{canonical} means that equal elements are in fact identical.
== LocalAlgebra(S, S, S) add
Rep:= Record(num:S, den:S)
coerce(d:S):% == [d,1]
zero?(x:%) == zero? x.num

if S has GcdDomain and S has canonicalUnitNormal then
retract(x:%):S ==
--
    one?(x.den) => x.num
    ((x.den) = 1) => x.num
    error "Denominator not equal to 1"

retractIfCan(x:%):Union(S, "failed") ==
--
    one?(x.den) => x.num
    ((x.den) = 1) => x.num
    "failed"
else
retract(x:%):S ==
    (a:= x.num exquo x.den) case "failed" =>
        error "Denominator not equal to 1"
    a
retractIfCan(x:%):Union(S,"failed") == x.num exquo x.den

if S has EuclideanDomain then
wholePart x ==
--
    one?(x.den) => x.num
    ((x.den) = 1) => x.num
    x.num quo x.den

if S has IntegerNumberSystem then

floor x ==
--
    one?(x.den) => x.num
    ((x.den) = 1) => x.num
    x < 0 => -ceiling(-x)
    wholePart x

ceiling x ==
--
    one?(x.den) => x.num
    ((x.den) = 1) => x.num
    x < 0 => -floor(-x)
    1 + wholePart x

if S has OpenMath then
-- TODO: somewhere this file does something which redefines the division
-- operator. Doh!

```

```

writeOMFrac(dev: OpenMathDevice, x: %): Void ==
  MputApp(dev)
  MputSymbol(dev, "nums1", "rational")
  Mwrite(dev, x.num, false)
  Mwrite(dev, x.den, false)
  MputEndApp(dev)

OMwrite(x: %): String ==
  s: String := ""
  sp := OM_-STRINGTOSTRINGPTR(s)$Lisp
  dev: OpenMathDevice := OMopenString(sp pretend String, OMencodingXML)
  MputObject(dev)
  writeOMFrac(dev, x)
  MputEndObject(dev)
  Mclose(dev)
  s := OM_-STRINGPTRTOSTRING(sp)$Lisp pretend String
  s

OMwrite(x: %, wholeObj: Boolean): String ==
  s: String := ""
  sp := OM_-STRINGTOSTRINGPTR(s)$Lisp
  dev: OpenMathDevice := OMopenString(sp pretend String, OMencodingXML)
  if wholeObj then
    MputObject(dev)
    writeOMFrac(dev, x)
  if wholeObj then
    MputEndObject(dev)
  Mclose(dev)
  s := OM_-STRINGPTRTOSTRING(sp)$Lisp pretend String
  s

OMwrite(dev: OpenMathDevice, x: %): Void ==
  MputObject(dev)
  writeOMFrac(dev, x)
  MputEndObject(dev)

OMwrite(dev: OpenMathDevice, x: %, wholeObj: Boolean): Void ==
  if wholeObj then
    MputObject(dev)
    writeOMFrac(dev, x)
  if wholeObj then
    MputEndObject(dev)

if S has GcdDomain then
  cancelGcd: % -> S
  normalize: % -> %

normalize x ==
  zero?(x.num) => 0

```



```

--      one?(x.den) => x
      ((x.den) = 1) => x
      uca := unitNormal(x.den)
      zero?(x.den := uca.canonical) => error "division by zero"
      x.num := x.num * uca.associate
      x

recip x ==
  zero?(x.num) => "failed"
  normalize [x.den, x.num]

cancelGcd x ==
--      one?(x.den) => x.den
      ((x.den) = 1) => x.den
      d := gcd(x.num, x.den)
      xn := x.num exquo d
      xn case "failed" =>
        error "gcd not gcd in QF cancelGcd (numerator)"
      xd := x.den exquo d
      xd case "failed" =>
        error "gcd not gcd in QF cancelGcd (denominator)"
      x.num := xn :: S
      x.den := xd :: S
      d

nn:S / dd:S ==
  zero? dd => error "division by zero"
  cancelGcd(z := [nn, dd])
  normalize z

x + y ==
  zero? y => x
  zero? x => y
  z := [x.den, y.den]
  d := cancelGcd z
  g := [z.den * x.num + z.num * y.num, d]
  cancelGcd g
  g.den := g.den * z.num * z.den
  normalize g

-- We can not rely on the defaulting mechanism
-- to supply a definition for -, even though this
-- definition would do, for the following reasons:
-- 1) The user could have defined a subtraction
--    in Localize, which would not work for
--    QuotientField;
-- 2) even if he doesn't, the system currently
--    places a default definition in Localize,
--    which uses Localize's +, which does not
--    cancel gcDs

```

```

x - y ==
  zero? y => x
  z := [x.den, y.den]
  d := cancelGcd z
  g := [z.den * x.num - z.num * y.num, d]
  cancelGcd g
  g.den := g.den * z.num * z.den
  normalize g

x:% * y:% ==
  zero? x or zero? y => 0
--   one? x => y
  (x = 1) => y
--   one? y => x
  (y = 1) => x
  (x, y) := ([x.num, y.den], [y.num, x.den])
  cancelGcd x; cancelGcd y;
  normalize [x.num * y.num, x.den * y.den]

n:Integer * x:% ==
  y := [n::S, x.den]
  cancelGcd y
  normalize [x.num * y.num, y.den]

nn:S * x:% ==
  y := [nn, x.den]
  cancelGcd y
  normalize [x.num * y.num, y.den]

differentiate(x:%, deriv:S -> S) ==
  y := [deriv(x.den), x.den]
  d := cancelGcd(y)
  y.num := deriv(x.num) * y.den - x.num * y.num
  (d, y.den) := (y.den, d)
  cancelGcd y
  y.den := y.den * d * d
  normalize y

if S has canonicalUnitNormal then
  x = y == (x.num = y.num) and (x.den = y.den)
--x / dd == (cancelGcd (z:=[x.num,dd*x.den])); normalize z)

--   one? x == one? (x.num) and one? (x.den)
  one? x == ((x.num) = 1) and ((x.den) = 1)
  -- again assuming canonical nature of representation

else
  nn:S/dd:S == if zero? dd then error "division by zero" else [nn,dd]

recip x ==

```

```

    zero?(x.num) => "failed"
    [x.den, x.num]

if (S has RetractableTo Fraction Integer) then
  retract(x:~):Fraction(Integer) == retract(retract(x)@S)

  retractIfCan(x:~):Union(Fraction Integer, "failed") ==
    (u := retractIfCan(x)@Union(S, "failed")) case "failed" => "failed"
    retractIfCan(u::S)

else if (S has RetractableTo Integer) then
  retract(x:~):Fraction(Integer) ==
    retract(number x) / retract(denom x)

  retractIfCan(x:~):Union(Fraction Integer, "failed") ==
    (n := retractIfCan number x) case "failed" => "failed"
    (d := retractIfCan denom x) case "failed" => "failed"
    (n::Integer) / (d::Integer)

QFP ==> SparseUnivariatePolynomial %
DP ==> SparseUnivariatePolynomial S
import UnivariatePolynomialCategoryFunctions2(%,QFP,S,DP)
import UnivariatePolynomialCategoryFunctions2(S,DP,%,QFP)

if S has GcdDomain then
  gcdPolynomial(pp,qq) ==
    zero? pp => qq
    zero? qq => pp
    zero? degree pp or zero? degree qq => 1
    denpp:="lcm"/[denom u for u in coefficients pp]
    ppD:DP:=map(x+>retract(x*denpp),pp)
    denqq:="lcm"/[denom u for u in coefficients qq]
    qqD:DP:=map(x+>retract(x*denqq),qq)
    g:=gcdPolynomial(ppD,qqD)
    zero? degree g => 1
--    one? (lc:=leadingCoefficient g) => map(#1::%,g)
    ((lc:=leadingCoefficient g) = 1) => map(x+>x::%,g)
    map(x+>x/lc,g)

if (S has PolynomialFactorizationExplicit) then
  -- we'll let the solveLinearPolynomialEquations operator
  -- default from Field
  pp,qq: QFP
  lpp: List QFP
  import Factored SparseUnivariatePolynomial %
  if S has CharacteristicNonZero then
    if S has canonicalUnitNormal and S has GcdDomain then
      charthRoot x ==
        n:= charthRoot x.num
        n case "failed" => "failed"

```

```

    d:=charthRoot x.den
    d case "failed" => "failed"
    n/d
else
    charthRoot x ==
    -- to find x = p-th root of n/d
    -- observe that xd is p-th root of n*d**(p-1)
    ans:=charthRoot(x.num *
        (x.den)**(characteristic()$%-1)::NonNegativeInteger)
    ans case "failed" => "failed"
    ans / x.den
clear: List % -> List S
clear l ==
    d:="lcm"/[x.den for x in l]
    [ x.num * (d exquo x.den)::S for x in l]
mat: Matrix %
conditionP mat ==
    matD: Matrix S
    matD:= matrix [ clear l for l in listOfLists mat ]
    ansD := conditionP matD
    ansD case "failed" => "failed"
    ansDD:=ansD :: Vector(S)
    [ ansDD(i)::% for i in 1..#ansDD]$Vector(%)

factorPolynomial(pp) ==
    zero? pp => 0
    denpp:="lcm"/[denom u for u in coefficients pp]
    ppD:DP:=map(x+>retract(x*denpp),pp)
    ff:=factorPolynomial ppD
    den1:%=denpp::%
    lfact>List Record(flg:Union("nil", "sqfr", "irred", "prime"),
        fctr:QFP, xpnt:Integer)
    lfact:= [[w.flg,
        if leadingCoefficient w.fctr =1 then
            map(x+>x::%,w.fctr)
        else (lc:=(leadingCoefficient w.fctr)::%;
            den1:=den1/lc**w.xpnt;
            map(x+>x::%/lc,w.fctr)),
        w.xpnt] for w in factorList ff]
    makeFR(map(x+>x::%/den1,unit(ff)),lfact)
factorSquareFreePolynomial(pp) ==
    zero? pp => 0
    degree pp = 0 => makeFR(pp,empty())
    lcpp:=leadingCoefficient pp
    pp:=pp/lcpp
    denpp:="lcm"/[denom u for u in coefficients pp]
    ppD:DP:=map(x+>retract(x*denpp),pp)
    ff:=factorSquareFreePolynomial ppD
    den1:%=denpp::%/lcpp
    lfact>List Record(flg:Union("nil", "sqfr", "irred", "prime"),

```

```

                                fctr:QFP, xpnt:Integer)
lfact:= [[w.flg,
          if leadingCoefficient w.fctr =1 then
            map(x+>x:=%,w.fctr)
          else (lc:=(leadingCoefficient w.fctr)::%;
               den1:=den1/lc**w.xpnt;
               map(x+>x:=%/lc,w.fctr)),
          w.xpnt] for w in factorList ff]
makeFR(map(x+>x:=%/den1,unit(ff)),lfact)

```

---

— FRAC.dotabb —

```

"FRAC" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FRAC"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"FRAC" -> "PFECAT"

```

---

## 7.25 domain FRIDEAL FractionalIdeal

— FractionalIdeal.input —

```

)set break resume
)sys rm -f FractionalIdeal.output
)spool FractionalIdeal.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FractionalIdeal
--R FractionalIdeal(R: EuclideanDomain,F: QuotientFieldCategory R,UP: UnivariatePolynomialCa
--R Abbreviation for FractionalIdeal is FRIDEAL
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FRIDEAL
--R
--R----- Operations -----
--R ?? : (% ,%) -> %
--R ??? : (% ,PositiveInteger) -> %
--R ?? : (% ,%) -> Boolean
--R ?? : (% ,Integer) -> %
--R basis : % -> Vector A
--R ??? : (% ,Integer) -> %
--R ?? : (% ,%) -> %
--R 1 : () -> %
--R ?? : (% ,PositiveInteger) -> %
--R coerce : % -> OutputForm

```

```

--R commutator : (% ,%) -> %
--R denom : % -> R
--R ideal : Vector A -> %
--R latex : % -> String
--R norm : % -> F
--R one? : % -> Boolean
--R sample : () -> %
--R ***? : (% ,NonNegativeInteger) -> %
--R ?? : (% ,NonNegativeInteger) -> %
--R randomLC : (NonNegativeInteger,Vector A) -> A
--R
--R conjugate : (% ,%) -> %
--R hash : % -> SingleInteger
--R inv : % -> %
--R minimize : % -> %
--R numer : % -> Vector A
--R recip : % -> Union(%, "failed")
--R ~=? : (% ,%) -> Boolean
--E 1

)spool
)lisp (bye)

```

---

— FractionalIdeal.help —

```

=====
FractionalIdeal examples
=====

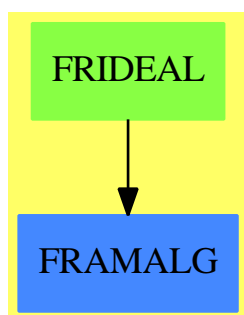
```

```

See Also:
o )show FractionalIdeal

```

### 7.25.1 FractionalIdeal (FRIDEAL)



See

- ⇒ “FramedModule” (FRMOD) 7.26.1 on page 967
- ⇒ “HyperellipticFiniteDivisor” (HELLFDIV) 9.11.1 on page 1149
- ⇒ “FiniteDivisor” (FDIV) 7.4.1 on page 781

**Exports:**

|          |        |        |            |           |
|----------|--------|--------|------------|-----------|
| 1        | basis  | coerce | commutator | conjugate |
| denom    | hash   | ideal  | inv        | latex     |
| minimize | norm   | numer  | one?       | randomLC  |
| recip    | sample | ?^=?   | ?**?       | ?^?       |
| ?*?      | ?**?   | ?/?    | ?=?        | ?^?       |

— domain **FRIDEAL** FractionalIdeal —

```

)abbrev domain FRIDEAL FractionalIdeal
++ Author: Manuel Bronstein
++ Date Created: 27 Jan 1989
++ Date Last Updated: 30 July 1993
++ Keywords: ideal, algebra, module.
++ Examples: )r FRIDEAL INPUT
++ Description:
++ Fractional ideals in a framed algebra.

FractionalIdeal(R, F, UP, A): Exports == Implementation where
  R : EuclideanDomain
  F : QuotientFieldCategory R
  UP: UnivariatePolynomialCategory F
  A : Join(FramedAlgebra(F, UP), RetractableTo F)

VF ==> Vector F
VA ==> Vector A
UPA ==> SparseUnivariatePolynomial A
QF ==> Fraction UP

Exports ==> Group with
  ideal   : VA -> %
    ++ ideal([f1,...,fn]) returns the ideal \spad{(f1,...,fn)}.
  basis   : % -> VA
    ++ basis((f1,...,fn)) returns the vector \spad{[f1,...,fn]}.
  norm    : % -> F
    ++ norm(I) returns the norm of the ideal I.
  numer   : % -> VA
    ++ numer(1/d * (f1,...,fn)) = the vector \spad{[f1,...,fn]}.
  denom   : % -> R
    ++ denom(1/d * (f1,...,fn)) returns d.
  minimize: % -> %
    ++ minimize(I) returns a reduced set of generators for \spad{I}.
  randomLC: (NonNegativeInteger, VA) -> A
    ++ randomLC(n,x) should be local but conditional.

Implementation ==> add
  import CommonDenominator(R, F, VF)
  import MatrixCommonDenominator(UP, QF)
  import InnerCommonDenominator(R, F, List R, List F)
  import MatrixCategoryFunctions2(F, Vector F, Vector F, Matrix F,

```

```

UP, Vector UP, Vector UP, Matrix UP)
import MatrixCategoryFunctions2(UP, Vector UP, Vector UP,
    Matrix UP, F, Vector F, Vector F, Matrix F)
import MatrixCategoryFunctions2(UP, Vector UP, Vector UP,
    Matrix UP, QF, Vector QF, Vector QF, Matrix QF)

Rep := Record(num:VA, den:R)

poly      : % -> UPA
invrep    : Matrix F -> A
upmat     : (A, NonNegativeInteger) -> Matrix UP
summat    : % -> Matrix UP
num20     : VA -> OutputForm
agcd      : List A -> R
vgcd      : VF -> R
mkIdeal   : (VA, R) -> %
intIdeal  : (List A, R) -> %
ret?      : VA -> Boolean
tryRange  : (NonNegativeInteger, VA, R, %) -> Union(%, "failed")

1          == [[1]$VA, 1]
numer i    == i.num
denom i    == i.den
mkIdeal(v, d) == [v, d]
invrep m    == represents(transpose(m) * coordinates(1$A))
upmat(x, i) == map(s +-> monomial(s, i)$UP, regularRepresentation x)
ret? v      == any?(s+>retractIfCan(s)@Union(F,"failed") case F, v)
x = y      == denom(x) = denom(y) and numer(x) = numer(y)
agcd l     == reduce("gcd", [vgcd coordinates a for a in l]$List(R), 0)

norm i ==
    ("gcd"/[retract(u)@R for u in coefficients determinant summat i])
    / denom(i) ** rank()$A

tryRange(range, nm, nrm, i) ==
    for j in 0..10 repeat
        a := randomLC(10 * range, nm)
        unit? gcd((retract(norm a)@R exquo nrm)::R, nrm) =>
            return intIdeal([nrm:F::A, a], denom i)
    "failed"

summat i ==
    m := minIndex(v := numer i)
    reduce("+",
        [upmat(qelt(v, j + m), j) for j in 0..#v-1]$List(Matrix UP))

inv i ==
    m := inverse(map(s+>s::QF, summat i))::Matrix(QF)
    cd := splitDenominator(denom(i)::F::UP::QF * m)
    cd2 := splitDenominator coefficients(cd.den)

```



```

invd:= cd2.den / reduce("gcd", cd2.num)
d := reduce("max", [degree p for p in parts(cd.num)])
ideal
  [invd * invrep map(s+>coefficient(s, j), cd.num) for j in 0..d]$VA

ideal v ==
  d := reduce("lcm", [commonDenominator coordinates qelt(v, i)
    for i in minIndex v .. maxIndex v]$List(R))
  intIdeal([d::F * qelt(v, i) for i in minIndex v .. maxIndex v], d)

intIdeal(l, d) ==
  lr := empty()$List(R)
  nr := empty()$List(A)
  for x in removeDuplicates l repeat
    if (u := retractIfCan(x)@Union(F, "failed")) case F
    then lr := concat(retract(u::F)@R, lr)
    else nr := concat(x, nr)
  r := reduce("gcd", lr, 0)
  g := agcd nr
  a := (r quo (b := gcd(gcd(d, r), g))):F::A
  d := d quo b
  r ^= 0 and ((g exquo r) case R) => mkIdeal([a], d)
  invb := inv(b::F)
  va:VA := [invb * m for m in nr]
  zero? a => mkIdeal(va, d)
  mkIdeal(concat(a, va), d)

vgcd v ==
  reduce("gcd",
    [retract(v.i)@R for i in minIndex v .. maxIndex v]$List(R))

poly i ==
  m := minIndex(v := numer i)
  +/[monomial(qelt(v, i + m), i) for i in 0..#v-1]

i1 * i2 ==
  intIdeal(coefficients(poly i1 * poly i2), denom i1 * denom i2)

i:$ ** m:Integer ==
  m < 0 => inv(i) ** (-m)
  n := m::NonNegativeInteger
  v := numer i
  intIdeal([qelt(v, j) ** n for j in minIndex v .. maxIndex v],
    denom(i) ** n)

num20 v ==
  paren [qelt(v, i)::OutputForm
    for i in minIndex v .. maxIndex v]$List(OutputForm)

basis i ==

```

```

v := numer i
d := inv(denom(i)::F)
[d * qelt(v, j) for j in minIndex v .. maxIndex v]

coerce(i:$):OutputForm ==
  nm := num20 numer i
--  one? denom i => nm
  (denom i = 1) => nm
  (1::Integer::OutputForm) / (denom(i)::OutputForm) * nm

if F has Finite then
  randomLC(m, v) ==
    +/[random()$F * qelt(v, j) for j in minIndex v .. maxIndex v]
else
  randomLC(m, v) ==
    +/[(random()$Integer rem m::Integer) * qelt(v, j)
      for j in minIndex v .. maxIndex v]

minimize i ==
  n := (#(nm := numer i))
--  one?(n) or (n < 3 and ret? nm) => i
  (n = 1) or (n < 3 and ret? nm) => i
  nrm := retract(norm mkIdeal(nm, 1))@R
  for range in 1..5 repeat
    (u := tryRange(range, nm, nrm, i)) case $ => return(u:$)
  i

```

---

— FRIDEAL.dotabb —

```

"FRIDEAL" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FRIDEAL"]
"FRAMALG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FRAMALG"]
"FRIDEAL" -> "FRAMALG"

```

## 7.26 domain FRMOD FramedModule

---

— FramedModule.input —

```

)set break resume
)sys rm -f FramedModule.output
)spool FramedModule.output

```

```

)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FramedModule
--R FramedModule(R: EuclideanDomain,F: QuotientFieldCategory R,UP: UnivariatePolynomialCategory R)
--R Abbreviation for FramedModule is FRMOD
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FRMOD
--R
--R----- Operations -----
--R ?? : (%,% ) -> %
--R ?? : (%,% ) -> Boolean
--R ?? : (% ,PositiveInteger) -> %
--R coerce : % -> OutputForm
--R latex : % -> String
--R norm : % -> F
--R recip : % -> Union(%, "failed")
--R ~=? : (%,% ) -> Boolean
--R ??? : (% ,NonNegativeInteger) -> %
--R ?? : (% ,NonNegativeInteger) -> %
--R module : FractionalIdeal(R,F,UP,A) -> % if A has RETRACT F
--R
--E 1

)spool
)lisp (bye)

```

---

— FramedModule.help —

```

=====
FramedModule examples
=====

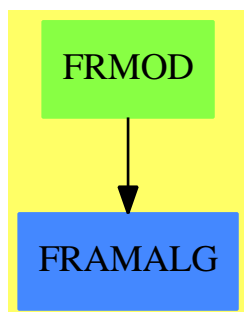
```

```

See Also:
o )show FramedModule

```

## 7.26.1 FramedModule (FRMOD)



See

⇒ “FractionalIdeal” (FRIDEAL) 7.25.1 on page 961

⇒ “HyperellipticFiniteDivisor” (HELLFDIV) 9.11.1 on page 1149

⇒ “FiniteDivisor” (FDIV) 7.4.1 on page 781

**Exports:**

|        |       |        |       |        |
|--------|-------|--------|-------|--------|
| 1      | basis | coerce | hash  | latex  |
| module | norm  | one?   | recip | sample |
| ?~=?   | ?**?  | ?^?    | ?*?   | ?**?   |
| ?=?    |       |        |       |        |

— domain FRMOD FramedModule —

```
)abbrev domain FRMOD FramedModule
++ Author: Manuel Bronstein
++ Date Created: 27 Jan 1989
++ Date Last Updated: 24 Jul 1990
++ Keywords: ideal, algebra, module.
++ Description:
++ Module representation of fractional ideals.
```

```
FramedModule(R, F, UP, A, ibasis): Exports == Implementation where
R      : EuclideanDomain
F      : QuotientFieldCategory R
UP     : UnivariatePolynomialCategory F
A      : FramedAlgebra(F, UP)
ibasis: Vector A
```

```
VR ==> Vector R
VF ==> Vector F
VA ==> Vector A
M  ==> Matrix F
```

```
Exports ==> Monoid with
basis : % -> VA
++ basis((f1,...,fn)) = the vector \spad{[f1,...,fn]}.
```

```

norm : % -> F
  ++ norm(f) returns the norm of the module f.
module: VA -> %
  ++ module([f1,...,fn]) = the module generated by \spad{(f1,...,fn)}
  ++ over R.
if A has RetractableTo F then
  module: FractionalIdeal(R, F, UP, A) -> %
    ++ module(I) returns I viewed has a module over R.

Implementation ==> add
import MatrixCommonDenominator(R, F)
import ModularHermitianRowReduction(R)

Rep := VA

iflag?:Reference(Boolean) := ref true
wflag?:Reference(Boolean) := ref true
imat := new(#ibasis, #ibasis, 0)$M
wmat := new(#ibasis, #ibasis, 0)$M

rowdiv      : (VR, R) -> VF
vectProd    : (VA, VA) -> VA
wmatrix     : VA -> M
W2A         : VF -> A
intmat      : () -> M
invintmat   : () -> M
getintmat   : () -> Boolean
getinvintmat: () -> Boolean

1 == ibasis
module(v:VA) == v
basis m == m pretend VA
rowdiv(r, f) == [r.i / f for i in minIndex r..maxIndex r]
coerce(m:%):OutputForm == coerce(basis m)$VA
W2A v == represents(v * intmat())
wmatrix v == coordinates(v) * invintmat()

getinvintmat() ==
  m := inverse(intmat()):M
  for i in minRowIndex m .. maxRowIndex m repeat
    for j in minColIndex m .. maxColIndex m repeat
      imat(i, j) := qelt(m, i, j)
  false

getintmat() ==
  m := coordinates ibasis
  for i in minRowIndex m .. maxRowIndex m repeat
    for j in minColIndex m .. maxColIndex m repeat
      wmat(i, j) := qelt(m, i, j)
  false

```

```

invintmat() ==
  if iflag?() then iflag?() := getinvintmat()
  imat

intmat() ==
  if wflag?() then wflag?() := getintmat()
  wmat

vectProd(v1, v2) ==
  k := minIndex(v := new(#v1 * #v2, 0)$VA)
  for i in minIndex v1 .. maxIndex v1 repeat
    for j in minIndex v2 .. maxIndex v2 repeat
      qsetelt_!(v, k, qelt(v1, i) * qelt(v2, j))
      k := k + 1
  v pretend VA

norm m ==
  #(basis m) ^= #ibasis => error "Module not of rank n"
  determinant(coordinates(basis m) * invintmat())

m1 * m2 ==
  m := rowEch((cd := splitDenominator wmatrix(
    vectProd(basis m1, basis m2))).num)
  module [u for i in minRowIndex m .. maxRowIndex m |
    (u := W2A rowdiv(row(m, i), cd.den)) ^= 0]$VA

if A has RetractableTo F then
  module(i:FractionalIdeal(R, F, UP, A)) ==
    module(basis i) * module(ibasis)

```

---

— FRMOD.dotabb —

```

"FRMOD" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FRMOD"]
"FRAMALG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FRAMALG"]
"FRMOD" -> "FRAMALG"

```

---

## 7.27 domain FAGROUP FreeAbelianGroup

— FreeAbelianGroup.input —

```

)set break resume
)sys rm -f FreeAbelianGroup.output
)spool FreeAbelianGroup.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FreeAbelianGroup
--R FreeAbelianGroup S: SetCategory is a domain constructor
--R Abbreviation for FreeAbelianGroup is FAGROUP
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FAGROUP
--R
--R----- Operations -----
--R ?? : (Integer,S) -> %           ?? : (%,Integer) -> %
--R ?? : (Integer,%) -> %          ?? : (PositiveInteger,%) -> %
--R +? : (S,%) -> %                +? : (%,% ) -> %
--R -? : (%,% ) -> %              -? : % -> %
--R ?? : (%,% ) -> Boolean         0 : () -> %
--R coefficient : (S,%) -> Integer  coerce : S -> %
--R coerce : % -> OutputForm        hash : % -> SingleInteger
--R latex : % -> String              mapGen : ((S -> S),%) -> %
--R nthCoef : (%,Integer) -> Integer nthFactor : (%,Integer) -> S
--R retract : % -> S                 sample : () -> %
--R size : % -> NonNegativeInteger  zero? : % -> Boolean
--R ~=? : (%,% ) -> Boolean
--R ?? : (NonNegativeInteger,%) -> %
--R <? : (%,% ) -> Boolean if S has ORDSET
--R <=? : (%,% ) -> Boolean if S has ORDSET
--R >? : (%,% ) -> Boolean if S has ORDSET
--R >=? : (%,% ) -> Boolean if S has ORDSET
--R highCommonTerms : (%,% ) -> % if Integer has OAMON
--R mapCoef : ((Integer -> Integer),%) -> %
--R max : (%,% ) -> % if S has ORDSET
--R min : (%,% ) -> % if S has ORDSET
--R retractIfCan : % -> Union(S,"failed")
--R subtractIfCan : (%,% ) -> Union(%, "failed")
--R terms : % -> List Record(gen: S,exp: Integer)
--R
--E 1

)spool
)lisp (bye)

```

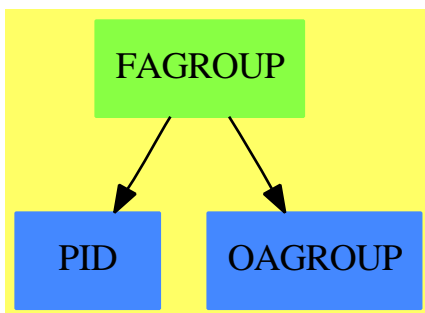
```
=====
FreeAbelianGroup examples
=====
```

See Also:

```
o )show FreeAbelianGroup
```

—————

### 7.27.1 FreeAbelianGroup (FAGROUP)



See

⇒ “ListMonoidOps” (LMOPS) 13.10.1 on page 1473

⇒ “FreeMonoid” (FMONOID) 7.32.1 on page 987

⇒ “FreeGroup” (FGROUP) 7.29.1 on page 976

⇒ “InnerFreeAbelianMonoid” (IFAMON) 10.22.1 on page 1250

⇒ “FreeAbelianMonoid” (FAMONOID) 7.28.1 on page 974

#### Exports:

|         |               |         |              |                 |
|---------|---------------|---------|--------------|-----------------|
| 0       | coefficient   | coerce  | hash         | highCommonTerms |
| latex   | mapCoef       | mapGen  | max          | min             |
| nthCoef | nthFactor     | retract | retractIfCan | sample          |
| size    | subtractIfCan | terms   | zero?        | ?~=?            |
| ?*?     | ?<?           | ?<=?    | ?>?          | ?>=?            |
| ?+?     | ?-?           | -?      | ?=?          |                 |

— domain FAGROUP FreeAbelianGroup —

```
)abbrev domain FAGROUP FreeAbelianGroup
++ Free abelian group on any set of generators
++ Author: Manuel Bronstein
++ Date Created: November 1989
++ Date Last Updated: 6 June 1991
++ Description:
++ The free abelian group on a set S is the monoid of finite sums of
```



```

++ the form \spad{reduce(+,[ni * si])} where the si's are in S, and the ni's
++ are integers. The operation is commutative.

FreeAbelianGroup(S:SetCategory): Exports == Implementation where
  Exports ==> Join(AbelianGroup, Module Integer,
    FreeAbelianMonoidCategory(S, Integer)) with
    if S has OrderedSet then OrderedSet

Implementation ==> InnerFreeAbelianMonoid(S, Integer, 1) add
  - f == mapCoef("-", f)

if S has OrderedSet then
  inmax: List Record(gen: S, exp: Integer) -> Record(gen: S, exp:Integer)

  inmax l ==
    mx := first l
    for t in rest l repeat
      if mx.gen < t.gen then mx := t
    mx

-- lexicographic order
a < b ==
  zero? a =>
    zero? b => false
    0 < (inmax terms b).exp
  ta := inmax terms a
  zero? b => ta.exp < 0
  tb := inmax terms b
  ta.gen < tb.gen => 0 < tb.exp
  tb.gen < ta.gen => ta.exp < 0
  ta.exp < tb.exp => true
  tb.exp < ta.exp => false
  lc := ta.exp * ta.gen
  (a - lc) < (b - lc)



---



— FAGROUP.dotabb —

"FAGROUP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FAGROUP"]
"PID" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PID"]
"OAGROUP" [color="#4488FF",href="bookvol10.2.pdf#nameddest=OAGROUP"]
"FAGROUP" -> "PID"
"FAGROUP" -> "OAGROUP"



---



```

## 7.28 domain FAMONOID FreeAbelianMonoid

— FreeAbelianMonoid.input —

```
)set break resume
)sys rm -f FreeAbelianMonoid.output
)spool FreeAbelianMonoid.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FreeAbelianMonoid
--R FreeAbelianMonoid S: SetCategory is a domain constructor
--R Abbreviation for FreeAbelianMonoid is FAMONOID
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FAMONOID
--R
--R----- Operations -----
--R ??? : (NonNegativeInteger,S) -> %      ??? : (PositiveInteger,%) -> %
--R ?+? : (S,%) -> %                        ?+? : (%,%) -> %
--R ?? : (%,%) -> Boolean                    0 : () -> %
--R coerce : S -> %                          coerce : % -> OutputForm
--R hash : % -> SingleInteger                 latex : % -> String
--R mapGen : ((S -> S),%) -> %                 nthFactor : (%,Integer) -> S
--R retract : % -> S                          sample : () -> %
--R size : % -> NonNegativeInteger             zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ??? : (NonNegativeInteger,%) -> %
--R coefficient : (S,%) -> NonNegativeInteger
--R highCommonTerms : (%,%) -> % if NonNegativeInteger has OAMON
--R mapCoef : ((NonNegativeInteger -> NonNegativeInteger),%) -> %
--R nthCoef : (%,Integer) -> NonNegativeInteger
--R retractIfCan : % -> Union(S,"failed")
--R subtractIfCan : (%,%) -> Union(%, "failed")
--R terms : % -> List Record(gen: S,exp: NonNegativeInteger)
--R
--E 1

)spool
)lisp (bye)
```

— FreeAbelianMonoid.help —

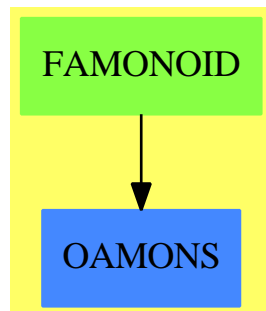
=====

FreeAbelianMonoid examples

See Also:

o )show FreeAbelianMonoid

### 7.28.1 FreeAbelianMonoid (FAMONOID)



See

⇒ “ListMonoidOps” (LMOPS) 13.10.1 on page 1473  
 ⇒ “FreeMonoid” (FMONOID) 7.32.1 on page 987  
 ⇒ “FreeGroup” (FGROUP) 7.29.1 on page 976  
 ⇒ “InnerFreeAbelianMonoid” (IFAMON) 10.22.1 on page 1250  
 ⇒ “FreeAbelianGroup” (FAGROUP) 7.27.1 on page 971

#### Exports:

|         |              |        |         |                 |
|---------|--------------|--------|---------|-----------------|
| 0       | coefficient  | coerce | hash    | highCommonTerms |
| latex   | mapCoef      | mapGen | nthCoef | nthFactor       |
| retract | retractIfCan | sample | size    | subtractIfCan   |
| terms   | zero?        | ?~=?   | ?*?     | ?+?             |
| ?=?     |              |        |         |                 |

— domain FAMONOID FreeAbelianMonoid —

```

)abbrev domain FAMONOID FreeAbelianMonoid
++ Free abelian monoid on any set of generators
++ Author: Manuel Bronstein
++ Date Created: November 1989
++ Date Last Updated: 6 June 1991
++ Description:
++ The free abelian monoid on a set S is the monoid of finite sums of
++ the form \spad{reduce(+,[ni * si])} where the si's are in S, and the ni's
++ are non-negative integers. The operation is commutative.
  
```

```
FreeAbelianMonoid(S: SetCategory):
  FreeAbelianMonoidCategory(S, NonNegativeInteger)
  == InnerFreeAbelianMonoid(S, NonNegativeInteger, 1)
```

---

— FAMONOID.dotabb —

```
"FAMONOID" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FAMONOID"]
"OAMONS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=OAMONS"]
"FAMONOID" -> "OAMONS"
```

---

## 7.29 domain FGROUPE FreeGroup

— FreeGroup.input —

```
)set break resume
)sys rm -f FreeGroup.output
)spool FreeGroup.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FreeGroup
--R FreeGroup S: SetCategory is a domain constructor
--R Abbreviation for FreeGroup is FGROUPE
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FGROUPE
--R
--R----- Operations -----
--R ?? : (S,S) -> %
--R ?? : (S,S) -> %
--R ??? : (S,Integer) -> %
--R ?? : (S,Integer) -> %
--R ?? : (S,Integer) -> %
--R 1 : () -> %
--R ?? : (S,PositiveInteger) -> %
--R coerce : % -> OutputForm
--R conjugate : (S,S) -> %
--R inv : % -> %
--R mapGen : ((S -> S),S) -> %
--R nthFactor : (S,Integer) -> S
--R ?? : (S,S) -> %
--R ??? : (S,Integer) -> %
--R ??? : (S,PositiveInteger) -> %
--R ?? : (S,S) -> Boolean
--R ?? : (S,Integer) -> %
--R coerce : S -> %
--R commutator : (S,S) -> %
--R hash : % -> SingleInteger
--R latex : % -> String
--R nthExpon : (S,Integer) -> Integer
--R one? : % -> Boolean
```

```

--R recip : % -> Union(%, "failed")      retract : % -> S
--R sample : () -> %                     size : % -> NonNegativeInteger
--R ?~=? : (%, %) -> Boolean
--R ??? : (%, NonNegativeInteger) -> %
--R ?? : (%, NonNegativeInteger) -> %
--R factors : % -> List Record(gen: S, exp: Integer)
--R mapExpon : ((Integer -> Integer), %) -> %
--R retractIfCan : % -> Union(S, "failed")
--R
--E 1

)spool
)lisp (bye)

```

---

— FreeGroup.help —

=====

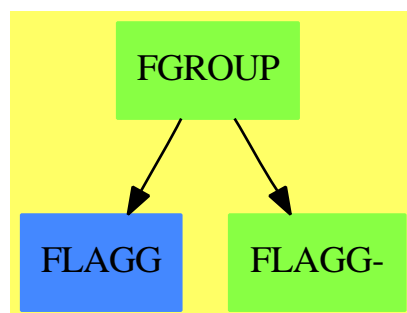
FreeGroup examples

=====

See Also:

- o )show FreeGroup

### 7.29.1 FreeGroup (FGROUP)



See

- ⇒ “ListMonoidOps” (LMOPS) 13.10.1 on page 1473
- ⇒ “FreeMonoid” (FMONOID) 7.32.1 on page 987
- ⇒ “InnerFreeAbelianMonoid” (IFAMON) 10.22.1 on page 1250
- ⇒ “FreeAbelianMonoid” (FAMONOID) 7.28.1 on page 974
- ⇒ “FreeAbelianGroup” (FAGROUP) 7.27.1 on page 971

**Exports:**

|              |           |            |           |         |
|--------------|-----------|------------|-----------|---------|
| 1            | coerce    | commutator | conjugate | factors |
| hash         | inv       | latex      | mapExpon  | mapGen  |
| nthExpon     | nthFactor | one?       | recip     | retract |
| retractIfCan | sample    | size       | ?~=?      | ?**?    |
| ?^?          | ?*?       | ?/?        | ?=?       |         |

## — domain FGROUPE FreeGroup —

```

)abbrev domain FGROUPE FreeGroup
++ Free group on any set of generators
++ Author: Stephen M. Watt
++ Date Created: ???
++ Date Last Updated: 6 June 1991
++ Description:
++ The free group on a set S is the group of finite products of
++ the form \spad{reduce(*,[si ** ni])} where the si's are in S, and the ni's
++ are integers. The multiplication is not commutative.

FreeGroup(S: SetCategory): Join(Group, RetractableTo S) with
  "(": (S, $) -> $
    ++ s * x returns the product of x by s on the left.
  "(": ($, S) -> $
    ++ x * s returns the product of x by s on the right.
  "**": (S, Integer) -> $
    ++ s ** n returns the product of s by itself n times.
  size : $ -> NonNegativeInteger
    ++ size(x) returns the number of monomials in x.
  nthExpon : ($, Integer) -> Integer
    ++ nthExpon(x, n) returns the exponent of the nth monomial of x.
  nthFactor : ($, Integer) -> S
    ++ nthFactor(x, n) returns the factor of the nth monomial of x.
  mapExpon : (Integer -> Integer, $) -> $
    ++ mapExpon(f, a1\^e1 ... an\^en) returns
    ++ \spad{a1\^f(e1) ... an\^f(en)}.
  mapGen : (S -> S, $) -> $
    ++ mapGen(f, a1\^e1 ... an\^en) returns
    ++ \spad{f(a1)\^e1 ... f(an)\^en}.
  factors : $ -> List Record(gen: S, exp: Integer)
    ++ factors(a1\^e1,...,an\^en) returns \spad{[[a1, e1],...,[an, en]]}.
== ListMonoidOps(S, Integer, 1) add
  Rep := ListMonoidOps(S, Integer, 1)

  1 == makeUnit()
  one? f == empty? listOfMonoms f
  s:S ** n:Integer == makeTerm(s, n)
  f:$ * s:S == rightMult(f, s)
  s:S * f:$ == leftMult(s, f)
  inv f == reverse_! mapExpon("-", f)
  factors f == copy listOfMonoms f

```

```

mapExpon(f, x)          == mapExpon(f, x)$Rep
mapGen(f, x)            == mapGen(f, x)$Rep
coerce(f:$):OutputForm == outputForm(f, "*", "**", 1)

f:$ * g:$ ==
  one? f => g
  one? g => f
  r := reverse listOfMonoms f
  q := copy listOfMonoms g
  while not empty? r and not empty? q and r.first.gen = q.first.gen
    and r.first.exp = -q.first.exp repeat
    r := rest r
    q := rest q
  empty? r => makeMulti q
  empty? q => makeMulti reverse_! r
  r.first.gen = q.first.gen =>
    setlast_!(h := reverse_! r,
              [q.first.gen, q.first.exp + r.first.exp])
  makeMulti concat_!(h, rest q)
  makeMulti concat_!(reverse_! r, q)

```

---

— FGROUP.dotabb —

```

"FGROUP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FGROUP"]
"FLAGG"  [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"FLAGG-" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FLAGG"]
"FGROUP" -> "FLAGG"
"FGROUP" -> "FLAGG-"

```

## 7.30 domain FM FreeModule

---

— FreeModule.input —

```

)set break resume
)sys rm -f FreeModule.output
)spool FreeModule.output
)set message test on
)set message auto off
)clear all

```

```

--S 1 of 1
)show FreeModule
--R FreeModule(R: Ring,S: OrderedSet) is a domain constructor
--R Abbreviation for FreeModule is FM
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FM
--R
--R----- Operations -----
--R ???: (R,% ) -> %          ??? : (R,% ) -> %
--R ??? : (Integer,% ) -> %    ??? : (PositiveInteger,% ) -> %
--R ??+? : (%,% ) -> %        ?-? : (%,% ) -> %
--R -? : % -> %               ?=? : (%,% ) -> Boolean
--R 0 : () -> %               coerce : % -> OutputForm
--R hash : % -> SingleInteger latex : % -> String
--R leadingCoefficient : % -> R leadingSupport : % -> S
--R map : ((R -> R),% ) -> %    monomial : (R,S) -> %
--R reductum : % -> %          sample : () -> %
--R zero? : % -> Boolean       ?~=? : (%,% ) -> Boolean
--R ??? : (NonNegativeInteger,% ) -> %
--R subtractIfCan : (%,% ) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)

```

---

— FreeModule.help —

```

=====
FreeModule examples
=====

```

See Also:

```

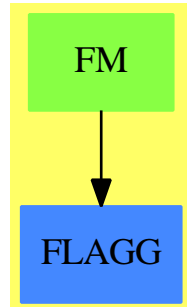
o )show FreeModule

```

---



### 7.30.1 FreeModule (FM)



See

⇒ “PolynomialRing” (PR) 17.27.1 on page 2052

⇒ “SparseUnivariatePolynomial” (SUP) 20.18.1 on page 2425

⇒ “UnivariatePolynomial” (UP) 22.4.1 on page 2784

#### Exports:

|                |        |          |          |                    |
|----------------|--------|----------|----------|--------------------|
| 0              | coerce | hash     | latex    | leadingCoefficient |
| leadingSupport | map    | monomial | reductum | sample             |
| subtractIfCan  | zero?  | ?~=?     | ?*?      | ?+?                |
| ?-?            | -?     | ?=?      |          |                    |

— domain FM FreeModule —

```
)abbrev domain FM FreeModule
++ Author: Dave Barton, James Davenport, Barry Trager
++ Date Created:
++ Date Last Updated:
++ Basic Functions: BiModule(R,R)
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A bi-module is a free module
++ over a ring with generators indexed by an ordered set.
++ Each element can be expressed as a finite linear combination of
++ generators. Only non-zero terms are stored.
```

```
FreeModule(R:Ring,S:OrderedSet):
  Join(BiModule(R,R),IndexedDirectProductCategory(R,S)) with
  if R has CommutativeRing then Module(R)
== IndexedDirectProductAbelianGroup(R,S) add
--representations
Term:= Record(k:S,c:R)
Rep:= List Term
```

```

--declarations
  x,y: %
  r: R
  n: Integer
  f: R -> R
  s: S
--define
  if R has EntireRing then
    r * x ==
      zero? r => 0
--      one? r => x
      (r = 1) => x
      --map(r*#1,x)
      [[u.k,r*u.c] for u in x ]
  else
    r * x ==
      zero? r => 0
--      one? r => x
      (r = 1) => x
      --map(r*#1,x)
      [[u.k,a] for u in x | (a:=r*u.c) ^= 0$R]
  if R has EntireRing then
    x * r ==
--      zero? r => 0
      one? r => x
      (r = 1) => x
      --map(r*#1,x)
      [[u.k,u.c*r] for u in x ]
  else
    x * r ==
--      zero? r => 0
      one? r => x
      (r = 1) => x
      --map(r*#1,x)
      [[u.k,a] for u in x | (a:=u.c*r) ^= 0$R]

coerce(x) : OutputForm ==
  null x => (0$R) :: OutputForm
  le : List OutputForm := nil
  for rec in reverse x repeat
    rec.c = 1 => le := cons(rec.k :: OutputForm, le)
    le := cons(rec.c :: OutputForm * rec.k :: OutputForm, le)
  reduce("+",le)

```

---

— FM.dotabb —

```

"FM" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FM"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"FM" -> "FLAGG"

```

## 7.31 domain FM1 FreeModule1

— FreeModule1.input —

```

)set break resume
)sys rm -f FreeModule1.output
)spool FreeModule1.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FreeModule1
--R FreeModule1(R: Ring,S: OrderedSet) is a domain constructor
--R Abbreviation for FreeModule1 is FM1
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FM1
--R
--R----- Operations -----
--R ???: (S,R) -> %               ??? : (R,S) -> %
--R ??? : (% ,R) -> %             ??? : (R,% ) -> %
--R ??? : (Integer,% ) -> %       ??? : (PositiveInteger,% ) -> %
--R ?+? : (% ,%) -> %             ?-? : (% ,%) -> %
--R -? : % -> %                   ?? : (% ,%) -> Boolean
--R 0 : () -> %                   coefficient : (% ,S) -> R
--R coefficients : % -> List R     coerce : S -> %
--R coerce : % -> OutputForm       hash : % -> SingleInteger
--R latex : % -> String            leadingCoefficient : % -> R
--R leadingMonomial : % -> S        map : ((R -> R),%) -> %
--R monom : (S,R) -> %             monomial? : % -> Boolean
--R monomials : % -> List %        reductum : % -> %
--R retract : % -> S               sample : () -> %
--R zero? : % -> Boolean           ?~=? : (% ,%) -> Boolean
--R ??? : (NonNegativeInteger,% ) -> %
--R leadingTerm : % -> Record(k: S,c: R)
--R listOfTerms : % -> List Record(k: S,c: R)
--R numberOfMonomials : % -> NonNegativeInteger
--R retractIfCan : % -> Union(S,"failed")
--R subtractIfCan : (% ,%) -> Union(% ,"failed")
--R

```

```
--E 1
```

```
)spool
)lisp (bye)
```

```
_____
```

```
— FreeModule1.help —
```

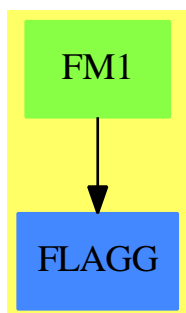
```
=====
FreeModule1 examples
=====
```

```
See Also:
```

```
o )show FreeModule1
```

```
_____
```

### 7.31.1 FreeModule1 (FM1)



#### Exports:

|          |                    |                 |             |                   |
|----------|--------------------|-----------------|-------------|-------------------|
| 0        | coefficient        | coefficients    | coerce      | hash              |
| latex    | leadingCoefficient | leadingMonomial | leadingTerm | listOfTerms       |
| map      | monom              | monomial?       | monomials   | numberOfMonomials |
| reductum | retract            | retractIfCan    | sample      | subtractIfCan     |
| zero?    | ?~=?               | ?*?             | ?+?         | ?-?               |
| -?       | ?=?                |                 |             |                   |

```
— domain FM1 FreeModule1 —
```

```
)abbrev domain FM1 FreeModule1
++ Author: Michel Petitot petitot@lifl.fr
++ Date Created: 91
++ Date Last Updated: 7 Juillet 92
```

```

++ Fix History: compilation v 2.1 le 13 dec 98
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This domain implements linear combinations
++ of elements from the domain \spad{S} with coefficients
++ in the domain \spad{R} where \spad{S} is an ordered set
++ and \spad{R} is a ring (which may be non-commutative).
++ This domain is used by domains of non-commutative algebra such as:
++ XDistributedPolynomial, XRecursivePolynomial.

FreeModule1(R:Ring,S:OrderedSet): FMcat == FMdef where
  EX ==> OutputForm
  TERM ==> Record(k:S,c:R)

FMcat == FreeModuleCat(R,S) with
  "":(S,R) -> %
  ++ \spad{s*r} returns the product \spad{r*s}
  ++ used by \spadtype{XRecursivePolynomial}
FMdef == FreeModule(R,S) add
  -- representation
  Rep := List TERM

  -- declarations
  lt: List TERM
  x : %
  r : R
  s : S

  -- define
  numberOfMonomials p ==
    # (p::Rep)

  listOfTerms(x) == x::List TERM

  leadingTerm x == x.first
  leadingMonomial x == x.first.k
  coefficients x == [t.c for t in x]
  monomials x == [ monom (t.k, t.c) for t in x]

  retractIfCan x ==
    numberOfMonomials(x) ^= 1 => "failed"
    x.first.c = 1 => x.first.k
    "failed"

  coerce(s:S):% == [[s,1$R]]

```

```

retract x ==
  (rr := retractIfCan x) case "failed" => error "FM1.retract impossible"
  rr :: S

if R has noZeroDivisors then
  r * x ==
    r = 0 => 0
    [[u.k,r * u.c]$TERM for u in x]
  x * r ==
    r = 0 => 0
    [[u.k,u.c * r]$TERM for u in x]
else
  r * x ==
    r = 0 => 0
    [[u.k,a] for u in x | not (a:=r*u.c)= 0$R]
  x * r ==
    r = 0 => 0
    [[u.k,a] for u in x | not (a:=u.c*r)= 0$R]

r * s ==
  r = 0 => 0
  [[s,r]$TERM]

s * r ==
  r = 0 => 0
  [[s,r]$TERM]

monom(b,r):% == [[b,r]$TERM]

outTerm(r:R, s:S):EX ==
  r=1 => s::EX
  r::EX * s::EX

coerce(a:%):EX ==
  empty? a => (0$R)::EX
  reduce(_+, reverse_! [outTerm(t.c, t.k) for t in a])$List(EX)

coefficient(x,s) ==
  null x => 0$R
  x.first.k > s => coefficient(rest x,s)
  x.first.k = s => x.first.c
  0$R

```

---

— FM1.dotabb —

"FM1" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FM1"]

```
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"FM1" -> "FLAGG"
```

## 7.32 domain FMONOID FreeMonoid

— FreeMonoid.input —

```
)set break resume
)sys rm -f FreeMonoid.output
)spool FreeMonoid.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FreeMonoid
--R FreeMonoid S: SetCategory is a domain constructor
--R Abbreviation for FreeMonoid is FMONOID
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FMONOID
--R
--R----- Operations -----
--R ?? : (S,S) -> %               ?? : (S,S) -> %
--R ?? : (S,S) -> %               ***? : (S,PositiveInteger) -> %
--R ?? : (S,S) -> Boolean         1 : () -> %
--R ?? : (S,PositiveInteger) -> % coerce : S -> %
--R coerce : % -> OutputForm      hash : % -> SingleInteger
--R hclf : (S,S) -> %             hcrf : (S,S) -> %
--R latex : % -> String           mapGen : ((S -> S),S) -> %
--R nthFactor : (S,Integer) -> S   one? : % -> Boolean
--R recip : % -> Union(%, "failed") retract : % -> S
--R sample : () -> %              size : % -> NonNegativeInteger
--R ~=? : (S,S) -> Boolean
--R ***? : (S,NonNegativeInteger) -> %
--R ***? : (S,NonNegativeInteger) -> %
--R ?<? : (S,S) -> Boolean if S has ORDSET
--R ?<=? : (S,S) -> Boolean if S has ORDSET
--R ?>? : (S,S) -> Boolean if S has ORDSET
--R ?>=? : (S,S) -> Boolean if S has ORDSET
--R ?? : (S,NonNegativeInteger) -> %
--R divide : (S,S) -> Union(Record(lm: %,rm: %), "failed")
--R factors : % -> List Record(gen: S,exp: NonNegativeInteger)
--R lquo : (S,S) -> Union(%, "failed")
--R mapExpon : ((NonNegativeInteger -> NonNegativeInteger),S) -> %
```

```

--R max : (%,%) -> % if S has ORDSET
--R min : (%,%) -> % if S has ORDSET
--R nthExpon : (%,Integer) -> NonNegativeInteger
--R overlap : (%,%) -> Record(lm: %,mm: %,rm: %)
--R retractIfCan : % -> Union(S,"failed")
--R rquo : (%,%) -> Union(%,"failed")
--R
--E 1

```

```

)spool
)lisp (bye)

```

---

— FreeMonoid.help —

```

=====
FreeMonoid examples
=====

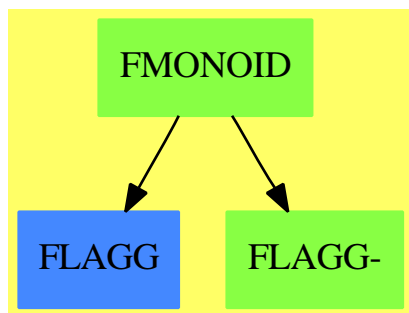
```

```

See Also:
o )show FreeMonoid

```

### 7.32.1 FreeMonoid (FMONOID)



See

- ⇒ “ListMonoidOps” (LMOPS) 13.10.1 on page 1473
- ⇒ “FreeGroup” (FGROUP) 7.29.1 on page 976
- ⇒ “InnerFreeAbelianMonoid” (IFAMON) 10.22.1 on page 1250
- ⇒ “FreeAbelianMonoid” (FAMONOID) 7.28.1 on page 974
- ⇒ “FreeAbelianGroup” (FAGROUP) 7.27.1 on page 971



**Exports:**

|              |         |        |          |           |
|--------------|---------|--------|----------|-----------|
| 1            | coerce  | divide | factors  | hash      |
| hclf         | hcrf    | latex  | lquo     | mapExpon  |
| mapGen       | max     | min    | nthExpon | nthFactor |
| one?         | overlap | recip  | rquo     | retract   |
| retractIfCan | sample  | size   | ?^=?     | ?**?      |
| ?<?          | ?<=?    | ?>?    | ?>=?     | ?^?       |
| ?*?          | ?=?     |        |          |           |

## — domain FMONOID FreeMonoid —

```

)abbrev domain FMONOID FreeMonoid
++ Free monoid on any set of generators
++ Author: Stephen M. Watt
++ Date Created: ???
++ Date Last Updated: 6 June 1991
++ Description:
++ The free monoid on a set S is the monoid of finite products of
++ the form \spad{reduce(*,[si ** ni])} where the si's are in S, and the ni's
++ are nonnegative integers. The multiplication is not commutative.

FreeMonoid(S: SetCategory): FMcategory == FMdefinition where
  NNI ==> NonNegativeInteger
  REC ==> Record(gen: S, exp: NonNegativeInteger)
  Ex ==> OutputForm

FMcategory ==> Join(Monoid, RetractableTo S) with
  "(": (S, $) -> $
  ++ s * x returns the product of x by s on the left.
  "(": ($, S) -> $
  ++ x * s returns the product of x by s on the right.
  "**": (S, NonNegativeInteger) -> $
  ++ s ** n returns the product of s by itself n times.
  hclf: ($, $) -> $
  ++ hclf(x, y) returns the highest common left factor of x and y,
  ++ i.e. the largest d such that \spad{x = d a} and \spad{y = d b}.
  hcrf: ($, $) -> $
  ++ hcrf(x, y) returns the highest common right factor of x and y,
  ++ i.e. the largest d such that \spad{x = a d} and \spad{y = b d}.
  lquo: ($, $) -> Union($, "failed")
  ++ lquo(x, y) returns the exact left quotient of x by y i.e.
  ++ q such that \spad{x = y * q},
  ++ "failed" if x is not of the form \spad{y * q}.
  rquo: ($, $) -> Union($, "failed")
  ++ rquo(x, y) returns the exact right quotient of x by y i.e.
  ++ q such that \spad{x = q * y},
  ++ "failed" if x is not of the form \spad{q * y}.
  divide: ($, $) -> Union(Record(lm: $, rm: $), "failed")
  ++ divide(x, y) returns the left and right exact quotients of

```

```

    ++ x by y, i.e. \spad{[l, r]} such that \spad{x = l * y * r},
    ++ "failed" if x is not of the form \spad{l * y * r}.
overlap: ($, $) -> Record(lm: $, mm: $, rm: $)
    ++ overlap(x, y) returns \spad{[l, m, r]} such that
    ++ \spad{x = l * m}, \spad{y = m * r} and l and r have no overlap,
    ++ i.e. \spad{overlap(l, r) = [l, 1, r]}.
size      : $ -> NNI
    ++ size(x) returns the number of monomials in x.
factors   : $ -> List Record(gen: S, exp: NonNegativeInteger)
    ++ factors(a1\^e1,...,an\^en) returns \spad{[[a1, e1],...,[an, en]]}.
nthExpon  : ($, Integer) -> NonNegativeInteger
    ++ nthExpon(x, n) returns the exponent of the nth monomial of x.
nthFactor : ($, Integer) -> S
    ++ nthFactor(x, n) returns the factor of the nth monomial of x.
mapExpon  : (NNI -> NNI, $) -> $
    ++ mapExpon(f, a1\^e1 ... an\^en) returns \spad{a1\^f(e1) ... an\^f(en)}.
mapGen    : (S -> S, $) -> $
    ++ mapGen(f, a1\^e1 ... an\^en) returns \spad{f(a1)\^e1 ... f(an)\^en}.
if S has OrderedSet then OrderedSet

FMdefinition ==> ListMonoidOps(S, NonNegativeInteger, 1) add
Rep := ListMonoidOps(S, NonNegativeInteger, 1)

1          == makeUnit()
one? f     == empty? listOfMonoms f
coerce(f:$): Ex == outputForm(f, "*", "**", 1)
hcrf(f, g) == reverse_! hclf(reverse f, reverse g)
f:$ * s:$  == rightMult(f, s)
s:$ * f:$  == leftMult(s, f)
factors f  == copy listOfMonoms f
mapExpon(f, x) == mapExpon(f, x)$Rep
mapGen(f, x)  == mapGen(f, x)$Rep
s:$ ** n:NonNegativeInteger == makeTerm(s, n)

f:$ * g:$ ==
--      one? f => g
      (f = 1) => g
--      one? g => f
      (g = 1) => f
      lg := listOfMonoms g
      ls := last(lf := listOfMonoms f)
      ls.gen = lg.first.gen =>
          setlast_!(h := copy lf, [lg.first.gen, lg.first.exp+ls.exp])
          makeMulti concat(h, rest lg)
      makeMulti concat(lf, lg)

overlap(la, ar) ==
--      one? la or one? ar => [la, 1, ar]
      (la = 1) or (ar = 1) => [la, 1, ar]
      lla := la0 := listOfMonoms la

```

```

lar := listOfMonoms ar
l:List(REC) := empty()
while not empty? lla repeat
  if lla.first.gen = lar.first.gen then
    if lla.first.exp < lar.first.exp and empty? rest lla then
      return [makeMulti l,
              makeTerm(la.first.gen, lla.first.exp),
              makeMulti concat([lar.first.gen,
                                (lar.first.exp - lla.first.exp)::NNI],
                                rest lar)]

    if lla.first.exp >= lar.first.exp then
      if (ru:= lquo(makeMulti rest lar,
                    makeMulti rest lla)) case $ then
        if lla.first.exp > lar.first.exp then
          l := concat_!(l, [lla.first.gen,
                            (lla.first.exp - lar.first.exp)::NNI])
          m := concat([lla.first.gen, lar.first.exp],
                      rest lla)

          else m := lla
          return [makeMulti l, makeMulti m, ru::$]
        l := concat_!(l, lla.first)
        lla := rest lla
      [makeMulti la0, 1, makeMulti lar]

divide(lar, a) ==
--
  one? a => [lar, 1]
  (a = 1) => [lar, 1]
  Na : Integer := #(la := listOfMonoms a)
  Nlar : Integer := #(llar := listOfMonoms lar)
  l:List(REC) := empty()
  while Na <= Nlar repeat
    if llar.first.gen = la.first.gen and
      llar.first.exp >= la.first.exp then
      -- Can match a portion of this lar factor.
      -- Now match tail.
      (q:=lquo(makeMulti rest llar,makeMulti rest la))case $ =>
        if llar.first.exp > la.first.exp then
          l := concat_!(l, [la.first.gen,
                            (llar.first.exp - la.first.exp)::NNI])
          return [makeMulti l, q::$]
        l := concat_!(l, first llar)
        llar := rest llar
        Nlar := Nlar - 1
      "failed"

hclf(f, g) ==
  h:List(REC) := empty()
  for f0 in listOfMonoms f for g0 in listOfMonoms g repeat
    f0.gen ^= g0.gen => return makeMulti h
    h := concat_!(h, [f0.gen, min(f0.exp, g0.exp)])

```

```

    f0.exp ^= g0.exp => return makeMulti h
  makeMulti h

lquo(aq, a) ==
  size a > #(laq := copy listOfMonoms aq) => "failed"
  for a0 in listOfMonoms a repeat
    a0.gen ^= laq.first.gen or a0.exp > laq.first.exp =>
      return "failed"
    if a0.exp = laq.first.exp then laq := rest laq
    else setfirst_!(laq, [laq.first.gen,
      (laq.first.exp - a0.exp)::NNI])
  makeMulti laq

rquo(qa, a) ==
  (u := lquo(reverse qa, reverse a)) case "failed" => "failed"
  reverse_!(u::)$

if S has OrderedSet then
  a < b ==
    la := listOfMonoms a
    lb := listOfMonoms b
    na: Integer := #la
    nb: Integer := #lb
    while na > 0 and nb > 0 repeat
      la.first.gen > lb.first.gen => return false
      la.first.gen < lb.first.gen => return true
      if la.first.exp = lb.first.exp then
        la:=rest la
        lb:=rest lb
        na:=na - 1
        nb:=nb - 1
      else if la.first.exp > lb.first.exp then
        la:=concat([la.first.gen,
          (la.first.exp - lb.first.exp)::NNI], rest lb)
        lb:=rest lb
        nb:=nb - 1
      else
        lb:=concat([lb.first.gen,
          (lb.first.exp-la.first.exp)::NNI], rest la)
        la:=rest la
        na:=na-1
    empty? la and not empty? lb

```

---

— FMONOID.dotabb —

"FMONOID" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FMONOID"]

```

"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"FLAGG-" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FLAGG"]
"FMONOID" -> "FLAGG-"
"FMONOID" -> "FLAGG"

```

### 7.33 domain FNLA FreeNilpotentLie

— FreeNilpotentLie.input —

```

)set break resume
)sys rm -f FreeNilpotentLie.output
)spool FreeNilpotentLie.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FreeNilpotentLie
--R FreeNilpotentLie(n: NonNegativeInteger,class: NonNegativeInteger,R: CommutativeRing) is
--R Abbreviation for FreeNilpotentLie is FNLA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FNLA
--R
--R----- Operations -----
--R ?? : (R,%) -> %           ?? : (% ,R) -> %
--R ?? : (% ,%) -> %         ?? : (Integer,%) -> %
--R ?? : (PositiveInteger,%) -> %   ??? : (% ,PositiveInteger) -> %
--R ?? : (% ,%) -> %         ?-? : (% ,%) -> %
--R -? : % -> %              ?=? : (% ,%) -> Boolean
--R 0 : () -> %              antiCommutator : (% ,%) -> %
--R associator : (% ,% ,%) -> %    coerce : % -> OutputForm
--R commutator : (% ,%) -> %       deepExpand : % -> OutputForm
--R hash : % -> SingleInteger     latex : % -> String
--R sample : () -> %              shallowExpand : % -> OutputForm
--R zero? : % -> Boolean          ?~=? : (% ,%) -> Boolean
--R ?? : (NonNegativeInteger,%) -> %
--R dimension : () -> NonNegativeInteger
--R generator : NonNegativeInteger -> %
--R leftPower : (% ,PositiveInteger) -> %
--R plenaryPower : (% ,PositiveInteger) -> %
--R rightPower : (% ,PositiveInteger) -> %
--R subtractIfCan : (% ,%) -> Union(% ,"failed")
--R
--E 1

```

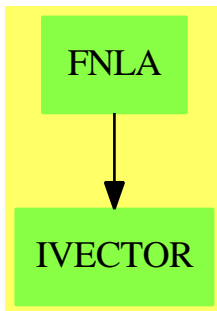
```
)spool
)lisp (bye)
```

— FreeNilpotentLie.help —

```
=====
FreeNilpotentLie examples
=====
```

```
See Also:
o )show FreeNilpotentLie
```

### 7.33.1 FreeNilpotentLie (FNLA)



See

⇒ “OrdSetInts” (OSI) 16.20.1 on page 1825  
 ⇒ “Commutator” (COMM) 4.7.1 on page 395

#### Exports:

|               |                |            |        |               |
|---------------|----------------|------------|--------|---------------|
| 0             | antiCommutator | associator | coerce | commutator    |
| deepExpand    | dimension      | generator  | hash   | latex         |
| leftPower     | plenaryPower   | rightPower | sample | shallowExpand |
| subtractIfCan | zero?          | ?~=?       | ?*?    | ?**?          |
| ?+?           | ?-?            | -?         | ?=?    |               |

— domain FNLA FreeNilpotentLie —

```
)abbrev domain FNLA FreeNilpotentLie
++ Author: Larry Lambe
++ Date Created: July 1988
```

```

++ Date Last Updated: March 13 1991
++ Related Constructors: OrderedSetInts, Commutator
++ AMS Classification: Primary 17B05, 17B30; Secondary 17A50
++ Keywords: free Lie algebra, Hall basis, basic commutators
++ Related Constructors: HallBasis, FreeMod, Commutator, OrdSetInts
++ Description:
++ Generate the Free Lie Algebra over a ring R with identity;
++ A P. Hall basis is generated by a package call to HallBasis.

FreeNilpotentLie(n:NNI,class:NNI,R: CommutativeRing): Export == Implement where
  B ==> Boolean
  Com ==> Commutator
  HB ==> HallBasis
  I ==> Integer
  NNI ==> NonNegativeInteger
  O ==> OutputForm
  OSI ==> OrdSetInts
  FM ==> FreeModule(R,OSI)
  VI ==> Vector Integer
  VLI ==> Vector List Integer
  lC ==> leadingCoefficient
  lS ==> leadingSupport

Export ==> NonAssociativeAlgebra(R) with
  dimension : () -> NNI
    ++ dimension() is the rank of this Lie algebra
  deepExpand : % -> 0
    ++ deepExpand(x) is not documented
  shallowExpand : % -> 0
    ++ shallowExpand(x) is not documented
  generator : NNI -> %
    ++ generator(i) is the ith Hall Basis element

Implement ==> FM add
  Rep := FM
  f,g : %

  coms:VLI
  coms := generate(n,class)$HB

  dimension == #coms

  have : (I,I) -> %
    -- have(left,right) is a lookup function for basic commutators
    -- already generated; if the nth basic commutator is
    -- [left,wt,right], then have(left,right) = n
  have(i,j) ==
    wt:I := coms(i).2 + coms(j).2
    wt > class => 0
    lo:I := 1

```

```

hi:I := dimension
while hi-lo > 1 repeat
  mid:I := (hi+lo) quo 2
  if coms(mid).2 < wt then lo := mid else hi := mid
while coms(hi).1 < i repeat hi := hi + 1
while coms(hi).3 < j repeat hi := hi + 1
monomial(1,hi::OSI)$FM

generator(i) ==
i > dimension => 0$Rep
monomial(1,i::OSI)$FM

putIn : I -> %
putIn(i) ==
monomial(1$R,i::OSI)$FM

brkt : (I,%) -> %
brkt(k,f) ==
f = 0 => 0
dg:I := value 1S f
reductum(f) = 0 =>
k = dg => 0
k > dg => -1C(f)*brkt(dg, putIn(k))
inHallBasis?(n,k,dg,coms(dg).1) => 1C(f)*have(k, dg)
1C(f)*( brkt(coms(dg).1, _
brkt(k,putIn coms(dg).3)) - brkt(coms(dg).3, _
brkt(k,putIn coms(dg).1) ))
brkt(k,monomial(1C f,1S f)$FM)+brkt(k,reductum f)

f*g ==
reductum(f) = 0 =>
1C(f)*brkt(value(1S f),g)
monomial(1C f,1S f)$FM*g + reductum(f)*g

Fac : I -> Com
-- an auxilliary function used for output of Free Lie algebra
-- elements (see expand)
Fac(m) ==
coms(m).1 = 0 => mkcomm(m)$Com
mkcomm(Fac coms(m).1, Fac coms(m).3)

shallowE : (R,OSI) -> 0
shallowE(r,s) ==
k := value s
r = 1 =>
k <= n => s::0
mkcomm(mkcomm(coms(k).1)$Com,mkcomm(coms(k).3)$Com)$Com::0
k <= n => r::0 * s::0
r::0 * mkcomm(mkcomm(coms(k).1)$Com,mkcomm(coms(k).3)$Com)$Com::0

```



```

shallowExpand(f) ==
  f = 0          => 0::0
  reductum(f) = 0 => shallowE(lC f,lS f)
  shallowE(lC f,lS f) + shallowExpand(reductum f)

deepExpand(f) ==
  f = 0          => 0::0
  reductum(f) = 0 =>
    lC(f)=1 => Fac(value(lS f))::0
    lC(f)::0 * Fac(value(lS f))::0
  lC(f)=1 => Fac(value(lS f))::0 + deepExpand(reductum f)
  lC(f)::0 * Fac(value(lS f))::0 + deepExpand(reductum f)

```

---

— FNLA.dotabb —

```

"FNLA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FNLA"]
"IVECTOR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IVECTOR"]
"FNLA" -> "IVECTOR"

```

---

## 7.34 domain FPARFRAC FullPartialFractionExpansion

— FullPartialFractionExpansion.input —

```

)set break resume
)sys rm -f FullPartialFractionExpansion.output
)spool FullPartialFractionExpansion.output
)set message test on
)set message auto off
)clear all
--S 1 of 16
Fx := FRAC UP(x, FRAC INT)
--R
--R
--R (1) Fraction UnivariatePolynomial(x,Fraction Integer)
--R
--E 1
Type: Domain

--S 2 of 16
f : Fx := 36 / (x**5-2*x**4-2*x**3+4*x**2+x-2)
--R

```

```

--R
--R
--R      36
--R  (2)  -----
--R      5      4      3      2
--R      x  - 2x  - 2x  + 4x  + x - 2
--R
--R      Type: Fraction UnivariatePolynomial(x,Fraction Integer)
--E 2

--S 3 of 16
g := fullPartialFraction f
--R
--R
--R      4      4      --+      - 3%A - 6
--R  (3)  ----- - ----- + > -----
--R      x - 2  x + 1  --+      2
--R      2      (x - %A)
--R      %A  - 1= 0
--R
--R      Type: FullPartialFractionExpansion(Fraction Integer,UnivariatePolynomial(x,Fraction Integer))
--E 3

--S 4 of 16
g :: Fx
--R
--R
--R      36
--R  (4)  -----
--R      5      4      3      2
--R      x  - 2x  - 2x  + 4x  + x - 2
--R
--R      Type: Fraction UnivariatePolynomial(x,Fraction Integer)
--E 4

--S 5 of 16
g5 := D(g, 5)
--R
--R
--R      480      480      --+      2160%A + 4320
--R  (5)  - ----- + ----- + > -----
--R      6      6      --+      7
--R      (x - 2)  (x + 1)  2      (x - %A)
--R      %A  - 1= 0
--R
--R      Type: FullPartialFractionExpansion(Fraction Integer,UnivariatePolynomial(x,Fraction Integer))
--E 5

--S 6 of 16
f5 := D(f, 5)
--R
--R
--R  (6)
--R      10      9      8      7      6
--R      - 544320x  + 4354560x  - 14696640x  + 28615680x  - 40085280x

```

```

--R      +
--R      5      4      3      2
--R      46656000x - 39411360x + 18247680x - 5870880x + 3317760x + 246240
--R      /
--R      20      19      18      17      16      15      14      13
--R      x - 12x + 53x - 76x - 159x + 676x - 391x - 1596x
--R      +
--R      12      11      10      9      8      7      6      5
--R      2527x + 1148x - 4977x + 1372x + 4907x - 3444x - 2381x + 2924x
--R      +
--R      4      3      2
--R      276x - 1184x + 208x + 192x - 64
--R      Type: Fraction UnivariatePolynomial(x,Fraction Integer)
--E 6

```

```

--S 7 of 16
g5::Fx - f5
--R
--R
--R      (7)  0
--R      Type: Fraction UnivariatePolynomial(x,Fraction Integer)
--E 7

```

```

--S 8 of 16
f : Fx := (x**5 * (x-1)) / ((x**2 + x + 1)**2 * (x-2)**3)
--R
--R
--R      6      5
--R      x - x
--R      (8)  -----
--R      7      6      5      3      2
--R      x - 4x + 3x + 9x - 6x - 4x - 8
--R      Type: Fraction UnivariatePolynomial(x,Fraction Integer)
--E 8

```

```

--S 9 of 16
g := fullPartialFraction f
--R
--R
--R      (9)
--R      1952      464      32      179      135
--R      ----      ---      --      - ---- %A + ----
--R      2401      343      49      2401      2401
--R      ----- + ----- + ----- + ----- > -----
--R      x - 2      2      3      2      x - %A
--R      (x - 2)      (x - 2)      2
--R      %A + %A + 1= 0
--R      +
--R      37      20
--R      ---- %A + ----

```

```

--R      +-+      1029      1029
--R      >      -----
--R      +-+      2
--R      2      (x - %A)
--R      %A + %A + 1= 0
--RType: FullPartialFractionExpansion(Fraction Integer,UnivariatePolynomial(x,Fraction Integer))
--E 9

--S 10 of 16
g :: Fx - f
--R
--R
--R      (10)  0
--R
--R      Type: Fraction UnivariatePolynomial(x,Fraction Integer)
--E 10

--S 11 of 16
f : Fx := (2*x**7-7*x**5+26*x**3+8*x) / (x**8-5*x**6+6*x**4+4*x**2-8)
--R
--R
--R      7      5      3
--R      2x - 7x + 26x + 8x
--R      (11) -----
--R      8      6      4      2
--R      x - 5x + 6x + 4x - 8
--R
--R      Type: Fraction UnivariatePolynomial(x,Fraction Integer)
--E 11

--S 12 of 16
g := fullPartialFraction f
--R
--R
--R      1      1
--R      -      -
--R      +-+      2      +-+      1      +-+      2
--R      (12)  > ----- + > ----- + > -----
--R      +-+      x - %A      +-+      3      +-+      x - %A
--R      2      2      (x - %A)      2
--R      %A - 2= 0      %A - 2= 0      %A + 1= 0
--RType: FullPartialFractionExpansion(Fraction Integer,UnivariatePolynomial(x,Fraction Integer))
--E 12

--S 13 of 16
g :: Fx - f
--R
--R
--R      (13)  0
--R
--R      Type: Fraction UnivariatePolynomial(x,Fraction Integer)
--E 13

```

--S 14 of 16

f:Fx := x\*\*3 / (x\*\*21 + 2\*x\*\*20 + 4\*x\*\*19 + 7\*x\*\*18 + 10\*x\*\*17 + 17\*x\*\*16 + 22\*x\*\*15 + 30\*x\*\*14 + 40\*x\*\*13 + 47\*x\*\*12 + 46\*x\*\*11 + 49\*x\*\*10 + 43\*x\*\*9 + 38\*x\*\*8 + 32\*x\*\*7 + 23\*x\*\*6 + 19\*x\*\*5 + 10\*x\*\*4 + 7\*x\*\*3 + 2\*x\*\*2 + x + 1)

--R

--R

--R (14)

--R 3

--R x

--R /

--R 21 20 19 18 17 16 15 14 13 12

--R x + 2x + 4x + 7x + 10x + 17x + 22x + 30x + 36x + 40x

--R +

--R 11 10 9 8 7 6 5 4 3 2

--R 47x + 46x + 49x + 43x + 38x + 32x + 23x + 19x + 10x + 7x + 2x

--R +

--R 1

--R Type: Fraction UnivariatePolynomial(x,Fraction Integer)

--E 14

--S 15 of 16

g := fullPartialFraction f

--R

--R

--R (15)

--R 1 1 19

--R - %A - %A - %A

--R ---+ 2 ---+ 9 27

--R > ----- + > -----

--R ---+ x - %A ---+ x - %A

--R 2 2

--R %A + 1 = 0 %A + %A + 1 = 0

--R +

--R 1 1

--R -- %A - %A

--R ---+ 27 27

--R > -----

--R ---+ (x - %A)

--R 2

--R %A + %A + 1 = 0

--R +

--R SIGMA

--R 5 2

--R %A + %A + 1 = 0

--R ,

--R 96556567040 4 420961732891 3 59101056149 2

--R - ----- %A + ----- %A - ----- %A

--R 912390759099 912390759099 912390759099

--R +

--R 373545875923 529673492498

--R - ----- %A + -----

--R 912390759099 912390759099

```

--R      /
--R      x - %A
--R      +
--R      SIGMA
--R      5      2
--R      %A  + %A  + 1= 0
--R      ,
--R      5580868  4      2024443  3      4321919  2      84614      5070620
--R      - ----- %A - ----- %A + ----- %A - ----- %A - -----
--R      94070601      94070601      94070601      1542141      94070601
--R      -----
--R      2
--R      (x - %A)
--R      +
--R      SIGMA
--R      5      2
--R      %A  + %A  + 1= 0
--R      ,
--R      1610957  4      2763014  3      2016775  2      266953      4529359
--R      ----- %A + ----- %A - ----- %A + ----- %A + -----
--R      94070601      94070601      94070601      94070601      94070601
--R      -----
--R      3
--R      (x - %A)
--RType: FullPartialFractionExpansion(Fraction Integer,UnivariatePolynomial(x,Fraction Integer))
--E 15

--S 16 of 16
g :: Fx - f
--R
--R
--R      (16)  0
--R
--R      Type: Fraction UnivariatePolynomial(x,Fraction Integer)
--E 16
)spool
)lisp (bye)

```

---

— FullPartialFractionExpansion.help —

```

=====
FullPartialFractionExpansion expansion
=====

```

The domain FullPartialFractionExpansion implements factor-free conversion of quotients to full partial fractions.

Our examples will all involve quotients of univariate polynomials

with rational number coefficients.

```
Fx := FRAC UP(x, FRAC INT)
      Fraction UnivariatePolynomial(x, Fraction Integer)
      Type: Domain
```

Here is a simple-looking rational function.

```
f : Fx := 36 / (x**5-2*x**4-2*x**3+4*x**2+x-2)
      36
      -----
      5      4      3      2
      x  - 2x  - 2x  + 4x  + x - 2
      Type: Fraction UnivariatePolynomial(x, Fraction Integer)
```

We use `fullPartialFraction` to convert it to an object of type `FullPartialFractionExpansion`.

```
g := fullPartialFraction f
      4      4      --+      - 3%A - 6
      ---- - ---- + > -----
      x - 2   x + 1   --+      2
      2      (x - %A)
      %A - 1= 0
Type: FullPartialFractionExpansion(Fraction Integer,
      UnivariatePolynomial(x, Fraction Integer))
```

Use a coercion to change it back into a quotient.

```
g :: Fx
      36
      -----
      5      4      3      2
      x  - 2x  - 2x  + 4x  + x - 2
      Type: Fraction UnivariatePolynomial(x, Fraction Integer)
```

Full partial fractions differentiate faster than rational functions.

```
g5 := D(g, 5)
      480      480      --+      2160%A + 4320
      - ---- + ---- + > -----
      6      6      --+      7
      (x - 2) (x + 1) 2      (x - %A)
      %A - 1= 0
Type: FullPartialFractionExpansion(Fraction Integer,
      UnivariatePolynomial(x, Fraction Integer))
```

```
f5 := D(f, 5)
      10      9      8      7      6
      - 544320x  + 4354560x  - 14696640x  + 28615680x  - 40085280x
```

```

+
      5      4      3      2
46656000x - 39411360x + 18247680x - 5870880x + 3317760x + 246240
/
      20      19      18      17      16      15      14      13
x - 12x + 53x - 76x - 159x + 676x - 391x - 1596x
+
      12      11      10      9      8      7      6      5
2527x + 1148x - 4977x + 1372x + 4907x - 3444x - 2381x + 2924x
+
      4      3      2
276x - 1184x + 208x + 192x - 64
Type: Fraction UnivariatePolynomial(x,Fraction Integer)

```

We can check that the two forms represent the same function.

```

g5::Fx - f5
0
Type: Fraction UnivariatePolynomial(x,Fraction Integer)

```

Here are some examples that are more complicated.

```

f : Fx := (x**5 * (x-1)) / ((x**2 + x + 1)**2 * (x-2)**3)
      6      5
      x - x
-----
      7      6      5      3      2
x - 4x + 3x + 9x - 6x - 4x - 8
Type: Fraction UnivariatePolynomial(x,Fraction Integer)

```

```

g := fullPartialFraction f
      1952      464      32      179      135
      ----      ---      --      - ---- %A + ----
      2401      343      49      ---+      2401      2401
      ----- + ----- + ----- + > -----
      x - 2      2      3      ---+      x - %A
      (x - 2)      (x - 2)      2
      %A + %A + 1= 0
+
      37      20
      ---- %A + ----
      ---+      1029      1029
      > -----
      ---+      2
      2      (x - %A)
      %A + %A + 1= 0

```

```

Type: FullPartialFractionExpansion(Fraction Integer,
UnivariatePolynomial(x,Fraction Integer))

```

```

g :: Fx - f

```



```

0
Type: Fraction UnivariatePolynomial(x,Fraction Integer)

f : Fx := (2*x**7-7*x**5+26*x**3+8*x) / (x**8-5*x**6+6*x**4+4*x**2-8)
      7      5      3
      2x  - 7x  + 26x  + 8x
      -----
      8      6      4      2
      x  - 5x  + 6x  + 4x  - 8
Type: Fraction UnivariatePolynomial(x,Fraction Integer)

g := fullPartialFraction f
      1
      -
      --+      2      --+      1      --+      2
      >      ----- + >      ----- + >      -----
      --+      x - %A      --+      3      --+      x - %A
      2      2      (x - %A)      2
      %A  - 2= 0      %A  - 2= 0      %A  + 1= 0
Type: FullPartialFractionExpansion(Fraction Integer,
UnivariatePolynomial(x,Fraction Integer))

g :: Fx - f
0
Type: Fraction UnivariatePolynomial(x,Fraction Integer)

f:Fx := x**3 / (x**21 + 2*x**20 + 4*x**19 + 7*x**18 + 10*x**17 + 17*x**16 + 22*x**15 + 30*x**14 + 40*x**13 + 47*x**12 + 46*x**11 + 49*x**10 + 43*x**9 + 38*x**8 + 32*x**7 + 23*x**6 + 19*x**5 + 10*x**4 + 7*x**3 + 2*x**2 + x + 1)
      3
      x
      /
      21      20      19      18      17      16      15      14      13      12
      x  + 2x  + 4x  + 7x  + 10x  + 17x  + 22x  + 30x  + 36x  + 40x
      +
      11      10      9      8      7      6      5      4      3      2
      47x  + 46x  + 49x  + 43x  + 38x  + 32x  + 23x  + 19x  + 10x  + 7x  + 2x
      +
      1
Type: Fraction UnivariatePolynomial(x,Fraction Integer)

g := fullPartialFraction f
      1      19
      - %A  - --
      --+      2      --+      9      27
      >      ----- + >      -----
      --+      x - %A      --+      x - %A
      2      2
      %A  + 1= 0      %A  + %A + 1= 0
      +
      1      1
      -- %A  - --

```

```

      ---+      27      27
      >      -----
      ---+      2
      2      (x - %A)
%A  + %A + 1= 0
+
SIGMA
  5      2
  %A  + %A  + 1= 0
,
      96556567040  4  420961732891  3  59101056149  2
      - ----- %A  + ----- %A  - ----- %A
      912390759099  912390759099  912390759099
+
      373545875923  529673492498
      - ----- %A  + -----
      912390759099  912390759099
/
x - %A
+
SIGMA
  5      2
  %A  + %A  + 1= 0
,
      5580868  4  2024443  3  4321919  2  84614  5070620
      - ----- %A  - ----- %A  + ----- %A  - ----- %A  - -----
      94070601  94070601  94070601  1542141  94070601
-----
                                     2
                                     (x - %A)
+
SIGMA
  5      2
  %A  + %A  + 1= 0
,
      1610957  4  2763014  3  2016775  2  266953  4529359
      ----- %A  + ----- %A  - ----- %A  + ----- %A  + -----
      94070601  94070601  94070601  94070601  94070601
-----
                                     3
                                     (x - %A)

```

Type: FullPartialFractionExpansion(Fraction Integer,UnivariatePolynomial(x,Fraction Integer))

This verification takes much longer than the conversion to partial fractions.

```

g :: Fx - f
0

```

Type: Fraction UnivariatePolynomial(x,Fraction Integer)

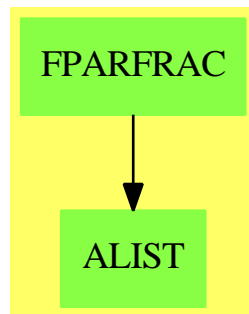
Use PartialFraction for standard partial fraction decompositions.

For more information, see the paper: Bronstein, M and Salvy, B.  
 "Full Partial Fraction Decomposition of Rational Functions,"  
 Proceedings of ISSAC'93, Kiev, ACM Press.

See Also:

- o )help PartialFraction
- o )show FullPartialFractionExpansion

### 7.34.1 FullPartialFractionExpansion (FPARFRAC)



#### Exports:

|        |           |          |          |                     |
|--------|-----------|----------|----------|---------------------|
| coerce | construct | convert  | D        | differentiate       |
| hash   | latex     | polyPart | fracPart | fullPartialFraction |
| ?~=?   | ?+?       | ?=?      |          |                     |

— domain FPARFRAC FullPartialFractionExpansion —

```

)abbrev domain FPARFRAC FullPartialFractionExpansion
++ Author: Manuel Bronstein
++ Date Created: 9 December 1992
++ Date Last Updated: 6 October 1993
++ References: M.Bronstein & B.Salvy,
++             Full Partial Fraction Decomposition of Rational Functions,
++             in Proceedings of ISSAC'93, Kiev, ACM Press.
++ Description:
++ Full partial fraction expansion of rational functions
  
```

```

FullPartialFractionExpansion(F, UP): Exports == Implementation where
  F : Join(Field, CharacteristicZero)
  UP : UnivariatePolynomialCategory F
  
```

```

N ==> NonNegativeInteger
Q ==> Fraction Integer
O ==> OutputForm
RF ==> Fraction UP
SUP ==> SparseUnivariatePolynomial RF
REC ==> Record(exponent: N, center: UP, num: UP)
ODV ==> OrderlyDifferentialVariable Symbol
ODP ==> OrderlyDifferentialPolynomial UP
ODF ==> Fraction ODP
FPF ==> Record(polyPart: UP, fracPart: List REC)

Exports ==> Join(SetCategory, ConvertibleTo RF) with
"+": (UP, $) -> $
  ++ p + x returns the sum of p and x
fullPartialFraction: RF -> $
  ++ fullPartialFraction(f) returns \spad{[p, [[j, Dj, Hj]...]]} such that
  ++ \spad{f = p(x) + sum_{[j,Dj,Hj] in l} sum_{Dj(a)=0} Hj(a)/(x - a)^j}.
polyPart: $ -> UP
  ++ polyPart(f) returns the polynomial part of f.
fracPart: $ -> List REC
  ++ fracPart(f) returns the list of summands of the fractional part of f.
construct: List REC -> $
  ++ construct(l) is the inverse of fracPart.
differentiate: $ -> $
  ++ differentiate(f) returns the derivative of f.
D: $ -> $
  ++ D(f) returns the derivative of f.
differentiate: ($, N) -> $
  ++ differentiate(f, n) returns the n-th derivative of f.
D: ($, NonNegativeInteger) -> $
  ++ D(f, n) returns the n-th derivative of f.

Implementation ==> add
Rep := FPF

fullParFrac: (UP, UP, UP, N) -> List REC
outputexp : (O, N) -> O
output : (N, UP, UP) -> O
REC2RF : (UP, UP, N) -> RF
UP2SUP : UP -> SUP
diffrec : REC -> REC
FP2O : List REC -> O

-- create a differential variable
u := new()$Symbol
u0 := makeVariable(u, 0)$ODV
alpha := u::O
x := monomial(1, 1)$UP
xx := x::O
zr := (O$N)::O

```

```

construct 1      == [0, 1]
D r              == differentiate r
D(r, n)          == differentiate(r,n)
polyPart f      == f.polyPart
fracPart f      == f.fracPart
p:UP + f:$       == [p + polyPart f, fracPart f]

differentiate f ==
  differentiate(polyPart f) + construct [diffrec rec for rec in fracPart f]

differentiate(r, n) ==
  for i in 1..n repeat r := differentiate r
  r

-- diffrec(sum_{rec.center(a) = 0} rec.num(a) / (x - a)^e) =
--      sum_{rec.center(a) = 0} -e rec.num(a) / (x - a)^{e+1}
--      where e = rec.exponent
diffrec rec ==
  e := rec.exponent
  [e + 1, rec.center, - e * rec.num]

convert(f:$):RF ==
  ans := polyPart(f)::RF
  for rec in fracPart f repeat
    ans := ans + REC2RF(rec.center, rec.num, rec.exponent)
  ans

UP2SUP p == map((z1:F):RF +-> z1::UP::RF, p)_
  $UnivariatePolynomialCategoryFunctions2(F, UP, RF, SUP)

-- returns Trace_k^k(a) (h(a) / (x - a)^n) where d(a) = 0
REC2RF(d, h, n) ==
  one?(m := degree d) =>
    ((m := degree d) = 1) =>
      a := - (leadingCoefficient reductum d) / (leadingCoefficient d)
      h(a)::UP / (x - a::UP)**n
    dd := UP2SUP d
    hh := UP2SUP h
    aa := monomial(1, 1)$SUP
    p := (x::RF::SUP - aa)**n rem dd
    rec := extendedEuclidean(p, dd, hh)::Record(coef1:SUP, coef2:SUP)
    t := rec.coef1 -- we want Trace_k^k(a)(t) now
    ans := coefficient(t, 0)
    for i in 1..degree(d)-1 repeat
      t := (t * aa) rem dd
      ans := ans + coefficient(t, i)
    ans

fullPartialFraction f ==

```

```

qr := divide(number f, d := denom f)
qr.quotient + construct concat
    [fullParFrac(qr.remainder, d, rec.factor, rec.exponent::N)
     for rec in factors squareFree denom f]

fullParFrac(a, d, q, n) ==
  ans:List REC := empty()
  em := e := d quo (q ** n)
  rec := extendedEuclidean(e, q, 1)::Record(coef1:UP,coef2:UP)
  bm := b := rec.coef1 -- b = inverse of e modulo q
  lvar:List(ODV) := [u0]
  um := 1::ODP
  un := (u1 := u0::ODP)**n
  lval:List(UP) := [q1 := q := differentiate(q0 := q)]
  h:ODF := a::ODP / (e * un)
  rec := extendedEuclidean(q1, q0, 1)::Record(coef1:UP,coef2:UP)
  c := rec.coef1 -- c = inverse of q' modulo q
  cm := 1::UP
  cn := (c ** n) rem q0
  for m in 1..n repeat
    p := retract(em * un * um * h)@ODP
    pp := retract(eval(p, lvar, lval))@UP
    h := inv(m::Q) * differentiate h
    q := differentiate q
    lvar := concat(makeVariable(u, m), lvar)
    lval := concat(inv((m+1)::F) * q, lval)
    qq := q0 quo gcd(pp, q0) -- new center
    if (degree(qq) > 0) then
      ans := concat([(n + 1 - m)::N, qq, (pp * bm * cn * cm) rem qq], ans)
    cm := (c * cm) rem q0 -- cm = c**m modulo q now
    um := u1 * um -- um = u**m now
    em := e * em -- em = e**{m+1} now
    bm := (b * bm) rem q0 -- bm = b**{m+1} modulo q now
  ans

coerce(f:$):0 ==
  ans := FP20(l := fracPart f)
  zero?(p := polyPart f) =>
    empty? l => (0$N)::0
    ans
  p::0 + ans

FP20 l ==
  empty? l => empty()
  rec := first l
  ans := output(rec.exponent, rec.center, rec.num)
  for rec in rest l repeat
    ans := ans + output(rec.exponent, rec.center, rec.num)
  ans

```

```

output(n, d, h) ==
--   one? degree d =>
      (degree d) = 1 =>
        a := - leadingCoefficient(reductum d) / leadingCoefficient(d)
        h(a)::0 / outputexp((x - a::UP)::0, n)
      sum(outputForm(makeSUP h, alpha) / outputexp(xx - alpha, n),
        outputForm(makeSUP d, alpha) = zr)

outputexp(f, n) ==
--   one? n => f
      (n = 1) => f
      f ** (n::0)

```

---

— FPARFRAC.dotabb —

```

"FPARFRAC" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FPARFRAC"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"FPARFRAC" -> "ALIST"

```

---

## 7.35 domain FUNCTION FunctionCalled

---

— FunctionCalled.input —

```

)set break resume
)sys rm -f FunctionCalled.output
)spool FunctionCalled.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FunctionCalled
--R FunctionCalled f: Symbol is a domain constructor
--R Abbreviation for FunctionCalled is FUNCTION
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FUNCTION
--R
--R----- Operations -----
--R ?=? : (% , %) -> Boolean          coerce : % -> OutputForm
--R hash : % -> SingleInteger        latex : % -> String

```

```
--R name : % -> Symbol                ?~=? : (%,% ) -> Boolean
--R
--E 1
```

```
)spool
)lisp (bye)
```

---

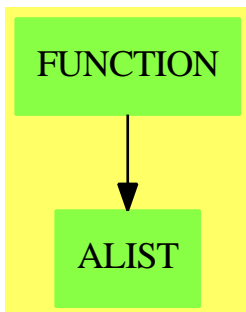
— FunctionCalled.help —

```
=====
FunctionCalled examples
=====
```

```
See Also:
o )show FunctionCalled
```

---

### 7.35.1 FunctionCalled (FUNCTION)



#### Exports:

```
coerce hash latex name ?=? ?~=?
```

— domain FUNCTION FunctionCalled —

```
)abbrev domain FUNCTION FunctionCalled
++ Author: Mark Botch
++ Description:
++ This domain implements named functions
```

```
FunctionCalled(f:Symbol): SetCategory with
```



```

name: % -> Symbol
++ name(x) returns the symbol
== add
name r                == f
coerce(r: %):OutputForm == f::OutputForm
x = y                 == true
latex(x: %):String    == latex f

```

---

— **FUNCTION.dotabb** —

```

"FUNCTION" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FUNCTION"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"FUNCTION" -> "ALIST"

```

---

## Chapter 8

# Chapter G

### 8.1 domain GDMP GeneralDistributedMultivariatePolynomial

— GeneralDistributedMultivariatePolynomial.input —

```
)set break resume
)sys rm -f GeneralDistributedMultivariatePolynomial.output
)spool GeneralDistributedMultivariatePolynomial.output
)set message test on
)set message auto off
)clear all
--S 1 of 10
(d1,d2,d3) : DMP([z,y,x],FRAC INT)
--R
--R
--R                                          Type: Void
--E 1

--S 2 of 10
d1 := -4*z + 4*y**2*x + 16*x**2 + 1
--R
--R
--R
--R          2      2
--R   (2)  - 4z + 4y x + 16x  + 1
--R
--R          Type: DistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 2

--S 3 of 10
d2 := 2*z*y**2 + 4*x + 1
--R
--R
```

```

--R      2
--R (3) 2z y + 4x + 1
--R      Type: DistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 3

--S 4 of 10
d3 := 2*z*x**2 - 2*y**2 - x
--R
--R
--R      2      2
--R (4) 2z x - 2y - x
--R      Type: DistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 4

--S 5 of 10
groebner [d1,d2,d3]
--R
--R
--R (5)
--R      1568 6 1264 5 6 4 182 3 2047 2 103 2857
--R [z - ---- x - ---- x + --- x + --- x - ---- x - ---- x - ----,
--R      2745 305 305 549 610 2745 10980
--R      2 112 6 84 5 1264 4 13 3 84 2 1772 2
--R y + ---- x - ---- x - ---- x - ---- x + ---- x + ----,
--R      2745 305 305 549 305 2745 2745
--R      7 29 6 17 4 11 3 1 2 15 1
--R x + -- x - -- x - -- x + -- x + -- x + -]
--R      4 16 8 32 16 4
--R      Type: List DistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 5

--S 6 of 10
(n1,n2,n3) : HDMP([z,y,x],FRAC INT)
--R
--R
--R                                          Type: Void
--E 6

--S 7 of 10
n1 := d1
--R
--R
--R      2      2
--R (7) 4y x + 16x - 4z + 1
--R      Type: HomogeneousDistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 7

--S 8 of 10
n2 := d2
--R
--R

```

```

--R      2
--R (8) 2z y + 4x + 1
--R Type: HomogeneousDistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 8

--S 9 of 10
n3 := d3
--R
--R
--R      2      2
--R (9) 2z x - 2y - x
--R Type: HomogeneousDistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 9

--S 10 of 10
groebner [n1,n2,n3]
--R
--R
--R (10)
--R      4      3      3      2      1      1      4      29      3      1      2      7      9      1
--R [y + 2x - - x + - z - -, x + - x - - y - - z x - - x - -,
--R      2      2      2      8      4      8      4      16      4
--R      2      1      2      2      1      2      2      1
--R z y + 2x + -, y x + 4x - z + -, z x - y - - x,
--R      2      2      4      2
--R z - 4y + 2x - - z - - x]
--R      4      2
--RType: List HomogeneousDistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 10
)spool
)lisp (bye)

```

---

— GeneralDistributedMultivariatePolynomial.help —

```

=====
MultivariatePolynomial
DistributedMultivariatePolynomial
HomogeneousDistributedMultivariatePolynomial
GeneralDistributedMultivariatePolynomial
=====

```

DistributedMultivariatePolynomial which is abbreviated as DMP and HomogeneousDistributedMultivariatePolynomial, which is abbreviated as HDMP, are very similar to MultivariatePolynomial except that they are represented and displayed in a non-recursive manner.

```
(d1,d2,d3) : DMP([z,y,x],FRAC INT)
              Type: Void
```

The constructor DMP orders its monomials lexicographically while HDMP orders them by total order refined by reverse lexicographic order.

```
d1 := -4*z + 4*y**2*x + 16*x**2 + 1
      2      2
      - 4z + 4y x + 16x + 1
      Type: DistributedMultivariatePolynomial([z,y,x],Fraction Integer)

d2 := 2*z*y**2 + 4*x + 1
      2
      2z y + 4x + 1
      Type: DistributedMultivariatePolynomial([z,y,x],Fraction Integer)

d3 := 2*z*x**2 - 2*y**2 - x
      2      2
      2z x - 2y - x
      Type: DistributedMultivariatePolynomial([z,y,x],Fraction Integer)
```

These constructors are mostly used in Groebner basis calculations.

```
groebner [d1,d2,d3]
      1568 6 1264 5 6 4 182 3 2047 2 103 2857
      [z - ---- x - ---- x + --- x + --- x - ---- x - ---- x - ----,
      2745 305 305 549 610 2745 10980
      2 112 6 84 5 1264 4 13 3 84 2 1772 2
      y + ---- x - --- x - ---- x - --- x + --- x + ---- x + ----,
      2745 305 305 549 305 2745 2745
      7 29 6 17 4 11 3 1 2 15 1
      x + -- x - -- x - -- x + -- x + -- x + -]
      4 16 8 32 16 4
      Type: List DistributedMultivariatePolynomial([z,y,x],Fraction Integer)
```

```
(n1,n2,n3) : HDMP([z,y,x],FRAC INT)
              Type: Void
```

```
n1 := d1
      2      2
      4y x + 16x - 4z + 1
      Type: HomogeneousDistributedMultivariatePolynomial([z,y,x],Fraction Integer)

n2 := d2
      2
      2z y + 4x + 1
      Type: HomogeneousDistributedMultivariatePolynomial([z,y,x],Fraction Integer)

n3 := d3
```

```

      2      2
2z x - 2y - x
Type: HomogeneousDistributedMultivariatePolynomial([z,y,x],Fraction Integer)

```

Note that we get a different Groebner basis when we use the HDMP polynomials, as expected.

```

groebner [n1,n2,n3]
      4      3      3      2      1      1      4      29      3      1      2      7      9      1
[y + 2x - - x + - z - -, x + - x - - y - - z x - - x - -,
      2      2      8      4      8      4      16      4
      2      1      2      2      1      2      2      1
z y + 2x + -, y x + 4x - z + -, z x - y - - x,
      2      4      2
      2      2      2      1      3
z - 4y + 2x - - z - - x]
      4      2
Type: List HomogeneousDistributedMultivariatePolynomial([z,y,x],
Fraction Integer)

```

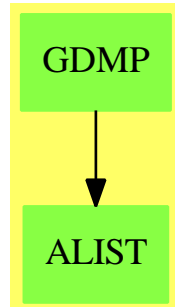
GeneralDistributedMultivariatePolynomial is somewhat more flexible in the sense that as well as accepting a list of variables to specify the variable ordering, it also takes a predicate on exponent vectors to specify the term ordering. With this polynomial type the user can experiment with the effect of using completely arbitrary term orderings. This flexibility is mostly important for algorithms such as Groebner basis calculations which can be very sensitive to term ordering.

See Also:

- o )help Polynomial
- o )help UnivariatePolynomial
- o )help MultivariatePolynomial
- o )help HomogeneousDistributedMultivariatePolynomial
- o )help DistributedMultivariatePolynomial
- o )show GeneralDistributedMultivariatePolynomial

---

### 8.1.1 GeneralDistributedMultivariatePolynomial (GDMP)



See

⇒ “DistributedMultivariatePolynomial” (DMP) 5.13.1 on page 557

⇒ “HomogeneousDistributedMultivariatePolynomial” (HDMP) 9.10.1 on page 1145

#### Exports:

|                    |                               |               |
|--------------------|-------------------------------|---------------|
| 0                  | 1                             | associates?   |
| binomThmExpt       | characteristic                | charthRoot    |
| coefficient        | coefficients                  | coerce        |
| conditionP         | content                       | D             |
| degree             | differentiate                 | discriminant  |
| eval               | exquo                         | factor        |
| factorPolynomial   | factorSquareFreePolynomial    | gcd           |
| gcdPolynomial      | ground                        | ground?       |
| hash               | isExpt                        | isPlus        |
| isTimes            | latex                         | lcm           |
| leadingCoefficient | leadingMonomial               | mainVariable  |
| map                | mapExponents                  | max           |
| min                | minimumDegree                 | monicDivide   |
| monomial           | monomial?                     | monomials     |
| multivariate       | numberOfMonomials             | one?          |
| patternMatch       | pomopo!                       | prime?        |
| primitiveMonomials | primitivePart                 | recip         |
| reducedSystem      | reductum                      | reorder       |
| resultant          | retract                       | retractIfCan  |
| sample             | solveLinearPolynomialEquation | squareFree    |
| squareFreePart     | squareFreePolynomial          | subtractIfCan |
| totalDegree        | unit?                         | unitCanonical |
| unitNormal         | univariate                    | variables     |
| zero?              | ?*?                           | ?**?          |
| ?+?                | ?-?                           | -?            |
| ?=?                | ?~=?                          | ?<?           |
| ?<=?               | ?>?                           | ?>=?          |
| ?^?                |                               |               |

— domain GDMP GeneralDistributedMultivariatePolynomial —

```

)abbrev domain GDMP GeneralDistributedMultivariatePolynomial
++ Author: Barry Trager
++ Date Created:
++ Date Last Updated:
++ Basic Functions: Ring, degree, eval, coefficient, monomial, differentiate,
++ resultant, gcd, leadingCoefficient
++ Related Constructors: DistributedMultivariatePolynomial,
++ HomogeneousDistributedMultivariatePolynomial
++ Also See: Polynomial
++ AMS Classifications:
++ Keywords: polynomial, multivariate, distributed
++ References:
++ Description:
++ This type supports distributed multivariate polynomials
++ whose variables are from a user specified list of symbols.
++ The coefficient ring may be non commutative,
++ but the variables are assumed to commute.
++ The term ordering is specified by its third parameter.
++ Suggested types which define term orderings include:
++ \spadtype{DirectProduct}, \spadtype{HomogeneousDirectProduct},
++ \spadtype{SplitHomogeneousDirectProduct} and finally
++ \spadtype{OrderedDirectProduct} which accepts an arbitrary user
++ function to define a term ordering.

GeneralDistributedMultivariatePolynomial(vl,R,E): public == private where
  vl: List Symbol
  R: Ring
  E: DirectProductCategory(#vl,NonNegativeInteger)
  OV ==> OrderedVariableList(vl)
  SUP ==> SparseUnivariatePolynomial
  NNI ==> NonNegativeInteger

  public == PolynomialCategory(R,E,OV) with
    reorder: (% ,List Integer) -> %
      ++ reorder(p, perm) applies the permutation perm to the variables
      ++ in a polynomial and returns the new correctly ordered polynomial

  private == PolynomialRing(R,E) add
    --representations
    Term := Record(k:E,c:R)
    Rep := List Term
    n := #vl
    Vec ==> Vector(NonNegativeInteger)
    zero?(p : %): Boolean == null(p : Rep)

    totalDegree p ==
      zero? p => 0

```



```

"max"/[reduce("+", (t.k)::(Vector NNI), 0) for t in p]

monomial(p:%, v: OV, e: NonNegativeInteger):% ==
  locv := lookup v
  p*monomial(1,
    directProduct [if z=locv then e else 0 for z in 1..n]$Vec)

coerce(v: OV):% == monomial(1,v,1)

listCoef(p : %): List R ==
  rec : Term
  [rec.c for rec in (p:Rep)]

mainVariable(p: %) ==
  zero?(p) => "failed"
  for v in vl repeat
    vv := variable(v)::OV
    if degree(p,vv)>0 then return vv
  "failed"

ground?(p) == mainVariable(p) case "failed"

retract(p : %): R ==
  not ground? p => error "not a constant"
  leadingCoefficient p

retractIfCan(p : %): Union(R, "failed") ==
  ground?(p) => leadingCoefficient p
  "failed"

degree(p: %, v: OV) == degree(univariate(p,v))
minimumDegree(p: %, v: OV) == minimumDegree(univariate(p,v))
differentiate(p: %, v: OV) ==
  multivariate(differentiate(univariate(p,v)), v)

degree(p: %, lv: List OV) == [degree(p,v) for v in lv]
minimumDegree(p: %, lv: List OV) == [minimumDegree(p,v) for v in lv]

numberOfMonomials(p:%) ==
  l : Rep := p : Rep
  null(l) => 1
  #l

monomial?(p : %): Boolean ==
  l : Rep := p : Rep
  null(l) or null rest(l)

if R has OrderedRing then
  maxNorm(p : %): R ==
    l : List R := nil

```

```

    r,m : R
    m := 0
    for r in listCoef(p) repeat
        if r > m then m := r
        else if (-r) > m then m := -r
    m

--trailingCoef(p : %) ==
-- l : Rep := p : Rep
-- null l => 0
-- r : Term := last l
-- r.c

--leadingPrimitiveMonomial(p : %) ==
-- ground?(p) => 1$%
-- r : Term := first(p:Rep)
-- r := [r.k,1$R]$Term      -- new cell
-- list(r)$Rep :: %

-- The following 2 defs are inherited from PolynomialRing

--leadingMonomial(p : %) ==
-- ground?(p) => p
-- r : Term := first(p:Rep)
-- r := [r.k,r.c]$Term      -- new cell
-- list(r)$Rep :: %

--reductum(p : %): % ==
-- ground? p => 0$%
-- (rest(p:Rep)):%

if R has Field then
    (p : %) / (r : R) == inv(r) * p

variables(p: %) ==
    maxdeg:Vector(NonNegativeInteger) := new(n,0)
    while not zero?(p) repeat
        tdeg := degree p
        p := reductum p
        for i in 1..n repeat
            maxdeg.i := max(maxdeg.i, tdeg.i)
        [index(i:PositiveInteger) for i in 1..n | maxdeg.i^=0]

reorder(p: %,perm: List Integer):% ==
    #perm ^= n => error "must be a complete permutation of all vars"
    q := [[directProduct [term.k.j for j in perm]$Vec,term.c]$Term
           for term in p]
    sort((z1,z2) +-> z1.k > z2.k,q)

--coerce(dp:DistributedMultivariatePolynomial(vl,R)):% ==

```

```

-- q:=dp>List(Term)
-- sort(#1.k > #2.k,q):%

univariate(p: %,v: OV):SUP(%) ==
  zero?(p) => 0
  exp := degree p
  locv := lookup v
  deg:NonNegativeInteger := 0
  nexp := directProduct [if i=locv then (deg :=exp.i;0) else exp.i
                        for i in 1..n]$Vec
  monomial(monomial(leadingCoefficient p,nexp),deg)+
    univariate(reductum p,v)

eval(p: %,v: OV,val:R):% == univariate(p,v)(val)

eval(p: %,v: OV,val:R):% == eval(p,v,val:R)$%

eval(p: %,lv: List OV,lval: List R):% ==
  lv = [] => p
  eval(eval(p,first lv,(first lval)::R)$%, rest lv, rest lval)$%

-- assume Lvar are sorted correctly
evalSortedVarlist(p: %,Lvar: List OV,Lpval: List %):% ==
  v := mainVariable p
  v case "failed" => p
  pv := v:: OV
  Lvar=[] or Lpval=[] => p
  mvar := Lvar.first
  mvar > pv => evalSortedVarlist(p,Lvar.rest,Lpval.rest)
  pval := Lpval.first
  pts:SUP(%) := map(x+>evalSortedVarlist(x,Lvar,Lpval),univariate(p,pv))
  mvar=pv => pts(pval)
  multivariate(pts,pv)

eval(p:%,Lvar:List OV,Lpval:List %) ==
  nlvar:List OV := sort((x,y) +> x > y,Lvar)
  nlpval :=
    Lvar = nlvar => Lpval
    nlpval := [Lpval.position(mvar,Lvar) for mvar in nlvar]
  evalSortedVarlist(p,nlvar,nlpval)

multivariate(p1:SUP(%),v: OV):% ==
  0=p1 => 0
  degree p1 = 0 => leadingCoefficient p1
  leadingCoefficient(p1)*(v::%)**degree(p1) +
    multivariate(reductum p1,v)

univariate(p: %):SUP(R) ==
  (v := mainVariable p) case "failed" =>
    monomial(leadingCoefficient p,0)

```

```

q := univariate(p,v:: OV)
ans:SUP(R) := 0
while q ^= 0 repeat
  ans := ans + monomial(ground leadingCoefficient q,degree q)
  q := reductum q
ans

multivariate(p:SUP(R),v: OV):% ==
0=p => 0
(leadingCoefficient p)*monomial(1,v,degree p) +
  multivariate(reductum p,v)

if R has GcdDomain then
content(p: %):R ==
zero?(p) => 0
"gcd"/[t.c for t in p]

if R has EuclideanDomain and not(R has FloatingPointSystem) then
gcd(p: %,q:%):% ==
gcd(p,q)$PolynomialGcdPackage(E,OV,R,%)

else gcd(p: %,q:%):% ==
r : R
(pv := mainVariable(p)) case "failed" =>
(r := leadingCoefficient p) = 0$R => q
gcd(r,content q)::%
(qv := mainVariable(q)) case "failed" =>
(r := leadingCoefficient q) = 0$R => p
gcd(r,content p)::%
pv<qv => gcd(p,content univariate(q,qv))
qv<pv => gcd(q,content univariate(p,pv))
multivariate(gcd(univariate(p,pv),univariate(q,qv)),pv)

coerce(p: %) : OutputForm ==
zero?(p) => (0$R) :: OutputForm
l,lt : List OutputForm
lt := nil
vl1 := [v::OutputForm for v in vl]
for t in reverse p repeat
  l := nil
  for i in 1..#vl1 repeat
    t.k.i = 0 => l
    t.k.i = 1 => l := cons(vl1.i,l)
    l := cons(vl1.i ** t.k.i ::OutputForm,l)
  l := reverse l
  if (t.c ^= 1) or (null l) then l := cons(t.c :: OutputForm,l)
  1 = #l => lt := cons(first l,lt)
lt := cons(reduce(" ",l),lt)

```

```
1 = #lt => first lt
reduce("+",lt)
```

---

— GDMP.dotabb —

```
"GDMP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=GDMP"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"GDMP" -> "ALIST"
```

---

## 8.2 domain GMODPOL GeneralModulePolynomial

— GeneralModulePolynomial.input —

```
)set break resume
)sys rm -f GeneralModulePolynomial.output
)spool GeneralModulePolynomial.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show GeneralModulePolynomial
--R GeneralModulePolynomial(v1: List Symbol,R: CommutativeRing,IS: OrderedSet,E: DirectProduct)
--R Abbreviation for GeneralModulePolynomial is GMODPOL
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for GMODPOL
--R
--R----- Operations -----
--R ??? : (R,%) -> %
--R ??? : (P,%) -> %
--R ??? : (Integer,%) -> %
--R ?+? : (%,%) -> %
--R -? : % -> %
--R 0 : () -> %
--R coerce : % -> OutputForm
--R latex : % -> String
--R leadingExponent : % -> E
--R multMonom : (R,E,%) -> %
--R sample : () -> %
--R zero? : % -> Boolean
--R ??? : (%,R) -> %
--R ??? : (P,%) -> %
--R ??? : (PositiveInteger,%) -> %
--R ?-? : (%,%) -> %
--R ?? : (%,%) -> Boolean
--R build : (R,IS,E) -> %
--R hash : % -> SingleInteger
--R leadingCoefficient : % -> R
--R leadingIndex : % -> IS
--R reductum : % -> %
--R unitVector : IS -> %
--R ?~=? : (%,%) -> Boolean
```

```

--R ???: (NonNegativeInteger,%) -> %
--R leadingMonomial : % -> ModuleMonomial(IS,E,ff)
--R monomial : (R,ModuleMonomial(IS,E,ff)) -> %
--R subtractIfCan : (%,%) -> Union(%, "failed")
--R
--E 1

)spool
)lisp (bye)

```

---

— GeneralModulePolynomial.help —

=====

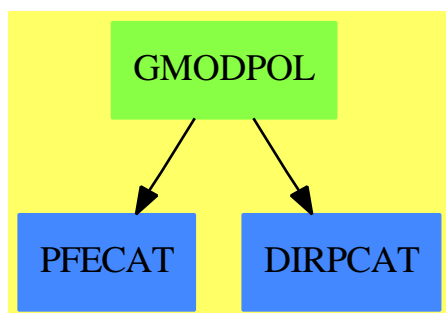
GeneralModulePolynomial examples

=====

See Also:

- o )show GeneralModulePolynomial

### 8.2.1 GeneralModulePolynomial (GMODPOL)



See

⇒ “ModuleMonomial” (MODMONOM) 14.11.1 on page 1608

**Exports:**

|                    |                 |              |                 |            |
|--------------------|-----------------|--------------|-----------------|------------|
| 0                  | build           | coerce       | hash            | latex      |
| leadingCoefficient | leadingExponent | leadingIndex | leadingMonomial | monomial   |
| multMonom          | reductum        | sample       | subtractIfCan   | unitVector |
| zero?              | ?~=?            | ?*?          | ?+?             | ?-?        |
| -?                 | ?=?             |              |                 |            |

## — domain GMODPOL GeneralModulePolynomial —

```

)abbrev domain GMODPOL GeneralModulePolynomial
++ Author: Mark Botch
++ Description:
++ This package is undocumented

GeneralModulePolynomial(vl, R, IS, E, ff, P): public == private where
  vl: List(Symbol)
  R: CommutativeRing
  IS: OrderedSet
  NNI ==> NonNegativeInteger
  E: DirectProductCategory(#vl, NNI)
  MM ==> Record(index:IS, exponent:E)
  ff: (MM, MM) -> Boolean
  OV ==> OrderedVariableList(vl)
  P: PolynomialCategory(R, E, OV)
  ModMonom ==> ModuleMonomial(IS, E, ff)

public == Join(Module(P), Module(R)) with
  leadingCoefficient: $ -> R
  ++ leadingCoefficient(x) is not documented
  leadingMonomial: $ -> ModMonom
  ++ leadingMonomial(x) is not documented
  leadingExponent: $ -> E
  ++ leadingExponent(x) is not documented
  leadingIndex: $ -> IS
  ++ leadingIndex(x) is not documented
  reductum: $ -> $
  ++ reductum(x) is not documented
  monomial: (R, ModMonom) -> $
  ++ monomial(r,x) is not documented
  unitVector: IS -> $
  ++ unitVector(x) is not documented
  build: (R, IS, E) -> $
  ++ build(r,i,e) is not documented
  multMonom: (R, E, $) -> $
  ++ multMonom(r,e,x) is not documented
  "*": (P,$) -> $
  ++ p*x is not documented

private == FreeModule(R, ModMonom) add
  Rep:= FreeModule(R, ModMonom)
  leadingMonomial(p:$):ModMonom == leadingSupport(p)$Rep
  leadingExponent(p:$):E == exponent(leadingMonomial p)
  leadingIndex(p:$):IS == index(leadingMonomial p)
  unitVector(i:IS):$ == monomial(1,[i, 0$E]$ModMonom)

```

```

-----

build(c:R, i:IS, e:E):$ == monomial(c, construct(i, e))

-----

---- WARNING: assumes c ^= 0

multMonom(c:R, e:E, mp:$):$ ==
  zero? mp => mp
  monomial(c * leadingCoefficient mp, [leadingIndex mp,
    e + leadingExponent mp]) + multMonom(c, e, reductum mp)

-----

((p:P) * (mp:$)): $ ==
  zero? p => 0
  multMonom(leadingCoefficient p, degree p, mp) +
    reductum(p) * mp

-----

— GMODPOL.dotabb —

"GMODPOL" [color="#88FF44",href="bookvol10.3.pdf#nameddest=GMODPOL"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"DIRPCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=DIRPCAT"]
"GMODPOL" -> "PFECAT"
"GMODPOL" -> "DIRPCAT"

-----

```

## 8.3 domain GCNAALG GenericNonAssociativeAlgebra

— GenericNonAssociativeAlgebra.input —

```

)set break resume
)sys rm -f GenericNonAssociativeAlgebra.output
)spool GenericNonAssociativeAlgebra.output
)set message test on
)set message auto off

```



```
)clear all
```

```
--S 1 of 1
```

```
)show GenericNonAssociativeAlgebra
```

```
--R GenericNonAssociativeAlgebra(R: CommutativeRing,n: PositiveInteger,ls: List Symbol,gamma
```

```
--R Abbreviation for GenericNonAssociativeAlgebra is GCNAALG
```

```
--R This constructor is not exposed in this frame.
```

```
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for GCNAALG
```

```
--R
```

```
--R----- Operations -----
```

```
--R ?? : (%,% ) -> %
```

```
?? : (Integer,% ) -> %
```

```
--R ?? : (PositiveInteger,% ) -> %
```

```
??? : (% ,PositiveInteger) -> %
```

```
--R ?? : (%,% ) -> %
```

```
?-? : (%,% ) -> %
```

```
--R -? : % -> %
```

```
?=? : (%,% ) -> Boolean
```

```
--R 0 : () -> %
```

```
alternative? : () -> Boolean
```

```
--R antiAssociative? : () -> Boolean
```

```
antiCommutative? : () -> Boolean
```

```
--R antiCommutator : (%,% ) -> %
```

```
associative? : () -> Boolean
```

```
--R associator : (%,%,% ) -> %
```

```
basis : () -> Vector %
```

```
--R coerce : % -> OutputForm
```

```
commutative? : () -> Boolean
```

```
--R commutator : (%,% ) -> %
```

```
flexible? : () -> Boolean
```

```
--R generic : (Symbol,Vector %) -> %
```

```
generic : Vector % -> %
```

```
--R generic : Vector Symbol -> %
```

```
generic : Symbol -> %
```

```
--R generic : () -> %
```

```
hash : % -> SingleInteger
```

```
--R jacobiIdentity? : () -> Boolean
```

```
jordanAdmissible? : () -> Boolean
```

```
--R jordanAlgebra? : () -> Boolean
```

```
latex : % -> String
```

```
--R leftAlternative? : () -> Boolean
```

```
lieAdmissible? : () -> Boolean
```

```
--R lieAlgebra? : () -> Boolean
```

```
powerAssociative? : () -> Boolean
```

```
--R rank : () -> PositiveInteger
```

```
rightAlternative? : () -> Boolean
```

```
--R sample : () -> %
```

```
someBasis : () -> Vector %
```

```
--R zero? : % -> Boolean
```

```
?~=? : (%,% ) -> Boolean
```

```
--R ?? : (SquareMatrix(n,Fraction Polynomial R),%) -> %
```

```
--R ?? : (Fraction Polynomial R,% ) -> %
```

```
--R ?? : (% ,Fraction Polynomial R) -> %
```

```
--R ?? : (NonNegativeInteger,% ) -> %
```

```
--R apply : (Matrix Fraction Polynomial R,% ) -> %
```

```
--R associatorDependence : () -> List Vector Fraction Polynomial R if Fraction Polynomial R
```

```
--R coerce : Vector Fraction Polynomial R -> %
```

```
--R conditionsForIdempotents : () -> List Polynomial R if R has INTDOM
```

```
--R conditionsForIdempotents : Vector % -> List Polynomial R if R has INTDOM
```

```
--R conditionsForIdempotents : () -> List Polynomial Fraction Polynomial R
```

```
--R conditionsForIdempotents : Vector % -> List Polynomial Fraction Polynomial R
```

```
--R convert : Vector Fraction Polynomial R -> %
```

```
--R convert : % -> Vector Fraction Polynomial R
```

```
--R coordinates : Vector % -> Matrix Fraction Polynomial R
```

```
--R coordinates : % -> Vector Fraction Polynomial R
```

```
--R coordinates : (Vector %,Vector %) -> Matrix Fraction Polynomial R
```

```
--R coordinates : (% ,Vector %) -> Vector Fraction Polynomial R
```

```
--R ?.? : (% ,Integer) -> Fraction Polynomial R
```

```
--R generic : (Vector Symbol,Vector %) -> %
```

```
--R genericLeftDiscriminant : () -> Fraction Polynomial R if R has INTDOM
```

```

--R genericLeftMinimalPolynomial : % -> SparseUnivariatePolynomial Fraction Polynomial R if R has INTDOM
--R genericLeftNorm : % -> Fraction Polynomial R if R has INTDOM
--R genericLeftTrace : % -> Fraction Polynomial R if R has INTDOM
--R genericLeftTraceForm : (%,%) -> Fraction Polynomial R if R has INTDOM
--R genericRightDiscriminant : () -> Fraction Polynomial R if R has INTDOM
--R genericRightMinimalPolynomial : % -> SparseUnivariatePolynomial Fraction Polynomial R if R has INTDOM
--R genericRightNorm : % -> Fraction Polynomial R if R has INTDOM
--R genericRightTrace : % -> Fraction Polynomial R if R has INTDOM
--R genericRightTraceForm : (%,%) -> Fraction Polynomial R if R has INTDOM
--R leftCharacteristicPolynomial : % -> SparseUnivariatePolynomial Fraction Polynomial R
--R leftDiscriminant : () -> Fraction Polynomial R
--R leftDiscriminant : Vector % -> Fraction Polynomial R
--R leftMinimalPolynomial : % -> SparseUnivariatePolynomial Fraction Polynomial R if Fraction Polynomial
--R leftNorm : % -> Fraction Polynomial R
--R leftPower : (%,PositiveInteger) -> %
--R leftRankPolynomial : () -> SparseUnivariatePolynomial Fraction Polynomial R if R has INTDOM
--R leftRankPolynomial : () -> SparseUnivariatePolynomial Polynomial Fraction Polynomial R if Fraction P
--R leftRecip : % -> Union(%, "failed") if Fraction Polynomial R has INTDOM
--R leftRegularRepresentation : % -> Matrix Fraction Polynomial R
--R leftRegularRepresentation : (%,Vector %) -> Matrix Fraction Polynomial R
--R leftTrace : % -> Fraction Polynomial R
--R leftTraceMatrix : () -> Matrix Fraction Polynomial R
--R leftTraceMatrix : Vector % -> Matrix Fraction Polynomial R
--R leftUnit : () -> Union(%, "failed") if Fraction Polynomial R has INTDOM
--R leftUnits : () -> Union(Record(particular: %,basis: List %), "failed")
--R noncommutativeJordanAlgebra? : () -> Boolean
--R plenaryPower : (%,PositiveInteger) -> %
--R recip : % -> Union(%, "failed") if Fraction Polynomial R has INTDOM
--R represents : Vector Fraction Polynomial R -> %
--R represents : (Vector Fraction Polynomial R,Vector %) -> %
--R rightCharacteristicPolynomial : % -> SparseUnivariatePolynomial Fraction Polynomial R
--R rightDiscriminant : () -> Fraction Polynomial R
--R rightDiscriminant : Vector % -> Fraction Polynomial R
--R rightMinimalPolynomial : % -> SparseUnivariatePolynomial Fraction Polynomial R if Fraction Polynomial
--R rightNorm : % -> Fraction Polynomial R
--R rightPower : (%,PositiveInteger) -> %
--R rightRankPolynomial : () -> SparseUnivariatePolynomial Fraction Polynomial R if R has INTDOM
--R rightRankPolynomial : () -> SparseUnivariatePolynomial Polynomial Fraction Polynomial R if Fraction
--R rightRecip : % -> Union(%, "failed") if Fraction Polynomial R has INTDOM
--R rightRegularRepresentation : % -> Matrix Fraction Polynomial R
--R rightRegularRepresentation : (%,Vector %) -> Matrix Fraction Polynomial R
--R rightTrace : % -> Fraction Polynomial R
--R rightTraceMatrix : () -> Matrix Fraction Polynomial R
--R rightTraceMatrix : Vector % -> Matrix Fraction Polynomial R
--R rightUnit : () -> Union(%, "failed") if Fraction Polynomial R has INTDOM
--R rightUnits : () -> Union(Record(particular: %,basis: List %), "failed")
--R structuralConstants : () -> Vector Matrix Fraction Polynomial R
--R structuralConstants : Vector % -> Vector Matrix Fraction Polynomial R
--R subtractIfCan : (%,%) -> Union(%, "failed")
--R unit : () -> Union(%, "failed") if Fraction Polynomial R has INTDOM

```

```
--R
--E 1

)spool
)lisp (bye)
```

---

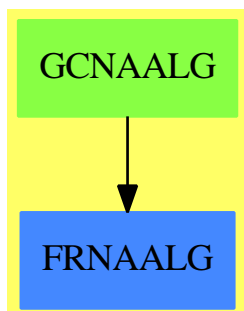
— GenericNonAssociativeAlgebra.help —

```
=====
GenericNonAssociativeAlgebra examples
=====
```

```
See Also:
o )show GenericNonAssociativeAlgebra
```

---

### 8.3.1 GenericNonAssociativeAlgebra (GCNAALG)



**Exports:**

|                               |                               |
|-------------------------------|-------------------------------|
| 0                             | alternative?                  |
| antiAssociative?              | antiCommutative?              |
| antiCommutator                | apply                         |
| associative?                  | associator                    |
| associatorDependence          | basis                         |
| coerce                        | commutative?                  |
| commutator                    | conditionsForIdempotents      |
| convert                       | convert                       |
| coordinates                   | coordinates                   |
| coordinates                   | coordinates                   |
| flexible?                     | generic                       |
| genericLeftDiscriminant       | genericLeftMinimalPolynomial  |
| genericLeftNorm               | genericLeftTrace              |
| genericLeftTraceForm          | genericRightDiscriminant      |
| genericRightMinimalPolynomial | genericRightNorm              |
| genericRightTrace             | genericRightTraceForm         |
| hash                          | jacobiIdentity?               |
| jordanAdmissible?             | jordanAlgebra?                |
| latex                         | leftAlternative?              |
| leftCharacteristicPolynomial  | leftDiscriminant              |
| leftDiscriminant              | leftMinimalPolynomial         |
| leftNorm                      | leftPower                     |
| leftRankPolynomial            | leftRankPolynomial            |
| leftRecip                     | leftRegularRepresentation     |
| leftRegularRepresentation     | leftTrace                     |
| leftTraceMatrix               | leftTraceMatrix               |
| leftUnit                      | leftUnits                     |
| lieAdmissible?                | lieAlgebra?                   |
| noncommutativeJordanAlgebra?  | plenaryPower                  |
| powerAssociative?             | rank                          |
| recip                         | represents                    |
| rightAlternative?             | rightCharacteristicPolynomial |
| rightDiscriminant             | rightDiscriminant             |
| rightMinimalPolynomial        | rightNorm                     |
| rightPower                    | rightRankPolynomial           |
| rightRankPolynomial           | rightRecip                    |
| rightRegularRepresentation    | rightRegularRepresentation    |
| rightTrace                    | rightTraceMatrix              |
| rightTraceMatrix              | rightUnit                     |
| rightUnits                    | sample                        |
| someBasis                     | structuralConstants           |
| structuralConstants           | subtractIfCan                 |
| unit                          | zero?                         |
| ?*?                           | ?**?                          |
| ?+?                           | ?-?                           |
| -?                            | ?=?                           |
| ?..?                          | ?~=?                          |

## — domain GCNAALG GenericNonAssociativeAlgebra —

```

)abbrev domain GCNAALG GenericNonAssociativeAlgebra
++ Authors: J. Grabmeier, R. Wisbauer
++ Date Created: 26 June 1991
++ Date Last Updated: 26 June 1991
++ Basic Operations: generic
++ Related Constructors: AlgebraPackage
++ Also See:
++ AMS Classifications:
++ Keywords: generic element. rank polynomial
++ Reference:
++ A. Woerz-Busekros: Algebra in Genetics
++ Lectures Notes in Biomathematics 36,
++ Springer-Verlag, Heidelberg, 1980
++ Description:
++ AlgebraGenericElementPackage allows you to create generic elements
++ of an algebra, i.e. the scalars are extended to include symbolic
++ coefficients

GenericNonAssociativeAlgebra(R : CommutativeRing, n : PositiveInteger, _
  ls : List Symbol, gamma: Vector Matrix R ): public == private where

NNI ==> NonNegativeInteger
V ==> Vector
PR ==> Polynomial R
FPR ==> Fraction Polynomial R
SUP ==> SparseUnivariatePolynomial
S ==> Symbol

public ==> Join(FramedNonAssociativeAlgebra(FPR), _
  LeftModule(SquareMatrix(n,FPR)) ) with

coerce : Vector FPR -> %
++ coerce(v) assumes that it is called with a vector
++ of length equal to the dimension of the algebra, then
++ a linear combination with the basis element is formed
leftUnits() -> Union(Record(particular: %, basis: List %), "failed")
++ leftUnits() returns the affine space of all left units of the
++ algebra, or \spad{"failed"} if there is none
rightUnits() -> Union(Record(particular: %, basis: List %), "failed")
++ rightUnits() returns the affine space of all right units of the
++ algebra, or \spad{"failed"} if there is none
generic : () -> %
++ generic() returns a generic element, i.e. the linear combination
++ of the fixed basis with the symbolic coefficients
++ \spad{%x1,%x2,...}
generic : Symbol -> %
++ generic(s) returns a generic element, i.e. the linear combination

```

```

    ++ of the fixed basis with the symbolic coefficients
    ++ \spad{s1,s2,..}
generic : Vector Symbol -> %
    ++ generic(vs) returns a generic element, i.e. the linear combination
    ++ of the fixed basis with the symbolic coefficients
    ++ \spad{vs};
    ++ error, if the vector of symbols is too short
generic : Vector % -> %
    ++ generic(ve) returns a generic element, i.e. the linear combination
    ++ of \spad{ve} basis with the symbolic coefficients
    ++ \spad{%x1,%x2,..}
generic : (Symbol, Vector %) -> %
    ++ generic(s,v) returns a generic element, i.e. the linear combination
    ++ of v with the symbolic coefficients
    ++ \spad{s1,s2,..}
generic : (Vector Symbol, Vector %) -> %
    ++ generic(vs,ve) returns a generic element, i.e. the linear combination
    ++ of \spad{ve} with the symbolic coefficients \spad{vs}
    ++ error, if the vector of symbols is shorter than the vector of
    ++ elements
if R has IntegralDomain then
    leftRankPolynomial : () -> SparseUnivariatePolynomial FPR
        ++ leftRankPolynomial() returns the left minimal polynomial
        ++ of the generic element
    genericLeftMinimalPolynomial : % -> SparseUnivariatePolynomial FPR
        ++ genericLeftMinimalPolynomial(a) substitutes the coefficients
        ++ of {em a} for the generic coefficients in
        ++ \spad{leftRankPolynomial()}
    genericLeftTrace : % -> FPR
        ++ genericLeftTrace(a) substitutes the coefficients
        ++ of \spad{a} for the generic coefficients into the
        ++ coefficient of the second highest term in
        ++ \spadfun{leftRankPolynomial} and changes the sign.
        ++ This is a linear form
    genericLeftNorm : % -> FPR
        ++ genericLeftNorm(a) substitutes the coefficients
        ++ of \spad{a} for the generic coefficients into the
        ++ coefficient of the constant term in \spadfun{leftRankPolynomial}
        ++ and changes the sign if the degree of this polynomial is odd.
        ++ This is a form of degree k
    rightRankPolynomial : () -> SparseUnivariatePolynomial FPR
        ++ rightRankPolynomial() returns the right minimal polynomial
        ++ of the generic element
    genericRightMinimalPolynomial : % -> SparseUnivariatePolynomial FPR
        ++ genericRightMinimalPolynomial(a) substitutes the coefficients
        ++ of \spad{a} for the generic coefficients in
        ++ \spadfun{rightRankPolynomial}
    genericRightTrace : % -> FPR
        ++ genericRightTrace(a) substitutes the coefficients
        ++ of \spad{a} for the generic coefficients into the

```

```

    ++ coefficient of the second highest term in
    ++ \spadfun{rightRankPolynomial} and changes the sign
genericRightNorm : % -> FPR
    ++ genericRightNorm(a) substitutes the coefficients
    ++ of \spad{a} for the generic coefficients into the
    ++ coefficient of the constant term in \spadfun{rightRankPolynomial}
    ++ and changes the sign if the degree of this polynomial is odd
genericLeftTraceForm : (%,%) -> FPR
    ++ genericLeftTraceForm (a,b) is defined to be
    ++ \spad{genericLeftTrace (a*b)}, this defines
    ++ a symmetric bilinear form on the algebra
genericLeftDiscriminant: () -> FPR
    ++ genericLeftDiscriminant() is the determinant of the
    ++ generic left trace forms of all products of basis element,
    ++ if the generic left trace form is associative, an algebra
    ++ is separable if the generic left discriminant is invertible,
    ++ if it is non-zero, there is some ring extension which
    ++ makes the algebra separable
genericRightTraceForm : (%,%) -> FPR
    ++ genericRightTraceForm (a,b) is defined to be
    ++ \spadfun{genericRightTrace (a*b)}, this defines
    ++ a symmetric bilinear form on the algebra
genericRightDiscriminant: () -> FPR
    ++ genericRightDiscriminant() is the determinant of the
    ++ generic left trace forms of all products of basis element,
    ++ if the generic left trace form is associative, an algebra
    ++ is separable if the generic left discriminant is invertible,
    ++ if it is non-zero, there is some ring extension which
    ++ makes the algebra separable
conditionsForIdempotents: Vector % -> List Polynomial R
    ++ conditionsForIdempotents([v1,...,vn]) determines a complete list
    ++ of polynomial equations for the coefficients of idempotents
    ++ with respect to the \spad{R}-module basis \spad{v1},...,\spad{vn}
conditionsForIdempotents: () -> List Polynomial R
    ++ conditionsForIdempotents() determines a complete list
    ++ of polynomial equations for the coefficients of idempotents
    ++ with respect to the fixed \spad{R}-module basis

private ==> AlgebraGivenByStructuralConstants(FPR,n,ls,_
    coerce(gamma)$CoerceVectorMatrixPackage(R) ) add

listOfNumbers : List String := [STRINGIMAGE(q)$Lisp for q in 1..n]
symbolsForCoef : V Symbol :=
    [concat("%", concat("x", i))::Symbol for i in listOfNumbers]
genericElement : % :=
    v : Vector PR :=
        [monomial(1$PR, [symbolsForCoef.i],[1]) for i in 1..n]
    convert map(coerce,v)$VectorFunctions2(PR,FPR)

eval : (FPR, %) -> FPR

```

```

eval(rf,a) ==
  -- for the moment we only substitute the numerators
  -- of the coefficients
  coefOfa : List PR :=
    map(number, entries coordinates a)$ListFunctions2(FPR,PR)
  ls : List PR :=[monomial(1$PR, [s],[1]) for s in entries symbolsForCoef]
  lEq : List Equation PR := []
  for i in 1..maxIndex ls repeat
    lEq := cons(equation(ls.i,coefOfa.i)$Equation(PR) , lEq)
  top : PR := eval(number(rf),lEq)$PR
  bot : PR := eval(number(rf),lEq)$PR
  top/bot

if R has IntegralDomain then

  genericLeftTraceForm(a,b) == genericLeftTrace(a*b)
  genericLeftDiscriminant() ==
    listBasis : List % := entries basis()$%
    m : Matrix FPR := matrix
      [[genericLeftTraceForm(a,b) for a in listBasis] for b in listBasis]
    determinant m

  genericRightTraceForm(a,b) == genericRightTrace(a*b)
  genericRightDiscriminant() ==
    listBasis : List % := entries basis()$%
    m : Matrix FPR := matrix
      [[genericRightTraceForm(a,b) for a in listBasis] for b in listBasis]
    determinant m

  leftRankPoly : SparseUnivariatePolynomial FPR := 0
  initLeft? : Boolean :=true

  initializeLeft: () -> Void
  initializeLeft() ==
    -- reset initialize flag
    initLeft?:=false
    leftRankPoly := leftMinimalPolynomial genericElement
    void()$Void

  rightRankPoly : SparseUnivariatePolynomial FPR := 0
  initRight? : Boolean :=true

  initializeRight: () -> Void
  initializeRight() ==
    -- reset initialize flag
    initRight?:=false
    rightRankPoly := rightMinimalPolynomial genericElement

```



```

void()$Void

leftRankPolynomial() ==
  if initLeft? then initializeLeft()
  leftRankPoly

rightRankPolynomial() ==
  if initRight? then initializeRight()
  rightRankPoly

genericLeftMinimalPolynomial a ==
  if initLeft? then initializeLeft()
  map(x+>eval(x,a),leftRankPoly)$SUP(FPR)

genericRightMinimalPolynomial a ==
  if initRight? then initializeRight()
  map(x+>eval(x,a),rightRankPoly)$SUP(FPR)

genericLeftTrace a ==
  if initLeft? then initializeLeft()
  d1 : NNI := (degree leftRankPoly - 1) :: NNI
  rf : FPR := coefficient(leftRankPoly, d1)
  rf := eval(rf,a)
  - rf

genericRightTrace a ==
  if initRight? then initializeRight()
  d1 : NNI := (degree rightRankPoly - 1) :: NNI
  rf : FPR := coefficient(rightRankPoly, d1)
  rf := eval(rf,a)
  - rf

genericLeftNorm a ==
  if initLeft? then initializeLeft()
  rf : FPR := coefficient(leftRankPoly, 1)
  if odd? degree leftRankPoly then rf := - rf
  rf

genericRightNorm a ==
  if initRight? then initializeRight()
  rf : FPR := coefficient(rightRankPoly, 1)
  if odd? degree rightRankPoly then rf := - rf
  rf

conditionsForIdempotents(b: V %) : List Polynomial R ==
  x : % := generic(b)
  map(enumer, entries coordinates(x*x-x,b))$ListFunctions2(FPR,PR)

conditionsForIdempotents(): List Polynomial R ==
  x : % := genericElement

```

```

    map(numer,entries coordinates(x*x-x))$ListFunctions2(FPR,PR)

generic() == genericElement

generic(vs:V S, ve: V %): % ==
  maxIndex v > maxIndex ve =>
    error "generic: too little symbols"
  v : Vector PR :=
    [monomial(1$PR, [vs.i],[1]) for i in 1..maxIndex ve]
  represents(map(coerce,v)$VectorFunctions2(PR,FPR),ve)

generic(s: S, ve: V %): % ==
  lON : List String := [STRINGIMAGE(q)$Lisp for q in 1..maxIndex ve]
  sFC : Vector Symbol :=
    [concat(s pretend String, i)::Symbol for i in lON]
  generic(sFC, ve)

generic(ve : V %) ==
  lON : List String := [STRINGIMAGE(q)$Lisp for q in 1..maxIndex ve]
  sFC : Vector Symbol :=
    [concat("%", concat("x", i))::Symbol for i in lON]
  v : Vector PR :=
    [monomial(1$PR, [sFC.i],[1]) for i in 1..maxIndex ve]
  represents(map(coerce,v)$VectorFunctions2(PR,FPR),ve)

generic(vs:V S): % == generic(vs, basis())$%)

generic(s: S): % == generic(s, basis())$%)

-- variations on eval
--coef0fa : List FPR := entries coordinates a
--ls : List Symbol := entries symbolsForCoef
-- a very dangerous sequential implementation for the moment,
-- because the compiler doesn't manage the parallel code
-- also doesn't run:
-- not known that (Fraction (Polynomial R)) has (has (Polynomial R)
-- (Evalable (Fraction (Polynomial R))))
--res : FPR := rf
--for eq in lEq repeat res := eval(res,eq)$FPR
--res
--rf
--eval(rf, le)$FPR
--eval(rf, entries symbolsForCoef, coef0fa)$FPR
--eval(rf, ls, coef0fa)$FPR
--le : List Equation PR := [equation(lh,rh) for lh in ls for rh in coef0fa]

```

---

— GCNAALG.dotabb —

```
"GCNAALG" [color="#88FF44",href="bookvol10.3.pdf#nameddest=GCNAALG"]
"FRNAALG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FRNAALG"]
"GCNAALG" -> "FRNAALG"
```

—

## 8.4 domain GPOLSET GeneralPolynomialSet

— GeneralPolynomialSet.input —

```
)set break resume
)sys rm -f GeneralPolynomialSet.output
)spool GeneralPolynomialSet.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show GeneralPolynomialSet
--R GeneralPolynomialSet(R: Ring,E: OrderedAbelianMonoidSup,VarSet: OrderedSet,P: RecursiveP
--R Abbreviation for GeneralPolynomialSet is GPOLSET
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for GPOLSET
--R
--R----- Operations -----
--R ?=? : (%,%) -> Boolean          coerce : % -> List P
--R coerce : % -> OutputForm        collect : (%,VarSet) -> %
--R collectUnder : (%,VarSet) -> %  collectUpper : (%,VarSet) -> %
--R construct : List P -> %         convert : List P -> %
--R copy : % -> %                  empty : () -> %
--R empty? : % -> Boolean          eq? : (%,%) -> Boolean
--R hash : % -> SingleInteger      latex : % -> String
--R mainVariables : % -> List VarSet map : ((P -> P),%) -> %
--R mvar : % -> VarSet             retract : List P -> %
--R sample : () -> %              trivialIdeal? : % -> Boolean
--R variables : % -> List VarSet   ?~=? : (%,%) -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R any? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
--R convert : % -> InputForm if P has KONVERT INFORM
--R count : ((P -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R count : (P,%) -> NonNegativeInteger if $ has finiteAggregate and P has SETCAT
--R eval : (%,List Equation P) -> % if P has EVALAB P and P has SETCAT
--R eval : (%,Equation P) -> % if P has EVALAB P and P has SETCAT
```

```

--R eval : (% , P , P) -> % if P has EVALAB P and P has SETCAT
--R eval : (% , List P , List P) -> % if P has EVALAB P and P has SETCAT
--R every? : ((P -> Boolean) , %) -> Boolean if $ has finiteAggregate
--R find : ((P -> Boolean) , %) -> Union(P , "failed")
--R headRemainder : (P , %) -> Record(num: P , den: R) if R has INTDOM
--R less? : (% , NonNegativeInteger) -> Boolean
--R mainVariable? : (VarSet , %) -> Boolean
--R map! : ((P -> P) , %) -> % if $ has shallowlyMutable
--R member? : (P , %) -> Boolean if $ has finiteAggregate and P has SETCAT
--R members : % -> List P if $ has finiteAggregate
--R more? : (% , NonNegativeInteger) -> Boolean
--R parts : % -> List P if $ has finiteAggregate
--R reduce : (((P , P) -> P) , %) -> P if $ has finiteAggregate
--R reduce : (((P , P) -> P) , % , P) -> P if $ has finiteAggregate
--R reduce : (((P , P) -> P) , % , P , P) -> P if $ has finiteAggregate and P has SETCAT
--R remainder : (P , %) -> Record(rnum: R , polnum: P , den: R) if R has INTDOM
--R remove : ((P -> Boolean) , %) -> % if $ has finiteAggregate
--R remove : (P , %) -> % if $ has finiteAggregate and P has SETCAT
--R removeDuplicates : % -> % if $ has finiteAggregate and P has SETCAT
--R retractIfCan : List P -> Union(% , "failed")
--R rewriteIdealWithHeadRemainder : (List P , %) -> List P if R has INTDOM
--R rewriteIdealWithRemainder : (List P , %) -> List P if R has INTDOM
--R roughBase? : % -> Boolean if R has INTDOM
--R roughEqualIdeals? : (% , %) -> Boolean if R has INTDOM
--R roughSubIdeal? : (% , %) -> Boolean if R has INTDOM
--R roughUnitIdeal? : % -> Boolean if R has INTDOM
--R select : ((P -> Boolean) , %) -> % if $ has finiteAggregate
--R size? : (% , NonNegativeInteger) -> Boolean
--R sort : (% , VarSet) -> Record(under: % , floor: % , upper: %)
--R triangular? : % -> Boolean if R has INTDOM
--R
--E 1

)spool
)lisp (bye)

```

---

— GeneralPolynomialSet.help —

```

=====
GeneralPolynomialSet examples
=====

```

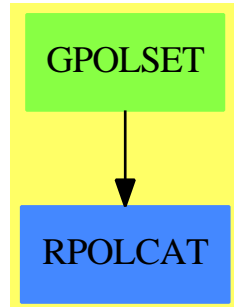
```

See Also:
o )show GeneralPolynomialSet

```

---

### 8.4.1 GeneralPolynomialSet (GPOLSET)



#### Exports:

|                               |                           |
|-------------------------------|---------------------------|
| any?                          | coerce                    |
| collect                       | collectUnder              |
| collectUpper                  | construct                 |
| convert                       | copy                      |
| count                         | empty                     |
| empty?                        | eq?                       |
| eval                          | every?                    |
| find                          | hash                      |
| headRemainder                 | latex                     |
| less?                         | mainVariables             |
| mainVariable?                 | map                       |
| map!                          | member?                   |
| members                       | more?                     |
| mvar                          | parts                     |
| reduce                        | remainder                 |
| remove                        | removeDuplicates          |
| retract                       | retractIfCan              |
| rewriteIdealWithHeadRemainder | rewriteIdealWithRemainder |
| roughBase?                    | roughEqualIdeals?         |
| roughSubIdeal?                | roughUnitIdeal?           |
| sample                        | select                    |
| size?                         | sort                      |
| triangular?                   | trivialIdeal?             |
| variables                     | #?                        |
| ?=?                           | ?~=?                      |

— domain GPOLSET GeneralPolynomialSet —

```

)abbrev domain GPOLSET GeneralPolynomialSet
++ Author: Marc Moreno Maza
++ Date Created: 04/26/1994
++ Date Last Updated: 12/15/1998
  
```

```

++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords: polynomial, multivariate, ordered variables set
++ References:
++ Description:
++ A domain for polynomial sets.

GeneralPolynomialSet(R,E,VarSet,P) : Exports == Implementation where

  R:Ring
  VarSet:OrderedSet
  E:OrderedAbelianMonoidSup
  P:RecursivePolynomialCategory(R,E,VarSet)
  LP ==> List P
  PtoP ==> P -> P

  Exports == PolynomialSetCategory(R,E,VarSet,P) with

    convert : LP -> $
      ++ \axiom{convert(lp)} returns the polynomial set whose members
      ++ are the polynomials of \axiom{lp}.

    finiteAggregate
    shallowlyMutable

Implementation == add

  Rep := List P

  construct lp ==
    (removeDuplicates(lp)$List(P))::$

  copy ps ==
    construct(copy(members(ps))$LP)$LP

  empty() ==
    []

  parts ps ==
    ps pretend LP

  map (f : PtoP, ps : $) : $ ==
    construct(map(f,members(ps))$LP)$LP

  map! (f : PtoP, ps : $) : $ ==
    construct(map!(f,members(ps))$LP)$LP

  member? (p,ps) ==

```

```

member?(p,members(ps))$LP

ps1 = ps2 ==
  {p for p in parts(ps1)} =$(Set P) {p for p in parts(ps2)}

coerce(ps:$) : OutputForm ==
  lp : List(P) := sort(infRittWu?,members(ps))$(List P)
  brace([p::OutputForm for p in lp]$List(OutputForm))$OutputForm

mvar ps ==
  empty? ps => error"Error from GPOLSET in mvar : #1 is empty"
  lv : List VarSet := variables(ps)
  empty? lv =>
    error "Error from GPOLSET in mvar : every polynomial in #1 is constant"
    reduce(max,lv)$(List VarSet)

retractIfCan(lp) ==
  (construct(lp))::Union($,"failed")

coerce(ps:$) : (List P) ==
  ps pretend (List P)

convert(lp:LP) : $ ==
  construct lp

```

---

— GPOLSET.dotabb —

```

"GPOLSET" [color="#88FF44",href="bookvol10.3.pdf#nameddest=GPOLSET"]
"RPOLCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=RPOLCAT"]
"GPOLSET" -> "RPOLCAT"

```

---

## 8.5 domain GSTBL GeneralSparseTable

---

— GeneralSparseTable.input —

```

)set break resume
)sys rm -f GeneralSparseTable.output
)spool GeneralSparseTable.output
)set message test on
)set message auto off

```

```

)set break resume
)clear all
--S 1 of 7
patrons: GeneralSparseTable(String, Integer, KeyedAccessFile(Integer), 0) := table() ;
--E 1

--S 2 of 7
patrons."Smith" := 10500
--E 2

--S 3 of 7
patrons."Jones" := 22000
--E 3

--S 4 of 7
patrons."Jones"
--E 4

--S 5 of 7
patrons."Stingy"
--E 5

--S 6 of 7
reduce(+, entries patrons)
--E 6

--S 7 of 7
)system rm -r kaf*.sdata
--E 7
)spool
)lisp (bye)

```

---

#### — GeneralSparseTable.help —

```

=====
GeneralSparseTable
=====

```

Sometimes when working with tables there is a natural value to use as the entry in all but a few cases. The GeneralSparseTable constructor can be used to provide any table type with a default value for entries.

Suppose we launched a fund-raising campaign to raise fifty thousand dollars. To record the contributions, we want a table with strings as keys (for the names) and integer entries (for the amount). In a data base of cash contributions, unless someone has been explicitly



entered, it is reasonable to assume they have made a zero dollar contribution.

This creates a keyed access file with default entry 0.

```
patrons: GeneralSparseTable(String, Integer, KeyedAccessFile(Integer), 0) := table() ;
```

Now patrons can be used just as any other table. Here we record two gifts.

```
patrons."Smith" := 10500
```

```
patrons."Jones" := 22000
```

Now let us look up the size of the contributions from Jones and Stingy.

```
patrons."Jones"
```

```
patrons."Stingy"
```

Have we met our seventy thousand dollar goal?

```
reduce(+, entries patrons)
```

So the project is cancelled and we can delete the data base:

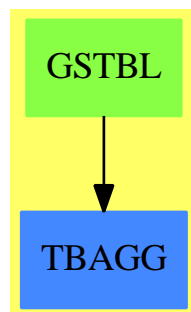
```
)system rm -r kaf*.sdata
```

See Also:

```
o )show GeneralSparseTable
```

—————▶

### 8.5.1 GeneralSparseTable (GSTBL)



See

⇒ “HashTable” (HASHTBL) 9.1.1 on page 1085  
 ⇒ “InnerTable” (INTABL) 10.27.1 on page 1299  
 ⇒ “Table” (TABLE) 21.1.1 on page 2621  
 ⇒ “EqTable” (EQTBL) 6.2.1 on page 667  
 ⇒ “StringTable” (STRTBL) 20.32.1 on page 2569  
 ⇒ “SparseTable” (STBL) 20.16.1 on page 2409

**Exports:**

|        |          |                  |           |          |
|--------|----------|------------------|-----------|----------|
| any?   | bag      | coerce           | construct | convert  |
| copy   | count    | dictionary       | elt       | empty    |
| empty? | entries  | entry?           | eq?       | eval     |
| every? | extract! | fill!            | find      | first    |
| hash   | index?   | indices          | insert!   | inspect  |
| key?   | keys     | latex            | less?     | map      |
| map!   | maxIndex | member?          | members   | minIndex |
| more?  | parts    | qelt             | qsetelt!  | reduce   |
| remove | remove!  | removeDuplicates | sample    | search   |
| select | select!  | setelt           | size?     | swap!    |
| table  | #?       | ?=?              | ?~=?      | ??       |

— domain GSTBL GeneralSparseTable —

```
)abbrev domain GSTBL GeneralSparseTable
++ Author: Stephen M. Watt
++ Date Created: 1986
++ Date Last Updated: June 21, 1991
++ Basic Operations:
++ Related Domains: Table
++ Also See:
++ AMS Classifications:
++ Keywords: equation
++ Examples:
++ References:
++ Description:
++ A sparse table has a default entry, which is returned if no other
++ value has been explicitly stored for a key.

GeneralSparseTable(Key, Entry, Tbl, dent): TableAggregate(Key, Entry) == Impl
  where
    Key, Entry: SetCategory
    Tbl: TableAggregate(Key, Entry)
    dent: Entry

    Impl ==> Tbl add
      Rep := Tbl

      elt(t:%, k:Key) ==
        (u := search(k, t)$Rep) case "failed" => dent
```

```

u::Entry

setelt(t:%, k:Key, e:Entry) ==
  e = dent => (remove_!(k, t); e)
  setelt(t, k, e)$Rep

search(k:Key, t:%) ==
  (u := search(k, t)$Rep) case "failed" => dent
  u

```

---

— GSTBL.dotabb —

```

"GSTBL" [color="#88FF44",href="bookvol10.3.pdf#nameddest=GSTBL"]
"TBAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=TBAGG"]
"GSTBL" -> "TBAGG"

```

---

## 8.6 domain GTSET GeneralTriangularSet

— GeneralTriangularSet.input —

```

)set break resume
)sys rm -f GeneralTriangularSet.output
)spool GeneralTriangularSet.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show GeneralTriangularSet
--R GeneralTriangularSet(R: IntegralDomain,E: OrderedAbelianMonoidSup,V: OrderedSet,P: Recur
--R Abbreviation for GeneralTriangularSet is GTSET
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for GTSET
--R
--R----- Operations -----
--R ?=? : (%,% ) -> Boolean          algebraic? : (V,% ) -> Boolean
--R algebraicVariables : % -> List V  coerce : % -> List P
--R coerce : % -> OutputForm          collect : (% ,V) -> %
--R collectQuasiMonic : % -> %        collectUnder : (% ,V) -> %
--R collectUpper : (% ,V) -> %        construct : List P -> %

```

```

--R copy : % -> %
--R empty : () -> %
--R eq? : (% , %) -> Boolean
--R first : % -> Union(P, "failed")
--R headReduce : (P, %) -> P
--R headReduced? : (P, %) -> Boolean
--R initiallyReduce : (P, %) -> P
--R initials : % -> List P
--R latex : % -> String
--R mainVariables : % -> List V
--R mvar : % -> V
--R normalized? : (P, %) -> Boolean
--R removeZero : (P, %) -> P
--R retract : List P -> %
--R stronglyReduce : (P, %) -> P
--R trivialIdeal? : % -> Boolean
--R zeroSetSplit : List P -> List %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R any? : ((P -> Boolean), %) -> Boolean if $ has finiteAggregate
--R autoReduced? : (% , ((P, List P) -> Boolean)) -> Boolean
--R basicSet : (List P, (P -> Boolean), ((P, P) -> Boolean)) -> Union(Record(bas: %, top: List P), "failed")
--R basicSet : (List P, ((P, P) -> Boolean)) -> Union(Record(bas: %, top: List P), "failed")
--R coHeight : % -> NonNegativeInteger if V has FINITE
--R convert : % -> InputForm if P has KONVERT INFORM
--R count : ((P -> Boolean), %) -> NonNegativeInteger if $ has finiteAggregate
--R count : (P, %) -> NonNegativeInteger if $ has finiteAggregate and P has SETCAT
--R eval : (% , List Equation P) -> % if P has EVALAB P and P has SETCAT
--R eval : (% , Equation P) -> % if P has EVALAB P and P has SETCAT
--R eval : (% , P, P) -> % if P has EVALAB P and P has SETCAT
--R eval : (% , List P, List P) -> % if P has EVALAB P and P has SETCAT
--R every? : ((P -> Boolean), %) -> Boolean if $ has finiteAggregate
--R extendIfCan : (% , P) -> Union(% , "failed")
--R find : ((P -> Boolean), %) -> Union(P, "failed")
--R headRemainder : (P, %) -> Record(num: P, den: R) if R has INTDOM
--R initiallyReduced? : (P, %) -> Boolean
--R less? : (% , NonNegativeInteger) -> Boolean
--R map! : ((P -> P), %) -> % if $ has shallowlyMutable
--R member? : (P, %) -> Boolean if $ has finiteAggregate and P has SETCAT
--R members : % -> List P if $ has finiteAggregate
--R more? : (% , NonNegativeInteger) -> Boolean
--R parts : % -> List P if $ has finiteAggregate
--R quasiComponent : % -> Record(close: List P, open: List P)
--R reduce : (P, % , ((P, P) -> P), ((P, P) -> Boolean)) -> P
--R reduce : (((P, P) -> P), %) -> P if $ has finiteAggregate
--R reduce : (((P, P) -> P), % , P) -> P if $ has finiteAggregate
--R reduce : (((P, P) -> P), % , P, P) -> P if $ has finiteAggregate and P has SETCAT
--R reduced? : (P, % , ((P, P) -> Boolean)) -> Boolean
--R remainder : (P, %) -> Record(rnum: R, polnum: P, den: R) if R has INTDOM
--R remove : ((P -> Boolean), %) -> % if $ has finiteAggregate
--R remove : (P, %) -> % if $ has finiteAggregate and P has SETCAT
--R degree : % -> NonNegativeInteger
--R empty? : % -> Boolean
--R extend : (% , P) -> %
--R hash : % -> SingleInteger
--R headReduced? : % -> Boolean
--R infRittWu? : (% , %) -> Boolean
--R initiallyReduced? : % -> Boolean
--R last : % -> Union(P, "failed")
--R mainVariable? : (V, %) -> Boolean
--R map : ((P -> P), %) -> %
--R normalized? : % -> Boolean
--R reduceByQuasiMonic : (P, %) -> P
--R rest : % -> Union(% , "failed")
--R sample : () -> %
--R stronglyReduced? : % -> Boolean
--R variables : % -> List V
--R ~=? : (% , %) -> Boolean

```

```

--R removeDuplicates : % -> % if $ has finiteAggregate and P has SETCAT
--R retractIfCan : List P -> Union(%, "failed")
--R rewriteIdealWithHeadRemainder : (List P, %) -> List P if R has INTDOM
--R rewriteIdealWithRemainder : (List P, %) -> List P if R has INTDOM
--R rewriteSetWithReduction : (List P, %, ((P, P) -> P), ((P, P) -> Boolean)) -> List P
--R roughBase? : % -> Boolean if R has INTDOM
--R roughEqualIdeals? : (%, %) -> Boolean if R has INTDOM
--R roughSubIdeal? : (%, %) -> Boolean if R has INTDOM
--R roughUnitIdeal? : % -> Boolean if R has INTDOM
--R select : (%, V) -> Union(P, "failed")
--R select : ((P -> Boolean), %) -> % if $ has finiteAggregate
--R size? : (%, NonNegativeInteger) -> Boolean
--R sort : (%, V) -> Record(under: %, floor: %, upper: %)
--R stronglyReduced? : (P, %) -> Boolean
--R triangular? : % -> Boolean if R has INTDOM
--R zeroSetSplitIntoTriangularSystems : List P -> List Record(close: %, open: List P)
--R
--E 1

)spool
)lisp (bye)

```

---

— GeneralTriangularSet.help —

```

=====
GeneralTriangularSet examples
=====

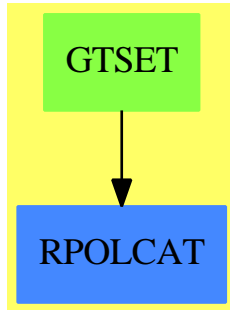
```

```

See Also:
o )show GeneralTriangularSet

```

### 8.6.1 GeneralTriangularSet (GTSET)



See

⇒ “WuWenTsunTriangularSet” (WUTSET) 24.2.1 on page 2884

**Exports:**

|                                   |                               |
|-----------------------------------|-------------------------------|
| algebraic?                        | algebraicVariables            |
| any?                              | autoReduced?                  |
| basicSet                          | coerce                        |
| collect                           | collectQuasiMonic             |
| collectUnder                      | collectUpper                  |
| coHeight                          | construct                     |
| convert                           | copy                          |
| count                             | degree                        |
| empty                             | empty?                        |
| eq?                               | eval                          |
| every?                            | extend                        |
| extendIfCan                       | find                          |
| first                             | hash                          |
| headReduce                        | headReduced?                  |
| headReduced?                      | headRemainder                 |
| infRittWu?                        | initiallyReduce               |
| initiallyReduced?                 | initials                      |
| last                              | latex                         |
| less?                             | mainVariable?                 |
| mainVariables                     | map                           |
| map!                              | member?                       |
| members                           | more?                         |
| mvar                              | normalized?                   |
| normalized?                       | parts                         |
| quasiComponent                    | reduce                        |
| reduceByQuasiMonic                | reduced?                      |
| remainder                         | remove                        |
| removeDuplicates                  | removeZero                    |
| rest                              | retract                       |
| retractIfCan                      | rewriteIdealWithHeadRemainder |
| rewriteIdealWithRemainder         | rewriteSetWithReduction       |
| roughBase?                        | roughEqualIdeals?             |
| roughSubIdeal?                    | roughUnitIdeal?               |
| sample                            | select                        |
| size?                             | sort                          |
| stronglyReduce                    | stronglyReduced?              |
| triangular?                       | trivialIdeal?                 |
| variables                         | zeroSetSplit                  |
| zeroSetSplitIntoTriangularSystems | #?                            |
| ?=?                               | ?~=?                          |

— domain GTSET GeneralTriangularSet —

```
)abbrev domain GTSET GeneralTriangularSet
++ Author: Marc Moreno Maza (marc@nag.co.uk)
```

```

++ Date Created: 10/06/1995
++ Date Last Updated: 06/12/1996
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References :
++ [1] P. AUBRY, D. LAZARD and M. MORENO MAZA "On the Theories
++      of Triangular Sets" Journal of Symbol. Comp. (to appear)
++ Description:
++ A domain constructor of the category \axiomType{TriangularSetCategory}.
++ The only requirement for a list of polynomials to be a member of such
++ a domain is the following: no polynomial is constant and two distinct
++ polynomials have distinct main variables. Such a triangular set may
++ not be auto-reduced or consistent. Triangular sets are stored
++ as sorted lists w.r.t. the main variables of their members but they
++ are displayed in reverse order.

```

```

GeneralTriangularSet(R,E,V,P) : Exports == Implementation where

```

```

R : IntegralDomain
E : OrderedAbelianMonoidSup
V : OrderedSet
P : RecursivePolynomialCategory(R,E,V)
N ==> NonNegativeInteger
Z ==> Integer
B ==> Boolean
LP ==> List P
PtoP ==> P -> P

```

```

Exports == TriangularSetCategory(R,E,V,P)

```

```

Implementation == add

```

```

Rep ==> LP

rep(s:$):Rep == s pretend Rep
per(l:Rep):$ == l pretend $

copy ts ==
  per(copy(rep(ts))$LP)
empty() ==
  per([])
empty?(ts:$) ==
  empty?(rep(ts))
parts ts ==
  rep(ts)
members ts ==
  rep(ts)

```



```

map (f : PtoP, ts : $) : $ ==
  construct(map(f,rep(ts))$LP)$
map! (f : PtoP, ts : $) : $ ==
  construct(map!(f,rep(ts))$LP)$
member? (p,ts) ==
  member?(p,rep(ts))$LP

unitIdealIfCan() ==
  "failed"::Union($,"failed")
roughUnitIdeal? ts ==
  false

-- the following assume that rep(ts) is decreasingly sorted
-- w.r.t. the main variavles of the polynomials in rep(ts)
coerce(ts:$) : OutputForm ==
  lp : List(P) := reverse(rep(ts))
  brace([p::OutputForm for p in lp]$List(OutputForm))$OutputForm
mvar ts ==
  empty? ts => error"failed in mvar : $ -> V from GTSET"
  mvar(first(rep(ts)))$P
first ts ==
  empty? ts => "failed"::Union(P,"failed")
  first(rep(ts))::Union(P,"failed")
last ts ==
  empty? ts => "failed"::Union(P,"failed")
  last(rep(ts))::Union(P,"failed")
rest ts ==
  empty? ts => "failed"::Union($,"failed")
  per(rest(rep(ts)))::Union($,"failed")
coerce(ts:$) : (List P) ==
  rep(ts)
collectUpper (ts,v) ==
  empty? ts => ts
  lp := rep(ts)
  newlp : Rep := []
  while (not empty? lp) and (mvar(first(lp)) > v) repeat
    newlp := cons(first(lp),newlp)
    lp := rest lp
  per(reverse(newlp))
collectUnder (ts,v) ==
  empty? ts => ts
  lp := rep(ts)
  while (not empty? lp) and (mvar(first(lp)) >= v) repeat
    lp := rest lp
  per(lp)

-- for another domain of TSETCAT build on this domain GTSET
-- the following operations must be redefined
extendIfCan(ts:$,p:P) ==
  ground? p => "failed"::Union($,"failed")

```

```

empty? ts => (per([unitCanonical(p)]$LP))::Union($,"failed")
not (mvar(ts) < mvar(p)) => "failed":Union($,"failed")
(per(cons(p,rep(ts))))::Union($,"failed")

```

---

— GTSET.dotabb —

```

"GTSET" [color="#88FF44",href="bookvol10.3.pdf#nameddest=GTSET"]
"RPOLCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=RPOLCAT"]
"GTSET" -> "RPOLCAT"

```

---

## 8.7 domain GSERIES GeneralUnivariatePowerSeries

— GeneralUnivariatePowerSeries.input —

```

)set break resume
)sys rm -f GeneralUnivariatePowerSeries.output
)spool GeneralUnivariatePowerSeries.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show GeneralUnivariatePowerSeries
--R GeneralUnivariatePowerSeries(Coef: Ring,var: Symbol,cen: Coef) is a domain constructor
--R Abbreviation for GeneralUnivariatePowerSeries is GSERIES
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for GSERIES
--R
--R----- Operations -----
--R ??? : (Coef,%) -> %          ??? : (%,Coef) -> %
--R ??? : (%,%) -> %           ??? : (Integer,%) -> %
--R ??? : (PositiveInteger,%) -> %   ??? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %           ?-? : (%,%) -> %
--R -? : % -> %                ?=? : (%,%) -> Boolean
--R 1 : () -> %                0 : () -> %
--R ?? : (%,PositiveInteger) -> %   center : % -> Coef
--R coerce : Variable var -> %       coerce : Integer -> %
--R coerce : % -> OutputForm         complete : % -> %
--R degree : % -> Fraction Integer   hash : % -> SingleInteger
--R latex : % -> String              leadingCoefficient : % -> Coef

```

```

--R leadingMonomial : % -> %                                map : ((Coef -> Coef),%) -> %
--R monomial? : % -> Boolean                                one? : % -> Boolean
--R order : % -> Fraction Integer                            pole? : % -> Boolean
--R recip : % -> Union(%, "failed")                          reductum : % -> %
--R sample : () -> %                                         variable : % -> Symbol
--R zero? : % -> Boolean                                     ~=? : (%,%) -> Boolean
--R ?? : (%, Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?? : (Fraction Integer, %) -> % if Coef has ALGEBRA FRAC INT
--R ?? : (NonNegativeInteger, %) -> %
--R ??? : (%, Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ??? : (%, %) -> % if Coef has ALGEBRA FRAC INT
--R ??? : (%, Integer) -> % if Coef has FIELD
--R ??? : (%, NonNegativeInteger) -> %
--R ??/? : (%, %) -> % if Coef has FIELD
--R ??/? : (%, Coef) -> % if Coef has FIELD
--R D : % -> % if Coef has *: (Fraction Integer, Coef) -> Coef
--R D : (%, NonNegativeInteger) -> % if Coef has *: (Fraction Integer, Coef) -> Coef
--R D : (%, Symbol) -> % if Coef has *: (Fraction Integer, Coef) -> Coef and Coef has PDRING S
--R D : (%, List Symbol) -> % if Coef has *: (Fraction Integer, Coef) -> Coef and Coef has PDR
--R D : (%, Symbol, NonNegativeInteger) -> % if Coef has *: (Fraction Integer, Coef) -> Coef and
--R D : (%, List Symbol, List NonNegativeInteger) -> % if Coef has *: (Fraction Integer, Coef)
--R ?? : (%, Integer) -> % if Coef has FIELD
--R ?? : (%, NonNegativeInteger) -> %
--R acos : % -> % if Coef has ALGEBRA FRAC INT
--R acosh : % -> % if Coef has ALGEBRA FRAC INT
--R acot : % -> % if Coef has ALGEBRA FRAC INT
--R acoth : % -> % if Coef has ALGEBRA FRAC INT
--R acsc : % -> % if Coef has ALGEBRA FRAC INT
--R acsch : % -> % if Coef has ALGEBRA FRAC INT
--R approximate : (%, Fraction Integer) -> Coef if Coef has **: (Coef, Fraction Integer) -> Co
--R asec : % -> % if Coef has ALGEBRA FRAC INT
--R asech : % -> % if Coef has ALGEBRA FRAC INT
--R asin : % -> % if Coef has ALGEBRA FRAC INT
--R asinh : % -> % if Coef has ALGEBRA FRAC INT
--R associates? : (%, %) -> Boolean if Coef has INTDOM
--R atan : % -> % if Coef has ALGEBRA FRAC INT
--R atanh : % -> % if Coef has ALGEBRA FRAC INT
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%, "failed") if Coef has CHARNZ
--R coefficient : (%, Fraction Integer) -> Coef
--R coerce : % -> % if Coef has INTDOM
--R coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
--R coerce : UnivariatePuisseuxSeries(Coef, var, cen) -> %
--R coerce : Coef -> % if Coef has COMRING
--R cos : % -> % if Coef has ALGEBRA FRAC INT
--R cosh : % -> % if Coef has ALGEBRA FRAC INT
--R cot : % -> % if Coef has ALGEBRA FRAC INT
--R coth : % -> % if Coef has ALGEBRA FRAC INT
--R csc : % -> % if Coef has ALGEBRA FRAC INT
--R csch : % -> % if Coef has ALGEBRA FRAC INT

```

```

--R differentiate : (% , Variable var) -> %
--R differentiate : % -> % if Coef has *: (Fraction Integer, Coef) -> Coef
--R differentiate : (% , NonNegativeInteger) -> % if Coef has *: (Fraction Integer, Coef) -> Coef
--R differentiate : (% , Symbol) -> % if Coef has *: (Fraction Integer, Coef) -> Coef and Coef has PDRING S
--R differentiate : (% , List Symbol) -> % if Coef has *: (Fraction Integer, Coef) -> Coef and Coef has PDR
--R differentiate : (% , Symbol, NonNegativeInteger) -> % if Coef has *: (Fraction Integer, Coef) -> Coef an
--R differentiate : (% , List Symbol, List NonNegativeInteger) -> % if Coef has *: (Fraction Integer, Coef)
--R divide : (% , %) -> Record(quotient: % , remainder: %) if Coef has FIELD
--R ?.? : (% , %) -> % if Fraction Integer has SGROUP
--R ?.? : (% , Fraction Integer) -> Coef
--R euclideanSize : % -> NonNegativeInteger if Coef has FIELD
--R eval : (% , Coef) -> Stream Coef if Coef has **: (Coef, Fraction Integer) -> Coef
--R exp : % -> % if Coef has ALGEBRA FRAC INT
--R expressIdealMember : (List % , %) -> Union(List % , "failed") if Coef has FIELD
--R exquo : (% , %) -> Union(% , "failed") if Coef has INTDOM
--R extend : (% , Fraction Integer) -> %
--R extendedEuclidean : (% , %) -> Record(coef1: % , coef2: % , generator: %) if Coef has FIELD
--R extendedEuclidean : (% , % , %) -> Union(Record(coef1: % , coef2: % ) , "failed") if Coef has FIELD
--R factor : % -> Factored % if Coef has FIELD
--R gcd : (% , %) -> % if Coef has FIELD
--R gcd : List % -> % if Coef has FIELD
--R gcdPolynomial : (SparseUnivariatePolynomial % , SparseUnivariatePolynomial %) -> SparseUnivariatePolym
--R integrate : (% , Variable var) -> % if Coef has ALGEBRA FRAC INT
--R integrate : (% , Symbol) -> % if Coef has integrate: (Coef, Symbol) -> Coef and Coef has variables: Coe
--R integrate : % -> % if Coef has ALGEBRA FRAC INT
--R inv : % -> % if Coef has FIELD
--R lcm : (% , %) -> % if Coef has FIELD
--R lcm : List % -> % if Coef has FIELD
--R log : % -> % if Coef has ALGEBRA FRAC INT
--R monomial : (% , List SingletonAsOrderedSet, List Fraction Integer) -> %
--R monomial : (% , SingletonAsOrderedSet, Fraction Integer) -> %
--R monomial : (Coef, Fraction Integer) -> %
--R multiEuclidean : (List % , %) -> Union(List % , "failed") if Coef has FIELD
--R multiplyExponents : (% , Fraction Integer) -> %
--R multiplyExponents : (% , PositiveInteger) -> %
--R nthRoot : (% , Integer) -> % if Coef has ALGEBRA FRAC INT
--R order : (% , Fraction Integer) -> Fraction Integer
--R pi : () -> % if Coef has ALGEBRA FRAC INT
--R prime? : % -> Boolean if Coef has FIELD
--R principalIdeal : List % -> Record(coef: List % , generator: %) if Coef has FIELD
--R ?quo? : (% , %) -> % if Coef has FIELD
--R ?rem? : (% , %) -> % if Coef has FIELD
--R sec : % -> % if Coef has ALGEBRA FRAC INT
--R sech : % -> % if Coef has ALGEBRA FRAC INT
--R series : (NonNegativeInteger, Stream Record(k: Fraction Integer, c: Coef)) -> %
--R sin : % -> % if Coef has ALGEBRA FRAC INT
--R sinh : % -> % if Coef has ALGEBRA FRAC INT
--R sizeLess? : (% , %) -> Boolean if Coef has FIELD
--R sqrt : % -> % if Coef has ALGEBRA FRAC INT
--R squareFree : % -> Factored % if Coef has FIELD

```

```

--R squareFreePart : % -> % if Coef has FIELD
--R subtractIfCan : (%,% ) -> Union(%, "failed")
--R tan : % -> % if Coef has ALGEBRA FRAC INT
--R tanh : % -> % if Coef has ALGEBRA FRAC INT
--R terms : % -> Stream Record(k: Fraction Integer, c: Coef)
--R truncate : (% , Fraction Integer, Fraction Integer) -> %
--R truncate : (% , Fraction Integer) -> %
--R unit? : % -> Boolean if Coef has INTDOM
--R unitCanonical : % -> % if Coef has INTDOM
--R unitNormal : % -> Record(unit: %, canonical: %, associate: %) if Coef has INTDOM
--R variables : % -> List SingletonAsOrderedSet
--R
--E 1

)spool
)lisp (bye)

```

— GeneralUnivariatePowerSeries.help —

```

=====
GeneralUnivariatePowerSeries examples
=====

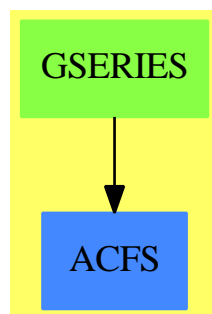
```

```

See Also:
o )show GeneralUnivariatePowerSeries

```

### 8.7.1 GeneralUnivariatePowerSeries (GSERIES)



**Exports:**

|                 |                |                   |                    |
|-----------------|----------------|-------------------|--------------------|
| 0               | 1              | acos              | acosh              |
| acot            | acoth          | acsc              | acsch              |
| approximate     | asec           | asech             | asin               |
| asinh           | associates?    | atan              | atanh              |
| center          | characteristic | charthRoot        | coefficient        |
| coerce          | complete       | cos               | cosh               |
| cot             | coth           | csc               | csch               |
| D               | degree         | differentiate     | divide             |
| euclideanSize   | eval           | exp               | expressIdealMember |
| exquo           | extend         | extendedEuclidean | factor             |
| gcd             | gcdPolynomial  | hash              | integrate          |
| inv             | latex          | lcm               | leadingCoefficient |
| leadingMonomial | log            | map               | monomial           |
| monomial?       | multiEuclidean | multiplyExponents | nthRoot            |
| one?            | order          | pi                | pole?              |
| prime?          | principalIdeal | recip             | reductum           |
| sample          | sec            | sech              | series             |
| sin             | sinh           | sizeLess?         | sqrt               |
| squareFree      | squareFreePart | subtractIfCan     | tan                |
| tanh            | terms          | truncate          | unit?              |
| unitCanonical   | unitNormal     | variable          | variables          |
| zero?           | ?+?            | ?-?               | -?                 |
| ?=?             | ?^?            | ?~=?              | ?*?                |
| ?**?            | ?/?            | ?..?              |                    |
| ?quo?           | ?rem?          |                   |                    |

— domain GSERIES GeneralUnivariatePowerSeries —

```

)abbrev domain GSERIES GeneralUnivariatePowerSeries
++ Author: Clifton J. Williamson
++ Date Created: 22 September 1993
++ Date Last Updated: 23 September 1993
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: series, Puiseux
++ Examples:
++ References:
++ Description:
++ This is a category of univariate Puiseux series constructed
++ from univariate Laurent series. A Puiseux series is represented
++ by a pair \spad{[r,f(x)]}, where r is a positive rational number and
++ \spad{f(x)} is a Laurent series. This pair represents the Puiseux
++ series \spad{f(x)\^r)}.

GeneralUnivariatePowerSeries(Coef,var,cen): Exports == Implementation where

```

```

Coef : Ring
var  : Symbol
cen  : Coef
I    ==> Integer
UTS  ==> UnivariateTaylorSeries
ULS  ==> UnivariateLaurentSeries
UPXS ==> UnivariatePuisseuxSeries
EFULS ==> ElementaryFunctionsUnivariateLaurentSeries
EFUPXS ==> ElementaryFunctionsUnivariatePuisseuxSeries
FS2UPS ==> FunctionSpaceToUnivariatePowerSeries

Exports ==> UnivariatePuisseuxSeriesCategory Coef with
coerce: Variable(var) -> %
    ++ coerce(var) converts the series variable \spad{var} into a
    ++ Puiseux series.
coerce: UPXS(Coef,var,cen) -> %
    ++ coerce(f) converts a Puiseux series to a general power series.
differentiate: (%,Variable(var)) -> %
    ++ \spad{differentiate(f(x),x)} returns the derivative of
    ++ \spad{f(x)} with respect to \spad{x}.
if Coef has Algebra Fraction Integer then
    integrate: (%,Variable(var)) -> %
        ++ \spad{integrate(f(x))} returns an anti-derivative of the power
        ++ series \spad{f(x)} with constant coefficient 0.
        ++ We may integrate a series when we can divide coefficients
        ++ by integers.

Implementation ==> UnivariatePuisseuxSeries(Coef,var,cen) add

coerce(upxs:UPXS(Coef,var,cen)) == upxs pretend %

puiseux: % -> UPXS(Coef,var,cen)
puiseux f == f pretend UPXS(Coef,var,cen)

if Coef has Algebra Fraction Integer then

    differentiate f ==
        str1 : String := "'differentiate' unavailable on this domain; "
        str2 : String := "use 'approximate' first"
        error concat(str1,str2)

    differentiate(f:%,v:Variable(var)) == differentiate f

if Coef has PartialDifferentialRing(Symbol) then
    differentiate(f:%,s:Symbol) ==
        (s = variable(f)) =>
            str1 : String := "'differentiate' unavailable on this domain; "
            str2 : String := "use 'approximate' first"
            error concat(str1,str2)
        dcds := differentiate(center f,s)

```

```

deriv := differentiate(puiseux f) :: %
map(x+>differentiate(x,s),f) - dcds * deriv

integrate f ==
  str1 : String := "'integrate' unavailable on this domain; "
  str2 : String := "use 'approximate' first"
  error concat(str1,str2)

integrate(f:%,v:Variable(var)) == integrate f

if Coef has integrate: (Coef,Symbol) -> Coef and _
  Coef has variables: Coef -> List Symbol then

  integrate(f:%,s:Symbol) ==
    (s = variable(f)) =>
      str1 : String := "'integrate' unavailable on this domain; "
      str2 : String := "use 'approximate' first"
      error concat(str1,str2)
      not entry?(s,variables center f) => map(x+>integrate(x,s),f)
      error "integrate: center is a function of variable of integration"

if Coef has TranscendentalFunctionCategory and _
  Coef has PrimitiveFunctionCategory and _
  Coef has AlgebraicallyClosedFunctionSpace Integer then

  integrateWithOneAnswer: (Coef,Symbol) -> Coef
  integrateWithOneAnswer(f,s) ==
    res := integrate(f,s)$FunctionSpaceIntegration(Integer,Coef)
    res case Coef => res :: Coef
    first(res :: List Coef)

  integrate(f:%,s:Symbol) ==
    (s = variable(f)) =>
      str1 : String := "'integrate' unavailable on this domain; "
      str2 : String := "use 'approximate' first"
      error concat(str1,str2)
      not entry?(s,variables center f) =>
        map(x+>integrateWithOneAnswer(x,s),f)
      error "integrate: center is a function of variable of integration"

```

---

— GSERIES.dotabb —

"GSERIES" [color="#88FF44",href="bookvol10.3.pdf#nameddest=GSERIES"]  
 "ACFS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ACFS"]  
 "GSERIES" -> "ACFS"



## 8.8 domain GRIMAGE GraphImage

— GraphImage.input —

```
)set break resume
)sys rm -f GraphImage.output
)spool GraphImage.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show GraphImage
--R GraphImage is a domain constructor
--R Abbreviation for GraphImage is GRIMAGE
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for GRIMAGE
--R
--R----- Operations -----
--R ?=? : (%,%) -> Boolean          coerce : % -> OutputForm
--R graphImage : () -> %            hash : % -> SingleInteger
--R key : % -> Integer              latex : % -> String
--R makeGraphImage : % -> %         ranges : % -> List Segment Float
--R units : % -> List Float         ~=?: (%,%) -> Boolean
--R appendPoint : (%,Point DoubleFloat) -> Void
--R coerce : List List Point DoubleFloat -> %
--R component : (%,Point DoubleFloat,Palette,Palette,PositiveInteger) -> Void
--R component : (%,Point DoubleFloat) -> Void
--R component : (%,List Point DoubleFloat,Palette,Palette,PositiveInteger) -> Void
--R figureUnits : List List Point DoubleFloat -> List DoubleFloat
--R makeGraphImage : (List List Point DoubleFloat,List Palette,List Palette,List PositiveInt
--R makeGraphImage : (List List Point DoubleFloat,List Palette,List Palette,List PositiveInt
--R makeGraphImage : List List Point DoubleFloat -> %
--R point : (%,Point DoubleFloat,Palette) -> Void
--R pointLists : % -> List List Point DoubleFloat
--R putColorInfo : (List List Point DoubleFloat,List Palette) -> List List Point DoubleFloat
--R ranges : (%,List Segment Float) -> List Segment Float
--R units : (%,List Float) -> List Float
--R
--E 1

)spool
)lisp (bye)
```

## — GraphImage.help —

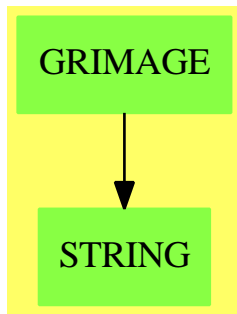
```
=====
GraphImage examples
=====
```

See Also:

o )show GraphImage

—————

## 8.8.1 GraphImage (GRIMAGE)

**Exports:**

|             |              |           |                |            |
|-------------|--------------|-----------|----------------|------------|
| appendPoint | coerce       | component | figureUnits    | graphImage |
| hash        | key          | latex     | makeGraphImage | point      |
| pointLists  | putColorInfo | ranges    | units          | ?~=?       |
| ?=?         |              |           |                |            |

## — domain GRIMAGE GraphImage —

```
)abbrev domain GRIMAGE GraphImage
++ Author: Jim Wen
++ Date Created: 27 April 1989
++ Date Last Updated: 1995 September 20, Mike Richardson (MGR)
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ TwoDimensionalGraph creates virtual two dimensional graphs
++ (to be displayed on TwoDimensionalViewports).
```

GraphImage (): Exports == Implementation where

```

VIEW    ==> VIEWPORTSERVER$Lisp
sendI   ==> SOCK_-SEND_-INT
sendSF  ==> SOCK_-SEND_-FLOAT
sendSTR ==> SOCK_-SEND_-STRING
getI    ==> SOCK_-GET_-INT
getSF   ==> SOCK_-GET_-FLOAT

typeGRAPH ==> 2
typeVIEW2D ==> 3

makeGRAPH ==> (-1)$SingleInteger
makeVIEW2D ==> (-1)$SingleInteger

I    ==> Integer
PI   ==> PositiveInteger
NNI  ==> NonNegativeInteger
SF   ==> DoubleFloat
F    ==> Float
L    ==> List
P    ==> Point(SF)
V    ==> Vector
SEG  ==> Segment
RANGESF ==> L SEG SF
RANGEF  ==> L SEG F
UNITSF  ==> L SF
UNITF   ==> L F
PAL     ==> Palette
E       ==> OutputForm
DROP    ==> DrawOption
PP      ==> PointPackage(SF)
COORDSYS ==> CoordinateSystems(SF)

Exports ==> SetCategory with
graphImage      : ()                                -> $
  ++ graphImage() returns an empty graph with 0 point lists
  ++ of the domain \spadtype{GraphImage}. A graph image contains
  ++ the graph data component of a two dimensional viewport.
makeGraphImage  : $                                -> $
  ++ makeGraphImage(gi) takes the given graph, \spad{gi} of the
  ++ domain \spadtype{GraphImage}, and sends it's data to the
  ++ viewport manager where it waits to be included in a two-dimensional
  ++ viewport window. \spad{gi} cannot be an empty graph, and it's
  ++ elements must have been created using the \spadfun{point} or
  ++ \spadfun{component} functions, not by a previous
  ++ \spadfun{makeGraphImage}.
makeGraphImage  : (L L P)                            -> $
  ++ makeGraphImage(llp) returns a graph of the domain

```

```

++ \spadtype{GraphImage} which is composed of the points and
++ lines from the list of lists of points, \spad{llp}, with
++ default point size and default point and line colours. The graph
++ data is then sent to the viewport manager where it waits to be
++ included in a two-dimensional viewport window.
makeGraphImage : (L L P,L PAL,L PAL,L PI) -> $
++ makeGraphImage(llp,lpal1,lpal2,lp) returns a graph of the
++ domain \spadtype{GraphImage} which is composed of the points
++ and lines from the list of lists of points, \spad{llp}, whose
++ point colors are indicated by the list of palette colors,
++ \spad{lpal1}, and whose lines are colored according to the list
++ of palette colors, \spad{lpal2}. The parameter lp is a list of
++ integers which denote the size of the data points. The graph
++ data is then sent to the viewport manager where it waits to be
++ included in a two-dimensional viewport window.
makeGraphImage : (L L P,L PAL,L PAL,L PI,L DROP) -> $
++ makeGraphImage(llp,lpal1,lpal2,lp,lopt) returns a graph of
++ the domain \spadtype{GraphImage} which is composed of the
++ points and lines from the list of lists of points, \spad{llp},
++ whose point colors are indicated by the list of palette colors,
++ \spad{lpal1}, and whose lines are colored according to the list
++ of palette colors, \spad{lpal2}. The parameter lp is a list of
++ integers which denote the size of the data points, and \spad{lopt}
++ is the list of draw command options. The graph data is then sent
++ to the viewport manager where it waits to be included in a
++ two-dimensional viewport window.
pointLists : $ -> L L P
++ pointLists(gi) returns the list of lists of points which compose
++ the given graph, \spad{gi}, of the domain \spadtype{GraphImage}.
key : $ -> I
++ key(gi) returns the process ID of the given graph, \spad{gi},
++ of the domain \spadtype{GraphImage}.
ranges : $ -> RANGEF
++ ranges(gi) returns the list of ranges of the point components from
++ the indicated graph, \spad{gi}, of the domain \spadtype{GraphImage}.
ranges : ($,RANGEF) -> RANGEF
++ ranges(gi,lr) modifies the list of ranges for the given graph,
++ \spad{gi} of the domain \spadtype{GraphImage}, to be that of the
++ list of range segments, \spad{lr}, and returns the new range list
++ for \spad{gi}.
units : $ -> UNITF
++ units(gi) returns the list of unit increments for the x and y
++ axes of the indicated graph, \spad{gi}, of the domain
++ \spadtype{GraphImage}.
units : ($,UNITF) -> UNITF
++ units(gi,lu) modifies the list of unit increments for the x and y
++ axes of the given graph, \spad{gi} of the domain
++ \spadtype{GraphImage}, to be that of the list of unit increments,
++ \spad{lu}, and returns the new list of units for \spad{gi}.
component : ($,L P,PAL,PAL,PI) -> Void

```

```

++ component(gi,lp,pal1,pal2,p) sets the components of the
++ graph, \spad{gi} of the domain \spadtype{GraphImage}, to the
++ values given. The point list for \spad{gi} is set to the list
++ \spad{lp}, the color of the points in \spad{lp} is set to
++ the palette color \spad{pal1}, the color of the lines which
++ connect the points \spad{lp} is set to the palette color
++ \spad{pal2}, and the size of the points in \spad{lp} is given
++ by the integer p.
component      : ($,P)                                -> Void
++ component(gi,pt) modifies the graph \spad{gi} of the domain
++ \spadtype{GraphImage} to contain one point component, \spad{pt}
++ whose point color, line color and point size are determined by
++ the default functions \spadfun{pointColorDefault},
++ \spadfun{lineColorDefault}, and \spadfun{pointSizeDefault}.
component      : ($,P,PAL,PAL,PI)                    -> Void
++ component(gi,pt,pal1,pal2,ps) modifies the graph \spad{gi} of
++ the domain \spadtype{GraphImage} to contain one point component,
++ \spad{pt} whose point color is set to the palette color \spad{pal1},
++ line color is set to the palette color \spad{pal2}, and point
++ size is set to the positive integer \spad{ps}.
appendPoint    : ($,P)                                -> Void
++ appendPoint(gi,pt) appends the point \spad{pt} to the end
++ of the list of points component for the graph, \spad{gi}, which is
++ of the domain \spadtype{GraphImage}.
point          : ($,P,PAL)                            -> Void
++ point(gi,pt,pal) modifies the graph \spad{gi} of the domain
++ \spadtype{GraphImage} to contain one point component, \spad{pt}
++ whose point color is set to be the palette color \spad{pal}, and
++ whose line color and point size are determined by the default
++ functions \spadfun{lineColorDefault} and \spadfun{pointSizeDefault}.
coerce         : L L P                                -> $
++ coerce(llp)
++ component(gi,pt) creates and returns a graph of the domain
++ \spadtype{GraphImage} which is composed of the list of list
++ of points given by \spad{llp}, and whose point colors, line colors
++ and point sizes are determined by the default functions
++ \spadfun{pointColorDefault}, \spadfun{lineColorDefault}, and
++ \spadfun{pointSizeDefault}. The graph data is then sent to the
++ viewport manager where it waits to be included in a two-dimensional
++ viewport window.
coerce         : $                                    -> E
++ coerce(gi) returns the indicated graph, \spad{gi}, of domain
++ \spadtype{GraphImage} as output of the domain \spadtype{OutputForm}.
putColorInfo   : (L L P,L PAL)                        -> L L P
++ putColorInfo(llp,lpal) takes a list of list of points, \spad{llp},
++ and returns the points with their hue and shade components
++ set according to the list of palette colors, \spad{lpal}.
figureUnits    : L L P                                -> UNITSF

```

Implementation ==> add

```

import Color()
import Palette()
import ViewDefaultsPackage()
import PlotTools()
import DrawOptionFunctions0
import P
import PP
import COORDSYS

Rep := Record(key: I, rangesField: RANGESF, unitsField: UNITSF, _
  llPoints: L L P, pointColors: L PAL, lineColors: L PAL, pointSizes: L PI, _
  optionsField: L DROP)

--%Internal Functions

graph      : RANGEF                                -> $
scaleStep  : SF                                    -> SF
makeGraph  : $                                     -> $

numberCheck(nums:Point SF):Void ==
  for i in minIndex(nums)..maxIndex(nums) repeat
    COMPLEXP(nums.(i::PositiveInteger))$Lisp =>
      error "An unexpected complex number was encountered in the calculations."

doOptions(g:Rep):Void ==
  lr : RANGEF := ranges(g.optionsField,ranges g)
  if (#lr > 1$I) then
    g.rangesField := [segment(convert(lo(lr.1))@SF,convert(hi(lr.1))@SF)$(Segment(SF)),
      segment(convert(lo(lr.2))@SF,convert(hi(lr.2))@SF)$(Segment(SF))]
  else
    g.rangesField := []
  lu : UNITF := units(g.optionsField,units g)
  if (#lu > 1$I) then
    g.unitsField := [convert(lu.1)@SF,convert(lu.2)@SF]
  else
    g.unitsField := []
-- etc - graphimage specific stuff...

putColorInfo(llp,listOfPalettes) ==
  llp2 : L L P := []
  for lp in llp for pal in listOfPalettes repeat
    lp2 : L L P := []
    daHue   := (hue(hue pal))::SF
    daShade := (shade pal)::SF
    for p in lp repeat
      if (d := dimension p) < 3 then
        p := extend(p,[daHue,daShade])
      else

```

```

    p.3 := daHue
    d < 4 => p := extend(p,[daShade])
    p.4 := daShade
    lp2 := cons(p,lp2)
    llp2 := cons(reverse_! lp2,llp2)
    reverse_! llp2

graph demRanges ==
    null demRanges => [ 0, [], [], [], [], [], [], [] ]
    demRangesSF : RANGESF := _
        [ segment(convert(lo demRanges.1)@SF,convert(hi demRanges.1)@SF)$(Segment(SF)), _
          segment(convert(lo demRanges.1)@SF,convert(hi demRanges.1)@SF)$(Segment(SF)) ]
    [ 0, demRangesSF, [], [], [], [], [], [] ]

scaleStep(range) ==                                -- MGR

    adjust:NNI
    tryStep:SF
    scaleDown:SF
    numerals:String
    adjust := 0
    while range < 100.0::SF repeat
        adjust := adjust + 1
        range := range * 10.0::SF -- might as well take big steps
    tryStep := range/10.0::SF
    numerals := string(((retract(ceiling(tryStep)$SF)$SF)@I))$String
    scaleDown := (10@I **$I (((#(numerals)@I) - 1$I) pretend PI))::SF
    scaleDown*ceiling(tryStep/scaleDown - 0.5::SF)/((10 **$I adjust)::SF)

figureUnits(listOfListsOfPoints) ==
    -- figure out the min/max and divide by 10 for unit markers
    xMin := xMax := xCoord first first listOfListsOfPoints
    yMin := yMax := yCoord first first listOfListsOfPoints
    if xMin ~= xMin then xMin:=max()
    if xMax ~= xMax then xMax:=min()
    if yMin ~= yMin then yMin:=max()
    if yMax ~= yMax then yMax:=min()
    for pL in listOfListsOfPoints repeat
        for p in pL repeat
            if ((px := (xCoord p)) < xMin) then
                xMin := px
            if px > xMax then
                xMax := px
            if ((py := (yCoord p)) < yMin) then
                yMin := py
            if py > yMax then
                yMax := py
    if xMin = xMax then
        xMin := xMin - convert(0.5)$Float
        xMax := xMax + convert(0.5)$Float

```

```

if yMin = yMax then
  yMin := yMin - convert(0.5)$Float
  yMax := yMax + convert(0.5)$Float
[scaleStep(xMax-xMin),scaleStep(yMax-yMin)]

plotLists(graf:Rep,listOfListsOfPoints:L L P,listOfPointColors:L PAL,listOfLineColors:L PAL,listOfPo
givenLen := #listOfListsOfPoints
-- take out point lists that are actually empty
listOfListsOfPoints := [ 1 for 1 in listOfListsOfPoints | ^null 1 ]
if (null listOfListsOfPoints) then
  error "GraphImage was given a list that contained no valid point lists"
if ((len := #listOfListsOfPoints) ^= givenLen) then
  sayBrightly([" Warning: Ignoring pointless point list":E]$List(E))$Lisp
graf.llPoints := listOfListsOfPoints
-- do point colors
if ((givenLen := #listOfPointColors) > len) then
  -- pad or discard elements if given list has length different from the point list
  graf.pointColors := concat(listOfPointColors,
    new((len - givenLen)::NonNegativeInteger + 1, pointColorDefault()))
else graf.pointColors := first(listOfPointColors, len)
-- do line colors
if ((givenLen := #listOfLineColors) > len) then
  graf.lineColors := concat(listOfLineColors,
    new((len - givenLen)::NonNegativeInteger + 1, lineColorDefault()))
else graf.lineColors := first(listOfLineColors, len)
-- do point sizes
if ((givenLen := #listOfPointSizes) > len) then
  graf.pointSizes := concat(listOfPointSizes,
    new((len - givenLen)::NonNegativeInteger + 1, pointSizeDefault()))
else graf.pointSizes := first(listOfPointSizes, len)
graf

makeGraph graf ==
doOptions(graf)
(s := #(graf.llPoints)) = 0 =>
  error "You are trying to make a graph with no points"
key graf ^= 0 =>
  error "You are trying to draw over an existing graph"
transform := coord(graf.optionsField,cartesian$COORDSYS)$DrawOptionFunctions0
graf.llPoints:= putColorInfo(graf.llPoints,graf.pointColors)
if null(ranges graf) then -- figure out best ranges for points
  graf.rangesField := calcRanges(graf.llPoints) --::V SEG SF
if null(units graf) then -- figure out best ranges for points
  graf.unitsField := figureUnits(graf.llPoints) --::V SEG SF
sayBrightly([" Graph data being transmitted to the viewport manager...":E]$List(E))$Lisp
sendI(VIEW,typeGRAPH)$Lisp
sendI(VIEW,makeGRAPH)$Lisp
tonto := (graf.rangesField)::RANGESF
sendSF(VIEW,lo(first tonto))$Lisp
sendSF(VIEW,hi(first tonto))$Lisp

```



```

sendSF(VIEW,lo(second tonto))$Lisp
sendSF(VIEW,hi(second tonto))$Lisp
sendSF(VIEW,first (graf.unitsField))$Lisp
sendSF(VIEW,second (graf.unitsField))$Lisp
sendI(VIEW,s)$Lisp      -- how many lists of points are being sent
for aList in graf.llPoints for pColor in graf.pointColors for lColor in graf.lineColors
  sendI(VIEW,#aList)$Lisp -- how many points in this list
  for p in aList repeat
    aPoint := transform p
    sendSF(VIEW,xCoord aPoint)$Lisp
    sendSF(VIEW,yCoord aPoint)$Lisp
    sendSF(VIEW,hue(p)$PP)$Lisp -- ?use aPoint as well...?
    sendSF(VIEW,shade(p)$PP)$Lisp
    hueShade := hue hue pColor + shade pColor * numberOfHues()
    sendI(VIEW,hueShade)$Lisp
    hueShade := (hue hue lColor -1)*5 + shade lColor
    sendI(VIEW,hueShade)$Lisp
    sendI(VIEW,s)$Lisp
  graf.key := getI(VIEW)$Lisp
graf

--%Exported Functions
makeGraphImage(graf:$) == makeGraph graf
key graf == graf.key
pointLists graf == graf.llPoints
ranges graf ==
  null graf.rangesField => []
  [segment(convert(lo graf.rangesField.1)@F,convert(hi graf.rangesField.1)@F), _
   segment(convert(lo graf.rangesField.2)@F,convert(hi graf.rangesField.2)@F)]
ranges(graf,rangesList) ==
  graf.rangesField :=
    [segment(convert(lo rangesList.1)@SF,convert(hi rangesList.1)@SF), _
     segment(convert(lo rangesList.2)@SF,convert(hi rangesList.2)@SF)]
  rangesList
units graf ==
  null(graf.unitsField) => []
  [convert(graf.unitsField.1)@F,convert(graf.unitsField.2)@F]
units (graf,unitsToBe) ==
  graf.unitsField := [convert(unitsToBe.1)@SF,convert(unitsToBe.2)@SF]
  unitsToBe
graphImage == graph []

makeGraphImage(llp) ==
  makeGraphImage(llp,
    [pointColorDefault() for i in 1..(1:=#llp)],
    [lineColorDefault() for i in 1..1],
    [pointSizeDefault() for i in 1..1])

makeGraphImage(llp,lpc,llc,lps) ==

```

```

makeGraphImage(llp,lpc,llc,lps,[])

makeGraphImage(llp,lpc,llc,lps,opts) ==
  graf := graph(ranges(opts,[]))
  graf.optionsField := opts
  graf := plotLists(graf,llp,lpc,llc,lps)
  transform := coord(graf.optionsField,cartesian$COORDSYS)$DrawOptionFunctions0
  for aList in graf.llPoints repeat
    for p in aList repeat
      aPoint := transform p
      numberCheck aPoint
  makeGraph graf

component (graf:$,ListOfPoints:L P,PointColor:PAL,LineColor:PAL,PointSize:PI) ==
  graf.llPoints      := append(graf.llPoints,[ListOfPoints])
  graf.pointColors   := append(graf.pointColors,[PointColor])
  graf.lineColors    := append(graf.lineColors,[LineColor])
  graf.pointSizes    := append(graf.pointSizes,[PointSize])

component (graf,aPoint) ==
  component(graf,aPoint,pointColorDefault(),lineColorDefault(),pointSizeDefault())

component (graf:$,aPoint:P,PointColor:PAL,LineColor:PAL,PointSize:PI) ==
  component (graf,[aPoint],PointColor,LineColor,PointSize)

appendPoint (graf,aPoint) ==
  num : I := #(graf.llPoints) - 1
  num < 0 => error "No point lists to append to!"
  (graf.llPoints.num) := append((graf.llPoints.num),[aPoint])

point (graf,aPoint,PointColor) ==
  component(graf,aPoint,PointColor,lineColorDefault(),pointSizeDefault())

coerce (llp : L L P) : $ ==
  makeGraphImage(llp,
    [pointColorDefault() for i in 1..(l:=#llp)],
    [lineColorDefault() for i in 1..l],
    [pointSizeDefault() for i in 1..l])

coerce (graf : $) : E ==
  hconcat( ["Graph with " :: E,(p := # pointLists graf) :: E,
    (p=1 => " point list"; " point lists") :: E])

```

---

— GRIMAGE.dotabb —

"GRIMAGE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=GRIMAGE"]

```
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"GRIMAGE" -> "STRING"
```

## 8.9 domain GOPT GuessOption

— GuessOption.input —

```
)set break resume
)sys rm -f GuessOption.output
)spool GuessOption.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show GuessOption
--R GuessOption is a domain constructor
--R Abbreviation for GuessOption is GOPT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for GOPT
--R
--R----- Operations -----
--R ?=? : (%,% ) -> Boolean          allDegrees : Boolean -> %
--R checkExtraValues : Boolean -> %   coerce : % -> OutputForm
--R debug : Boolean -> %              displayKind : Symbol -> %
--R functionName : Symbol -> %        functionName : List Symbol -> %
--R hash : % -> SingleInteger         indexName : Symbol -> %
--R latex : % -> String               one : Boolean -> %
--R safety : NonNegativeInteger -> %  variableName : Symbol -> %
--R ?~=? : (%,% ) -> Boolean
--R Somos : Union(PositiveInteger,Boolean) -> %
--R check : Union(skip,MonteCarlo,deterministic) -> %
--R homogeneous : Union(PositiveInteger,Boolean) -> %
--R maxDegree : Union(NonNegativeInteger,arbitrary) -> %
--R maxDerivative : Union(NonNegativeInteger,arbitrary) -> %
--R maxLevel : Union(NonNegativeInteger,arbitrary) -> %
--R maxMixedDegree : NonNegativeInteger -> %
--R maxPower : Union(PositiveInteger,arbitrary) -> %
--R maxShift : Union(NonNegativeInteger,arbitrary) -> %
--R maxSubst : Union(PositiveInteger,arbitrary) -> %
--R option : (List %,Symbol) -> Union(Any,"failed")
--R
--E 1
```

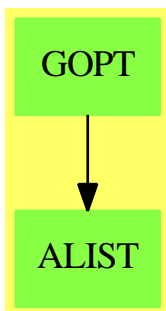
```
)spool
)lisp (bye)
```

— GuessOption.help —

```
=====
GuessOption examples
=====
```

```
See Also:
o )show GuessOption
```

### 8.9.1 GuessOption (GOPT)



#### Exports:

|                               |                            |                          |                             |                                  |
|-------------------------------|----------------------------|--------------------------|-----------------------------|----------------------------------|
| <code>?=?</code>              | <code>?~=?</code>          | <code>Somos</code>       | <code>allDegrees</code>     | <code>check</code>               |
| <code>checkExtraValues</code> | <code>coerce</code>        | <code>debug</code>       | <code>displayKind</code>    | <code>functionName</code>        |
| <code>functionNames</code>    | <code>hash</code>          | <code>homogeneous</code> | <code>indexName</code>      | <code>latex</code>               |
| <code>maxDegree</code>        | <code>maxDerivative</code> | <code>maxLevel</code>    | <code>maxMixedDegree</code> | <code>maxPower</code>            |
| <code>maxShift</code>         | <code>maxSubst</code>      | <code>one</code>         | <code>option</code>         | <code>safety variableName</code> |

— domain GOPT GuessOption —

```
)abbrev domain GOPT GuessOption
++ Author: Martin Rubey
++ Description: GuessOption is a domain whose elements are various options used
++ by Guess.
GuessOption(): Exports == Implementation where

Exports == SetCategory with
```

```

maxDerivative: Union(NonNegativeInteger, "arbitrary") -> %
++ maxDerivative(d) specifies the maximum derivative in an algebraic
++ differential equation. This option is expressed in the form
++ \spad{maxDerivative == d}.

maxShift: Union(NonNegativeInteger, "arbitrary") -> %
++ maxShift(d) specifies the maximum shift in a recurrence
++ equation. This option is expressed in the form \spad{maxShift == d}.

maxSubst: Union(PositiveInteger, "arbitrary") -> %
++ maxSubst(d) specifies the maximum degree of the monomial substituted
++ into the function we are looking for. That is, if \spad{maxSubst ==
++ d}, we look for polynomials such that  $p(f(x), f(x^2), \dots,$ 
++  $f(x^d))=0$ . equation. This option is expressed in the form
++ \spad{maxSubst == d}.

maxPower: Union(PositiveInteger, "arbitrary") -> %
++ maxPower(d) specifies the maximum degree in an algebraic differential
++ equation. For example, the degree of  $(f'')^3 f'$  is 4. maxPower(-1)
++ specifies that the maximum exponent can be arbitrary. This option is
++ expressed in the form \spad{maxPower == d}.

homogeneous: Union(PositiveInteger, Boolean) -> %
++ homogeneous(d) specifies whether we allow only homogeneous algebraic
++ differential equations. This option is expressed in the form
++ \spad{homogeneous == d}. If true, then maxPower must be
++ set, too, and ADEs with constant total degree are allowed.
++ If a PositiveInteger is given, only ADE's with this total degree are
++ allowed.

Somos: Union(PositiveInteger, Boolean) -> %
++ Somos(d) specifies whether we want that the total degree of the
++ differential operators is constant, and equal to d, or maxDerivative
++ if true. If true, maxDerivative must be set, too.

maxLevel: Union(NonNegativeInteger, "arbitrary") -> %
++ maxLevel(d) specifies the maximum number of recursion levels operators
++ guessProduct and guessSum will be applied. This option is expressed in
++ the form \spad{maxLevel == d}.

maxDegree: Union(NonNegativeInteger, "arbitrary") -> %
++ maxDegree(d) specifies the maximum degree of the coefficient
++ polynomials in an algebraic differential equation or a recursion with
++ polynomial coefficients. For rational functions with an exponential
++ term, \spad{maxDegree} bounds the degree of the denominator
++ polynomial.
++ This option is expressed in the form \spad{maxDegree == d}.

maxMixedDegree: NonNegativeInteger -> %

```

```

++ maxMixedDegree(d) specifies the maximum q-degree of the coefficient
++ polynomials in a recurrence with polynomial coefficients, in the case
++ of mixed shifts. Although slightly inconsistent, maxMixedDegree(0)
++ specifies that no mixed shifts are allowed. This option is expressed
++ in the form \spad{maxMixedDegree == d}.

allDegrees: Boolean -> %
++ allDegrees(d) specifies whether all possibilities of the degree vector
++ - taking into account maxDegree - should be tried. This is mainly
++ interesting for rational interpolation. This option is expressed in
++ the form \spad{allDegrees == d}.

safety: NonNegativeInteger -> %
++ safety(d) specifies the number of values reserved for testing any
++ solutions found. This option is expressed in the form \spad{safety ==
++ d}.

check: Union("skip", "MonteCarlo", "deterministic") -> %
++ check(d) specifies how we want to check the solution. If
++ the value is "skip", we return the solutions found by the
++ interpolation routine without checking. If the value is
++ "MonteCarlo", we use a probabilistic check. This option is
++ expressed in the form \spad{check == d}

checkExtraValues: Boolean -> %
++ checkExtraValues(d) specifies whether we want to check the
++ solution beyond the order given by the degree bounds. This
++ option is expressed in the form \spad{checkExtraValues == d}

one: Boolean -> %
++ one(d) specifies whether we are happy with one solution. This option
++ is expressed in the form \spad{one == d}.

debug: Boolean -> %
++ debug(d) specifies whether we want additional output on the
++ progress. This option is expressed in the form \spad{debug == d}.

functionName: Symbol -> %
++ functionName(d) specifies the name of the function given by the
++ algebraic differential equation or recurrence. This option is
++ expressed in the form \spad{functionName == d}.

functionNames: List(Symbol) -> %
++ functionNames(d) specifies the names for the function in
++ algebraic dependence. This option is
++ expressed in the form \spad{functionNames == d}.

variableName: Symbol -> %
++ variableName(d) specifies the variable used in by the algebraic
++ differential equation. This option is expressed in the form

```

```

++ \spad{variableName == d}.

indexName: Symbol -> %
++ indexName(d) specifies the index variable used for the formulas. This
++ option is expressed in the form \spad{indexName == d}.

displayKind: Symbol -> %
++ displayKind(d) specifies kind of the result: generating function,
++ recurrence or equation. This option should not be set by the
++ user, but rather by the HP-specification.

option : (List %, Symbol) -> Union(Any, "failed")
++ option(l, option) returns which options are given.

Implementation ==> add
import AnyFunctions1(Boolean)
import AnyFunctions1(Symbol)
import AnyFunctions1(NonNegativeInteger)
import AnyFunctions1(Union(NonNegativeInteger, "arbitrary"))
import AnyFunctions1(Union(PositiveInteger, "arbitrary"))
import AnyFunctions1(Union(PositiveInteger, Boolean))
import AnyFunctions1(Union("skip", "MonteCarlo", "deterministic"))

Rep := Record(keyword: Symbol, value: Any)

maxLevel d      == ['maxLevel,      d::Any]
maxDerivative d == ['maxDerivative, d::Any]
maxShift d      == maxDerivative d
maxSubst d      ==
    if d case PositiveInteger
    then maxDerivative((d::Integer-1)::NonNegativeInteger)
    else maxDerivative d
maxDegree d      == ['maxDegree,      d::Any]
maxMixedDegree d == ['maxMixedDegree, d::Any]
allDegrees d      == ['allDegrees,      d::Any]
maxPower d       == ['maxPower,        d::Any]
safety d         == ['safety,          d::Any]
homogeneous d    == ['homogeneous,     d::Any]
Somos d          == ['Somos,            d::Any]
debug d          == ['debug,            d::Any]
check d          == ['check,            d::Any]
checkExtraValues d == ['checkExtraValues, d::Any]
one d            == ['one,              d::Any]
functionName d   == ['functionName,     d::Any]
functionNames d ==
    ['functionNames, coerce(d)$AnyFunctions1(List(Symbol))]
variableName d   == ['variableName,     d::Any]
indexName d      == ['indexName,        d::Any]
displayKind d    == ['displayKind,      d::Any]

```

```
coerce(x: %): OutputForm == x.keyword::OutputForm = x.value::OutputForm
x: % = y: % == x.keyword = y.keyword and x.value = y.value
```

```
option(l, s) ==
  for x in l repeat
    x.keyword = s => return(x.value)
  "failed"
```

---

— GOPT.dotabb —

```
"GOPT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=GOPT"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"GOPT" -> "ALIST"
```

---

## 8.10 domain GOPT0 GuessOptionFunctions0

— GuessOptionFunctions0.input —

```
)set break resume
)sys rm -f GuessOptionFunctions0.output
)spool GuessOptionFunctions0.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show GuessOptionFunctions0
--R GuessOptionFunctions0 is a domain constructor
--R Abbreviation for GuessOptionFunctions0 is GOPT0
--R This constructor is not exposed in this frame.
--R----- Operations -----
--R ?? : (%,% ) -> Boolean          coerce : % -> OutputForm
--R hash : % -> SingleInteger      latex : % -> String
--R one : List(GuessOption) -> Boolean  ?~=? : (%,% ) -> Boolean
--R MonteCarlo : List(GuessOption) -> Boolean
--R Somos : List(GuessOption) -> Union(PositiveInteger, Boolean)
--R allDegrees : List(GuessOption) -> Boolean
--R check : List(GuessOption) -> Boolean
--R checkOptions : List(GuessOption) -> Void
--R debug : List(GuessOption) -> Boolean
```



```

--R displayAsGF : List(GuessOption) -> Boolean
--R functionName : List(GuessOption) -> Symbol
--R homogeneous : List(GuessOption) -> Union(PositiveInteger,Boolean)
--R indexName : List(GuessOption) -> Symbol
--R maxDegree : List(GuessOption) -> Union(NonNegativeInteger,arbitrary)
--R maxDerivative : List(GuessOption) -> Union(NonNegativeInteger,arbitrary)
--R maxLevel : List(GuessOption) -> Union(NonNegativeInteger,arbitrary)
--R maxMixedDegree : List(GuessOption) -> NonNegativeInteger
--R maxPower : List(GuessOption) -> Union(PositiveInteger,arbitrary)
--R maxShift : List(GuessOption) -> Union(NonNegativeInteger,arbitrary)
--R maxSubst : List(GuessOption) -> Union(PositiveInteger,arbitrary)
--R safety : List(GuessOption) -> NonNegativeInteger
--R variableName : List(GuessOption) -> Symbol
--R
--E 1

)spool
)lisp (bye)

```

---

— GuessOptionFunctions0.help —

```

=====
GuessOptionFunctions0 examples
=====

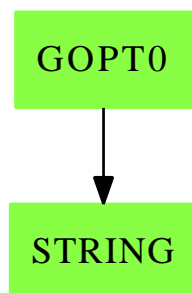
```

```

See Also:
o )show GuessOptionFunctions0

```

### 8.10.1 GuessOptionFunctions0 (GOPT0)



**Exports:**

|              |               |             |                |              |
|--------------|---------------|-------------|----------------|--------------|
| ?=?          | ?~=?          | MonteCarlo  | Somos          | allDegrees   |
| check        | checkOptions  | coerce      | debug          | displayAsGF  |
| functionName | hash          | homogeneous | indexName      | latex        |
| maxDegree    | maxDerivative | maxLevel    | maxMixedDegree | maxPower     |
| maxShift     | maxSubst      | one         | safety         | variableName |

**— domain GOPT0 GuessOptionFunctions0 —**

```

)abbrev domain GOPT0 GuessOptionFunctions0
++ Author: Martin Rubey
++ Description:
++ GuessOptionFunctions0 provides operations that extract the
++ values of options for Guess.
GuessOptionFunctions0(): Exports == Implementation where

LGOPT ==> List GuessOption

Exports == SetCategory with

maxDerivative: LGOPT -> Union(NonNegativeInteger, "arbitrary")
++ maxDerivative returns the specified maxDerivative.

maxShift: LGOPT -> Union(NonNegativeInteger, "arbitrary")
++ maxShift returns the specified maxShift.

maxSubst: LGOPT -> Union(PositiveInteger, "arbitrary")
++ maxSubst returns the specified maxSubst.

maxPower: LGOPT -> Union(PositiveInteger, "arbitrary")
++ maxPower returns the specified maxPower.

homogeneous: LGOPT -> Union(PositiveInteger, Boolean)
++ homogeneous returns whether we allow only homogeneous algebraic
++ differential equations, default being false

Somos: LGOPT -> Union(PositiveInteger, Boolean)
++ Somos returns whether we allow only Somos-like operators, default
++ being false

maxLevel: LGOPT -> Union(NonNegativeInteger, "arbitrary")
++ maxLevel returns the specified maxLevel.

maxDegree: LGOPT -> Union(NonNegativeInteger, "arbitrary")
++ maxDegree returns the specified maxDegree.

maxMixedDegree: LGOPT -> NonNegativeInteger
++ maxMixedDegree returns the specified maxMixedDegree.

allDegrees: LGOPT -> Boolean

```

```

++ allDegrees returns whether all possibilities of the degree vector
++ should be tried, the default being false.

safety: LGOPT -> NonNegativeInteger
++ safety returns the specified safety or 1 as default.

check: LGOPT -> Union("skip", "MonteCarlo", "deterministic")
++ check(d) specifies how we want to check the solution. If
++ the value is "skip", we return the solutions found by the
++ interpolation routine without checking. If the value is
++ "MonteCarlo", we use a probabilistic check. The default is
++ "deterministic".

checkExtraValues: LGOPT -> Boolean
++ checkExtraValues(d) specifies whether we want to check the
++ solution beyond the order given by the degree bounds. The
++ default is true.

one: LGOPT -> Boolean
++ one returns whether we need only one solution, default being true.

functionName: LGOPT -> Symbol
++ functionName returns the name of the function given by the algebraic
++ differential equation, default being f

variableName: LGOPT -> Symbol
++ variableName returns the name of the variable used in by the
++ algebraic differential equation, default being x

indexName: LGOPT -> Symbol
++ indexName returns the name of the index variable used for the
++ formulas, default being n

displayAsGF: LGOPT -> Boolean
++ displayAsGF specifies whether the result is a generating function
++ or a recurrence. This option should not be set by the user, but rather
++ by the HP-specification, therefore, there is no default.

debug: LGOPT -> Boolean
++ debug returns whether we want additional output on the progress,
++ default being false

checkOptions: LGOPT -> Void
++ checkOptions checks whether the given options are consistent, and
++ yields an error otherwise

Implementation == add

maxLevel 1 ==
  if (opt := option(1, 'maxLevel)) case "failed" then

```

```

    "arbitrary"
  else
    retract(opt::Any)$AnyFunctions1(Union(NonNegativeInteger, "arbitrary"))

maxDerivative l ==
  if (opt := option(l, 'maxDerivative)) case "failed" then
    "arbitrary"
  else
    retract(opt::Any)$AnyFunctions1(Union(NonNegativeInteger, "arbitrary"))

maxShift l == maxDerivative l

maxSubst l ==
  d := maxDerivative l
  if d case NonNegativeInteger
  then (d+1)::PositiveInteger
  else d

maxDegree l ==
  if (opt := option(l, 'maxDegree)) case "failed" then
    "arbitrary"
  else
    retract(opt::Any)$AnyFunctions1(Union(NonNegativeInteger, "arbitrary"))

maxMixedDegree l ==
  if (opt := option(l, 'maxMixedDegree)) case "failed" then
    0
  else
    retract(opt :: Any)$AnyFunctions1(NonNegativeInteger)

allDegrees l ==
  if (opt := option(l, 'allDegrees)) case "failed" then
    false
  else
    retract(opt :: Any)$AnyFunctions1(Boolean)

maxPower l ==
  if (opt := option(l, 'maxPower)) case "failed" then
    "arbitrary"
  else
    retract(opt :: Any)$AnyFunctions1(Union(PositiveInteger, "arbitrary"))

safety l ==
  if (opt := option(l, 'safety)) case "failed" then
    1$NonNegativeInteger
  else
    retract(opt :: Any)$AnyFunctions1(NonNegativeInteger)

check l ==
  if (opt := option(l, 'check)) case "failed" then

```

```

        "deterministic"
    else
        retract(opt::Any)$AnyFunctions1(_
            Union("skip", "MonteCarlo", "deterministic"))

checkExtraValues l ==
    if (opt := option(l, 'checkExtraValues)) case "failed" then
        true
    else
        retract(opt :: Any)$AnyFunctions1(Boolean)

one l ==
    if (opt := option(l, 'one)) case "failed" then
        true
    else
        retract(opt :: Any)$AnyFunctions1(Boolean)

debug l ==
    if (opt := option(l, 'debug)) case "failed" then
        false
    else
        retract(opt :: Any)$AnyFunctions1(Boolean)

homogeneous l ==
    if (opt := option(l, 'homogeneous)) case "failed" then
        false
    else
        retract(opt :: Any)$AnyFunctions1(Union(PositiveInteger, Boolean))

Somos l ==
    if (opt := option(l, 'Somos)) case "failed" then
        false
    else
        retract(opt :: Any)$AnyFunctions1(Union(PositiveInteger, Boolean))

variableName l ==
    if (opt := option(l, 'variableName)) case "failed" then
        'x
    else
        retract(opt :: Any)$AnyFunctions1(Symbol)

functionName l ==
    if (opt := option(l, 'functionName)) case "failed" then
        'f
    else
        retract(opt :: Any)$AnyFunctions1(Symbol)

indexName l ==
    if (opt := option(l, 'indexName)) case "failed" then
        'n

```

```

else
  retract(opt :: Any)$AnyFunctions1(Symbol)

displayAsGF l ==
  if (opt := option(l, 'displayAsGF)) case "failed" then
    error "GuessOption: displayAsGF not set"
  else
    retract(opt :: Any)$AnyFunctions1(Boolean)

NNI ==> NonNegativeInteger
PI ==> PositiveInteger

checkOptions l ==
  maxD := maxDerivative l
  maxP := maxPower l
  homo := homogeneous l
  Somo := Somos l

  if Somo case PI then
    if one? Somo then
      error "Guess: Somos must be Boolean or at least two"

    if maxP case PI and one? maxP then
      error "Guess: Somos requires that maxPower is at least two"

    if maxD case NNI and maxD > Somo then
      err:String:=concat [_
        "Guess: if Somos is an integer, it should be larger than ",_
        "maxDerivative/maxShift or at least as big as maxSubst" ]
      error err
  else
    if Somo then
      if maxP case PI and one? maxP then
        error "Guess: Somos requires that maxPower is at least two"

      if not (maxD case NNI) or zero? maxD or one? maxD then
        err:String:= concat [_
          "Guess: Somos==true requires that maxDerivative/maxShift",_
          " is an integer, at least two, or maxSubst is an ",_
          "integer, at least three" ]
        error err

      if not (maxP case PI) and homo case Boolean and not homo then
        err:String:= concat [_
          "Guess: Somos requires that maxPower is set or ", _
          "homogeneous is not false" ]
        error err

  if homo case PI then
    if maxP case PI and maxP ~= homo then

```

```

err:String:= _
    "Guess: only one of homogeneous and maxPower may be an integer"
error err

if maxD case NNI and zero? maxD then
    err:String:= concat [_
        "Guess: homogeneous requires that maxShift/maxDerivative ",_
        "is at least one or maxSubst is at least two" ]
    error err
else
    if homo then
        if not maxP case PI then
            err:String:= concat [_
                "Guess: homogeneous==true requires that maxPower is ", _
                "an integer" ]
            error err

        if maxD case NNI and zero? maxD then
            err:String:= concat [_
                "Guess: homogeneous requires that maxShift/maxDerivative",_
                " is at least one or maxSubst is at least two" ]
            error err

```

---

— GOPT0.dotabb —

```

"GOPT0" [color="#88FF44",href="bookvol10.3.pdf#nameddest=GOPT0"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"GOPT0" -> "STRING"

```

---

## Chapter 9

# Chapter H

### 9.1 domain HASHTBL HashTable

— HashTable.input —

```
)set break resume
)sys rm -f HashTable.output
)spool HashTable.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show HashTable
--R HashTable(Key: SetCategory,Entry: SetCategory,hashfn: String) is a domain constructor
--R Abbreviation for HashTable is HASHTBL
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for HASHTBL
--R
--R----- Operations -----
--R copy : % -> % dictionary : () -> %
--R elt : (% ,Key,Entry) -> Entry ?.? : (% ,Key) -> Entry
--R empty : () -> % empty? : % -> Boolean
--R entries : % -> List Entry eq? : (% ,%) -> Boolean
--R index? : (Key,% ) -> Boolean indices : % -> List Key
--R key? : (Key,% ) -> Boolean keys : % -> List Key
--R map : ((Entry -> Entry),%) -> % qelt : (% ,Key) -> Entry
--R sample : () -> % setelt : (% ,Key,Entry) -> Entry
--R table : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ==? : (% ,%) -> Boolean if Record(key: Key,entry: Entry) has SETCAT or Entry has SETCAT
--R any? : ((Entry -> Boolean),%) -> Boolean if $ has finiteAggregate
```



```

--R any? : ((Record(key: Key,entry: Entry) -> Boolean),%) -> Boolean if $ has finiteAggregate
--R bag : List Record(key: Key,entry: Entry) -> %
--R coerce : % -> OutputForm if Record(key: Key,entry: Entry) has SETCAT or Entry has SETCAT
--R construct : List Record(key: Key,entry: Entry) -> %
--R convert : % -> InputForm if Record(key: Key,entry: Entry) has KONVERT INFORM
--R count : ((Entry -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R count : (Entry,%) -> NonNegativeInteger if $ has finiteAggregate and Entry has SETCAT
--R count : (Record(key: Key,entry: Entry),%) -> NonNegativeInteger if $ has finiteAggregate
--R count : ((Record(key: Key,entry: Entry) -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R dictionary : List Record(key: Key,entry: Entry) -> %
--R entry? : (Entry,%) -> Boolean if $ has finiteAggregate and Entry has SETCAT
--R eval : (%,List Equation Entry) -> % if Entry has EVALAB Entry and Entry has SETCAT
--R eval : (%,Equation Entry) -> % if Entry has EVALAB Entry and Entry has SETCAT
--R eval : (%,Entry,Entry) -> % if Entry has EVALAB Entry and Entry has SETCAT
--R eval : (%,List Entry,List Entry) -> % if Entry has EVALAB Entry and Entry has SETCAT
--R eval : (%,List Record(key: Key,entry: Entry),List Record(key: Key,entry: Entry)) -> % if Entry has SETCAT
--R eval : (%,Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> % if Record(key: Key,entry: Entry) has SETCAT
--R eval : (%,Equation Record(key: Key,entry: Entry)) -> % if Record(key: Key,entry: Entry) has SETCAT
--R eval : (%,List Equation Record(key: Key,entry: Entry)) -> % if Record(key: Key,entry: Entry) has SETCAT
--R every? : ((Entry -> Boolean),%) -> Boolean if $ has finiteAggregate
--R every? : ((Record(key: Key,entry: Entry) -> Boolean),%) -> Boolean if $ has finiteAggregate
--R extract! : % -> Record(key: Key,entry: Entry)
--R fill! : (%,Entry) -> % if $ has shallowlyMutable
--R find : ((Record(key: Key,entry: Entry) -> Boolean),%) -> Union(Record(key: Key,entry: Entry),%)
--R first : % -> Entry if Key has ORDSET
--R hash : % -> SingleInteger if Record(key: Key,entry: Entry) has SETCAT or Entry has SETCAT
--R insert! : (Record(key: Key,entry: Entry),%) -> %
--R inspect : % -> Record(key: Key,entry: Entry)
--R latex : % -> String if Record(key: Key,entry: Entry) has SETCAT or Entry has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map : ((Entry,Entry) -> Entry),%,%) -> %
--R map : ((Record(key: Key,entry: Entry) -> Record(key: Key,entry: Entry)),%,%) -> %
--R map! : ((Entry -> Entry),%) -> % if $ has shallowlyMutable
--R map! : ((Record(key: Key,entry: Entry) -> Record(key: Key,entry: Entry)),%,%) -> % if $ has shallowlyMutable
--R maxIndex : % -> Key if Key has ORDSET
--R member? : (Entry,%) -> Boolean if $ has finiteAggregate and Entry has SETCAT
--R member? : (Record(key: Key,entry: Entry),%) -> Boolean if $ has finiteAggregate and Record(key: Key,entry: Entry) has SETCAT
--R members : % -> List Entry if $ has finiteAggregate
--R members : % -> List Record(key: Key,entry: Entry) if $ has finiteAggregate
--R minIndex : % -> Key if Key has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List Entry if $ has finiteAggregate
--R parts : % -> List Record(key: Key,entry: Entry) if $ has finiteAggregate
--R qsetelt! : (%,Key,Entry) -> Entry if $ has shallowlyMutable
--R reduce : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> Record(key: Key,entry: Entry)),%,%) -> %
--R reduce : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> Record(key: Key,entry: Entry)),%,%) -> %
--R reduce : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> Record(key: Key,entry: Entry)),%,%) -> %
--R remove : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (Record(key: Key,entry: Entry),%) -> % if $ has finiteAggregate and Record(key: Key,entry: Entry) has SETCAT
--R remove! : (Key,%) -> Union(Entry,"failed")

```

```

--R remove! : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finiteAggregate
--R remove! : (Record(key: Key,entry: Entry),%) -> % if $ has finiteAggregate
--R removeDuplicates : % -> % if $ has finiteAggregate and Record(key: Key,entry: Entry) has SETCAT
--R search : (Key,%) -> Union(Entry,"failed")
--R select : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finiteAggregate
--R select! : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R swap! : (%,Key,Key) -> Void if $ has shallowlyMutable
--R table : List Record(key: Key,entry: Entry) -> %
--R ?~=? : (%,%) -> Boolean if Record(key: Key,entry: Entry) has SETCAT or Entry has SETCAT
--R
--E 1

```

```

)spool
)lisp (bye)

```

---

— HashTable.help —

```

=====
HashTable examples
=====

```

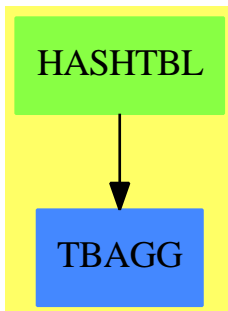
```

See Also:
o )show HashTable

```

---

### 9.1.1 HashTable (HASHTBL)



See

⇒ “InnerTable” (INTABL) 10.27.1 on page 1299  
 ⇒ “Table” (TABLE) 21.1.1 on page 2621

⇒ “EqTable” (EQTBL) 6.2.1 on page 667  
 ⇒ “StringTable” (STRTBL) 20.32.1 on page 2569  
 ⇒ “GeneralSparseTable” (GSTBL) 8.5.1 on page 1044  
 ⇒ “SparseTable” (STBL) 20.16.1 on page 2409

**Exports:**

|        |          |                  |           |          |
|--------|----------|------------------|-----------|----------|
| any?   | bag      | coerce           | construct | convert  |
| copy   | count    | dictionary       | entry?    | elt      |
| empty  | empty?   | entries          | eq?       | eval     |
| every? | extract! | fill!            | find      | first    |
| hash   | index?   | indices          | insert!   | inspect  |
| key?   | keys     | latex            | less?     | map      |
| map!   | maxIndex | member?          | members   | minIndex |
| more?  | parts    | qelt             | qsetelt!  | reduce   |
| remove | remove!  | removeDuplicates | sample    | search   |
| select | select!  | setelt           | size?     | swap!    |
| table  | #?       | ?=?              | ?~=?      | ?..?     |

— domain HASHTBL HashTable —

```
)abbrev domain HASHTBL HashTable
++ Author: Stephen M. Watt
++ Date Created: 1985
++ Date Last Updated: June 21, 1991
++ Basic Operations:
++ Related Domains: Table, EqTable, StringTable
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ This domain provides access to the underlying Lisp hash tables.
++ By varying the hashfn parameter, tables suited for different
++ purposes can be obtained.
```

```
HashTable(Key, Entry, hashfn): Exports == Implementation where
  Key, Entry: SetCategory
  hashfn: String -- Union("EQ", "UEQUAL", "CVEC", "ID")
```

```
Exports ==> TableAggregate(Key, Entry) with
  finiteAggregate
```

```
Implementation ==> add
  Pair ==> Record(key: Key, entry: Entry)
  Ex ==> OutputForm
  failMsg := GENSYM()$Lisp
```

```
t1 = t2 == EQ(t1, t2)$Lisp
```

```

keys t                == HKEYS(t)$Lisp
# t                   == HASH_-TABLE_-COUNT(t)$Lisp
setelt(t, k, e)       == HPUT(t,k,e)$Lisp
remove_!(k:Key, t:%) ==
  r := HGET(t,k,failMsg)$Lisp
  not EQ(r,failMsg)$Lisp =>
    HREM(t, k)$Lisp
    r pretend Entry
    "failed"

empty() ==
  MAKE_-HASHTABLE(INTERN(hashfn)$Lisp,
    INTERN("STRONG")$Lisp)$Lisp

search(k:Key, t:%) ==
  r := HGET(t, k, failMsg)$Lisp
  not EQ(r, failMsg)$Lisp => r pretend Entry
  "failed"

```

---

— HASHTBL.dotabb —

```

"HASHTBL" [color="#88FF44",href="bookvol10.3.pdf#nameddest=HASHTBL"]
"TBAGG"   [color="#4488FF",href="bookvol10.2.pdf#nameddest=TBAGG"]
"HASHTBL" -> "TBAGG"

```

---

## 9.2 domain HEAP Heap

---

— Heap.input —

```

)set break resume
)sys rm -f Heap.output
)spool Heap.output
)set message test on
)set message auto off
)clear all

--S 1 of 42
a:Heap INT:= heap [1,2,3,4,5]
--R
--R

```

```
--R (1) [5,4,2,1,3]
```

```
--R
```

Type: Heap Integer

```
--E 1
```

```
--S 2 of 42
```

```
bag([1,2,3,4,5])$Heap(INT)
```

```
--R
```

```
--R
```

```
--R (2) [5,4,3,1,2]
```

```
--R
```

Type: Heap Integer

```
--E 2
```

```
--S 3 of 42
```

```
c:=copy a
```

```
--R
```

```
--R
```

```
--R (3) [5,4,2,1,3]
```

```
--R
```

Type: Heap Integer

```
--E 3
```

```
--S 4 of 42
```

```
empty? a
```

```
--R
```

```
--R
```

```
--R (4) false
```

```
--R
```

Type: Boolean

```
--E 4
```

```
--S 5 of 42
```

```
b:=empty()$(Heap INT)
```

```
--R
```

```
--R
```

```
--R (5) []
```

```
--R
```

Type: Heap Integer

```
--E 5
```

```
--S 6 of 42
```

```
empty? b
```

```
--R
```

```
--R
```

```
--R (6) true
```

```
--R
```

Type: Boolean

```
--E 6
```

```
--S 7 of 42
```

```
eq?(a,c)
```

```
--R
```

```
--R
```

```
--R (7) false
```

```
--R
```

Type: Boolean

[illegible]

```

(a~=c)
--R
--R
--R (14) true
--R
--R                                          Type: Boolean
--E 13

--S 14 of 42
a
--R
--R
--R (15) [4,3,2,1]
--R
--R                                          Type: Heap Integer
--E 14

--S 15 of 42
inspect a
--R
--R
--R (16) 4
--R
--R                                          Type: PositiveInteger
--E 15

--S 16 of 42
insert!(9,a)
--R
--R
--R (17) [9,4,2,1,3]
--R
--R                                          Type: Heap Integer
--E 16

--S 17 of 42
map(x+>x+10,a)
--R
--R
--R (18) [19,14,12,11,13]
--R
--R                                          Type: Heap Integer
--E 17

--S 18 of 42
a
--R
--R
--R (19) [9,4,2,1,3]
--R
--R                                          Type: Heap Integer
--E 18

--S 19 of 42
map!(x+>x+10,a)
--R

```

```

--R
--R (20) [19,14,12,11,13]
--R
--R                                          Type: Heap Integer
--E 19

--S 20 of 42
a
--R
--R
--R (21) [19,14,12,11,13]
--R
--R                                          Type: Heap Integer
--E 20

--S 21 of 42
max a
--R
--R
--R (22) 19
--R
--R                                          Type: PositiveInteger
--E 21

--S 22 of 42
merge(a,c)
--R
--R
--R (23) [19,14,12,11,13,5,4,2,1,3]
--R
--R                                          Type: Heap Integer
--E 22

--S 23 of 42
a
--R
--R
--R (24) [19,14,12,11,13]
--R
--R                                          Type: Heap Integer
--E 23

--S 24 of 42
merge!(a,c)
--R
--R
--R (25) [19,14,12,11,13,5,4,2,1,3]
--R
--R                                          Type: Heap Integer
--E 24

--S 25 of 42
a
--R
--R
--R (26) [19,14,12,11,13,5,4,2,1,3]

```



```

--R
--E 25
Type: Heap Integer

--S 26 of 42
c
--R
--R
--R (27) [5,4,2,1,3]
--R
--R
--E 26
Type: Heap Integer

--S 27 of 42
sample()$Heap(INT)
--R
--R
--R (28) []
--R
--R
--E 27
Type: Heap Integer

--S 28 of 42
#a
--R
--R
--R (29) 10
--R
--R
--E 28
Type: PositiveInteger

--S 29 of 42
any?(x+-(x=14),a)
--R
--R
--R (30) true
--R
--R
--E 29
Type: Boolean

--S 30 of 42
every?(x+-(x=11),a)
--R
--R
--R (31) false
--R
--R
--E 30
Type: Boolean

--S 31 of 42
parts a
--R
--R
--R (32) [19,14,12,11,13,5,4,2,1,3]
--R
--R
--E 31
Type: List Integer

```

```
--S 32 of 42
size?(a,9)
--R
--R
--R (33) false
--R
--R                                         Type: Boolean
--E 32
```

```
--S 33 of 42
more?(a,9)
--R
--R
--R (34) true
--R
--R                                         Type: Boolean
--E 33
```

```
--S 34 of 42
less?(a,9)
--R
--R
--R (35) false
--R
--R                                         Type: Boolean
--E 34
```

```
--S 35 of 42
members a
--R
--R
--R (36) [19,14,12,11,13,5,4,2,1,3]
--R
--R                                         Type: List Integer
--E 35
```

```
--S 36 of 42
member?(14,a)
--R
--R
--R (37) true
--R
--R                                         Type: Boolean
--E 36
```

```
--S 37 of 42
latex a
--R
--R
--R (38) "\mbox{\bf Unimplemented}"
--R
--R                                         Type: String
--E 37
```

```
--S 38 of 42
```

```

hash a
--R
--R
--I (39) 36647017
--R
--R                                          Type: SingleInteger
--E 38

--S 39 of 42
count(14,a)
--R
--R
--R (40) 1
--R
--R                                          Type: PositiveInteger
--E 39

--S 40 of 42
count(x+-(x>13),a)
--R
--R
--R (41) 2
--R
--R                                          Type: PositiveInteger
--E 40

--S 41 of 42
coerce a
--R
--R
--R (42) [19,14,12,11,13,5,4,2,1,3]
--R
--R                                          Type: OutputForm
--E 41

--S 42 of 42
)show Heap
--R
--R Heap S: OrderedSet is a domain constructor
--R Abbreviation for Heap is HEAP
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for HEAP
--R
--R----- Operations -----
--R bag : List S -> %                copy : % -> %
--R empty : () -> %                  empty? : % -> Boolean
--R eq? : (%,% ) -> Boolean          extract! : % -> S
--R heap : List S -> %              insert! : (S,% ) -> %
--R inspect : % -> S                map : ((S -> S),%) -> %
--R max : % -> S                    merge : (%,% ) -> %
--R merge! : (%,% ) -> %            sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ==? : (%,% ) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate

```

```

--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if $ has finiteAggregate and S has SETCAT
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R members : % -> List S if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List S if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R ~=?: (%,%) -> Boolean if S has SETCAT
--R
--E 42

```

```

)spool
)lisp (bye)

```

---

— Heap.help —

=====

Heap examples

=====

The domain Heap(S) implements a priority queue of objects of type S such that the operation extract! removes and returns the maximum element. The implementation represents heaps as flexible arrays. The representation and algorithms give complexity of  $O(\log(n))$  for insertion and extractions, and  $O(n)$  for construction.

Create a heap of five elements:

```

a:Heap INT:= heap [1,2,3,4,5]
               [5,4,2,1,3]

```

Use bag to convert a Bag into a Heap:

```

bag([1,2,3,4,5])$Heap(INT)
               [5,4,3,1,2]

```

The operation copy can be used to copy a Heap:

```
c:=copy a
    [5,4,2,1,3]
```

Use `empty?` to check if the heap is empty:

```
empty? a
    false
```

Use `empty` to create a new, empty heap:

```
b:=empty()$(Heap INT)
    []
```

and we can see that the newly created heap is empty:

```
empty? b
    true
```

The `eq?` function compares the reference of one heap to another:

```
eq?(a,c)
    false
```

The `extract!` function removes largest element of the heap:

```
extract! a
    5
```

Now `extract!` elements repeatedly until none are left, collecting the elements in a list.

```
[extract!(h) while not empty?(h)]
    [9,7,3,2,- 4,- 7]
                                Type: List Integer
```

Another way to produce the same result is by defining a `heapsort` function.

```
heapsort(x) == (empty? x => []; cons(extract!(x),heapsort x))
                                Type: Void
```

Create another sample heap.

```
h1 := heap [17,-4,9,-11,2,7,-7]
    [17,2,9,- 11,- 4,7,- 7]
                                Type: Heap Integer
```

Apply `heapsort` to present elements in order.

```
heapsort h1
```

```
[17,9,7,2,- 4,- 7,- 11]
      Type: List Integer
```

Heaps can be compared with =

```
(a=c)@Boolean
      false
```

and ~=

```
(a~=c)
      true
```

The inspect function shows the largest element in the heap:

```
inspect a
      4
```

The insert! function adds an element to the heap:

```
insert!(9,a)
      [9,4,2,1,3]
```

The map function applies a function to every element of the heap and returns a new heap:

```
map(x+>x+10,a)
      [19,14,12,11,13]
```

The original heap is unchanged:

```
a
      [9,4,2,1,3]
```

The map! function applies a function to every element of the heap and returns the original heap with modifications:

```
map!(x+>x+10,a)
      [19,14,12,11,13]
```

The original heap has been modified:

```
a
      [19,14,12,11,13]
```

The max function returns the largest element in the heap:

```
max a
      19
```

The merge function takes two heaps and creates a new heap with all of the elements:

```
merge(a,c)
[19,14,12,11,13,5,4,2,1,3]
```

Notice that the original heap is unchanged:

```
a
[19,14,12,11,13]
```

The merge! function takes two heaps and modifies the first heap argument to contain all of the elements:

```
merge!(a,c)
[19,14,12,11,13,5,4,2,1,3]
```

Notice that the first argument was modified:

```
a
[19,14,12,11,13,5,4,2,1,3]
```

but the second argument was not:

```
c
[5,4,2,1,3]
```

A new, empty heap can be created with sample:

```
sample()$Heap(INT)
[]
```

The # function gives the size of the heap:

```
#a
10
```

The any? function tests each element against a predicate function and returns true if any pass:

```
any?(x-->(x=14),a)
true
```

The every? function tests each element against a predicate function and returns true if they all pass:

```
every?(x-->(x=11),a)
false
```

The parts function returns a list of the elements in the heap:

```
parts a
      [19,14,12,11,13,5,4,2,1,3]
```

The size? predicate compares the size of the heap to a value:

```
size?(a,9)
      false
```

The more? predicate asks if the heap size is larger than a value:

```
more?(a,9)
      true
```

The less? predicate asks if the heap size is smaller than a value:

```
less?(a,9)
      false
```

The members function returns a list of the elements of the heap:

```
members a
      [19,14,12,11,13,5,4,2,1,3]
```

The member? predicate asks if an element is in the heap:

```
member?(14,a)
      true
```

The count function has two forms, one of which counts the number of copies of an element in the heap:

```
count(14,a)
      1
```

The second form of the count function accepts a predicate to test against each member of the heap and counts the number of true results:

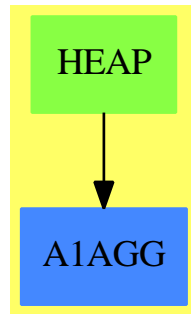
```
count(x+>(x>13),a)
      2
```

See Also:

- o )show Stack
- o )show ArrayStack
- o )show Queue
- o )show Dequeue
- o )show Heap
- o )show BagAggregate



### 9.2.1 Heap (HEAP)



See

- ⇒ “Stack” (STACK) 20.28.1 on page 2521
- ⇒ “ArrayStack” (ASTACK) 2.10.1 on page 65
- ⇒ “Queue” (QUEUE) 18.5.1 on page 2143
- ⇒ “Dequeue” (DEQUEUE) 5.5.1 on page 497

#### Exports:

|          |         |        |         |         |
|----------|---------|--------|---------|---------|
| any?     | bag     | coerce | copy    | count   |
| empty    | empty?  | eq?    | eval    | every?  |
| extract! | hash    | heap   | insert! | inspect |
| latex    | less?   | map    | map!    | max     |
| member?  | members | merge  | merge!  | more?   |
| parts    | sample  | size?  | #?      | ?=?     |
| ?~=?     |         |        |         |         |

— domain **HEAP** Heap —

```

)abbrev domain HEAP Heap
++ Author: Michael Monagan and Stephen Watt
++ Date Created: June 86 and July 87
++ Date Last Updated: Feb 92
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ Heap implemented in a flexible array to allow for insertions
++ Complexity:  $O(\log n)$  insertion, extraction and  $O(n)$  construction

```

```

--% Dequeue and Heap data types

Heap(S:OrderedSet): Exports == Implementation where
  Exports == PriorityQueueAggregate S with
    heap : List S -> %
      ++ heap(ls) creates a heap of elements consisting of the
      ++ elements of ls.
      ++
      ++E i:Heap INT := heap [1,6,3,7,5,2,4]

-- Inherited Signatures repeated for examples documentation

bag : List S -> %
  ++
  ++X bag([1,2,3,4,5])$Heap(INT)
copy : % -> %
  ++
  ++X a:Heap INT:= heap [1,2,3,4,5]
  ++X copy a
empty? : % -> Boolean
  ++
  ++X a:Heap INT:= heap [1,2,3,4,5]
  ++X empty? a
empty : () -> %
  ++
  ++X b:=empty()$(Heap INT)
eq? : (%,% ) -> Boolean
  ++
  ++X a:Heap INT:= heap [1,2,3,4,5]
  ++X b:=copy a
  ++X eq?(a,b)
extract_! : % -> S
  ++
  ++X a:Heap INT:= heap [1,2,3,4,5]
  ++X extract! a
  ++X a
insert_! : (S,% ) -> %
  ++
  ++X a:Heap INT:= heap [1,2,3,4,5]
  ++X insert!(8,a)
  ++X a
inspect : % -> S
  ++
  ++X a:Heap INT:= heap [1,2,3,4,5]
  ++X inspect a
map : ((S -> S),%) -> %
  ++
  ++X a:Heap INT:= heap [1,2,3,4,5]
  ++X map(x+>x+10,a)
  ++X a

```

```

max : % -> S
++
++X a:Heap INT:= heap [1,2,3,4,5]
++X max a
merge : (%,%) -> %
++
++X a:Heap INT:= heap [1,2,3,4,5]
++X b:Heap INT:= heap [6,7,8,9,10]
++X merge(a,b)
merge! : (%,%) -> %
++
++X a:Heap INT:= heap [1,2,3,4,5]
++X b:Heap INT:= heap [6,7,8,9,10]
++X merge!(a,b)
++X a
++X b
sample : () -> %
++
++X sample()$Heap(INT)
less? : (%,NonNegativeInteger) -> Boolean
++
++X a:Heap INT:= heap [1,2,3,4,5]
++X less?(a,9)
more? : (%,NonNegativeInteger) -> Boolean
++
++X a:Heap INT:= heap [1,2,3,4,5]
++X more?(a,9)
size? : (%,NonNegativeInteger) -> Boolean
++
++X a:Heap INT:= heap [1,2,3,4,5]
++X size?(a,5)
if $ has shallowlyMutable then
  map! : ((S -> S),%) -> %
  ++
  ++X a:Heap INT:= heap [1,2,3,4,5]
  ++X map!(x+>x+10,a)
  ++X a
if S has SetCategory then
  latex : % -> String
  ++
  ++X a:Heap INT:= heap [1,2,3,4,5]
  ++X latex a
  hash : % -> SingleInteger
  ++
  ++X a:Heap INT:= heap [1,2,3,4,5]
  ++X hash a
  coerce : % -> OutputForm
  ++
  ++X a:Heap INT:= heap [1,2,3,4,5]
  ++X coerce a

```

```

"=": (%,% ) -> Boolean
++
++X a:Heap INT:= heap [1,2,3,4,5]
++X b:Heap INT:= heap [1,2,3,4,5]
++X (a=b)@Boolean
"~=" : (%,% ) -> Boolean
++
++X a:Heap INT:= heap [1,2,3,4,5]
++X b:=copy a
++X (a~=b)
if % has finiteAggregate then
every? : ((S -> Boolean),%) -> Boolean
++
++X a:Heap INT:= heap [1,2,3,4,5]
++X every?(x+-(x=4),a)
any? : ((S -> Boolean),%) -> Boolean
++
++X a:Heap INT:= heap [1,2,3,4,5]
++X any?(x+-(x=4),a)
count : ((S -> Boolean),%) -> NonNegativeInteger
++
++X a:Heap INT:= heap [1,2,3,4,5]
++X count(x+-(x>2),a)
_# : % -> NonNegativeInteger
++
++X a:Heap INT:= heap [1,2,3,4,5]
++X #a
parts : % -> List S
++
++X a:Heap INT:= heap [1,2,3,4,5]
++X parts a
members : % -> List S
++
++X a:Heap INT:= heap [1,2,3,4,5]
++X members a
if % has finiteAggregate and S has SetCategory then
member? : (S,%) -> Boolean
++
++X a:Heap INT:= heap [1,2,3,4,5]
++X member?(3,a)
count : (S,%) -> NonNegativeInteger
++
++X a:Heap INT:= heap [1,2,3,4,5]
++X count(4,a)

Implementation == IndexedFlexibleArray(S,0) add
Rep := IndexedFlexibleArray( S,0)
empty() == empty()$Rep
heap 1 ==
n := #1

```

```

h := empty()
n = 0 => h
for x in l repeat insert_!(x,h)
h
siftUp: (%,Integer,Integer) -> Void
siftUp(r,i,n) ==
  -- assertion 0 <= i < n
  t := r.i
  while (j := 2*i+1) < n repeat
    if (k := j+1) < n and r.j < r.k then j := k
    if t < r.j then (r.i := r.j; r.j := t; i := j) else leave

extract_! r ==
  -- extract the maximum from the heap O(log n)
  n := #r :: Integer
  n = 0 => error "empty heap"
  t := r(0)
  r(0) := r(n-1)
  delete_!(r,n-1)
  n = 1 => t
  siftUp(r,0,n-1)
  t

insert_!(x,r) ==
  -- Williams' insertion algorithm O(log n)
  j := (#r) :: Integer
  r:=concat_!(r,concat(x,empty()$Rep))
  while j > 0 repeat
    i := (j-1) quo 2
    if r(i) >= x then leave
    r(j) := r(i)
    j := i
  r(j):=x
  r

max r == if #r = 0 then error "empty heap" else r.0
inspect r == max r

makeHeap(r:%):% ==
  -- Floyd's heap construction algorithm O(n)
  n := #r
  for k in n quo 2 -1 .. 0 by -1 repeat siftUp(r,k,n)
  r
bag l == makeHeap construct(l)$Rep
merge(a,b) == makeHeap concat(a,b)
merge_!(a,b) == makeHeap concat_!(a,b)

```

---

```
"HEAP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=HEAP"]
"A1AGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=A1AGG"]
"HEAP" -> "A1AGG"
```

— HexadecimalExpansion.input —

[illegible]

```

--E 3

--S 4 of 7
hex(1/1007)
--R
--R
--R (4)
--R 0.
--R OVERBAR
--R 0041149783F0BF2C7D13933192AF6980619EE345E91EC2BB9D5CCA5C071E40926E54E8D
--R DAE24196C0B2F8A0AAD60DBA57F5D4C8536262210C74F1
--R                                         Type: HexadecimalExpansion
--E 4

--S 5 of 7
p := hex(1/4)*x**2 + hex(2/3)*x + hex(4/9)
--R
--R
--R (5)  $0.4x^2 + 0.Ax + 0.71C$ 
--R                                         Type: Polynomial HexadecimalExpansion
--E 5

--S 6 of 7
q := D(p, x)
--R
--R
--R (6)  $0.8x + 0.A$ 
--R                                         Type: Polynomial HexadecimalExpansion
--E 6

--S 7 of 7
g := gcd(p, q)
--R
--R
--R (7)  $x + 1.5$ 
--R                                         Type: Polynomial HexadecimalExpansion
--E 7
)spool
)lisp (bye)

```

---

— HexadecimalExpansion.help —

```

=====
HexadecimalExpansion

```

=====

All rationals have repeating hexadecimal expansions. The operation `hex` returns these expansions of type `HexadecimalExpansion`. Operations to access the individual numerals of a hexadecimal expansion can be obtained by converting the value to `RadixExpansion(16)`. More examples of expansions are available in the `DecimalExpansion`, `BinaryExpansion`, and `RadixExpansion`.

This is a hexadecimal expansion of a rational number.

```
r := hex(22/7)
    ---
    3.249
                                Type: HexadecimalExpansion
```

Arithmetic is exact.

```
r + hex(6/7)
    4
                                Type: HexadecimalExpansion
```

The period of the expansion can be short or long ...

```
[hex(1/i) for i in 350..354]
    -----
    [0.00BB3EE721A54D88, 0.00BAB6561, 0.00BA2E8, 0.00B9A7862A0FF465879D5F,
    -----
    0.00B92143FA36F5E02E4850FE8DBD78]
                                Type: List HexadecimalExpansion
```

or very long!

```
hex(1/1007)
    -----
    0.0041149783F0BF2C7D13933192AF6980619EE345E91EC2BB9D5CCA5C071E40926E54E8D
    -----
    DAE24196C0B2F8A0AAD60DBA57F5D4C8536262210C74F1
                                Type: HexadecimalExpansion
```

These numbers are bona fide algebraic objects.

```
p := hex(1/4)*x**2 + hex(2/3)*x + hex(4/9)
    2      -      ---
    0.4x  + 0.Ax + 0.71C
                                Type: Polynomial HexadecimalExpansion
```

```
q := D(p, x)
    -
    0.8x + 0.A
```



Type: Polynomial HexadecimalExpansion

$g := \gcd(p, q)$

$x + 1.5$

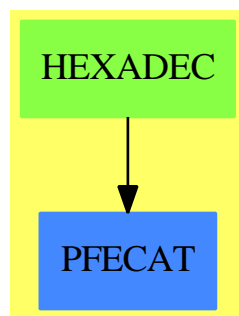
Type: Polynomial HexadecimalExpansion

See Also:

- o )help RadixExpansion
- o )help BinaryExpansion
- o )help DecimalExpansion
- o )show HexadecimalExpansion

—————→

### 9.3.1 HexadecimalExpansion (HEXADEC)



See

- ⇒ “RadixExpansion” (RADIX) 19.2.1 on page 2165
- ⇒ “BinaryExpansion” (BINARY) 3.7.1 on page 274
- ⇒ “DecimalExpansion” (DECIMAL) 5.3.1 on page 451

**Exports:**

|                               |                      |
|-------------------------------|----------------------|
| 0                             | 1                    |
| abs                           | associates?          |
| ceiling                       | characteristic       |
| charthRoot                    | coerce               |
| conditionP                    | convert              |
| D                             | denom                |
| denominator                   | differentiate        |
| divide                        | euclideanSize        |
| eval                          | expressIdealMember   |
| exquo                         | extendedEuclidean    |
| factor                        | factorPolynomial     |
| factorSquareFreePolynomial    | floor                |
| fractionPart                  | gcd                  |
| gcdPolynomial                 | hash                 |
| hex                           | init                 |
| inv                           | latex                |
| lcm                           | map                  |
| max                           | min                  |
| multiEuclidean                | negative?            |
| nextItem                      | numer                |
| numerator                     | one?                 |
| patternMatch                  | positive?            |
| prime?                        | principalIdeal       |
| random                        | recip                |
| reducedSystem                 | retract              |
| retractIfCan                  | sample               |
| sign                          | sizeLess?            |
| solveLinearPolynomialEquation | squareFree           |
| squareFreePart                | squareFreePolynomial |
| subtractIfCan                 | unit?                |
| unitCanonical                 | unitNormal           |
| wholePart                     | zero?                |
| ?*?                           | ?**?                 |
| ?+?                           | ?-?                  |
| -?                            | ?/?                  |
| ?=?                           | ?^?                  |
| ?~=?                          | ?<?                  |
| ?<=?                          | ?>?                  |
| ?>=?                          | ?..?                 |
| ?quo?                         | ?rem?                |

— domain HEXADEC HexadecimalExpansion —

```
)abbrev domain HEXADEC HexadecimalExpansion
++ Author: Clifton J. Williamson
```

```

++ Date Created: April 26, 1990
++ Date Last Updated: May 15, 1991
++ Basic Operations:
++ Related Domains: RadixExpansion
++ Also See:
++ AMS Classifications:
++ Keywords: radix, base, hexadecimal
++ Examples:
++ References:
++ Description:
++ This domain allows rational numbers to be presented as repeating
++ hexadecimal expansions.

HexadecimalExpansion(): Exports == Implementation where
  Exports ==> QuotientFieldCategory(Integer) with
    coerce: % -> Fraction Integer
      ++ coerce(h) converts a hexadecimal expansion to a rational number.
    coerce: % -> RadixExpansion(16)
      ++ coerce(h) converts a hexadecimal expansion to a radix expansion
      ++ with base 16.
    fractionPart: % -> Fraction Integer
      ++ fractionPart(h) returns the fractional part of a hexadecimal expansion.
    hex: Fraction Integer -> %
      ++ hex(r) converts a rational number to a hexadecimal expansion.

Implementation ==> RadixExpansion(16) add
  hex r == r :: %
  coerce(x:%): RadixExpansion(16) == x pretend RadixExpansion(16)



---


— HEXADEC.dotabb —

"HEXADEC" [color="#88FF44",href="bookvol10.3.pdf#nameddest=HEXADEC"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"HEXADEC" -> "PFECAT"

```

---

## 9.4 package HTMLFORM HTMLFormat

Here I have put some information about 'how to use' and 'the benefits of' this HTML formatter. Also some information for programmers if they want to extend this package.

If you want information about creating output formatters in general then, rather than duplicating content here I refer you to `mathml.spad.pamphlet` containing the `MathMLFormat`

domain by Arthur C. Ralfs. This contains useful information for writers of output formatters.

### 9.4.1 Overview

This package allows users to cut and paste output from the Axiom command line to a HTML page. This output is enabled by typing:

```
)set output html on
```

After this the command line will output html (in addition to other formats that are enabled) and this html code can then be copied and pasted into a HTML document.

The HTML produced is well formed XML, that is, all tags have equivalent closing tags.

### 9.4.2 Why output to HTML?

In some ways HTMLFormat is a compromise between the standard text output and specialised formats like MathMLFormat. The potential quality is never going to be as good as output to a specialised maths renderer but on the other hand it is a lot better than the clunky fixed width font text output. The quality is not the only issue though, the direct output in any format is unlikely to be exactly what the user wants, so possibly more important than quality is the ability to edit the output.

HTMLFormat has advantages that the other output formats don't, for instance,

- It works with any browser without the need for plugins (as far as I know most computers should have the required fonts)
- Users can easily annotate and add comments using colour, bold, underline and so on.
- Annotations can easily be done with whatever html editor or text editor you are familiar with.
- Edits to the output will cause the width of columns and so on to be automatically adjusted, no need to try to insert spaces to get the superscripts to line up again!
- It is very easy to customise output so, for instance, we can fit a lot of information in a compact space on the page.

## 9.5 Using the formatter

We can cause the command line interpreter to output in html by typing the following:

```
)set output html on
```

After this the command line will output html (in addition to other formats that are enabled) and this html code can then be copied and pasted into an existing HTML document.

If you do not already have an html page to copy the output to then you can create one with a text editor and entering the following:

```
<!DOCTYPE html PUBLIC "-//W3C//DTD XHTML 1.1//EN"
"http://www.w3.org/TR/xhtml11/DTD/xhtml11.dtd">
<html xmlns="http://www.w3.org/1999/xhtml" >
  <head>
    <title>Enter Your Title Here</title>
  </head>
  <body>
    Copy and paste the output from command line here.
  </body>
</html>
```

Or using any program that will export to html such as OpenOffice.org writer.

## 9.6 Form of the output

HTMLFormat does not try to interpret syntax, for instance in an example like:

```
(1) -> integral(x^x,x)
```

it just takes what OutputForm provides and does not try to replace %A with the bound variable x.

## 9.7 Matrix Formatting

A big requirement for me is to fit big matrices on ordinary web pages.

At the moment the default output for a matrix is a grid, however it easy to modify this for a single matrix, or a whole page or whole site by using css (cascading style sheets). For instance we can get a more conventional looking matrix by adding the following style to the top of the page after the `<head>` tag:

```
<style type="text/css">
#matl {border-left-style:solid}
#matr {border-right-style:solid}
#matlt {border-left-style:solid;border-top-style:solid}
#matrt {border-right-style:solid;border-top-style:solid}
#matlb {border-left-style:solid;border-bottom-style:solid}
#matrb {border-right-style:solid;border-bottom-style:solid}
</style>
```

There are many other possibilities, for instance we can generate a matrix with bars either side to indicate a determinant. All we have to do is change the css for the site, page or individual element.

## 9.8 Programmers Guide

This package converts from `OutputForm`, which is a hierarchical tree structure, to `html` which uses tags arranged in a hierarchical tree structure. So the package converts from one tree (graph) structure to another.

This conversion is done in two stages using an intermediate `Tree String` structure. This `Tree String` structure represents `HTML` where:

- leafs represents unstructured text
- string in leafs contains the text
- non-leafs represents xml elements
- string in non-leafs represents xml attributes

This is created by traversing `OutputForm` while building up the `Tree String` structure.

The second stage is to convert the `Tree Structure` to text. All text output is done using:

```
sayTeX$Lisp
```

I have not produced an output to `String` as I don't know a way to append to a long string efficiently and I don't know how to insert carriage- returns into a `String`.

### 9.8.1 Future Developments

There would be some benefits in creating a `XMLFormat` category which would contain common elements for all xml formatted outputs such as `HTMLFormat`, `MathMLFormat`, `SVGFormat` and `X3DFormat`. However programming effort might be better spent creating a version of `OutputForm` which has better syntax information.

— `HTMLFormat.input` —

```
)set break resume
)sys rm -f HTMLFormat.output
)spool HTMLFormat.output
)set message test on
)set message auto off
)clear all

--S 1 of 9
)show HTMLFormat
--R HTMLFormat is a domain constructor
--R Abbreviation for HTMLFormat is HTMLFORM
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for HTMLFORM
```

```

--R
--R----- Operations -----
--R ==? : (% ,%) -> Boolean          coerce : OutputForm -> String
--R coerce : % -> OutputForm          coerceL : OutputForm -> String
--R coerceS : OutputForm -> String    display : String -> Void
--R exprex : OutputForm -> String      hash : % -> SingleInteger
--R latex : % -> String                ?~=? : (% ,%) -> Boolean
--R
--E 1

--S 2 of 9
coerce("3+4"::OutputForm)$HTMLFORM
--R
--R"3+4"
--R
--R (1) " "
--R
--R                                          Type: String
--E 2

--S 3 of 9
coerce("sqrt(3+4)"::OutputForm)$HTMLFORM
--R
--R"sqrt(3+4)"
--R
--R (2) " "
--R
--R                                          Type: String
--E 3

--S 4 of 9
coerce(sqrt(3+4)::OutputForm)$HTMLFORM
--R
--R&radic;7
--R
--R (3) " "
--R
--R                                          Type: String
--E 4

--S 5 of 9
coerce(sqrt(3+x)::OutputForm)$HTMLFORM
--R
--R<table border='0' id='root'>
--R<tr id='root'>
--R<td id='root'>
--R&radic;
--R</td>
--R<td id='root' style='border-top-style:solid'>
--Rx+3
--R</td>
--R</tr>
--R</table>

```

```
--R
--R (4) " "
--R
--R                                         Type: String
--E 5
```

```
--S 6 of 9
coerceS(sqrt(3+x)::OutputForm)$HTMLFORM
--R
--R<table border='0' id='root'>
--R<tr id='root'>
--R<td id='root'>
--R&radic;
--R</td>
--R<td id='root' style='border-top-style:solid'>
--Rx+3
--R</td>
--R</tr>
--R</table>
--R
--R (5) " "
--R
--R                                         Type: String
--E 6
```

```
--S 7 of 9
coerceL(sqrt(3+x)::OutputForm)$HTMLFORM
--R
--R<table border='0' id='root'>
--R<tr id='root'>
--R<td id='root'>
--R&radic;
--R</td>
--R<td id='root' style='border-top-style:solid'>
--Rx+3
--R</td>
--R</tr>
--R</table>
--R
--R (6) " "
--R
--R                                         Type: String
--E 7
```

```
--S 8 of 9
expres(sqrt(3+x)::OutputForm)$HTMLFORM
--R
--R
--R (7) "{{ROOT}}{{+}}{{x}}{{3}}}"
--R
--R                                         Type: String
--E 8
```

```
--S 9 of 9
```



```

display(coerce(sqrt(3+x)::OutputForm)$HTMLFORM)$HTMLFORM
--R
--R<table border='0' id='root'>
--R<tr id='root'>
--R<td id='root'>
--R&radic;
--R</td>
--R<td id='root' style='border-top-style:solid'>
--Rx+3
--R</td>
--R</tr>
--R</table>
--R
--R
--E 9
)spool
)lisp (bye)

```

Type: Void

---

— HTMLFormat.help —

=====

HTMLFormat examples

=====

```

coerce("3+4"::OutputForm)$HTMLFORM
"3+4"

```

```

coerce("sqrt(3+4)"::OutputForm)$HTMLFORM
"sqrt(3+4)"

```

```

coerce(sqrt(3+4)::OutputForm)$HTMLFORM
&radic;7

```

```

coerce(sqrt(3+x)::OutputForm)$HTMLFORM
<table border='0' id='root'>
<tr id='root'>
<td id='root'>
&radic;
</td>
<td id='root' style='border-top-style:solid'>
x+3
</td>
</tr>
</table>

```

```

coerceS(sqrt(3+x)::OutputForm)$HTMLFORM
<table border='0' id='root'>

```

```

<tr id='root'>
<td id='root'>
&radic;
</td>
<td id='root' style='border-top-style:solid'>
x+3
</td>
</tr>
</table>

coerceL(sqrt(3+x)::OutputForm)$HTMLFORM
<table border='0' id='root'>
<tr id='root'>
<td id='root'>
&radic;
</td>
<td id='root' style='border-top-style:solid'>
x+3
</td>
</tr>
</table>

expres(sqrt(3+x)::OutputForm)$HTMLFORM
"{{ROOT}}{{+}}{{x}}{{3}}}"

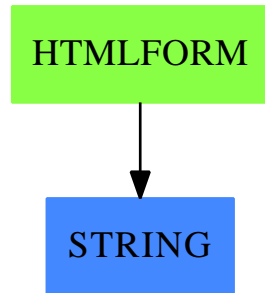
display(coerce(sqrt(3+x)::OutputForm)$HTMLFORM)$HTMLFORM
<table border='0' id='root'>
<tr id='root'>
<td id='root'>
&radic;
</td>
<td id='root' style='border-top-style:solid'>
x+3
</td>
</tr>
</table>

See Also:
o )show HTMLFormat

```

---

## 9.8.2 HTMLFormat (HTMLFORM)



See

← “SetCategory” (SETCAT) ?? on page ??

### Exports:

?=?    ?=?    coerce   coerceL   coerceS  
 display   exprex   hash   latex

— domain HTMLFORM HTMLFormat —

```

)abbrev domain HTMLFORM HTMLFormat
++ Author: Martin J Baker, Arthur C. Ralfs, Robert S. Sutor
++ Date: January 2010
++ Description:
++ HtmlFormat provides a coercion from OutputForm to html.
HTMLFormat(): public == private where
  E      ==> OutputForm
  I      ==> Integer
  L      ==> List
  S      ==> String

public == SetCategory with
  coerce:   E -> S
    ++ coerce(o) changes o in the standard output format to html format.
    ++
    ++X coerce(sqrt(3+x)::OutputForm)$HTMLFORM
  coerceS:  E -> S
    ++ coerceS(o) changes o in the standard output format to html
    ++ format and displays formatted result.
    ++
    ++X coerceS(sqrt(3+x)::OutputForm)$HTMLFORM
  coerceL:  E -> S
    ++ coerceL(o) changes o in the standard output format to html
    ++ format and displays result as one long string.
    ++
    ++X coerceL(sqrt(3+x)::OutputForm)$HTMLFORM
  exprex:   E -> S
  
```

```

    ++ exprex(o) coverts \spadtype{OutputForm} to \spadtype{String}
    ++
    ++X exprex(sqrt(3+x)::OutputForm)$HTMLFORM
display: S -> Void
    ++ display(o) prints the string returned by coerce.
    ++
    ++X display(coerce(sqrt(3+x)::OutputForm)$HTMLFORM)$HTMLFORM

private == add
import OutputForm
import Character
import Integer
import List OutputForm
import List String

expr: E
prec,opPrec: I
str: S
blank      : S := " \ "

maxPrec    : I := 1000000
minPrec    : I := 0

unaryOps   : L S := ["-"]$(L S)
unaryPrecs : L I := [700]$(L I)

-- the precedence of / in the following is relatively low because
-- the bar obviates the need for parentheses.
binaryOps  : L S := ["+->","|","^","/","<",">","=","OVER"]$(L S)
binaryPrecs : L I := [0,0,900,700,400,400,400,700]$(L I)
naryOps    : L S := ["-","+","*","blank",";",";", " ", "ROW", "",
    " \cr ","&","/","\","/"]$(L S)
naryPrecs  : L I := [700,700,800,800,110,110,0,0,0,0,0,600,600]$(L I)
naryNGOps  : L S := ["ROW","&"]$(L S)
plexOps    : L S := ["SIGMA","SIGMA2","PI","PI2","INTSIGN",_
    "INDEFINTEGRAL"]$(L S)
plexPrecs  : L I := [700,800,700,800,700,700]$(L I)
specialOps : L S := ["MATRIX","BRACKET","BRACE","CONCATB","VCONCAT",_
    "AGGLST","CONCAT","OVERBAR","ROOT","SUB","TAG",_
    "SUPERSUB","ZAG","AGGSET","SC","PAREN",_
    "SEGMENT","QUOTE","theMap", "SLASH"]

-- the next two lists provide translations for some strings for
-- which HTML has some special character codes.
specialStrings : L S :=
    ["cos", "cot", "csc", "log", "sec", "sin", "tan", _
    "cosh", "coth", "csch", "sech", "sinh", "tanh", _
    "acos","asin","atan","erf","...","$","infinity","Gamma", _
    "%pi","%e","%i"]
specialStringsInHTML : L S :=

```

```

["cos","cot","csc","log","sec","sin","tan", _
 "cosh","coth","csch","sech","sinh","tanh", _
 "arccos","arcsin","arctan","erf",&#x2026;","$",&#x221E;","_
 "&#x0413;","&#x003C0;","&#x02147;","&#x02148;"]

debug := false

atomize:E -> L E

formatBinary:(S,L E, I) -> Tree S

formatFunction:(Tree S,L E, I) -> Tree S

formatMatrix:L E -> Tree S

formatNary:(S,L E, I) -> Tree S

formatNaryNoGroup:(S,L E, I) -> Tree S

formatNullary:S -> Tree S

formatPlex:(S,L E, I) -> Tree S

formatSpecial:(S,L E, I) -> Tree S

formatUnary:(S, E, I) -> Tree S

formatHtml:(E,I) -> Tree S

precondition:E -> E
-- this function is applied to the OutputForm expression before
-- doing anything else.

outputTree:Tree S -> Void
-- This function traverses the tree and linierises it into a string.
-- To get the formatting we use a nested set of tables. It also checks
-- for +- and removes the +. it may also need to remove the outer
-- set of brackets.

stringify:E -> S

coerce(expr : E): S ==
  outputTree formatHtml(precondition expr, minPrec)
  " "

coerceS(expr : E): S ==
  outputTree formatHtml(precondition expr, minPrec)
  " "

coerceL(expr : E): S ==

```

```

    outputTree formatHtml(precondition expr, minPrec)
    " "

display(html : S): Void ==
    sayTeX$Lisp html
    void()$Void

newNode(tag:S,node: Tree S): (Tree S) ==
    t := tree(S,[node])
    setvalue!(t,tag)
    t

newNodes(tag:S,nodes: L Tree S): (Tree S) ==
    t := tree(S,nodes)
    setvalue!(t,tag)
    t

-- returns true if this can be represented without a table
notTable?(node: Tree S): Boolean ==
    empty?(node) => true
    leaf?(node) => true
    prefix?("table",value(node))$String => false
    c := children(node)
    for a in c repeat
        if not notTable?(a) then return false
    true

-- this returns a string representation of OutputForm arguments
-- it is used when debug is true to trace the calling of functions
-- in this package
argsToString(args : L E): S ==
    sop : S := exprex first args
    args := rest args
    s : S := concat ["{",sop]
    for a in args repeat
        s1 : S := exprex a
        s := concat [s,s1]
    s := concat [s,"}"]

exprex(expr : E): S ==
    -- This breaks down an expression into atoms and returns it as
    -- a string. It's for developmental purposes to help understand
    -- the expressions.
    a : E
    expr := precondition expr
    (ATOM(expr)$Lisp@Boolean) or (stringify expr = "NOTHING") =>
        concat ["{",stringify expr,"}"]
    le : L E := (expr pretend L E)
    op := first le
    sop : S := exprex op

```

```

args : L E := rest le
nargs : I := #args
s : S := concat ["{",sop]
if nargs > 0 then
  for a in args repeat
    s1 : S := exprex a
    s := concat [s,s1]
s := concat [s,""]

atomize(expr : E): L E ==
-- This breaks down an expression into a flat list of atomic
-- expressions.
-- expr should be preconditioned.
le : L E := nil()
a : E
letmp : L E
(ATOM(expr)$Lisp@Boolean) or (stringify expr = "NOTHING") =>
  le := append(le,list(expr))
letmp := expr pretend L E
for a in letmp repeat
  le := append(le,atomize a)
le

-- output html test using tables and
-- remove unnecessary '+' at end of first string
-- when second string starts with '-'
outputTree(t: Tree S): Void ==
  endWithPlus:Boolean := false -- if the last string ends with '+'
  -- and the next string starts with '-' then the '+' needs to be
  -- removed
  if empty?(t) then
    --if debug then sayTeX$Lisp "outputTree empty"
    return void()$Void
  if leaf?(t) then
    --if debug then sayTeX$Lisp concat("outputTree leaf:",value(t))
    sayTeX$Lisp value(t)
    return void()$Void
  tagName := copy value(t)
  tagPos := position(char(" "),tagName,1)$String
  if tagPos > 1 then
    tagName := split(tagName,char(" ")).1
    --sayTeX$Lisp "outputTree: tagPos="string(tagPos)" "tagName
  if value(t) ~= "" then sayTeX$Lisp concat ["<",value(t),">"]
  c := children(t)
  enableGrid:Boolean := (#c > 1) and not notTable?(t)
  if enableGrid then
    if tagName = "table" then enableGrid := false
    if tagName = "tr" then enableGrid := false
  b:List Boolean := [leaf?(c1) for c1 in c]
  -- if all children are strings then no need to wrap in table

```

```

allString: Boolean := true
for c1 in c repeat if not leaf?(c1) then allString := false
if allString then
  s:String := ""
  for c1 in c repeat s := concat(s,value(c1))
  sayTeX$Lisp s
  if value(t) ~= "" then sayTeX$Lisp concat ["</",tagName,">"]
  return void()$Void
if enableGrid then
  sayTeX$Lisp "<table border='0'>"
  sayTeX$Lisp "<tr>"
  for c1 in c repeat
    if enableGrid then sayTeX$Lisp "<td>"
    outputTree(c1)
    if enableGrid then sayTeX$Lisp "</td>"
  if enableGrid then
    sayTeX$Lisp "</tr>"
    sayTeX$Lisp "</table>"
  if value(t) ~= "" then sayTeX$Lisp concat ["</",tagName,">"]
  void()$Void

stringify expr == (mathObject2String$Lisp expr)@S

precondition expr ==
  outputTran$Lisp expr

-- I dont know what SC is so put it in a table for now
formatSC(args : L E, prec : I) : Tree S ==
  if debug then sayTeX$Lisp "formatSC: "concat [" args=",
    argsToString(args)," prec=",string(prec)$S]
  null args => tree("")
  cells:L Tree S := [_
    newNode("td id='sc' style='border-bottom-style:solid'",_
      formatHtml(a,prec)) for a in args]
  row:Tree S := newNodes("tr id='sc'",cells)
  newNode("table border='0' id='sc'",row)

-- to build an overbar we put it in a single column,
-- single row table and set the top border to solid
buildOverbar(content : Tree S) : Tree S ==
  if debug then sayTeX$Lisp "buildOverbar"
  cell:Tree S := _
    newNode("td id='overbar' style='border-top-style:solid'",content)
  row:Tree S := newNode("tr id='overbar'",cell)
  newNode("table border='0' id='overbar'",row)

-- to build an square root we put it in a double column,
-- single row table and set the top border of the second column to
-- solid
buildRoot(content : Tree S) : Tree S ==

```



```

if debug then sayTeX$Lisp "buildRoot"
if leaf?(content) then
  -- root of a single term so no need for overbar
  return newNodes("", [tree("&radic;"), content])
cell1:Tree S := newNode("td id='root'", tree("&radic;"))
cell2:Tree S := _
  newNode("td id='root' style='border-top-style:solid'", content)
row:Tree S := newNodes("tr id='root'", [cell1, cell2])
newNode("table border='0' id='root'", row)

-- to build an 'n'th root we put it in a double column,
-- single row table and set the top border of the second column to
-- solid
buildNRoot(content : Tree S, nth: Tree S) : Tree S ==
  if debug then sayTeX$Lisp "buildNRoot"
  power:Tree S := newNode("sup", nth)
  if leaf?(content) then
    -- root of a single term so no need for overbar
    return newNodes("", [power, tree("&radic;"), content])
  cell1:Tree S := newNodes("td id='nroot'", [power, tree("&radic;")])
  cell2:Tree S := _
    newNode("td id='nroot' style='border-top-style:solid'", content)
  row:Tree S := newNodes("tr id='nroot'", [cell1, cell2])
  newNode("table border='0' id='nroot'", row)

-- formatSpecial handles "theMap", "AGGLST", "AGGSET", "TAG", "SLASH",
-- "VCONCAT", "CONCATB", "CONCAT", "QUOTE", "BRACKET", "BRACE", "PAREN",
-- "OVERBAR", "ROOT", "SEGMENT", "SC", "MATRIX", "ZAG"
-- note "SUB" and "SUPERSUB" are handled directly by formatHtml
formatSpecial(op : S, args : L E, prec : I) : Tree S ==
  if debug then sayTeX$Lisp _
    "formatSpecial: " concat ["op=", op, " args=", argsToString(args), _
      " prec=", string(prec)$S]
  arg : E
  prescript : Boolean := false
  op = "theMap" => tree("theMap(...)")
  op = "AGGLST" =>
    formatNary(" ", args, prec)
  op = "AGGSET" =>
    formatNary(";", args, prec)
  op = "TAG" =>
    newNodes("", [formatHtml(first args, prec), tree("&#x02192;"), _
      formatHtml(second args, prec)])
  --RightArrow
  op = "SLASH" =>
    newNodes("", [formatHtml(first args, prec), tree("/"), _
      formatHtml(second args, prec)])
  op = "VCONCAT" =>
    newNodes("table", [newNode("td", formatHtml(u, minPrec)) _
      for u in args]::L Tree S)

```

```

op = "CONCATB" =>
  formatNary(" ",args,prec)
op = "CONCAT" =>
  formatNary("",args,minPrec)
op = "QUOTE" =>
  newNodes("",[tree("'"),formatHtml(first args, minPrec)])
op = "BRACKET" =>
  newNodes("",[tree("("),formatHtml(first args, minPrec),tree(")")]])
op = "BRACE" =>
  newNodes("",[tree("{"),formatHtml(first args, minPrec),tree("}")]])
op = "PAREN" =>
  newNodes("",[tree("("),formatHtml(first args, minPrec),tree(")")]])
op = "OVERBAR" =>
  null args => tree("")
  buildOverbar(formatHtml(first args,minPrec))
op = "ROOT" and #args < 1 => tree("")
op = "ROOT" and #args = 1 => _
  buildRoot(formatHtml(first args, minPrec))
op = "ROOT" and #args > 1 => _
  buildNRoot(formatHtml(first args, minPrec),_
    formatHtml(second args, minPrec))
op = "SEGMENT" =>
  -- '..' indicates a range in a list for example
  tmp : Tree S := newNodes("",[formatHtml(first args, minPrec),_
    tree("..")])
  null rest args => tmp
  newNodes("",[tmp,formatHtml(first rest args, minPrec)])
op = "SC" => formatSC(args,minPrec)
op = "MATRIX" => formatMatrix rest args
op = "ZAG" =>
  -- {{+}}{3}{{ZAG}}{1}{7}}{{ZAG}}{1}{15}}{{ZAG}}{1}{1}}{{ZAG}}{1}{25}}_
  -- {{ZAG}}{1}{1}}{{ZAG}}{1}{7}}{{ZAG}}{1}{4}}
  -- to format continued fraction traditionally need to intercept
  -- it at the formatNary of the "+"
  newNodes("",[tree(" \zag{"),formatHtml(first args, minPrec),
    tree("{}{"),
    formatHtml(first rest args,minPrec),tree("{}")])])
  tree("formatSpecial not implemented:"op)

formatSuperSub(expr : E, args : L E, opPrec : I) : Tree S ==
  -- This one produces ordinary derivatives with differential notation,
  -- it needs a little more work yet.
  -- first have to divine the semantics, add cases as needed
  if debug then sayTeX$Lisp _
    "formatSuperSub: " concat ["expr=",stringify expr," args=",_
      argsToString(args)," prec=",string(opPrec)$S]
  atomE : L E := atomize(expr)
  op : S := stringify first atomE
  op ~ = "SUPERSUB" => tree("Mistake in formatSuperSub: no SUPERSUB")
  #args ~ = 1 => tree("Mistake in SuperSub: #args <> 1")

```

```

var : E := first args
-- should be looking at something like {{SUPERSUB}{var}{ }{,...,}}
-- for example here's the second derivative of y w.r.t. x
-- {{{{SUPERSUB}{y}{ }{,,}{x}}, expr is the first {} and args is the
-- {x}
funcS : S := stringify first rest atomE
bvarS : S := stringify first args
-- count the number of commas
commaS : S := stringify first rest rest rest atomE
commaTest : S := ","
ndiffs : I := 0
while position(commaTest,commaS,1) > 0 repeat
  ndiffs := ndiffs+1
  commaTest := commaTest","
res:Tree S := newNodes("",_
  [tree("&#x02146;"string(ndiffs)"funcS"&#x02146;"),_
    formatHtml(first args,minPrec),tree("&#x02061;"string(ndiffs)"&#x02061;"),_
    formatHtml(first args,minPrec),tree("")]
  res

-- build structure such as integral as a table
buildPlex3(main:Tree S,supsc:Tree S,op:Tree S,subsc:Tree S) : Tree S ==
  if debug then sayTeX$Lisp "buildPlex"
  ssup:Tree S := newNode("td id='plex'",supsc)
  sop:Tree S := newNode("td id='plex'",op)
  ssub:Tree S := newNode("td id='plex'",subsc)
  m:Tree S := newNode("td rowspan='3' id='plex'",main)
  rows:(List Tree S) := [newNodes("tr id='plex'",[ssup,m]),_
    newNode("tr id='plex'",sop),newNode("tr id='plex'",ssub)]
  newNodes("table border='0' id='plex'",rows)

-- build structure such as integral as a table
buildPlex2(main : Tree S,supsc : Tree S,op : Tree S) : Tree S ==
  if debug then sayTeX$Lisp "buildPlex"
  ssup:Tree S := newNode("td id='plex'",supsc)
  sop:Tree S := newNode("td id='plex'",op)
  m:Tree S := newNode("td rowspan='2' id='plex'",main)
  rows:(List Tree S) := [newNodes("tr id='plex'",[sop,m]),_
    newNode("tr id='plex'",ssup)]
  newNodes("table border='0' id='plex'",rows)

-- format an integral
-- args.1 = "NOTHING"
-- args.2 = bound variable
-- args.3 = body, thing being integrated
--
-- axiom replaces the bound variable with something like
-- %A and puts the original variable used
-- in the input command as a superscript on the integral sign.
formatIntSign(args : L E, opPrec : I) : Tree S ==

```

```

-- the original OutputForm expression looks something like this:
-- {{INTSIGN}{NOTHING or lower limit?}
-- {bvar or upper limit?}{*}{integrand}{{CONCAT}{d}{axiom var}}}}
-- the args list passed here consists of the rest of this list, i.e.
-- starting at the NOTHING or ...
if debug then sayTeX$Lisp "formatIntSign: " concat [" args=",
  argsToString(args)," prec=",string(opPrec)$S]
(stringify first args) = "NOTHING" =>
  buildPlex2(formatHtml(args.3,opPrec),tree("&int;"),_
    formatHtml(args.2,opPrec)) -- could use &#x0222B; or &int;
buildPlex3(formatHtml(first args,opPrec),formatHtml(args.3,opPrec),_
  tree("&int;"),formatHtml(args.2,opPrec))

-- plex ops are "SIGMA","SIGMA2","PI","PI2","INTSIGN","INDEFINTEGRAL"
-- expects 2 or 3 args
formatPlex(op : S, args : L E, prec : I) : Tree S ==
  if debug then sayTeX$Lisp "formatPlex: " concat ["op=",op," args=",
    argsToString(args)," prec=",string(prec)$S]
  checkarg:Boolean := false
  hold : S
  p : I := position(op,plexOps)
  p < 1 => error "unknown plex op"
  op = "INTSIGN" => formatIntSign(args,minPrec)
  opPrec := plexPrecs.p
  n : I := #args
  (n ~= 2) and (n ~= 3) => error "wrong number of arguments for plex"
  s : Tree S :=
    op = "SIGMA"    =>
      checkarg := true
      tree("&#x02211;")
    -- Sum
    op = "SIGMA2"   =>
      checkarg := true
      tree("&#x02211;")
    -- Sum
    op = "PI"       =>
      checkarg := true
      tree("&#x0220F;")
    -- Product
    op = "PI2"      =>
      checkarg := true
      tree("&#x0220F;")
    -- Product
    op = "INTSIGN" => tree("&#x0222B;")
    -- Integral, int
    op = "INDEFINTEGRAL" => tree("&#x0222B;")
    -- Integral, int
    tree("formatPlex: unexpected op:"op)
-- if opPrec < prec then perhaps we should parenthesize?
-- but we need to be careful we don't get loads of unnecessary

```

```

-- brackets
if n=2 then return buildPlex2(formatHtml(first args,minPrec),_
  formatHtml(args.2,minPrec),s)
buildPlex3(formatHtml(first args,minPrec),formatHtml(args.2,minPrec),_
  s,formatHtml(args.3,minPrec))

-- an example is: op=ROW arg={{ROW}{1}{2}}
formatMatrixRow(op : S, arg : E, prec : I,y:I,h:I) : L Tree S ==
  if debug then sayTeX$Lisp "formatMatrixRow: " concat ["op=",op,_
    " args=",stringify arg," prec=",string(prec)$S]
  ATOM(arg)$Lisp@Boolean => [_
    tree("formatMatrixRow does not contain row")]
  l : L E := (arg pretend L E)
  op : S := stringify first l
  args : L E := rest l
  --sayTeX$Lisp "formatMatrixRow op="op" args="argsToString(args)
  w:I := #args
  cells:(List Tree S) := empty()
  for x in 1..w repeat
    --sayTeX$Lisp "formatMatrixRow: x="string(x)$S" width="string(w)$S
    attrib:S := "td id='mat'"
    if x=1 then attrib := "td id='matl'"
    if x=w then attrib := "td id='matr'"
    if y=1 then attrib := "td id='matt'"
    if y=h then attrib := "td id='matb'"
    if x=1 and y=1 then attrib := "td id='matlt'"
    if x=1 and y=h then attrib := "td id='matlb'"
    if x=w and y=1 then attrib := "td id='matrt'"
    if x=w and y=h then attrib := "td id='matrb'"
    cells := append(cells,[newNode(attrib,formatHtml(args.(x),prec))])
  cells

-- an example is: op=MATRIX args={{ROW}{1}{2}}{{ROW}{3}{4}}
formatMatrixContent(op : S, args : L E, prec : I) : L Tree S ==
  if debug then sayTeX$Lisp "formatMatrixContent: " concat ["op=",op,_
    " args=",argsToString(args)," prec=",string(prec)$S]
  y:I := 0
  rows:(List Tree S) := [newNodes("tr id='mat'",_
    formatMatrixRow("ROW",e,prec,y:=y+1,#args)) for e in args]
  rows

formatMatrix(args : L E) : Tree S ==
  -- format for args is [[ROW ...],[ROW ...],[ROW ...]]
  -- generate string for formatting columns (centered)
  if debug then sayTeX$Lisp "formatMatrix: " concat ["args=",_
    argsToString(args)]
  newNodes("table border='1' id='mat'",_
    formatMatrixContent("MATRIX",args,minPrec))

-- output arguments in column table

```

```

buildColumnTable(elements : List Tree S) : Tree S ==
  if debug then sayTeX$Lisp "buildColumnTable"
  cells:(List Tree S) := [newNode("td id='col'",j) for j in elements]
  rows:(List Tree S) := [newNode("tr id='col'",i) for i in cells]
  newNodes("table border='0' id='col'",rows)

-- build superscript structure as either sup tag or
-- if it contains anything that won't go into a
-- sup tag then build it as a table
buildSuperscript(main : Tree S,super : Tree S) : Tree S ==
  if debug then sayTeX$Lisp "buildSuperscript"
  notTable?(super) => newNodes("",[main,newNode("sup",super)])
  m:Tree S := newNode("td rowspan='2' id='sup'",main)
  su:Tree S := newNode("td id='sup'",super)
  e:Tree S := newNode("td id='sup'",tree("&nbsp;"))
  rows:(List Tree S) := [newNodes("tr id='sup'",[m,su]),_
    newNode("tr id='sup'",e)]
  newNodes("table border='0' id='sup'",rows)

-- build subscript structure as either sub tag or
-- if it contains anything that won't go into a
-- sub tag then build it as a table
buildSubscript(main : Tree S,subsc : Tree S) : Tree S ==
  if debug then sayTeX$Lisp "buildSubscript"
  notTable?(subsc) => newNodes("",[main,newNode("sub",subsc)])
  m:Tree S := newNode("td rowspan='2' id='sub'",main)
  su:Tree S := newNode("td id='sub'",subsc)
  e:Tree S := newNode("td id='sub'",tree("&nbsp;"))
  rows:(List Tree S) := [newNodes("tr id='sub'",[m,e]),_
    newNode("tr id='sub'",su)]
  newNodes("table border='0' id='sub'",rows)

formatSub(expr : E, args : L E, opPrec : I) : Tree S ==
  -- format subscript
  -- this function expects expr to start with SUB
  -- it expects first args to be the operator or value that
  -- the subscript is applied to
  -- and the rest args to be the subscript
  if debug then sayTeX$Lisp "formatSub: " concat ["expr=",_
    stringify expr," args=",argsToString(args)," prec=",_
    string(opPrec)$S]
  atomE : L E := atomize(expr)
  if empty?(atomE) then
    if debug then sayTeX$Lisp "formatSub: expr=empty"
    return tree("formatSub: expr=empty")
  op : S := stringify first atomE
  op ~= "SUB" =>
    if debug then sayTeX$Lisp "formatSub: expr~=SUB"
    tree("formatSub: expr~=SUB")
  -- assume args.1 is the expression and args.2 is its subscript

```

```

if #args < 2 then
  if debug then sayTeX$Lisp concat("formatSub: num args=",_
    string(#args)$String)$String
  return tree(concat("formatSub: num args=",_
    string(#args)$String)$String)
if #args > 2 then
  if debug then sayTeX$Lisp concat("formatSub: num args=",_
    string(#args)$String)$String
  return buildSubscript(formatHtml(first args,opPrec),_
    newNodes("",[formatHtml(e,opPrec) for e in rest args]))
buildSubscript(formatHtml(first args,opPrec),_
  formatHtml(args.2,opPrec))

formatFunction(op : Tree S, args : L E, prec : I) : Tree S ==
  if debug then sayTeX$Lisp "formatFunction: " concat ["args=",_
    argsToString(args)," prec=",string(prec)$S]
  newNodes("",[op,tree("("),formatNary("",args,minPrec),tree(")")]])

formatNullary(op : S) : Tree S ==
  if debug then sayTeX$Lisp "formatNullary: " concat ["op=",op]
  op = "NOTHING" => empty()$Tree(S)
  tree(op"()")

-- implement operation with single argument
-- an example is minus '-'
-- prec is precedence of operator, used to force brackets where
-- more tightly bound operation is next to less tightly bound operation
formatUnary(op : S, arg : E, prec : I) : Tree S ==
  if debug then sayTeX$Lisp "formatUnary: " concat ["op=",op," arg=",_
    stringify arg," prec=",string(prec)$S]
  p : I := position(op,unaryOps)
  p < 1 => error "unknown unary op"
  opPrec := unaryPrecs.p
  s : Tree S := newNodes("",[tree(op),formatHtml(arg,opPrec)])
  opPrec < prec => newNodes("",[tree("("),s,tree(")")]])
  s

-- output division with numerator above the denominator
-- implemented as a table
buildOver(top : Tree S,bottom : Tree S) : Tree S ==
  if debug then sayTeX$Lisp "buildOver"
  topCell:Tree S := newNode("td",top)
  bottomCell:Tree S := newNode("td style='border-top-style:solid',_
    bottom)
  rows:(List Tree S) := [newNode("tr id='col'",topCell),_
    newNode("tr id='col'",bottomCell)]
  newNodes("table border='0' id='col'",rows)

-- op may be: "|","^","/","OVER","+->"
-- note: "+" and "*" are n-ary ops

```

```

formatBinary(op : S, args : L E, prec : I) : Tree S ==
  if debug then sayTeX$Lisp "formatBinary: " concat ["op=",op,_,
    " args=",argsToString(args)," prec=",string(prec)$S]
  p : I := position(op,binaryOps)
  p < 1 => error "unknown binary op"
  opPrec := binaryPrecs.p
  -- if base op is product or sum need to add parentheses
  if ATOM(first args)$Lisp@Boolean then
    opa:S := stringify first args
  else
    la : L E := (first args pretend L E)
    opa : S := stringify first la
  if (opa = "SIGMA" or opa = "SIGMA2" or opa = "PI" or opa = "PI2")_
    and op = "^" then
    s1 : Tree S := newNodes("",[tree("("),formatHtml(first args,_,
      opPrec),tree(")")]])
  else
    s1 : Tree S := formatHtml(first args, opPrec)
    s2 : Tree S := formatHtml(first rest args, opPrec)
    op = "|" => newNodes("",[s1,tree(op),s2])
    op = "^" => buildSuperscript(s1,s2)
    op = "/" => newNodes("",[s1,tree(op),s2])
    op = "OVER" => buildOver(s1,s2)
    op = "+->" => newNodes("",[s1,tree("&mdash;&rsaquo;"),s2])
    newNodes("",[s1,tree(op),s2])

-- build a zag from a table with a right part and a
-- upper and lower left part
buildZag(top:Tree S,lowerLeft:Tree S,lowerRight:Tree S) : Tree S ==
  if debug then sayTeX$Lisp "buildZag"
  cellTop:Tree S := _
    newNode("td colspan='2' id='zag' style='border-bottom-style:solid',_"
      top)
  cellLowerLeft:Tree S := newNodes("td id='zag',[lowerLeft,tree("+")]])
  cellLowerRight:Tree S := newNode("td id='zag',lowerRight)
  row1:Tree S := newNodes("tr id='zag',[cellTop])
  row2:Tree S := newNodes("tr id='zag',[cellLowerLeft,cellLowerRight])
  newNodes("table border='0' id='zag',[row1,row2])

formatZag(args : L E,nestLevel:I) : Tree S ==
  -- args will be a list of things like this {{ZAG}{1}{7}}, the ZAG
  -- must be there, the '1' and '7' could conceivably be more complex
  -- expressions
  --
  -- ex 1. continuedFraction(314159/100000)
  -- {{+}{{3}}{{ZAG}{1}{7}}{{ZAG}{1}{15}}{{ZAG}{1}{1}}{{ZAG}{1}{25}}
  -- {{ZAG}{1}{1}}{{ZAG}{1}{7}}{{ZAG}{1}{4}}}
  -- this is the preconditioned output form
  -- including "op", the args list would be the rest of this
  -- i.e op = '+' and args = {{3}}{{ZAG}{1}{7}}{{ZAG}{1}{15}}

```



```

-- {{ZAG}{1}{1}}{{ZAG}{1}{25}}{{ZAG}{1}{1}}{{ZAG}{1}{7}}{{ZAG}{1}{4}}}
--
-- ex 2. continuedFraction(14159/100000)
-- this one doesn't have the leading integer
-- {{+}}{{ZAG}{1}{7}}{{ZAG}{1}{15}}{{ZAG}{1}{1}}{{ZAG}{1}{25}}
-- {{ZAG}{1}{1}}{{ZAG}{1}{7}}{{ZAG}{1}{4}}}
--
-- ex 3. continuedFraction(3,repeating [1], repeating [3,6])
-- {{+}{3}}{{ZAG}{1}{3}}{{ZAG}{1}{6}}{{ZAG}{1}{3}}{{ZAG}{1}{6}}
-- {{ZAG}{1}{3}}{{ZAG}{1}{6}}{{ZAG}{1}{3}}{{ZAG}{1}{6}}
-- {{ZAG}{1}{3}}{{ZAG}{1}{6}}{...}
--
-- In each of these examples the args list consists of the terms
-- following the '+' op
-- so the first arg could be a "ZAG" or something
-- else, but the second arg looks like it has to be "ZAG", so maybe
-- test for #args > 1 and args.2 contains "ZAG".
-- Note that since the resulting tables are nested we need
-- to handle the whole continued fraction at once, i.e. we can't
-- just look for, e.g., {{ZAG}{1}{6}}
--
-- we will assume that the font starts at 16px and reduce it by 4
-- <span style='font-size:16px'>outer zag</span>
-- <span style='font-size:14px'>next zag</span>
-- <span style='font-size:12px'>next zag</span>
-- <span style='font-size:10px'>next zag</span>
-- <span style='font-size:9px'>lowest zag</span>
if debug then sayTeX$Lisp "formatZag: " concat ["args=",_
  argsToString(args)]
tmpZag : L E := first args pretend L E
fontAttrib : S :=
  nestLevel < 2 => "span style='font-size:16px'"
  nestLevel = 2 => "span style='font-size:14px'"
  nestLevel = 3 => "span style='font-size:12px'"
  nestLevel = 4 => "span style='font-size:10px'"
  "span style='font-size:9px'"
-- may want to test that tmpZag contains 'ZAG'
#args > 1 =>
  newNode(fontAttrib,buildZag(formatHtml(first rest tmpZag,minPrec),_
    formatHtml(first rest rest tmpZag,minPrec),_
    formatZag(rest args,nestLevel+1)))
(first args = "...": E)@Boolean => tree("&#x2026;")
op:S := stringify first args
position("ZAG",op,1) > 0 =>
  newNode(fontAttrib,buildOver(formatHtml(first rest tmpZag,minPrec),_
    formatHtml(first rest rest tmpZag,minPrec)))
tree("formatZag: Last argument in ZAG construct unknown operator: "op)

-- returns true if this term starts with a minus '-' sign
-- this is used so that we can suppress any plus '+' in front

```

```

-- of the - so we dont get terms like +-
neg?(arg : E) : Boolean ==
  if debug then sayTeX$Lisp "neg?: " concat ["arg=",argsToString([arg])]
  ATOM(arg)$Lisp@Boolean => false
  l : L E := (arg pretend L E)
  op : S := stringify first l
  op = "-" => true
  false

formatNary(op : S, args : L E, prec : I) : Tree S ==
  if debug then sayTeX$Lisp "formatNary: " concat ["op=",op," args=",_
    argsToString(args)," prec=",string(prec)$S]
  formatNaryNoGroup(op, args, prec)

-- possible op values are:
-- ",", ";", "*", " ", "ROW", "+", "-"
-- an example is content of matrix which gives:
-- {{ROW}{1}{2}}{{ROW}{3}{4}}
-- or AGGLST which gives op=, args={{1}{2}}
--
-- need to:
-- format ZAG
-- check for +-
-- add brackets for sigma or pi or root ("SIGMA","SIGMA2","PI","PI2")
formatNaryNoGroup(op : S, args : L E, prec : I) : Tree S ==
  if debug then sayTeX$Lisp "formatNaryNoGroup: " concat ["op=",op,_
    " args=",argsToString(args)," prec=",string(prec)$S]
  checkargs:Boolean := false
  null args => empty()$Tree(S)
  p : I := position(op,naryOps)
  p < 1 => error "unknown nary op"
  -- need to test for "ZAG" case and divert it here
  (#args > 1) and (position("ZAG",stringify first rest args,1) > 0) =>
    tmpS : S := stringify first args
    position("ZAG",tmpS,1) > 0 => formatZag(args,1)
    newNodes("",[formatHtml(first args,minPrec),tree("+"),_
      formatZag(rest args,1)])
  -- At least for the ops "*", "+", "-" we need to test to see if a
  -- sigma or pi is one of their arguments because we might need
  -- parentheses as indicated
  -- by the problem with summation(operator(f)(i),i=1..n)+1 versus
  -- summation(operator(f)(i)+1,i=1..n) having identical displays as of
  -- 2007-12-21
  l := empty()$Tree(S)
  opPrec := naryPrecs.p
  -- if checkargs is true check each arg except last one to see if it's
  -- a sigma or pi and if so add parentheses. Other op's may have to be
  -- checked for in future
  count:I := 1
  tags : (L Tree S)

```

```

if opPrec < prec then tags := [tree("("),formatHtml(args.1,opPrec)]
if opPrec >= prec then tags := [formatHtml(args.1,opPrec)]
for a in rest args repeat
  if op ~= "+" or not neg?(a) then tags := append(tags,[tree(op)])
  tags := append(tags,[formatHtml(a,opPrec)])
if opPrec < prec then tags := append(tags,[tree(")")]
newNodes("",tags)

-- expr is a tree structure
-- prec is the precision of integers
-- formatHtml returns a string for this node in the tree structure
-- and calls recursively to evaluate sub expressions
formatHtml(arg : E,prec : I) : Tree S ==
  if debug then sayTeX$Lisp "formatHtml: " concat ["arg=",_
    argsToString([arg])," prec=",string(prec)$S]
  i,len : Integer
  intSplitLen : Integer := 20
  ATOM(arg)$Lisp@Boolean =>
    if debug then sayTeX$Lisp "formatHtml atom: " concat ["expr=",_
      stringify arg," prec=",string(prec)$S]
    str := stringify arg
    (i := position(str,specialStrings)) > 0 =>
      tree(specialStringsInHTML.i)
    tree(str)
  l : L E := (arg pretend L E)
  null l => tree(blank)
  op : S := stringify first l
  args : L E := rest l
  nargs : I := #args
  -- need to test here in case first l is SUPERSUB case and then
  -- pass first l and args to formatSuperSub.
  position("SUPERSUB",op,1) > 0 =>
    formatSuperSub(first l,args,minPrec)
  -- now test for SUB
  position("SUB",op,1) > 0 =>
    formatSub(first l,args,minPrec)
  -- special cases
  -- specialOps are:
  -- MATRIX, BRACKET, BRACE, CONCATB, VCONCAT
  -- AGGLST, CONCAT, OVERBAR, ROOT, SUB, TAG
  -- SUPERSUB, ZAG, AGGSET, SC, PAREN
  -- SEGMENT, QUOTE, theMap, SLASH
  member?(op, specialOps) => formatSpecial(op,args,prec)
  -- specialOps are:
  -- SIGMA, SIGMA2, PI, PI2, INTSIGN, INDEFINTEGRAL
  member?(op, plexOps) => formatPlex(op,args,prec)
  -- nullary case: function with no arguments
  0 = nargs => formatNullary op
  -- unary case: function with one argument such as '-'
  (1 = nargs) and member?(op, unaryOps) =>

```



```

--R ?.? : (%,Integer) -> S                               elt : (%,Integer,S) -> S
--R empty : () -> %                                       empty? : % -> Boolean
--R entries : % -> List S                                 eq? : (%,% ) -> Boolean
--R index? : (Integer,% ) -> Boolean                     indices : % -> List Integer
--R map : ((S -> S),%) -> %                               qelt : (%,Integer) -> S
--R sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?? : (PositiveInteger,% ) -> % if S has ABELSG
--R ?? : (NonNegativeInteger,% ) -> % if S has CABMON
--R ?? : (S,% ) -> % if S has RING
--R ?? : (% ,S) -> % if S has RING
--R ?? : (% ,%) -> % if S has MONOID
--R ?? : (Integer,% ) -> % if S has RING
--R ??? : (% ,PositiveInteger) -> % if S has MONOID
--R ??? : (% ,NonNegativeInteger) -> % if S has MONOID
--R ?+? : (% ,%) -> % if S has ABELSG
--R ?-? : (% ,%) -> % if S has RING
--R ?/? : (% ,S) -> % if S has FIELD
--R ?<? : (% ,%) -> Boolean if S has OAMONS or S has ORDRING
--R ?<=? : (% ,%) -> Boolean if S has OAMONS or S has ORDRING
--R ?=? : (% ,%) -> Boolean if S has SETCAT
--R ?>? : (% ,%) -> Boolean if S has OAMONS or S has ORDRING
--R ?>=? : (% ,%) -> Boolean if S has OAMONS or S has ORDRING
--R D : (% ,(S -> S)) -> % if S has RING
--R D : (% ,(S -> S),NonNegativeInteger) -> % if S has RING
--R D : (% ,List Symbol,List NonNegativeInteger) -> % if S has PDRING SYMBOL and S has RING
--R D : (% ,Symbol,NonNegativeInteger) -> % if S has PDRING SYMBOL and S has RING
--R D : (% ,List Symbol) -> % if S has PDRING SYMBOL and S has RING
--R D : (% ,Symbol) -> % if S has PDRING SYMBOL and S has RING
--R D : (% ,NonNegativeInteger) -> % if S has DIFRING and S has RING
--R D : % -> % if S has DIFRING and S has RING
--R ?? : (% ,PositiveInteger) -> % if S has MONOID
--R ?? : (% ,NonNegativeInteger) -> % if S has MONOID
--R abs : % -> % if S has ORDRING
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R characteristic : () -> NonNegativeInteger if S has RING
--R coerce : S -> % if S has SETCAT
--R coerce : Fraction Integer -> % if S has RETRACT FRAC INT and S has SETCAT
--R coerce : Integer -> % if S has RETRACT INT and S has SETCAT or S has RING
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,% ) -> NonNegativeInteger if $ has finiteAggregate and S has SETCAT
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R differentiate : (% ,(S -> S)) -> % if S has RING
--R differentiate : (% ,(S -> S),NonNegativeInteger) -> % if S has RING
--R differentiate : (% ,List Symbol,List NonNegativeInteger) -> % if S has PDRING SYMBOL and S has RING
--R differentiate : (% ,Symbol,NonNegativeInteger) -> % if S has PDRING SYMBOL and S has RING
--R differentiate : (% ,List Symbol) -> % if S has PDRING SYMBOL and S has RING
--R differentiate : (% ,Symbol) -> % if S has PDRING SYMBOL and S has RING
--R differentiate : (% ,NonNegativeInteger) -> % if S has DIFRING and S has RING
--R differentiate : % -> % if S has DIFRING and S has RING

```

```

--R dimension : () -> CardinalNumber if S has FIELD
--R dot : (% , %) -> S if S has RING
--R entry? : (S , %) -> Boolean if $ has finiteAggregate and S has SETCAT
--R eval : (% , List S , List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (% , S , S) -> % if S has EVALAB S and S has SETCAT
--R eval : (% , Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (% , List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean) , %) -> Boolean if $ has finiteAggregate
--R fill! : (% , S) -> % if $ has shallowlyMutable
--R first : % -> S if Integer has ORDSET
--R hash : % -> SingleInteger if S has SETCAT
--R index : PositiveInteger -> % if S has FINITE
--R latex : % -> String if S has SETCAT
--R less? : (% , NonNegativeInteger) -> Boolean
--R lookup : % -> PositiveInteger if S has FINITE
--R map! : ((S -> S) , %) -> % if $ has shallowlyMutable
--R max : (% , %) -> % if S has OAMONS or S has ORDRING
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (S , %) -> Boolean if $ has finiteAggregate and S has SETCAT
--R members : % -> List S if $ has finiteAggregate
--R min : (% , %) -> % if S has OAMONS or S has ORDRING
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (% , NonNegativeInteger) -> Boolean
--R negative? : % -> Boolean if S has ORDRING
--R one? : % -> Boolean if S has MONOID
--R parts : % -> List S if $ has finiteAggregate
--R positive? : % -> Boolean if S has ORDRING
--R qsetelt! : (% , Integer , S) -> S if $ has shallowlyMutable
--R random : () -> % if S has FINITE
--R recip : % -> Union(% , "failed") if S has MONOID
--R reducedSystem : Matrix % -> Matrix S if S has RING
--R reducedSystem : (Matrix % , Vector %) -> Record(mat: Matrix S , vec: Vector S) if S has RING
--R reducedSystem : (Matrix % , Vector %) -> Record(mat: Matrix Integer , vec: Vector Integer) if S has LINE
--R reducedSystem : Matrix % -> Matrix Integer if S has LINEXP INT and S has RING
--R retract : % -> S if S has SETCAT
--R retract : % -> Fraction Integer if S has RETRACT FRAC INT and S has SETCAT
--R retract : % -> Integer if S has RETRACT INT and S has SETCAT
--R retractIfCan : % -> Union(S , "failed") if S has SETCAT
--R retractIfCan : % -> Union(Fraction Integer , "failed") if S has RETRACT FRAC INT and S has SETCAT
--R retractIfCan : % -> Union(Integer , "failed") if S has RETRACT INT and S has SETCAT
--R setelt : (% , Integer , S) -> S if $ has shallowlyMutable
--R sign : % -> Integer if S has ORDRING
--R size : () -> NonNegativeInteger if S has FINITE
--R size? : (% , NonNegativeInteger) -> Boolean
--R subtractIfCan : (% , %) -> Union(% , "failed") if S has CABMON
--R sup : (% , %) -> % if S has OAMONS
--R swap! : (% , Integer , Integer) -> Void if $ has shallowlyMutable
--R unitVector : PositiveInteger -> % if S has RING
--R zero? : % -> Boolean if S has CABMON
--R ?~=?: (% , %) -> Boolean if S has SETCAT

```

```
--R
--E 1

)spool
)lisp (bye)
```

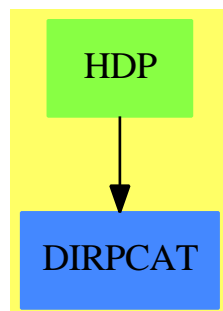
---

— HomogeneousDirectProduct.help —

```
=====
HomogeneousDirectProduct examples
=====
```

```
See Also:
o )show HomogeneousDirectProduct
```

### 9.9.1 HomogeneousDirectProduct (HDP)



See

- ⇒ “OrderedDirectProduct” (ODP) 16.13.1 on page 1778
- ⇒ “SplitHomogeneousDirectProduct” (SHDP) 20.23.1 on page 2467

**Exports:**

|               |               |               |            |                |
|---------------|---------------|---------------|------------|----------------|
| 0             | 1             | abs           | any?       | characteristic |
| coerce        | copy          | count         | D          | differentiate  |
| dimension     | directProduct | dot           | elt        | empty          |
| empty?        | entries       | entry?        | eq?        | eval           |
| every?        | fill!         | first         | hash       | index          |
| index?        | indices       | latex         | less?      | lookup         |
| map           | map!          | max           | maxIndex   | member?        |
| members       | min           | minIndex      | more?      | negative?      |
| one?          | parts         | positive?     | qelt       | qsetelt!       |
| random        | recip         | reducedSystem | retract    | retractIfCan   |
| sample        | setelt        | sign          | size       | size?          |
| subtractIfCan | sup           | swap!         | unitVector | zero?          |
| #?            | ?*?           | ?**?          | ?+?        | ?-?            |
| ?/?           | ?<?           | ?<=?          | ?=?        | ?>?            |
| ?>=?          | ?^?           | ?~=?          | -?         | ?..?           |

— domain HDP HomogeneousDirectProduct —

```
)abbrev domain HDP HomogeneousDirectProduct
++ Author: Mark Botch
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors: Vector, DirectProduct
++ Also See: OrderedDirectProduct, SplitHomogeneousDirectproduct
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This type represents the finite direct or cartesian product of an
++ underlying ordered component type. The vectors are ordered first
++ by the sum of their components, and then refined using a reverse
++ lexicographic ordering. This type is a suitable third argument for
++ \spadtype{GeneralDistributedMultivariatePolynomial}.
```

```
HomogeneousDirectProduct(dim,S) : T == C where
```

```
  dim : NonNegativeInteger
  S      : OrderedAbelianMonoidSup
```

```
  T == DirectProductCategory(dim,S)
```

```
  C == DirectProduct(dim,S) add
```

```
    Rep:=Vector(S)
```

```
    v1:% < v2:% ==
```

```
-- reverse lexicographical ordering
```

```
  n1:S:=0
```

```
  n2:S:=0
```

```
  for i in 1..dim repeat
```

```
    n1:= n1+qelt(v1,i)
```



---

---

— HomogeneousDistributedMultivariatePolynomial.input —

```

)set break resume
)sys rm -f HomogeneousDistributedMultivariatePolynomial.output
)spool HomogeneousDistributedMultivariatePolynomial.output
)set message test on
)set message auto off
)clear all
--S 1 of 10
(d1,d2,d3) : DMP([z,y,x],FRAC INT)
--R
--R
--R                                          Type: Void
--E 1

--S 2 of 10
d1 := -4*z + 4*y**2*x + 16*x**2 + 1
--R
--R
--R
--R          2      2
--R      (2)  - 4z + 4y x + 16x + 1
--R
--R      Type: DistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 2

```

```

--S 3 of 10
d2 := 2*z*y**2 + 4*x + 1
--R
--R
--R      2
--R (3) 2z y + 4x + 1
--R      Type: DistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 3

--S 4 of 10
d3 := 2*z*x**2 - 2*y**2 - x
--R
--R
--R      2      2
--R (4) 2z x - 2y - x
--R      Type: DistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 4

--S 5 of 10
groebner [d1,d2,d3]
--R
--R
--R (5)
--R      1568 6 1264 5 6 4 182 3 2047 2 103 2857
--R [z - ---- x - ---- x + --- x + --- x - ---- x - ---- x - ----,
--R      2745 305 305 549 610 2745 10980
--R      2 112 6 84 5 1264 4 13 3 84 2 1772 2
--R y + ---- x - --- x - ---- x - --- x + --- x + ---- x + ----,
--R      2745 305 305 549 305 2745 2745
--R      7 29 6 17 4 11 3 1 2 15 1
--R x + -- x - -- x - -- x + -- x + -- x + -]
--R      4 16 8 32 16 4
--R      Type: List DistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 5

--S 6 of 10
(n1,n2,n3) : HDMP([z,y,x],FRAC INT)
--R
--R
--R                                          Type: Void
--E 6

--S 7 of 10
n1 := d1
--R
--R
--R      2      2
--R (7) 4y x + 16x - 4z + 1
--R      Type: HomogeneousDistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 7

```

```

--S 8 of 10
n2 := d2
--R
--R
--R      2
--R (8) 2z y + 4x + 1
--R Type: HomogeneousDistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 8

--S 9 of 10
n3 := d3
--R
--R
--R      2      2
--R (9) 2z x - 2y - x
--R Type: HomogeneousDistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 9

--S 10 of 10
groebner [n1,n2,n3]
--R
--R
--R (10)
--R      4      3      3      2      1      1      4      29      3      1      2      7      9      1
--R [y + 2x - - x + - z - -, x + - x - - y - - z x - - x - -,
--R      2      2      8      4      8      4      16      4
--R      2      1      2      2      1      2      2      1
--R z y + 2x + -, y x + 4x - z + -, z x - y - - x,
--R      2      4      2
--R      2      2      2      1      3
--R z - 4y + 2x - - z - - x]
--R      4      2
--RType: List HomogeneousDistributedMultivariatePolynomial([z,y,x],Fraction Integer)
--E 10
)spool
)lisp (bye)

```

---

— HomogeneousDistributedMultivariatePolynomial.help —

```

=====
MultivariatePolynomial
DistributedMultivariatePolynomial
HomogeneousDistributedMultivariatePolynomial
GeneralDistributedMultivariatePolynomial
=====

```

### 9.10. DOMAIN HDMP HOMOGENEOUSDISTRIBUTEDMULTIVARIATEPOLYNOMIAL1143

DistributedMultivariatePolynomial which is abbreviated as DMP and HomogeneousDistributedMultivariatePolynomial, which is abbreviated as HDMP, are very similar to MultivariatePolynomial except that they are represented and displayed in a non-recursive manner.

```
(d1,d2,d3) : DMP([z,y,x],FRAC INT)
              Type: Void
```

The constructor DMP orders its monomials lexicographically while HDMP orders them by total order refined by reverse lexicographic order.

```
d1 := -4*z + 4*y**2*x + 16*x**2 + 1
      2      2
    - 4z + 4y x + 16x + 1
      Type: DistributedMultivariatePolynomial([z,y,x],Fraction Integer)

d2 := 2*z*y**2 + 4*x + 1
      2
    2z y + 4x + 1
      Type: DistributedMultivariatePolynomial([z,y,x],Fraction Integer)

d3 := 2*z*x**2 - 2*y**2 - x
      2      2
    2z x - 2y - x
      Type: DistributedMultivariatePolynomial([z,y,x],Fraction Integer)
```

These constructors are mostly used in Groebner basis calculations.

```
groebner [d1,d2,d3]
      1568 6 1264 5 6 4 182 3 2047 2 103 2857
[z - ---- x - ---- x + --- x + --- x - ---- x - ---- x - ----,
 2745 305 305 549 610 2745 10980
 2 112 6 84 5 1264 4 13 3 84 2 1772 2
y + ---- x - --- x - ---- x - --- x + --- x + ---- x + ----,
 2745 305 305 549 305 2745 2745
 7 29 6 17 4 11 3 1 2 15 1
x + -- x - -- x - -- x + -- x + -- x + -]
 4 16 8 32 16 4
      Type: List DistributedMultivariatePolynomial([z,y,x],Fraction Integer)
```

```
(n1,n2,n3) : HDMP([z,y,x],FRAC INT)
              Type: Void
```

```
n1 := d1
      2      2
    4y x + 16x - 4z + 1
      Type: HomogeneousDistributedMultivariatePolynomial([z,y,x],Fraction Integer)
```

```
n2 := d2
```

```

      2
    2z y  + 4x + 1
Type: HomogeneousDistributedMultivariatePolynomial([z,y,x],Fraction Integer)

n3 := d3
      2      2
    2z x  - 2y  - x
Type: HomogeneousDistributedMultivariatePolynomial([z,y,x],Fraction Integer)

```

Note that we get a different Groebner basis when we use the HDMP polynomials, as expected.

```

groebner [n1,n2,n3]
      4      3      3      2      1      1      4      29      3      1      2      7      9      1
[y  + 2x  - - x  + - z - -, x  + -- x  - - y  - - z x - -- x - -,
      2      2      8      4      8      4      16      4
      2      1      2      2      1      2      2      1
z y  + 2x + -, y x + 4x  - z + -, z x  - y  - - x,
      2      4      2
      2      2      2      1      3
z  - 4y  + 2x  - - z - - x]
      4      2
Type: List HomogeneousDistributedMultivariatePolynomial([z,y,x],
Fraction Integer)

```

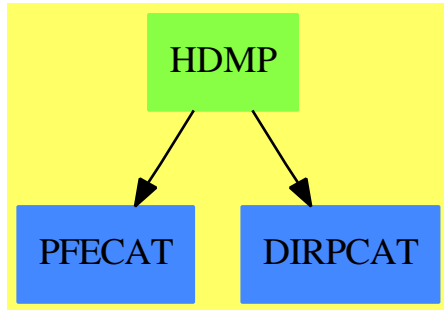
GeneralDistributedMultivariatePolynomial is somewhat more flexible in the sense that as well as accepting a list of variables to specify the variable ordering, it also takes a predicate on exponent vectors to specify the term ordering. With this polynomial type the user can experiment with the effect of using completely arbitrary term orderings. This flexibility is mostly important for algorithms such as Groebner basis calculations which can be very sensitive to term ordering.

See Also:

- o )help Polynomial
- o )help UnivariatePolynomial
- o )help MultivariatePolynomial
- o )help DistributedMultivariatePolynomial
- o )help GeneralDistributedMultivariatePolynomial
- o )show HomogeneousDistributedMultivariatePolynomial

---

### 9.10.1 HomogeneousDistributedMultivariatePolynomial (HDMP)



See

⇒ “GeneralDistributedMultivariatePolynomial” (GDMP) 8.1.1 on page 1018

⇒ “DistributedMultivariatePolynomial” (DMP) 5.13.1 on page 557

**Exports:**

|               |                    |                               |
|---------------|--------------------|-------------------------------|
| 0             | 1                  | associates?                   |
| binomThmExpt  | characteristic     | charthRoot                    |
| coefficient   | coefficients       | coerce                        |
| conditionP    | content            | convert                       |
| D             | degree             | differentiate                 |
| discriminant  | eval               | exquo                         |
| factor        | factorPolynomial   | factorSquareFreePolynomial    |
| gcd           | gcdPolynomial      | ground                        |
| ground?       | hash               | isExpt                        |
| isPlus        | isTimes            | latex                         |
| lcm           | leadingCoefficient | leadingMonomial               |
| mainVariable  | map                | mapExponents                  |
| max           | min                | minimumDegree                 |
| monicDivide   | monomial           | monomial?                     |
| monomials     | multivariate       | numberOfMonomials             |
| one?          | patternMatch       | pomopo!                       |
| prime?        | primitiveMonomials | primitivePart                 |
| recip         | reducedSystem      | reductum                      |
| reorder       | resultant          | retract                       |
| retractIfCan  | sample             | solveLinearPolynomialEquation |
| squareFree    | squareFreePart     | squareFreePolynomial          |
| subtractIfCan | totalDegree        | unit?                         |
| unitCanonical | unitNormal         | univariate                    |
| variables     | zero?              | ?*?                           |
| ?**?          | ?+?                | ?-?                           |
| -?            | ?=?                | ?^?                           |
| ?~=?          | ?/?                | ?<?                           |
| ?<=?          | ?>?                | ?>=?                          |
| ?^?           |                    |                               |

— **domain HDMP HomogeneousDistributedMultivariatePolynomial**

```

)abbrev domain HDMP HomogeneousDistributedMultivariatePolynomial
++ Author: Barry Trager
++ Date Created:
++ Date Last Updated:
++ Basic Functions: Ring, degree, eval, coefficient, monomial, differentiate,
++ resultant, gcd, leadingCoefficient
++ Related Constructors: DistributedMultivariatePolynomial,
++ GeneralDistributedMultivariatePolynomial
++ Also See: Polynomial
++ AMS Classifications:
++ Keywords: polynomial, multivariate, distributed
++ References:
++ Description:

```

```

++ This type supports distributed multivariate polynomials
++ whose variables are from a user specified list of symbols.
++ The coefficient ring may be non commutative,
++ but the variables are assumed to commute.
++ The term ordering is total degree ordering refined by reverse
++ lexicographic ordering with respect to the position that the variables
++ appear in the list of variables parameter.

HomogeneousDistributedMultivariatePolynomial(vl,R): public == private where
  vl : List Symbol
  R : Ring
  E ==> HomogeneousDirectProduct(#vl,NonNegativeInteger)
  OV ==> OrderedVariableList(vl)
  public == PolynomialCategory(R,E,OV) with
    reorder: (%,List Integer) -> %
    ++ reorder(p, perm) applies the permutation perm to the variables
    ++ in a polynomial and returns the new correctly ordered polynomial
  private ==
    GeneralDistributedMultivariatePolynomial(vl,R,E)

```

---

— HDMP.dotabb —

```

"HDMP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=HDMP"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"DIRPCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=DIRPCAT"]
"HDMP" -> "PFECAT"
"HDMP" -> "DIRPCAT"

```

---

## 9.11 domain HELLFDIV HyperellipticFiniteDivisor

— HyperellipticFiniteDivisor.input —

```

)set break resume
)sys rm -f HyperellipticFiniteDivisor.output
)spool HyperellipticFiniteDivisor.output
)set message test on
)set message auto off
)clear all

```

--S 1 of 1



```

)show HyperellipticFiniteDivisor
--R HyperellipticFiniteDivisor(F: Field,UP: UnivariatePolynomialCategory F,UPUP: UnivariatePolynomialCategory F)
--R Abbreviation for HyperellipticFiniteDivisor is HELLFDIV
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for HELLFDIV
--R
--R----- Operations -----
--R ?? : (Integer,%) -> %           ?? : (PositiveInteger,%) -> %
--R ?+? : (%,%) -> %               ?-? : (%,%) -> %
--R -? : % -> %                     ?? : (%,%) -> Boolean
--R 0 : () -> %                     coerce : % -> OutputForm
--R divisor : (R,UP,UP,UP,F) -> %   divisor : (F,F,Integer) -> %
--R divisor : (F,F) -> %           divisor : R -> %
--R hash : % -> SingleInteger       latex : % -> String
--R principal? : % -> Boolean       reduce : % -> %
--R sample : () -> %               zero? : % -> Boolean
--R ~=? : (%,%) -> Boolean
--R ?? : (NonNegativeInteger,%) -> %
--R decompose : % -> Record(id: FractionalIdeal(UP,Fraction UP,UPUP,R),principalPart: R)
--R divisor : FractionalIdeal(UP,Fraction UP,UPUP,R) -> %
--R generator : % -> Union(R,"failed")
--R ideal : % -> FractionalIdeal(UP,Fraction UP,UPUP,R)
--R subtractIfCan : (%,%) -> Union(%, "failed")
--R
--E 1

)spool
)lisp (bye)

```

---

— HyperellipticFiniteDivisor.help —

```

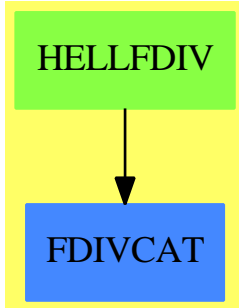
=====
HyperellipticFiniteDivisor examples
=====

```

See Also:

- o )show HyperellipticFiniteDivisor

## 9.11.1 HyperellipticFiniteDivisor (HELLFDIV)



See

⇒ “FractionalIdeal” (FRIDEAL) 7.25.1 on page 961

⇒ “FramedModule” (FRMOD) 7.26.1 on page 967

⇒ “FiniteDivisor” (FDIV) 7.4.1 on page 781

**Exports:**

|        |               |           |            |        |
|--------|---------------|-----------|------------|--------|
| 0      | coerce        | decompose | divisor    | hash   |
| ideal  | generator     | latex     | principal? | reduce |
| sample | subtractIfCan | zero?     | ?~=?       | ?*?    |
| ?+?    | ?-?           | -?        | ?=?        |        |

— domain HELLFDIV HyperellipticFiniteDivisor —

```
)abbrev domain HELLFDIV HyperellipticFiniteDivisor
++ Author: Manuel Bronstein
++ Date Created: 19 May 1993
++ Date Last Updated: 20 July 1998
++ Keywords: divisor, algebraic, curve.
++ Description:
++ This domains implements finite rational divisors on an hyperelliptic curve,
++ that is finite formal sums SUM(n * P) where the n's are integers and the
++ P's are finite rational points on the curve.
++ The equation of the curve must be y^2 = f(x) and f must have odd degree.
```

```
HyperellipticFiniteDivisor(F, UP, UPUP, R): Exports == Implementation where
```

```
  F  : Field
  UP  : UnivariatePolynomialCategory F
  UPUP: UnivariatePolynomialCategory Fraction UP
  R   : FunctionFieldCategory(F, UP, UPUP)
```

```
  O  ==> OutputForm
  Z  ==> Integer
  RF ==> Fraction UP
  ID ==> FractionalIdeal(UP, RF, UPUP, R)
  ERR ==> error "divisor: incomplete implementation for hyperelliptic curves"
```

```

Exports ==> FiniteDivisorCategory(F, UP, UPUP, R)

Implementation ==> add
  if (uhyper:Union(UP, "failed") := hyperelliptic()$R) case "failed" then
    error "HyperellipticFiniteDivisor: curve must be hyperelliptic"

-- we use the semi-reduced representation from D.Cantor, "Computing in the
-- Jacobian of a HyperellipticCurve", Mathematics of Computation, vol 48,
-- no.177, January 1987, 95-101.
-- The representation [a,b,f] for D means  $D = [a,b] + \text{div}(f)$ 
-- and [a,b] is a semi-reduced representative on the Jacobian
Rep := Record(center:UP, polyPart:UP, principalPart:R, reduced?:Boolean)

hyper:UP := uhyper::UP
gen:Z     := ((degree(hyper)::Z - 1) exquo 2)::Z      -- genus of the curve
dvd:0     := "div"::Symbol::0
zer:0     := 0::Z::0

makeDivisor : (UP, UP, R) -> %
intReduc    : (R, UP) -> R
princ?      : % -> Boolean
polyIfCan   : R -> Union(UP, "failed")
redpolyIfCan : (R, UP) -> Union(UP, "failed")
intReduce   : (R, UP) -> R
mkIdeal     : (UP, UP) -> ID
reducedTimes : (Z, UP, UP) -> %
reducedDouble: (UP, UP) -> %

0           == divisor(1$R)
divisor(g:R) == [1, 0, g, true]
makeDivisor(a, b, g) == [a, b, g, false]
-- princ? d      == one?(d.center) and zero?(d.polyPart)
princ? d      == (d.center = 1) and zero?(d.polyPart)
ideal d       == ideal([d.principalPart]) * mkIdeal(d.center, d.polyPart)
decompose d == [ideal makeDivisor(d.center, d.polyPart, 1), d.principalPart]
mkIdeal(a, b) == ideal [a::RF::R, reduce(monomial(1, 1)$UPUP-b::RF::UPUP)]

-- keep the sum reduced if d1 and d2 are both reduced at the start
d1 + d2 ==
  a1 := d1.center; a2 := d2.center
  b1 := d1.polyPart; b2 := d2.polyPart
  rec := principalIdeal [a1, a2, b1 + b2]
  d := rec.generator
  h := rec.coef          -- d = h1 a1 + h2 a2 + h3(b1 + b2)
  a := ((a1 * a2) exquo d**2)::UP
  b:UP:= first(h) * a1 * b2
  b := b + second(h) * a2 * b1
  b := b + third(h) * (b1*b2 + hyper)
  b := (b exquo d)::UP rem a
  dd := makeDivisor(a, b, d::RF * d1.principalPart * d2.principalPart)

```

```

d1.reduced? and d2.reduced? => reduce dd
dd

-- if is cheaper to keep on reducing as we exponentiate if d is already reduced
n:Z * d:% ==
  zero? n => 0
  n < 0 => (-n) * (-d)
  divisor(d.principalPart ** n) + divisor(mkIdeal(d.center,d.polyPart)**n)

divisor(i:ID) ==
--   one?(n := #(v := basis minimize i)) => divisor v minIndex v
  (n := #(v := basis minimize i)) = 1 => divisor v minIndex v
  n ^= 2 => ERR
  a := v minIndex v
  h := v maxIndex v
  (u := polyIfCan a) case UP =>
    (w := redpolyIfCan(h, u::UP)) case UP => makeDivisor(u::UP, w::UP, 1)
  ERR
  (u := polyIfCan h) case UP =>
    (w := redpolyIfCan(a, u::UP)) case UP => makeDivisor(u::UP, w::UP, 1)
  ERR
  ERR

polyIfCan a ==
  (u := retractIfCan(a)@Union(RF, "failed")) case "failed" => "failed"
  (v := retractIfCan(u::RF)@Union(UP, "failed")) case "failed" => "failed"
  v::UP

redpolyIfCan(h, a) ==
  degree(p := lift h) ^= 1 => "failed"
  q := - coefficient(p, 0) / coefficient(p, 1)
  rec := extendedEuclidean(denom q, a)
  not ground?(rec.generator) => "failed"
  ((numer(q) * rec.coef1) exquo rec.generator)::UP rem a

coerce(d:%):0 ==
  r := bracket [d.center::0, d.polyPart::0]
  g := prefix(dvd, [d.principalPart::0])
--   z := one?(d.principalPart)
  z := (d.principalPart = 1)
  princ? d => (z => zer; g)
  z => r
  r + g

reduce d ==
  d.reduced? => d
  degree(a := d.center) <= gen => (d.reduced? := true; d)
  b := d.polyPart
  a0 := ((hyper - b**2) exquo a)::UP
  b0 := (-b) rem a0

```

```

g := d.principalPart * reduce(b::RF::UPUP-monomial(1,1)$UPUP)/a0::RF::R
reduce makeDivisor(a0, b0, g)

generator d ==
  d := reduce d
  princ? d => d.principalPart
  "failed"

- d ==
  a := d.center
  makeDivisor(a, - d.polyPart, inv(a::RF * d.principalPart))

d1 = d2 ==
  d1 := reduce d1
  d2 := reduce d2
  d1.center = d2.center and d1.polyPart = d2.polyPart
  and d1.principalPart = d2.principalPart

divisor(a, b) ==
  x := monomial(1, 1)$UP
  not ground? gcd(d := x - a::UP, retract(discriminant())@UP) =>
    error "divisor: point is singular"
  makeDivisor(d, b::UP, 1)

intReduce(h, b) ==
  v := integralCoordinates(h).num
  integralRepresents(
    [qelt(v, i) rem b for i in minIndex v .. maxIndex v], 1)

-- with hyperelliptic curves, it is cheaper to keep divisors in reduced form
divisor(h, a, dp, g, r) ==
  h := h - (r * dp)::RF::R
  a := gcd(a, retract(norm h)@UP)
  h := intReduce(h, a)
  if not ground? gcd(g, a) then h := intReduce(h ** rank(), a)
  hh := lift h
  b := - coefficient(hh, 0) / coefficient(hh, 1)
  rec := extendedEuclidean(denom b, a)
  not ground?(rec.generator) => ERR
  bb := ((numer(b) * rec.coef1) exquo rec.generator)::UP rem a
  reduce makeDivisor(a, bb, 1)

```

---

— HELLFDIV.dotabb —

"HELLFDIV" [color="#88FF44",href="bookvol10.3.pdf#nameddest=HELLFDIV"]  
 "FDIVCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FDIVCAT"]

"HELLFDIV" -> "FDIVCAT"

---



## Chapter 10

# Chapter I

### 10.1 domain ICP InfClsPt

— InfClsPt.input —

```
)set break resume
)sys rm -f InfClsPt.output
)spool InfClsPt.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show InfClsPt
--R InfClsPt(K: Field,symb: List Symbol,BLMET: BlowUpMethodCategory) is a domain constructor
--R Abbreviation for InfClsPt is ICP
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ICP
--R
--R----- Operations -----
--R ==? : (%,% ) -> Boolean          actualExtensionV : % -> K
--R chartV : % -> BLMET              coerce : % -> OutputForm
--R degree : % -> PositiveInteger    excpDivV : % -> Divisor Places K
--R fullOut : % -> OutputForm        fullOutput : () -> Boolean
--R fullOutput : Boolean -> Boolean  hash : % -> SingleInteger
--R latex : % -> String              localPointV : % -> AffinePlane K
--R multV : % -> NonNegativeInteger  pointV : % -> ProjectivePlane K
--R setchart! : (% ,BLMET) -> BLMET  symbNameV : % -> Symbol
--R ~=? : (%,% ) -> Boolean
--R create : (ProjectivePlane K,DistributedMultivariatePolynomial(symb,K)) -> %
--R create : (ProjectivePlane K,DistributedMultivariatePolynomial([construct,QUOTEX,QUOTEY],K),AffinePlane K) -> %
--R curveV : % -> DistributedMultivariatePolynomial([construct,QUOTEX,QUOTEY],K)
```



```

--R localParamV : % -> List NeitherSparseOrDensePowerSeries K
--R setcurve! : (% , DistributedMultivariatePolynomial([construct, QUOTE $\mathcal{O}$ , QUOTE $\mathcal{O}$ ], K)) -> Distrib
--R setexcpDiv! : (% , Divisor Places K) -> Divisor Places K
--R setlocalParam! : (% , List NeitherSparseOrDensePowerSeries K) -> List NeitherSparseOrDense
--R setlocalPoint! : (% , AffinePlane K) -> AffinePlane K
--R setmult! : (% , NonNegativeInteger) -> NonNegativeInteger
--R setpoint! : (% , ProjectivePlane K) -> ProjectivePlane K
--R setsubmult! : (% , NonNegativeInteger) -> NonNegativeInteger
--R setsymbName! : (% , Symbol) -> Symbol
--R subMultV : % -> NonNegativeInteger
--R
--E 1

)spool
)lisp (bye)

```

—————

— InfClsPt.help —

=====

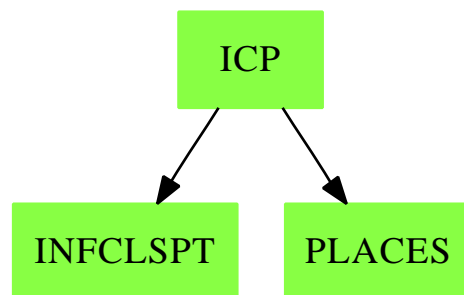
InfClsPt examples

=====

See Also:

- o )show InfClsPt

### 10.1.1 InfClsPt (ICP)



|                |              |                  |
|----------------|--------------|------------------|
| ?=?            | ?~=?         | actualExtensionV |
| chartV         | coerce       | create           |
| curveV         | degree       | excpDivV         |
| fullOut        | fullOutput   | hash             |
| latex          | localParamV  | localPointV      |
| multV          | pointV       | setchart!        |
| setcurve!      | setexcpDiv!  | setlocalParam!   |
| setlocalPoint! | setmult!     | setpoint!        |
| setsubmult!    | setsymbName! | subMultV         |
| symbNameV      |              |                  |

[illegible]

---

— ICP.dotabb —

```
"ICP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ICP"]
"INFCLSPT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=INFCLSPT"]
"PLACES" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PLACES"]
"ICP" -> "INFCLSPT"
"ICP" -> "PLACES"
```

---

## 10.2 domain ICARD IndexCard

---

— IndexCard.input —

```
)set break resume
)sys rm -f IndexCard.output
)spool IndexCard.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show IndexCard
--R IndexCard is a domain constructor
--R Abbreviation for IndexCard is ICARD
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ICARD
--R
--R----- Operations -----
--R ?<? : (% ,%) -> Boolean          ?<=? : (% ,%) -> Boolean
--R ?=? : (% ,%) -> Boolean          ?>? : (% ,%) -> Boolean
--R ?>=? : (% ,%) -> Boolean          coerce : String -> %
--R coerce : % -> OutputForm          display : % -> Void
--R ?.? : (% ,Symbol) -> String        fullDisplay : % -> Void
--R hash : % -> SingleInteger          latex : % -> String
--R max : (% ,%) -> %                  min : (% ,%) -> %
--R ?~=? : (% ,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

---

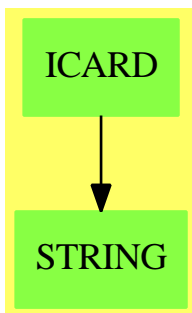
— IndexCard.help —

```
=====
IndexCard examples
=====
```

See Also:

o )show IndexCard

### 10.2.1 IndexCard (ICARD)



See

⇒ “DataList” (DLIST) 5.2.1 on page 445  
 ⇒ “Database” (DBASE) 5.1.1 on page 440  
 ⇒ “QueryEquation” (QEQUAT) 18.4.1 on page 2129

#### Exports:

|        |         |             |      |       |
|--------|---------|-------------|------|-------|
| coerce | display | fullDisplay | hash | latex |
| max    | min     | ?~=?        | ?<?  | ?<=?  |
| ?=?    | ?>?     | ?>=?        | ?..? |       |

— domain ICARD IndexCard —

```
)abbrev domain ICARD IndexCard
++ Author: Mark Botch
++ Description:
++ This domain implements a container of information about the AXIOM library

IndexCard() : Exports == Implementation where
  Exports == OrderedSet with
    elt: (%,Symbol) -> String
    ++ elt(ic,s) selects a particular field from \axiom{ic}. Valid fields
```

```

++ are \axiom{name, nargs, exposed, type, abbreviation, kind, origin,
++ params, condition, doc}.
display: % -> Void
++ display(ic) prints a summary of information contained in \axiom{ic}.
fullDisplay: % -> Void
++ fullDisplay(ic) prints all of the information contained in \axiom{ic}.
coerce: String -> %
++ coerce(s) converts \axiom{s} into an \axiom{IndexCard}. Warning: if
++ \axiom{s} is not of the right format then an error will occur
Implementation == add
x<y==(x pretend String) < (y pretend String)
x==y==(x pretend String) = (y pretend String)
display(x) ==
  name : OutputForm := dbName(x)$Lisp
  type : OutputForm := dbPart(x,4,1$Lisp)$Lisp
  output(hconcat(name,hconcat(" : ",type)))$OutputPackage
fullDisplay(x) ==
  name : OutputForm := dbName(x)$Lisp
  type : OutputForm := dbPart(x,4,1$Lisp)$Lisp
  origin:OutputForm :=
    hconcat(alqlGetOrigin(x$Lisp)$Lisp,alqlGetParams(x$Lisp)$Lisp)
  fromPart : OutputForm := hconcat(" from ",origin)
  condition : String := dbPart(x,6,1$Lisp)$Lisp
  ifPart : OutputForm :=
    condition = "" => empty()
    hconcat(" if ",condition::OutputForm)
  exposed? : String := SUBSTRING(dbPart(x,3,1)$Lisp,0,1)$Lisp
  exposedPart : OutputForm :=
    exposed? = "n" => " (unexposed)"
    empty()
  firstPart := hconcat(name,hconcat(" : ",type))
  secondPart := hconcat(fromPart,hconcat(ifPart,exposedPart))
  output(hconcat(firstPart,secondPart))$OutputPackage
coerce(s:String): % == (s pretend %)
coerce(x): OutputForm == (x pretend String)::OutputForm
elt(x,sel) ==
  s := PNAME(sel)$Lisp pretend String
  s = "name" => dbName(x)$Lisp
  s = "nargs" => dbPart(x,2,1$Lisp)$Lisp
  s = "exposed" => SUBSTRING(dbPart(x,3,1)$Lisp,0,1)$Lisp
  s = "type" => dbPart(x,4,1$Lisp)$Lisp
  s = "abbreviation" => dbPart(x,5,1$Lisp)$Lisp
  s = "kind" => alqlGetKindString(x)$Lisp
  s = "origin" => alqlGetOrigin(x)$Lisp
  s = "params" => alqlGetParams(x)$Lisp
  s = "condition" => dbPart(x,6,1$Lisp)$Lisp
  s = "doc" => dbComments(x)$Lisp
  error "unknown selector"

```

### 10.3 domain IBITS IndexedBits

[illegible]

[illegible]

---

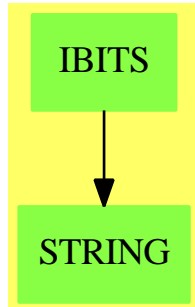
```
=====
IndexedBits
=====
```

[illegible]





## 10.3.1 IndexedBits (IBITS)



See

- ⇒ “Reference” (REF) 19.5.1 on page 2209
- ⇒ “Boolean” (BOOLEAN) 3.15.1 on page 304
- ⇒ “Bits” (BITS) 3.12.1 on page 297

**Exports:**

|          |          |           |         |                  |
|----------|----------|-----------|---------|------------------|
| And      | any?     | coerce    | concat  | construct        |
| convert  | copy     | copyInto! | count   | count            |
| delete   | elt      | empty     | empty?  | entries          |
| entry?   | eq?      | eval      | every?  | fill!            |
| find     | first    | hash      | index?  | indices          |
| insert   | latex    | less?     | map     | map!             |
| max      | maxIndex | member?   | members | merge            |
| min      | minIndex | more?     | nand    | new              |
| nor      | Not      | not?      | Or      | parts            |
| position | qelt     | qsetelt!  | reduce  | removeDuplicates |
| reverse  | reverse! | sample    | select  | size?            |
| sort     | sort!    | sorted?   | swap!   | xor              |
| #?       | ?.?      | ?/\?      | ?<?     | ?<=?             |
| ?=?      | ?>?      | ?>=?      | ?\/?    | ^?               |
| ?.?      | ~?       | ?~=?      | ?or?    | ?and?            |

— domain IBITS IndexedBits —

```

)abbrev domain IBITS IndexedBits
++ Author: Stephen Watt and Michael Monagan
++ Date Created: July 86
++ Change History: Oct 87
++ Basic Operations: range
++ Related Constructors:
++ Keywords: indexed bits
++ Description:
++ \spadtype{IndexedBits} is a domain to compactly represent
++ large quantities of Boolean data.
  
```

```

IndexedBits(mn:Integer): BitAggregate() with
-- temporaries until parser gets better
Not: % -> %
    ++ Not(n) returns the bit-by-bit logical Not of n.
Or : (% , %) -> %
    ++ Or(n,m) returns the bit-by-bit logical Or of
    ++ n and m.
And: (% , %) -> %
    ++ And(n,m) returns the bit-by-bit logical And of
    ++ n and m.
== add

range: (% , Integer) -> Integer
    --++ range(j,i) returns the range i of the boolean j.

minIndex u == mn

range(v, i) ==
    i >= 0 and i < #v => i
    error "Index out of range"

coerce(v):OutputForm ==
    t:Character := char "1"
    f:Character := char "0"
    s := new(#v, space()$Character)$String
    for i in minIndex(s)..maxIndex(s) for j in mn.. repeat
        s.i := if v.j then t else f
    s::OutputForm

new(n, b)      == BVEC_-MAKE_-FULL(n,TRUTH_-TO_-BIT(b)$Lisp)$Lisp
empty()        == BVEC_-MAKE_-FULL(0,0)$Lisp
copy v        == BVEC_-COPY(v)$Lisp
#v            == BVEC_-SIZE(v)$Lisp
v = u         == BVEC_-EQUAL(v, u)$Lisp
v < u         == BVEC_-GREATER(u, v)$Lisp
_and(u, v)    == (#v=#u => BVEC_-AND(v,u)$Lisp; map("and",v,u))
_or(u, v)     == (#v=#u => BVEC_-OR(v, u)$Lisp; map("or", v,u))
xor(v,u)      == (#v=#u => BVEC_-XOR(v,u)$Lisp; map("xor",v,u))
setelt(v:%, i:Integer, f:Boolean) ==
    BVEC_-SETELT(v, range(v, abs(i-mn)), TRUTH_-TO_-BIT(f)$Lisp)$Lisp
elt(v:%, i:Integer) ==
    BIT_-TO_-TRUTH(BVEC_-ELT(v, range(v, abs(i-mn)))$Lisp)$Lisp

Not v          == BVEC_-NOT(v)$Lisp
And(u, v)      == (#v=#u => BVEC_-AND(v,u)$Lisp; map("and",v,u))
Or(u, v)       == (#v=#u => BVEC_-OR(v, u)$Lisp; map("or", v,u))

```

---

— IBITS.dotabb —

```
"IBITS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IBITS"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"IBITS" -> "STRING"
```

## 10.4 domain IDPAG IndexedDirectProductAbelianGroup

— IndexedDirectProductAbelianGroup.input —

```
)set break resume
)sys rm -f IndexedDirectProductAbelianGroup.output
)spool IndexedDirectProductAbelianGroup.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show IndexedDirectProductAbelianGroup
--R IndexedDirectProductAbelianGroup(A: AbelianGroup,S: OrderedSet) is a domain constructor
--R Abbreviation for IndexedDirectProductAbelianGroup is IDPAG
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for IDPAG
--R
--R----- Operations -----
--R ??? : (Integer,%) -> %          ??? : (PositiveInteger,%) -> %
--R ?+? : (%,%) -> %              ?-? : (%,%) -> %
--R -? : % -> %                   ?=? : (%,%) -> Boolean
--R 0 : () -> %                   coerce : % -> OutputForm
--R hash : % -> SingleInteger      latex : % -> String
--R leadingCoefficient : % -> A     leadingSupport : % -> S
--R map : ((A -> A),%) -> %         monomial : (A,S) -> %
--R reductum : % -> %              sample : () -> %
--R zero? : % -> Boolean           ?~=? : (%,%) -> Boolean
--R ??? : (NonNegativeInteger,%) -> %
--R subtractIfCan : (%,%) -> Union(%, "failed")
--R
--E 1

)spool
)lisp (bye)
```

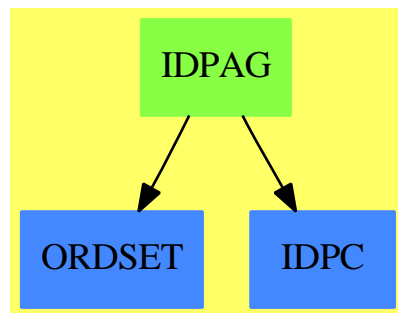
— IndexedDirectProductAbelianGroup.help —

```
=====
IndexedDirectProductAbelianGroup examples
=====
```

See Also:

```
o )show IndexedDirectProductAbelianGroup
```

### 10.4.1 IndexedDirectProductAbelianGroup (IDPAG)



See

⇒ “IndexedDirectProductObject” (IDPO) 10.6.1 on page 1175

⇒ “IndexedDirectProductAbelianMonoid” (IDPAM) 10.5.1 on page 1171

⇒ “IndexedDirectProductOrderedAbelianMonoid” (IDPOAM) 10.7.1 on page 1178

⇒ “IndexedDirectProductOrderedAbelianMonoidSup” (IDPOAMS) 10.8.1 on page 1180

**Exports:**

|                |        |          |          |                    |
|----------------|--------|----------|----------|--------------------|
| 0              | coerce | hash     | latex    | leadingCoefficient |
| leadingSupport | map    | monomial | reductum | sample             |
| subtractIfCan  | zero?  | ?~=?     | ?*?      | ?+?                |
| ?-?            | -?     | ?=?      |          |                    |

— domain IDPAG IndexedDirectProductAbelianGroup —

```
)abbrev domain IDPAG IndexedDirectProductAbelianGroup
```

```
++ Author: Mark Botch
```

```
++ Description:
```

```
++ Indexed direct products of abelian groups over an abelian group \spad{A} of
++ generators indexed by the ordered set S.
```

```
++ All items have finite support: only non-zero terms are stored.
```

```

IndexedDirectProductAbelianGroup(A:AbelianGroup,S:OrderedSet):
  Join(AbelianGroup,IndexedDirectProductCategory(A,S))
== IndexedDirectProductAbelianMonoid(A,S) add
--representations
  Term:= Record(k:S,c:A)
  Rep:= List Term
  x,y: %
  r: A
  n: Integer
  f: A -> A
  s: S
  -x == [[u.k,-u.c] for u in x]
  n * x ==
    n = 0 => 0
    n = 1 => x
    [[u.k,a] for u in x | (a:=n*u.c) ^= 0$A]

qsetrest!: (Rep, Rep) -> Rep
qsetrest!(l: Rep, e: Rep): Rep == RPLACD(l, e)$Lisp

x - y ==
  null x => -y
  null y => x
  endcell: Rep := empty()
  res: Rep := empty()
  while not empty? x and not empty? y repeat
    newcell := empty()
    if x.first.k = y.first.k then
      r:= x.first.c - y.first.c
      if not zero? r then
        newcell := cons([x.first.k, r], empty())
      x := rest x
      y := rest y
    else if x.first.k > y.first.k then
      newcell := cons(x.first, empty())
      x := rest x
    else
      newcell := cons([y.first.k,-y.first.c], empty())
      y := rest y
    if not empty? newcell then
      if not empty? endcell then
        qsetrest!(endcell, newcell)
        endcell := newcell
      else
        res := newcell;
        endcell := res
  if empty? x then end := - y
  else end := x
  if empty? res then res := end
  else qsetrest!(endcell, end)

```

```

res

--      x - y ==
--      empty? x => - y
--      empty? y => x
--      y.first.k > x.first.k => cons([y.first.k,-y.first.c],(x - y.rest))
--      x.first.k > y.first.k => cons(x.first,(x.rest - y))
--      r:= x.first.c - y.first.c
--      r = 0 => x.rest - y.rest
--      cons([x.first.k,r],(x.rest - y.rest))

```

— IDPAG.dotabb —

```

"IDPAG" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IDPAG"]
"ORDSET" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ORDSET"]
"IDPC" [color="#4488FF",href="bookvol10.2.pdf#nameddest=IDPC"]
"IDPAG" -> "IDPC"
"IDPAG" -> "ORDSET"

```

## 10.5 domain IDPAM IndexedDirectProductAbelianMonoid

— IndexedDirectProductAbelianMonoid.input —

```

)set break resume
)sys rm -f IndexedDirectProductAbelianMonoid.output
)spool IndexedDirectProductAbelianMonoid.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show IndexedDirectProductAbelianMonoid
--R IndexedDirectProductAbelianMonoid(A: AbelianMonoid,S: OrderedSet) is a domain construct
--R Abbreviation for IndexedDirectProductAbelianMonoid is IDPAM
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for IDPAM
--R
--R----- Operations -----
--R ?? : (PositiveInteger,%) -> %      ??? : (%,%) -> %
--R ?? : (%,%) -> Boolean              0 : () -> %

```

```

--R coerce : % -> OutputForm          hash : % -> SingleInteger
--R latex : % -> String                leadingCoefficient : % -> A
--R leadingSupport : % -> S            map : ((A -> A),%) -> %
--R monomial : (A,S) -> %              reductum : % -> %
--R sample : () -> %                  zero? : % -> Boolean
--R ?~=? : (%,% ) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R
--E 1

)spool
)lisp (bye)

```

---

— IndexedDirectProductAbelianMonoid.help —

```

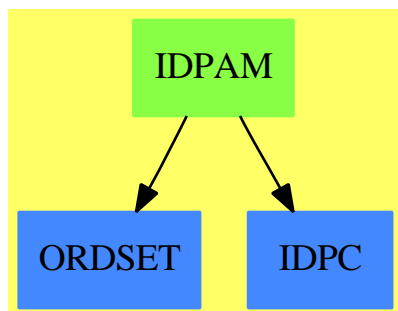
=====
IndexedDirectProductAbelianMonoid examples
=====

```

See Also:

- o )show IndexedDirectProductAbelianMonoid

### 10.5.1 IndexedDirectProductAbelianMonoid (IDPAM)



See

- ⇒ “IndexedDirectProductObject” (IDPO) 10.6.1 on page 1175
- ⇒ “IndexedDirectProductOrderedAbelianMonoid” (IDPOAM) 10.7.1 on page 1178
- ⇒ “IndexedDirectProductOrderedAbelianMonoidSup” (IDPOAMS) 10.8.1 on page 1180
- ⇒ “IndexedDirectProductAbelianGroup” (IDPAG) 10.4.1 on page 1168



**Exports:**

|                |        |          |          |                    |
|----------------|--------|----------|----------|--------------------|
| 0              | coerce | hash     | latex    | leadingCoefficient |
| leadingSupport | map    | monomial | reductum | sample             |
| zero?          | ?~=?   | ?*?      | ?+?      | ?=?                |

— domain IDPAM IndexedDirectProductAbelianMonoid —

```

)abbrev domain IDPAM IndexedDirectProductAbelianMonoid
++ Author: Mark Botch
++ Description:
++ Indexed direct products of abelian monoids over an abelian monoid
++ \spad{A} of generators indexed by the ordered set S. All items have
++ finite support. Only non-zero terms are stored.

IndexedDirectProductAbelianMonoid(A:AbelianMonoid,S:OrderedSet):
  Join(AbelianMonoid,IndexedDirectProductCategory(A,S))
== IndexedDirectProductObject(A,S) add
--representations
  Term:= Record(k:S,c:A)
  Rep:= List Term
  x,y: %
  r: A
  n: NonNegativeInteger
  f: A -> A
  s: S
  0 == []
  zero? x == null x

  -- PERFORMANCE CRITICAL; Should build list up
  -- by merging 2 sorted lists. Doing this will
  -- avoid the recursive calls (very useful if there is a
  -- large number of vars in a polynomial.
  x + y ==
  null x => y
  null y => x
  y.first.k > x.first.k => cons(y.first,(x + y.rest))
  x.first.k > y.first.k => cons(x.first,(x.rest + y))
  r:= x.first.c + y.first.c
  r = 0 => x.rest + y.rest
  cons([x.first.k,r],(x.rest + y.rest))
qsetrest!: (Rep, Rep) -> Rep
qsetrest!(l: Rep, e: Rep): Rep == RPLACD(l, e)$Lisp

x + y ==
null x => y
null y => x
endcell: Rep := empty()
res: Rep := empty()
while not empty? x and not empty? y repeat
  newcell := empty()

```

```

    if x.first.k = y.first.k then
      r := x.first.c + y.first.c
      if not zero? r then
        newcell := cons([x.first.k, r], empty())
        x := rest x
        y := rest y
      else if x.first.k > y.first.k then
        newcell := cons(x.first, empty())
        x := rest x
      else
        newcell := cons(y.first, empty())
        y := rest y
    if not empty? newcell then
      if not empty? endcell then
        qsetrest!(endcell, newcell)
        endcell := newcell
      else
        res := newcell;
        endcell := res
    if empty? x then end := y
    else end := x
    if empty? res then res := end
    else qsetrest!(endcell, end)
    res

n * x ==
  n = 0 => 0
  n = 1 => x
  [[u.k,a] for u in x | (a:=n*u.c) ^= 0$A]

monomial(r,s) == (r = 0 => 0; [[s,r]])
map(f,x) == [[tm.k,a] for tm in x | (a:=f(tm.c)) ^= 0$A]

reductum x == (null x => 0; rest x)
leadingCoefficient x == (null x => 0; x.first.c)

```

---

— IDPAM.dotabb —

```

"IDPAM" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IDPAM"]
"ORDSET" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ORDSET"]
"IDPC" [color="#4488FF",href="bookvol10.2.pdf#nameddest=IDPC"]
"IDPAM" -> "IDPC"
"IDPAM" -> "ORDSET"

```

---

## 10.6 domain IDPO IndexedDirectProductObject

— IndexedDirectProductObject.input —

```
)set break resume
)sys rm -f IndexedDirectProductObject.output
)spool IndexedDirectProductObject.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show IndexedDirectProductObject
--R IndexedDirectProductObject(A: SetCategory,S: OrderedSet) is a domain constructor
--R Abbreviation for IndexedDirectProductObject is IDPO
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for IDPO
--R
--R----- Operations -----
--R ?=? : (%,% ) -> Boolean          coerce : % -> OutputForm
--R hash : % -> SingleInteger        latex : % -> String
--R leadingCoefficient : % -> A      leadingSupport : % -> S
--R map : ((A -> A),%) -> %          monomial : (A,S) -> %
--R reductum : % -> %               ?~=? : (%,% ) -> Boolean
--R
--E 1

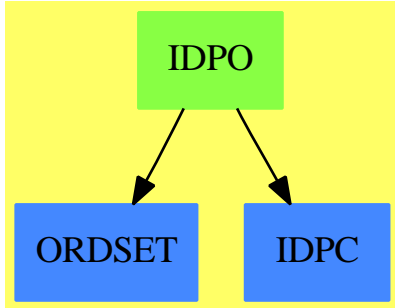
)spool
)lisp (bye)
```

— IndexedDirectProductObject.help —

```
=====
IndexedDirectProductObject examples
=====
```

```
See Also:
o )show IndexedDirectProductObject
```

## 10.6.1 IndexedDirectProductObject (IDPO)



See

- ⇒ “IndexedDirectProductAbelianMonoid” (IDPAM) 10.5.1 on page 1171
- ⇒ “IndexedDirectProductOrderedAbelianMonoid” (IDPOAM) 10.7.1 on page 1178
- ⇒ “IndexedDirectProductOrderedAbelianMonoidSup” (IDPOAMS) 10.8.1 on page 1180
- ⇒ “IndexedDirectProductAbelianGroup” (IDPAG) 10.4.1 on page 1168

**Exports:**

|        |          |          |                    |                |
|--------|----------|----------|--------------------|----------------|
| coerce | hash     | latex    | leadingCoefficient | leadingSupport |
| map    | monomial | reductum | ?=?                | ?~=?           |

— domain IDPO IndexedDirectProductObject —

```

)abbrev domain IDPO IndexedDirectProductObject
++ Author: Mark Botch
++ Description:
++ Indexed direct products of objects over a set \spad{A}
++ of generators indexed by an ordered set S. All items have finite support.

```

```

IndexedDirectProductObject(A:SetCategory,S:OrderedSet): _
  IndexedDirectProductCategory(A,S)
== add
  --representations
  Term:= Record(k:S,c:A)
  Rep:= List Term
  --declarations
  x,y: %
  f: A -> A
  s: S
  --define
  x = y ==
    while not null x and _^ null y repeat
      x.first.k ^= y.first.k => return false
      x.first.c ^= y.first.c => return false
      x:=x.rest
      y:=y.rest
    null x and null y

```

```

coerce(x:%):OutputForm ==
  bracket [rarrow(t.k :: OutputForm, t.c :: OutputForm) for t in x]

-- sample():% == [[sample()$S,sample()$A]$Term]$Rep

monomial(r,s) == [[s,r]]
map(f,x) == [[tm.k,f(tm.c)] for tm in x]

reductum x      ==
  rest x
leadingCoefficient x ==
  null x => error "Can't take leadingCoefficient of empty product element"
  x.first.c
leadingSupport x ==
  null x => error "Can't take leadingCoefficient of empty product element"
  x.first.k

```

— IDPO.dotabb —

```

"IDPO" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IDPO"]
"ORDSET" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ORDSET"]
"IDPC" [color="#4488FF",href="bookvol10.2.pdf#nameddest=IDPC"]
"IDPO" -> "IDPC"
"IDPO" -> "ORDSET"

```

## 10.7 domain IDPOAM IndexedDirectProductOrdered-AbelianMonoid

— IndexedDirectProductOrderedAbelianMonoid.input —

```

)set break resume
)sys rm -f IndexedDirectProductOrderedAbelianMonoid.output
)spool IndexedDirectProductOrderedAbelianMonoid.output
)set message test on
)set message auto off
)clear all

--S 1 of 1

```

## 10.7. DOMAIN IDPOAM INDEXEDDIRECTPRODUCTORDEREDABELIANMONOID1177

```

)show IndexedDirectProductOrderedAbelianMonoid
--R IndexedDirectProductOrderedAbelianMonoid(A: OrderedAbelianMonoid,S: OrderedSet) is a domain constr
--R Abbreviation for IndexedDirectProductOrderedAbelianMonoid is IDPOAM
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for IDPOAM
--R
--R----- Operations -----
--R ?? : (PositiveInteger,%) -> %          ?+? : (%,%) -> %
--R ?<? : (%,%) -> Boolean                ?<=? : (%,%) -> Boolean
--R ==? : (%,%) -> Boolean                ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean              0 : () -> %
--R coerce : % -> OutputForm             hash : % -> SingleInteger
--R latex : % -> String                  leadingCoefficient : % -> A
--R leadingSupport : % -> S              map : ((A -> A),%) -> %
--R max : (%,%) -> %                     min : (%,%) -> %
--R monomial : (A,S) -> %                reductum : % -> %
--R sample : () -> %                     zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?? : (NonNegativeInteger,%) -> %
--R
--E 1

)spool
)lisp (bye)

```

### — IndexedDirectProductOrderedAbelianMonoid.help —

```

=====
IndexedDirectProductOrderedAbelianMonoid examples
=====

```

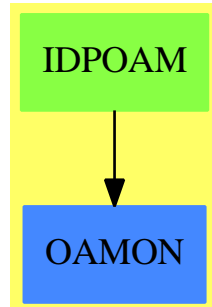
See Also:

```

o )show IndexedDirectProductOrderedAbelianMonoid

```

### 10.7.1 IndexedDirectProductOrderedAbelianMonoid (IDPOAM)



See

- ⇒ “IndexedDirectProductObject” (IDPO) 10.6.1 on page 1175
- ⇒ “IndexedDirectProductAbelianMonoid” (IDPAM) 10.5.1 on page 1171
- ⇒ “IndexedDirectProductOrderedAbelianMonoidSup” (IDPOAMS) 10.8.1 on page 1180
- ⇒ “IndexedDirectProductAbelianGroup” (IDPAG) 10.4.1 on page 1168

#### Exports:

|                |        |       |       |                    |
|----------------|--------|-------|-------|--------------------|
| 0              | coerce | hash  | latex | leadingCoefficient |
| leadingSupport | map    | max   | min   | monomial           |
| reductum       | sample | zero? | ?~=?  | ?*?                |
| ?+?            | ?<?    | ?<=?  | ?=?   | ?>?                |
| ?>=?           |        |       |       |                    |

— domain IDPOAM IndexedDirectProductOrderedAbelianMonoid

```

)abbrev domain IDPOAM IndexedDirectProductOrderedAbelianMonoid
++ Author: Mark Botch
++ Description:
++ Indexed direct products of ordered abelian monoids \spad{A} of
++ generators indexed by the ordered set S.
++ The inherited order is lexicographical.
++ All items have finite support: only non-zero terms are stored.

IndexedDirectProductOrderedAbelianMonoid(A:OrderedAbelianMonoid,S:OrderedSet):
  Join(OrderedAbelianMonoid,IndexedDirectProductCategory(A,S))
== IndexedDirectProductAbelianMonoid(A,S) add
--representations
  Term:= Record(k:S,c:A)
  Rep:= List Term
  x,y: %
  x<y ==
    empty? y => false
    empty? x => true -- note careful order of these two lines
    y.first.k > x.first.k => true

```

```

y.first.k < x.first.k => false
y.first.c > x.first.c => true
y.first.c < x.first.c => false
x.rest < y.rest

```

---

— IDPOAM.dotabb —

```

"IDPOAM" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IDPOAM"]
"OAMON" [color="#4488FF",href="bookvol10.2.pdf#nameddest=OAMON"]
"IDPOAM" -> "OAMON"

```

---

## 10.8 domain IDPOAMS IndexedDirectProductOrderedAbelianMonoidSup

— IndexedDirectProductOrderedAbelianMonoidSup.input —

```

)set break resume
)sys rm -f IndexedDirectProductOrderedAbelianMonoidSup.output
)spool IndexedDirectProductOrderedAbelianMonoidSup.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show IndexedDirectProductOrderedAbelianMonoidSup
--R IndexedDirectProductOrderedAbelianMonoidSup(A: OrderedAbelianMonoidSup,S: OrderedSet) is a domain c
--R Abbreviation for IndexedDirectProductOrderedAbelianMonoidSup is IDPOAMS
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for IDPOAMS
--R
--R----- Operations -----
--R ?*? : (PositiveInteger,%) -> %      ?+? : (%,%) -> %
--R ?<? : (%,%) -> Boolean              ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean              ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean             0 : () -> %
--R coerce : % -> OutputForm           hash : % -> SingleInteger
--R latex : % -> String                 leadingCoefficient : % -> A
--R leadingSupport : % -> S             map : ((A -> A),%) -> %
--R max : (%,%) -> %                   min : (%,%) -> %

```



```

--R monomial : (A,S) -> %                reductum : % -> %
--R sample : () -> %                    sup : (%,% ) -> %
--R zero? : % -> Boolean                ~=? : (%,% ) -> Boolean
--R ?? : (NonNegativeInteger,% ) -> %
--R subtractIfCan : (%,% ) -> Union(%, "failed")
--R
--E 1

)spool
)lisp (bye)

```

— IndexedDirectProductOrderedAbelianMonoidSup.help —

```

=====
IndexedDirectProductOrderedAbelianMonoidSup examples
=====

```

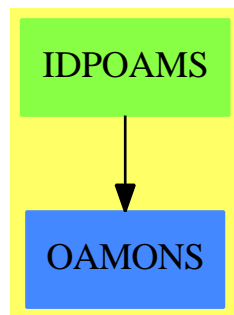
See Also:

```

o )show IndexedDirectProductOrderedAbelianMonoidSup

```

### 10.8.1 IndexedDirectProductOrderedAbelianMonoidSup (IDPOAMS)



See

- ⇒ “IndexedDirectProductObject” (IDPO) 10.6.1 on page 1175
- ⇒ “IndexedDirectProductAbelianMonoid” (IDPAM) 10.5.1 on page 1171
- ⇒ “IndexedDirectProductOrderedAbelianMonoid” (IDPOAM) 10.7.1 on page 1178
- ⇒ “IndexedDirectProductAbelianGroup” (IDPAG) 10.4.1 on page 1168

**Exports:**

|                |        |               |       |                    |
|----------------|--------|---------------|-------|--------------------|
| 0              | coerce | hash          | latex | leadingCoefficient |
| leadingSupport | map    | max           | min   | monomial           |
| reductum       | sample | subtractIfCan | sup   | zero?              |
| ?~=?           | ?*?    | ?+?           | ?<?   | ?<=?               |
| ?=?            | ?>?    | ?>=?          |       |                    |

— domain IDPOAMS IndexedDirectProductOrderedAbelianMonoid-Sup —

```
)abbrev domain IDPOAMS IndexedDirectProductOrderedAbelianMonoidSup
++ Author: Mark Botch
++ Description:
++ Indexed direct products of ordered abelian monoid sups \spad{A},
++ generators indexed by the ordered set S.
++ All items have finite support: only non-zero terms are stored.
```

```
IndexedDirectProductOrderedAbelianMonoidSup(A:OrderedAbelianMonoidSup,S:OrderedSet):
  Join(OrderedAbelianMonoidSup,IndexedDirectProductCategory(A,S))
== IndexedDirectProductOrderedAbelianMonoid(A,S) add
--representations
  Term:= Record(k:S,c:A)
  Rep:= List Term
  x,y: %
  r: A
  s: S

  subtractIfCan(x,y) ==
    empty? y => x
    empty? x => "failed"
    x.first.k < y.first.k => "failed"
    x.first.k > y.first.k =>
      t:= subtractIfCan(x.rest, y)
      t case "failed" => "failed"
      cons( x.first, t)
    u:=subtractIfCan(x.first.c, y.first.c)
    u case "failed" => "failed"
    zero? u => subtractIfCan(x.rest, y.rest)
    t:= subtractIfCan(x.rest, y.rest)
    t case "failed" => "failed"
    cons([x.first.k,u],t)

  sup(x,y) ==
    empty? y => x
    empty? x => y
    x.first.k < y.first.k => cons(y.first,sup(x,y.rest))
    x.first.k > y.first.k => cons(x.first,sup(x.rest,y))
    u:=sup(x.first.c, y.first.c)
    cons([x.first.k,u],sup(x.rest,y.rest))
```

---

— IDPOAMS.dotabb —

```
"IDPOAMS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IDPOAMS"]
"OAMONS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=OAMONS"]
"IDPOAMS" -> "OAMONS"
```

---

## 10.9 domain INDE IndexedExponents

— IndexedExponents.input —

```
)set break resume
)sys rm -f IndexedExponents.output
)spool IndexedExponents.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show IndexedExponents
--R IndexedExponents Varset: OrderedSet is a domain constructor
--R Abbreviation for IndexedExponents is INDE
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for INDE
--R
--R----- Operations -----
--R ?? : (PositiveInteger,%) -> %          ?? : (%,%) -> %
--R <? : (%,%) -> Boolean                  ?<=? : (%,%) -> Boolean
--R ==? : (%,%) -> Boolean                 ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean                0 : () -> %
--R coerce : % -> OutputForm              hash : % -> SingleInteger
--R latex : % -> String                    leadingSupport : % -> Varset
--R max : (%,%) -> %                       min : (%,%) -> %
--R reductum : % -> %                      sample : () -> %
--R sup : (%,%) -> %                       zero? : % -> Boolean
--R ~=? : (%,%) -> Boolean
--R ?? : (NonNegativeInteger,%) -> %
--R leadingCoefficient : % -> NonNegativeInteger
--R map : ((NonNegativeInteger -> NonNegativeInteger),%) -> %
--R monomial : (NonNegativeInteger,Varset) -> %
```

```
--R subtractIfCan : (%,% ) -> Union(%,"failed")
--R
--E 1
```

```
)spool
)lisp (bye)
```

---

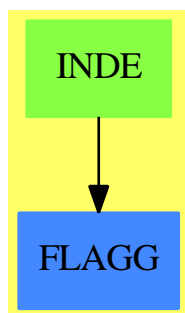
— IndexedExponents.help —

```
=====
IndexedExponents examples
=====
```

```
See Also:
o )show IndexedExponents
```

---

### 10.9.1 IndexedExponents (INDE)



See

⇒ “Polynomial” (POLY) 17.25.1 on page 2037  
 ⇒ “MultivariatePolynomial” (MPOLY) 14.16.1 on page 1645  
 ⇒ “SparseMultivariatePolynomial” (SMP) 20.14.1 on page 2381

**Exports:**

|                |        |               |       |                    |
|----------------|--------|---------------|-------|--------------------|
| 0              | coerce | hash          | latex | leadingCoefficient |
| leadingSupport | map    | max           | min   | monomial           |
| reductum       | sample | subtractIfCan | sup   | zero?              |
| ?~=?           | ?*?    | ?+?           | ?<?   | ?<=?               |
| ?=?            | ?>?    | ?>=?          |       |                    |

— domain INDE IndexedExponents —

```

)abbrev domain INDE IndexedExponents
++ Author: James Davenport
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ IndexedExponents of an ordered set of variables gives a representation
++ for the degree of polynomials in commuting variables. It gives an ordered
++ pairing of non negative integer exponents with variables

IndexedExponents(Varset:OrderedSet): C == T where
  C == Join(OrderedAbelianMonoidSup,
            IndexedDirectProductCategory(NonNegativeInteger,Varset))
  T == IndexedDirectProductOrderedAbelianMonoidSup(NonNegativeInteger,Varset) add
    Term:= Record(k:Varset,c:NonNegativeInteger)
    Rep:= List Term
    x:%
    t:Term
    coerceOF(t):OutputForm ==      --++ converts term to OutputForm
      t.c = 1 => (t.k)::OutputForm
      (t.k)::OutputForm ** (t.c)::OutputForm
    coerce(x):OutputForm == ++ converts entire exponents to OutputForm
      null x => 1::Integer::OutputForm
      null rest x => coerceOF(first x)
      reduce("",[coerceOF t for t in x])

```

---

— INDE.dotabb —

```

"INDE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=INDE"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"INDE" -> "FLAGG"

```

---

## 10.10 domain IFARRAY IndexedFlexibleArray

— IndexedFlexibleArray.input —

```

)set break resume
)sys rm -f IndexedFlexibleArray.output
)spool IndexedFlexibleArray.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show IndexedFlexibleArray
--R IndexedFlexibleArray(S: Type,mn: Integer) is a domain constructor
--R Abbreviation for IndexedFlexibleArray is IFARRAY
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for IFARRAY
--R
--R----- Operations -----
--R concat : List % -> %                concat : (%,% ) -> %
--R concat : (S,% ) -> %                concat : (% ,S) -> %
--R concat! : (% ,S) -> %               concat! : (% ,%) -> %
--R construct : List S -> %             copy : % -> %
--R delete : (% ,Integer) -> %          delete! : (% ,Integer) -> %
--R ?.? : (% ,Integer) -> S             elt : (% ,Integer,S) -> S
--R empty : () -> %                     empty? : % -> Boolean
--R entries : % -> List S               eq? : (% ,%) -> Boolean
--R flexibleArray : List S -> %         index? : (Integer,% ) -> Boolean
--R indices : % -> List Integer         insert : (% ,% ,Integer) -> %
--R insert : (S,% ,Integer) -> %        insert! : (S,% ,Integer) -> %
--R insert! : (% ,% ,Integer) -> %      map : (((S,S) -> S),% ,%) -> %
--R map : ((S -> S),% ) -> %            new : (NonNegativeInteger,S) -> %
--R qelt : (% ,Integer) -> S            reverse : % -> %
--R sample : () -> %                   shrinkable : Boolean -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?<? : (% ,%) -> Boolean if S has ORDSET
--R ?<=? : (% ,%) -> Boolean if S has ORDSET
--R ?=? : (% ,%) -> Boolean if S has SETCAT
--R ?>? : (% ,%) -> Boolean if S has ORDSET
--R ?>=? : (% ,%) -> Boolean if S has ORDSET
--R any? : ((S -> Boolean),% ) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R convert : % -> InputForm if S has KONVERT INFORM
--R copyInto! : (% ,% ,Integer) -> % if $ has shallowlyMutable
--R count : (S,% ) -> NonNegativeInteger if $ has finiteAggregate and S has SETCAT
--R count : ((S -> Boolean),% ) -> NonNegativeInteger if $ has finiteAggregate
--R delete : (% ,UniversalSegment Integer) -> %
--R delete! : (% ,UniversalSegment Integer) -> %
--R ?.? : (% ,UniversalSegment Integer) -> %
--R entry? : (S,% ) -> Boolean if $ has finiteAggregate and S has SETCAT
--R eval : (% ,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (% ,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (% ,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (% ,List Equation S) -> % if S has EVALAB S and S has SETCAT

```

```

--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,S) -> % if $ has shallowlyMutable
--R find : ((S -> Boolean),%) -> Union(S,"failed")
--R first : % -> S if Integer has ORDSET
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R max : (%,%) -> % if S has ORDSET
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R members : % -> List S if $ has finiteAggregate
--R merge : (%,%) -> % if S has ORDSET
--R merge : (((S,S) -> Boolean),%,%) -> %
--R merge! : (((S,S) -> Boolean),%,%) -> %
--R merge! : (%,%) -> % if S has ORDSET
--R min : (%,%) -> % if S has ORDSET
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List S if $ has finiteAggregate
--R physicalLength : % -> NonNegativeInteger
--R physicalLength! : (%,Integer) -> %
--R position : (S,%,Integer) -> Integer if S has SETCAT
--R position : (S,%) -> Integer if S has SETCAT
--R position : ((S -> Boolean),%) -> Integer
--R qsetelt! : (%,Integer,S) -> S if $ has shallowlyMutable
--R reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S,S) -> S if $ has finiteAggregate and S has SETCAT
--R remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (S,%) -> % if $ has finiteAggregate and S has SETCAT
--R remove! : ((S -> Boolean),%) -> %
--R remove! : (S,%) -> % if S has SETCAT
--R removeDuplicates : % -> % if $ has finiteAggregate and S has SETCAT
--R removeDuplicates! : % -> % if S has SETCAT
--R reverse! : % -> % if $ has shallowlyMutable
--R select : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R select! : ((S -> Boolean),%) -> %
--R setelt : (%,UniversalSegment Integer,S) -> S if $ has shallowlyMutable
--R setelt : (%,Integer,S) -> S if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : % -> % if S has ORDSET
--R sort : (((S,S) -> Boolean),%) -> %
--R sort! : % -> % if $ has shallowlyMutable and S has ORDSET
--R sort! : (((S,S) -> Boolean),%) -> % if $ has shallowlyMutable
--R sorted? : % -> Boolean if S has ORDSET
--R sorted? : (((S,S) -> Boolean),%) -> Boolean
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R ~=? : (%,%) -> Boolean if S has SETCAT
--R

```

```
--E 1
```

```
)spool
)lisp (bye)
```

```
_____
```

```
— IndexedFlexibleArray.help —
```

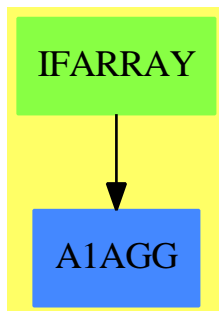
```
=====
IndexedFlexibleArray examples
=====
```

```
See Also:
```

```
o )show IndexedFlexibleArray
```

```
_____
```

### 10.10.1 IndexedFlexibleArray (IFARRAY)



See

- ⇒ “PrimitiveArray” (PRIMARR) 17.30.1 on page 2069
- ⇒ “Tuple” (TUPLE) 21.12.1 on page 2711
- ⇒ “FlexibleArray” (FARRAY) 7.14.1 on page 853
- ⇒ “IndexedOneDimensionalArray” (IARRAY1) 10.13.1 on page 1208
- ⇒ “OneDimensionalArray” (ARRAY1) 16.3.1 on page 1736



**Exports:**

|                  |                   |                 |               |
|------------------|-------------------|-----------------|---------------|
| concat           | concat!           | construct       | copy          |
| delete           | delete!           | elt             | empty         |
| empty?           | entries           | eq?             | flexibleArray |
| index?           | indices           | insert          | insert!       |
| map              | new               | qelt            | reverse       |
| sample           | shrinkable        | any?            | coerce        |
| convert          | copyInto!         | count           | delete        |
| delete!          | entry?            | eval            | every?        |
| fill!            | find              | first           | hash          |
| latex            | less?             | map!            | max           |
| maxIndex         | member?           | members         | merge         |
| merge!           | min               | minIndex        | more?         |
| parts            | physicalLength    | physicalLength! | position      |
| qsetelt!         | reduce            | remove          | remove!       |
| removeDuplicates | removeDuplicates! | reverse!        | select        |
| select!          | setelt            | size?           | sort          |
| sort!            | sorted?           | swap!           | #?            |
| ?<?              | ?<=?              | ?=?             | ?>?           |
| ?>=?             | ?~=?              | ?..?            |               |

— domain IFARRAY IndexedFlexibleArray —

```

)abbrev domain IFARRAY IndexedFlexibleArray
++ Author: Michael Monagan July/87, modified SMW June/91
++ Description:
++ A FlexibleArray is the notion of an array intended to allow for growth
++ at the end only. Hence the following efficient operations\br
++ \spad{append(x,a)} meaning append item x at the end of the array \spad{a}\br
++ \spad{delete(a,n)} meaning delete the last item from the array \spad{a}\br
++ Flexible arrays support the other operations inherited from
++ \spadtype{ExtensibleLinearAggregate}. However, these are not efficient.
++ Flexible arrays combine the \spad{0(1)} access time property of arrays
++ with growing and shrinking at the end in \spad{0(1)} (average) time.
++ This is done by using an ordinary array which may have zero or more
++ empty slots at the end. When the array becomes full it is copied
++ into a new larger (50% larger) array. Conversely, when the array
++ becomes less than 1/2 full, it is copied into a smaller array.
++ Flexible arrays provide for an efficient implementation of many
++ data structures in particular heaps, stacks and sets.

IndexedFlexibleArray(S:Type, mn: Integer): Exports == Implementation where
  A ==> PrimitiveArray S
  I ==> Integer
  N ==> NonNegativeInteger
  U ==> UniversalSegment Integer
Exports ==
  Join(OneDimensionalArrayAggregate S,ExtensibleLinearAggregate S) with

```

```

flexibleArray : List S -> %
++ flexibleArray(l) creates a flexible array from the list of elements l
++
++X T1:=IndexedFlexibleArray(Integer,20)
++X flexibleArray([i for i in 1..10])$T1

physicalLength : % -> NonNegativeInteger
++ physicalLength(x) returns the number of elements x can
++ accomodate before growing
++
++X T1:=IndexedFlexibleArray(Integer,20)
++X t2:=flexibleArray([i for i in 1..10])$T1
++X physicalLength t2

physicalLength_! : (% , I) -> %
++ physicalLength!(x,n) changes the physical length of x to be n and
++ returns the new array.
++
++X T1:=IndexedFlexibleArray(Integer,20)
++X t2:=flexibleArray([i for i in 1..10])$T1
++X physicalLength!(t2,15)

shrinkable: Boolean -> Boolean
++ shrinkable(b) sets the shrinkable attribute of flexible arrays to b
++ and returns the previous value
++
++X T1:=IndexedFlexibleArray(Integer,20)
++X shrinkable(false)$T1

Implementation == add
Rep := Record(physLen:I, logLen:I, f:A)
shrinkable? : Boolean := true
growAndFill : (% , I, S) -> %
growWith    : (% , I, S) -> %
growAdding  : (% , I, %) -> %
shrink      : (% , I)   -> %
newa       : (N, A) -> A

physicalLength(r) == (r.physLen) pretend NonNegativeInteger
physicalLength_!(r, n) ==
  r.physLen = 0 => error "flexible array must be non-empty"
  growWith(r, n, r.f.0)

empty()      == [0, 0, empty()]
#r           == (r.logLen)::N
fill_!(r, x) == (fill_!(r.f, x); r)
maxIndex r   == r.logLen - 1 + mn
minIndex r   == mn
new(n, a)    == [n, n, new(n, a)]

```

```

shrinkable(b) ==
  oldval := shrinkable?
  shrinkable? := b
  oldval

flexibleArray l ==
  n := #l
  n = 0 => empty()
  x := l.1
  a := new(n,x)
  for i in mn + 1..mn + n-1 for y in rest l repeat a.i := y
  a

-- local utility operations
newa(n, a) ==
  zero? n => empty()
  new(n, a.0)

growAdding(r, b, s) ==
  b = 0 => r
  #r > 0 => growAndFill(r, b, (r.f).0)
  #s > 0 => growAndFill(r, b, (s.f).0)
  error "no default filler element"

growAndFill(r, b, x) ==
  (r.logLen := r.logLen + b) <= r.physLen => r
  -- enlarge by 50% + b
  n := r.physLen + r.physLen quo 2 + 1
  if r.logLen > n then n := r.logLen
  growWith(r, n, x)

growWith(r, n, x) ==
  y := new(n:N, x)$PrimitiveArray(S)
  a := r.f
  for k in 0 .. r.physLen-1 repeat y.k := a.k
  r.physLen := n
  r.f := y
  r

shrink(r, i) ==
  r.logLen := r.logLen - i
  negative?(n := r.logLen) => error "internal bug in flexible array"
  2*n+2 > r.physLen => r
  not shrinkable? => r
  if n < r.logLen
    then error "cannot shrink flexible array to indicated size"
  n = 0 => empty()
  r.physLen := n
  y := newa(n:N, a := r.f)
  for k in 0 .. n-1 repeat y.k := a.k

```

```

    r.f := y
    r

copy r ==
    n := #r
    a := r.f
    v := newa(n, a := r.f)
    for k in 0..n-1 repeat v.k := a.k
    [n, n, v]

elt(r:%, i:I) ==
    i < mn or i >= r.logLen + mn =>
        error "index out of range"
    r.f.(i-mn)

setelt(r:%, i:I, x:S) ==
    i < mn or i >= r.logLen + mn =>
        error "index out of range"
    r.f.(i-mn) := x

-- operations inherited from extensible aggregate
merge(g, a, b) == merge_!(g, copy a, b)
concat(x:S, r:%) == insert_!(x, r, mn)

concat_!(r:%, x:S) ==
    growAndFill(r, 1, x)
    r.f.(r.logLen-1) := x
    r

concat_!(a:%, b:%) ==
    if eq?(a, b) then b := copy b
    n := #a
    growAdding(a, #b, b)
    copyInto_!(a, b, n + mn)

remove_!(g:(S->Boolean), a:%) ==
    k:I := 0
    for i in 0..maxIndex a - mn repeat
        if not g(a.i) then (a.k := a.i; k := k+1)
    shrink(a, #a - k)

delete_!(r:%, i1:I) ==
    i := i1 - mn
    i < 0 or i > r.logLen => error "index out of range"
    for k in i..r.logLen-2 repeat r.f.k := r.f.(k+1)
    shrink(r, 1)

delete_!(r:%, i:U) ==
    l := lo i - mn; m := maxIndex r - mn

```

```

h := (hasHi i => hi i - mn; m)
l < 0 or h > m => error "index out of range"
for j in l.. for k in h+1..m repeat r.f.j := r.f.k
shrink(r, max(0,h-l+1))

insert_!(x:S, r:%, i1:I):% ==
  i := i1 - mn
  n := r.logLen
  i < 0 or i > n => error "index out of range"
  growAndFill(r, 1, x)
  for k in n-1 .. i by -1 repeat r.f.(k+1) := r.f.k
  r.f.i := x
  r

insert_!(a:%, b:%, i1:I):% ==
  i := i1 - mn
  if eq?(a, b) then b := copy b
  m := #a; n := #b
  i < 0 or i > n => error "index out of range"
  growAdding(b, m, a)
  for k in n-1 .. i by -1 repeat b.f.(m+k) := b.f.k
  for k in m-1 .. 0 by -1 repeat b.f.(i+k) := a.f.k
  b

merge_!(g, a, b) ==
  m := #a; n := #b; growAdding(a, n, b)
  for i in m-1..0 by -1 for j in m+n-1.. by -1 repeat a.f.j := a.f.i
  i := n; j := 0
  for k in 0.. while i < n+m and j < n repeat
    if g(a.f.i,b.f.j) then (a.f.k := a.f.i; i := i+1)
    else (a.f.k := b.f.j; j := j+1)
  for k in k.. for j in j..n-1 repeat a.f.k := b.f.j
  a

select_!(g:(S->Boolean), a:%) ==
  k:I := 0
  for i in 0..maxIndex a - mn repeat_
    if g(a.f.i) then (a.f.k := a.f.i;k := k+1)
  shrink(a, #a - k)

if S has SetCategory then
  removeDuplicates_! a ==
    ct := #a
    ct < 2 => a

    i := mn
    nlim := mn + ct
    nlim0 := nlim
    while i < nlim repeat
      j := i+1

```

```

    for k in j..nlim-1 | a.k ^= a.i repeat
      a.j := a.k
      j := j+1
    nlim := j
    i := i+1
  nlim ^= nlim0 => delete_!(a, i..)
a

```

---

— IFARRAY.dotabb —

```

"IFARRAY" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IFARRAY"]
"A1AGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=A1AGG"]
"IFARRAY" -> "A1AGG"

```

---

## 10.11 domain ILIST IndexedList

---

— IndexedList.input —

```

)set break resume
)sys rm -f IndexedList.output
)spool IndexedList.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show IndexedList
--R IndexedList(S: Type,mn: Integer) is a domain constructor
--R Abbreviation for IndexedList is ILIST
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ILIST
--R
--R----- Operations -----
--R children : % -> List %          concat : (%,S) -> %
--R concat : List % -> %            concat : (S,%) -> %
--R concat : (%,%) -> %             concat! : (%,S) -> %
--R concat! : (%,%) -> %            construct : List S -> %
--R copy : % -> %                   cycleEntry : % -> %
--R cycleTail : % -> %              cyclic? : % -> Boolean
--R delete : (%,Integer) -> %       delete! : (%,Integer) -> %

```

```

--R distance : (%,%) -> Integer
--R ?.? : (%,Integer) -> S
--R ?.rest : (%,rest) -> %
--R ?.value : (%,value) -> S
--R empty? : % -> Boolean
--R eq? : (%,%) -> Boolean
--R first : % -> S
--R indices : % -> List Integer
--R insert : (%,%,Integer) -> %
--R insert! : (%,%,Integer) -> %
--R leaf? : % -> Boolean
--R list : S -> %
--R map : ((S -> S),%) -> %
--R nodes : % -> List %
--R qelt : (%,Integer) -> S
--R reverse : % -> %
--R second : % -> S
--R third : % -> S
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R <? : (%,%) -> Boolean if S has ORDSET
--R <=? : (%,%) -> Boolean if S has ORDSET
--R ==? : (%,%) -> Boolean if S has SETCAT
--R >? : (%,%) -> Boolean if S has ORDSET
--R >=? : (%,%) -> Boolean if S has ORDSET
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R child? : (%,%) -> Boolean if S has SETCAT
--R coerce : % -> OutputForm if S has SETCAT
--R convert : % -> InputForm if S has KONVERT INFORM
--R copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
--R count : (S,%) -> NonNegativeInteger if $ has finiteAggregate and S has SETCAT
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R cycleLength : % -> NonNegativeInteger
--R cycleSplit! : % -> % if $ has shallowlyMutable
--R delete : (%,UniversalSegment Integer) -> %
--R delete! : (%,UniversalSegment Integer) -> %
--R ?.? : (%,UniversalSegment Integer) -> %
--R entry? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,S) -> % if $ has shallowlyMutable
--R find : ((S -> Boolean),%) -> Union(S,"failed")
--R first : (%,NonNegativeInteger) -> %
--R hash : % -> SingleInteger if S has SETCAT
--R last : (%,NonNegativeInteger) -> %
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
elt : (%,Integer,S) -> S
?.last : (%,last) -> S
?.first : (%,first) -> S
empty : () -> %
entries : % -> List S
explicitlyFinite? : % -> Boolean
index? : (Integer,%) -> Boolean
insert : (S,%,Integer) -> %
insert! : (S,%,Integer) -> %
last : % -> S
leaves : % -> List S
map : (((S,S) -> S),%,%) -> %
new : (NonNegativeInteger,S) -> %
possiblyInfinite? : % -> Boolean
rest : % -> %
sample : () -> %
tail : % -> %
value : % -> S

```

```

--R max : (%,% ) -> % if S has ORDSET
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (S,% ) -> Boolean if $ has finiteAggregate and S has SETCAT
--R members : % -> List S if $ has finiteAggregate
--R merge : (((S,S) -> Boolean),%,%) -> %
--R merge : (%,% ) -> % if S has ORDSET
--R merge! : (((S,S) -> Boolean),%,%) -> %
--R merge! : (%,% ) -> % if S has ORDSET
--R min : (%,% ) -> % if S has ORDSET
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (% ,NonNegativeInteger) -> Boolean
--R node? : (%,% ) -> Boolean if S has SETCAT
--R parts : % -> List S if $ has finiteAggregate
--R position : ((S -> Boolean),%) -> Integer
--R position : (S,% ) -> Integer if S has SETCAT
--R position : (S,% ,Integer) -> Integer if S has SETCAT
--R qsetelt! : (% ,Integer,S) -> S if $ has shallowlyMutable
--R reduce : (((S,S) -> S),%,S,S) -> S if $ has finiteAggregate and S has SETCAT
--R reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
--R remove : (S,% ) -> % if $ has finiteAggregate and S has SETCAT
--R remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove! : ((S -> Boolean),%) -> %
--R remove! : (S,% ) -> % if S has SETCAT
--R removeDuplicates : % -> % if $ has finiteAggregate and S has SETCAT
--R removeDuplicates! : % -> % if S has SETCAT
--R rest : (% ,NonNegativeInteger) -> %
--R reverse! : % -> % if $ has shallowlyMutable
--R select : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R select! : ((S -> Boolean),%) -> %
--R setchildren! : (% ,List %) -> % if $ has shallowlyMutable
--R setelt : (% ,Integer,S) -> S if $ has shallowlyMutable
--R setelt : (% ,UniversalSegment Integer,S) -> S if $ has shallowlyMutable
--R setelt : (% ,last,S) -> S if $ has shallowlyMutable
--R setelt : (% ,rest,%) -> % if $ has shallowlyMutable
--R setelt : (% ,first,S) -> S if $ has shallowlyMutable
--R setelt : (% ,value,S) -> S if $ has shallowlyMutable
--R setfirst! : (% ,S) -> S if $ has shallowlyMutable
--R setlast! : (% ,S) -> S if $ has shallowlyMutable
--R setrest! : (% ,%) -> % if $ has shallowlyMutable
--R setvalue! : (% ,S) -> S if $ has shallowlyMutable
--R size? : (% ,NonNegativeInteger) -> Boolean
--R sort : (((S,S) -> Boolean),%) -> %
--R sort : % -> % if S has ORDSET
--R sort! : (((S,S) -> Boolean),%) -> % if $ has shallowlyMutable
--R sort! : % -> % if $ has shallowlyMutable and S has ORDSET
--R sorted? : (((S,S) -> Boolean),%) -> Boolean
--R sorted? : % -> Boolean if S has ORDSET
--R split! : (% ,Integer) -> % if $ has shallowlyMutable
--R swap! : (% ,Integer,Integer) -> Void if $ has shallowlyMutable

```



```
--R ?~=? : (%,% ) -> Boolean if S has SETCAT
--R
--E 1
```

```
)spool
)lisp (bye)
```

---

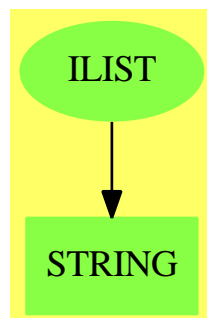
— IndexedList.help —

```
=====
IndexedList examples
=====
```

```
See Also:
o )show IndexedList
```

---

### 10.11.1 IndexedList (ILIST)



See

⇒ “List” (LIST) 13.9.1 on page 1468

⇒ “AssociationList” (ALIST) 2.42.1 on page 218

**Exports:**

|                  |                   |                   |                   |
|------------------|-------------------|-------------------|-------------------|
| any?             | child?            | children          | coerce            |
| concat           | convert           | concat!           | copyInto!         |
| construct        | copy              | count             | cycleEntry        |
| cycleLength      | cycleSplit!       | cycleTail         | cyclic?           |
| delete           | delete!           | distance          | elt               |
| empty            | empty?            | entries           | entry?            |
| eq?              | eval              | every?            | explicitlyFinite? |
| fill!            | find              | first             | hash              |
| index?           | indices           | insert            | insert!           |
| last             | latex             | leaf?             | leaves            |
| less?            | list              | map               | map!              |
| max              | maxIndex          | member?           | members           |
| merge            | merge!            | min               | minIndex          |
| more?            | new               | node?             | nodes             |
| parts            | position          | possiblyInfinite? | qelt              |
| qsetelt!         | reduce            | remove            | remove!           |
| removeDuplicates | removeDuplicates! | rest              | reverse           |
| reverse!         | sample            | second            | select            |
| select!          | setchildren!      | setelt            | setfirst!         |
| setlast!         | setrest!          | setvalue!         | size?             |
| sort             | sort!             | sorted?           | split!            |
| swap!            | tail              | third             | value             |
| #?               | ?<?               | ?<=?              | ?=?               |
| ?>?              | ?>=?              | ?~=?              | ?..?              |
| ?..last          | ?..rest           | ?..first          | ?..value          |

— domain ILIST IndexedList —

```

)abbrev domain ILIST IndexedList
++ Author: Michael Monagan
++ Date Created: Sep 1987
++ Change History:
++ Basic Operations:
++ \#, concat, concat!, construct, copy, elt, elt, empty,
++ empty?, eq?, first, member?, merge!, mergeSort, minIndex,
++ parts, removeDuplicates!, rest, rest, reverse, reverse!,
++ setelt, setfirst!, setrest!, sort!, split!
++ Related Constructors: List
++ Also See:
++ AMS Classification:
++ Keywords: list, aggregate, index
++ Description:
++ \spadtype{IndexedList} is a basic implementation of the functions
++ in \spadtype{ListAggregate}, often using functions in the underlying
++ LISP system. The second parameter to the constructor (\spad{mn})
++ is the beginning index of the list. That is, if \spad{l} is a
++ list, then \spad{elt(l,mn)} is the first value. This constructor

```

++ is probably best viewed as the implementation of singly-linked  
 ++ lists that are addressable by index rather than as a mere wrapper  
 ++ for LISP lists.

```

IndexedList(S:Type, mn:Integer): Exports == Implementation where
  cycleMax ==> 1000          -- value used in checking for cycles

-- The following seems to be a bit out of date, but is kept in case
-- a knowledgeable person wants to update it:
-- The following LISP dependencies are divided into two groups
-- Those that are required
-- CONS, EQ, NIL, NULL, QCAR, QCDR, RPLACA, RPLACD
-- Those that are included for efficiency only
-- NEQ, LIST, CAR, CDR, NCONC2, NREVERSE, LENGTH
-- Also REVERSE, since it's called in Polynomial Ring

Qfirst ==> QCAR$Lisp
Qrest  ==> QCDR$Lisp
Qnull  ==> NULL$Lisp
Qeq     ==> EQ$Lisp
Qneq    ==> NEQ$Lisp
Qcons   ==> CONS$Lisp
Qpush   ==> PUSH$Lisp

Exports ==> ListAggregate S
Implementation ==>
  add
    #x                == LENGTH(x)$Lisp
    concat(s:S,x:%)   == CONS(s,x)$Lisp
    eq?(x,y)           == EQ(x,y)$Lisp
    first x            == SPADfirst(x)$Lisp
    elt(x,"first")     == SPADfirst(x)$Lisp
    empty()            == NIL$Lisp
    empty? x           == NULL(x)$Lisp
    rest x             == CDR(x)$Lisp
    elt(x,"rest")      == CDR(x)$Lisp
    setfirst_!(x,s)    ==
      empty? x => error "Cannot update an empty list"
      Qfirst RPLACA(x,s)$Lisp
    setelt(x,"first",s) ==
      empty? x => error "Cannot update an empty list"
      Qfirst RPLACA(x,s)$Lisp
    setrest_!(x,y)     ==
      empty? x => error "Cannot update an empty list"
      Qrest RPLACD(x,y)$Lisp
    setelt(x,"rest",y) ==
      empty? x => error "Cannot update an empty list"
      Qrest RPLACD(x,y)$Lisp
    construct l        == l pretend %
    parts s            == s pretend List S

```

```

reverse_! x      == NREVERSE(x)$Lisp
reverse x        == REVERSE(x)$Lisp
minIndex x       == mn

rest(x, n) ==
  for i in 1..n repeat
    if Qnull x then error "index out of range"
    x := Qrest x
  x

copy x ==
  y := empty()
  for i in 0.. while not Qnull x repeat
    if Qeq(i,cycleMax) and cyclic? x then error "cyclic list"
    y := Qcons(Qfirst x,y)
    x := Qrest x
  (NREVERSE(y)$Lisp)%

if S has SetCategory then
  coerce(x):OutputForm ==
    -- displays cycle with overbar over the cycle
    y := empty()$List(OutputForm)
    s := cycleEntry x
    while Qneq(x, s) repeat
      y := concat((first x)::OutputForm, y)
      x := rest x
    y := reverse_! y
    empty? s => bracket y
    -- cyclic case: z is cyclic part
    z := list((first x)::OutputForm)
    while Qneq(s, rest x) repeat
      x := rest x
      z := concat((first x)::OutputForm, z)
    bracket concat_!(y, overbar commaSeparate reverse_! z)

x = y ==
  Qeq(x,y) => true
  while not Qnull x and not Qnull y repeat
    Qfirst x ^=$S Qfirst y => return false
    x := Qrest x
    y := Qrest y
  Qnull x and Qnull y

latex(x : %): String ==
  s : String := "\left["
  while not Qnull x repeat
    s := concat(s, latex(Qfirst x)$S)$String
    x := Qrest x
    if not Qnull x then s := concat(s, ", ")$String
  concat(s, " \right]")$String

```

```

member?(s,x) ==
  while not Qnull x repeat
    if s = Qfirst x then return true else x := Qrest x
  false

-- Lots of code from parts of AGGCAT, repeated here to
-- get faster compilation
concat_!(x:%,y:%) ==
  Qnull x =>
    Qnull y => x
    Qpush(first y,x)
    QRPLACD(x,rest y)$Lisp
  x
  z:=x
  while not Qnull Qrest z repeat
    z:=Qrest z
    QRPLACD(z,y)$Lisp
  x

-- Then a quicky:
if S has SetCategory then
  removeDuplicates_! l ==
    p := l
    while not Qnull p repeat
--      p := setrest_!(p, remove_!(#1 = Qfirst p, Qrest p))
-- far too expensive - builds closures etc.
    pp:=p
    f:S:=Qfirst p
    p:=Qrest p
    while not Qnull (pr:=Qrest pp) repeat
      if (Qfirst pr)@S = f then QRPLACD(pp,Qrest pr)$Lisp
      else pp:=pr
    l

-- then sorting
mergeSort: ((S, S) -> Boolean, %, Integer) -> %

sort_!(f, l)          == mergeSort(f, l, #1)

merge_!(f, p, q) ==
  Qnull p => q
  Qnull q => p
  Qeq(p, q) => error "cannot merge a list into itself"
  if f(Qfirst p, Qfirst q)
    then (r := t := p; p := Qrest p)
    else (r := t := q; q := Qrest q)
  while not Qnull p and not Qnull q repeat
    if f(Qfirst p, Qfirst q)
      then (QRPLACD(t, p)$Lisp; t := p; p := Qrest p)

```

```

      else (QRPLACD(t, q)$Lisp; t := q; q := Qrest q)
    QRPLACD(t, if Qnull p then q else p)$Lisp
  r

split_!(p, n) ==
  n < 1 => error "index out of range"
  p := rest(p, (n - 1)::NonNegativeInteger)
  q := Qrest p
  QRPLACD(p, NIL$Lisp)$Lisp
  q

mergeSort(f, p, n) ==
  if n = 2 and f(first rest p, first p) then p := reverse_! p
  n < 3 => p
  l := (n quo 2)::NonNegativeInteger
  q := split_!(p, l)
  p := mergeSort(f, p, l)
  q := mergeSort(f, q, n - l)
  merge_!(f, p, q)

```

---

— ILIST.dotabb —

```

"ILIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ILIST",
        shape=ellipse]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"ILIST" -> "STRING"

```

---

## 10.12 domain IMATRIX IndexedMatrix

— IndexedMatrix.input —

```

)set break resume
)sys rm -f IndexedMatrix.output
)spool IndexedMatrix.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show IndexedMatrix

```

```

--R IndexedMatrix(R: Ring,mnRow: Integer,mnCol: Integer) is a domain constructor
--R Abbreviation for IndexedMatrix is IMATRIX
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for IMATRIX
--R
--R----- Operations -----
--R ?? : (Integer,%) -> %           ?? : (% ,R) -> %
--R ?? : (R,%) -> %               ?? : (% ,%) -> %
--R +? : (% ,%) -> %             -? : % -> %
--R -? : (% ,%) -> %             antisymmetric? : % -> Boolean
--R copy : % -> %                 diagonal? : % -> Boolean
--R diagonalMatrix : List % -> %   diagonalMatrix : List R -> %
--R elt : (% ,Integer,Integer,R) -> R elt : (% ,Integer,Integer) -> R
--R empty : () -> %              empty? : % -> Boolean
--R eq? : (% ,%) -> Boolean       fill! : (% ,R) -> %
--R horizConcat : (% ,%) -> %    listOfLists : % -> List List R
--R map : (((R,R) -> R),% ,% ,R) -> % map : (((R,R) -> R),% ,%) -> %
--R map : ((R -> R),%) -> %       map! : ((R -> R),%) -> %
--R matrix : List List R -> %    maxColIndex : % -> Integer
--R maxRowIndex : % -> Integer   minColIndex : % -> Integer
--R minRowIndex : % -> Integer   ncols : % -> NonNegativeInteger
--R nrows : % -> NonNegativeInteger parts : % -> List R
--R qelt : (% ,Integer,Integer) -> R sample : () -> %
--R square? : % -> Boolean       squareTop : % -> %
--R symmetric? : % -> Boolean    transpose : % -> %
--R vertConcat : (% ,%) -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?? : (IndexedVector(R,mnCol),%) -> IndexedVector(R,mnCol)
--R ?? : (% ,IndexedVector(R,mnRow)) -> IndexedVector(R,mnRow)
--R ***? : (% ,Integer) -> % if R has FIELD
--R ***? : (% ,NonNegativeInteger) -> %
--R ?/? : (% ,R) -> % if R has FIELD
--R ==? : (% ,%) -> Boolean if R has SETCAT
--R any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : IndexedVector(R,mnRow) -> %
--R coerce : % -> OutputForm if R has SETCAT
--R column : (% ,Integer) -> IndexedVector(R,mnRow)
--R columnSpace : % -> List IndexedVector(R,mnRow) if R has EUCDOM
--R count : (R,%) -> NonNegativeInteger if $ has finiteAggregate and R has SETCAT
--R count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R determinant : % -> R if R has commutative *
--R elt : (% ,List Integer,List Integer) -> %
--R eval : (% ,List R,List R) -> % if R has EVALAB R and R has SETCAT
--R eval : (% ,R,R) -> % if R has EVALAB R and R has SETCAT
--R eval : (% ,Equation R) -> % if R has EVALAB R and R has SETCAT
--R eval : (% ,List Equation R) -> % if R has EVALAB R and R has SETCAT
--R every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R exquo : (% ,R) -> Union(% ,"failed") if R has INTDOM
--R hash : % -> SingleInteger if R has SETCAT
--R inverse : % -> Union(% ,"failed") if R has FIELD

```

```

--R latex : % -> String if R has SETCAT
--R less? : (% , NonNegativeInteger) -> Boolean
--R member? : (R, %) -> Boolean if $ has finiteAggregate and R has SETCAT
--R members : % -> List R if $ has finiteAggregate
--R minordet : % -> R if R has commutative *
--R more? : (% , NonNegativeInteger) -> Boolean
--R new : (NonNegativeInteger, NonNegativeInteger, R) -> %
--R nullSpace : % -> List IndexedVector(R, mnRow) if R has INTDOM
--R nullity : % -> NonNegativeInteger if R has INTDOM
--R pfaffian : % -> R if R has COMRING
--R qsetelt! : (% , Integer, Integer, R) -> R
--R rank : % -> NonNegativeInteger if R has INTDOM
--R row : (% , Integer) -> IndexedVector(R, mnCol)
--R rowEchelon : % -> % if R has EUCDOM
--R scalarMatrix : (NonNegativeInteger, R) -> %
--R setColumn! : (% , Integer, IndexedVector(R, mnRow)) -> %
--R setRow! : (% , Integer, IndexedVector(R, mnCol)) -> %
--R setelt : (% , List Integer, List Integer, %) -> %
--R setelt : (% , Integer, Integer, R) -> R
--R setsubMatrix! : (% , Integer, Integer, %) -> %
--R size? : (% , NonNegativeInteger) -> Boolean
--R subMatrix : (% , Integer, Integer, Integer, Integer) -> %
--R swapColumns! : (% , Integer, Integer) -> %
--R swapRows! : (% , Integer, Integer) -> %
--R transpose : IndexedVector(R, mnCol) -> %
--R zero : (NonNegativeInteger, NonNegativeInteger) -> %
--R ?~=? : (% , %) -> Boolean if R has SETCAT
--R
--E 1

```

```

)spool
)lisp (bye)

```

---

— IndexedMatrix.help —

```

=====
IndexedMatrix examples
=====

```

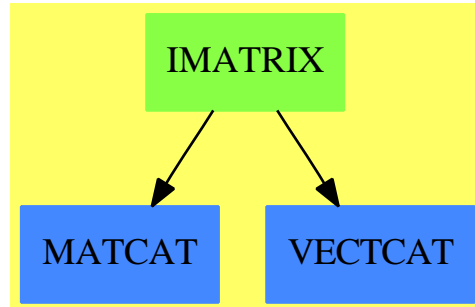
```

See Also:
o )show IndexedMatrix

```



### 10.12.1 IndexedMatrix (IMATRIX)



See

- ⇒ “Matrix” (MATRIX) 14.7.1 on page 1586
- ⇒ “RectangularMatrix” (RMATRIX) 19.4.1 on page 2205
- ⇒ “SquareMatrix” (SQMATRIX) 20.27.1 on page 2505

#### Exports:

|               |                |             |                |            |
|---------------|----------------|-------------|----------------|------------|
| any?          | antisymmetric? | coerce      | column         | copy       |
| count         | determinant    | diagonal?   | diagonalMatrix | elt        |
| empty         | empty?         | eq?         | eval           | every?     |
| exquo         | fill!          | hash        | horizConcat    | inverse    |
| latex         | less?          | listOfLists | map            | map!       |
| matrix        | maxColIndex    | maxRowIndex | member?        | members    |
| minColIndex   | minordet       | minRowIndex | more?          | ncols      |
| new           | nrows          | nullSpace   | nullity        | parts      |
| qelt          | qsetelt!       | rank        | row            | rowEchelon |
| sample        | scalarMatrix   | setColumn!  | setRow!        | setelt     |
| setsubMatrix! | size?          | square?     | squareTop      | subMatrix  |
| swapColumns!  | swapRows!      | symmetric?  | transpose      | vertConcat |
| zero          | #?             | ?*?         | ***?           | ?/?        |
| ?=?           | ?~=?           | ?+?         | -?             | ?-?        |

— domain IMATRIX IndexedMatrix —

```

)abbrev domain IMATRIX IndexedMatrix
++ Author: Grabmeier, Gschnitzer, Williamson
++ Date Created: 1987
++ Date Last Updated: July 1990
++ Basic Operations:
++ Related Domains: Matrix, RectangularMatrix, SquareMatrix,
++   StorageEfficientMatrixOperations
++ Also See:
++ AMS Classifications:
++ Keywords: matrix, linear algebra
++ Examples:
++ References:
  
```

```

++ Description:
++ An \spad{IndexedMatrix} is a matrix where the minimal row and column
++ indices are parameters of the type. The domains Row and Col
++ are both IndexedVectors.
++ The index of the 'first' row may be obtained by calling the
++ function \spadfun{minRowIndex}. The index of the 'first' column may
++ be obtained by calling the function \spadfun{minColIndex}. The index of
++ the first element of a 'Row' is the same as the index of the
++ first column in a matrix and vice versa.

```

```

IndexedMatrix(R,mnRow,mnCol): Exports == Implementation where

```

```

  R : Ring
  mnRow, mnCol : Integer
  Row ==> IndexedVector(R,mnCol)
  Col ==> IndexedVector(R,mnRow)
  MATLIN ==> MatrixLinearAlgebraFunctions(R,Row,Col,$)

```

```

Exports ==> MatrixCategory(R,Row,Col)

```

```

Implementation ==>

```

```

  InnerIndexedTwoDimensionalArray(R,mnRow,mnCol,Row,Col) add

```

```

  swapRows_!(x,i1,i2) ==
    (i1 < minRowIndex(x)) or (i1 > maxRowIndex(x)) or _
    (i2 < minRowIndex(x)) or (i2 > maxRowIndex(x)) =>
      error "swapRows!: index out of range"
    i1 = i2 => x
    minRow := minRowIndex x
    xx := x pretend PrimitiveArray(PrimitiveArray(R))
    n1 := i1 - minRow; n2 := i2 - minRow
    row1 := qelt(xx,n1)
    qsetelt_!(xx,n1,qelt(xx,n2))
    qsetelt_!(xx,n2,row1)
    xx pretend $

```

```

  if R has commutative("*") then

```

```

    determinant x == determinant(x)$MATLIN
    minordet    x == minordet(x)$MATLIN

```

```

  if R has EuclideanDomain then

```

```

    rowEchelon x == rowEchelon(x)$MATLIN

```

```

  if R has IntegralDomain then

```

```

    rank      x == rank(x)$MATLIN
    nullity   x == nullity(x)$MATLIN
    nullSpace x == nullSpace(x)$MATLIN

```

if R has Field then

```
inverse      x == inverse(x)$MATLIN
```

---

— IMATRIX.dotabb —

```
"IMATRIX" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IMATRIX"]
"MATCAT"  [color="#4488FF",href="bookvol10.2.pdf#nameddest=MATCAT"]
"VECTCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=VECTCAT"]
"IMATRIX" -> "MATCAT"
"IMATRIX" -> "VECTCAT"
```

---

## 10.13 domain IARRAY1 IndexedOneDimensionalArray

— IndexedOneDimensionalArray.input —

```
)set break resume
)sys rm -f IndexedOneDimensionalArray.output
)spool IndexedOneDimensionalArray.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show IndexedOneDimensionalArray
--R IndexedOneDimensionalArray(S: Type,mn: Integer) is a domain constructor
--R Abbreviation for IndexedOneDimensionalArray is IARRAY1
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for IARRAY1
--R
--R----- Operations -----
--R concat : List % -> %                concat : (%,% ) -> %
--R concat : (S,% ) -> %                concat : (% ,S) -> %
--R construct : List S -> %             copy : % -> %
--R delete : (% ,Integer) -> %          ?.? : (% ,Integer) -> S
--R elt : (% ,Integer,S) -> S           empty : () -> %
--R empty? : % -> Boolean               entries : % -> List S
--R eq? : (% ,%) -> Boolean             index? : (Integer,% ) -> Boolean
--R indices : % -> List Integer         insert : (% ,% ,Integer) -> %
--R insert : (S,% ,Integer) -> %       map : (((S,S) -> S),% ,%) -> %
```

```

--R map : ((S -> S),%) -> %                                new : (NonNegativeInteger,S) -> %
--R qelt : (%,Integer) -> S                                reverse : % -> %
--R sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?<? : (%,%) -> Boolean if S has ORDSET
--R ?<=? : (%,%) -> Boolean if S has ORDSET
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R ?>? : (%,%) -> Boolean if S has ORDSET
--R ?>=? : (%,%) -> Boolean if S has ORDSET
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R convert : % -> InputForm if S has KONVERT INFORM
--R copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
--R count : (S,%) -> NonNegativeInteger if $ has finiteAggregate and S has SETCAT
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R delete : (%,UniversalSegment Integer) -> %
--R ?.? : (%,UniversalSegment Integer) -> %
--R entry? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,S) -> % if $ has shallowlyMutable
--R find : ((S -> Boolean),%) -> Union(S,"failed")
--R first : % -> S if Integer has ORDSET
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R max : (%,%) -> % if S has ORDSET
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R members : % -> List S if $ has finiteAggregate
--R merge : (%,%) -> % if S has ORDSET
--R merge : (((S,S) -> Boolean),%,%) -> %
--R min : (%,%) -> % if S has ORDSET
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List S if $ has finiteAggregate
--R position : (S,%,Integer) -> Integer if S has SETCAT
--R position : (S,%) -> Integer if S has SETCAT
--R position : ((S -> Boolean),%) -> Integer
--R qsetelt! : (%,Integer,S) -> S if $ has shallowlyMutable
--R reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S,S) -> S if $ has finiteAggregate and S has SETCAT
--R remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (S,%) -> % if $ has finiteAggregate and S has SETCAT
--R removeDuplicates : % -> % if $ has finiteAggregate and S has SETCAT

```

```

--R reverse! : % -> % if $ has shallowlyMutable
--R select : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R setelt : (%,UniversalSegment Integer,S) -> S if $ has shallowlyMutable
--R setelt : (%,Integer,S) -> S if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : % -> % if S has ORDSET
--R sort : (((S,S) -> Boolean),%) -> %
--R sort! : % -> % if $ has shallowlyMutable and S has ORDSET
--R sort! : (((S,S) -> Boolean),%) -> % if $ has shallowlyMutable
--R sorted? : % -> Boolean if S has ORDSET
--R sorted? : (((S,S) -> Boolean),%) -> Boolean
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R ~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)

```

---

— IndexedOneDimensionalArray.help —

```

=====
IndexedOneDimensionalArray examples
=====

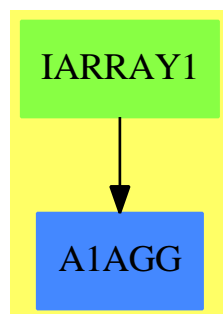
```

```

See Also:
o )show IndexedOneDimensionalArray

```

### 10.13.1 IndexedOneDimensionalArray (IARRAY1)



See

- ⇒ “PrimitiveArray” (PRIMARR) 17.30.1 on page 2069
- ⇒ “Tuple” (TUPLE) 21.12.1 on page 2711
- ⇒ “IndexedFlexibleArray” (IFARRAY) 10.10.1 on page 1187
- ⇒ “FlexibleArray” (FARRAY) 7.14.1 on page 853
- ⇒ “OneDimensionalArray” (ARRAY1) 16.3.1 on page 1736

**Exports:**

|          |           |           |                  |          |
|----------|-----------|-----------|------------------|----------|
| concat   | construct | copy      | delete           | elt      |
| empty    | empty?    | entries   | eq?              | index?   |
| indices  | insert    | insert    | map              | map      |
| new      | qelt      | reverse   | sample           | any?     |
| coerce   | convert   | copyInto! | count            | count    |
| delete   | entry?    | eval      | eval             | eval     |
| eval     | every?    | fill!     | find             | first    |
| hash     | latex     | less?     | map!             | max      |
| maxIndex | member?   | members   | merge            | merge    |
| min      | minIndex  | more?     | parts            | position |
| position | position  | qsetelt!  | reduce           | reduce   |
| reduce   | remove    | remove    | removeDuplicates | reverse! |
| select   | setelt    | setelt    | size?            | sort     |
| sort     | sort!     | sort!     | sorted?          | sorted?  |
| swap!    | #?        | ?<?       | ?<=?             | ?=?      |
| ?>?      | ?>=?      | ?~=?      | ?..?             |          |

— domain IARRAY1 IndexedOneDimensionalArray —

```
)abbrev domain IARRAY1 IndexedOneDimensionalArray
++ Author Micheal Monagan Aug/87
++ Description:
++ This is the basic one dimensional array data type.
```

```
IndexedOneDimensionalArray(S:Type, mn:Integer):
  OneDimensionalArrayAggregate S == add
    Qmax ==> QVMAXINDEX$Lisp
    Qsize ==> QVSIZE$Lisp
  -- Qelt ==> QVELT$Lisp
  -- Qsetelt ==> QSETVELT$Lisp
    Qelt ==> ELT$Lisp
    Qsetelt ==> SETELT$Lisp
  -- Qelt1 ==> QVELT_-1$Lisp
  -- Qsetelt1 ==> QSETVELT_-1$Lisp
    Qnew ==> MAKE_-ARRAY$Lisp
    I ==> Integer

  #x == Qsize x
  fill_!(x, s) == (for i in 0..Qmax x repeat Qsetelt(x, i, s); x)
  minIndex x == mn

  empty() == Qnew(0$Lisp)
```

```

new(n, s)          == fill_!(Qnew n,s)

map_!(f, s1) ==
  n:Integer := Qmax(s1)
  n < 0 => s1
  for i in 0..n repeat Qsetelt(s1, i, f(Qelt(s1,i)))
  s1

map(f, s1) ==
  n:Integer := Qmax(s1)
  n < 0 => s1
  ss2:% := Qnew(n+1)
  for i in 0..n repeat Qsetelt(ss2, i, f(Qelt(s1,i)))
  ss2

map(f, a, b) ==
  maxind:Integer := min(Qmax a, Qmax b)
  maxind < 0 => empty()
  c:% := Qnew(maxind+1)
  for i in 0..maxind repeat
    Qsetelt(c, i, f(Qelt(a,i),Qelt(b,i)))
  c

if zero? mn then
  qelt(x, i) == Qelt(x, i)
  qsetelt_!(x, i, s) == Qsetelt(x, i, s)

elt(x:%, i:I) ==
  negative? i or i > maxIndex(x) => error "index out of range"
  qelt(x, i)

setelt(x:%, i:I, s:S) ==
  negative? i or i > maxIndex(x) => error "index out of range"
  qsetelt_!(x, i, s)

-- else if one? mn then
else if (mn = 1) then
  maxIndex x == Qsize x
  qelt(x, i) == Qelt(x, i-1)
  qsetelt_!(x, i, s) == Qsetelt(x, i-1, s)

elt(x:%, i:I) ==
  QSLESSP(i,1$Lisp)$Lisp or QSLESSP(Qsize x,i)$Lisp =>
    error "index out of range"
  Qelt(x, i-1)

setelt(x:%, i:I, s:S) ==
  QSLESSP(i,1$Lisp)$Lisp or QSLESSP(Qsize x,i)$Lisp =>
    error "index out of range"
  Qsetelt(x, i-1, s)

```

```

else
  qelt(x, i)      == Qelt(x, i - mn)
  qsetelt_!(x, i, s) == Qsetelt(x, i - mn, s)

  elt(x:%, i:I) ==
    i < mn or i > maxIndex(x) => error "index out of range"
    qelt(x, i)

  setelt(x:%, i:I, s:S) ==
    i < mn or i > maxIndex(x) => error "index out of range"
    qsetelt_!(x, i, s)

```

---

— IARRAY1.dotabb —

```

"IARRAY1" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IARRAY1"]
"A1AGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=A1AGG"]
"IARRAY1" -> "A1AGG"

```

---

## 10.14 domain ISTRING IndexedString

---

— IndexedString.input —

```

)set break resume
)sys rm -f IndexedString.output
)spool IndexedString.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show IndexedString
--R IndexedString mn: Integer is a domain constructor
--R Abbreviation for IndexedString is ISTRING
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ISTRING
--R
--R----- Operations -----
--R coerce : Character -> %          concat : List % -> %
--R concat : (%,% ) -> %            concat : (Character,%) -> %

```



```

--R concat : (% ,Character) -> %
--R copy : % -> %
--R ?.? : (% ,%) -> %
--R empty : () -> %
--R entries : % -> List Character
--R hash : % -> Integer
--R indices : % -> List Integer
--R leftTrim : (% ,Character) -> %
--R lowerCase! : % -> %
--R qelt : (% ,Integer) -> Character
--R rightTrim : (% ,Character) -> %
--R split : (% ,Character) -> List %
--R trim : (% ,CharacterClass) -> %
--R upperCase : % -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?<? : (% ,%) -> Boolean if Character has ORDSET
--R ?<=? : (% ,%) -> Boolean if Character has ORDSET
--R ?=? : (% ,%) -> Boolean if Character has SETCAT
--R ?>? : (% ,%) -> Boolean if Character has ORDSET
--R ?>=? : (% ,%) -> Boolean if Character has ORDSET
--R any? : ((Character -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if Character has SETCAT
--R convert : % -> InputForm if Character has KONVERT INFORM
--R copyInto! : (% ,%,Integer) -> % if $ has shallowlyMutable
--R count : (Character,%) -> NonNegativeInteger if $ has finiteAggregate and Character has SI
--R count : ((Character -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R delete : (% ,UniversalSegment Integer) -> %
--R ?.? : (% ,UniversalSegment Integer) -> %
--R elt : (% ,Integer,Character) -> Character
--R entry? : (Character,%) -> Boolean if $ has finiteAggregate and Character has SETCAT
--R eval : (% ,List Character,List Character) -> % if Character has EVALAB CHAR and Character
--R eval : (% ,Character,Character) -> % if Character has EVALAB CHAR and Character has SETCAT
--R eval : (% ,Equation Character) -> % if Character has EVALAB CHAR and Character has SETCAT
--R eval : (% ,List Equation Character) -> % if Character has EVALAB CHAR and Character has SI
--R every? : ((Character -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (% ,Character) -> % if $ has shallowlyMutable
--R find : ((Character -> Boolean),%) -> Union(Character,"failed")
--R first : % -> Character if Integer has ORDSET
--R hash : % -> SingleInteger if Character has SETCAT
--R insert : (Character,%,Integer) -> %
--R latex : % -> String if Character has SETCAT
--R leftTrim : (% ,CharacterClass) -> %
--R less? : (% ,NonNegativeInteger) -> Boolean
--R map : (((Character,Character) -> Character),%,%) -> %
--R map : ((Character -> Character),%) -> %
--R map! : ((Character -> Character),%) -> % if $ has shallowlyMutable
--R match : (% ,%,Character) -> NonNegativeInteger
--R match? : (% ,%,Character) -> Boolean
--R max : (% ,%) -> % if Character has ORDSET
--R maxIndex : % -> Integer if Integer has ORDSET
--R construct : List Character -> %
--R delete : (% ,Integer) -> %
--R ?.? : (% ,Integer) -> Character
--R empty? : % -> Boolean
--R eq? : (% ,%) -> Boolean
--R index? : (Integer,%) -> Boolean
--R insert : (% ,%,Integer) -> %
--R lowerCase : % -> %
--R prefix? : (% ,%) -> Boolean
--R reverse : % -> %
--R sample : () -> %
--R suffix? : (% ,%) -> Boolean
--R trim : (% ,Character) -> %
--R upperCase! : % -> %

```

```

--R member? : (Character,%) -> Boolean if $ has finiteAggregate and Character has SETCAT
--R members : % -> List Character if $ has finiteAggregate
--R merge : (%,%) -> % if Character has ORDSET
--R merge : (((Character,Character) -> Boolean),%,%) -> %
--R min : (%,%) -> % if Character has ORDSET
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R new : (NonNegativeInteger,Character) -> %
--R parts : % -> List Character if $ has finiteAggregate
--R position : (CharacterClass,%,Integer) -> Integer
--R position : (%,%,Integer) -> Integer
--R position : (Character,%,Integer) -> Integer if Character has SETCAT
--R position : (Character,%) -> Integer if Character has SETCAT
--R position : ((Character -> Boolean),%) -> Integer
--R qsetelt! : (%,Integer,Character) -> Character if $ has shallowlyMutable
--R reduce : (((Character,Character) -> Character),%) -> Character if $ has finiteAggregate
--R reduce : (((Character,Character) -> Character),%,Character) -> Character if $ has finiteAggregate
--R reduce : (((Character,Character) -> Character),%,Character,Character) -> Character if $ has finiteAg
--R remove : ((Character -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (Character,%) -> % if $ has finiteAggregate and Character has SETCAT
--R removeDuplicates : % -> % if $ has finiteAggregate and Character has SETCAT
--R replace : (%,UniversalSegment Integer,%) -> %
--R reverse! : % -> % if $ has shallowlyMutable
--R rightTrim : (%,CharacterClass) -> %
--R select : ((Character -> Boolean),%) -> % if $ has finiteAggregate
--R setelt : (%,UniversalSegment Integer,Character) -> Character if $ has shallowlyMutable
--R setelt : (%,Integer,Character) -> Character if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : % -> % if Character has ORDSET
--R sort : (((Character,Character) -> Boolean),%) -> %
--R sort! : % -> % if $ has shallowlyMutable and Character has ORDSET
--R sort! : (((Character,Character) -> Boolean),%) -> % if $ has shallowlyMutable
--R sorted? : % -> Boolean if Character has ORDSET
--R sorted? : (((Character,Character) -> Boolean),%) -> Boolean
--R split : (%,CharacterClass) -> List %
--R substring? : (%,%,Integer) -> Boolean
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R ?~=? : (%,%) -> Boolean if Character has SETCAT
--R
--E 1

```

```

)spool
)lisp (bye)

```

---

— IndexedString.help —

=====

IndexedString examples

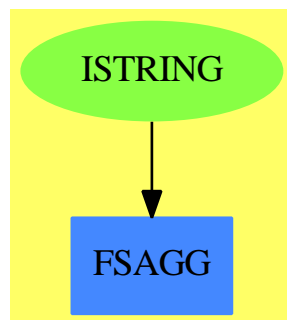
=====

See Also:

o )show IndexedString

—————

### 10.14.1 IndexedString (ISTRING)



See

⇒ “Character” (CHAR) 4.3.1 on page 357

⇒ “CharacterClass” (CCLASS) 4.4.1 on page 365

⇒ “String” (STRING) 20.31.1 on page 2565

#### Exports:

|          |           |            |            |                  |
|----------|-----------|------------|------------|------------------|
| any?     | coerce    | concat     | construct  | convert          |
| copy     | copyInto! | count      | delete     | elt              |
| empty    | empty?    | entries    | entry?     | eq?              |
| eval     | every?    | fill!      | find       | first            |
| hash     | index?    | indices    | insert     | latex            |
| leftTrim | less?     | lowerCase  | lowerCase! | map              |
| map!     | match     | match?     | max        | maxIndex         |
| member?  | members   | merge      | min        | minIndex         |
| more?    | new       | parts      | prefix?    | position         |
| qelt     | qsetelt!  | reduce     | remove     | removeDuplicates |
| replace  | reverse   | reverse!   | rightTrim  | sample           |
| select   | setelt    | size?      | sort       | sort!            |
| sorted?  | split     | suffix?    | substring? | swap!            |
| trim     | upperCase | upperCase! | #?         | ?<?              |
| ?<=?     | ?=?       | ?>?        | ?>=?       | ?~=?             |
| ??       |           |            |            |                  |

— domain ISTRING IndexedString —

```

)abbrev domain ISTRING IndexedString
++ Authors: Stephen Watt, Michael Monagan, Manuel Bronstein 1986 .. 1991
-- The following Lisp dependencies are divided into two groups
-- Those that are required
-- QENUM QESSET QCSIZE MAKE-FULL-CVEC EQ QSLESSP QSGREATERP
-- Those that can be included for efficiency only
-- COPY STRCONC SUBSTRING STRPOS RPLACSTR DOWNCASE UPCASE CGREATERP
++ Description:
++ This domain implements low-level strings

IndexedString(mn:Integer): Export == Implementation where
  B ==> Boolean
  C ==> Character
  I ==> Integer
  N ==> NonNegativeInteger
  U ==> UniversalSegment Integer

  Export ==> StringAggregate() with
    hash: % -> I
    ++ hash(x) provides a hashing function for strings

  Implementation ==> add
    -- These assume Character's Rep is Small I
    Qelt ==> QENUM$Lisp
    Qequal ==> EQUAL$Lisp
    Qsetelt ==> QESSET$Lisp
    Qsize ==> QCSIZE$Lisp
    Cheq ==> EQL$Lisp
    Chlt ==> QSLESSP$Lisp
    Chgt ==> QSGREATERP$Lisp

    c: Character
    cc: CharacterClass

-- new n == MAKE_-FULL_-CVEC(n, space$C)$Lisp
new(n, c) == MAKE_-FULL_-CVEC(n, c)$Lisp
empty() == MAKE_-FULL_-CVEC(0$Lisp)$Lisp
empty?(s) == Qsize(s) = 0
#s == Qsize(s)
s = t == Qequal(s, t)
s < t == CGREATERP(t,s)$Lisp
concat(s:%,t:%) == STRCONC(s,t)$Lisp
copy s == COPY_-SEQ(s)$Lisp
insert(s:%, t:%, i:I) == concat(concat(s(mn..i-1), t), s(i..))
coerce(s:%)::OutputForm == outputForm(s pretend String)
minIndex s == mn
upperCase_! s == map_!(upperCase, s)
lowerCase_! s == map_!(lowerCase, s)

latex s == concat("\mbox{'", concat(s pretend String, "'}"))

```

```

replace(s, sg, t) ==
  l := lo(sg) - mn
  m := #s
  n := #t
  h:I := if hasHi sg then hi(sg) - mn else maxIndex s - mn
  l < 0 or h >= m or h < l-1 => error "index out of range"
  r := new((m-(h-l+1)+n)::N, space$C)
  for k in 0.. for i in 0..l-1 repeat Qsetelt(r, k, Qelt(s, i))
  for k in k.. for i in 0..n-1 repeat Qsetelt(r, k, Qelt(t, i))
  for k in k.. for i in h+1..m-1 repeat Qsetelt(r, k, Qelt(s, i))
  r

setelt(s:%, i:I, c:C) ==
  i < mn or i > maxIndex(s) => error "index out of range"
  Qsetelt(s, i - mn, c)
  c

substring?(part, whole, startpos) ==
  np:I := Qsize part
  nw:I := Qsize whole
  (startpos := startpos - mn) < 0 => error "index out of bounds"
  np > nw - startpos => false
  for ip in 0..np-1 for iw in startpos.. repeat
    not Cheq(Qelt(part, ip), Qelt(whole, iw)) => return false
  true

position(s:%, t:%, startpos:I) ==
  (startpos := startpos - mn) < 0 => error "index out of bounds"
  startpos >= Qsize t => mn - 1
  r:I := STRPOS(s, t, startpos, NIL$Lisp)$Lisp
  EQ(r, NIL$Lisp)$Lisp => mn - 1
  r + mn

position(c: Character, t: %, startpos: I) ==
  (startpos := startpos - mn) < 0 => error "index out of bounds"
  startpos >= Qsize t => mn - 1
  for r in startpos..Qsize t - 1 repeat
    if Cheq(Qelt(t, r), c) then return r + mn
  mn - 1

position(cc: CharacterClass, t: %, startpos: I) ==
  (startpos := startpos - mn) < 0 => error "index out of bounds"
  startpos >= Qsize t => mn - 1
  for r in startpos..Qsize t - 1 repeat
    if member?(Qelt(t,r), cc) then return r + mn
  mn - 1

suffix?(s, t) ==
  (m := maxIndex s) > (n := maxIndex t) => false

```

```

substring?(s, t, mn + n - m)

split(s, c) ==
  n := maxIndex s
  for i in mn..n while s.i = c repeat 0
  l := empty()$List(%)
  j:Integer -- j is conditionally intialized
  while i <= n and (j := position(c, s, i)) >= mn repeat
    l := concat(s(i..j-1), l)
    for i in j..n while s.i = c repeat 0
  if i <= n then l := concat(s(i..n), l)
  reverse_! l

split(s, cc) ==
  n := maxIndex s
  for i in mn..n while member?(s.i,cc) repeat 0
  l := empty()$List(%)
  j:Integer -- j is conditionally intialized
  while i <= n and (j := position(cc, s, i)) >= mn repeat
    l := concat(s(i..j-1), l)
    for i in j..n while member?(s.i,cc) repeat 0
  if i <= n then l := concat(s(i..n), l)
  reverse_! l

leftTrim(s, c) ==
  n := maxIndex s
  for i in mn .. n while s.i = c repeat 0
  s(i..n)

leftTrim(s, cc) ==
  n := maxIndex s
  for i in mn .. n while member?(s.i,cc) repeat 0
  s(i..n)

rightTrim(s, c) ==
  for j in maxIndex s .. mn by -1 while s.j = c repeat 0
  s(minIndex(s)..j)

rightTrim(s, cc) ==
  for j in maxIndex s .. mn by -1 while member?(s.j, cc) repeat 0
  s(minIndex(s)..j)

concat l ==
  t := new(+/#s for s in l, space$C)
  i := mn
  for s in l repeat
    copyInto_!(t, s, i)
    i := i + #s
  t

```

```

copyInto_!(y, x, s) ==
  m := #x
  n := #y
  s := s - mn
  s < 0 or s+m > n => error "index out of range"
  RPLACSTR(y, s, m, x, 0, m)$Lisp
  y

elt(s:%, i:I) ==
  i < mn or i > maxIndex(s) => error "index out of range"
  Qelt(s, i - mn)

elt(s:%, sg:U) ==
  l := lo(sg) - mn
  h := if hasHi sg then hi(sg) - mn else maxIndex s - mn
  l < 0 or h >= #s => error "index out of bound"
  SUBSTRING(s, l, max(0, h-l+1))$Lisp

hash(s:$):Integer ==
  n:I := Qsize s
  zero? n => 0
--   one? n => ord(s.mn)
  (n = 1) => ord(s.mn)
  ord(s.mn) * ord s(mn+n-1) * ord s(mn + n quo 2)

match(pattern,target,wildcard) ==
  stringMatch(pattern,target,CHARACTER(wildcard)$Lisp)$Lisp

match?(pattern, target, dontcare) ==
  n := maxIndex pattern
  p := position(dontcare, pattern, m := minIndex pattern)::N
  p = m-1 => pattern = target
  (p ^= m) and not prefix?(pattern(m..p-1), target) => false
  i := p      -- index into target
  q := position(dontcare, pattern, p + 1)::N
  while q ^= m-1 repeat
    s := pattern(p+1..q-1)
    i := position(s, target, i)::N
    i = m-1 => return false
    i := i + #s
    p := q
    q := position(dontcare, pattern, q + 1)::N
  (p ^= n) and not suffix?(pattern(p+1..n), target) => false
  true

```

```
"ISTRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ISTRING",
           shape=ellipse]
"FSAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FSAGG"]
"ISTRING" -> "FSAGG"
```

## 10.15 domain IARRAY2 IndexedTwoDimensionalArray

An IndexedTwoDimensionalArray is a 2-dimensional array where the minimal row and column indices are parameters of the type. Rows and columns are returned as IndexedOneDimensionalArray's with minimal indices matching those of the IndexedTwoDimensionalArray. The index of the 'first' row may be obtained by calling the function 'minRowIndex'. The index of the 'first' column may be obtained by calling the function 'minColIndex'. The index of the first element of a 'Row' is the same as the index of the first column in an array and vice versa.

### — IndexedTwoDimensionalArray.input —

```
)set break resume
)sys rm -f IndexedTwoDimensionalArray.output
)spool IndexedTwoDimensionalArray.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show IndexedTwoDimensionalArray
--R IndexedTwoDimensionalArray(R: Type,mnRow: Integer,mnCol: Integer) is a domain constructor
--R Abbreviation for IndexedTwoDimensionalArray is IARRAY2
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for IARRAY2
--R
--R----- Operations -----
--R copy : % -> %                elt : (%,Integer,Integer,R) -> R
--R elt : (%,Integer,Integer) -> R    empty : () -> %
--R empty? : % -> Boolean          eq? : (%,% ) -> Boolean
--R fill! : (% ,R) -> %            map : ((R,R) -> R),%,%,R) -> %
--R map : ((R,R) -> R),%,%,R) -> %    map : ((R -> R),%) -> %
--R map! : ((R -> R),%) -> %        maxColIndex : % -> Integer
--R maxRowIndex : % -> Integer      minColIndex : % -> Integer
--R minRowIndex : % -> Integer      ncols : % -> NonNegativeInteger
--R nrows : % -> NonNegativeInteger parts : % -> List R
--R qelt : (% ,Integer,Integer) -> R sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?? : (% ,%) -> Boolean if R has SETCAT
```



```

--R any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if R has SETCAT
--R column : (%,Integer) -> IndexedOneDimensionalArray(R,mnRow)
--R count : (R,%) -> NonNegativeInteger if $ has finiteAggregate and R has SETCAT
--R count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List R,List R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,R,R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,Equation R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,List Equation R) -> % if R has EVALAB R and R has SETCAT
--R every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if R has SETCAT
--R latex : % -> String if R has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R member? : (R,%) -> Boolean if $ has finiteAggregate and R has SETCAT
--R members : % -> List R if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R new : (NonNegativeInteger,NonNegativeInteger,R) -> %
--R qsetelt! : (%,Integer,Integer,R) -> R
--R row : (%,Integer) -> IndexedOneDimensionalArray(R,mnCol)
--R setColumn! : (%,Integer,IndexedOneDimensionalArray(R,mnRow)) -> %
--R setRow! : (%,Integer,IndexedOneDimensionalArray(R,mnCol)) -> %
--R setelt : (%,Integer,Integer,R) -> R
--R size? : (%,NonNegativeInteger) -> Boolean
--R ~=? : (%,%) -> Boolean if R has SETCAT
--R
--E 1

)spool
)lisp (bye)

```

---

— IndexedTwoDimensionalArray.help —

```

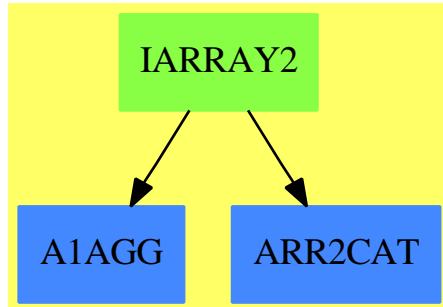
=====
IndexedTwoDimensionalArray examples
=====

```

See Also:

- o )show IndexedTwoDimensionalArray

## 10.15.1 IndexedTwoDimensionalArray (IARRAY2)



See

⇒ “InnerIndexedTwoDimensionalArray” (IIARRAY2) 10.23.1 on page 1254

⇒ “TwoDimensionalArray” (ARRAY2) 21.13.1 on page 2722

**Exports:**

|          |             |             |             |         |
|----------|-------------|-------------|-------------|---------|
| any?     | coerce      | column      | copy        | count   |
| count    | elt         | empty       | empty?      | eq?     |
| eval     | every?      | fill!       | hash        | latex   |
| less?    | maxColIndex | maxRowIndex | map         | map!    |
| member?  | members     | minColIndex | minRowIndex | more?   |
| ncols    | new         | nrows       | parts       | qelt    |
| qsetelt! | row         | sample      | setColumn!  | setRow! |
| setelt   | size?       | #?          | ?=?         | ?~=?    |

— domain IARRAY2 IndexedTwoDimensionalArray —

```

)abbrev domain IARRAY2 IndexedTwoDimensionalArray
++ Author: Mark Botch
++ Description:
++ This domain implements two dimensional arrays
  
```

```

IndexedTwoDimensionalArray(R,mnRow,mnCol):Exports == Implementation where
  R : Type
  mnRow, mnCol : Integer
  Row ==> IndexedOneDimensionalArray(R,mnCol)
  Col ==> IndexedOneDimensionalArray(R,mnRow)
  
```

```

Exports ==> TwoDimensionalArrayCategory(R,Row,Col)
  
```

```

Implementation ==>
  InnerIndexedTwoDimensionalArray(R,mnRow,mnCol,Row,Col)
  
```

—

## — IARRAY2.dotabb —

```

"IARRAY2" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IARRAY2"]
"A1AGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=A1AGG"]
"ARR2CAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ARR2CAT"]
"IARRAY2" -> "ARR2CAT"
"IARRAY2" -> "A1AGG"

```

## 10.16 domain IVECTOR IndexedVector

## — IndexedVector.input —

```

)set break resume
)sys rm -f IndexedVector.output
)spool IndexedVector.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show IndexedVector
--R IndexedVector(R: Type,mn: Integer) is a domain constructor
--R Abbreviation for IndexedVector is IVECTOR
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for IVECTOR
--R
--R----- Operations -----
--R -? : % -> % if R has ABELGRP          concat : List % -> %
--R concat : (%,%) -> %                  concat : (R,%) -> %
--R concat : (%,R) -> %                  construct : List R -> %
--R copy : % -> %                        delete : (%,Integer) -> %
--R ?.? : (%,Integer) -> R               elt : (%,Integer,R) -> R
--R empty : () -> %                      empty? : % -> Boolean
--R entries : % -> List R                 eq? : (%,%) -> Boolean
--R index? : (Integer,%) -> Boolean        indices : % -> List Integer
--R insert : (%,%,Integer) -> %           insert : (R,%,Integer) -> %
--R map : ((R,R) -> R),%,%) -> %         map : ((R -> R),%) -> %
--R new : (NonNegativeInteger,R) -> %     qelt : (%,Integer) -> R
--R reverse : % -> %                     sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?? : (%,R) -> % if R has MONOID
--R ?? : (R,%) -> % if R has MONOID
--R ?? : (Integer,%) -> % if R has ABELGRP

```

```

--R ?+? : (%,% ) -> % if R has ABELSG
--R ?-? : (%,% ) -> % if R has ABELGRP
--R ?<? : (%,% ) -> Boolean if R has ORDSET
--R ?<=? : (%,% ) -> Boolean if R has ORDSET
--R ?=? : (%,% ) -> Boolean if R has SETCAT
--R ?>? : (%,% ) -> Boolean if R has ORDSET
--R ?>=? : (%,% ) -> Boolean if R has ORDSET
--R any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if R has SETCAT
--R convert : % -> InputForm if R has KONVERT INFORM
--R copyInto! : (%,% ,Integer) -> % if $ has shallowlyMutable
--R count : (R,% ) -> NonNegativeInteger if $ has finiteAggregate and R has SETCAT
--R count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R cross : (%,% ) -> % if R has RING
--R delete : (% ,UniversalSegment Integer) -> %
--R dot : (%,% ) -> R if R has RING
--R ?.? : (% ,UniversalSegment Integer) -> %
--R entry? : (R,% ) -> Boolean if $ has finiteAggregate and R has SETCAT
--R eval : (% ,List R,List R) -> % if R has EVALAB R and R has SETCAT
--R eval : (% ,R,R) -> % if R has EVALAB R and R has SETCAT
--R eval : (% ,Equation R) -> % if R has EVALAB R and R has SETCAT
--R eval : (% ,List Equation R) -> % if R has EVALAB R and R has SETCAT
--R every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (% ,R) -> % if $ has shallowlyMutable
--R find : ((R -> Boolean),%) -> Union(R,"failed")
--R first : % -> R if Integer has ORDSET
--R hash : % -> SingleInteger if R has SETCAT
--R latex : % -> String if R has SETCAT
--R length : % -> R if R has RADCAT and R has RING
--R less? : (% ,NonNegativeInteger) -> Boolean
--R magnitude : % -> R if R has RADCAT and R has RING
--R map! : ((R -> R),%) -> % if $ has shallowlyMutable
--R max : (%,% ) -> % if R has ORDSET
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (R,% ) -> Boolean if $ has finiteAggregate and R has SETCAT
--R members : % -> List R if $ has finiteAggregate
--R merge : (%,% ) -> % if R has ORDSET
--R merge : (((R,R) -> Boolean),%,%) -> %
--R min : (%,% ) -> % if R has ORDSET
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (% ,NonNegativeInteger) -> Boolean
--R outerProduct : (%,% ) -> Matrix R if R has RING
--R parts : % -> List R if $ has finiteAggregate
--R position : (R,% ,Integer) -> Integer if R has SETCAT
--R position : (R,% ) -> Integer if R has SETCAT
--R position : ((R -> Boolean),%) -> Integer
--R qsetelt! : (% ,Integer,R) -> R if $ has shallowlyMutable
--R reduce : (((R,R) -> R),%) -> R if $ has finiteAggregate
--R reduce : (((R,R) -> R),%,R) -> R if $ has finiteAggregate
--R reduce : (((R,R) -> R),%,R,R) -> R if $ has finiteAggregate and R has SETCAT

```

```

--R remove : ((R -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (R,%) -> % if $ has finiteAggregate and R has SETCAT
--R removeDuplicates : % -> % if $ has finiteAggregate and R has SETCAT
--R reverse! : % -> % if $ has shallowlyMutable
--R select : ((R -> Boolean),%) -> % if $ has finiteAggregate
--R setelt : (%,UniversalSegment Integer,R) -> R if $ has shallowlyMutable
--R setelt : (%,Integer,R) -> R if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : % -> % if R has ORDSET
--R sort : ((R,R) -> Boolean),%) -> %
--R sort! : % -> % if $ has shallowlyMutable and R has ORDSET
--R sort! : ((R,R) -> Boolean),%) -> % if $ has shallowlyMutable
--R sorted? : % -> Boolean if R has ORDSET
--R sorted? : ((R,R) -> Boolean),%) -> Boolean
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R zero : NonNegativeInteger -> % if R has ABELMON
--R ~=? : (%,%) -> Boolean if R has SETCAT
--R
--E 1

)spool
)lisp (bye)

```

---

— IndexedVector.help —

```

=====
IndexedVector examples
=====

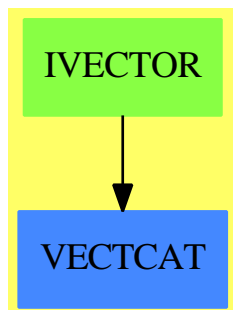
```

```

See Also:
o )show IndexedVector

```

## 10.16.1 IndexedVector (IVECTOR)

**Exports:**

|              |           |                  |           |           |
|--------------|-----------|------------------|-----------|-----------|
| any?         | coerce    | concat           | construct | convert   |
| copy         | copyInto! | count            | cross     | delete    |
| dot          | elt       | empty            | empty?    | entries   |
| entry?       | eq?       | eval             | every?    | fill!     |
| find         | first     | hash             | index?    | indices   |
| insert       | latex     | length           | less?     | magnitude |
| map!         | max       | maxIndex         | member?   | members   |
| merge        | min       | minIndex         | more?     | new       |
| outerProduct | parts     | position         | qelt      | qsetelt!  |
| reduce       | remove    | removeDuplicates | reverse   | reverse!  |
| sample       | select    | setelt           | size?     | sort      |
| sort!        | sorted?   | swap!            | zero      | #?        |
| ?*?          | ?+?       | ?-?              | ?<?       | ?<=?      |
| ?=?          | ?>?       | ?>=?             | ?~=?      | -?        |
| ?.?          |           |                  |           |           |

— domain IVECTOR IndexedVector —

```

)abbrev domain IVECTOR IndexedVector
++ Author: Mark Botch
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors: Vector, DirectProduct
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This type represents vector like objects with varying lengths
++ and a user-specified initial index.

```

```
IndexedVector(R:Type, mn:Integer):
```

```
VectorCategory R == IndexedOneDimensionalArray(R, mn)
```

---

— IVECTOR.dotabb —

```
"IVECTOR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IVECTOR"]
"VECTCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=VECTCAT"]
"IVECTOR" -> "VECTCAT"
```

---

## 10.17 domain ITUPLE InfiniteTuple

— InfiniteTuple.input —

```
)set break resume
)sys rm -f InfiniteTuple.output
)spool InfiniteTuple.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show InfiniteTuple
--R InfiniteTuple S: Type is a domain constructor
--R Abbreviation for InfiniteTuple is ITUPLE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ITUPLE
--R
--R----- Operations -----
--R coerce : % -> OutputForm          construct : % -> Stream S
--R generate : ((S -> S),S) -> %      map : ((S -> S),%) -> %
--R select : ((S -> Boolean),%) -> %
--R filterUntil : ((S -> Boolean),%) -> %
--R filterWhile : ((S -> Boolean),%) -> %
--R
--E 1

)spool
)lisp (bye)
```

---

## — InfiniteTuple.help —

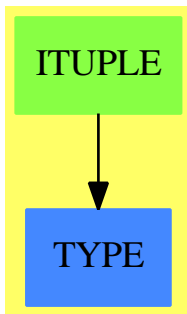
```
=====
InfiniteTuple examples
=====
```

See Also:

```
o )show InfiniteTuple
```

---

## 10.17.1 InfiniteTuple (ITUPLE)

**Exports:**

```
coerce construct filterUntil filterWhile generate map select
```

## — domain ITUPLE InfiniteTuple —

```
)abbrev domain ITUPLE InfiniteTuple
++ Author: Clifton J. Williamson
++ Date Created: 16 February 1990
++ Date Last Updated: 16 February 1990
++ Keywords:
++ Examples:
++ References:
++ Description:
++ This package implements 'infinite tuples' for the interpreter.
++ The representation is a stream.
```

```
InfiniteTuple(S:Type): Exports == Implementation where
```

```
Exports ==> CoercibleTo OutputForm with
  map: (S -> S, %) -> %
```



```

++ map(f,t) replaces the tuple t
++ by \spad{[f(x) for x in t]}.
filterWhile: (S -> Boolean, %) -> %
++ filterWhile(p,t) returns \spad{[x for x in t while p(x)]}.
filterUntil: (S -> Boolean, %) -> %
++ filterUntil(p,t) returns \spad{[x for x in t while not p(x)]}.
select: (S -> Boolean, %) -> %
++ select(p,t) returns \spad{[x for x in t | p(x)]}.
generate: (S -> S,S) -> %
++ generate(f,s) returns \spad{[s,f(s),f(f(s)),...]}.
construct: % -> Stream S
++ construct(t) converts an infinite tuple to a stream.

Implementation ==> Stream S add
generate(f,x) == generate(f,x)$Stream(S) pretend %
filterWhile(f, x) == filterWhile(f,x pretend Stream(S))$Stream(S) pretend %
filterUntil(f, x) == filterUntil(f,x pretend Stream(S))$Stream(S) pretend %
select(f, x) == select(f,x pretend Stream(S))$Stream(S) pretend %
construct x == x pretend Stream(S)
--      coerce x ==
--      coerce(x)$Stream(S)

```

---

— ITUPLE.dotabb —

```

"ITUPLE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ITUPLE"]
"TYPE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=TYPE"]
"ITUPLE" -> "TYPE"

```

---

## 10.18 domain INFCLSPT InfinitelyClosePoint

— InfinitelyClosePoint.input —

```

)set break resume
)sys rm -f InfinitelyClosePoint.output
)spool InfinitelyClosePoint.output
)set message test on
)set message auto off
)clear all

--S 1 of 1

```

```

)show InfinitelyClosePoint
--R InfinitelyClosePoint(K: Field,symb: List Symbol,PolyRing: PolynomialCategory(K,E,OrderedVariableList
--R Abbreviation for InfinitelyClosePoint is INFCLSPT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for INFCLSPT
--R
--R----- Operations -----
--R ?=? : (%,% ) -> Boolean          actualExtensionV : % -> K
--R chartV : % -> BLMET              coerce : % -> OutputForm
--R create : (ProjPt,PolyRing) -> %   degree : % -> PositiveInteger
--R excpDivV : % -> DIVISOR           fullOut : % -> OutputForm
--R fullOutput : () -> Boolean        fullOutput : Boolean -> Boolean
--R hash : % -> SingleInteger         latex : % -> String
--R localParamV : % -> List PCS       localPointV : % -> AffinePlane K
--R multV : % -> NonNegativeInteger   pointV : % -> ProjPt
--R setchart! : (% ,BLMET) -> BLMET   setpoint! : (% ,ProjPt) -> ProjPt
--R symbNameV : % -> Symbol           ?~=? : (%,% ) -> Boolean
--R create : (ProjPt,DistributedMultivariatePolynomial([construct,QUOTEX,QUOTEY],K),AffinePlane K,NonNeg
--R curveV : % -> DistributedMultivariatePolynomial([construct,QUOTEX,QUOTEY],K)
--R setcurve! : (% ,DistributedMultivariatePolynomial([construct,QUOTEX,QUOTEY],K)) -> DistributedMultiva
--R setexcpDiv! : (% ,DIVISOR) -> DIVISOR
--R setlocalParam! : (% ,List PCS) -> List PCS
--R setlocalPoint! : (% ,AffinePlane K) -> AffinePlane K
--R setmult! : (% ,NonNegativeInteger) -> NonNegativeInteger
--R setsubmult! : (% ,NonNegativeInteger) -> NonNegativeInteger
--R setsymbName! : (% ,Symbol) -> Symbol
--R subMultV : % -> NonNegativeInteger
--R
--E 1

)spool
)lisp (bye)

```

---

— InfinitelyClosePoint.help —

```

=====
InfinitelyClosePoint examples
=====

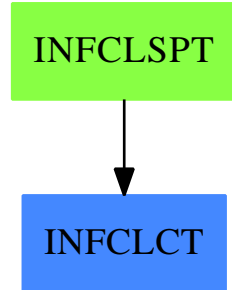
```

```

See Also:
o )show InfinitelyClosePoint

```

## 10.18.1 InfinitelyClosePoint (INFCLSPT)

**Exports:**

|                |                |                  |
|----------------|----------------|------------------|
| ?=?            | ?~=?           | actualExtensionV |
| chartV         | coerce         | create           |
| curveV         | degree         | excpDivV         |
| fullOut        | fullOutput     | fullOutput       |
| hash           | latex          | localParamV      |
| localPointV    | multV          | pointV           |
| setchart!      | setcurve!      | setexcpDiv!      |
| setlocalParam! | setlocalPoint! | setmult!         |
| setpoint!      | setsubmult!    | setsymbName!     |
| subMultV       | symbNameV      |                  |

— domain INFCLSPT InfinitelyClosePoint —

```

)abbrev domain INFCLSPT InfinitelyClosePoint
++ Authors: Gaetan Hache
++ Date Created: june 1996
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
++ This domain is part of the PAFF package
InfinitelyClosePoint(K,symb,PolyRing,E,ProjPt,PCS,Plc,DIVISOR,BLMET):Exports == Implementation
  K:Field
  symb: List Symbol
  E:DirectProductCategory(#symb,NonNegativeInteger)
  OV ==> OrderedVariableList(symb)
  PolyRing: PolynomialCategory(K,E,OV)

  bls      ==> ['X,'Y]
  BlUpRing ==> DistributedMultivariatePolynomial(bls , K)
  E2       ==> DirectProduct( #bls , NonNegativeInteger )
  outRec   ==> Record(name:Symbol,mult:NonNegativeInteger)
  AFP      ==> AffinePlane(K)
  OV2      ==> OrderedVariableList( bls )

  PCS: LocalPowerSeriesCategory(K)

```

```

ProjPt:ProjectiveSpaceCategory(K)
Plc: PlacesCategory(K,PCS)
DIVISOR: DivisorCategory(Plc)
BLMET : BlowUpMethodCategory

bigoutRecBLQT ==> Record(dominate:ProjPt,_
                        name:Symbol,_
                        mult:NonNegativeInteger,_
                        defCurve:BlUpRing,_
                        localPoint:AFP,_
                        chart:BLMET,_
                        expD:DIVISOR)

bigoutRecHN ==> Record(dominate:ProjPt,_
                      name:Symbol,_
                      mult:NonNegativeInteger,_
                      defCurve:BlUpRing,_
                      localPoint:AFP,_
                      chart:BLMET,_
                      subMultip: NonNegativeInteger,_
                      expD:DIVISOR)

representation ==> Record(point:ProjPt,_
                          curve:BlUpRing,_
                          localPoint:AFP,_
                          mult:NonNegativeInteger,_
                          chrt:BLMET,_
                          subMultiplicity:NonNegativeInteger,_
                          excpDiv:DIVISOR,_
                          localParam:List(PCS),_
                          actualExtension:K,_
                          symbName:Symbol)

Exports == InfinitelyClosePointCategory(K,symb,PolyRing,E,ProjPt,PCS,Plc,DIVISOR,BLMET) with

fullOut: % -> OutputForm
  ++ fullOut(tr) yields a full output of tr (see function fullOutput).

fullOutput: Boolean -> Boolean
  ++ fullOutput(b) sets a flag such that when true, a coerce to
  ++ OutputForm yields the full output of tr, otherwise encode(tr) is
  ++ output (see encode function). The default is false.

fullOutput: () -> Boolean
  ++ fullOutput returns the value of the flag set by fullOutput(b).

Implementation == representation add
Rep := representation

```

```

polyRing2BiRing: (PolyRing, Integer) -> BLUpRing
polyRing2BiRing(pol,nV)==
  zero? pol => 0$BLUpRing
  d:= degree pol
  lc:= leadingCoefficient pol
  dd: List NonNegativeInteger := entries d
  ddr:=vector([dd.i for i in 1..#dd | ^(i=nV)])$Vector(NonNegativeInteger)
  ddre:E2 := directProduct( ddr )$E2
  monomial(lc,ddre)$BLUpRing + polyRing2BiRing( reductum pol , nV )

projPt2affPt: (ProjPt, Integer) -> AFP
projPt2affPt(pt,nV)==
  ll:= pt :: List(K)
  l:= [ ll.i for i in 1..#ll | ^(i = nV )]
  affinePoint( l)

fullOut(a)==
  oo: bigoutRecBLQT
  oo2: bigoutRecHN
  BLMET has BlowUpWithQuadTrans =>
    oo:= [ pointV(a), symbNameV(a), multV(a), curveV(a), _
      localPointV(a), chartV(a), excpDivV(a) ]$bigoutRecBLQT
  oo :: OutputForm
  BLMET has BlowUpWithHamburgerNoether =>
    oo2:= [ pointV(a), symbNameV(a), multV(a), curveV(a), _
      localPointV(a), chartV(a), subMultV(a), excpDivV(a) ]$bigoutRecHN
  oo2 :: OutputForm

fullOutputFlag:Boolean:=false()

fullOutput(f)== fullOutputFlag:=f

fullOutput == fullOutputFlag

coerce(a:%):OutputForm==
  fullOutput() => fullOut(a)
  oo:outRec:= [ symbNameV(a) , multV(a) ]$outRec
  oo :: OutputForm

degree(a)==
  K has PseudoAlgebraicClosureOfPerfectFieldCategory => extDegree actualExtensionV a
  1

create(pointA,curveA,localPointA,multA,chartA,subM,excpDivA,atcL,aName)== -- CHH
  ([pointA,curveA,localPointA,multA,chartA,subM,excpDivA,empty()$List(PCS),atcL,aName]$R

create(pointA,curveA)==
  nV := lastNonNul pointA
  localPointA := projPt2affPt(pointA,nV)

```

```

multA:NonNegativeInteger:=0$NonNegativeInteger
chartA:BLMET
if BLMET has QuadraticTransform then chartA:=( [0,0, nV] :: List Integer ) :: BLMET  -- CHH
if BLMET has HamburgerNoether then
  chartA := createHN( 0,0,nV,0,0,true,"right")  -- A changer le "right"
excpDivA:DIVISOR:= 0$DIVISOR
actL:K:=definingField pointA
aName:Symbol:=new(P)$Symbol
affCurvA : BUpRing := polyRing2BiRing(curveA,nV)
create(pointA,affCurvA,localPointA,multA,chartA,0$NonNegativeInteger,excpDivA,actL,aName)

subMultV(a:%)== (a:Rep)(subMultiplicity)

setsubmult_!(a:%,sm:NonNegativeInteger)== (a:Rep)(subMultiplicity) := sm

pointV(a:%)      ==(a:Rep)(point)

symbNameV(a:%)    ==(a:Rep)(symbName)

curveV(a:%)      ==(a:Rep)(curve)

localPointV(a:%)  ==(a:Rep)(localPoint)

multV(a:%)        ==(a:Rep)(mult)

chartV(a:%)       ==(a:Rep)(chrt)  -- CHH

excpDivV(a:%)     ==(a:Rep)(excpDiv)

localParamV(a:%)  ==(a:Rep)(localParam)

actualExtensionV(a:%) == (a:Rep)(actualExtension)

setpoint_!(a:%,n:ProjPt)      ==(a:Rep)(point):=n

setcurve_!(a:%,n:BUpRing)     ==(a:Rep)(curve):=n

setlocalPoint_!(a:%,n:AFP)    ==(a:Rep)(localPoint):=n

setmult_!(a:%,n:NonNegativeInteger)      ==(a:Rep)(mult):=n

setchart_!(a:%,n:BLMET)  ==(a:Rep)(chrt):=n  -- CHH

setlocalParam_!(a:%,n:List(PCS)) ==(a:Rep)(localParam):=n

setexcpDiv_!(a:%,n:DIVISOR) ==(a:Rep)(excpDiv):=n

setsymbName_!(a:%,n:Symbol) ==(a:Rep)(symbName):=n

```

---

— INFCLSPT.dotabb —

```
"INFCLSPT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=INFCLSPT"]
"INFCLCT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=INFCLCT"]
"INFCLSPT" -> "INFCLCT"
```

---

## 10.19 domain INFCLSPS InfinitelyClosePointOverPseudoAlgebraicClosureOfFiniteField

— InfinitelyClosePointOverPseudoAlgebraicClosureOfFiniteField.input

---

```
)set break resume
)sys rm -f InfinitelyClosePointOverPseudoAlgebraicClosureOfFiniteField.output
)spool InfinitelyClosePointOverPseudoAlgebraicClosureOfFiniteField.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show InfinitelyClosePointOverPseudoAlgebraicClosureOfFiniteField
--R InfinitelyClosePointOverPseudoAlgebraicClosureOfFiniteField(K: FiniteFieldCategory,symb: I
--R Abbreviation for InfinitelyClosePointOverPseudoAlgebraicClosureOfFiniteField is INFCLSPS
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for INFCLSPS
--R
--R----- Operations -----
--R ?=? : (% ,%) -> Boolean          chartV : % -> BLMET
--R coerce : % -> OutputForm         degree : % -> PositiveInteger
--R fullOut : % -> OutputForm         fullOutput : () -> Boolean
--R fullOutput : Boolean -> Boolean    hash : % -> SingleInteger
--R latex : % -> String               multV : % -> NonNegativeInteger
--R setchart! : (% ,BLMET) -> BLMET   symbNameV : % -> Symbol
--R ?~=? : (% ,%) -> Boolean
--R actualExtensionV : % -> PseudoAlgebraicClosureOfFiniteField K
--R create : (ProjectivePlaneOverPseudoAlgebraicClosureOfFiniteField K,DistributedMultivariat
--R create : (ProjectivePlaneOverPseudoAlgebraicClosureOfFiniteField K,DistributedMultivariat
--R curveV : % -> DistributedMultivariatePolynomial([construct,QUOTEX,QUOTEY],PseudoAlgebraic
--R excpDivV : % -> Divisor PlacesOverPseudoAlgebraicClosureOfFiniteField K
--R localParamV : % -> List NeitherSparseOrDensePowerSeries PseudoAlgebraicClosureOfFiniteFi
--R localPointV : % -> AffinePlane PseudoAlgebraicClosureOfFiniteField K
```

## 10.19. DOMAIN INFCLSPS INFINITLYCLOSEPOINTOVERPSEUDOALGEBRAICCLOSUREOFFINITEFI

```
--R pointV : % -> ProjectivePlaneOverPseudoAlgebraicClosureOfFiniteField K
--R setcurve! : (% ,DistributedMultivariatePolynomial([construct,QUOTEX,QUOTEY],PseudoAlgebraicClosureOfF
--R setexcpDiv! : (% ,Divisor PlacesOverPseudoAlgebraicClosureOfFiniteField K) -> Divisor PlacesOverPseud
--R setlocalParam! : (% ,List NeitherSparseOrDensePowerSeries PseudoAlgebraicClosureOfFiniteField K) -> L
--R setlocalPoint! : (% ,AffinePlane PseudoAlgebraicClosureOfFiniteField K) -> AffinePlane PseudoAlgebrai
--R setmult! : (% ,NonNegativeInteger) -> NonNegativeInteger
--R setpoint! : (% ,ProjectivePlaneOverPseudoAlgebraicClosureOfFiniteField K) -> ProjectivePlaneOverPseud
--R setsubmult! : (% ,NonNegativeInteger) -> NonNegativeInteger
--R setsymbName! : (% ,Symbol) -> Symbol
--R subMultV : % -> NonNegativeInteger
--R
--E 1
```

```
)spool
)lisp (bye)
```

\_\_\_\_\_

— InfinitlyClosePointOverPseudoAlgebraicClosureOfFiniteField.help

\_\_\_\_\_

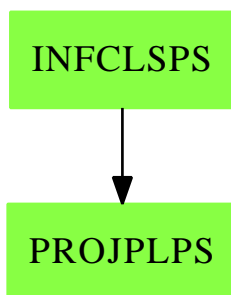
```
=====
InfinitlyClosePointOverPseudoAlgebraicClosureOfFiniteField examples
=====
```

See Also:

o )show InfinitlyClosePointOverPseudoAlgebraicClosureOfFiniteField

\_\_\_\_\_

### 10.19.1 InfinitlyClosePointOverPseudoAlgebraicClosureOfFiniteField (INFCLSPS)





**Exports:**

|                |              |                  |
|----------------|--------------|------------------|
| ?=?            | ?~=?         | actualExtensionV |
| chartV         | coerce       | create           |
| curveV         | degree       | excpDivV         |
| fullOut        | fullOutput   | hash             |
| latex          | localParamV  | localPointV      |
| multV          | pointV       | setchart!        |
| setcurve!      | setexcpDiv!  | setlocalParam!   |
| setlocalPoint! | setmult!     | setpoint!        |
| setsubmult!    | setsymbName! | subMultV         |
| symbNameV      |              |                  |

— domain INFCLSPS InfinitelyClosePointOverPseudoAlgebraicClosureOfFiniteField —

```

)abbrev domain INFCLSPS InfinitelyClosePointOverPseudoAlgebraicClosureOfFiniteField
++ Authors: Gaetan Hache
++ Date Created: june 1996
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
++ This domain is part of the PAFF package
InfinitelyClosePointOverPseudoAlgebraicClosureOfFiniteField(K,symb,BLMET):_
Exports == Implementation where

K:FiniteFieldCategory
symb: List Symbol
BLMET : BlowUpMethodCategory

E      ==> DirectProduct(#symb,NonNegativeInteger)
KK     ==> PseudoAlgebraicClosureOfFiniteField(K)
PolyRing ==> DistributedMultivariatePolynomial(symb,KK)
ProjPt ==> ProjectivePlaneOverPseudoAlgebraicClosureOfFiniteField(K)
PCS    ==> NeitherSparseOrDensePowerSeries(KK)
Plc    ==> PlacesOverPseudoAlgebraicClosureOfFiniteField(K)
DIVISOR ==> Divisor(Plc)

Exports == InfinitelyClosePointCategory(KK,symb,PolyRing,E,ProjPt,_
                                           PCS,Plc,DIVISOR,BLMET) with

fullOut: % -> OutputForm
++ fullOut(tr) yields a full output of tr (see function fullOutput).

fullOutput: Boolean -> Boolean

++ fullOutput(b) sets a flag such that when true, a coerce to OutputForm
++ yields the full output of tr, otherwise encode(tr) is output
++ (see encode function). The default is false.

fullOutput: () -> Boolean

```



```

--R ?? : (%,Integer) -> %
--R belong? : BasicOperator -> Boolean
--R box : % -> %
--R coerce : % -> %
--R coerce : Kernel % -> %
--R convert : % -> Complex Float
--R convert : % -> Float
--R distribute : (%,% ) -> %
--R elt : (BasicOperator,%,% ) -> %
--R eval : (% ,List % ,List % ) -> %
--R eval : (% ,Equation % ) -> %
--R eval : (% ,Kernel % ,%) -> %
--R freeOf? : (% ,Symbol) -> Boolean
--R gcd : (%,% ) -> %
--R hash : % -> SingleInteger
--R inv : % -> %
--R kernel : (BasicOperator,% ) -> %
--R latex : % -> String
--R lcm : List % -> %
--R max : (%,% ) -> %
--R norm : (% ,List Kernel % ) -> %
--R nthRoot : (% ,Integer) -> %
--R paren : List % -> %
--R prime? : % -> Boolean
--R recip : % -> Union(% ,"failed")
--R ?rem? : (%,% ) -> %
--R retract : % -> Integer
--R rootOf : Polynomial % -> %
--R sample : () -> %
--R sqrt : % -> %
--R squareFreePart : % -> %
--R tower : % -> List Kernel %
--R unit? : % -> Boolean
--R zero? : % -> Boolean
--R zerosOf : Polynomial % -> List %
--R ?? : (NonNegativeInteger,% ) -> %
--R ??? : (% ,NonNegativeInteger) -> %
--R ?? : (% ,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R coerce : SparseMultivariatePolynomial(Integer,Kernel % ) -> %
--R definingPolynomial : % -> % if $ has RING
--R denom : % -> SparseMultivariatePolynomial(Integer,Kernel % )
--R differentiate : (% ,NonNegativeInteger) -> %
--R divide : (%,% ) -> Record(quotient: %,remainder: %)
--R elt : (BasicOperator,List % ) -> %
--R elt : (BasicOperator,% ,% ,% ,%) -> %
--R elt : (BasicOperator,% ,% ,% ,%) -> %
--R euclideanSize : % -> NonNegativeInteger
--R eval : (% ,BasicOperator,(% -> % )) -> %
--R eval : (% ,BasicOperator,(List % -> % )) -> %
--R associates? : (%,% ) -> Boolean
--R box : List % -> %
--R coerce : Integer -> %
--R coerce : Fraction Integer -> %
--R coerce : % -> OutputForm
--R convert : % -> DoubleFloat
--R differentiate : % -> %
--R distribute : % -> %
--R elt : (BasicOperator,% ) -> %
--R eval : (% ,% ,%) -> %
--R eval : (% ,List Equation % ) -> %
--R factor : % -> Factored %
--R freeOf? : (%,% ) -> Boolean
--R gcd : List % -> %
--R height : % -> NonNegativeInteger
--R is? : (% ,Symbol) -> Boolean
--R kernels : % -> List Kernel %
--R lcm : (%,% ) -> %
--R map : ((% -> % ),Kernel % ) -> %
--R min : (%,% ) -> %
--R norm : (% ,Kernel % ) -> %
--R one? : % -> Boolean
--R paren : % -> %
--R ?quo? : (%,% ) -> %
--R reduce : % -> %
--R retract : % -> Fraction Integer
--R retract : % -> Kernel %
--R rootsOf : Polynomial % -> List %
--R sizeLess? : (%,% ) -> Boolean
--R squareFree : % -> Factored %
--R subst : (% ,Equation % ) -> %
--R trueEqual : (%,% ) -> Boolean
--R unitCanonical : % -> %
--R zeroOf : Polynomial % -> %
--R ?~=? : (%,% ) -> Boolean

```

```

--R eval : (% ,List BasicOperator,List (List % -> %)) -> %
--R eval : (% ,List BasicOperator,List (% -> %)) -> %
--R eval : (% ,Symbol,(% -> %)) -> %
--R eval : (% ,Symbol,(List % -> %)) -> %
--R eval : (% ,List Symbol,List (List % -> %)) -> %
--R eval : (% ,List Symbol,List (% -> %)) -> %
--R eval : (% ,List Kernel %,List %) -> %
--R even? : % -> Boolean if $ has RETRACT INT
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (% ,%) -> Union(%,"failed")
--R extendedEuclidean : (% ,%) -> Record(coef1: %,coef2: %,generator: %)
--R extendedEuclidean : (% ,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolynomial %
--R is? : (% ,BasicOperator) -> Boolean
--R kernel : (BasicOperator,List %) -> %
--R mainKernel : % -> Union(Kernel %,"failed")
--R minPoly : Kernel % -> SparseUnivariatePolynomial % if $ has RING
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R norm : (SparseUnivariatePolynomial %,List Kernel %) -> SparseUnivariatePolynomial %
--R norm : (SparseUnivariatePolynomial %,Kernel %) -> SparseUnivariatePolynomial %
--R numer : % -> SparseMultivariatePolynomial(Integer,Kernel %)
--R odd? : % -> Boolean if $ has RETRACT INT
--R operator : BasicOperator -> BasicOperator
--R operators : % -> List BasicOperator
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R reducedSystem : Matrix % -> Matrix Fraction Integer
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Fraction Integer,vec: Vector Fraction Integer)
--R reducedSystem : Matrix % -> Matrix Integer
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer)
--R retractIfCan : % -> Union(Fraction Integer,"failed")
--R retractIfCan : % -> Union(Integer,"failed")
--R retractIfCan : % -> Union(Kernel %,"failed")
--R rootOf : SparseUnivariatePolynomial % -> %
--R rootOf : (SparseUnivariatePolynomial %,Symbol) -> %
--R rootsOf : SparseUnivariatePolynomial % -> List %
--R rootsOf : (SparseUnivariatePolynomial %,Symbol) -> List %
--R subst : (% ,List Kernel %,List %) -> %
--R subst : (% ,List Equation %) -> %
--R subtractIfCan : (% ,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R zeroOf : SparseUnivariatePolynomial % -> %
--R zeroOf : (SparseUnivariatePolynomial %,Symbol) -> %
--R zerosOf : SparseUnivariatePolynomial % -> List %
--R zerosOf : (SparseUnivariatePolynomial %,Symbol) -> List %
--R
--E 1

)spool
)lisp (bye)

```

---

— InnerAlgebraicNumber.help —

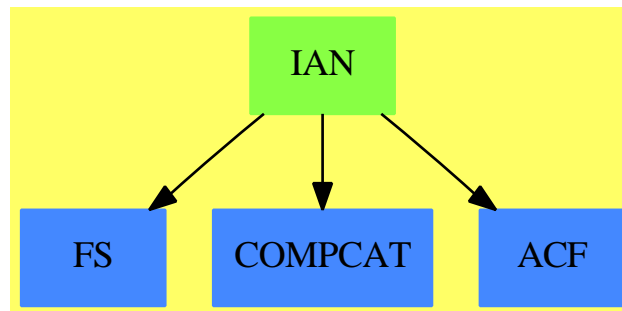
```
=====
InnerAlgebraicNumber examples
=====
```

See Also:

```
o )show InnerAlgebraicNumber
```

---

### 10.20.1 InnerAlgebraicNumber (IAN)



See

⇒ “AlgebraicNumber” (AN) 2.6.1 on page 35

**Exports:**

|                   |                    |                    |               |
|-------------------|--------------------|--------------------|---------------|
| 0                 | 1                  | associates?        | belong?       |
| box               | characteristic     | coerce             | convert       |
| D                 | definingPolynomial | denom              | differentiate |
| distribute        | divide             | elt                | euclideanSize |
| eval              | even?              | expressIdealMember | exquo         |
| extendedEuclidean | factor             | freeOf?            | gcd           |
| gcdPolynomial     | hash               | height             | inv           |
| is?               | kernel             | kernels            | latex         |
| lcm               | mainKernel         | map                | max           |
| min               | minPoly            | multiEuclidean     | norm          |
| nthRoot           | numer              | odd?               | one?          |
| operator          | operators          | paren              | prime?        |
| principalIdeal    | recip              | reduce             | reducedSystem |
| retract           | retractIfCan       | rootOf             | rootsOf       |
| sample            | sizeLess?          | sqrt               | squareFree    |
| squareFreePart    | subst              | subtractIfCan      | tower         |
| trueEqual         | unit?              | unitCanonical      | unitNormal    |
| zero?             | zeroOf             | zerosOf            | ?*?           |
| ?**?              | ?+?                | -?                 | ?-?           |
| ?/?               | ?<?                | ?<=?               | ?=?           |
| ?>?               | ?>=?               | ?^?                | ?~=?          |
| ?*?               | ?**?               | ?quo?              | ?rem?         |

— domain IAN InnerAlgebraicNumber —

```

)abbrev domain IAN InnerAlgebraicNumber
++ Author: Manuel Bronstein
++ Date Created: 22 March 1988
++ Date Last Updated: 4 October 1995 (JHD)
++ Keywords: algebraic, number.
++ Description:
++ Algebraic closure of the rational numbers.

InnerAlgebraicNumber(): Exports == Implementation where
  Z ==> Integer
  FE ==> Expression Z
  K ==> Kernel %
  P ==> SparseMultivariatePolynomial(Z, K)
  ALGOP ==> "%alg"
  SUP ==> SparseUnivariatePolynomial

Exports ==> Join(ExpressionSpace, AlgebraicallyClosedField,
  RetractableTo Z, RetractableTo Fraction Z,
  LinearlyExplicitRingOver Z, RealConstant,
  LinearlyExplicitRingOver Fraction Z,
  CharacteristicZero,
  ConvertibleTo Complex Float, DifferentialRing) with

```

```

coerce : P -> %
  ++ coerce(p) returns p viewed as an algebraic number.
numer  : % -> P
  ++ numer(f) returns the numerator of f viewed as a
  ++ polynomial in the kernels over Z.
denom  : % -> P
  ++ denom(f) returns the denominator of f viewed as a
  ++ polynomial in the kernels over Z.
reduce : % -> %
  ++ reduce(f) simplifies all the unreduced algebraic numbers
  ++ present in f by applying their defining relations.
trueEqual : (%,%) -> Boolean
  ++ trueEqual(x,y) tries to determine if the two numbers are equal
norm : (SUP(%),Kernel %) -> SUP(%)
  ++ norm(p,k) computes the norm of the polynomial p
  ++ with respect to the extension generated by kernel k
norm : (SUP(%),List Kernel %) -> SUP(%)
  ++ norm(p,l) computes the norm of the polynomial p
  ++ with respect to the extension generated by kernels l
norm : (%,Kernel %) -> %
  ++ norm(f,k) computes the norm of the algebraic number f
  ++ with respect to the extension generated by kernel k
norm : (%,List Kernel %) -> %
  ++ norm(f,l) computes the norm of the algebraic number f
  ++ with respect to the extension generated by kernels l
Implementation ==> FE add

Rep := FE

-- private
mainRatDenom(f:%):% ==
  ratDenom(f::Rep::FE)$AlgebraicManipulations(Integer, FE)::Rep::%
--      mv:= mainVariable denom f
--      mv case "failed" => f
--      algv:=mv::K
--      q:=univariate(f, algv, minPoly(algv))_
--      $PolynomialCategoryQuotientFunctions(IndexedExponents K,K,Integer,P,%)
--      q(algv::%)

findDenominator(z:SUP %):Record(num:SUP %,den:%) ==
  zz:=z
  while not(zz=0) repeat
    dd:=(denom leadingCoefficient zz)::%
    not(dd=1) =>
      rec:=findDenominator(dd*z)
      return [rec.num,rec.den*dd]
  zz:=reductum zz
  [z,1]
makeUnivariate(p:P,k:Kernel %):SUP % ==
  map(x+>x::%,univariate(p,k))$SparseUnivariatePolynomialFunctions2(P,%)

```

```

-- public
a,b:%
differentiate(x:%):% == 0
zero? a == zero? numer a
-- one? a == one? numer a and one? denom a
one? a == (numer a = 1) and (denom a = 1)
x:% / y:% == mainRatDenom(x /$Rep y)
x:% ** n:Integer ==
  n < 0 => mainRatDenom (x **$Rep n)
  x **$Rep n
trueEqual(a,b) ==
  -- if two algebraic numbers have the same norm (after deleting repeated
  -- roots, then they are certainly conjugates. Note that we start with a
  -- monic polynomial, so don't have to check for constant factors.
  -- this will be fooled by sqrt(2) and -sqrt(2), but the = in
  -- AlgebraicNumber knows what to do about this.
  ka:=reverse tower a
  kb:=reverse tower b
  empty? ka and empty? kb => retract(a)@Fraction Z = retract(b)@Fraction Z
  pa,pb: SparseUnivariatePolynomial %
  pa:=monomial(1,1)-monomial(a,0)
  pb:=monomial(1,1)-monomial(b,0)
  na:=map(retract,norm(pa,ka))_
    $SparseUnivariatePolynomialFunctions2(% ,Fraction Z)
  nb:=map(retract,norm(pb,kb))_
    $SparseUnivariatePolynomialFunctions2(% ,Fraction Z)
  (sa:=squareFreePart(na)) = (sb:=squareFreePart(nb)) => true
  g:=gcd(sa,sb)
  (dg:=degree g) = 0 => false
  -- of course, if these have a factor in common, then the
  -- answer is really ambiguous, so we ought to be using Duval-type
  -- technology
  dg = degree sa or dg = degree sb => true
  false
norm(z:%,k:Kernel %): % ==
  p:=minPoly k
  n:=makeUnivariate(numer z,k)
  d:=makeUnivariate(denom z,k)
  resultant(n,p)/resultant(d,p)
norm(z:%,l:List Kernel %): % ==
  for k in l repeat
    z:=norm(z,k)
  z
norm(z:SUP %,k:Kernel %): SUP % ==
  p:=map(x +-> x::SUP %,minPoly k)_
    $SparseUnivariatePolynomialFunctions2(% ,SUP %)
  f:=findDenominator z
  zz:=map(x +-> makeUnivariate(numer x,k),f.num)_
    $SparseUnivariatePolynomialFunctions2(% ,SUP %)
  zz:=swap(zz)$CommuteUnivariatePolynomialCategory(% ,SUP %,SUP SUP %)

```



```

    resultant(p,zz)/norm(f.den,k)
norm(z:SUP %,l:List Kernel %): SUP % ==
  for k in l repeat
    z:=norm(z,k)
  z
belong? op          == belong?(op)$ExpressionSpace_&%(%) or has?(op, ALGOP)

convert(x:):Float ==
  retract map(y +-> y::Float, x pretend FE)$ExpressionFunctions2(Z,Float)

convert(x:):DoubleFloat ==
  retract map(y +-> y::DoubleFloat,x pretend FE)_
    $ExpressionFunctions2(Z, DoubleFloat)

convert(x:):Complex(Float) ==
  retract map(y +-> y::Complex(Float),x pretend FE)_
    $ExpressionFunctions2(Z, Complex Float)

```

---

— IAN.dotabb —

```

"IAN" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IAN"]
"FS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FS"]
"COMPCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=COMPCAT"]
"ACF" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ACF"]
"IAN" -> "ACF"
"IAN" -> "FS"
"IAN" -> "COMPCAT"

```

---

## 10.21 domain IFF InnerFiniteField

— InnerFiniteField.input —

```

)set break resume
)sys rm -f InnerFiniteField.output
)spool InnerFiniteField.output
)set message test on
)set message auto off
)clear all

```

--S 1 of 1

```

)show InnerFiniteField
--R InnerFiniteField(p: PositiveInteger,n: PositiveInteger) is a domain constructor
--R Abbreviation for InnerFiniteField is IFF
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for IFF
--R
--R----- Operations -----
--R ?? : (InnerPrimeField p,%) -> %      ?? : (%,InnerPrimeField p) -> %
--R ?? : (Fraction Integer,%) -> %      ?? : (%,Fraction Integer) -> %
--R ?? : (%,%) -> %                      ?? : (Integer,%) -> %
--R ?? : (PositiveInteger,%) -> %        ??? : (%,Integer) -> %
--R ??? : (%,PositiveInteger) -> %      +? : (%,%) -> %
--R ?-? : (%,%) -> %                    -? : % -> %
--R ?/? : (%,InnerPrimeField p) -> %    ?/? : (%,%) -> %
--R ?? : (%,%) -> Boolean                1 : () -> %
--R 0 : () -> %                          ?? : (%,Integer) -> %
--R ?? : (%,PositiveInteger) -> %        algebraic? : % -> Boolean
--R associates? : (%,%) -> Boolean        basis : () -> Vector %
--R coerce : InnerPrimeField p -> %      coerce : Fraction Integer -> %
--R coerce : % -> %                      coerce : Integer -> %
--R coerce : % -> OutputForm              degree : % -> PositiveInteger
--R dimension : () -> CardinalNumber      factor : % -> Factored %
--R gcd : List % -> %                    gcd : (%,%) -> %
--R hash : % -> SingleInteger             inGroundField? : % -> Boolean
--R inv : % -> %                          latex : % -> String
--R lcm : List % -> %                    lcm : (%,%) -> %
--R norm : % -> InnerPrimeField p         one? : % -> Boolean
--R prime? : % -> Boolean                 ?quo? : (%,%) -> %
--R recip : % -> Union(%, "failed")        ?rem? : (%,%) -> %
--R retract : % -> InnerPrimeField p      sample : () -> %
--R sizeLess? : (%,%) -> Boolean           squareFree : % -> Factored %
--R squareFreePart : % -> %               trace : % -> InnerPrimeField p
--R transcendent? : % -> Boolean           unit? : % -> Boolean
--R unitCanonical : % -> %                zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?? : (NonNegativeInteger,%) -> %
--R ??? : (%,NonNegativeInteger) -> %
--R D : (%,NonNegativeInteger) -> % if InnerPrimeField p has FINITE
--R D : % -> % if InnerPrimeField p has FINITE
--R Frobenius : (%,NonNegativeInteger) -> % if InnerPrimeField p has FINITE
--R Frobenius : % -> % if InnerPrimeField p has FINITE
--R ?? : (%,NonNegativeInteger) -> %
--R basis : PositiveInteger -> Vector %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%, "failed") if InnerPrimeField p has CHARNZ or InnerPrimeField p has FINITE
--R charthRoot : % -> % if InnerPrimeField p has FINITE
--R conditionP : Matrix % -> Union(Vector %, "failed") if InnerPrimeField p has FINITE
--R coordinates : Vector % -> Matrix InnerPrimeField p
--R coordinates : % -> Vector InnerPrimeField p
--R createNormalElement : () -> % if InnerPrimeField p has FINITE

```

```

--R createPrimitiveElement : () -> % if InnerPrimeField p has FINITE
--R definingPolynomial : () -> SparseUnivariatePolynomial InnerPrimeField p
--R degree : % -> OnePointCompletion PositiveInteger
--R differentiate : (% , NonNegativeInteger) -> % if InnerPrimeField p has FINITE
--R differentiate : % -> % if InnerPrimeField p has FINITE
--R discreteLog : (% , %) -> Union(NonNegativeInteger, "failed") if InnerPrimeField p has CHARNZ
--R discreteLog : % -> NonNegativeInteger if InnerPrimeField p has FINITE
--R divide : (% , %) -> Record(quotient: %, remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List % , %) -> Union(List % , "failed")
--R exquo : (% , %) -> Union(% , "failed")
--R extendedEuclidean : (% , % , %) -> Union(Record(coef1: %, coef2: %), "failed")
--R extendedEuclidean : (% , %) -> Record(coef1: %, coef2: %, generator: %)
--R extensionDegree : () -> PositiveInteger
--R extensionDegree : () -> OnePointCompletion PositiveInteger
--R factorsOfCyclicGroupSize : () -> List Record(factor: Integer, exponent: Integer) if InnerPrimeField p has FINITE
--R gcdPolynomial : (SparseUnivariatePolynomial % , SparseUnivariatePolynomial %) -> SparseUnivariatePolynomial %
--R generator : () -> % if InnerPrimeField p has FINITE
--R index : PositiveInteger -> % if InnerPrimeField p has FINITE
--R init : () -> % if InnerPrimeField p has FINITE
--R linearAssociatedExp : (% , SparseUnivariatePolynomial InnerPrimeField p) -> % if InnerPrimeField p has FINITE
--R linearAssociatedLog : (% , %) -> Union(SparseUnivariatePolynomial InnerPrimeField p, "failed")
--R linearAssociatedLog : % -> SparseUnivariatePolynomial InnerPrimeField p if InnerPrimeField p has FINITE
--R linearAssociatedOrder : % -> SparseUnivariatePolynomial InnerPrimeField p if InnerPrimeField p has FINITE
--R lookup : % -> PositiveInteger if InnerPrimeField p has FINITE
--R minimalPolynomial : (% , PositiveInteger) -> SparseUnivariatePolynomial % if InnerPrimeField p has FINITE
--R minimalPolynomial : % -> SparseUnivariatePolynomial InnerPrimeField p
--R multiEuclidean : (List % , %) -> Union(List % , "failed")
--R nextItem : % -> Union(% , "failed") if InnerPrimeField p has FINITE
--R norm : (% , PositiveInteger) -> % if InnerPrimeField p has FINITE
--R normal? : % -> Boolean if InnerPrimeField p has FINITE
--R normalElement : () -> % if InnerPrimeField p has FINITE
--R order : % -> OnePointCompletion PositiveInteger if InnerPrimeField p has CHARNZ or InnerPrimeField p has FINITE
--R order : % -> PositiveInteger if InnerPrimeField p has FINITE
--R primeFrobenius : % -> % if InnerPrimeField p has CHARNZ or InnerPrimeField p has FINITE
--R primeFrobenius : (% , NonNegativeInteger) -> % if InnerPrimeField p has CHARNZ or InnerPrimeField p has FINITE
--R primitive? : % -> Boolean if InnerPrimeField p has FINITE
--R primitiveElement : () -> % if InnerPrimeField p has FINITE
--R principalIdeal : List % -> Record(coef: List % , generator: %)
--R random : () -> % if InnerPrimeField p has FINITE
--R representationType : () -> Union("prime", polynomial, normal, cyclic) if InnerPrimeField p has FINITE
--R represents : Vector InnerPrimeField p -> %
--R retractIfCan : % -> Union(InnerPrimeField p, "failed")
--R size : () -> NonNegativeInteger if InnerPrimeField p has FINITE
--R subtractIfCan : (% , %) -> Union(% , "failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger, NonNegativeInteger) if InnerPrimeField p has FINITE
--R trace : (% , PositiveInteger) -> % if InnerPrimeField p has FINITE
--R transcendenceDegree : () -> NonNegativeInteger
--R unitNormal : % -> Record(unit: %, canonical: %, associate: %)
--R

```

```
--E 1
```

```
)spool
)lisp (bye)
```

---

— InnerFiniteField.help —

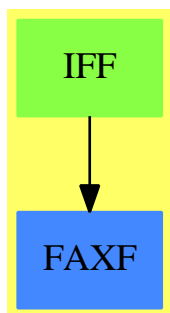
```
=====
InnerFiniteField examples
=====
```

See Also:

```
o )show InnerFiniteField
```

---

### 10.21.1 InnerFiniteField (IFF)



See

⇒ “FiniteFieldExtensionByPolynomial” (FFP) 7.10.1 on page 818

⇒ “FiniteFieldExtension” (FFX) 7.9.1 on page 813

⇒ “FiniteField” (FF) 7.5.1 on page 787

**Exports:**

|                           |                          |                        |
|---------------------------|--------------------------|------------------------|
| 0                         | 1                        | algebraic?             |
| associates?               | basis                    | characteristic         |
| charthRoot                | coerce                   | conditionP             |
| coordinates               | createNormalElement      | createPrimitiveElement |
| D                         | definingPolynomial       | degree                 |
| dimension                 | differentiate            | discreteLog            |
| divide                    | euclideanSize            | expressIdealMember     |
| exquo                     | extendedEuclidean        | extensionDegree        |
| factor                    | factorsOfCyclicGroupSize | Frobenius              |
| gcd                       | gcdPolynomial            | generator              |
| hash                      | index                    | inGroundField?         |
| init                      | inv                      | latex                  |
| lcm                       | linearAssociatedExp      | linearAssociatedLog    |
| linearAssociatedOrder     | lookup                   | minimalPolynomial      |
| multiEuclidean            | nextItem                 | norm                   |
| normal?                   | normalElement            | one?                   |
| order                     | prime?                   | primeFrobenius         |
| primitive?                | primitiveElement         | principalIdeal         |
| random                    | recip                    | representationType     |
| represents                | retract                  | retractIfCan           |
| sample                    | size                     | sizeLess?              |
| squareFree                | squareFreePart           | subtractIfCan          |
| tableForDiscreteLogarithm | trace                    | transcendenceDegree    |
| transcendent?             | unit?                    | unitCanonical          |
| unitNormal                | zero?                    | ?*                     |
| ?**?                      | ?+?                      | ?-                     |
| -?                        | ?/?                      | ?=?                    |
| ?^?                       | ?~=?                     | ?quo?                  |
| ?rem?                     |                          |                        |

— domain IFF InnerFiniteField —

```

)abbrev domain IFF InnerFiniteField
++ Author: Mark Botch
++ Date Created: ???
++ Date Last Updated: 29 May 1990
++ Basic Operations:
++ Related Constructors: FiniteFieldExtensionByPolynomial,
++ FiniteFieldPolynomialPackage
++ Also See: FiniteFieldCyclicGroup, FiniteFieldNormalBasis
++ AMS Classifications:
++ Keywords: field, extension field, algebraic extension,
++ finite extension, finite field, Galois field
++ Reference:
++ R.Lidl, H.Niederreiter: Finite Field, Encyclopedia of Mathematics an
++ Its Applications, Vol. 20, Cambridge Univ. Press, 1983, ISBN 0 521 30240 4

```

```

++ J. Grabmeier, A. Scheerhorn: Finite Fields in AXIOM.
++ AXIOM Technical Report Series, ATR/5 NP2522.
++ Description:
++ InnerFiniteField(p,n) implements finite fields with \spad{p**n} elements
++ where p is assumed prime but does not check.
++ For a version which checks that p is prime, see \spadtype{FiniteField}.

InnerFiniteField(p:PositiveInteger, n:PositiveInteger) ==
  FiniteFieldExtension(InnerPrimeField p, n)

```

---

— IFF.dotabb —

```

"IFF" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IFF"]
"FAXF" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FAXF"]
"IFF" -> "FAXF"

```

---

## 10.22 domain IFAMON InnerFreeAbelianMonoid

---

— InnerFreeAbelianMonoid.input —

```

)set break resume
)sys rm -f InnerFreeAbelianMonoid.output
)spool InnerFreeAbelianMonoid.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show InnerFreeAbelianMonoid
--R InnerFreeAbelianMonoid(S: SetCategory,E: CancellationAbelianMonoid,un: E) is a domain constructor
--R Abbreviation for InnerFreeAbelianMonoid is IFAMON
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for IFAMON
--R
--R----- Operations -----
--R ?? : (E,S) -> %           ?? : (PositiveInteger,%) -> %
--R ?? : (S,%) -> %           ?? : (%,%) -> %
--R ?? : (%,%) -> Boolean     0 : () -> %
--R coefficient : (S,%) -> E   coerce : S -> %
--R coerce : % -> OutputForm   hash : % -> SingleInteger

```

```

--R latex : % -> String
--R mapGen : ((S -> S),%) -> %
--R nthFactor : (%,Integer) -> S
--R sample : () -> %
--R zero? : % -> Boolean
--R ?? : (NonNegativeInteger,%) -> %
--R highCommonTerms : (%,%) -> % if E has OAMON
--R retractIfCan : % -> Union(S,"failed")
--R subtractIfCan : (%,%) -> Union(%, "failed")
--R terms : % -> List Record(gen: S,exp: E)
--R
--E 1

)spool
)lisp (bye)

```

---

— InnerFreeAbelianMonoid.help —

```

=====
InnerFreeAbelianMonoid examples
=====

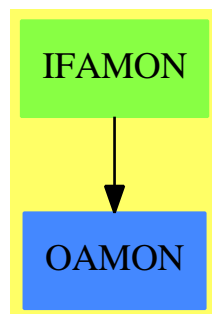
```

```

See Also:
o )show InnerFreeAbelianMonoid

```

### 10.22.1 InnerFreeAbelianMonoid (IFAMON)



See

⇒ “ListMonoidOps” (LMOPS) 13.10.1 on page 1473  
 ⇒ “FreeMonoid” (FMONOID) 7.32.1 on page 987

⇒ “FreeGroup” (FGROUP) 7.29.1 on page 976  
 ⇒ “FreeAbelianMonoid” (FAMONOID) 7.28.1 on page 974  
 ⇒ “FreeAbelianGroup” (FAGROUP) 7.27.1 on page 971

**Exports:**

|         |              |        |         |                 |
|---------|--------------|--------|---------|-----------------|
| 0       | coefficient  | coerce | hash    | highCommonTerms |
| latex   | mapCoef      | mapGen | nthCoef | nthFactor       |
| retract | retractIfCan | sample | size    | subtractIfCan   |
| terms   | zero?        | ?~=?   | ?*?     | ?+?             |
| ?=?     |              |        |         |                 |

— domain IFAMON InnerFreeAbelianMonoid —

```

)abbrev domain IFAMON InnerFreeAbelianMonoid
++ Author: Manuel Bronstein
++ Date Created: November 1989
++ Date Last Updated: 6 June 1991
++ Description:
++ Internal implementation of a free abelian monoid on any set of generators

InnerFreeAbelianMonoid(S: SetCategory, E:CancellationAbelianMonoid, un:E):
  FreeAbelianMonoidCategory(S, E) == ListMonoidOps(S, E, un) add
    Rep := ListMonoidOps(S, E, un)

    0 == makeUnit()
    zero? f == empty? listOfMonoms f
    terms f == copy listOfMonoms f
    nthCoef(f, i) == nthExpon(f, i)
    nthFactor(f, i) == nthFactor(f, i)$Rep
    s:S + f:$ == plus(s, un, f)
    f:$ + g:$ == plus(f, g)
    (f:$ = g:$):Boolean == commutativeEquality(f,g)
    n:E * s:S == makeTerm(s, n)
    n:NonNegativeInteger * f:$ == mapExpon(x +-> n*x, f)
    coerce(f:$):OutputForm == outputForm(f, "+", (x,y) +-> y*x, 0)
    mapCoef(f, x) == mapExpon(f, x)
    mapGen(f, x) == mapGen(f, x)$Rep

    coefficient(s, f) ==
      for x in terms f repeat
        x.gen = s => return(x.exp)
      0

    if E has OrderedAbelianMonoid then
      highCommonTerms(f, g) ==
        makeMulti [[x.gen, min(x.exp, n)] for x in listOfMonoms f |
          (n := coefficient(x.gen, g)) > 0]

```



— IFAMON.dotabb —

```
"IFAMON" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IFAMON"]
"OAMON" [color="#4488FF",href="bookvol10.2.pdf#nameddest=OAMON"]
"IFAMON" -> "OAMON"
```

## 10.23 domain IIARRAY2 InnerIndexedTwoDimensionalArray

This is an internal type which provides an implementation of 2-dimensional arrays as PrimitiveArray's of PrimitiveArray's.

— InnerIndexedTwoDimensionalArray.input —

```
)set break resume
)sys rm -f InnerIndexedTwoDimensionalArray.output
)spool InnerIndexedTwoDimensionalArray.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show InnerIndexedTwoDimensionalArray
--R InnerIndexedTwoDimensionalArray(R: Type,mnRow: Integer,mnCol: Integer,Row: FiniteLinearA
--R Abbreviation for InnerIndexedTwoDimensionalArray is IIARRAY2
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for IIARRAY2
--R
--R----- Operations -----
--R column : (%,Integer) -> Col          copy : % -> %
--R elt : (%,Integer,Integer,R) -> R      elt : (%,Integer,Integer) -> R
--R empty : () -> %                      empty? : % -> Boolean
--R eq? : (%,% ) -> Boolean              fill! : (% ,R) -> %
--R map : (((R,R) -> R),%,%,R) -> %      map : (((R,R) -> R),%,%) -> %
--R map : ((R -> R),%) -> %              map! : ((R -> R),%) -> %
--R maxColIndex : % -> Integer            maxRowIndex : % -> Integer
--R minColIndex : % -> Integer            minRowIndex : % -> Integer
--R ncols : % -> NonNegativeInteger      nrows : % -> NonNegativeInteger
--R parts : % -> List R                  qelt : (% ,Integer,Integer) -> R
--R row : (% ,Integer) -> Row             sample : () -> %
--R setRow! : (% ,Integer,Row) -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (% ,%) -> Boolean if R has SETCAT
```

```

--R any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if R has SETCAT
--R count : (R,%) -> NonNegativeInteger if $ has finiteAggregate and R has SETCAT
--R count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (% ,List R,List R) -> % if R has EVALAB R and R has SETCAT
--R eval : (% ,R,R) -> % if R has EVALAB R and R has SETCAT
--R eval : (% ,Equation R) -> % if R has EVALAB R and R has SETCAT
--R eval : (% ,List Equation R) -> % if R has EVALAB R and R has SETCAT
--R every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if R has SETCAT
--R latex : % -> String if R has SETCAT
--R less? : (% ,NonNegativeInteger) -> Boolean
--R member? : (R,%) -> Boolean if $ has finiteAggregate and R has SETCAT
--R members : % -> List R if $ has finiteAggregate
--R more? : (% ,NonNegativeInteger) -> Boolean
--R new : (NonNegativeInteger,NonNegativeInteger,R) -> %
--R qsetelt! : (% ,Integer,Integer,R) -> R
--R setColumn! : (% ,Integer,Col) -> %
--R setelt : (% ,Integer,Integer,R) -> R
--R size? : (% ,NonNegativeInteger) -> Boolean
--R ?~=? : (% ,%) -> Boolean if R has SETCAT
--R
--E 1

```

```

)spool
)lisp (bye)

```

---

— InnerIndexedTwoDimensionalArray.help —

```

=====
InnerIndexedTwoDimensionalArray examples
=====

```

See Also:

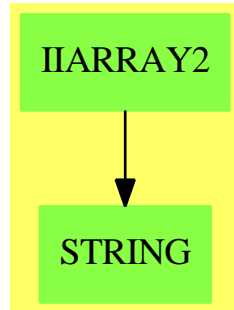
```

o )show InnerIndexedTwoDimensionalArray

```

---

### 10.23.1 InnerIndexedTwoDimensionalArray (IIARRAY2)



See

⇒ “IndexedTwoDimensionalArray” (IARRAY2) 10.15.1 on page 1221

⇒ “TwoDimensionalArray” (ARRAY2) 21.13.1 on page 2722

#### Exports:

|         |             |             |             |          |
|---------|-------------|-------------|-------------|----------|
| any?    | coerce      | column      | copy        | count    |
| elt     | empty       | empty?      | eq?         | eval     |
| every?  | fill!       | hash        | latex       | less?    |
| map     | map!        | maxColIndex | maxRowIndex | member?  |
| members | minColIndex | minRowIndex | more?       | ncols    |
| new     | nrows       | parts       | qelt        | qsetelt! |
| row     | sample      | setColumn!  | setelt      | setRow!  |
| size?   | #?          | ?=?         | ?~=?        |          |

— domain IIARRAY2 InnerIndexedTwoDimensionalArray —

```

)abbrev domain IIARRAY2 InnerIndexedTwoDimensionalArray
++ Author: Mark Botch
++ Description:
++ There is no description for this domain

InnerIndexedTwoDimensionalArray(R,mnRow,mnCol,Row,Col):_
  Exports == Implementation where
    R : Type
    mnRow, mnCol : Integer
    Row : FiniteLinearAggregate R
    Col : FiniteLinearAggregate R

  Exports ==> TwoDimensionalArrayCategory(R,Row,Col)

  Implementation ==> add

    Rep := PrimitiveArray PrimitiveArray R

--% Predicates
  
```

```

empty? m == empty?(m)$Rep

--% Primitive array creation

empty() == empty()$Rep

new(rows,cols,a) ==
  rows = 0 =>
    error "new: arrays with zero rows are not supported"
  cols = 0 =>
    error "new: arrays with zero columns are not supported"
  --
  arr : PrimitiveArray PrimitiveArray R := new(rows,empty())
  for i in minIndex(arr)..maxIndex(arr) repeat
    qsetelt_!(arr,i,new(cols,a))
  arr

--% Size inquiries

minRowIndex m == mnRow
minColIndex m == mnCol
maxRowIndex m == nrow m + mnRow - 1
maxColIndex m == ncol m + mnCol - 1

nrow m == (# m)$Rep

ncol m ==
  empty? m => 0
  # m(minIndex(m)$Rep)

--% Part selection/assignment

qelt(m,i,j) ==
  qelt(qelt(m,i - minRowIndex m)$Rep,j - minColIndex m)

elt(m:%,i:Integer,j:Integer) ==
  i < minRowIndex(m) or i > maxRowIndex(m) =>
    error "elt: index out of range"
  j < minColIndex(m) or j > maxColIndex(m) =>
    error "elt: index out of range"
  qelt(m,i,j)

qsetelt_!(m,i,j,r) ==
  setelt(qelt(m,i - minRowIndex m)$Rep,j - minColIndex m,r)

setelt(m:%,i:Integer,j:Integer,r:R) ==
  i < minRowIndex(m) or i > maxRowIndex(m) =>
    error "setelt: index out of range"
  j < minColIndex(m) or j > maxColIndex(m) =>
    error "setelt: index out of range"

```

```

qsetelt_!(m,i,j,r)

if R has SetCategory then
  latex(m : %) : String ==
    s : String := "\left[ \begin{array}{l}"
    j : Integer
    for j in minColIndex(m)..maxColIndex(m) repeat
      s := concat(s,"c")$String
    s := concat(s,"} ")$String
    i : Integer
    for i in minRowIndex(m)..maxRowIndex(m) repeat
      for j in minColIndex(m)..maxColIndex(m) repeat
        s := concat(s, latex(qelt(m,i,j))$R)$String
        if j < maxColIndex(m) then s := concat(s, " & ")$String
        if i < maxRowIndex(m) then s := concat(s, " \\" )$String
      concat(s, "\end{array} \right]")$String

```

— IIARRAY2.dotabb —

```

"IIARRAY2" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IIARRAY2"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"IIARRAY2" -> "STRING"

```

## 10.24 domain IPADIC InnerPAdicInteger

— InnerPAdicInteger.input —

```

)set break resume
)sys rm -f InnerPAdicInteger.output
)spool InnerPAdicInteger.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show InnerPAdicInteger
--R InnerPAdicInteger(p: Integer,unBalanced?: Boolean) is a domain constructor
--R Abbreviation for InnerPAdicInteger is IPADIC
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for IPADIC

```

```

--R
--R----- Operations -----
--R ?? : (% , %) -> %                ?? : (Integer , %) -> %
--R ?? : (PositiveInteger , %) -> %    ??? : (% , PositiveInteger) -> %
--R +? : (% , %) -> %                ?-? : (% , %) -> %
--R -? : % -> %                      ?=? : (% , %) -> Boolean
--R 1 : () -> %                      0 : () -> %
--R ?? : (% , PositiveInteger) -> %    associates? : (% , %) -> Boolean
--R coerce : % -> %                   coerce : Integer -> %
--R coerce : % -> OutputForm           complete : % -> %
--R digits : % -> Stream Integer       extend : (% , Integer) -> %
--R gcd : List % -> %                  gcd : (% , %) -> %
--R hash : % -> SingleInteger          latex : % -> String
--R lcm : List % -> %                  lcm : (% , %) -> %
--R moduloP : % -> Integer             modulus : () -> Integer
--R one? : % -> Boolean                order : % -> NonNegativeInteger
--R ?quo? : (% , %) -> %               quotientByP : % -> %
--R recip : % -> Union(% , "failed")   ?rem? : (% , %) -> %
--R sample : () -> %                  sizeLess? : (% , %) -> Boolean
--R sqrt : (% , Integer) -> %          unit? : % -> Boolean
--R unitCanonical : % -> %             zero? : % -> Boolean
--R ~=? : (% , %) -> Boolean
--R *? : (NonNegativeInteger , %) -> %
--R **? : (% , NonNegativeInteger) -> %
--R ^? : (% , NonNegativeInteger) -> %
--R approximate : (% , Integer) -> Integer
--R characteristic : () -> NonNegativeInteger
--R divide : (% , %) -> Record(quotient: % , remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List % , %) -> Union(List % , "failed")
--R exquo : (% , %) -> Union(% , "failed")
--R extendedEuclidean : (% , % , %) -> Union(Record(coef1: % , coef2: % ) , "failed")
--R extendedEuclidean : (% , %) -> Record(coef1: % , coef2: % , generator: %)
--R gcdPolynomial : (SparseUnivariatePolynomial % , SparseUnivariatePolynomial %) -> SparseUnivariatePolynomial
--R multiEuclidean : (List % , %) -> Union(List % , "failed")
--R principalIdeal : List % -> Record(coef: List % , generator: %)
--R root : (SparseUnivariatePolynomial Integer , Integer) -> %
--R subtractIfCan : (% , %) -> Union(% , "failed")
--R unitNormal : % -> Record(unit: % , canonical: % , associate: %)
--R
--E 1

)spool
)lisp (bye)

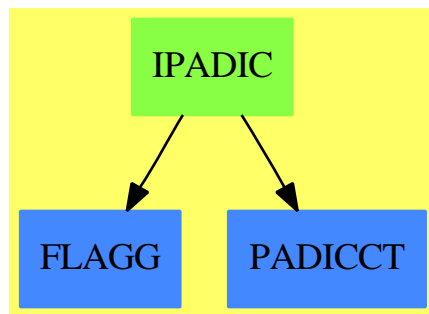
```

```
=====
InnerPAdicInteger examples
=====
```

See Also:

```
o )show InnerPAdicInteger
```

### 10.24.1 InnerPAdicInteger (IPADIC)



See

- ⇒ “PAdicInteger” (PADIC) 17.1.1 on page 1841
- ⇒ “BalancedPAdicInteger” (BPADIC) 3.2.1 on page 240
- ⇒ “PAdicRationalConstructor” (PADICRC) 17.3.1 on page 1850
- ⇒ “PAdicRational” (PADICRAT) 17.2.1 on page 1845
- ⇒ “BalancedPAdicRational” (BPADICRT) 3.3.1 on page 244

#### Exports:

|                |                   |                    |                |
|----------------|-------------------|--------------------|----------------|
| 0              | 1                 | approximate        | associates?    |
| characteristic | coerce            | complete           | digits         |
| divide         | euclideanSize     | expressIdealMember | exquo          |
| extend         | extendedEuclidean | gcd                | gcdPolynomial  |
| hash           | latex             | lcm                | multiEuclidean |
| moduloP        | modulus           | one?               | order          |
| principalIdeal | quotientByP       | recip              | root           |
| sample         | sizeLess?         | sqrt               | subtractIfCan  |
| unit?          | unitCanonical     | unitNormal         | zero?          |
| ?~=?           | ?*?               | ?**?               | ?^?            |
| ?+?            | ?-?               | -?                 | ?=?            |
| ?quo?          | ?rem?             |                    |                |

— domain IPADIC InnerPAdicInteger —

```
)abbrev domain IPADIC InnerPAdicInteger
```

```

++ Author: Clifton J. Williamson
++ Date Created: 20 August 1989
++ Date Last Updated: 15 May 1990
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Keywords: p-adic, completion
++ Examples:
++ References:
++ Description:
++ This domain implements  $\mathbb{Z}_p$ , the p-adic completion of the integers.
++ This is an internal domain.

```

```

InnerPAdicInteger(p,unBalanced?): Exports == Implementation where

```

```

    p          : Integer
    unBalanced? : Boolean
    I    ==> Integer
    NNI ==> NonNegativeInteger
    OUT ==> OutputForm
    L    ==> List
    ST   ==> Stream
    SUP ==> SparseUnivariatePolynomial

```

```

Exports ==> PAdicIntegerCategory p

```

```

Implementation ==> add

```

```

    PEXPR := p :: OUT

```

```

    Rep := ST I

```

```

    characteristic() == 0
    euclideanSize(x) == order(x)

```

```

    stream(x:%):ST I == x pretend ST(I)
    padic(x:ST I):% == x pretend %
    digits x == stream x

```

```

    extend(x,n) == extend(x,n + 1)$Rep
    complete x == complete(x)$Rep

```

```

--      notBalanced?:() -> Boolean
--      notBalanced?() == unBalanced?

```

```

modP:I -> I
modP n ==
    unBalanced? or (p = 2) => positiveRemainder(n,p)
    symmetricRemainder(n,p)

```



```

modPInfo: I -> Record(digit:I, carry:I)
modPInfo n ==
  dv := divide(n,p)
  r0 := dv.remainder; q := dv.quotient
  if (r := modP r0) ^= r0 then q := q + ((r0 - r) quo p)
  [r,q]

invModP: I -> I
invModP n == invmod(n,p)

modulus()      == p
moduloP x      == (empty? x => 0; frst x)
quotientByP x == (empty? x => x; rst x)

approximate(x,n) ==
  n <= 0 or empty? x => 0
  frst x + p * approximate(rst x, n - 1)

x = y ==
  st : ST I := stream(x - y)
  n : I := _$streamCount$Lisp
  for i in 0..n repeat
    empty? st => return true
    frst st ^= 0 => return false
    st := rst st
  empty? st

order x ==
  st := stream x
  for i in 0..1000 repeat
    empty? st => return 0
    frst st ^= 0 => return i
  st := rst st
  error "order: series has more than 1000 leading zero coefs"

0 == padic concat(0$I, empty())
1 == padic concat(1$I, empty())

intToPAAdic: I -> ST I
intToPAAdic n == delay
  n = 0 => empty()
  modp := modPInfo n
  concat(modp.digit, intToPAAdic modp.carry)

intPlusPAAdic: (I, ST I) -> ST I
intPlusPAAdic(n,x) == delay
  empty? x => intToPAAdic n
  modp := modPInfo(n + frst x)
  concat(modp.digit, intPlusPAAdic(modp.carry, rst x))

```

```

intMinusPAdic: (I,ST I) -> ST I
intMinusPAdic(n,x) == delay
  empty? x => intToPAdic n
  modp := modPInfo(n - first x)
  concat(modp.digit,intMinusPAdic(modp.carry,rst x))

plusAux: (I,ST I,ST I) -> ST I
plusAux(n,x,y) == delay
  empty? x and empty? y => intToPAdic n
  empty? x => intPlusPAdic(n,y)
  empty? y => intPlusPAdic(n,x)
  modp := modPInfo(n + first x + first y)
  concat(modp.digit,plusAux(modp.carry,rst x,rst y))

minusAux: (I,ST I,ST I) -> ST I
minusAux(n,x,y) == delay
  empty? x and empty? y => intToPAdic n
  empty? x => intMinusPAdic(n,y)
  empty? y => intPlusPAdic(n,x)
  modp := modPInfo(n + first x - first y)
  concat(modp.digit,minusAux(modp.carry,rst x,rst y))

x + y == padic plusAux(0,stream x,stream y)
x - y == padic minusAux(0,stream x,stream y)
- y == padic intMinusPAdic(0,stream y)
coerce(n:I) == padic intToPAdic n

intMult: (I,ST I) -> ST I
intMult(n,x) == delay
  empty? x => empty()
  modp := modPInfo(n * first x)
  concat(modp.digit,intPlusPAdic(modp.carry,intMult(n,rst x)))

(n:I) * (x:%) ==
  padic intMult(n,stream x)

timesAux: (ST I,ST I) -> ST I
timesAux(x,y) == delay
  empty? x or empty? y => empty()
  modp := modPInfo(first x * first y)
  car := modp.digit
  cdr : ST I --!!
  cdr := plusAux(modp.carry,intMult(first x,rst y),timesAux(rst x,y))
  concat(car,cdr)

(x:%) * (y:%) == padic timesAux(stream x,stream y)

quotientAux: (ST I,ST I) -> ST I
quotientAux(x,y) == delay

```

```

empty? x => error "quotientAux: first argument"
empty? y => empty()
modP first x = 0 =>
  modP first y = 0 => quotientAux(rst x,rst y)
  error "quotient: quotient not integral"
z0 := modP(invModP first x * first y)
yy : ST I --!!
yy := rest minusAux(0,y,intMult(z0,x))
concat(z0,quotientAux(x,yy))

recip x ==
  empty? x or modP first x = 0 => "failed"
  padic quotientAux(stream x,concat(1,empty()))

iExquo: (%,% ,I) -> Union(%,"failed")
iExquo(xx,yy,n) ==
  n > 1000 =>
    error "exquo: quotient by series with many leading zero coefs"
  empty? yy => "failed"
  empty? xx => 0
  zero? first yy =>
    zero? first xx => iExquo(rst xx,rst yy,n + 1)
    "failed"
  (rec := recip yy) case "failed" => "failed"
  xx * (rec :: %)

x exquo y == iExquo(stream x,stream y,0)

divide(x,y) ==
  (z:=x exquo y) case "failed" => [0,x]
  [z, 0]

iSqrt: (I,I,I,% ) -> %
iSqrt(pn,an,bn,bSt) == delay
  bn1 := (empty? bSt => bn; bn + pn * first(bSt))
  c := (bn1 - an*an) quo pn
  aa := modP(c * invmod(2*an,p))
  nSt := (empty? bSt => bSt; rst bSt)
  concat(aa,iSqrt(pn*p,an + pn*aa,bn1,nSt))

sqrt(b,a) ==
  p = 2 =>
    error "sqrt: no square roots in Z2 yet"
  not zero? modP(a*a - (bb := moduloP b)) =>
    error "sqrt: not a square root (mod p)"
  b := (empty? b => b; rst b)
  a := modP a
  concat(a,iSqrt(p,a,bb,b))

iRoot: (SUP I,I,I,I) -> ST I

```

```

iRoot(f,alpha,invFpx0,pPow) == delay
  num := -((f(alpha) exquo pPow) :: I)
  digit := modP(num * invFpx0)
  concat(digit,iRoot(f,alpha + digit * pPow,invFpx0,p * pPow))

root(f,x0) ==
  x0 := modP x0
  not zero? modP f(x0) =>
    error "root: not a root (mod p)"
  fpx0 := modP (differentiate f)(x0)
  zero? fpx0 =>
    error "root: approximate root must be a simple root (mod p)"
  invFpx0 := modP invModP fpx0
  padic concat(x0,iRoot(f,x0,invFpx0,p))

termOutput:(I,I) -> OUT
termOutput(k,c) ==
  k = 0 => c :: OUT
  mon := (k = 1 => PEXPR; PEXPR ** (k :: OUT))
  c = 1 => mon
  c = -1 => -mon
  (c :: OUT) * mon

showAll?:() -> Boolean
-- check a global Lisp variable
showAll?() == true

coerce(x:%):OUT ==
  empty?(st := stream x) => 0 :: OUT
  n : NNI ; count : NNI := _$streamCount$Lisp
  l : L OUT := empty()
  for n in 0..count while not empty? st repeat
    if first(st) ^= 0 then
      l := concat(termOutput(n :: I,first st),l)
      st := rst st
  if showAll?() then
    for n in (count + 1).. while explicitEntries? st and _
      not eq?(st,rst st) repeat
        if first(st) ^= 0 then
          l := concat(termOutput(n pretend I,first st),l)
          st := rst st
  l :=
    explicitlyEmpty? st => l
    eq?(st,rst st) and first st = 0 => l
    concat(prefix("0" :: OUT,[PEXPR ** (n :: OUT)]),l)
  empty? l => 0 :: OUT
  reduce("+",reverse_! l)

```

---

— IPADIC.dotabb —

```
"IPADIC" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IPADIC"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"PADICCT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PADICCT"]
"IPADIC" -> "PADICCT"
"IPADIC" -> "FLAGG"
```

## 10.25 domain IPF InnerPrimeField

— InnerPrimeField.input —

```
)set break resume
)sys rm -f InnerPrimeField.output
)spool InnerPrimeField.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show InnerPrimeField
--R InnerPrimeField p: PositiveInteger is a domain constructor
--R Abbreviation for InnerPrimeField is IPF
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for IPF
--R
--R----- Operations -----
--R ?? : (Fraction Integer,%) -> %      ?? : (%,Fraction Integer) -> %
--R ?? : (%,%) -> %                    ?? : (Integer,%) -> %
--R ?? : (PositiveInteger,%) -> %      ??? : (%,Integer) -> %
--R ??? : (%,PositiveInteger) -> %    ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                  -? : % -> %
--R ?/? : (%,%) -> %                  ?? : (%,%) -> Boolean
--R D : % -> %                        D : (%,NonNegativeInteger) -> %
--R 1 : () -> %                       0 : () -> %
--R ?? : (%,Integer) -> %             ?? : (%,PositiveInteger) -> %
--R algebraic? : % -> Boolean         associates? : (%,%) -> Boolean
--R basis : () -> Vector %            charthRoot : % -> %
--R coerce : Fraction Integer -> %    coerce : % -> %
--R coerce : Integer -> %             coerce : % -> OutputForm
--R convert : % -> Integer            coordinates : % -> Vector %
--R createPrimitiveElement : () -> %   degree : % -> PositiveInteger
--R differentiate : % -> %            dimension : () -> CardinalNumber
```

```

--R factor : % -> Factored %
--R gcd : (% , %) -> %
--R inGroundField? : % -> Boolean
--R init : () -> %
--R latex : % -> String
--R lcm : (% , %) -> %
--R norm : % -> %
--R order : % -> PositiveInteger
--R primeFrobenius : % -> %
--R primitiveElement : () -> %
--R random : () -> %
--R ?rem? : (% , %) -> %
--R retract : % -> %
--R size : () -> NonNegativeInteger
--R squareFree : % -> Factored %
--R trace : % -> %
--R unit? : % -> Boolean
--R zero? : % -> Boolean
--R ?? : (NonNegativeInteger , %) -> %
--R ??? : (% , NonNegativeInteger) -> %
--R Frobenius : % -> % if $ has FINITE
--R Frobenius : (% , NonNegativeInteger) -> % if $ has FINITE
--R ?? : (% , NonNegativeInteger) -> %
--R basis : PositiveInteger -> Vector %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%, "failed")
--R conditionP : Matrix % -> Union(Vector %, "failed")
--R coordinates : Vector % -> Matrix %
--R createNormalElement : () -> % if $ has FINITE
--R definingPolynomial : () -> SparseUnivariatePolynomial %
--R degree : % -> OnePointCompletion PositiveInteger
--R differentiate : (% , NonNegativeInteger) -> %
--R discreteLog : % -> NonNegativeInteger
--R discreteLog : (% , %) -> Union(NonNegativeInteger, "failed")
--R divide : (% , %) -> Record(quotient: %, remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %, %) -> Union(List %, "failed")
--R exquo : (% , %) -> Union(%, "failed")
--R extendedEuclidean : (% , %, %) -> Union(Record(coef1: %, coef2: %), "failed")
--R extendedEuclidean : (% , %) -> Record(coef1: %, coef2: %, generator: %)
--R extensionDegree : () -> OnePointCompletion PositiveInteger
--R extensionDegree : () -> PositiveInteger
--R factorsOfCyclicGroupSize : () -> List Record(factor: Integer, exponent: Integer)
--R gcdPolynomial : (SparseUnivariatePolynomial %, SparseUnivariatePolynomial %) -> SparseUnivariatePolynomial %
--R generator : () -> % if $ has FINITE
--R linearAssociatedExp : (% , SparseUnivariatePolynomial %) -> % if $ has FINITE
--R linearAssociatedLog : % -> SparseUnivariatePolynomial % if $ has FINITE
--R linearAssociatedLog : (% , %) -> Union(SparseUnivariatePolynomial %, "failed") if $ has FINITE
--R linearAssociatedOrder : % -> SparseUnivariatePolynomial % if $ has FINITE
--R minimalPolynomial : % -> SparseUnivariatePolynomial %
gcd : List % -> %
hash : % -> SingleInteger
index : PositiveInteger -> %
inv : % -> %
lcm : List % -> %
lookup : % -> PositiveInteger
one? : % -> Boolean
prime? : % -> Boolean
primitive? : % -> Boolean
?quo? : (% , %) -> %
recip : % -> Union(%, "failed")
represents : Vector % -> %
sample : () -> %
sizeLess? : (% , %) -> Boolean
squareFreePart : % -> %
transcendent? : % -> Boolean
unitCanonical : % -> %
?~=?: (% , %) -> Boolean

```

```

--R minimalPolynomial : (% , PositiveInteger) -> SparseUnivariatePolynomial % if $ has FINITE
--R multiEuclidean : (List % , %) -> Union(List % , "failed")
--R nextItem : % -> Union(% , "failed")
--R norm : (% , PositiveInteger) -> % if $ has FINITE
--R normal? : % -> Boolean if $ has FINITE
--R normalElement : () -> % if $ has FINITE
--R order : % -> OnePointCompletion PositiveInteger
--R primeFrobenius : (% , NonNegativeInteger) -> %
--R principalIdeal : List % -> Record(coef: List % , generator: %)
--R representationType : () -> Union("prime" , polynomial , normal , cyclic)
--R retractIfCan : % -> Union(% , "failed")
--R subtractIfCan : (% , %) -> Union(% , "failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger , NonNegativeInteger)
--R trace : (% , PositiveInteger) -> % if $ has FINITE
--R transcendenceDegree : () -> NonNegativeInteger
--R unitNormal : % -> Record(unit: % , canonical: % , associate: %)
--R
--E 1

)spool
)lisp (bye)

```

---

### — InnerPrimeField.help —

```

=====
InnerPrimeField examples
=====

```

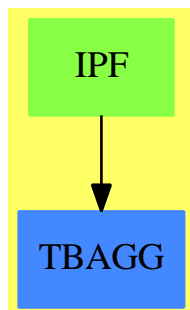
```

See Also:
o )show InnerPrimeField

```

---

## 10.25.1 InnerPrimeField (IPF)



See

⇒ “PrimeField” (PF) 17.29.1 on page 2064

**Exports:**

|                     |                           |                          |
|---------------------|---------------------------|--------------------------|
| 0                   | 1                         | algebraic?               |
| associates?         | basis                     | characteristic           |
| charthRoot          | coerce                    | conditionP               |
| convert             | coordinates               | createPrimitiveElement   |
| createNormalElement | D                         | definingPolynomial       |
| degree              | differentiate             | dimension                |
| discreteLog         | divide                    | euclideanSize            |
| expressIdealMember  | exquo                     | extendedEuclidean        |
| extensionDegree     | factor                    | factorsOfCyclicGroupSize |
| Frobenius           | gcd                       | gcdPolynomial            |
| generator           | hash                      | inGroundField?           |
| index               | init                      | inv                      |
| latex               | lcm                       | linearAssociatedExp      |
| linearAssociatedLog | linearAssociatedOrder     | lookup                   |
| minimalPolynomial   | multiEuclidean            | nextItem                 |
| norm                | normal?                   | normalElement            |
| one?                | order                     | prime?                   |
| primeFrobenius      | primitive?                | primitiveElement         |
| principalIdeal      | random                    | recip                    |
| representationType  | represents                | retract                  |
| retractIfCan        | sample                    | size                     |
| sizeLess?           | squareFree                | squareFreePart           |
| subtractIfCan       | tableForDiscreteLogarithm | trace                    |
| transcendenceDegree | transcendent?             | unit?                    |
| unitCanonical       | unitNormal                | zero?                    |
| ?*?                 | ?**?                      | ?+?                      |
| ?-?                 | -?                        | ?/?                      |
| ?=?                 | ?^?                       | ?~=?                     |
| ?quo?               | ?rem?                     |                          |



## — domain IPF InnerPrimeField —

```

)abbrev domain IPF InnerPrimeField
++ Authors: N.N., J.Grabmeier, A.Scheerhorn
++ Date Created: ?, November 1990, 26.03.1991
++ Date Last Updated: 12 April 1991
++ Basic Operations:
++ Related Constructors: PrimeField
++ Also See:
++ AMS Classifications:
++ Keywords: prime characteristic, prime field, finite field
++ References:
++ R.Lidl, H.Niederreiter: Finite Field, Encycoldia of Mathematics and
++ Its Applications, Vol. 20, Cambridge Univ. Press, 1983, ISBN 0 521 30240 4
++ AXIOM Technical Report Series, to appear.
++ Description:
++ InnerPrimeField(p) implements the field with p elements.
++ Note: argument p MUST be a prime (this domain does not check).
++ See \spadtype{PrimeField} for a domain that does check.

InnerPrimeField(p:PositiveInteger): Exports == Implementation where

  I ==> Integer
  NNI ==> NonNegativeInteger
  PI ==> PositiveInteger
  TBL ==> Table(PI,NNI)
  R ==> Record(key:PI,entry:NNI)
  SUP ==> SparseUnivariatePolynomial
  OUT ==> OutputForm

Exports ==> Join(FiniteFieldCategory,FiniteAlgebraicExtensionField($),_
  ConvertibleTo(Integer))

Implementation ==> IntegerMod p add

  initializeElt:() -> Void
  initializeLog:() -> Void

-- global variables =====

primitiveElt:PI:=1
-- for the lookup the primitive Element computed by createPrimitiveElement()

sizeCG :=(p-1) pretend NonNegativeInteger
-- the size of the cyclic group

facOfGroupSize := nil()$(List Record(factor:Integer,exponent:Integer))
-- the factorization of the cyclic group size

```

```

initlog?:Boolean:=true
-- gets false after initialization of the logarithm table

initelt?:Boolean:=true
-- gets false after initialization of the primitive Element

discLogTable:Table(PI,TBL):=table()$Table(PI,TBL)
-- tables indexed by the factors of the size q of the cyclic group
-- discLogTable.factor is a table of with keys
-- primitiveElement() ** (i * (q quo factor)) and entries i for
-- i in 0..n-1, n computed in initialize() in order to use
-- the minimal size limit 'limit' optimal.

-- functions =====

generator() == 1

-- This uses x**(p-1)=1 (mod p), so x**(q(p-1)+r) = x**r (mod p)
x:$ ** n:Integer ==
  zero?(n) => 1
  zero?(x) => 0
  r := positiveRemainder(n,p-1)::NNI
  ((x pretend IntegerMod p) **$IntegerMod(p) r) pretend $

if p <= convert(max())$SingleInteger@Integer then
  q := p::SingleInteger

  recip x ==
    zero?(y := convert(x)@Integer :: SingleInteger) => "failed"
    invmod(y, q)::Integer::$
else
  recip x ==
    zero?(y := convert(x)@Integer) => "failed"
    invmod(y, p)::$

convert(x:$) == x pretend I

normalElement() == 1

createNormalElement() == 1

characteristic() == p

factorsOfCyclicGroupSize() ==
  p=2 => facOfGroupSize -- this fixes an infinite loop of functions
                        -- calls, problem was that factors factor(1)
                        -- is the empty list
  if empty? facOfGroupSize then initializeElt()
  facOfGroupSize

```

```

representationType() == "prime"

tableForDiscreteLogarithm(fac) ==
  if initlog? then initializeLog()
  tbl:=search(fac::PI, discLogTable)$Table(PI, TBL)
  tbl case "failed" =>
    error "tableForDiscreteLogarithm: argument must be prime divisor_
of the order of the multiplicative group"
  tbl pretend TBL

primitiveElement() ==
  if initelt? then initializeElt()
  index(primitiveElt)

initializeElt() ==
  facOfGroupSize:=factors(factor(sizeCG)$I)$(Factored I)
  -- get a primitive element
  primitiveElt:=lookup(createPrimitiveElement())
  -- set initialization flag
  initelt? := false
  void$Void

initializeLog() ==
  if initelt? then initializeElt()
  -- set up tables for discrete logarithm
  limit:Integer:=30
  -- the minimum size for the discrete logarithm table
  for f in facOfGroupSize repeat
    fac:=f.factor
    base:=$:=primitiveElement() ** (sizeCG quo fac)
    l:Integer:=length(fac)$Integer
    n:Integer:=0
    if odd?(l)$Integer then n:=shift(fac, -(l quo 2))
                        else n:=shift(1, (l quo 2))

    if n < limit then
      d:=(fac-1) quo limit + 1
      n:=(fac-1) quo d + 1
    tbl:TBL:=table()$TBL
    a:=$:=1
    for i in (0::NNI)..(n-1)::NNI repeat
      insert_!([lookup(a), i::NNI]$R, tbl)$TBL
      a:=a*base
    insert_!([fac::PI, copy(tbl)$TBL]_
      $Record(key:PI, entry:TBL), discLogTable)$Table(PI, TBL)
  -- tell user about initialization
  -- print("discrete logarithm table initialized":OUT)
  -- set initialization flag
  initlog? := false
  void$Void

```

```

degree(x):PI == 1::PositiveInteger
extensionDegree():PI == 1::PositiveInteger

--    sizeOfGroundField() == p::NonNegativeInteger

inGroundField?(x) == true

coordinates(x) == new(1,x)$(Vector $)

represents(v) == v.1

retract(x) == x

retractIfCan(x) == x

basis() == new(1,1::$)$(Vector $)
basis(n:PI) ==
  n = 1 => basis()
  error("basis: argument must divide extension degree")

definingPolynomial() ==
  monomial(1,1)$(SUP $) - monomial(1,0)$(SUP $)

minimalPolynomial(x) ==
  monomial(1,1)$(SUP $) - monomial(x,0)$(SUP $)

charthRoot x == x

```

---

— IPF.dotabb —

```

"IPF" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IPF"]
"TBAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=TBAGG"]
"IPF" -> "TBAGG"

```

---

## 10.26 domain ISUPS InnerSparseUnivariatePowerSeries

— InnerSparseUnivariatePowerSeries.input —

```

)set break resume

```

```

)sys rm -f InnerSparseUnivariatePowerSeries.output
)spool InnerSparseUnivariatePowerSeries.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show InnerSparseUnivariatePowerSeries
--R InnerSparseUnivariatePowerSeries Coef: Ring is a domain constructor
--R Abbreviation for InnerSparseUnivariatePowerSeries is ISUPS
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ISUPS
--R
--R----- Operations -----
--R ??? : (Coef,%) -> %          ??? : (%,Coef) -> %
--R ??? : (%,%) -> %           ??? : (Integer,%) -> %
--R ??? : (PositiveInteger,%) -> %   ??? : (%,PositiveInteger) -> %
--R ??+ : (%,%) -> %           ?-? : (%,%) -> %
--R -? : % -> %               ?=? : (%,%) -> Boolean
--R 1 : () -> %               0 : () -> %
--R ??^ : (%,PositiveInteger) -> %   center : % -> Coef
--R coefficient : (%,Integer) -> Coef   coerce : Integer -> %
--R coerce : % -> OutputForm           complete : % -> %
--R degree : % -> Integer              ?..? : (%,Integer) -> Coef
--R extend : (%,Integer) -> %          hash : % -> SingleInteger
--R iCompose : (%,%) -> %              latex : % -> String
--R leadingCoefficient : % -> Coef      leadingMonomial : % -> %
--R map : ((Coef -> Coef),%) -> %       monomial : (Coef,Integer) -> %
--R monomial? : % -> Boolean            one? : % -> Boolean
--R order : (%,Integer) -> Integer       order : % -> Integer
--R pole? : % -> Boolean                recip : % -> Union(%, "failed")
--R reductum : % -> %                  sample : () -> %
--R taylorQuoByVar : % -> %            truncate : (%,Integer) -> %
--R variable : % -> Symbol              zero? : % -> Boolean
--R ?^=? : (%,%) -> Boolean
--R ??? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ??? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
--R ??? : (NonNegativeInteger,%) -> %
--R ??? : (%,NonNegativeInteger) -> %
--R ?/? : (%,Coef) -> % if Coef has FIELD
--R D : % -> % if Coef has *: (Integer,Coef) -> Coef
--R D : (%,NonNegativeInteger) -> % if Coef has *: (Integer,Coef) -> Coef
--R D : (%,Symbol) -> % if Coef has *: (Integer,Coef) -> Coef and Coef has PDRING SYMBOL
--R D : (%,List Symbol) -> % if Coef has *: (Integer,Coef) -> Coef and Coef has PDRING SYMBOL
--R D : (%,Symbol,NonNegativeInteger) -> % if Coef has *: (Integer,Coef) -> Coef and Coef has PDRING SYMBOL
--R D : (%,List Symbol,List NonNegativeInteger) -> % if Coef has *: (Integer,Coef) -> Coef and Coef has PDRING SYMBOL
--R ??^ : (%,NonNegativeInteger) -> %
--R approximate : (%,Integer) -> Coef if Coef has **: (Coef,Integer) -> Coef and Coef has co
--R associates? : (%,%) -> Boolean if Coef has INTDOM
--R cAcos : % -> % if Coef has ALGEBRA FRAC INT

```

```

--R cAcosh : % -> % if Coef has ALGEBRA FRAC INT
--R cAcot : % -> % if Coef has ALGEBRA FRAC INT
--R cAcoth : % -> % if Coef has ALGEBRA FRAC INT
--R cAcsc : % -> % if Coef has ALGEBRA FRAC INT
--R cAcsch : % -> % if Coef has ALGEBRA FRAC INT
--R cAsec : % -> % if Coef has ALGEBRA FRAC INT
--R cAsech : % -> % if Coef has ALGEBRA FRAC INT
--R cAsin : % -> % if Coef has ALGEBRA FRAC INT
--R cAsinh : % -> % if Coef has ALGEBRA FRAC INT
--R cAtan : % -> % if Coef has ALGEBRA FRAC INT
--R cAtanh : % -> % if Coef has ALGEBRA FRAC INT
--R cCos : % -> % if Coef has ALGEBRA FRAC INT
--R cCosh : % -> % if Coef has ALGEBRA FRAC INT
--R cCot : % -> % if Coef has ALGEBRA FRAC INT
--R cCoth : % -> % if Coef has ALGEBRA FRAC INT
--R cCsc : % -> % if Coef has ALGEBRA FRAC INT
--R cCsch : % -> % if Coef has ALGEBRA FRAC INT
--R cExp : % -> % if Coef has ALGEBRA FRAC INT
--R cLog : % -> % if Coef has ALGEBRA FRAC INT
--R cPower : (% , Coef) -> % if Coef has ALGEBRA FRAC INT
--R cRationalPower : (% , Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R cSec : % -> % if Coef has ALGEBRA FRAC INT
--R cSech : % -> % if Coef has ALGEBRA FRAC INT
--R cSin : % -> % if Coef has ALGEBRA FRAC INT
--R cSinh : % -> % if Coef has ALGEBRA FRAC INT
--R cTan : % -> % if Coef has ALGEBRA FRAC INT
--R cTanh : % -> % if Coef has ALGEBRA FRAC INT
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(% , "failed") if Coef has CHARNZ
--R coerce : Coef -> % if Coef has COMRING
--R coerce : % -> % if Coef has INTDOM
--R coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
--R differentiate : % -> % if Coef has *: (Integer, Coef) -> Coef
--R differentiate : (% , NonNegativeInteger) -> % if Coef has *: (Integer, Coef) -> Coef
--R differentiate : (% , Symbol) -> % if Coef has *: (Integer, Coef) -> Coef and Coef has PDRING SYMBOL
--R differentiate : (% , List Symbol) -> % if Coef has *: (Integer, Coef) -> Coef and Coef has PDRING SYMBOL
--R differentiate : (% , Symbol, NonNegativeInteger) -> % if Coef has *: (Integer, Coef) -> Coef and Coef has PDRING SYMBOL
--R differentiate : (% , List Symbol, List NonNegativeInteger) -> % if Coef has *: (Integer, Coef) -> Coef and Coef has PDRING SYMBOL
--R ?.? : (% , %) -> % if Integer has SGROUP
--R eval : (% , Coef) -> Stream Coef if Coef has **: (Coef, Integer) -> Coef
--R exquo : (% , %) -> Union(% , "failed") if Coef has INTDOM
--R getRef : % -> Reference OrderedCompletion Integer
--R getStream : % -> Stream Record(k: Integer, c: Coef)
--R iExquo : (% , %, Boolean) -> Union(% , "failed")
--R integrate : % -> % if Coef has ALGEBRA FRAC INT
--R makeSeries : (Reference OrderedCompletion Integer, Stream Record(k: Integer, c: Coef)) -> %
--R monomial : (% , List SingletonAsOrderedSet, List Integer) -> %
--R monomial : (% , SingletonAsOrderedSet, Integer) -> %
--R multiplyCoefficients : ((Integer -> Coef), %) -> %
--R multiplyExponents : (% , PositiveInteger) -> %

```

```

--R series : Stream Record(k: Integer,c: Coef) -> %
--R seriesToOutputForm : (Stream Record(k: Integer,c: Coef),Reference OrderedCompletion Integer) -> %
--R subtractIfCan : (%,% ) -> Union(%,"failed")
--R terms : % -> Stream Record(k: Integer,c: Coef)
--R truncate : (%,Integer,Integer) -> %
--R unit? : % -> Boolean if Coef has INTDOM
--R unitCanonical : % -> % if Coef has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if Coef has INTDOM
--R variables : % -> List SingletonAsOrderedSet
--R
--E 1

)spool
)lisp (bye)

```

---

### — InnerSparseUnivariatePowerSeries.help —

```

=====
InnerSparseUnivariatePowerSeries examples
=====

```

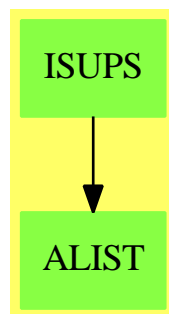
```

See Also:
o )show InnerSparseUnivariatePowerSeries

```

---

## 10.26.1 InnerSparseUnivariatePowerSeries (ISUPS)



**Exports:**

|                 |                      |                    |                    |
|-----------------|----------------------|--------------------|--------------------|
| 0               | 1                    | approximate        | associates?        |
| cAcos           | cAcosh               | cAcot              | cAcoth             |
| cAcsc           | cAcsch               | cAsec              | cAsech             |
| cAsin           | cAsinh               | cAtan              | cAtanh             |
| cCos            | cCosh                | cCot               | cCoth              |
| cCsc            | cCsch                | center             | cExp               |
| cLog            | coefficient          | cPower             | cRationalPower     |
| cSec            | cSech                | cSin               | cSinh              |
| cTan            | cTanh                | characteristic     | charthRoot         |
| coerce          | complete             | D                  | differentiate      |
| degree          | eval                 | exquo              | extend             |
| getRef          | getStream            | hash               | iCompose           |
| iExquo          | integrate            | latex              | leadingCoefficient |
| leadingMonomial | makeSeries           | map                | monomial           |
| monomial?       | multiplyCoefficients | multiplyExponents  | one?               |
| order           | pole?                | recip              | reductum           |
| sample          | series               | seriesToOutputForm | subtractIfCan      |
| taylorQuoByVar  | terms                | truncate           | unit?              |
| unitCanonical   | unitNormal           | variable           | variables          |
| zero?           | ?*                   | ?**?               | ?+?                |
| ?-?             | -?                   | ?=?                | ?^?                |
| ??              | ?~=?                 | ?/?                | ?^?                |
| ??              |                      |                    |                    |

— domain ISUPS InnerSparseUnivariatePowerSeries —

```

)abbrev domain ISUPS InnerSparseUnivariatePowerSeries
++ Author: Clifton J. Williamson
++ Date Created: 28 October 1994
++ Date Last Updated: 9 March 1995
++ Basic Operations:
++ Related Domains: SparseUnivariateTaylorSeries, SparseUnivariateLaurentSeries
++   SparseUnivariatePuisseuxSeries
++ Also See:
++ AMS Classifications:
++ Keywords: sparse, series
++ Examples:
++ References:
++ Description:
++ InnerSparseUnivariatePowerSeries is an internal domain
++ used for creating sparse Taylor and Laurent series.

```

```

InnerSparseUnivariatePowerSeries(Coef): Exports == Implementation where
  Coef  : Ring
  B     ==> Boolean
  COM   ==> OrderedCompletion Integer
  I     ==> Integer

```



```

L      ==> List
NNI    ==> NonNegativeInteger
OUT    ==> OutputForm
PI     ==> PositiveInteger
REF    ==> Reference OrderedCompletion Integer
RN     ==> Fraction Integer
Term   ==> Record(k:Integer,c:Coef)
SG     ==> String
ST     ==> Stream Term

Exports ==> UnivariatePowerSeriesCategory(Coef,Integer) with
makeSeries: (REF,ST) -> %
  ++ makeSeries(refer,str) creates a power series from the reference
  ++ \spad{refer} and the stream \spad{str}.
getRef: % -> REF
  ++ getRef(f) returns a reference containing the order to which the
  ++ terms of f have been computed.
getStream: % -> ST
  ++ getStream(f) returns the stream of terms representing the series f.
series: ST -> %
  ++ series(st) creates a series from a stream of non-zero terms,
  ++ where a term is an exponent-coefficient pair. The terms in the
  ++ stream should be ordered by increasing order of exponents.
monomial?: % -> B
  ++ monomial?(f) tests if f is a single monomial.
multiplyCoefficients: (I -> Coef,%) -> %
  ++ multiplyCoefficients(fn,f) returns the series
  ++ \spad{sum(fn(n) * an * x^n,n = n0..)},
  ++ where f is the series \spad{sum(an * x^n,n = n0..)}.
iExquo: (%,%,B) -> Union(%, "failed")
  ++ iExquo(f,g,taylor?) is the quotient of the power series f and g.
  ++ If \spad{taylor?} is \spad{true}, then we must have
  ++ \spad{order(f) >= order(g)}.
taylorQuoByVar: % -> %
  ++ taylorQuoByVar(a0 + a1 x + a2 x**2 + ...)
  ++ returns \spad{a1 + a2 x + a3 x**2 + ...}
iCompose: (%,%) -> %
  ++ iCompose(f,g) returns \spad{f(g(x))}. This is an internal function
  ++ which should only be called for Taylor series \spad{f(x)} and
  ++ \spad{g(x)} such that the constant coefficient of \spad{g(x)} is zero.
seriesToOutputForm: (ST,REF,Symbol,Coef,RN) -> OutputForm
  ++ seriesToOutputForm(st,refer,var,cen,r) prints the series
  ++ \spad{f((var - cen)^r)}.
if Coef has Algebra Fraction Integer then
  integrate: % -> %
    ++ integrate(f(x)) returns an anti-derivative of the power series
    ++ \spad{f(x)} with constant coefficient 0.
    ++ Warning: function does not check for a term of degree -1.
  cPower: (%,Coef) -> %
    ++ cPower(f,r) computes \spad{f^r}, where f has constant coefficient 1.

```

```

    ++ For use when the coefficient ring is commutative.
cRationalPower: (% ,RN) -> %
    ++ cRationalPower(f,r) computes \spad{f^r}.
    ++ For use when the coefficient ring is commutative.
cExp: % -> %
    ++ cExp(f) computes the exponential of the power series f.
    ++ For use when the coefficient ring is commutative.
cLog: % -> %
    ++ cLog(f) computes the logarithm of the power series f.
    ++ For use when the coefficient ring is commutative.
cSin: % -> %
    ++ cSin(f) computes the sine of the power series f.
    ++ For use when the coefficient ring is commutative.
cCos: % -> %
    ++ cCos(f) computes the cosine of the power series f.
    ++ For use when the coefficient ring is commutative.
cTan: % -> %
    ++ cTan(f) computes the tangent of the power series f.
    ++ For use when the coefficient ring is commutative.
cCot: % -> %
    ++ cCot(f) computes the cotangent of the power series f.
    ++ For use when the coefficient ring is commutative.
cSec: % -> %
    ++ cSec(f) computes the secant of the power series f.
    ++ For use when the coefficient ring is commutative.
cCsc: % -> %
    ++ cCsc(f) computes the cosecant of the power series f.
    ++ For use when the coefficient ring is commutative.
cAsin: % -> %
    ++ cAsin(f) computes the arcsine of the power series f.
    ++ For use when the coefficient ring is commutative.
cAcos: % -> %
    ++ cAcos(f) computes the arccosine of the power series f.
    ++ For use when the coefficient ring is commutative.
cAtan: % -> %
    ++ cAtan(f) computes the arctangent of the power series f.
    ++ For use when the coefficient ring is commutative.
cAcot: % -> %
    ++ cAcot(f) computes the arccotangent of the power series f.
    ++ For use when the coefficient ring is commutative.
cAsec: % -> %
    ++ cAsec(f) computes the arcsecant of the power series f.
    ++ For use when the coefficient ring is commutative.
cAcsc: % -> %
    ++ cAcsc(f) computes the arccosecant of the power series f.
    ++ For use when the coefficient ring is commutative.
cSinh: % -> %
    ++ cSinh(f) computes the hyperbolic sine of the power series f.
    ++ For use when the coefficient ring is commutative.
cCosh: % -> %

```

```

    ++ cCosh(f) computes the hyperbolic cosine of the power series f.
    ++ For use when the coefficient ring is commutative.
cTanh: % -> %
    ++ cTanh(f) computes the hyperbolic tangent of the power series f.
    ++ For use when the coefficient ring is commutative.
cCoth: % -> %
    ++ cCoth(f) computes the hyperbolic cotangent of the power series f.
    ++ For use when the coefficient ring is commutative.
cSech: % -> %
    ++ cSech(f) computes the hyperbolic secant of the power series f.
    ++ For use when the coefficient ring is commutative.
cCsch: % -> %
    ++ cCsch(f) computes the hyperbolic cosecant of the power series f.
    ++ For use when the coefficient ring is commutative.
cAsinh: % -> %
    ++ cAsinh(f) computes the inverse hyperbolic sine of the power
    ++ series f. For use when the coefficient ring is commutative.
cAcosh: % -> %
    ++ cAcosh(f) computes the inverse hyperbolic cosine of the power
    ++ series f. For use when the coefficient ring is commutative.
cAtanh: % -> %
    ++ cAtanh(f) computes the inverse hyperbolic tangent of the power
    ++ series f. For use when the coefficient ring is commutative.
cAcoth: % -> %
    ++ cAcoth(f) computes the inverse hyperbolic cotangent of the power
    ++ series f. For use when the coefficient ring is commutative.
cAsech: % -> %
    ++ cAsech(f) computes the inverse hyperbolic secant of the power
    ++ series f. For use when the coefficient ring is commutative.
cAcsch: % -> %
    ++ cAcsch(f) computes the inverse hyperbolic cosecant of the power
    ++ series f. For use when the coefficient ring is commutative.

Implementation ==> add
import REF

Rep := Record(%ord: REF,%str: Stream Term)
-- when the value of 'ord' is n, this indicates that all non-zero
-- terms of order up to and including n have been computed;
-- when 'ord' is plusInfinity, all terms have been computed;
-- lazy evaluation of 'str' has the side-effect of modifying the value
-- of 'ord'

--% Local functions

makeTerm:      (Integer,Coef) -> Term
getCoef:       Term -> Coef
getExpon:      Term -> Integer
iSeries:       (ST,REF) -> ST
iExtend:       (ST,COM,REF) -> ST

```

```

iTruncate0:      (ST,REF,REF,COM,I,I) -> ST
iTruncate:       (% ,COM,I) -> %
iCoefficient:    (ST,Integer) -> Coef
iOrder:          (ST,COM,REF) -> I
iMap1:           ((Coef,I) -> Coef,I -> I,B,ST,REF,REF,Integer) -> ST
iMap2:           ((Coef,I) -> Coef,I -> I,B,%) -> %
iPlus1:          ((Coef,Coef) -> Coef,ST,REF,ST,REF,REF,I) -> ST
iPlus2:          ((Coef,Coef) -> Coef,%,%) -> %
productByTerm:   (Coef,I,ST,REF,REF,I) -> ST
productLazyEval: (ST,REF,ST,REF,COM) -> Void
iTimes:          (ST,REF,ST,REF,REF,I) -> ST
iDivide:         (ST,REF,ST,REF,Coef,I,REF,I) -> ST
divide:          (% ,I,%,I,Coef) -> %
compose0:        (ST,REF,ST,REF,I,%,%,I,REF,I) -> ST
factorials?:     () -> Boolean
termOutput:      (RN,Coef,OUT) -> OUT
showAll?:        () -> Boolean

--% macros

makeTerm(exp,coef) == [exp,coef]
getCoef term == term.c
getExpon term == term.k

makeSeries(refer,x) == [refer,x]
getRef ups == ups.%ord
getStream ups == ups.%str

--% creation and destruction of series

monomial(coef,expon) ==
  nix : ST := empty()
  st :=
    zero? coef => nix
    concat(makeTerm(expon,coef),nix)
  makeSeries(ref plusInfinity(),st)

monomial? ups == (not empty? getStream ups) and (empty? rst getStream ups)

coerce(n:I)      == n :: Coef :: %
coerce(r:Coef) == monomial(r,0)

iSeries(x,refer) ==
  empty? x => (setelt(refer,plusInfinity()); empty())
  setelt(refer,(getExpon first x) :: COM)
  concat(first x,iSeries(rst x,refer))

series(x:ST) ==
  empty? x => 0
  n := getExpon first x; refer := ref(n :: COM)

```

```

makeSeries(refer,iSeries(x,refer))

--% values

characteristic() == characteristic()$Coef

0 == monomial(0,0)
1 == monomial(1,0)

iExtend(st,n,refer) ==
  (elt refer) < n =>
    explicitlyEmpty? st => (setelt(refer,plusInfinity()); st)
    explicitEntries? st => iExtend(rst st,n,refer)
    iExtend(lazyEvaluate st,n,refer)
  st

extend(x,n) == (iExtend(getStream x,n :: COM,getRef x); x)
complete x == (iExtend(getStream x,plusInfinity(),getRef x); x)

iTruncate0(x,xRefer,refer,minExp,maxExp,n) == delay
  explicitlyEmpty? x => (setelt(refer,plusInfinity()); empty())
  nn := n :: COM
  while (elt xRefer) < nn repeat lazyEvaluate x
  explicitEntries? x =>
    (nx := getExpon(xTerm := frst x)) > maxExp =>
      (setelt(refer,plusInfinity()); empty())
    setelt(refer,nx :: COM)
    (nx :: COM) >= minExp =>
      concat(makeTerm(nx,getCoef xTerm),_
        iTruncate0(rst x,xRefer,refer,minExp,maxExp,nx + 1))
    iTruncate0(rst x,xRefer,refer,minExp,maxExp,nx + 1)
  -- can't have elt(xRefer) = infy unless all terms have been computed
  degr := retract(elt xRefer)@I
  setelt(refer,degr :: COM)
  iTruncate0(x,xRefer,refer,minExp,maxExp,degr + 1)

iTruncate(ups,minExp,maxExp) ==
  x := getStream ups; xRefer := getRef ups
  explicitlyEmpty? x => 0
  explicitEntries? x =>
    deg := getExpon frst x
    refer := ref((deg - 1) :: COM)
    makeSeries(refer,iTruncate0(x,xRefer,refer,minExp,maxExp,deg))
  -- can't have elt(xRefer) = infy unless all terms have been computed
  degr := retract(elt xRefer)@I
  refer := ref(degr :: COM)
  makeSeries(refer,iTruncate0(x,xRefer,refer,minExp,maxExp,degr + 1))

truncate(ups,n) == iTruncate(ups,minusInfinity(),n)
truncate(ups,n1,n2) ==

```

```

    if n1 > n2 then (n1,n2) := (n2,n1)
    iTruncate(ups,n1 :: COM,n2)

iCoefficient(st,n) ==
  explicitEntries? st =>
    term := frst st
    (expon := getExpon term) > n => 0
    expon = n => getCof term
    iCoefficient(rst st,n)
  0

coefficient(x,n) == (extend(x,n); iCoefficient(getStream x,n))
elt(x:%,n:Integer) == coefficient(x,n)

iOrder(st,n,refer) ==
  explicitlyEmpty? st => error "order: series has infinite order"
  explicitEntries? st =>
    ((r := getExpon frst st) :: COM) >= n => retract(n)@Integer
    r
  -- can't have elt(xRefer) = infty unless all terms have been computed
  degr := retract(elt refer)@I
  (degr :: COM) >= n => retract(n)@Integer
  iOrder(lazyEvaluate st,n,refer)

order x == iOrder(getStream x,plusInfinity(),getRef x)
order(x,n) == iOrder(getStream x,n :: COM,getRef x)

terms x == getStream x

--% predicates

zero? ups ==
  x := getStream ups; ref := getRef ups
  whatInfinity(n := elt ref) = 1 => explicitlyEmpty? x
  count : NNI := _$streamCount$Lisp
  for i in 1..count repeat
    explicitlyEmpty? x => return true
    explicitEntries? x => return false
  lazyEvaluate x
  false

ups1 = ups2 == zero?(ups1 - ups2)

--% arithmetic

iMap1(cFcn,eFcn,check?,x,xRefer,refer,n) == delay
  -- when this function is called, all terms in 'x' of order < n have been
  -- computed and we compute the eFcn(n)th order coefficient of the result
  explicitlyEmpty? x => (setelt(refer,plusInfinity()); empty())
  -- if terms in 'x' up to order n have not been computed,

```

```

-- apply lazy evaluation
nn := n :: COM
while (elt xRefer) < nn repeat lazyEvaluate x
-- 'x' may now be empty: retest
explicitlyEmpty? x => (setelt(refer,plusInfinity()); empty())
-- must have nx >= n
explicitEntries? x =>
  xCoef := getCoef(xTerm := frst x); nx := getExpon xTerm
  newCoef := cFcn(xCoef,nx); m := eFcn nx
  setelt(refer,m :: COM)
  not check? =>
    concat(makeTerm(m,newCoef),_
      iMap1(cFcn,eFcn,check?,rst x,xRefer,refer,nx + 1))
  zero? newCoef => iMap1(cFcn,eFcn,check?,rst x,xRefer,refer,nx + 1)
  concat(makeTerm(m,newCoef),_
    iMap1(cFcn,eFcn,check?,rst x,xRefer,refer,nx + 1))
-- can't have elt(xRefer) = infty unless all terms have been computed
degr := retract(elt xRefer)@I
setelt(refer,eFcn(degr) :: COM)
iMap1(cFcn,eFcn,check?,x,xRefer,refer,degr + 1)

iMap2(cFcn,eFcn,check?,ups) ==
-- 'eFcn' must be a strictly increasing function,
-- i.e. i < j => eFcn(i) < eFcn(j)
xRefer := getRef ups; x := getStream ups
explicitlyEmpty? x => 0
explicitEntries? x =>
  deg := getExpon frst x
  refer := ref(eFcn(deg - 1) :: COM)
  makeSeries(refer,iMap1(cFcn,eFcn,check?,x,xRefer,refer,deg))
-- can't have elt(xRefer) = infty unless all terms have been computed
degr := retract(elt xRefer)@I
refer := ref(eFcn(degr) :: COM)
makeSeries(refer,iMap1(cFcn,eFcn,check?,x,xRefer,refer,degr + 1))

map(fcn,x) == iMap2((y,n) +-> fcn(y), z +-> z, true, x)
differentiate x == iMap2((y,n) +-> n*y, z +-> z - 1, true, x)
multiplyCoefficients(f,x) == iMap2((y,n) +-> f(n)*y, z +-> z, true, x)
multiplyExponents(x,n) == iMap2((y,m) +-> y, z +-> n*z, false, x)

iPlus1(op,x,xRefer,y,yRefer,refer,n) == delay
-- when this function is called, all terms in 'x' and 'y' of order < n
-- have been computed and we are computing the nth order coefficient of
-- the result; note the 'op' is either '+' or '-'
explicitlyEmpty? x =>
  iMap1((x1,m) +-> op(0,x1), z +-> z, false, y, yRefer, refer, n)
explicitlyEmpty? y =>
  iMap1((x1,m) +-> op(x1,0), z +-> z, false, x, xRefer, refer, n)
-- if terms up to order n have not been computed,
-- apply lazy evaluation

```

```

nn := n :: COM
while (elt xRefer) < nn repeat lazyEvaluate x
while (elt yRefer) < nn repeat lazyEvaluate y
-- 'x' or 'y' may now be empty: retest
explicitlyEmpty? x =>
  iMap1((x1,m) +-> op(0,x1), z +-> z, false, y, yRefer, refer, n)
explicitlyEmpty? y =>
  iMap1((x1,m) +-> op(x1,0), z +-> z, false, x, xRefer, refer, n)
-- must have nx >= n, ny >= n
-- both x and y have explicit terms
explicitEntries?(x) and explicitEntries?(y) =>
  xCoef := getCoef(xTerm := frst x); nx := getExpon xTerm
  yCoef := getCoef(yTerm := frst y); ny := getExpon yTerm
  nx = ny =>
    setelt(refer,nx :: COM)
    zero? (coef := op(xCoef,yCoef)) =>
      iPlus1(op,rst x,xRefer,rst y,yRefer,refer,nx + 1)
      concat(makeTerm(nx,coef),_
        iPlus1(op,rst x,xRefer,rst y,yRefer,refer,nx + 1))
  nx < ny =>
    setelt(refer,nx :: COM)
    concat(makeTerm(nx,op(xCoef,0)),_
      iPlus1(op,rst x,xRefer,y,yRefer,refer,nx + 1))
  setelt(refer,ny :: COM)
  concat(makeTerm(ny,op(0,yCoef)),_
    iPlus1(op,x,xRefer,rst y,yRefer,refer,ny + 1))
-- y has no term of degree n
explicitEntries? x =>
  xCoef := getCoef(xTerm := frst x); nx := getExpon xTerm
  -- can't have elt(yRefer) = infy unless all terms have been computed
  (degr := retract(elt yRefer)@I) < nx =>
    setelt(refer,elt yRefer)
    iPlus1(op,x,xRefer,y,yRefer,refer,degr + 1)
  setelt(refer,nx :: COM)
  concat(makeTerm(nx,op(xCoef,0)),_
    iPlus1(op,rst x,xRefer,y,yRefer,refer,nx + 1))
-- x has no term of degree n
explicitEntries? y =>
  yCoef := getCoef(yTerm := frst y); ny := getExpon yTerm
  -- can't have elt(xRefer) = infy unless all terms have been computed
  (degr := retract(elt xRefer)@I) < ny =>
    setelt(refer,elt xRefer)
    iPlus1(op,x,xRefer,y,yRefer,refer,degr + 1)
  setelt(refer,ny :: COM)
  concat(makeTerm(ny,op(0,yCoef)),_
    iPlus1(op,x,xRefer,rst y,yRefer,refer,ny + 1))
-- neither x nor y has a term of degree n
setelt(refer,xyRef := min(elt xRefer,elt yRefer))
-- can't have xyRef = infy unless all terms have been computed
iPlus1(op,x,xRefer,y,yRefer,refer,retract(xyRef)@I + 1)

```



```

iPlus2(op,ups1,ups2) ==
  xRefer := getRef ups1; x := getStream ups1
  xDeg :=
    explicitlyEmpty? x => return map(z +-> op(0$Coef,z),ups2)
    explicitEntries? x => (getExpon first x) - 1
    -- can't have elt(xRefer) = infy unless all terms have been computed
    retract(elt xRefer)@I
  yRefer := getRef ups2; y := getStream ups2
  yDeg :=
    explicitlyEmpty? y => return map(z +-> op(z,0$Coef),ups1)
    explicitEntries? y => (getExpon first y) - 1
    -- can't have elt(yRefer) = infy unless all terms have been computed
    retract(elt yRefer)@I
  deg := min(xDeg,yDeg); refer := ref(deg :: COM)
  makeSeries(refer,iPlus1(op,x,xRefer,y,yRefer,refer,deg + 1))

x + y == iPlus2((xi,yi) +-> xi + yi, x, y)
x - y == iPlus2((xi,yi) +-> xi - yi, x, y)
- y == iMap2((x,n) +-> -x, z +-> z, false, y)

-- gives correct defaults for I, NNI and PI
n:I * x:% == (zero? n => 0; map(z +-> n*z, x))
n:NNI * x:% == (zero? n => 0; map(z +-> n*z, x))
n:PI * x:% == (zero? n => 0; map(z +-> n*z, x))

productByTerm(coef,expon,x,xRefer,refer,n) ==
  iMap1((y,m) +-> coef*y, z +-> z+expon, true, x, xRefer, refer, n)

productLazyEval(x,xRefer,y,yRefer,nn) ==
  explicitlyEmpty?(x) or explicitlyEmpty?(y) => void()
  explicitEntries? x =>
    explicitEntries? y => void()
    xDeg := (getExpon first x) :: COM
    while (xDeg + elt(yRefer)) < nn repeat lazyEvaluate y
    void()
  explicitEntries? y =>
    yDeg := (getExpon first y) :: COM
    while (yDeg + elt(xRefer)) < nn repeat lazyEvaluate x
    void()
  lazyEvaluate x
  -- if x = y, then y may now have explicit entries
  if lazy? y then lazyEvaluate y
  productLazyEval(x,xRefer,y,yRefer,nn)

iTimes(x,xRefer,y,yRefer,refer,n) == delay
  -- when this function is called, we are computing the nth order
  -- coefficient of the product
  productLazyEval(x,xRefer,y,yRefer,n :: COM)
  explicitlyEmpty?(x) or explicitlyEmpty?(y) =>

```

```

    (setelt(refer,plusInfinity()); empty())
-- must have nx + ny >= n
explicitEntries?(x) and explicitEntries?(y) =>
  xCoef := getCoef(xTerm := first x); xExpon := getExpon xTerm
  yCoef := getCoef(yTerm := first y); yExpon := getExpon yTerm
  expon := xExpon + yExpon
  setelt(refer,expon :: COM)
  scRefer := ref(expon :: COM)
  scMult := productByTerm(xCoef,xExpon,rst y,yRefer,scRefer,yExpon + 1)
  prRefer := ref(expon :: COM)
  pr := iTimes(rst x,xRefer,y,yRefer,prRefer,expon + 1)
  sm := iPlus1((a,b) +-> a+b,scMult,scRefer,pr,prRefer,refer,expon + 1)
  zero?(coef := xCoef * yCoef) => sm
  concat(makeTerm(expon,coef),sm)
explicitEntries? x =>
  xExpon := getExpon first x
  -- can't have elt(yRefer) = infy unless all terms have been computed
  degr := retract(elt yRefer)@I
  setelt(refer,(xExpon + degr) :: COM)
  iTimes(x,xRefer,y,yRefer,refer,xExpon + degr + 1)
explicitEntries? y =>
  yExpon := getExpon first y
  -- can't have elt(xRefer) = infy unless all terms have been computed
  degr := retract(elt xRefer)@I
  setelt(refer,(yExpon + degr) :: COM)
  iTimes(x,xRefer,y,yRefer,refer,yExpon + degr + 1)
-- can't have elt(xRefer) = infy unless all terms have been computed
xDegr := retract(elt xRefer)@I
yDegr := retract(elt yRefer)@I
setelt(refer,(xDegr + yDegr) :: COM)
iTimes(x,xRefer,y,yRefer,refer,xDegr + yDegr + 1)

ups1:% * ups2:% ==
  xRefer := getRef ups1; x := getStream ups1
  xDeg :=
    explicitlyEmpty? x => return 0
    explicitEntries? x => (getExpon first x) - 1
    -- can't have elt(xRefer) = infy unless all terms have been computed
    retract(elt xRefer)@I
  yRefer := getRef ups2; y := getStream ups2
  yDeg :=
    explicitlyEmpty? y => return 0
    explicitEntries? y => (getExpon first y) - 1
    -- can't have elt(yRefer) = infy unless all terms have been computed
    retract(elt yRefer)@I
  deg := xDeg + yDeg + 1; refer := ref(deg :: COM)
  makeSeries(refer,iTimes(x,xRefer,y,yRefer,refer,deg + 1))

iDivide(x,xRefer,y,yRefer,rym,m,refer,n) == delay
  -- when this function is called, we are computing the nth order

```

```

-- coefficient of the result
explicitlyEmpty? x => (setelt(refer,plusInfinity()); empty())
-- if terms up to order n - m have not been computed,
-- apply lazy evaluation
nm := (n + m) :: COM
while (elt xRefer) < nm repeat lazyEvaluate x
-- 'x' may now be empty: retest
explicitlyEmpty? x => (setelt(refer,plusInfinity()); empty())
-- must have nx >= n + m
explicitEntries? x =>
  newCoef := getCoef(xTerm := first x) * rym; nx := getExpon xTerm
  prodRefer := ref(nx :: COM)
  prod := productByTerm(-newCoef,nx - m,rst y,yRefer,prodRefer,1)
  sumRefer := ref(nx :: COM)
  sum := iPlus1((a,b)+->a+b,rst x,xRefer,prod,prodRefer,sumRefer,nx + 1)
  setelt(refer,(nx - m) :: COM); term := makeTerm(nx - m,newCoef)
  concat(term,iDivide(sum,sumRefer,y,yRefer,rym,m,refer,nx - m + 1))
-- can't have elt(xRefer) = infy unless all terms have been computed
degr := retract(elt xRefer)@I
setelt(refer,(degr - m) :: COM)
iDivide(x,xRefer,y,yRefer,rym,m,refer,degr - m + 1)

divide(ups1,deg1,ups2,deg2,r) ==
  xRefer := getRef ups1; x := getStream ups1
  yRefer := getRef ups2; y := getStream ups2
  refer := ref((deg1 - deg2) :: COM)
  makeSeries(refer,iDivide(x,xRefer,y,yRefer,r,deg2,refer,deg1 - deg2 + 1))

iExquo(ups1,ups2,taylor?) ==
  xRefer := getRef ups1; x := getStream ups1
  yRefer := getRef ups2; y := getStream ups2
  n : I := 0
  -- try to find first non-zero term in y
  -- give up after 1000 lazy evaluations
  while not explicitEntries? y repeat
    explicitlyEmpty? y => return "failed"
    lazyEvaluate y
    (n := n + 1) > 1000 => return "failed"
  yCoef := getCoef(yTerm := first y); ny := getExpon yTerm
  (ry := recip yCoef) case "failed" => "failed"
  nn := ny :: COM
  if taylor? then
    while (elt(xRefer) < nn) repeat
      explicitlyEmpty? x => return 0
      explicitEntries? x => return "failed"
      lazyEvaluate x
  -- check if ups2 is a monomial
  empty? rst y => iMap2((y1,m) +-> y1*(ry::Coef),z +->z-ny, false, ups1)
  explicitlyEmpty? x => 0
  nx :=

```

```

explicitEntries? x =>
  ((deg := getExpon first x) < ny) and taylor? => return "failed"
  deg - 1
-- can't have elt(xRefer) = infty unless all terms have been computed
retract(elt xRefer)@I
divide(ups1,nx,ups2,ny,ry :: Coef)

taylorQuoByVar ups ==
  iMap2((y,n) +-> y, z +-> z-1,false,ups - monomial(coefficient(ups,0),0))

compose0(x,xRefer,y,yRefer,y0rd,y1,yn0,n0,refer,n) == delay
-- when this function is called, we are computing the nth order
-- coefficient of the composite
explicitlyEmpty? x => (setelt(refer,plusInfinity()); empty())
-- if terms in 'x' up to order n have not been computed,
-- apply lazy evaluation
nn := n :: COM; yy0rd := y0rd :: COM
while (yy0rd * elt(xRefer)) < nn repeat lazyEvaluate x
explicitEntries? x =>
  xCoef := getCoef(xTerm := first x); n1 := getExpon xTerm
  zero? n1 =>
    setelt(refer,n1 :: COM)
    concat(makeTerm(n1,xCoef),_
      compose0(rst x,xRefer,y,yRefer,y0rd,y1,yn0,n0,refer,n1 + 1))
  yn1 := yn0 * y1 ** ((n1 - n0) :: NNI)
  z := getStream yn1; zRefer := getRef yn1
  degr := y0rd * n1; prodRefer := ref((degr - 1) :: COM)
  prod := iMap1((s,k)+->xCoef*s,m+>m,true,z,zRefer,prodRefer,degr)
  coRefer := ref((degr + y0rd - 1) :: COM)
  co := compose0(rst x,xRefer,y,yRefer,y0rd,y1,yn1,n1,coRefer,degr+y0rd)
  setelt(refer,(degr - 1) :: COM)
  iPlus1((a,b)+->a+b,prod,prodRefer,co,coRefer,refer,degr)
-- can't have elt(xRefer) = infty unless all terms have been computed
degr := y0rd * (retract(elt xRefer)@I + 1)
setelt(refer,(degr - 1) :: COM)
compose0(x,xRefer,y,yRefer,y0rd,y1,yn0,n0,refer,degr)

iCompose(ups1,ups2) ==
  x := getStream ups1; xRefer := getRef ups1
  y := getStream ups2; yRefer := getRef ups2
  -- try to compute the order of 'ups2'
  n : I := _$streamCount$Lisp
  for i in 1..n while not explicitEntries? y repeat
    explicitlyEmpty? y => coefficient(ups1,0) :: %
    lazyEvaluate y
  explicitlyEmpty? y => coefficient(ups1,0) :: %
  y0rd : I :=
    explicitEntries? y => getExpon first y
    retract(elt yRefer)@I
  compRefer := ref((-1) :: COM)

```

```

makeSeries(compRefer, _
            compose0(x, xRefer, y, yRefer, yOrd, ups2, 1, 0, compRefer, 0))

if Coef has Algebra Fraction Integer then

    integrate x == iMap2((y, n) +-> 1/(n+1)*y, z +-> z+1, true, x)

--% Fixed point computations

Ys ==> Y$ParadoxicalCombinatorsForStreams(Term)

integ0: (ST, REF, REF, I) -> ST
integ0(x, intRef, ansRef, n) == delay
    nLess1 := (n - 1) :: COM
    while (elt intRef) < nLess1 repeat lazyEvaluate x
    explicitlyEmpty? x => (setelt(ansRef, plusInfinity()); empty())
    explicitEntries? x =>
        xCoef := getCoef(xTerm := frst x); nx := getExpon xTerm
        setelt(ansRef, (n1 := (nx + 1)) :: COM)
        concat(makeTerm(n1, inv(n1 :: RN) * xCoef), _
                integ0(rst x, intRef, ansRef, n1))
    -- can't have elt(intRef) = infy unless all terms have been computed
    degr := retract(elt intRef)@I; setelt(ansRef, (degr + 1) :: COM)
    integ0(x, intRef, ansRef, degr + 2)

integ1: (ST, REF, REF) -> ST
integ1(x, intRef, ansRef) == integ0(x, intRef, ansRef, 1)

lazyInteg: (Coef, () -> ST, REF, REF) -> ST
lazyInteg(a, xf, intRef, ansRef) ==
    ansStr : ST := integ1(delay xf, intRef, ansRef)
    concat(makeTerm(0, a), ansStr)

cPower(f, r) ==
    -- computes f^r. f should have constant coefficient 1.
    fp := differentiate f
    fInv := iExquo(1, f, false) :: %; y := r * fp * fInv
    yRef := getRef y; yStr := getStream y
    intRef := ref((-1) :: COM); ansRef := ref(0 :: COM)
    ansStr :=
        Ys(s+>lazyInteg(1, iTimes(s, ansRef, yStr, yRef, intRef, 0), intRef, ansRef))
    makeSeries(ansRef, ansStr)

iExp: (% , Coef) -> %
iExp(f, cc) ==
    -- computes exp(f). cc = exp coefficient(f, 0)
    fp := differentiate f
    fpRef := getRef fp; fpStr := getStream fp
    intRef := ref((-1) :: COM); ansRef := ref(0 :: COM)
    ansStr :=

```

```

    Ys(s+>lazyInteg(cc,
        iTimes(s,ansRef,fpStr,fpRef,intRef,0),intRef,ansRef))
    makeSeries(ansRef,ansStr)

sincos0: (Coef,Coef,L ST,REF,REF,ST,REF,ST,REF) -> L ST
sincos0(sinc,cosc,list,sinRef,cosRef,fpStr,fpRef,fpStr2,fpRef2) ==
    sinStr := first list; cosStr := second list
    prodRef1 := ref((-1) :: COM); prodRef2 := ref((-1) :: COM)
    prodStr1 := iTimes(cosStr,cosRef,fpStr,fpRef,prodRef1,0)
    prodStr2 := iTimes(sinStr,sinRef,fpStr2,fpRef2,prodRef2,0)
    [lazyInteg(sinc,prodStr1,prodRef1,sinRef),_
    lazyInteg(cosc,prodStr2,prodRef2,cosRef)]

iSincos: (%,Coef,Coef,I) -> Record(%sin: %, %cos: %)
iSincos(f,sinc,cosc,sign) ==
    fp := differentiate f
    fpRef := getRef fp; fpStr := getStream fp
--    fp2 := (one? sign => fp; -fp)
    fp2 := ((sign = 1) => fp; -fp)
    fpRef2 := getRef fp2; fpStr2 := getStream fp2
    sinRef := ref(0 :: COM); cosRef := ref(0 :: COM)
    sincos :=
        Ys(s+>sincos0(sinc,cosc,s,sinRef,cosRef,fpStr,fpRef,fpStr2,fpRef2),2)
    sinStr := (zero? sinc => rst first sincos; first sincos)
    cosStr := (zero? cosc => rst second sincos; second sincos)
    [makeSeries(sinRef,sinStr),makeSeries(cosRef,cosStr)]

tan0: (Coef,ST,REF,ST,REF,I) -> ST
tan0(cc,ansStr,ansRef,fpStr,fpRef,sign) ==
    sqRef := ref((-1) :: COM)
    sqStr := iTimes(ansStr,ansRef,ansStr,ansRef,sqRef,0)
    one : % := 1; oneStr := getStream one; oneRef := getRef one
    yRef := ref((-1) :: COM)
    yStr : ST :=
--    one? sign => iPlus1(#1 + #2,oneStr,oneRef,sqStr,sqRef,yRef,0)
        (sign = 1) => iPlus1((a,b)+->a+b,oneStr,oneRef,sqStr,sqRef,yRef,0)
        iPlus1((a,b)+->a-b,oneStr,oneRef,sqStr,sqRef,yRef,0)
    intRef := ref((-1) :: COM)
    lazyInteg(cc,iTimes(yStr,yRef,fpStr,fpRef,intRef,0),intRef,ansRef)

iTan: (%,%,Coef,I) -> %
iTan(f,fp,cc,sign) ==
    -- computes the tangent (and related functions) of f.
    fpRef := getRef fp; fpStr := getStream fp
    ansRef := ref(0 :: COM)
    ansStr := Ys(s+>tan0(cc,s,ansRef,fpStr,fpRef,sign))
    zero? cc => makeSeries(ansRef,rst ansStr)
    makeSeries(ansRef,ansStr)

--% Error Reporting

```

```

TRCONST : SG := "series expansion involves transcendental constants"
NPOWERS : SG := "series expansion has terms of negative degree"
FPOWERS : SG := "series expansion has terms of fractional degree"
MAYFPOW : SG := "series expansion may have terms of fractional degree"
LOGS : SG := "series expansion has logarithmic term"
NPOWLOG : SG :=
    "series expansion has terms of negative degree or logarithmic term"
NOTINV : SG := "leading coefficient not invertible"

--% Rational powers and transcendental functions

orderOrFailed : % -> Union(I,"failed")
orderOrFailed uts ==
-- returns the order of x or "failed"
-- if -1 is returned, the series is identically zero
x := getStream uts
for n in 0..1000 repeat
    explicitlyEmpty? x => return -1
    explicitEntries? x => return getExpon first x
    lazyEvaluate x
"failed"

RATPOWERS : Boolean := Coef has "**": (Coef,RN) -> Coef
TRANSFCN : Boolean := Coef has TranscendentalFunctionCategory

cRationalPower(uts,r) ==
(ord0 := orderOrFailed uts) case "failed" =>
    error "**: series with many leading zero coefficients"
order := ord0 :: I
(n := order exquo denom(r)) case "failed" =>
    error "**: rational power does not exist"
cc := coefficient(uts,order)
(ccInv := recip cc) case "failed" => error concat("**: ",NOTINV)
ccPow :=
--    one? cc => cc
(cc = 1) => cc
--    one? denom r =>
(denom r) = 1 =>
    not negative?(num := numer r) => cc ** (num :: NNI)
    (ccInv :: Coef) ** ((-num) :: NNI)
RATPOWERS => cc ** r
error "** rational power of coefficient undefined"
uts1 := (ccInv :: Coef) * uts
uts2 := uts1 * monomial(1,-order)
monomial(ccPow,(n :: I) * numer(r)) * cPower(uts2,r :: Coef)

cExp uts ==
zero?(cc := coefficient(uts,0)) => iExp(uts,1)
TRANSFCN => iExp(uts,exp cc)

```

```

    error concat("exp: ",TRCONST)

cLog uts ==
    zero?(cc := coefficient(uts,0)) =>
        error "log: constant coefficient should not be 0"
--    one? cc => integrate(differentiate(uts) * (iExquo(1,uts,true) :: %))
    (cc = 1) => integrate(differentiate(uts) * (iExquo(1,uts,true) :: %))
    TRANSFCN =>
        y := iExquo(1,uts,true) :: %
        (log(cc) :: %) + integrate(y * differentiate(uts))
    error concat("log: ",TRCONST)

sincos: % -> Record(%sin: %, %cos: %)
sincos uts ==
    zero?(cc := coefficient(uts,0)) => iSincos(uts,0,1,-1)
    TRANSFCN => iSincos(uts,sin cc,cos cc,-1)
    error concat("sincos: ",TRCONST)

cSin uts == sincos(uts).%sin
cCos uts == sincos(uts).%cos

cTan uts ==
    zero?(cc := coefficient(uts,0)) => iTan(uts,differentiate uts,0,1)
    TRANSFCN => iTan(uts,differentiate uts,tan cc,1)
    error concat("tan: ",TRCONST)

cCot uts ==
    zero? uts => error "cot: cot(0) is undefined"
    zero?(cc := coefficient(uts,0)) => error error concat("cot: ",NPOWERS)
    TRANSFCN => iTan(uts,-differentiate uts,cot cc,1)
    error concat("cot: ",TRCONST)

cSec uts ==
    zero?(cc := coefficient(uts,0)) => iExquo(1,cCos uts,true) :: %
    TRANSFCN =>
        cosUts := cCos uts
        zero? coefficient(cosUts,0) => error concat("sec: ",NPOWERS)
        iExquo(1,cosUts,true) :: %
    error concat("sec: ",TRCONST)

cCsc uts ==
    zero? uts => error "csc: csc(0) is undefined"
    TRANSFCN =>
        sinUts := cSin uts
        zero? coefficient(sinUts,0) => error concat("csc: ",NPOWERS)
        iExquo(1,sinUts,true) :: %
    error concat("csc: ",TRCONST)

cAsin uts ==
    zero?(cc := coefficient(uts,0)) =>

```



```

    integrate(cRationalPower(1 - uts*uts,-1/2) * differentiate(uts))
TRANSFCN =>
  x := 1 - uts * uts
  cc = 1 or cc = -1 =>
    -- compute order of 'x'
    (ord := orderOrFailed x) case "failed" =>
      error concat("asin: ",MAYFPOW)
    (order := ord :: I) = -1 => return asin(cc) :: %
    odd? order => error concat("asin: ",FPOWERS)
    c0 := asin(cc) :: %
    c0 + integrate(cRationalPower(x,-1/2) * differentiate(uts))
  c0 := asin(cc) :: %
  c0 + integrate(cRationalPower(x,-1/2) * differentiate(uts))
  error concat("asin: ",TRCONST)

cAcos uts ==
zero? uts =>
  TRANSFCN => acos(0)$Coef :: %
  error concat("acos: ",TRCONST)
TRANSFCN =>
  x := 1 - uts * uts
  cc := coefficient(uts,0)
  cc = 1 or cc = -1 =>
    -- compute order of 'x'
    (ord := orderOrFailed x) case "failed" =>
      error concat("acos: ",MAYFPOW)
    (order := ord :: I) = -1 => return acos(cc) :: %
    odd? order => error concat("acos: ",FPOWERS)
    c0 := acos(cc) :: %
    c0 + integrate(-cRationalPower(x,-1/2) * differentiate(uts))
  c0 := acos(cc) :: %
  c0 + integrate(-cRationalPower(x,-1/2) * differentiate(uts))
  error concat("acos: ",TRCONST)

cAtan uts ==
zero?(cc := coefficient(uts,0)) =>
  y := iExquo(1,(1 :: %) + uts*uts,true) :: %
  integrate(y * (differentiate uts))
TRANSFCN =>
  (y := iExquo(1,(1 :: %) + uts*uts,true)) case "failed" =>
    error concat("atan: ",LOGS)
  (atan(cc) :: %) + integrate((y :: %) * (differentiate uts))
  error concat("atan: ",TRCONST)

cAcot uts ==
TRANSFCN =>
  (y := iExquo(1,(1 :: %) + uts*uts,true)) case "failed" =>
    error concat("acot: ",LOGS)
  cc := coefficient(uts,0)
  (acot(cc) :: %) + integrate(-(y :: %) * (differentiate uts))

```

```

error concat("acot: ",TRCONST)

cAsec uts ==
zero?(cc := coefficient(uts,0)) =>
  error "asec: constant coefficient should not be 0"
TRANSFCN =>
  x := uts * uts - 1
  y :=
    cc = 1 or cc = -1 =>
      -- compute order of 'x'
      (ord := orderOrFailed x) case "failed" =>
        error concat("asec: ",MAYFPOW)
      (order := ord :: I) = -1 => return asec(cc) :: %
      odd? order => error concat("asec: ",FPOWERS)
      cRationalPower(x,-1/2) * differentiate(uts)
      cRationalPower(x,-1/2) * differentiate(uts)
      (z := iExquo(y,uts,true)) case "failed" =>
        error concat("asec: ",NOTINV)
      (asec(cc) :: %) + integrate(z :: %)
  error concat("asec: ",TRCONST)

cAcsc uts ==
zero?(cc := coefficient(uts,0)) =>
  error "acsc: constant coefficient should not be 0"
TRANSFCN =>
  x := uts * uts - 1
  y :=
    cc = 1 or cc = -1 =>
      -- compute order of 'x'
      (ord := orderOrFailed x) case "failed" =>
        error concat("acsc: ",MAYFPOW)
      (order := ord :: I) = -1 => return acsc(cc) :: %
      odd? order => error concat("acsc: ",FPOWERS)
      -cRationalPower(x,-1/2) * differentiate(uts)
      -cRationalPower(x,-1/2) * differentiate(uts)
      (z := iExquo(y,uts,true)) case "failed" =>
        error concat("acsc: ",NOTINV)
      (acsc(cc) :: %) + integrate(z :: %)
  error concat("acsc: ",TRCONST)

sinhcosh: % -> Record(%sinh: %, %cosh: %)
sinhcosh uts ==
zero?(cc := coefficient(uts,0)) =>
  tmp := iSincos(uts,0,1,1)
  [tmp.%sin,tmp.%cos]
TRANSFCN =>
  tmp := iSincos(uts,sinh cc,cosh cc,1)
  [tmp.%sin,tmp.%cos]
error concat("sinhcosh: ",TRCONST)

```

```

cSinh uts == sinhcosh(uts).%sinh
cCosh uts == sinhcosh(uts).%cosh

cTanh uts ==
  zero?(cc := coefficient(uts,0)) => iTan(uts,differentiate uts,0,-1)
  TRANSFCN => iTan(uts,differentiate uts,tanh cc,-1)
  error concat("tanh: ",TRCONST)

cCoth uts ==
  tanhUts := cTanh uts
  zero? tanhUts => error "coth: coth(0) is undefined"
  zero? coefficient(tanhUts,0) => error concat("coth: ",NPOWERS)
  iExquo(1,tanhUts,true) :: %

cSech uts ==
  coshUts := cCosh uts
  zero? coefficient(coshUts,0) => error concat("sech: ",NPOWERS)
  iExquo(1,coshUts,true) :: %

cCsch uts ==
  sinhUts := cSinh uts
  zero? coefficient(sinhUts,0) => error concat("csch: ",NPOWERS)
  iExquo(1,sinhUts,true) :: %

cAsinh uts ==
  x := 1 + uts * uts
  zero?(cc := coefficient(uts,0)) => cLog(uts + cRationalPower(x,1/2))
  TRANSFCN =>
    (ord := orderOrFailed x) case "failed" =>
      error concat("asinh: ",MAYFPOW)
    (order := ord :: I) = -1 => return asinh(cc) :: %
    odd? order => error concat("asinh: ",FPOWERS)
    -- the argument to 'log' must have a non-zero constant term
    cLog(uts + cRationalPower(x,1/2))
  error concat("asinh: ",TRCONST)

cAcosh uts ==
  zero? uts =>
    TRANSFCN => acosh(0)$Coef :: %
    error concat("acosh: ",TRCONST)
  TRANSFCN =>
    cc := coefficient(uts,0); x := uts*uts - 1
    cc = 1 or cc = -1 =>
      -- compute order of 'x'
      (ord := orderOrFailed x) case "failed" =>
        error concat("acosh: ",MAYFPOW)
      (order := ord :: I) = -1 => return acosh(cc) :: %
      odd? order => error concat("acosh: ",FPOWERS)
      -- the argument to 'log' must have a non-zero constant term
      cLog(uts + cRationalPower(x,1/2))

```

```

    cLog(uts + cRationalPower(x,1/2))
    error concat("acosh: ",TRCONST)

cAtanh uts ==
  half := inv(2 :: RN) :: Coef
  zero?(cc := coefficient(uts,0)) =>
    half * (cLog(1 + uts) - cLog(1 - uts))
  TRANSFCN =>
    cc = 1 or cc = -1 => error concat("atanh: ",LOGS)
    half * (cLog(1 + uts) - cLog(1 - uts))
  error concat("atanh: ",TRCONST)

cAcoth uts ==
  zero? uts =>
    TRANSFCN => acoth(0)$Coef :: %
    error concat("acoth: ",TRCONST)
  TRANSFCN =>
    cc := coefficient(uts,0); half := inv(2 :: RN) :: Coef
    cc = 1 or cc = -1 => error concat("acoth: ",LOGS)
    half * (cLog(uts + 1) - cLog(uts - 1))
  error concat("acoth: ",TRCONST)

cAsech uts ==
  zero? uts => error "asech: asech(0) is undefined"
  TRANSFCN =>
    zero?(cc := coefficient(uts,0)) =>
      error concat("asech: ",NPOWLOG)
    x := 1 - uts * uts
    cc = 1 or cc = -1 =>
      -- compute order of 'x'
      (ord := orderOrFailed x) case "failed" =>
        error concat("asech: ",MAYFPOW)
      (order := ord :: I) = -1 => return asech(cc) :: %
      odd? order => error concat("asech: ",FPOWERS)
      (utsInv := iExquo(1,uts,true)) case "failed" =>
        error concat("asech: ",NOTINV)
      cLog((1 + cRationalPower(x,1/2)) * (utsInv :: %))
      (utsInv := iExquo(1,uts,true)) case "failed" =>
        error concat("asech: ",NOTINV)
      cLog((1 + cRationalPower(x,1/2)) * (utsInv :: %))
    error concat("asech: ",TRCONST)

cAcsch uts ==
  zero? uts => error "acsch: acsch(0) is undefined"
  TRANSFCN =>
    zero?(cc := coefficient(uts,0)) => error concat("acsch: ",NPOWLOG)
    x := uts * uts + 1
    -- compute order of 'x'
    (ord := orderOrFailed x) case "failed" =>
      error concat("acsc: ",MAYFPOW)

```

```

        (order := ord :: I) = -1 => return acsch(cc) :: %
        odd? order => error concat("acsch: ",FPOWERS)
        (utsInv := iExquo(1,uts,true)) case "failed" =>
            error concat("acsch: ",NOTINV)
        cLog((1 + cRationalPower(x,1/2)) * (utsInv :: %))
        error concat("acsch: ",TRCONST)

--% Output forms

-- check a global Lisp variable
factorials?() == false

termOutput(k,c,vv) ==
-- creates a term c * vv ** k
    k = 0 => c :: OUT
    mon := (k = 1 => vv; vv ** (k :: OUT))
--      if factorials?() and k > 1 then
--          c := factorial(k)$IntegerCombinatoricFunctions * c
--          mon := mon / hconcat(k :: OUT,"!" :: OUT)
    c = 1 => mon
    c = -1 => -mon
    (c :: OUT) * mon

-- check a global Lisp variable
showAll?() == true

seriesToOutputForm(st,refer,var,cen,r) ==
    vv :=
        zero? cen => var :: OUT
        paren(var :: OUT - cen :: OUT)
    l : L OUT := empty()
    while explicitEntries? st repeat
        term := frst st
        l := concat(termOutput(getExpon(term) * r,getCoef term,vv),l)
        st := rst st
    l :=
        explicitlyEmpty? st => l
        (deg := retractIfCan(elt refer)@Union(I,"failed")) case I =>
            concat(prefix("0" :: OUT,[vv ** (((deg :: I) + 1) * r) :: OUT]),l)
    l
    empty? l => (0$Coef) :: OUT
    reduce("+",reverse_! l)

```

---

— ISUPS.dotabb —

"ISUPS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ISUPS"]

"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]  
 "ISUPS" -> "ALIST"

## 10.27 domain INTABL InnerTable

— InnerTable.input —

```
)set break resume
)sys rm -f InnerTable.output
)spool InnerTable.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show InnerTable
--R InnerTable(Key: SetCategory,Entry: SetCategory,addDom) where
--R   addDom: TableAggregate(Key,Entry) with
--R       finiteAggregate is a domain constructor
--R Abbreviation for InnerTable is INTABL
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for INTABL
--R
--R----- Operations -----
--R copy : % -> %                dictionary : () -> %
--R elt : (% ,Key,Entry) -> Entry    ?.? : (% ,Key) -> Entry
--R empty : () -> %                empty? : % -> Boolean
--R entries : % -> List Entry        eq? : (% ,%) -> Boolean
--R index? : (Key,%) -> Boolean      indices : % -> List Key
--R key? : (Key,%) -> Boolean        keys : % -> List Key
--R map : ((Entry -> Entry),%) -> %   qelt : (% ,Key) -> Entry
--R sample : () -> %                setelt : (% ,Key,Entry) -> Entry
--R table : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?? : (% ,%) -> Boolean if Record(key: Key,entry: Entry) has SETCAT or Entry has SETCAT
--R any? : ((Entry -> Boolean),%) -> Boolean if $ has finiteAggregate
--R any? : ((Record(key: Key,entry: Entry) -> Boolean),%) -> Boolean if $ has finiteAggregate
--R bag : List Record(key: Key,entry: Entry) -> %
--R coerce : % -> OutputForm if Record(key: Key,entry: Entry) has SETCAT or Entry has SETCAT
--R construct : List Record(key: Key,entry: Entry) -> %
--R convert : % -> InputForm if Record(key: Key,entry: Entry) has KONVERT INFORM
--R count : ((Entry -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R count : (Entry,%) -> NonNegativeInteger if $ has finiteAggregate and Entry has SETCAT
--R count : (Record(key: Key,entry: Entry),%) -> NonNegativeInteger if $ has finiteAggregate and Record
```

```

--R count : ((Record(key: Key,entry: Entry) -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R dictionary : List Record(key: Key,entry: Entry) -> %
--R entry? : (Entry,%) -> Boolean if $ has finiteAggregate and Entry has SETCAT
--R eval : (%,List Equation Entry) -> % if Entry has EVALAB Entry and Entry has SETCAT
--R eval : (%,Equation Entry) -> % if Entry has EVALAB Entry and Entry has SETCAT
--R eval : (%,Entry,Entry) -> % if Entry has EVALAB Entry and Entry has SETCAT
--R eval : (%,List Entry,List Entry) -> % if Entry has EVALAB Entry and Entry has SETCAT
--R eval : (%,List Record(key: Key,entry: Entry),List Record(key: Key,entry: Entry)) -> % if $ has finiteAggregate
--R eval : (%,Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> % if Record(key: Key,entry: Entry) has SETCAT
--R eval : (%,Equation Record(key: Key,entry: Entry)) -> % if Record(key: Key,entry: Entry) has SETCAT
--R eval : (%,List Equation Record(key: Key,entry: Entry)) -> % if Record(key: Key,entry: Entry) has SETCAT
--R every? : ((Entry -> Boolean),%) -> Boolean if $ has finiteAggregate
--R every? : ((Record(key: Key,entry: Entry) -> Boolean),%) -> Boolean if $ has finiteAggregate
--R extract! : % -> Record(key: Key,entry: Entry)
--R fill! : (%,Entry) -> % if $ has shallowlyMutable
--R find : ((Record(key: Key,entry: Entry) -> Boolean),%) -> Union(Record(key: Key,entry: Entry))
--R first : % -> Entry if Key has ORDSET
--R hash : % -> SingleInteger if Record(key: Key,entry: Entry) has SETCAT or Entry has SETCAT
--R insert! : (Record(key: Key,entry: Entry),%) -> %
--R inspect : % -> Record(key: Key,entry: Entry)
--R latex : % -> String if Record(key: Key,entry: Entry) has SETCAT or Entry has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map : ((Entry,Entry) -> Entry),%,%) -> %
--R map : ((Record(key: Key,entry: Entry) -> Record(key: Key,entry: Entry)),%) -> %
--R map! : ((Entry -> Entry),%) -> % if $ has shallowlyMutable
--R map! : ((Record(key: Key,entry: Entry) -> Record(key: Key,entry: Entry)),%) -> % if $ has finiteAggregate
--R maxIndex : % -> Key if Key has ORDSET
--R member? : (Entry,%) -> Boolean if $ has finiteAggregate and Entry has SETCAT
--R member? : (Record(key: Key,entry: Entry),%) -> Boolean if $ has finiteAggregate and Record(key: Key,entry: Entry) has SETCAT
--R members : % -> List Entry if $ has finiteAggregate
--R members : % -> List Record(key: Key,entry: Entry) if $ has finiteAggregate
--R minIndex : % -> Key if Key has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List Entry if $ has finiteAggregate
--R parts : % -> List Record(key: Key,entry: Entry) if $ has finiteAggregate
--R qsetelt! : (%,Key,Entry) -> Entry if $ has shallowlyMutable
--R reduce : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> Record(key: Key,entry: Entry)),%) -> %
--R reduce : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> Record(key: Key,entry: Entry)),%) -> %
--R reduce : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> Record(key: Key,entry: Entry)),%) -> %
--R remove : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (Record(key: Key,entry: Entry),%) -> % if $ has finiteAggregate and Record(key: Key,entry: Entry) has SETCAT
--R remove! : (Key,%) -> Union(Entry,"failed")
--R remove! : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finiteAggregate
--R remove! : (Record(key: Key,entry: Entry),%) -> % if $ has finiteAggregate
--R removeDuplicates : % -> % if $ has finiteAggregate and Record(key: Key,entry: Entry) has SETCAT
--R search : (Key,%) -> Union(Entry,"failed")
--R select : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finiteAggregate
--R select! : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R swap! : (%,Key,Key) -> Void if $ has shallowlyMutable

```

```

--R table : List Record(key: Key,entry: Entry) -> %
--R ?~=? : (%,% ) -> Boolean if Record(key: Key,entry: Entry) has SETCAT or Entry has SETCAT
--R
--E 1

)spool
)lisp (bye)

```

— InnerTable.help —

```

=====
InnerTable examples
=====

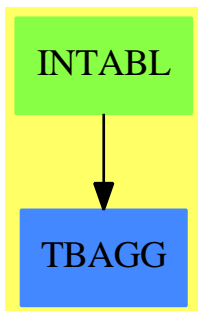
```

```

See Also:
o )show InnerTable

```

### 10.27.1 InnerTable (INTABL)



See

- ⇒ “HashTable” (HASHTBL) 9.1.1 on page 1085
- ⇒ “Table” (TABLE) 21.1.1 on page 2621
- ⇒ “EqTable” (EQTBL) 6.2.1 on page 667
- ⇒ “StringTable” (STRTBL) 20.32.1 on page 2569
- ⇒ “GeneralSparseTable” (GSTBL) 8.5.1 on page 1044
- ⇒ “SparseTable” (STBL) 20.16.1 on page 2409



**Exports:**

|          |        |          |            |                  |
|----------|--------|----------|------------|------------------|
| any?     | any?   | bag      | coerce     | construct        |
| convert  | copy   | count    | dictionary | entry?           |
| elt      | empty  | empty?   | entries    | eq?              |
| eval     | every? | extract! | fill!      | find             |
| first    | hash   | index?   | indices    | insert!          |
| inspect  | key?   | keys     | latex      | less?            |
| map      | map!   | maxIndex | member?    | members          |
| minIndex | more?  | parts    | sample     | qelt             |
| qsetelt! | reduce | remove   | remove!    | removeDuplicates |
| search   | select | select!  | setelt     | size?            |
| swap!    | table  | ?.?      | #?         | ?=?              |
| ?~=?     |        |          |            |                  |

— domain INTABL InnerTable —

```

)abbrev domain INTABL InnerTable
++ Author: Barry Trager
++ Date Created: 1992
++ Date Last Updated: Sept 15, 1992
++ Basic Operations:
++ Related Domains: HashTable, AssociationList, Table
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ This domain is used to provide a conditional "add" domain
++ for the implementation of \spadtype{Table}.

InnerTable(Key: SetCategory, Entry: SetCategory, addDom):Exports == Implementation where
  addDom : TableAggregate(Key, Entry) with
    finiteAggregate
  Exports ==> TableAggregate(Key, Entry) with
    finiteAggregate
  Implementation ==> addDom

```

— INTABL.dotabb —

```

"INTABL" [color="#88FF44",href="bookvol10.3.pdf#nameddest=INTABL"]
"TBAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=TBAGG"]
"INTABL" -> "TBAGG"

```

## 10.28 domain ITAYLOR InnerTaylorSeries

— InnerTaylorSeries.input —

```
)set break resume
)sys rm -f InnerTaylorSeries.output
)spool InnerTaylorSeries.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show InnerTaylorSeries
--R InnerTaylorSeries Coef: Ring is a domain constructor
--R Abbreviation for InnerTaylorSeries is ITAYLOR
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ITAYLOR
--R
--R----- Operations -----
--R ??? : (%,Integer) -> %          ??? : (%,Coef) -> %
--R ??? : (Coef,%) -> %            ??? : (%,%) -> %
--R ??? : (Integer,%) -> %         ??? : (PositiveInteger,%) -> %
--R ??? : (%,PositiveInteger) -> % ?+? : (%,%) -> %
--R ?-? : (%,%) -> %              -? : % -> %
--R ?? : (%,%) -> Boolean         1 : () -> %
--R 0 : () -> %                   ?? : (%,PositiveInteger) -> %
--R coefficients : % -> Stream Coef  coerce : Integer -> %
--R coerce : % -> OutputForm        hash : % -> SingleInteger
--R latex : % -> String             one? : % -> Boolean
--R order : % -> NonNegativeInteger pole? : % -> Boolean
--R recip : % -> Union(%, "failed") sample : () -> %
--R series : Stream Coef -> %       zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ??? : (NonNegativeInteger,%) -> %
--R ??? : (%,NonNegativeInteger) -> %
--R ?? : (%,NonNegativeInteger) -> %
--R associates? : (%,%) -> Boolean if Coef has INTDOM
--R characteristic : () -> NonNegativeInteger
--R coerce : % -> % if Coef has INTDOM
--R exquo : (%,%) -> Union(%, "failed") if Coef has INTDOM
--R order : (%,NonNegativeInteger) -> NonNegativeInteger
--R subtractIfCan : (%,%) -> Union(%, "failed")
--R unit? : % -> Boolean if Coef has INTDOM
--R unitCanonical : % -> % if Coef has INTDOM
--R unitNormal : % -> Record(unit: %, canonical: %, associate: %) if Coef has INTDOM
--R
--E 1
```

```
)spool
)lisp (bye)
```

---

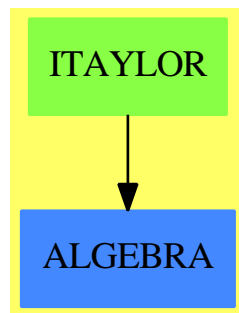
— InnerTaylorSeries.help —

```
=====
InnerTaylorSeries examples
=====
```

See Also:

```
o )show InnerTaylorSeries
```

### 10.28.1 InnerTaylorSeries (ITAYLOR)



See

⇒ “UnivariateTaylorSeries” (UTS) 22.9.1 on page 2834

**Exports:**

|               |       |               |                |              |
|---------------|-------|---------------|----------------|--------------|
| 0             | 1     | associates?   | characteristic | coefficients |
| coerce        | exquo | hash          | latex          | one?         |
| order         | pole? | recip         | sample         | series       |
| subtractIfCan | unit? | unitCanonical | unitNormal     | zero?        |
| ?~=?          | ?*?   | ?**?          | ?^?            | ?+?          |
| ?-?           | -?    | ?=?           |                |              |

— domain ITAYLOR InnerTaylorSeries —

```
)abbrev domain ITAYLOR InnerTaylorSeries
++ Author: Clifton J. Williamson
++ Date Created: 21 December 1989
++ Date Last Updated: 25 February 1989
```

```

++ Basic Operations:
++ Related Domains: UnivariateTaylorSeries(Coef,var,cen)
++ Also See:
++ AMS Classifications:
++ Keywords: stream, dense Taylor series
++ Examples:
++ References:
++ Description:
++ Internal package for dense Taylor series.
++ This is an internal Taylor series type in which Taylor series
++ are represented by a \spadtype{Stream} of \spadtype{Ring} elements.
++ For univariate series, the \spad{Stream} elements are the Taylor
++ coefficients. For multivariate series, the \spad{n}th Stream element
++ is a form of degree n in the power series variables.

InnerTaylorSeries(Coef): Exports == Implementation where
  Coef : Ring
  I ==> Integer
  NNI ==> NonNegativeInteger
  ST ==> Stream Coef
  STT ==> StreamTaylorSeriesOperations Coef

Exports ==> Ring with
  coefficients: % -> Stream Coef
    ++\spad{coefficients(x)} returns a stream of ring elements.
    ++ When x is a univariate series, this is a stream of Taylor
    ++ coefficients. When x is a multivariate series, the
    ++ \spad{n}th element of the stream is a form of
    ++ degree n in the power series variables.
  series: Stream Coef -> %
    ++\spad{series(s)} creates a power series from a stream of
    ++ ring elements.
    ++ For univariate series types, the stream s should be a stream
    ++ of Taylor coefficients. For multivariate series types, the
    ++ stream s should be a stream of forms the \spad{n}th element
    ++ of which is a
    ++ form of degree n in the power series variables.
  pole?: % -> Boolean
    ++\spad{pole?(x)} tests if the series x has a pole.
    ++ Note: this is false when x is a Taylor series.
  order: % -> NNI
    ++\spad{order(x)} returns the order of a power series x,
    ++ i.e. the degree of the first non-zero term of the series.
  order: (% ,NNI) -> NNI
    ++\spad{order(x,n)} returns the minimum of n and the order of x.
  "*" : (Coef,%)->%
    ++\spad{c*x} returns the product of c and the series x.
  "*" : (% ,Coef)->%
    ++\spad{x*c} returns the product of c and the series x.
  "*" : (% ,Integer)->%

```

```

++\spad{x*i} returns the product of integer i and the series x.
if Coef has IntegralDomain then IntegralDomain
---+ An IntegralDomain provides 'exquo'

Implementation ==> add

Rep := Stream Coef

--% declarations
x,y: %

--% definitions

-- In what follows, we will be calling operations on Streams
-- which are NOT defined in the package Stream. Thus, it is
-- necessary to explicitly pass back and forth between Rep and %.
-- This will be done using the functions 'stream' and 'series'.

stream : % -> Stream Coef
stream x == x pretend Stream(Coef)
series st == st pretend %

0 == coerce(0)$STT
1 == coerce(1)$STT

x = y ==
-- tests if two power series are equal
-- difference must be a finite stream of zeroes of length <= n + 1,
-- where n = $streamCount$Lisp
st : ST := stream(x - y)
n : I := _$streamCount$Lisp
for i in 0..n repeat
  empty? st => return true
  first st ^= 0 => return false
  st := rst st
empty? st

coefficients x == stream x

x + y == stream(x) +$STT stream(y)
x - y == stream(x) -$STT stream(y)
(x:%) * (y:%) == stream(x) *$STT stream(y)
- x == -$STT (stream x)
(i:I) * (x:%) == (i::Coef) *$STT stream x
(x:%) * (i:I) == stream(x) *$STT (i::Coef)
(c:Coef) * (x:%) == c *$STT stream x
(x:%) * (c:Coef) == stream(x) *$STT c

recip x ==
(rec := recip$STT stream x) case "failed" => "failed"

```

```

series(rec :: ST)

if Coef has IntegralDomain then

  x exquo y ==
    (quot := stream(x) exquo$STT stream(y)) case "failed" => "failed"
    series(quot :: ST)

x:% ** n:NNI ==
  n = 0 => 1
  expt(x,n :: PositiveInteger)$RepeatedSquaring(%)

characteristic() == characteristic()$Coef
pole? x == false

iOrder: (ST,NNI,NNI) -> NNI
iOrder(st,n,n0) ==
  (n = n0) or (empty? st) => n0
  zero? first st => iOrder(rst st,n + 1,n0)
  n

order(x,n) == iOrder(stream x,0,n)

iOrder2: (ST,NNI) -> NNI
iOrder2(st,n) ==
  empty? st => error "order: series has infinite order"
  zero? first st => iOrder2(rst st,n + 1)
  n

order x == iOrder2(stream x,0)

```

---

— ITAYLOR.dotabb —

```

"ITAYLOR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ITAYLOR"]
"ALGEBRA" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ALGEBRA"]
"ITAYLOR" -> "ALGEBRA"

```

---

## 10.29 domain INFORM InputForm

— InputForm.input —

```

)set break resume
)sys rm -f InputForm.output
)spool InputForm.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show InputForm
--R InputForm is a domain constructor
--R Abbreviation for InputForm is INFORM
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for INFORM
--R
--R----- Operations -----
--R #? : % -> Integer          ??? : (%,%) -> %
--R ??? : (%,Integer) -> %    ?+? : (%,%) -> %
--R ?/? : (%,%) -> %          ?=? : (%,%) -> Boolean
--R 1 : () -> %               0 : () -> %
--R atom? : % -> Boolean      binary : (%,List %) -> %
--R car : % -> %              cdr : % -> %
--R coerce : % -> OutputForm  convert : SExpression -> %
--R convert : % -> SExpression convert : OutputForm -> %
--R convert : DoubleFloat -> % convert : Integer -> %
--R convert : Symbol -> %      convert : String -> %
--R convert : List % -> %      declare : List % -> Symbol
--R destruct : % -> List %    ?.? : (%,List Integer) -> %
--R ?.? : (%,Integer) -> %    eq : (%,%) -> Boolean
--R expr : % -> OutputForm    flatten : % -> %
--R float : % -> DoubleFloat  float? : % -> Boolean
--R hash : % -> SingleInteger integer : % -> Integer
--R integer? : % -> Boolean    interpret : % -> Any
--R lambda : (%,List Symbol) -> % latex : % -> String
--R list? : % -> Boolean       null? : % -> Boolean
--R pair? : % -> Boolean       parse : String -> %
--R string : % -> String       string? : % -> Boolean
--R symbol : % -> Symbol       symbol? : % -> Boolean
--R unparse : % -> String      ?~=? : (%,%) -> Boolean
--R ??? : (%,NonNegativeInteger) -> %
--R compile : (Symbol,List %) -> Symbol
--R function : (%,List Symbol,Symbol) -> %
--R
--E 1

)spool
)lisp (bye)

```

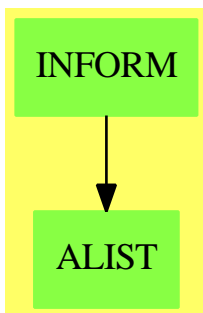
---

— **InputForm.help** —

```
=====
InputForm examples
=====
```

See Also:

o )show InputForm

10.29.1 **InputForm (INFORM)****Exports:**

|           |          |         |         |          |
|-----------|----------|---------|---------|----------|
| 0         | 1        | atom?   | binary  | car      |
| cdr       | coerce   | compile | convert | declare  |
| destruct  | eq       | expr    | flatten | float    |
| float?    | function | hash    | integer | integer? |
| interpret | lambda   | latex   | list?   | null?    |
| pair?     | parse    | string  | string? | symbol   |
| symbol?   | unparse  | #?      | ?~=?    | ?**?     |
| ?*?       | ?+?      | ?/?     | ?=?     | ?..?     |

— **domain INFORM InputForm** —

```
)abbrev domain INFORM InputForm
++ Author: Manuel Bronstein
++ Date Created: ???
++ Date Last Updated: 19 April 1991
++ Description:
++ Domain of parsed forms which can be passed to the interpreter.
++ This is also the interface between algebra code and facilities
++ in the interpreter.
```



```

--)boot $noSubsumption := true

InputForm():
  Join(SExpressionCategory(String,Symbol,Integer,DoubleFloat,OutputForm),
    ConvertibleTo SExpression) with
  interpret: % -> Any
    ++ interpret(f) passes f to the interpreter.
  convert : SExpression -> %
    ++ convert(s) makes s into an input form.
  binary : (%, List %) -> %
    ++ \spad{binary(op, [a1,...,an])} returns the input form
    ++ corresponding to \spad{a1 op a2 op ... op an}.
    ++
    ++X a:=[1,2,3]::List(InputForm)
    ++X binary(_+::InputForm,a)

function : (%, List Symbol, Symbol) -> %
  ++ \spad{function(code, [x1,...,xn], f)} returns the input form
  ++ corresponding to \spad{f(x1,...,xn) == code}.
lambda : (%, List Symbol) -> %
  ++ \spad{lambda(code, [x1,...,xn])} returns the input form
  ++ corresponding to \spad{(x1,...,xn) +-> code} if \spad{n > 1},
  ++ or to \spad{x1 +-> code} if \spad{n = 1}.
"+" : (%, %) -> %
  ++ \spad{a + b} returns the input form corresponding to \spad{a + b}.
"*" : (%, %) -> %
  ++ \spad{a * b} returns the input form corresponding to \spad{a * b}.
"/" : (%, %) -> %
  ++ \spad{a / b} returns the input form corresponding to \spad{a / b}.
"***" : (%, NonNegativeInteger) -> %
  ++ \spad{a ** b} returns the input form corresponding to \spad{a ** b}.
"***" : (%, Integer) -> %
  ++ \spad{a ** b} returns the input form corresponding to \spad{a ** b}.
0 : constant -> %
  ++ \spad{0} returns the input form corresponding to 0.
1 : constant -> %
  ++ \spad{1} returns the input form corresponding to 1.
flatten : % -> %
  ++ flatten(s) returns an input form corresponding to s with
  ++ all the nested operations flattened to triples using new
  ++ local variables.
  ++ If s is a piece of code, this speeds up
  ++ the compilation tremendously later on.
unparse : % -> String
  ++ unparse(f) returns a string s such that the parser
  ++ would transform s to f.
  ++ Error: if f is not the parsed form of a string.
parse : String -> %
  ++ parse is the inverse of unparse. It parses a string to InputForm.

```

```

declare : List % -> Symbol
  ++ declare(t) returns a name f such that f has been
  ++ declared to the interpreter to be of type t, but has
  ++ not been assigned a value yet.
  ++ Note: t should be created as \spad{devaluate(T)$Lisp} where T is the
  ++ actual type of f (this hack is required for the case where
  ++ T is a mapping type).
compile : (Symbol, List %) -> Symbol
  ++ \spad{compile(f, [t1,...,tn])} forces the interpreter to compile
  ++ the function f with signature \spad{(t1,...,tn) -> ?}.
  ++ returns the symbol f if successful.
  ++ Error: if f was not defined beforehand in the interpreter,
  ++ or if the ti's are not valid types, or if the compiler fails.
== SExpression add
Rep := SExpression

mkProperOp: Symbol -> %
strsym : % -> String
tuplify : List Symbol -> %
flatten0 : (% , Symbol, NonNegativeInteger) ->
  Record(lst: List %, symb:%)

0 == convert(0::Integer)
1 == convert(1::Integer)
convert(x:%):SExpression == x pretend SExpression
convert(x:SExpression):% == x

conv(ll : List %): % ==
  convert(ll pretend List SExpression)$SExpression pretend %

lambda(f,l) == conv([convert("+-> "::Symbol),tuplify l,f]$List(%))

interpret x ==
  v := interpret(x)$Lisp
  mkObj(unwrap(objVal(v)$Lisp)$Lisp, objMode(v)$Lisp)$Lisp

convert(x:DoubleFloat):% ==
  zero? x => 0
--  one? x => 1
  (x = 1) => 1
  convert(x)$Rep

flatten s ==
  -- will not compile if I use 'or'
  atom? s => s
  every?(atom?,destruct s)$List(%) => s
  sy := new()$Symbol
  n:NonNegativeInteger := 0
  l2 := [flatten0(x, sy, n := n + 1) for x in rest(l := destruct s)]
  conv(concat(convert("SEQ"::Symbol)@%,

```

```

concat(concat [u.lst for u in l2], conv(
  [convert("exit"::Symbol)@%, 1$%, conv(concat(first l,
    [u.symb for u in l2]))@%]$List(%)@%))@%)
flatten0(s, sy, n) ==
atom? s => [nil(), s]
a := convert(concat(string sy, convert(n)@String)::Symbol)@%
l2 := [flatten0(x, sy, n := n+1) for x in rest(l := destruct s)]
[concat(concat [u.lst for u in l2], conv([convert(
  "LET"::Symbol)@%, a, conv(concat(first l,
    [u.symb for u in l2]))@%]$List(%)@%), a]
strsym s ==
string? s => string s
symbol? s => string symbol s
error "strsym: form is neither a string or symbol"
unparse x ==
atom?(s:% := form2String(x)$Lisp) => strsym s
concat [strsym a for a in destruct s]
parse(s:String):% ==
ncParseFromString(s)$Lisp
declare signature ==
declare(name := new()$Symbol, signature)$Lisp
name
compile(name, types) ==
symbol car cdr car
selectLocalMms(mkProperOp name, convert(name)@%,
  types, nil$List(%)$Lisp
mkProperOp name ==
op := mkAtree(nme := convert(name)@%)$Lisp
transferPropsToNode(nme, op)$Lisp
convert op
binary(op, args) ==
(n := #args) < 2 => error "Need at least 2 arguments"
n = 2 => convert([op, first args, last args]$List(%)
  convert([op, first args, binary(op, rest args)]$List(%)
tuplify l ==
empty? rest l => convert first l
conv
concat(convert("Tuple"::Symbol), [convert x for x in l]$List(%)
function(f, l, name) ==
nn := convert(new(1 + #l, convert(nil()$List(%)$List(%)@%

```

```

conv([convert("DEF"::Symbol), conv(cons(convert(name)@%,
                                     [convert(x)@% for x in l])), nn, nn, f]$List(%))

s1 + s2 ==
  s1 = 0 => s2
  s2 = 0 => s1
  conv [convert("+ "::Symbol), s1, s2]$List(%)

s1 * s2 ==
  s1 = 0 or s2 = 0 => 0
  s1 = 1 => s2
  s2 = 1 => s1
  conv [convert("* "::Symbol), s1, s2]$List(%)

s1:% ** n:Integer ==
  s1 = 0 and n > 0 => 0
  s1 = 1 or zero? n => 1
--   one? n => s1
  (n = 1) => s1
  conv [convert("*** "::Symbol), s1, convert n]$List(%)

s1:% ** n:NonNegativeInteger == s1 ** (n::Integer)

s1 / s2 ==
  s2 = 1 => s1
  conv [convert("/ "::Symbol), s1, s2]$List(%)

```

---

— INFORM.dotabb —

```

"INFORM" [color="#88FF44",href="bookvol10.3.pdf#nameddest=INFORM"]
"ALIST"  [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"INFORM" -> "ALIST"

```

---

## 10.30 domain INT Integer

The function **one?** has been rewritten back to its original form. The NAG version called a lisp primitive that exists only in Codemist Common Lisp and is not defined in Common Lisp.

---

— Integer.input —

[illegible]

```
--S 6 of 42
x <= -1
--R
--R
--R (6) true
--R
--R                                         Type: Boolean
--E 6
```

```
--S 7 of 42
negative?(x)
--R
--R
--R (7) true
--R
--R                                         Type: Boolean
--E 7
```

```
--S 8 of 42
x > 0
--R
--R
--R (8) false
--R
--R                                         Type: Boolean
--E 8
```

```
--S 9 of 42
x >= 1
--R
--R
--R (9) false
--R
--R                                         Type: Boolean
--E 9
```

```
--S 10 of 42
positive?(x)
--R
--R
--R (10) false
--R
--R                                         Type: Boolean
--E 10
```

```
--S 11 of 42
zero?(x)
--R
--R
--R (11) false
--R
--R                                         Type: Boolean
--E 11
```

```
--S 12 of 42
one?(x)
```

```

--R
--R
--R (12) false
--R
--R                                         Type: Boolean
--E 12

--S 13 of 42
(x = -101)@Boolean
--R
--R
--R (13) true
--R
--R                                         Type: Boolean
--E 13

--S 14 of 42
odd?(x)
--R
--R
--R (14) true
--R
--R                                         Type: Boolean
--E 14

--S 15 of 42
even?(x)
--R
--R
--R (15) false
--R
--R                                         Type: Boolean
--E 15

--S 16 of 42
gcd(56788,43688)
--R
--R
--R (16) 4
--R
--R                                         Type: PositiveInteger
--E 16

--S 17 of 42
lcm(56788,43688)
--R
--R
--R (17) 620238536
--R
--R                                         Type: PositiveInteger
--E 17

--S 18 of 42
max(678,567)
--R
--R

```

```

--R (18) 678
--R                                         Type: PositiveInteger
--E 18

--S 19 of 42
min(678,567)
--R
--R
--R (19) 567
--R                                         Type: PositiveInteger
--E 19

--S 20 of 42
reduce(max,[2,45,-89,78,100,-45])
--R
--R
--R (20) 100
--R                                         Type: PositiveInteger
--E 20

--S 21 of 42
reduce(min,[2,45,-89,78,100,-45])
--R
--R
--R (21) - 89
--R                                         Type: Integer
--E 21

--S 22 of 42
reduce(gcd,[2,45,-89,78,100,-45])
--R
--R
--R (22) 1
--R                                         Type: PositiveInteger
--E 22

--S 23 of 42
reduce(lcm,[2,45,-89,78,100,-45])
--R
--R
--R (23) 1041300
--R                                         Type: PositiveInteger
--E 23

--S 24 of 42
13 / 4
--R
--R
--R (24) 13
--R (24) --

```



[illegible]

[illegible]

[illegible]

)lisp (bye)

---

— Integer.help —

=====

Integer examples

=====

Axiom provides many operations for manipulating arbitrary precision integers. In this section we will show some of those that come from Integer itself plus some that are implemented in other packages.

\subsection{Basic Functions}

The size of an integer in Axiom is only limited by the amount of computer storage you have available. The usual arithmetic operations are available.

```
2**(5678 - 4856 + 2 * 17)
4804810770435008147181540925125924391239526139871682263473855610088084200076_
308293086342527091412083743074572278211496076276922026433435687527334980249_
539302425425230458177649495442143929053063884787051467457680738771416988598_
1549563293528878334250628775936
```

Type: PositiveInteger

There are a number of ways of working with the sign of an integer. Let's use this x as an example.

```
x := -101
- 101
```

Type: Integer

First of all, there is the absolute value function.

```
abs(x)
101
```

Type: PositiveInteger

The sign operation returns -1 if its argument is negative, 0 if zero and 1 if positive.

```
sign(x)
- 1
```

Type: Integer

You can determine if an integer is negative in several other ways.

```
x < 0
true
Type: Boolean
```

```
x <= -1
true
Type: Boolean
```

```
negative?(x)
true
Type: Boolean
```

Similarly, you can find out if it is positive.

```
x > 0
false
Type: Boolean
```

```
x >= 1
false
Type: Boolean
```

```
positive?(x)
false
Type: Boolean
```

This is the recommended way of determining whether an integer is zero.

```
zero?(x)
false
Type: Boolean
```

Use the `zero?` operation whenever you are testing any mathematical object for equality with zero. This is usually more efficient than using `=` (think of matrices: it is easier to tell if a matrix is zero by just checking term by term than constructing another "zero" matrix and comparing the two matrices term by term) and also avoids the problem that `=` is usually used for creating equations.

This is the recommended way of determining whether an integer is equal to one.

```
one?(x)
false
Type: Boolean
```

This syntax is used to test equality using `=`. It says that you want a Boolean (true or false) answer rather than an equation.

```
(x = -101)@Boolean
```

```
true
```

```
Type: Boolean
```

The operations `odd?` and `even?` determine whether an integer is odd or even, respectively. They each return a Boolean object.

```
odd?(x)
```

```
true
```

```
Type: Boolean
```

```
even?(x)
```

```
false
```

```
Type: Boolean
```

The operation `gcd` computes the greatest common divisor of two integers.

```
gcd(56788,43688)
```

```
4
```

```
Type: PositiveInteger
```

The operation `lcm` computes their least common multiple.

```
lcm(56788,43688)
```

```
620238536
```

```
Type: PositiveInteger
```

To determine the maximum of two integers, use `max`.

```
max(678,567)
```

```
678
```

```
Type: PositiveInteger
```

To determine the minimum, use `min`.

```
min(678,567)
```

```
567
```

```
Type: PositiveInteger
```

The `reduce` operation is used to extend binary operations to more than two arguments. For example, you can use `reduce` to find the maximum integer in a list or compute the least common multiple of all integers in the list.

```
reduce(max,[2,45,-89,78,100,-45])
```

```
100
```

```
Type: PositiveInteger
```

```
reduce(min,[2,45,-89,78,100,-45])
```

```
- 89
```

```
Type: Integer
```

```
reduce(gcd,[2,45,-89,78,100,-45])
1
Type: PositiveInteger
```

```
reduce(lcm,[2,45,-89,78,100,-45])
1041300
Type: PositiveInteger
```

The infix operator "/" is not used to compute the quotient of integers. Rather, it is used to create rational numbers as described in Fraction.

```
13 / 4
13
--
4
Type: Fraction Integer
```

The infix operation quo computes the integer quotient.

```
13 quo 4
3
Type: PositiveInteger
```

The infix operation rem computes the integer remainder.

```
13 rem 4
1
Type: PositiveInteger
```

One integer is evenly divisible by another if the remainder is zero. The operation exquo can also be used.

```
zero?(167604736446952 rem 2003644)
true
Type: Boolean
```

The operation divide returns a record of the quotient and remainder and thus is more efficient when both are needed.

```
d := divide(13,4)
[quotient= 3,remainder= 1]
Type: Record(quotient: Integer,remainder: Integer)

d.quotient
3
Type: PositiveInteger
```

See help on Records for details on Records.

```
d.remainder
1
```

Type: PositiveInteger

```
=====
Primes and Factorization
=====
```

Use the operation factor to factor integers. It returns an object of type Factored Integer.

```
factor 102400
12 2
2 5
```

Type: Factored Integer

The operation prime? returns true or false depending on whether its argument is a prime.

```
prime? 7
true
```

Type: Boolean

```
prime? 8
false
```

Type: Boolean

The operation nextPrime returns the least prime number greater than its argument.

```
nextPrime 100
101
```

Type: PositiveInteger

The operation prevPrime returns the greatest prime number less than its argument.

```
prevPrime 100
97
```

Type: PositiveInteger

To compute all primes between two integers (inclusively), use the operation primes.

```
primes(100,175)
[173,167,163,157,151,149,139,137,131,127,113,109,107,103,101]
```

Type: List Integer

You might sometimes want to see the factorization of an integer when it is considered a Gaussian integer.



```
factor(2 :: Complex Integer)
      2
- %i (1 + %i)
Type: Factored Complex Integer
```

=====

Some Number Theoretic Functions

=====

Axiom provides several number theoretic operations for integers.

The operation `fibonacci` computes the Fibonacci numbers. The algorithm has running time  $O(\log^3 n)$  for argument  $n$ .

```
[fibonacci(k) for k in 0..]
[0,1,1,2,3,5,8,13,21,34,...]
Type: Stream Integer
```

The operation `legendre` computes the Legendre symbol for its two integer arguments where the second one is prime. If you know the second argument to be prime, use `jacobi` instead where no check is made.

```
[legendre(i,11) for i in 0..10]
[0,1,- 1,1,1,1,- 1,- 1,- 1,1,- 1]
Type: List Integer
```

The operation `jacobi` computes the Jacobi symbol for its two integer arguments. By convention, 0 is returned if the greatest common divisor of the numerator and denominator is not 1.

```
[jacobi(i,15) for i in 0..9]
[0,1,1,0,1,0,0,- 1,1,0]
Type: List Integer
```

The operation `eulerPhi` computes the values of Euler's  $\phi$ -function where  $\phi(n)$  equals the number of positive integers less than or equal to  $n$  that are relatively prime to the positive integer  $n$ .

```
[eulerPhi i for i in 1..]
[1,1,2,2,4,2,6,4,6,4,...]
Type: Stream Integer
```

The operation `moebiusMu` computes the Moebius  $\mu$  function.

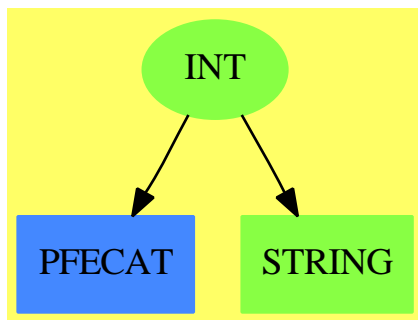
```
[moebiusMu i for i in 1..]
[1,- 1,- 1,0,- 1,1,- 1,0,0,1,...]
Type: Stream Integer
```

See Also:

- o )help Complex
- o )help Factored
- o )help Records
- o )help Fraction
- o )help RadixExpansion
- o )help HexadecimalExpansion
- o )help BinaryExpansion
- o )help DecimalExpansion
- o )help IntegerNumberTheoryFunctions
- o )help RomanNumeral
- o )show Integer

—————→

### 10.30.1 Integer (INT)



See

- ⇒ “NonNegativeInteger” (NNI) 15.5.1 on page 1702
- ⇒ “PositiveInteger” (PI) 17.28.1 on page 2060
- ⇒ “RomanNumeral” (ROMAN) 19.12.1 on page 2286

**Exports:**

|                   |                   |                    |                |
|-------------------|-------------------|--------------------|----------------|
| 0                 | 1                 | abs                | addmod         |
| associates?       | base              | binomial           | bit?           |
| characteristic    | coerce            | convert            | copy           |
| D                 | dec               | differentiate      | divide         |
| euclideanSize     | even?             | expressIdealMember | exquo          |
| extendedEuclidean | extendedEuclidean | factor             | factorial      |
| gcd               | gcdPolynomial     | hash               | inc            |
| init              | invmod            | latex              | lcm            |
| length            | mask              | max                | min            |
| mulmod            | multiEuclidean    | negative?          | nextItem       |
| odd?              | OMwrite           | one?               | patternMatch   |
| permutation       | positive?         | positiveRemainder  | powmod         |
| prime?            | principalIdeal    | random             | rational       |
| rational?         | rationalIfCan     | recip              | reducedSystem  |
| retract           | retractIfCan      | sample             | shift          |
| sign              | sizeLess?         | squareFree         | squareFreePart |
| submod            | subtractIfCan     | symmetricRemainder | unit?          |
| unitCanonical     | unitNormal        | zero?              | ?*?            |
| ?**?              | ?+?               | ?-?                | -?             |
| ?<?               | ?<=?              | ?=?                | ?>?            |
| ?>=?              | ?^?               | ?~=?               | ?quo?          |
| ?rem?             |                   |                    |                |

## — domain INT Integer —

```

)abbrev domain INT Integer
++ Author: Mark Botch
++ Date Created:
++ Change History:
++ Basic Operations:
++ Related Constructors:
++ Keywords: integer
++ Description:
++ \spadtype{Integer} provides the domain of arbitrary precision integers.

```

```

Integer: Join(IntegerNumberSystem, ConvertibleTo String, OpenMath) with
  random : % -> %
    ++ random(n) returns a random integer from 0 to \spad{n-1}.
  canonical
    ++ mathematical equality is data structure equality.
  canonicalsClosed
    ++ two positives multiply to give positive.
  noetherian
    ++ ascending chain condition on ideals.
  infinite
    ++ nextItem never returns "failed".
== add

```

```

ZP ==> SparseUnivariatePolynomial %
ZZP ==> SparseUnivariatePolynomial Integer
x,y: %
n: NonNegativeInteger

writeOMInt(dev: OpenMathDevice, x: %): Void ==
  if x < 0 then
    OMputApp(dev)
    OMputSymbol(dev, "arith1", "unary__minus")
    OMputInteger(dev, (-x) pretend Integer)
    OMputEndApp(dev)
  else
    OMputInteger(dev, x pretend Integer)

OMwrite(x: %): String ==
  s: String := ""
  sp := OM_-STRINGTOSTRINGPTR(s)$Lisp
  dev: OpenMathDevice := OMopenString(sp pretend String, OMencodingXML)
  OMputObject(dev)
  writeOMInt(dev, x)
  OMputEndObject(dev)
  OMclose(dev)
  s := OM_-STRINGPTRTOSTRING(sp)$Lisp pretend String
  s

OMwrite(x: %, wholeObj: Boolean): String ==
  s: String := ""
  sp := OM_-STRINGTOSTRINGPTR(s)$Lisp
  dev: OpenMathDevice := OMopenString(sp pretend String, OMencodingXML)
  if wholeObj then
    OMputObject(dev)
    writeOMInt(dev, x)
  if wholeObj then
    OMputEndObject(dev)
  OMclose(dev)
  s := OM_-STRINGPTRTOSTRING(sp)$Lisp pretend String
  s

OMwrite(dev: OpenMathDevice, x: %): Void ==
  OMputObject(dev)
  writeOMInt(dev, x)
  OMputEndObject(dev)

OMwrite(dev: OpenMathDevice, x: %, wholeObj: Boolean): Void ==
  if wholeObj then
    OMputObject(dev)
  writeOMInt(dev, x)
  if wholeObj then
    OMputEndObject(dev)

```

```

zero? x == ZEROP(x)$Lisp
-- one? x == ONEP(x)$Lisp
one? x == x = 1
0 == 0$Lisp
1 == 1$Lisp
base() == 2$Lisp
copy x == x
inc x == x + 1
dec x == x - 1
hash x == SXHASH(x)$Lisp
negative? x == MINUSP(x)$Lisp
coerce(x):OutputForm == outputForm(x pretend Integer)
coerce(m:Integer):% == m pretend %
convert(x:%):Integer == x pretend Integer
length a == INTEGER-LENGTH(a)$Lisp
addmod(a, b, p) ==
  (c:=a + b) >= p => c - p
  c
submod(a, b, p) ==
  (c:=a - b) < 0 => c + p
  c
mulmod(a, b, p) == (a * b) rem p
convert(x:%):Float == coerce(x pretend Integer)$Float
convert(x:%):DoubleFloat == coerce(x pretend Integer)$DoubleFloat
convert(x:%):InputForm == convert(x pretend Integer)$InputForm
convert(x:%):String == string(x pretend Integer)$String

latex(x:%):String ==
  s : String := string(x pretend Integer)$String
  (-1 < (x pretend Integer)) and ((x pretend Integer) < 10) => s
  concat("{", concat(s, "}")$String)$String

positiveRemainder(a, b) ==
  negative?(r := a rem b) =>
    negative? b => r - b
    r + b
  r

reducedSystem(m:Matrix %):Matrix(Integer) ==
  m pretend Matrix(Integer)

reducedSystem(m:Matrix %, v:Vector %):
  Record(mat:Matrix(Integer), vec:Vector(Integer)) ==
  [m pretend Matrix(Integer), vec pretend Vector(Integer)]

abs(x) == ABS(x)$Lisp
random() == random()$Lisp
random(x) == RANDOM(x)$Lisp
x = y == EQL(x,y)$Lisp
x < y == (x<y)$Lisp

```

```

- x == (-x)$Lisp
x + y == (x+y)$Lisp
x - y == (x-y)$Lisp
x * y == (x*y)$Lisp
(m:Integer) * (y:%) == (m*y)$Lisp -- for subsumption problem
x ** n == EXPT(x,n)$Lisp
odd? x == ODDP(x)$Lisp
max(x,y) == MAX(x,y)$Lisp
min(x,y) == MIN(x,y)$Lisp
divide(x,y) == DIVIDE2(x,y)$Lisp
x quo y == QUOTIENT2(x,y)$Lisp
x rem y == REMAINDER2(x,y)$Lisp
shift(x, y) == ASH(x,y)$Lisp
x exquo y ==
  zero? y => "failed"
  zero?(x rem y) => x quo y
  "failed"
-- recip(x) == if one? x or x=-1 then x else "failed"
recip(x) == if (x = 1) or x=-1 then x else "failed"
gcd(x,y) == GCD(x,y)$Lisp
UCA ==> Record(unit:%,canonical:%,associate:%)
unitNormal x ==
  x < 0 => [-1,-x,-1]$UCA
  [1,x,1]$UCA
unitCanonical x == abs x
solveLinearPolynomialEquation(lp>List ZP,p:ZP):Union(List ZP,"failed") ==
  solveLinearPolynomialEquation(lp pretend List ZZP,
    p pretend ZZP)$IntegerSolveLinearPolynomialEquation pretend
    Union(List ZP,"failed")
squareFreePolynomial(p:ZP):Factored ZP ==
  squareFree(p)$UnivariatePolynomialSquareFree(%,ZP)
factorPolynomial(p:ZP):Factored ZP ==
  -- GaloisGroupFactorizer doesn't factor the content
  -- so we have to do this by hand
  pp:=primitivePart p
  leadingCoefficient pp = leadingCoefficient p =>
    factor(p)$GaloisGroupFactorizer(ZP)
  mergeFactors(factor(pp)$GaloisGroupFactorizer(ZP),
    map((x1:%):ZP+>x1::ZP,
      factor((leadingCoefficient p exquo
        leadingCoefficient pp)
        ::%))$FactoredFunctions2(%,ZP)
    )$FactoredFunctionUtilities(ZP)
factorSquareFreePolynomial(p:ZP):Factored ZP ==
  factorSquareFree(p)$GaloisGroupFactorizer(ZP)
gcdPolynomial(p:ZP, q:ZP):ZP ==
  zero? p => unitCanonical q
  zero? q => unitCanonical p
  gcd([p,q])$HeuGcd(ZP)
-- myNextPrime: (%,NonNegativeInteger) -> %

```

```
-- myNextPrime(x,n) ==
--   nextPrime(x)$IntegerPrimesPackage(%)
--   TT:=InnerModularGcd(%,ZP,67108859 pretend %,myNextPrime)
--   gcdPolynomial(p,q) == modularGcd(p,q)$TT
```

---

— INT.dotabb —

```
"INT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=INT",
       shape=ellipse]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"INT" -> "STRING"
"INT" -> "PFECAT"
```

---

## 10.31 domain ZMOD IntegerMod

— IntegerMod.input —

```
)set break resume
)sys rm -f IntegerMod.output
)spool IntegerMod.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show IntegerMod
--R IntegerMod p: PositiveInteger is a domain constructor
--R Abbreviation for IntegerMod is ZMOD
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ZMOD
--R
--R----- Operations -----
--R ??? : (%,%) -> %               ??? : (Integer,%) -> %
--R ??? : (PositiveInteger,%) -> %   ??? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %               ?-? : (%,%) -> %
--R -? : % -> %                     ?=? : (%,%) -> Boolean
--R 1 : () -> %                     0 : () -> %
--R ?? : (%,PositiveInteger) -> %   coerce : Integer -> %
--R coerce : % -> OutputForm        convert : % -> Integer
```

```

--R hash : % -> SingleInteger          index : PositiveInteger -> %
--R init : () -> %                     latex : % -> String
--R lookup : % -> PositiveInteger       one? : % -> Boolean
--R random : () -> %                   recip : % -> Union(%, "failed")
--R sample : () -> %                   size : () -> NonNegativeInteger
--R zero? : % -> Boolean                ?~=? : (%,%) -> Boolean
--R ?? : (NonNegativeInteger,%) -> %
--R *** : (%,NonNegativeInteger) -> %
--R ^? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R nextItem : % -> Union(%, "failed")
--R subtractIfCan : (%,%) -> Union(%, "failed")
--R
--E 1

)spool
)lisp (bye)

```

— IntegerMod.help —

=====

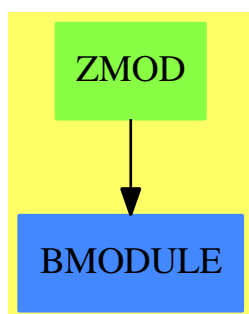
IntegerMod examples

=====

See Also:

- o )show IntegerMod

### 10.31.1 IntegerMod (ZMOD)





**Exports:**

|          |               |                |        |         |
|----------|---------------|----------------|--------|---------|
| 0        | 1             | characteristic | coerce | convert |
| hash     | index         | init           | latex  | lookup  |
| nextItem | one?          | random         | recip  | sample  |
| size     | subtractIfCan | zero?          | ?^=?   | ?*?     |
| ?**?     | ?^?           | ?+?            | ?-?    | -?      |
| ?=?      |               |                |        |         |

— domain ZMOD IntegerMod —

```

)abbrev domain ZMOD IntegerMod
++ Author: Mark Botch
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ IntegerMod(n) creates the ring of integers reduced modulo the integer n.

IntegerMod(p:PositiveInteger):
  Join(CommutativeRing, Finite, ConvertibleTo Integer, StepThrough) == add
    size() == p
    characteristic() == p
    lookup x == (zero? x => p; (convert(x)@Integer) :: PositiveInteger)

-- Code is duplicated for the optimizer to kick in.
if p <= convert(max())$SingleInteger@Integer then
  Rep:= SingleInteger
  q := p::SingleInteger

  bloodyCompiler: Integer -> %
  bloodyCompiler n == positiveRemainder(n, p)$Integer :: Rep

  convert(x:%):Integer == convert(x)$Rep
  coerce(x):OutputForm == coerce(x)$Rep
  coerce(n:Integer):% == bloodyCompiler n
  0 == 0$Rep
  1 == 1$Rep
  init == 0$Rep
  nextItem(n) ==
    m:=n+1
    m=0 => "failed"
    m

  x = y == x =$Rep y
  x:% * y:% == mulmod(x, y, q)

```

```

n:Integer * x:%      == mulmod(bloodyCompiler n, x, q)
x + y                == addmod(x, y, q)
x - y                == submod(x, y, q)
random()              == random(q)$Rep
index a               == positiveRemainder(a:%, q)
- x                   == (zero? x => 0; q -$Rep x)

x:% ** n:NonNegativeInteger ==
  n < p => powmod(x, n::Rep, q)
  powmod(convert(x)@Integer, n, p)$Integer :: Rep

recip x ==
  (c1, c2, g) := extendedEuclidean(x, q)$Rep
--   not one? g => "failed"
   not (g = 1) => "failed"
   positiveRemainder(c1, q)

else
  Rep:= Integer

  convert(x:%):Integer == convert(x)$Rep
  coerce(n:Integer):%  == positiveRemainder(n::Rep, p)
  coerce(x):OutputForm == coerce(x)$Rep
  0                     == 0$Rep
  1                     == 1$Rep
  init                  == 0$Rep
  nextItem(n)           ==
                        m:=n+1
                        m=0 => "failed"
                        m

  x = y                 == x =$Rep y
  x:% * y:%             == mulmod(x, y, p)
  n:Integer * x:%       == mulmod(positiveRemainder(n::Rep, p), x, p)
  x + y                 == addmod(x, y, p)
  x - y                 == submod(x, y, p)
  random()              == random(p)$Rep
  index a               == positiveRemainder(a::Rep, p)
  - x                   == (zero? x => 0; p -$Rep x)
  x:% ** n:NonNegativeInteger == powmod(x, n::Rep, p)

  recip x ==
    (c1, c2, g) := extendedEuclidean(x, p)$Rep
--   not one? g => "failed"
   not (g = 1) => "failed"
   positiveRemainder(c1, p)

```

---

— ZMOD.dotabb —

```
"ZMOD" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ZMOD"]
"BMODULE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=BMODULE"]
"ZMOD" -> "BMODULE"
```

## 10.32 domain INTFTBL IntegrationFunctionsTable

— IntegrationFunctionsTable.input —

```
)set break resume
)sys rm -f IntegrationFunctionsTable.output
)spool IntegrationFunctionsTable.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show IntegrationFunctionsTable
--R IntegrationFunctionsTable is a domain constructor
--R Abbreviation for IntegrationFunctionsTable is INTFTBL
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for INTFTBL
--R
--R----- Operations -----
--R clearTheFTable : () -> Void          showTheFTable : () -> %
--R entries : % -> List Record(key: Record(var: Symbol,fn: Expression DoubleFloat,range: Segment OrderedCompletion D
--R entry : Record(var: Symbol,fn: Expression DoubleFloat,range: Segment OrderedCompletion D
--R fTable : List Record(key: Record(var: Symbol,fn: Expression DoubleFloat,range: Segment OrderedCompletion D
--R insert! : Record(key: Record(var: Symbol,fn: Expression DoubleFloat,range: Segment OrderedCompletion D
--R keys : % -> List Record(var: Symbol,fn: Expression DoubleFloat,range: Segment OrderedCompletion D
--R showAttributes : Record(var: Symbol,fn: Expression DoubleFloat,range: Segment OrderedCompletion D
--R
--E 1

)spool
)lisp (bye)
```

— IntegrationFunctionsTable.help —

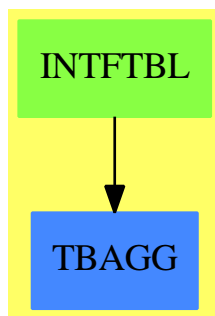
```
=====
IntegrationFunctionsTable examples
=====
```

See Also:

```
o )show IntegrationFunctionsTable
```

—————

### 10.32.1 IntegrationFunctionsTable (INTFTBL)



#### Exports:

```
clearTheFTable  entries  entry  fTable  insert!  keys  showAttributes  showTheFTable
```

— domain INTFTBL IntegrationFunctionsTable —

```
)abbrev domain INTFTBL IntegrationFunctionsTable
++ Author: Brian Dupee
++ Date Created: March 1995
++ Date Last Updated: June 1995
++ Description:
++ There is no description for this domain
```

```
IntegrationFunctionsTable(): E == I where
  EF2 ==> ExpressionFunctions2
  EFI ==> Expression Fraction Integer
  FI ==> Fraction Integer
  LEDF ==> List Expression DoubleFloat
  KEDF ==> Kernel Expression DoubleFloat
  EEDF ==> Equation Expression DoubleFloat
  EDF ==> Expression DoubleFloat
  PDF ==> Polynomial DoubleFloat
  LDF ==> List DoubleFloat
  SDF ==> Stream DoubleFloat
```

```

DF ==> DoubleFloat
F ==> Float
ST ==> String
LST ==> List String
SI ==> SingleInteger
SOCDF ==> Segment OrderedCompletion DoubleFloat
OCDF ==> OrderedCompletion DoubleFloat
OCEDF ==> OrderedCompletion Expression DoubleFloat
EOCEFI ==> Equation OrderedCompletion Expression Fraction Integer
OCEFI ==> OrderedCompletion Expression Fraction Integer
OCFI ==> OrderedCompletion Fraction Integer
NIA ==> Record(var:Symbol,fn:EDF,range:SOCDF,abserr:DF,relerr:DF)
INT ==> Integer
CTYPE ==> Union(continuous: "Continuous at the end points",
    lowerSingular: "There is a singularity at the lower end point",
    upperSingular: "There is a singularity at the upper end point",
    bothSingular: "There are singularities at both end points",
    notEvaluated: "End point continuity not yet evaluated")
RTYPE ==> Union(finite: "The range is finite",
    lowerInfinite: "The bottom of range is infinite",
    upperInfinite: "The top of range is infinite",
    bothInfinite: "Both top and bottom points are infinite",
    notEvaluated: "Range not yet evaluated")
STYPE ==> Union(str:SDF,
    notEvaluated:"Internal singularities not yet evaluated")
ATT ==> Record(endPointContinuity:CTYPE,
    singularitiesStream:STYPE,range:RTYPE)
ROA ==> Record(key:NIA,entry:ATT)

E ==> with

    showTheFTable:() -> $
    ++ showTheFTable() returns the current table of functions.
    clearTheFTable : () -> Void
    ++ clearTheFTable() clears the current table of functions.
    keys : $ -> List(NIA)
    ++ keys(f) returns the list of keys of f
    fTable: List Record(key:NIA,entry:ATT) -> $
    ++ fTable(l) creates a functions table from the elements of l.
    insert!:Record(key:NIA,entry:ATT) -> $
    ++ insert!(r) inserts an entry r into theIFTable
    showAttributes:NIA -> Union(ATT,"failed")
    ++ showAttributes(x) is not documented
    entries : $ -> List Record(key:NIA,entry:ATT)
    ++ entries(x) is not documented
    entry:NIA -> ATT
    ++ entry(n) is not documented
I ==> add

Rep := Table(NIA,ATT)

```

```

import Rep

theFTable:$ := empty()$Rep

showTheFTable():$ ==
  theFTable

clearTheFTable():Void ==
  theFTable := empty()$Rep
  void()$Void

fTable(l>List Record(key:NIA,entry:ATT)):$ ==
  theFTable := table(l)$Rep

insert!(r:Record(key:NIA,entry:ATT)):$ ==
  insert!(r,theFTable)$Rep

keys(t:$):List NIA ==
  keys(t)$Rep

showAttributes(k:NIA):Union(ATT,"failed") ==
  search(k,theFTable)$Rep

entries(t:$):List Record(key:NIA,entry:ATT) ==
  members(t)$Rep

entry(k:NIA):ATT ==
  qelt(theFTable,k)$Rep

```

---

— INTFTBL.dotabb —

```

"INTFTBL" [color="#88FF44",href="bookvol10.3.pdf#nameddest=INTFTBL"]
"TBAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=TBAGG"]
"INTFTBL" -> "TBAGG"

```

---

## 10.33 domain IR IntegrationResult

— IntegrationResult.input —

```

)set break resume

```

```

)sys rm -f IntegrationResult.output
)spool IntegrationResult.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show IntegrationResult
--R IntegrationResult F: Field is a domain constructor
--R Abbreviation for IntegrationResult is IR
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for IR
--R
--R----- Operations -----
--R ?? : (%,Fraction Integer) -> %      ?? : (Fraction Integer,%) -> %
--R ?? : (Integer,%) -> %                ?? : (PositiveInteger,%) -> %
--R +? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                          ?? : (%,%) -> Boolean
--R 0 : () -> %                          coerce : F -> %
--R coerce : % -> OutputForm              differentiate : (%,(F -> F)) -> F
--R elem? : % -> Boolean                  hash : % -> SingleInteger
--R integral : (F,F) -> %                 latex : % -> String
--R ratpart : % -> F                      retract : % -> F
--R sample : () -> %                     zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?? : (NonNegativeInteger,%) -> %
--R differentiate : (%,Symbol) -> F if F has PDRING SYMBOL
--R integral : (F,Symbol) -> % if F has RETRACT SYMBOL
--R logpart : % -> List Record(scalar: Fraction Integer,coeff: SparseUnivariatePolynomial F,l
--R mkAnswer : (F,List Record(scalar: Fraction Integer,coeff: SparseUnivariatePolynomial F,l
--R notelem : % -> List Record(integrand: F,intvar: F)
--R retractIfCan : % -> Union(F,"failed")
--R subtractIfCan : (%,%) -> Union(%, "failed")
--R
--E 1

)spool
)lisp (bye)

```

---

— IntegrationResult.help —

```

=====
IntegrationResult examples
=====

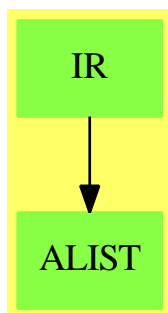
```

```

See Also:
o )show IntegrationResult

```

## 10.33.1 IntegrationResult (IR)

**Exports:**

|          |         |               |               |         |
|----------|---------|---------------|---------------|---------|
| 0        | coerce  | differentiate | elem?         | hash    |
| integral | latex   | logpart       | mkAnswer      | notelem |
| ratpart  | retract | retractIfCan  | subtractIfCan | sample  |
| zero?    | ?~=?    | ?*?           | ?+?           | ?-?     |
| -?       | ?=?     |               |               |         |

— domain IR IntegrationResult —

```

)abbrev domain IR IntegrationResult
++ Author: Barry Trager, Manuel Bronstein
++ Date Created: 1987
++ Date Last Updated: 12 August 1992
++ Keywords: integration.
++ Description:
++ The result of a transcendental integration.
++ If a function f has an elementary integral g, then g can be written
++ in the form \spad{g = h + c1 log(u1) + c2 log(u2) + ... + cn log(un)}
++ where h, which is in the same field than f, is called the rational
++ part of the integral, and \spad{c1 log(u1) + ... cn log(un)} is called the
++ logarithmic part of the integral. This domain manipulates integrals
++ represented in that form, by keeping both parts separately. The logs
++ are not explicitly computed.

```

```

IntegrationResult(F:Field): Exports == Implementation where
  O ==> OutputForm
  B ==> Boolean
  Z ==> Integer
  Q ==> Fraction Integer
  SE ==> Symbol

```



```

UP ==> SparseUnivariatePolynomial F
LOG ==> Record(scalar:Q, coeff:UP, logand:UP)
NE ==> Record(integrand:F, intvar:F)

Exports ==> (Module Q, RetractableTo F) with
mkAnswer: (F, List LOG, List NE) -> %
  ++ mkAnswer(r,l,ne) creates an integration result from
  ++ a rational part r, a logarithmic part l, and a non-elementary part ne.
ratpart : % -> F
  ++ ratpart(ir) returns the rational part of an integration result
logpart : % -> List LOG
  ++ logpart(ir) returns the logarithmic part of an integration result
notelem : % -> List NE
  ++ notelem(ir) returns the non-elementary part of an integration result
elem? : % -> B
  ++ elem?(ir) tests if an integration result is elementary over F?
integral: (F, F) -> %
  ++ integral(f,x) returns the formal integral of f with respect to x
differentiate: (% , F -> F) -> F
  ++ differentiate(ir,D) differentiates ir with respect to the derivation D.
if F has PartialDifferentialRing(SE) then
  differentiate: (% , Symbol) -> F
  ++ differentiate(ir,x) differentiates ir with respect to x
if F has RetractableTo Symbol then
  integral: (F, Symbol) -> %
  ++ integral(f,x) returns the formal integral of f with respect to x

Implementation ==> add
Rep := Record(ratp: F, logp: List LOG, nelem: List NE)

timelog : (Q, LOG) -> LOG
timene : (Q, NE) -> NE
LOG2O : LOG -> O
NE2O : NE -> O
Q2F : Q -> F
nesimp : List NE -> List NE
neselect: (List NE, F) -> F
pLogDeriv: (LOG, F -> F) -> F
pNeDeriv : (NE, F -> F) -> F

alpha:O := new()$Symbol :: O

- u == (-1$Z) * u
0 == mkAnswer(0, empty(), empty())
coerce(x:F):% == mkAnswer(x, empty(), empty())
ratpart u == u.ratp
logpart u == u.logp
notelem u == u.nelem
elem? u == empty? notelem u

```

```

mkAnswer(x, l, n) == [x, l, nesimp n]
timelog(r, lg)    == [r * lg.scalar, lg.coeff, lg.logand]
integral(f:F,x:F) == (zero? f => 0; mkAnswer(0, empty(), [[f, x]]))
timene(r, ne)     == [Q2F(r) * ne.integrand, ne.intvar]
n:Z * u:%         == (n:Q) * u
Q2F r             == numer(r)::F / denom(r)::F
neselect(l, x)    == _+/[ne.integrand for ne in l | ne.intvar = x]

if F has RetractableTo Symbol then
  integral(f:F, x:Symbol):% == integral(f, x:F)

LOG20 rec ==
--  one? degree rec.coeff =>
  (degree rec.coeff) = 1 =>
    -- deg 1 minimal poly doesn't get sigma
    lastc := - coefficient(rec.coeff, 0) / coefficient(rec.coeff, 1)
    lg    := (rec.logand) lastc
    logandp := prefix("log"::Symbol::0, [lg::0])
    (cc := Q2F(rec.scalar) * lastc) = 1 => logandp
    cc = -1 => - logandp
    cc::0 * logandp
    coeffp:0 := (outputForm(rec.coeff, alpha) = 0::Z::0)@0
    logandp :=
      alpha * prefix("log"::Symbol::0, [outputForm(rec.logand, alpha)])
    if (cc := Q2F(rec.scalar)) ^= 1 then
      logandp := cc::0 * logandp
    sum(logandp, coeffp)

nesimp l ==
  [[u,x] for x in removeDuplicates_!([ne.intvar for ne in l]$List(F))
    | (u := neselect(l, x)) ^= 0]

if (F has LiouvillianFunctionCategory) and (F has RetractableTo Symbol) then
  retractIfCan u ==
    empty? logpart u =>
      ratpart u +
        _+/[integral(ne.integrand, retract(ne.intvar)@Symbol)$F
          for ne in notelem u]
    "failed"

else
  retractIfCan u ==
    elem? u and empty? logpart u => ratpart u
    "failed"

r:Q * u:% ==
  r = 0 => 0
  mkAnswer(Q2F(r) * ratpart u, map(x1+>timelog(r, x1), logpart u),
    map(x2+>timene(r, x2), notelem u))

```

```

-- Initial attempt, quick and dirty, no simplification
u + v ==
  mkAnswer(ratpart u + ratpart v, concat(logpart u, logpart v),
            nesimp concat(notelem u, notelem v))

if F has PartialDifferentialRing(Symbol) then
  differentiate(u:%, x:Symbol):F ==
    differentiate(u, x1-->differentiate(x1, x))

differentiate(u:%, derivation:F -> F):F ==
  derivation ratpart u +
    _+/[pLogDeriv(log, derivation) for log in logpart u]
    + _+/[pNeDeriv(ne, derivation) for ne in notelem u]

pNeDeriv(ne, derivation) ==
--   one? derivation(ne.intvar) => ne.integrand
   (derivation(ne.intvar) = 1) => ne.integrand
  zero? derivation(ne.integrand) => 0
  error "pNeDeriv: cannot differentiate not elementary part into F"

pLogDeriv(log, derivation) ==
  map(derivation, log.coeff) ^= 0 =>
    error "pLogDeriv: can only handle logs with constant coefficients"
--   one?(n := degree(log.coeff)) =>
   ((n := degree(log.coeff)) = 1) =>
     c := - (leadingCoefficient reductum log.coeff)
           / (leadingCoefficient log.coeff)

   ans := (log.logand) c
   Q2F(log.scalar) * c * derivation(ans) / ans
  numlog := map(derivation, log.logand)
  diflog := extendedEuclidean(log.logand, log.coeff,
                               numlog)::Record(coef1:UP, coef2:UP)

  algans := diflog.coef1
  ans:F := 0
  for i in 0..(n-1) repeat
    algans := algans * monomial(1, 1) rem log.coeff
    ans := ans + coefficient(algans, i)
  Q2F(log.scalar) * ans

coerce(u:%):0 ==
  (r := retractIfCan u) case F => r::F::0
  l := reverse_! [LOG20 f for f in logpart u]$List(0)
  if ratpart u ^= 0 then l := concat(ratpart(u)::0, l)
  if not elem? u then l := concat([NE20 f for f in notelem u], l)
  null l => 0::0
  reduce("+", l)

NE20 ne ==
  int((ne.integrand)::0 * hconcat ["d "::Symbol::0, (ne.intvar)::0])

```

— IR.dotabb —

---

### 10.34 domain INTRVL Interval

— Interval.input —

```

)set break resume
)sys rm -f Interval.output
)spool Interval.output
)set message test on
)set message auto off
)clear all

--S 1 of 13
t1:=0::Interval(Float)
--R
--R
--R (1) [0.0,0.0]
--R
--R Type: Interval Float
--E 1

--S 2 of 13
t2:=1::Interval(Float)
--R
--R
--R (2) [1.0,1.0]
--R
--R Type: Interval Float
--E 2

--S 3 of 13
t3:=3*t1
--R
--R
--R (3) [- 0.3 E -20,0.3 E -20]
--R
--R Type: Interval Float
--E 3

```

```

--S 4 of 13
t4:=1/4*t2
--R
--R
--R      [1.0,1.0]
--R      (4)  -----
--R      [4.0,4.0]
--R
--R                                          Type: Fraction Interval Float
--E 4

--S 5 of 13
acos(t3)
--R
--R
--R      (5)  [1.5707963267 948966192,1.5707963267 948966192]
--R
--R                                          Type: Interval Float
--E 5

--S 6 of 13
t6:=t4*t4
--R
--R
--R      [1.0,1.0]
--R      (6)  -----
--R      [16.0,16.0]
--R
--R                                          Type: Fraction Interval Float
--E 6

--S 7 of 13
sup(t6)
--R
--R
--R      (7)  0.0625000000 0000000000 1
--R
--R                                          Type: Float
--E 7

--S 8 of 13
inf(t6)
--R
--R
--R      (8)  0.0624999999 9999999999 9
--R
--R                                          Type: Float
--E 8

--S 9 of 13
width(t6)
--R
--R
--R      (9)  0.1 E -20
--R
--R                                          Type: Float

```

```

--E 9

--S 10 of 13
positive? t3
--R
--R
--R (10) false
--R
--R                                          Type: Boolean
--E 10

--S 11 of 13
negative? t3
--R
--R
--R (11) false
--R
--R                                          Type: Boolean
--E 11

--S 12 of 13
contains?(t3,0.3)
--R
--R
--R (12) false
--R
--R                                          Type: Boolean
--E 12

--S 13 of 13
)show Interval
--R
--R Interval R: Join(FloatingPointSystem,TranscendentalFunctionCategory) is a domain constructor
--R Abbreviation for Interval is INTRVL
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for INTRVL
--R
--R----- Operations -----
--R ?? : (%,% ) -> %
--R ?? : (PositiveInteger,% ) -> %
--R ??? : (%,% ) -> %
--R ??+? : (%,% ) -> %
--R -? : % -> %
--R ?<=? : (%,% ) -> Boolean
--R ?>? : (%,% ) -> Boolean
--R 1 : () -> %
--R ?^? : (% ,PositiveInteger) -> %
--R acosh : % -> %
--R acoth : % -> %
--R acsch : % -> %
--R asech : % -> %
--R asinh : % -> %
--R atan : % -> %
--R ??? : (Integer,% ) -> %
--R ??? : (% ,Fraction Integer) -> %
--R ??? : (% ,PositiveInteger) -> %
--R ?-? : (%,% ) -> %
--R ?<? : (%,% ) -> Boolean
--R ?=? : (%,% ) -> Boolean
--R ?>=? : (%,% ) -> Boolean
--R 0 : () -> %
--R acos : % -> %
--R acot : % -> %
--R acsc : % -> %
--R asec : % -> %
--R asin : % -> %
--R associates? : (%,% ) -> Boolean
--R atanh : % -> %

```

```

--R coerce : Integer -> %
--R coerce : Integer -> %
--R contains? : (% , R) -> Boolean
--R cosh : % -> %
--R coth : % -> %
--R csch : % -> %
--R gcd : List % -> %
--R hash : % -> SingleInteger
--R interval : Fraction Integer -> %
--R interval : (R , R) -> %
--R lcm : List % -> %
--R log : % -> %
--R min : (% , %) -> %
--R nthRoot : (% , Integer) -> %
--R pi : () -> %
--R qinterval : (R , R) -> %
--R retract : % -> Integer
--R sec : % -> %
--R sin : % -> %
--R sqrt : % -> %
--R tan : % -> %
--R unit? : % -> Boolean
--R width : % -> R
--R ~=? : (% , %) -> Boolean
--R ?? : (NonNegativeInteger , %) -> %
--R ***? : (% , NonNegativeInteger) -> %
--R ??^? : (% , NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R exquo : (% , %) -> Union(% , "failed")
--R gcdPolynomial : (SparseUnivariatePolynomial % , SparseUnivariatePolynomial %) -> SparseUni
--R retractIfCan : % -> Union(Integer , "failed")
--R subtractIfCan : (% , %) -> Union(% , "failed")
--R unitNormal : % -> Record(unit : % , canonical : % , associate : %)
--R
--E 13

)spool
)lisp (bye)

```

---

— Interval.help —

Interval examples

```

t1:=0::Interval(Float)

```

```

[0.0,0.0]

t2:=1::Interval(Float)
[1.0,1.0]

t3:=3*t1
[- 0.3 E -20,0.3 E -20]

t4:=1/4*t2
[1.0,1.0]
-----
[4.0,4.0]

acos(t3)
[1.5707963267 948966192,1.5707963267 948966192]

t6:=t4*t4
[1.0,1.0]
-----
[16.0,16.0]

sup(t6)
0.0625000000 0000000000 1

inf(t6)
0.0624999999 9999999999 9

width(t6)
0.1 E -20

positive? t3
false

negative? t3
false

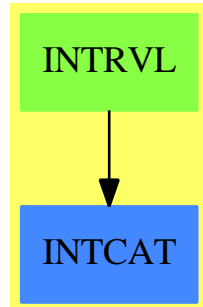
contains?(t3,0.3)
false

```

---



## 10.34.1 Interval (INTRVL)

**Exports:**

|                |               |               |         |              |
|----------------|---------------|---------------|---------|--------------|
| 0              | 1             | acos          | acosh   | acot         |
| acoth          | acsc          | acsch         | asec    | asech        |
| asin           | asinh         | associates?   | atan    | atanh        |
| characteristic | coerce        | contains?     | cos     | cosh         |
| cot            | coth          | csc           | csch    | exp          |
| exquo          | ged           | gcdPolynomial | hash    | inf          |
| interval       | latex         | lcm           | log     | max          |
| min            | negative?     | nthRoot       | one?    | pi           |
| positive?      | qinterval     | recip         | retract | retractIfCan |
| sample         | sec           | sech          | sin     | sinh         |
| sqrt           | subtractIfCan | sup           | tan     | tanh         |
| unit?          | unitCanonical | unitNormal    | width   | zero?        |
| ?*?            | ?**?          | ?+?           | ?-?     | -?           |
| ?<?            | ?<=?          | ?=?           | ?>?     | ?>=?         |
| ?^?            | ?~=?          |               |         |              |

— domain INTRVL Interval —

```

)abbrev domain INTRVL Interval
++ Author: Mike Dewar
++ Date Created: November 1996
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This domain is an implementation of interval arithmetic and transcendental
++ functions over intervals.

Interval(R:Join(FloatingPointSystem,TranscendentalFunctionCategory)): IntervalCategory(R) ==

```

```

import Integer
-- import from R

Rep := Record(Inf:R, Sup:R)

roundDown(u:R):R ==
  if zero?(u) then float(-1, -(bits() pretend Integer))
  else float(mantissa(u) - 1, exponent(u))

roundUp(u:R):R ==
  if zero?(u) then float(1, -(bits()) pretend Integer)
  else float(mantissa(u) + 1, exponent(u))

-- Sometimes the float representation does not use all the bits (e.g. when
-- representing an integer in software using arbitrary-length Integers as
-- your mantissa it is convenient to keep them exact). This function
-- normalises things so that rounding etc. works as expected. It is only
-- called when creating new intervals.
normaliseFloat(u:R):R ==
  zero? u => u
  m : Integer := mantissa u
  b : Integer := bits() pretend Integer
  l : Integer := length(m)
  if l < b then
    BASE : Integer := base()$R pretend Integer
    float(m*BASE**((b-l) pretend PositiveInteger), exponent(u)-b+1)
  else
    u

interval(i:R, s:R):% ==
  i > s => [roundDown normaliseFloat s, roundUp normaliseFloat i]
  [roundDown normaliseFloat i, roundUp normaliseFloat s]

interval(f:R):% ==
  zero?(f) => 0
  one?(f) => 1
  -- This next part is necessary to allow e.g. mapping between Expressions:
  -- AXIOM assumes that Integers stay as Integers!
-- import from Union(value1:Integer, failed:"failed")
fnew : R := normaliseFloat f
retractIfCan(f)@Union(Integer, "failed") case "failed" =>
  [roundDown fnew, roundUp fnew]
  [fnew, fnew]

qinterval(i:R, s:R):% ==
  [roundDown normaliseFloat i, roundUp normaliseFloat s]

exactInterval(i:R, s:R):% == [i, s]
exactSupInterval(i:R, s:R):% == [roundDown i, s]

```

```

exactInfInterval(i:R,s:R):% == [i,roundUp s]

inf(u:%):R == u.Inf
sup(u:%):R == u.Sup
width(u:%):R == u.Sup - u.Inf

contains?(u:%,f:R):Boolean == (f > inf(u)) and (f < sup(u))

positive?(u:%):Boolean == inf(u) > 0
negative?(u:%):Boolean == sup(u) < 0

_< (a:%,b:%):Boolean ==
  if inf(a) < inf(b) then
    true
  else if inf(a) > inf(b) then
    false
  else
    sup(a) < sup(b)

_+ (a:%,b:%):% ==
  -- A couple of blatant hacks to preserve the Ring Axioms!
  if zero?(a) then return(b) else if zero?(b) then return(a)
  if a = b then return qinterval(2*inf(a),2*sup(a))
  qinterval(inf(a) + inf(b), sup(a) + sup(b))

_- (a:%,b:%):% ==
  if zero?(a) then return(-b) else if zero?(b) then return(a)
  if a = b then 0 else qinterval(inf(a) - sup(b), sup(a) - inf(b))

_* (a:%,b:%):% ==
  -- A couple of blatant hacks to preserve the Ring Axioms!
  if one?(a) then return(b) else if one?(b) then return(a)
  if zero?(a) then return(0) else if zero?(b) then return(0)
  prods : List R := sort [inf(a)*inf(b),sup(a)*sup(b),
                          inf(a)*sup(b),sup(a)*inf(b)]
  qinterval(first prods, last prods)

_* (a:Integer,b:%):% ==
  if (a > 0) then
    qinterval(a*inf(b),a*sup(b))
  else if (a < 0) then
    qinterval(a*sup(b),a*inf(b))
  else
    0

_* (a:PositiveInteger,b:%):% == qinterval(a*inf(b),a*sup(b))

```

```

_** (a:%,n:PositiveInteger):% ==
  contains?(a,0) and zero?((n pretend Integer) rem 2) =>
    interval(0,max(inf(a)**n,sup(a)**n))
  interval(inf(a)**n,sup(a)**n)

_^ (a:%,n:PositiveInteger):% ==
  contains?(a,0) and zero?((n pretend Integer) rem 2) =>
    interval(0,max(inf(a)**n,sup(a)**n))
  interval(inf(a)**n,sup(a)**n)

_- (a:%):% == exactInterval(-sup(a),-inf(a))

_= (a:%,b:%):Boolean == (inf(a)=inf(b)) and (sup(a)=sup(b))
~_= (a:%,b:%):Boolean == (inf(a)~=inf(b)) or (sup(a)~=sup(b))

1 ==
  one : R := normaliseFloat 1
  [one,one]

0 == [0,0]

recip(u:%):Union(%, "failed") ==
  contains?(u,0) => "failed"
  vals:List R := sort [1/inf(u),1/sup(u)]$List(R)
  qinterval(first vals, last vals)

unit?(u:%):Boolean == contains?(u,0)

_exquo(u:%,v:%):Union(%, "failed") ==
  contains?(v,0) => "failed"
  one?(v) => u
  u=v => 1
  u=-v => -1
  vals:List R := sort [inf(u)/inf(v),inf(u)/sup(v),sup(u)/inf(v),sup(u)/sup(v)]$List(R)
  qinterval(first vals, last vals)

gcd(u:%,v:%):% == 1

coerce(u:Integer):% ==
  ur := normaliseFloat(u::R)
  exactInterval(ur,ur)

interval(u:Fraction Integer):% ==
--   import    log2 : % -> %
--           coerce : Integer -> %
--           retractIfCan : % -> Union(value1:Integer,failed:"failed")

```

```

--      from Float
      flt := u::R

      -- Test if the representation in R is exact
      --den := denom(u)::Float
      bin : Union(Integer,"failed") := retractIfCan(log2(denom(u)::Float))
      bin case Integer and length(numer u)$Integer < (bits() pretend Integer) =>
        flt := normaliseFloat flt
        exactInterval(flt,flt)

      qinterval(flt,flt)

      retractIfCan(u:%):Union(Integer,"failed") ==
        not zero? width(u) => "failed"
        retractIfCan inf u

      retract(u:%):Integer ==
        not zero? width(u) =>
          error "attempt to retract a non-Integer interval to an Integer"
          retract inf u

      coerce(u:%):OutputForm ==
        bracket([coerce inf(u), coerce sup(u)]$List(OutputForm))

      characteristic():NonNegativeInteger == 0

      -- Explicit export from TranscendentalFunctionCategory
      pi():% == qinterval(pi(),pi())

      -- From ElementaryFunctionCategory
      log(u:%):% ==
        positive?(u) => qinterval(log inf u, log sup u)
        error "negative logs in interval"

      exp(u:%):% == qinterval(exp inf u, exp sup u)

      *_* (u:%,v:%):% ==
        zero?(v) => if zero?(u) then error "0**0 is undefined" else 1
        one?(u)  => 1
        expts : List R := sort [inf(u)**inf(v),sup(u)**sup(v),
                               inf(u)**sup(v),sup(u)**inf(v)]
        qinterval(first expts, last expts)

      -- From TrigonometricFunctionCategory

```

```

-- This function checks whether an interval contains a value of the form
-- 'offset + 2 n pi'.
hasTwoPiMultiple(offset:R,ipi:R,i:%):Boolean ==
  next : Integer := retract ceiling( (inf(i) - offset)/(2*ipi) )
  contains?(i,offset+2*next*ipi)

-- This function checks whether an interval contains a value of the form
-- 'offset + n pi'.
hasPiMultiple(offset:R,ipi:R,i:%):Boolean ==
  next : Integer := retract ceiling( (inf(i) - offset)/ipi )
  contains?(i,offset+next*ipi)

sin(u:%):% ==
  ipi : R := pi()$R
  hasOne? : Boolean := hasTwoPiMultiple(ipi/(2::R),ipi,u)
  hasMinusOne? : Boolean := hasTwoPiMultiple(3*ipi/(2::R),ipi,u)

  if hasOne? and hasMinusOne? then
    exactInterval(-1,1)
  else
    vals : List R := sort [sin inf u, sin sup u]
    if hasOne? then
      exactSupInterval(first vals, 1)
    else if hasMinusOne? then
      exactInfInterval(-1,last vals)
    else
      qinterval(first vals, last vals)

cos(u:%):% ==
  ipi : R := pi()
  hasOne? : Boolean := hasTwoPiMultiple(0,ipi,u)
  hasMinusOne? : Boolean := hasTwoPiMultiple(ipi,ipi,u)

  if hasOne? and hasMinusOne? then
    exactInterval(-1,1)
  else
    vals : List R := sort [cos inf u, cos sup u]
    if hasOne? then
      exactSupInterval(first vals, 1)
    else if hasMinusOne? then
      exactInfInterval(-1,last vals)
    else
      qinterval(first vals, last vals)

```

```

tan(u:~):~ ==
  ipi : R := pi()
  if width(u) > ipi then
    error "Interval contains a singularity"
  else
    -- Since we know the interval is less than pi wide, monotonicity implies
    -- that there is no singularity. If there is a singularity on a endpoint
    -- of the interval the user will see the error generated by R.
    lo : R := tan inf u
    hi : R := tan sup u

    lo > hi => error "Interval contains a singularity"
    qinterval(lo,hi)

csc(u:~):~ ==
  ipi : R := pi()
  if width(u) > ipi then
    error "Interval contains a singularity"
  else
    -- import from Integer
    -- singularities are at multiples of Pi
    if hasPiMultiple(0,ipi,u) then error "Interval contains a singularity"
    vals : List R := sort [csc inf u, csc sup u]
    if hasTwoPiMultiple(ipi/(2::R),ipi,u) then
      exactInfInterval(1,last vals)
    else if hasTwoPiMultiple(3*ipi/(2::R),ipi,u) then
      exactSupInterval(first vals,-1)
    else
      qinterval(first vals, last vals)

sec(u:~):~ ==
  ipi : R := pi()
  if width(u) > ipi then
    error "Interval contains a singularity"
  else
    -- import from Integer
    -- singularities are at Pi/2 + n Pi
    if hasPiMultiple(ipi/(2::R),ipi,u) then
      error "Interval contains a singularity"
    vals : List R := sort [sec inf u, sec sup u]
    if hasTwoPiMultiple(0,ipi,u) then
      exactInfInterval(1,last vals)
    else if hasTwoPiMultiple(ipi,ipi,u) then
      exactSupInterval(first vals,-1)
    else
      qinterval(first vals, last vals)

```

```

cot(u:%) :=
  ipi : R := pi()
  if width(u) > ipi then
    error "Interval contains a singularity"
  else
    -- Since we know the interval is less than pi wide, monotonicity implies
    -- that there is no singularity. If there is a singularity on a endpoint
    -- of the interval the user will see the error generated by R.
    hi : R := cot inf u
    lo : R := cot sup u

    lo > hi => error "Interval contains a singularity"
    qinterval(lo,hi)

-- From ArcTrigonometricFunctionCategory

asin(u:%) :=
  lo : R := inf(u)
  hi : R := sup(u)
  if (lo < -1) or (hi > 1) then error "asin only defined on the region -1..1"
  qinterval(asin lo, asin hi)

acos(u:%) :=
  lo : R := inf(u)
  hi : R := sup(u)
  if (lo < -1) or (hi > 1) then error "acos only defined on the region -1..1"
  qinterval(acos hi, acos lo)

atan(u:%) := qinterval(atan inf u, atan sup u)

acot(u:%) := qinterval(acot sup u, acot inf u)

acsc(u:%) :=
  lo : R := inf(u)
  hi : R := sup(u)
  if ((lo <= -1) and (hi >= -1)) or ((lo <= 1) and (hi >= 1)) then
    error "acsc not defined on the region -1..1"
  qinterval(acsc hi, acsc lo)

asec(u:%) :=
  lo : R := inf(u)

```



```

    hi : R := sup(u)
    if ((lo < -1) and (hi > -1)) or ((lo < 1) and (hi > 1)) then
        error "asec not defined on the region -1..1"
    qinterval(asec lo, asec hi)

-- From HyperbolicFunctionCategory

tanh(u:~):~ == qinterval(tanh inf u, tanh sup u)

sinh(u:~):~ == qinterval(sinh inf u, sinh sup u)

sech(u:~):~ ==
    negative? u => qinterval(sech inf u, sech sup u)
    positive? u => qinterval(sech sup u, sech inf u)
    vals : List R := sort [sech inf u, sech sup u]
    exactSupInterval(first vals,1)

cosh(u:~):~ ==
    negative? u => qinterval(cosh sup u, cosh inf u)
    positive? u => qinterval(cosh inf u, cosh sup u)
    vals : List R := sort [cosh inf u, cosh sup u]
    exactInfInterval(1,last vals)

csch(u:~):~ ==
    contains?(u,0) => error "csch: singularity at zero"
    qinterval(csch sup u, csch inf u)

coth(u:~):~ ==
    contains?(u,0) => error "coth: singularity at zero"
    qinterval(coth sup u, coth inf u)

-- From ArcHyperbolicFunctionCategory

acosh(u:~):~ ==
    inf(u)<1 => error "invalid argument: acosh only defined on the region 1.."
    qinterval(acosh inf u, acosh sup u)

acoth(u:~):~ ==
    lo : R := inf(u)
    hi : R := sup(u)
    if ((lo <= -1) and (hi >= -1)) or ((lo <= 1) and (hi >= 1)) then
        error "acoth not defined on the region -1..1"
    qinterval(acoth hi, acoth lo)

```

```

acsch(u:):% ==
  contains?(u,0) => error "acsch: singularity at zero"
  qinterval(acsch sup u, acsch inf u)

asech(u:):% ==
  lo : R := inf(u)
  hi : R := sup(u)
  if (lo <= 0) or (hi > 1) then
    error "asech only defined on the region 0 < x <= 1"
  qinterval(asech hi, asech lo)

asinh(u:):% == qinterval(asinh inf u, asinh sup u)

atanh(u:):% ==
  lo : R := inf(u)
  hi : R := sup(u)
  if (lo <= -1) or (hi >= 1) then
    error "atanh only defined on the region -1 < x < 1"
  qinterval(atanh lo, atanh hi)

-- From RadicalCategory
_** (u:%,n:Fraction Integer):% == interval(inf(u)**n,sup(u)**n)

-----

— INTRVL.dotabb —

"INTRVL" [color="#88FF44",href="bookvol10.3.pdf#nameddest=INTRVL"]
"INTCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=INTCAT"]
"INTRVL" -> "INTCAT"

-----

```



**Chapter 11**

**Chapter J**



## Chapter 12

# Chapter K

### 12.1 domain KERNEL Kernel

— Kernel.input —

```
)set break resume
)sys rm -f Kernel.output
)spool Kernel.output
)set message test on
)set message auto off
)clear all
--S 1 of 19
x :: Expression Integer
--R
--R
--R (1) x
--R
--R                                          Type: Expression Integer
--E 1

--S 2 of 19
kernel x
--R
--R
--R (2) x
--R
--R                                          Type: Kernel Expression Integer
--E 2

--S 3 of 19
sin(x) + cos(x)
--R
--R
--R (3) sin(x) + cos(x)
```

```

--R
--E 3
Type: Expression Integer

--S 4 of 19
kernels %
--R
--R
--R (4) [sin(x),cos(x)]
--R
--E 4
Type: List Kernel Expression Integer

--S 5 of 19
sin(x)**2 + sin(x) + cos(x)
--R
--R
--R (5) sin(x)2 + sin(x) + cos(x)
--R
--E 5
Type: Expression Integer

--S 6 of 19
kernels %
--R
--R
--R (6) [sin(x),cos(x)]
--R
--E 6
Type: List Kernel Expression Integer

--S 7 of 19
kernels(1 :: Expression Integer)
--R
--R
--R (7) []
--R
--E 7
Type: List Kernel Expression Integer

--S 8 of 19
mainKernel(cos(x) + tan(x))
--R
--R
--R (8) tan(x)
--R
--E 8
Type: Union(Kernel Expression Integer,...)

--S 9 of 19
height kernel x
--R
--R
--R (9) 1
--R
--E 9
Type: PositiveInteger

```

```
--E 9

--S 10 of 19
height mainKernel(sin x)
--R 
--R 
--R (10)  2
--R 
Type: PositiveInteger
--E 10

--S 11 of 19
height mainKernel(sin cos x)
--R 
--R 
--R (11)  3
--R 
Type: PositiveInteger
--E 11

--S 12 of 19
height mainKernel(sin cos (tan x + sin x))
--R 
--R 
--R (12)  4
--R 
Type: PositiveInteger
--E 12

--S 13 of 19
operator mainKernel(sin cos (tan x + sin x))
--R 
--R 
--R (13)  sin
--R 
Type: BasicOperator
--E 13

--S 14 of 19
name mainKernel(sin cos (tan x + sin x))
--R 
--R 
--R (14)  sin
--R 
Type: Symbol
--E 14

--S 15 of 19
f := operator 'f
--R 
--R 
--R (15)  f
--R 
Type: BasicOperator
--E 15
```



```

--S 16 of 19
e := f(x, y, 10)
--R
--R
--R (16) f(x,y,10)
--R
--R                                          Type: Expression Integer
--E 16

--S 17 of 19
is?(e, f)
--R
--R
--R (17) true
--R
--R                                          Type: Boolean
--E 17

--S 18 of 19
is?(e, 'f)
--R
--R
--R (18) true
--R
--R                                          Type: Boolean
--E 18

--S 19 of 19
argument mainKernel e
--R
--R
--R (19) [x,y,10]
--R
--R                                          Type: List Expression Integer
--E 19
)spool
)lisp (bye)

```

---

— Kernel.help —

---

#### Kernel examples

---

A kernel is a symbolic function application (such as  $\sin(x+y)$ ) or a symbol (such as  $x$ ). More precisely, a non-symbol kernel over a set  $S$  is an operator applied to a given list of arguments from  $S$ . The operator has type `BasicOperator` and the kernel object is usually part of an `Expression` object.

Kernels are created implicitly for you when you create expressions.

```
x :: Expression Integer
x
```

Type: Expression Integer

You can directly create a "symbol" kernel by using the kernel operation.

```
kernel x
x
```

Type: Kernel Expression Integer

This expression has two different kernels.

```
sin(x) + cos(x)
sin(x) + cos(x)
```

Type: Expression Integer

The operator kernels returns a list of the kernels in an object of type Expression.

```
kernels %
[sin(x),cos(x)]
```

Type: List Kernel Expression Integer

This expression also has two different kernels.

```
sin(x)**2 + sin(x) + cos(x)
      2
sin(x)  + sin(x) + cos(x)
```

Type: Expression Integer

The sin(x) kernel is used twice.

```
kernels %
[sin(x),cos(x)]
```

Type: List Kernel Expression Integer

An expression need not contain any kernels.

```
kernels(1 :: Expression Integer)
[]
```

Type: List Kernel Expression Integer

If one or more kernels are present, one of them is designated the main kernel.

```
mainKernel(cos(x) + tan(x))
tan(x)
```

Type: Union(Kernel Expression Integer,...)

Kernels can be nested. Use `height` to determine the nesting depth.

```
height kernel x
1
Type: PositiveInteger
```

This has height 2 because the `x` has height 1 and then we apply an operator to that.

```
height mainKernel(sin x)
2
Type: PositiveInteger
```

```
height mainKernel(sin cos x)
3
Type: PositiveInteger
```

```
height mainKernel(sin cos (tan x + sin x))
4
Type: PositiveInteger
```

Use the `operator` operation to extract the operator component of the kernel. The operator has type `BasicOperator`.

```
operator mainKernel(sin cos (tan x + sin x))
sin
Type: BasicOperator
```

Use the `name` operation to extract the name of the operator component of the kernel. The name has type `Symbol`. This is really just a shortcut for a two-step process of extracting the operator and then calling `name` on the operator.

```
name mainKernel(sin cos (tan x + sin x))
sin
Type: Symbol
```

Axiom knows about functions such as `sin`, `cos` and so on and can make kernels and then expressions using them. To create a kernel and expression using an arbitrary operator, use `operator`.

Now `f` can be used to create symbolic function applications.

```
f := operator 'f
f
Type: BasicOperator

e := f(x, y, 10)
f(x,y,10)
Type: Expression Integer
```

Use the `is?` operation to learn if the operator component of a kernel is equal to a given operator.

```
is?(e, f)
true
Type: Boolean
```

You can also use a symbol or a string as the second argument to `is?`.

```
is?(e, 'f')
true
Type: Boolean
```

Use the `argument` operation to get a list containing the argument component of a kernel.

```
argument mainKernel e
[x,y,10]
Type: List Expression Integer
```

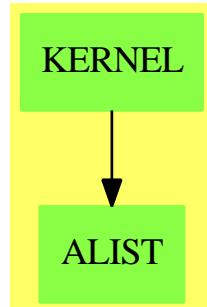
Conceptually, an object of type `Expression` can be thought of a quotient of multivariate polynomials, where the "variables" are kernels. The arguments of the kernels are again expressions and so the structure recurses. See `Expression` for examples of using kernels to take apart expression objects.

See Also:

- o `)help Expression`
- o `)help BasicOperator`
- o `)show Kernel`

---

### 12.1.1 Kernel (KERNEL)



See

⇒ “MakeCachableSet” (MKCHSET) 14.5.1 on page 1534

#### Exports:

|          |          |          |             |             |
|----------|----------|----------|-------------|-------------|
| argument | coerce   | convert  | hash        | height      |
| is?      | kernel   | latex    | max         | min         |
| name     | operator | position | setPosition | symbolIfCan |
| ?~=?     | ?<?      | ?<=?     | ?=?         | ?>?         |
| ?>=?     |          |          |             |             |

— domain **KERNEL** Kernel —

```

)abbrev domain KERNEL Kernel
++ Author: Manuel Bronstein
++ Date Created: 22 March 1988
++ Date Last Updated: 10 August 1994
++ Description:
++ A kernel over a set S is an operator applied to a given list
++ of arguments from S.

```

```

Kernel(S:OrderedSet): Exports == Implementation where
  O ==> OutputForm
  N ==> NonNegativeInteger
  OP ==> BasicOperator

```

```

SYMBOL ==> "%symbol"
PMPRED ==> "%pmpredicate"
PMOPT ==> "%pmoptional"
PMMULT ==> "%pmmultiple"
PMCONST ==> "%pmconstant"
SPECIALDISP ==> "%specialDisp"
SPECIALEQUAL ==> "%specialEqual"
SPECIALINPUT ==> "%specialInput"

```

```

Exports ==> Join(CachableSet, Patternable S) with
  name      : % -> Symbol

```

```

    ++ name(op(a1,...,an)) returns the name of op.
operator: % -> OP
    ++ operator(op(a1,...,an)) returns the operator op.
argument: % -> List S
    ++ argument(op(a1,...,an)) returns \spad{[a1,...,an]}.
height : % -> N
    ++ height(k) returns the nesting level of k.
kernel : (OP, List S, N) -> %
    ++ kernel(op, [a1,...,an], m) returns the kernel \spad{op(a1,...,an)}
    ++ of nesting level m.
    ++ Error: if op is k-ary for some k not equal to m.
kernel : Symbol -> %
    ++ kernel(x) returns x viewed as a kernel.
symbolIfCan: % -> Union(Symbol, "failed")
    ++ symbolIfCan(k) returns k viewed as a symbol if k is a symbol, and
    ++ "failed" otherwise.
is?      : (%, OP) -> Boolean
    ++ is?(op(a1,...,an), f) tests if op = f.
is?      : (%, Symbol) -> Boolean
    ++ is?(op(a1,...,an), s) tests if the name of op is s.
if S has ConvertibleTo InputForm then ConvertibleTo InputForm

Implementation ==> add
import SortedCache(%)

Rep := Record(op:OP, arg:List S, nest:N, posit:N)

clearCache()

B2Z : Boolean -> Integer
triage: (%, %) -> Integer
preds : OP -> List Any

is?(k:%, s:Symbol) == is?(operator k, s)
is?(k:%, o:OP) == (operator k) = o
name k == name operator k
height k == k.nest
operator k == k.op
argument k == k.arg
position k == k.posit
setPosition(k, n) == k.posit := n
B2Z flag == (flag => -1; 1)
kernel s == kernel(assert(operator(s,0),SYMBOL), nil(), 1)

preds o ==
  (u := property(o, PMPRED)) case "failed" => nil()
  (u::None) pretend List(Any)

symbolIfCan k ==
  has?(operator k, SYMBOL) => name operator k

```

```

"failed"

k1 = k2 ==
  if k1.posit = 0 then enterInCache(k1, triage)
  if k2.posit = 0 then enterInCache(k2, triage)
  k1.posit = k2.posit

k1 < k2 ==
  if k1.posit = 0 then enterInCache(k1, triage)
  if k2.posit = 0 then enterInCache(k2, triage)
  k1.posit < k2.posit

kernel(fn, x, n) ==
  ((u := arity fn) case N) and (#x ^= u::N)
  => error "Wrong number of arguments"
  enterInCache([fn, x, n, 0]$Rep, triage)

-- SPECIALDISP contains a map List S -> OutputForm
-- it is used when the converting the arguments first is not good,
-- for instance with formal derivatives.
coerce(k:%):OutputForm ==
  (v := symbolIfCan k) case Symbol => v::Symbol::OutputForm
  (f := property(o := operator k, SPECIALDISP)) case None =>
    ((f::None) pretend (List S -> OutputForm)) (argument k)
  l := [x::OutputForm for x in argument k]$List(OutputForm)
  (u := display o) case "failed" => prefix(name(o)::OutputForm, l)
  (u::(List OutputForm -> OutputForm)) l

triage(k1, k2) ==
  k1.nest ^= k2.nest => B2Z(k1.nest < k2.nest)
  k1.op ^= k2.op => B2Z(k1.op < k2.op)
  (n1 := #(argument k1)) ^= (n2 := #(argument k2)) => B2Z(n1 < n2)
  ((func := property(operator k1, SPECIALEQUAL)) case None) and
  (((func::None) pretend ((%, %) -> Boolean)) (k1, k2)) => 0
  for x1 in argument(k1) for x2 in argument(k2) repeat
    x1 ^= x2 => return B2Z(x1 < x2)
  0

if S has ConvertibleTo InputForm then
  convert(k:%):InputForm ==
    (v := symbolIfCan k) case Symbol => convert(v::Symbol)@InputForm
    (f := property(o := operator k, SPECIALINPUT)) case None =>
      ((f::None) pretend (List S -> InputForm)) (argument k)
    l := [convert x for x in argument k]$List(InputForm)
    (u := input operator k) case "failed" =>
      convert concat(convert name operator k, l)
    (u::(List InputForm -> InputForm)) l

if S has ConvertibleTo Pattern Integer then
  convert(k:%):Pattern(Integer) ==

```

```

o := operator k
(v := symbolIfCan k) case Symbol =>
  s := patternVariable(v::Symbol,
    has?(o, PMCONST), has?(o, PMOPT), has?(o, PMMULT))
  empty?(l := preds o) => s
  setPredicates(s, l)
o [convert x for x in k.arg]$List(Pattern Integer)

if S has ConvertibleTo Pattern Float then
convert(k:%):Pattern(Float) ==
o := operator k
(v := symbolIfCan k) case Symbol =>
  s := patternVariable(v::Symbol,
    has?(o, PMCONST), has?(o, PMOPT), has?(o, PMMULT))
  empty?(l := preds o) => s
  setPredicates(s, l)
o [convert x for x in k.arg]$List(Pattern Float)

```

---

— KERNEL.dotabb —

```

"KERNEL" [color="#88FF44",href="bookvol10.3.pdf#nameddest=KERNEL"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"KERNEL" -> "ALIST"

```

---

## 12.2 domain KAFILE KeyedAccessFile

— KeyedAccessFile.input —

```

)set break resume
)sys rm -f KeyedAccessFile.output
)spool KeyedAccessFile.output
)set message test on
)set message auto off
)clear all

--S 1 of 20
ey: KeyedAccessFile(Integer) := open("editor.year", "output")
--R
--R
--R (1) "editor.year"

```



[illegible]

```

--S 8 of 20
search("Char", ey)
--R
--R
--R   (8)  1986
--R
--R                                          Type: Union(Integer,...)
--E 8

```

```

--S 9 of 20
search("Smith", ey)
--R
--R
--R   (9)  "failed"
--R
--R                                          Type: Union("failed",...)
--E 9

```

```

--S 10 of 20
remove!("Char", ey)
--R
--R
--R   (10)  1986
--R
--R                                          Type: Union(Integer,...)
--E 10

```

```

--S 11 of 20
keys ey
--R
--R
--R   (11)  ["Fitch","Caviness"]
--R
--R                                          Type: List String
--E 11

```

```

--S 12 of 20
#ey
--R
--R
--R   (12)  2
--R
--R                                          Type: PositiveInteger
--E 12

```

```

--S 13 of 20
KE := Record(key: String, entry: Integer)
--R
--R
--R   (13)  Record(key: String,entry: Integer)
--R
--R                                          Type: Domain
--E 13

```

```

--S 14 of 20

```

```

reopen!(ey, "output")
--R
--R
--R (14) "editor.year"
--R
--R                                          Type: KeyedAccessFile Integer
--E 14

--S 15 of 20
write!(ey, ["van Hulzen", 1983]$KE)
--R
--R
--R (15) [key= "van Hulzen",entry= 1983]
--R
--R                                          Type: Record(key: String,entry: Integer)
--E 15

--S 16 of 20
write!(ey, ["Calmet", 1982]$KE)
--R
--R
--R (16) [key= "Calmet",entry= 1982]
--R
--R                                          Type: Record(key: String,entry: Integer)
--E 16

--S 17 of 20
write!(ey, ["Wang", 1981]$KE)
--R
--R
--R (17) [key= "Wang",entry= 1981]
--R
--R                                          Type: Record(key: String,entry: Integer)
--E 17

--S 18 of 20
close! ey
--R
--R
--R (18) "editor.year"
--R
--R                                          Type: KeyedAccessFile Integer
--E 18

--S 19 of 20
keys ey
--R
--R
--R (19) ["Wang","Calmet","van Hulzen","Fitch","Caviness"]
--R
--R                                          Type: List String
--E 19

--S 20 of 20
members ey
--R

```

```

--R
--R (20) [1981,1982,1983,1984,1985]
--R
--R                                          Type: List Integer
--E 20

)system rm -r editor.year

)spool
)lisp (bye)

```

---

— KeyedAccessFile.help —

```

=====
KeyedAccessFile examples
=====

```

The domain KeyedAccessFile(S) provides files which can be used as associative tables. Data values are stored in these files and can be retrieved according to their keys. The keys must be strings so this type behaves very much like the StringTable(S) domain. The difference is that keyed access files reside in secondary storage while string tables are kept in memory.

Before a keyed access file can be used, it must first be opened. A new file can be created by opening it for output.

```
ey: KeyedAccessFile(Integer) := open("editor.year", "output")
```

Just as for vectors, tables or lists, values are saved in a keyed access file by setting elements.

```
ey."Char" := 1986
```

```
ey."Caviness" := 1985
```

```
ey."Fitch" := 1984
```

Values are retrieved using application, in any of its syntactic forms.

```
ey."Char"
```

```
ey("Char")
```

```
ey "Char"
```

Attempting to retrieve a non-existent element in this way causes an error. If it is not known whether a key exists, you should use the

search operation.

```
search("Char", ey)

search("Smith", ey)
```

When an entry is no longer needed, it can be removed from the file.

```
remove!("Char", ey)
```

The keys operation returns a list of all the keys for a given file.

```
keys ey
```

The # operation gives the number of entries.

```
#ey
```

The table view of keyed access files provides safe operations. That is, if the Axiom program is terminated between file operations, the file is left in a consistent, current state. This means, however, that the operations are somewhat costly. For example, after each update the file is closed.

Here we add several more items to the file, then check its contents.

```
KE := Record(key: String, entry: Integer)

reopen!(ey, "output")
```

If many items are to be added to a file at the same time, then it is more efficient to use the write operation.

```
write!(ey, ["van Hulzen", 1983]$KE)

write!(ey, ["Calmet", 1982]$KE)

write!(ey, ["Wang", 1981]$KE)

close! ey
```

The read operation is also available from the file view, but it returns elements in a random order. It is generally clearer and more efficient to use the keys operation and to extract elements by key.

```
keys ey

members ey

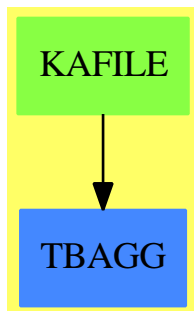
)system rm -r editor.year
```

See Also:

- o )help Table
- o )help StringTable
- o )help File
- o )help TextFile
- o )help Library
- o )show KeyedAccessFile

—————▶

### 12.2.1 KeyedAccessFile (KAFILE)



See

- ⇒ “File” (FILE) 7.2.1 on page 770
- ⇒ “TextFile” (TEXTFILE) 21.5.1 on page 2651
- ⇒ “BinaryFile” (BINFILE) 3.8.1 on page 277
- ⇒ “Library” (LIB) 13.2.1 on page 1392

**Exports:**

|            |          |                  |         |          |
|------------|----------|------------------|---------|----------|
| any?       | any?     | bag              | close!  | coerce   |
| construct  | convert  | copy             | count   | count    |
| dictionary | elt      | empty            | empty?  | entries  |
| entry?     | eq?      | eval             | every?  | every?   |
| extract!   | fill!    | find             | first   | hash     |
| index?     | indices  | insert!          | inspect | iomode   |
| key?       | keys     | latex            | less?   | map      |
| map!       | maxIndex | member?          | members | minIndex |
| more?      | name     | open             | pack!   | parts    |
| parts      | qelt     | qsetelt!         | read!   | reduce   |
| remove     | remove!  | removeDuplicates | reopen! | sample   |
| search     | select   | select!          | setelt  | size?    |
| swap!      | table    | write!           | #?      | ?~=?     |
| ?=?        | ?.?      |                  |         |          |

— domain **KAFfile** **KeyedAccessFile** —

```
)abbrev domain KAFfile KeyedAccessFile
++ Author: Stephen M. Watt
++ Date Created: 1985
++ Date Last Updated: June 4, 1991
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ This domain allows a random access file to be viewed both as a table
++ and as a file object. The KeyedAccessFile format is a directory
++ containing a single file called 'index.kaf'. This file is a random
++ access file. The first thing in the file is an integer which is the
++ byte offset of an association list (the dictionary) at the end of
++ the file. The association list is of the form
++ ((key . byteoffset) (key . byteoffset)...)
++ where the byte offset is the number of bytes from the beginning of
++ the file. This offset contains an s-expression for the value of the key.
```

```
KeyedAccessFile(Entry): KAFcategory == KAFcapsule where
  Name ==> FileName
  Key ==> String
  Entry : SetCategory

KAFcategory ==
  Join(FileCategory(Name, Record(key: Key, entry: Entry)),
    TableAggregate(Key, Entry)) with
    finiteAggregate
```

```

        pack_!: % -> %
        ++ pack!(f) reorganizes the file f on disk to recover
        ++ unused space.

KAFcapsule == add

CLASS      ==> 131  -- an arbitrary no. greater than 127
FileState ==> SExpression
IOMode     ==> String

Cons:= Record(car: SExpression, cdr: SExpression)
Rep  := Record(fileName:  Name,      _
                fileState:  FileState, _
                fileIOMode: IOMode)

defstream(fn: Name, mode: IOMode): FileState ==
    kafstring:=concat(fn::String, "/index.kaf")::FileName
    mode = "input" =>
        not readable? kafstring => error ["File is not readable", fn]
        RDEFINSTREAM(fn)$Lisp
    mode = "output" =>
        not writable? fn => error ["File is not writable", fn]
        RDEFOUTSTREAM(fn)$Lisp
    error ["IO mode must be input or output", mode]

---- From Set ----
f1 = f2 ==
    f1.fileName = f2.fileName
coerce(f: %): OutputForm ==
    f.fileName::OutputForm

---- From FileCategory ----
open fname ==
    open(fname, "either")
open(fname, mode) ==
    mode = "either" =>
        exists? fname =>
            open(fname, "input")
        writable? fname =>
            reopen_!(open(fname, "output"), "input")
        error "File does not exist and cannot be created"
    [fname, defstream(fname, mode), mode]
reopen_!(f, mode) ==
    close_! f
    if mode ~= "closed" then
        f.fileState := defstream(f.fileName, mode)
        f.fileIOMode := mode
    f
close_! f ==

```



```

        if f.fileIOmode ^= "closed" then
            RSHUT(f.fileState)$Lisp
            f.fileIOmode := "closed"
        f
    read_! f ==
        f.fileIOmode ^= "input" => error ["File not in read state",f]
        ks: List Symbol := RKEYIDS(f.fileName)$Lisp
        null ks => error ["Attempt to read empty file", f]
        ix := random()$Integer rem #ks
        k: String := PNAME(ks.ix)$Lisp
        [k, SPADRREAD(k, f.fileState)$Lisp]
    write_!(f, pr) ==
        f.fileIOmode ^= "output" => error ["File not in write state",f]
        SPADRWRITE(pr.key, pr.entry, f.fileState)$Lisp
        pr
    name f ==
        f.fileName
    iomode f ==
        f.fileIOmode

---- From TableAggregate ----
empty() ==
    fn := new("", "kaf", "sdata")$Name
    open fn
    keys f ==
        close_! f
        l: List SExpression := RKEYIDS(f.fileName)$Lisp
        [PNAME(n)$Lisp for n in l]
    # f ==
        # keys f
    elt(f,k) ==
        reopen_!(f, "input")
        SPADRREAD(k, f.fileState)$Lisp
    setelt(f,k,e) ==
        -- Leaves f in a safe, closed state. For speed use "write".
        reopen_!(f, "output")
        UNWIND_-PROTECT(write_!(f, [k,e]), close_! f)$Lisp
        close_! f
        e
    search(k,f) ==
        not member?(k, keys f) => "failed" -- can't trap RREAD error
        reopen_!(f, "input")
        (SPADRREAD(k, f.fileState)$Lisp)@Entry
    remove_!(k:String,f:%) ==
        result := search(k,f)
        result case "failed" => result
        close_! f
        RDROPITEMS(NAMESTRING(f.fileName)$Lisp, LIST(k)$Lisp)$Lisp
        result
    pack_! f ==

```

```
close_! f
RPACKFILE(f.fileName)$Lisp
f
```

---

— KAFILE.dotabb —

```
"KAFILE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=KAFILE"]
"TBAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=TBAGG"]
"KAFILE" -> "TBAGG"
```

---



## Chapter 13

# Chapter L

### 13.1 domain LAUPOL LaurentPolynomial

— LaurentPolynomial.input —

```
)set break resume
)sys rm -f LaurentPolynomial.output
)spool LaurentPolynomial.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show LaurentPolynomial
--R LaurentPolynomial(R: IntegralDomain,UP: UnivariatePolynomialCategory R) is a domain constructor
--R Abbreviation for LaurentPolynomial is LAUPOL
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for LAUPOL
--R
--R----- Operations -----
--R ??? : (%,% ) -> %
--R ??? : (PositiveInteger,% ) -> %
--R ?+? : (%,% ) -> %
--R -? : % -> %
--R D : % -> % if UP has DIFRING
--R 1 : () -> %
--R ?? : (% ,PositiveInteger) -> %
--R coefficient : (% ,Integer) -> R
--R coerce : R -> %
--R coerce : Integer -> %
--R convert : % -> Fraction UP
--R hash : % -> SingleInteger
--R ??? : (Integer,% ) -> %
--R ??? : (% ,PositiveInteger) -> %
--R ?-? : (%,% ) -> %
--R ?=? : (%,% ) -> Boolean
--R D : (% ,(UP -> UP)) -> %
--R 0 : () -> %
--R associates? : (%,% ) -> Boolean
--R coerce : UP -> %
--R coerce : % -> %
--R coerce : % -> OutputForm
--R degree : % -> Integer
--R latex : % -> String
```

```

--R leadingCoefficient : % -> R
--R monomial? : % -> Boolean
--R order : % -> Integer
--R reductum : % -> %
--R retract : % -> R
--R trailingCoefficient : % -> R
--R unitCanonical : % -> %
--R ~=? : (%,% ) -> Boolean
--R ?? : (NonNegativeInteger,% ) -> %
--R ??? : (% ,NonNegativeInteger) -> %
--R D : (% ,NonNegativeInteger) -> % if UP has DIFRING
--R D : (% ,Symbol) -> % if UP has PDRING SYMBOL
--R D : (% ,List Symbol) -> % if UP has PDRING SYMBOL
--R D : (% ,Symbol,NonNegativeInteger) -> % if UP has PDRING SYMBOL
--R D : (% ,List Symbol,List NonNegativeInteger) -> % if UP has PDRING SYMBOL
--R D : (% ,(UP -> UP),NonNegativeInteger) -> %
--R ?? : (% ,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(% ,"failed") if R has CHARNZ
--R coerce : Fraction Integer -> % if R has RETRACT FRAC INT
--R differentiate : % -> % if UP has DIFRING
--R differentiate : (% ,NonNegativeInteger) -> % if UP has DIFRING
--R differentiate : (% ,Symbol) -> % if UP has PDRING SYMBOL
--R differentiate : (% ,List Symbol) -> % if UP has PDRING SYMBOL
--R differentiate : (% ,Symbol,NonNegativeInteger) -> % if UP has PDRING SYMBOL
--R differentiate : (% ,List Symbol,List NonNegativeInteger) -> % if UP has PDRING SYMBOL
--R differentiate : (% ,(UP -> UP),NonNegativeInteger) -> %
--R differentiate : (% ,(UP -> UP)) -> %
--R divide : (% ,%) -> Record(quotient: %,remainder: %) if R has FIELD
--R euclideanSize : % -> NonNegativeInteger if R has FIELD
--R expressIdealMember : (List % ,%) -> Union(List % ,"failed") if R has FIELD
--R exquo : (% ,%) -> Union(% ,"failed")
--R extendedEuclidean : (% ,%) -> Record(coef1: %,coef2: %,generator: %) if R has FIELD
--R extendedEuclidean : (% ,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if R has FIELD
--R gcd : (% ,%) -> % if R has FIELD
--R gcd : List % -> % if R has FIELD
--R gcdPolynomial : (SparseUnivariatePolynomial % ,SparseUnivariatePolynomial %) -> SparseUni
--R lcm : (% ,%) -> % if R has FIELD
--R lcm : List % -> % if R has FIELD
--R multiEuclidean : (List % ,%) -> Union(List % ,"failed") if R has FIELD
--R principalIdeal : List % -> Record(coef: List % ,generator: %) if R has FIELD
--R ?quo? : (% ,%) -> % if R has FIELD
--R ?rem? : (% ,%) -> % if R has FIELD
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retract : % -> Integer if R has RETRACT INT
--R retractIfCan : % -> Union(UP ,"failed")
--R retractIfCan : % -> Union(R ,"failed")
--R retractIfCan : % -> Union(Fraction Integer ,"failed") if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(Integer ,"failed") if R has RETRACT INT
--R separate : Fraction UP -> Record(polyPart: %,fracPart: Fraction UP) if R has FIELD

```

```

--R sizeLess? : (%,% ) -> Boolean if R has FIELD
--R subtractIfCan : (%,% ) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

```

```

)spool
)lisp (bye)

```

---

### — LaurentPolynomial.help —

```

=====
LaurentPolynomial examples
=====

```

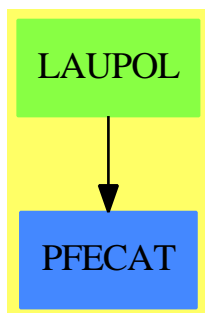
```

See Also:
o )show LaurentPolynomial

```

---

#### 13.1.1.1 LaurentPolynomial (LAUPOL)



**Exports:**

|                     |                    |               |
|---------------------|--------------------|---------------|
| 0                   | 1                  | associates?   |
| characteristic      | charthRoot         | coefficient   |
| coerce              | convert            | D             |
| degree              | differentiate      | divide        |
| euclideanSize       | expressIdealMember | exquo         |
| extendedEuclidean   | gcd                | gcdPolynomial |
| hash                | latex              | lcm           |
| leadingCoefficient  | monomial           | monomial?     |
| multiEuclidean      | one?               | order         |
| principalIdeal      | recip              | reductum      |
| retract             | retractIfCan       | sample        |
| separate            | sizeLess?          | subtractIfCan |
| trailingCoefficient | unit?              | unitCanonical |
| unitNormal          | zero?              | ?*?           |
| ?**?                | ?+?                | ?-?           |
| -?                  | ?=?                | ?^?           |
| ?~=?                | ?quo?              | ?rem?         |

— domain LAUPOL LaurentPolynomial —

```

)abbrev domain LAUPOL LaurentPolynomial
++ Author: Manuel Bronstein
++ Date Created: May 1988
++ Date Last Updated: 26 Apr 1990
++ Description:
++ Univariate polynomials with negative and positive exponents.

LaurentPolynomial(R, UP): Exports == Implementation where
  R : IntegralDomain
  UP: UnivariatePolynomialCategory R

O  ==> OutputForm
B  ==> Boolean
N  ==> NonNegativeInteger
Z  ==> Integer
RF ==> Fraction UP

Exports ==> Join(DifferentialExtension UP, IntegralDomain,
  ConvertibleTo RF, FullyRetractableTo R, RetractableTo UP) with
monomial?      : % -> B
  ++ monomial?(x) is not documented
degree         : % -> Z
  ++ degree(x) is not documented
order          : % -> Z
  ++ order(x) is not documented
reductum       : % -> %
  ++ reductum(x) is not documented

```

```

leadingCoefficient : % -> R
  ++ leadingCoefficient is not documented
trailingCoefficient: % -> R
  ++ trailingCoefficient is not documented
coefficient        : (% , Z) -> R
  ++ coefficient(x,n) is not documented
monomial           : (R, Z) -> %
  ++ monomial(x,n) is not documented
if R has CharacteristicZero then CharacteristicZero
if R has CharacteristicNonZero then CharacteristicNonZero
if R has Field then
  EuclideanDomain
  separate: RF -> Record(polyPart:%, fracPart:RF)
  ++ separate(x) is not documented

Implementation ==> add
Rep := Record(polypart: UP, order0: Z)

poly   : % -> UP
check0 : (Z, UP) -> %
mkgpol : (Z, UP) -> %
gpoly  : (UP, Z) -> %
toutput: (R, Z, 0) -> 0
monTerm: (R, Z, 0) -> 0

0 == [0, 0]
1 == [1, 0]
p = q == p.order0 = q.order0 and p.polypart = q.polypart
poly p == p.polypart
order p == p.order0
gpoly(p, n) == [p, n]
monomial(r, n) == check0(n, r::UP)
coerce(p:UP):% == mkgpoly(0, p)
reductum p == check0(order p, reductum poly p)
n:Z * p:% == check0(order p, n * poly p)
characteristic() == characteristic()$R
coerce(n:Z):% == n::R:%
degree p == degree(poly p)::Z + order p
monomial? p == monomial? poly p
coerce(r:R):% == gpoly(r::UP, 0)
convert(p:%):RF == poly(p) * (monomial(1, 1)$UP)::RF ** order p
p:% * q:% == check0(order p + order q, poly p * poly q)
- p == gpoly(- poly p, order p)
check0(n, p) == (zero? p => 0; gpoly(p, n))
trailingCoefficient p == coefficient(poly p, 0)
leadingCoefficient p == leadingCoefficient poly p

coerce(p:%):0 ==
  zero? p => 0::Z::0
  1 := nil()$List(0)

```



```

v := monomial(1, 1)$UP :: 0
while p ^= 0 repeat
  l := concat(l, toutput(leadingCoefficient p, degree p, v))
  p := reductum p
reduce("+", l)

coefficient(p, n) ==
  (m := n - order p) < 0 => 0
  coefficient(poly p, m::N)

differentiate(p:%, derivation:UP -> UP) ==
  t := monomial(1, 1)$UP
  mkgpol(order(p) - 1,
    derivation(poly p) * t + order(p) * poly(p) * derivation t)

monTerm(r, n, v) ==
  zero? n => r::0
--   one? n => v
  (n = 1) => v
  v ** (n::0)

toutput(r, n, v) ==
  mon := monTerm(r, n, v)
--   zero? n or one? r => mon
  zero? n or (r = 1) => mon
  r = -1 => - mon
  r::0 * mon

recip p ==
  (q := recip poly p) case "failed" => "failed"
  gpoly(q::UP, - order p)

p + q ==
  zero? q => p
  zero? p => q
  (d := order p - order q) > 0 =>
    gpoly(poly(p) * monomial(1, d::N) + poly q, order q)
  d < 0 => gpoly(poly(p) + poly(q) * monomial(1, (-d)::N), order p)
  mkgpoly(order p, poly(p) + poly q)

mkgpoly(n, p) ==
  zero? p => 0
  d := order(p, monomial(1, 1)$UP)
  gpoly((p exquo monomial(1, d))::UP, n + d::Z)

p exquo q ==
  (r := poly(p) exquo poly q) case "failed" => "failed"
  check0(order p - order q, r::UP)

retractIfCan(p:%):Union(UP, "failed") ==

```

```

order(p) < 0 => error "Not retractable"
poly(p) * monomial(1, order(p)::N)$UP

retractIfCan(p:%):Union(R, "failed") ==
  order(p) ^= 0 => "failed"
  retractIfCan poly p

if R has Field then
  gcd(p, q) == gcd(poly p, poly q)::%

separate f ==
  n := order(q := denom f, monomial(1, 1))
  q := (q exquo (tn := monomial(1, n)$UP))::UP
  bc := extendedEuclidean(tn,q,numer f)::Record(coef1:UP,coef2:UP)
  qr := divide(bc.coef1, q)
  [mkgpol(-n, bc.coef2 + tn * qr.quotient), qr.remainder / q]

-- returns (z, r) s.t. p = q z + r,
-- and degree(r) < degree(q), order(r) >= min(order(p), order(q))
divide(p, q) ==
  c := min(order p, order q)
  qr := divide(poly(p) * monomial(1, (order p - c)::N)$UP, poly q)
  [mkgpol(c - order q, qr.quotient), mkgpol(c, qr.remainder)]

euclideanSize p == degree poly p

extendedEuclidean(a, b, c) ==
  (bc := extendedEuclidean(poly a, poly b, poly c)) case "failed"
  => "failed"
  [mkgpol(order c - order a, bc.coef1),
   mkgpol(order c - order b, bc.coef2)]

-----

— LAUPOL.dotabb —

"LAUPOL" [color="#88FF44",href="bookvol10.3.pdf#nameddest=LAUPOL"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"LAUPOL" -> "PFECAT"

-----

```

## 13.2 domain LIB Library

— Library.input —

```

)set break resume
)sys rm -f Library.output
)spool Library.output
)set message test on
)set message auto off
)clear all

```

```

--S 1 of 7
stuff := library "Neat.stuff"
--R
--R
--R (1) "Neat.stuff"
--R
--E 1

```

Type: Library

```

--S 2 of 7
stuff.int := 32**2
--R
--R
--R (2) 1024
--R
--E 2

```

Type: PositiveInteger

```

--S 3 of 7
stuff."poly" := x**2 + 1
--R
--R
--R      2
--R (3) x  + 1
--R
--E 3

```

Type: Polynomial Integer

```

--S 4 of 7
stuff.str := "Hello"
--R
--R
--R (4) "Hello"
--R
--E 4

```

Type: String

```

--S 5 of 7
keys stuff
--R
--R
--R (5) ["str","poly","int"]
--R
--E 5

```

Type: List String

```

--S 6 of 7
stuff.poly

```

```
--R
--R
--R      2
--R (6)  x  + 1
--R
--R                                          Type: Polynomial Integer
--E 6
```

```
--S 7 of 7
stuff("poly")
--R
--R
--R      2
--R (7)  x  + 1
--R
--R                                          Type: Polynomial Integer
--E 7
```

```
)system rm -rf Neat.stuff
)spool
)lisp (bye)
```

---

— Library.help —

```
=====
Library examples
=====
```

The Library domain provides a simple way to store Axiom values in a file. This domain is similar to KeyedAccessFile but fewer declarations are needed and items of different types can be saved together in the same file.

To create a library, you supply a file name.

```
stuff := library "Neat.stuff"
```

Now values can be saved by key in the file. The keys should be mnemonic, just as the field names are for records. They can be given either as strings or symbols.

```
stuff.int := 32**2

stuff."poly" := x**2 + 1

stuff.str := "Hello"
```

You obtain the set of available keys using the keys operation.

```
keys stuff
```

You extract values by giving the desired key in this way.

```
stuff.poly
```

```
stuff("poly")
```

When the file is no longer needed, you should remove it from the file system.

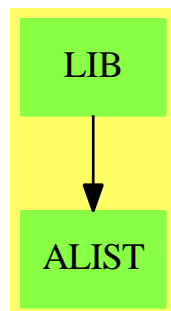
```
)system rm -rf Neat.stuff
```

See Also:

- o )help File
- o )help TextFile
- o )help KeyedAccessFile
- o )show Library

---

### 13.2.1 Library (LIB)



See

- ⇒ “File” (FILE) 7.2.1 on page 770
- ⇒ “TextFile” (TEXTFILE) 21.5.1 on page 2651
- ⇒ “BinaryFile” (BINFILE) 3.8.1 on page 277
- ⇒ “KeyedAccessFile” (KAFILE) 12.2.1 on page 1377

**Exports:**

|          |           |                  |          |            |
|----------|-----------|------------------|----------|------------|
| any?     | any?      | bag              | close!   | coerce     |
| copy     | construct | convert          | count    | dictionary |
| elt      | empty     | empty?           | entries  | entry?     |
| eq?      | eval      | every?           | every?   | extract!   |
| fill!    | find      | first            | hash     | index?     |
| indices  | insert!   | inspect          | key?     | keys       |
| latex    | less?     | library          | map      | map!       |
| maxIndex | member?   | members          | minIndex | more?      |
| pack!    | parts     | qelt             | qsetelt! | reduce     |
| remove   | remove!   | removeDuplicates | sample   | search     |
| select   | select!   | setelt           | size?    | swap!      |
| table    | #?        | ?=?              | ?~=?     | ?..?       |

— domain LIB Library —

```

)abbrev domain LIB Library
++ Author: Stephen M. Watt
++ Date Created: 1985
++ Date Last Updated: June 4, 1991
++ Basic Operations:
++ Related Domains: KeyedAccessFile
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ This domain provides a simple way to save values in files.

Library(): TableAggregate(String, Any) with
  library: FileName -> %
    ++ library(ln) creates a new library file.
  pack_!: % -> %
    ++ pack!(f) reorganizes the file f on disk to recover
    ++ unused space.

  elt : (% , Symbol) -> Any
    ++ elt(lib,k) or lib.k extracts the value corresponding to
    ++ the key \spad{k} from the library \spad{lib}.

  setelt : (% , Symbol, Any) -> Any
    ++ \spad{lib.k := v} saves the value \spad{v} in the library
    ++ \spad{lib}. It can later be extracted using the key \spad{k}.

  close_!: % -> %
    ++ close!(f) returns the library f closed to input and output.

== KeyedAccessFile(Any) add

```

```
Rep := KeyedAccessFile(Any)
library f == open f
elt(f:%,v:Symbol) == elt(f, string v)
setelt(f:%, v:Symbol, val:Any) == setelt(f, string v, val)
```

---

— LIB.dotabb —

```
"LIB" [color="#88FF44",href="bookvol10.3.pdf#nameddest=LIB"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"LIB" -> "ALIST"
```

---

### 13.3 domain LEXP LieExponentials

— LieExponentials.input —

```
)set break resume
)sys rm -f LieExponentials.output
)spool LieExponentials.output
)set message test on
)set message auto off
)clear all
--S 1 of 13
a: Symbol := 'a
--R
--R
--R (1) a
--R
--R                                          Type: Symbol
--E 1

--S 2 of 13
b: Symbol := 'b
--R
--R
--R (2) b
--R
--R                                          Type: Symbol
--E 2

--S 3 of 13
coef := Fraction(Integer)
--R
```

```

--R
--R (3) Fraction Integer
--R
--R                                          Type: Domain
--E 3

--S 4 of 13
group := LieExponentials(Symbol, coef, 3)
--R
--R
--R (4) LieExponentials(Symbol,Fraction Integer,3)
--R
--R                                          Type: Domain
--E 4

--S 5 of 13
lpoly := LiePolynomial(Symbol, coef)
--R
--R
--R (5) LiePolynomial(Symbol,Fraction Integer)
--R
--R                                          Type: Domain
--E 5

--S 6 of 13
poly := XPBWPolynomial(Symbol, coef)
--R
--R
--R (6) XPBWPolynomial(Symbol,Fraction Integer)
--R
--R                                          Type: Domain
--E 6

--S 7 of 13
ea := exp(a::lpoly)$group
--R
--R
--R [a]
--R (7) e
--R
--R                                          Type: LieExponentials(Symbol,Fraction Integer,3)
--E 7

--S 8 of 13
eb := exp(b::lpoly)$group
--R
--R
--R [b]
--R (8) e
--R
--R                                          Type: LieExponentials(Symbol,Fraction Integer,3)
--E 8

--S 9 of 13
g: group := ea*eb
--R

```



```

--R
--R      1      2      1      2
--R      - [a b ] - [a b ]
--R      [b] 2      [a b] 2      [a]
--R      (9) e e      e e      e
--R                                     Type: LieExponentials(Symbol,Fraction Integer,3)
--E 9

```

```

--S 10 of 13

```

```

g :: poly

```

```

--R
--R
--R      (10)
--R      1      1      1
--R      1 + [a] + [b] + - [a][a] + [a b] + [b][a] + - [b][b] + - [a][a][a]
--R      2      2      6
--R      +
--R      1      2      1      2      1      1
--R      - [a b] + [a b][a] + - [a b ] + - [b][a][a] + [b][a b] + - [b][b][a]
--R      2      2      2      2
--R      +
--R      1
--R      - [b][b][b]
--R      6
--R                                     Type: XPBWPolynomial(Symbol,Fraction Integer)
--E 10

```

```

--S 11 of 13

```

```

log(g)$group

```

```

--R
--R
--R      1      1      2      1      2
--R      (11) [a] + [b] + - [a b] + -- [a b] + -- [a b ]
--R      2      12      12
--R                                     Type: LiePolynomial(Symbol,Fraction Integer)
--E 11

```

```

--S 12 of 13

```

```

g1: group := inv(g)

```

```

--R
--R
--R      - [b] - [a]
--R      (12) e      e
--R                                     Type: LieExponentials(Symbol,Fraction Integer,3)
--E 12

```

```

--S 13 of 13

```

```

g*g1

```

```

--R

```

```

--R

```

```
--R (13) 1
--R                                         Type: LieExponentials(Symbol,Fraction Integer,3)
--E 13
)spool
)lisp (bye)
```

---

— LieExponentials.help —

=====

LieExponentials examples

=====

```
a: Symbol := 'a
a
                                         Type: Symbol

b: Symbol := 'b
b
                                         Type: Symbol
```

=====

Declarations of domains

=====

```
coef := Fraction(Integer)
Fraction Integer
                                         Type: Domain

group := LieExponentials(Symbol, coef, 3)
LieExponentials(Symbol,Fraction Integer,3)
                                         Type: Domain

lpoly := LiePolynomial(Symbol, coef)
LiePolynomial(Symbol,Fraction Integer)
                                         Type: Domain

poly := XPBWPolynomial(Symbol, coef)
XPBWPolynomial(Symbol,Fraction Integer)
                                         Type: Domain
```

=====

Calculations

=====

```
ea := exp(a::lpoly)$group
[a]
e
```

```

Type: LieExponentials(Symbol,Fraction Integer,3)

eb := exp(b::lpoly)$group
    [b]
    e
Type: LieExponentials(Symbol,Fraction Integer,3)

g: group := ea*eb
    1      2      1      2
    - [a b ] - [a b ]
    [b] 2      [a b] 2      [a]
    e  e      e  e      e
Type: LieExponentials(Symbol,Fraction Integer,3)

g :: poly
    1      1      1
    + [a] + [b] + - [a][a] + [a b] + [b][a] + - [b][b] + - [a][a][a]
    2      2      6
+
    1      2      1      2      1
    - [a b] + [a b][a] + - [a b] + - [b][a][a] + [b][a b] + - [b][b][a]
    2      2      2      2      2
+
    1
    - [b][b][b]
    6
Type: XPBWPolynomial(Symbol,Fraction Integer)

log(g)$group
    1      1      2      1      2
    [a] + [b] + - [a b] + -- [a b] + -- [a b]
    2      12      12
Type: LiePolynomial(Symbol,Fraction Integer)

g1: group := inv(g)
    - [b] - [a]
    e      e
Type: LieExponentials(Symbol,Fraction Integer,3)

g*g1
    1
Type: LieExponentials(Symbol,Fraction Integer,3)

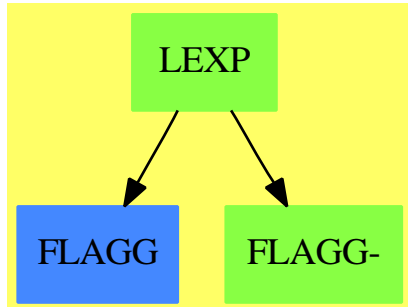
```

See Also:

o )show LieExponentials

---

## 13.3.1 LieExponentials (LEXP)

**Exports:**

|                   |         |                |             |
|-------------------|---------|----------------|-------------|
| 1                 | coerce  | commutator     | conjugate   |
| exp               | hash    | identification | inv         |
| latex             | log     | listOfTerms    | LyndonBasis |
| LyndonCoordinates | mirror  | one?           | recip       |
| sample            | varList | ?~=?           | ?^?         |
| ?*?               | ?**?    | ?/?            | ?=?         |

— domain **LEXP** LieExponentials —

```

)abbrev domain LEXP LieExponentials
++ Author: Michel Petitot (petitot@lifl.fr).
++ Date Created: 91
++ Date Last Updated: 7 Juillet 92
++ Fix History: compilation v 2.1 le 13 dec 98
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ Management of the Lie Group associated with a
++ free nilpotent Lie algebra. Every Lie bracket with
++ length greater than \axiom{Order} are assumed to be null.
++ The implementation inherits from the \spadtype{XPBWPolynomial}
++ domain constructor: Lyndon coordinates are exponential coordinates
++ of the second kind.

```

```

LieExponentials(VarSet, R, Order): XDPcat == XDPdef where

```

```

EX    ==> OutputForm
PI    ==> PositiveInteger
NNI   ==> NonNegativeInteger
I     ==> Integer

```

```

RN      ==> Fraction(I)
R       : Join(CommutativeRing, Module RN)
Order  : PI
VarSet  : OrderedSet
LWORD   ==> LyndonWord(VarSet)
LWORDS  ==> List LWORD
BASIS   ==> PoincareBirkhoffWittLyndonBasis(VarSet)
TERM    ==> Record(k:BASIS, c:R)
LTERMS  ==> List(TERM)
LPOLY   ==> LiePolynomial(VarSet,R)
XDPOLY  ==> XDistributedPolynomial(VarSet,R)
PBWPOLY==> XPBWPolynomial(VarSet, R)
TERM1   ==> Record(k:LWORD, c:R)
EQ       ==> Equation(R)

XDPcat == Group with
  exp      : LPOLY -> $
  ++ \axiom{exp(p)} returns the exponential of \axiom{p}.
  log      : $ -> LPOLY
  ++ \axiom{log(p)} returns the logarithm of \axiom{p}.
  listOfTerms : $ -> LTERMS
  ++ \axiom{listOfTerms(p)} returns the internal representation of \axiom{p}.
  coerce    : $ -> XDPOLY
  ++ \axiom{coerce(g)} returns the internal representation of \axiom{g}.
  coerce    : $ -> PBWPOLY
  ++ \axiom{coerce(g)} returns the internal representation of \axiom{g}.
  mirror    : $ -> $
  ++ \axiom{mirror(g)} is the mirror of the internal representation of \axiom{g}.
  varList   : $ -> List VarSet
  ++ \axiom{varList(g)} returns the list of variables of \axiom{g}.
  LyndonBasis : List VarSet -> List LPOLY
  ++ \axiom{LyndonBasis(lv)} returns the Lyndon basis of the nilpotent free
  ++ Lie algebra.
  LyndonCoordinates: $ -> List TERM1
  ++ \axiom{LyndonCoordinates(g)} returns the exponential coordinates of \axiom{g}.
  identification: ($,$) -> List EQ
  ++ \axiom{identification(g,h)} returns the list of equations \axiom{g_i = h_i},
  ++ where \axiom{g_i} (resp. \axiom{h_i}) are exponential coordinates
  ++ of \axiom{g} (resp. \axiom{h}).

XDPdef == PBWPOLY add

-- Representation
Rep := PBWPOLY

-- local functions
compareTerm1s: (TERM1, TERM1) -> Boolean
out: TERM1 -> EX
ident: (List TERM1, List TERM1) -> List EQ

```

```

-- functions locales
ident(l1, l2) ==
  import(TERM1)
  null l1 => [equation(0$R,t.c)$EQ for t in l2]
  null l2 => [equation(t.c, 0$R)$EQ for t in l1]
  u1 : LWORD := l1.first.k; c1 :R := l1.first.c
  u2 : LWORD := l2.first.k; c2 :R := l2.first.c
  u1 = u2 =>
    r: R := c1 - c2
    r = 0 => ident(rest l1, rest l2)
    cons(equation(c1,c2)$EQ , ident(rest l1, rest l2))
  lexico(u1, u2)$LWORD =>
    cons(equation(0$R,c2)$EQ , ident(l1, rest l2))
    cons(equation(c1,0$R)$EQ , ident(rest l1, l2))

-- ordre lexico decroissant
compareTerms(u:TERM1, v:TERM1):Boolean == lexico(v.k, u.k)$LWORD

out(t:TERM1):EX ==
  t.c =$R 1 => char("e")$Character :: EX ** t.k ::EX
  char("e")$Character :: EX ** (t.c::EX * t.k::EX)

-- definitions
identification(x,y) ==
  l1: List TERM1 := LyndonCoordinates x
  l2: List TERM1 := LyndonCoordinates y
  ident(l1, l2)

LyndonCoordinates x ==
  lt: List TERM1 := [[l::LWORD, t.c]$TERM1 for t in listOfTerms x | _
    (l := retractIfCan(t.k)$BASIS) case LWORD ]
  lt := sort(compareTerms,lt)

x:$ * y:$ == product(x::Rep, y::Rep, Order::I::NNI)$Rep

exp p == exp(p::Rep , Order::I::NNI)$Rep

log p == LiePolyIfCan(log(p,Order::I::NNI))$Rep :: LPOLY

coerce(p:$):EX ==
  p = 1$$ => 1$R :: EX
  lt : List TERM1 := LyndonCoordinates p
  reduce(*, [out t for t in lt])$List(EX)

LyndonBasis(lv) ==
  [LiePoly(1)$LPOLY for l in LyndonWordsList(lv,Order)$LWORD]

coerce(p:$):PBWPOLY == p::Rep

```

— LEXP.dotabb —

---

— LiePolynomial.input —

```
)set break resume
)sys rm -f LiePolynomial.output
)spool LiePolynomial.output
)set message test on
)set message auto off
)clear all
--S 1 of 28
RN := Fraction Integer
--R
--R
--R (1) Fraction Integer
--R
--R
--R Type: Domain
--E 1

--S 2 of 28
Lpoly := LiePolynomial(Symbol,RN)
--R
--R
--R (2) LiePolynomial(Symbol,Fraction Integer)
--R
--R
--R Type: Domain
--E 2
```

```

--S 3 of 28
Dpoly := XDPOLY(Symbol,RN)
--R
--R
--R (3) XDistributedPolynomial(Symbol,Fraction Integer)
--R
--R                                         Type: Domain
--E 3

```

```

--S 4 of 28
Lword := LyndonWord Symbol
--R
--R
--R (4) LyndonWord Symbol
--R
--R                                         Type: Domain
--E 4

```

```

--S 5 of 28
a:Symbol := 'a
--R
--R
--R (5) a
--R
--R                                         Type: Symbol
--E 5

```

```

--S 6 of 28
b:Symbol := 'b
--R
--R
--R (6) b
--R
--R                                         Type: Symbol
--E 6

```

```

--S 7 of 28
c:Symbol := 'c
--R
--R
--R (7) c
--R
--R                                         Type: Symbol
--E 7

```

```

--S 8 of 28
aa: Lpoly := a
--R
--R
--R (8) [a]
--R
--R                                         Type: LiePolynomial(Symbol,Fraction Integer)
--E 8

```

```

--S 9 of 28
bb: Lpoly := b

```



```

--R
--R
--R (9) [b]
--R
--R                                         Type: LiePolynomial(Symbol,Fraction Integer)
--E 9

--S 10 of 28
cc: Lpoly := c
--R
--R
--R (10) [c]
--R
--R                                         Type: LiePolynomial(Symbol,Fraction Integer)
--E 10

--S 11 of 28
p : Lpoly := [aa,bb]
--R
--R
--R (11) [a b]
--R
--R                                         Type: LiePolynomial(Symbol,Fraction Integer)
--E 11

--S 12 of 28
q : Lpoly := [p,bb]
--R
--R
--R
--R          2
--R (12) [a b ]
--R
--R                                         Type: LiePolynomial(Symbol,Fraction Integer)
--E 12

--S 13 of 28
liste : List Lword := LyndonWordsList([a,b], 4)
--R
--R
--R
--R          2      2      3      2 2      3
--R (13) [[a],[b],[a b],[a b],[a b ],[a b],[a b ],[a b ]]
--R
--R                                         Type: List LyndonWord Symbol
--E 13

--S 14 of 28
r: Lpoly := p + q + 3*LiePoly(liste.4)$Lpoly
--R
--R
--R
--R          2      2
--R (14) [a b] + 3[a b] + [a b ]
--R
--R                                         Type: LiePolynomial(Symbol,Fraction Integer)
--E 14

--S 15 of 28

```

```

s:Lpoly := [p,r]
--R
--R
--R      2      2
--R      (15)  - 3[a b a b] + [a b a b]
--R                                          Type: LiePolynomial(Symbol,Fraction Integer)
--E 15

--S 16 of 28
t:Lpoly := s + 2*LiePoly(liste.3) - 5*LiePoly(liste.5)
--R
--R
--R      2      2      2
--R      (16)  2[a b] - 5[a b] - 3[a b a b] + [a b a b]
--R                                          Type: LiePolynomial(Symbol,Fraction Integer)
--E 16

--S 17 of 28
degree t
--R
--R
--R      (17)  5
--R                                          Type: PositiveInteger
--E 17

--S 18 of 28
mirror t
--R
--R
--R      2      2      2
--R      (18)  - 2[a b] - 5[a b] - 3[a b a b] + [a b a b]
--R                                          Type: LiePolynomial(Symbol,Fraction Integer)
--E 18

--S 19 of 28
Jacobi(p: Lpoly, q: Lpoly, r: Lpoly): Lpoly == _
  [ [p,q]$Lpoly, r] + [ [q,r]$Lpoly, p] + [ [r,p]$Lpoly, q]
--R
--R      Function declaration Jacobi : (LiePolynomial(Symbol,Fraction Integer
--R      ),LiePolynomial(Symbol,Fraction Integer),LiePolynomial(Symbol,
--R      Fraction Integer)) -> LiePolynomial(Symbol,Fraction Integer) has
--R      been added to workspace.
--R                                          Type: Void
--E 19

--S 20 of 28
test: Lpoly := Jacobi(a,b,b)
--R
--R      Compiling function Jacobi with type (LiePolynomial(Symbol,Fraction
--R      Integer),LiePolynomial(Symbol,Fraction Integer),LiePolynomial(

```

```

--R      Symbol,Fraction Integer)) -> LiePolynomial(Symbol,Fraction
--R      Integer)
--R
--R      (20)  0
--R
--R                                          Type: LiePolynomial(Symbol,Fraction Integer)
--E 20

--S 21 of 28
test: Lpoly := Jacobi(p,q,r)
--R
--R
--R      (21)  0
--R
--R                                          Type: LiePolynomial(Symbol,Fraction Integer)
--E 21

--S 22 of 28
test: Lpoly := Jacobi(r,s,t)
--R
--R
--R      (22)  0
--R
--R                                          Type: LiePolynomial(Symbol,Fraction Integer)
--E 22

--S 23 of 28
eval(p, a, p)$Lpoly
--R
--R
--R      (23)  [a b ]
--R
--R                                          Type: LiePolynomial(Symbol,Fraction Integer)
--E 23

--S 24 of 28
eval(p, [a,b], [2*bb, 3*aa])$Lpoly
--R
--R
--R      (24)  - 6[a b]
--R
--R                                          Type: LiePolynomial(Symbol,Fraction Integer)
--E 24

--S 25 of 28
r: Lpoly := [p,c]
--R
--R
--R      (25)  [a b c] + [a c b]
--R
--R                                          Type: LiePolynomial(Symbol,Fraction Integer)
--E 25

--S 26 of 28
r1: Lpoly := eval(r, [a,b,c], [bb, cc, aa])$Lpoly

```

```

--R
--R
--R (26) - [a b c]
--R                                         Type: LiePolynomial(Symbol,Fraction Integer)
--E 26

--S 27 of 28
r2: Lpoly := eval(r, [a,b,c], [cc, aa, bb])$Lpoly
--R
--R
--R (27) - [a c b]
--R                                         Type: LiePolynomial(Symbol,Fraction Integer)
--E 27

--S 28 of 28
r + r1 + r2
--R
--R
--R (28)  0
--R                                         Type: LiePolynomial(Symbol,Fraction Integer)
--E 28
)spool

)lisp (bye)

```

---

— LiePolynomial.help —

```

=====
LiePolynomial examples
=====

=====
Declaration of domains
=====

RN := Fraction Integer
    Fraction Integer
                                Type: Domain

Lpoly := LiePolynomial(Symbol,RN)
    LiePolynomial(Symbol,Fraction Integer)
                                Type: Domain

Dpoly := XDPOLY(Symbol,RN)
    XDistributedPolynomial(Symbol,Fraction Integer)
                                Type: Domain

```

```

Lword := LyndonWord Symbol
LyndonWord Symbol
Type: Domain

```

```

=====
Initialisation
=====

```

```

a:Symbol := 'a
a
Type: Symbol

b:Symbol := 'b
b
Type: Symbol

c:Symbol := 'c
c
Type: Symbol

aa: Lpoly := a
[a]
Type: LiePolynomial(Symbol,Fraction Integer)

bb: Lpoly := b
[b]
Type: LiePolynomial(Symbol,Fraction Integer)

cc: Lpoly := c
[c]
Type: LiePolynomial(Symbol,Fraction Integer)

p : Lpoly := [aa,bb]
[a b]
Type: LiePolynomial(Symbol,Fraction Integer)

q : Lpoly := [p,bb]
2
[a b ]
Type: LiePolynomial(Symbol,Fraction Integer)

```

All the Lyndon words of order 4

```

liste : List Lword := LyndonWordsList([a,b], 4)
2      2      3      2 2      3
[[a],[b],[a b],[a b],[a b ],[a b],[a b ],[a b ]]
Type: List LyndonWord Symbol

```

```

r: Lpoly := p + q + 3*LiePoly(liste.4)$Lpoly
2      2

```

```

[a b] + 3[a b] + [a b]
      Type: LiePolynomial(Symbol,Fraction Integer)

s:Lpoly := [p,r]
      2      2
- 3[a b a b] + [a b a b]
      Type: LiePolynomial(Symbol,Fraction Integer)

t:Lpoly := s + 2*LiePoly(liste.3) - 5*LiePoly(liste.5)
      2      2      2
2[a b] - 5[a b] - 3[a b a b] + [a b a b]
      Type: LiePolynomial(Symbol,Fraction Integer)

degree t
5
      Type: PositiveInteger

mirror t
      2      2      2
- 2[a b] - 5[a b] - 3[a b a b] + [a b a b]
      Type: LiePolynomial(Symbol,Fraction Integer)

=====
Jacobi Relation
=====

Jacobi(p: Lpoly, q: Lpoly, r: Lpoly): Lpoly == _
[ [p,q]\$Lpoly, r] + [ [q,r]\$Lpoly, p] + [ [r,p]\$Lpoly, q]
      Type: Void

=====
Tests
=====

test: Lpoly := Jacobi(a,b,b)
0
      Type: LiePolynomial(Symbol,Fraction Integer)

test: Lpoly := Jacobi(p,q,r)
0
      Type: LiePolynomial(Symbol,Fraction Integer)

test: Lpoly := Jacobi(r,s,t)
0
      Type: LiePolynomial(Symbol,Fraction Integer)

=====
Evaluation
=====

```

```

eval(p, a, p)$Lpoly
      2
    [a b ]
                                     Type: LiePolynomial(Symbol,Fraction Integer)

eval(p, [a,b], [2*bb, 3*aa])$Lpoly
- 6[a b]
                                     Type: LiePolynomial(Symbol,Fraction Integer)

r: Lpoly := [p,c]
    [a b c] + [a c b]
                                     Type: LiePolynomial(Symbol,Fraction Integer)

r1: Lpoly := eval(r, [a,b,c], [bb, cc, aa])$Lpoly
- [a b c]
                                     Type: LiePolynomial(Symbol,Fraction Integer)

r2: Lpoly := eval(r, [a,b,c], [cc, aa, bb])$Lpoly
- [a c b]
                                     Type: LiePolynomial(Symbol,Fraction Integer)

r + r1 + r2
      0
                                     Type: LiePolynomial(Symbol,Fraction Integer)

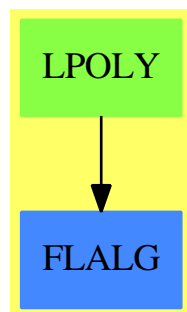
```

See Also:

- o )help LyndonWord
- o )help XDistributedPolynomial
- o )show LiePolynomial

—————▶

### 13.4.1 LiePolynomial (LPOLY)



**Exports:**

|               |              |                    |                 |
|---------------|--------------|--------------------|-----------------|
| 0             | coef         | coefficient        | coefficients    |
| coerce        | construct    | degree             | eval            |
| hash          | latex        | leadingCoefficient | leadingMonomial |
| leadingTerm   | LiePoly      | LiePolyIfCan       | listOfTerms     |
| lquo          | map          | mirror             | monom           |
| monomial?     | monomials    | numberOfMonomials  | reductum        |
| retract       | retractIfCan | rquo               | sample          |
| subtractIfCan | trunc        | varList            | zero?           |
| ?~=?          | ?*?          | ?/?                | ?+?             |
| ?-?           | -?           | ?=?                |                 |

— domain LPOLY LiePolynomial —

```

)abbrev domain LPOLY LiePolynomial
++ Author: Michel Petitot (petitot@lifl.fr).
++ Date Created: 91
++ Date Last Updated: 7 Juillet 92
++ Fix History: compilation v 2.1 le 13 dec 98
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Free Lie Algebras by C. Reutenauer (Oxford science publications).
++ Description:
++ This type supports Lie polynomials in Lyndon basis
++ see Free Lie Algebras by C. Reutenauer
++ (Oxford science publications).

LiePolynomial(VarSet:OrderedSet, R:CommutativeRing) : Public == Private where
  MAGMA ==> Magma(VarSet)
  LWORD ==> LyndonWord(VarSet)
  WORD ==> OrderedFreeMonoid(VarSet)
  XDPOLY ==> XDistributedPolynomial(VarSet,R)
  XRPOLY ==> XRecursivePolynomial(VarSet,R)
  NNI ==> NonNegativeInteger
  RN ==> Fraction Integer
  EX ==> OutputForm
  TERM ==> Record(k: LWORD, c: R)

Public == Join(FreeLieAlgebra(VarSet,R), FreeModuleCat(R,LWORD)) with
  LiePolyIfCan: XDPOLY -> Union($, "failed")
  ++ \axiom{LiePolyIfCan(p)} returns \axiom{p} in Lyndon basis
  ++ if \axiom{p} is a Lie polynomial, otherwise \axiom{"failed"}
  ++ is returned.
  construct: (LWORD, LWORD) -> $
  ++ \axiom{construct(x,y)} returns the Lie bracket \axiom{[x,y]}.

```



```

construct: (LWORD, $) -> $
  ++ \axiom{construct(x,y)} returns the Lie bracket \axiom{[x,y]}.
construct: ($, LWORD) -> $
  ++ \axiom{construct(x,y)} returns the Lie bracket \axiom{[x,y]}.

```

```

Private == FreeModule1(R, LWORD) add
  import(TERM)

```

```

--representation
Rep := List TERM

```

```

-- fonctions locales
cr1 : (LWORD, $ ) -> $
cr2 : ($, LWORD ) -> $
crw : (LWORD, LWORD) -> $      -- crochet de 2 mots de Lyndon
DPoly: LWORD -> XDPOLY
lquo1: (XRPOLY , LWORD) -> XRPOLY
lyndon: (LWORD, LWORD) -> $
makeLyndon: (LWORD, LWORD) -> LWORD
rquo1: (XRPOLY , LWORD) -> XRPOLY
RPoly: LWORD -> XRPOLY
eval1: (LWORD, VarSet, $) -> $      -- 08/03/98
eval2: (LWORD, List VarSet, List $) -> $      -- 08/03/98

```

```

-- Evaluation
eval1(lw,v,nv) ==      -- 08/03/98
  not member?(v, varList(lw)$LWORD) => LiePoly lw
  (s := retractIfCan(lw)$LWORD) case VarSet =>
    if (s::VarSet) = v then nv else LiePoly lw
  l: LWORD := left lw
  r: LWORD := right lw
  construct(eval1(l,v,nv), eval1(r,v,nv))

eval2(lw,lv,lnv) ==      -- 08/03/98
  p: Integer
  (s := retractIfCan(lw)$LWORD) case VarSet =>
    p := position(s::VarSet, lv)$List(VarSet)
    if p=0 then lw::$ else elt(lnv,p)$List($
  l: LWORD := left lw
  r: LWORD := right lw
  construct(eval2(l,lv,lnv), eval2(r,lv,lnv))

eval(p:$, v: VarSet, nv: $): $ ==      -- 08/03/98
  +/ [t.c * eval1(t.k, v, nv) for t in p]

eval(p:$, lv: List(VarSet), lnv: List($)): $ ==      -- 08/03/98
  +/ [t.c * eval2(t.k, lv, lnv) for t in p]

lquo1(p,lw) ==

```

```

constant? p => 0$XRPOLY
retractable? lw => lquo(p, retract lw)$XRPOLY
lquo1(lquo1(p, left lw),right lw) - lquo1(lquo1(p, right lw),left lw)
rquo1(p,lw) ==
constant? p => 0$XRPOLY
retractable? lw => rquo(p, retract lw)$XRPOLY
rquo1(rquo1(p, left lw),right lw) - rquo1(rquo1(p, right lw),left lw)

coef(p, lp) == coef(p, lp::XRPOLY)$XRPOLY

lquo(p, lp) ==
lp = 0 => 0$XRPOLY
+ / [t.c * lquo1(p,t.k) for t in lp]

rquo(p, lp) ==
lp = 0 => 0$XRPOLY
+ / [t.c * rquo1(p,t.k) for t in lp]

LiePolyIfCan p ==          -- inefficace a cause de la rep. de XDPOLY
not quasiRegular? p => "failed"
p1: XDPOLY := p ; r:$ := 0
while p1 ^= 0 repeat
  t: Record(k:WORD, c:R) := mindegTerm p1
  w: WORD := t.k; coef:R := t.c
  (l := lyndonIfCan(w)$LWORD) case "failed" => return "failed"
  lp:$ := coef * LiePoly(l::LWORD)
  r := r + lp
  p1 := p1 - lp::XDPOLY
r

--definitions locales
makeLyndon(u,v) == (u::MAGMA * v::MAGMA) pretend LWORD

crw(u,v) ==                -- u et v sont des mots de Lyndon
u = v => 0
lexico(u,v) => lyndon(u,v)
- lyndon (v,u)

lyndon(u,v) ==             -- u et v sont des mots de Lyndon tq u < v
retractable? u => monom(makeLyndon(u,v),1)
u1: LWORD := left u
u2: LWORD := right u
lexico(u2,v) => cr1(u1, lyndon(u2,v)) + cr2(lyndon(u1,v), u2)
monom(makeLyndon(u,v),1)

cr1 (l, p) ==
+ / [t.c * crw(l, t.k) for t in p]

cr2 (p, l) ==
+ / [t.c * crw(t.k, l) for t in p]

```

```

DPoly w ==
  retractable? w => retract(w) :: XDPOLY
  l:XDPOLY := DPoly left w
  r:XDPOLY := DPoly right w
  l*r - r*l

RPoly w ==
  retractable? w => retract(w) :: XRPOLY
  l:XRPOLY := RPoly left w
  r:XRPOLY := RPoly right w
  l*r - r*l

-- definitions

coerce(v:VarSet) == monom(v::LWORD , 1)

construct(x:$ , y:$):$ ==
  +/[t.c * cr1(t.k, y) for t in x]

construct(l:LWORD , p:$):$ == cr1(l,p)
construct(p:$ , l:LWORD):$ == cr2(p,l)
construct(u:LWORD , v:LWORD):$ == crw(u,v)

coerce(p:$):XDPOLY ==
  +/ [t.c * DPoly(t.k) for t in p]

coerce(p:$):XRPOLY ==
  +/ [t.c * RPoly(t.k) for t in p]

LiePoly(l) == monom(l,1)

varList p ==
  le : List VarSet := "setUnion"/[varList(t.k)$LWORD for t in p]
  sort(le)$List(VarSet)

mirror p ==
  [[t.k, (odd? length t.k => t.c; -t.c)]$TERM for t in p]

trunc(p, n) ==
  degree(p) > n => trunc( reductum p , n)
  p

degree p ==
  null p => 0
  length( p.first.k)$LWORD

-- listOfTerms p == p pretend List TERM

--
coerce(x) : EX ==

```

```

--      null x => (0$R) :: EX
--      le : List EX := nil
--      for rec in x repeat
--          rec.c = 1$R => le := cons(rec.k :: EX, le)
--          le := cons(mkBinary("*"::EX, rec.c :: EX, rec.k :: EX), le)
--      1 = #le => first le
--      mkNary("+" :: EX,le)

```

---

— LPOLY.dotabb —

```

"LPOLY" [color="#88FF44",href="bookvol10.3.pdf#nameddest=LPOLY"]
"FLALG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLALG"]
"LPOLY" -> "FLALG"

```

---

## 13.5 domain LSQM LieSquareMatrix

— LieSquareMatrix.input —

```

)set break resume
)sys rm -f LieSquareMatrix.output
)spool LieSquareMatrix.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show LieSquareMatrix
--R LieSquareMatrix(n: PositiveInteger,R: CommutativeRing) is a domain constructor
--R Abbreviation for LieSquareMatrix is LSQM
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for LSQM
--R
--R----- Operations -----
--R ??? : (R,%) -> %          ??? : (%,R) -> %
--R ??? : (%,%) -> %          ??? : (Integer,%) -> %
--R ??? : (PositiveInteger,%) -> %    ??? : (%,PositiveInteger) -> %
--R ??? : (%,%) -> %          ?-? : (%,%) -> %
--R -? : % -> %                ?? : (%,%) -> Boolean
--R D : % -> % if R has DIFRING      D : (%,(R -> R)) -> %
--R 1 : () -> %                  0 : () -> %

```

```

--R ??? : (% , PositiveInteger) -> %
--R antiAssociative? : () -> Boolean
--R antiCommutator : (% , %) -> %
--R apply : (Matrix R , %) -> %
--R associator : (% , % , %) -> %
--R coerce : % -> Matrix R
--R coerce : Integer -> %
--R commutative? : () -> Boolean
--R convert : % -> Vector R
--R coordinates : % -> Vector R
--R diagonal? : % -> Boolean
--R diagonalProduct : % -> R
--R elt : (% , Integer , Integer) -> R
--R empty : () -> %
--R eq? : (% , %) -> Boolean
--R hash : % -> SingleInteger
--R jordanAdmissible? : () -> Boolean
--R latex : % -> String
--R leftDiscriminant : Vector % -> R
--R leftNorm : % -> R
--R leftTraceMatrix : () -> Matrix R
--R lieAlgebra? : () -> Boolean
--R map : ((R -> R) , %) -> %
--R matrix : List List R -> %
--R maxRowIndex : % -> Integer
--R minRowIndex : % -> Integer
--R nrows : % -> NonNegativeInteger
--R powerAssociative? : () -> Boolean
--R rank : () -> PositiveInteger
--R represents : Vector R -> %
--R rightAlternative? : () -> Boolean
--R rightDiscriminant : () -> R
--R rightTrace : % -> R
--R sample : () -> %
--R someBasis : () -> Vector %
--R symmetric? : % -> Boolean
--R zero? : % -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?? : (DirectProduct(n,R) , %) -> DirectProduct(n,R)
--R ?? : (% , DirectProduct(n,R)) -> DirectProduct(n,R)
--R ?? : (NonNegativeInteger , %) -> %
--R ??? : (% , Integer) -> % if R has FIELD
--R ??? : (% , NonNegativeInteger) -> %
--R ?/? : (% , R) -> % if R has FIELD
--R D : (% , NonNegativeInteger) -> % if R has DIFRING
--R D : (% , Symbol) -> % if R has PDRING SYMBOL
--R D : (% , List Symbol) -> % if R has PDRING SYMBOL
--R D : (% , Symbol , NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (% , List Symbol , List NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (% , (R -> R) , NonNegativeInteger) -> %
alternative? : () -> Boolean
antiCommutative? : () -> Boolean
antisymmetric? : % -> Boolean
associative? : () -> Boolean
basis : () -> Vector %
coerce : R -> %
coerce : % -> OutputForm
commutator : (% , %) -> %
convert : Vector R -> %
copy : % -> %
diagonalMatrix : List R -> %
?.? : (% , Integer) -> R
elt : (% , Integer , Integer , R) -> R
empty? : % -> Boolean
flexible? : () -> Boolean
jacobiIdentity? : () -> Boolean
jordanAlgebra? : () -> Boolean
leftAlternative? : () -> Boolean
leftDiscriminant : () -> R
leftTrace : % -> R
lieAdmissible? : () -> Boolean
listOfLists : % -> List List R
map : ((R , R) -> R) , % , % -> %
maxColIndex : % -> Integer
minColIndex : % -> Integer
ncols : % -> NonNegativeInteger
one? : % -> Boolean
qelt : (% , Integer , Integer) -> R
recip : % -> Union(% , "failed")
retract : % -> R
rightDiscriminant : Vector % -> R
rightNorm : % -> R
rightTraceMatrix : () -> Matrix R
scalarMatrix : R -> %
square? : % -> Boolean
trace : % -> R
?~=? : (% , %) -> Boolean

```

```

--R ??? : (% , NonNegativeInteger) -> %
--R any? : ((R -> Boolean), %) -> Boolean if $ has finiteAggregate
--R associatorDependence : () -> List Vector R if R has INTDOM
--R characteristic : () -> NonNegativeInteger
--R coerce : Fraction Integer -> % if R has RETRACT FRAC INT
--R column : (% , Integer) -> DirectProduct(n, R)
--R conditionsForIdempotents : Vector % -> List Polynomial R
--R conditionsForIdempotents : () -> List Polynomial R
--R coordinates : (% , Vector %) -> Vector R
--R coordinates : (Vector % , Vector %) -> Matrix R
--R coordinates : Vector % -> Matrix R
--R count : (R, %) -> NonNegativeInteger if $ has finiteAggregate and R has SETCAT
--R count : ((R -> Boolean), %) -> NonNegativeInteger if $ has finiteAggregate
--R determinant : % -> R if R has commutative *
--R diagonal : % -> DirectProduct(n, R)
--R differentiate : % -> % if R has DIFRING
--R differentiate : (% , NonNegativeInteger) -> % if R has DIFRING
--R differentiate : (% , Symbol) -> % if R has PDRING SYMBOL
--R differentiate : (% , List Symbol) -> % if R has PDRING SYMBOL
--R differentiate : (% , Symbol, NonNegativeInteger) -> % if R has PDRING SYMBOL
--R differentiate : (% , List Symbol, List NonNegativeInteger) -> % if R has PDRING SYMBOL
--R differentiate : (% , (R -> R), NonNegativeInteger) -> %
--R differentiate : (% , (R -> R)) -> %
--R eval : (% , List R, List R) -> % if R has EVALAB R and R has SETCAT
--R eval : (% , R, R) -> % if R has EVALAB R and R has SETCAT
--R eval : (% , Equation R) -> % if R has EVALAB R and R has SETCAT
--R eval : (% , List Equation R) -> % if R has EVALAB R and R has SETCAT
--R every? : ((R -> Boolean), %) -> Boolean if $ has finiteAggregate
--R exquo : (% , R) -> Union(% , "failed") if R has INTDOM
--R inverse : % -> Union(% , "failed") if R has FIELD
--R leftCharacteristicPolynomial : % -> SparseUnivariatePolynomial R
--R leftMinimalPolynomial : % -> SparseUnivariatePolynomial R if R has INTDOM
--R leftPower : (% , PositiveInteger) -> %
--R leftRankPolynomial : () -> SparseUnivariatePolynomial Polynomial R if R has FIELD
--R leftRecip : % -> Union(% , "failed") if R has INTDOM
--R leftRegularRepresentation : (% , Vector %) -> Matrix R
--R leftRegularRepresentation : % -> Matrix R
--R leftTraceMatrix : Vector % -> Matrix R
--R leftUnit : () -> Union(% , "failed") if R has INTDOM
--R leftUnits : () -> Union(Record(particular: % , basis: List %), "failed") if R has INTDOM
--R less? : (% , NonNegativeInteger) -> Boolean
--R map! : ((R -> R), %) -> % if $ has shallowlyMutable
--R member? : (R, %) -> Boolean if $ has finiteAggregate and R has SETCAT
--R members : % -> List R if $ has finiteAggregate
--R minordet : % -> R if R has commutative *
--R more? : (% , NonNegativeInteger) -> Boolean
--R noncommutativeJordanAlgebra? : () -> Boolean
--R nullSpace : % -> List DirectProduct(n, R) if R has INTDOM
--R nullity : % -> NonNegativeInteger if R has INTDOM
--R parts : % -> List R if $ has finiteAggregate

```

```

--R plenaryPower : (% ,PositiveInteger) -> %
--R rank : % -> NonNegativeInteger if R has INTDOM
--R reducedSystem : Matrix % -> Matrix R
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) i
--R reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT
--R represents : (Vector R,Vector %) -> %
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retract : % -> Integer if R has RETRACT INT
--R retractIfCan : % -> Union(R,"failed")
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
--R rightCharacteristicPolynomial : % -> SparseUnivariatePolynomial R
--R rightMinimalPolynomial : % -> SparseUnivariatePolynomial R if R has INTDOM
--R rightPower : (% ,PositiveInteger) -> %
--R rightRankPolynomial : () -> SparseUnivariatePolynomial Polynomial R if R has FIELD
--R rightRecip : % -> Union(%,"failed") if R has INTDOM
--R rightRegularRepresentation : (% ,Vector %) -> Matrix R
--R rightRegularRepresentation : % -> Matrix R
--R rightTraceMatrix : Vector % -> Matrix R
--R rightUnit : () -> Union(%,"failed") if R has INTDOM
--R rightUnits : () -> Union(Record(particular: %,basis: List %),"failed") if R has INTDOM
--R row : (% ,Integer) -> DirectProduct(n,R)
--R rowEchelon : % -> % if R has EUCDOM
--R size? : (% ,NonNegativeInteger) -> Boolean
--R structuralConstants : Vector % -> Vector Matrix R
--R structuralConstants : () -> Vector Matrix R
--R subtractIfCan : (% ,%) -> Union(%,"failed")
--R unit : () -> Union(%,"failed") if R has INTDOM
--R
--E 1

)spool
)lisp (bye)

```

---

— LieSquareMatrix.help —

```

=====
LieSquareMatrix examples
=====

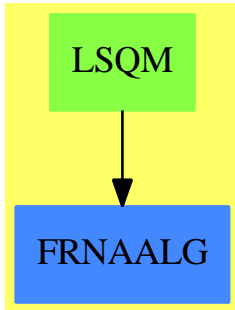
```

```

See Also:
o )show LieSquareMatrix

```

### 13.5.1 LieSquareMatrix (LSQM)



See

⇒ “AssociatedLieAlgebra” (LIE) 2.41.1 on page 211

⇒ “AssociatedJordanAlgebra” (JORDAN) 2.40.1 on page 206



**Exports:**

|                               |                            |                              |
|-------------------------------|----------------------------|------------------------------|
| 0                             | 1                          | alternative?                 |
| antiAssociative?              | antiCommutator             | antisymmetric?               |
| any?                          | apply                      | associative?                 |
| associator                    | associatorDependence       | basis                        |
| characteristic                | coerce                     | column                       |
| commutative?                  | commutator                 | conditionsForIdempotents     |
| convert                       | coordinates                | copy                         |
| count                         | D                          | determinant                  |
| diagonal                      | diagonal?                  | diagonalMatrix               |
| diagonalProduct               | differentiate              | elt                          |
| empty                         | empty?                     | eq?                          |
| eval                          | every?                     | exquo                        |
| flexible?                     | hash                       | inverse                      |
| jacobiIdentity?               | jordanAdmissible?          | jordanAlgebra?               |
| latex                         | leftAlternative?           | leftCharacteristicPolynomial |
| leftDiscriminant              | leftDiscriminant           | leftMinimalPolynomial        |
| leftNorm                      | leftPower                  | leftRankPolynomial           |
| leftRecip                     | leftRegularRepresentation  | leftRegularRepresentation    |
| leftTrace                     | leftTraceMatrix            | leftUnit                     |
| leftUnits                     | less?                      | lieAdmissible?               |
| lieAlgebra?                   | listOfLists                | map                          |
| map!                          | matrix                     | maxColIndex                  |
| maxRowIndex                   | member?                    | members                      |
| minColIndex                   | minordet                   | minRowIndex                  |
| more?                         | ncols                      | noncommutativeJordanAlgebra? |
| nrows                         | nullSpace                  | nullity                      |
| one?                          | parts                      | plenaryPower                 |
| powerAssociative?             | qelt                       | rank                         |
| recip                         | reducedSystem              | represents                   |
| retract                       | retractIfCan               | rightAlternative?            |
| rightCharacteristicPolynomial | rightDiscriminant          | rightMinimalPolynomial       |
| rightNorm                     | rightPower                 | rightRankPolynomial          |
| rightRecip                    | rightRegularRepresentation | rightTrace                   |
| rightTraceMatrix              | rightUnit                  | rightUnits                   |
| row                           | rowEchelon                 | sample                       |
| scalarMatrix                  | size?                      | someBasis                    |
| square?                       | structuralConstants        | structuralConstants          |
| subtractIfCan                 | symmetric?                 | trace                        |
| unit                          | zero?                      | #?                           |
| ?~=?                          | ?*?                        | ?**?                         |
| ?+?                           | ?-?                        | -?                           |
| ?/?                           | ?=?                        | ?^?                          |
| ?..?                          |                            |                              |

```

)abbrev domain LSQM LieSquareMatrix
++ Author: J. Grabmeier
++ Date Created: 07 March 1991
++ Date Last Updated: 08 March 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ LieSquareMatrix(n,R) implements the Lie algebra of the n by n
++ matrices over the commutative ring R.
++ The Lie bracket (commutator) of the algebra is given by\br
++ \spad{a*b := (a *$SQMATRIX(n,R) b - b *$SQMATRIX(n,R) a)},\br
++ where \spadfun{*$SQMATRIX(n,R)} is the usual matrix multiplication.

LieSquareMatrix(n,R): Exports == Implementation where

  n      : PositiveInteger
  R      : CommutativeRing

  Row ==> DirectProduct(n,R)
  Col ==> DirectProduct(n,R)

Exports ==> Join(SquareMatrixCategory(n,R,Row,Col), CoercibleTo Matrix R,
  FramedNonAssociativeAlgebra R) --with

Implementation ==> AssociatedLieAlgebra (R,SquareMatrix(n, R)) add

  Rep := AssociatedLieAlgebra (R,SquareMatrix(n, R))
  -- local functions
  n2 : PositiveInteger := n*n

  convDM : DirectProduct(n2,R) -> %
  conv : DirectProduct(n2,R) -> SquareMatrix(n,R)
  --++ converts n2-vector to (n,n)-matrix row by row
  conv v ==
    cond : Matrix(R) := new(n,n,0$R)$Matrix(R)
    z : Integer := 0
    for i in 1..n repeat
      for j in 1..n repeat
        z := z+1
        setelt(cond,i,j,v.z)
    squareMatrix(cond)$SquareMatrix(n, R)

  coordinates(a:%,b:Vector(%)):Vector(R) ==
    -- only valid for b canonicalBasis
    res : Vector R := new(n2,0$R)

```

```

z : Integer := 0
for i in 1..n repeat
  for j in 1..n repeat
    z := z+1
    res.z := elt(a,i,j)$%
  res

convDM v ==
  sq := conv v
  coerce(sq)$Rep :: %

basis() ==
  n2 : PositiveInteger := n*n
  ldp : List DirectProduct(n2,R) :=
    [unitVector(i::PositiveInteger)$DirectProduct(n2,R) for i in 1..n2]
  res:Vector % := vector map(convDM,_
    ldp)$ListFunctions2(DirectProduct(n2,R), %)

someBasis() == basis()
rank() == n*n

-- transpose: % -> %
--   ++ computes the transpose of a matrix
-- squareMatrix: Matrix R -> %
--   ++ converts a Matrix to a LieSquareMatrix
-- coerce: % -> Matrix R
--   ++ converts a LieSquareMatrix to a Matrix
-- symdecomp : % -> Record(sym:%,antisym:%)
-- if R has commutative("*") then
--   minorsVect: -> Vector(Union(R,"uncomputed")) --range: 1..2**n-1
-- if R has commutative("*") then central
-- if R has commutative("*") and R has unitsKnown then unitsKnown

```

---

— LSQM.dotabb —

```

"LSQM" [color="#88FF44",href="bookvol10.3.pdf#nameddest=LSQM"]
"FRNAALG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FRNAALG"]
"LSQM" -> "FRNAALG"

```

---



```

--S 6 of 16
phi1 == Dop(phi) / exp x
--R
--R
--R                                          Type: Void
--E 6

--S 7 of 16
phi2 == phi1 *x**(n+3)
--R
--R
--R                                          Type: Void
--E 7

--S 8 of 16
phi3 == retract(phi2)@(POLY INT)
--R
--R
--R                                          Type: Void
--E 8

--S 9 of 16
pans == phi3 ::UP(x,POLY INT)
--R
--R
--R                                          Type: Void
--E 9

--S 10 of 16
pans1 == [coefficient(pans, (n+3-i) :: NNI) for i in 2..n+1]
--R
--R
--R                                          Type: Void
--E 10

--S 11 of 16
leq == solve(pans1,[subscript(s,[i]) for i in 1..n])
--R
--R
--R                                          Type: Void
--E 11

--S 12 of 16
leq
--R
--R   Compiling body of rule n to compute value of type PositiveInteger
--R   Compiling body of rule phi to compute value of type Expression
--R   Integer
--R   Compiling body of rule phi1 to compute value of type Expression
--R   Integer
--R   Compiling body of rule phi2 to compute value of type Expression
--R   Integer
--R   Compiling body of rule phi3 to compute value of type Polynomial
--R   Integer
--R   Compiling body of rule pans to compute value of type
--R   UnivariatePolynomial(x,Polynomial Integer)

```

```

--R Compiling body of rule pans1 to compute value of type List
--R Polynomial Integer
--R Compiling body of rule leq to compute value of type List List
--R Equation Fraction Polynomial Integer
--I Compiling function G3349 with type Integer -> Boolean
--R
--R (12)
--R
--R      2      3      2
--R      s G      3s H + s G + 6s G      (9s G + 54s )H + s G + 18s G + 72s G
--R      0      0      0      0      0      0      0      0      0
--R      [[s = ---, s = -----, s = -----]]
--R      1      3      2      18      3      162
--R
--R      Type: List List Equation Fraction Polynomial Integer
--E 12

--S 13 of 16
n==4
--R
--R Compiled code for n has been cleared.
--R Compiled code for leq has been cleared.
--R Compiled code for pans1 has been cleared.
--R Compiled code for phi2 has been cleared.
--R Compiled code for phi has been cleared.
--R Compiled code for phi3 has been cleared.
--R Compiled code for phi1 has been cleared.
--R Compiled code for pans has been cleared.
--R 1 old definition(s) deleted for function or rule n
--R
--R
--R Type: Void
--E 13

--S 14 of 16
leq
--R
--R Compiling body of rule n to compute value of type PositiveInteger
--R Compiling body of rule phi to compute value of type Expression
--R Integer
--R Compiling body of rule phi1 to compute value of type Expression
--R Integer
--R Compiling body of rule phi2 to compute value of type Expression
--R Integer
--R Compiling body of rule phi3 to compute value of type Polynomial
--R Integer
--R Compiling body of rule pans to compute value of type
--R UnivariatePolynomial(x,Polynomial Integer)
--R Compiling body of rule pans1 to compute value of type List
--R Polynomial Integer
--R Compiling body of rule leq to compute value of type List List
--R Equation Fraction Polynomial Integer
--R
--R (14)

```

```

--R  [
--R      2
--R      s G      3s H + s G + 6s G
--R      0      0      0      0
--R      [s = ---, s = -----,
--R      1 3 2      18
--R      3      2
--R      (9s G + 54s )H + s G + 18s G + 72s G
--R      0      0      0      0      0
--R      s = -----,
--R      3      162
--R
--R      s =
--R      4
--R      2      2      4      3      2
--R      27s H + (18s G + 378s G + 1296s )H + s G + 36s G + 396s G
--R      0      0      0      0      0      0      0
--R      +
--R      1296s G
--R      0
--R      /
--R      1944
--R      ]
--R  ]
--R
--R      Type: List List Equation Fraction Polynomial Integer
--E 14

```

```

--S 15 of 16

```

```

n==7

```

```

--R
--R  Compiled code for n has been cleared.
--R  Compiled code for leq has been cleared.
--R  Compiled code for pans1 has been cleared.
--R  Compiled code for phi2 has been cleared.
--R  Compiled code for phi has been cleared.
--R  Compiled code for phi3 has been cleared.
--R  Compiled code for phi1 has been cleared.
--R  Compiled code for pans has been cleared.
--R  1 old definition(s) deleted for function or rule n

```

```

--R
--R      Type: Void

```

```

--E 15

```

```

--S 16 of 16

```

```

leq

```

```

--R
--R  Compiling body of rule n to compute value of type PositiveInteger
--R  Compiling body of rule phi to compute value of type Expression
--R      Integer
--R  Compiling body of rule phi1 to compute value of type Expression
--R      Integer

```

```

--R Compiling body of rule phi2 to compute value of type Expression
--R Integer
--R Compiling body of rule phi3 to compute value of type Polynomial
--R Integer
--R Compiling body of rule pans to compute value of type
--R UnivariatePolynomial(x,Polynomial Integer)
--R Compiling body of rule pans1 to compute value of type List
--R Polynomial Integer
--R Compiling body of rule leq to compute value of type List List
--R Equation Fraction Polynomial Integer
--R
--R (16)
--R [
--R
--R 
$$s = \frac{s^3 G^2 + 3s^2 H G + 3s H^2 + 6s^2 G^2}{18},$$

--R
--R 
$$s = \frac{(9s^3 G + 54s^2)H + s^3 G^2 + 18s^2 G^2 + 72s G^2}{162},$$

--R
--R 
$$s = \frac{27s^2 H^2 + (18s^2 G + 378s G + 1296s)H + s^4 G^4 + 36s^3 G^3 + 396s^2 G^2 + 1296s G}{1944},$$

--R
--R 
$$s = \frac{(135s^2 G + 2268s)H^2 + (30s^3 G + 1350s^2 G + 16416s G + 38880s)H + s^5 G^5 + 60s^4 G^4 + 1188s^3 G^3 + 9504s^2 G^2 + 25920s G}{29160}$$

--R
--R ]

```



```

--R      6
--R      3      2      2
--R      405s H + (405s G + 18468s G + 174960s )H
--R      0      0      0      0
--R      +
--R      4      3      2      6
--R      (45s G + 3510s G + 88776s G + 777600s G + 1166400s )H + s G
--R      0      0      0      0      0      0
--R      +
--R      5      4      3      2
--R      90s G + 2628s G + 27864s G + 90720s G
--R      0      0      0      0
--R      /
--R      524880
--R      ,
--R      s =
--R      7
--R      3
--R      (2835s G + 91854s )H
--R      0      0
--R      +
--R      3      2      2
--R      (945s G + 81648s G + 2082996s G + 14171760s )H
--R      0      0      0      0
--R      +
--R      5      4      3      2
--R      (63s G + 7560s G + 317520s G + 5554008s G + 34058880s G)H
--R      0      0      0      0      0
--R      +
--R      7      6      5      4      3      2
--R      s G + 126s G + 4788s G + 25272s G - 1744416s G - 26827200s G
--R      0      0      0      0      0      0
--R      +
--R      - 97977600s G
--R      0
--R      /
--R      11022480
--R      ]
--R      ]
--R
--R      Type: List List Equation Fraction Polynomial Integer
--E 16
)spool

```

```
=====
LinearOrdinaryDifferentialOperator examples
=====
```

LinearOrdinaryDifferentialOperator(A, diff) is the domain of linear ordinary differential operators with coefficients in a ring A with a given derivation.

```
=====
Differential Operators with Series Coefficients
=====
```

Problem:

Find the first few coefficients of  $\exp(x)/x^i$  of Dop phi where

```
Dop := D^3 + G/x^2 * D + H/x^3 - 1
phi := sum(s[i]*exp(x)/x^i, i = 0..)
```

Solution:

Define the differential.

```
Dx: LODO(EXPR INT, f +-> D(f, x))
      Type: Void
```

```
Dx := D()
D
Type: LinearOrdinaryDifferentialOperator(Expression Integer,
theMap LAMBDA-CLOSURE(NIL,NIL,NIL,G1404 envArg,
SPADCALL(G1404,QUOTE x,
ELT(*1;anonymousFunction;0;frame0;internal;MV,0))))
```

Now define the differential operator Dop.

```
Dop:= Dx^3 + G/x^2*Dx + H/x^3 - 1
      3
      G      - x  + H
D  + -- D + -----
      2      3
      x      x
Type: LinearOrdinaryDifferentialOperator(Expression Integer,
theMap LAMBDA-CLOSURE(NIL,NIL,NIL,G1404 envArg,
SPADCALL(G1404,QUOTE x,
ELT(*1;anonymousFunction;0;frame0;internal;MV,0))))
```

```
n == 3
      Type: Void
```

```
phi == reduce(+,[subscript(s,[i])*exp(x)/x^i for i in 0..n])
      Type: Void
```

```

phi1 == Dop(phi) / exp x
                                     Type: Void

phi2 == phi1 *x**(n+3)
                                     Type: Void

phi3 == retract(phi2)@(POLY INT)
                                     Type: Void

pans == phi3 ::UP(x,POLY INT)
                                     Type: Void

pans1 == [coefficient(pans, (n+3-i) :: NNI) for i in 2..n+1]
                                     Type: Void

leq == solve(pans1,[subscript(s,[i]) for i in 1..n])
                                     Type: Void

```

Evaluate this for several values of n.

```

leq
      2
      s G      3s H + s G + 6s G      (9s G + 54s )H + s G + 18s G + 72s G
      0         0   0   0         0         0         0         0         0
[[s = ---, s = -----, s = -----]]
  1  3  2         18         3         162
                                     Type: List List Equation Fraction Polynomial Integer

```

```

n==4
                                     Type: Void

```

```

leq
[
      2
      s G      3s H + s G + 6s G
      0         0   0   0
[s = ---, s = -----,
  1  3  2         18
      3         2
      (9s G + 54s )H + s G + 18s G + 72s G
      0         0   0         0         0
s = -----,
  3         162
s =
  4
      2         2         4         3         2
      27s H + (18s G + 378s G + 1296s )H + s G + 36s G + 396s G
      0         0         0         0         0         0         0

```

```

      +
      1296s G
      0
    /
    1944
  ]
]
Type: List List Equation Fraction Polynomial Integer

n==7
Type: Void

leq
[
      2
      s G      3s H + s G + 6s G
      0        0      0      0
[s = ---, s = -----,
 1  3      2      18
      3      2
      (9s G + 54s )H + s G + 18s G + 72s G
      0      0      0      0      0
s = -----,
 3      162
s =
4
      2      2      4      3      2
      27s H + (18s G + 378s G + 1296s )H + s G + 36s G + 396s G
      0      0      0      0      0      0
+
      1296s G
      0
/
1944
,
s =
5
      2      3      2
      (135s G + 2268s )H + (30s G + 1350s G + 16416s G + 38880s )H
      0      0      0      0      0      0
+
      5      4      3      2
      s G + 60s G + 1188s G + 9504s G + 25920s G
      0      0      0      0      0
/
29160
,

```

```

s =
6
      3      2      2
    405s H + (405s G + 18468s G + 174960s )H
      0      0      0      0
+
      4      3      2      6
    (45s G + 3510s G + 88776s G + 777600s G + 1166400s )H + s G
      0      0      0      0      0      0
+
      5      4      3      2
    90s G + 2628s G + 27864s G + 90720s G
      0      0      0      0
/
524880
,
s =
7
      3
    (2835s G + 91854s )H
      0      0
+
      3      2      2
    (945s G + 81648s G + 2082996s G + 14171760s )H
      0      0      0      0
+
      5      4      3      2
    (63s G + 7560s G + 317520s G + 5554008s G + 34058880s G)H
      0      0      0      0      0
+
      7      6      5      4      3      2
    s G + 126s G + 4788s G + 25272s G - 1744416s G - 26827200s G
      0      0      0      0      0      0
+
    - 97977600s G
      0
/
11022480
]
]

```

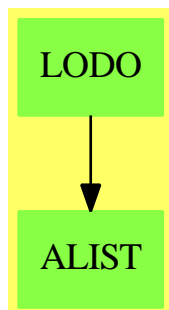
Type: List List Equation Fraction Polynomial Integer

See Also:

o )show LinearOrdinaryDifferentialOperator

\_\_\_\_\_

## 13.6.1 LinearOrdinaryDifferentialOperator (LODO)



See

⇒ “LinearOrdinaryDifferentialOperator1” (LODO1) 13.7.1 on page 1443

⇒ “LinearOrdinaryDifferentialOperator2” (LODO2) 13.8.1 on page 1455

**Exports:**

|                    |                    |                   |
|--------------------|--------------------|-------------------|
| 0                  | 1                  | adjoint           |
| apply              | characteristic     | coefficient       |
| coefficients       | coerce             | content           |
| D                  | degree             | directSum         |
| exquo              | hash               | latex             |
| leadingCoefficient | leftDivide         | leftExactQuotient |
| leftExtendedGcd    | leftGcd            | leftLcm           |
| leftQuotient       | leftRemainder      | minimumDegree     |
| monicLeftDivide    | monicRightDivide   | monomial          |
| one?               | primitivePart      | recip             |
| reductum           | retract            | retractIfCan      |
| rightDivide        | rightExactQuotient | rightExtendedGcd  |
| rightGcd           | rightLcm           | rightQuotient     |
| rightRemainder     | sample             | subtractIfCan     |
| symmetricPower     | symmetricProduct   | symmetricSquare   |
| zero?              | ?*?                | ?**?              |
| ?+?                | ?-?                | -?                |
| ?=?                | ?^?                | ?..?              |
| ?~=?               |                    |                   |

— domain LODO LinearOrdinaryDifferentialOperator —

```

)abbrev domain LODO LinearOrdinaryDifferentialOperator
++ Author: Manuel Bronstein
++ Date Created: 9 December 1993
++ Date Last Updated: 15 April 1994
++ Keywords: differential operator
++ Description:
++ \spad{LinearOrdinaryDifferentialOperator} defines a ring of

```

```

++ differential operators with coefficients in a ring A with a given
++ derivation.
++ Multiplication of operators corresponds to functional composition:\br
++ \spad{(L1 * L2).(f) = L1 L2 f}

```

```

LinearOrdinaryDifferentialOperator(A:Ring, diff: A -> A):
  LinearOrdinaryDifferentialOperatorCategory A
  == SparseUnivariateSkewPolynomial(A, 1, diff) add
  Rep := SparseUnivariateSkewPolynomial(A, 1, diff)

  outputD := "D"@String :: Symbol :: OutputForm

  coerce(l:%):OutputForm == outputForm(l, outputD)
  elt(p:%, a:A):A == apply(p, 0, a)

  if A has Field then
    import LinearOrdinaryDifferentialOperatorsOps(A, %)

    symmetricProduct(a, b) == symmetricProduct(a, b, diff)
    symmetricPower(a, n) == symmetricPower(a, n, diff)
    directSum(a, b) == directSum(a, b, diff)

```

---

— LODO.dotabb —

```

"LODO" [color="#88FF44",href="bookvol10.3.pdf#nameddest=LODO"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"LODO" -> "ALIST"

```

---

## 13.7 domain LODO1 LinearOrdinaryDifferentialOperator1

— LinearOrdinaryDifferentialOperator1.input —

```

)set break resume
)sys rm -f LinearOrdinaryDifferentialOperator1.output
)spool LinearOrdinaryDifferentialOperator1.output
)set message test on
)set message auto off
)clear all

```

```

--S 1 of 20
RFZ := Fraction UnivariatePolynomial('x, Integer)
--R
--R
--R (1) Fraction UnivariatePolynomial(x,Integer)
--R
--R                                          Type: Domain
--E 1

--S 2 of 20
x : RFZ := 'x
--R
--R
--R (2) x
--R
--R                                          Type: Fraction UnivariatePolynomial(x,Integer)
--E 2

--S 3 of 20
Dx : LODO1 RFZ := D()
--R
--R
--R (3) D
--RType: LinearOrdinaryDifferentialOperator1 Fraction UnivariatePolynomial(x,Integer)
--E 3

--S 4 of 20
b : LODO1 RFZ := 3*x**2*Dx**2 + 2*Dx + 1/x
--R
--R
--R
--R      2 2      1
--R (4) 3x D  + 2D + -
--R                               x
--RType: LinearOrdinaryDifferentialOperator1 Fraction UnivariatePolynomial(x,Integer)
--E 4

--S 5 of 20
a : LODO1 RFZ := b*(5*x*Dx + 7)
--R
--R
--R
--R      3 3      2      2      7
--R (5) 15x D  + (51x  + 10x)D  + 29D + -
--R                               x
--RType: LinearOrdinaryDifferentialOperator1 Fraction UnivariatePolynomial(x,Integer)
--E 5

--S 6 of 20
p := x**2 + 1/x**2
--R
--R
--R
--R      4
--R x  + 1

```



```

--R (6) -----
--R      2
--R     x
--R
--R                                          Type: Fraction UnivariatePolynomial(x,Integer)
--E 6

--S 7 of 20
(a*b - b*a) p
--R
--R
--R      4
--R     - 75x  + 540x - 75
--R (7) -----
--R      4
--R     x
--R
--R                                          Type: Fraction UnivariatePolynomial(x,Integer)
--E 7

--S 8 of 20
ld := leftDivide(a,b)
--R
--R
--R (8) [quotient= 5x D + 7,remainder= 0]
--RType: Record(quotient: LinearOrdinaryDifferentialOperator1 Fraction UnivariatePolynomial(x,Integer),remainder: Fraction UnivariatePolynomial(x,Integer))
--E 8

--S 9 of 20
a = b * ld.quotient + ld.remainder
--R
--R
--R      3 3      2      2      7      3 3      2      2      7
--R (9) 15x D  + (51x  + 10x)D  + 29D + - 15x D  + (51x  + 10x)D  + 29D + -
--R                                     x                                     x
--RType: Equation LinearOrdinaryDifferentialOperator1 Fraction UnivariatePolynomial(x,Integer)
--E 9

--S 10 of 20
rd := rightDivide(a,b)
--R
--R
--R      5
--R (10) [quotient= 5x D + 7,remainder= 10D + -]
--R                                     x
--RType: Record(quotient: LinearOrdinaryDifferentialOperator1 Fraction UnivariatePolynomial(x,Integer),remainder: Fraction UnivariatePolynomial(x,Integer))
--E 10

--S 11 of 20
a = rd.quotient * b + rd.remainder
--R
--R

```

```

--R      3 3      2      2      7      3 3      2      2      7
--R (11) 15x D + (51x + 10x)D + 29D + -= 15x D + (51x + 10x)D + 29D + -
--R      x      x
--RType: Equation LinearOrdinaryDifferentialOperator1 Fraction UnivariatePolynomial(x,Integer)
--E 11

--S 12 of 20
rightQuotient(a,b)
--R
--R
--R (12) 5x D + 7
--RType: LinearOrdinaryDifferentialOperator1 Fraction UnivariatePolynomial(x,Integer)
--E 12

--S 13 of 20
rightRemainder(a,b)
--R
--R
--R      5
--R (13) 10D + -
--R      x
--RType: LinearOrdinaryDifferentialOperator1 Fraction UnivariatePolynomial(x,Integer)
--E 13

--S 14 of 20
leftExactQuotient(a,b)
--R
--R
--R (14) 5x D + 7
--RType: Union(LinearOrdinaryDifferentialOperator1 Fraction UnivariatePolynomial(x,Integer),...)
--E 14

--S 15 of 20
e := leftGcd(a,b)
--R
--R
--R      2 2      1
--R (15) 3x D + 2D + -
--R      x
--RType: LinearOrdinaryDifferentialOperator1 Fraction UnivariatePolynomial(x,Integer)
--E 15

--S 16 of 20
leftRemainder(a, e)
--R
--R
--R (16) 0
--RType: LinearOrdinaryDifferentialOperator1 Fraction UnivariatePolynomial(x,Integer)
--E 16

```

```

--S 17 of 20
rightRemainder(a, e)
--R
--R
--R      5
--R (17) 10D + -
--R      x
--RType: LinearOrdinaryDifferentialOperator1 Fraction UnivariatePolynomial(x,Integer)
--E 17

--S 18 of 20
f := rightLcm(a,b)
--R
--R
--R      3 3      2      2      7
--R (18) 15x D + (51x + 10x)D + 29D + -
--R      x
--RType: LinearOrdinaryDifferentialOperator1 Fraction UnivariatePolynomial(x,Integer)
--E 18

--S 19 of 20
rightRemainder(f, b)
--R
--R
--R      5
--R (19) 10D + -
--R      x
--RType: LinearOrdinaryDifferentialOperator1 Fraction UnivariatePolynomial(x,Integer)
--E 19

--S 20 of 20
leftRemainder(f, b)
--R
--R
--R (20) 0
--RType: LinearOrdinaryDifferentialOperator1 Fraction UnivariatePolynomial(x,Integer)
--E 20
)spool
)lisp (bye)

```

---

— LinearOrdinaryDifferentialOperator1.help —

```

=====
LinearOrdinaryDifferentialOperator1 example
=====

```

LinearOrdinaryDifferentialOperator1(A) is the domain of linear

ordinary differential operators with coefficients in the differential ring A.

```
=====
Differential Operators with Rational Function Coefficients
=====
```

This example shows differential operators with rational function coefficients. In this case operator multiplication is non-commutative and, since the coefficients form a field, an operator division algorithm exists.

We begin by defining RFZ to be the rational functions in x with integer coefficients and Dx to be the differential operator for d/dx.

```
RFZ := Fraction UnivariatePolynomial('x, Integer)
      Fraction UnivariatePolynomial(x,Integer)
      Type: Domain

x : RFZ := 'x
x
                                     Type: Fraction UnivariatePolynomial(x,Integer)

Dx : LODO1 RFZ := D()
D
                                     Type: LinearOrdinaryDifferentialOperator1
                                     Fraction UnivariatePolynomial(x,Integer)
```

Operators are created using the usual arithmetic operations.

```
b : LODO1 RFZ := 3*x**2*Dx**2 + 2*Dx + 1/x
      2 2      1
      3x D  + 2D + -
      x
                                     Type: LinearOrdinaryDifferentialOperator1
                                     Fraction UnivariatePolynomial(x,Integer)

a : LODO1 RFZ := b*(5*x*Dx + 7)
      3 3      2      2      7
      15x D  + (51x  + 10x)D  + 29D + -
      x
                                     Type: LinearOrdinaryDifferentialOperator1
                                     Fraction UnivariatePolynomial(x,Integer)
```

Operator multiplication corresponds to functional composition.

```
p := x**2 + 1/x**2
      4
      x  + 1
      -----
      2
```

x

Type: Fraction UnivariatePolynomial(x,Integer)

Since operator coefficients depend on x, the multiplication is not commutative.

$$\frac{(a*b - b*a) p}{x^4 - 75x^3 + 540x^2 - 75x}$$

Type: Fraction UnivariatePolynomial(x,Integer)

When the coefficients of operator polynomials come from a field, as in this case, it is possible to define operator division. Division on the left and division on the right yield different results when the multiplication is non-commutative.

The results of leftDivide and rightDivide are quotient-remainder pairs satisfying:

leftDivide(a,b) = [q, r] such that a = b\*q + r  
 rightDivide(a,b) = [q, r] such that a = q\*b + r

In both cases, the degree of the remainder, r, is less than the degree of b.

```
ld := leftDivide(a,b)
[quotient= 5x D + 7,remainder= 0]
Type: Record(quotient: LinearOrdinaryDifferentialOperator1
             Fraction UnivariatePolynomial(x,Integer),
             remainder: LinearOrdinaryDifferentialOperator1
             Fraction UnivariatePolynomial(x,Integer))

a = b * ld.quotient + ld.remainder
3 3      2      2      7      3 3      2      2      7
15x D + (51x + 10x)D + 29D + -= 15x D + (51x + 10x)D + 29D + -
Type: Equation LinearOrdinaryDifferentialOperator1
Fraction UnivariatePolynomial(x,Integer)
```

The operations of left and right division are so-called because the quotient is obtained by dividing a on that side by b.

```
rd := rightDivide(a,b)
[quotient= 5x D + 7,remainder= 10D + -]
Type: Record(quotient: LinearOrdinaryDifferentialOperator1
             Fraction UnivariatePolynomial(x,Integer),
             remainder: LinearOrdinaryDifferentialOperator1
             Fraction UnivariatePolynomial(x,Integer))
```

```

a = rd.quotient * b + rd.remainder
      3 3      2      2      7      3 3      2      2      7
15x D + (51x + 10x)D + 29D + -= 15x D + (51x + 10x)D + 29D + -
Type: Equation LinearOrdinaryDifferentialOperator1
      Fraction UnivariatePolynomial(x,Integer)

```

Operations `rightQuotient` and `rightRemainder` are available if only one of the quotient or remainder are of interest to you. This is the quotient from right division.

```

rightQuotient(a,b)
5x D + 7
Type: LinearOrdinaryDifferentialOperator1
      Fraction UnivariatePolynomial(x,Integer)

```

This is the remainder from right division. The corresponding "left" functions, `leftQuotient` and `leftRemainder` are also available.

```

rightRemainder(a,b)
      5
10D + -
      x
Type: LinearOrdinaryDifferentialOperator1
      Fraction UnivariatePolynomial(x,Integer)

```

For exact division, operations `leftExactQuotient` and `rightExactQuotient` are supplied. These return the quotient but only if the remainder is zero. The call `rightExactQuotient(a,b)` would yield an error.

```

leftExactQuotient(a,b)
5x D + 7
Type: Union(LinearOrdinaryDifferentialOperator1
      Fraction UnivariatePolynomial(x,Integer),...)

```

The division operations allow the computation of left and right greatest common divisors, `leftGcd` and `rightGcd` via remainder sequences, and consequently the computation of left and right least common multiples, `rightLcm` and `leftLcm`.

```

e := leftGcd(a,b)
      2 2      1
3x D + 2D + -
      x
Type: LinearOrdinaryDifferentialOperator1
      Fraction UnivariatePolynomial(x,Integer)

```

Note that a greatest common divisor doesn't necessarily divide `a` and `b` on both sides. Here the left greatest common divisor does not divide `a` on the right.

```

leftRemainder(a, e)
0
Type: LinearOrdinaryDifferentialOperator1
      Fraction UnivariatePolynomial(x,Integer)

```

```

rightRemainder(a, e)
5
10D + -
      x
Type: LinearOrdinaryDifferentialOperator1
      Fraction UnivariatePolynomial(x,Integer)

```

Similarly, a least common multiple is not necessarily divisible from both sides.

```

f := rightLcm(a,b)
3 3      2      2      7
15x D + (51x + 10x)D + 29D + -
Type: LinearOrdinaryDifferentialOperator1
      Fraction UnivariatePolynomial(x,Integer)

```

```

rightRemainder(f, b)
5
10D + -
      x
Type: LinearOrdinaryDifferentialOperator1
      Fraction UnivariatePolynomial(x,Integer)

```

```

leftRemainder(f, b)
0
Type: LinearOrdinaryDifferentialOperator1
      Fraction UnivariatePolynomial(x,Integer)

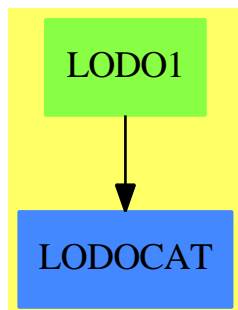
```

See Also:

```
o )show LinearOrdinaryDifferentialOperator1
```

---

## 13.7.1 LinearOrdinaryDifferentialOperator1 (LODO1)



See

⇒ “LinearOrdinaryDifferentialOperator” (LODO) 13.6.1 on page 1433

⇒ “LinearOrdinaryDifferentialOperator2” (LODO2) 13.8.1 on page 1455

**Exports:**

|                 |                   |                    |                    |
|-----------------|-------------------|--------------------|--------------------|
| 0               | 1                 | adjoint            | apply              |
| characteristic  | coefficient       | coefficients       | coerce             |
| content         | D                 | degree             | directSum          |
| exquo           | hash              | latex              | leadingCoefficient |
| leftDivide      | leftExactQuotient | leftExtendedGcd    | leftGcd            |
| leftLcm         | leftQuotient      | leftRemainder      | minimumDegree      |
| monicLeftDivide | monicRightDivide  | monomial           | one?               |
| primitivePart   | recip             | reductum           | retract            |
| retractIfCan    | rightDivide       | rightExactQuotient | rightExtendedGcd   |
| rightGcd        | rightLcm          | rightQuotient      | rightRemainder     |
| sample          | subtractIfCan     | symmetricPower     | symmetricProduct   |
| symmetricSquare | zero?             | ?*?                | ?**?               |
| ?+?             | ?-?               | -?                 | ?=?                |
| ?^?             | ?.?               | ?~=?               |                    |

— domain LODO1 LinearOrdinaryDifferentialOperator1 —

```

)abbrev domain LODO1 LinearOrdinaryDifferentialOperator1
++ Author: Manuel Bronstein
++ Date Created: 9 December 1993
++ Date Last Updated: 31 January 1994
++ Keywords: differential operator
++ Description:
++ \spad{LinearOrdinaryDifferentialOperator1} defines a ring of
++ differential operators with coefficients in a differential ring A.
++ Multiplication of operators corresponds to functional composition:\br
++ \spad{(L1 * L2).(f) = L1 L2 f}

```

```

LinearOrdinaryDifferentialOperator1(A:DifferentialRing) ==
  LinearOrdinaryDifferentialOperator(A, differentiate$A)

```



---

— LODO1.dotabb —

```
"LOD01" [color="#88FF44",href="bookvol10.3.pdf#nameddest=LOD01"]
"LODOCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=LODOCAT"]
"LOD01" -> "LODOCAT"
```

---

## 13.8 domain LODO2 LinearOrdinaryDifferentialOperator2

— LinearOrdinaryDifferentialOperator2.input —

```
)set break resume
)sys rm -f LinearOrdinaryDifferentialOperator2.output
)spool LinearOrdinaryDifferentialOperator2.output
)set message test on
)set message auto off
)clear all
--S 1 of 26
Q := Fraction Integer
--R
--R
--R (1) Fraction Integer
--R
--R                                          Type: Domain
--E 1

--S 2 of 26
PQ := UnivariatePolynomial('x, Q)
--R
--R
--R (2) UnivariatePolynomial(x,Fraction Integer)
--R
--R                                          Type: Domain
--E 2

--S 3 of 26
x: PQ := 'x
--R
--R
--R (3) x
```

```

--R                                     Type: UnivariatePolynomial(x,Fraction Integer)
--E 3

--S 4 of 26
Dx: LODO2(Q, PQ) := D()
--R
--R
--R      (4)  D
--RType: LinearOrdinaryDifferentialOperator2(Fraction Integer,UnivariatePolynomial(x,Fraction Integer))
--E 4

--S 5 of 26
a := Dx + 1
--R
--R
--R      (5)  D + 1
--RType: LinearOrdinaryDifferentialOperator2(Fraction Integer,UnivariatePolynomial(x,Fraction Integer))
--E 5

--S 6 of 26
b := a + 1/2*Dx**2 - 1/2
--R
--R
--R      (6)   $-\frac{1}{2}D^2 + D + -\frac{1}{2}$ 
--RType: LinearOrdinaryDifferentialOperator2(Fraction Integer,UnivariatePolynomial(x,Fraction Integer))
--E 6

--S 7 of 26
p := 4*x**2 + 2/3
--R
--R
--R      (7)   $4x^2 + -\frac{2}{3}$ 
--R                                     Type: UnivariatePolynomial(x,Fraction Integer)
--E 7

--S 8 of 26
a p
--R
--R
--R      (8)   $4x^2 + 8x + -\frac{2}{3}$ 
--R                                     Type: UnivariatePolynomial(x,Fraction Integer)
--E 8

--S 9 of 26

```

```

(a * b) p = a b p
--R
--R
--R      2      37      2      37
--R      (9)  2x  + 12x + --= 2x  + 12x + --
--R              3              3
--R                                     Type: Equation UnivariatePolynomial(x,Fraction Integer)
--E 9

--S 10 of 26
c := (1/9)*b*(a + b)^2
--R
--R
--R      1 6      5 5      13 4      19 3      79 2      7      1
--R      (10)  -- D  + -- D  + -- D  + -- D  + -- D  + -- D + -
--R          72      36      24      18      72      12      8
--RType: LinearOrdinaryDifferentialOperator2(Fraction Integer,UnivariatePolynomial(x,Fraction Integer))
--E 10

--S 11 of 26
(a**2 - 3/4*b + c) (p + 1)
--R
--R
--R      2      44      541
--R      (11)  3x  + -- x + ---
--R          3      36
--R                                     Type: UnivariatePolynomial(x,Fraction Integer)
--E 11
)clear all
--S 12 of 26
PZ := UnivariatePolynomial(x,Integer)
--R
--R
--R      (1)  UnivariatePolynomial(x,Integer)
--R
--R                                     Type: Domain
--E 12

--S 13 of 26
x:PZ := 'x
--R
--R
--R      (2)  x
--R
--R                                     Type: UnivariatePolynomial(x,Integer)
--E 13

--S 14 of 26
Mat := SquareMatrix(3,PZ)
--R
--R
--R      (3)  SquareMatrix(3,UnivariatePolynomial(x,Integer))

```

```
--R                                                    Type: Domain
--E 14
```

```
--S 15 of 26
```

```
Vect := DPMM(3, PZ, Mat, PZ)
```

```
--R
--R
--R (4)
--R DirectProductMatrixModule(3,UnivariatePolynomial(x,Integer),SquareMatrix(3,Un
--R ivariatePolynomial(x,Integer)),UnivariatePolynomial(x,Integer))
--R                                                    Type: Domain
--E 15
```

```
--S 16 of 26
```

```
Modo := LODO2(Mat, Vect)
```

```
--R
--R
--R (5)
--R LinearOrdinaryDifferentialOperator2(SquareMatrix(3,UnivariatePolynomial(x,Int
--R eger)),DirectProductMatrixModule(3,UnivariatePolynomial(x,Integer),SquareMatr
--R ix(3,UnivariatePolynomial(x,Integer)),UnivariatePolynomial(x,Integer)))
--R                                                    Type: Domain
--E 16
```

```
--S 17 of 26
```

```
m:Mat := matrix [ [x^2,1,0],[1,x^4,0],[0,0,4*x^2] ]
```

```
--R
--R
--R      + 2      +
--R      |x   1   0 |
--R      |       |
--R (6)  |   4   |
--R      |1   x   0 |
--R      |       |
--R      |       2|
--R      +0   0   4x +
--R
--R                                                    Type: SquareMatrix(3,UnivariatePolynomial(x,Integer))
--E 17
```

```
--S 18 of 26
```

```
p:Vect := directProduct [3*x^2+1,2*x,7*x^3+2*x]
```

```
--R
--R
--R      2      3
--R (7)  [3x  + 1,2x,7x  + 2x]
--RType: DirectProductMatrixModule(3,UnivariatePolynomial(x,Integer),SquareMatrix(3,UnivariatePolynomial
--E 18
```

```
--S 19 of 26
```

```
q: Vect := m * p
```

```

--R
--R
--R      4      2      5      2      5      3
--R      (8) [3x + x + 2x, 2x + 3x + 1, 28x + 8x ]
--RType: DirectProductMatrixModule(3,UnivariatePolynomial(x,Integer),SquareMatrix(3,UnivariatePolynomial(x,Integer)))
--E 19

--S 20 of 26
Dx : Mod0 := D()
--R
--R
--R      (9) D
--RType: LinearOrdinaryDifferentialOperator2(SquareMatrix(3,UnivariatePolynomial(x,Integer)))
--E 20

--S 21 of 26
a : Mod0 := Dx + m
--R
--R
--R      + 2      +
--R      |x  1  0 |
--R      |      |
--R      (10) D + | 4  |
--R      |1  x  0 |
--R      |      |
--R      |      2|
--R      +0  0  4x +
--RType: LinearOrdinaryDifferentialOperator2(SquareMatrix(3,UnivariatePolynomial(x,Integer)))
--E 21

--S 22 of 26
b : Mod0 := m*Dx + 1
--R
--R
--R      + 2      +
--R      |x  1  0 | +1  0  0+
--R      |      | |      |
--R      (11) | 4  |D + |0  1  0|
--R      |1  x  0 | |      |
--R      |      | +0  0  1+
--R      |      2|
--R      +0  0  4x +
--RType: LinearOrdinaryDifferentialOperator2(SquareMatrix(3,UnivariatePolynomial(x,Integer)))
--E 22

--S 23 of 26
c := a*b
--R
--R
--R      (12)

```

```

--R      + 2      +      + 4      4      2      +      + 2      +
--R      |x  1      0 |      |x  + 2x + 2      x  + x      0      |      |x  1      0 |
--R      |      |      | 2      |      |      |      |      |      |      |
--R      |      4      |D + |      4      2      8      3      |D + |      4      |
--R      |1  x      0 |      | x  + x      x  + 4x  + 2      0      |      |1  x      0 |
--R      |      |      |      |      |      |      |      |      |      |
--R      |      2|      |      |      4      |      |      2|
--R      +0  0  4x +      +      0      0      16x  + 8x + 1+      +0  0  4x +
--RType: LinearOrdinaryDifferentialOperator2(SquareMatrix(3,UnivariatePolynomial(x,Integer)),DirectProduct
--E 23

--S 24 of 26
a p
--R
--R
--R      4      2      5      2      5      3      2
--R      (13) [3x  + x  + 8x, 2x  + 3x  + 3, 28x  + 8x  + 21x  + 2]
--RType: DirectProductMatrixModule(3,UnivariatePolynomial(x,Integer),SquareMatrix(3,UnivariatePolynomial
--E 24

--S 25 of 26
b p
--R
--R
--R      3      2      4      4      3      2
--R      (14) [6x  + 3x  + 3, 2x  + 8x, 84x  + 7x  + 8x  + 2x]
--RType: DirectProductMatrixModule(3,UnivariatePolynomial(x,Integer),SquareMatrix(3,UnivariatePolynomial
--E 25

--S 26 of 26
(a + b + c) (p + q)
--R
--R
--R      (15)
--R      8      7      6      5      4      3      2
--R      [10x  + 12x  + 16x  + 30x  + 85x  + 94x  + 40x  + 40x + 17,
--R      12      9      8      7      6      5      4      3      2
--R      10x  + 10x  + 12x  + 92x  + 6x  + 32x  + 72x  + 28x  + 49x  + 32x + 19,
--R      8      7      6      5      4      3      2
--R      2240x  + 224x  + 1280x  + 3508x  + 492x  + 751x  + 98x  + 18x + 4]
--RType: DirectProductMatrixModule(3,UnivariatePolynomial(x,Integer),SquareMatrix(3,UnivariatePolynomial
--E 26
)spool
)lisp (bye)

```

```
=====
LinearOrdinaryDifferentialOperator2
=====
```

LinearOrdinaryDifferentialOperator2(A, M) is the domain of linear ordinary differential operators with coefficients in the differential ring A and operating on M, an A-module. This includes the cases of operators which are polynomials in D acting upon scalar or vector expressions of a single variable. The coefficients of the operator polynomials can be integers, rational functions, matrices or elements of other domains.

```
=====
Differential Operators with Constant Coefficients
=====
```

This example shows differential operators with rational number coefficients operating on univariate polynomials.

We begin by making type assignments so we can conveniently refer to univariate polynomials in x over the rationals.

```
Q := Fraction Integer
    Fraction Integer
                                Type: Domain

PQ := UnivariatePolynomial('x, Q)
     UnivariatePolynomial(x, Fraction Integer)
                                Type: Domain

x: PQ := 'x
   x
                                Type: UnivariatePolynomial(x, Fraction Integer)
```

Now we assign Dx to be the differential operator D corresponding to d/dx.

```
Dx: LOD02(Q, PQ) := D()
    D
    Type: LinearOrdinaryDifferentialOperator2(Fraction Integer,
        UnivariatePolynomial(x, Fraction Integer))
```

New operators are created as polynomials in D().

```
a := Dx + 1
    D + 1
    Type: LinearOrdinaryDifferentialOperator2(Fraction Integer,
        UnivariatePolynomial(x, Fraction Integer))

b := a + 1/2*Dx**2 - 1/2
    1 2      1
```

```

- D  + D + -
 2      2
Type: LinearOrdinaryDifferentialOperator2(Fraction Integer,
      UnivariatePolynomial(x,Fraction Integer))

```

To apply the operator  $a$  to the value  $p$  the usual function call syntax is used.

```

p := 4*x**2 + 2/3
      2      2
      4x  + -
          3
Type: UnivariatePolynomial(x,Fraction Integer)

```

```

a p
      2      2
      4x  + 8x + -
          3
Type: UnivariatePolynomial(x,Fraction Integer)

```

Operator multiplication is defined by the identity  $(a*b)p = a(b(p))$

```

(a * b) p = a b p
      2      37      2      37
      2x  + 12x + --- 2x  + 12x + --
          3          3
Type: Equation UnivariatePolynomial(x,Fraction Integer)

```

Exponentiation follows from multiplication.

```

c := (1/9)*b*(a + b)^2
      1 6    5 5    13 4    19 3    79 2    7    1
      -- D  + -- D  + -- D  + -- D  + -- D  + -- D  + -
      72    36    24    18    72    12    8
Type: LinearOrdinaryDifferentialOperator2(Fraction Integer,
      UnivariatePolynomial(x,Fraction Integer))

```

Finally, note that operator expressions may be applied directly.

```

(a**2 - 3/4*b + c) (p + 1)
      2    44    541
      3x  + -- x + ---
          3      36
Type: UnivariatePolynomial(x,Fraction Integer)

```

```

=====
Differential Operators with Matrix Coefficients Operating on Vectors}
=====

```

This is another example of linear ordinary differential operators with



non-commutative multiplication. Unlike the rational function case, the differential ring of square matrices (of a given dimension) with univariate polynomial entries does not form a field. Thus the number of operations available is more limited.

In this section, the operators have three by three matrix coefficients with polynomial entries.

```
PZ := UnivariatePolynomial(x,Integer)
      UnivariatePolynomial(x,Integer)
      Type: Domain

x:PZ := 'x
      x
      Type: UnivariatePolynomial(x,Integer)

Mat := SquareMatrix(3,PZ)
      SquareMatrix(3,UnivariatePolynomial(x,Integer))
      Type: Domain
```

The operators act on the vectors considered as a Mat-module.

```
Vect := DPMM(3, PZ, Mat, PZ)
DirectProductMatrixModule(3,UnivariatePolynomial(x,Integer),SquareMatrix(3,UnivariatePolynomial(x,Integer)),UnivariatePolynomial(x,Integer))
      Type: Domain

Modo := LOD02(Mat, Vect)
LinearOrdinaryDifferentialOperator2(SquareMatrix(3,UnivariatePolynomial(x,Integer)),DirectProductMatrixModule(3,UnivariatePolynomial(x,Integer),SquareMatrix(3,UnivariatePolynomial(x,Integer)),UnivariatePolynomial(x,Integer)))
      Type: Domain
```

The matrix  $m$  is used as a coefficient and the vectors  $p$  and  $q$  are operated upon.

```
m:Mat := matrix [ [x^2,1,0],[1,x^4,0],[0,0,4*x^2] ]
      + 2      +
      |x  1  0 |
      |      |
      |  4  |
      |1  x  0 |
      |      |
      |      2|
      +0  0  4x +
      Type: SquareMatrix(3,UnivariatePolynomial(x,Integer))

p:Vect := directProduct [3*x^2+1,2*x,7*x^3+2*x]
      2      3
      [3x  + 1,2x,7x  + 2x]
```

```

Type: DirectProductMatrixModule(3,UnivariatePolynomial(x,Integer),
    SquareMatrix(3,UnivariatePolynomial(x,Integer)),
    UnivariatePolynomial(x,Integer))

```

```

q: Vect := m * p
      4      2      5      2      5      3
[3x  + x  + 2x, 2x  + 3x  + 1, 28x  + 8x ]
Type: DirectProductMatrixModule(3,UnivariatePolynomial(x,Integer),
    SquareMatrix(3,UnivariatePolynomial(x,Integer)),
    UnivariatePolynomial(x,Integer))

```

Now form a few operators.

```

Dx : Modo := D()
D
Type: LinearOrdinaryDifferentialOperator2(
    SquareMatrix(3,UnivariatePolynomial(x,Integer)),
    DirectProductMatrixModule(3,UnivariatePolynomial(x,Integer),
    SquareMatrix(3,UnivariatePolynomial(x,Integer)),
    UnivariatePolynomial(x,Integer)))

```

```

a : Modo := Dx + m
      + 2      +
      |x  1    0 |
      |          |
D + |          4 |
    |1  x    0 |
    |          |
    |          2|
    +0  0  4x +
Type: LinearOrdinaryDifferentialOperator2(
    SquareMatrix(3,UnivariatePolynomial(x,Integer)),
    DirectProductMatrixModule(3,UnivariatePolynomial(x,Integer),
    SquareMatrix(3,UnivariatePolynomial(x,Integer)),
    UnivariatePolynomial(x,Integer)))

```

```

b : Modo := m*Dx + 1
      + 2      +
      |x  1    0 |      +1  0  0+
      |          |      |   |   |
      |          4 |D + |0  1  0|
      |1  x    0 |      |   |   |
      |          |      +0  0  1+
      |          2|
      +0  0  4x +
Type: LinearOrdinaryDifferentialOperator2(
    SquareMatrix(3,UnivariatePolynomial(x,Integer)),
    DirectProductMatrixModule(3,UnivariatePolynomial(x,Integer),
    SquareMatrix(3,UnivariatePolynomial(x,Integer)),
    UnivariatePolynomial(x,Integer)))

```

```

c := a*b
+ 2      +      + 4      4      2      +      + 2      +
|x  1    0 |      |x  + 2x + 2    x  + x      0      |      |x  1    0 |
|          | 2    |          8      3          |          |          |
|      4    |D + |      4      2      x  + 4x  + 2      0      |D + |      4    |
|1   x    0 |      |   x  + x      x  + 4x  + 2      0      |      |1   x    0 |
|          |      |          |          |          |          |          |
|          |      |          |          |          |          |          |
+0   0   4x +      +      0      0      16x  + 8x + 1+      +0   0   4x +
Type: LinearOrdinaryDifferentialOperator2(
  SquareMatrix(3,UnivariatePolynomial(x,Integer)),
  DirectProductMatrixModule(3,UnivariatePolynomial(x,Integer),
    SquareMatrix(3,UnivariatePolynomial(x,Integer)),
    UnivariatePolynomial(x,Integer)))

```

These operators can be applied to vector values.

```

a p
      4      2      5      2      5      3      2
[3x  + x  + 8x, 2x  + 3x  + 3, 28x  + 8x  + 21x  + 2]
Type: DirectProductMatrixModule(3,UnivariatePolynomial(x,Integer),
  SquareMatrix(3,UnivariatePolynomial(x,Integer)),
  UnivariatePolynomial(x,Integer))

```

```

b p
      3      2      4      4      3      2
[6x  + 3x  + 3, 2x  + 8x, 84x  + 7x  + 8x  + 2x]
Type: DirectProductMatrixModule(3,UnivariatePolynomial(x,Integer),
  SquareMatrix(3,UnivariatePolynomial(x,Integer)),
  UnivariatePolynomial(x,Integer))

```

```

(a + b + c) (p + q)
      8      7      6      5      4      3      2
[10x  + 12x  + 16x  + 30x  + 85x  + 94x  + 40x  + 40x + 17,
      12      9      8      7      6      5      4      3      2
10x  + 10x  + 12x  + 92x  + 6x  + 32x  + 72x  + 28x  + 49x  + 32x + 19,
      8      7      6      5      4      3      2
2240x  + 224x  + 1280x  + 3508x  + 492x  + 751x  + 98x  + 18x + 4]
Type: DirectProductMatrixModule(3,UnivariatePolynomial(x,Integer),
  SquareMatrix(3,UnivariatePolynomial(x,Integer)),
  UnivariatePolynomial(x,Integer))

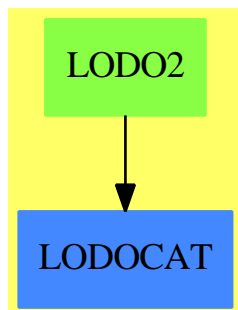
```

See Also:

o )show LinearOrdinaryDifferentialOperator2

---

## 13.8.1 LinearOrdinaryDifferentialOperator2 (LODO2)



See

⇒ “LinearOrdinaryDifferentialOperator” (LODO) 13.6.1 on page 1433

⇒ “LinearOrdinaryDifferentialOperator1” (LODO1) 13.7.1 on page 1443

**Exports:**

|                 |                   |                    |                    |
|-----------------|-------------------|--------------------|--------------------|
| 0               | 1                 | adjoint            | apply              |
| characteristic  | coefficient       | coefficients       | coerce             |
| content         | D                 | degree             | directSum          |
| exquo           | hash              | latex              | leadingCoefficient |
| leftDivide      | leftExactQuotient | leftExtendedGcd    | leftGcd            |
| leftLcm         | leftQuotient      | leftRemainder      | minimumDegree      |
| monicLeftDivide | monicRightDivide  | monomial           | one?               |
| primitivePart   | recip             | reductum           | retract            |
| retractIfCan    | rightDivide       | rightExactQuotient | rightExtendedGcd   |
| rightGcd        | rightLcm          | rightQuotient      | rightRemainder     |
| sample          | subtractIfCan     | symmetricPower     | symmetricProduct   |
| symmetricSquare | zero?             | ?*?                | ?**?               |
| ?+?             | ?-?               | -?                 | ?=?                |
| ?^?             | ?..?              | ?~=?               |                    |

— domain LODO2 LinearOrdinaryDifferentialOperator2 —

```

)abbrev domain LODO2 LinearOrdinaryDifferentialOperator2
++ Author: Stephen M. Watt, Manuel Bronstein
++ Date Created: 1986
++ Date Last Updated: 1 February 1994
++ Keywords: differential operator
++ Description:
++ \spad{LinearOrdinaryDifferentialOperator2} defines a ring of
++ differential operators with coefficients in a differential ring A
++ and acting on an A-module M.
++ Multiplication of operators corresponds to functional composition:\br
++ \spad{(L1 * L2).(f) = L1 L2 f}

```

LinearOrdinaryDifferentialOperator2(A, M): Exports == Implementation where

```

A: DifferentialRing
M: LeftModule A with
  differentiate: $ -> $
  ++ differentiate(x) returns the derivative of x

Exports ==> Join(LinearOrdinaryDifferentialOperatorCategory A, Eltable(M, M))

Implementation ==> LinearOrdinaryDifferentialOperator(A, differentiate$A) add
elt(p:%, m:M):M ==
  apply(p, differentiate, m)$ApplyUnivariateSkewPolynomial(A, M, %)

```

---

— LODO2.dotabb —

```

"LODO2" [color="#88FF44",href="bookvol10.3.pdf#nameddest=LODO2"]
"LODOCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=LODOCAT"]
"LODO2" -> "LODOCAT"

```

---

## 13.9 domain LIST List

— List.input —

```

)set break resume
)sys rm -f List.output
)spool List.output
)set message test on
)set message auto off
)clear all
--S 1 of 34
[2, 4, 5, 6]
--R
--R
--R (1) [2,4,5,6]
--R
--R                                          Type: List PositiveInteger
--E 1

--S 2 of 34
[1]
--R
--R
--R (2) [1]

```



```

--S 9 of 34
first k
--R
--R
--R (9)  4
--R
--R                                         Type: PositiveInteger
--E 9

--S 10 of 34
k.first
--R
--R
--R (10)  4
--R
--R                                         Type: PositiveInteger
--E 10

--S 11 of 34
k.1
--R
--R
--R (11)  4
--R
--R                                         Type: PositiveInteger
--E 11

--S 12 of 34
k(1)
--R
--R
--R (12)  4
--R
--R                                         Type: PositiveInteger
--E 12

--S 13 of 34
n := #k
--R
--R
--R (13)  8
--R
--R                                         Type: PositiveInteger
--E 13

--S 14 of 34
last k
--R
--R
--R (14)  2
--R
--R                                         Type: PositiveInteger
--E 14

--S 15 of 34

```

```

k.last
--R
--R
--R (15)  2
--R
--R                                          Type: PositiveInteger
--E 15

--S 16 of 34
k.(#k)
--R
--R
--R (16)  2
--R
--R                                          Type: PositiveInteger
--E 16

--S 17 of 34
k := [4,3,7,3,8,5,9,2]
--R
--R
--R (17)  [4,3,7,3,8,5,9,2]
--R
--R                                          Type: List PositiveInteger
--E 17

--S 18 of 34
k.1 := 999
--R
--R
--R (18)  999
--R
--R                                          Type: PositiveInteger
--E 18

--S 19 of 34
k
--R
--R
--R (19)  [999,3,7,3,8,5,9,2]
--R
--R                                          Type: List PositiveInteger
--E 19

--S 20 of 34
k := [1,2]
--R
--R
--R (20)  [1,2]
--R
--R                                          Type: List PositiveInteger
--E 20

--S 21 of 34
m := cons(0,k)
--R

```



```

--R
--R (21) [0,1,2]
--R
--R                                         Type: List Integer
--E 21

--S 22 of 34
m.2 := 99
--R
--R
--R (22) 99
--R
--R                                         Type: PositiveInteger
--E 22

--S 23 of 34
m
--R
--R
--R (23) [0,99,2]
--R
--R                                         Type: List Integer
--E 23

--S 24 of 34
k
--R
--R
--R (24) [99,2]
--R
--R                                         Type: List PositiveInteger
--E 24

--S 25 of 34
k := [1,2,3]
--R
--R
--R (25) [1,2,3]
--R
--R                                         Type: List PositiveInteger
--E 25

--S 26 of 34
rest k
--R
--R
--R (26) [2,3]
--R
--R                                         Type: List PositiveInteger
--E 26

--S 27 of 34
removeDuplicates [4,3,4,3,5,3,4]
--R
--R
--R (27) [4,3,5]

```



```

--S 34 of 34
expand [1..]
--R
--R
--R (34) [1,2,3,4,5,6,7,8,9,10,...]
--R
--R                                          Type: Stream Integer
--E 34
)spool
)lisp (bye)

```

---

— List.help —

```

=====
List examples
=====

```

A list is a finite collection of elements in a specified order that can contain duplicates. A list is a convenient structure to work with because it is easy to add or remove elements and the length need not be constant. There are many different kinds of lists in Axiom, but the default types (and those used most often) are created by the List constructor. For example, there are objects of type List Integer, List Float and List Polynomial Fraction Integer. Indeed, you can even have List List List Boolean (that is, lists of lists of lists of Boolean values). You can have lists of any type of Axiom object.

```

=====
Creating Lists
=====

```

The easiest way to create a list with, for example, the elements 2, 4, 5, 6 is to enclose the elements with square brackets and separate the elements with commas.

The spaces after the commas are optional, but they do improve the readability.

```

[2, 4, 5, 6]
[2,4,5,6]

```

Type: List PositiveInteger

To create a list with the single element 1, you can use either [1] or the operation list.

```

[1]
list 1

```

Type: List PositiveInteger

```
list(1)
[1]
```

Type: List PositiveInteger

Once created, two lists *k* and *m* can be concatenated by issuing `append(k,m)`. `append` does not physically join the lists, but rather produces a new list with the elements coming from the two arguments.

```
append([1,2,3],[5,6,7])
[1,2,3,5,6,7]
```

Type: List PositiveInteger

Use `cons` to append an element onto the front of a list.

```
cons(10,[9,8,7])
[10,9,8,7]
```

Type: List PositiveInteger

```
=====
Accessing List Elements
=====
```

To determine whether a list has any elements, use the operation `empty?`.

```
empty? [x+1]
false
```

Type: Boolean

Alternatively, equality with the list constant `nil` can be tested.

```
([] = nil)@Boolean
true
```

Type: Boolean

We'll use this in some of the following examples.

```
k := [4,3,7,3,8,5,9,2]
[4,3,7,3,8,5,9,2]
```

Type: List PositiveInteger

Each of the next four expressions extracts the first element of *k*.

```
first k
4
```

Type: PositiveInteger

```
k.first
4
```

Type: PositiveInteger

```
k.1
 4
```

Type: PositiveInteger

```
k(1)
 4
```

Type: PositiveInteger

The last two forms generalize to `k.i` and `k(i)`, respectively, where  $1 \leq i \leq n$  and  $n$  equals the length of `k`.

This length is calculated by `#`.

```
n := #k
 8
```

Type: PositiveInteger

Performing an operation such as `k.i` is sometimes referred to as indexing into `k` or elting into `k`. The latter phrase comes about because the name of the operation that extracts elements is called `elt`. That is, `k.3` is just alternative syntax for `elt(k,3)`. It is important to remember that list indices begin with 1. If we issue `k := [1,3,2,9,5]` then `k.4` returns 9. It is an error to use an index that is not in the range from 1 to the length of the list.

The last element of a list is extracted by any of the following three expressions.

```
last k
 2
```

Type: PositiveInteger

```
k.last
 2
```

Type: PositiveInteger

This form computes the index of the last element and then extracts the element from the list.

```
k.(#k)
 2
```

Type: PositiveInteger

#### =====

#### Changing List Elements

#### =====

We'll use this in some of the following examples.

```
k := [4,3,7,3,8,5,9,2]
      [4,3,7,3,8,5,9,2]
```

Type: List PositiveInteger

List elements are reset by using the `k.i` form on the left-hand side of an assignment. This expression resets the first element of `k` to 999.

```
k.1 := 999
      999
```

Type: PositiveInteger

As with indexing into a list, it is an error to use an index that is not within the proper bounds. Here you see that `k` was modified.

```
k
      [999,3,7,3,8,5,9,2]
```

Type: List PositiveInteger

The operation that performs the assignment of an element to a particular position in a list is called `setelt`. This operation is destructive in that it changes the list. In the above example, the assignment returned the value 999 and `k` was modified. For this reason, lists are called mutable objects: it is possible to change part of a list (mutate it) rather than always returning a new list reflecting the intended modifications.

Moreover, since lists can share structure, changes to one list can sometimes affect others.

```
k := [1,2]
      [1,2]
```

Type: List PositiveInteger

```
m := cons(0,k)
      [0,1,2]
```

Type: List Integer

Change the second element of `m`.

```
m.2 := 99
      99
```

Type: PositiveInteger

See, `m` was altered.

```
m
      [0,99,2]
```

Type: List Integer

But what about `k`? It changed too!

```
k
[99,2]
Type: List PositiveInteger
```

---

#### Other Functions

---

An operation that is used frequently in list processing is that which returns all elements in a list after the first element.

```
k := [1,2,3]
[1,2,3]
Type: List PositiveInteger
```

Use the `rest` operation to do this.

```
rest k
[2,3]
Type: List PositiveInteger
```

To remove duplicate elements in a list `k`, use `removeDuplicates`.

```
removeDuplicates [4,3,4,3,5,3,4]
[4,3,5]
Type: List PositiveInteger
```

To get a list with elements in the order opposite to those in a list `k`, use `reverse`.

```
reverse [1,2,3,4,5,6]
[6,5,4,3,2,1]
Type: List PositiveInteger
```

To test whether an element is in a list, use `member?`: `member?(a,k)` returns true or false depending on whether `a` is in `k` or not.

```
member?(1/2, [3/4,5/6,1/2])
true
Type: Boolean
```

```
member?(1/12, [3/4,5/6,1/2])
false
Type: Boolean
```

We can get a list containing all but the last of the elements in a given non-empty list `k`.

```
reverse(rest(reverse(k)))
[1,2]
```

Type: List PositiveInteger

```
=====  
Dot, Dot  
=====
```

Certain lists are used so often that Axiom provides an easy way of constructing them. If  $n$  and  $m$  are integers, then `expand [n..m]` creates a list containing  $n, n+1, \dots, m$ . If  $n > m$  then the list is empty. It is actually permissible to leave off the  $m$  in the dot-dot construction (see below).

The dot-dot notation can be used more than once in a list construction and with specific elements being given. Items separated by dots are called segments.

```
[1..3,10,20..23]
[1..3,10..10,20..23]
```

Type: List Segment PositiveInteger

Segments can be expanded into the range of items between the endpoints by using `expand`.

```
expand [1..3,10,20..23]
[1,2,3,10,20,21,22,23]
```

Type: List Integer

What happens if we leave off a number on the right-hand side of `..`?

```
expand [1..]
[1,2,3,4,5,6,7,8,9,10,...]
```

Type: Stream Integer

What is created in this case is a Stream which is a generalization of a list.

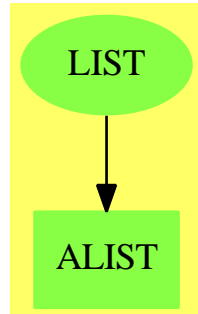
See Also:

- o )help Stream
- o )show List

---



### 13.9.1 List (LIST)



See

⇒ “IndexedList” (ILIST) 10.11.1 on page 1196

⇒ “AssociationList” (ALIST) 2.42.1 on page 218

#### Exports:

|                   |              |          |                   |                   |
|-------------------|--------------|----------|-------------------|-------------------|
| any?              | append       | child?   | children          | coerce            |
| concat            | concat!      | cons     | construct         | convert           |
| copy              | copyInto!    | count    | cycleEntry        | cycleLength       |
| cycleSplit!       | cycleTail    | cyclic?  | delete            | delete!           |
| distance          | elt          | empty    | empty?            | entries           |
| entry?            | eq?          | eval     | every?            | explicitlyFinite? |
| fill!             | find         | first    | hash              | index?            |
| indices           | insert       | insert!  | last              | latex             |
| leaf?             | leaves       | less?    | list              | map               |
| map!              | max          | maxIndex | member?           | members           |
| merge             | merge!       | min      | minIndex          | more?             |
| node?             | new          | nil      | nodes             | null              |
| OMwrite           | parts        | position | possiblyInfinite? | qelt              |
| qsetelt!          | reduce       | remove   | remove!           | removeDuplicates  |
| removeDuplicates! | rest         | reverse  | reverse!          | sample            |
| second            | select       | select!  | setDifference     | setIntersection   |
| setUnion          | setchildren! | setelt   | setfirst!         | setlast!          |
| setrest!          | setvalue!    | size?    | sort              | sort!             |
| sorted?           | split!       | swap!    | tail              | third             |
| value             | #?           | ?<?      | ?<=?              | ?=?               |
| ?>?               | ?>=?         | ?..?     | ?~=?              | ?..?              |
| ?..last           | ?..rest      | ?..first | ?..value          |                   |

— domain LIST List —

```

)abbrev domain LIST List
++ Author: Michael Monagan
++ Date Created: Sep 1987
++ Change History:
  
```

```

++ Related Constructors: ListFunctions2, ListFunctions3, ListToMap
++ Also See: IndexList, ListAggregate
++ AMS Classification:
++ Keywords: list, index, aggregate, lisp
++ Description:
++ \spadtype{List} implements singly-linked lists that are
++ addressable by indices; the index of the first element
++ is 1. In addition to the operations provided by
++ \spadtype{IndexedList}, this constructor provides some
++ LISP-like functions such as \spadfun{null} and \spadfun{cons}.

List(S:Type): Exports == Implementation where
LISTMININDEX ==> 1      -- this is the minimum list index

Exports ==> ListAggregate S with
nil      : ()      -> %
  ++ nil() returns the empty list.
null     : %      -> Boolean
  ++ null(u) tests if list \spad{u} is the
  ++ empty list.
cons     : (S, %) -> %
  ++ cons(element,u) appends \spad{element} onto the front
  ++ of list \spad{u} and returns the new list. This new list
  ++ and the old one will share some structure.
append   : (%, %) -> %
  ++ append(u1,u2) appends the elements of list \spad{u1}
  ++ onto the front of list \spad{u2}. This new list
  ++ and \spad{u2} will share some structure.
if S has SetCategory then
  setUnion : (%, %) -> %
    ++ setUnion(u1,u2) appends the two lists u1 and u2, then
    ++ removes all duplicates. The order of elements in the
    ++ resulting list is unspecified.
  setIntersection : (%, %) -> %
    ++ setIntersection(u1,u2) returns a list of the elements
    ++ that lists \spad{u1} and \spad{u2} have in common.
    ++ The order of elements in the resulting list is unspecified.
  setDifference : (%, %) -> %
    ++ setDifference(u1,u2) returns a list of the elements
    ++ of \spad{u1} that are not also in \spad{u2}.
    ++ The order of elements in the resulting list is unspecified.
if S has OpenMath then OpenMath

Implementation ==>
IndexedList(S, LISTMININDEX) add
  nil() == NIL$Lisp
  null l == NULL(l)$Lisp
  cons(s, l) == CONS(s, l)$Lisp
  append(l:%, t:%) == APPEND(l, t)$Lisp

```

```

if S has OpenMath then
  writeOMList(dev: OpenMathDevice, x: %): Void ==
    OmputApp(dev)
    OmputSymbol(dev, "list1", "list")
    -- The following didn't compile because the compiler isn't
    -- convinced that 'xval' is a S.  Duhhh! MCD.
    --for xval in x repeat
    --  OMwrite(dev, xval, false)
    while not null x repeat
      OMwrite(dev,first x,false)
      x := rest x
    OmputEndApp(dev)

  OMwrite(x: %): String ==
    s: String := ""
    sp := OM_-STRINGTOSTRINGPTR(s)$Lisp
    dev: OpenMathDevice := OMOpenString(sp pretend String, OMencodingXML)
    OmputObject(dev)
    writeOMList(dev, x)
    OmputEndObject(dev)
    OMclose(dev)
    s := OM_-STRINGPTRTOSTRING(sp)$Lisp pretend String
    s

  OMwrite(x: %, wholeObj: Boolean): String ==
    s: String := ""
    sp := OM_-STRINGTOSTRINGPTR(s)$Lisp
    dev: OpenMathDevice := OMOpenString(sp pretend String, OMencodingXML)
    if wholeObj then
      OmputObject(dev)
    writeOMList(dev, x)
    if wholeObj then
      OmputEndObject(dev)
    OMclose(dev)
    s := OM_-STRINGPTRTOSTRING(sp)$Lisp pretend String
    s

  OMwrite(dev: OpenMathDevice, x: %): Void ==
    OmputObject(dev)
    writeOMList(dev, x)
    OmputEndObject(dev)

  OMwrite(dev: OpenMathDevice, x: %, wholeObj: Boolean): Void ==
    if wholeObj then
      OmputObject(dev)
    writeOMList(dev, x)
    if wholeObj then
      OmputEndObject(dev)

if S has SetCategory then

```

```

setUnion(l1:%,l2:%)      == removeDuplicates concat(l1,l2)

setIntersection(l1:%,l2:%) ==
  u :% := empty()
  l1 := removeDuplicates l1
  while not empty? l1 repeat
    if member?(first l1,l2) then u := cons(first l1,u)
    l1 := rest l1
  u

setDifference(l1:%,l2:%) ==
  l1 := removeDuplicates l1
  lu:% := empty()
  while not empty? l1 repeat
    l11:=l1.1
    if not member?(l11,l2) then lu := concat(l11,lu)
    l1 := rest l1
  lu

if S has ConvertibleTo InputForm then
  convert(x:%):InputForm ==
    convert concat(convert("construct"::Symbol)@InputForm,
      [convert a for a in (x pretend List S)]$List(InputForm))

```

---

— LIST.dotabb —

```

"LIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=LIST",
  shape=ellipse]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"LIST" -> "ALIST"

```

---

## 13.10 domain LMOPS ListMonoidOps

— ListMonoidOps.input —

```

)set break resume
)sys rm -f ListMonoidOps.output
)spool ListMonoidOps.output
)set message test on
)set message auto off

```

```

)clear all

--S 1 of 1
)show ListMonoidOps
--R ListMonoidOps(S: SetCategory,E: AbelianMonoid,un: E) is a domain constructor
--R Abbreviation for ListMonoidOps is LMOPS
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for LMOPS
--R
--R----- Operations -----
--R ?=? : (%,% ) -> Boolean               coerce : S -> %
--R coerce : % -> OutputForm              hash : % -> SingleInteger
--R latex : % -> String                   leftMult : (S,%) -> %
--R makeTerm : (S,E) -> %                 makeUnit : () -> %
--R mapExpon : ((E -> E),%) -> %          mapGen : ((S -> S),%) -> %
--R nthExpon : (%,Integer) -> E           nthFactor : (%,Integer) -> S
--R plus : (%,% ) -> %                   plus : (S,E,%) -> %
--R retract : % -> S                     reverse : % -> %
--R reverse! : % -> %                    rightMult : (% ,S) -> %
--R size : % -> NonNegativeInteger        ?~=? : (%,% ) -> Boolean
--R commutativeEquality : (%,% ) -> Boolean
--R listOfMonoms : % -> List Record(gen: S,exp: E)
--R makeMulti : List Record(gen: S,exp: E) -> %
--R outputForm : (% ,((OutputForm,OutputForm) -> OutputForm),((OutputForm,OutputForm) -> OutputForm)) -> OutputForm
--R retractIfCan : % -> Union(S,"failed")
--R
--E 1

)spool
)lisp (bye)

```

---

— ListMonoidOps.help —

```

=====
ListMonoidOps examples
=====

```

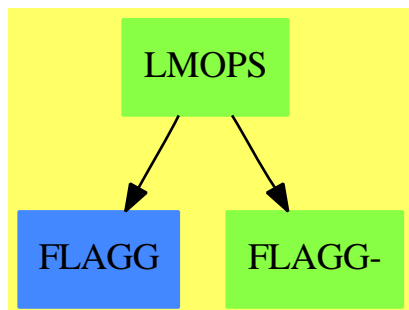
```

See Also:
o )show ListMonoidOps

```

---

## 13.10.1 ListMonoidOps (LMOPS)



See

- ⇒ “FreeMonoid” (FMONOID) 7.32.1 on page 987
- ⇒ “FreeGroup” (FGROUP) 7.29.1 on page 976
- ⇒ “InnerFreeAbelianMonoid” (IFAMON) 10.22.1 on page 1250
- ⇒ “FreeAbelianMonoid” (FAMONOID) 7.28.1 on page 974
- ⇒ “FreeAbelianGroup” (FAGROUP) 7.27.1 on page 971

**Exports:**

|              |                     |           |            |           |
|--------------|---------------------|-----------|------------|-----------|
| coerce       | commutativeEquality | hash      | latex      | leftMult  |
| listOfMonoms | makeTerm            | makeUnit  | mapExpon   | mapGen    |
| makeMulti    | nthExpon            | nthFactor | outputForm | plus      |
| retract      | retractIfCan        | reverse   | reverse!   | rightMult |
| size         | ?=?                 | ?~=?      |            |           |

— domain LMOPS ListMonoidOps —

```

)abbrev domain LMOPS ListMonoidOps
++ Author: Manuel Bronstein
++ Date Created: November 1989
++ Date Last Updated: 6 June 1991
++ Description:
++ This internal package represents monoid (abelian or not, with or
++ without inverses) as lists and provides some common operations
++ to the various flavors of monoids.

```

```
ListMonoidOps(S, E, un): Exports == Implementation where
```

```

S : SetCategory
E : AbelianMonoid
un: E

```

```

REC ==> Record(gen:S, exp: E)
0    ==> OutputForm

```

```

Exports ==> Join(SetCategory, RetractableTo S) with
  outputForm : ($, (0, 0) -> 0, (0, 0) -> 0, Integer) -> 0

```

```

++ outputForm(l, fop, fexp, unit) converts the monoid element
++ represented by l to an \spadtype{OutputForm}.
++ Argument unit is the output form
++ for the \spadignore{unit} of the monoid (e.g. 0 or 1),
++ \spad{fop(a, b)} is the
++ output form for the monoid operation applied to \spad{a} and b
++ (e.g. \spad{a + b}, \spad{a * b}, \spad{ab}),
++ and \spad{fexp(a, n)} is the output form
++ for the exponentiation operation applied to \spad{a} and n
++ (e.g. \spad{n a}, \spad{n * a}, \spad{a ** n}, \spad{a\^n}).
listOfMonoms : $ -> List REC
++ listOfMonoms(l) returns the list of the monomials forming l.
makeTerm      : (S, E) -> $
++ makeTerm(s, e) returns the monomial s exponentiated by e
++ (e.g. s^e or e * s).
makeMulti     : List REC -> $
++ makeMulti(l) returns the element whose list of monomials is l.
nthExpon      : ($, Integer) -> E
++ nthExpon(l, n) returns the exponent of the n^th monomial of l.
nthFactor     : ($, Integer) -> S
++ nthFactor(l, n) returns the factor of the n^th monomial of l.
reverse       : $ -> $
++ reverse(l) reverses the list of monomials forming l. This
++ has some effect if the monoid is non-abelian, i.e.
++ \spad{reverse(a1\^e1 ... an\^en) = an\^en ... a1\^e1} which is different.
reverse!      : $ -> $
++ reverse!(l) reverses the list of monomials forming l, destroying
++ the element l.
size          : $ -> NonNegativeInteger
++ size(l) returns the number of monomials forming l.
makeUnit      : () -> $
++ makeUnit() returns the unit element of the monomial.
rightMult     : ($, S) -> $
++ rightMult(a, s) returns \spad{a * s} where \spad{*}
++ is the monoid operation,
++ which is assumed non-commutative.
leftMult      : (S, $) -> $
++ leftMult(s, a) returns \spad{s * a} where
++ \spad{*} is the monoid operation,
++ which is assumed non-commutative.
plus          : (S, E, $) -> $
++ plus(s, e, x) returns \spad{e * s + x} where \spad{+}
++ is the monoid operation,
++ which is assumed commutative.
plus          : ($, $) -> $
++ plus(x, y) returns \spad{x + y} where \spad{+}
++ is the monoid operation,
++ which is assumed commutative.
commutativeEquality: ($, $) -> Boolean
++ commutativeEquality(x,y) returns true if x and y are equal

```

```

    ++ assuming commutativity
mapExpon      : (E -> E, $) -> $
    ++ mapExpon(f, a1\^e1 ... an\^en) returns \spad{a1\^f(e1) ... an\^f(en)}.
mapGen        : (S -> S, $) -> $
    ++ mapGen(f, a1\^e1 ... an\^en) returns \spad{f(a1)\^e1 ... f(an)\^en}.

Implementation ==> add
Rep := List REC

localplus: ($, $) -> $

makeUnit()      == empty()$Rep
size l           == # listOfMonoms l
coerce(s:$):$    == [[s, un]]
coerce(l:$):0    == coerce(l)$Rep
makeTerm(s, e)   == (zero? e => makeUnit(); [[s, e]])
makeMulti l      == l
f = g            == f = $Rep g
listOfMonoms l   == l pretend List(REC)
nthExpon(f, i)   == f.(i-1+minIndex f).exp
nthFactor(f, i)  == f.(i-1+minIndex f).gen
reverse l        == reverse(l)$Rep
reverse_! l      == reverse_!(l)$Rep
mapGen(f, l)     == [[f(x.gen), x.exp] for x in l]

mapExpon(f, l) ==
  ans:List(REC) := empty()
  for x in l repeat
    if (a := f(x.exp)) ^= 0 then ans := concat([x.gen, a], ans)
  reverse_! ans

outputForm(l, op, opexp, id) ==
  empty? l => id::OutputForm
  l:List(0) :=
    [(p.exp = un => p.gen::0; opexp(p.gen::0, p.exp::0)) for p in l]
  reduce(op, l)

retractIfCan(l:$):Union(S, "failed") ==
  not empty? l and empty? rest l and l.first.exp = un => l.first.gen
  "failed"

rightMult(f, s) ==
  empty? f => s::$
  s = f.last.gen => (setlast_!(h := copy f, [s, f.last.exp + un]); h)
  concat(f, [s, un])

leftMult(s, f) ==
  empty? f => s::$
  s = f.first.gen => concat([s, f.first.exp + un], rest f)
  concat([s, un], f)

```



```

commutativeEquality(s1:$, s2:$):Boolean ==
  #s1 ^= #s2 => false
  for t1 in s1 repeat
    if not member?(t1,s2) then return false
  true

plus_!(s:S, n:E, f:$):$ ==
  h := g := concat([s, n], f)
  h1 := rest h
  while not empty? h1 repeat
    s = h1.first.gen =>
      l :=
        zero?(m := n + h1.first.exp) => rest h1
        concat([s, m], rest h1)
      setrest_!(h, l)
      return rest g
    h := h1
    h1 := rest h1
  g

plus(s, n, f) == plus_!(s,n,copy f)

plus(f, g) ==
  #f < #g => localplus(f, g)
  localplus(g, f)

localplus(f, g) ==
  g := copy g
  for x in f repeat
    g := plus(x.gen, x.exp, g)
  g

```

---

— LMOPS.dotabb —

```

"LMOPS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=LMOPS"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"FLAGG-" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FLAGG"]
"LMOPS" -> "FLAGG"
"LMOPS" -> "FLAGG-"

```

---

## 13.11 domain LMDICT ListMultiDictionary

— ListMultiDictionary.input —

```
)set break resume
)sys rm -f ListMultiDictionary.output
)spool ListMultiDictionary.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ListMultiDictionary
--R ListMultiDictionary S: SetCategory is a domain constructor
--R Abbreviation for ListMultiDictionary is LMDICT
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for LMDICT
--R
--R----- Operations -----
--R bag : List S -> %                construct : List S -> %
--R copy : % -> %                    dictionary : List S -> %
--R dictionary : () -> %             duplicates? : % -> Boolean
--R empty : () -> %                 empty? : % -> Boolean
--R eq? : (%,% ) -> Boolean          extract! : % -> S
--R insert! : (S,% ) -> %            inspect : % -> S
--R map : ((S -> S),%) -> %          removeDuplicates! : % -> %
--R sample : () -> %                substitute : (S,S,% ) -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?? : (%,% ) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R convert : % -> InputForm if S has KONVERT INFORM
--R count : (S,% ) -> NonNegativeInteger if $ has finiteAggregate and S has SETCAT
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R duplicates : % -> List Record(entry: S,count: NonNegativeInteger)
--R eval : (% ,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (% ,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (% ,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (% ,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R find : ((S -> Boolean),%) -> Union(S,"failed")
--R hash : % -> SingleInteger if S has SETCAT
--R insert! : (S,% ,NonNegativeInteger) -> %
--R latex : % -> String if S has SETCAT
--R less? : (% ,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,% ) -> Boolean if $ has finiteAggregate and S has SETCAT
--R members : % -> List S if $ has finiteAggregate
```

```

--R more? : (% , NonNegativeInteger) -> Boolean
--R parts : % -> List S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S,S) -> S if $ has finiteAggregate and S has SETCAT
--R remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (S,%) -> % if $ has finiteAggregate and S has SETCAT
--R remove! : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove! : (S,%) -> % if $ has finiteAggregate
--R removeDuplicates : % -> % if $ has finiteAggregate and S has SETCAT
--R select : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R select! : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R size? : (% , NonNegativeInteger) -> Boolean
--R ?~=? : (% ,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)

```

---

— ListMultiDictionary.help —

```

=====
ListMultiDictionary examples
=====

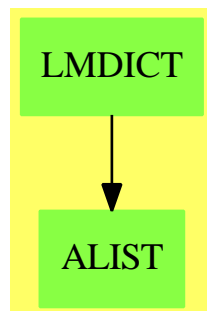
```

```

See Also:
o )show ListMultiDictionary

```

### 13.11.1 ListMultiDictionary (LMDICT)



**Exports:**

|             |         |                  |                   |            |
|-------------|---------|------------------|-------------------|------------|
| any?        | bag     | coerce           | construct         | convert    |
| copy        | count   | dictionary       | dictionary        | duplicates |
| duplicates? | empty   | empty?           | eq?               | eval       |
| extract!    | every?  | find             | hash              | insert!    |
| inspect     | latex   | less?            | map               | map!       |
| member?     | members | more?            | parts             | reduce     |
| remove      | remove! | removeDuplicates | removeDuplicates! | sample     |
| select      | select! | size?            | substitute        | #?         |
| ?~=?        | ?=?     |                  |                   |            |

— domain LMDICT ListMultiDictionary —

```

)abbrev domain LMDICT ListMultiDictionary
++ Author: Mark Botch
++ Date Created:
++ Date Last Updated: 13 June 1994 Frederic Lehobey
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The \spadtype{ListMultiDictionary} domain implements a
++ dictionary with duplicates
++ allowed. The representation is a list with duplicates represented
++ explicitly. Hence most operations will be relatively inefficient
++ when the number of entries in the dictionary becomes large.
++ If the objects in the dictionary belong to an ordered set,
++ the entries are maintained in ascending order.

ListMultiDictionary(S:SetCategory): EE == II where

NNI ==> NonNegativeInteger
D ==> Record(entry:S, count:NonNegativeInteger)

EE ==> MultiDictionary(S) with
  finiteAggregate
  duplicates?: % -> Boolean
  ++ duplicates?(d) tests if dictionary d has duplicate entries.
  substitute : (S, S, %) -> %
  ++ substitute(x,y,d) replace x's with y's in dictionary d.
II ==> add
  Rep := Reference List S

  sub: (S, S, S) -> S

  coerce(s:%):OutputForm ==

```

```

    prefix("dictionary"::OutputForm, [x::OutputForm for x in parts s])

#s                == # parts s
copy s            == dictionary copy parts s
empty? s          == empty? parts s
bag l             == dictionary l
dictionary()      == dictionary empty()

empty():% == ref empty()

dictionary(ls:List S):% ==
  empty? ls => empty()
  lmd := empty()
  for x in ls repeat insert_!(x,lmd)
  lmd

if S has ConvertibleTo InputForm then
  convert(lmd:%):InputForm ==
    convert [convert("dictionary"::Symbol)@InputForm,
             convert(parts lmd)@InputForm]

map(f, s)         == dictionary map(f, parts s)
map_!(f, s)       == dictionary map_!(f, parts s)
parts s           == deref s
sub(x, y, z)      == (z = x => y; z)
insert_!(x, s, n) == (for i in 1..n repeat insert_!(x, s); s)
substitute(x, y, s) == dictionary map(z1 +-> sub(x, y, z1), parts s)
removeDuplicates_! s == dictionary removeDuplicates_! parts s

inspect s ==
  empty? s => error "empty dictionary"
  first parts s

extract_! s ==
  empty? s => error "empty dictionary"
  x := first(p := parts s)
  setref(s, rest p)
  x

duplicates? s ==
  empty?(p := parts s) => false
  q := rest p
  while not empty? q repeat
    first p = first q => return true
    p := q
    q := rest q
  false

remove_!(p: S->Boolean, lmd:%):% ==
  for x in removeDuplicates parts lmd | p(x) repeat remove_!(x,lmd)

```

```

lmd

select_!(p: S->Boolean, lmd:%):% == remove_!((z:S):Boolean+>not p(z), lmd)

duplicates(lmd:%):List D ==
  ld: List D := empty()
  for x in removeDuplicates parts lmd | (n := count(x, lmd)) >
    1$NonNegativeInteger repeat
      ld := cons([x, n], ld)
  ld

if S has OrderedSet then
  s = t == parts s = parts t

remove_!(x:S, s:%) ==
  p := deref s
  while not empty? p and x = first p repeat p := rest p
  setref(s, p)
  empty? p => s
  q := rest p
  while not empty? q and x > first q repeat (p := q; q := rest q)
  while not empty? q and x = first q repeat q := rest q
  p.rest := q
  s

insert_!(x, s) ==
  p := deref s
  empty? p or x < first p =>
    setref(s, concat(x, p))
  s
  q := rest p
  while not empty? q and x > first q repeat (p := q; q := rest q)
  p.rest := concat(x, q)
  s

else
  remove_!(x:S, s:%) == (setref(s, remove_!(x, parts s)); s)

s = t ==
  a := copy s
  while not empty? a repeat
    x := inspect a
    count(x, s) ^!= count(x, t) => return false
    remove_!(x, a)
  true

insert_!(x, s) ==
  p := deref s
  while not empty? p repeat
    x = first p =>

```

```

      p.rest := concat(x, rest p)
      s
    p := rest p
    setref(s, concat(x, deref s))
  s

```

---

— LMDICT.dotabb —

```

"LMDICT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=LMDICT"]
"ALIST"  [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"LMDICT" -> "ALIST"

```

---

## 13.12 domain LA LocalAlgebra

— LocalAlgebra.input —

```

)set break resume
)sys rm -f LocalAlgebra.output
)spool LocalAlgebra.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show LocalAlgebra
--R LocalAlgebra(A: Algebra R,R: CommutativeRing,S: SubsetCategory(Monoid,R)) is a domain c
--R Abbreviation for LocalAlgebra is LA
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for LA
--R
--R----- Operations -----
--R ?? : (R,%) -> %          ?? : (% ,R) -> %
--R ?? : (% ,%) -> %        ?? : (Integer,%) -> %
--R ?? : (PositiveInteger,%) -> %   ??? : (% ,PositiveInteger) -> %
--R +? : (% ,%) -> %        ?-? : (% ,%) -> %
--R -? : % -> %             ?/? : (A,S) -> %
--R ?/? : (% ,S) -> %       ?? : (% ,%) -> Boolean
--R 1 : () -> %             0 : () -> %
--R ?? : (% ,PositiveInteger) -> %   coerce : R -> %
--R coerce : Integer -> %           coerce : % -> OutputForm

```

```

--R denom : % -> S
--R latex : % -> String
--R one? : % -> Boolean
--R sample : () -> %
--R ?~=? : (%,% ) -> Boolean
--R ?*? : (NonNegativeInteger,% ) -> %
--R ?**? : (% ,NonNegativeInteger) -> %
--R ?<? : (%,% ) -> Boolean if A has ORDRING
--R ?<=? : (%,% ) -> Boolean if A has ORDRING
--R ?>? : (%,% ) -> Boolean if A has ORDRING
--R ?>=? : (%,% ) -> Boolean if A has ORDRING
--R ?^? : (% ,NonNegativeInteger) -> %
--R abs : % -> % if A has ORDRING
--R characteristic : () -> NonNegativeInteger
--R max : (%,% ) -> % if A has ORDRING
--R min : (%,% ) -> % if A has ORDRING
--R negative? : % -> Boolean if A has ORDRING
--R positive? : % -> Boolean if A has ORDRING
--R sign : % -> Integer if A has ORDRING
--R subtractIfCan : (%,% ) -> Union(% ,"failed")
--R
--E 1

)spool
)lisp (bye)

```

---

— LocalAlgebra.help —

```

=====
LocalAlgebra examples
=====

```

```

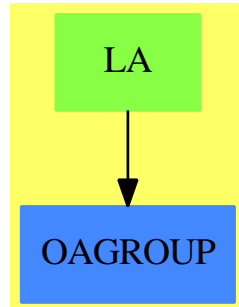
See Also:
o )show LocalAlgebra

```

---



### 13.12.1 LocalAlgebra (LA)



See

⇒ “Localize” (LO) 13.13.1 on page 1486

⇒ “Fraction” (FRAC) 7.24.1 on page 952

#### Exports:

|           |       |               |                |        |
|-----------|-------|---------------|----------------|--------|
| 0         | 1     | abs           | characteristic | coerce |
| denom     | hash  | latex         | max            | min    |
| negative? | numer | one?          | positive?      | recip  |
| sample    | sign  | subtractIfCan | zero?          | ?~=?   |
| ?*?       | ?**?  | ?<?           | ?<=?           | ?>?    |
| ?>=?      | ?^?   | ?+?           | ?-?            | -?     |
| ?/?       | ?=?   |               |                |        |

— domain LA LocalAlgebra —

```

)abbrev domain LA LocalAlgebra
++ Author: Dave Barton, Barry Trager
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ LocalAlgebra produces the localization of an algebra, i.e.
++ fractions whose numerators come from some R algebra.

```

```

LocalAlgebra(A: Algebra R,
             R: CommutativeRing,
             S: SubsetCategory(Monoid, R)): Algebra R with
if A has OrderedRing then OrderedRing
_/ : (%,S) -> %
++ x / d divides the element x by d.
_/ : (A,S) -> %

```

```

    ++ a / d divides the element \spad{a} by d.
numer: % -> A
    ++ numer x returns the numerator of x.
denom: % -> S
    ++ denom x returns the denominator of x.
== Localize(A, R, S) add
1 == 1$A / 1$S
x:% * y:% == (numer(x) * numer(y)) / (denom(x) * denom(y))
characteristic() == characteristic()$A

```

---

— LA.dotabb —

```

"LA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=LA"]
"OAGROUP" [color="#4488FF",href="bookvol10.2.pdf#nameddest=OAGROUP"]
"LA" -> "OAGROUP"

```

---

## 13.13 domain LO Localize

— Localize.input —

```

)set break resume
)sys rm -f Localize.output
)spool Localize.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Localize
--R Localize(M: Module R,R: CommutativeRing,S: SubsetCategory(Monoid,R)) is a domain constructor
--R Abbreviation for Localize is LO
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for LO
--R
--R----- Operations -----
--R ??? : (%,R) -> %          ??? : (R,%) -> %
--R ??? : (Integer,%) -> %    ??? : (PositiveInteger,%) -> %
--R ??? : (%,%) -> %         ?-? : (%,%) -> %
--R -? : % -> %              ?/? : (M,S) -> %
--R ?/? : (%,S) -> %         ?=? : (%,%) -> Boolean

```

```

--R 0 : () -> %
--R denom : % -> S
--R latex : % -> String
--R sample : () -> %
--R ?~=? : (%,% ) -> Boolean
--R ?*? : (NonNegativeInteger,% ) -> %
--R ?<? : (%,% ) -> Boolean if M has OAGROUP
--R ?<=? : (%,% ) -> Boolean if M has OAGROUP
--R ?>? : (%,% ) -> Boolean if M has OAGROUP
--R ?>=? : (%,% ) -> Boolean if M has OAGROUP
--R max : (%,% ) -> % if M has OAGROUP
--R min : (%,% ) -> % if M has OAGROUP
--R subtractIfCan : (%,% ) -> Union(%, "failed")
--R
--E 1

)spool
)lisp (bye)

```

---

— Localize.help —

```

=====
Localize examples
=====

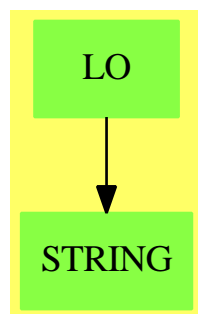
```

```

See Also:
o )show Localize

```

### 13.13.1 Localize (LO)



See

⇒ “LocalAlgebra” (LA) 13.12.1 on page 1484

⇒ “Fraction” (FRAC) 7.24.1 on page 952

### Exports:

|       |        |       |        |               |
|-------|--------|-------|--------|---------------|
| 0     | coerce | denom | hash   | latex         |
| max   | min    | numer | sample | subtractIfCan |
| zero? | ?~=?   | ?*?   | ?<?    | ?<=?          |
| ?>?   | ?>=?   | ?+?   | ?-?    | -?            |
| ?/?   | ?=?    |       |        |               |

— domain LO Localize —

```
)abbrev domain LO Localize
++ Author: Dave Barton, Barry Trager
++ Date Created:
++ Date Last Updated:
++ Basic Functions: + - / numer denom
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords: localization
++ References:
++ Description:
++ Localize(M,R,S) produces fractions with numerators
++ from an R module M and denominators from some multiplicative subset D of R.
```

```
Localize(M:Module R,
         R:CommutativeRing,
         S:SubsetCategory(Monoid, R)): Module R with
if M has OrderedAbelianGroup then OrderedAbelianGroup
_ / : (% , S) -> %
++ x / d divides the element x by d.
_ / : (M, S) -> %
++ m / d divides the element m by d.
numer: % -> M
++ numer x returns the numerator of x.
denom: % -> S
++ denom x returns the denominator of x.
==
add
--representation
Rep:= Record(num:M,den:S)
--declarations
x,y: %
n: Integer
m: M
r: R
d: S
--definitions
0 == [0,1]
```

```

zero? x == zero? (x.num)
-x== [-x.num,x.den]
x=y == y.den*x.num = x.den*y.num
numer x == x.num
denom x == x.den
if M has OrderedAbelianGroup then
  x < y ==
--      if y.den::R < 0 then (x,y):=(y,x)
--      if x.den::R < 0 then (x,y):=(y,x)
      y.den*x.num < x.den*y.num
x+y == [y.den*x.num+x.den*y.num, x.den*y.den]
n*x == [n*x.num,x.den]
r*x == if r=x.den then [x.num,1] else [r*x.num,x.den]
x/d ==
      zero?(u:S:=d*x.den) => error "division by zero"
      [x.num,u]
m/d == if zero? d then error "division by zero" else [m,d]
coerce(x:%):OutputForm ==
--      one?(xd:=x.den) => (x.num)::OutputForm
      ((xd:=x.den) = 1) => (x.num)::OutputForm
      (x.num)::OutputForm / (xd::OutputForm)
latex(x:%): String ==
--      one?(xd:=x.den) => latex(x.num)
      ((xd:=x.den) = 1) => latex(x.num)
      nl : String := concat("{", concat(latex(x.num), "}")$String)$String
      dl : String := concat("{", concat(latex(x.den), "}")$String)$String
      concat("{ ", concat(nl, concat(" \over ", concat(dl, " }"))$String)$String)$String)$String

```

---

— LO.dotabb —

```

"LO" [color="#88FF44",href="bookvol10.3.pdf#nameddest=LO"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"LO" -> "STRING"

```

---

## 13.14 domain LWORD LyndonWord

— LyndonWord.input —

```

)set break resume
)sys rm -f LyndonWord.output

```



--E 6

--S 7 of 22

LyndonWordsList1([a,b,c],3)\$lword

--R

--R

--R (7)

--R [[a],[b],[c]], [[a b],[a c],[b c]],

--R  $\begin{matrix} 2 & 2 & 2 & & 2 & 2 & 2 \\ & & & & & & \end{matrix}$

--R [[a b],[a c],[a b ],[a b c],[a c b],[a c ],[b c],[b c ]]

--R Type: OneDimensionalArray List LyndonWord Symbol

--E 7

--S 8 of 22

LyndonWordsList([a,b,c],3)\$lword

--R

--R

--R (8)

--R  $\begin{matrix} & & & & 2 & 2 & 2 \\ & & & & & & \end{matrix}$

--R [[a], [b], [c], [a b], [a c], [b c], [a b ], [a c ], [a b ], [a b c], [a c b],

--R  $\begin{matrix} 2 & 2 & 2 \\ & & \end{matrix}$

--R [a c ], [b c ], [b c ]]

--R

Type: List LyndonWord Symbol

--E 8

--S 9 of 22

lw := LyndonWordsList([a,b],5)\$lword

--R

--R

--R (9)

--R  $\begin{matrix} & & 2 & 2 & 3 & 2 & 2 & 3 & 4 & 3 & 2 \\ & & & & & & & & & & \end{matrix}$

--R [[a], [b], [a b], [a b ], [a b ], [a b ], [a b ], [a b ], [a b ], [a b ], [a b ],

--R  $\begin{matrix} 2 & 2 & 3 & & 2 & 4 \\ & & & & & \end{matrix}$

--R [a b a b ], [a b ], [a b a b ], [a b ]]

--R

Type: List LyndonWord Symbol

--E 9

--S 10 of 22

w1 : word := lw.4 :: word

--R

--R

--R  $\begin{matrix} 2 \\ & \end{matrix}$

--R (10) a b

--R

Type: OrderedFreeMonoid Symbol

--E 10

--S 11 of 22

w2 : word := lw.5 :: word

--R

--R

```

--R      2
--R (11) a b
--R
--R                                         Type: OrderedFreeMonoid Symbol
--E 11

--S 12 of 22
factor(a::word)$lword
--R
--R
--R (12) [[a]]
--R
--R                                         Type: List LyndonWord Symbol
--E 12

--S 13 of 22
factor(w1*w2)$lword
--R
--R
--R      2      2
--R (13) [[a b a b]]
--R
--R                                         Type: List LyndonWord Symbol
--E 13

--S 14 of 22
factor(w2*w2)$lword
--R
--R
--R      2      2
--R (14) [[a b ],[a b ]]
--R
--R                                         Type: List LyndonWord Symbol
--E 14

--S 15 of 22
factor(w2*w1)$lword
--R
--R
--R      2      2
--R (15) [[a b ],[a b]]
--R
--R                                         Type: List LyndonWord Symbol
--E 15

--S 16 of 22
lyndon?(w1)$lword
--R
--R
--R (16) true
--R
--R                                         Type: Boolean
--E 16

--S 17 of 22
lyndon?(w1*w2)$lword

```



```

--R
--R
--R   (17)  true
--R
--R                                          Type: Boolean
--E 17

--S 18 of 22
lyndon?(w2*w1)$lword
--R
--R
--R   (18)  false
--R
--R                                          Type: Boolean
--E 18

--S 19 of 22
lyndonIfCan(w1)$lword
--R
--R
--R           2
--R   (19)  [a b]
--R
--R                                          Type: Union(LyndonWord Symbol,...)
--E 19

--S 20 of 22
lyndonIfCan(w2*w1)$lword
--R
--R
--R   (20)  "failed"
--R
--R                                          Type: Union("failed",...)
--E 20

--S 21 of 22
lyndon(w1)$lword
--R
--R
--R           2
--R   (21)  [a b]
--R
--R                                          Type: LyndonWord Symbol
--E 21

--S 22 of 22
lyndon(w1*w2)$lword
--R
--R
--R           2      2
--R   (22)  [a b a b ]
--R
--R                                          Type: LyndonWord Symbol
--E 22
)spool
)lisp (bye)

```

---

— LyndonWord.help —

=====

LyndonWord examples

=====

A function  $f$  in  $[0,1]$  is called acyclic if  $C(f)$  consists of  $n$  different objects. The canonical representative of the orbit of an acyclic function is usually called a Lyndon Word. If  $f$  is acyclic, then all elements in the orbit  $C(f)$  are acyclic as well, and we call  $C(f)$  an acyclic orbit.

=====

Initialisations

=====

```
a:Symbol := 'a
a
                                Type: Symbol
```

```
b:Symbol := 'b
b
                                Type: Symbol
```

```
c:Symbol := 'c
c
                                Type: Symbol
```

```
lword:= LyndonWord(Symbol)
LyndonWord Symbol
                                Type: Domain
```

```
magma := Magma(Symbol)
Magma Symbol
                                Type: Domain
```

```
word := OrderedFreeMonoid(Symbol)
OrderedFreeMonoid Symbol
                                Type: Domain
```

All Lyndon words of with a, b, c to order 3

```
LyndonWordsList1([a,b,c],3)$lword
[[[a],[b],[c]], [[a b],[a c],[b c]],
  2      2      2      2      2      2
 [[a b],[a c],[a b ],[a b c],[a c b],[a c ],[b c],[b c ]]]
                                Type: OneDimensionalArray List LyndonWord Symbol
```

All Lyndon words of with a, b, c to order 3 in flat list

```
LyndonWordsList([a,b,c],3)$lword
      2      2      2
[[a], [b], [c], [a b], [a c], [b c], [a b], [a c], [a b ], [a b c], [a c b],
      2      2      2
[a c ], [b c], [b c ]]
```

Type: List LyndonWord Symbol

All Lyndon words of with a, b to order 5

```
lw := LyndonWordsList([a,b],5)$lword
      2      2      3      2 2      3      4      3 2
[[a], [b], [a b], [a b], [a b ], [a b], [a b ], [a b ], [a b], [a b ],
      2      2 3      2      4
[a b a b], [a b ], [a b a b ], [a b ]]
```

Type: List LyndonWord Symbol

```
w1 : word := lw.4 :: word
      2
a b
```

Type: OrderedFreeMonoid Symbol

```
w2 : word := lw.5 :: word
      2
a b
```

Type: OrderedFreeMonoid Symbol

Let's try factoring

```
factor(a::word)$lword
[[a]]
```

Type: List LyndonWord Symbol

```
factor(w1*w2)$lword
      2      2
[[a b a b ]]
```

Type: List LyndonWord Symbol

```
factor(w2*w2)$lword
      2      2
[[a b ],[a b ]]
```

Type: List LyndonWord Symbol

```
factor(w2*w1)$lword
      2      2
[[a b ],[a b ]]
```

Type: List LyndonWord Symbol

```
=====
Checks and coercions
=====
```

```

lyndon?(w1)$lword
  true
                                Type: Boolean

lyndon?(w1*w2)$lword
  true
                                Type: Boolean

lyndon?(w2*w1)$lword
  false
                                Type: Boolean

lyndonIfCan(w1)$lword
  2
  [a b]
                                Type: Union(LyndonWord Symbol,...)

lyndonIfCan(w2*w1)$lword
  "failed"
                                Type: Union("failed",...)

lyndon(w1)$lword
  2
  [a b]
                                Type: LyndonWord Symbol

lyndon(w1*w2)$lword
  2      2
  [a b a b ]
                                Type: LyndonWord Symbol

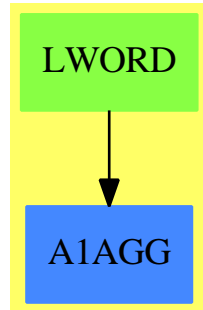
```

See Also:

o )show LyndonWord

---

### 13.14.1 LyndonWord (LWORD)



#### Exports:

|                 |                  |        |         |             |
|-----------------|------------------|--------|---------|-------------|
| coerce          | factor           | hash   | latex   | left        |
| length          | lexico           | lyndon | lyndon? | lyndonIfCan |
| LyndonWordsList | LyndonWordsList1 | max    | min     | retract     |
| retractIfCan    | retractable?     | right  | varList | ?<?         |
| ?<=?            | ?=?              | ?>?    | ?>=?    | ?~=?        |

— domain LWORD LyndonWord —

```

)abbrev domain LWORD LyndonWord
++ Author: Michel Petitot (petitot@lifl.fr).
++ Date Created: 91
++ Date Last Updated: 7 Juillet 92
++ Fix History: compilation v 2.1 le 13 dec 98
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Free Lie Algebras by C. Reutenauer (Oxford science publications).
++ Description:
++ Lyndon words over arbitrary (ordered) symbols:
++ see Free Lie Algebras by C. Reutenauer (Oxford science publications).
++ A Lyndon word is a word which is smaller than any of its right factors
++ w.r.t. the pure lexicographical ordering.
++ If \axiom{a} and \axiom{b} are two Lyndon words such that \axiom{a < b}
++ holds w.r.t lexicographical ordering then \axiom{a*b} is a Lyndon word.
++ Parenthesized Lyndon words can be generated from symbols by using the
++ following rule:\br
++ \axiom{[[a,b],c]} is a Lyndon word iff \axiom{a*b < c <= b} holds.\br
++ Lyndon words are internally represented by binary trees using the
++ \spadtype{Magma} domain constructor.
++ Two ordering are provided: lexicographic and
++ length-lexicographic.
  
```

```

LyndonWord(VarSet:OrderedSet):Public == Private where
  OFMON ==> OrderedFreeMonoid(VarSet)
  PI     ==> PositiveInteger
  NNI    ==> NonNegativeInteger
  I      ==> Integer
  OF     ==> OutputForm
  ARRAY1==> OneDimensionalArray

Public == Join(OrderedSet,RetractableTo VarSet) with
  retractable? : $ -> Boolean
    ++ \axiom{retractable?(x)} tests if \axiom{x} is a tree
    ++ with only one entry.
  left         : $ -> $
    ++ \axiom{left(x)} returns left subtree of \axiom{x} or
    ++ error if retractable?(x) is true.
  right        : $ -> $
    ++ \axiom{right(x)} returns right subtree of \axiom{x} or
    ++ error if retractable?(x) is true.
  length       : $ -> PI
    ++ \axiom{length(x)} returns the number of entries in \axiom{x}.
  lexico       : ($,$) -> Boolean
    ++ \axiom{lexico(x,y)} returns \axiom{true} iff \axiom{x} is smaller than
    ++ \axiom{y} w.r.t. the lexicographical ordering induced by \axiom{VarSet}.
  coerce       : $ -> OFMON
    ++ \axiom{coerce(x)} returns the element of \axiomType{OrderedFreeMonoid}(VarSet)
    ++ corresponding to \axiom{x}.
  coerce       : $ -> Magma VarSet
    ++ \axiom{coerce(x)} returns the element of \axiomType{Magma}(VarSet)
    ++ corresponding to \axiom{x}.
  factor       : OFMON -> List $
    ++ \axiom{factor(x)} returns the decreasing factorization into Lyndon words.
  lyndon?      : OFMON -> Boolean
    ++ \axiom{lyndon?(w)} test if \axiom{w} is a Lyndon word.
  lyndon       : OFMON -> $
    ++ \axiom{lyndon(w)} convert \axiom{w} into a Lyndon word,
    ++ error if \axiom{w} is not a Lyndon word.
  lyndonIfCan  : OFMON -> Union($, "failed")
    ++ \axiom{lyndonIfCan(w)} convert \axiom{w} into a Lyndon word.
  varList      : $ -> List VarSet
    ++ \axiom{varList(x)} returns the list of distinct entries of \axiom{x}.
  LyndonWordsList1: (List VarSet, PI) -> ARRAY1 List $
    ++ \axiom{LyndonWordsList1(vl, n)} returns an array of lists of Lyndon
    ++ words over the alphabet \axiom{vl}, up to order \axiom{n}.
  LyndonWordsList : (List VarSet, PI) -> List $
    ++ \axiom{LyndonWordsList(vl, n)} returns the list of Lyndon
    ++ words over the alphabet \axiom{vl}, up to order \axiom{n}.

Private == Magma(VarSet) add
  -- Representation

```

```

Rep:= Magma(VarSet)

-- Fonctions locales
LetterList : OFMON -> List VarSet
factor1    : (List $, $, List $) -> List $

-- Definitions
lyndon? w ==
  w = 1$OFMON => false
  f: OFMON := rest w
  while f ^= 1$OFMON repeat
    not lexico(w,f) => return false
    f := rest f
  true

lyndonIfCan w ==
  l: List $ := factor w
  # l = 1 => first l
  "failed"

lyndon w ==
  l: List $ := factor w
  # l = 1 => first l
  error "This word is not a Lyndon word"

LetterList w ==
  w = 1 => []
  cons(first w , LetterList rest w)

factor1 (gauche, x, droite) ==
  g: List $ := gauche; d: List $ := droite
  while not null g repeat      ++ (l in g or l=x) et u in d
    lexico( g.first , x ) =>    ++ => right(l) >= u
    x := g.first *$Rep x        -- crochetage
    null(d) => g := rest g
    g := cons( x, rest g)       -- mouvement a droite
    x := first d
    d := rest d
    d := cons( x , d)           -- mouvement a gauche
    x := first g
    g := rest g
  return cons(x, d)

factor w ==
  w = 1 => []
  l : List $ := reverse [ u::$ for u in LetterList w]
  factor1( rest l, first l , [] )

x < y ==                      -- lexicographique par longueur
  lx,ly: PI

```

```

lx:= length x ; ly:= length y
lx = ly => lexico(x,y)
lx < ly

coerce(x:$):OF == bracket(x::OFMON::OF)
coerce(x:$):Magma VarSet == x::Rep

LyndonWordsList1 (vl,n) ==    -- a ameliorer !!!!!!!!!!!
    null vl => error "empty list"
    base: ARRAY1 List $ := new(n::I::NNI ,[])

    -- mots de longueur 1
    lbase1:List $ := [w::$ for w in sort(vl)]
    base.1 := lbase1

    -- calcul des mots de longueur ll
    for ll in 2..n:I repeat
        lbase1 := []
        for a in base(1) repeat          -- lettre + mot
            for b in base(ll-1) repeat
                if lexico(a,b) then lbase1:=cons(a*b,lbase1)

            for i in 2..ll-1 repeat        -- mot + mot
                for a in base(i) repeat
                    for b in base(ll-i) repeat
                        if lexico(a,b) and (lexico(b,right a) or b = right a )
                            then lbase1:=cons(a*b,lbase1)

            base(ll):= sort_!(lexico, lbase1)
    return base

LyndonWordsList (vl,n) ==
    v:ARRAY1 List $ := LyndonWordsList1(vl,n)
    "append"/ [v.i for i in 1..n]

```

---

— LWORD.dotabb —

```

"LWORD" [color="#88FF44",href="bookvol10.3.pdf#nameddest=LWORD"]
"A1AGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=A1AGG"]
"LWORD" -> "A1AGG"

```

---





## Chapter 14

# Chapter M

### 14.1 domain MCMPLX MachineComplex

— MachineComplex.input —

```
)set break resume
)sys rm -f MachineComplex.output
)spool MachineComplex.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show MachineComplex
--R MachineComplex is a domain constructor
--R Abbreviation for MachineComplex is MCMPLX
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for MCMPLX
--R
--R----- Operations -----
--R ??? : (%,MachineFloat) -> %          ??? : (MachineFloat,%) -> %
--R ??? : (%,%) -> %                    ??? : (Integer,%) -> %
--R ??? : (PositiveInteger,%) -> %      ??? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                    ?-? : (%,%) -> %
--R -? : % -> %                        ?<? : (%,%) -> Boolean
--R ?<=? : (%,%) -> Boolean             ?=? : (%,%) -> Boolean
--R ?>? : (%,%) -> Boolean             ?>=? : (%,%) -> Boolean
--R 1 : () -> %                        0 : () -> %
--R ^^? : (%,PositiveInteger) -> %     associates? : (%,%) -> Boolean
--R basis : () -> Vector %              coerce : % -> Complex Float
--R coerce : Complex Integer -> %        coerce : Complex Float -> %
--R coerce : MachineFloat -> %          coerce : Integer -> %
```

```

--R coerce : % -> %
--R coerce : % -> OutputForm
--R discriminant : () -> MachineFloat
--R hash : % -> SingleInteger
--R imaginary : () -> %
--R max : (%,%) -> %
--R norm : % -> MachineFloat
--R rank : () -> PositiveInteger
--R recip : % -> Union(%, "failed")
--R retract : % -> Integer
--R trace : % -> MachineFloat
--R unitCanonical : % -> %
--R ~=? : (%,%) -> Boolean
--R ?? : (%, Fraction Integer) -> % if MachineFloat has FIELD
--R ?? : (Fraction Integer, %) -> % if MachineFloat has FIELD
--R ?? : (NonNegativeInteger, %) -> %
--R ??? : (%, Integer) -> % if MachineFloat has FIELD
--R ??? : (%, Fraction Integer) -> % if MachineFloat has RADCAT and MachineFloat has TRANFUN
--R ??? : (%, %) -> % if MachineFloat has TRANFUN
--R ??? : (%, NonNegativeInteger) -> %
--R ?/? : (%, %) -> % if MachineFloat has FIELD
--R D : % -> % if MachineFloat has DIFRING
--R D : (%, NonNegativeInteger) -> % if MachineFloat has DIFRING
--R D : (%, Symbol) -> % if MachineFloat has PDRING SYMBOL
--R D : (%, List Symbol) -> % if MachineFloat has PDRING SYMBOL
--R D : (%, Symbol, NonNegativeInteger) -> % if MachineFloat has PDRING SYMBOL
--R D : (%, List Symbol, List NonNegativeInteger) -> % if MachineFloat has PDRING SYMBOL
--R D : (%, (MachineFloat -> MachineFloat), NonNegativeInteger) -> %
--R D : (%, (MachineFloat -> MachineFloat)) -> %
--R ?? : (%, Integer) -> % if MachineFloat has FIELD
--R ?? : (%, NonNegativeInteger) -> %
--R abs : % -> % if MachineFloat has RNS
--R acos : % -> % if MachineFloat has TRANFUN
--R acosh : % -> % if MachineFloat has TRANFUN
--R acot : % -> % if MachineFloat has TRANFUN
--R acoth : % -> % if MachineFloat has TRANFUN
--R acsc : % -> % if MachineFloat has TRANFUN
--R acsch : % -> % if MachineFloat has TRANFUN
--R argument : % -> MachineFloat if MachineFloat has TRANFUN
--R asec : % -> % if MachineFloat has TRANFUN
--R asech : % -> % if MachineFloat has TRANFUN
--R asin : % -> % if MachineFloat has TRANFUN
--R asinh : % -> % if MachineFloat has TRANFUN
--R atan : % -> % if MachineFloat has TRANFUN
--R atanh : % -> % if MachineFloat has TRANFUN
--R characteristic : () -> NonNegativeInteger
--R characteristicPolynomial : % -> SparseUnivariatePolynomial MachineFloat
--R charthRoot : % -> Union(%, "failed") if $ has CHARNZ and MachineFloat has EUCDOM and Mach
--R charthRoot : % -> % if MachineFloat has FFIELDC
--R coerce : Fraction Integer -> % if MachineFloat has FIELD or MachineFloat has RETRACT FRA

```

```

--R coerce : Complex MachineInteger -> %
--R coerce : Complex MachineFloat -> %
--R complex : (MachineFloat,MachineFloat) -> %
--R conditionP : Matrix % -> Union(Vector %,"failed") if $ has CHARNZ and MachineFloat has EUCDOM and Ma
--R convert : % -> Vector MachineFloat
--R convert : Vector MachineFloat -> %
--R convert : % -> SparseUnivariatePolynomial MachineFloat
--R convert : SparseUnivariatePolynomial MachineFloat -> %
--R convert : % -> Pattern Integer if MachineFloat has KONVERT PATTERN INT
--R convert : % -> Pattern Float if MachineFloat has KONVERT PATTERN FLOAT
--R convert : % -> Complex Float if MachineFloat has REAL
--R convert : % -> Complex DoubleFloat if MachineFloat has REAL
--R convert : % -> InputForm if MachineFloat has KONVERT INFORM
--R coordinates : (% ,Vector %) -> Vector MachineFloat
--R coordinates : (Vector % ,Vector %) -> Matrix MachineFloat
--R coordinates : % -> Vector MachineFloat
--R coordinates : Vector % -> Matrix MachineFloat
--R cos : % -> % if MachineFloat has TRANFUN
--R cosh : % -> % if MachineFloat has TRANFUN
--R cot : % -> % if MachineFloat has TRANFUN
--R coth : % -> % if MachineFloat has TRANFUN
--R createPrimitiveElement : () -> % if MachineFloat has FFIELDC
--R csc : % -> % if MachineFloat has TRANFUN
--R csch : % -> % if MachineFloat has TRANFUN
--R definingPolynomial : () -> SparseUnivariatePolynomial MachineFloat
--R derivationCoordinates : (Vector % ,(MachineFloat -> MachineFloat)) -> Matrix MachineFloat if MachineF
--R differentiate : % -> % if MachineFloat has DIFRING
--R differentiate : (% ,NonNegativeInteger) -> % if MachineFloat has DIFRING
--R differentiate : (% ,Symbol) -> % if MachineFloat has PDRING SYMBOL
--R differentiate : (% ,List Symbol) -> % if MachineFloat has PDRING SYMBOL
--R differentiate : (% ,Symbol,NonNegativeInteger) -> % if MachineFloat has PDRING SYMBOL
--R differentiate : (% ,List Symbol,List NonNegativeInteger) -> % if MachineFloat has PDRING SYMBOL
--R differentiate : (% ,(MachineFloat -> MachineFloat),NonNegativeInteger) -> %
--R differentiate : (% ,(MachineFloat -> MachineFloat)) -> %
--R discreteLog : % -> NonNegativeInteger if MachineFloat has FFIELDC
--R discreteLog : (% ,%) -> Union(NonNegativeInteger,"failed") if MachineFloat has FFIELDC
--R discriminant : Vector % -> MachineFloat
--R divide : (% ,%) -> Record(quotient: % ,remainder: %) if MachineFloat has EUCDOM
--R ?.? : (% ,MachineFloat) -> % if MachineFloat has ELTAB(MFLOAT,MFLOAT)
--R euclideanSize : % -> NonNegativeInteger if MachineFloat has EUCDOM
--R eval : (% ,List MachineFloat,List MachineFloat) -> % if MachineFloat has EVALAB MFLOAT
--R eval : (% ,MachineFloat,MachineFloat) -> % if MachineFloat has EVALAB MFLOAT
--R eval : (% ,Equation MachineFloat) -> % if MachineFloat has EVALAB MFLOAT
--R eval : (% ,List Equation MachineFloat) -> % if MachineFloat has EVALAB MFLOAT
--R eval : (% ,List Symbol,List MachineFloat) -> % if MachineFloat has IEVALAB(SYMBOL,MFLOAT)
--R eval : (% ,Symbol,MachineFloat) -> % if MachineFloat has IEVALAB(SYMBOL,MFLOAT)
--R exp : % -> % if MachineFloat has TRANFUN
--R expressIdealMember : (List % ,%) -> Union(List % ,"failed") if MachineFloat has EUCDOM
--R exquo : (% ,MachineFloat) -> Union(% ,"failed") if MachineFloat has INTDOM
--R exquo : (% ,%) -> Union(% ,"failed")

```

```

--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if MachineFloat has L
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if MachineFloat
--R factor : % -> Factored % if MachineFloat has EUCDOM and MachineFloat has PFECAT or Machi
--R factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial %
--R factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePo
--R factorsOfCyclicGroupSize : () -> List Record(factor: Integer,exponent: Integer) if Machi
--R gcd : (%,%) -> % if MachineFloat has EUCDOM
--R gcd : List % -> % if MachineFloat has EUCDOM
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUni
--R index : PositiveInteger -> % if MachineFloat has FINITE
--R init : () -> % if MachineFloat has FFIELDC
--R inv : % -> % if MachineFloat has FIELD
--R lcm : (%,%) -> % if MachineFloat has EUCDOM
--R lcm : List % -> % if MachineFloat has EUCDOM
--R lift : % -> SparseUnivariatePolynomial MachineFloat
--R log : % -> % if MachineFloat has TRANFUN
--R lookup : % -> PositiveInteger if MachineFloat has FINITE
--R map : ((MachineFloat -> MachineFloat),%) -> %
--R minimalPolynomial : % -> SparseUnivariatePolynomial MachineFloat if MachineFloat has FIE
--R multiEuclidean : (List %,%) -> Union(List %,"failed") if MachineFloat has EUCDOM
--R nextItem : % -> Union(%,"failed") if MachineFloat has FFIELDC
--R nthRoot : (%,Integer) -> % if MachineFloat has RADCAT and MachineFloat has TRANFUN
--R order : % -> PositiveInteger if MachineFloat has FFIELDC
--R order : % -> OnePointCompletion PositiveInteger if MachineFloat has FFIELDC
--R patternMatch : (%,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMatchResult(I
--R patternMatch : (%,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchResult(Float
--R pi : () -> % if MachineFloat has TRANFUN
--R polarCoordinates : % -> Record(r: MachineFloat,phi: MachineFloat) if MachineFloat has RN
--R prime? : % -> Boolean if MachineFloat has EUCDOM and MachineFloat has PFECAT or MachineF
--R primeFrobenius : (%,NonNegativeInteger) -> % if MachineFloat has FFIELDC
--R primeFrobenius : % -> % if MachineFloat has FFIELDC
--R primitive? : % -> Boolean if MachineFloat has FFIELDC
--R primitiveElement : () -> % if MachineFloat has FFIELDC
--R principalIdeal : List % -> Record(coef: List %,generator: %) if MachineFloat has EUCDOM
--R ?quo? : (%,%) -> % if MachineFloat has EUCDOM
--R random : () -> % if MachineFloat has FINITE
--R rational : % -> Fraction Integer if MachineFloat has INS
--R rational? : % -> Boolean if MachineFloat has INS
--R rationalIfCan : % -> Union(Fraction Integer,"failed") if MachineFloat has INS
--R reduce : SparseUnivariatePolynomial MachineFloat -> %
--R reduce : Fraction SparseUnivariatePolynomial MachineFloat -> Union(%,"failed") if Machin
--R reducedSystem : Matrix % -> Matrix Integer if MachineFloat has LINEXP INT
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) i
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix MachineFloat,vec: Vector Machi
--R reducedSystem : Matrix % -> Matrix MachineFloat
--R regularRepresentation : (%,Vector %) -> Matrix MachineFloat
--R regularRepresentation : % -> Matrix MachineFloat
--R ?rem? : (%,%) -> % if MachineFloat has EUCDOM
--R representationType : () -> Union("prime",polynomial,normal,cyclic) if MachineFloat has F
--R represents : (Vector MachineFloat,Vector %) -> %

```

```

--R represents : Vector MachineFloat -> %
--R retract : % -> Fraction Integer if MachineFloat has RETRACT FRAC INT
--R retractIfCan : % -> Union(Fraction Integer,"failed") if MachineFloat has RETRACT FRAC INT
--R retractIfCan : % -> Union(MachineFloat,"failed")
--R retractIfCan : % -> Union(Integer,"failed")
--R sec : % -> % if MachineFloat has TRANFUN
--R sech : % -> % if MachineFloat has TRANFUN
--R sin : % -> % if MachineFloat has TRANFUN
--R sinh : % -> % if MachineFloat has TRANFUN
--R size : () -> NonNegativeInteger if MachineFloat has FINITE
--R sizeLess? : (%,%) -> Boolean if MachineFloat has EUCDOM
--R solveLinearPolynomialEquation : (List SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) ->
--R sqrt : % -> % if MachineFloat has RADCAT and MachineFloat has TRANFUN
--R squareFree : % -> Factored % if MachineFloat has EUCDOM and MachineFloat has PFECAT or MachineFloat
--R squareFreePart : % -> % if MachineFloat has EUCDOM and MachineFloat has PFECAT or MachineFloat has F
--R squareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % if Mach
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger,NonNegativeInteger) if MachineFloat has
--R tan : % -> % if MachineFloat has TRANFUN
--R tanh : % -> % if MachineFloat has TRANFUN
--R traceMatrix : Vector % -> Matrix MachineFloat
--R traceMatrix : () -> Matrix MachineFloat
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

```

```

)spool
)lisp (bye)

```

---

— MachineComplex.help —

```

=====
MachineComplex examples
=====

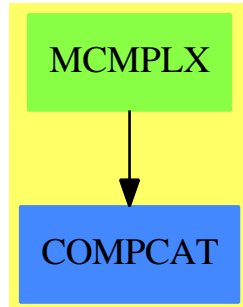
```

```

See Also:
o )show MachineComplex

```

### 14.1.1 MachineComplex (MCMPLX)



See

⇒ “MachineInteger” (MINT) 14.3.1 on page 1521

⇒ “MachineFloat” (MFLOAT) 14.2.1 on page 1511

**Exports:**

|                       |                               |                       |
|-----------------------|-------------------------------|-----------------------|
| 0                     | 1                             | abs                   |
| acos                  | acosh                         | acot                  |
| acoth                 | acsc                          | acsch                 |
| argument              | asec                          | asech                 |
| asin                  | asinh                         | associates?           |
| atan                  | atanh                         | basis                 |
| characteristic        | characteristicPolynomial      | charthRoot            |
| coerce                | complex                       | conditionP            |
| conjugate             | convert                       | coordinates           |
| cos                   | cosh                          | cot                   |
| coth                  | createPrimitiveElement        | csc                   |
| csch                  | D                             | definingPolynomial    |
| derivationCoordinates | differentiate                 | discreteLog           |
| discriminant          | divide                        | euclideanSize         |
| eval                  | exp                           | expressIdealMember    |
| exquo                 | extendedEuclidean             | factor                |
| factorPolynomial      | factorSquareFreePolynomial    | gcdPolynomial         |
| gcd                   | factorsOfCyclicGroupSize      | generator             |
| hash                  | imag                          | imaginary             |
| index                 | init                          | inv                   |
| latex                 | lcm                           | lift                  |
| log                   | lookup                        | map                   |
| max                   | min                           | minimalPolynomial     |
| multiEuclidean        | nextItem                      | norm                  |
| nthRoot               | one?                          | order                 |
| patternMatch          | pi                            | polarCoordinates      |
| prime?                | primeFrobenius                | primitive?            |
| primitiveElement      | principalIdeal                | random                |
| rank                  | rational                      | rational?             |
| rationalIfCan         | real                          | recip                 |
| reduce                | reducedSystem                 | regularRepresentation |
| representationType    | represents                    | retract               |
| retractIfCan          | sample                        | sec                   |
| sech                  | sin                           | sinh                  |
| size                  | solveLinearPolynomialEquation | sizeLess?             |
| sqrt                  | squareFree                    | squareFreePart        |
| squareFreePolynomial  | tableForDiscreteLogarithm     | subtractIfCan         |
| tan                   | tanh                          | trace                 |
| traceMatrix           | traceMatrix                   | unit?                 |
| unitCanonical         | unitNormal                    | zero?                 |
| ?*?                   | ?**?                          | ?+?                   |
| ?-?                   | -?                            | ?<?                   |
| ?<=?                  | ?=?                           | ?>?                   |
| ?>=?                  | ?^?                           | ?~=?                  |
| ?/?                   | ?..?                          | ?quo?                 |
| ?rem?                 |                               |                       |



## — domain MCMPLX MachineComplex —

```

)abbrev domain MCMPLX MachineComplex
++ Date Created:  December 1993
++ Date Last Updated:
++ Basic Operations:
++ Related Domains:
++ Also See: FortranExpression, FortranMachineTypeCategory, MachineInteger,
++ MachineFloat
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ A domain which models the complex number representation
++ used by machines in the AXIOM-NAG link.

```

```

MachineComplex():Exports == Implementation where

```

```

Exports ==> Join (FortranMachineTypeCategory,
                  ComplexCategory(MachineFloat)) with
  coerce : Complex Float -> $
    ++ coerce(u) transforms u into a MachineComplex
  coerce : Complex Integer -> $
    ++ coerce(u) transforms u into a MachineComplex
  coerce : Complex MachineFloat -> $
    ++ coerce(u) transforms u into a MachineComplex
  coerce : Complex MachineInteger -> $
    ++ coerce(u) transforms u into a MachineComplex
  coerce : $ -> Complex Float
    ++ coerce(u) transforms u into a CComplex Float

```

```

Implementation ==> Complex MachineFloat add

```

```

  coerce(u:Complex Float):$ ==
    complex(real(u)::MachineFloat,imag(u)::MachineFloat)

  coerce(u:Complex Integer):$ ==
    complex(real(u)::MachineFloat,imag(u)::MachineFloat)

  coerce(u:Complex MachineInteger):$ ==
    complex(real(u)::MachineFloat,imag(u)::MachineFloat)

  coerce(u:Complex MachineFloat):$ ==
    complex(real(u),imag(u))

  coerce(u:$):Complex Float ==
    complex(real(u)::Float,imag(u)::Float)

```

---

— MCMPLX.dotabb —

```
"MCMPLX" [color="#88FF44",href="bookvol10.3.pdf#nameddest=MCMPLX"]
"COMPCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=COMPCAT"]
"MCMPLX" -> "COMPCAT"
```

---

## 14.2 domain MFLOAT MachineFloat

— MachineFloat.input —

```
)set break resume
)sys rm -f MachineFloat.output
)spool MachineFloat.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show MachineFloat
--R MachineFloat is a domain constructor
--R Abbreviation for MachineFloat is MFLOAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for MFLOAT
--R
--R----- Operations -----
--R ???: (Fraction Integer,%) -> %      ??? : (%,Fraction Integer) -> %
--R ??? : (%,%) -> %                  ??? : (Integer,%) -> %
--R ??? : (PositiveInteger,%) -> %     ??? : (%,Fraction Integer) -> %
--R ??? : (%,Integer) -> %           ??? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                 ?-? : (%,%) -> %
--R -? : % -> %                      ?/? : (%,%) -> %
--R ?<? : (%,%) -> Boolean            ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean            ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean           1 : () -> %
--R 0 : () -> %                      ???: (%,Integer) -> %
--R ???: (%,PositiveInteger) -> %     abs : % -> %
--R associates? : (%,%) -> Boolean     base : () -> PositiveInteger
--R bits : () -> PositiveInteger        ceiling : % -> %
--R coerce : MachineInteger -> %       coerce : % -> Float
--R coerce : Float -> %                coerce : Fraction Integer -> %
--R coerce : Integer -> %              coerce : Fraction Integer -> %
```

```

--R coerce : % -> %
--R coerce : % -> OutputForm
--R convert : % -> DoubleFloat
--R digits : () -> PositiveInteger
--R factor : % -> Factored %
--R floor : % -> %
--R gcd : List % -> %
--R hash : % -> SingleInteger
--R latex : % -> String
--R lcm : (% , %) -> %
--R max : (% , %) -> %
--R min : (% , %) -> %
--R negative? : % -> Boolean
--R nthRoot : (% , Integer) -> %
--R order : % -> Integer
--R precision : () -> PositiveInteger
--R ?quo? : (% , %) -> %
--R ?rem? : (% , %) -> %
--R retract : % -> Fraction Integer
--R round : % -> %
--R sign : % -> Integer
--R sqrt : % -> %
--R squareFreePart : % -> %
--R unit? : % -> Boolean
--R wholePart : % -> Integer
--R ?~=?: (% , %) -> Boolean
--R ?? : (NonNegativeInteger , %) -> %
--R ??? : (% , NonNegativeInteger) -> %
--R ??^? : (% , NonNegativeInteger) -> %
--R base : PositiveInteger -> PositiveInteger
--R bits : PositiveInteger -> PositiveInteger if $ has arbitraryPrecision
--R changeBase : (Integer , Integer , PositiveInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R decreasePrecision : Integer -> PositiveInteger if $ has arbitraryPrecision
--R digits : PositiveInteger -> PositiveInteger if $ has arbitraryPrecision
--R divide : (% , %) -> Record(quotient: % , remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List % , %) -> Union(List % , "failed")
--R exquo : (% , %) -> Union(% , "failed")
--R extendedEuclidean : (% , % , %) -> Union(Record(coef1: % , coef2: % ) , "failed")
--R extendedEuclidean : (% , %) -> Record(coef1: % , coef2: % , generator: %)
--R float : (Integer , Integer , PositiveInteger) -> %
--R gcdPolynomial : (SparseUnivariatePolynomial % , SparseUnivariatePolynomial % ) -> SparseUni
--R increasePrecision : Integer -> PositiveInteger if $ has arbitraryPrecision
--R max : () -> % if not has($ , arbitraryExponent) and not has($ , arbitraryPrecision)
--R maximumExponent : Integer -> Integer
--R min : () -> % if not has($ , arbitraryExponent) and not has($ , arbitraryPrecision)
--R minimumExponent : Integer -> Integer
--R multiEuclidean : (List % , %) -> Union(List % , "failed")
--R patternMatch : (% , Pattern Float , PatternMatchResult(Float , %)) -> PatternMatchResult(Float

```

```

--R precision : PositiveInteger -> PositiveInteger
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R retractIfCan : % -> Union(Float,"failed")
--R retractIfCan : % -> Union(Fraction Integer,"failed")
--R retractIfCan : % -> Union(Integer,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

```

```

)spool
)lisp (bye)

```

---

— MachineFloat.help —

```

=====
MachineFloat examples
=====

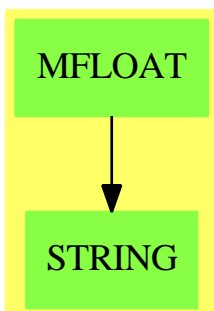
```

```

See Also:
o )show MachineFloat

```

### 14.2.1 MachineFloat (MFLOAT)



See

⇒ “MachineInteger” (MINT) 14.3.1 on page 1521  
 ⇒ “MachineComplex” (MCMLPX) 14.1.1 on page 1506

**Exports:**

|                    |                |                   |                   |
|--------------------|----------------|-------------------|-------------------|
| 1                  | 0              | abs               | associates?       |
| base               | bits           | ceiling           | coerce            |
| changeBase         | characteristic | convert           | decreasePrecision |
| digits             | exponent       | divide            | euclideanSize     |
| expressIdealMember | exquo          | extendedEuclidean | factor            |
| float              | floor          | fractionPart      | gcd               |
| gcdPolynomial      | hash           | increasePrecision | inv               |
| latex              | lcm            | mantissa          | max               |
| maximumExponent    | min            | minimumExponent   | multiEuclidean    |
| negative?          | norm           | nthRoot           | one?              |
| order              | patternMatch   | positive?         | precision         |
| prime?             | principalIdeal | recip             | retract           |
| retractIfCan       | round          | sample            | sign              |
| sizeLess?          | sqrt           | squareFree        | squareFreePart    |
| subtractIfCan      | truncate       | unit?             | unitCanonical     |
| unitNormal         | wholePart      | zero?             | ?*?               |
| ?**?               | ?+?            | ?-?               | -?                |
| ?/?                | ?<?            | ?<=?              | ?=?               |
| ?>?                | ?>=?           | ?~=?              | ?^?               |
| ?quo?              | ?rem?          |                   |                   |

— domain MFLOAT MachineFloat —

```

)abbrev domain MFLOAT MachineFloat
++ Author: Mike Dewar
++ Date Created: December 1993
++ Date Last Updated:
++ Basic Operations:
++ Related Domains:
++ Also See: FortranExpression, FortranMachineTypeCategory, MachineInteger,
++ MachineComplex
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ A domain which models the floating point representation
++ used by machines in the AXIOM-NAG link.

```

```
MachineFloat(): Exports == Implementation where
```

```

PI ==> PositiveInteger
NNI ==> NonNegativeInteger
F ==> Float
I ==> Integer
S ==> String
FI ==> Fraction Integer

```

```

SUP ==> SparseUnivariatePolynomial
SF ==> DoubleFloat

Exports ==> Join(FloatingPointSystem, FortranMachineTypeCategory, Field,
  RetractableTo(Float), RetractableTo(Fraction(Integer)), CharacteristicZero) with
precision : PI -> PI
  ++ precision(p) sets the number of digits in the model to p
precision : () -> PI
  ++ precision() returns the number of digits in the model
base : PI -> PI
  ++ base(b) sets the base of the model to b
base : () -> PI
  ++ base() returns the base of the model
maximumExponent : I -> I
  ++ maximumExponent(e) sets the maximum exponent in the model to e
maximumExponent : () -> I
  ++ maximumExponent() returns the maximum exponent in the model
minimumExponent : I -> I
  ++ minimumExponent(e) sets the minimum exponent in the model to e
minimumExponent : () -> I
  ++ minimumExponent() returns the minimum exponent in the model
coerce : $ -> F
  ++ coerce(u) transforms a MachineFloat to a standard Float
coerce : MachineInteger -> $
  ++ coerce(u) transforms a MachineInteger into a MachineFloat
mantissa : $ -> I
  ++ mantissa(u) returns the mantissa of u
exponent : $ -> I
  ++ exponent(u) returns the exponent of u
changeBase : (I,I,PI) -> $
  ++ changeBase(exp,man,base) is not documented

Implementation ==> add

import F
import FI

Rep := Record(mantissa:I,exponent:I)

-- Parameters of the Floating Point Representation
P : PI := 16      -- Precision
B : PI := 2       -- Base
EMIN : I := -1021 -- Minimum Exponent
EMAX : I := 1024  -- Maximum Exponent

-- Useful constants
POWER : PI := 53 -- The maximum power of B which will yield P
              -- decimal digits.
MMAX : PI := B**POWER

```

```

-- locals
locRound: (FI)->I
checkExponent: ($) -> $
normalise: ($) -> $
newPower: (PI,PI)->Void

retractIfCan(u:$):Union(FI,"failed") ==
  mantissa(u)*(B/1)**(exponent(u))

wholePart(u:$):Integer ==
  man:I:=mantissa u
  exp:I:=exponent u
  f:=
    positive? exp => man*B**(exp pretend PI)
    zero? exp => man
    wholePart(man/B**((-exp) pretend PI))
normalise(u:$):$ ==
  -- We want the largest possible mantissa, to ensure a canonical
  -- representation.
  exp : I := exponent u
  man : I := mantissa u
  BB : I := B pretend I
  sgn : I := sign man ; man := abs man
  zero? man => [0,0]$Rep
  if man < MMAX then
    while man < MMAX repeat
      exp := exp - 1
      man := man * BB
  if man > MMAX then
    q1:FI:= man/1
    BBF:FI:=BB/1
    while wholePart(q1) > MMAX repeat
      q1:= q1 / BBF
      exp:=exp + 1
    man := locRound(q1)
  positive?(sgn) => checkExponent [man,exp]$Rep
  checkExponent [-man,exp]$Rep

mantissa(u:$):I == elt(u,mantissa)$Rep
exponent(u:$):I == elt(u,exponent)$Rep

newPower(base:PI,prec:PI):Void ==
  power : PI := 1
  target : PI := 10**prec
  current : PI := base
  while (current := current*base) < target repeat power := power+1
  POWER := power
  MMAX := B**POWER
  void()

```

```

changeBase(exp:I,man:I,base:PI):$ ==
  newExp : I := 0
  f      : FI := man*(base pretend I)::FI**exp
  sign   : I := sign f
  f      : FI := abs f
  newMan : I := wholePart f
  zero? f => [0,0]$Rep
  BB     : FI := (B pretend I)::FI
  if newMan < MMAX then
    while newMan < MMAX repeat
      newExp := newExp - 1
      f := f*BB
      newMan := wholePart f
  if newMan > MMAX then
    while newMan > MMAX repeat
      newExp := newExp + 1
      f := f/BB
      newMan := wholePart f
  [sign*newMan,newExp]$Rep

checkExponent(u:$):$ ==
  exponent(u) < EMIN or exponent(u) > EMAX =>
    message :S := concat(["Exponent out of range: ",
      convert(EMIN)@S, "..", convert(EMAX)@S])$S
    error message
  u

coerce(u:$):OutputForm ==
  coerce(u:F)

coerce(u:MachineInteger):$ ==
  checkExponent changeBase(0,retract(u)@Integer,10)

coerce(u:$):F ==
  oldDigits : PI := digits(P)$F
  r : F := float(mantissa u,exponent u,B)$Float
  digits(oldDigits)$F
  r

coerce(u:F):$ ==
  checkExponent changeBase(exponent(u)$F,mantissa(u)$F,base())$F

coerce(u:I):$ ==
  checkExponent changeBase(0,u,10)

coerce(u:FI):$ == (numer u)::$/ (denom u)::

retract(u:$):FI ==
  value : Union(FI,"failed") := retractIfCan(u)

```



```

    value case "failed" => error "Cannot retract to a Fraction Integer"
    value::FI

retract(u:$):F == u::F

retractIfCan(u:$):Union(F,"failed") == u::F::Union(F,"failed")

retractIfCan(u:$):Union(I,"failed") ==
    value:FI := mantissa(u)*(B pretend I)::FI**exponent(u)
    zero? fractionPart(value) => wholePart(value)::Union(I,"failed")
    "failed":Union(I,"failed")

retract(u:$):I ==
    result : Union(I,"failed") := retractIfCan u
    result = "failed" => error "Not an Integer"
    result::I

precision(p: PI):PI ==
    old : PI := P
    newPower(B,p)
    P := p
    old

precision():PI == P

base(b:PI):PI ==
    old : PI := b
    newPower(b,P)
    B := b
    old

base():PI == B

maximumExponent(u:I):I ==
    old : I := EMAX
    EMAX := u
    old

maximumExponent():I == EMAX

minimumExponent(u:I):I ==
    old : I := EMIN
    EMIN := u
    old

minimumExponent():I == EMIN

0 == [0,0]$Rep
1 == changeBase(0,1,10)

```

```

zero?(u:$):Boolean == u=[0,0]$Rep

f1:$
f2:$

locRound(x:FI):I ==
  abs(fractionPart(x)) >= 1/2 => wholePart(x)+sign(x)
  wholePart(x)

recip f1 ==
  zero? f1 => "failed"
  normalise [ locRound(B**(2*POWER)/mantissa f1),-(exponent f1 + 2*POWER)]

f1 * f2 ==
  normalise [mantissa(f1)*mantissa(f2),exponent(f1)+exponent(f2)]$Rep

f1 **(p:FI) ==
  ((f1::F)**p)::%

--inline
f1 / f2 ==
  zero? f2 => error "division by zero"
  zero? f1 => 0
  f1=f2 => 1
  normalise [locRound(mantissa(f1)*B**(2*POWER)/mantissa(f2)),
    exponent(f1)-(exponent f2 + 2*POWER)]

inv(f1) == 1/f1

f1 exquo f2 == f1/f2

divide(f1,f2) == [ f1/f2,0]

f1 quo f2 == f1/f2
f1 rem f2 == 0
u:I * f1 ==
  normalise [u*mantissa(f1),exponent(f1)]$Rep

f1 = f2 == mantissa(f1)=mantissa(f2) and exponent(f1)=exponent(f2)

f1 + f2 ==
  m1 : I := mantissa f1
  m2 : I := mantissa f2
  e1 : I := exponent f1
  e2 : I := exponent f2
  e1 > e2 =>
--insignificance

```

```

e1 > e2 + POWER + 2 =>
  zero? f1 => f2
  f1
  normalise [m1*(B pretend I)**((e1-e2) pretend NNI)+m2,e2]$Rep
e2 > e1 + POWER +2 =>
  zero? f2 => f1
  f2
  normalise [m2*(B pretend I)**((e2-e1) pretend NNI)+m1,e1]$Rep

- f1 == [- mantissa f1,exponent f1]$Rep

f1 - f2 == f1 + (-f2)

f1 < f2 ==
m1 : I := mantissa f1
m2 : I := mantissa f2
e1 : I := exponent f1
e2 : I := exponent f2
sign(m1) = sign(m2) =>
  e1 < e2 => true
  e1 = e2 and m1 < m2 => true
  false
sign(m1) = 1 => false
sign(m1) = 0 and sign(m2) = -1 => false
true

characteristic():NNI == 0

```

---

— MFLOAT.dotabb —

```

"MFLOAT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=MFLOAT"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"MFLOAT" -> "STRING"

```

---

## 14.3 domain MINT MachineInteger

— MachineInteger.input —

```

)set break resume
)sys rm -f MachineInteger.output

```

```

)spool MachineInteger.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show MachineInteger
--R MachineInteger is a domain constructor
--R Abbreviation for MachineInteger is MINT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for MINT
--R
--R----- Operations -----
--R ?? : (%,% ) -> %
--R ?? : (PositiveInteger,% ) -> %
--R +? : (%,% ) -> %
--R -? : % -> %
--R ?<=? : (%,% ) -> Boolean
--R ?>? : (%,% ) -> Boolean
--R D : (% ,NonNegativeInteger) -> %
--R 1 : () -> %
--R ?? : (% ,PositiveInteger) -> %
--R addmod : (% ,% ,% ) -> %
--R base : () -> %
--R bit? : (% ,% ) -> Boolean
--R coerce : % -> %
--R coerce : % -> OutputForm
--R convert : % -> InputForm
--R convert : % -> Float
--R copy : % -> %
--R differentiate : % -> %
--R factor : % -> Factored %
--R gcd : (% ,% ) -> %
--R hash : % -> %
--R inc : % -> %
--R invmod : (% ,% ) -> %
--R lcm : (% ,% ) -> %
--R length : % -> %
--R max : (% ,% ) -> %
--R min : (% ,% ) -> %
--R negative? : % -> Boolean
--R one? : % -> Boolean
--R positive? : % -> Boolean
--R powmod : (% ,% ,% ) -> %
--R ?quo? : (% ,% ) -> %
--R random : % -> %
--R rational? : % -> Boolean
--R ?rem? : (% ,% ) -> %
--R sample : () -> %
--R sign : % -> Integer
--R ?? : (Integer,% ) -> %
--R ***? : (% ,PositiveInteger) -> %
--R -? : (% ,% ) -> %
--R ?<? : (% ,% ) -> Boolean
--R ?=? : (% ,% ) -> Boolean
--R ?>=? : (% ,% ) -> Boolean
--R D : % -> %
--R 0 : () -> %
--R abs : % -> %
--R associates? : (% ,% ) -> Boolean
--R binomial : (% ,% ) -> %
--R coerce : Integer -> %
--R coerce : Integer -> %
--R convert : % -> Integer
--R convert : % -> Pattern Integer
--R convert : % -> DoubleFloat
--R dec : % -> %
--R even? : % -> Boolean
--R factorial : % -> %
--R gcd : List % -> %
--R hash : % -> SingleInteger
--R init : () -> %
--R latex : % -> String
--R lcm : List % -> %
--R mask : % -> %
--R maxint : () -> PositiveInteger
--R mulmod : (% ,% ,% ) -> %
--R odd? : % -> Boolean
--R permutation : (% ,% ) -> %
--R positiveRemainder : (% ,% ) -> %
--R prime? : % -> Boolean
--R random : () -> %
--R rational : % -> Fraction Integer
--R recip : % -> Union(% ,"failed")
--R retract : % -> Integer
--R shift : (% ,% ) -> %
--R sizeLess? : (% ,% ) -> Boolean

```

```

--R squareFree : % -> Factored %          squareFreePart : % -> %
--R submod : (%,%,% ) -> %                symmetricRemainder : (%,% ) -> %
--R unit? : % -> Boolean                  unitCanonical : % -> %
--R zero? : % -> Boolean                  ~=? : (%,% ) -> Boolean
--R ?? : (NonNegativeInteger,% ) -> %
--R ***? : (% ,NonNegativeInteger) -> %
--R ??^? : (% ,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R coerce : Expression Integer -> Expression %
--R differentiate : (% ,NonNegativeInteger) -> %
--R divide : (% ,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (% ,%) -> Union(%,"failed")
--R extendedEuclidean : (% ,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (% ,%) -> Record(coef1: %,coef2: %,generator: %)
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUni
--R maxint : PositiveInteger -> PositiveInteger
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R nextItem : % -> Union(%,"failed")
--R patternMatch : (% ,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMatchResult(I
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R rationalIfCan : % -> Union(Fraction Integer,"failed")
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer)
--R reducedSystem : Matrix % -> Matrix Integer
--R retractIfCan : % -> Union(Integer,"failed")
--R subtractIfCan : (% ,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)

```

---

— MachineInteger.help —

```

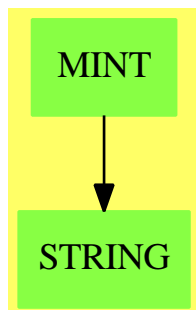
=====
MachineInteger examples
=====

```

See Also:

- o )show MachineInteger

## 14.3.1 MachineInteger (MINT)



See

⇒ “MachineFloat” (MFLOAT) 14.2.1 on page 1511

⇒ “MachineComplex” (MCMLPX) 14.1.1 on page 1506

**Exports:**

|                    |               |                |                    |               |
|--------------------|---------------|----------------|--------------------|---------------|
| 0                  | 1             | abs            | addmod             | associates?   |
| base               | binomial      | bit?           | characteristic     | coerce        |
| convert            | copy          | D              | dec                | differentiate |
| divide             | euclideanSize | even?          | expressIdealMember | exquo         |
| extendedEuclidean  | factor        | factorial      | gcd                | gcdPolynomial |
| hash               | inc           | init           | invmod             | latex         |
| lcm                | length        | mask           | max                | maxint        |
| min                | mulmod        | multiEuclidean | negative?          | nextItem      |
| odd?               | one?          | patternMatch   | permutation        | positive?     |
| positiveRemainder  | powmod        | prime?         | principalIdeal     | random        |
| rational           | rationalIfCan | rational?      | recip              | reducedSystem |
| retract            | retractIfCan  | sample         | shift              | sign          |
| sizeLess?          | squareFree    | squareFreePart | submod             | subtractIfCan |
| symmetricRemainder | unit?         | unitCanonical  | unitNormal         | zero?         |
| ?~=?               | ?*?           | ?**?           | ?^?                | ?+?           |
| ?-?                | -?            | ?<?            | ?<=?               | ?=?           |
| ?>?                | ?>=?          | ?quo?          | ?rem?              |               |

— domain MINT MachineInteger —

```

)abbrev domain MINT MachineInteger
++ Author: Mike Dewar
++ Date Created: December 1993
++ Date Last Updated:
++ Basic Operations:
++ Related Domains:
++ Also See: FortranExpression, FortranMachineTypeCategory, MachineFloat,
++ MachineComplex
++ AMS Classifications:
++ Keywords:

```

```

++ Examples:
++ References:
++ Description:
++ A domain which models the integer representation
++ used by machines in the AXIOM-NAG link.

```

```

MachineInteger(): Exports == Implementation where

```

```

S ==> String

```

```

Exports ==> Join(FortranMachineTypeCategory,IntegerNumberSystem) with
  maxint : PositiveInteger -> PositiveInteger
    ++ maxint(u) sets the maximum integer in the model to u
  maxint : () -> PositiveInteger
    ++ maxint() returns the maximum integer in the model
  coerce : Expression Integer -> Expression $
    ++ coerce(x) returns x with coefficients in the domain

```

```

Implementation ==> Integer add

```

```

MAXINT : PositiveInteger := 2**32

```

```

maxint():PositiveInteger == MAXINT

```

```

maxint(new:PositiveInteger):PositiveInteger ==
  old := MAXINT
  MAXINT := new
  old

```

```

coerce(u:Expression Integer):Expression($) ==
  map(coerce,u)$ExpressionFunctions2(Integer,$)

```

```

coerce(u:Integer):$ ==
  import S
  abs(u) > MAXINT =>
    message: S := concat [convert(u)@S," > MAXINT(",convert(MAXINT)@S,")"]
    error message
  u pretend $

```

```

retract(u:$):Integer == u pretend Integer

```

```

retractIfCan(u:$):Union(Integer,"failed") == u pretend Integer

```

---

— MINT.dotabb —

```

"MINT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=MINT"]

```





```

--S 5 of 22
tree := Magma(Symbol)
--R
--R
--R (5) Magma Symbol
--R
--E 5

```

Type: Domain

```

--S 6 of 22
a:tree := x*x
--R
--R
--R (6) [x,x]
--R
--E 6

```

Type: Magma Symbol

```

--S 7 of 22
b:tree := y*y
--R
--R
--R (7) [y,y]
--R
--E 7

```

Type: Magma Symbol

```

--S 8 of 22
c:tree := a*b
--R
--R
--R (8) [[x,x],[y,y]]
--R
--E 8

```

Type: Magma Symbol

```

--S 9 of 22
left c
--R
--R
--R (9) [x,x]
--R
--E 9

```

Type: Magma Symbol

```

--S 10 of 22
right c
--R
--R
--R (10) [y,y]
--R
--E 10

```

Type: Magma Symbol

```

--S 11 of 22

```

```

length c
--R
--R
--R (11)  4
--R
--R                                          Type: PositiveInteger
--E 11

--S 12 of 22
c::word
--R
--R
--R          2 2
--R (12)  x y
--R
--R                                          Type: OrderedFreeMonoid Symbol
--E 12

--S 13 of 22
a < b
--R
--R
--R (13)  true
--R
--R                                          Type: Boolean
--E 13

--S 14 of 22
a < c
--R
--R
--R (14)  true
--R
--R                                          Type: Boolean
--E 14

--S 15 of 22
b < c
--R
--R
--R (15)  true
--R
--R                                          Type: Boolean
--E 15

--S 16 of 22
first c
--R
--R
--R (16)  x
--R
--R                                          Type: Symbol
--E 16

--S 17 of 22
rest c

```

```

--R
--R
--R (17) [x,[y,y]]
--R
--R                                         Type: Magma Symbol
--E 17

--S 18 of 22
rest rest c
--R
--R
--R (18) [y,y]
--R
--R                                         Type: Magma Symbol
--E 18

--S 19 of 22
ax:tree := a*x
--R
--R
--R (19) [[x,x],x]
--R
--R                                         Type: Magma Symbol
--E 19

--S 20 of 22
xa:tree := x*a
--R
--R
--R (20) [x,[x,x]]
--R
--R                                         Type: Magma Symbol
--E 20

--S 21 of 22
xa < ax
--R
--R
--R (21) true
--R
--R                                         Type: Boolean
--E 21

--S 22 of 22
lexico(xa,ax)
--R
--R
--R (22) false
--R
--R                                         Type: Boolean
--E 22
)spool
)lisp (bye)

```

---

## — Magma.help —

## =====

## Magma examples

=====

## Initialisations

```
x:Symbol := 'x
x
Type: Symbol
```

```
y:Symbol := 'y
y
Type: Symbol
```

```
z:Symbol := 'z
z
Type: Symbol
```

```
word := OrderedFreeMonoid(Symbol)
OrderedFreeMonoid Symbol
Type: Domain
```

```
tree := Magma(Symbol)
Magma Symbol
Type: Domain
```

## Let's make some trees

```
a:tree := x*x
[x,x]
Type: Magma Symbol
```

```
b:tree := y*y
[y,y]
Type: Magma Symbol
```

```
c:tree := a*b
[[x,x],[y,y]]
Type: Magma Symbol
```

## Query the trees

```
left c
[x,x]
Type: Magma Symbol
```

```
right c
```

```

[y,y]
Type: Magma Symbol

length c
4
Type: PositiveInteger

Coerce to the monoid

c::word
2 2
x y
Type: OrderedFreeMonoid Symbol

Check ordering

a < b
true
Type: Boolean

a < c
true
Type: Boolean

b < c
true
Type: Boolean

Navigate the tree

first c
x
Type: Symbol

rest c
[x,[y,y]]
Type: Magma Symbol

rest rest c
[y,y]
Type: Magma Symbol

Check ordering

ax:tree := a*x
[[x,x],x]
Type: Magma Symbol

xa:tree := x*a
[x,[x,x]]

```

Type: Magma Symbol

```
xa < ax
true
```

Type: Boolean

```
lexico(xa,ax)
false
```

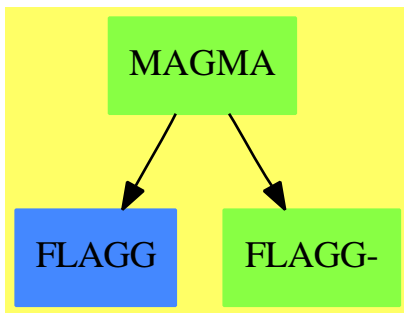
Type: Boolean

See Also:

- o )show Magma

---

#### 14.4.1 Magma (MAGMA)



Exports:

|         |         |              |              |        |
|---------|---------|--------------|--------------|--------|
| coerce  | first   | hash         | latex        | left   |
| length  | lexico  | max          | min          | mirror |
| rest    | retract | retractIfCan | retractable? | right  |
| varList | ?~=?    | ?*?          | ?<?          | ?<=?   |
| ?=?     | ?>?     | ?>=?         |              |        |

— domain MAGMA Magma —

```
)abbrev domain MAGMA Magma
++ Author: Michel Petitot (petitot@lifl.fr).
++ Date Created: 91
++ Date Last Updated: 7 Juillet 92
++ Fix History: compilation v 2.1 le 13 dec 98
++ Basic Functions:
++ Related Constructors:
++ Also See:
```

```

++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This type is the basic representation of
++ parenthesized words (binary trees over arbitrary symbols)
++ useful in \spadtype{LiePolynomial}.

Magma(VarSet:OrderedSet):Public == Private where
  WORD ==> OrderedFreeMonoid(VarSet)
  EX ==> OutputForm

Public == Join(OrderedSet,RetractableTo VarSet) with
  "*"          : ($,$) -> $
  ++ \axiom{x*y} returns the tree \axiom{[x,y]}.
  coerce       : $ -> WORD
  ++ \axiom{coerce(x)} returns the element of
  ++ \axiomType{OrderedFreeMonoid}(VarSet)
  ++ corresponding to \axiom{x} by removing parentheses.
  first        : $ -> VarSet
  ++ \axiom{first(x)} returns the first entry of the tree \axiom{x}.
  left         : $ -> $
  ++ \axiom{left(x)} returns left subtree of \axiom{x} or
  ++ error if retractable?(x) is true.
  length       : $ -> PositiveInteger
  ++ \axiom{length(x)} returns the number of entries in \axiom{x}.
  lexico       : ($,$) -> Boolean
  ++ \axiom{lexico(x,y)} returns \axiom{true} iff \axiom{x} is smaller
  ++ than \axiom{y} w.r.t. the lexicographical ordering induced by
  ++ \axiom{VarSet}.
  ++ N.B. This operation does not take into account the tree structure of
  ++ its arguments. Thus this is not a total ordering.
  mirror       : $ -> $
  ++ \axiom{mirror(x)} returns the reversed word of \axiom{x}.
  ++ That is \axiom{x} itself if retractable?(x) is true and
  ++ \axiom{mirror(z) * mirror(y)} if \axiom{x} is \axiom{y*z}.
  rest        : $ -> $
  ++ \axiom{rest(x)} return \axiom{x} without the first entry or
  ++ error if retractable?(x) is true.
  retractable? : $ -> Boolean
  ++ \axiom{retractable?(x)} tests if \axiom{x} is a tree with
  ++ only one entry.
  right       : $ -> $
  ++ \axiom{right(x)} returns right subtree of \axiom{x} or
  ++ error if retractable?(x) is true.
  varList     : $ -> List VarSet
  ++ \axiom{varList(x)} returns the list of distinct entries of \axiom{x}.

Private == add
-- representation

```

```

VWORD := Record(left:$ ,right:$)
Rep:= Union(VarSet,VWORD)

recursif: ($,$) -> Boolean

-- define
x:$ = y:$ ==
  x case VarSet =>
    y case VarSet => x::VarSet = y::VarSet
    false
  y case VWORD => x::VWORD = y::VWORD
  false

varList x ==
  x case VarSet => [x::VarSet]
  lv: List VarSet := setUnion(varList x.left, varList x.right)
  sort_!(lv)

left x ==
  x case VarSet => error "x has only one entry"
  x.left

right x ==
  x case VarSet => error "x has only one entry"
  x.right

retractable? x == (x case VarSet)

retract x ==
  x case VarSet => x::VarSet
  error "Not retractable"

retractIfCan x == (retractable? x => x::VarSet ; "failed")
coerce(l:VarSet):$ == l

mirror x ==
  x case VarSet => x
  [mirror x.right, mirror x.left]$VWORD

coerce(x:$): WORD ==
  x case VarSet => x::VarSet::WORD
  x.left::WORD * x.right::WORD

coerce(x:$):EX ==
  x case VarSet => x::VarSet::EX
  bracket [x.left::EX, x.right::EX]

x * y == [x,y]$VWORD

first x ==
  x case VarSet => x::VarSet

```



```

first x.left

rest x ==
  x case VarSet => error "rest$Magma: inexistant rest"
  lx:$ := x.left
  lx case VarSet => x.right
  [rest lx , x.right]$VWORD

length x ==
  x case VarSet => 1
  length(x.left) + length(x.right)

recursif(x,y) ==
  x case VarSet =>
    y case VarSet => x::VarSet < y::VarSet
    true
  y case VarSet => false
  x.left = y.left => x.right < y.right
  x.left < y.left

lexico(x,y) ==      -- peut etre amelioree !!!!!!!!!!!
  x case VarSet =>
    y case VarSet => x::VarSet < y::VarSet
    x::VarSet <= first y
  y case VarSet => first x < retract y
  fx:VarSet := first x ; fy:VarSet := first y
  fx = fy => lexico(rest x , rest y)
  fx < fy

x < y ==      -- recursif par longueur
  lx,ly: PositiveInteger
  lx:= length x ; ly:= length y
  lx = ly => recursif(x,y)
  lx < ly

```

---

— MAGMA.dotabb —

```

"MAGMA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=MAGMA"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"FLAGG-" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FLAGG"]
"MAGMA" -> "FLAGG"
"MAGMA" -> "FLAGG-"

```

---

## 14.5 domain MKCHSET MakeCachableSet

### — MakeCachableSet.input —

```

)set break resume
)sys rm -f MakeCachableSet.output
)spool MakeCachableSet.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show MakeCachableSet
--R MakeCachableSet S: SetCategory is a domain constructor
--R Abbreviation for MakeCachableSet is MKCHSET
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for MKCHSET
--R
--R----- Operations -----
--R ?<? : (%,% ) -> Boolean          ?<=? : (%,% ) -> Boolean
--R ?=? : (%,% ) -> Boolean          ?>? : (%,% ) -> Boolean
--R ?>=? : (%,% ) -> Boolean          coerce : S -> %
--R coerce : % -> S                  coerce : % -> OutputForm
--R hash : % -> SingleInteger         latex : % -> String
--R max : (%,% ) -> %                 min : (%,% ) -> %
--R ?~=? : (%,% ) -> Boolean
--R position : % -> NonNegativeInteger
--R setPosition : (% ,NonNegativeInteger) -> Void
--R
--E 1

)spool
)lisp (bye)

```

### — MakeCachableSet.help —

```

=====
MakeCachableSet examples
=====

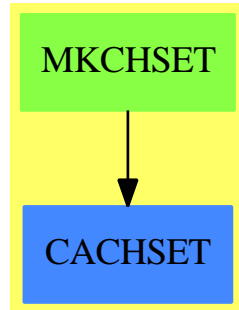
```

```

See Also:
o )show MakeCachableSet

```

### 14.5.1 MakeCachableSet (MKCHSET)



See

⇒ “Kernel” (KERNEL) 12.1.1 on page 1368

#### Exports:

|             |      |       |      |     |          |
|-------------|------|-------|------|-----|----------|
| coerce      | hash | latex | max  | min | position |
| setPosition | ?~=? | ?<?   | ?<=? | ?=? | ?>?      |
| ?>=?        |      |       |      |     |          |

— domain MKCHSET MakeCachableSet —

```

)abbrev domain MKCHSET MakeCachableSet
++ Author: Manuel Bronstein
++ Date Created: ???
++ Date Last Updated: 14 May 1991
++ Description:
++ MakeCachableSet(S) returns a cachable set which is equal to S as a set.

```

```

MakeCachableSet(S:SetCategory): Exports == Implementation where
  Exports ==> Join(CachableSet, CoercibleTo S) with
    coerce: S -> %
    ++ coerce(s) returns s viewed as an element of %.

```

```

Implementation ==> add
  import SortedCache(%)

```

```

Rep := Record(setpart: S, pos: NonNegativeInteger)

```

```

clearCache()

```

```

position x                == x.pos
setPosition(x, n)         == (x.pos := n; void)
coerce(x:%):S             == x.setpart
coerce(x:%):OutputForm    == x::S::OutputForm
coerce(s:S):%             == enterInCache([s, 0]$Rep, x+-(s = x::S))

```

```

x < y ==

```

```

    if position(x) = 0 then enterInCache(x, x1+-(x::S = x1::S))
    if position(y) = 0 then enterInCache(y, x1+-(y::S = x1::S))
    position(x) < position(y)

x = y ==
    if position(x) = 0 then enterInCache(x, x1+-(x::S = x1::S))
    if position(y) = 0 then enterInCache(y, x1+-(y::S = x1::S))
    position(x) = position(y)

```

---

— MKCHSET.dotabb —

```

"MKCHSET" [color="#88FF44",href="bookvol10.3.pdf#nameddest=MKCHSET"]
"CACHSET" [color="#4488FF",href="bookvol10.2.pdf#nameddest=CACHSET"]
"MKCHSET" -> "CACHSET"

```

---

## 14.6 domain MMLFORM MathMLFormat

— MathMLFormat.input —

```

)set break resume
)sys rm -f MathMLFormat.output
)spool MathMLFormat.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show MathMLFormat
--R MathMLFormat is a domain constructor
--R Abbreviation for MathMLFormat is MMLFORM
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for MMLFORM
--R
--R----- Operations -----
--R ?=? : (%,% ) -> Boolean          coerce : OutputForm -> String
--R coerce : % -> OutputForm        coerceL : OutputForm -> String
--R coerces : OutputForm -> String  display : String -> Void
--R exprex : OutputForm -> String   hash : % -> SingleInteger
--R latex : % -> String             ?~=? : (%,% ) -> Boolean
--R

```

```
--E 1

)spool
)lisp (bye)

-----

— MathMLFormat.help —

=====
MathMLFormat examples
=====

See Also:
o )show MathMLFormat

-----
```

Both this code and documentation are still under development and I don't pretend they are anywhere close to perfect or even finished. However the code does work and I hope it might be useful to somebody both for its ability to output MathML from Axiom and as an example of how to write a new output form.

### 14.6.1 Introduction to Mathematical Markup Language

MathML exists in two forms: presentation and content. At this time (2007-02-11) the package only has a presentation package. A content package is in the works however it is more difficult. Unfortunately Axiom does not make its semantics easily available. The **OutputForm** domain mediates between the individual Axiom domains and the user visible output but **OutputForm** does not provide full semantic information. From my currently incomplete understanding of Axiom it appears that remedying this would entail going back to the individual domains and rewriting a lot of code. However some semantics are conveyed directly by **OutputForm** and other things can be deduced from **OutputForm** or from the original user command.

### 14.6.2 Displaying MathML

The MathML string produced by ")set output mathml on" can be pasted directly into an appropriate xhtml page and then viewed in Firefox or some other MathML aware browser. The boiler plate code needed for a test page, testmathml.xml, is:

```
<?xml version="1.0" ?>

<!DOCTYPE html PUBLIC "-//W3C//DTD XHTML 1.1 plus MathML 2.0//EN"
    "http://www.w3.org/Math/DTD/mathml2/xhtml-math11-f.dtd" [
```

```

<!ENTITY mathml "http://www.w3.org/1998/Math/MathML">
]>

<html xmlns="http://www.w3.org/1999/xhtml"
      xmlns:xlink="http://www.w3.org/1999/xlink" >

  <head>
    <title>MathML Test </title>
  </head>

  <body>

    </body>
</html>

```

Paste the MathML string into the body element and it should display nicely in Firefox.

### 14.6.3 Test Cases

Here's a list of test cases that currently format correctly:

1.  $(x+y)^{**2}$
2.  $\text{integrate}(x^{**x}, x)$
3.  $\text{integral}(x^{**x}, x)$
4.  $(5 + \sqrt{63} + \sqrt{847})^{**}(1/3)$
5.  $\text{set } [1, 2, 3]$
6.  $\text{multiset } [x \bmod 5 \text{ for } x \text{ in primes}(2, 1000)]$
7.  $\text{series}(\sin(a^x), x=0)$
8.  $\text{matrix } [ [x^{**i} + y^{**j} \text{ for } i \text{ in } 1..10] \text{ for } j \text{ in } 1..10]$
9.  $y := \text{operator 'y a. } D(y(x, z), [x, x, z, x]) \text{ b. } D(y \ x, x, 2)$
10.  $x := \text{series 'x a. } \sin(1+x)$
11.  $\text{series}(1/\log(y), y=1)$
12.  $y:\text{UTS}(\text{FLOAT}, 'z, 0) := \exp(z)$
13. a.  $c := \text{continuedFraction}(314159/100000)$  b.  $c := \text{continuedFraction}(314159/100000)$

The **TexFormat** domain has the capability to format an object with subscripts, superscripts, presubscripts and presuperscripts however I don't know of any Axiom command that produces such an object. In fact at present I see the case of "SUPERSUB" being used for putting primes in the superscript position to denote ordinary differentiation. I also only see the "SUB" case being used to denote partial derivatives.

### 14.6.4 )set output mathml on

Making mathml appear as output during a normal Axiom session by invoking `)set output mathml on` proved to be a bit tedious and seems to be undocumented. I document my experience here in case it proves useful to somebody else trying to get a new output format from Axiom.

In **MathMLFormat** the functions `coerce(expr : OutputForm) : String` and `display(s : String) : Void` provide the desired mathml output. Note that this package was constructed by close examination of Robert Sutor's **TexFormat** domain and much remains from that source. To have mathml displayed as output we need to get Axiom to call `display(coerce(expr))` at the appropriate place. Here's what I did to get that to happen. Note that my starting point here was an attempt by Andrey Grozin to do the same. To figure things out I searched through files for "tex" to see what was done for the **TexFormat** domain, and used `grep` to find which files had mention of **TexFormat**.

### 14.6.5 File src/interp/setvars.boot.pamphlet

Create an output mathml section by analogy to the tex section. Remember to add the code chunk "outputmathmlCode" at the end.

setvars.boot is a bootstrap file which means that it has to be precompiled into lisp code and then that code has to be inserted back into setvars.boot. To do this extract the boot code by running "notangle" on it. I did this from the "tmp" directory. From inside axiom run `)lisp (boottran::boottocl "tmp/setvars.boot")` which put "setvars.clisp" into "int/interp/setvars.clisp". Then replace the lisp in "setvars.boot.pamphlet" with that in the newly generated "setvars.clisp".

The relevant code chunks appearing in "setvars.boot.pamphlet" are:

```
outputmathmlCode
setOutputMathml
describeSetOutputMathml
```

and the relevant variables are:

```
setOutputMathml
$mathmlOutputStream
$mathmlOutputFile
$mathmlFormat
describeSetOutputMathml
```

### 14.6.6 File setvar.boot.pamphlet

Create an output mathml section in "setvar.boot.pamphlet" again patterned after the tex section. I changed the default file extension from ".stex" to ".smml".

To the "sectionoutput" table I added the line

```
mathml                                created output in MathML style      Off:CONSOLE
```

Added the code chunk "outputmathml" to the code chunk "output" in "sectionoutput".

Relevant code chunks:

```
outputmathml
```

Relevant variables:

```
setOutputMathml
$mathmlFormat
$mathmlOutputFile
```

Note when copying the tex stuff I changed occurrences of "tex" to "mathml", "Tex" to "Mathml" and "TeX" to "MathML".

### 14.6.7 File src/algebra/Makefile.pamphlet

The file "src/algebra/tex.spad.pamphlet" contains the domain **TexFormat** (TEX) and the package **TexFormat1** (TEX1). However the sole function of **TexFormat1** is to *coerce* objects from a domain into **OutputForm** and then apply **TexFormat** to them. It is to save programmers the trouble of doing the coercion themselves from inside spad code. It does not appear to be used for the main purpose of delivering Axiom output in TeX format. In order to keep the mathml package as simple as possible, and because I didn't see much use for this, I didn't copy the **TexFormat1** package. So no analog of the TEX1 entries in "Makefile.pamphlet" were needed. One curiosity I don't understand is why TEX1 appears in layer 4 when it seems to depend on TEX which appears in layer 14.

Initially I added "\$OUT/MMLFORM.o" to layer 14 and "mathml.spad.pamphlet" to completed spad files in layer 14. When trying to compile the build failed at MMLFORM. It left "MMLFORM.erlib" in "int/algebra" instead of "MMLFORM.NRLIB" which confused me at first because mathml.spad compiled under a running axiom. By examining the file "obj/tmp/trace" I saw that a new dependency had been introduced, compared to TexFormat, with the function eltName depending on the domain FSAGG in layer 16. So the lines had to be moved from layer 14 to layer 17.

Added appropriate lines to "SPADFILES" and "DOCFILES".

### 14.6.8 File src/algebra/exposed.lsp.pamphlet

Add the line "(|MathMLFormat| . MMLFORM)"

### 14.6.9 File src/algebra/Lattice.pamphlet

I don't see that this file is used anywhere but I made the appropriate changes anyway by searching for "TEX" and mimicing everything for MMLFORM.



### 14.6.10 File `src/doc/axiom.bib.pamphlet`

Added `mathml.spad` subsection to "`src/doc/axiom.bib.pamphlet`".

### 14.6.11 File `interp/i-output.boot.pamphlet`

This is where the *coerce* and *display* functions from `MathMLFormat` actually get called. The following was added:

```
mathmlFormat expr ==
  mml := '(MathMLFormat)
  mmlrep := '(String)
  formatFn := getFunctionFromDomain("coerce",mml,[$OutputForm])
  displayFn := getFunctionFromDomain("display",mml,[mmlrep])
  SPADCALL(SPADCALL(expr,formatFn),displayFn)
  TERPRI $mathmlOutputStream
  FORCE_-OUTPUT $mathmlOutputStream
  NIL
```

Note that compared to the `texFormat` function there are a couple of differences. Since **MathMLFormat** is currently a package rather than a domain there is the "mmlrep" variable whereas in `texFormat` the argument of the "display" function is an instance of the domain. Also the *coerce* function here only has one argument, namely "\$OutputForm".

Also for the function "`output(expr,domain)`" add lines for `mathml`, e.g. "if \$mathmlFormat then `mathmlFormat expr`".

After these changes Axiom compiled with `mathml` enabled under `)set output`.

### 14.6.12 Public Declarations

The declarations

```
E      ==> OutputForm
I      ==> Integer
L      ==> List
S      ==> String
US     ==> UniversalSegment(Integer)
```

provide abbreviations for domains used heavily in the code. The publicly exposed functions are:

*coerce*:  $E \rightarrow S$  This function is the main one for converting an expression in domain `OutputForm` into a MathML string.

*coerceS*:  $E \rightarrow S$  This function is for use from the command line. It converts an `OutputForm` expression into a MathML string and does some formatting so that the output is not one long line. If you take the output from this function, stick it in an emacs buffer in `nxml-mode`

and then indent according to mode, you'll get something that's nicer to look at than what comes from `coerce`. Note that `coerceS` returns the same value as `coerce` but invokes a display function as well so that the result will be printed twice in different formats. The need for this is that the output from `coerce` is automatically formatted with line breaks by Axiom's output routine that are not in the right place.

*coerceL*:  $E \rightarrow S$  Similar to `coerceS` except that the displayed result is the MathML string in one long line. These functions can be used, for instance, to get the MathML for the previous result by typing `coerceL`(

*expres*:  $E \rightarrow S$  Converts **OutputForm** to **String** with the structure preserved with braces. This is useful in developing this package. Actually this is not quite accurate. The function *precondition* is first applied to the **OutputForm** expression before *expres*. Raw **OutputForm** and the nature of the *precondition* function is still obscure to me at the time of this writing (2007-02-14), however I probably need to understand it to make sure I'm not missing any semantics. The `spad` function *precondition* is just a wrapper for the `lisp` function `outputTranSLisp`, which I guess is compiled from `boot`.

*display*:  $S \rightarrow \text{Void}$  This one prints the string returned by `coerce` as one long line, adding "math" tags: `<math ...> ... </math>`. Thus the output from this can be stuck directly into an appropriate `html/xhtml` page and will be displayed nicely by a MathML aware browser.

*displayF*:  $S \rightarrow \text{Void}$  This function doesn't exist yet but it would be nice to have a humanly readable formatted output as well. The basics do exist in the `coerceS` function however the formatting still needs some work to be really good.

— public declarations —

```
)abbrev domain MMLFORM MathMLFormat
++ Author: Arthur C. Ralfs
++ Date: January 2007
++ Basic Operations: coerce, coerceS, coerceL, expres, display
++ Description:
++ This package is based on the TeXFormat domain by Robert S. Sutor
++ \spadtype{MathMLFormat} provides a coercion from \spadtype{OutputForm}
++ to MathML format.
```

```
MathMLFormat(): public == private where
E      ==> OutputForm
I      ==> Integer
L      ==> List
S      ==> String
US     ==> UniversalSegment(Integer)
```

```
public == SetCategory with
coerce:  E -> S
++ coerceS(o) changes o in the standard output format to MathML
++ format.
coerceS: E -> S
++ coerceS(o) changes o in the standard output format to MathML
```

```

    ++ format and displays formatted result.
coerceL:  E -> S
    ++ coerceS(o) changes o in the standard output format to MathML
    ++ format and displays result as one long string.
expres:   E -> S
    ++ converts \spadtype{OutputForm} to \spadtype{String} with the
    ++ structure preserved with braces.  Actually this is not quite
    ++ accurate.  The function \spadfun{precondition} is first
    ++ applied to the
    ++ \spadtype{OutputForm} expression before \spadfun{expres}.
    ++ The raw \spadtype{OutputForm} and
    ++ the nature of the \spadfun{precondition} function is
    ++ still obscure to me
    ++ at the time of this writing (2007-02-14).
display:  S -> Void
    ++ prints the string returned by coerce, adding <math ...> tags.

```

### 14.6.13 Private Constant Declarations

— private constant declarations —

```

private == add
import OutputForm
import Character
import Integer
import List OutputForm
import List String

-- local variable declarations and definitions

expr: E
prec,opPrec: I
str: S
blank      : S := " \ "

maxPrec    : I := 1000000
minPrec    : I := 0

unaryOps   : L S := ["-", "^"]$(L S)
unaryPrecs : L I := [700, 260]$(L I)

-- the precedence of / in the following is relatively low because
-- the bar obviates the need for parentheses.
binaryOps  : L S := ["+->", "|", "***", "/", "<", ">", "=", "OVER"]$(L S)
binaryPrecs : L I := [0, 0, 900, 700, 400, 400, 400, 700]$(L I)

```

```

naryOps      : L S := ["-", "+", "*", blank, ",", ";", " ", "ROW", "",
  " \cr ", "&", "</mtd></mtr><mtr><mtd>"]$(L S)
naryPrecs    : L I := [700,700,800, 800,110,110, 0, 0, 0,
  0, 0, 0]$(L I)
naryNGOps    : L S := ["ROW", "&"]$(L S)

plexOps      : L S := ["SIGMA", "SIGMA2", "PI", "PI2", "INTSIGN", "INDEFINTEGRAL"]$(L S)
plexPrecs    : L I := [ 700, 800, 700, 800 , 700, 700]$(L I)

specialOps   : L S := ["MATRIX", "BRACKET", "BRACE", "CONCATB", "VCONCAT", _
  "AGGLST", "CONCAT", "OVERBAR", "ROOT", "SUB", "TAG", _
  "SUPERSUB", "ZAG", "AGGSET", "SC", "PAREN", _
  "SEGMENT", "QUOTE", "theMap", "SLASH" ]

-- the next two lists provide translations for some strings for
-- which MML provides special macros.

specialStrings : L S :=
  ["cos", "cot", "csc", "log", "sec", "sin", "tan",
   "cosh", "coth", "csch", "sech", "sinh", "tanh",
   "acos", "asin", "atan", "erf", "...", "$", "infinity"]
specialStringsInMML : L S :=
  ["<mo>cos</mo>", "<mo>cot</mo>", "<mo>csc</mo>", "<mo>log</mo>", "<mo>sec</mo>", "<mo>sin</mo>", "<mo>tan</mo>",
   "<mo>cosh</mo>", "<mo>coth</mo>", "<mo>csch</mo>", "<mo>sech</mo>", "<mo>sinh</mo>", "<mo>tanh</mo>",
   "<mo>arccos</mo>", "<mo>arcsin</mo>", "<mo>arctan</mo>", "<mo>erf</mo>", "<mo>&#x2026;</mo>", "<mo>$</mo>", "<mo>infinity</mo>"]

```

#### 14.6.14 Private Function Declarations

These are the local functions:

addBraces:S -> S

addBrackets:S -> S

atomize:E -> L E

displayElt:S -> Void function for recursively displaying mathml nicely formatted

eltLimit:(S,I,S) -> I demarcates end postion of mathml element with name:S starting at position i:I in mathml string s:S and returns end of end tag as i:I position in mathml string, i.e. find start and end of substring: <name ...>...</name>

eltName:(I,S) -> S find name of mathml element starting at position i:I in string s:S

group:S -> S

formatBinary:(S,L E, I) -> S

formatFunction:(S,L E, I) -> S

```

formatMatrix:L E -> S
formatNary:(S,L E, I) -> S
formatNaryNoGroup:(S,L E, I) -> S
formatNullary:S -> S
formatPlex:(S,L E, I) -> S
formatSpecial:(S,L E, I) -> S
formatUnary:(S, E, I) -> S
formatMml:(E,I) -> S
newWithNum:I -> $ this is a relic from tex.spad and is not used here so far. I'll probably
remove it.
parenthesize:S -> S
precondition:E -> E this function is applied to the OutputForm expression before doing
anything else.
postcondition:S -> S this function is applied after all other OutputForm -> MathML trans-
formations. In the TexFormat domain the ungroup function first peels off the outermost set
of braces however I have replaced braces with <mrow>s here and sometimes the outermost
set of <mrow>s is necessary to get proper display in Firefox. For instance with getting the
correct size of brackets on a matrix the whole expression needs to be enclosed in a mrow
element. It also checks for +- and removes the +.
stringify:E -> S
tagEnd:(S,I,S) -> I finds closing ">" of start or end tag for mathML element for formatting
MathML string for human readability. No analog in TexFormat.
ungroup:S -> S

```

— private function declarations —

```

-- local function signatures

addBraces:      S -> S
addBrackets:    S -> S
atomize:        E -> L E
displayElt:     S -> Void
    ++ function for recursively displaying mathml nicely formatted
eltLimit:       (S,I,S) -> I
    ++ demarcates end position of mathml element with name:S starting at
    ++ position i:I in mathml string s:S and returns end of end tag as
    ++ i:I position in mathml string, i.e. find start and end of
    ++ substring: <name ...>...</name>
eltName:        (I,S) -> S
    ++ find name of mathml element starting at position i:I in string s:S
group:          S -> S
formatBinary:   (S,L E, I) -> S

```

```

formatFunction: (S,L E, I) -> S
formatIntSign:  (L E, I) -> S
formatMatrix:   L E -> S
formatNary:      (S,L E, I) -> S
formatNaryNoGroup: (S,L E, I) -> S
formatNullary:  S -> S
formatPlex:      (S,L E, I) -> S
formatSpecial:   (S,L E, I) -> S
formatSub:       (E, L E, I) -> S
formatSuperSub:  (E, L E, I) -> S
formatSuperSub1: (E, L E, I) -> S
formatUnary:     (S, E, I) -> S
formatMml:       (E,I) -> S
formatZag:       L E -> S
formatZag1:      L E -> S
newWithNum:      I -> $
parenthesize:    S -> S
precondition:    E -> E
postcondition:   S -> S
stringify:       E -> S
tagEnd:          (S,I,S) -> I
  ++ finds closing ">" of start or end tag for mathML element
ungroup:         S -> S

```

---

### 14.6.15 Public Function Definitions

Note that I use the function `sayTeX$Lisp` much as I would `printf` in a C program. I've noticed in grepping the code that there are other "say" functions, `sayBrightly` and `sayMessage` for instance, but I have no idea what the difference is between them at this point. `sayTeX$Lisp` does the job so for the time being I'll use that until I learn more.

The functions `coerceS` and `coerceL` should probably be changed to display functions, *i.e.* `displayS` and `display L`, returning `Void`. I really only need the one `coerce` function.

#### — public function definitions —

```

-- public function definitions

coerce(expr : E): S ==
  s : S := postcondition formatMml(precondition expr, minPrec)
  s

coerceS(expr : E): S ==
  s : S := postcondition formatMml(precondition expr, minPrec)
  sayTeX$Lisp "<math xmlns=_\"http://www.w3.org/1998/Math/MathML_\" mathsize=_\"big_\" display=_\"block_\"

```

```

displayElt(s)
sayTeX$Lisp "</math>"
s

coerceL(expr : E): S ==
s : S := postcondition formatMml(precondition expr, minPrec)
sayTeX$Lisp "<math xmlns=_\"http://www.w3.org/1998/Math/MathML_\" mathsize=_\"big_\" displ
sayTeX$Lisp s
sayTeX$Lisp "</math>"
s

display(mathml : S): Void ==
sayTeX$Lisp "<math xmlns=_\"http://www.w3.org/1998/Math/MathML_\" mathsize=_\"big_\" displ
sayTeX$Lisp mathml
sayTeX$Lisp "</math>"
void()$Void

exprex(expr : E): S ==
-- This breaks down an expression into atoms and returns it as
-- a string. It's for developmental purposes to help understand
-- the expressions.
a : E
expr := precondition expr
-- sayTeX$Lisp "0: \"stringify expr
(ATOM(expr)$Lisp@Boolean) or (stringify expr = "NOTHING") =>
concat ["{\",stringify expr,\"}"]
le : L E := (expr pretend L E)
op := first le
sop : S := exprex op
args : L E := rest le
nargs : I := #args
-- sayTeX$Lisp concat ["1: \",stringify first le,\" : \",string(nargs)$S]
s : S := concat ["{\",sop]
if nargs > 0 then
for a in args repeat
-- sayTeX$Lisp concat ["2: \",stringify a]
s1 : S := exprex a
s := concat [s,s1]
s := concat [s,\"}"]

```

---

### 14.6.16 Private Function Definitions

#### Display Functions

```
displayElt(mathml:S):Void
eltName(pos:I,mathml:S):S
eltLimit(name:S,pos:I,mathml:S):I
tagEnd(name:S,pos:I,mathml:S):I
```

— display functions —

```
displayElt(mathML:S): Void ==
  -- Takes a string of syntactically complete mathML
  -- and formats it for display.
  --   sayTeX$Lisp "****displayElt1****"
  --   sayTeX$Lisp mathML
  enT:I -- marks end of tag, e.g. "<name>"
  enE:I -- marks end of element, e.g. "<name> ... </name>"
  end:I -- marks end of mathML string
  u:US
  end := #mathML
  length:I := 60
  --   sayTeX$Lisp "****displayElt1.1****"
  name:S := eltName(1,mathML)
  --   sayTeX$Lisp name
  --   sayTeX$Lisp concat("****displayElt1.2****",name)
  enE := eltLimit(name,2+#name,mathML)
  --   sayTeX$Lisp "****displayElt2****"
  if enE < length then
  --   sayTeX$Lisp "****displayElt3****"
    u := segment(1,enE)$US
    sayTeX$Lisp mathML.u
  else
  --   sayTeX$Lisp "****displayElt4****"
    enT := tagEnd(name,1,mathML)
    u := segment(1,enT)$US
    sayTeX$Lisp mathML.u
    u := segment(enT+1,enE-#name-3)$US
    displayElt(mathML.u)
    u := segment(enE-#name-2,enE)$US
    sayTeX$Lisp mathML.u
  if end > enE then
  --   sayTeX$Lisp "****displayElt5****"
    u := segment(enE+1,end)$US
    displayElt(mathML.u)

  void()$Void
```



```

eltName(pos:I,mathML:S): S ==
  -- Assuming pos is the position of "<" for a start tag of a mathML
  -- element finds and returns the element's name.
  i:I := pos+1
  --sayTeX$Lisp "eltName:mathmML string: "mathML
  while member?(mathML.i,lowerCase()$CharacterClass)$CharacterClass repeat
    i := i+1
  u:US := segment(pos+1,i-1)
  name:S := mathML.u

eltLimit(name:S,pos:I,mathML:S): I ==
  -- Finds the end of a mathML element like "<name ...> ... </name>"
  -- where pos is the position of the space after name in the start tag
  -- although it could point to the closing ">". Returns the position
  -- of the ">" in the end tag.
  pI:I := pos
  startI:I
  endI:I
  startS:S := concat ["<",name]
  endS:S := concat ["</",name,">"]
  level:I := 1
  --sayTeX$Lisp "eltLimit: element name: "name
  while (level > 0) repeat
    startI := position(startS,mathML,pI)$String

    endI := position(endS,mathML,pI)$String

    if (startI = 0) then
      level := level-1
      --sayTeX$Lisp "****eltLimit 1*****"
      pI := tagEnd(name,endI,mathML)
    else
      if (startI < endI) then
        level := level+1
        pI := tagEnd(name,startI,mathML)
      else
        level := level-1
        pI := tagEnd(name,endI,mathML)
  pI

tagEnd(name:S,pos:I,mathML:S):I ==
  -- Finds the closing ">" for either a start or end tag of a mathML
  -- element, so the return value is the position of ">" in mathML.
  pI:I := pos
  while (mathML.pI ^= char ">") repeat
    pI := pI+1
  u:US := segment(pos,pI)$US
  --sayTeX$Lisp "tagEnd: "mathML.u
  pI

```

## Formatting Functions

Still need to format `\zag` in `formatSpecial`!

In `formatPlex` the case `op = "INTSIGN"` is now passed off to `formatIntSign` which is a change from the `TexFormat` domain. This is done here for presentation mark up to replace the ugly bound variable that Axiom delivers. For content mark up this has to be done anyway.

The `formatPlex` function also allows for `op = "INDEFINTEGRAL"`. However I don't know what Axiom command gives rise to this case. The `INTSIGN` case already allows for both definite and indefinite integrals.

In the function `formatSpecial` various cases are handled including `SUB` and `SUPERSUB`. These cases are now caught in `formatMml` and so the code in `formatSpecial` doesn't get executed. The only cases I know of using these are partial derivatives for `SUB` and ordinary derivatives or `SUPERSUB` however in `TexFormat` the capability is there to handle multi-scripts, i.e. an object with subscripts, superscripts, pre-subscripts and pre-superscripts but I am so far unaware of any Axiom command that produces such a multiscripted object.

Another question is how to represent derivatives. At present I have differential notation for partials and prime notation for ordinary derivatives, but it would be nice to allow for different derivative notations in different circumstances, maybe some options to `)set output mathml on`.

Ordinary derivatives are formatted in `formatSuperSub` and there are 2 versions, `formatSuperSub` and `formatSuperSub1`, which at this point have to be switched by swapping names.

### — formatting functions —

```
atomize(expr : E): L E ==
-- This breaks down an expression into a flat list of atomic expressions.
-- expr should be preconditioned.
le : L E := nil()
a : E
letmp : L E
(ATOM(expr)$Lisp@Boolean) or (stringify expr = "NOTHING") =>
  le := append(le,list(expr))
letmp := expr pretend L E
for a in letmp repeat
  le := append(le,atomize a)
le

ungroup(str: S): S ==
len : I := #str
len < 14 => str
lrow : S := "<mrow>"
```

```

rrow : S := "</mrow>"
-- drop leading and trailing mrows
u1 : US := segment(1,6)$US
u2 : US := segment(len-6,len)$US
if (str.u1 = $S lrow) and (str.u2 = $S rrow) then
  u : US := segment(7,len-7)$US
  str := str.u
str

postcondition(str: S): S ==
--
  str := ungroup str
  len : I := #str
  plusminus : S := "<mo>+</mo><mo>-</mo>"
  pos : I := position(plusminus,str,1)
  if pos > 0 then
    ustart:US := segment(1,pos-1)$US
    uend:US := segment(pos+20,len)$US
    str := concat [str.ustrart,"<mo>-</mo>",str.uend]
    if pos < len-18 then
      str := postcondition(str)
  str

stringify expr == (mathObject2String$Lisp expr)@S

group str ==
  concat ["<mrow>",str,"</mrow>"]

addBraces str ==
  concat ["<mo>{</mo>",str,"<mo>}</mo>"]

addBrackets str ==
  concat ["<mo>[</mo>",str,"<mo>]</mo>"]

parenthesize str ==
  concat ["<mo>(</mo>",str,"<mo>)</mo>"]

precondition expr ==
  outputTran$Lisp expr

formatSpecial(op : S, args : L E, prec : I) : S ==
  arg : E
  prescript : Boolean := false
  op = "theMap" => "<mtext>theMap(...)</mtext>"
  op = "AGGLST" =>
    formatNary(",",args,prec)
  op = "AGGSET" =>
    formatNary(";",args,prec)
  op = "TAG" =>
    group concat [formatMml(first args,prec),
      "<mo>&#x02192;</mo>",

```

```

        formatMml(second args,prec)]
        --RightArrow
op = "SLASH" =>
    group concat [formatMml(first args,prec),
        "<mo>/</mo>",formatMml(second args,prec)]
op = "VCONCAT" =>
    group concat("<table><mtr>",
        concat(concat([concat("<td>",concat(formatMml(u, minPrec),"</td>"))
            for u in args]::L S),
            "</mtr></table>"))
op = "CONCATB" =>
    formatNary(" ",args,prec)
op = "CONCAT" =>
    formatNary("",args,minPrec)
op = "QUOTE" =>
    group concat("<mo>'</mo>",formatMml(first args, minPrec))
op = "BRACKET" =>
    group addBrackets ungroup formatMml(first args, minPrec)
op = "BRACE" =>
    group addBraces ungroup formatMml(first args, minPrec)
op = "PAREN" =>
    group parenthesize ungroup formatMml(first args, minPrec)
op = "OVERBAR" =>
    null args => ""
    group concat ["<mover accent='true'><mrow>",<mo> stretchy='true'>&#x000AF;</mo></mover>"]
        --OverBar
op = "ROOT" =>
    null args => ""
    tmp : S := group formatMml(first args, minPrec)
    null rest args => concat ["<msqrt>",tmp,"</msqrt>"]
    group concat
        ["<mroot><mrow>",tmp,"</mrow>",<mo> stretchy='true'>&#x000AF;</mo></mroot>"]
        formatMml(first rest args, minPrec),"</mroot>"]
op = "SEGMENT" =>
    tmp : S := concat [formatMml(first args, minPrec),"<mo>..</mo>"]
    group
        null rest args => tmp
        concat [tmp,formatMml(first rest args, minPrec)]
-- SUB should now be diverted in formatMml although I'll leave
-- the code here for now.
op = "SUB" =>
    group concat ["<msub>",formatMml(first args, minPrec),
        formatSpecial("AGGLST",rest args,minPrec),"</msub>"]
-- SUPERSUB should now be diverted in formatMml although I'll leave
-- the code here for now.
op = "SUPERSUB" =>
    base:S := formatMml(first args, minPrec)
    args := rest args

```

```

if #args = 1 then
  "<msub><mrow>"base"</mrow><mrow>"_
  formatMml(first args, minPrec)"</mrow></msub>"
else if #args = 2 then
  -- it would be nice to substitute &#x2032; for , in the case of
  -- an ordinary derivative, it looks a lot better.
  "<msubsup><mrow>"base"</mrow><mrow>"_
  formatMml(first args,minPrec)_
  "</mrow><mrow>"_
  formatMml(first rest args, minPrec)_
  "</mrow></msubsup>"
else if #args = 3 then
  "<mmultiscripts><mrow>"base"</mrow><mrow>"_
  formatMml(first args,minPrec)"</mrow><mrow>"_
  formatMml(first rest args,minPrec)"</mrow><mprescripts/><mrow>"_
  formatMml(first rest rest args,minPrec)_
  "</mrow><none/></mmultiscripts>"
else if #args = 4 then
  "<mmultiscripts><mrow>"base"</mrow><mrow>"_
  formatMml(first args,minPrec)"</mrow><mrow>"_
  formatMml(first rest args,minPrec)"</mrow><mprescripts/><mrow>"_
  formatMml(first rest rest args,minPrec)_
  "</mrow><mrow>"formatMml(first rest rest rest args,minPrec)_
  "</mrow></mmultiscripts>"
else
  "<mtext>Problem with multiscript object</mtext>"
op = "SC" =>
  -- need to handle indentation someday
  null args => ""
  tmp := formatNaryNoGroup("</mtd></mtr><mtr><mtd>", args, minPrec)
  group concat ["<mtable><mtr><mtd>",tmp,"</mtd></mtr></mtable>"]
op = "MATRIX" => formatMatrix rest args
op = "ZAG" =>
  -- {{+}}{3}{{ZAG}}{1}{7}}{{ZAG}}{1}{15}}{{ZAG}}{1}{1}}
  -- {{ZAG}}{1}{25}}{{ZAG}}{1}{1}}{{ZAG}}{1}{7}}{{ZAG}}{1}{4}}
  -- to format continued fraction traditionally need to intercept it at the
  -- formatNary of the "+"
  concat [" \zag{",formatMml(first args, minPrec),"}{",
    formatMml(first rest args,minPrec),"}"]
  concat ["<mtext>not done yet for: ",op,"</mtext>"]

formatSub(expr : E, args : L E, opPrec : I) : S ==
  -- This one produces differential notation partial derivatives.
  -- It doesn't work in all cases and may not be workable, use
  -- formatSub1 below for now.
  -- At this time this is only to handle partial derivatives.
  -- If the SUB case handles anything else I'm not aware of it.
  -- This an example of the 4th partial of y(x,z) w.r.t. x,x,z,x
  -- {{SUB}}{y}{{CONCAT}}{CONCAT}{{CONCAT}}{CONCAT}{,}{1}}
  -- {{CONCAT}}{,}{1}}{{CONCAT}}{,}{2}}{{CONCAT}}{,}{1}}{{x}}{z}}

```

```

atomE : L E := atomize(expr)
op : S := stringify first atomE
op ^= "SUB" => "<mtext>Mistake in formatSub: no SUB</mtext>"
stringify first rest rest atomE ^= "CONCAT" => _
    "<mtext>Mistake in formatSub: no CONCAT</mtext>"
-- expecting form for atomE like
-- [{SUB}{func}{CONCAT}...{CONCAT}{},{n}{CONCAT}{},{n}...{CONCAT}{},{n}],
-- counting the first CONCATs before the comma gives the number of
-- derivatives
ndiffs : I := 0
tmpLE : L E := rest rest atomE
while stringify first tmpLE = "CONCAT" repeat
    ndiffs := ndiffs+1
    tmpLE := rest tmpLE
numLS : L S := nil
i : I := 1
while i < ndiffs repeat
    numLS := append(numLS,list(stringify first rest tmpLE))
    tmpLE := rest rest rest tmpLE
    i := i+1
numLS := append(numLS,list(stringify first rest tmpLE))
-- numLS contains the numbers of the bound variables as strings
-- for the differentiations, thus for the differentiation [x,x,z,x]
-- for y(x,z) numLS = ["1","1","2","1"]
posLS : L S := nil
i := 0
-- sayTeX$Lisp "formatSub: nargs = "string(#args)
while i < #args repeat
    posLS := append(posLS,list(string(i+1)))
    i := i+1
-- posLS contains the positions of the bound variables in args
-- as a list of strings, e.g. for the above example ["1","2"]
tmpS: S := stringify atomE.2
if ndiffs = 1 then
    s : S := "<mfrac><mo>&#x02202;</mo><mi>"tmpS"</mi><mrow>"
else
    s : S := "<mfrac><mrow><msup><mo>&#x02202;</mo><mn>"string(ndiffs)"</mn></msup><mi>"tmpS"</mi></>
-- need to find the order of the differentiation w.r.t. the i-th
-- variable
i := 1
j : I
k : I
tmpS: S
while i < #posLS+1 repeat
    j := 0
    k := 1
    while k < #numLS + 1 repeat
        if numLS.k = string i then j := j + 1
        k := k+1
    if j > 0 then

```

```

    tmpS := stringify args.i
    if j = 1 then
      s := s"<mo>&#x02202;</mo><mi>"tmpS"</mi>"
    else
      s := s"<mo>&#x02202;</mo><msup><mi>"tmpS_
        "</mi><mn>"string(j)"</mn></msup>"
    i := i + 1
  s := s"</mrow></mfrac><mo>(</mo>"
  i := 1
  while i < #posLS+1 repeat
    tmpS := stringify args.i
    s := s"<mi>"tmpS"</mi>"
    if i < #posLS then s := s"<mo>,</mo>"
    i := i+1
  s := s"<mo>></mo>"

formatSub1(expr : E, args : L E, opPrec : I) : S ==
-- This one produces partial derivatives notated by ",n" as
-- subscripts.
-- At this time this is only to handle partial derivatives.
-- If the SUB case handles anything else I'm not aware of it.
-- This an example of the 4th partial of y(x,z) w.r.t. x,x,z,x
-- {{{SUB}{y}{{CONCAT}}{{CONCAT}}{{CONCAT}}{{CONCAT}{{,}}{1}}
-- {{CONCAT}{{,}}{1}}}{{CONCAT}{{,}}{2}}}{{CONCAT}{{,}}{1}}}{x}{z}},
-- here expr is everything in the first set of braces and
-- args is {{x}{z}}
atomE : L E := atomize(expr)
op : S := stringify first atomE
op ^= "SUB" => "<mtext>Mistake in formatSub: no SUB</mtext>"
stringify first rest rest atomE ^= "CONCAT" => "<mtext>Mistake in formatSub: no CONCAT"
-- expecting form for atomE like
-- [{SUB}{func}{CONCAT}...{CONCAT}{{,}}{n}{{CONCAT}{{,}}{n}}...{CONCAT}{{,}}{n}],
--counting the first CONCATs before the comma gives the number of
--derivatives
ndiffs : I := 0
tmpLE : L E := rest rest atomE
while stringify first tmpLE = "CONCAT" repeat
  ndiffs := ndiffs+1
  tmpLE := rest tmpLE
numLS : L S := nil
i : I := 1
while i < ndiffs repeat
  numLS := append(numLS,list(stringify first rest tmpLE))
  tmpLE := rest rest rest tmpLE
  i := i+1
numLS := append(numLS,list(stringify first rest tmpLE))
-- numLS contains the numbers of the bound variables as strings
-- for the differentiations, thus for the differentiation [x,x,z,x]
-- for y(x,z) numLS = ["1","1","2","1"]
posLS : L S := nil

```

```

i := 0
-- sayTeX$Lisp "formatSub: nargs = "string(#args)
while i < #args repeat
  posLS := append(posLS,list(string(i+1)))
  i := i+1
-- posLS contains the positions of the bound variables in args
-- as a list of strings, e.g. for the above example ["1","2"]
funcS: S := stringify atomE.2
s : S := "<msub><mi>"funcS"</mi><mrow>"
i := 1
while i < #numLS+1 repeat
  s := s"<mo>,</mo><mn>"numLS.i"</mn>"
  i := i + 1
s := s"</mrow></msub><mo>(</mo>"
i := 1
while i < #posLS+1 repeat
--   tmpS := stringify args.i
  tmpS := formatMml(first args,minPrec)
  args := rest args
  s := s"<mi>"tmpS"</mi>"
  if i < #posLS then s := s"<mo>,</mo>"
  i := i+1
s := s"<mo>></mo>"

formatSuperSub(expr : E, args : L E, opPrec : I) : S ==
-- this produces prime notation ordinary derivatives.
-- first have to divine the semantics, add cases as needed
-- WriteLine$Lisp "SuperSub1 begin"
atomE : L E := atomize(expr)
op : S := stringify first atomE
-- WriteLine$Lisp "op: "op
op ^= "SUPERSUB" => _
  "<mtext>Mistake in formatSuperSub: no SUPERSUB1</mtext>"
#args ^= 1 => "<mtext>Mistake in SuperSub1: #args <> 1</mtext>"
var : E := first args
-- should be looking at something like {{SUPERSUB}{var}{ }{,,...}} for
-- example here's the second derivative of y w.r.t. x
-- {{SUPERSUB}{y}{ }{,,}}{x}}, expr is the first {} and args is the
-- {x}
funcS : S := stringify first rest atomE
-- WriteLine$Lisp "funcS: "funcS
bvarS : S := stringify first args
-- WriteLine$Lisp "bvarS: "bvarS
-- count the number of commas
commaS : S := stringify first rest rest rest atomE
commaTest : S := ","
i : I := 0
while position(commaTest,commaS,1) > 0 repeat
  i := i+1
  commaTest := commaTest","

```



```

s : S := "<msup><mi>"funcS"</mi><mrow>"
-- WriteLine$Lisp "s: "s
j : I := 0
while j < i repeat
  s := s"<mo>&#x02032;</mo>"
  j := j + 1
s := s"</mrow></msup><mo>&#x02061;</mo><mo></mo>"_
  formatMml(first args,minPrec)"<mo></mo>"

formatSuperSub1(expr : E, args : L E, opPrec : I) : S ==
-- This one produces ordinary derivatives with differential notation,
-- it needs a little more work yet.
-- first have to divine the semantics, add cases as needed
-- WriteLine$Lisp "SuperSub begin"
atomE : L E := atomize(expr)
op : S := stringify first atomE
op ^ = "SUPERSUB" => _
  "<mtext>Mistake in formatSuperSub: no SUPERSUB</mtext>"
#args ^ = 1 => "<mtext>Mistake in SuperSub: #args <> 1</mtext>"
var : E := first args
-- should be looking at something like {{SUPERSUB}{var}{ }{,,...}} for
-- example here's the second derivative of y w.r.t. x
-- {{SUPERSUB}{y}{ }{,,}{x}}, expr is the first {} and args is the
-- {x}
funcS : S := stringify first rest atomE
bvarS : S := stringify first args
-- count the number of commas
commaS : S := stringify first rest rest rest atomE
commaTest : S := ","
ndiffs : I := 0
while position(commaTest,commaS,1) > 0 repeat
  ndiffs := ndiffs+1
  commaTest := commaTest","
s : S := "<mfrac><mrow><msup><mo>&#x02146;</mo><mn>"string(ndiffs)_
  "</mn></msup><mi>"funcS"</mi></mrow><mrow><mo>&#x02146;</mo><msup><mi>"_
  formatMml(first args,minPrec)"</mi><mn>"string(ndiffs)_
  "</mn></msup></mrow></mfrac><mo>&#x02061;</mo><mo></mo><mi>"_
  formatMml(first args,minPrec)"</mi><mo></mo>"

formatPlex(op : S, args : L E, prec : I) : S ==
  checkarg:Boolean := false
  hold : S
  p : I := position(op,plexOps)
  p < 1 => error "unknown plex op"
  op = "INTSIGN" => formatIntSign(args,minPrec)
  opPrec := plexPrecs.p
  n : I := #args
  (n ^ = 2) and (n ^ = 3) => error "wrong number of arguments for plex"
  s : S :=
    op = "SIGMA" =>

```

```

        checkarg := true
        "<mo>&#x02211;</mo>"
-- Sum
op = "SIGMA2" =>
    checkarg := true
    "<mo>&#x02211;</mo>"
-- Sum
op = "PI" =>
    checkarg := true
    "<mo>&#x0220F;</mo>"
-- Product
op = "PI2" =>
    checkarg := true
    "<mo>&#x0220F;</mo>"
-- Product
-- op = "INTSIGN" => "<mo>&#x0222B;</mo>"
-- Integral, int
op = "INDEFINTEGRAL" => "<mo>&#x0222B;</mo>"
-- Integral, int
"???"
hold := formatMml(first args,minPrec)
args := rest args
if op ^= "INDEFINTEGRAL" then
    if hold ^= "" then
        s := concat ["<munderover>",s,group hold]
    else
        s := concat ["<munderover>",s,group " "]
    if not null rest args then
        hold := formatMml(first args,minPrec)
        if hold ^= "" then
            s := concat [s,group hold,"</munderover>"]
        else
            s := concat [s,group " ","</munderover>"]
        args := rest args
-- if checkarg true need to test op arg for "+" at least
-- and wrap parentheses if so
if checkarg then
    la : L E := (first args pretend L E)
    opa : S := stringify first la
    if opa = "+" then
        s :=
            concat [s,"<mo>(</mo>",formatMml(first args,minPrec),"<mo>)</mo>"]
        else s := concat [s,formatMml(first args,minPrec)]
    else s := concat [s,formatMml(first args,minPrec)]
else
    hold := group concat [hold,formatMml(first args,minPrec)]
    s := concat [s,hold]
-- if opPrec < prec then s := parenthesize s
-- getting ugly parentheses on fractions
group s

```

```

formatIntSign(args : L E, opPrec : I) : S ==
-- the original OutputForm expression looks something like this:
-- {{INTSIGN}}{NOTHING or lower limit?}
-- {bvar or upper limit?}{{*}}{integrand}{{CONCAT}}{d}{axiom var}}}}
-- the args list passed here consists of the rest of this list, i.e.
-- starting at the NOTHING or ...
(stringify first args) = "NOTHING" =>
-- the bound variable is the second one in the argument list
bvar : E := first rest args
bvarS : S := stringify bvar
tmpS : S
i : I := 0
u1 : US
u2 : US
-- this next one atomizes the integrand plus differential
atomE : L E := atomize(first rest rest args)
-- pick out the bound variable used by axiom
varRS : S := stringify last(atomE)
tmpLE : L E := ((first rest rest args) pretend L E)
integrand : S := formatMml(first rest tmpLE,minPrec)
-- replace the bound variable, i.e. axiom uses something of the form
-- %A for the bound variable and puts the original variable used
-- in the input command as a superscript on the integral sign.
-- I'm assuming that the axiom variable is 2 characters.
while (i := position(varRS,integrand,i+1)) > 0 repeat
  u1 := segment(1,i-1)$US
  u2 := segment(i+2,#integrand)$US
  integrand := concat [integrand.u1,bvarS,integrand.u2]
concat ["<mrow><mo>&#x0222B;</mo>" integrand _
        "<mo>&#x02146;</mo><mi>" bvarS "</mi></mrow>"]

lowlim : S := stringify first args
highlim : S := stringify first rest args
bvar : E := last atomize(first rest rest args)
bvarS : S := stringify bvar
tmpLE : L E := ((first rest rest args) pretend L E)
integrand : S := formatMml(first rest tmpLE,minPrec)
concat ["<mrow><munderover><mo>&#x0222B;</mo><mi>" lowlim "</mi><mi>" highlim "</mi></mrow>"]

formatMatrix(args : L E) : S ==
-- format for args is [[ROW ...],[ROW ...],[ROW ...]]
-- generate string for formatting columns (centered)
group addBrackets concat
  ["<mtable><mtr><td>",formatNaryNoGroup("</td></mtr><mtr><td>",args,minPrec),
   "</td></mtr></mtable>"]

formatFunction(op : S, args : L E, prec : I) : S ==
group concat ["<mo>",op,"</mo>",parenthesize formatNary(",",args,minPrec)]

```

```

formatNullary(op : S) ==
  op = "NOTHING" => ""
  group concat ["<mo>",op,"</mo><mo>(</mo><mo>)</mo>"]

formatUnary(op : S, arg : E, prec : I) ==
  p : I := position(op,unaryOps)
  p < 1 => error "unknown unary op"
  opPrec := unaryPrecs.p
  s : S := concat ["<mo>",op,"</mo>",formatMml(arg,opPrec)]
  opPrec < prec => group parenthesize s
  op = "-" => s
  group s

formatBinary(op : S, args : L E, prec : I) : S ==
  p : I := position(op,binaryOps)
  p < 1 => error "unknown binary op"
  opPrec := binaryPrecs.p
  -- if base op is product or sum need to add parentheses
  if ATOM(first args)$Lisp@Boolean then
    opa:S := stringify first args
  else
    la : L E := (first args pretend L E)
    opa : S := stringify first la
  if (opa = "SIGMA" or opa = "SIGMA2" or opa = "PI" or opa = "PI2") _
    and op = "***" then
    s1:S:=concat ["<mo>(</mo>",formatMml(first args, opPrec),"<mo>)</mo>"]
  else
    s1 : S := formatMml(first args, opPrec)
    s2 : S := formatMml(first rest args, opPrec)
  op :=
    op = "|"      => s := concat ["<mrow>",s1,"</mrow><mo>",op,"</mo><mrow>",s2,"</mrow>"]
    op = "***"    => s := concat ["<msup><mrow>",s1,"</mrow><mrow>",s2,"</mrow></msup>"]
    op = "/"      => s := concat ["<mfrac><mrow>",s1,"</mrow><mrow>",s2,"</mrow></mfrac>"]
    op = "OVER"   => s := concat ["<mfrac><mrow>",s1,"</mrow><mrow>",s2,"</mrow></mfrac>"]
    op = "+->"    => s := concat ["<mrow>",s1,"</mrow><mo>",op,"</mo><mrow>",s2,"</mrow>"]
    s := concat ["<mrow>",s1,"</mrow><mo>",op,"</mo><mrow>",s2,"</mrow>"]
  group
  op = "OVER" => s
  --      opPrec < prec => parenthesize s
  -- ugly parentheses?
  s

formatNary(op : S, args : L E, prec : I) : S ==
  group formatNaryNoGroup(op, args, prec)

formatNaryNoGroup(op : S, args : L E, prec : I) : S ==
  checkargs:Boolean := false
  null args => ""
  p : I := position(op,naryOps)

```

```

p < 1 => error "unknown nary op"
-- need to test for "ZAG" case and divert it here
-- ex 1. continuedFraction(314159/100000)
-- {{+}}{3}{{ZAG}}{1}{7}}{{ZAG}}{1}{15}}{{ZAG}}{1}{1}}{{ZAG}}{1}{25}}
-- {{ZAG}}{1}{1}}{{ZAG}}{1}{7}}{{ZAG}}{1}{4}}
-- this is the preconditioned output form
-- including "op", the args list would be the rest of this
-- i.e op = '+' and args = {{3}}{{ZAG}}{1}{7}}{{ZAG}}{1}{15}}
-- {{ZAG}}{1}{1}}{{ZAG}}{1}{25}}{{ZAG}}{1}{1}}{{ZAG}}{1}{7}}{{ZAG}}{1}{4}}
-- ex 2. continuedFraction(14159/100000)
-- this one doesn't have the leading integer
-- {{+}}{{ZAG}}{1}{7}}{{ZAG}}{1}{15}}{{ZAG}}{1}{1}}{{ZAG}}{1}{25}}
-- {{ZAG}}{1}{1}}{{ZAG}}{1}{7}}{{ZAG}}{1}{4}}
--
-- ex 3. continuedFraction(3, repeating [1], repeating [3,6])
-- {{+}}{3}{{ZAG}}{1}{3}}{{ZAG}}{1}{6}}{{ZAG}}{1}{3}}{{ZAG}}{1}{6}}
-- {{ZAG}}{1}{3}}{{ZAG}}{1}{6}}{{ZAG}}{1}{3}}{{ZAG}}{1}{6}}
-- {{ZAG}}{1}{3}}{{ZAG}}{1}{6}}{...}
-- In each of these examples the args list consists of the terms
-- following the '+' op
-- so the first arg could be a "ZAG" or something
-- else, but the second arg looks like it has to be "ZAG", so maybe
-- test for #args > 1 and args.2 contains "ZAG".
-- Note that since the resulting MathML <mfrac>s are nested we need
-- to handle the whole continued fraction at once, i.e. we can't
-- just look for, e.g., {{ZAG}}{1}{6}}
(#args > 1) and (position("ZAG",stringify first rest args,1) > 0) =>
  tmpS : S := stringify first args
  position("ZAG",tmpS,1) > 0 => formatZag(args)
-- position("ZAG",tmpS,1) > 0 => formatZag1(args)
  concat [formatMml(first args,minPrec) "<mo>+</mo>" _
    formatZag(rest args)]
-- At least for the ops "*", "+", "-" we need to test to see if a sigma
-- or pi is one of their arguments because we might need parentheses
-- as indicated by the problem with
-- summation(operator(f)(i),i=1..n)+1 versus
-- summation(operator(f)(i)+1,i=1..n) having identical displays as
-- of 2007-21-21
op :=
  op = ","      => "<mo>,</mo>" --originally , \:
  op = ";"      => "<mo>;</mo>" --originally ; \: should figure these out
  op = "*"      => "<mspace width='0.3em'/>"
-- InvisibleTimes
  op = " "      => "<mspace width='0.5em'/>"
  op = "ROW"    => "</mtd><mtd>"
  op = "+"      =>
    checkargs := true
    "<mo>+</mo>"
  op = "-"      =>
    checkargs := true

```

```

        "<mo>-</mo>"
    op
    l : L S := nil
    opPrec := naryPrecs.p
    -- if checkargs is true check each arg except last one to see if it's
    -- a sigma or pi and if so add parentheses. Other op's may have to be
    -- checked for in future
    count:I := 1
    for a in args repeat
--      WriteLine$Lisp "checking args"
      if checkargs then
        if count < #args then
          -- check here for sum or product
          if ATOM(a)$Lisp@Boolean then
            opa:S := stringify a
          else
            la : L E := (a pretend L E)
            opa : S := stringify first la
          if opa = "SIGMA" or opa = "SIGMA2" or _
            opa = "PI" or opa = "PI2" then
            l := concat(op,concat(_
              concat ["<mo>(</mo>",formatMml(a,opPrec),_
                "<mo></mo>"],1)$L(S))$L(S)
              else l := concat(op,concat(formatMml(a,opPrec),1)$L(S))$L(S)
            else l := concat(op,concat(formatMml(a,opPrec),1)$L(S))$L(S)
            else l := concat(op,concat(formatMml(a,opPrec),1)$L(S))$L(S)
          count := count + 1
        s : S := concat reverse rest l
        opPrec < prec => parenthesize s
      s

formatZag(args : L E) : S ==
-- args will be a list of things like this {{ZAG}}{1}{7}}, the ZAG
-- must be there, the '1' and '7' could conceivably be more complex
-- expressions
tmpZag : L E := first args pretend L E
-- may want to test that tmpZag contains 'ZAG'
#args > 1 => "<mfrac>"formatMml(first rest tmpZag,minPrec)"<mrow><mn>"formatMml(first rest rest tmpZag,minPrec)"</mrow></mfrac>"
-- EQUAL(tmpZag, "...")$Lisp => "<mo>&#x2026;</mo>"
(first args = "...":E)@Boolean => "<mo>&#x2026;</mo>"
position("ZAG",stringify first args,1) > 0 =>
  "<mfrac>"formatMml(first rest tmpZag,minPrec)formatMml(first rest rest tmpZag,minPrec)"</mfrac>"
"<mtext>formatZag: Unexpected kind of ZAG</mtext>"

formatZag1(args : L E) : S ==
-- make alternative ZAG format without diminishing fonts, maybe
-- use a table
-- {{ZAG}}{1}{7}}
tmpZag : L E := first args pretend L E

```

```
#args > 1 => "<mfrac>"formatMml(first rest tmpZag,minPrec)"<mrow><mn>"formatMml(first
(first args = "...": E)@Boolean => "<mo>&#x2026;</mo>"
error "formatZag1: Unexpected kind of ZAG"
```

```
formatMml(expr : E,prec : I) ==
  i,len : Integer
  intSplitLen : Integer := 20
  ATOM(expr)$Lisp@Boolean =>
    str := stringify expr
    len := #str
    -- this bit seems to deal with integers
  INTEGERP$Lisp expr =>
    i := expr pretend Integer
    if (i < 0) or (i > 9)
    then
      group
        nstr : String := ""
        -- insert some blanks into the string, if too long
        while ((len := #str) > intSplitLen) repeat
          nstr := concat [nstr," ",
            elt(str,segment(1,intSplitLen)$US)]
          str := elt(str,segment(intSplitLen+1)$US)
        empty? nstr => concat ["<mn>",str,"</mn>"]
        nstr :=
          empty? str => nstr
          concat [nstr," ",str]
          concat ["<mn>",elt(nstr,segment(2)$US),"</mn>"]
        else str := concat ["<mn>",str,"</mn>"]
    str = "%pi" => "<mi>&#x003C0;</mi>"
    -- pi
    str = "%e" => "<mi>&#x02147;</mi>"
    -- ExponentialE
    str = "%i" => "<mi>&#x02148;</mi>"
    -- ImaginaryI
    len > 0 and str.1 = char "%" => concat(concat("<mi>",str),"</mi>")
    -- should handle floats
    len > 1 and digit? str.1 => concat ["<mn>",str,"</mn>"]
    -- presumably this is a literal string
    len > 0 and str.1 = char "_" =>
      concat(concat("<mtext>",str),"</mtext>")
    len = 1 and str.1 = char " " => " "
    (i := position(str,specialStrings)) > 0 =>
      specialStringsInMML.i
    (i := position(char " ",str)) > 0 =>
      -- We want to preserve spacing, so use a roman font.
      -- What's this for? Leave the \rm in for now so I can see
      -- where it arises. Removed 2007-02-14
      concat(concat("<mtext>",str),"</mtext>")
    -- if we get to here does that mean it's a variable?
```

```

    concat ["<mi>",str,"</mi>"]
l : L E := (expr pretend L E)
null l => blank
op : S := stringify first l
args : L E := rest l
nargs : I := #args
-- need to test here in case first l is SUPERSUB case and then
-- pass first l and args to formatSuperSub.
position("SUPERSUB",op,1) > 0 =>
    formatSuperSub(first l,args,minPrec)
-- now test for SUB
position("SUB",op,1) > 0 =>
    formatSub1(first l,args,minPrec)

-- special cases
member?(op, specialOps) => formatSpecial(op,args,prec)
member?(op, plexOps)    => formatPlex(op,args,prec)

-- nullary case
0 = nargs => formatNullary op

-- unary case
(1 = nargs) and member?(op, unaryOps) =>
    formatUnary(op, first args, prec)

-- binary case
(2 = nargs) and member?(op, binaryOps) =>
    formatBinary(op, args, prec)

-- nary case
member?(op,naryNGOps) => formatNaryNoGroup(op,args, prec)
member?(op,naryOps)  => formatNary(op,args, prec)

op := formatMml(first l,minPrec)
formatFunction(op,args,prec)

```

---

### 14.6.17 Mathematical Markup Language Form

— MathMLFormat.input —

```

)set break resume
)spool MathMLFormat.output
)set message test on
)set message auto off
)clear all

```



```
--S 1 of 5
)set output mathml on
```

```
--R
--E 1
```

```
--S 2 of 5
```

```
1/2
```

```
--R
```

```
--R
```

```
--R      1
```

```
--R (1)  -
```

```
--R      2
```

```
--R<math xmlns="http://www.w3.org/1998/Math/MathML" mathsize="big" display="block">
```

```
--R<math><mfrac><mrow><mn>1</mn></mrow><mrow><mn>2</mn></mrow></mfrac></math>
```

```
--R</math>
```

```
--R
```

```
--R
```

Type: Fraction Integer

```
--E 2
```

```
--S 3 of 5
```

```
1/(x+5)
```

```
--R
```

```
--R
```

```
--R      1
```

```
--R (2)  ----
```

```
--R      x + 5
```

```
--R<math xmlns="http://www.w3.org/1998/Math/MathML" mathsize="big" display="block">
```

```
--R<math><mfrac><mrow><mn>1</mn></mrow><mrow><mrow><mi>x</mi><mo>+</mo><mn>5</mn></mrow></mrow></mfrac></math>
```

```
--R</math>
```

```
--R
```

```
--R
```

Type: Fraction Polynomial Integer

```
--E 3
```

```
--S 4 of 5
```

```
(x+3)/(y-5)
```

```
--R
```

```
--R
```

```
--R      x + 3
```

```
--R (3)  ----
```

```
--R      y - 5
```

```
--R<math xmlns="http://www.w3.org/1998/Math/MathML" mathsize="big" display="block">
```

```
--R<math><mfrac><mrow><mrow><mi>x</mi><mo>+</mo><mn>3</mn></mrow></mrow><mrow><mrow><mi>y</mi><mo>-</mo><mn>5</mn></mrow></mrow></mfrac></math>
```

```
--R</math>
```

```
--R
```

```
--R
```

Type: Fraction Polynomial Integer

```
--E 4
```

```
--S 5 of 5
```

```

)show MathMLFormat
--R MathMLFormat is a domain constructor
--R Abbreviation for MathMLFormat is MMLFORM
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for MMLFORM
--R
--R----- Operations -----
--R ?=? : (% ,%) -> Boolean          coerce : OutputForm -> String
--R coerce : % -> OutputForm         coerceL : OutputForm -> String
--R coerceS : OutputForm -> String   display : String -> Void
--R exprex : OutputForm -> String    hash : % -> SingleInteger
--R latex : % -> String              ?~=? : (% ,%) -> Boolean
--R
--E 5

)spool
)lisp (bye)

```

---

— MathMLFormat.help —

=====

MathMLFormat examples

=====

MathML is an HTML-like markup language for mathematics. It uses the "knuckle" syntax of HTML such as "<mo>" to introduce a math operator and "</mo>" to mark the end of the operator. Axiom can generate MathML output and does so when it communicates to the browser front end.

This output is enabled by

```
)set output mathml on
```

after which you'll see the MathML markup as well as the algebra.  
Note that you can turn off the algebra output with

```
)set output algebra off
```

but we don't do that here so you can compare the output.

1/2

1/2

```
<math xmlns="http://www.w3.org/1998/Math/MathML" mathsize="big"
display="block">
```

```

<mrow>
  <mn>1</mn>
  <mo>/</mo>
  <mn>2</mn>
</mrow>
</math>

```

$$1/(x+5)$$

$$1/(x + 5)$$

```

<math xmlns="http://www.w3.org/1998/Math/MathML" mathsize="big"
  display="block">
  <mrow>
    <mn>1</mn>
    <mo>/</mo>
    <mrow>
      <mo>(</mo>
      <mi>x</mi>
      <mo>+</mo>
      <mn>5</mn>
      <mo>)</mo>
    </mrow>
  </mrow>
</math>

```

$$(x+3)/(y-5)$$

$$(x + 3)/(y - 5)$$

```

<math xmlns="http://www.w3.org/1998/Math/MathML" mathsize="big"
  display="block">
  <mrow>
    <mrow>
      <mo>(</mo>
      <mi>x</mi>
      <mo>+</mo>
      <mn>3</mn>
      <mo>)</mo>
    </mrow>
    <mo>/</mo>
    <mrow>
      <mo>(</mo>
      <mi>y</mi>
      <mo>-</mo>
      <mn>5</mn>
      <mo>)</mo>
    </mrow>
  </mrow>
</math>

```

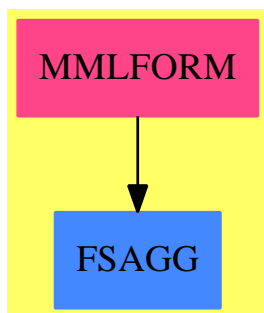
`R</math>`

See Also:

o `)show MathMLFormat`

---

### 14.6.18 MathMLForm (MMLFORM)



**Exports:**

`coerce` `coerceL` `coerceS` `display` `expres`  
`hash` `latex` `?=?` `?~=?`

— domain MMLFORM MathMLFormat —

```

\getchunk{public declarations}
\getchunk{private constant declarations}
\getchunk{private function declarations}
\getchunk{public function definitions}
\getchunk{display functions}
\getchunk{formatting functions}
  
```

---

— MMLFORM.dotabb —

```

"MMLFORM" [color="#FF4488",href="bookvol10.4.pdf#nameddest=MMLFORM"]
"FSAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FSAGG"]
"MMLFORM" -> "FSAGG"
  
```

---

## 14.7 domain MATRIX Matrix

— Matrix.input —

```
)set break resume
)sys rm -f Matrix.output
)spool Matrix.output
)set message test on
)set message auto off
)clear all
--S 1 of 38
```

```
m : Matrix(Integer) := new(3,3,0)
```

```
--R
```

```
--R
```

```
--R      +0  0  0+
```

```
--R      |      |
```

```
--R      (1) |0  0  0|
```

```
--R      |      |
```

```
--R      +0  0  0+
```

```
--R
```

Type: Matrix Integer

```
--E 1
```

```
--S 2 of 38
```

```
setelt(m,2,3,5)
```

```
--R
```

```
--R
```

```
--R      (2)  5
```

```
--R
```

Type: PositiveInteger

```
--E 2
```

```
--S 3 of 38
```

```
m(1,2) := 10
```

```
--R
```

```
--R
```

```
--R      (3)  10
```

```
--R
```

Type: PositiveInteger

```
--E 3
```

```
--S 4 of 38
```

```
m
```

```
--R
```

```
--R
```

```
--R      +0  10  0+
```

```
--R      |      |
```

```
--R      (4) |0  0  5|
```

```
--R      |      |
```

```
--R      +0  0  0+
```

```
--R
```

Type: Matrix Integer

--E 4

--S 5 of 38

matrix [ [1,2,3,4],[0,9,8,7] ]

--R

--R

--R +1 2 3 4+

--R (5) | |

--R +0 9 8 7+

--R

Type: Matrix Integer

--E 5

--S 6 of 38

dm := diagonalMatrix [1,x\*\*2,x\*\*3,x\*\*4,x\*\*5]

--R

--R

--R +1 0 0 0 0 +

--R | |

--R | 2 |

--R |0 x 0 0 0 |

--R | |

--R | 3 |

--R (6) |0 0 x 0 0 |

--R | |

--R | 4 |

--R |0 0 0 x 0 |

--R | |

--R | 5 |

--R +0 0 0 0 x +

--R

Type: Matrix Polynomial Integer

--E 6

--S 7 of 38

setRow!(dm,5,vector [1,1,1,1,1])

--R

--R

--R +1 0 0 0 0+

--R | |

--R | 2 |

--R |0 x 0 0 0 |

--R | |

--R (7) | 3 |

--R |0 0 x 0 0 |

--R | |

--R | 4 |

--R |0 0 0 x 0 |

--R | |

--R +1 1 1 1 1+

--R

Type: Matrix Polynomial Integer

--E 7

```

--S 8 of 38
setColumn!(dm,2,vector [y,y,y,y])
--R
--R
--R      +1  y  0  0  0+
--R      |
--R      |0  y  0  0  0|
--R      |
--R      |      3
--R      |0  y  x  0  0|
--R      |
--R      |      4
--R      |0  y  0  x  0|
--R      |
--R      +1  y  1  1  1+
--R
--E 8

```

Type: Matrix Polynomial Integer

```

--S 9 of 38
cdm := copy(dm)
--R
--R
--R      +1  y  0  0  0+
--R      |
--R      |0  y  0  0  0|
--R      |
--R      |      3
--R      |0  y  x  0  0|
--R      |
--R      |      4
--R      |0  y  0  x  0|
--R      |
--R      +1  y  1  1  1+
--R
--E 9

```

Type: Matrix Polynomial Integer

```

--S 10 of 38
setelt(dm,4,1,1-x**7)
--R
--R
--R      7
--R      (10) - x  + 1
--R
--E 10

```

Type: Polynomial Integer

```

--S 11 of 38
[dm,cdm]
--R
--R

```

```

--R      + 1      y 0 0 0+ +1 y 0 0 0+
--R      |      |      |
--R      | 0      y 0 0 0| |0 y 0 0 0|
--R      |      |      |
--R      |      3      |      3      |
--R  (11) [| 0      y x 0 0|,|0 y x 0 0|]
--R      |      |      |
--R      | 7      4      |      4      |
--R      |- x + 1 y 0 x 0| |0 y 0 x 0|
--R      |      |      |
--R      + 1      y 1 1 1+ +1 y 1 1 1+
--R
--R                                          Type: List Matrix Polynomial Integer
--E 11

```

```

--S 12 of 38
subMatrix(dm,2,3,2,4)
--R
--R
--R      +y 0 0+
--R  (12) |      |
--R      | 3  |
--R      +y x 0+
--R
--R                                          Type: Matrix Polynomial Integer
--E 12

```

```

--S 13 of 38
d := diagonalMatrix [1.2,-1.3,1.4,-1.5]
--R
--R
--R      +1.2  0.0  0.0  0.0 +
--R      |      |      |
--R      |0.0 - 1.3  0.0  0.0 |
--R  (13) |      |      |
--R      |0.0  0.0  1.4  0.0 |
--R      |      |      |
--R      +0.0  0.0  0.0 - 1.5+
--R
--R                                          Type: Matrix Float
--E 13

```

```

--S 14 of 38
e := matrix [ [6.7,9.11],[-31.33,67.19] ]
--R
--R
--R      + 6.7  9.11 +
--R  (14) |      |
--R      +- 31.33 67.19+
--R
--R                                          Type: Matrix Float
--E 14

```

```

--S 15 of 38

```



```
setsubMatrix!(d,1,2,e)
```

```
--R
--R
--R      +1.2    6.7    9.11    0.0 +
--R      |
--R      |0.0 - 31.33  67.19    0.0 |
--R  (15) |
--R      |0.0    0.0    1.4    0.0 |
--R      |
--R      +0.0    0.0    0.0 - 1.5+
```

Type: Matrix Float

```
--E 15
```

```
--S 16 of 38
```

```
d
--R
--R
--R      +1.2    6.7    9.11    0.0 +
--R      |
--R      |0.0 - 31.33  67.19    0.0 |
--R  (16) |
--R      |0.0    0.0    1.4    0.0 |
--R      |
--R      +0.0    0.0    0.0 - 1.5+
```

Type: Matrix Float

```
--E 16
```

```
--S 17 of 38
```

```
a := matrix [ [1/2,1/3,1/4],[1/5,1/6,1/7] ]
```

```
--R
--R
--R      +1  1  1+
--R      |- - -|
--R      |2  3  4|
--R  (17) |
--R      |1  1  1|
--R      |- - -|
--R      +5  6  7+
```

Type: Matrix Fraction Integer

```
--E 17
```

```
--S 18 of 38
```

```
b := matrix [ [3/5,3/7,3/11],[3/13,3/17,3/19] ]
```

```
--R
--R
--R      +3  3  3+
--R      |- - --|
--R      |5  7  11|
--R  (18) |
--R      | 3  3  3|
```

```

--R      |--  --  --|
--R      +13  17  19+
--R
--R                                          Type: Matrix Fraction Integer
--E 18

```

```

--S 19 of 38
horizConcat(a,b)
--R
--R
--R      +1  1  1  3  3  3+
--R      |-  -  -  -  -  --|
--R      |2  3  4  5  7  11|
--R      (19) |              |
--R      |1  1  1  3  3  3|
--R      |-  -  -  --  --  --|
--R      +5  6  7  13  17  19+
--R
--R                                          Type: Matrix Fraction Integer
--E 19

```

```

--S 20 of 38
vab := vertConcat(a,b)
--R
--R
--R      +1  1  1  +
--R      |-  -  -  |
--R      |2  3  4  |
--R      |              |
--R      |1  1  1  |
--R      |-  -  -  |
--R      |5  6  7  |
--R      (20) |              |
--R      |3  3  3|
--R      |-  -  --|
--R      |5  7  11|
--R      |              |
--R      | 3  3  3|
--R      |--  --  --|
--R      +13  17  19+
--R
--R                                          Type: Matrix Fraction Integer
--E 20

```

```

--S 21 of 38
transpose vab
--R
--R
--R      +1  1  3  3+
--R      |-  -  -  --|
--R      |2  5  5  13|
--R      |              |
--R      |1  1  3  3|

```

```

--R (21) |- - - --|
--R      |3 6 7 17|
--R      |      |
--R      |1 1 3 3|
--R      |- - -- --|
--R      +4 7 11 19+
--R
--E 21

```

Type: Matrix Fraction Integer

```

--S 22 of 38
m := matrix [ [1,2],[3,4] ]
--R
--R
--R      +1 2+
--R (22) |  |
--R      +3 4+
--R
--E 22

```

Type: Matrix Integer

```

--S 23 of 38
4 * m * (-5)
--R
--R
--R      +- 20 - 40+
--R (23) |      |
--R      +- 60 - 80+
--R
--E 23

```

Type: Matrix Integer

```

--S 24 of 38
n := matrix([ [1,0,-2],[-3,5,1] ])
--R
--R
--R      + 1 0 - 2+
--R (24) |      |
--R      +- 3 5 1 +
--R
--E 24

```

Type: Matrix Integer

```

--S 25 of 38
m * n
--R
--R
--R      +- 5 10 0 +
--R (25) |      |
--R      +- 9 20 - 2+
--R
--E 25

```

Type: Matrix Integer

```

--S 26 of 38

```

```

vec := column(n,3)
--R
--R
--R (26) [- 2,1]
--R
--R                                          Type: Vector Integer
--E 26

--S 27 of 38
vec * m
--R
--R
--R (27) [1,0]
--R
--R                                          Type: Vector Integer
--E 27

--S 28 of 38
m * vec
--R
--R
--R (28) [0,- 2]
--R
--R                                          Type: Vector Integer
--E 28

--S 29 of 38
hilb := matrix([ [1/(i + j) for i in 1..3] for j in 1..3])
--R
--R
--R
--R      +1  1  1+
--R      |-  -  -|
--R      |2  3  4|
--R      |      |
--R      |1  1  1|
--R (29) |-  -  -|
--R      |3  4  5|
--R      |      |
--R      |1  1  1|
--R      |-  -  -|
--R      +4  5  6+
--R
--R                                          Type: Matrix Fraction Integer
--E 29

--S 30 of 38
inverse(hilb)
--R
--R
--R
--R      + 72      - 240      180 +
--R      |          |
--R (30) |- 240      900      - 720|
--R      |          |
--R      + 180      - 720      600 +

```

```

--R                                     Type: Union(Matrix Fraction Integer,...)
--E 30

--S 31 of 38
mm := matrix([ [1,2,3,4], [5,6,7,8], [9,10,11,12], [13,14,15,16] ])
--R
--R
--R      +1   2   3   4 +
--R      |           |
--R      |5   6   7   8 |
--R  (31) |           |
--R      |9   10  11  12|
--R      |           |
--R      +13  14  15  16+
--R
--R                                     Type: Matrix Integer
--E 31

--S 32 of 38
inverse(mm)
--R
--R
--R  (32) "failed"
--R
--R                                     Type: Union("failed",...)
--E 32

--S 33 of 38
determinant(mm)
--R
--R
--R  (33) 0
--R
--R                                     Type: NonNegativeInteger
--E 33

--S 34 of 38
trace(mm)
--R
--R
--R  (34) 34
--R
--R                                     Type: PositiveInteger
--E 34

--S 35 of 38
rank(mm)
--R
--R
--R  (35) 2
--R
--R                                     Type: PositiveInteger
--E 35

--S 36 of 38

```

```

nullity(mm)
--R
--R
--R (36)  2
--R
--R                                          Type: PositiveInteger
--E 36

--S 37 of 38
nullSpace(mm)
--R
--R
--R (37)  [[1,- 2,1,0],[2,- 3,0,1]]
--R
--R                                          Type: List Vector Integer
--E 37

--S 38 of 38
rowEchelon(mm)
--R
--R
--R
--R      +1  2  3  4  +
--R      |      |
--R      |0  4  8 12|
--R (38) |      |
--R      |0  0  0  0 |
--R      |      |
--R      +0  0  0  0 +
--R
--R                                          Type: Matrix Integer
--E 38
)spool
)lisp (bye)

```

— Matrix.help —

```

=====
Matrix examples
=====

```

The Matrix domain provides arithmetic operations on matrices and standard functions from linear algebra. This domain is similar to the TwoDimensionalArray domain, except that the entries for Matrix must belong to a Ring.

```

=====
Creating Matrices
=====

```

There are many ways to create a matrix from a collection of values or

from existing matrices.

If the matrix has almost all items equal to the same value, use `new` to create a matrix filled with that value and then reset the entries that are different.

```
m : Matrix(Integer) := new(3,3,0)
+0  0  0+
|      |
|0  0  0|
|      |
+0  0  0+
Type: Matrix Integer
```

To change the entry in the second row, third column to 5, use `setelt`.

```
setelt(m,2,3,5)
5
Type: PositiveInteger
```

An alternative syntax is to use assignment.

```
m(1,2) := 10
10
Type: PositiveInteger
```

The matrix was destructively modified.

```
m
+0  10  0+
|      |
|0  0  5|
|      |
+0  0  0+
Type: Matrix Integer
```

If you already have the matrix entries as a list of lists, use `matrix`.

```
matrix [ [1,2,3,4],[0,9,8,7] ]
+1  2  3  4+
|      |
+0  9  8  7+
Type: Matrix Integer
```

If the matrix is diagonal, use `diagonalMatrix`.

```
dm := diagonalMatrix [1,x**2,x**3,x**4,x**5]
+1  0  0  0  0 +
|      |
|  2      |
|      |
```

```

|0 x 0 0 0|
|          |
|      3   |
|0 0 x 0 0|
|          |
|      4   |
|0 0 0 x 0|
|          |
|      5   |
+0 0 0 0 x +
Type: Matrix Polynomial Integer

```

Use `setRow` and `setColumn` to change a row or column of a matrix.

```

setRow!(dm,5,vector [1,1,1,1,1])
+1 0 0 0 0+
|          |
|      2   |
|0 x 0 0 0|
|          |
|      3   |
|0 0 x 0 0|
|          |
|      4   |
|0 0 0 x 0|
|          |
+1 1 1 1 1+
Type: Matrix Polynomial Integer

```

```

setColumn!(dm,2,vector [y,y,y,y,y])
+1 y 0 0 0+
|          |
|0 y 0 0 0|
|          |
|      3   |
|0 y x 0 0|
|          |
|      4   |
|0 y 0 x 0|
|          |
+1 y 1 1 1+
Type: Matrix Polynomial Integer

```

Use `copy` to make a copy of a matrix.

```

cdm := copy(dm)
+1 y 0 0 0+
|          |
|0 y 0 0 0|
|          |

```



```

|      3      |
|0  y  x  0  0|
|      |
|      4      |
|0  y  0  x  0|
|      |
+1  y  1  1  1+
Type: Matrix Polynomial Integer

```

This is useful if you intend to modify a matrix destructively but want a copy of the original.

```

setelt(dm,4,1,1-x**7)
7
- x  + 1
Type: Polynomial Integer

```

```

[dm,cdm]
+ 1      y 0  0  0+ +1 y 0  0  0+
|      |      |
| 0      y 0  0  0| |0 y 0  0  0|
|      |      |
|      3      |      3      |
[| 0      y x  0  0|,|0 y x  0  0|]
|      |      |
| 7      4      |      4      |
|- x + 1 y 0  x  0| |0 y 0  x  0|
|      |      |
+ 1      y 1  1  1+ +1 y 1  1  1+
Type: List Matrix Polynomial Integer

```

Use `subMatrix` to extract part of an existing matrix. The syntax is `subMatrix(m, firstrow, lastrow, firstcol, lastcol)`.

```

subMatrix(dm,2,3,2,4)
+y 0  0+
|      |
| 3      |
+y x  0+
Type: Matrix Polynomial Integer

```

To change a submatrix, use `setsubMatrix`.

```

d := diagonalMatrix [1.2,-1.3,1.4,-1.5]
+1.2  0.0  0.0  0.0 +
|      |
|0.0  - 1.3  0.0  0.0 |
|      |
|0.0  0.0  1.4  0.0 |
|      |

```

```
+0.0  0.0  0.0  - 1.5+
      Type: Matrix Float
```

If  $e$  is too big to fit where you specify, an error message is displayed. Use `subMatrix` to extract part of  $e$ , if necessary.

```
e := matrix [ [6.7,9.11],[-31.33,67.19] ]
      + 6.7    9.11 +
      |          |
      +- 31.33 67.19+
      Type: Matrix Float
```

This changes the submatrix of  $d$  whose upper left corner is at the first row and second column and whose size is that of  $e$ .

```
setsubMatrix!(d,1,2,e)
      +1.2    6.7    9.11    0.0 +
      |          |          |
      |0.0 - 31.33 67.19    0.0 |
      |          |          |
      |0.0    0.0    1.4    0.0 |
      |          |          |
      +0.0    0.0    0.0 - 1.5+
      Type: Matrix Float
```

```
d
      +1.2    6.7    9.11    0.0 +
      |          |          |
      |0.0 - 31.33 67.19    0.0 |
      |          |          |
      |0.0    0.0    1.4    0.0 |
      |          |          |
      +0.0    0.0    0.0 - 1.5+
      Type: Matrix Float
```

Matrices can be joined either horizontally or vertically to make new matrices.

```
a := matrix [ [1/2,1/3,1/4],[1/5,1/6,1/7] ]
      +1  1  1+
      |-  -  -|
      |2  3  4|
      |    |
      |1  1  1|
      |-  -  -|
      +5  6  7+
      Type: Matrix Fraction Integer
```

```
b := matrix [ [3/5,3/7,3/11],[3/13,3/17,3/19] ]
      +3  3  3+
```

```

|- - --|
|5 7 11|
|    |
| 3 3 3|
|-- -- --|
+13 17 19+

```

Type: Matrix Fraction Integer

Use `horizConcat` to append them side to side. The two matrices must have the same number of rows.

```

horizConcat(a,b)
+1 1 1 3 3 3+
|- - - - - --|
|2 3 4 5 7 11|
|    |
|1 1 1 3 3 3|
|- - - -- -- --|
+5 6 7 13 17 19+

```

Type: Matrix Fraction Integer

Use `vertConcat` to stack one upon the other. The two matrices must have the same number of columns.

```

vab := vertConcat(a,b)
+1 1 1 +
|- - - |
|2 3 4 |
|    |
|1 1 1 |
|- - - |
|5 6 7 |
|    |
|3 3 3|
|- - --|
|5 7 11|
|    |
| 3 3 3|
|-- -- --|
+13 17 19+

```

Type: Matrix Fraction Integer

The operation `transpose` is used to create a new matrix by reflection across the main diagonal.

```

transpose vab
+1 1 3 3+
|- - - --|
|2 5 5 13|
|    |

```

```

|1  1  3  3|
|-  -  -  --|
|3  6  7  17|
|
|1  1  3  3|
|-  -  --  --|
+4  7  11  19+

```

Type: Matrix Fraction Integer

#### Operations on Matrices

Axiom provides both left and right scalar multiplication.

```

m := matrix [ [1,2],[3,4] ]
      +1  2+
      |   |
      +3  4+

```

Type: Matrix Integer

```

4 * m * (-5)
+- 20  - 40+
|           |
+- 60  - 80+

```

Type: Matrix Integer

You can add, subtract, and multiply matrices provided, of course, that the matrices have compatible dimensions. If not, an error message is displayed.

```

n := matrix([ [1,0,-2],[-3,5,1] ])
      + 1  0  - 2+
      |           |
      +- 3  5  1 +

```

Type: Matrix Integer

This following product is defined but  $n * m$  is not.

```

m * n
+- 5  10  0 +
|           |
+- 9  20  - 2+

```

Type: Matrix Integer

The operations `nrows` and `ncols` return the number of rows and columns of a matrix. You can extract a row or a column of a matrix using the operations `row` and `column`. The object returned is a Vector.

Here is the third column of the matrix `n`.

```
vec := column(n,3)
      [- 2,1]
```

Type: Vector Integer

You can multiply a matrix on the left by a "row vector" and on the right by a "column vector".

```
vec * m
      [1,0]
```

Type: Vector Integer

Of course, the dimensions of the vector and the matrix must be compatible or an error message is returned.

```
m * vec
      [0,- 2]
```

Type: Vector Integer

The operation `inverse` computes the inverse of a matrix if the matrix is invertible, and returns "failed" if not.

This Hilbert matrix is invertible.

```
hilb := matrix([ [1/(i + j) for i in 1..3] for j in 1..3])
      +1  1  1+
      |-  -  -|
      |2  3  4|
      |   |
      |1  1  1|
      |-  -  -|
      |3  4  5|
      |   |
      |1  1  1|
      |-  -  -|
      +4  5  6+
```

Type: Matrix Fraction Integer

```
inverse(hilb)
      + 72    - 240    180 +
      |      |
      |- 240    900    - 720|
      |      |
      + 180    - 720    600 +
      Type: Union(Matrix Fraction Integer,...)
```

This matrix is not invertible.

```
mm := matrix([ [1,2,3,4], [5,6,7,8], [9,10,11,12], [13,14,15,16] ])
      +1  2  3  4 +
```

```

|      |
|5   6   7   8|
|      |
|9   10  11  12|
|      |
+13  14  15  16+

```

Type: Matrix Integer

```

inverse(mm)
"failed"

```

Type: Union("failed",...)

The operation `determinant` computes the determinant of a matrix provided that the entries of the matrix belong to a `CommutativeRing`.

The above matrix `mm` is not invertible and, hence, must have determinant 0.

```

determinant(mm)
0

```

Type: NonNegativeInteger

The operation `trace` computes the trace of a square matrix.

```

trace(mm)
34

```

Type: PositiveInteger

The operation `rank` computes the rank of a matrix: the maximal number of linearly independent rows or columns.

```

rank(mm)
2

```

Type: PositiveInteger

The operation `nullity` computes the nullity of a matrix: the dimension of its null space.

```

nullity(mm)
2

```

Type: PositiveInteger

The operation `nullSpace` returns a list containing a basis for the null space of a matrix. Note that the nullity is the number of elements in a basis for the null space.

```

nullSpace(mm)
[[1,- 2,1,0],[2,- 3,0,1]]

```

Type: List Vector Integer

The operation `rowEchelon` returns the row echelon form of a matrix. It

is easy to see that the rank of this matrix is two and that its nullity is also two.

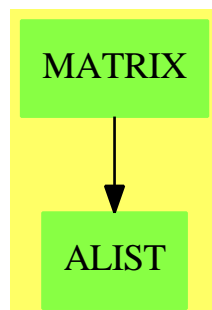
```
rowEchelon(mm)
+1  2  3  4 +
|      |
|0  4  8 12|
|      |
|0  0  0  0|
|      |
+0  0  0  0 +
Type: Matrix Integer
```

See Also

- o )help OneDimensionalArray
- o )help TwoDimensionalArray
- o )help Vector
- o )help Permanent
- o )show Matrix

—————→

### 14.7.1 Matrix (MATRIX)



See

- ⇒ “IndexedMatrix” (IMATRIX) 10.12.1 on page 1204
- ⇒ “RectangularMatrix” (RMATRIX) 19.4.1 on page 2205
- ⇒ “SquareMatrix” (SQMATRIX) 20.27.1 on page 2505

**Exports:**

|                |             |               |             |                |
|----------------|-------------|---------------|-------------|----------------|
| antisymmetric? | any?        | coerce        | column      | convert        |
| copy           | count       | determinant   | diagonal?   | diagonalMatrix |
| elt            | empty       | empty?        | eq?         | eval           |
| every?         | exquo       | fill!         | hash        | horizConcat    |
| inverse        | latex       | less?         | listOfLists | map            |
| map!           | matrix      | maxColIndex   | maxRowIndex | member?        |
| members        | minColIndex | minordet      | minRowIndex | more?          |
| ncols          | new         | nrows         | nullSpace   | nullity        |
| parts          | qelt        | qsetelt!      | rank        | row            |
| rowEchelon     | sample      | scalarMatrix  | setColumn!  | setRow!        |
| setelt         | setelt      | setsubMatrix! | size?       | square?        |
| squareTop      | subMatrix   | swapColumns!  | swapRows!   | symmetric?     |
| transpose      | vertConcat  | zero          | #?          | ?**?           |
| ?/?            | ?=?         | ?~=?          | ?*?         | ?+?            |
| -?             | ?-?         |               |             |                |

## — domain MATRIX Matrix —

```

)abbrev domain MATRIX Matrix
++ Author: Grabmeier, Gschnitzer, Williamson
++ Date Created: 1987
++ Date Last Updated: July 1990
++ Basic Operations:
++ Related Domains: IndexedMatrix, RectangularMatrix, SquareMatrix
++ Also See:
++ AMS Classifications:
++ Keywords: matrix, linear algebra
++ Examples:
++ References:
++ Description:
++ \spadtype{Matrix} is a matrix domain where 1-based indexing is used
++ for both rows and columns.

Matrix(R): Exports == Implementation where
  R : Ring
  Row ==> Vector R
  Col ==> Vector R
  mnRow ==> 1
  mnCol ==> 1
  MATLIN ==> MatrixLinearAlgebraFunctions(R,Row,Col,$)
  MATSTOR ==> StorageEfficientMatrixOperations(R)

Exports ==> MatrixCategory(R,Row,Col) with
  diagonalMatrix: Vector R -> $
  ++ \spad{diagonalMatrix(v)} returns a diagonal matrix where the elements
  ++ of v appear on the diagonal.

if R has ConvertibleTo InputForm then ConvertibleTo InputForm

```



```

if R has Field then
  inverse: $ -> Union($,"failed")
    ++ \spad{inverse(m)} returns the inverse of the matrix m.
    ++ If the matrix is not invertible, "failed" is returned.
    ++ Error: if the matrix is not square.
-- matrix: Vector Vector R -> $
--   ++ \spad{matrix(v)} converts the vector of vectors v to a matrix, where
--   ++ the vector of vectors is viewed as a vector of the rows of the
--   ++ matrix
-- diagonalMatrix: Vector $ -> $
--   ++ \spad{diagonalMatrix([m1,...,mk])} creates a block diagonal matrix
--   ++ M with block matrices m1,...,mk down the diagonal,
--   ++ with 0 block matrices elsewhere.
-- vectorOfVectors: $ -> Vector Vector R
--   ++ \spad{vectorOfVectors(m)} returns the rows of the matrix m as a
--   ++ vector of vectors

```

```

Implementation ==>

```

```

InnerIndexedTwoDimensionalArray(R,mnRow,mnCol,Row,Col) add
minr ==> minRowIndex
maxr ==> maxRowIndex
minc ==> minColIndex
maxc ==> maxColIndex
mini ==> minIndex
maxi ==> maxIndex

```

```

minRowIndex x == mnRow
minColIndex x == mnCol

```

```

swapRows_!(x,i1,i2) ==
  (i1 < minRowIndex(x)) or (i1 > maxRowIndex(x)) or _
  (i2 < minRowIndex(x)) or (i2 > maxRowIndex(x)) =>
    error "swapRows!: index out of range"
  i1 = i2 => x
  minRow := minRowIndex x
  xx := x pretend PrimitiveArray(PrimitiveArray(R))
  n1 := i1 - minRow; n2 := i2 - minRow
  row1 := qelt(xx,n1)
  qsetelt_!(xx,n1,qelt(xx,n2))
  qsetelt_!(xx,n2,row1)
  xx pretend $

```

```

positivePower($,Integer,NonNegativeInteger) -> $
positivePower(x,n,nn) ==
-- one? n => x
  (n = 1) => x
  -- no need to allocate space for 3 additional matrices
  n = 2 => x * x
  n = 3 => x * x * x
  n = 4 => (y := x * x; y * y)

```

```

a := new(nn,nn,0) pretend Matrix(R)
b := new(nn,nn,0) pretend Matrix(R)
c := new(nn,nn,0) pretend Matrix(R)
xx := x pretend Matrix(R)
power_!(a,b,c,xx,n :: NonNegativeInteger)$MATSTOR pretend $

x:$ ** n:NonNegativeInteger ==
  not((nn := nrows x) = ncols x) =>
    error "***: matrix must be square"
  zero? n => scalarMatrix(nn,1)
  positivePower(x,n,nn)

if R has commutative("*") then

  determinant x == determinant(x)$MATLIN
  minordet    x == minordet(x)$MATLIN

if R has EuclideanDomain then

  rowEchelon x == rowEchelon(x)$MATLIN

if R has IntegralDomain then

  rank      x == rank(x)$MATLIN
  nullity   x == nullity(x)$MATLIN
  nullSpace x == nullSpace(x)$MATLIN

if R has Field then

  inverse    x == inverse(x)$MATLIN

x:$ ** n:Integer ==
  nn := nrows x
  not(nn = ncols x) =>
    error "***: matrix must be square"
  zero? n => scalarMatrix(nn,1)
  positive? n => positivePower(x,n,nn)
  (xInv := inverse x) case "failed" =>
    error "***: matrix must be invertible"
  positivePower(xInv :: $,-n,nn)

-- matrix(v: Vector Vector R) ==
--   (rows := # v) = 0 => new(0,0,0)
--   -- error check: this is a top level function
--   cols := # v.mini(v)
--   for k in (mini(v) + 1)..maxi(v) repeat
--     cols ^= # v.k => error "matrix: rows of different lengths"
--   ans := new(rows,cols,0)
--   for i in minr(ans)..maxr(ans) for k in mini(v)..maxi(v) repeat
--     vv := v.k

```

```

--      for j in minc(ans)..maxc(ans) for l in mini(vv)..maxi(vv) repeat
--      ans(i,j) := vv.l
--      ans

diagonalMatrix(v: Vector R) ==
  n := #v; ans := zero(n,n)
  for i in minr(ans)..maxr(ans) for j in minc(ans)..maxc(ans) _
    for k in mini(v)..maxi(v) repeat qsetelt_!(ans,i,j,qelt(v,k))
  ans

diagonalMatrix(vec: Vector $) ==
  rows : NonNegativeInteger := 0
  cols : NonNegativeInteger := 0
  for r in mini(vec)..maxi(vec) repeat
  mat := vec.r
  rows := rows + nrows mat; cols := cols + ncols mat
  ans := zero(rows,cols)
  loR := minr ans; loC := minc ans
  for r in mini(vec)..maxi(vec) repeat
  mat := vec.r
  hiR := loR + nrows(mat) - 1; hiC := loC + ncols(mat) - 1
  for i in loR..hiR for k in minr(mat)..maxr(mat) repeat
  for j in loC..hiC for l in minc(mat)..maxc(mat) repeat
  ans(i,j) := mat(k,l)
  loR := hiR + 1; loC := hiC + 1
  ans

vectorOfVectors x ==
  vv : Vector Vector R := new(nrows x,0)
  cols := ncols x
  for k in mini(vv)..maxi(vv) repeat
  vv.k := new(cols,0)
  for i in minr(x)..maxr(x) for k in mini(vv)..maxi(vv) repeat
  v := vv.k
  for j in minc(x)..maxc(x) for l in mini(v)..maxi(v) repeat
  v.l := x(i,j)
  vv

if R has ConvertibleTo InputForm then
  convert(x:$):InputForm ==
    convert [convert("matrix"::Symbol)@InputForm,
      convert listOfLists x]$List(InputForm)

```

---

— MATRIX.dotabb —

"MATRIX" [color="#88FF44",href="bookvol10.3.pdf#nameddest=MATRIX"]

"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]  
 "MATRIX" -> "ALIST"

## 14.8 domain MODMON ModMonic

— ModMonic.input —

```
)set break resume
)sys rm -f ModMonic.output
)spool ModMonic.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ModMonic
--R ModMonic(R: Ring,Rep: UnivariatePolynomialCategory R) is a domain constructor
--R Abbreviation for ModMonic is MODMON
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for MODMON
--R
--R----- Operations -----
--R ?? : (R,%) -> %
--R ?? : (Integer,%) -> %
--R ?? : (PositiveInteger,%) -> %
--R ?? : (%,%) -> %
--R -? : (%,%) -> Boolean
--R An : % -> Vector R
--R D : (%,(R -> R)) -> %
--R D : (%,NonNegativeInteger) -> %
--R 1 : () -> %
--R UnVectorise : Vector R -> %
--R 0 : () -> %
--R Vectorise : % -> Vector R
--R coefficients : % -> List R
--R ?? : (%,PositiveInteger) -> %
--R coerce : R -> %
--R coerce : Integer -> %
--R coerce : % -> OutputForm
--R degree : % -> NonNegativeInteger
--R differentiate : % -> %
--R ?? : (%,R) -> R
--R eval : (%,List %,List %) -> %
--R eval : (%,Equation %) -> %
--R eval : (%,List Equation %) -> %
--R ground? : % -> Boolean
--R hash : % -> SingleInteger
--R init : () -> % if R has STEP
--R latex : % -> String
--R leadingCoefficient : % -> R
--R lift : % -> Rep
--R map : ((R -> R),%) -> %
--R modulus : () -> Rep
--R monomial? : % -> Boolean
--R monomials : % -> List %
```

```

--R one? : % -> Boolean
--R primitiveMonomials : % -> List %
--R recip : % -> Union(%, "failed")
--R reductum : % -> %
--R sample : () -> %
--R zero? : % -> Boolean
--R ?? : (Fraction Integer, %) -> % if R has ALGEBRA FRAC INT
--R ?? : (%, Fraction Integer) -> % if R has ALGEBRA FRAC INT
--R ?? : (NonNegativeInteger, %) -> %
--R ??? : (%, NonNegativeInteger) -> %
--R ?/? : (%, R) -> % if R has FIELD
--R ?<? : (%, %) -> Boolean if R has ORDSET
--R ?<=? : (%, %) -> Boolean if R has ORDSET
--R ?>? : (%, %) -> Boolean if R has ORDSET
--R ?>=? : (%, %) -> Boolean if R has ORDSET
--R D : (%, (R -> R), NonNegativeInteger) -> %
--R D : (%, List Symbol, List NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (%, Symbol, NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (%, List Symbol) -> % if R has PDRING SYMBOL
--R D : (%, Symbol) -> % if R has PDRING SYMBOL
--R D : (%, List SingletonAsOrderedSet, List NonNegativeInteger) -> %
--R D : (%, SingletonAsOrderedSet, NonNegativeInteger) -> %
--R D : (%, List SingletonAsOrderedSet) -> %
--R D : (%, SingletonAsOrderedSet) -> %
--R ?? : (%, NonNegativeInteger) -> %
--R associates? : (%, %) -> Boolean if R has INTDOM
--R binomThmExpt : (%, %, NonNegativeInteger) -> % if R has COMRING
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%, "failed") if $ has CHARNZ and R has PFECAT or R has CHARNZ
--R coefficient : (%, List SingletonAsOrderedSet, List NonNegativeInteger) -> %
--R coefficient : (%, SingletonAsOrderedSet, NonNegativeInteger) -> %
--R coefficient : (%, NonNegativeInteger) -> R
--R coerce : % -> % if R has INTDOM
--R coerce : Fraction Integer -> % if R has ALGEBRA FRAC INT or R has RETRACT FRAC INT
--R coerce : SingletonAsOrderedSet -> %
--R composite : (Fraction %, %) -> Union(Fraction %, "failed") if R has INTDOM
--R composite : (%, %) -> Union(%, "failed") if R has INTDOM
--R computePowers : () -> PrimitiveArray %
--R conditionP : Matrix % -> Union(Vector %, "failed") if $ has CHARNZ and R has PFECAT
--R content : (%, SingletonAsOrderedSet) -> % if R has GCDDOM
--R content : % -> R if R has GCDDOM
--R convert : % -> InputForm if SingletonAsOrderedSet has KONVERT INFORM and R has KONVERT I
--R convert : % -> Pattern Integer if SingletonAsOrderedSet has KONVERT PATTERN INT and R has
--R convert : % -> Pattern Float if SingletonAsOrderedSet has KONVERT PATTERN FLOAT and R has
--R degree : (%, List SingletonAsOrderedSet) -> List NonNegativeInteger
--R degree : (%, SingletonAsOrderedSet) -> NonNegativeInteger
--R differentiate : (%, (R -> R), %) -> %
--R differentiate : (%, (R -> R)) -> %
--R differentiate : (%, (R -> R), NonNegativeInteger) -> %
--R differentiate : (%, List Symbol, List NonNegativeInteger) -> % if R has PDRING SYMBOL

```

```

--R differentiate : (% , Symbol , NonNegativeInteger) -> % if R has PDRING SYMBOL
--R differentiate : (% , List Symbol) -> % if R has PDRING SYMBOL
--R differentiate : (% , Symbol) -> % if R has PDRING SYMBOL
--R differentiate : (% , NonNegativeInteger) -> %
--R differentiate : (% , List SingletonAsOrderedSet , List NonNegativeInteger) -> %
--R differentiate : (% , SingletonAsOrderedSet , NonNegativeInteger) -> %
--R differentiate : (% , List SingletonAsOrderedSet) -> %
--R differentiate : (% , SingletonAsOrderedSet) -> %
--R discriminant : % -> R if R has COMRING
--R discriminant : (% , SingletonAsOrderedSet) -> % if R has COMRING
--R divide : (% , %) -> Record(quotient: % , remainder: %) if R has FIELD
--R divideExponents : (% , NonNegativeInteger) -> Union(% , "failed")
--R ?.? : (% , Fraction %) -> Fraction % if R has INTDOM
--R elt : (Fraction % , R) -> R if R has FIELD
--R elt : (Fraction % , Fraction %) -> Fraction % if R has INTDOM
--R euclideanSize : % -> NonNegativeInteger if R has FIELD
--R eval : (% , List SingletonAsOrderedSet , List %) -> %
--R eval : (% , SingletonAsOrderedSet , %) -> %
--R eval : (% , List SingletonAsOrderedSet , List R) -> %
--R eval : (% , SingletonAsOrderedSet , R) -> %
--R expressIdealMember : (List % , %) -> Union(List % , "failed") if R has FIELD
--R exquo : (% , %) -> Union(% , "failed") if R has INTDOM
--R exquo : (% , R) -> Union(% , "failed") if R has INTDOM
--R extendedEuclidean : (% , %) -> Record(coef1: % , coef2: % , generator: %) if R has FIELD
--R extendedEuclidean : (% , % , %) -> Union(Record(coef1: % , coef2: % ) , "failed") if R has FIELD
--R factor : % -> Factored % if R has PFECAT
--R factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % if R has PF
--R factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % if R has PF
--R frobenius : % -> % if R has FFIELDC
--R gcd : (% , %) -> % if R has GCDDOM
--R gcd : List % -> % if R has GCDDOM
--R gcdPolynomial : (SparseUnivariatePolynomial % , SparseUnivariatePolynomial %) -> SparseUnivariatePolynomial % if R has GCDDOM
--R index : PositiveInteger -> % if R has FINITE
--R integrate : % -> % if R has ALGEBRA FRAC INT
--R isExpt : % -> Union(Record(var: SingletonAsOrderedSet , exponent: NonNegativeInteger) , "failed")
--R isPlus : % -> Union(List % , "failed")
--R isTimes : % -> Union(List % , "failed")
--R karatsubaDivide : (% , NonNegativeInteger) -> Record(quotient: % , remainder: %)
--R lcm : (% , %) -> % if R has GCDDOM
--R lcm : List % -> % if R has GCDDOM
--R lookup : % -> PositiveInteger if R has FINITE
--R mainVariable : % -> Union(SingletonAsOrderedSet , "failed")
--R makeSUP : % -> SparseUnivariatePolynomial R
--R mapExponents : ((NonNegativeInteger -> NonNegativeInteger) , %) -> %
--R max : (% , %) -> % if R has ORDSET
--R min : (% , %) -> % if R has ORDSET
--R minimumDegree : (% , List SingletonAsOrderedSet) -> List NonNegativeInteger
--R minimumDegree : (% , SingletonAsOrderedSet) -> NonNegativeInteger
--R minimumDegree : % -> NonNegativeInteger
--R monicDivide : (% , %) -> Record(quotient: % , remainder: %)

```

```

--R monicDivide : (%,%,SingletonAsOrderedSet) -> Record(quotient: %,remainder: %)
--R monomial : (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R monomial : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R monomial : (R,NonNegativeInteger) -> %
--R multiEuclidean : (List %,%) -> Union(List %,"failed") if R has FIELD
--R multiplyExponents : (%,NonNegativeInteger) -> %
--R multivariate : (SparseUnivariatePolynomial %,SingletonAsOrderedSet) -> %
--R multivariate : (SparseUnivariatePolynomial R,SingletonAsOrderedSet) -> %
--R nextItem : % -> Union(%,"failed") if R has STEP
--R numberOfMonomials : % -> NonNegativeInteger
--R order : (%,%) -> NonNegativeInteger if R has INTDOM
--R patternMatch : (%,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMatchResult(Integer,%)
--R patternMatch : (%,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchResult(Float,%)
--R pomopo! : (%,R,NonNegativeInteger,%) -> %
--R prime? : % -> Boolean if R has PFECAT
--R primitivePart : (%,SingletonAsOrderedSet) -> % if R has GCDDOM
--R primitivePart : % -> % if R has GCDDOM
--R principalIdeal : List % -> Record(coef: List %,generator: %) if R has FIELD
--R pseudoDivide : (%,%) -> Record(coef: R,quotient: %,remainder: %) if R has INTDOM
--R pseudoQuotient : (%,%) -> % if R has INTDOM
--R ?quo? : (%,%) -> % if R has FIELD
--R random : () -> % if R has FINITE
--R reducedSystem : Matrix % -> Matrix R
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) if R has INTDOM
--R reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT
--R ?rem? : (%,%) -> % if R has FIELD
--R resultant : (%,%) -> R if R has COMRING
--R resultant : (%,%,SingletonAsOrderedSet) -> % if R has COMRING
--R retract : % -> SingletonAsOrderedSet
--R retract : % -> Integer if R has RETRACT INT
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(SingletonAsOrderedSet,"failed")
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(R,"failed")
--R separate : (%,%) -> Record(primePart: %,commonPart: %) if R has GCDDOM
--R shiftLeft : (%,NonNegativeInteger) -> %
--R shiftRight : (%,NonNegativeInteger) -> %
--R size : () -> NonNegativeInteger if R has FINITE
--R sizeLess? : (%,%) -> Boolean if R has FIELD
--R solveLinearPolynomialEquation : (List SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> %
--R squareFree : % -> Factored % if R has GCDDOM
--R squareFreePart : % -> % if R has GCDDOM
--R squareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial %
--R subResultantGcd : (%,%) -> % if R has INTDOM
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R totalDegree : (%,List SingletonAsOrderedSet) -> NonNegativeInteger
--R totalDegree : % -> NonNegativeInteger
--R unit? : % -> Boolean if R has INTDOM

```

```

--R unitCanonical : % -> % if R has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has INTDOM
--R univariate : % -> SparseUnivariatePolynomial R
--R univariate : (% ,SingletonAsOrderedSet) -> SparseUnivariatePolynomial %
--R unmakeSUP : SparseUnivariatePolynomial R -> %
--R variables : % -> List SingletonAsOrderedSet
--R vectorise : (% ,NonNegativeInteger) -> Vector R
--R
--E 1

```

```

)spool
)lisp (bye)

```

---

— ModMonic.help —

```

=====
ModMonic examples
=====

```

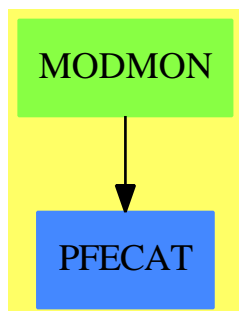
```

See Also:
o )show ModMonic

```

---

### 14.8.1 ModMonic (MODMON)



Exports:



|                      |                               |                   |                |
|----------------------|-------------------------------|-------------------|----------------|
| 0                    | 1                             | An                | associates?    |
| binomThmExpt         | characteristic                | charthRoot        | coefficient    |
| coefficients         | coerce                        | composite         | computePowers  |
| conditionP           | content                       | convert           | D              |
| degree               | differentiate                 | discriminant      | divide         |
| divideExponents      | elt                           | euclideanSize     | eval           |
| expressIdealMember   | exquo                         | extendedEuclidean | factor         |
| factorPolynomial     | factorSquareFreePolynomial    | frobenius         | gcd            |
| gcdPolynomial        | ground                        | ground?           | hash           |
| index                | init                          | integrate         | isExpt         |
| isPlus               | isTimes                       | karatsubaDivide   | latex          |
| lcm                  | leadingCoefficient            | leadingMonomial   | lift           |
| lookup               | mainVariable                  | makeSUP           | map            |
| mapExponents         | max                           | min               | minimumDegree  |
| modulus              | monicDivide                   | monomial          | monomial?      |
| monomials            | multiEuclidean                | multiplyExponents | multivariate   |
| nextItem             | numberOfMonomials             | one?              | order          |
| patternMatch         | pomopo!                       | pow               | prime?         |
| primitiveMonomials   | primitivePart                 | principalIdeal    | pseudoDivide   |
| pseudoQuotient       | pseudoRemainder               | random            | recip          |
| reduce               | reducedSystem                 | reductum          | resultant      |
| retract              | retractIfCan                  | sample            | separate       |
| setPoly              | shiftLeft                     | shiftRight        | size           |
| sizeLess?            | solveLinearPolynomialEquation | squareFree        | squareFreePart |
| squareFreePolynomial | subResultantGcd               | subtractIfCan     | totalDegree    |
| unit?                | unitCanonical                 | unitNormal        | univariate     |
| unmakeSUP            | UnVectorise                   | variables         | vectorise      |
| Vectorise            | zero?                         | ?*?               | ?**?           |
| ?+?                  | ?-?                           | -?                | ?=?            |
| ?^?                  | ?..?                          | ?~=?              | ?/?            |
| ?<?                  | ?<=?                          | ?>?               | ?>=?           |
| ?quo?                | ?rem?                         |                   |                |

— domain MODMON ModMonic —

```

)abbrev domain MODMON ModMonic
++ Author: Mark Botch
++ Description:
++ This package has not been documented
-- following line prevents caching ModMonic
)bo PUSH('ModMonic, $mutableDomains)

ModMonic(R,Rep): C == T
where
  R: Ring
  Rep: UnivariatePolynomialCategory(R)
  C == UnivariatePolynomialCategory(R) with

```

```

--operations
  setPoly : Rep -> Rep
    ++ setPoly(x) is not documented
  modulus : -> Rep
    ++ modulus() is not documented
  reduce: Rep -> %
    ++ reduce(x) is not documented
  lift: % -> Rep --reduce lift = identity
    ++ lift(x) is not documented
  coerce: Rep -> %
    ++ coerce(x) is not documented
  Vectorise: % -> Vector(R)
    ++ Vectorise(x) is not documented
  UnVectorise: Vector(R) -> %
    ++ UnVectorise(v) is not documented
  An: % -> Vector(R)
    ++ An(x) is not documented
  pow : -> PrimitiveArray(%)
    ++ pow() is not documented
  computePowers : -> PrimitiveArray(%)
    ++ computePowers() is not documented
  if R has FiniteFieldCategory then
    frobenius: % -> %
      ++ frobenius(x) is not documented
  --LinearTransf: (% , Vector(R)) -> SquareMatrix<deg> R
--assertions
  if R has Finite then Finite
T == add
--constants
  m:Rep := monomial(1,1)$Rep --| degree(m) > 0 and LeadingCoef(m) = R$1
  d := degree(m)$Rep
  d1 := (d-1):NonNegativeInteger
  twod := 2*d1+1
  frobenius?:Boolean := R has FiniteFieldCategory
  --VectorRep:= DirectProduct(d:NonNegativeInteger,R)
--declarations
  x,y: %
  p: Rep
  d,n: Integer
  e,k1,k2: NonNegativeInteger
  c: R
  --vect: Vector(R)
  power:PrimitiveArray(%)
  frobeniusPower:PrimitiveArray(%)
  computeFrobeniusPowers : () -> PrimitiveArray(%)
--representations
--mutable m    --take this out??
--define
  power := new(0,0)
  frobeniusPower := new(0,0)

```

```

setPoly (mon : Rep) ==
  mon = $Rep m => mon
  oldm := m
  leadingCoefficient mon ^= 1 => error "polynomial must be monic"
  -- following copy code needed since FFPOLY can modify mon
  copymon:Rep:= 0
  while not zero? mon repeat
    copymon := monomial(leadingCoefficient mon, degree mon)$Rep + copymon
    mon := reductum mon
  m := copymon
  d := degree(m)$Rep
  d1 := (d-1)::NonNegativeInteger
  twod := 2*d1+1
  power := computePowers()
  if frobenius? then
    degree(oldm)>1 and not((oldm exquo$Rep m) case "failed") =>
      for i in 1..d1 repeat
        frobeniusPower(i) := reduce lift frobeniusPower(i)
      frobeniusPower := computeFrobeniusPowers()
  m
modulus == m
if R has Finite then
  size == d * size$R
  random == UnVectorise([random()$R for i in 0..d1])
0 == 0$Rep
1 == 1$Rep
c * x == c * $Rep x
n * x == (n::R) * $Rep x
coerce(c:R):% == monomial(c,0)$Rep
coerce(x:%):OutputForm == coerce(x)$Rep
coefficient(x,e):R == coefficient(x,e)$Rep
reductum(x) == reductum(x)$Rep
leadingCoefficient x == (leadingCoefficient x)$Rep
degree x == (degree x)$Rep
lift(x) == x pretend Rep
reduce(p) == (monicDivide(p,m)$Rep).remainder
coerce(p) == reduce(p)
x = y == x = $Rep y
x + y == x + $Rep y
- x == -$Rep x
x * y ==
  p := x * $Rep y
  ans:=0$Rep
  while (n:=degree p)>d1 repeat
    ans:=ans + leadingCoefficient(p)*power.(n-d)
    p := reductum p
  ans+p
Vectorise(x) == [coefficient(lift(x),i) for i in 0..d1]
UnVectorise(vect) ==
  reduce(+/[monomial(vect.(i+1),i) for i in 0..d1])

```

```

computePowers ==
  mat : PrimitiveArray(%) := new(d,0)
  mat.0 := reductum(-m)$Rep
  w: % := monomial$Rep (1,1)
  for i in 1..d1 repeat
    mat.i := w *$Rep mat.(i-1)
    if degree mat.i=d then
      mat.i:= reductum mat.i + leadingCoefficient mat.i * mat.0
  mat
if frobenius? then
  computeFrobeniusPowers() ==
    mat : PrimitiveArray(%) := new(d,1)
    mat.1:= mult := monomial(1, size$R)$%
    for i in 2..d1 repeat
      mat.i := mult * mat.(i-1)
    mat

  frobenius(a:%) := % ==
    aq: % := 0
    while a^=0 repeat
      aq:= aq + leadingCoefficient(a)*frobeniusPower(degree a)
      a := reductum a
    aq

pow == power
monomial(c,e)==
  if e<d then monomial(c,e)$Rep
  else
    if e<=twod then
      c * power.(e-d)
    else
      k1:=e quo twod
      k2 := (e-k1*twod)::NonNegativeInteger
      reduce((power.d1 **k1)*monomial(c,k2))
if R has Field then

  (x:% exquo y: %):Union(%, "failed") ==
    uv := extendedEuclidean(y, modulus(), x)$Rep
    uv case "failed" => "failed"
    return reduce(uv.coef1)

  recip(y: %):Union(%, "failed") == 1 exquo y
  divide(x: %, y: %) ==
    (q := (x exquo y)) case "failed" => error "not divisible"
    [q, 0]

-- An(MM) == Vectorise(-(reduce(reductum(m))::MM))
-- LinearTransf(vect,MM) ==
--   ans:= 0::SquareMatrix<d>(R)
--   for i in 1..d do setelt(ans,i,1,vect.i)

```

```

--      for j in 2..d do
--          setelt(ans,1,j, elt(ans,d,j-1) * An(MM).1)
--      for i in 2..d do
--          setelt(ans,i,j, elt(ans,i-1,j-1) + elt(ans,d,j-1) * An(MM).i)
--      ans

```

---

— MODMON.dotabb —

```

"MODMON" [color="#88FF44",href="bookvol10.3.pdf#nameddest=MODMON"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"MODMON" -> "PFECAT"

```

---

## 14.9 domain MODFIELD ModularField

— ModularField.input —

```

)set break resume
)sys rm -f ModularField.output
)spool ModularField.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ModularField
--R ModularField(R: CommutativeRing,Mod: AbelianMonoid,reduction: ((R,Mod) -> R),merge: ((Mod,Mod) -> Mod))
--R Abbreviation for ModularField is MODFIELD
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for MODFIELD
--R
--R----- Operations -----
--R ?? : (Fraction Integer,%) -> %      ??? : (%,Fraction Integer) -> %
--R ?? : (%,%) -> %                    ??? : (Integer,%) -> %
--R ?? : (PositiveInteger,%) -> %      ??? : (%,Integer) -> %
--R ??? : (%,PositiveInteger) -> %    ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                  -? : % -> %
--R ?/? : (%,%) -> %                  ?=? : (%,%) -> Boolean
--R 1 : () -> %                       0 : () -> %
--R ?? : (%,Integer) -> %             ?? : (%,PositiveInteger) -> %
--R associates? : (%,%) -> Boolean    coerce : % -> R

```

```

--R coerce : Fraction Integer -> %      coerce : % -> %
--R coerce : Integer -> %              coerce : % -> OutputForm
--R factor : % -> Factored %           gcd : List % -> %
--R gcd : (%,% ) -> %                 hash : % -> SingleInteger
--R inv : % -> %                      latex : % -> String
--R lcm : List % -> %                 lcm : (%,% ) -> %
--R modulus : % -> Mod                one? : % -> Boolean
--R prime? : % -> Boolean              ?quo? : (%,% ) -> %
--R recip : % -> Union(%, "failed")    reduce : (R, Mod) -> %
--R ?rem? : (%,% ) -> %               sample : () -> %
--R sizeLess? : (%,% ) -> Boolean      squareFree : % -> Factored %
--R squareFreePart : % -> %           unit? : % -> Boolean
--R unitCanonical : % -> %            zero? : % -> Boolean
--R ?~=? : (%,% ) -> Boolean
--R ?*? : (NonNegativeInteger,% ) -> %
--R ?**? : (% , NonNegativeInteger) -> %
--R ?^? : (% , NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R divide : (%,% ) -> Record(quotient: %, remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R exQuo : (%,% ) -> Union(%, "failed")
--R expressIdealMember : (List %, %) -> Union(List %, "failed")
--R exquo : (%,% ) -> Union(%, "failed")
--R extendedEuclidean : (%,% , %) -> Union(Record(coef1: %, coef2: %), "failed")
--R extendedEuclidean : (%,% ) -> Record(coef1: %, coef2: %, generator: %)
--R gcdPolynomial : (SparseUnivariatePolynomial %, SparseUnivariatePolynomial %) -> SparseUnivariatePolynomial
--R multiEuclidean : (List %, %) -> Union(List %, "failed")
--R principalIdeal : List % -> Record(coef: List %, generator: %)
--R subtractIfCan : (%,% ) -> Union(%, "failed")
--R unitNormal : % -> Record(unit: %, canonical: %, associate: %)
--R
--E 1

)spool
)lisp (bye)

```

---

— ModularField.help —

```

=====
ModularField examples
=====

```

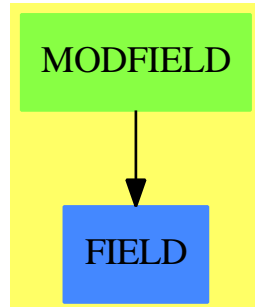
```

See Also:
o )show ModularField

```

---

### 14.9.1 ModularField (MODFIELD)



See

⇒ “ModularRing” (MODRING) 14.10.1 on page 1604

⇒ “EuclideanModularRing” (EMR) 6.3.1 on page 670

#### Exports:

|                   |               |                    |                |                |
|-------------------|---------------|--------------------|----------------|----------------|
| 0                 | 1             | associates?        | characteristic | coerce         |
| divide            | euclideanSize | expressIdealMember | exquo          | exQuo          |
| extendedEuclidean | factor        | gcd                | gcdPolynomial  | hash           |
| inv               | latex         | lcm                | modulus        | multiEuclidean |
| one?              | prime?        | principalIdeal     | recip          | reduce         |
| sample            | sizeLess?     | squareFree         | squareFreePart | subtractIfCan  |
| unit?             | unitCanonical | unitNormal         | zero?          | ?*?            |
| ?**?              | ?+?           | ?-?                | -?             | ?/?            |
| ?=?               | ?^?           | ?~=?               | ?quo?          | ?rem?          |

— domain MODFIELD ModularField —

```

)abbrev domain MODFIELD ModularField
++ Author: Mark Botch
++ Description:
++ These domains are used for the factorization and gcds
++ of univariate polynomials over the integers in order to work modulo
++ different primes.
++ See \spadtype{ModularRing}, \spadtype{EuclideanModularRing}

ModularField(R,Mod,reduction:(R,Mod) -> R,
              merge:(Mod,Mod) -> Union(Mod,"failed"),
              exactQuo : (R,R,Mod) -> Union(R,"failed")) : C == T

where
  R    : CommutativeRing
  Mod  : AbelianMonoid

  C == Field with
    modulus: % -> Mod
    ++ modulus(x) is not documented
  
```

```

coerce: % -> R
  ++ coerce(x) is not documented
reduce: (R,Mod) -> %
  ++ reduce(r,m) is not documented
exQuo: (%,% ) -> Union(%, "failed")
  ++ exQuo(x,y) is not documented

T == ModularRing(R,Mod,reduction,merge,exactQuo)

-----

-- MODFIELD.dotabb --

"MODFIELD" [color="#88FF44",href="bookvol10.3.pdf#nameddest=MODFIELD"]
"FIELD" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FIELD"]
"MODFIELD" -> "FIELD"

```

## 14.10 domain MODRING ModularRing

— ModularRing.input —

```

)set break resume
)sys rm -f ModularRing.output
)spool ModularRing.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ModularRing
--R ModularRing(R: CommutativeRing,Mod: AbelianMonoid,reduction: ((R,Mod) -> R),merge: ((Mod,Mod) -> Uni
--R Abbreviation for ModularRing is MODRING
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for MODRING
--R
--R----- Operations -----
--R ?? : (%,% ) -> %                ?? : (Integer,%) -> %
--R ?? : (PositiveInteger,%) -> %   ??? : (% ,PositiveInteger) -> %
--R ?? : (%,% ) -> %                ?-? : (%,% ) -> %
--R -? : % -> %                     ?? : (%,% ) -> Boolean
--R 1 : () -> %                     0 : () -> %
--R ?? : (% ,PositiveInteger) -> %   coerce : % -> R

```



```

--R coerce : Integer -> %
--R hash : % -> SingleInteger
--R latex : % -> String
--R one? : % -> Boolean
--R reduce : (R,Mod) -> %
--R zero? : % -> Boolean
--R ?? : (NonNegativeInteger,%) -> %
--R ??? : (%,NonNegativeInteger) -> %
--R ?? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R exQuo : (%,%) -> Union(%, "failed")
--R subtractIfCan : (%,%) -> Union(%, "failed")
--R
--E 1

)spool
)lisp (bye)

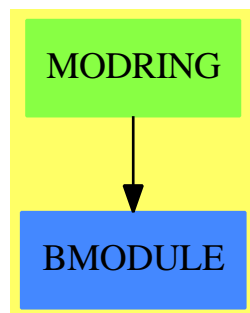
```

— ModularRing.help —

=====  
ModularRing examples  
=====

See Also:  
o )show ModularRing

### 14.10.1 ModularRing (MODRING)



See

⇒ “EuclideanModularRing” (EMR) 6.3.1 on page 670

⇒ “ModularField” (MODFIELD) 14.9.1 on page 1602

### Exports:

|       |        |                |               |       |
|-------|--------|----------------|---------------|-------|
| 0     | 1      | characteristic | coerce        | exQuo |
| hash  | inv    | latex          | modulus       | one?  |
| recip | reduce | sample         | subtractIfCan | zero? |
| ?~=?  | ?*?    | ?**?           | ?^?           | ?+?   |
| ?-?   | -?     | ?=?            |               |       |

— domain MODRING ModularRing —

```
)abbrev domain MODRING ModularRing
++ Author: P.Gianni, B.Trager
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ These domains are used for the factorization and gcds
++ of univariate polynomials over the integers in order to work modulo
++ different primes.
++ See \spadtype{EuclideanModularRing} ,\spadtype{ModularField}

ModularRing(R,Mod,reduction:(R,Mod) -> R,
            merge:(Mod,Mod) -> Union(Mod,"failed"),
            exactQuo : (R,R,Mod) -> Union(R,"failed")) : C == T

where
  R    : CommutativeRing
  Mod  : AbelianMonoid

C == Ring with
  modulus: % -> Mod
    ++ modulus(x) is not documented
  coerce: % -> R
    ++ coerce(x) is not documented
  reduce: (R,Mod) -> %
    ++ reduce(r,m) is not documented
  exQuo: (%,%)-> Union(%, "failed")
    ++ exQuo(x,y) is not documented
  recip: % -> Union(%, "failed")
    ++ recip(x) is not documented
  inv: % -> %
    ++ inv(x) is not documented

T == add
```

```

--representation
Rep:= Record(val:R,module:Mod)
--declarations
x,y: %

--define
modulus(x) == x.module
coerce(x) == x.val
coerce(i:Integer):% == [i::R,0]$Rep
i:Integer * x:% == (i::%)*x
coerce(x):OutputForm == (x.val)::OutputForm
reduce (a:R,m:Mod) == [reduction(a,m),m]$Rep

characteristic():NonNegativeInteger == characteristic()$R
0 == [0$R,0$Mod]$Rep
1 == [1$R,0$Mod]$Rep
zero? x == zero? x.val
-- one? x == one? x.val
one? x == (x.val = 1)

newmodulo(m1:Mod,m2:Mod) : Mod ==
  r:=merge(m1,m2)
  r case "failed" => error "incompatible moduli"
  r::Mod

x=y ==
  x.val = y.val => true
  x.module = y.module => false
  (x-y).val = 0
x+y == reduce((x.val +$R y.val),newmodulo(x.module,y.module))
x-y == reduce((x.val -$R y.val),newmodulo(x.module,y.module))
-x == reduce ((-$R x.val),x.module)
x*y == reduce((x.val *$R y.val),newmodulo(x.module,y.module))

exQuo(x,y) ==
  xm:=x.module
  if xm ~=$Mod y.module then xm:=newmodulo(xm,y.module)
  r:=exactQuo(x.val,y.val,xm)
  r case "failed"=> "failed"
  [r::R,xm]$Rep

--if R has EuclideanDomain then
recip x ==
  r:=exactQuo(1$R,x.val,x.module)
  r case "failed" => "failed"
  [r,x.module]$Rep

inv x ==
  if (u:=recip x) case "failed" then error("not invertible")
  else u::%

```

---

— MODRING.dotabb —

```
"MODRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=MODRING"]
"BMODULE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=BMODULE"]
"MODRING" -> "BMODULE"
```

---

## 14.11 domain MODMONOM ModuleMonomial

— ModuleMonomial.input —

```
)set break resume
)sys rm -f ModuleMonomial.output
)spool ModuleMonomial.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ModuleMonomial
--R ModuleMonomial(IS: OrderedSet,E: SetCategory,ff: ((Record(index: IS,exponent: E),Record(index: IS,exponent: E)),Record(index: IS,exponent: E)))
--R Abbreviation for ModuleMonomial is MODMONOM
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for MODMONOM
--R
--R----- Operations -----
--R ?<? : (%,% ) -> Boolean          ?<=? : (%,% ) -> Boolean
--R ?=? : (%,% ) -> Boolean          ?>? : (%,% ) -> Boolean
--R ?>=? : (%,% ) -> Boolean          coerce : % -> OutputForm
--R construct : (IS,E) -> %          exponent : % -> E
--R hash : % -> SingleInteger        index : % -> IS
--R latex : % -> String              max : (%,% ) -> %
--R min : (%,% ) -> %                ?~=? : (%,% ) -> Boolean
--R coerce : % -> Record(index: IS,exponent: E)
--R coerce : Record(index: IS,exponent: E) -> %
--R
--E 1

)spool
)lisp (bye)
```

---

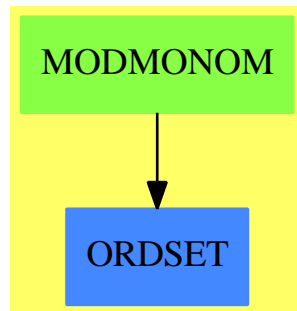
— ModuleMonomial.help —

```
=====
ModuleMonomial examples
=====
```

```
See Also:
o )show ModuleMonomial
```

---

### 14.11.1 ModuleMonomial (MODMONOM)



See

⇒ “GeneralModulePolynomial” (GMODPOL) 8.2.1 on page 1025

**Exports:**

|        |           |          |      |       |
|--------|-----------|----------|------|-------|
| coerce | construct | exponent | hash | index |
| latex  | max       | min      | ?~=? | ?<?   |
| ?<=?   | ?=?       | ?>?      | ?>=? |       |

— domain MODMONOM ModuleMonomial —

```
)abbrev domain MODMONOM ModuleMonomial
```

```
++ Author: Mark Botch
```

```
++ Description:
```

```
++ This package has no documentation
```

```
ModuleMonomial(IS: OrderedSet,
               E: SetCategory,
               ff:(MM, MM) -> Boolean): T == C where
```

```
MM ==> Record(index:IS, exponent:E)
```

```

T == OrderedSet with
  exponent: $ -> E
    ++ exponent(x) is not documented
  index: $ -> IS
    ++ index(x) is not documented
  coerce: MM -> $
    ++ coerce(x) is not documented
  coerce: $ -> MM
    ++ coerce(x) is not documented
  construct: (IS, E) -> $
    ++ construct(i,e) is not documented
C == MM add
  Rep:= MM
  x:$ < y:$ == ff(x::Rep, y::Rep)
  exponent(x:$):E == x.exponent
  index(x:$): IS == x.index
  coerce(x:$):MM == x::Rep::MM
  coerce(x:MM):$ == x::Rep::$
  construct(i:IS, e:E):$ == [i, e]$MM::Rep::$

```

---

— MODMONOM.dotabb —

```

"MODMONOM" [color="#88FF44",href="bookvol10.3.pdf#nameddest=MODMONOM"]
"ORDSET" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ORDSET"]
"MODMONOM" -> "ORDSET"

```

---

## 14.12 domain MODOP ModuleOperator

— ModuleOperator.input —

```

)set break resume
)sys rm -f ModuleOperator.output
)spool ModuleOperator.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ModuleOperator

```

```

--R ModuleOperator(R: Ring,M: LeftModule R) is a domain constructor
--R Abbreviation for ModuleOperator is MODOP
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for MODOP
--R
--R----- Operations -----
--R ?? : (%,% ) -> %                ?? : (Integer,% ) -> %
--R ?? : (PositiveInteger,% ) -> %   ??? : (% ,Integer) -> %
--R ??? : (% ,PositiveInteger) -> %  ?+? : (% ,%) -> %
--R ?-? : (% ,%) -> %                -? : % -> %
--R ?=? : (% ,%) -> Boolean          1 : () -> %
--R 0 : () -> %                     ?? : (% ,PositiveInteger) -> %
--R coerce : BasicOperator -> %     coerce : R -> %
--R coerce : Integer -> %           coerce : % -> OutputForm
--R ?.? : (% ,M) -> M               evaluate : (% ,(M -> M)) -> %
--R hash : % -> SingleInteger       latex : % -> String
--R one? : % -> Boolean             opeval : (BasicOperator,M) -> M
--R recip : % -> Union(% ,"failed") retract : % -> BasicOperator
--R retract : % -> R                sample : () -> %
--R zero? : % -> Boolean            ?~=? : (% ,%) -> Boolean
--R ?? : (% ,R) -> % if R has COMRING
--R ?? : (R ,%) -> % if R has COMRING
--R ?? : (NonNegativeInteger,% ) -> %
--R ??? : (BasicOperator,Integer) -> %
--R ??? : (% ,NonNegativeInteger) -> %
--R ?? : (% ,NonNegativeInteger) -> %
--R adjoint : (% ,%) -> % if R has COMRING
--R adjoint : % -> % if R has COMRING
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(% ,"failed") if R has CHARNZ
--R conjug : R -> R if R has COMRING
--R evaluateInverse : (% ,(M -> M)) -> %
--R makeop : (R,FreeGroup BasicOperator) -> %
--R retractIfCan : % -> Union(BasicOperator ,"failed")
--R retractIfCan : % -> Union(R ,"failed")
--R subtractIfCan : (% ,%) -> Union(% ,"failed")
--R
--E 1

)spool
)lisp (bye)

```

---

— ModuleOperator.help —

```

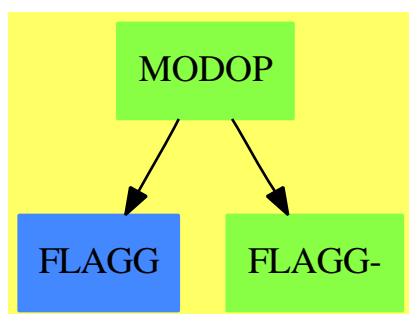
=====
ModuleOperator examples
=====

```

See Also:

o )show ModuleOperator

### 14.12.1 ModuleOperator (MODOP)



See

⇒ “Operator” (OP) 16.10.1 on page 1766

**Exports:**

|         |              |          |                 |            |
|---------|--------------|----------|-----------------|------------|
| 0       | 1            | adjoint  | characteristic  | charthRoot |
| coerce  | conjug       | evaluate | evaluateInverse | hash       |
| latex   | makeop       | one?     | opeval          | recip      |
| retract | retractIfCan | sample   | subtractIfCan   | zero?      |
| ?*?     | ?**?         | ?+?      | ?-?             | -?         |
| ?=?     | ?^?          | ?..?     | ?~=?            |            |

— domain MODOP ModuleOperator —

```

)abbrev domain MODOP ModuleOperator
++ Author: Manuel Bronstein
++ Date Created: 15 May 1990
++ Date Last Updated: 17 June 1993
++ Description:
++ Algebra of ADDITIVE operators on a module.
  
```

```

ModuleOperator(R: Ring, M:LeftModule(R)): Exports == Implementation where
  O  ==> OutputForm
  OP ==> BasicOperator
  FG ==> FreeGroup OP
  RM ==> Record(coef:R, monom:FG)
  TERM ==> List RM
  FAB ==> FreeAbelianGroup TERM
  
```



```

OPADJ ==> "%opAdjoint"
OPEVAL ==> "%opEval"
INVEVAL ==> "%invEval"

Exports ==> Join(Ring, RetractableTo R, RetractableTo OP,
                Eltable(M, M)) with
if R has CharacteristicZero then CharacteristicZero
if R has CharacteristicNonZero then CharacteristicNonZero
if R has CommutativeRing then
  Algebra(R)
  adjoint: $ -> $
  ++ adjoint(op) returns the adjoint of the operator \spad{op}.
  adjoint: ($, $) -> $
  ++ adjoint(op1, op2) sets the adjoint of op1 to be op2.
  ++ op1 must be a basic operator
  conjug : R -> R
  ++ conjug(x) should be local but conditional
  evaluate: ($, M -> M) -> $
  ++ evaluate(f, u +-> g u) attaches the map g to f.
  ++ f must be a basic operator
  ++ g MUST be additive, i.e. \spad{g(a + b) = g(a) + g(b)} for
  ++ any \spad{a}, \spad{b} in M.
  ++ This implies that \spad{g(n a) = n g(a)} for
  ++ any \spad{a} in M and integer \spad{n > 0}.
  evaluateInverse: ($, M -> M) -> $
  ++ evaluateInverse(x,f) is not documented
  "**": (OP, Integer) -> $
  ++ op**n is not documented
  "**": ($, Integer) -> $
  ++ op**n is not documented
  opeval : (OP, M) -> M
  ++ opeval should be local but conditional
  makeop : (R, FG) -> $
  ++ makeop should be local but conditional

Implementation ==> FAB add
import NoneFunctions1($)
import BasicOperatorFunctions1(M)

Rep := FAB

inv : TERM -> $
termeval : (TERM, M) -> M
rmeval : (RM, M) -> M
monomeval: (FG, M) -> M
opInvEval: (OP, M) -> M
mkop : (R, FG) -> $
termprod0: (Integer, TERM, TERM) -> $
termprod : (Integer, TERM, TERM) -> TERM
termcopy : TERM -> TERM

```

```

trm20      : (Integer, TERM) -> 0
term20     : TERM -> 0
rm20       : (R, FG) -> 0
nocopy     : OP -> $

1          == makeop(1, 1)
coerce(n:Integer):$ == n::R::$
coerce(r:R):$      == (zero? r => 0; makeop(r, 1))
coerce(op:OP):$    == nocopy copy op
nocopy(op:OP):$    == makeop(1, op::FG)
elt(x:$, r:M)      == +/[t.exp * termeval(t.gen, r) for t in terms x]
rmeval(t, r)       == t.coef * monomeval(t.monom, r)
termcopy t         == [[rm.coef, rm.monom] for rm in t]
characteristic()   == characteristic()$R
mkop(r, fg)        == [[r, fg]$RM]$TERM :: $
evaluate(f, g)     == nocopy setProperty(retract(f)@OP,OPEVAL,g pretend None)

if R has OrderedSet then
  makeop(r, fg) == (r >= 0 => mkop(r, fg); - mkop(-r, fg))
else makeop(r, fg) == mkop(r, fg)

inv(t:TERM):$ ==
  empty? t => 1
  c := first(t).coef
  m := first(t).monom
  inv(rest t) * makeop(1, inv m) * (recip(c)::R::$)

x:$ ** i:Integer ==
  i = 0 => 1
  i > 0 => expt(x,i pretend PositiveInteger)$RepeatedSquaring($
    (inv(retract(x)@TERM)) ** (-i))

evaluateInverse(f, g) ==
  nocopy setProperty(retract(f)@OP, INVEVAL, g pretend None)

coerce(x:$):0 ==
  zero? x => (0$R)::0
  reduce(_+, [trm20(t.exp, t.gen) for t in terms x])$List(0)

trm20(c, t) ==
--   one? c => term20 t
  (c = 1) => term20 t
  c = -1 => - term20 t
  c::0 * term20 t

term20 t ==
  reduce(_*, [rm20(rm.coef, rm.monom) for rm in t])$List(0)

rm20(c, m) ==
--   one? c => m::0

```

```

(c = 1) => m::0
--      one? m => c::0
(m = 1) => c::0
c::0 * m::0

x:$ * y:$ ==
+/[ +/[termprod0(t.exp * s.exp, t.gen, s.gen) for s in terms y]
    for t in terms x]

termprod0(n, x, y) ==
  n >= 0 => termprod(n, x, y)::$
  - (termprod(-n, x, y)::$)

termprod(n, x, y) ==
  lc := first(xx := termcopy x)
  lc.coef := n * lc.coef
  rm := last xx
--      one?(first(y).coef) =>
--      ((first(y).coef) = 1) =>
--          rm.monom := rm.monom * first(y).monom
--          concat_!(xx, termcopy rest y)
--      one?(rm.monom) =>
--      ((rm.monom) = 1) =>
--          rm.coef := rm.coef * first(y).coef
--          rm.monom := first(y).monom
--          concat_!(xx, termcopy rest y)
--      concat_!(xx, termcopy y)

if M has ExpressionSpace then
  opeval(op, r) ==
    (func := property(op, OPEVAL)) case "failed" => kernel(op, r)
    ((func::None) pretend (M -> M)) r
else
  opeval(op, r) ==
    (func := property(op, OPEVAL)) case "failed" =>
      error "eval: operator has no evaluation function"
    ((func::None) pretend (M -> M)) r

opInvEval(op, r) ==
  (func := property(op, INVEVAL)) case "failed" =>
    error "eval: operator has no inverse evaluation function"
  ((func::None) pretend (M -> M)) r

termeval(t, r) ==
  for rm in reverse t repeat r := rmeval(rm, r)
  r

monomeval(m, r) ==
  for rec in reverse_! factors m repeat

```

```

    e := rec.exp
    g := rec.gen
    e > 0 =>
        for i in 1..e repeat r := opeval(g, r)
    e < 0 =>
        for i in 1..(-e) repeat r := opInvEval(g, r)
    r

recip x ==
    (r := retractIfCan(x)@Union(R, "failed")) case "failed" => "failed"
    (r1 := recip(r::R)) case "failed" => "failed"
    r1::R::$

retractIfCan(x:$):Union(R, "failed") ==
    (r:= retractIfCan(x)@Union(TERM,"failed")) case "failed" => "failed"
    empty?(t := r::TERM) => 0$R
    empty? rest t =>
        rm := first t
--        one?(rm.monom) => rm.coef
        (rm.monom = 1) => rm.coef
        "failed"
    "failed"

retractIfCan(x:$):Union(OP, "failed") ==
    (r:= retractIfCan(x)@Union(TERM,"failed")) case "failed" => "failed"
    empty?(t := r::TERM) => "failed"
    empty? rest t =>
        rm := first t
--        one?(rm.coef) => retractIfCan(rm.monom)
        (rm.coef = 1) => retractIfCan(rm.monom)
        "failed"
    "failed"

if R has CommutativeRing then
    termadj : TERM -> $
    rmadj : RM -> $
    monomadj : FG -> $
    opadj : OP -> $

    r:R * x:$ == r::$ * x
    x:$ * r:R == x * (r::$)
    adjoint x == +/[t.exp * termadj(t.gen) for t in terms x]
    rmadj t == conjug(t.coef) * monomadj(t.monom)
    adjoint(op, adj) == nocopy setProperty(retract(op)@OP, OPADJ, adj::None)

termadj t ==
    ans:$ := 1
    for rm in t repeat ans := rmadj(rm) * ans
    ans

```

```

monomadj m ==
  ans:$ := 1
  for rec in factors m repeat ans := (opadj(rec.gen) ** rec.exp) * ans
  ans

opadj op ==
  (adj := property(op, OPADJ)) case "failed" =>
    error "adjoint: operator does not have a defined adjoint"
  (adj::None) pretend $

if R has conjugate:R -> R then conjug r == conjugate r else conjug r == r

```

---

— MODOP.dotabb —

```

"MODOP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=MODOP"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"FLAGG-" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FLAGG"]
"MODOP" -> "FLAGG"
"MODOP" -> "FLAGG-"

```

---

## 14.13 domain MOEBIUS MoebiusTransform

— MoebiusTransform.input —

```

)set break resume
)sys rm -f MoebiusTransform.output
)spool MoebiusTransform.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show MoebiusTransform
--R MoebiusTransform F: Field is a domain constructor
--R Abbreviation for MoebiusTransform is MOEBIUS
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for MOEBIUS
--R
--R----- Operations -----
--R ???: (% ,%) -> %
--R ??? : (% ,Integer) -> %

```

```

--R ??? : (% , PositiveInteger) -> %      ?/? : (% , %) -> %
--R ?=? : (% , %) -> Boolean              1 : () -> %
--R ?? : (% , Integer) -> %              ?? : (% , PositiveInteger) -> %
--R coerce : % -> OutputForm             commutator : (% , %) -> %
--R conjugate : (% , %) -> %             eval : (% , F) -> F
--R hash : % -> SingleInteger            inv : % -> %
--R latex : % -> String                  moebius : (F , F , F , F) -> %
--R one? : % -> Boolean                  recip : % -> %
--R recip : () -> %                     recip : % -> Union(%, "failed")
--R sample : () -> %                    scale : (% , F) -> %
--R scale : F -> %                       shift : (% , F) -> %
--R shift : F -> %                       ?~=? : (% , %) -> Boolean
--R ??? : (% , NonNegativeInteger) -> %
--R ?? : (% , NonNegativeInteger) -> %
--R eval : (% , OnePointCompletion F) -> OnePointCompletion F
--R
--E 1

)spool
)lisp (bye)

```

---

— MoebiusTransform.help —

=====

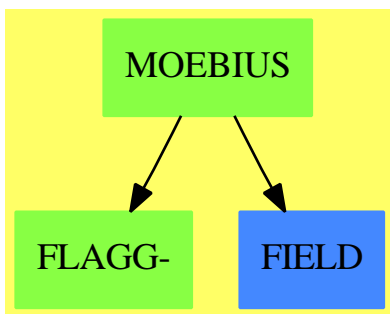
MoebiusTransform examples

=====

See Also:

- o )show MoebiusTransform

### 14.13.1 MoebiusTransform (MOEBIUS)



**Exports:**

|       |        |            |           |      |
|-------|--------|------------|-----------|------|
| 1     | coerce | commutator | conjugate | eval |
| hash  | inv    | latex      | moebius   | one? |
| recip | sample | scale      | shift     | ?~=? |
| ?**?  | ?^?    | ?*?        | ?/?       | ?=?  |
| ?^?   |        |            |           |      |

— domain MOEBIUS MoebiusTransform —

```

)abbrev domain MOEBIUS MoebiusTransform
++ Author: Stephen "Say" Watt
++ Date Created: January 1987
++ Date Last Updated: 11 April 1990
++ Keywords:
++ Examples:
++ References:
++ Description:
++ MoebiusTransform(F) is the domain of fractional linear (Moebius)
++ transformations over F. This a domain of 2-by-2 matrices acting on P1(F).

```

```

MoebiusTransform(F): Exports == Implementation where

```

```

  F : Field
  OUT ==> OutputForm
  P1F ==> OnePointCompletion F          -- projective 1-space over F

```

```

Exports ==> Group with

```

```

  moebius: (F,F,F,F) -> %
    ++ moebius(a,b,c,d) returns \spad{matrix [[a,b],[c,d]]}.
  shift: F -> %
    ++ shift(k) returns \spad{matrix [[1,k],[0,1]]} representing the map
    ++ \spad{x -> x + k}.
  scale: F -> %
    ++ scale(k) returns \spad{matrix [[k,0],[0,1]]} representing the map
    ++ \spad{x -> k * x}.
  recip: () -> %
    ++ recip() returns \spad{matrix [[0,1],[1,0]]} representing the map
    ++ \spad{x -> 1 / x}.
  shift: (% ,F) -> %
    ++ shift(m,h) returns \spad{shift(h) * m}
    ++ (see shift from MoebiusTransform).
  scale: (% ,F) -> %
    ++ scale(m,h) returns \spad{scale(h) * m}
    ++ (see shift from MoebiusTransform).
  recip: % -> %
    ++ recip(m) = recip() * m
  eval: (% ,F) -> F
    ++ eval(m,x) returns \spad{(a*x + b)/(c*x + d)}

```

```

    ++ where \spad{m = moebius(a,b,c,d)}
    ++ (see moebius from MoebiusTransform).
eval: (%,P1F) -> P1F
    ++ eval(m,x) returns \spad{(a*x + b)/(c*x + d)}
    ++ where \spad{m = moebius(a,b,c,d)}
    ++ (see moebius from MoebiusTransform).

Implementation ==> add

Rep := Record(a: F,b: F,c: F,d: F)

moebius(aa,bb,cc,dd) == [aa,bb,cc,dd]

a(t: %): F == t.a
b(t: %): F == t.b
c(t: %): F == t.c
d(t: %): F == t.d

1 == moebius(1,0,0,1)
t * s ==
    moebius(b(t)*c(s) + a(t)*a(s), b(t)*d(s) + a(t)*b(s),
            d(t)*c(s) + c(t)*a(s), d(t)*d(s) + c(t)*b(s))
inv t == moebius(d(t),-b(t),-c(t),a(t))

shift f == moebius(1,f,0,1)
scale f == moebius(f,0,0,1)
recip() == moebius(0,1,1,0)

shift(t,f) == moebius(a(t) + f*c(t), b(t) + f*d(t), c(t), d(t))
scale(t,f) == moebius(f*a(t),f*b(t),c(t),d(t))
recip t == moebius(c(t),d(t),a(t),b(t))

eval(t: %,f: F) == (a(t)*f + b(t))/(c(t)*f + d(t))
eval(t: %,f: P1F) ==
    (ff := retractIfCan(f)@Union(F,"failed")) case "failed" =>
        (a(t)/c(t)) :: P1F
    zero?(den := c(t) * (fff := ff :: F) + d(t)) => infinity()
    ((a(t) * fff + b(t))/den) :: P1F

coerce t ==
    var := "%x" :: OUT
    num := (a(t) :: OUT) * var + (b(t) :: OUT)
    den := (c(t) :: OUT) * var + (d(t) :: OUT)
    rarrow(var,num/den)

proportional?: (List F,List F) -> Boolean
proportional?(list1,list2) ==
    empty? list1 => empty? list2
    empty? list2 => false
    zero? (x1 := first list1) =>

```



```

      (zero? first list2) and proportional?(rest list1,rest list2)
zero? (x2 := first list2) => false
map((f1:F):F +-> f1/x1, list1) = map((g1:F):F +-> g1/x2, list2)

t = s ==
list1 : List F := [a(t),b(t),c(t),d(t)]
list2 : List F := [a(s),b(s),c(s),d(s)]
proportional?(list1,list2)

```

---

— MOEBIUS.dotabb —

```

"MOEBIUS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=MOEBIUS"]
"FLAGG-" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FLAGG"]
"FIELD" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FIELD"]
"MOEBIUS" -> "FIELD"
"MOEBIUS" -> "FLAGG-"

```

## 14.14 domain MRING MonoidRing

---

— MonoidRing.input —

```

)set break resume
)sys rm -f MonoidRing.output
)spool MonoidRing.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show MonoidRing
--R MonoidRing(R: Ring,M: Monoid) is a domain constructor
--R Abbreviation for MonoidRing is MRING
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for MRING
--R
--R----- Operations -----
--R ?? : (% ,%) -> %
--R ?? : (PositiveInteger,% ) -> %
--R ?? : (% ,%) -> %
--R -? : % -> %
--R ?? : (Integer,% ) -> %
--R ??? : (% ,PositiveInteger) -> %
--R -? : (% ,%) -> %
--R ?? : (% ,%) -> Boolean

```

```

--R 1 : () -> %
--R ??? : (% , PositiveInteger) -> %
--R coefficients : % -> List R
--R coerce : M -> %
--R coerce : % -> OutputForm
--R latex : % -> String
--R monomial : (R,M) -> %
--R monomials : % -> List %
--R recip : % -> Union(%,"failed")
--R retract : % -> M
--R zero? : % -> Boolean
--R ?? : (% , R) -> % if R has COMRING
--R ?? : (R , %) -> % if R has COMRING
--R ?? : (NonNegativeInteger , %) -> %
--R ***? : (% , NonNegativeInteger) -> %
--R ?? : (% , NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if R has CHARNZ
--R coerce : List Record(coef: R, monom: M) -> %
--R index : PositiveInteger -> % if M has FINITE and R has FINITE
--R leadingCoefficient : % -> R if M has ORDSET
--R leadingMonomial : % -> M if M has ORDSET
--R lookup : % -> PositiveInteger if M has FINITE and R has FINITE
--R numberOfMonomials : % -> NonNegativeInteger
--R random : () -> % if M has FINITE and R has FINITE
--R reductum : % -> % if M has ORDSET
--R retractIfCan : % -> Union(R,"failed")
--R retractIfCan : % -> Union(M,"failed")
--R size : () -> NonNegativeInteger if M has FINITE and R has FINITE
--R subtractIfCan : (% , %) -> Union(%,"failed")
--R terms : % -> List Record(coef: R, monom: M)
--R
--E 1

```

```

)spool
)lisp (bye)

```

---

— MonoidRing.help —

```

=====
MonoidRing examples
=====

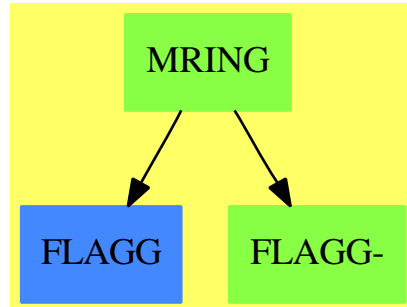
```

```

See Also:
o )show MonoidRing

```

## 14.14.1 MonoidRing (MRING)

**Exports:**

|                    |                 |                   |              |             |
|--------------------|-----------------|-------------------|--------------|-------------|
| 0                  | 1               | characteristic    | charthRoot   | coefficient |
| coefficients       | coerce          | hash              | index        | latex       |
| leadingCoefficient | leadingMonomial | lookup            | map          | monomial    |
| monomial?          | monomials       | numberOfMonomials | one?         | random      |
| recip              | reductum        | retract           | retractIfCan | sample      |
| size               | subtractIfCan   | terms             | zero?        | ?*?         |
| ?**?               | ?+?             | ?-?               | -?           | ?=?         |
| ?^?                | ?~=?            |                   |              |             |

— domain MRING MonoidRing —

```

)abbrev domain MRING MonoidRing
++ Authors: Stephan M. Watt; revised by Johannes Grabmeier
++ Date Created: January 1986
++ Date Last Updated: 14 December 1995, Mike Dewar
++ Basic Operations: *, +, monomials, coefficients
++ Related Constructors: Polynomial
++ Also See:
++ AMS Classifications:
++ Keywords: monoid ring, group ring, polynomials in non-commuting
++ indeterminates
++ References:
++ Description:
++ \spadtype{MonoidRing}(R,M), implements the algebra
++ of all maps from the monoid M to the commutative ring R with
++ finite support.
++ Multiplication of two maps f and g is defined
++ to map an element c of M to the (convolution) sum over f(a)g(b)
++ such that ab = c. Thus M can be identified with a canonical
++ basis and the maps can also be considered as formal linear combinations
++ of the elements in M. Scalar multiples of a basis element are called
++ monomials. A prominent example is the class of polynomials
++ where the monoid is a direct product of the natural numbers
++ with pointwise addition. When M is
  
```

```

++ \spadtype{FreeMonoid Symbol}, one gets polynomials
++ in infinitely many non-commuting variables. Another application
++ area is representation theory of finite groups G, where modules
++ over \spadtype{MonoidRing}(R,G) are studied.

```

```

MonoidRing(R: Ring, M: Monoid): MRcategory == MRdefinition where
  Term ==> Record(coef: R, monom: M)

```

```

MRcategory ==> Join(Ring, RetractableTo M, RetractableTo R) with
  monomial      : (R, M) -> %
  ++ monomial(r,m) creates a scalar multiple of the basis element m.
  coefficient : (% , M) -> R
  ++ coefficient(f,m) extracts the coefficient of m in f with respect
  ++ to the canonical basis M.
  coerce:      List Term -> %
  ++ coerce(lt) converts a list of terms and
  ++ coefficients to a member of the domain.
  terms        : % -> List Term
  ++ terms(f) gives the list of non-zero coefficients combined
  ++ with their corresponding basis element as records.
  ++ This is the internal representation.
  map          : (R -> R, %) -> %
  ++ map(fn,u) maps function fn onto the coefficients
  ++ of the non-zero monomials of u.
  monomial?    : % -> Boolean
  ++ monomial?(f) tests if f is a single monomial.
  coefficients: % -> List R
  ++ coefficients(f) lists all non-zero coefficients.
  monomials: % -> List %
  ++ monomials(f) gives the list of all monomials whose
  ++ sum is f.
  numberOfMonomials: % -> NonNegativeInteger
  ++ numberOfMonomials(f) is the number of non-zero coefficients
  ++ with respect to the canonical basis.
  if R has CharacteristicZero then CharacteristicZero
  if R has CharacteristicNonZero then CharacteristicNonZero
  if R has CommutativeRing then Algebra(R)
  if (R has Finite and M has Finite) then Finite
  if M has OrderedSet then
    leadingMonomial : % -> M
    ++ leadingMonomial(f) gives the monomial of f whose
    ++ corresponding monoid element is the greatest
    ++ among all those with non-zero coefficients.
    leadingCoefficient: % -> R
    ++ leadingCoefficient(f) gives the coefficient of f, whose
    ++ corresponding monoid element is the greatest
    ++ among all those with non-zero coefficients.
    reductum          : % -> %
    ++ reductum(f) is f minus its leading monomial.

```

```

MRdefinition ==> add
  Ex ==> OutputForm
  Cf ==> coef
  Mn ==> monom

Rep := List Term

coerce(x: List Term): % == x :: %

monomial(r:R, m:M) ==
  r = 0 => empty()
  [[r, m]]

if (R has Finite and M has Finite) then
  size() == size()$R ** size()$M

index k ==
  -- use p-adic decomposition of k
  -- coefficient of p**j determines coefficient of index(i+p)$M
  i:Integer := k rem size()
  p:Integer := size()$R
  n:Integer := size()$M
  ans:% := 0
  for j in 0.. while i > 0 repeat
    h := i rem p
    -- we use index(p) = 0$R
    if h ^= 0 then
      c : R := index(h :: PositiveInteger)$R
      m : M := index((j+n) :: PositiveInteger)$M
      --ans := ans + c *$% m
      ans := ans + monomial(c, m)$%
    i := i quo p
  ans

lookup(z : %) : PositiveInteger ==
  -- could be improved, if M has OrderedSet
  -- z = index lookup z, n = lookup index n
  -- use p-adic decomposition of k
  -- coefficient of p**j determines coefficient of index(i+p)$M
  zero?(z) => size()$% pretend PositiveInteger
  liTe : List Term := terms z -- all non-zero coefficients
  p : Integer := size()$R
  n : Integer := size()$M
  res : Integer := 0
  for te in liTe repeat
    -- assume that lookup(p)$R = 0
    l:NonNegativeInteger:=lookup(te.Mn)$M
    ex : NonNegativeInteger := (n=l => 0;l)
    co : Integer := lookup(te.Cf)$R
    res := res + co * p ** ex

```

```

    res pretend PositiveInteger

    random() == index( (1+(random())$Integer rem size())% )_
    pretend PositiveInteger)%%

0          == empty()
1          == [[1, 1]]
terms a    == (copy a) pretend List(Term)
monomials a == [[t] for t in a]
coefficients a == [t.Cf for t in a]
coerce(m:M):% == [[1, m]]
coerce(r:R): % ==
-- coerce of ring
  r = 0 => 0
  [[r, 1]]
coerce(n:Integer): % ==
-- coerce of integers
  n = 0 => 0
  [[n::R, 1]]
- a == [[ -t.Cf, t.Mn] for t in a]
if R has noZeroDivisors
  then
    (r:R) * (a:%) ==
      r = 0 => 0
      [[r*t.Cf, t.Mn] for t in a]
  else
    (r:R) * (a:%) ==
      r = 0 => 0
      [[rt, t.Mn] for t in a | (rt:=r*t.Cf) ^= 0]
if R has noZeroDivisors
  then
    (n:Integer) * (a:%) ==
      n = 0 => 0
      [[n*t.Cf, t.Mn] for t in a]
  else
    (n:Integer) * (a:%) ==
      n = 0 => 0
      [[nt, t.Mn] for t in a | (nt:=n*t.Cf) ^= 0]
map(f, a) == [[ft, t.Mn] for t in a | (ft:=f(t.Cf)) ^= 0]
numberOfMonomials a == #a

retractIfCan(a:%):Union(M, "failed") ==
--   one?(#a) and one?(a.first.Cf) => a.first.Mn
   ((#a) = 1) and ((a.first.Cf) = 1) => a.first.Mn
   "failed"

retractIfCan(a:%):Union(R, "failed") ==
--   one?(#a) and one?(a.first.Mn) => a.first.Cf
   ((#a) = 1) and ((a.first.Mn) = 1) => a.first.Cf
   "failed"

```

```

if R has noZeroDivisors then
  if M has Group then
    recip a ==
      lt := terms a
      #lt ^= 1 => "failed"
      (u := recip lt.first.Cf) case "failed" => "failed"
      --(u::R) * inv lt.first.Mn
      monomial((u::R), inv lt.first.Mn)$%
  else
    recip a ==
      #a ^= 1 or a.first.Mn ^= 1 => "failed"
      (u := recip a.first.Cf) case "failed" => "failed"
      u::R::%

mkTerm(r:R, m:M):Ex ==
  r=1 => m::Ex
  r=0 or m=1 => r::Ex
  r::Ex * m::Ex

coerce(a:%):Ex ==
  empty? a => (0$Integer)::Ex
  empty? rest a => mkTerm(a.first.Cf, a.first.Mn)
  reduce(+, [mkTerm(t.Cf, t.Mn) for t in a])$List(Ex)

if M has OrderedSet then -- we mean totally ordered
  -- Terms are stored in decending order.
  leadingCoefficient a == (empty? a => 0; a.first.Cf)
  leadingMonomial a == (empty? a => 1; a.first.Mn)
  reductum a == (empty? a => a; rest a)

a = b ==
  #a ^= #b => false
  for ta in a for tb in b repeat
    ta.Cf ^= tb.Cf or ta.Mn ^= tb.Mn => return false
  true

a + b ==
  c:% := empty()
  while not empty? a and not empty? b repeat
    ta := first a; tb := first b
    ra := rest a; rb := rest b
    c :=
      ta.Mn > tb.Mn => (a := ra; concat_!(c, ta))
      ta.Mn < tb.Mn => (b := rb; concat_!(c, tb))
      a := ra; b := rb
      not zero?(r := ta.Cf+tb.Cf) =>
        concat_!(c, [r, ta.Mn])
    c
  concat_!(c, concat(a, b))

```

```

coefficient(a, m) ==
  for t in a repeat
    if t.Mn = m then return t.Cf
    if t.Mn < m then return 0
  0

if M has OrderedMonoid then

-- we use that multiplying an ordered list of monoid elements
-- by a single element respects the ordering

if R has noZeroDivisors then
  a:% * b:% ==
    +/[[[ta.Cf*tb.Cf, ta.Mn*tb.Mn]$Term
      for tb in b ] for ta in reverse a]
else
  a:% * b:% ==
    +/[[[r, ta.Mn*tb.Mn]$Term
      for tb in b | not zero?(r := ta.Cf*tb.Cf)]
      for ta in reverse a]
else -- M hasn't OrderedMonoid

-- we cannot assume that mutiplying an ordered list of
-- monoid elements by a single element respects the ordering:
-- we have to order and to collect equal terms
ge : (Term,Term) -> Boolean
ge(s,t) == t.Mn <= s.Mn

sortAndAdd : List Term -> List Term
sortAndAdd(liTe) == -- assume liTe not empty
  liTe := sort(ge,liTe)
  m : M := (first liTe).Mn
  cf : R := (first liTe).Cf
  res : List Term := []
  for te in rest liTe repeat
    if m = te.Mn then
      cf := cf + te.Cf
    else
      if not zero? cf then res := cons([cf,m]$Term, res)
      m := te.Mn
      cf := te.Cf
  if not zero? cf then res := cons([cf,m]$Term, res)
  reverse res

if R has noZeroDivisors then
  a:% * b:% ==
    zero? a => a

```



```

        zero? b => b -- avoid calling sortAndAdd with []
        +/[sortAndAdd [[ta.Cf*tb.Cf, ta.Mn*tb.Mn]$Term
          for tb in b ] for ta in reverse a]
    else
      a:% * b:% ==
      zero? a => a
      zero? b => b -- avoid calling sortAndAdd with []
      +/[sortAndAdd [[r, ta.Mn*tb.Mn]$Term
        for tb in b | not zero?(r := ta.Cf*tb.Cf)]
        for ta in reverse a]

else -- M hasn't OrderedSet
  -- Terms are stored in random order.
  a = b ==
  #a ^= #b => false
  brace(a pretend List(Term)) = $Set(Term) brace(b pretend List(Term))

coefficient(a, m) ==
  for t in a repeat
    t.Mn = m => return t.Cf
  0

addterm(Tabl: AssociationList(M,R), r:R, m:M):R ==
  (u := search(m, Tabl)) case "failed" => Tabl.m := r
  zero?(r := r + u::R) => (remove_!(m, Tabl); 0)
  Tabl.m := r

a + b ==
  Tabl := table()$AssociationList(M,R)
  for t in a repeat
    Tabl t.Mn := t.Cf
  for t in b repeat
    addterm(Tabl, t.Cf, t.Mn)
  [[Tabl m, m]$Term for m in keys Tabl]

a:% * b:% ==
  Tabl := table()$AssociationList(M,R)
  for ta in a repeat
    for tb in (b pretend List(Term)) repeat
      addterm(Tabl, ta.Cf*tb.Cf, ta.Mn*tb.Mn)
  [[Tabl.m, m]$Term for m in keys Tabl]

```

---

— MRING.dotabb —

"MRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=MRING"]

```
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"FLAGG-" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FLAGG"]
"MRING" -> "FLAGG"
"MRING" -> "FLAGG-"
```

## 14.15 domain MSET Multiset

— Multiset.input —

```
)set break resume
)sys rm -f Multiset.output
)spool Multiset.output
)set message test on
)set message auto off
)clear all
--S 1 of 14
s := multiset [1,2,3,4,5,4,3,2,3,4,5,6,7,4,10]
--R
--R
--R (1) {1,2: 2,3: 3,4: 4,2: 5,6,7,10}
--R
--R                                          Type: Multiset PositiveInteger
--E 1

--S 2 of 14
insert!(3,s)
--R
--R
--R (2) {1,2: 2,4: 3,4: 4,2: 5,6,7,10}
--R
--R                                          Type: Multiset PositiveInteger
--E 2

--S 3 of 14
remove!(3,s,1)
--R
--R
--R (3) {1,2: 2,3: 3,4: 4,2: 5,6,7,10}
--R
--R                                          Type: Multiset PositiveInteger
--E 3

--S 4 of 14
s
--R
--R
--R (4) {1,2: 2,3: 3,4: 4,2: 5,6,7,10}
```

[illegible]

```

--S 11 of 14
difference(s,t)
--R
--R
--R (11) {1,3: 3,4: 4,6,7,10}
--R
--R                                          Type: Multiset Integer
--E 11

--S 12 of 14
S := symmetricDifference(s,t)
--R
--R
--R (12) {1,3: 3,4: 4,6,7,10,- 9}
--R
--R                                          Type: Multiset Integer
--E 12

--S 13 of 14
(U = union(S,I))@Boolean
--R
--R
--R (13) true
--R
--R                                          Type: Boolean
--E 13

--S 14 of 14
t1 := multiset [1,2,2,3]; [t1 < t, t1 < s, t < s, t1 <= s]
--R
--R
--R (14) [false,true,false,true]
--R
--R                                          Type: List Boolean
--E 14
)spool
)lisp (bye)

```

---

— Multiset.help —

=====

Multiset examples

=====

The domain Multiset(R) is similar to Set(R) except that multiplicities (counts of duplications) are maintained and displayed. Use the operation multiset to create multisets from lists. All the standard operations from sets are available for multisets. An element with multiplicity greater than one has the multiplicity displayed first, then a colon, and then the element.

Create a multiset of integers.

```
s := multiset [1,2,3,4,5,4,3,2,3,4,5,6,7,4,10]
      {1,2: 2,3: 3,4: 4,2: 5,6,7,10}
      Type: Multiset PositiveInteger
```

The operation `insert!` adds an element to a multiset.

```
insert!(3,s)
      {1,2: 2,4: 3,4: 4,2: 5,6,7,10}
      Type: Multiset PositiveInteger
```

Use `remove!` to remove an element. If a third argument is present, it specifies how many instances to remove. Otherwise all instances of the element are removed. Display the resulting multiset.

```
remove!(3,s,1); s
      {1,2: 2,3: 3,4: 4,2: 5,6,7,10}
      Type: Multiset PositiveInteger

remove!(5,s); s
      {1,2: 2,3: 3,4: 4,6,7,10}
      Type: Multiset PositiveInteger
```

The operation `count` returns the number of copies of a given value.

```
count(5,s)
      0
      Type: NonNegativeInteger
```

A second multiset.

```
t := multiset [2,2,2,-9]
      {3: 2,- 9}
      Type: Multiset Integer
```

The union of two multisets is additive.

```
U := union(s,t)
      {1,5: 2,3: 3,4: 4,6,7,10,- 9}
      Type: Multiset Integer
```

The `intersect` operation gives the elements that are in common, with additive multiplicity.

```
I := intersect(s,t)
      {5: 2}
      Type: Multiset Integer
```

The difference of  $s$  and  $t$  consists of the elements that  $s$  has but  $t$  does not. Elements are regarded as indistinguishable, so that if  $s$  and  $t$  have any element in common, the difference does not contain that element.

```
difference(s,t)
{1,3: 3,4: 4,6,7,10}
Type: Multiset Integer
```

The `symmetricDifference` is the union of `difference(s, t)` and `difference(t, s)`.

```
S := symmetricDifference(s,t)
{1,3: 3,4: 4,6,7,10,- 9}
Type: Multiset Integer
```

Check that the union of the `symmetricDifference` and the `intersect` equals the union of the elements.

```
(U = union(S,I))@Boolean
true
Type: Boolean
```

Check some inclusion relations.

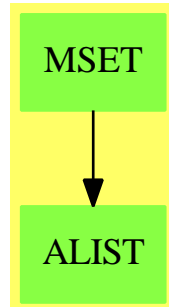
```
t1 := multiset [1,2,2,3]; [t1 < t, t1 < s, t < s, t1 <= s]
[false,true,false,true]
Type: List Boolean
```

See Also:

o `)show Multiset`

---

## 14.15.1 Multiset (MSET)

**Exports:**

|            |                     |         |            |                  |
|------------|---------------------|---------|------------|------------------|
| any?       | bag                 | brace   | coerce     | construct        |
| convert    | copy                | count   | dictionary | difference       |
| duplicates | empty               | empty?  | eq?        | eval             |
| every?     | extract!            | find    | hash       | insert!          |
| inspect    | intersect           | latex   | less?      | map              |
| map!       | members             | member? | more?      | multiset         |
| parts      | reduce              | remove  | remove!    | removeDuplicates |
| sample     | select              | select! | set        | size?            |
| subset?    | symmetricDifference | union   | #?         | ?<?              |
| ?=?        | ?~=?                |         |            |                  |

— domain MSET Multiset —

```

)abbrev domain MSET Multiset
++ Author:Stephen M. Watt, William H. Burge, Richard D. Jenks, Frederic Lehobey
++ Date Created:NK
++ Date Last Updated: 14 June 1994
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ A multiset is a set with multiplicities.

```

```

Multiset(S: SetCategory): MultisetAggregate S with
  finiteAggregate
  shallowlyMutable
  multiset: () -> %
    ++ multiset()$D creates an empty multiset of domain D.
  multiset: S -> %
    ++ multiset(s) creates a multiset with singleton s.

```

```

multiset: List S -> %
  ++ multiset(ls) creates a multiset with elements from \spad{ls}.
members: % -> List S
  ++ members(ms) returns a list of the elements of \spad{ms}
  ++ without their multiplicity. See also \spadfun{parts}.
remove: (S,%,Integer) -> %
  ++ remove(x,ms,number) removes at most \spad{number} copies of
  ++ element x if \spad{number} is positive, all of them if
  ++ \spad{number} equals zero, and all but at most \spad{-number} if
  ++ \spad{number} is negative.
remove: ( S -> Boolean ,%,Integer) -> %
  ++ remove(p,ms,number) removes at most \spad{number} copies of
  ++ elements x such that \spad{p(x)} is \spadfun{true}
  ++ if \spad{number} is positive, all of them if
  ++ \spad{number} equals zero, and all but at most \spad{-number} if
  ++ \spad{number} is negative.
remove_!: (S,%,Integer) -> %
  ++ remove!(x,ms,number) removes destructively at most \spad{number}
  ++ copies of element x if \spad{number} is positive, all
  ++ of them if \spad{number} equals zero, and all but at most
  ++ \spad{-number} if \spad{number} is negative.
remove_!: ( S -> Boolean ,%,Integer) -> %
  ++ remove!(p,ms,number) removes destructively at most \spad{number}
  ++ copies of elements x such that \spad{p(x)} is
  ++ \spadfun{true} if \spad{number} is positive, all of them if
  ++ \spad{number} equals zero, and all but at most \spad{-number} if
  ++ \spad{number} is negative.

== add

Tbl ==> Table(S, Integer)
tbl ==> table$Tbl
Rep := Record(count: Integer, table: Tbl)

n: Integer
ms, m1, m2: %
t, t1, t2: Tbl
D ==> Record(entry: S, count: NonNegativeInteger)
K ==> Record(key: S, entry: Integer)

elt(t:Tbl, s:S):Integer ==
  a := search(s,t)$Tbl
  a case "failed" => 0
  a::Integer

empty():% == [0,tbl()]
multiset():% == empty()
dictionary():% == empty() -- DictionaryOperations
set():% == empty()
brace():% == empty()

```



```

construct(l>List S):% ==
  t := tbl()
  n := 0
  for e in l repeat
    t.e := inc t.e
    n := inc n
  [n, t]
multiset(l>List S):% == construct l
bag(l>List S):% == construct l      -- BagAggregate
dictionary(l>List S):% == construct l -- DictionaryOperations
set(l>List S):% == construct l
brace(l>List S):% == construct l

multiset(s:S):% == construct [s]

if S has ConvertibleTo InputForm then
  convert(ms:%):InputForm ==
    convert [convert("multiset"::Symbol)@InputForm,
      convert(parts ms)@InputForm]

members(ms:%):List S == keys ms.table

coerce(ms:%):OutputForm ==
  l: List OutputForm := empty()
  t := ms.table
  colon := ": " :: OutputForm
  for e in keys t repeat
    ex := e::OutputForm
    n := t.e
    item :=
      n > 1 => hconcat [n :: OutputForm,colon, ex]
    ex
  l := cons(item,l)
  brace l

duplicates(ms:%):List D == -- MultiDictionary
  ld : List D := empty()
  t := ms.table
  for e in keys t | (n := t.e) > 1 repeat
    ld := cons([e,n::NonNegativeInteger],ld)
  ld

extract_!(ms:%):S ==      -- BagAggregate
  empty? ms => error "extract: Empty multiset"
  ms.count := dec ms.count
  t := ms.table
  e := inspect(t).key
  if (n := t.e) > 1 then t.e := dec n
  else remove_!(e,t)

```

```

e

inspect(ms:%):S == inspect(ms.table).key -- BagAggregate

insert_!(e:S,ms:%):% == -- BagAggregate
  ms.count := inc ms.count
  ms.table.e := inc ms.table.e
  ms

member?(e:S,ms:%):Boolean == member?(e,keys ms.table)

empty?(ms:%):Boolean == ms.count = 0

#(ms:%):NonNegativeInteger == ms.count::NonNegativeInteger

count(e:S, ms:%):NonNegativeInteger == ms.table.e::NonNegativeInteger

remove_!(e:S, ms:%, max:Integer):% ==
  zero? max => remove_!(e,ms)
  t := ms.table
  if member?(e, keys t) then
    ((n := t.e) <= max) =>
      remove_!(e,t)
      ms.count := ms.count-n
  max > 0 =>
    t.e := n-max
    ms.count := ms.count-max
  (n := n+max) > 0 =>
    t.e := -max
    ms.count := ms.count-n
  ms

remove_!(p: S -> Boolean, ms:%, max:Integer):% ==
  zero? max => remove_!(p,ms)
  t := ms.table
  for e in keys t | p(e) repeat
    ((n := t.e) <= max) =>
      remove_!(e,t)
      ms.count := ms.count-n
  max > 0 =>
    t.e := n-max
    ms.count := ms.count-max
  (n := n+max) > 0 =>
    t.e := -max
    ms.count := ms.count-n
  ms

remove(e:S, ms:%, max:Integer):% == remove_!(e, copy ms, max)

remove(p: S -> Boolean,ms:%,max:Integer):% == remove_!(p, copy ms, max)

```

```

remove_!(e:S, ms:%):% == -- DictionaryOperations
  t := ms.table
  if member?(e, keys t) then
    ms.count := ms.count-t.e
    remove_!(e, t)
  ms

remove_!(p:S ->Boolean, ms:%):% == -- DictionaryOperations
  t := ms.table
  for e in keys t | p(e) repeat
    ms.count := ms.count-t.e
    remove_!(e, t)
  ms

select_!(p: S -> Boolean, ms:%):% == -- DictionaryOperations
  remove_!((s1:S):Boolean+->not p(s1), ms)

removeDuplicates_!(ms:%):% == -- MultiDictionary
  t := ms.table
  l := keys t
  for e in l repeat t.e := 1
  ms.count := #l
  ms

insert_!(e:S,ms:%,more:NonNegativeInteger):% == -- MultiDictionary
  ms.count := ms.count+more
  ms.table.e := ms.table.e+more
  ms

map_!(f: S->S, ms:%):% == -- HomogeneousAggregate
  t := ms.table
  t1 := tbl()
  for e in keys t repeat
    t1.f(e) := t.e
    remove_!(e, t)
  ms.table := t1
  ms

map(f: S -> S, ms:%):% == map_!(f, copy ms) -- HomogeneousAggregate

parts(m:%):List S ==
  l := empty()$List(S)
  t := m.table
  for e in keys t repeat
    for i in 1..t.e repeat
      l := cons(e,l)
  l

union(m1:%, m2:%):% ==

```

```

t := tbl()
t1:= m1.table
t2:= m2.table
for e in keys t1 repeat t.e := t1.e
for e in keys t2 repeat t.e := t2.e + t.e
[m1.count + m2.count, t]

-- intersect(m1:%, m2:%):% ==
if #m1 > #m2 then intersect(m2, m1)
t := tbl()
t1:= m1.table
t2:= m2.table
n := 0
for e in keys t1 repeat
  m := min(t1.e,t2.e)
  m > 0 =>
    m := t1.e + t2.e
    t.e := m
    n := n + m
[n, t]

difference(m1:%, m2:%):% ==
t := tbl()
t1:= m1.table
t2:= m2.table
n := 0
for e in keys t1 repeat
  k1 := t1.e
  k2 := t2.e
  k1 > 0 and k2 = 0 =>
    t.e := k1
    n := n + k1
n = 0 => empty()
[n, t]

symmetricDifference(m1:%, m2:%):% ==
union(difference(m1,m2), difference(m2,m1))

m1 = m2 ==
m1.count ^= m2.count => false
t1 := m1.table
t2 := m2.table
for e in keys t1 repeat
  t1.e ^= t2.e => return false
for e in keys t2 repeat
  t1.e ^= t2.e => return false
true

m1 < m2 ==
m1.count >= m2.count => false

```

```

t1 := m1.table
t2 := m2.table
for e in keys t1 repeat
  t1.e > t2.e => return false
m1.count < m2.count

subset?(m1:%, m2:%):Boolean ==
  m1.count > m2.count => false
  t1 := m1.table
  t2 := m2.table
  for e in keys t1 repeat t1.e > t2.e => return false
  true

```

---

— MSET.dotabb —

```

"MSET" [color="#88FF44",href="bookvol10.3.pdf#nameddest=MSET"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"MSET" -> "ALIST"

```

---

## 14.16 domain MPOLY MultivariatePolynomial

— MultivariatePolynomial.input —

```

)set break resume
)sys rm -f MultivariatePolynomial.output
)spool MultivariatePolynomial.output
)set message test on
)set message auto off
)clear all
--S 1 of 9
m : MPOLY([x,y],INT) := (x^2 - x*y^3 +3*y)^2
--R
--R
--R
--R      4      3 3      6      2      4      2
--R      (1)  x  - 2y x  + (y  + 6y)x  - 6y x + 9y
--R
--R                                          Type: MultivariatePolynomial([x,y],Integer)
--E 1

--S 2 of 9
m :: MPOLY([y,x],INT)

```

```

--R
--R
--R      2 6      4      3 3      2      2      4
--R  (2)  x y - 6x y - 2x y + 9y + 6x y + x
--R                                          Type: MultivariatePolynomial([y,x],Integer)
--E 2

--S 3 of 9
p : MPOLY([x,y],POLY INT)
--R
--R
--R                                          Type: Void
--E 3

--S 4 of 9
p :: POLY INT
--R
--R
--R  (4)  p
--R
--R                                          Type: Polynomial Integer
--E 4

--S 5 of 9
% :: MPOLY([a,b],POLY INT)
--R
--R
--R  (5)  p
--R
--R                                          Type: MultivariatePolynomial([a,b],Polynomial Integer)
--E 5

--S 6 of 9
q : UP(x, FRAC MPOLY([y,z],INT))
--R
--R
--R                                          Type: Void
--E 6

--S 7 of 9
q := (x^2 - x*(z+1)/y + 2)^2
--R
--R
--R      2      2
--R      4      - 2z - 2      3      4y + z + 2z + 1      2      - 4z - 4
--R  (7)  x + ----- x + ----- x + ----- x + 4
--R              y              2              y
--R              y
--R  --R Type: UnivariatePolynomial(x,Fraction MultivariatePolynomial([y,z],Integer))
--E 7

--S 8 of 9
q :: UP(z, FRAC MPOLY([x,y],INT))
--R

```

```

--R
--R (8)
--R      2      3      2      2 4      3      2      2      2
--R      x  - 2y x + 2x - 4y x  y x - 2y x + (4y + 1)x - 4y x + 4y
--R      -- z + ----- z + -----
--R      2      2      2
--R      y      y      y
--R Type: UnivariatePolynomial(z,Fraction MultivariatePolynomial([x,y],Integer))
--E 8

--S 9 of 9
q :: MPOLY([x,z], FRAC UP(y,INT))
--R
--R
--R
--R      2
--R      4      2      2 3      1 2      2      4y + 1 2      4      4
--R      (9) x + (- - z - -)x + (- z + - z + -----)x + (- - z - -)x + 4
--R      y      y      2      2      2      y      y
--R      y      y      y
--R Type: MultivariatePolynomial([x,z],Fraction UnivariatePolynomial(y,Integer))
--E 9
)spool
)lisp (bye)

```

---

— MultivariatePolynomial.help —

=====

MultivariatePolynomial examples

=====

The domain constructor `MultivariatePolynomial` is similar to `Polynomial` except that it specifies the variables to be used. `Polynomial` are available for `MultivariatePolynomial`. The abbreviation for `MultivariatePolynomial` is `MPOLY`. The type expressions

```

MultivariatePolynomial([x,y],Integer)
MPOLY([x,y],INT)

```

refer to the domain of multivariate polynomials in the variables `x` and `y` where the coefficients are restricted to be integers. The first variable specified is the main variable and the display of the polynomial reflects this.

This polynomial appears with terms in descending powers of the variable `x`.

```

m : MPOLY([x,y],INT) := (x^2 - x*y^3 + 3*y)^2
      4      3 3      6      2      4      2

```

```

x^2 - 2y x + (y^2 + 6y)x - 6y x + 9y
Type: MultivariatePolynomial([x,y],Integer)

```

It is easy to see a different variable ordering by doing a conversion.

```

m :: MPOLY([y,x],INT)
      2 6      4      3 3      2      2      4
x y - 6x y - 2x y + 9y + 6x y + x
Type: MultivariatePolynomial([y,x],Integer)

```

You can use other, unspecified variables, by using Polynomial in the coefficient type of MPOLY.

```

p : MPOLY([x,y],POLY INT)
Type: Void

```

Conversions can be used to re-express such polynomials in terms of the other variables. For example, you can first push all the variables into a polynomial with integer coefficients.

```

p :: POLY INT
p
Type: Polynomial Integer

```

Now pull out the variables of interest.

```

% :: MPOLY([a,b],POLY INT)
%
Type: MultivariatePolynomial([a,b],Polynomial Integer)

```

Restriction:

Axiom does not allow you to create types where MultivariatePolynomial is contained in the coefficient type of Polynomial. Therefore, MPOLY([x,y],POLY INT) is legal but POLY MPOLY([x,y],INT) is not.

Multivariate polynomials may be combined with univariate polynomials to create types with special structures.

```

q : UP(x, FRAC MPOLY([y,z],INT))
Type: Void

```

This is a polynomial in x whose coefficients are quotients of polynomials in y and z.

```

q := (x^2 - x*(z+1)/y + 2)^2
      2      2
      4      - 2z - 2      3      4y + z + 2z + 1      2      - 4z - 4
(7)  x + ----- x + ----- x + ----- x + 4
      y              2              y

```



Type: UnivariatePolynomial(x,Fraction MultivariatePolynomial([y,z],Integer))

Use conversions for structural rearrangements. z does not appear in a denominator and so it can be made the main variable.

```
q :: UP(z, FRAC MPOLY([x,y],INT))
      2      3      2      2 4      3      2      2      2
      x  2  - 2y x  + 2x  - 4y x  y x  - 2y x  + (4y  + 1)x  - 4y x  + 4y
      -- z  + ----- z + -----
      2      2      2
      y      y      y
Type: UnivariatePolynomial(z,Fraction MultivariatePolynomial([x,y],Integer))
```

Or you can make a multivariate polynomial in x and z whose coefficients are fractions in polynomials in y.

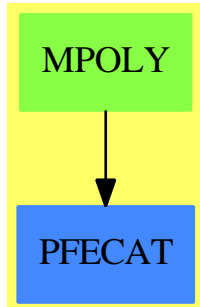
```
q :: MPOLY([x,z], FRAC UP(y,INT))
      4      2      2 3      1 2      2      4y  + 1 2      4      4
      x  + (- - z - -)x  + (-- z  + -- z  + -----)x  + (- - z - -)x  + 4
      y      y      2      2      2      y      y
      y      y      y
Type: MultivariatePolynomial([x,z],Fraction UnivariatePolynomial(y,Integer))
```

A conversion like `q :: MPOLY([x,y], FRAC UP(z,INT))` is not possible in this example because y appears in the denominator of a fraction. As you can see, Axiom provides extraordinary flexibility in the manipulation and display of expressions via its conversion facility.

See Also:

- o )help DistributedMultivariatePolynomial
- o )help UnivariatePolynomial
- o )help Polynomial
- o )show MultivariatePolynomial

---

**14.16.1 MultivariatePolynomial (MPOLY)**

**See**

⇒ “Polynomial” (POLY) 17.25.1 on page 2037

⇒ “SparseMultivariatePolynomial” (SMP) 20.14.1 on page 2381

⇒ “IndexedExponents” (INDE) 10.9.1 on page 1183

**Exports:**

|                |                               |                            |
|----------------|-------------------------------|----------------------------|
| 0              | 1                             | associates?                |
| binomThmExpt   | characteristic                | charthRoot                 |
| coefficient    | coefficients                  | coerce                     |
| conditionP     | content                       | convert                    |
| D              | degree                        | differentiate              |
| discriminant   | eval                          | exquo                      |
| factor         | factorPolynomial              | factorSquareFreePolynomial |
| gcd            | gcdPolynomial                 | ground                     |
| ground?        | hash                          | isExpt                     |
| isPlus         | isTimes                       | latex                      |
| lcm            | leadingCoefficient            | leadingMonomial            |
| mainVariable   | map                           | mapExponents               |
| max            | min                           | minimumDegree              |
| monicDivide    | monomial                      | monomial?                  |
| monomials      | multivariate                  | numberOfMonomials          |
| one?           | patternMatch                  | pomopo!                    |
| prime?         | primitivePart                 | primitiveMonomials         |
| recip          | reducedSystem                 | reductum                   |
| resultant      | retract                       | retractIfCan               |
| sample         | solveLinearPolynomialEquation | squareFree                 |
| squareFreePart | squareFreePolynomial          | subtractIfCan              |
| totalDegree    | unit?                         | unitCanonical              |
| unitNormal     | univariate                    | variables                  |
| zero?          | ?*?                           | ?**?                       |
| ?+?            | ?-?                           | -?                         |
| ?=?            | ?^?                           | ?~=?                       |
| ?/?            | ?<?                           | ?<=?                       |
| ?>?            | ?>=?                          |                            |

— domain MPOLY MultivariatePolynomial —

```

)abbrev domain MPOLY MultivariatePolynomial
++ Author: Dave Barton, Barry Trager
++ Date Created:
++ Date Last Updated:
++ Basic Functions: Ring, degree, eval, coefficient, monomial, differentiate,
++ resultant, gcd
++ Related Constructors: SparseMultivariatePolynomial, Polynomial
++ Also See:
++ AMS Classifications:
++ Keywords: polynomial, multivariate
++ References:
++ Description:
++ This type is the basic representation of sparse recursive multivariate
++ polynomials whose variables are from a user specified list of symbols.
++ The ordering is specified by the position of the variable in the list.
++ The coefficient ring may be non commutative,
++ but the variables are assumed to commute.

```

```
MultivariatePolynomial(vl:List Symbol, R:Ring)
== SparseMultivariatePolynomial(--SparseUnivariatePolynomial,
    R, OrderedVariableList vl)
```

---

— MPOLY.dotabb —

```
"MPOLY" [color="#88FF44",href="bookvol10.3.pdf#nameddest=MPOLY"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"MPOLY" -> "PFECAT"
```

---

## 14.17 domain MYEXPR MyExpression

— MyExpression.input —

```
)set break resume
)sys rm -f MyExpression.output
)spool MyExpression.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show MyExpression
--R MyExpression(q: Symbol,R: Join(Ring,OrderedSet,IntegralDomain)) is a domain constructor
--R Abbreviation for MyExpression is MYEXPR
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for MYEXPR
--R
--R----- Operations -----
--R ?? : (PositiveInteger,%) -> %      ?? : (Integer,%) -> %
--R ?? : (%,%) -> %                  ***? : (%,%) -> %
--R ***? : (%,PositiveInteger) -> %  ?+? : (%,%) -> %
--R -? : % -> %                      ?-? : (%,%) -> %
--R ?/? : (%,%) -> %                 ?<? : (%,%) -> Boolean
--R ?<=? : (%,%) -> Boolean           ?=? : (%,%) -> Boolean
--R ?>? : (%,%) -> Boolean           ?>=? : (%,%) -> Boolean
--R D : (%,Symbol) -> %              D : (%,List Symbol) -> %
--R 1 : () -> %                      0 : () -> %
--R ?? : (%,PositiveInteger) -> %    applyQuote : (Symbol,%,%) -> %
```

```

--R applyQuote : (Symbol,%) -> %
--R belong? : BasicOperator -> Boolean
--R box : List % -> %
--R coerce : Integer -> %
--R coerce : R -> %
--R coerce : Kernel % -> %
--R denominator : % -> %
--R distribute : (%,%) -> %
--R elt : (BasicOperator,%,%) -> %
--R eval : (%,List %,List %) -> %
--R eval : (%,Equation %) -> %
--R eval : (%,Kernel %,%) -> %
--R factorials : % -> %
--R freeOf? : (%,Symbol) -> Boolean
--R ground : % -> R
--R hash : % -> SingleInteger
--R is? : (%,Symbol) -> Boolean
--R kernels : % -> List Kernel %
--R map : ((% -> %),Kernel %) -> %
--R min : (%,%) -> %
--R one? : % -> Boolean
--R paren : % -> %
--R product : (%,Symbol) -> %
--R retract : % -> Integer
--R retract : % -> Symbol
--R sample : () -> %
--R summation : (%,Symbol) -> %
--R unit? : % -> Boolean
--R variables : % -> List Symbol
--R ~=? : (%,%) -> Boolean
--R ?? : (NonNegativeInteger,%) -> %
--R ?? : (%,R) -> % if R has COMRING
--R ?? : (R,%) -> % if R has COMRING
--R ?? : (%,Fraction Integer) -> % if R has INTDOM
--R ?? : (Fraction Integer,%) -> % if R has INTDOM
--R ??? : (%,Integer) -> % if R has GROUP or R has INTDOM
--R ??? : (%,NonNegativeInteger) -> %
--R ?/? : (SparseMultivariatePolynomial(R,Kernel %),SparseMultivariatePolynomial(R,Kernel %)) -> %
--R D : (%,Symbol,NonNegativeInteger) -> %
--R D : (%,List Symbol,List NonNegativeInteger) -> %
--R ?? : (%,Integer) -> % if R has GROUP or R has INTDOM
--R ?? : (%,NonNegativeInteger) -> %
--R applyQuote : (Symbol,List %) -> %
--R applyQuote : (Symbol,%,%,%) -> %
--R applyQuote : (Symbol,%,%,%) -> %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%, "failed") if R has CHARNZ
--R coerce : Fraction Integer -> % if R has INTDOM or R has RETRACT FRAC INT
--R coerce : Fraction MyUnivariatePolynomial(q,R) -> %
--R coerce : MyUnivariatePolynomial(q,R) -> %
--R associates? : (%,%) -> Boolean
--R binomial : (%,%) -> %
--R box : % -> %
--R coerce : % -> %
--R coerce : Symbol -> %
--R coerce : % -> OutputForm
--R differentiate : (%,Symbol) -> %
--R distribute : % -> %
--R elt : (BasicOperator,%) -> %
--R eval : (%,%,%) -> %
--R eval : (%,List Equation %) -> %
--R factorial : % -> %
--R factorials : (%,Symbol) -> %
--R freeOf? : (%,%) -> Boolean
--R ground? : % -> Boolean
--R height : % -> NonNegativeInteger
--R kernel : (BasicOperator,%) -> %
--R latex : % -> String
--R max : (%,%) -> %
--R numerator : % -> %
--R paren : List % -> %
--R permutation : (%,%) -> %
--R recip : % -> Union(%, "failed")
--R retract : % -> R
--R retract : % -> Kernel %
--R subst : (%,Equation %) -> %
--R tower : % -> List Kernel %
--R unitCanonical : % -> %
--R zero? : % -> Boolean

```

```

--R coerce : Polynomial R -> % if R has RING
--R coerce : Fraction Polynomial R -> % if R has INTDOM
--R coerce : Fraction Polynomial Fraction R -> % if R has INTDOM
--R coerce : Polynomial Fraction R -> % if R has INTDOM
--R coerce : Fraction R -> % if R has INTDOM
--R coerce : SparseMultivariatePolynomial(R,Kernel %) -> % if R has RING
--R commutator : (% ,%) -> % if R has GROUP
--R conjugate : (% ,%) -> % if R has GROUP
--R convert : % -> InputForm if R has KONVERT INFORM
--R convert : Factored % -> % if R has INTDOM
--R convert : % -> Pattern Float if R has KONVERT PATTERN FLOAT
--R convert : % -> Pattern Integer if R has KONVERT PATTERN INT
--R definingPolynomial : % -> % if $ has RING
--R denom : % -> SparseMultivariatePolynomial(R,Kernel %) if R has INTDOM
--R differentiate : (% ,List Symbol) -> %
--R differentiate : (% ,Symbol,NonNegativeInteger) -> %
--R differentiate : (% ,List Symbol,List NonNegativeInteger) -> %
--R divide : (% ,%) -> Record(quotient: %,remainder: %) if R has INTDOM
--R elt : (BasicOperator,List %) -> %
--R elt : (BasicOperator,% ,% ,% ,%) -> %
--R elt : (BasicOperator,% ,% ,% ,%) -> %
--R euclideanSize : % -> NonNegativeInteger if R has INTDOM
--R eval : (% ,Symbol,NonNegativeInteger,(% -> %)) -> % if R has RING
--R eval : (% ,Symbol,NonNegativeInteger,(List % -> %)) -> % if R has RING
--R eval : (% ,List Symbol,List NonNegativeInteger,List (List % -> %)) -> % if R has RING
--R eval : (% ,List Symbol,List NonNegativeInteger,List (% -> %)) -> % if R has RING
--R eval : (% ,List BasicOperator,List %,Symbol) -> % if R has KONVERT INFORM
--R eval : (% ,BasicOperator,%,Symbol) -> % if R has KONVERT INFORM
--R eval : % -> % if R has KONVERT INFORM
--R eval : (% ,List Symbol) -> % if R has KONVERT INFORM
--R eval : (% ,Symbol) -> % if R has KONVERT INFORM
--R eval : (% ,BasicOperator,(% -> %)) -> %
--R eval : (% ,BasicOperator,(List % -> %)) -> %
--R eval : (% ,List BasicOperator,List (List % -> %)) -> %
--R eval : (% ,List BasicOperator,List (% -> %)) -> %
--R eval : (% ,Symbol,(% -> %)) -> %
--R eval : (% ,Symbol,(List % -> %)) -> %
--R eval : (% ,List Symbol,List (List % -> %)) -> %
--R eval : (% ,List Symbol,List (% -> %)) -> %
--R eval : (% ,List Kernel %,List %) -> %
--R even? : % -> Boolean if $ has RETRACT INT
--R expressIdealMember : (List % ,%) -> Union(List % ,"failed") if R has INTDOM
--R exquo : (% ,%) -> Union(% ,"failed")
--R extendedEuclidean : (% ,%) -> Record(coef1: %,coef2: %,generator: %) if R has INTDOM
--R extendedEuclidean : (% ,% ,%) -> Union(Record(coef1: %,coef2: %),"failed") if R has INTDOM
--R factor : % -> Factored % if R has INTDOM
--R gcd : (% ,%) -> % if R has INTDOM
--R gcd : List % -> % if R has INTDOM
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolynomial %
--R inv : % -> % if R has GROUP or R has INTDOM

```

```

--R is? : (% , BasicOperator) -> Boolean
--R isExpt : (% , Symbol) -> Union(Record(var: Kernel %, exponent: Integer), "failed") if R has I
--R isExpt : (% , BasicOperator) -> Union(Record(var: Kernel %, exponent: Integer), "failed") if
--R isExpt : % -> Union(Record(var: Kernel %, exponent: Integer), "failed") if R has SGROUP
--R isMult : % -> Union(Record(coef: Integer, var: Kernel %), "failed") if R has ABELSG
--R isPlus : % -> Union(List %, "failed") if R has ABELSG
--R isPower : % -> Union(Record(val: %, exponent: Integer), "failed") if R has RING
--R isTimes : % -> Union(List %, "failed") if R has SGROUP
--R kernel : (BasicOperator, List %) -> %
--R lcm : (% , %) -> % if R has INTDOM
--R lcm : List % -> % if R has INTDOM
--R mainKernel : % -> Union(Kernel %, "failed")
--R minPoly : Kernel % -> SparseUnivariatePolynomial % if $ has RING
--R multiEuclidean : (List %, %) -> Union(List %, "failed") if R has INTDOM
--R numer : % -> SparseMultivariatePolynomial(R, Kernel %) if R has RING
--R odd? : % -> Boolean if $ has RETRACT INT
--R operator : BasicOperator -> BasicOperator
--R operators : % -> List BasicOperator
--R patternMatch : (% , Pattern Float, PatternMatchResult(Float, %)) -> PatternMatchResult(Float
--R patternMatch : (% , Pattern Integer, PatternMatchResult(Integer, %)) -> PatternMatchResult(I
--R prime? : % -> Boolean if R has INTDOM
--R principalIdeal : List % -> Record(coef: List %, generator: %) if R has INTDOM
--R product : (% , SegmentBinding %) -> %
--R ?quo? : (% , %) -> % if R has INTDOM
--R reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT and R has RING
--R reducedSystem : (Matrix %, Vector %) -> Record(mat: Matrix Integer, vec: Vector Integer) i
--R reducedSystem : Matrix % -> Matrix R if R has RING
--R reducedSystem : (Matrix %, Vector %) -> Record(mat: Matrix R, vec: Vector R) if R has RING
--R ?rem? : (% , %) -> % if R has INTDOM
--R retract : % -> Fraction Integer if R has INTDOM and R has RETRACT INT or R has RETRACT FI
--R retract : % -> Fraction MyUnivariatePolynomial(q, R)
--R retract : % -> MyUnivariatePolynomial(q, R)
--R retract : % -> Polynomial R if R has RING
--R retract : % -> Fraction Polynomial R if R has INTDOM
--R retractIfCan : % -> Union(Fraction Integer, "failed") if R has INTDOM and R has RETRACT I
--R retractIfCan : % -> Union(Integer, "failed")
--R retractIfCan : % -> Union(MyUnivariatePolynomial(q, R), "failed")
--R retractIfCan : % -> Union(Polynomial R, "failed") if R has RING
--R retractIfCan : % -> Union(Fraction Polynomial R, "failed") if R has INTDOM
--R retractIfCan : % -> Union(R, "failed")
--R retractIfCan : % -> Union(Symbol, "failed")
--R retractIfCan : % -> Union(Kernel %, "failed")
--R sizeLess? : (% , %) -> Boolean if R has INTDOM
--R squareFree : % -> Factored % if R has INTDOM
--R squareFreePart : % -> % if R has INTDOM
--R subst : (% , List Kernel %, List %) -> %
--R subst : (% , List Equation %) -> %
--R subtractIfCan : (% , %) -> Union(% , "failed")
--R summation : (% , SegmentBinding %) -> %
--R unitNormal : % -> Record(unit: %, canonical: %, associate: %)

```

```
--R univariate : (% ,Kernel %) -> Fraction SparseUnivariatePolynomial % if R has INTDOM
--R
--E 1
```

```
)spool
)lisp (bye)
```

---

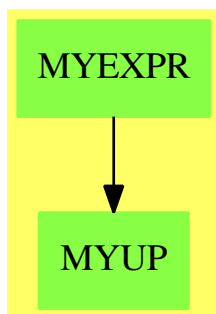
— MyExpression.help —

```
=====
MyExpression examples
=====
```

```
See Also:
o )show MyExpression
```

---

### 14.17.1 MyExpression (MYEXPR)



See

⇒ “MyUnivariatePolynomial” (MYUP) 14.18.1 on page 1658

**Exports:**



|                    |                |                    |                |
|--------------------|----------------|--------------------|----------------|
| 0                  | 1              | applyQuote         | associates?    |
| belong?            | binomial       | box                | characteristic |
| charthRoot         | coerce         | commutator         | conjugate      |
| convert            | D              | definingPolynomial | denom          |
| denominator        | differentiate  | distribute         | divide         |
| elt                | euclideanSize  | eval               | even?          |
| expressIdealMember | exquo          | extendedEuclidean  | factor         |
| factorial          | factorials     | freeOf?            | gcd            |
| gcdPolynomial      | ground         | ground?            | hash           |
| height             | inv            | is?                | is?            |
| isExpt             | isMult         | isPlus             | isPower        |
| isTimes            | kernel         | kernels            | latex          |
| lcm                | mainKernel     | map                | max            |
| min                | minPoly        | multiEuclidean     | numer          |
| numerator          | odd?           | one?               | operator       |
| operators          | paren          | patternMatch       | permutation    |
| prime?             | principalIdeal | product            | recip          |
| reducedSystem      | retract        | retractIfCan       | sample         |
| sizeLess?          | squareFree     | squareFreePart     | subst          |
| subtractIfCan      | summation      | tower              | unit?          |
| unitCanonical      | unitNormal     | univariate         | variables      |
| zero?              | ?*?            | ?**?               | ?+?            |
| -?                 | ?-?            | ?/?                | ?<?            |
| ?<=?               | ?=?            | ?>?                | ?>=?           |
| ?^?                | ?~=?           | ?quo?              | ?rem?          |

— domain MYEXPR MyExpression —

```
)abbrev domain MYEXPR MyExpression
```

```
++ Author: Mark Botch
```

```
++ Description:
```

```
++ This domain has no description
```

```
MyExpression(q: Symbol, R): Exports == Implementation where
```

```
R: Join(Ring, OrderedSet, IntegralDomain)
```

```
UP ==> MyUnivariatePolynomial(q, R)
```

```
Exports == Join(FunctionSpace R, IntegralDomain,
  RetractableTo UP, RetractableTo Symbol,
  RetractableTo Integer, CombinatorialOpsCategory,
  PartialDifferentialRing Symbol) with
```

```
_* : (%,% ) -> %
```

```
_/: (%,% ) -> %
```

```
_*_ : (%,% ) -> %
```

```
numerator : % -> %
```

```
denominator : % -> %
```

```
ground? : % -> Boolean
```

```

coerce: Fraction UP -> %
retract: % -> Fraction UP

Implementation == Expression R add
Rep := Expression R

iunivariate(p: Polynomial R): UP ==
  poly: SparseUnivariatePolynomial(Polynomial R)
  := univariate(p, q)$(Polynomial R)
  map((z1:Polynomial R):R +-> retract(z1), poly)_
  $UnivariatePolynomialCategoryFunctions2(Polynomial R,
    SparseUnivariatePolynomial Polynomial R,
    R, UP)

retract(p: %): Fraction UP ==
  poly: Fraction Polynomial R := retract p
  upoly: UP := iunivariate numer poly
  vpoly: UP := iunivariate denom poly

  upoly / vpoly

retract(p: %): UP == iunivariate retract p

coerce(r: Fraction UP): % ==
  num: SparseUnivariatePolynomial R := makeSUP numer r
  den: SparseUnivariatePolynomial R := makeSUP denom r
  u: Polynomial R := multivariate(num, q)
  v: Polynomial R := multivariate(den, q)

  quot: Fraction Polynomial R := u/v

  quot::(Expression R)

```

---

— MYEXPR.dotabb —

```

"MYEXPR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=MYEXPR"]
"MYUP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=MYUP"]
"MYEXPR" -> "MYUP"

```

---

## 14.18 domain MYUP MyUnivariatePolynomial

## — MyUnivariatePolynomial.input —

```

)set break resume
)sys rm -f MyUnivariatePolynomial.output
)spool MyUnivariatePolynomial.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show MyUnivariatePolynomial
--R MyUnivariatePolynomial(x: Symbol,R: Ring) is a domain constructor
--R Abbreviation for MyUnivariatePolynomial is MYUP
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for MYUP
--R
--R----- Operations -----
--R ??? : (%,R) -> %
--R ??? : (%,%) -> %
--R ??? : (PositiveInteger,%) -> %
--R ?+? : (%,%) -> %
--R -? : % -> %
--R D : (%,(R -> R)) -> %
--R D : (%,NonNegativeInteger) -> %
--R 0 : () -> %
--R coefficients : % -> List R
--R coerce : Polynomial R -> %
--R coerce : R -> %
--R coerce : % -> OutputForm
--R differentiate : % -> %
--R ?.? : (%,R) -> R
--R eval : (%,%,%) -> %
--R eval : (%,List Equation %) -> %
--R ground? : % -> Boolean
--R init : () -> % if R has STEP
--R leadingCoefficient : % -> R
--R map : ((R -> R),%) -> %
--R monomials : % -> List %
--R primitiveMonomials : % -> List %
--R recip : % -> Union(%, "failed")
--R retract : % -> Symbol
--R sample : () -> %
--R ?~=? : (%,%) -> Boolean
--R ??? : (Fraction Integer,%) -> % if R has ALGEBRA FRAC INT
--R ??? : (%,Fraction Integer) -> % if R has ALGEBRA FRAC INT
--R ??? : (NonNegativeInteger,%) -> %
--R ??? : (%,NonNegativeInteger) -> %
--R ?/? : (%,R) -> % if R has FIELD
--R ?<? : (%,%) -> Boolean if R has ORDSET
--R ?<=? : (%,%) -> Boolean if R has ORDSET
--R ??? : (R,%) -> %
--R ??? : (Integer,%) -> %
--R ??? : (%,PositiveInteger) -> %
--R ?-? : (%,%) -> %
--R ?=? : (%,%) -> Boolean
--R D : % -> %
--R 1 : () -> %
--R ?? : (%,PositiveInteger) -> %
--R coerce : Symbol -> %
--R coerce : Variable x -> %
--R coerce : Integer -> %
--R degree : % -> NonNegativeInteger
--R ?.? : (%,%) -> %
--R eval : (%,List %,List %) -> %
--R eval : (%,Equation %) -> %
--R ground : % -> R
--R hash : % -> SingleInteger
--R latex : % -> String
--R leadingMonomial : % -> %
--R monomial? : % -> Boolean
--R one? : % -> Boolean
--R pseudoRemainder : (%,%) -> %
--R reductum : % -> %
--R retract : % -> R
--R zero? : % -> Boolean

```

```

--R ??? : (%,%) -> Boolean if R has ORDSET
--R ?>=? : (%,%) -> Boolean if R has ORDSET
--R D : (%,(R -> R),NonNegativeInteger) -> %
--R D : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (%,List Symbol) -> % if R has PDRING SYMBOL
--R D : (%,Symbol) -> % if R has PDRING SYMBOL
--R D : (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R D : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R D : (%,List SingletonAsOrderedSet) -> %
--R D : (%,SingletonAsOrderedSet) -> %
--R ??? : (%,NonNegativeInteger) -> %
--R associates? : (%,%) -> Boolean if R has INTDOM
--R binomThmExpt : (%,%,NonNegativeInteger) -> % if R has COMRING
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%, "failed") if $ has CHARNZ and R has PFECAT or R has CHARNZ
--R coefficient : (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R coefficient : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R coefficient : (%,NonNegativeInteger) -> R
--R coerce : % -> % if R has INTDOM
--R coerce : Fraction Integer -> % if R has ALGEBRA FRAC INT or R has RETRACT FRAC INT
--R coerce : SingletonAsOrderedSet -> %
--R composite : (Fraction %,%) -> Union(Fraction %, "failed") if R has INTDOM
--R composite : (%,%) -> Union(%, "failed") if R has INTDOM
--R conditionP : Matrix % -> Union(Vector %, "failed") if $ has CHARNZ and R has PFECAT
--R content : (%,SingletonAsOrderedSet) -> % if R has GCDDOM
--R content : % -> R if R has GCDDOM
--R convert : % -> InputForm if SingletonAsOrderedSet has KONVERT INFORM and R has KONVERT INFORM
--R convert : % -> Pattern Integer if SingletonAsOrderedSet has KONVERT PATTERN INT and R has KONVERT PA
--R convert : % -> Pattern Float if SingletonAsOrderedSet has KONVERT PATTERN FLOAT and R has KONVERT PA
--R degree : (%,List SingletonAsOrderedSet) -> List NonNegativeInteger
--R degree : (%,SingletonAsOrderedSet) -> NonNegativeInteger
--R differentiate : (%,(R -> R),%) -> %
--R differentiate : (%,(R -> R)) -> %
--R differentiate : (%,(R -> R),NonNegativeInteger) -> %
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
--R differentiate : (%,List Symbol) -> % if R has PDRING SYMBOL
--R differentiate : (%,Symbol) -> % if R has PDRING SYMBOL
--R differentiate : (%,NonNegativeInteger) -> %
--R differentiate : (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R differentiate : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R differentiate : (%,List SingletonAsOrderedSet) -> %
--R differentiate : (%,SingletonAsOrderedSet) -> %
--R discriminant : % -> R if R has COMRING
--R discriminant : (%,SingletonAsOrderedSet) -> % if R has COMRING
--R divide : (%,%) -> Record(quotient: %,remainder: %) if R has FIELD
--R divideExponents : (%,NonNegativeInteger) -> Union(%, "failed")
--R ?.? : (%,Fraction %) -> Fraction % if R has INTDOM
--R elt : (Fraction %,R) -> R if R has FIELD

```

```

--R elt : (Fraction %,Fraction %) -> Fraction % if R has INTDOM
--R euclideanSize : % -> NonNegativeInteger if R has FIELD
--R eval : (%,List SingletonAsOrderedSet,List %) -> %
--R eval : (%,SingletonAsOrderedSet,%) -> %
--R eval : (%,List SingletonAsOrderedSet,List R) -> %
--R eval : (%,SingletonAsOrderedSet,R) -> %
--R expressIdealMember : (List %,%) -> Union(List %,"failed") if R has FIELD
--R exquo : (%,%) -> Union(%,"failed") if R has INTDOM
--R exquo : (%,R) -> Union(%,"failed") if R has INTDOM
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if R has FIELD
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if R has FIELD
--R factor : % -> Factored % if R has PFECAT
--R factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial %
--R factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial %
--R fmecc : (%,NonNegativeInteger,R,%) -> %
--R gcd : (%,%) -> % if R has GCDDOM
--R gcd : List % -> % if R has GCDDOM
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolynomial %
--R integrate : % -> % if R has ALGEBRA FRAC INT
--R isExpt : % -> Union(Record(var: SingletonAsOrderedSet,exponent: NonNegativeInteger),"failed")
--R isPlus : % -> Union(List %,"failed")
--R isTimes : % -> Union(List %,"failed")
--R karatsubaDivide : (%,NonNegativeInteger) -> Record(quotient: %,remainder: %)
--R lcm : (%,%) -> % if R has GCDDOM
--R lcm : List % -> % if R has GCDDOM
--R mainVariable : % -> Union(SingletonAsOrderedSet,"failed")
--R makeSUP : % -> SparseUnivariatePolynomial R
--R mapExponents : ((NonNegativeInteger -> NonNegativeInteger),%) -> %
--R max : (%,%) -> % if R has ORDSET
--R min : (%,%) -> % if R has ORDSET
--R minimumDegree : (%,List SingletonAsOrderedSet) -> List NonNegativeInteger
--R minimumDegree : (%,SingletonAsOrderedSet) -> NonNegativeInteger
--R minimumDegree : % -> NonNegativeInteger
--R monicDivide : (%,%) -> Record(quotient: %,remainder: %)
--R monicDivide : (%,%,SingletonAsOrderedSet) -> Record(quotient: %,remainder: %)
--R monomial : (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R monomial : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R monomial : (R,NonNegativeInteger) -> %
--R multiEuclidean : (List %,%) -> Union(List %,"failed") if R has FIELD
--R multiplyExponents : (%,NonNegativeInteger) -> %
--R multivariate : (SparseUnivariatePolynomial %,SingletonAsOrderedSet) -> %
--R multivariate : (SparseUnivariatePolynomial R,SingletonAsOrderedSet) -> %
--R nextItem : % -> Union(%,"failed") if R has STEP
--R numberOfMonomials : % -> NonNegativeInteger
--R order : (%,%) -> NonNegativeInteger if R has INTDOM
--R patternMatch : (%,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMatchResult(Integer,%)
--R patternMatch : (%,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchResult(Float,%)
--R pomopo! : (%,R,NonNegativeInteger,%) -> %
--R prime? : % -> Boolean if R has PFECAT
--R primitivePart : (%,SingletonAsOrderedSet) -> % if R has GCDDOM

```

```

--R primitivePart : % -> % if R has GCDDOM
--R principalIdeal : List % -> Record(coef: List %,generator: %) if R has FIELD
--R pseudoDivide : (%,%)-> Record(coef: R,quotient: %,remainder: %) if R has INTDOM
--R pseudoQuotient : (%,%)-> % if R has INTDOM
--R ?quo? : (%,%)-> % if R has FIELD
--R reducedSystem : Matrix % -> Matrix R
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) if R has LINE
--R reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT
--R ?rem? : (%,%)-> % if R has FIELD
--R resultant : (%,%)-> R if R has COMRING
--R resultant : (%%,SingletonAsOrderedSet) -> % if R has COMRING
--R retract : % -> SingletonAsOrderedSet
--R retract : % -> Integer if R has RETRACT INT
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(Symbol,"failed")
--R retractIfCan : % -> Union(SingletonAsOrderedSet,"failed")
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(R,"failed")
--R separate : (%,%)-> Record(primePart: %,commonPart: %) if R has GCDDOM
--R shiftLeft : (%,NonNegativeInteger) -> %
--R shiftRight : (%,NonNegativeInteger) -> %
--R sizeLess? : (%,%)-> Boolean if R has FIELD
--R solveLinearPolynomialEquation : (List SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) ->
--R squareFree : % -> Factored % if R has GCDDOM
--R squareFreePart : % -> % if R has GCDDOM
--R squareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % if R ha
--R subResultantGcd : (%,%)-> % if R has INTDOM
--R subtractIfCan : (%,%)-> Union(%,"failed")
--R totalDegree : (%,List SingletonAsOrderedSet) -> NonNegativeInteger
--R totalDegree : % -> NonNegativeInteger
--R unit? : % -> Boolean if R has INTDOM
--R unitCanonical : % -> % if R has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has INTDOM
--R univariate : % -> SparseUnivariatePolynomial R
--R univariate : (%,SingletonAsOrderedSet) -> SparseUnivariatePolynomial %
--R unmakeSUP : SparseUnivariatePolynomial R -> %
--R variables : % -> List SingletonAsOrderedSet
--R vectorise : (%,NonNegativeInteger) -> Vector R
--R
--E 1

)spool
)lisp (bye)

```

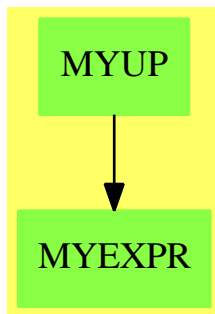
```
=====
MyUnivariatePolynomial examples
=====
```

See Also:

```
o )show MyUnivariatePolynomial
```

—————

### 14.18.1 MyUnivariatePolynomial (MYUP)



See

⇒ “MyExpression” (MYEXPR) 14.17.1 on page 1651

**Exports:**

|                            |                               |                    |
|----------------------------|-------------------------------|--------------------|
| 0                          | 1                             | associates?        |
| binomThmExpt               | characteristic                | charthRoot         |
| coefficient                | coefficients                  | coerce             |
| composite                  | conditionP                    | content            |
| convert                    | D                             | degree             |
| differentiate              | discriminant                  | divide             |
| divideExponents            | elt                           | euclideanSize      |
| eval                       | expressIdealMember            | exquo              |
| extendedEuclidean          | factor                        | factorPolynomial   |
| factorSquareFreePolynomial | finecg                        | gcd                |
| gcdPolynomial              | ground                        | ground?            |
| hash                       | init                          | integrate          |
| isExpt                     | isPlus                        | isTimes            |
| karatsubaDivide            | latex                         | lcm                |
| leadingCoefficient         | leadingMonomial               | mainVariable       |
| makeSUP                    | map                           | mapExponents       |
| max                        | min                           | minimumDegree      |
| monicDivide                | monomial                      | monomial?          |
| monomials                  | multiEuclidean                | multiplyExponents  |
| multivariate               | nextItem                      | numberOfMonomials  |
| one?                       | order                         | patternMatch       |
| pomopo!                    | prime?                        | primitiveMonomials |
| primitivePart              | principalIdeal                | pseudoDivide       |
| pseudoQuotient             | pseudoRemainder               | recip              |
| reducedSystem              | reductum                      | resultant          |
| retract                    | retractIfCan                  | sample             |
| separate                   | shiftLeft                     | shiftRight         |
| sizeLess?                  | solveLinearPolynomialEquation | squareFree         |
| squareFreePart             | squareFreePolynomial          | subResultantGcd    |
| subtractIfCan              | totalDegree                   | unit?              |
| unitCanonical              | unitNormal                    | univariate         |
| unmakeSUP                  | variables                     | vectorise          |
| zero?                      | ?*?                           | ?**?               |
| ?+?                        | ?-?                           | -?                 |
| ?=?                        | ?^?                           | ?..?               |
| ?~=?                       | ?/?                           | ?<?                |
| ?<=?                       | ?>?                           | ?>=?               |
| ?^?                        | ?quo?                         | ?rem?              |

— domain MYUP MyUnivariatePolynomial —

```
)abbrev domain MYUP MyUnivariatePolynomial
```

```
++ Author: Mark Botch
```

```
++ Description:
```

```
++ This domain has no description
```

```
MyUnivariatePolynomial(x:Symbol, R:Ring):
```



```

UnivariatePolynomialCategory(R) with
  RetractableTo Symbol;
  coerce: Variable(x) -> %
    ++ coerce(x) converts the variable x to a univariate polynomial.
  fmeq: (% , NonNegativeInteger, R, %) -> %
    ++ fmeq(p1, e, r, p2) finds x : p1 - r * x**e * p2
  if R has univariate: (R, Symbol) -> SparseUnivariatePolynomial R
  then coerce: R -> %
  coerce: Polynomial R -> %
== SparseUnivariatePolynomial(R) add
Rep := SparseUnivariatePolynomial(R)
coerce(p: %): OutputForm == outputForm(p, outputForm x)
coerce(x: Symbol): % == monomial(1, 1)
coerce(v: Variable(x)): % == monomial(1, 1)
retract(p: %): Symbol ==
  retract(p)@SingletonAsOrderedSet
  x
if R has univariate: (R, Symbol) -> SparseUnivariatePolynomial R
then coerce(p: R): % == univariate(p, x)$R

coerce(p: Polynomial R): % ==
  poly: SparseUnivariatePolynomial(Polynomial R)
  := univariate(p, x)$(Polynomial R)
  map((z1: Polynomial R): R +-> retract(z1), poly)_
  $UnivariatePolynomialCategoryFunctions2(Polynomial R,
    SparseUnivariatePolynomial Polynomial R, R, %)

```

---

— MYUP.dotabb —

```

"MYUP" [color="#88FF44", href="bookvol10.3.pdf#nameddest=MYUP"]
"MYEXPR" [color="#88FF44", href="bookvol10.3.pdf#nameddest=MYEXPR"]
"MYUP" -> "MYEXPR"

```

---

## Chapter 15

## Chapter N

### 15.1 domain NSDPS NeitherSparseOrDensePowerSeries

— NeitherSparseOrDensePowerSeries.input —

```
)set break resume
)sys rm -f NeitherSparseOrDensePowerSeries.output
)spool NeitherSparseOrDensePowerSeries.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show NeitherSparseOrDensePowerSeries
--R NeitherSparseOrDensePowerSeries K: Field is a domain constructor
--R Abbreviation for NeitherSparseOrDensePowerSeries is NSDPS
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for NSDPS
--R
--R----- Operations -----
--R ??? : (%,K) -> %                ??? : (K,%) -> %
--R ??? : (Fraction Integer,%) -> %  ??? : (%,Fraction Integer) -> %
--R ??? : (%,%) -> %                ??? : (Integer,%) -> %
--R ??? : (PositiveInteger,%) -> %    ??? : (%,Integer) -> %
--R ??? : (%,PositiveInteger) -> %    ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                -? : % -> %
--R ?/? : (%,%) -> %                ?? : (%,%) -> Boolean
--R 1 : () -> %                      0 : () -> %
--R ?? : (%,Integer) -> %            ?? : (%,PositiveInteger) -> %
--R associates? : (%,%) -> Boolean    center : % -> K
--R children : % -> List %            coefOfFirstNonZeroTerm : % -> K
--R coefficient : (%,Integer) -> K    coerce : Fraction Integer -> %
```

```

--R coerce : % -> %
--R coerce : % -> OutputForm
--R concat : (%,%) -> %
--R copy : % -> %
--R cycleTail : % -> %
--R degree : % -> Integer
--R delete : (%,Integer) -> %
--R ?.rest : (%,rest) -> %
--R empty : () -> %
--R eq? : (%,%) -> Boolean
--R explicitlyEmpty? : % -> Boolean
--R extend : (%,Integer) -> %
--R filterUpTo : (%,Integer) -> %
--R gcd : List % -> %
--R hash : % -> SingleInteger
--R indices : % -> List Integer
--R inv : % -> %
--R lazy? : % -> Boolean
--R lcm : List % -> %
--R leadingCoefficient : % -> K
--R leaf? : % -> Boolean
--R monomial : (K,Integer) -> %
--R nodes : % -> List %
--R order : % -> Integer
--R order : (%,Integer) -> Integer
--R posExpnPart : % -> %
--R prime? : % -> Boolean
--R printInfo : Boolean -> Boolean
--R recip : % -> Union(%, "failed")
--R ?rem? : (%,%) -> %
--R removeZeroes : % -> %
--R rest : % -> %
--R sample : () -> %
--R series : (Integer,K,%) -> %
--R sizeLess? : (%,%) -> Boolean
--R squareFreePart : % -> %
--R truncate : (%,Integer) -> %
--R unitCanonical : % -> %
--R zero? : % -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?*? : (NonNegativeInteger,%) -> %
--R ??? : (%,NonNegativeInteger) -> %
--R ?/? : (%,K) -> % if K has FIELD
--R D : (%,List Symbol,List NonNegativeInteger) -> % if K has *: (Integer,K) -> K and K has PDRING SYMBOL
--R D : (%,Symbol,NonNegativeInteger) -> % if K has *: (Integer,K) -> K and K has PDRING SYMBOL
--R D : (%,List Symbol) -> % if K has *: (Integer,K) -> K and K has PDRING SYMBOL
--R D : (%,Symbol) -> % if K has *: (Integer,K) -> K and K has PDRING SYMBOL
--R D : (%,NonNegativeInteger) -> % if K has *: (Integer,K) -> K
--R D : % -> % if K has *: (Integer,K) -> K
--R ?? : (%,NonNegativeInteger) -> %

coerce : Integer -> %
complete : % -> %
concat : List % -> %
cycleEntry : % -> %
cyclic? : % -> Boolean
delay : (() -> %) -> %
distance : (%,%) -> Integer
?.? : (%,Integer) -> K
empty? : % -> Boolean
explicitEntries? : % -> Boolean
explicitlyFinite? : % -> Boolean
factor : % -> Factored %
findCoef : (%,Integer) -> K
gcd : (%,%) -> %
index? : (Integer,%) -> Boolean
insert : (%,%,Integer) -> %
latex : % -> String
lazyEvaluate : % -> %
lcm : (%,%) -> %
leadingMonomial : % -> %
map : ((K -> K),%) -> %
monomial? : % -> Boolean
one? : % -> Boolean
order : % -> Integer
pole? : % -> Boolean
possiblyInfinite? : % -> Boolean
printInfo : () -> Boolean
?quo? : (%,%) -> %
reductum : % -> %
removeFirstZeroes : % -> %
removeZeroes : (Integer,%) -> %
rst : % -> %
sbt : (%,%) -> %
shift : (%,Integer) -> %
squareFree : % -> Factored %
tail : % -> %
unit? : % -> Boolean
variable : % -> Symbol
?~=? : (%,%) -> Boolean

```

```

--R any? : ((Record(k: Integer,c: K) -> Boolean),%) -> Boolean if $ has finiteAggregate
--R approximate : (%,Integer) -> K if K has **: (K,Integer) -> K and K has coerce: Symbol -> K
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%, "failed") if K has CHARNZ
--R child? : (%,% ) -> Boolean if Record(k: Integer,c: K) has SETCAT
--R coerce : % -> Stream Record(k: Integer,c: K)
--R coerce : Stream Record(k: Integer,c: K) -> %
--R coerce : K -> % if K has COMRING
--R concat : (Record(k: Integer,c: K),%) -> %
--R concat : (% ,Record(k: Integer,c: K)) -> %
--R concat! : (% ,%) -> % if $ has shallowlyMutable
--R concat! : (% ,Record(k: Integer,c: K)) -> % if $ has shallowlyMutable
--R construct : List Record(k: Integer,c: K) -> %
--R convert : % -> InputForm if Record(k: Integer,c: K) has KONVERT INFORM
--R count : ((Record(k: Integer,c: K) -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R count : (Record(k: Integer,c: K),%) -> NonNegativeInteger if $ has finiteAggregate and Record(k: Int
--R cycleLength : % -> NonNegativeInteger
--R cycleSplit! : % -> % if $ has shallowlyMutable
--R delete : (% ,UniversalSegment Integer) -> %
--R differentiate : (% ,List Symbol,List NonNegativeInteger) -> % if K has *: (Integer,K) -> K and K has
--R differentiate : (% ,Symbol,NonNegativeInteger) -> % if K has *: (Integer,K) -> K and K has PDRING SYM
--R differentiate : (% ,List Symbol) -> % if K has *: (Integer,K) -> K and K has PDRING SYMBOL
--R differentiate : (% ,Symbol) -> % if K has *: (Integer,K) -> K and K has PDRING SYMBOL
--R differentiate : (% ,NonNegativeInteger) -> % if K has *: (Integer,K) -> K
--R differentiate : % -> % if K has *: (Integer,K) -> K
--R divide : (% ,%) -> Record(quotient: %,remainder: %)
--R ?.value : (% ,value) -> Record(k: Integer,c: K)
--R ?.first : (% ,first) -> Record(k: Integer,c: K)
--R ?.last : (% ,last) -> Record(k: Integer,c: K)
--R ?.? : (% ,UniversalSegment Integer) -> %
--R ?.? : (% ,Integer) -> Record(k: Integer,c: K)
--R elt : (% ,Integer,Record(k: Integer,c: K)) -> Record(k: Integer,c: K)
--R ?.? : (% ,%) -> % if Integer has SGROUP
--R entries : % -> List Record(k: Integer,c: K)
--R entry? : (Record(k: Integer,c: K),%) -> Boolean if $ has finiteAggregate and Record(k: Integer,c: K)
--R euclideanSize : % -> NonNegativeInteger
--R eval : (% ,List Equation Record(k: Integer,c: K)) -> % if Record(k: Integer,c: K) has EVALAB Record(k:
--R eval : (% ,Equation Record(k: Integer,c: K)) -> % if Record(k: Integer,c: K) has EVALAB Record(k: Int
--R eval : (% ,Record(k: Integer,c: K),Record(k: Integer,c: K)) -> % if Record(k: Integer,c: K) has EVALA
--R eval : (% ,List Record(k: Integer,c: K),List Record(k: Integer,c: K)) -> % if Record(k: Integer,c: K)
--R eval : (% ,K) -> Stream K if K has **: (K,Integer) -> K
--R every? : ((Record(k: Integer,c: K) -> Boolean),%) -> Boolean if $ has finiteAggregate
--R expressIdealMember : (List %,%) -> Union(List %, "failed")
--R exquo : (% ,%) -> Union(%, "failed")
--R extendedEuclidean : (% ,%,%) -> Union(Record(coef1: %,coef2: %), "failed")
--R extendedEuclidean : (% ,%) -> Record(coef1: %,coef2: %,generator: %)
--R fill! : (% ,Record(k: Integer,c: K)) -> % if $ has shallowlyMutable
--R find : ((Record(k: Integer,c: K) -> Boolean),%) -> Union(Record(k: Integer,c: K), "failed")
--R findTerm : (% ,Integer) -> Record(k: Integer,c: K)
--R first : % -> Record(k: Integer,c: K)

```

```

--R first : (% , NonNegativeInteger) -> %
--R first : % -> Record(k: Integer, c: K)
--R gcdPolynomial : (SparseUnivariatePolynomial % , SparseUnivariatePolynomial %) -> SparseUni
--R insert : (Record(k: Integer, c: K) , % , Integer) -> %
--R last : % -> Record(k: Integer, c: K)
--R last : (% , NonNegativeInteger) -> %
--R leaves : % -> List Record(k: Integer, c: K)
--R less? : (% , NonNegativeInteger) -> Boolean
--R map : ((Record(k: Integer, c: K) -> Record(k: Integer, c: K)) , %) -> %
--R map : (((Record(k: Integer, c: K) , Record(k: Integer, c: K)) -> Record(k: Integer, c: K)) , % , %)
--R map! : ((Record(k: Integer, c: K) -> Record(k: Integer, c: K)) , %) -> % if $ has shallowlyMutabl
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (Record(k: Integer, c: K) , %) -> Boolean if $ has finiteAggregate and Record(k: Integer, c: K)
--R members : % -> List Record(k: Integer, c: K) if $ has finiteAggregate
--R minIndex : % -> Integer if Integer has ORDSET
--R monomial : (% , SingletonAsOrderedSet, Integer) -> %
--R monomial : (% , List SingletonAsOrderedSet, List Integer) -> %
--R monomial2series : (List % , List NonNegativeInteger, Integer) -> %
--R more? : (% , NonNegativeInteger) -> Boolean
--R multiEuclidean : (List % , %) -> Union(List % , "failed")
--R multiplyExponents : (% , PositiveInteger) -> %
--R new : (NonNegativeInteger, Record(k: Integer, c: K)) -> %
--R node? : (% , %) -> Boolean if Record(k: Integer, c: K) has SETCAT
--R numberOfComputedEntries : % -> NonNegativeInteger
--R orderIfNegative : % -> Union(Integer, "failed")
--R parts : % -> List Record(k: Integer, c: K) if $ has finiteAggregate
--R principalIdeal : List % -> Record(coef: List % , generator: %)
--R qelt : (% , Integer) -> Record(k: Integer, c: K)
--R qsetelt! : (% , Integer, Record(k: Integer, c: K)) -> Record(k: Integer, c: K) if $ has shallowlyMutabl
--R reduce : (((Record(k: Integer, c: K) , Record(k: Integer, c: K)) -> Record(k: Integer, c: K)) , %)
--R reduce : (((Record(k: Integer, c: K) , Record(k: Integer, c: K)) -> Record(k: Integer, c: K)) , %)
--R reduce : (((Record(k: Integer, c: K) , Record(k: Integer, c: K)) -> Record(k: Integer, c: K)) , %)
--R remove : (Record(k: Integer, c: K) , %) -> % if $ has finiteAggregate and Record(k: Integer, c: K)
--R remove : ((Record(k: Integer, c: K) -> Boolean) , %) -> %
--R removeDuplicates : % -> % if $ has finiteAggregate and Record(k: Integer, c: K) has SETCAT
--R rest : (% , NonNegativeInteger) -> %
--R second : % -> Record(k: Integer, c: K)
--R select : ((Record(k: Integer, c: K) -> Boolean) , %) -> %
--R setchildren! : (% , List %) -> % if $ has shallowlyMutable
--R setelt : (% , value, Record(k: Integer, c: K)) -> Record(k: Integer, c: K) if $ has shallowlyMutable
--R setelt : (% , first, Record(k: Integer, c: K)) -> Record(k: Integer, c: K) if $ has shallowlyMutable
--R setelt : (% , rest, %) -> % if $ has shallowlyMutable
--R setelt : (% , last, Record(k: Integer, c: K)) -> Record(k: Integer, c: K) if $ has shallowlyMutable
--R setelt : (% , UniversalSegment Integer, Record(k: Integer, c: K)) -> Record(k: Integer, c: K)
--R setelt : (% , Integer, Record(k: Integer, c: K)) -> Record(k: Integer, c: K) if $ has shallowlyMutable
--R setfirst! : (% , Record(k: Integer, c: K)) -> Record(k: Integer, c: K) if $ has shallowlyMutable
--R setlast! : (% , Record(k: Integer, c: K)) -> Record(k: Integer, c: K) if $ has shallowlyMutable
--R setrest! : (% , %) -> % if $ has shallowlyMutable
--R setvalue! : (% , Record(k: Integer, c: K)) -> Record(k: Integer, c: K) if $ has shallowlyMutable
--R size? : (% , NonNegativeInteger) -> Boolean

```

```

--R split! : (%,Integer) -> % if $ has shallowlyMutable
--R subtractIfCan : (%,% ) -> Union(%,"failed")
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R terms : % -> Stream Record(k: Integer,c: K)
--R third : % -> Record(k: Integer,c: K)
--R truncate : (%,Integer,Integer) -> %
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R value : % -> Record(k: Integer,c: K)
--R variables : % -> List SingletonAsOrderedSet
--R
--E 1

```

```

)spool
)lisp (bye)

```

---

### — NeitherSparseOrDensePowerSeries.help —

```

=====
NeitherSparseOrDensePowerSeries examples
=====

```

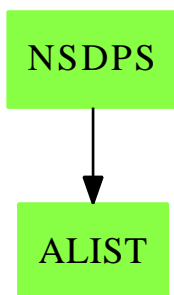
```

See Also:
o )show NeitherSparseOrDensePowerSeries

```

---

#### 15.1.1 NeitherSparseOrDensePowerSeries (NSDPS)



Exports:

|                         |                        |                   |
|-------------------------|------------------------|-------------------|
| 0                       | 1                      | #?                |
| -?                      | ***?                   | ?*?               |
| ?+?                     | ?-?                    | ?..?              |
| ?.first                 | ?.last                 | ?.rest            |
| ?.value                 | ?/?                    | ?=?               |
| ?^?                     | ?~=?                   | ?quo?             |
| ?rem?                   | D                      | any?              |
| approximate             | associates?            | center            |
| characteristic          | charthRoot             | child?            |
| children                | coefOfFirstNonZeroTerm | coefficient       |
| coerce                  | complete               | concat            |
| concat!                 | construct              | convert           |
| copy                    | count                  | cycleEntry        |
| cycleLength             | cycleSplit!            | cycleTail         |
| cyclic?                 | degree                 | delay             |
| delete                  | differentiate          | distance          |
| divide                  | elt                    | empty             |
| empty?                  | entries                | entry?            |
| eq?                     | euclideanSize          | eval              |
| every?                  | explicitEntries?       | explicitlyEmpty?  |
| explicitlyFinite?       | expressIdealMember     | exquo             |
| extend                  | extendedEuclidean      | factor            |
| fill!                   | filterUpTo             | find              |
| findCoef                | findTerm               | first             |
| frst                    | gcd                    | gcdPolynomial     |
| hash                    | index?                 | indices           |
| insert                  | inv                    | last              |
| latex                   | lazy?                  | lazyEvaluate      |
| lcm                     | leadingCoefficient     | leadingMonomial   |
| leaf?                   | leaves                 | less?             |
| map                     | map!                   | maxIndex          |
| member?                 | members                | minIndex          |
| monomial                | monomial2series        | monomial?         |
| more?                   | multiEuclidean         | multiplyExponents |
| new                     | node?                  | nodes             |
| numberOfComputedEntries | one?                   | order             |
| orderIfNegative         | parts                  | pole?             |
| posExpnPart             | possiblyInfinite?      | prime?            |
| principalIdeal          | printInfo              | qelt              |
| qsetelt!                | recip                  | reduce            |
| reductum                | remove                 | removeDuplicates  |
| removeFirstZeroes       | removeZeroes           | rest              |
| rst                     | sample                 | sbt               |
| second                  | select                 | series            |
| setchildren!            | setelt                 | setfirst!         |
| setlast!                | setrest!               | setvalue!         |
| shift                   | size?                  | sizeLess?         |
| split!                  | squareFree             | squareFreePart    |
| subtractIfCan           | swap!                  | tail              |
| terms                   | third                  | truncate          |
| unit?                   | unitCanonical          | unitNormal        |
| value                   | variable               | variables         |
| zero?                   |                        |                   |

## — domain NSDPS NeitherSparseOrDensePowerSeries —

```

)abbrev domain NSDPS NeitherSparseOrDensePowerSeries
++ Authors: Gaetan Hache
++ Date Created: june 1996
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
++ This domain is part of the PAFF package
NeitherSparseOrDensePowerSeries(K):Exports == Implementation where
  K:Field

SI  ==> SingleInteger
INT ==> Integer
TERM ==> Record(k:INT,c:K)
SER  ==> Stream(TERM)
NNI  ==> NonNegativeInteger

Exports ==> Join(LocalPowerSeriesCategory(K),LazyStreamAggregate(TERM)) with

  findTerm: (% ,Integer) -> TERM

Implementation ==> SER add

  Rep:=SER

  var : Symbol := 't

  multC: (K,INT,%) -> %

  orderIfNegative(s:%)==
    zero?(s) => "failed"
    f:=first(s)
    f.k >= 0 => "failed"
    zero?(f.c) => orderIfNegative(rest(s))
    f.k

  posExpnPart(s)==
    zero?(s) => 0
    o:=order s
    (o >= 0) => s
    posExpnPart(rst s)

  findTerm(s,n)==
    empty?(s) => [n,0]$TERM
    f:=first(s)
    f.k > n => [n,0]$TERM
    f.k = n => f
    findTerm(rst(s),n)

```



```

findCoef(s,i)==findTerm(s,i).c

coerce(s:%):SER == s::Rep

coerce(s:SER):%==s

localVarForPrintInfo:Boolean:=false()

printInfo==localVarForPrintInfo

printInfo(flag)==localVarForPrintInfo:=flag

outTerm: TERM -> OutputForm

removeZeroes(s)== delay
  zero?(s) => 0
  f:=first(s)
  zero?(f.c) => removeZeroes(rst(s))
  concat(f,removeZeroes(rst(s)))

inv(ra)==
  a:=removeFirstZeroes ra
  o:=-order(a)
  aa:=shift(a,o)
  aai:=recip aa
  aai case "failed" => _
    error "Big problem in inv function from CreateSeries"
  shift(aai,o)

iDiv: (%,%,K) -> %
iDiv(x,y,ry0) == delay
  empty? x => 0$%
  sx:TERM:=first x
  c0:K:=ry0 * sx.c
  nT:TERM:=[sx.k, c0]
  tc0:%:=series(sx.k,c0,0$%)
  concat(nT,iDiv(rst x - tc0 * rst y,y,ry0))

recip x ==
  empty? x => "failed"
  rh1:TERM:=first x
  ^zero?(rh1.k) => "failed"
  ic:K:= inv(rh1.c)
  delay
    concat([0,ic]$TERM,iDiv(- ic * rst x,x,ic))

removeFirstZeroes(s)==
  zero?(s) => 0
  f:=first(s)
  zero?(f.c) => removeFirstZeroes(rst(s))

```

```

s

sbt(sa,sbb)== delay
  sb:=removeFirstZeroes(sbb)
  o:=order sb
  ^ (o > 0) => _
    error "Cannot substitute by a series of order less than 1  !!!!!"
  empty?(sa) or empty?(sb) => 0
  fa:TERM:=first(sa)
  fb:TERM:=first(sb)
  firstElem:TERM:=[fa.k*fb.k, fa.c*(fb.c**fa.k)]
  zero?(fa.c) => sbt(rst(sa),sb)
  concat(firstElem, rest((fa.c) * sb ** (fa.k)) + sbt(rst(sa),sb) )

coerce(s:%):OutputForm==
  zero?(s) => "0" :: OutputForm
  count:SI:= _$streamCount$Lisp
  lstTerm:List TERM:=empty()
  rs:%:= s
  for i in 1..count while ^empty?(rs) repeat
    fs:=first rs
    rs:=rst rs
    lstTerm:=concat(lstTerm,fs)
  listOfOutTerm:List OutputForm:=_
    [outTerm(t) for t in lstTerm | ^zero?(t.c) ]
  out:OutputForm:=
    if empty?(listOfOutTerm) then
      "0" :: OutputForm
    else
      reduce("+", listOfOutTerm)
  empty?(rs) => out
  out + ("..." :: OutputForm)

outTerm(t)==
  ee:=t.k
  cc:=t.c
  oe:OutputForm:=ee::OutputForm
  oc:OutputForm:=cc::OutputForm
  symb:OutputForm:= var :: OutputForm
  one?(cc) and one?(ee) => symb
  zero?(ee) => oc
  one?(cc) => symb ** oe
  one?(ee) => oc * symb
  oc * symb ** oe

removeZeroes(n,s)== delay
  n < 0 => s
  zero?(s) => 0
  f:=first(s)
  zero?(f.c) => removeZeroes(n-1, rst(s))

```

```

concat(f,removeZeroes(n-1, rst(s)))

order(s:%)==
  zero?(s) => error _
  "From order (PlaneCurveLocalPowerSeries): cannot compute the order of 0"
f:=first(s)
zero?(f.c) => order(rest(s))
f.k

monomial2series(lpar,lexp,sh)==
  shift(reduce("*",[s**e for s in lpar for e in lexp]),sh)

coefOfFirstNonZeroTerm(s:%)==
  zero?(s) => error _
  "From order (PlaneCurveLocalPowerSeries): cannot find the coefOfFirstNonZeroTerm"
f:=first(s)
zero?(f.c) => coefOfFirstNonZeroTerm(rest(s))
f.c

degreeOfTermLower?: (TERM,INT) -> Boolean
degreeOfTermLower?(t,n)== t.k < n

filterUpTo(s,n)==filterWhile(degreeOfTermLower?(#1,n),s)

series(exp,coef,s)==cons([exp,coef]$TERM,s)

a:% ** n:NNI == -- delay
  zero?(n) => 1
  exp(a,n :: PositiveInteger)$RepeatedSquaring(%)

0 == empty()

1 == construct([[0,1]$TERM])

zero?(a)==empty?(a::Rep)

shift(s,n)== delay
  zero?(s) => 0
  fs:=first(s)
  es:=fs.k
  concat([es+n,fs.c]$TERM,shift(rest(s),n))

a:% + b:% == delay
  zero?(a) => b
  zero?(b) => a
  fa:=first(a)
  fb:=first(b)
  ea:=fa.k
  eb:=fb.k
  nc:K

```

```

    ea = eb => concat([ea,fa.c+fb.c]$TERM,rest(a) + rest(b))
    ea > eb => concat([eb,fb.c]$TERM,a + rest(b))
    eb > ea => concat([ea,fa.c]$TERM,rest(a) + b)

- a:% == --delay
  multC( (-1) :: K , 0 , a)

a:% - b:% == --delay
  a+(-b)

multC(coef,n,s)== delay
  zero?(coef) => 0
  zero?(s) => 0
  f:=first(s)
  concat([f.k+n,coef*f.c]$TERM,multC(coef,n,rest(s)))

coef:K * s:% == delay
  zero?(coef) => 0
  zero?(s) => 0
  f:=first(s)
  concat([f.k,coef*f.c]$TERM, coef *$% rest(s))

s:% * coef:K == coef * s

s1:% * s2:%== delay
  zero?(s1) or zero?(s2) => 0
  f1:TERM:=first(s1)
  f2:TERM:=first(s2)
  e1:INT:=f1.k; e2:INT:=f2.k
  c1:K:=f1.c;   c2:K:=f2.c
  concat([e1+e2,c1*c2]$TERM,_
    multC(c1,e1,rest(s2))+multC(c2,e2,rest(s1))+rest(s1)*rest(s2))

```

---

— NSDPS.dotabb —

```

"NSDPS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=NSDPS"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"NSDPS" -> "ALIST"

```

---

## 15.2 domain NSMP NewSparseMultivariatePolynomial

Based on the **PseudoRemainderSequence** package, the domain constructor **NewSparseMultivariatePolynomial** extends the constructor **SparseMultivariatePolynomial**. It also provides some additional operations related to polynomial system solving by means of triangular sets.

— NewSparseMultivariatePolynomial.input —

```
)set break resume
)sys rm -f NewSparseMultivariatePolynomial.output
)spool NewSparseMultivariatePolynomial.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show NewSparseMultivariatePolynomial
--R NewSparseMultivariatePolynomial(R: Ring,VarSet: OrderedSet) is a domain constructor
--R Abbreviation for NewSparseMultivariatePolynomial is NSMP
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for NSMP
--R
--R----- Operations -----
--R ?? : (R,R) -> %
--R ?? : (R,R) -> %
--R ?? : (Integer,R) -> %
--R ?? : (PositiveInteger,R) -> %
--R ?? : (R,R) -> %
--R -? : % -> %
--R D : (R,List VarSet) -> %
--R 1 : () -> %
--R ?? : (R,PositiveInteger) -> %
--R coerce : VarSet -> %
--R coerce : Integer -> %
--R deepestInitial : % -> %
--R differentiate : (R,VarSet) -> %
--R eval : (R,VarSet,R) -> %
--R eval : (R,R,R) -> %
--R eval : (R,List Equation %) -> %
--R ground? : % -> Boolean
--R head : % -> %
--R headReduced? : (R,R) -> Boolean
--R init : % -> %
--R iteratedInitials : % -> List %
--R lazyPquo : (R,R,VarSet) -> %
--R lazyPrem : (R,R,VarSet) -> %
--R leadingCoefficient : % -> R
--R leastMonomial : % -> %
--R mainMonomial : % -> %
--R ?? : (R,R) -> %
--R ?? : (Integer,R) -> %
--R ?? : (R,PositiveInteger) -> %
--R -? : (R,R) -> %
--R ?? : (R,R) -> Boolean
--R D : (R,VarSet) -> %
--R 0 : () -> %
--R coefficients : % -> List R
--R coerce : R -> %
--R coerce : % -> OutputForm
--R deepestTail : % -> %
--R eval : (R,VarSet,R) -> %
--R eval : (R,List %,List %) -> %
--R eval : (R,Equation %) -> %
--R ground : % -> R
--R hash : % -> SingleInteger
--R headReduce : (R,R) -> %
--R infRittWu? : (R,R) -> Boolean
--R initiallyReduce : (R,R) -> %
--R latex : % -> String
--R lazyPquo : (R,R) -> %
--R lazyPrem : (R,R) -> %
--R leadingMonomial : % -> %
--R mainCoefficients : % -> List %
--R mainMonomials : % -> List %
```

```

--R map : ((R -> R),%) -> %
--R monic? : % -> Boolean
--R monomial? : % -> Boolean
--R mvar : % -> VarSet
--R one? : % -> Boolean
--R pquo : (%,%) -> %
--R prem : (%,%) -> %
--R quasiMonic? : % -> Boolean
--R reduced? : (%,List %) -> Boolean
--R reductum : (%,VarSet) -> %
--R retract : % -> VarSet
--R sample : () -> %
--R tail : % -> %
--R zero? : % -> Boolean
--R ?? : (Fraction Integer,%) -> % if R has ALGEBRA FRAC INT
--R ?? : (%,Fraction Integer) -> % if R has ALGEBRA FRAC INT
--R ?? : (NonNegativeInteger,%) -> %
--R ***? : (%,NonNegativeInteger) -> %
--R ?/? : (%,R) -> % if R has FIELD
--R ?<? : (%,%) -> Boolean if R has ORDSET
--R ?<=? : (%,%) -> Boolean if R has ORDSET
--R ?>? : (%,%) -> Boolean if R has ORDSET
--R ?>=? : (%,%) -> Boolean if R has ORDSET
--R D : (%,List VarSet,List NonNegativeInteger) -> %
--R D : (%,VarSet,NonNegativeInteger) -> %
--R LazardQuotient : (%,%,NonNegativeInteger) -> % if R has INTDOM
--R LazardQuotient2 : (%,%,%,NonNegativeInteger) -> % if R has INTDOM
--R RittWuCompare : (%,%) -> Union(Boolean,"failed")
--R ?? : (%,NonNegativeInteger) -> %
--R associates? : (%,%) -> Boolean if R has INTDOM
--R binomThmExpt : (%,%,NonNegativeInteger) -> % if R has COMRING
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%, "failed") if $ has CHARNZ and R has PFECAT or R has CHARNZ
--R coefficient : (%,List VarSet,List NonNegativeInteger) -> %
--R coefficient : (%,VarSet,NonNegativeInteger) -> %
--R coefficient : (%,IndexedExponents VarSet) -> R
--R coerce : % -> % if R has INTDOM
--R coerce : Fraction Integer -> % if R has ALGEBRA FRAC INT or R has RETRACT FRAC INT
--R coerce : SparseMultivariatePolynomial(R,VarSet) -> %
--R coerce : % -> SparseMultivariatePolynomial(R,VarSet)
--R coerce : % -> Polynomial R if VarSet has KONVERT SYMBOL
--R conditionP : Matrix % -> Union(Vector %, "failed") if $ has CHARNZ and R has PFECAT
--R content : (%,VarSet) -> % if R has GCDDOM
--R content : % -> R if R has GCDDOM
--R convert : % -> Polynomial R if VarSet has KONVERT SYMBOL
--R convert : % -> String if R has RETRACT INT and VarSet has KONVERT SYMBOL
--R convert : Polynomial R -> % if VarSet has KONVERT SYMBOL
--R convert : Polynomial Integer -> % if R has ALGEBRA INT and VarSet has KONVERT SYMBOL and not has(R,A
--R convert : Polynomial Fraction Integer -> % if R has ALGEBRA FRAC INT and VarSet has KONVERT SYMBOL
--R convert : % -> InputForm if R has KONVERT INFORM and VarSet has KONVERT INFORM

```

```

--R convert : % -> Pattern Integer if R has KONVERT PATTERN INT and VarSet has KONVERT PATTE
--R convert : % -> Pattern Float if R has KONVERT PATTERN FLOAT and VarSet has KONVERT PATTE
--R degree : (% ,List VarSet) -> List NonNegativeInteger
--R degree : (% ,VarSet) -> NonNegativeInteger
--R degree : % -> IndexedExponents VarSet
--R differentiate : (% ,List VarSet,List NonNegativeInteger) -> %
--R differentiate : (% ,VarSet,NonNegativeInteger) -> %
--R differentiate : (% ,List VarSet) -> %
--R discriminant : (% ,VarSet) -> % if R has COMRING
--R eval : (% ,List VarSet,List %) -> %
--R eval : (% ,List VarSet,List R) -> %
--R exactQuotient : (% ,%) -> % if R has INTDOM
--R exactQuotient : (% ,R) -> % if R has INTDOM
--R exactQuotient! : (% ,%) -> % if R has INTDOM
--R exactQuotient! : (% ,R) -> % if R has INTDOM
--R exquo : (% ,%) -> Union(% ,"failed") if R has INTDOM
--R exquo : (% ,R) -> Union(% ,"failed") if R has INTDOM
--R extendedSubResultantGcd : (% ,%) -> Record(gcd: % ,coef1: % ,coef2: % ) if R has INTDOM
--R factor : % -> Factored % if R has PFECAT
--R factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial %
--R factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePo
--R gcd : (% ,%) -> % if R has GCDDOM
--R gcd : List % -> % if R has GCDDOM
--R gcd : (R ,%) -> R if R has GCDDOM
--R gcdPolynomial : (SparseUnivariatePolynomial % ,SparseUnivariatePolynomial %) -> SparseUni
--R halfExtendedSubResultantGcd1 : (% ,%) -> Record(gcd: % ,coef1: % ) if R has INTDOM
--R halfExtendedSubResultantGcd2 : (% ,%) -> Record(gcd: % ,coef2: % ) if R has INTDOM
--R headReduced? : (% ,List %) -> Boolean
--R initiallyReduced? : (% ,List %) -> Boolean
--R initiallyReduced? : (% ,%) -> Boolean
--R isExpt : % -> Union(Record(var: VarSet ,exponent: NonNegativeInteger) ,"failed")
--R isPlus : % -> Union(List % ,"failed")
--R isTimes : % -> Union(List % ,"failed")
--R lastSubResultant : (% ,%) -> % if R has INTDOM
--R lazyPremWithDefault : (% ,% ,VarSet) -> Record(coef: % ,gap: NonNegativeInteger ,remainder: % )
--R lazyPremWithDefault : (% ,%) -> Record(coef: % ,gap: NonNegativeInteger ,remainder: % )
--R lazyPseudoDivide : (% ,% ,VarSet) -> Record(coef: % ,gap: NonNegativeInteger ,quotient: % ,re
--R lazyPseudoDivide : (% ,%) -> Record(coef: % ,gap: NonNegativeInteger ,quotient: % ,remainder
--R lazyResidueClass : (% ,%) -> Record(polnum: % ,polden: % ,power: NonNegativeInteger)
--R lcm : (% ,%) -> % if R has GCDDOM
--R lcm : List % -> % if R has GCDDOM
--R leadingCoefficient : (% ,VarSet) -> %
--R mainContent : % -> % if R has GCDDOM
--R mainPrimitivePart : % -> % if R has GCDDOM
--R mainSquareFreePart : % -> % if R has GCDDOM
--R mainVariable : % -> Union(VarSet ,"failed")
--R mapExponents : ((IndexedExponents VarSet -> IndexedExponents VarSet) ,%) -> %
--R max : (% ,%) -> % if R has ORDSET
--R min : (% ,%) -> % if R has ORDSET
--R minimumDegree : (% ,List VarSet) -> List NonNegativeInteger

```

```

--R minimumDegree : (% , VarSet) -> NonNegativeInteger
--R minimumDegree : % -> IndexedExponents VarSet
--R monicDivide : (% , % , VarSet) -> Record(quotient: % , remainder: %)
--R monomial : (% , List VarSet , List NonNegativeInteger) -> %
--R monomial : (% , VarSet , NonNegativeInteger) -> %
--R monomial : (R , IndexedExponents VarSet) -> %
--R multivariate : (SparseUnivariatePolynomial % , VarSet) -> %
--R multivariate : (SparseUnivariatePolynomial R , VarSet) -> %
--R nextsubResultant2 : (% , % , % , %) -> % if R has INTDOM
--R normalized? : (% , List %) -> Boolean
--R numberOfMonomials : % -> NonNegativeInteger
--R patternMatch : (% , Pattern Integer , PatternMatchResult(Integer , %)) -> PatternMatchResult(Integer , %) if
--R patternMatch : (% , Pattern Float , PatternMatchResult(Float , %)) -> PatternMatchResult(Float , %) if R has
--R pomopo! : (% , R , IndexedExponents VarSet , %) -> %
--R primPartElseUnitCanonical : % -> % if R has INTDOM
--R primPartElseUnitCanonical! : % -> % if R has INTDOM
--R prime? : % -> Boolean if R has PFECAT
--R primitivePart : (% , VarSet) -> % if R has GCDDOM
--R primitivePart : % -> % if R has GCDDOM
--R primitivePart! : % -> % if R has GCDDOM
--R pseudoDivide : (% , %) -> Record(quotient: % , remainder: %)
--R reducedSystem : Matrix % -> Matrix R
--R reducedSystem : (Matrix % , Vector %) -> Record(mat: Matrix R , vec: Vector R)
--R reducedSystem : (Matrix % , Vector %) -> Record(mat: Matrix Integer , vec: Vector Integer) if R has LINE
--R reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT
--R resultant : (% , %) -> % if R has INTDOM
--R resultant : (% , % , VarSet) -> % if R has COMRING
--R retract : % -> SparseMultivariatePolynomial(R , VarSet)
--R retract : Polynomial R -> % if VarSet has KONVERT SYMBOL and not has(R , Algebra Fraction Integer) and
--R retract : Polynomial Integer -> % if R has ALGEBRA INT and VarSet has KONVERT SYMBOL and not has(R , A
--R retract : Polynomial Fraction Integer -> % if R has ALGEBRA FRAC INT and VarSet has KONVERT SYMBOL
--R retract : % -> Integer if R has RETRACT INT
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(SparseMultivariatePolynomial(R , VarSet) , "failed")
--R retractIfCan : Polynomial R -> Union(% , "failed") if VarSet has KONVERT SYMBOL and not has(R , Algebra
--R retractIfCan : Polynomial Integer -> Union(% , "failed") if R has ALGEBRA INT and VarSet has KONVERT S
--R retractIfCan : Polynomial Fraction Integer -> Union(% , "failed") if R has ALGEBRA FRAC INT and VarSet
--R retractIfCan : % -> Union(VarSet , "failed")
--R retractIfCan : % -> Union(Integer , "failed") if R has RETRACT INT
--R retractIfCan : % -> Union(Fraction Integer , "failed") if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(R , "failed")
--R solveLinearPolynomialEquation : (List SparseUnivariatePolynomial % , SparseUnivariatePolynomial %) ->
--R squareFree : % -> Factored % if R has GCDDOM
--R squareFreePart : % -> % if R has GCDDOM
--R squareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % if R ha
--R subResultantChain : (% , %) -> List % if R has INTDOM
--R subResultantGcd : (% , %) -> % if R has INTDOM
--R subtractIfCan : (% , %) -> Union(% , "failed")
--R totalDegree : (% , List VarSet) -> NonNegativeInteger
--R totalDegree : % -> NonNegativeInteger

```



```

--R unit? : % -> Boolean if R has INTDOM
--R unitCanonical : % -> % if R has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has INTDOM
--R univariate : % -> SparseUnivariatePolynomial R
--R univariate : (% ,VarSet) -> SparseUnivariatePolynomial %
--R
--E 1

)spool
)lisp (bye)

```

---

— NewSparseMultivariatePolynomial.help —

```

=====
NewSparseMultivariatePolynomial examples
=====

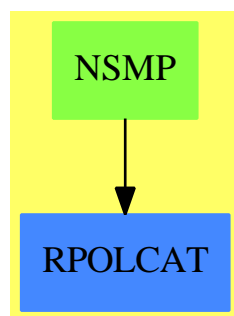
```

```

See Also:
o )show NewSparseMultivariatePolynomial

```

### 15.2.1 NewSparseMultivariatePolynomial (NSMP)



See

⇒ “NewSparseUnivariatePolynomial” (NSUP) 15.3.1 on page 1691

**Exports:**

|                              |                               |                     |
|------------------------------|-------------------------------|---------------------|
| 0                            | 1                             | associates?         |
| binomThmExpt                 | characteristic                | charthRoot          |
| coefficient                  | coefficients                  | coerce              |
| conditionP                   | content                       | D                   |
| degree                       | deepestInitial                | deepestTail         |
| differentiate                | discriminant                  | eval                |
| exactQuotient                | exactQuotient!                | exquo               |
| extendedSubResultantGcd      | factor                        | factorPolynomial    |
| factorSquareFreePolynomial   | gcd                           | gcdPolynomial       |
| halfExtendedSubResultantGcd1 | ground                        | ground?             |
| halfExtendedSubResultantGcd2 | hash                          | head                |
| headReduce                   | headReduced?                  | infRittWu?          |
| init                         | initiallyReduce               | initiallyReduced?   |
| isExpt                       | isPlus                        | isTimes             |
| iteratedInitials             | lastSubResultant              | latex               |
| LazardQuotient               | LazardQuotient2               | lazyPremWithDefault |
| lazyPquo                     | lazyPrem                      | lazyPseudoDivide    |
| lazyResidueClass             | lcm                           | leadingCoefficient  |
| leadingMonomial              | leastMonomial                 | mainCoefficients    |
| mainContent                  | mainMonomial                  | mainMonomials       |
| mainPrimitivePart            | mainSquareFreePart            | mainVariable        |
| map                          | mapExponents                  | max                 |
| mdeg                         | min                           | minimumDegree       |
| monic?                       | monicDivide                   | monicModulo         |
| monomial                     | monomial?                     | monomials           |
| multivariate                 | mvar                          | nextsubResultant2   |
| normalized?                  | numberOfMonomials             | one?                |
| patternMatch                 | popopo!                       | pquo                |
| primPartElseUnitCanonical!   | primPartElseUnitCanonical     | prem                |
| prime?                       | primitiveMonomials            | primitivePart       |
| primitivePart!               | pseudoDivide                  | reducedSystem       |
| resultant                    | retract                       | retractIfCan        |
| RittWuCompare                | quasiMonic?                   | recip               |
| reduced?                     | reductum                      | retract             |
| sample                       | solveLinearPolynomialEquation | squareFree          |
| squareFreePart               | squareFreePolynomial          | subResultantChain   |
| subResultantGcd              | supRittWu?                    | subtractIfCan       |
| tail                         | totalDegree                   | unit?               |
| unitCanonical                | unitNormal                    | univariate          |
| univariate                   | variables                     | zero?               |
| ?*?                          | ***?                          | ?+?                 |
| ?-?                          | -?                            | ?=?                 |
| ?^?                          | ?~=?                          | ?/?                 |
| ?<?                          | ?<=?                          | ?>?                 |
| ?>=?                         |                               |                     |

## — domain NSMP NewSparseMultivariatePolynomial —

```

)abbrev domain NSMP NewSparseMultivariatePolynomial
++ Author: Marc Moreno Maza
++ Date Created: 22/04/94
++ Date Last Updated: 14/12/1998
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ A post-facto extension for \axiomType{SMP} in order
++ to speed up operations related to pseudo-division and gcd.
++ This domain is based on the \axiomType{NSUP} constructor which is
++ itself a post-facto extension of the \axiomType{SUP} constructor.

NewSparseMultivariatePolynomial(R,VarSet) : Exports == Implementation where
  R:Ring
  VarSet:OrderedSet
  N ==> NonNegativeInteger
  Z ==> Integer
  SUP ==> NewSparseUnivariatePolynomial
  SMPR ==> SparseMultivariatePolynomial(R, VarSet)
  SUP2 ==> NewSparseUnivariatePolynomialFunctions2($,$)

Exports == Join(RecursivePolynomialCategory(R,IndexedExponents VarSet,
  VarSet), CoercibleTo(SMPR),RetractableTo(SMPR))

Implementation == SparseMultivariatePolynomial(R, VarSet) add

  D := NewSparseUnivariatePolynomial($)
  VPoly:= Record(v:VarSet,ts:D)
  Rep:= Union(R,VPoly)

--local function
PSimp: (D,VarSet) -> %

PSimp(up,mv) ==
  if degree(up) = 0 then leadingCoefficient(up) else [mv,up]$VPoly

coerce (p:$):SMPR ==
  p pretend SMPR

coerce (p:SMPR):$ ==
  p pretend $

```

```

retractIfCan (p:$) : Union(SMPR,"failed") ==
  (p pretend SMPR)::Union(SMPR,"failed")

mvar p ==
  p case R => error" Error in mvar from NSMP : #1 has no variables."
  p.v

mdeg p ==
  p case R => 0$N
  degree(p.ts)$D

init p ==
  p case R => error" Error in init from NSMP : #1 has no variables."
  leadingCoefficient(p.ts)$D

head p ==
  p case R => p
  ([p.v,leadingMonomial(p.ts)$D]$VPoly)::Rep

tail p ==
  p case R => 0$$$
  red := reductum(p.ts)$D
  ground?(red)$D => (ground(red)$D)::Rep
  ([p.v,red]$VPoly)::Rep

iteratedInitials p ==
  p case R => []
  p := leadingCoefficient(p.ts)$D
  cons(p,iteratedInitials(p))

localDeepestInitial (p : $) : $ ==
  p case R => p
  localDeepestInitial leadingCoefficient(p.ts)$D

deepestInitial p ==
  p case R =>
    error"Error in deepestInitial from NSMP : #1 has no variables."
    localDeepestInitial leadingCoefficient(p.ts)$D

mainMonomial p ==
  zero? p =>
    error"Error in mainMonomial from NSMP : the argument is zero"
  p case R => 1$$$
  monomial(1$$,p.v,degree(p.ts)$D)

leastMonomial p ==
  zero? p =>
    error"Error in leastMonomial from NSMP : the argument is zero"
  p case R => 1$$$
  monomial(1$$,p.v,minimumDegree(p.ts)$D)

```

```

mainCoefficients p ==
  zero? p =>
    error"Error in mainCoefficients from NSMP : the argument is zero"
  p case R => [p]
  coefficients(p.ts)$D

leadingCoefficient(p:$,x:VarSet):$ ==
  (p case R) => p
  p.v = x => leadingCoefficient(p.ts)$D
  zero? (d := degree(p,x)) => p
  coefficient(p,x,d)

localMonicModulo(a:$,b:$):$ ==
  -- b is assumed to have initial 1
  a case R => a
  a.v < b.v => a
  mM: $
  if a.v > b.v
  then
    m : D := map((a1:%):% +-> localMonicModulo(a1,b),a.ts)$SUP2
  else
    m : D := monicModulo(a.ts,b.ts)$D
  if ground?(m)$D
  then
    mM := (ground(m)$D)::Rep
  else
    mM := ([a.v,m]$VPoly)::Rep
  mM

monicModulo (a,b) ==
  b case R => error"Error in monicModulo from NSMP : #2 is constant"
  ib : $ := init(b)@$
  not ground?(ib)$ =>
    error"Error in monicModulo from NSMP : #2 is not monic"
  mM : $
  -- if not one?(ib)$
  if not ((ib) = 1)$
  then
    r : R := ground(ib)$
    rec : Union(R,"failed"):= recip(r)$R
    (rec case "failed") =>
      error"Error in monicModulo from NSMP : #2 is not monic"
    a case R => a
    a := (rec::R) * a
    b := (rec::R) * b
    mM := ib * localMonicModulo (a,b)
  else
    mM := localMonicModulo (a,b)
  mM

```

```

prem(a:$, b:$): $ ==
  -- with pseudoRemainder$NSUP
  b case R =>
    error "in prem$NSMP: ground? #2"
  db: N := degree(b.ts)$D
  lcb: $ := leadingCoefficient(b.ts)$D
  test: Z := degree(a,b.v)::Z - db
  delta: Z := max(test + 1$Z, 0$Z)
  (a case R) or (a.v < b.v) => lcb ** (delta::N) * a
  a.v = b.v =>
    r: D := pseudoRemainder(a.ts,b.ts)$D
    ground?(r) => return (ground(r)$D)::Rep
    ([a.v,r]$VPoly)::Rep
  while not zero?(a) and not negative?(test) repeat
    term := monomial(leadingCoefficient(a,b.v),b.v,test::N)
    a := lcb * a - term * b
    delta := delta - 1$Z
    test := degree(a,b.v)::Z - db
  lcb ** (delta::N) * a

pquo (a:$, b:$) : $ ==
  cPS := lazyPseudoDivide (a,b)
  c := (cPS.coef) ** (cPS.gap)
  c * cPS.quotient

pseudoDivide(a:$, b:$): Record (quotient : $, remainder : $) ==
  -- from RPOLCAT
  cPS := lazyPseudoDivide(a,b)
  c := (cPS.coef) ** (cPS.gap)
  [c * cPS.quotient, c * cPS.remainder]

lazyPrem(a:$, b:$): $ ==
  -- with lazyPseudoRemainder$NSUP
  -- Uses leadingCoefficient: ($, V) -> $
  b case R =>
    error "in lazyPrem$NSMP: ground? #2"
  (a case R) or (a.v < b.v) => a
  a.v = b.v => PSimp(lazyPseudoRemainder(a.ts,b.ts)$D,a.v)
  db: N := degree(b.ts)$D
  lcb: $ := leadingCoefficient(b.ts)$D
  test: Z := degree(a,b.v)::Z - db
  while not zero?(a) and not negative?(test) repeat
    term := monomial(leadingCoefficient(a,b.v),b.v,test::N)
    a := lcb * a - term * b
    test := degree(a,b.v)::Z - db
  a

lazyPquo (a:$, b:$) : $ ==
  -- with lazyPseudoQuotient$NSUP

```

```

b case R =>
  error " in lazyPquo$NSMP: #2 is conctant"
(a case R) or (a.v < b.v) => 0
a.v = b.v => PSimp(lazyPseudoQuotient(a.ts,b.ts)$D,a.v)
db: N := degree(b.ts)$D
lcb: $ := leadingCoefficient(b.ts)$D
test: Z := degree(a,b.v)::Z - db
q := 0$$
test: Z := degree(a,b.v)::Z - db
while not zero?(a) and not negative?(test) repeat
  term := monomial(leadingCoefficient(a,b.v),b.v,test::N)
  a := lcb * a - term * b
  q := lcb * q + term
  test := degree(a,b.v)::Z - db
q

lazyPseudoDivide(a:$, b:$): Record(coef:$, gap: N,quotient:$, remainder:$) ==
-- with lazyPseudoDivide$NSUP
b case R =>
  error " in lazyPseudoDivide$NSMP: #2 is conctant"
(a case R) or (a.v < b.v) => [1$$,0$N,0$$,a]
a.v = b.v =>
  cgqr := lazyPseudoDivide(a.ts,b.ts)
  [cgqr.coef, cgqr.gap, PSimp(cgqr.quotient,a.v), PSimp(cgqr.remainder,a.v)]
db: N := degree(b.ts)$D
lcb: $ := leadingCoefficient(b.ts)$D
test: Z := degree(a,b.v)::Z - db
q := 0$$
delta: Z := max(test + 1$Z, 0$Z)
while not zero?(a) and not negative?(test) repeat
  term := monomial(leadingCoefficient(a,b.v),b.v,test::N)
  a := lcb * a - term * b
  q := lcb * q + term
  delta := delta - 1$Z
  test := degree(a,b.v)::Z - db
[lcb, (delta::N), q, a]

lazyResidueClass(a:$, b:$): Record(polnum:$, polden:$, power:N) ==
-- with lazyResidueClass$NSUP
b case R =>
  error " in lazyResidueClass$NSMP: #2 is conctant"
lcb: $ := leadingCoefficient(b.ts)$D
(a case R) or (a.v < b.v) => [a,lcb,0]
a.v = b.v =>
  lrc := lazyResidueClass(a.ts,b.ts)$D
  [PSimp(lrc.polnum,a.v), lrc.polden, lrc.power]
db: N := degree(b.ts)$D
test: Z := degree(a,b.v)::Z - db
pow: N := 0
while not zero?(a) and not negative?(test) repeat

```

```

    term := monomial(leadingCoefficient(a,b.v),b.v,test::N)
    a := lcb * a - term * b
    pow := pow + 1
    test := degree(a,b.v)::Z - db
    [a, lcb, pow]

if R has IntegralDomain
then

    packD := PseudoRemainderSequence($,D)

    exactQuo(x:$, y:$):$ ==
        ex: Union($,"failed") := x exquo$$ y
        (ex case $) => ex::$
        error "in exactQuotient$NSMP: bad args"

    LazardQuotient(x:$, y:$, n: N):$ ==
        zero?(n) => error("LazardQuotient$NSMP : n = 0")
--      one?(n) => x
        (n = 1) => x
        a: N := 1
        while n >= (b := 2*a) repeat a := b
        c: $ := x
        n := (n - a)::N
        repeat
--      one?(a) => return c
        (a = 1) => return c
        a := a quo 2
        c := exactQuo(c*c,y)
        if n >= a then ( c := exactQuo(c*x,y) ; n := (n - a)::N )

    LazardQuotient2(p:$, a:$, b:$, n: N) ==
        zero?(n) => error " in LazardQuotient2$NSMP: bad #4"
--      one?(n) => p
        (n = 1) => p
        c: $ := LazardQuotient(a,b,(n-1)::N)
        exactQuo(c*p,b)

    next_subResultant2(p:$, q:$, z:$, s:$) ==
        PSimp(next_sousResultant2(p.ts,q.ts,z.ts,s)$packD,p.v)

    subResultantGcd(a:$, b:$): $ ==
        (a case R) or (b case R) =>
            error "subResultantGcd$NSMP: one arg is constant"
        a.v ~= b.v =>
            error "subResultantGcd$NSMP: mvar(#1) ~= mvar(#2)"
        PSimp(subResultantGcd(a.ts,b.ts),a.v)

    halfExtendedSubResultantGcd1(a:$,b:$): Record (gcd: $, coef1: $) ==
        (a case R) or (b case R) =>

```



```

    error "halfExtendedSubResultantGcd1$NSMP: one arg is constant"
  a.v ~= b.v =>
    error "halfExtendedSubResultantGcd1$NSMP: mvar(#1) ~= mvar(#2)"
  hesrg := halfExtendedSubResultantGcd1(a.ts,b.ts)$D
  [PSimp(hesrg.gcd,a.v), PSimp(hesrg.coef1,a.v)]

halfExtendedSubResultantGcd2(a:$,b:$): Record (gcd: $, coef2: $) ==
  (a case R) or (b case R) =>
    error "halfExtendedSubResultantGcd2$NSMP: one arg is constant"
  a.v ~= b.v =>
    error "halfExtendedSubResultantGcd2$NSMP: mvar(#1) ~= mvar(#2)"
  hesrg := halfExtendedSubResultantGcd2(a.ts,b.ts)$D
  [PSimp(hesrg.gcd,a.v), PSimp(hesrg.coef2,a.v)]

extendedSubResultantGcd(a:$,b:$): Record (gcd: $, coef1: $, coef2: $) ==
  (a case R) or (b case R) =>
    error "extendedSubResultantGcd$NSMP: one arg is constant"
  a.v ~= b.v =>
    error "extendedSubResultantGcd$NSMP: mvar(#1) ~= mvar(#2)"
  esrg := extendedSubResultantGcd(a.ts,b.ts)$D
  [PSimp(esrg.gcd,a.v),PSimp(esrg.coef1,a.v),PSimp(esrg.coef2,a.v)]

resultant(a:$, b:$): $ ==
  (a case R) or (b case R) =>
    error "resultant$NSMP: one arg is constant"
  a.v ~= b.v =>
    error "resultant$NSMP: mvar(#1) ~= mvar(#2)"
  resultant(a.ts,b.ts)$D

subResultantChain(a:$, b:$): List $ ==
  (a case R) or (b case R) =>
    error "subResultantChain$NSMP: one arg is constant"
  a.v ~= b.v =>
    error "subResultantChain$NSMP: mvar(#1) ~= mvar(#2)"
  [PSimp(up,a.v) for up in subResultantsChain(a.ts,b.ts)]

lastSubResultant(a:$, b:$): $ ==
  (a case R) or (b case R) =>
    error "lastSubResultant$NSMP: one arg is constant"
  a.v ~= b.v =>
    error "lastSubResultant$NSMP: mvar(#1) ~= mvar(#2)"
  PSimp(lastSubResultant(a.ts,b.ts),a.v)

if R has EuclideanDomain
then

  exactQuotient (a:$,b:R) ==
    -- one? b => a
    (b = 1) => a
    a case R => (a::R quo$R b)::R

```

```

      ([a.v, map((a1:%)::% +-> exactQuotient(a1,b),a.ts)$SUP2]$VPoly)::Rep

exactQuotient! (a:$,b:R) ==
--   one? b => a
    (b = 1) => a
    a case R => (a::R quo$R b)::R
    a.ts := map((a1:%)::% +-> exactQuotient!(a1,b),a.ts)$SUP2
    a

else

    exactQuotient (a:$,b:R) ==
--   one? b => a
    (b = 1) => a
    a case R => ((a::R exquo$R b)::R)::R
    ([a.v, map((a1:%)::% +-> exactQuotient(a1,b),a.ts)$SUP2]$VPoly)::Rep

    exactQuotient! (a:$,b:R) ==
--   one? b => a
    (b = 1) => a
    a case R => ((a::R exquo$R b)::R)::R
    a.ts := map((a1:%)::% +-> exactQuotient!(a1,b),a.ts)$SUP2
    a

if R has GcdDomain
then

    localGcd(r:R,p:$):R ==
    p case R => gcd(r,p::R)$R
    gcd(r,content(p))$R

    gcd(r:R,p:$):R ==
--   one? r => r
    (r = 1) => r
    zero? p => r
    localGcd(r,p)

    content p ==
    p case R => p
    up : D := p.ts
    r := 0$R
--   while (not zero? up) and (not one? r) repeat
    while (not zero? up) and (not (r = 1)) repeat
        r := localGcd(r,leadingCoefficient(up))
        up := reductum up
    r

    primitivePart! p ==
    zero? p => p
    p case R => 1$$

```

```

cp := content(p)
p.ts :=
  unitCanonical(map((a1:%) :% +-> exactQuotient!(a1,cp),p.ts)$SUP2)$D
p

```

---

— NSMP.dotabb —

```

"NSMP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=NSMP"]
"RPOLCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=RPOLCAT"]
"NSMP" -> "RPOLCAT"

```

---

### 15.3 domain NSUP NewSparseUnivariatePolynomial

Based on the **PseudoRemainderSequence** package, the domain constructor **NewSparseUnivariatePolynomial** extends the constructor **SparseUnivariatePolynomial**.

— NewSparseUnivariatePolynomial.input —

```

)set break resume
)sys rm -f NewSparseUnivariatePolynomial.output
)spool NewSparseUnivariatePolynomial.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show NewSparseUnivariatePolynomial
--R NewSparseUnivariatePolynomial R: Ring is a domain constructor
--R Abbreviation for NewSparseUnivariatePolynomial is NSUP
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for NSUP
--R
--R----- Operations -----
--R ?? : (%,R) -> %           ?? : (R,%) -> %
--R ?? : (%,%) -> %           ?? : (Integer,%) -> %
--R ?? : (PositiveInteger,%) -> %   ??? : (%,PositiveInteger) -> %
--R ?? : (%,%) -> %           ?-? : (%,%) -> %
--R -? : % -> %               ?=? : (%,%) -> Boolean
--R D : (%,(R -> R)) -> %       D : % -> %
--R D : (%,NonNegativeInteger) -> %   1 : () -> %

```

```

--R 0 : () -> %
--R coefficients : % -> List R
--R coerce : Integer -> %
--R degree : % -> NonNegativeInteger
--R ?.? : (%,% ) -> %
--R eval : (% ,List % ,List % ) -> %
--R eval : (% ,Equation % ) -> %
--R ground : % -> R
--R hash : % -> SingleInteger
--R latex : % -> String
--R leadingCoefficient : % -> R
--R map : ((R -> R),%) -> %
--R monomial? : % -> Boolean
--R one? : % -> Boolean
--R pseudoRemainder : (% ,%) -> %
--R reductum : % -> %
--R sample : () -> %
--R ?~=? : (% ,%) -> Boolean
--R ?*? : (Fraction Integer,% ) -> % if R has ALGEBRA FRAC INT
--R ?*? : (% ,Fraction Integer) -> % if R has ALGEBRA FRAC INT
--R ?*? : (NonNegativeInteger,% ) -> %
--R ?**? : (% ,NonNegativeInteger) -> %
--R ?/? : (% ,R) -> % if R has FIELD
--R ?<? : (% ,%) -> Boolean if R has ORDSET
--R ?<=? : (% ,%) -> Boolean if R has ORDSET
--R ?>? : (% ,%) -> Boolean if R has ORDSET
--R ?>=? : (% ,%) -> Boolean if R has ORDSET
--R D : (% ,(R -> R),NonNegativeInteger) -> %
--R D : (% ,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (% ,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (% ,List Symbol) -> % if R has PDRING SYMBOL
--R D : (% ,Symbol) -> % if R has PDRING SYMBOL
--R D : (% ,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R D : (% ,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R D : (% ,List SingletonAsOrderedSet) -> %
--R D : (% ,SingletonAsOrderedSet) -> %
--R ?^? : (% ,NonNegativeInteger) -> %
--R associates? : (% ,%) -> Boolean if R has INTDOM
--R binomThmExpt : (% ,%,NonNegativeInteger) -> % if R has COMRING
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(% ,"failed") if $ has CHARNZ and R has PFECAT or R has CHARNZ
--R coefficient : (% ,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R coefficient : (% ,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R coefficient : (% ,NonNegativeInteger) -> R
--R coerce : % -> % if R has INTDOM
--R coerce : Fraction Integer -> % if R has ALGEBRA FRAC INT or R has RETRACT FRAC INT
--R coerce : SparseUnivariatePolynomial R -> %
--R coerce : % -> SparseUnivariatePolynomial R
--R coerce : SingletonAsOrderedSet -> %
--R composite : (Fraction % ,%) -> Union(Fraction % ,"failed") if R has INTDOM
???: (% ,PositiveInteger) -> %
coerce : R -> %
coerce : % -> OutputForm
differentiate : % -> %
?.? : (% ,R) -> R
eval : (% ,%,%) -> %
eval : (% ,List Equation % ) -> %
ground? : % -> Boolean
init : () -> % if R has STEP
lazyPseudoQuotient : (% ,%) -> %
leadingMonomial : % -> %
monicModulo : (% ,%) -> %
monomials : % -> List %
primitiveMonomials : % -> List %
recip : % -> Union(% ,"failed")
retract : % -> R
zero? : % -> Boolean

```

```

--R composite : (% , %) -> Union(% , "failed") if R has INTDOM
--R conditionP : Matrix % -> Union(Vector % , "failed") if $ has CHARNZ and R has PFECAT
--R content : (% , SingletonAsOrderedSet) -> % if R has GCDDOM
--R content : % -> R if R has GCDDOM
--R convert : % -> InputForm if SingletonAsOrderedSet has KONVERT INFORM and R has KONVERT I
--R convert : % -> Pattern Integer if SingletonAsOrderedSet has KONVERT PATTERN INT and R has
--R convert : % -> Pattern Float if SingletonAsOrderedSet has KONVERT PATTERN FLOAT and R has
--R degree : (% , List SingletonAsOrderedSet) -> List NonNegativeInteger
--R degree : (% , SingletonAsOrderedSet) -> NonNegativeInteger
--R differentiate : (% , (R -> R) , %) -> %
--R differentiate : (% , (R -> R)) -> %
--R differentiate : (% , (R -> R) , NonNegativeInteger) -> %
--R differentiate : (% , List Symbol , List NonNegativeInteger) -> % if R has PDRING SYMBOL
--R differentiate : (% , Symbol , NonNegativeInteger) -> % if R has PDRING SYMBOL
--R differentiate : (% , List Symbol) -> % if R has PDRING SYMBOL
--R differentiate : (% , Symbol) -> % if R has PDRING SYMBOL
--R differentiate : (% , NonNegativeInteger) -> %
--R differentiate : (% , List SingletonAsOrderedSet , List NonNegativeInteger) -> %
--R differentiate : (% , SingletonAsOrderedSet , NonNegativeInteger) -> %
--R differentiate : (% , List SingletonAsOrderedSet) -> %
--R differentiate : (% , SingletonAsOrderedSet) -> %
--R discriminant : % -> R if R has COMRING
--R discriminant : (% , SingletonAsOrderedSet) -> % if R has COMRING
--R divide : (% , %) -> Record(quotient: % , remainder: %) if R has FIELD
--R divideExponents : (% , NonNegativeInteger) -> Union(% , "failed")
--R ?.? : (% , Fraction %) -> Fraction % if R has INTDOM
--R elt : (Fraction % , R) -> R if R has FIELD
--R elt : (Fraction % , Fraction %) -> Fraction % if R has INTDOM
--R euclideanSize : % -> NonNegativeInteger if R has FIELD
--R eval : (% , List SingletonAsOrderedSet , List %) -> %
--R eval : (% , SingletonAsOrderedSet , %) -> %
--R eval : (% , List SingletonAsOrderedSet , List R) -> %
--R eval : (% , SingletonAsOrderedSet , R) -> %
--R expressIdealMember : (List % , %) -> Union(List % , "failed") if R has FIELD
--R exquo : (% , %) -> Union(% , "failed") if R has INTDOM
--R exquo : (% , R) -> Union(% , "failed") if R has INTDOM
--R extendedEuclidean : (% , %) -> Record(coef1: % , coef2: % , generator: %) if R has FIELD
--R extendedEuclidean : (% , % , %) -> Union(Record(coef1: % , coef2: % ) , "failed") if R has FIELD
--R extendedResultant : (% , %) -> Record(resultant: R , coef1: % , coef2: %) if R has INTDOM
--R extendedSubResultantGcd : (% , %) -> Record(gcd: % , coef1: % , coef2: %) if R has INTDOM
--R factor : % -> Factored % if R has PFECAT
--R factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial %
--R factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePo
--R fmeeg : (% , NonNegativeInteger , R , %) -> %
--R gcd : (% , %) -> % if R has GCDDOM
--R gcd : List % -> % if R has GCDDOM
--R gcdPolynomial : (SparseUnivariatePolynomial % , SparseUnivariatePolynomial %) -> SparseUni
--R halfExtendedResultant1 : (% , %) -> Record(resultant: R , coef1: %) if R has INTDOM
--R halfExtendedResultant2 : (% , %) -> Record(resultant: R , coef2: %) if R has INTDOM
--R halfExtendedSubResultantGcd1 : (% , %) -> Record(gcd: % , coef1: %) if R has INTDOM

```

```

--R halfExtendedSubResultantGcd2 : (% , %) -> Record(gcd: %,coef2: %) if R has INTDOM
--R integrate : % -> % if R has ALGEBRA FRAC INT
--R isExpt : % -> Union(Record(var: SingletonAsOrderedSet,exponent: NonNegativeInteger),"failed")
--R isPlus : % -> Union(List %,"failed")
--R isTimes : % -> Union(List %,"failed")
--R karatsubaDivide : (% , NonNegativeInteger) -> Record(quotient: %,remainder: %)
--R lastSubResultant : (% , %) -> % if R has INTDOM
--R lazyPseudoDivide : (% , %) -> Record(coef: R,gap: NonNegativeInteger,quotient: %,remainder: %)
--R lazyPseudoRemainder : (% , %) -> %
--R lazyResidueClass : (% , %) -> Record(polnum: %,polden: R,power: NonNegativeInteger)
--R lcm : (% , %) -> % if R has GCDDOM
--R lcm : List % -> % if R has GCDDOM
--R mainVariable : % -> Union(SingletonAsOrderedSet,"failed")
--R makeSUP : % -> SparseUnivariatePolynomial R
--R mapExponents : ((NonNegativeInteger -> NonNegativeInteger),%) -> %
--R max : (% , %) -> % if R has ORDSET
--R min : (% , %) -> % if R has ORDSET
--R minimumDegree : (% , List SingletonAsOrderedSet) -> List NonNegativeInteger
--R minimumDegree : (% , SingletonAsOrderedSet) -> NonNegativeInteger
--R minimumDegree : % -> NonNegativeInteger
--R monicDivide : (% , %) -> Record(quotient: %,remainder: %)
--R monicDivide : (% , %,SingletonAsOrderedSet) -> Record(quotient: %,remainder: %)
--R monomial : (% , List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R monomial : (% , SingletonAsOrderedSet,NonNegativeInteger) -> %
--R monomial : (R,NonNegativeInteger) -> %
--R multiEuclidean : (List % , %) -> Union(List %,"failed") if R has FIELD
--R multiplyExponents : (% , NonNegativeInteger) -> %
--R multivariate : (SparseUnivariatePolynomial %,SingletonAsOrderedSet) -> %
--R multivariate : (SparseUnivariatePolynomial R,SingletonAsOrderedSet) -> %
--R nextItem : % -> Union(%,"failed") if R has STEP
--R numberOfMonomials : % -> NonNegativeInteger
--R order : (% , %) -> NonNegativeInteger if R has INTDOM
--R patternMatch : (% , Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMatchResult(Integer,% ) if
--R patternMatch : (% , Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchResult(Float,% ) if Singl
--R pomopo! : (% , R,NonNegativeInteger,% ) -> %
--R prime? : % -> Boolean if R has PFECAT
--R primitivePart : (% , SingletonAsOrderedSet) -> % if R has GCDDOM
--R primitivePart : % -> % if R has GCDDOM
--R principalIdeal : List % -> Record(coef: List %,generator: %) if R has FIELD
--R pseudoDivide : (% , %) -> Record(coef: R,quotient: %,remainder: %) if R has INTDOM
--R pseudoQuotient : (% , %) -> % if R has INTDOM
--R ?quo? : (% , %) -> % if R has FIELD
--R reducedSystem : Matrix % -> Matrix R
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) if R has LINE
--R reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT
--R ?rem? : (% , %) -> % if R has FIELD
--R resultant : (% , %) -> R if R has COMRING
--R resultant : (% , %,SingletonAsOrderedSet) -> % if R has COMRING
--R retract : % -> SparseUnivariatePolynomial R

```

```

--R retract : % -> SingletonAsOrderedSet
--R retract : % -> Integer if R has RETRACT INT
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(SparseUnivariatePolynomial R,"failed")
--R retractIfCan : % -> Union(SingletonAsOrderedSet,"failed")
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(R,"failed")
--R separate : (%,% ) -> Record(primePart: %,commonPart: %) if R has GCDDOM
--R shiftLeft : (% ,NonNegativeInteger) -> %
--R shiftRight : (% ,NonNegativeInteger) -> %
--R sizeLess? : (%,% ) -> Boolean if R has FIELD
--R solveLinearPolynomialEquation : (List SparseUnivariatePolynomial % ,SparseUnivariatePolynomial % ) -> List SparseUnivariatePolynomial %
--R squareFree : % -> Factored % if R has GCDDOM
--R squareFreePart : % -> % if R has GCDDOM
--R squareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial %
--R subResultantGcd : (%,% ) -> % if R has INTDOM
--R subResultantsChain : (%,% ) -> List % if R has INTDOM
--R subtractIfCan : (%,% ) -> Union(%,"failed")
--R totalDegree : (% ,List SingletonAsOrderedSet) -> NonNegativeInteger
--R totalDegree : % -> NonNegativeInteger
--R unit? : % -> Boolean if R has INTDOM
--R unitCanonical : % -> % if R has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has INTDOM
--R univariate : % -> SparseUnivariatePolynomial R
--R univariate : (% ,SingletonAsOrderedSet) -> SparseUnivariatePolynomial %
--R unmakeSUP : SparseUnivariatePolynomial R -> %
--R variables : % -> List SingletonAsOrderedSet
--R vectorise : (% ,NonNegativeInteger) -> Vector R
--R
--E 1

)spool
)lisp (bye)

```

---

— NewSparseUnivariatePolynomial.help —

```

=====
NewSparseUnivariatePolynomial examples
=====

```

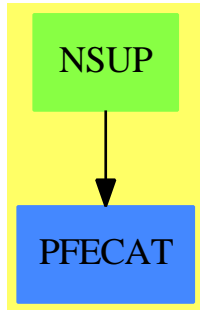
See Also:

```

o )show NewSparseUnivariatePolynomial

```

### 15.3.1 NewSparseUnivariatePolynomial (NSUP)



See

⇒ “NewSparseMultivariatePolynomial” (NSMP) 15.2.1 on page 1676



**Exports:**

|                        |                               |                              |
|------------------------|-------------------------------|------------------------------|
| 0                      | 1                             | associates?                  |
| binomThmExpt           | characteristic                | charthRoot                   |
| coefficient            | coefficients                  | coerce                       |
| composite              | conditionP                    | content                      |
| convert                | D                             | degree                       |
| differentiate          | discriminant                  | divide                       |
| divideExponents        | elt                           | euclideanSize                |
| eval                   | expressIdealMember            | exquo                        |
| extendedEuclidean      | extendedResultant             | extendedSubResultantGcd      |
| factor                 | factorPolynomial              | factorSquareFreePolynomial   |
| fmecg                  | gcd                           | gcdPolynomial                |
| ground                 | ground?                       | halfExtendedResultant1       |
| halfExtendedResultant2 | halfExtendedSubResultantGcd1  | halfExtendedSubResultantGcd2 |
| hash                   | init                          | integrate                    |
| isExpt                 | isPlus                        | isTimes                      |
| karatsubaDivide        | lastSubResultant              | latex                        |
| lazyPseudoDivide       | lazyPseudoQuotient            | lazyPseudoRemainder          |
| lazyResidueClass       | lcm                           | leadingCoefficient           |
| leadingMonomial        | mainVariable                  | makeSUP                      |
| map                    | mapExponents                  | max                          |
| min                    | minimumDegree                 | monicDivide                  |
| monicModulo            | monomial                      | monomial?                    |
| monomials              | multiEuclidean                | multiplyExponents            |
| multivariate           | nextItem                      | numberOfMonomials            |
| one?                   | order                         | patternMatch                 |
| popopo!                | prime?                        | primitiveMonomials           |
| primitivePart          | principalIdeal                | pseudoDivide                 |
| pseudoQuotient         | pseudoRemainder               | recip                        |
| reducedSystem          | reductum                      | resultant                    |
| retract                | retractIfCan                  | sample                       |
| separate               | shiftLeft                     | shiftRight                   |
| sizeLess?              | solveLinearPolynomialEquation | squareFree                   |
| squareFreePart         | squareFreePolynomial          | subResultantGcd              |
| subResultantsChain     | subtractIfCan                 | totalDegree                  |
| totalDegree            | unit?                         | unitCanonical                |
| unitNormal             | univariate                    | unmakeSUP                    |
| variables              | vectorise                     | zero?                        |
| ?*?                    | ?**?                          | ?+?                          |
| ?-?                    | -?                            | ?=?                          |
| ?^?                    | ?..?                          | ?~=?                         |
| ?/?                    | ?<?                           | ?<=?                         |
| ?>?                    | ?>=?                          | ?quo?                        |
| ?rem?                  |                               |                              |

```

)abbrev domain NSUP NewSparseUnivariatePolynomial
++ Author: Marc Moreno Maza
++ Date Created: 23/07/98
++ Date Last Updated: 14/12/98
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ A post-facto extension for \axiomType{SUP} in order
++ to speed up operations related to pseudo-division and gcd for
++ both \axiomType{SUP} and, consequently, \axiomType{NSMP}.

NewSparseUnivariatePolynomial(R): Exports == Implementation where

R:Ring
NNI ==> NonNegativeInteger
SUPR ==> SparseUnivariatePolynomial R

Exports == Join(UnivariatePolynomialCategory(R),
CoercibleTo(SUPR),RetractableTo(SUPR)) with
  fme cg : (% ,NNI,R,%) -> %
    ++ \axiom{fme cg}(p1,e,r,p2)} returns \axiom{p1 - r * x**e * p2}
    ++ where \axiom{x} is \axiom{monomial(1,1)}
  monicModulo : ($, $) -> $
    ++ \axiom{monicModulo(a,b)} returns \axiom{r} such that \axiom{r} is
    ++ reduced w.r.t. \axiom{b} and \axiom{b} divides \axiom{a - r}
    ++ where \axiom{b} is monic.
  lazyResidueClass: ($,$) -> Record(polnum:$, polden:R, power:NNI)
    ++ \axiom{lazyResidueClass(a,b)} returns \axiom{[r,c,n]} such that
    ++ \axiom{r} is reduced w.r.t. \axiom{b} and \axiom{b} divides
    ++ \axiom{c^n * a - r} where \axiom{c} is \axiom{leadingCoefficient(b)}
    ++ and \axiom{n} is as small as possible with the previous properties.
  lazyPseudoRemainder: ($,$) -> $
    ++ \axiom{lazyPseudoRemainder(a,b)} returns \axiom{r} if
    ++ \axiom{lazyResidueClass(a,b)} returns \axiom{[r,c,n]}.
    ++ This lazy pseudo-remainder is computed by means of the
    ++ fme cg from NewSparseUnivariatePolynomial operation.
  lazyPseudoDivide: ($,$) -> Record(coef:R,gap:NNI,quotient:$,remainder:$)
    ++ \axiom{lazyPseudoDivide(a,b)} returns \axiom{[c,g,q,r]} such that
    ++ \axiom{c^n * a = q*b + r} and \axiom{lazyResidueClass(a,b)} returns
    ++ \axiom{[r,c,n]} where
    ++ \axiom{n + g = max(0, degree(b) - degree(a) + 1)}.
  lazyPseudoQuotient: ($,$) -> $
    ++ \axiom{lazyPseudoQuotient(a,b)} returns \axiom{q} if
    ++ \axiom{lazyPseudoDivide(a,b)} returns \axiom{[c,g,q,r]}
  if R has IntegralDomain

```

```

then
  subResultantsChain: ($, $) -> List $
    ++ \axiom{subResultantsChain(a,b)} returns the list of the non-zero
    ++ sub-resultants of \axiom{a} and \axiom{b} sorted by increasing
    ++ degree.
  lastSubResultant: ($, $) -> $
    ++ \axiom{lastSubResultant(a,b)} returns \axiom{resultant(a,b)}
    ++ if \axiom{a} and \axiom{b} has no non-trivial gcd
    ++ in \axiom{R-1 P}
    ++ otherwise the non-zero sub-resultant with smallest index.
  extendedSubResultantGcd: ($, $) -> Record(gcd: $, coef1: $, coef2: $)
    ++ \axiom{extendedSubResultantGcd(a,b)} returns \axiom{[g,ca, cb]}
    ++ such that \axiom{g} is a gcd of \axiom{a} and \axiom{b} in
    ++ \axiom{R-1 P} and \axiom{g = ca * a + cb * b}
  halfExtendedSubResultantGcd1: ($, $) -> Record(gcd: $, coef1: $)
    ++ \axiom{halfExtendedSubResultantGcd1(a,b)} returns \axiom{[g,ca]}
    ++ such that \axiom{extendedSubResultantGcd(a,b)} returns
    ++ \axiom{[g,ca, cb]}
  halfExtendedSubResultantGcd2: ($, $) -> Record(gcd: $, coef2: $)
    ++ \axiom{halfExtendedSubResultantGcd2(a,b)} returns \axiom{[g,cb]}
    ++ such that \axiom{extendedSubResultantGcd(a,b)} returns
    ++ \axiom{[g,ca, cb]}
  extendedResultant: ($, $) -> Record(resultant: R, coef1: $, coef2: $)
    ++ \axiom{extendedResultant(a,b)} returns \axiom{[r,ca,cb]} such that
    ++ \axiom{r} is the resultant of \axiom{a} and \axiom{b} and
    ++ \axiom{r = ca * a + cb * b}
  halfExtendedResultant1: ($, $) -> Record(resultant: R, coef1: $)
    ++ \axiom{halfExtendedResultant1(a,b)} returns \axiom{[r,ca]}
    ++ such that \axiom{extendedResultant(a,b)} returns
    ++ \axiom{[r,ca, cb]}
  halfExtendedResultant2: ($, $) -> Record(resultant: R, coef2: $)
    ++ \axiom{halfExtendedResultant2(a,b)} returns \axiom{[r,ca]} such
    ++ that \axiom{extendedResultant(a,b)} returns \axiom{[r,ca, cb]}

Implementation == SparseUnivariatePolynomial(R) add

Term == Record(k:NonNegativeInteger,c:R)
Rep ==> List Term

rep(s:$):Rep == s pretend Rep
per(l:Rep):$ == l pretend $

coerce (p:$):SUPR ==
  p pretend SUPR

coerce (p:SUPR):$ ==
  p pretend $

retractIfCan (p:$) : Union(SUPR,"failed") ==
  (p pretend SUPR)::Union(SUPR,"failed")

```

```

monicModulo(x,y) ==
  zero? y =>
    error "in monicModulo$NSUP: division by 0"
  ground? y =>
    error "in monicModulo$NSUP: ground? #2"
  yy := rep y
--  not one? (yy.first.c) =>
  not ((yy.first.c) = 1) =>
    error "in monicModulo$NSUP: not monic #2"
  xx := rep x; empty? xx => x
  e := yy.first.k; y := per(yy.rest)
  -- while (not empty? xx) repeat
  repeat
    if (u:=subtractIfCan(xx.first.k,e)) case "failed" then break
    xx:= rep fmeCG(per rest(xx), u, xx.first.c, y)
    if empty? xx then break
  per xx

lazyResidueClass(x,y) ==
  zero? y =>
    error "in lazyResidueClass$NSUP: division by 0"
  ground? y =>
    error "in lazyResidueClass$NSUP: ground? #2"
  yy := rep y; co := yy.first.c; xx: Rep := rep x
  empty? xx => [x, co, 0]
  pow: NNI := 0; e := yy.first.k; y := per(yy.rest);
  repeat
    if (u:=subtractIfCan(xx.first.k,e)) case "failed" then break
    xx:= rep fmeCG(co * per rest(xx), u, xx.first.c, y)
    pow := pow + 1
    if empty? xx then break
  [per xx, co, pow]

lazyPseudoRemainder(x,y) ==
  zero? y =>
    error "in lazyPseudoRemainder$NSUP: division by 0"
  ground? y =>
    error "in lazyPseudoRemainder$NSUP: ground? #2"
  ground? x => x
  yy := rep y; co := yy.first.c
--  one? co => monicModulo(x,y)
  (co = 1) => monicModulo(x,y)
  (co = -1) => - monicModulo(-x,-y)
  xx:= rep x; e := yy.first.k; y := per(yy.rest)
  repeat
    if (u:=subtractIfCan(xx.first.k,e)) case "failed" then break
    xx:= rep fmeCG(co * per rest(xx), u, xx.first.c, y)
    if empty? xx then break
  per xx

```

```

lazyPseudoDivide(x,y) ==
  zero? y =>
    error "in lazyPseudoDivide$NSUP: division by 0"
  ground? y =>
    error "in lazyPseudoDivide$NSUP: ground? #2"
  yy := rep y; e := yy.first.k;
  xx: Rep := rep x; co := yy.first.c
  (empty? xx) or (xx.first.k < e) => [co,0,0,x]
  pow: NNI := subtractIfCan(xx.first.k,e)::NNI + 1
  qq: Rep := []; y := per(yy.rest)
  repeat
    if (u:=subtractIfCan(xx.first.k,e)) case "failed" then break
    qq := cons([u::NNI, xx.first.c]$Term, rep (co * per qq))
    xx := rep fmeqg(co * per rest(xx), u, xx.first.c, y)
    pow := subtractIfCan(pow,1)::NNI
    if empty? xx then break
  [co, pow, per reverse qq, per xx]

lazyPseudoQuotient(x,y) ==
  zero? y =>
    error "in lazyPseudoQuotient$NSUP: division by 0"
  ground? y =>
    error "in lazyPseudoQuotient$NSUP: ground? #2"
  yy := rep y; e := yy.first.k; xx: Rep := rep x
  (empty? xx) or (xx.first.k < e) => 0
  qq: Rep := []; co := yy.first.c; y := per(yy.rest)
  repeat
    if (u:=subtractIfCan(xx.first.k,e)) case "failed" then break
    qq := cons([u::NNI, xx.first.c]$Term, rep (co * per qq))
    xx := rep fmeqg(co * per rest(xx), u, xx.first.c, y)
    if empty? xx then break
  per reverse qq

if R has IntegralDomain
then
  pack ==> PseudoRemainderSequence(R, %)

  subResultantGcd(p1,p2) == subResultantGcd(p1,p2)$pack

  subResultantsChain(p1,p2) == chainSubResultants(p1,p2)$pack

  lastSubResultant(p1,p2) == lastSubResultant(p1,p2)$pack

  resultant(p1,p2) == resultant(p1,p2)$pack

  extendedResultant(p1,p2) ==
    re: Record(coef1: $, coef2: $, resultant: R) := resultantEuclidean(p1,p2)$pack
    [re.resultant, re.coef1, re.coef2]

```

```

halfExtendedResultant1(p1:$, p2:$): Record(resultant: R, coef1: $) ==
  re: Record(coef1: $, resultant: R) := semiResultantEuclidean1(p1, p2)$pack
  [re.resultant, re.coef1]

halfExtendedResultant2(p1:$, p2:$): Record(resultant: R, coef2: $) ==
  re: Record(coef2: $, resultant: R) := semiResultantEuclidean2(p1, p2)$pack
  [re.resultant, re.coef2]

extendedSubResultantGcd(p1,p2) ==
  re: Record(coef1: $, coef2: $, gcd: $) := subResultantGcdEuclidean(p1,p2)$pack
  [re.gcd, re.coef1, re.coef2]

halfExtendedSubResultantGcd1(p1:$, p2:$): Record(gcd: $, coef1: $) ==
  re: Record(coef1: $, gcd: $) := semiSubResultantGcdEuclidean1(p1, p2)$pack
  [re.gcd, re.coef1]

halfExtendedSubResultantGcd2(p1:$, p2:$): Record(gcd: $, coef2: $) ==
  re: Record(coef2: $, gcd: $) := semiSubResultantGcdEuclidean2(p1, p2)$pack
  [re.gcd, re.coef2]

pseudoDivide(x,y) ==
  zero? y =>
    error "in pseudoDivide$NSUP: division by 0"
  ground? y =>
    error "in pseudoDivide$NSUP: ground? #2"
  yy := rep y; e := yy.first.k
  xx: Rep := rep x; co := yy.first.c
  (empty? xx) or (xx.first.k < e) => [co,0,x]
  pow: NNI := subtractIfCan(xx.first.k,e)::NNI + 1
  qq: Rep := []; y := per(yy.rest)
  repeat
    if (u:=subtractIfCan(xx.first.k,e)) case "failed" then break
    qq := cons([u::NNI, xx.first.c]$Term, rep (co * per qq))
    xx := rep fmecg(co * per rest(xx), u, xx.first.c, y)
    pow := subtractIfCan(pow,1)::NNI
    if empty? xx then break
  zero? pow => [co, per reverse qq, per xx]
  default: R := co ** pow
  q := default * (per reverse qq)
  x := default * (per xx)
  [co, q, x]

pseudoQuotient(x,y) ==
  zero? y =>
    error "in pseudoDivide$NSUP: division by 0"
  ground? y =>
    error "in pseudoDivide$NSUP: ground? #2"
  yy := rep y; e := yy.first.k; xx: Rep := rep x
  (empty? xx) or (xx.first.k < e) => 0
  pow: NNI := subtractIfCan(xx.first.k,e)::NNI + 1

```

```

qq: Rep := []; co := yy.first.c; y := per(yy.rest)
repeat
  if (u:=subtractIfCan(xx.first.k,e)) case "failed" then break
  qq := cons([u::NNI, xx.first.c]$Term, rep (co * per qq))
  xx := rep fmeeg(co * per rest(xx), u, xx.first.c, y)
  pow := subtractIfCan(pow,1)::NNI
  if empty? xx then break
zero? pow => per reverse qq
(co ** pow) * (per reverse qq)

```

---

— NSUP.dotabb —

```

"NSUP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=NSUP"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"NSUP" -> "PFECAT"

```

---

## 15.4 domain NONE None

— None.input —

```

)set break resume
)sys rm -f None.output
)spool None.output
)set message test on
)set message auto off
)clear all
--S 1 of 3
[ ]
--R
--R
--R (1) []
--R
--E 1

```

Type: List None

```

--S 2 of 3
[ ] :: List Float
--R
--R
--R (2) []
--R

```

Type: List Float

```

--E 2

--S 3 of 3
[ ]$List(NonNegativeInteger)
--R
--R
--R (3) []
--R
--R                                         Type: List NonNegativeInteger
--E 3
)spool
)lisp (bye)

```

---

— None.help —

=====

None examples

=====

The None domain is not very useful for interactive work but it is provided nevertheless for completeness of the Axiom type system.

Probably the only place you will ever see it is if you enter an empty list with no type information.

```

[ ]
[]
                                     Type: List None

```

Such an empty list can be converted into an empty list of any other type.

```

[ ] :: List Float
[]
                                     Type: List Float

```

If you wish to produce an empty list of a particular type directly, such as List NonNegativeInteger, do it this way.

```

[ ]$List(NonNegativeInteger)
[]
                                     Type: List NonNegativeInteger

```

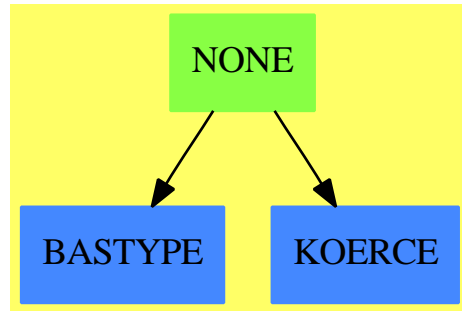
See Also:

- o )show None

---



### 15.4.1 None (NONE)



See

⇒ “Any” (ANY) 2.9.1 on page 50

**Exports:**

coerce hash latex ?? ?~=?

— domain NONE None —

```

)abbrev domain NONE None
++ Author: Mark Botch
++ Date Created:
++ Change History:
++ Basic Functions: coerce
++ Related Constructors: NoneFunctions1
++ Also See: Any
++ AMS Classification:
++ Keywords: none, empty
++ Description:
++ \spadtype{None} implements a type with no objects. It is mainly
++ used in technical situations where such a thing is needed (e.g.
++ the interpreter and some of the internal \spadtype{Expression} code).

```

```

None():SetCategory == add
  coerce(none:%):OutputForm == "NONE" :: OutputForm
  x:% = y:% == EQ(x,y)$Lisp

```

— —

— NONE.dotabb —

"NONE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=NONE"]

```

"BASTYPE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=BASTYPE"]
"KOERCE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=KOERCE"]
"NONE" -> "BASTYPE"
"NONE" -> "KOERCE"

```

## 15.5 domain NNI NonNegativeInteger

— NonNegativeInteger.input —

```

)set break resume
)sys rm -f NonNegativeInteger.output
)spool NonNegativeInteger.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show NonNegativeInteger
--R NonNegativeInteger is a domain constructor
--R Abbreviation for NonNegativeInteger is NNI
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for NNI
--R
--R----- Operations -----
--R ?? : (%,%) -> %                ?? : (PositiveInteger,%) -> %
--R ??? : (%,PositiveInteger) -> %    ?+? : (%,%) -> %
--R ?<? : (%,%) -> Boolean           ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean           ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean          1 : () -> %
--R 0 : () -> %                     ??^? : (%,PositiveInteger) -> %
--R coerce : % -> OutputForm         gcd : (%,%) -> %
--R hash : % -> SingleInteger         latex : % -> String
--R max : (%,%) -> %                 min : (%,%) -> %
--R one? : % -> Boolean              ?quo? : (%,%) -> %
--R random : % -> %                  recip : % -> Union(%, "failed")
--R ?rem? : (%,%) -> %               sample : () -> %
--R shift : (%,Integer) -> %         sup : (%,%) -> %
--R zero? : % -> Boolean             ?~=? : (%,%) -> Boolean
--R ?? : (NonNegativeInteger,%) -> %
--R ??? : (%,NonNegativeInteger) -> %
--R ??^? : (%,NonNegativeInteger) -> %
--R divide : (%,%) -> Record(quotient: %, remainder: %)
--R exquo : (%,%) -> Union(%, "failed")
--R subtractIfCan : (%,%) -> Union(%, "failed")

```

```
--R
--E 1

)spool
)lisp (bye)
```

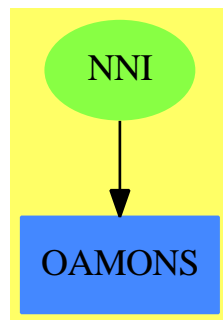
---

— NonNegativeInteger.help —

```
=====
NonNegativeInteger examples
=====
```

```
See Also:
o )show NonNegativeInteger
```

### 15.5.1 NonNegativeInteger (NNI)



**See**

⇒ “Integer” (INT) 10.30.1 on page 1325  
 ⇒ “PositiveInteger” (PI) 17.28.1 on page 2060  
 ⇒ “RomanNumeral” (ROMAN) 19.12.1 on page 2286

**Exports:**

|               |        |        |        |       |
|---------------|--------|--------|--------|-------|
| 0             | 1      | coerce | divide | exquo |
| gcd           | hash   | latex  | max    | min   |
| one?          | random | recip  | sample | shift |
| subtractIfCan | sup    | zero?  | ?~=?   | ?*?   |
| ?**?          | ?^?    | ?+?    | ?<?    | ?<=?  |
| ?=?           | ?>?    | ?>=?   | ?quo?  | ?rem? |

— domain NNI NonNegativeInteger —

```

)abbrev domain NNI NonNegativeInteger
++ Author: Mark Botch
++ Date Created:
++ Change History:
++ Basic Operations:
++ Related Constructors:
++ Keywords: integer
++ Description:
++ \spadtype{NonNegativeInteger} provides functions for non-negative integers.

NonNegativeInteger: Join(OrderedAbelianMonoidSup,Monoid) with
  _quo : (%,% ) -> %
    ++ a quo b returns the quotient of \spad{a} and b, forgetting
    ++ the remainder.
  _rem : (%,% ) -> %
    ++ a rem b returns the remainder of \spad{a} and b.
  gcd : (%,% ) -> %
    ++ gcd(a,b) computes the greatest common divisor of two
    ++ non negative integers \spad{a} and b.
  divide: (%,% ) -> Record(quotient:%,remainder:%)
    ++ divide(a,b) returns a record containing both
    ++ remainder and quotient.
  _exquo: (%,% ) -> Union(%, "failed")
    ++ exquo(a,b) returns the quotient of \spad{a} and b, or "failed"
    ++ if b is zero or \spad{a} rem b is zero.
  shift: (% , Integer) -> %
    ++ shift(a,i) shift \spad{a} by i bits.
  random : % -> %
    ++ random(n) returns a random integer from 0 to \spad{n-1}.
  commutative("*")
    ++ commutative("*") means multiplication is commutative,
    ++ that is, \spad{x*y = y*x}.

== SubDomain(Integer,#1 >= 0) add
  x,y:%
  sup(x,y) == MAX(x,y)$Lisp
  shift(x:% , n:Integer):% == ASH(x,n)$Lisp
  subtractIfCan(x, y) ==
    c:Integer := (x pretend Integer) - (y pretend Integer)
    c < 0 => "failed"
    c pretend %

```

---

— NNI.dotabb —

"NNI" [color="#88FF44",href="bookvol10.3.pdf#nameddest=NNI",shape=ellipse]  
 "OAMONS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=OAMONS"]

"NNI" -> "OAMONS"

## 15.6 domain NOTTING NottinghamGroup

— NottinghamGroup.input —

```

)set break resume
)sys rm -f NottinghamGroup.output
)spool NottinghamGroup.output
)set message test on
)set message auto off
)clear all

--S 1 of 8
x:=monomial(1,1)$UFPS PF 1783
--R
--R
--R (1) x
--R
--R                                         Type: UnivariateFormalPowerSeries PrimeField 1783
--E 1

--S 2 of 8
s:=retract(sin x)$NOTTING PF 1783
--R
--R
--R          3      5      7      9      11
--R (2) x + 297x + 1679x + 427x + 316x + 0(x )
--R
--R                                         Type: NottinghamGroup PrimeField 1783
--E 2

--S 3 of 8
s^2
--R
--R
--R          3      5      7      9      11
--R (3) x + 594x + 535x + 1166x + 1379x + 0(x )
--R
--R                                         Type: NottinghamGroup PrimeField 1783
--E 3

--S 4 of 8
s^-1
--R
--R
--R          3      5      7      9      11

```

```

--R (4)  $x + 1486x + 847x + 207x + 1701x + 0(x)$ 
--R                                         Type: NottinghamGroup PrimeField 1783
--E 4

--S 5 of 8
s^-1*s
--R
--R
--R (5)  $x + 0(x)$ 
--R                                         Type: NottinghamGroup PrimeField 1783
--E 5

--S 6 of 8
s*s^-1
--R
--R
--R (6)  $x + 0(x)$ 
--R                                         Type: NottinghamGroup PrimeField 1783
--E 6

--S 7 of 8
sample()$NOTTING(PF(1783))
--R
--R (7)  $x$ 
--R                                         Type: NottinghamGroup PrimeField 1783
--E 7

--S 8 of 8
)show NottinghamGroup
--R
--R NottinghamGroup F: FiniteFieldCategory is a domain constructor
--R Abbreviation for NottinghamGroup is NOTTING
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for NOTTING
--R
--R----- Operations -----
--R ?? : (% , %) -> %
--R ??? : (% , PositiveInteger) -> %
--R ?? : (% , %) -> Boolean
--R ?? : (% , Integer) -> %
--R coerce : % -> OutputForm
--R conjugate : (% , %) -> %
--R inv : % -> %
--R one? : % -> Boolean
--R sample : () -> %
--R ??? : (% , NonNegativeInteger) -> %
--R ?? : (% , NonNegativeInteger) -> %
--R retract : UnivariateFormalPowerSeries F -> %
--R ??? : (% , Integer) -> %
--R ?/? : (% , %) -> %
--R 1 : () -> %
--R ?? : (% , PositiveInteger) -> %
--R commutator : (% , %) -> %
--R hash : % -> SingleInteger
--R latex : % -> String
--R recip : % -> Union(%, "failed")
--R ~=? : (% , %) -> Boolean

```

```
--R
--E 8
```

```
)spool
)lisp (bye)
```

---

— NottinghamGroup.help —

```
=====
NottinghamGroup examples
=====
```

If  $F$  is a finite field with  $p^n$  elements, then we may form the group of all formal power series  $\{t(1+a_1t+a_2t^2+\dots)\}$  where  $u(0)=0$  and  $u'(0)=1$  and  $a_i$  is an element of  $F$ .

The group operation is substitution (composition). This is called the Nottingham Group.

The Nottingham Group is the projective limit of finite  $p$ -groups. Every finite  $p$ -group can be embedded in the Nottingham Group.

```
x:=monomial(1,1)$UFPS PF 1783
x
```

```
s:=retract(sin x)$NOTTING PF 1783
      3      5      7      9      11
x + 297x + 1679x + 427x + 316x + 0(x )
```

```
s^2
      3      5      7      9      11
x + 594x + 535x + 1166x + 1379x + 0(x )
```

```
s^-1
      3      5      7      9      11
x + 1486x + 847x + 207x + 1701x + 0(x )
```

```
s^-1*s
      11
x + 0(x )
```

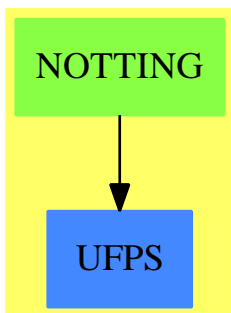
```
s*s^-1
      11
x + 0(x )
```

See Also:

- o )show NottinghamGroup
- o )show UnivariateFormalPowerSeries

—————

### 15.6.1 NottinghamGroup (NOTTING)



**Exports:**

|     |         |            |           |        |
|-----|---------|------------|-----------|--------|
| 1   | coerce  | commutator | conjugate | hash   |
| inv | latex   | one?       | recip     | sample |
| ^=  | retract | *          | **        | /      |
| =   | ^       |            |           |        |

— domain NOTTING NottinghamGroup —

```

)abbrev domain NOTTING NottinghamGroup
++ Author: Mark Botch
++ Description:
++ This is an implmenentation of the Nottingham Group

NottinghamGroup(F:FiniteFieldCategory): Group with
  retract: UnivariateFormalPowerSeries F -> %
  == add
  Rep:=UnivariateFormalPowerSeries F

  coerce f == coerce(f::Rep)$UnivariateFormalPowerSeries(F)

  retract f ==
    if zero? coefficient(f,0) and one? coefficient(f,1)
    then f::Rep
    else error"The leading term must be x"

  1 == monomial(1,1)
  
```



— NOTTING.dottabb —

— NumericalIntegrationProblem.input —

```

)set break resume
)sys rm -f NumericalIntegrationProblem.output
)spool NumericalIntegrationProblem.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show NumericalIntegrationProblem
--R NumericalIntegrationProblem is a domain constructor
--R Abbreviation for NumericalIntegrationProblem is NIPROB
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for NIPROB
--R
--R----- Operations -----
--R ?=? : (%,%) -> Boolean               coerce : % -> OutputForm
--R hash : % -> SingleInteger           latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R coerce : Union(nia: Record(var: Symbol,fn: Expression DoubleFloat,range: Segment OrderedCompletion DoubleFloat),
--R               Record(fn: Expression DoubleFloat,range: List Segment OrderedCompletion DoubleFloat))
--R coerce : Record(var: Symbol,fn: Expression DoubleFloat,range: Segment OrderedCompletion DoubleFloat)
--R retract : % -> Union(nia: Record(var: Symbol,fn: Expression DoubleFloat,range: Segment OrderedCompletion DoubleFloat),
--R                     Record(fn: Expression DoubleFloat,range: List Segment OrderedCompletion DoubleFloat))
--R
--E 1

```

```
)spool
)lisp (bye)
```

---

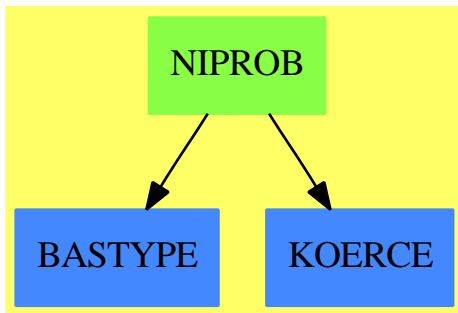
— NumericalIntegrationProblem.help —

```
=====
NumericalIntegrationProblem examples
=====
```

See Also:

```
o )show NumericalIntegrationProblem
```

### 15.7.1 NumericalIntegrationProblem (NIPROB)



**Exports:**

```
coerce hash latex retract ==? ?~=?
```

---

— domain NIPROB NumericalIntegrationProblem —

```
)abbrev domain NIPROB NumericalIntegrationProblem
++ Author: Brian Dupee
++ Date Created: December 1997
++ Date Last Updated: December 1997
++ Basic Operations: coerce, retract
++ Related Constructors: Union
++ Description:
++ \axiomType{NumericalIntegrationProblem} is a \axiom{domain}
++ for the representation of Numerical Integration problems for use
++ by ANNA.
```

```

++
++ The representation is a Union of two record types - one for integration of
++ a function of one variable:
++
++ \axiomType{Record}(var:\axiomType{Symbol},\br
++ fn:\axiomType{Expression DoubleFloat},\br
++ range:\axiomType{Segment OrderedCompletion DoubleFloat},\br
++ abserr:\axiomType{DoubleFloat},\br
++ relerr:\axiomType{DoubleFloat},)
++
++ and one for multivariate integration:
++
++ \axiomType{Record}(fn:\axiomType{Expression DoubleFloat},\br
++ range:\axiomType{List Segment OrderedCompletion DoubleFloat},\br
++ abserr:\axiomType{DoubleFloat},\br
++ relerr:\axiomType{DoubleFloat},).
++

NumericalIntegrationProblem(): EE == II where

EDFA ==> Expression DoubleFloat
SOCDFFA ==> Segment OrderedCompletion DoubleFloat
DFA ==> DoubleFloat
NIAA ==> Record(var:Symbol,fn:EDFA,range:SOCDFFA,abserr:DFA,relerr:DFA)
MDNIAA ==> Record(fn:EDFA,range:List SOCDFFA,abserr:DFA,relerr:DFA)

EE ==> SetCategory with
  coerce: NIAA -> %
    ++ coerce(x) is not documented
  coerce: MDNIAA -> %
    ++ coerce(x) is not documented
  coerce: Union(nia:NIAA,mdnia:MDNIAA) -> %
    ++ coerce(x) is not documented
  coerce: % -> OutputForm
    ++ coerce(x) is not documented
  retract: % -> Union(nia:NIAA,mdnia:MDNIAA)
    ++ retract(x) is not documented

II ==> add
  Rep := Union(nia:NIAA,mdnia:MDNIAA)

  coerce(s:NIAA) == [s]
  coerce(s:MDNIAA) == [s]
  coerce(s:Union(nia:NIAA,mdnia:MDNIAA)) == s
  coerce(x:%):OutputForm ==
    (x) case nia => (x.nia)::OutputForm
    (x.mdnia)::OutputForm
  retract(x:%):Union(nia:NIAA,mdnia:MDNIAA) ==
    (x) case nia => [x.nia]
    [x.mdnia]

```

---

— NIPROB.dotabb —

```
"NIPROB" [color="#88FF44",href="bookvol10.3.pdf#nameddest=NIPROB"]
"BASTYPE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=BASTYPE"]
"KOERCE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=KOERCE"]
"NIPROB" -> "BASTYPE"
"NIPROB" -> "KOERCE"
```

---

## 15.8 domain ODEPROB NumericalODEProblem

— NumericalODEProblem.input —

```
)set break resume
)sys rm -f NumericalODEProblem.output
)spool NumericalODEProblem.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show NumericalODEProblem
--R NumericalODEProblem is a domain constructor
--R Abbreviation for NumericalODEProblem is ODEPROB
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ODEPROB
--R
--R----- Operations -----
--R ?? : (% ,%) -> Boolean          coerce : % -> OutputForm
--R hash : % -> SingleInteger      latex : % -> String
--R ?~=? : (% ,%) -> Boolean
--R coerce : Record(xinit: DoubleFloat,xend: DoubleFloat,fn: Vector Expression DoubleFloat,yinit: List D
--R retract : % -> Record(xinit: DoubleFloat,xend: DoubleFloat,fn: Vector Expression DoubleFloat,yinit:
--R
--E 1

)spool
)lisp (bye)
```

---

## — NumericalODEProblem.help —

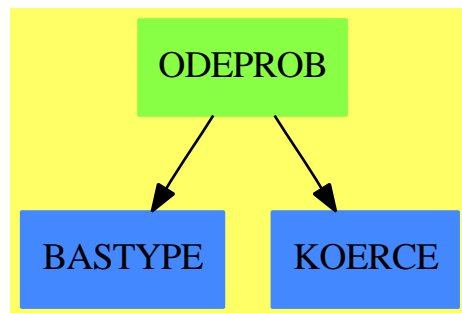
```
=====
NumericalODEProblem examples
=====
```

See Also:

```
o )show NumericalODEProblem
```

---

## 15.8.1 NumericalODEProblem (ODEPROB)

**Exports:**

```
coerce hash latex retract ==? ?~=?
```

## — domain ODEPROB NumericalODEProblem —

```
)abbrev domain ODEPROB NumericalODEProblem
++ Author: Brian Dupee
++ Date Created: December 1997
++ Date Last Updated: December 1997
++ Basic Operations: coerce, retract
++ Related Constructors: Union
++ Description:
++ \axiomType{NumericalODEProblem} is a \axiom{domain}
++ for the representation of Numerical ODE problems for use
++ by ANNA.
++
++ The representation is of type:
++
++ \axiomType{Record}(xinit:\axiomType{DoubleFloat},\br
++ xend:\axiomType{DoubleFloat},\br
```

```

++ fn:\axiomType{Vector Expression DoubleFloat},\br
++ yinit:\axiomType{List DoubleFloat},intvals:\axiomType{List DoubleFloat},\br
++ g:\axiomType{Expression DoubleFloat},abserr:\axiomType{DoubleFloat},\br
++ relerr:\axiomType{DoubleFloat})
++

NumericalODEProblem(): EE == II where

DFB ==> DoubleFloat
VEDFB ==> Vector Expression DoubleFloat
LDLB ==> List DoubleFloat
EDFB ==> Expression DoubleFloat
ODEAB ==> Record(xinit:DFB,xend:DFB,fn:VEDFB,yinit:LDLB,intvals:LDLB,g:EDFB,abserr:DFB,relerr:DFB)

EE ==> SetCategory with
  coerce: ODEAB -> %
    ++ coerce(x) is not documented
  coerce: % -> OutputForm
    ++ coerce(x) is not documented
  retract: % -> ODEAB
    ++ retract(x) is not documented

II ==> add
  Rep := ODEAB

  coerce(s:ODEAB) == s
  coerce(x:%):OutputForm ==
    (retract(x))::OutputForm
  retract(x:%):ODEAB == x :: Rep

```

---

— ODEPROB.dotabb —

```

"ODEPROB" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ODEPROB"]
"BASTYPE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=BASTYPE"]
"KOERCE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=KOERCE"]
"ODEPROB" -> "BASTYPE"
"ODEPROB" -> "KOERCE"

```

---

## 15.9 domain OPTPROB NumericalOptimizationProblem

— NumericalOptimizationProblem.input —

```
)set break resume
)sys rm -f NumericalOptimizationProblem.output
)spool NumericalOptimizationProblem.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show NumericalOptimizationProblem
--R NumericalOptimizationProblem is a domain constructor
--R Abbreviation for NumericalOptimizationProblem is OPTPROB
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for OPTPROB
--R
--R----- Operations -----
--R ==? : (% ,%) -> Boolean          coerce : % -> OutputForm
--R hash : % -> SingleInteger       latex : % -> String
--R ==~? : (% ,%) -> Boolean
--R coerce : Union(noa: Record(fn: Expression DoubleFloat,init: List DoubleFloat,lb: List Or
--R coerce : Record(lfn: List Expression DoubleFloat,init: List DoubleFloat) -> %
--R coerce : Record(fn: Expression DoubleFloat,init: List DoubleFloat,lb: List OrderedComple
--R retract : % -> Union(noa: Record(fn: Expression DoubleFloat,init: List DoubleFloat,lb: L
--R
--E 1

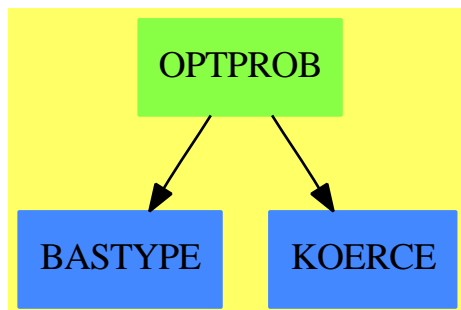
)spool
)lisp (bye)
```

— NumericalOptimizationProblem.help —

```
=====
NumericalOptimizationProblem examples
=====
```

See Also:  
o )show NumericalOptimizationProblem

## 15.9.1 NumericalOptimizationProblem (OPTPROB)

**Exports:**

coerce hash latex retract  $==?$   $\sim=?$

— domain OPTPROB NumericalOptimizationProblem —

```

)abbrev domain OPTPROB NumericalOptimizationProblem
++ Author: Brian Dupee
++ Date Created: December 1997
++ Date Last Updated: December 1997
++ Basic Operations: coerce, retract
++ Related Constructors: Union
++ Description:
++ \axiomType{NumericalOptimizationProblem} is a \axiom{domain}
++ for the representation of Numerical Optimization problems for use
++ by ANNA.
++
++ The representation is a Union of two record types - one for otimization of
++ a single function of one or more variables:
++
++ \axiomType{Record}{\br
++ fn:\axiomType{Expression DoubleFloat},\br
++ init:\axiomType{List DoubleFloat},\br
++ lb:\axiomType{List OrderedCompletion DoubleFloat},\br
++ cf:\axiomType{List Expression DoubleFloat},\br
++ ub:\axiomType{List OrderedCompletion DoubleFloat})
++
++ and one for least-squares problems i.e. optimization of a set of
++ observations of a data set:
++
++ \axiomType{Record}{lfn:\axiomType{List Expression DoubleFloat},\br
++ init:\axiomType{List DoubleFloat}).

NumericalOptimizationProblem(): EE == II where

  LDFD      ==> List DoubleFloat
  
```



```

LEDFD    ==> List Expression DoubleFloat
LSAD     ==> Record(lfn:LEDFD, init:LDFD)
UNOALSAD ==> Union(noa:NOAD,lsa:LSAD)
EDFD     ==> Expression DoubleFloat
LOCDFD   ==> List OrderedCompletion DoubleFloat
NOAD     ==> Record(fn:EDFD, init:LDFD, lb:LOCDFD, cf:LEDFD, ub:LOCDFD)

```

```

EE ==> SetCategory with
  coerce: NOAD -> %
    ++ coerce(x) is not documented
  coerce: LSAD -> %
    ++ coerce(x) is not documented
  coerce: UNOALSAD -> %
    ++ coerce(x) is not documented
  coerce: % -> OutputForm
    ++ coerce(x) is not documented
  retract: % -> UNOALSAD
    ++ retract(x) is not documented

```

```

II ==> add
  Rep := UNOALSAD

  coerce(s:NOAD) == [s]
  coerce(s:LSAD) == [s]
  coerce(x:UNOALSAD) == x
  coerce(x:%):OutputForm ==
    (x) case noa => (x.noa)::OutputForm
    (x.lsa)::OutputForm
  retract(x:%):UNOALSAD ==
    (x) case noa => [x.noa]
    [x.lsa]

```

---

— OPTPROB.dotabb —

```

"OPTPROB" [color="#88FF44",href="bookvol10.3.pdf#nameddest=OPTPROB"]
"BASTYPE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=BASTYPE"]
"KOERCE"  [color="#4488FF",href="bookvol10.2.pdf#nameddest=KOERCE"]
"OPTPROB" -> "BASTYPE"
"OPTPROB" -> "KOERCE"

```

---

## 15.10 domain PDEPROB NumericalPDEProblem

### — NumericalPDEProblem.input —

```
)set break resume
)sys rm -f NumericalPDEProblem.output
)spool NumericalPDEProblem.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show NumericalPDEProblem
--R NumericalPDEProblem is a domain constructor
--R Abbreviation for NumericalPDEProblem is PDEPROB
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PDEPROB
--R
--R----- Operations -----
--R ==? : (% ,%) -> Boolean          coerce : % -> OutputForm
--R hash : % -> SingleInteger       latex : % -> String
--R ==~? : (% ,%) -> Boolean
--R coerce : Record(pde: List Expression DoubleFloat,constraints: List Record(start: DoubleFloat,finish:
--R retract : % -> Record(pde: List Expression DoubleFloat,constraints: List Record(start: DoubleFloat,f
--R
--E 1

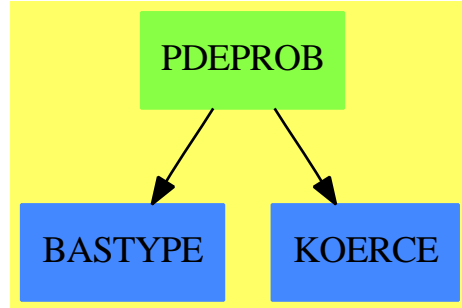
)spool
)lisp (bye)
```

### — NumericalPDEProblem.help —

```
=====
NumericalPDEProblem examples
=====
```

```
See Also:
o )show NumericalPDEProblem
```

## 15.10.1 NumericalPDEProblem (PDEPROB)

**Exports:**

coerce hash latex retract  $?=?$   $?^{\sim}=?$

— domain PDEPROB NumericalPDEProblem —

```

)abbrev domain PDEPROB NumericalPDEProblem
++ Author: Brian Dupee
++ Date Created: December 1997
++ Date Last Updated: December 1997
++ Basic Operations: coerce, retract
++ Related Constructors: Union
++ Description:
++ \axiomType{NumericalPDEProblem} is a \axiom{domain}
++ for the representation of Numerical PDE problems for use
++ by ANNA.
++
++ The representation is of type:
++
++ \axiomType{Record}(pde:\axiomType{List Expression DoubleFloat}, \br
++ constraints:\axiomType{List PDEC}, \br
++ f:\axiomType{List List Expression DoubleFloat},\br
++ st:\axiomType{String},\br
++ tol:\axiomType{DoubleFloat})
++
++ where \axiomType{PDEC} is of type:
++
++ \axiomType{Record}(start:\axiomType{DoubleFloat}, \br
++ finish:\axiomType{DoubleFloat},\br
++ grid:\axiomType{NonNegativeInteger},\br
++ boundaryType:\axiomType{Integer},\br
++ dStart:\axiomType{Matrix DoubleFloat}, \br
++ dFinish:\axiomType{Matrix DoubleFloat})

NumericalPDEProblem(): EE == II where

```

```

DFC ==> DoubleFloat
NNIC ==> NonNegativeInteger
INTC ==> Integer
MDFC ==> Matrix DoubleFloat
PDECC ==> Record(start:DFC, finish:DFC, grid:NNIC, boundaryType:INTC,
                 dStart:MDFC, dFinish:MDFC)
LEDFC ==> List Expression DoubleFloat
PDEBC ==> Record(pde:LEDFC, constraints:List PDECC, f:List LEDFC,
                 st:String, tol:DFC)

EE ==> SetCategory with
  coerce: PDEBC -> %
    ++ coerce(x) is not documented
  coerce: % -> OutputForm
    ++ coerce(x) is not documented
  retract: % -> PDEBC
    ++ retract(x) is not documented

II ==> add
  Rep := PDEBC

  coerce(s:PDEBC) == s
  coerce(x:%):OutputForm ==
    (retract(x))::OutputForm
  retract(x:%):PDEBC == x :: Rep

```

---

— PDEPROB.dotabb —

```

"PDEPROB" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PDEPROB"]
"BASTYPE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=BASTYPE"]
"KOERCE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=KOERCE"]
"PDEPROB" -> "BASTYPE"
"PDEPROB" -> "KOERCE"

```

---



## Chapter 16

# Chapter O

### 16.1 domain OCT Octonion

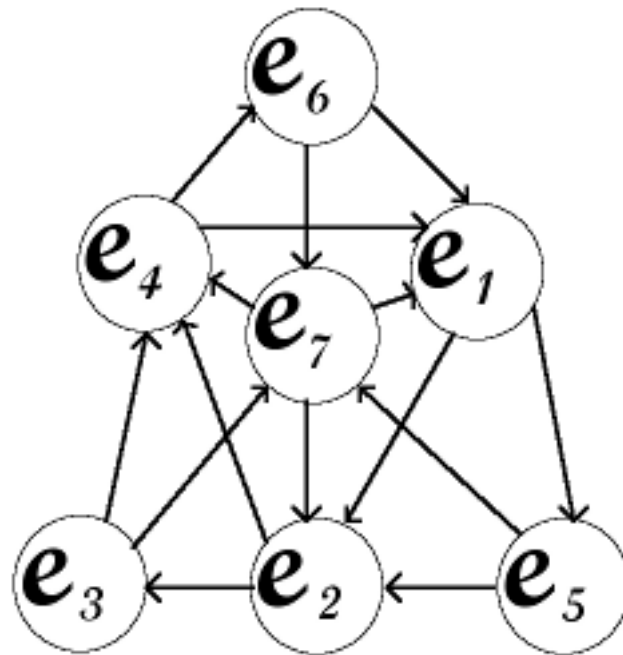
The octonions have the following multiplication table:

|     |      |      |      |      |      |      |      |
|-----|------|------|------|------|------|------|------|
| 1   | $i$  | $j$  | $k$  | $E$  | $I$  | $J$  | $K$  |
| $i$ | -1   | $k$  | $-j$ | $I$  | $-E$ | $-K$ | $J$  |
| $j$ | $-k$ | -1   | $i$  | $J$  | $K$  | $-E$ | $-I$ |
| $k$ | $j$  | $-i$ | -1   | $K$  | $-J$ | $I$  | $-E$ |
| $E$ | $-I$ | $-J$ | $-K$ | -1   | $i$  | $j$  | $k$  |
| $I$ | $E$  | $-K$ | $J$  | $-i$ | -1   | $-k$ | $j$  |
| $J$ | $K$  | $E$  | $-I$ | $-j$ | $k$  | -1   | $-i$ |
| $K$ | $-J$ | $I$  | $E$  | $-k$ | $-j$ | $i$  | -1   |

There are 3 different kinds of associativity. An algebra is

- **power-associative** if the subalgebra generated by any one element is associative. That is, given any element  $e$  then  $e * (e * e) = (e * e) * e$ .
- **alternative** if the subalgebra generated by any two elements is associative. That is, given any two elements,  $a$  and  $b$  then  $a * (a * b) = (a * a) * b$  and  $a * (b * a) = (a * b) * a$  and  $b * (a * a) = (b * a) * a$ .
- **associative** if the subalgebra generated by any three elements is associative. That is, given any three elements  $a$ ,  $b$ , and  $c$  then  $a * (b * c) = (a * b) * c$ .

The Octonions are power-associative and alternative but not associative, since  $I * (J * K) \neq (I * J) * K$ .



— Octonion.input —

```

)set break resume
)sys rm -f Octonion.output
)spool Octonion.output
)set message test on
)set message auto off
)clear all
--S 1 of 15
oci1 := octon(1,2,3,4,5,6,7,8)
--R
--R
--R (1) 1 + 2i + 3j + 4k + 5E + 6I + 7J + 8K
--R
--E 1

--S 2 of 15
oci2 := octon(7,2,3,-4,5,6,-7,0)
--R
--R
--R (2) 7 + 2i + 3j - 4k + 5E + 6I - 7J
--R
--E 2

--S 3 of 15
oci3 := octon(quatern(-7,-12,3,-10), quatern(5,6,9,0))

```

Type: Octonion Integer

Type: Octonion Integer

```

--R
--R
--R (3)  $-7 - 12i + 3j - 10k + 5E + 6I + 9J$ 
--R
--R                                         Type: Octonion Integer
--E 3

--S 4 of 15
(oci1 * oci2) * oci3 - oci1 * (oci2 * oci3)
--R
--R
--R (4)  $2696i - 2928j - 4072k + 16E - 1192I + 832J + 2616K$ 
--R
--R                                         Type: Octonion Integer
--E 4

--S 5 of 15
[real oci1, imagi oci1, imagj oci1, imagk oci1, _
 imagE oci1, imagI oci1, imagJ oci1, imagK oci1]
--R
--R
--R (5) [1,2,3,4,5,6,7,8]
--R
--R                                         Type: List PositiveInteger
--E 5

--S 6 of 15
q : Quaternion Polynomial Integer := quatern(q1, qi, qj, qk)
--R
--R
--R (6)  $q1 + qi i + qj j + qk k$ 
--R
--R                                         Type: Quaternion Polynomial Integer
--E 6

--S 7 of 15
E : Octonion Polynomial Integer:= octon(0,0,0,0,1,0,0,0)
--R
--R
--R (7) E
--R
--R                                         Type: Octonion Polynomial Integer
--E 7

--S 8 of 15
q * E
--R
--R
--R (8)  $q1 E + qi I + qj J + qk K$ 
--R
--R                                         Type: Octonion Polynomial Integer
--E 8

--S 9 of 15
E * q
--R

```



```

--R
--R (9)  $q_1 E - q_i I - q_j J - q_k K$ 
--R
--R                                         Type: Octonion Polynomial Integer
--E 9

--S 10 of 15
q * 1$(Octonion Polynomial Integer)
--R
--R
--R (10)  $q_1 + q_i i + q_j j + q_k k$ 
--R
--R                                         Type: Octonion Polynomial Integer
--E 10

--S 11 of 15
1$(Octonion Polynomial Integer) * q
--R
--R
--R (11)  $q_1 + q_i i + q_j j + q_k k$ 
--R
--R                                         Type: Octonion Polynomial Integer
--E 11

--S 12 of 15
o : Octonion Polynomial Integer := octon(o1, oi, oj, ok, oE, oI, oJ, oK)
--R
--R
--R (12)  $o_1 + o_i i + o_j j + o_k k + o_E E + o_I I + o_J J + o_K K$ 
--R
--R                                         Type: Octonion Polynomial Integer
--E 12

--S 13 of 15
norm o
--R
--R
--R (13)  $o_k^2 + o_j^2 + o_i^2 + o_K^2 + o_J^2 + o_I^2 + o_E^2 + o_1^2$ 
--R
--R                                         Type: Polynomial Integer
--E 13

--S 14 of 15
p : Octonion Polynomial Integer := octon(p1, pi, pj, pk, pE, pI, pJ, pK)
--R
--R
--R (14)  $p_1 + p_i i + p_j j + p_k k + p_E E + p_I I + p_J J + p_K K$ 
--R
--R                                         Type: Octonion Polynomial Integer
--E 14

--S 15 of 15
norm(o*p)-norm(p)*norm(o)
--R
--R

```

```
--R (15) 0
--R
--R                                          Type: Polynomial Integer
--E 15
)spool
)lisp (bye)
```

---

— Octonion.help —

=====

Octonion examples

=====

The Octonions, also called the Cayley-Dixon algebra, defined over a commutative ring are an eight-dimensional non-associative algebra. Their construction from quaternions is similar to the construction of quaternions from complex numbers.

As Octonion creates an eight-dimensional algebra, you have to give eight components to construct an octonion.

```
oci1 := octon(1,2,3,4,5,6,7,8)
      1 + 2i + 3j + 4k + 5E + 6I + 7J + 8K
                                Type: Octonion Integer
```

```
oci2 := octon(7,2,3,-4,5,6,-7,0)
      7 + 2i + 3j - 4k + 5E + 6I - 7J
                                Type: Octonion Integer
```

Or you can use two quaternions to create an octonion.

```
oci3 := octon(quatern(-7,-12,3,-10), quatern(5,6,9,0))
      - 7 - 12i + 3j - 10k + 5E + 6I + 9J
                                Type: Octonion Integer
```

You can easily demonstrate the non-associativity of multiplication.

```
(oci1 * oci2) * oci3 - oci1 * (oci2 * oci3)
      2696i - 2928j - 4072k + 16E - 1192I + 832J + 2616K
                                Type: Octonion Integer
```

As with the quaternions, we have a real part, the imaginary parts  $i$ ,  $j$ ,  $k$ , and four additional imaginary parts  $E$ ,  $I$ ,  $J$  and  $K$ . These parts correspond to the canonical basis  $(1, i, j, k, E, I, J, K)$ .

For each basis element there is a component operation to extract the coefficient of the basis element for a given octonion.

```
[real oci1, imagi oci1, imagj oci1, imagk oci1, _
  imagE oci1, imagI oci1, imagJ oci1, imagK oci1]
[1,2,3,4,5,6,7,8]
Type: List PositiveInteger
```

A basis with respect to the quaternions is given by (1,E). However, you might ask, what then are the commuting rules? To answer this, we create some generic elements.

We do this in Axiom by simply changing the ground ring from Integer to Polynomial Integer.

```
q : Quaternion Polynomial Integer := quatern(q1, qi, qj, qk)
q1 + qi i + qj j + qk k
Type: Quaternion Polynomial Integer

E : Octonion Polynomial Integer := octon(0,0,0,0,1,0,0,0)
E
Type: Octonion Polynomial Integer
```

Note that quaternions are automatically converted to octonions in the obvious way.

```
q * E
q1 E + qi I + qj J + qk K
Type: Octonion Polynomial Integer

E * q
q1 E - qi I - qj J - qk K
Type: Octonion Polynomial Integer

q * 1$(Octonion Polynomial Integer)
q1 + qi i + qj j + qk k
Type: Octonion Polynomial Integer

1$(Octonion Polynomial Integer) * q
q1 + qi i + qj j + qk k
Type: Octonion Polynomial Integer
```

Finally, we check that the norm, defined as the sum of the squares of the coefficients, is a multiplicative map.

```
o : Octonion Polynomial Integer := octon(o1, oi, oj, ok, oE, oI, oJ, oK)
o1 + oi i + oj j + ok k + oE E + oI I + oJ J + oK K
Type: Octonion Polynomial Integer

norm o
      2      2      2      2      2      2      2      2
ok  + oj  + oi  + oK  + oJ  + oI  + oE  + o1
Type: Polynomial Integer
```

```

p : Octonion Polynomial Integer := octon(p1, pi, pj, pk, pE, pI, pJ, pK)
  p1 + pi i + pj j + pk k + pE E + pI I + pJ J + pK K
                                Type: Octonion Polynomial Integer

```

Since the result is 0, the norm is multiplicative.

```

norm(o*p)-norm(p)*norm(o)
0
                                Type: Polynomial Integer

```

See Also:

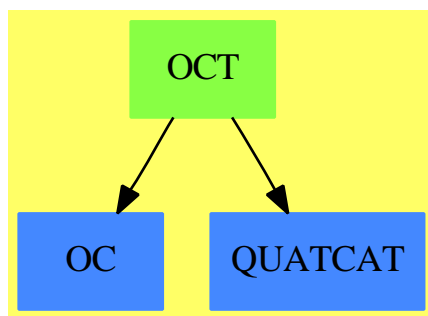
```

o )help Quaternion
o )show Octonion

```

—

### 16.1.1 Octonion (OCT)



#### Exports:

|               |           |               |                |              |
|---------------|-----------|---------------|----------------|--------------|
| 0             | 1         | abs           | characteristic | charthRoot   |
| coerce        | conjugate | convert       | eval           | hash         |
| imagE         | imagI     | imagJ         | imagK          | imagi        |
| imagj         | imagk     | index         | inv            | latex        |
| lookup        | map       | max           | min            | norm         |
| octon         | one?      | random        | rational       | rational?    |
| rationalIfCan | real      | recip         | retract        | retractIfCan |
| sample        | size      | subtractIfCan | zero?          | ?*?          |
| ?**?          | ?+?       | ?-?           | -?             | ?=?          |
| ?^?           | ?~=?      | ?<?           | ?<=?           | ?>?          |
| ?>=?          | ?..?      |               |                |              |

— domain OCT Octonion —

```

)abbrev domain OCT Octonion
++ Author: R. Wisbauer, J. Grabmeier
++ Date Created: 05 September 1990
++ Date Last Updated: 20 September 1990
++ Basic Operations: _+,_*,octon,image,imagi,imagj,imagk,
++  imagE,imagI,imagJ,imagK
++ Related Constructors: Quaternion
++ Also See: AlgebraGivenByStructuralConstants
++ AMS Classifications:
++ Keywords: octonion, non-associative algebra, Cayley-Dixon
++ References: e.g. I.L Kantor, A.S. Solodovnikov:
++  Hypercomplex Numbers, Springer Verlag Heidelberg, 1989,
++  ISBN 0-387-96980-2
++ Description:
++ Octonion implements octonions (Cayley-Dixon algebra) over a
++ commutative ring, an eight-dimensional non-associative
++ algebra, doubling the quaternions in the same way as doubling
++ the complex numbers to get the quaternions
++ the main constructor function is octon which takes 8
++ arguments: the real part, the i imaginary part, the j
++ imaginary part, the k imaginary part, (as with quaternions)
++ and in addition the imaginary parts E, I, J, K.

-->boot $noSubsumption := true
Octonion(R:CommutativeRing): export == impl where

QR ==> Quaternion R

export ==> Join(OctonionCategory R, FullyRetractableTo QR) with
  octon: (QR,QR) -> %
    ++ octon(qe,qE) constructs an octonion from two quaternions
    ++ using the relation  $0 = Q + QE$ .
impl ==> add
  Rep := Record(e: QR,E: QR)

  0 == [0,0]
  1 == [1,0]

  a,b,c,d,f,g,h,i : R
  p,q : QR
  x,y : %

  real x == real (x.e)
  imagi x == imagI (x.e)
  imagj x == imagJ (x.e)
  imagk x == imagK (x.e)
  imagE x == real (x.E)
  imagI x == imagI (x.E)
  imagJ x == imagJ (x.E)
  imagK x == imagK (x.E)

```

```

octon(a,b,c,d,f,g,h,i) == [quatern(a,b,c,d)$QR,quatern(f,g,h,i)$QR]
octon(p,q) == [p,q]
coerce(q) == [q,0$QR]
retract(x):QR ==
  not(zero? imagE x and zero? imagI x and zero? imagJ x and zero? imagK x)=>
    error "Cannot retract octonion to quaternion."
  quatern(real x, imagi x,imagj x, imagk x)$QR
retractIfCan(x):Union(QR,"failed") ==
  not(zero? imagE x and zero? imagI x and zero? imagJ x and zero? imagK x)=>
    "failed"
  quatern(real x, imagi x,imagj x, imagk x)$QR
x * y == [x.e*y.e-(conjugate y.E)*x.E, y.E*x.e + x.E*(conjugate y.e)]

```

---

— OCT.dotabb —

```

"OCT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=OCT"]
"OC" [color="#4488FF",href="bookvol10.2.pdf#nameddest=OC"]
"QUATCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=QUATCAT"]
"OCT" -> "OC"
"OCT" -> "QUATCAT"

```

---

## 16.2 domain ODEIFTBL ODEIntensityFunctionsTable

— ODEIntensityFunctionsTable.input —

```

)set break resume
)sys rm -f ODEIntensityFunctionsTable.output
)spool ODEIntensityFunctionsTable.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ODEIntensityFunctionsTable
--R ODEIntensityFunctionsTable is a domain constructor
--R Abbreviation for ODEIntensityFunctionsTable is ODEIFTBL
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ODEIFTBL
--R
--R----- Operations -----

```

```

--R clearTheIFTable : () -> Void          showTheIFTable : () -> %
--R iFTable : List Record(key: Record(xinit: DoubleFloat,xend: DoubleFloat,fn: Vector Expression I
--R insert! : Record(key: Record(xinit: DoubleFloat,xend: DoubleFloat,fn: Vector Expression I
--R keys : % -> List Record(xinit: DoubleFloat,xend: DoubleFloat,fn: Vector Expression Double
--R showIntensityFunctions : Record(xinit: DoubleFloat,xend: DoubleFloat,fn: Vector Expressi
--R
--E 1

```

```

)spool
)lisp (bye)

```

— ODEIntensityFunctionsTable.help —

```

=====
ODEIntensityFunctionsTable examples
=====

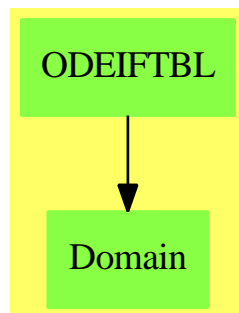
```

```

See Also:
o )show ODEIntensityFunctionsTable

```

### 16.2.1 ODEIntensityFunctionsTable (ODEIFTBL)



#### Exports:

```

clearTheIFTable iFTable insert! keys showIntensityFunctions showTheIFTable

```

— domain ODEIFTBL ODEIntensityFunctionsTable —

```

)abbrev domain ODEIFTBL ODEIntensityFunctionsTable
++ Author: Brian Dupee

```

```

++ Date Created: May 1994
++ Date Last Updated: January 1996
++ Basic Operations: showTheIFTable, insert!
++ Description:
++ \axiom{ODEIntensityFunctionsTable()} provides a dynamic table and a set of
++ functions to store details found out about sets of ODE's.

ODEIntensityFunctionsTable(): E == I where
  LEDF ==> List Expression DoubleFloat
  LEEDF ==> List Equation Expression DoubleFloat
  EEDF ==> Equation Expression DoubleFloat
  VEDF ==> Vector Expression DoubleFloat
  MEDF ==> Matrix Expression DoubleFloat
  MDF ==> Matrix DoubleFloat
  EDF ==> Expression DoubleFloat
  DF ==> DoubleFloat
  F ==> Float
  INT ==> Integer
  CDF ==> Complex DoubleFloat
  LDF ==> List DoubleFloat
  LF ==> List Float
  S ==> Symbol
  LS ==> List Symbol
  MFI ==> Matrix Fraction Integer
  LFI ==> List Fraction Integer
  FI ==> Fraction Integer
  ODEA ==> Record(xinit:DF,xend:DF,fn:VEDF,yinit:LDF,intvals:LDF,g:EDF,abserr:DF,relerr:DF)
  ON ==> Record(additions:INT,multiplications:INT,exponentiations:INT,functionCalls:INT)
  RVE ==> Record(val:EDF,exponent:INT)
  RSS ==> Record(stiffnessFactor:F,stabilityFactor:F)
  ATT ==> Record(stiffness:F,stability:F,expense:F,accuracy:F,intermediateResults:F)
  ROA ==> Record(key:ODEA,entry:ATT)

E ==> with
  showTheIFTable:() -> $
    ++ showTheIFTable() returns the current table of intensity functions.
  clearTheIFTable : () -> Void
    ++ clearTheIFTable() clears the current table of intensity functions.
  keys : $ -> List(ODEA)
    ++ keys(tab) returns the list of keys of f
  iFTable: List Record(key:ODEA,entry:ATT) -> $
    ++ iFTable(l) creates an intensity-functions table from the elements
    ++ of l.
  insert!:Record(key:ODEA,entry:ATT) -> $
    ++ insert!(r) inserts an entry r into theIFTable
  showIntensityFunctions:ODEA -> Union(ATT,"failed")
    ++ showIntensityFunctions(k) returns the entries in the
    ++ table of intensity functions k.

I ==> add

```



```

Rep := Table(ODEA,ATT)
import Rep

theIFTable:$ := empty()$Rep

showTheIFTable():$ ==
  theIFTable

clearTheIFTable():Void ==
  theIFTable := empty()$Rep
  void()$Void

iFTable(l:List Record(key:ODEA,entry:ATT)):$ ==
  theIFTable := table(l)$Rep

insert!(r:Record(key:ODEA,entry:ATT)):$ ==
  insert!(r,theIFTable)$Rep

keys(t:$):List ODEA ==
  keys(t)$Rep

showIntensityFunctions(k:ODEA):Union(ATT,"failed") ==
  search(k,theIFTable)$Rep

```

---

— ODEIFTBL.dotabb —

```

"ODEIFTBL" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ODEIFTBL"]
"Domain" [color="#88FF44"]
"ODEIFTBL" -> "Domain"

```

---

## 16.3 domain ARRAY1 OneDimensionalArray

— OneDimensionalArray.input —

```

)set break resume
)sys rm -f OneDimensionalArray.output
)spool OneDimensionalArray.output
)set message test on
)set message auto off
)clear all

```

```

--S 1 of 9
oneDimensionalArray [i**2 for i in 1..10]
--R
--R
--R (1) [1,4,9,16,25,36,49,64,81,100]
--R                                         Type: OneDimensionalArray PositiveInteger
--E 1

--S 2 of 9
a : ARRAY1 INT := new(10,0)
--R
--R
--R (2) [0,0,0,0,0,0,0,0,0,0]
--R                                         Type: OneDimensionalArray Integer
--E 2

--S 3 of 9
for i in 1..10 repeat a.i := i; a
--R
--R
--R (3) [1,2,3,4,5,6,7,8,9,10]
--R                                         Type: OneDimensionalArray Integer
--E 3

--S 4 of 9
map!(i +-> i ** 2,a); a
--R
--R
--R (4) [1,4,9,16,25,36,49,64,81,100]
--R                                         Type: OneDimensionalArray Integer
--E 4

--S 5 of 9
reverse! a
--R
--R
--R (5) [100,81,64,49,36,25,16,9,4,1]
--R                                         Type: OneDimensionalArray Integer
--E 5

--S 6 of 9
swap!(a,4,5); a
--R
--R
--R (6) [100,81,64,36,49,25,16,9,4,1]
--R                                         Type: OneDimensionalArray Integer
--E 6

--S 7 of 9
sort! a

```

```

--R
--R
--R (7) [1,4,9,16,25,36,49,64,81,100]
--R                                         Type: OneDimensionalArray Integer
--E 7

--S 8 of 9
b := a(6..10)
--R
--R
--R (8) [36,49,64,81,100]
--R                                         Type: OneDimensionalArray Integer
--E 8

--S 9 of 9
copyInto!(a,b,1)
--R
--R
--R (9) [36,49,64,81,100,36,49,64,81,100]
--R                                         Type: OneDimensionalArray Integer
--E 9
)spool
)lisp (bye)

```

---

— OneDimensionalArray.help —

=====

OneDimensionalArray examples

=====

The OneDimensionalArray domain is used for storing data in a one-dimensional indexed data structure. Such an array is a homogeneous data structure in that all the entries of the array must belong to the same Axiom domain. Each array has a fixed length specified by the user and arrays are not extensible. The indexing of one-dimensional arrays is one-based. This means that the "first" element of an array is given the index 1.

To create a one-dimensional array, apply the operation `oneDimensionalArray` to a list.

```

oneDimensionalArray [i**2 for i in 1..10]
[1,4,9,16,25,36,49,64,81,100]
                        Type: OneDimensionalArray PositiveInteger

```

Another approach is to first create `a`, a one-dimensional array of 10 0's. `OneDimensionalArray` has the convenient abbreviation `ARRAY1`.

```

a : ARRAY1 INT := new(10,0)
  [0,0,0,0,0,0,0,0,0,0]
                                Type: OneDimensionalArray Integer

Set each i-th element to i, then display the result.

for i in 1..10 repeat a.i := i; a
  [1,2,3,4,5,6,7,8,9,10]
                                Type: OneDimensionalArray Integer

Square each element by mapping the function i +-> i^2 onto each element.

map!(i +-> i ** 2,a); a
  [1,4,9,16,25,36,49,64,81,100]
                                Type: OneDimensionalArray Integer

Reverse the elements in place.

reverse! a
  [100,81,64,49,36,25,16,9,4,1]
                                Type: OneDimensionalArray Integer

Swap the 4th and 5th element.

swap!(a,4,5); a
  [100,81,64,36,49,25,16,9,4,1]
                                Type: OneDimensionalArray Integer

Sort the elements in place.

sort! a
  [1,4,9,16,25,36,49,64,81,100]
                                Type: OneDimensionalArray Integer

Create a new one-dimensional array b containing the last 5 elements of a.

b := a(6..10)
  [36,49,64,81,100]
                                Type: OneDimensionalArray Integer

Replace the first 5 elements of a with those of b.

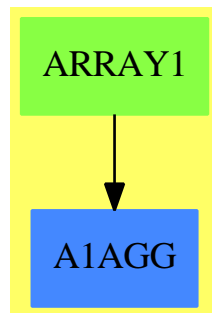
copyInto!(a,b,1)
  [36,49,64,81,100,36,49,64,81,100]
                                Type: OneDimensionalArray Integer

See Also:
o )help Vector
o )help FlexibleArray

```

```
o )show OneDimensionalArray
```

### 16.3.1 OneDimensionalArray (ARRAY1)



See

- ⇒ “PrimitiveArray” (PRIMARR) 17.30.1 on page 2069
- ⇒ “Tuple” (TUPLE) 21.12.1 on page 2711
- ⇒ “IndexedFlexibleArray” (IFARRAY) 10.10.1 on page 1187
- ⇒ “FlexibleArray” (FARRAY) 7.14.1 on page 853
- ⇒ “IndexedOneDimensionalArray” (IARRAY1) 10.13.1 on page 1208

#### Exports:

|         |           |                     |           |                  |
|---------|-----------|---------------------|-----------|------------------|
| any?    | coerce    | concat              | construct | convert          |
| copy    | copyInto! | count               | delete    | elt              |
| empty   | empty?    | entries             | entry?    | eq?              |
| eval    | every?    | fill!               | find      | first            |
| hash    | index?    | indices             | insert    | latex            |
| less?   | map       | map!                | max       | maxIndex         |
| member? | members   | merge               | min       | minIndex         |
| more?   | new       | oneDimensionalArray | parts     | position         |
| qelt    | qsetelt!  | reduce              | remove    | removeDuplicates |
| reverse | reverse!  | sample              | select    | setelt           |
| size?   | sort      | sort!               | sorted?   | swap!            |
| #?      | ?<?       | ?<=?                | ?=?       | ?>?              |
| ?>=?    | ?~=?      | ?..?                |           |                  |

— domain ARRAY1 OneDimensionalArray —

```

)abbrev domain ARRAY1 OneDimensionalArray
++ Author: Mark Botch
++ Description:
  
```

```

++ This is the domain of 1-based one dimensional arrays

OneDimensionalArray(S:Type): Exports == Implementation where
  ARRAYMININDEX ==> 1      -- if you want to change this, be my guest
  Exports == OneDimensionalArrayAggregate S with
    oneDimensionalArray: List S -> %
      ++ oneDimensionalArray(1) creates an array from a list of elements 1
      ++
      ++X oneDimensionalArray [i**2 for i in 1..10]

  oneDimensionalArray: (NonNegativeInteger, S) -> %
      ++ oneDimensionalArray(n,s) creates an array from n copies of element s
      ++
      ++X oneDimensionalArray(10,0.0)

Implementation == IndexedOneDimensionalArray(S, ARRAYMININDEX) add
  oneDimensionalArray(u) ==
    n := #u
    n = 0 => empty()
    a := new(n, first u)
    for i in 2..n for x in rest u repeat a.i := x
    a
  oneDimensionalArray(n,s) == new(n,s)

```

---

— ARRAY1.dotabb —

```

"ARRAY1" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ARRAY1"]
"A1AGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=A1AGG"]
"ARRAY1" -> "A1AGG"

```

---

## 16.4 domain ONECOMP OnePointCompletion

---

— OnePointCompletion.input —

```

)set break resume
)sys rm -f OnePointCompletion.output
)spool OnePointCompletion.output
)set message test on
)set message auto off
)clear all

```

```

--S 1 of 1
)show OnePointCompletion
--R OnePointCompletion R: SetCategory is a domain constructor
--R Abbreviation for OnePointCompletion is ONECOMP
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ONECOMP
--R
--R----- Operations -----
--R -? : % -> % if R has ABELGRP          ?? : (%,%) -> Boolean
--R 1 : () -> % if R has ORDRING          0 : () -> % if R has ABELGRP
--R coerce : R -> %                      coerce : % -> OutputForm
--R finite? : % -> Boolean                hash : % -> SingleInteger
--R infinite? : % -> Boolean              infinity : () -> %
--R latex : % -> String                  retract : % -> R
--R ~=? : (%,%) -> Boolean
--R ?? : (PositiveInteger,%) -> % if R has ABELGRP
--R ?? : (NonNegativeInteger,%) -> % if R has ABELGRP
--R ?? : (Integer,%) -> % if R has ABELGRP
--R ?? : (%,%) -> % if R has ORDRING
--R ??? : (%,NonNegativeInteger) -> % if R has ORDRING
--R ??? : (%,PositiveInteger) -> % if R has ORDRING
--R ?+? : (%,%) -> % if R has ABELGRP
--R ?-? : (%,%) -> % if R has ABELGRP
--R ?<? : (%,%) -> Boolean if R has ORDRING
--R ?<=? : (%,%) -> Boolean if R has ORDRING
--R ?>? : (%,%) -> Boolean if R has ORDRING
--R ?>=? : (%,%) -> Boolean if R has ORDRING
--R ??? : (%,NonNegativeInteger) -> % if R has ORDRING
--R ??? : (%,PositiveInteger) -> % if R has ORDRING
--R abs : % -> % if R has ORDRING
--R characteristic : () -> NonNegativeInteger if R has ORDRING
--R coerce : Integer -> % if R has ORDRING or R has RETRACT INT
--R coerce : Fraction Integer -> % if R has RETRACT FRAC INT
--R max : (%,%) -> % if R has ORDRING
--R min : (%,%) -> % if R has ORDRING
--R negative? : % -> Boolean if R has ORDRING
--R one? : % -> Boolean if R has ORDRING
--R positive? : % -> Boolean if R has ORDRING
--R rational : % -> Fraction Integer if R has INS
--R rational? : % -> Boolean if R has INS
--R rationalIfCan : % -> Union(Fraction Integer,"failed") if R has INS
--R recip : % -> Union(%, "failed") if R has ORDRING
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retract : % -> Integer if R has RETRACT INT
--R retractIfCan : % -> Union(R,"failed")
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
--R sample : () -> % if R has ABELGRP
--R sign : % -> Integer if R has ORDRING

```

```
--R subtractIfCan : (%,% ) -> Union(%, "failed") if R has ABELGRP
--R zero? : % -> Boolean if R has ABELGRP
--R
--E 1
```

```
)spool
)lisp (bye)
```

---

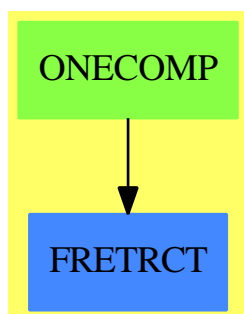
### — OnePointCompletion.help —

```
=====
OnePointCompletion examples
=====
```

```
See Also:
o )show OnePointCompletion
```

---

## 16.4.1 OnePointCompletion (ONECOMP)



See

⇒ “OrderedCompletion” (ORDCOMP) 16.12.1 on page 1772

### Exports:

|           |              |           |                |               |
|-----------|--------------|-----------|----------------|---------------|
| 0         | 1            | abs       | characteristic | coerce        |
| coerce    | finite?      | hash      | infinite?      | infinity      |
| latex     | max          | min       | negative?      | one?          |
| positive? | rational     | rational? | rationalIfCan  | recip         |
| retract   | retractIfCan | sample    | sign           | subtractIfCan |
| zero?     | -?           | ?=?       | ?^=?           | ?*?           |
| ?**?      | ?+?          | ?-?       | ?<?            | ?<=?          |
| ?>?       | ?>=?         | ?^?       |                |               |



## — domain ONECOMP OnePointCompletion —

```

)abbrev domain ONECOMP OnePointCompletion
++ Author: Manuel Bronstein
++ Date Created: 4 Oct 1989
++ Date Last Updated: 1 Nov 1989
++ Description:
++ Completion with infinity.
++ Adjunction of a complex infinity to a set.

OnePointCompletion(R:SetCategory): Exports == Implementation where
  B ==> Boolean

Exports ==> Join(SetCategory, FullyRetractableTo R) with
  infinity : () -> %
    ++ infinity() returns infinity.
  finite? : % -> B
    ++ finite?(x) tests if x is finite.
  infinite?: % -> B
    ++ infinite?(x) tests if x is infinite.
  if R has AbelianGroup then AbelianGroup
  if R has OrderedRing then OrderedRing
  if R has IntegerNumberSystem then
    rational?: % -> Boolean
      ++ rational?(x) tests if x is a finite rational number.
    rational : % -> Fraction Integer
      ++ rational(x) returns x as a finite rational number.
      ++ Error: if x is not a rational number.
    rationalIfCan: % -> Union(Fraction Integer, "failed")
      ++ rationalIfCan(x) returns x as a finite rational number if
      ++ it is one, "failed" otherwise.

Implementation ==> add
  Rep := Union(R, "infinity")

  coerce(r:R):% == r
  retract(x:%):R == (x case R => x::R; error "Not finite")
  finite? x == x case R
  infinite? x == x case "infinity"
  infinity() == "infinity"
  retractIfCan(x:%):Union(R, "failed") == (x case R => x::R; "failed")

  coerce(x:%):OutputForm ==
    x case "infinity" => "infinity":OutputForm
    x::R::OutputForm

  x = y ==
    x case "infinity" => y case "infinity"
    y case "infinity" => false

```

```

x::R = y::R

if R has AbelianGroup then
  0 == 0$R

  n:Integer * x:% ==
    x case "infinity" =>
      zero? n => error "Undefined product"
      infinity()
    n * x::R

  - x ==
    x case "infinity" => error "Undefined inverse"
    - (x::R)

  x + y ==
    x case "infinity" => x
    y case "infinity" => y
    x::R + y::R

if R has OrderedRing then
  fininf: R -> %

  1 == 1$R
  characteristic() == characteristic()$R

  fininf r ==
    zero? r => error "Undefined product"
    infinity()

  x:% * y:% ==
    x case "infinity" =>
      y case "infinity" => y
      fininf(y::R)
    y case "infinity" => fininf(x::R)
    x::R * y::R

  recip x ==
    x case "infinity" => 0
    zero?(x::R) => infinity()
    (u := recip(x::R)) case "failed" => "failed"
    u::R::%

  x < y ==
    x case "infinity" => false      -- do not change the order
    y case "infinity" => true       -- of those two tests
    x::R < y::R

if R has IntegerNumberSystem then
  rational? x == finite? x

```

```

rational x == rational(retract(x)@R)

rationalIfCan x ==
  (r:= retractIfCan(x)@Union(R,"failed")) case "failed" =>"failed"
  rational(r::R)

```

---

— ONECOMP.dotabb —

```

"ONECOMP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ONECOMP"]
"FRETRCT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FRETRCT"]
"ONECOMP" -> "FRETRCT"

```

---

## 16.5 domain OMCONN OpenMathConnection

— OpenMathConnection.input —

```

)set break resume
)sys rm -f OpenMathConnection.output
)spool OpenMathConnection.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OpenMathConnection
--R OpenMathConnection is a domain constructor
--R Abbreviation for OpenMathConnection is OMCONN
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for OMCONN
--R
--R----- Operations -----
--R OMcloseConn : % -> Void          OMmakeConn : SingleInteger -> %
--R OMbindTCP : (%,SingleInteger) -> Boolean
--R OMconnInDevice : % -> OpenMathDevice
--R OMconnOutDevice : % -> OpenMathDevice
--R OMconnectTCP : (%,String,SingleInteger) -> Boolean
--R
--E 1

)spool
)lisp (bye)

```

---

— OpenMathConnection.help —

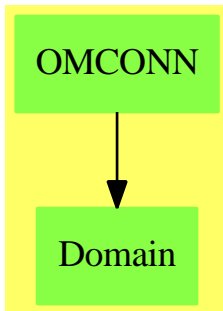
```
=====
OpenMathConnection examples
=====
```

See Also:

```
o )show OpenMathConnection
```

---

### 16.5.1 OpenMathConnection (OMCONN)



See

⇒ “OpenMathEncoding” (OMENC) 16.7.1 on page 1751

⇒ “OpenMathDevice” (OMDEV) 16.6.1 on page 1746

**Exports:**

```
OMbindTCP      OMcloseConn  OMconnectTCP  OMconnInDevice
OMconnOutDevice OMmakeConn
```

— domain OMCONN OpenMathConnection —

```
)abbrev domain OMCONN OpenMathConnection
++ Author: Vilya Harvey
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
```

```

++ Description:
++ \spadtype{OpenMathConnection} provides low-level functions
++ for handling connections to and from \spadtype{OpenMathDevice}s.

OpenMathConnection(): with
  OMmakeConn    : SingleInteger -> % ++ \spad{OMmakeConn}
  OMcloseConn   : % -> Void ++ \spad{OMcloseConn}
  OMconnInDevice: %-> OpenMathDevice ++ \spad{OMconnInDevice:}
  OMconnOutDevice: %-> OpenMathDevice ++ \spad{OMconnOutDevice:}
  OMconnectTCP  : (%, String, SingleInteger) -> Boolean ++ \spad{OMconnectTCP}
  OMbindTCP     : (%, SingleInteger) -> Boolean ++ \spad{OMbindTCP}
== add
  OMmakeConn(timeout: SingleInteger): % == OM_-MAKECONN(timeout)$Lisp
  OMcloseConn(conn: %): Void == OM_-CLOSECONN(conn)$Lisp

  OMconnInDevice(conn: %): OpenMathDevice ==
    OM_-GETCONNINDEV(conn)$Lisp
  OMconnOutDevice(conn: %): OpenMathDevice ==
    OM_-GETCONNOUTDEV(conn)$Lisp

  OMconnectTCP(conn: %, host: String, port: SingleInteger): Boolean ==
    OM_-CONNECTTCP(conn, host, port)$Lisp
  OMbindTCP(conn: %, port: SingleInteger): Boolean ==
    OM_-BINDTCP(conn, port)$Lisp

```

---

— OMCONN.dotabb —

```

"OMCONN" [color="#88FF44",href="bookvol10.3.pdf#nameddest=OMCONN"]
"Domain" [color="#88FF44"]
"OMCONN" -> "Domain"

```

---

## 16.6 domain OMDEV OpenMathDevice

— OpenMathDevice.input —

```

)set break resume
)sys rm -f OpenMathDevice.output
)spool OpenMathDevice.output
)set message test on
)set message auto off

```

```

)clear all

--S 1 of 1
)show OpenMathDevice
--R OpenMathDevice is a domain constructor
--R Abbreviation for OpenMathDevice is OMDEV
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for OMDEV
--R
--R----- Operations -----
--R OMclose : % -> Void
--R OMgetAtp : % -> Void
--R OMgetBVar : % -> Void
--R OMgetEndApp : % -> Void
--R OMgetEndAttr : % -> Void
--R OMgetEndBind : % -> Void
--R OMgetEndObject : % -> Void
--R OMgetFloat : % -> DoubleFloat
--R OMgetObject : % -> Void
--R OMgetType : % -> Symbol
--R OMputApp : % -> Void
--R OMputAttr : % -> Void
--R OMputBind : % -> Void
--R OMputEndAtp : % -> Void
--R OMputEndBVar : % -> Void
--R OMputEndError : % -> Void
--R OMputError : % -> Void
--R OMputString : (% ,String) -> Void
--R OMgetSymbol : % -> Record(cd: String,name: String)
--R OMopenFile : (String,String,OpenMathEncoding) -> %
--R OMopenString : (String,OpenMathEncoding) -> %
--R OMputFloat : (% ,DoubleFloat) -> Void
--R OMputInteger : (% ,Integer) -> Void
--R OMputSymbol : (% ,String,String) -> Void
--R OMputVariable : (% ,Symbol) -> Void
--R OMsetEncoding : (% ,OpenMathEncoding) -> Void
--R
--E 1

)spool
)lisp (bye)

```

---

— OpenMathDevice.help —

```

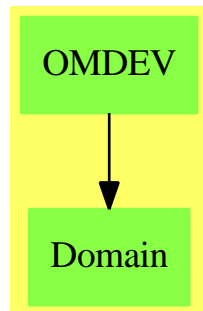
=====
OpenMathDevice examples
=====

```

See Also:

o `)show OpenMathDevice`

### 16.6.1 OpenMathDevice (OMDEV)



See

⇒ “OpenMathEncoding” (OMENC) 16.7.1 on page 1751

⇒ “OpenMathConnection” (OMCONN) 16.5.1 on page 1743

#### Exports:

|                |               |               |                |
|----------------|---------------|---------------|----------------|
| OMclose        | OMgetApp      | OMgetAtp      | OMgetAttr      |
| OMgetBVar      | OMgetBind     | OMgetEndApp   | OMgetEndAtp    |
| OMgetEndAttr   | OMgetEndBVar  | OMgetEndBind  | OMgetEndError  |
| OMgetEndObject | OMgetError    | OMgetFloat    | OMgetInteger   |
| OMgetObject    | OMgetString   | OMgetType     | OMgetVariable  |
| OMputApp       | OMputAtp      | OMputAttr     | OMputBVar      |
| OMputBind      | OMputEndApp   | OMputEndAtp   | OMputEndAttr   |
| OMputEndBVar   | OMputEndBind  | OMputEndError | OMputEndObject |
| OMputError     | OMputObject   | OMputString   | OMgetSymbol    |
| OMopenFile     | OMopenString  | OMputFloat    | OMputInteger   |
| OMputSymbol    | OMputVariable | OMsetEncoding |                |

— domain OMDEV OpenMathDevice —

```

)abbrev domain OMDEV OpenMathDevice
++ Author: Vilya Harvey
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
  
```

```

++ Keywords:
++ References:
++ Description:
++ \spadtype{OpenMathDevice} provides support for reading
++ and writing openMath objects to files, strings etc. It also provides
++ access to low-level operations from within the interpreter.

OpenMathDevice(): with
  OMopenFile      : (String, String, OpenMathEncoding) -> %
  ++ OMopenFile(f,mode,enc) opens file \axiom{f} for reading or writing
  ++ OpenMath objects (depending on \axiom{mode} which can be "r", "w"
  ++ or "a" for read, write and append respectively), in the encoding
  ++ \axiom{enc}.
  OMopenString   : (String, OpenMathEncoding) -> %
  ++ OMopenString(s,mode) opens the string \axiom{s} for reading or writing
  ++ OpenMath objects in encoding \axiom{enc}.
  OMclose        : % -> Void
  ++ OMclose(dev) closes \axiom{dev}, flushing output if necessary.
  OMsetEncoding  : (% , OpenMathEncoding) -> Void
  ++ OMsetEncoding(dev,enc) sets the encoding used for reading or writing
  ++ OpenMath objects to or from \axiom{dev} to \axiom{enc}.
  OMputApp       : % -> Void
  ++ OMputApp(dev) writes a begin application token to \axiom{dev}.
  OMputAtp       : % -> Void
  ++ OMputAtp(dev) writes a begin attribute pair token to \axiom{dev}.
  OMputAttr      : % -> Void
  ++ OMputAttr(dev) writes a begin attribute token to \axiom{dev}.
  OMputBind      : % -> Void
  ++ OMputBind(dev) writes a begin binder token to \axiom{dev}.
  OMputBVar      : % -> Void
  ++ OMputBVar(dev) writes a begin bound variable list token to \axiom{dev}.
  OMputError     : % -> Void
  ++ OMputError(dev) writes a begin error token to \axiom{dev}.
  OMputObject    : % -> Void
  ++ OMputObject(dev) writes a begin object token to \axiom{dev}.
  OMputEndApp    : % -> Void
  ++ OMputEndApp(dev) writes an end application token to \axiom{dev}.
  OMputEndAtp    : % -> Void
  ++ OMputEndAtp(dev) writes an end attribute pair token to \axiom{dev}.
  OMputEndAttr   : % -> Void
  ++ OMputEndAttr(dev) writes an end attribute token to \axiom{dev}.
  OMputEndBind   : % -> Void
  ++ OMputEndBind(dev) writes an end binder token to \axiom{dev}.
  OMputEndBVar   : % -> Void
  ++ OMputEndBVar(dev) writes an end bound variable list token to \axiom{dev}.
  OMputEndError  : % -> Void
  ++ OMputEndError(dev) writes an end error token to \axiom{dev}.
  OMputEndObject : % -> Void
  ++ OMputEndObject(dev) writes an end object token to \axiom{dev}.
  OMputInteger   : (% , Integer) -> Void

```



```

++ OMputInteger(dev,i) writes the integer \axiom{i} to \axiom{dev}.
OMputFloat    : (% , DoubleFloat) -> Void
++ OMputFloat(dev,i) writes the float \axiom{i} to \axiom{dev}.
OMputVariable : (% , Symbol) -> Void
++ OMputVariable(dev,i) writes the variable \axiom{i} to \axiom{dev}.
OMputString   : (% , String) -> Void
++ OMputString(dev,i) writes the string \axiom{i} to \axiom{dev}.
OMputSymbol   : (% , String, String) -> Void
++ OMputSymbol(dev,cd,s) writes the symbol \axiom{s} from CD \axiom{cd}
++ to \axiom{dev}.

OMgetApp      : % -> Void
++ OMgetApp(dev) reads a begin application token from \axiom{dev}.
OMgetAtp      : % -> Void
++ OMgetAtp(dev) reads a begin attribute pair token from \axiom{dev}.
OMgetAttr     : % -> Void
++ OMgetAttr(dev) reads a begin attribute token from \axiom{dev}.
OMgetBind     : % -> Void
++ OMgetBind(dev) reads a begin binder token from \axiom{dev}.
OMgetBVar     : % -> Void
++ OMgetBVar(dev) reads a begin bound variable list token from \axiom{dev}.
OMgetError    : % -> Void
++ OMgetError(dev) reads a begin error token from \axiom{dev}.
OMgetObject   : % -> Void
++ OMgetObject(dev) reads a begin object token from \axiom{dev}.
OMgetEndApp   : % -> Void
++ OMgetEndApp(dev) reads an end application token from \axiom{dev}.
OMgetEndAtp   : % -> Void
++ OMgetEndAtp(dev) reads an end attribute pair token from \axiom{dev}.
OMgetEndAttr  : % -> Void
++ OMgetEndAttr(dev) reads an end attribute token from \axiom{dev}.
OMgetEndBind  : % -> Void
++ OMgetEndBind(dev) reads an end binder token from \axiom{dev}.
OMgetEndBVar  : % -> Void
++ OMgetEndBVar(dev) reads an end bound variable list token from \axiom{dev}.
OMgetEndError : % -> Void
++ OMgetEndError(dev) reads an end error token from \axiom{dev}.
OMgetEndObject : % -> Void
++ OMgetEndObject(dev) reads an end object token from \axiom{dev}.
OMgetInteger  : % -> Integer
++ OMgetInteger(dev) reads an integer from \axiom{dev}.
OMgetFloat    : % -> DoubleFloat
++ OMgetFloat(dev) reads a float from \axiom{dev}.
OMgetVariable : % -> Symbol
++ OMgetVariable(dev) reads a variable from \axiom{dev}.
OMgetString   : % -> String
++ OMgetString(dev) reads a string from \axiom{dev}.
OMgetSymbol   : % -> Record(cd:String, name:String)
++ OMgetSymbol(dev) reads a symbol from \axiom{dev}.

```

```

OMgetType      : % -> Symbol
++ OMgetType(dev) returns the type of the next object on \axiom{dev}.
== add
OMopenFile(fname: String, fmode: String, enc: OpenMathEncoding): % ==
    OM_-OPENFILEDEV(fname, fmode, enc)$Lisp
OMopenString(str: String, enc: OpenMathEncoding): % ==
    OM_-OPENSTRINGDEV(str, enc)$Lisp
OMclose(dev: %): Void ==
    OM_-CLOSEDEV(dev)$Lisp
OMsetEncoding(dev: %, enc: OpenMathEncoding): Void ==
    OM_-SETDEVENCODING(dev, enc)$Lisp

OMputApp(dev: %): Void == OM_-PUTAPP(dev)$Lisp
OMputAtp(dev: %): Void == OM_-PUTATP(dev)$Lisp
OMputAttr(dev: %): Void == OM_-PUTATTR(dev)$Lisp
OMputBind(dev: %): Void == OM_-PUTBIND(dev)$Lisp
OMputBVar(dev: %): Void == OM_-PUTBVAR(dev)$Lisp
OMputError(dev: %): Void == OM_-PUTERROR(dev)$Lisp
OMputObject(dev: %): Void == OM_-PUTOBJECT(dev)$Lisp
OMputEndApp(dev: %): Void == OM_-PUTENDAPP(dev)$Lisp
OMputEndAtp(dev: %): Void == OM_-PUTENDATP(dev)$Lisp
OMputEndAttr(dev: %): Void == OM_-PUTENDATTR(dev)$Lisp
OMputEndBind(dev: %): Void == OM_-PUTENDBIND(dev)$Lisp
OMputEndBVar(dev: %): Void == OM_-PUTENDBVAR(dev)$Lisp
OMputEndError(dev: %): Void == OM_-PUTENDERROR(dev)$Lisp
OMputEndObject(dev: %): Void == OM_-PUTENDOBJECT(dev)$Lisp
OMputInteger(dev: %, i: Integer): Void == OM_-PUTINT(dev, i)$Lisp
OMputFloat(dev: %, f: DoubleFloat): Void == OM_-PUTFLOAT(dev, f)$Lisp
--OMputByteArray(dev: %, b: Array Byte): Void == OM_-PUTBYTEARRAY(dev, b)$Lisp
OMputVariable(dev: %, v: Symbol): Void == OM_-PUTVAR(dev, v)$Lisp
OMputString(dev: %, s: String): Void == OM_-PUTSTRING(dev, s)$Lisp
OMputSymbol(dev: %, cd: String, nm: String): Void == OM_-PUTSYMBOL(dev, cd, nm)$Lisp

OMgetApp(dev: %): Void == OM_-GETAPP(dev)$Lisp
OMgetAtp(dev: %): Void == OM_-GETATP(dev)$Lisp
OMgetAttr(dev: %): Void == OM_-GETATTR(dev)$Lisp
OMgetBind(dev: %): Void == OM_-GETBIND(dev)$Lisp
OMgetBVar(dev: %): Void == OM_-GETBVAR(dev)$Lisp
OMgetError(dev: %): Void == OM_-GETERROR(dev)$Lisp
OMgetObject(dev: %): Void == OM_-GETOBJECT(dev)$Lisp
OMgetEndApp(dev: %): Void == OM_-GETENDAPP(dev)$Lisp
OMgetEndAtp(dev: %): Void == OM_-GETENDATP(dev)$Lisp
OMgetEndAttr(dev: %): Void == OM_-GETENDATTR(dev)$Lisp
OMgetEndBind(dev: %): Void == OM_-GETENDBIND(dev)$Lisp
OMgetEndBVar(dev: %): Void == OM_-GETENDBVAR(dev)$Lisp
OMgetEndError(dev: %): Void == OM_-GETENDERROR(dev)$Lisp
OMgetEndObject(dev: %): Void == OM_-GETENDOBJECT(dev)$Lisp
OMgetInteger(dev: %): Integer == OM_-GETINT(dev)$Lisp
OMgetFloat(dev: %): DoubleFloat == OM_-GETFLOAT(dev)$Lisp
--OMgetByteArray(dev: %): Array Byte == OM_-GETBYTEARRAY(dev)$Lisp

```

```

OMgetVariable(dev: %): Symbol == OM_-GETVAR(dev)$Lisp
OMgetString(dev: %): String == OM_-GETSTRING(dev)$Lisp
OMgetSymbol(dev: %): Record(cd:String, name:String) == OM_-GETSYMBOL(dev)$Lisp

OMgetType(dev: %): Symbol == OM_-GETTYPE(dev)$Lisp

```

---

— OMDEV.dotabb —

```

"OMDEV" [color="#88FF44",href="bookvol10.3.pdf#nameddest=OMDEV"]
"Domain" [color="#88FF44"]
"OMDEV" -> "Domain"

```

---

## 16.7 domain OMENC OpenMathEncoding

— OpenMathEncoding.input —

```

)set break resume
)sys rm -f OpenMathEncoding.output
)spool OpenMathEncoding.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OpenMathEncoding
--R OpenMathEncoding is a domain constructor
--R Abbreviation for OpenMathEncoding is OMENC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for OMENC
--R
--R----- Operations -----
--R ?? : (%,% ) -> Boolean           OMencodingBinary : () -> %
--R OMencodingSGML : () -> %         OMencodingUnknown : () -> %
--R OMencodingXML : () -> %          coerce : % -> OutputForm
--R hash : % -> SingleInteger        latex : % -> String
--R ~=? : (%,% ) -> Boolean
--R
--E 1

)spool
)lisp (bye)

```

---

— OpenMathEncoding.help —

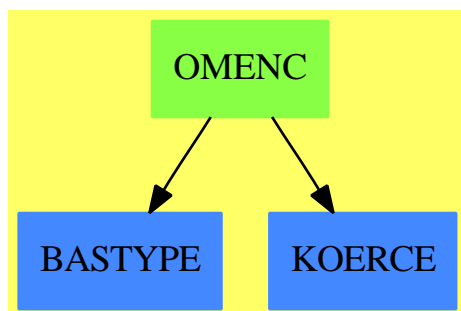
```
=====
OpenMathEncoding examples
=====
```

See Also:

o )show OpenMathEncoding

---

### 16.7.1 OpenMathEncoding (OMENC)



See

⇒ “OpenMathDevice” (OMDEV) 16.6.1 on page 1746

⇒ “OpenMathConnection” (OMCONN) 16.5.1 on page 1743

#### Exports:

|                   |               |       |                  |                |
|-------------------|---------------|-------|------------------|----------------|
| coerce            | hash          | latex | OMencodingBinary | OMencodingSGML |
| OMencodingUnknown | OMencodingXML | ?=?   | ?~=?             |                |

— domain OMENC OpenMathEncoding —

```
)abbrev domain OMENC OpenMathEncoding
```

```
++ Author: Vilya Harvey
```

```
++ Date Created:
```

```
++ Date Last Updated:
```

```
++ Basic Functions:
```

```
++ Related Constructors:
```

```
++ Also See:
```

```
++ AMS Classifications:
```

```
++ Keywords:
```

```
++ References:
```

```

++ Description:
++ \spadtype{OpenMathEncoding} is the set of valid OpenMath encodings.

OpenMathEncoding(): SetCategory with
  OMencodingUnknown : () -> %
  ++ OMencodingUnknown() is the constant for unknown encoding types. If this
  ++ is used on an input device, the encoding will be autodetected.
  ++ It is invalid to use it on an output device.
  OMencodingXML      : () -> %
  ++ OMencodingXML() is the constant for the OpenMath XML encoding.
  OMencodingSGML     : () -> %
  ++ OMencodingSGML() is the constant for the deprecated OpenMath SGML encoding.
  OMencodingBinary   : () -> %
  ++ OMencodingBinary() is the constant for the OpenMath binary encoding.
== add
Rep := SingleInteger

=(u,v) == (u=v)$Rep

import Rep

coerce(u) ==
  u::Rep = 0$Rep => "Unknown"::OutputForm
  u::Rep = 1$Rep => "Binary"::OutputForm
  u::Rep = 2::Rep => "XML"::OutputForm
  u::Rep = 3::Rep => "SGML"::OutputForm
  error "Bogus OpenMath Encoding Type"

OMencodingUnknown(): % == 0::Rep
OMencodingBinary(): % == 1::Rep
OMencodingXML(): % == 2::Rep
OMencodingSGML(): % == 3::Rep

```

---

— OMENC.dotabb —

```

"OMENC" [color="#88FF44",href="bookvol10.3.pdf#nameddest=OMENC"]
"BASTYPE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=BASTYPE"]
"KOERCE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=KOERCE"]
"OMENC" -> "BASTYPE"
"OMENC" -> "KOERCE"

```

---

## 16.8 domain OMERR OpenMathError

### — OpenMathError.input —

```

)set break resume
)sys rm -f OpenMathError.output
)spool OpenMathError.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OpenMathError
--R OpenMathError is a domain constructor
--R Abbreviation for OpenMathError is OMERR
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for OMERR
--R
--R----- Operations -----
--R ?=? : (% ,%) -> Boolean          coerce : % -> OutputForm
--R errorInfo : % -> List Symbol      hash : % -> SingleInteger
--R latex : % -> String              ?~=? : (% ,%) -> Boolean
--R errorKind : % -> OpenMathErrorKind
--R omError : (OpenMathErrorKind,List Symbol) -> %
--R
--E 1

)spool
)lisp (bye)

```

### — OpenMathError.help —

```

=====
OpenMathError examples
=====

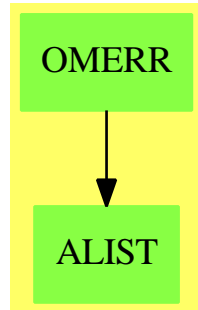
```

```

See Also:
o )show OpenMathError

```

### 16.8.1 OpenMathError (OMERR)



See

⇒ “OpenMathErrorKind” (OMERRK) 16.9.1 on page 1756

#### Exports:

coerce      errorInfo   errorKind   hash   latex  
 omError    ?=?            ?=?

— domain OMERR OpenMathError —

```

)abbrev domain OMERR OpenMathError
++ Author: Vilya Harvey
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ \spadtype{OpenMathError} is the domain of OpenMath errors.

OpenMathError() : SetCategory with
  errorKind : % -> OpenMathErrorKind
  ++ errorKind(u) returns the type of error which u represents.
  errorInfo : % -> List Symbol
  ++ errorInfo(u) returns information about the error u.
  omError    : (OpenMathErrorKind, List Symbol) -> %
  ++ omError(k,l) creates an instance of OpenMathError.
== add
Rep := Record(err:OpenMathErrorKind, info:List Symbol)

import List String

coerce(e:%):OutputForm ==
  OMParseError? e.err => message "Error parsing OpenMath object"
  
```

```

infoSize := #(e.info)
OMUnknownCD? e.err =>
--      not one? infoSize => error "Malformed info list in OMUnknownCD"
      not (infoSize = 1) => error "Malformed info list in OMUnknownCD"
      message concat("Cannot handle CD ",string first e.info)
OMUnknownSymbol? e.err =>
      not 2=infoSize => error "Malformed info list in OMUnknownSymbol"
      message concat ["Cannot handle Symbol ",
                      string e.info.2, " from CD ", string e.info.1]
OMReadError? e.err =>
      message "OpenMath read error"
      error "Malformed OpenMath Error"

omError(e:OpenMathErrorKind,i>List Symbol):% == [e,i]$Rep

errorKind(e:%):OpenMathErrorKind == e.err
errorInfo(e:%):List Symbol == e.info

```

---

— OMERR.dotabb —

```

"OMERR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=OMERR"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"OMERR" -> "ALIST"

```

---

## 16.9 domain OMERRK OpenMathErrorKind

— OpenMathErrorKind.input —

```

)set break resume
)sys rm -f OpenMathErrorKind.output
)spool OpenMathErrorKind.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OpenMathErrorKind
--R OpenMathErrorKind is a domain constructor
--R Abbreviation for OpenMathErrorKind is OMERRK
--R This constructor is exposed in this frame.

```



```

--R Issue )edit bookvol10.3.pamphlet to see algebra source code for OMERRK
--R
--R----- Operations -----
--R ?? : (% , %) -> Boolean          OMParseError? : % -> Boolean
--R OMReadError? : % -> Boolean      OMUnknownCD? : % -> Boolean
--R OMUnknownSymbol? : % -> Boolean  coerce : Symbol -> %
--R coerce : % -> OutputForm         hash : % -> SingleInteger
--R latex : % -> String              ?~=? : (% , %) -> Boolean
--R
--E 1

)spool
)lisp (bye)

```

— OpenMathErrorKind.help —

```

=====
OpenMathErrorKind examples
=====

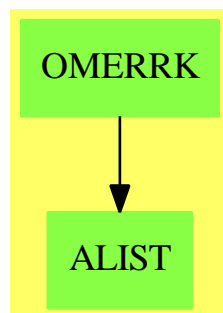
```

```

See Also:
o )show OpenMathErrorKind

```

### 16.9.1 OpenMathErrorKind (OMERRK)



See

⇒ “OpenMathError” (OMERR) 16.8.1 on page 1754

**Exports:**

OMParseError? OMReadError? OMUnknownCD? OMUnknownSymbol? coerce hash

## — domain OMERRK OpenMathErrorKind —

```

)abbrev domain OMERRK OpenMathErrorKind
++ Author: Vilya Harvey
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ \spadtype{OpenMathErrorKind} represents different kinds
++ of OpenMath errors: specifically parse errors, unknown CD or symbol
++ errors, and read errors.

OpenMathErrorKind() : SetCategory with
  coerce          : Symbol -> %
  ++ coerce(u) creates an OpenMath error object of an appropriate type if
  ++ \axiom{u} is one of \axiom{OMParseError}, \axiom{OMReadError},
  ++ \axiom{OMUnknownCD} or \axiom{OMUnknownSymbol}, otherwise it
  ++ raises a runtime error.
  OMParseError?    : % -> Boolean
  ++ OMParseError?(u) tests whether u is an OpenMath parsing error.
  OMUnknownCD?     : % -> Boolean
  ++ OMUnknownCD?(u) tests whether u is an OpenMath unknown CD error.
  OMUnknownSymbol? : % -> Boolean
  ++ OMUnknownSymbol?(u) tests whether u is an OpenMath unknown symbol error.
  OMReadError?     : % -> Boolean
  ++ OMReadError?(u) tests whether u is an OpenMath read error.
== add
Rep := Union(parseError:"OMParseError", unknownCD:"OMUnknownCD",
             unknownSymbol:"OMUnknownSymbol",readError:"OMReadError")

OMParseError?(u) == (u case parseError)$Rep
OMUnknownCD?(u) == (u case unknownCD)$Rep
OMUnknownSymbol?(u) == (u case unknownSymbol)$Rep
OMReadError?(u) == (u case readError)$Rep

coerce(s:Symbol):% ==
  s = OMParseError    => ["OMParseError"]$Rep
  s = OMUnknownCD     => ["OMUnknownCD"]$Rep
  s = OMUnknownSymbol => ["OMUnknownSymbol"]$Rep
  s = OMReadError     => ["OMReadError"]$Rep
  error concat(string s, " is not a valid OpenMathErrorKind.")

a = b == (a=b)$Rep

```

```
coerce(e:%):OutputForm == coerce(e)$Rep
```

---

— OMERRK.dotabb —

```
"OMERRK" [color="#88FF44",href="bookvol10.3.pdf#nameddest=OMERRK"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"OMERRK" -> "ALIST"
```

---

## 16.10 domain OP Operator

— Operator.input —

```
)set break resume
)sys rm -f Operator.output
)spool Operator.output
)set message test on
)set message auto off
)clear all
--S 1 of 21
R := SQMATRIX(2, INT)
--R
--R
--R (1) SquareMatrix(2,Integer)
--R
--R                                          Type: Domain
--E 1

--S 2 of 21
t := operator("tilde") :: OP(R)
--R
--R
--R (2) tilde
--R
--R                                          Type: Operator SquareMatrix(2,Integer)
--E 2

--S 3 of 21
)set expose add constructor Operator
--R
--I Operator is now explicitly exposed in frame frame0
--E 3
```

```

--S 4 of 21
evaluate(t, m +-> transpose m)
--R
--R
--R (3) tilde
--R
--R                                         Type: Operator SquareMatrix(2,Integer)
--E 4

--S 5 of 21
s : R := matrix [ [0, 1], [1, 0] ]
--R
--R
--R      +0  1+
--R (4)  |    |
--R      +1  0+
--R
--R                                         Type: SquareMatrix(2,Integer)
--E 5

--S 6 of 21
rho := t * s
--R
--R
--R      +0  1+
--R (5) tilde|    |
--R      +1  0+
--R
--R                                         Type: Operator SquareMatrix(2,Integer)
--E 6

--S 7 of 21
z := rho**4 - 1
--R
--R
--R      +0  1+      +0  1+      +0  1+      +0  1+
--R (6) - 1 + tilde|    |tilde|    |tilde|    |tilde|    |
--R      +1  0+      +1  0+      +1  0+      +1  0+
--R
--R                                         Type: Operator SquareMatrix(2,Integer)
--E 7

--S 8 of 21
m:R := matrix [ [1, 2], [3, 4] ]
--R
--R
--R      +1  2+
--R (7)  |    |
--R      +3  4+
--R
--R                                         Type: SquareMatrix(2,Integer)
--E 8

--S 9 of 21
z m

```

```

--R
--R
--R      +0  0+
--R  (8)  |   |
--R      +0  0+
--R
--R                                          Type: SquareMatrix(2,Integer)
--E 9

--S 10 of 21
rho m
--R
--R
--R      +3  1+
--R  (9)  |   |
--R      +4  2+
--R
--R                                          Type: SquareMatrix(2,Integer)
--E 10

--S 11 of 21
rho rho m
--R
--R
--R      +4  3+
--R  (10) |   |
--R      +2  1+
--R
--R                                          Type: SquareMatrix(2,Integer)
--E 11

--S 12 of 21
(rho^3) m
--R
--R
--R      +2  4+
--R  (11) |   |
--R      +1  3+
--R
--R                                          Type: SquareMatrix(2,Integer)
--E 12

--S 13 of 21
b := t * s - s * t
--R
--R
--R      +0  1+      +0  1+
--R  (12) - |   |tilde + tilde|   |
--R      +1  0+      +1  0+
--R
--R                                          Type: Operator SquareMatrix(2,Integer)
--E 13

--S 14 of 21
b m

```

```

--R
--R
--R      +1  - 3+
--R  (13) |    |
--R      +3  - 1+
--R
--R                                          Type: SquareMatrix(2,Integer)
--E 14

--S 15 of 21
L n ==
  n = 0 => 1
  n = 1 => x
  (2*n-1)/n * x * L(n-1) - (n-1)/n * L(n-2)
--R
--R                                          Type: Void
--E 15

--S 16 of 21
dx := operator("D") :: OP(POLY FRAC INT)
--R
--R
--R  (15) D
--R
--R                                          Type: Operator Polynomial Fraction Integer
--E 16

--S 17 of 21
evaluate(dx, p +-> D(p, 'x))
--R
--R
--R  (16) D
--R
--R                                          Type: Operator Polynomial Fraction Integer
--E 17

--S 18 of 21
E n == (1 - x**2) * dx**2 - 2 * x * dx + n*(n+1)
--R
--R
--R                                          Type: Void
--E 18

--S 19 of 21
L 15
--R
--R  Compiling function L with type Integer -> Polynomial Fraction
--R      Integer
--R  Compiling function L as a recurrence relation.
--R
--R  (18)
--R      9694845 15 35102025 13 50702925 11 37182145 9 14549535 7
--R      ----- x - ----- x + ----- x - ----- x + ----- x
--R      2048      2048      2048      2048      2048

```

```

--R      +
--R      2909907 5    255255 3    6435
--R      - ----- x + ----- x - ----- x
--R      2048      2048      2048
--R
--R                                          Type: Polynomial Fraction Integer
--E 19

--S 20 of 21
E 15
--R
--R      Compiling function E with type PositiveInteger -> Operator
--R      Polynomial Fraction Integer
--R
--R      2      2
--R      (19) 240 - 2x D - (x - 1)D
--R
--R                                          Type: Operator Polynomial Fraction Integer
--E 20

--S 21 of 21
(E 15)(L 15)
--R
--R
--R      (20) 0
--R
--R                                          Type: Polynomial Fraction Integer
--E 21
)spool
)lisp (bye)

```

---

— Operator.help —

=====

Operator examples

=====

Given any ring R, the ring of the Integer-linear operators over R is called `Operator(R)`. To create an operator over R, first create a basic operator using the operation operator, and then convert it to `Operator(R)` for the R you want.

We choose R to be the two by two matrices over the integers.

```

R := SQMATRIX(2, INT)
SquareMatrix(2,Integer)

```

Type: Domain

Create the operator tilde on R.

```
t := operator("tilde") :: OP(R)
tilde
Type: Operator SquareMatrix(2,Integer)
```

Since `Operator` is unexposed we must either package-call operations from it, or expose it explicitly. For convenience we will do the latter.

Expose `Operator`.

```
)set expose add constructor Operator
```

To attach an evaluation function (from  $R$  to  $R$ ) to an operator over  $R$ , use `evaluate(op, f)` where `op` is an operator over  $R$  and `f` is a function  $R \rightarrow R$ . This needs to be done only once when the operator is defined. Note that `f` must be Integer-linear (that is,  $f(ax+y) = a f(x) + f(y)$  for any integer  $a$ , and any  $x$  and  $y$  in  $R$ ).

We now attach the transpose map to the above operator `t`.

```
evaluate(t, m +-> transpose m)
tilde
Type: Operator SquareMatrix(2,Integer)
```

Operators can be manipulated formally as in any ring: `+` is the pointwise addition and `*` is composition. Any element  $x$  of  $R$  can be converted to an operator `op(x)` over  $R$ , and the evaluation function of `op(x)` is left-multiplication by  $x$ .

Multiplying on the left by this matrix swaps the two rows.

```
s : R := matrix [ [0, 1], [1, 0] ]
+0 1+
|   |
+1 0+
Type: SquareMatrix(2,Integer)
```

Can you guess what is the action of the following operator?

```
rho := t * s
+0 1+
tilde|   |
+1 0+
Type: Operator SquareMatrix(2,Integer)
```

Hint: applying `rho` four times gives the identity, so `rho^4-1` should return 0 when applied to any two by two matrix.

```
z := rho**4 - 1
+0 1+ +0 1+ +0 1+ +0 1+
- 1 + tilde| |tilde| |tilde| |tilde| |
```



```

+1 0+      +1 0+      +1 0+      +1 0+
Type: Operator SquareMatrix(2,Integer)

```

Now check with this matrix.

```

m:R := matrix [ [1, 2], [3, 4] ]
      +1 2+
      |  |
      +3 4+
Type: SquareMatrix(2,Integer)

```

```

z m
      +0 0+
      |  |
      +0 0+
Type: SquareMatrix(2,Integer)

```

As you have probably guessed by now, rho acts on matrices by rotating the elements clockwise.

```

rho m
      +3 1+
      |  |
      +4 2+
Type: SquareMatrix(2,Integer)

```

```

rho rho m
      +4 3+
      |  |
      +2 1+
Type: SquareMatrix(2,Integer)

```

```

(rho^3) m
      +2 4+
      |  |
      +1 3+
Type: SquareMatrix(2,Integer)

```

Do the swapping of rows and transposition commute? We can check by computing their bracket.

```

b := t * s - s * t
      +0 1+      +0 1+
      - |  |tilde + tilde|  |
      +1 0+      +1 0+
Type: Operator SquareMatrix(2,Integer)

```

Now apply it to m.

```

b m

```

$$\begin{array}{cc} +1 & - 3+ \\ | & | \\ +3 & - 1+ \end{array}$$

Type: SquareMatrix(2,Integer)

Next we demonstrate how to define a differential operator on a polynomial ring.

This is the recursive definition of the n-th Legendre polynomial.

```
L n ==
  n = 0 => 1
  n = 1 => x
  (2*n-1)/n * x * L(n-1) - (n-1)/n * L(n-2)
Type: Void
```

Create the differential operator d/dx on polynomials in x over the rational numbers.

```
dx := operator("D") :: OP(POLY FRAC INT)
D
Type: Operator Polynomial Fraction Integer
```

Now attach the map to it.

```
evaluate(dx, p +-> D(p, 'x))
D
Type: Operator Polynomial Fraction Integer
```

This is the differential equation satisfied by the n-th Legendre polynomial.

```
E n == (1 - x**2) * dx**2 - 2 * x * dx + n*(n+1)
Type: Void
```

Now we verify this for n = 15. Here is the polynomial.

```
L 15
  9694845 15 35102025 13 50702925 11 37182145 9 14549535 7
  ----- x - ----- x + ----- x - ----- x + ----- x
    2048      2048      2048      2048      2048
+
  2909907 5 255255 3 6435
  ----- x + ----- x - ----- x
    2048      2048      2048
Type: Polynomial Fraction Integer
```

Here is the operator.

```
E 15
  240 - 2x D - (x - 1)D
Type: Operator Polynomial Fraction Integer
```

Here is the evaluation.

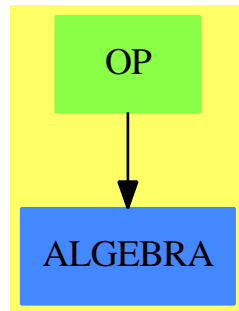
```
(E 15)(L 15)
0
```

Type: Polynomial Fraction Integer

See Also:

o )show Operator

### 16.10.1 Operator (OP)



See

⇒ “ModuleOperator” (MODOP) 14.12.1 on page 1611

**Exports:**

|         |              |          |                 |            |
|---------|--------------|----------|-----------------|------------|
| 0       | 1            | adjoint  | characteristic  | charthRoot |
| coerce  | conjug       | evaluate | evaluateInverse | makeop     |
| hash    | latex        | one?     | opeval          | recip      |
| retract | retractIfCan | sample   | subtractIfCan   | zero?      |
| ?~=?    | ?*?          | ?**?     | ?^?             | ?..?       |
| ?+?     | ?-?          | -?       | ?=?             |            |

— domain OP Operator —

```
)abbrev domain OP Operator
++ Author: Manuel Bronstein
++ Date Created: 15 May 1990
++ Date Last Updated: 12 February 1993
++ Description:
++ Algebra of ADDITIVE operators over a ring.
```

```
Operator(R: Ring) == ModuleOperator(R,R)
```

---

— OP.dotabb —

```
"OP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=OP"]
"ALGEBRA" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ALGEBRA"]
"OP" -> "ALGEBRA"
```

---

## 16.11 domain OMLO OppositeMonogenicLinearOperator

— OppositeMonogenicLinearOperator.input —

```
)set break resume
)sys rm -f OppositeMonogenicLinearOperator.output
)spool OppositeMonogenicLinearOperator.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OppositeMonogenicLinearOperator
--R OppositeMonogenicLinearOperator(P: MonogenicLinearOperator R,R: Ring) is a domain constructor
--R Abbreviation for OppositeMonogenicLinearOperator is OMLO
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for OMLO
--R
--R----- Operations -----
--R ??? : (R,%) -> %          ??? : (%,R) -> %
--R ??? : (%,%) -> %          ??? : (Integer,%) -> %
--R ??? : (PositiveInteger,%) -> %    ??? : (%,PositiveInteger) -> %
--R ??? : (%,%) -> %          ?-? : (%,%) -> %
--R -? : % -> %               ?=? : (%,%) -> Boolean
--R D : % -> % if P has DIFRING      1 : () -> %
--R 0 : () -> %                ?^? : (%,PositiveInteger) -> %
--R coerce : Integer -> %          coerce : % -> OutputForm
--R degree : % -> NonNegativeInteger hash : % -> SingleInteger
--R latex : % -> String            leadingCoefficient : % -> R
--R one? : % -> Boolean            op : P -> %
```

```

--R po : % -> P                                recip : % -> Union(%, "failed")
--R reductum : % -> %                            sample : () -> %
--R zero? : % -> Boolean                        ?~=? : (%, %) -> Boolean
--R ?? : (NonNegativeInteger, %) -> %
--R ***? : (%, NonNegativeInteger) -> %
--R D : (%, NonNegativeInteger) -> % if P has DIFRING
--R ?? : (%, NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R coefficient : (%, NonNegativeInteger) -> R
--R coerce : R -> % if R has COMRING
--R differentiate : % -> % if P has DIFRING
--R differentiate : (%, NonNegativeInteger) -> % if P has DIFRING
--R minimumDegree : % -> NonNegativeInteger
--R monomial : (R, NonNegativeInteger) -> %
--R subtractIfCan : (%, %) -> Union(%, "failed")
--R
--E 1

)spool
)lisp (bye)

```

---

— OppositeMonogenicLinearOperator.help —

```

=====
OppositeMonogenicLinearOperator examples
=====

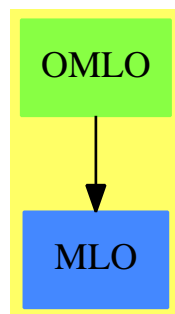
```

```

See Also:
o )show OppositeMonogenicLinearOperator

```

### 16.11.1 OppositeMonogenicLinearOperator (OMLO)



See

⇒ “OrdinaryDifferentialRing” (ODR) 16.18.1 on page 1820

⇒ “DirectProductModule” (DPMO) 5.11.1 on page 542

⇒ “DirectProductMatrixModule” (DPMM) 5.10.1 on page 538

#### Exports:

|                    |               |                |             |               |
|--------------------|---------------|----------------|-------------|---------------|
| 0                  | 1             | characteristic | coefficient | coerce        |
| D                  | degree        | differentiate  | hash        | latex         |
| leadingCoefficient | minimumDegree | monomial       | one?        | op            |
| po                 | recip         | reductum       | sample      | subtractIfCan |
| zero?              | ?^?           | ?~=?           | ?*?         | ?**?          |
| ?+?                | ?-?           | -?             | ?=?         |               |

— domain OMLO OppositeMonogenicLinearOperator —

```
)abbrev domain OMLO OppositeMonogenicLinearOperator
++ Author: Stephen M. Watt
++ Date Created: 1986
++ Date Last Updated: May 30, 1991
++ Basic Operations:
++ Related Domains: MonogenicLinearOperator
++ Also See:
++ AMS Classifications:
++ Keywords: opposite ring
++ Examples:
++ References:
++ Description:
++ This constructor creates the \spadtype{MonogenicLinearOperator} domain
++ which is ‘‘opposite’’ in the ring sense to P.
++ That is, as sets \spad{P = $} but \spad{a * b} in \spad{$} is equal to
++ \spad{b * a} in P.
```

OppositeMonogenicLinearOperator(P, R): OPRcat == OPRdef where

P: MonogenicLinearOperator(R)

R: Ring

OPRcat == MonogenicLinearOperator(R) with

if P has DifferentialRing then DifferentialRing

op: P -> \$ ++ op(p) creates a value in \$ equal to p in P.

po: \$ -> P ++ po(q) creates a value in P equal to q in \$.

OPRdef == P add

Rep := P

x, y: \$

a: P

op a == a: \$

po x == x: P

x\*y == (y:P) \*\$P (x:P)

coerce(x): OutputForm == prefix(op::OutputForm, [coerce(x:P)\$P])

---

— OMLO.dotabb —

```
"OMLO" [color="#88FF44",href="bookvol10.3.pdf#nameddest=OMLO"]
"ML0" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ML0"]
"OMLO" -> "ML0"
```

---

## 16.12 domain ORDCOMP OrderedCompletion

— OrderedCompletion.input —

```
)set break resume
)sys rm -f OrderedCompletion.output
)spool OrderedCompletion.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OrderedCompletion
--R OrderedCompletion R: SetCategory is a domain constructor
--R Abbreviation for OrderedCompletion is ORDCOMP
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ORDCOMP
--R
--R----- Operations -----
--R -? : % -> % if R has ABELGRP          ?? : (%,% ) -> Boolean
--R 1 : () -> % if R has ORDRING          0 : () -> % if R has ABELGRP
--R coerce : R -> %                      coerce : % -> OutputForm
--R finite? : % -> Boolean                hash : % -> SingleInteger
--R infinite? : % -> Boolean              latex : % -> String
--R minusInfinity : () -> %               plusInfinity : () -> %
--R retract : % -> R                     whatInfinity : % -> SingleInteger
--R ?~=? : (%,% ) -> Boolean
--R ?? : (PositiveInteger,% ) -> % if R has ABELGRP
--R ?? : (NonNegativeInteger,% ) -> % if R has ABELGRP
--R ?? : (Integer,% ) -> % if R has ABELGRP
--R ?? : (%,% ) -> % if R has ORDRING
--R ??? : (% ,NonNegativeInteger) -> % if R has ORDRING
--R ??? : (% ,PositiveInteger) -> % if R has ORDRING
```

```

--R ?+? : (%,% ) -> % if R has ABELGRP
--R ?-? : (%,% ) -> % if R has ABELGRP
--R ?<? : (%,% ) -> Boolean if R has ORDRING
--R ?<=? : (%,% ) -> Boolean if R has ORDRING
--R ?>? : (%,% ) -> Boolean if R has ORDRING
--R ?>=? : (%,% ) -> Boolean if R has ORDRING
--R ?^? : (% ,NonNegativeInteger) -> % if R has ORDRING
--R ?^? : (% ,PositiveInteger) -> % if R has ORDRING
--R abs : % -> % if R has ORDRING
--R characteristic : () -> NonNegativeInteger if R has ORDRING
--R coerce : Integer -> % if R has ORDRING or R has RETRACT INT
--R coerce : Fraction Integer -> % if R has RETRACT FRAC INT
--R max : (%,% ) -> % if R has ORDRING
--R min : (%,% ) -> % if R has ORDRING
--R negative? : % -> Boolean if R has ORDRING
--R one? : % -> Boolean if R has ORDRING
--R positive? : % -> Boolean if R has ORDRING
--R rational : % -> Fraction Integer if R has INS
--R rational? : % -> Boolean if R has INS
--R rationalIfCan : % -> Union(Fraction Integer,"failed") if R has INS
--R recip : % -> Union(%,"failed") if R has ORDRING
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retract : % -> Integer if R has RETRACT INT
--R retractIfCan : % -> Union(R,"failed")
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
--R sample : () -> % if R has ABELGRP
--R sign : % -> Integer if R has ORDRING
--R subtractIfCan : (%,% ) -> Union(%,"failed") if R has ABELGRP
--R zero? : % -> Boolean if R has ABELGRP
--R
--E 1

)spool
)lisp (bye)

```

---

— OrderedCompletion.help —

```

=====
OrderedCompletion examples
=====

```

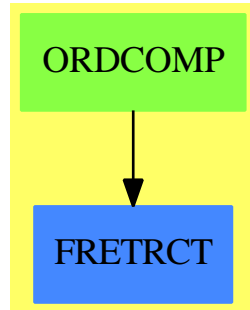
```

See Also:
o )show OrderedCompletion

```



### 16.12.1 OrderedCompletion (ORDCOMP)



See

⇒ “OnePointCompletion” (ONECOMP) 16.4.1 on page 1739

#### Exports:

|              |               |           |                |               |
|--------------|---------------|-----------|----------------|---------------|
| 0            | 1             | abs       | characteristic | coerce        |
| finite?      | hash          | infinite? | latex          | max           |
| min          | minusInfinity | negative? | one?           | plusInfinity  |
| positive?    | rational      | rational? | rationalIfCan  | recip         |
| retract      | retractIfCan  | sample    | sign           | subtractIfCan |
| whatInfinity | zero?         | ?~=?      | ?*?            | ?**?          |
| ?+?          | ?-?           | ?<?       | ?<=?           | ?>?           |
| ?>=?         | ?^?           | -?        | ?=?            |               |

— domain ORDCOMP OrderedCompletion —

```

)abbrev domain ORDCOMP OrderedCompletion
++ Author: Manuel Bronstein
++ Date Created: 4 Oct 1989
++ Date Last Updated: 1 Nov 1989
++ Description:
++ Completion with + and - infinity.
++ Adjunction of two real infinities quantities to a set.

OrderedCompletion(R:SetCategory): Exports == Implementation where
  B ==> Boolean

Exports ==> Join(SetCategory, FullyRetractableTo R) with
  plusInfinity : () -> %      ++ plusInfinity() returns +infinity.
  minusInfinity: () -> %      ++ minusInfinity() returns -infinity.
  finite?      : % -> B
    ++ finite?(x) tests if x is finite.
  infinite?    : % -> B
    ++ infinite?(x) tests if x is +infinity or -infinity,
  whatInfinity : % -> SingleInteger
    ++ whatInfinity(x) returns 0 if x is finite,
    ++ 1 if x is +infinity, and -1 if x is -infinity.

```

```

if R has AbelianGroup then AbelianGroup
if R has OrderedRing then OrderedRing
if R has IntegerNumberSystem then
  rational?: % -> Boolean
  ++ rational?(x) tests if x is a finite rational number.
  rational : % -> Fraction Integer
  ++ rational(x) returns x as a finite rational number.
  ++ Error: if x cannot be so converted.
  rationalIfCan: % -> Union(Fraction Integer, "failed")
  ++ rationalIfCan(x) returns x as a finite rational number if
  ++ it is one and "failed" otherwise.

Implementation ==> add
Rep := Union(fin:R, inf:B) -- true = +infinity, false = -infinity

coerce(r:R):%      == [r]
retract(x:%):R     == (x case fin => x.fin; error "Not finite")
finite? x          == x case fin
infinite? x        == x case inf
plusInfinity()     == [true]
minusInfinity()    == [false]

retractIfCan(x:%):Union(R, "failed") ==
  x case fin => x.fin
  "failed"

coerce(x:%):OutputForm ==
  x case fin => (x.fin)::OutputForm
  e := "infinity"::OutputForm
  x.inf => empty() + e
  - e

whatInfinity x ==
  x case fin => 0
  x.inf => 1
  -1

x = y ==
  x case inf =>
    y case inf => not xor(x.inf, y.inf)
    false
  y case inf => false
  x.fin = y.fin

if R has AbelianGroup then
  0 == [0$R]

n:Integer * x:% ==
  x case inf =>
    n > 0 => x

```

```

        n < 0 => [not(x.inf)]
        error "Undefined product"
    [n * x.fin]

- x ==
  x case inf => [not(x.inf)]
  [- (x.fin)]

x + y ==
  x case inf =>
    y case fin => x
    xor(x.inf, y.inf) => error "Undefined sum"
    x
  y case inf => y
  [x.fin + y.fin]

if R has OrderedRing then
  fininf: (B, R) -> %

  1 == [1$R]
  characteristic() == characteristic()$R

  fininf(b, r) ==
    r > 0 => [b]
    r < 0 => [not b]
    error "Undefined product"

x:% * y:% ==
  x case inf =>
    y case inf =>
      xor(x.inf, y.inf) => minusInfinity()
      plusInfinity()
      fininf(x.inf, y.fin)
    y case inf => fininf(y.inf, x.fin)
  [x.fin * y.fin]

recip x ==
  x case inf => 0
  (u := recip(x.fin)) case "failed" => "failed"
  [u:$R]

x < y ==
  x case inf =>
    y case inf =>
      xor(x.inf, y.inf) => y.inf
      false
    not(x.inf)
  y case inf => y.inf
  x.fin < y.fin

```

```

if R has IntegerNumberSystem then
  rational? x == finite? x
  rational x == rational(retract(x)@R)

rationalIfCan x ==
  (r:= retractIfCan(x)@Union(R,"failed")) case "failed" =>"failed"
  rational(r::R)

```

---

— ORDCOMP.dotabb —

```

"ORDCOMP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ORDCOMP"]
"FRETRCT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FRETRCT"]
"ORDCOMP" -> "FRETRCT"

```

---

## 16.13 domain ODP OrderedDirectProduct

— OrderedDirectProduct.input —

```

)set break resume
)sys rm -f OrderedDirectProduct.output
)spool OrderedDirectProduct.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OrderedDirectProduct
--R OrderedDirectProduct(dim: NonNegativeInteger,S: OrderedAbelianMonoidSup,f: ((Vector S,Vector S) -> B
--R Abbreviation for OrderedDirectProduct is ODP
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ODP
--R
--R----- Operations -----
--R -? : % -> % if S has RING          1 : () -> % if S has MONOID
--R 0 : () -> % if S has CABMON        coerce : % -> Vector S
--R copy : % -> %                      directProduct : Vector S -> %
--R ?.? : (%,Integer) -> S              elt : (%,Integer,S) -> S
--R empty : () -> %                    empty? : % -> Boolean
--R entries : % -> List S               eq? : (%,% ) -> Boolean
--R index? : (Integer,%) -> Boolean      indices : % -> List Integer

```

```

--R map : ((S -> S),%) -> %                                qelt : (%,Integer) -> S
--R sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?? : (PositiveInteger,%) -> % if S has ABELSG
--R ?? : (NonNegativeInteger,%) -> % if S has CABMON
--R ?? : (S,%) -> % if S has RING
--R ?? : (%,S) -> % if S has RING
--R ?? : (%,%) -> % if S has MONOID
--R ?? : (Integer,%) -> % if S has RING
--R ??? : (%,PositiveInteger) -> % if S has MONOID
--R ??? : (%,NonNegativeInteger) -> % if S has MONOID
--R +? : (%,%) -> % if S has ABELSG
--R -? : (%,%) -> % if S has RING
--R ?/? : (%,S) -> % if S has FIELD
--R ?<? : (%,%) -> Boolean if S has OAMONS or S has ORDRING
--R ?<=? : (%,%) -> Boolean if S has OAMONS or S has ORDRING
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R ?>? : (%,%) -> Boolean if S has OAMONS or S has ORDRING
--R ?>=? : (%,%) -> Boolean if S has OAMONS or S has ORDRING
--R D : (%,(S -> S)) -> % if S has RING
--R D : (%,(S -> S),NonNegativeInteger) -> % if S has RING
--R D : (%,List Symbol,List NonNegativeInteger) -> % if S has PDRING SYMBOL and S has RING
--R D : (%,Symbol,NonNegativeInteger) -> % if S has PDRING SYMBOL and S has RING
--R D : (%,List Symbol) -> % if S has PDRING SYMBOL and S has RING
--R D : (%,Symbol) -> % if S has PDRING SYMBOL and S has RING
--R D : (%,NonNegativeInteger) -> % if S has DIFRING and S has RING
--R D : % -> % if S has DIFRING and S has RING
--R ?? : (%,PositiveInteger) -> % if S has MONOID
--R ?? : (%,NonNegativeInteger) -> % if S has MONOID
--R abs : % -> % if S has ORDRING
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R characteristic : () -> NonNegativeInteger if S has RING
--R coerce : S -> % if S has SETCAT
--R coerce : Fraction Integer -> % if S has RETRACT FRAC INT and S has SETCAT
--R coerce : Integer -> % if S has RETRACT INT and S has SETCAT or S has RING
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if $ has finiteAggregate and S has SETCAT
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R differentiate : (%,(S -> S)) -> % if S has RING
--R differentiate : (%,(S -> S),NonNegativeInteger) -> % if S has RING
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if S has PDRING SYMBOL and S has RING
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if S has PDRING SYMBOL and S has RING
--R differentiate : (%,List Symbol) -> % if S has PDRING SYMBOL and S has RING
--R differentiate : (%,Symbol) -> % if S has PDRING SYMBOL and S has RING
--R differentiate : (%,NonNegativeInteger) -> % if S has DIFRING and S has RING
--R differentiate : % -> % if S has DIFRING and S has RING
--R dimension : () -> CardinalNumber if S has FIELD
--R dot : (%,%) -> S if S has RING
--R entry? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT

```

```

--R eval : (% , S , S) -> % if S has EVALAB S and S has SETCAT
--R eval : (% , Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (% , List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean) , %) -> Boolean if $ has finiteAggregate
--R fill! : (% , S) -> % if $ has shallowlyMutable
--R first : % -> S if Integer has ORDSET
--R hash : % -> SingleInteger if S has SETCAT
--R index : PositiveInteger -> % if S has FINITE
--R latex : % -> String if S has SETCAT
--R less? : (% , NonNegativeInteger) -> Boolean
--R lookup : % -> PositiveInteger if S has FINITE
--R map! : ((S -> S) , %) -> % if $ has shallowlyMutable
--R max : (% , %) -> % if S has OAMONS or S has ORDRING
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (S , %) -> Boolean if $ has finiteAggregate and S has SETCAT
--R members : % -> List S if $ has finiteAggregate
--R min : (% , %) -> % if S has OAMONS or S has ORDRING
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (% , NonNegativeInteger) -> Boolean
--R negative? : % -> Boolean if S has ORDRING
--R one? : % -> Boolean if S has MONOID
--R parts : % -> List S if $ has finiteAggregate
--R positive? : % -> Boolean if S has ORDRING
--R qsetelt! : (% , Integer , S) -> S if $ has shallowlyMutable
--R random : () -> % if S has FINITE
--R recip : % -> Union(% , "failed") if S has MONOID
--R reducedSystem : Matrix % -> Matrix S if S has RING
--R reducedSystem : (Matrix % , Vector %) -> Record(mat: Matrix S , vec: Vector S) if S has RING
--R reducedSystem : (Matrix % , Vector %) -> Record(mat: Matrix Integer , vec: Vector Integer) if S has LINE
--R reducedSystem : Matrix % -> Matrix Integer if S has LINEXP INT and S has RING
--R retract : % -> S if S has SETCAT
--R retract : % -> Fraction Integer if S has RETRACT FRAC INT and S has SETCAT
--R retract : % -> Integer if S has RETRACT INT and S has SETCAT
--R retractIfCan : % -> Union(S , "failed") if S has SETCAT
--R retractIfCan : % -> Union(Fraction Integer , "failed") if S has RETRACT FRAC INT and S has SETCAT
--R retractIfCan : % -> Union(Integer , "failed") if S has RETRACT INT and S has SETCAT
--R setelt : (% , Integer , S) -> S if $ has shallowlyMutable
--R sign : % -> Integer if S has ORDRING
--R size : () -> NonNegativeInteger if S has FINITE
--R size? : (% , NonNegativeInteger) -> Boolean
--R subtractIfCan : (% , %) -> Union(% , "failed") if S has CABMON
--R sup : (% , %) -> % if S has OAMONS
--R swap! : (% , Integer , Integer) -> Void if $ has shallowlyMutable
--R unitVector : PositiveInteger -> % if S has RING
--R zero? : % -> Boolean if S has CABMON
--R ?~=? : (% , %) -> Boolean if S has SETCAT
--R
--E 1

```

)spool

```
)lisp (bye)
```

```
_____
```

```
— OrderedDirectProduct.help —
```

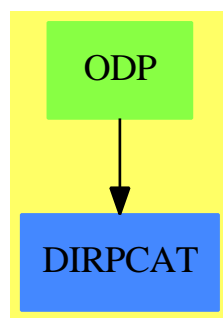
```
=====
OrderedDirectProduct examples
=====
```

See Also:

```
o )show OrderedDirectProduct
```

```
_____
```

### 16.13.1 OrderedDirectProduct (ODP)



See

⇒ “HomogeneousDirectProduct” (HDP) 9.9.1 on page 1138

⇒ “SplitHomogeneousDirectProduct” (SHDP) 20.23.1 on page 2467

**Exports:**

|               |               |               |            |                |
|---------------|---------------|---------------|------------|----------------|
| 0             | 1             | abs           | any?       | characteristic |
| coerce        | copy          | count         | D          | differentiate  |
| dimension     | directProduct | dot           | elt        | empty          |
| empty?        | entries       | entry?        | eq?        | eval           |
| every?        | fill!         | first         | hash       | index          |
| index?        | indices       | latex         | less?      | lookup         |
| map           | map!          | max           | maxIndex   | member?        |
| members       | min           | minIndex      | more?      | negative?      |
| one?          | parts         | positive?     | qelt       | qsetelt!       |
| random        | recip         | reducedSystem | retract    | retractIfCan   |
| sample        | setelt        | sign          | size       | size?          |
| subtractIfCan | sup           | swap!         | unitVector | zero?          |
| #?            | ?*?           | ?**?          | ?+?        | ?-?            |
| ?/?           | ?<?           | ?<=?          | ?=?        | ?>?            |
| ?>=?          | ?^?           | ?~=?          | -?         | ?..?           |

— domain ODP OrderedDirectProduct —

```
)abbrev domain ODP OrderedDirectProduct
-- all direct product category domains must be compiled
-- without subsumption, set SourceLevelSubset to EQUAL
-->bo $noSubsumption := true

++ Author: Mark Botch
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors: Vector, DirectProduct
++ Also See: HomogeneousDirectProduct, SplitHomogeneousDirectProduct
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This type represents the finite direct or cartesian product of an
++ underlying ordered component type. The ordering on the type is determined
++ by its third argument which represents the less than function on
++ vectors. This type is a suitable third argument for
++ \spadtype{GeneralDistributedMultivariatePolynomial}.

OrderedDirectProduct(dim:NonNegativeInteger,
                     S:OrderedAbelianMonoidSup,
                     f:(Vector(S),Vector(S))->Boolean):T
    == C where
T == DirectProductCategory(dim,S)
C == DirectProduct(dim,S) add
Rep:=Vector(S)
x:% < y:% == f(x::Rep,y::Rep)
```



---

— ODP.dotabb —

```
"ODP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ODP"]
"DIRPCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=DIRPCAT"]
"ODP" -> "DIRPCAT"
```

---

## 16.14 domain OFMONOID OrderedFreeMonoid

— OrderedFreeMonoid.input —

```
)set break resume
)sys rm -f OrderedFreeMonoid.output
)spool OrderedFreeMonoid.output
)set message test on
)set message auto off
)clear all

--S 1 of 24
m1:=(x*y*y*z)$OFMONOID(Symbol)
--R
--R
--R      2
--R   (1)  x y z
--R
--R                                          Type: OrderedFreeMonoid Symbol
--E 1

--S 2 of 24
m2:=(x*y)$OFMONOID(Symbol)
--R
--R
--R   (2)  x y
--R
--R                                          Type: OrderedFreeMonoid Symbol
--E 2

--S 3 of 24
lquo(m1,m2)
--R
--R
--R   (3)  y z
--R
--R                                          Type: Union(OrderedFreeMonoid Symbol,...)
--E 3
```

```

--S 4 of 24
m3:=(y*y)$OFMONOID(Symbol)
--R
--R
--R      2
--R   (4)  y
--R
--R                                          Type: OrderedFreeMonoid Symbol
--E 4

--S 5 of 24
divide(m1,m2)
--R
--R
--R   (5)  [lm= y z,rm= "failed"]
--RType: Union(Record(lm: Union(OrderedFreeMonoid Symbol,"failed"),rm: Union(OrderedFreeMonoid Symbol,"f
--E 5

--S 6 of 24
divide(m1,m3)
--R
--R
--R   (6)  [lm= "failed",rm= "failed"]
--RType: Union(Record(lm: Union(OrderedFreeMonoid Symbol,"failed"),rm: Union(OrderedFreeMonoid Symbol,"f
--E 6

--S 7 of 24
m4:=(y^3)$OFMONOID(Symbol)
--R
--R
--R      3
--R   (7)  y
--R
--R                                          Type: OrderedFreeMonoid Symbol
--E 7

--S 8 of 24
divide(m1,m4)
--R
--R
--R   (8)  [lm= "failed",rm= "failed"]
--RType: Union(Record(lm: Union(OrderedFreeMonoid Symbol,"failed"),rm: Union(OrderedFreeMonoid Symbol,"f
--E 8

)set function compile on

-- Build the non-commutative algebra h=k[x,y] and then make computations
-- in h using some predefined rules for x and y. For example, giving
--   x*y*x=y*x*y
--   x*x=a*x+b

```



```

-- Substitution rules are applied to words from the monoid over the
-- variables and return polynomials

--S 14 of 24
subs(w:M):H ==
  -- x*y*x = y*x*x
  n1:=lquo(w,(x::V*y::V*x::V)$M)$M
  n1 case "failed" => monom(w,1)$H
  -- x*x = a*x+b
  n2:=lquo(w,(x::V^2)$M)$M
  n2 case "failed" => monom(w,1)$H
  -- y*y = a*y+b
  n3:=lquo(w,(y::V^2)$M)$M
  n3 case "failed" => monom(w,1)$H
  monom(n3,1)$H * (a::K*y::V+b::K)$M * monom(n3,1)$H
  monom(n2,1)$H * (a::K*x::V+b::K)$H * monom(n2,1)$H
  monom(n1,1)$H * (y::V*x::V*y::V)$H * monom(n1,1)$H

--R
--R Function declaration subs : OrderedFreeMonoid OrderedVariableList [x
--R ,y] -> XDistributedPolynomial(OrderedVariableList [x,y],
--R SparseMultivariatePolynomial(Fraction Integer,OrderedVariableList
--R [a,b])) has been added to workspace.
--R
--R Type: Void
--E 14

-- Apply rules to a term. Keep coefficients
--S 15 of 24
newterm(x:Record(k:M,c:K)):H == x.c*subs(x,k)
--R
--R Function declaration newterm : Record(k: OrderedFreeMonoid
--R OrderedVariableList [x,y],c: SparseMultivariatePolynomial(
--R Fraction Integer,OrderedVariableList [a,b])) ->
--R XDistributedPolynomial(OrderedVariableList [x,y],
--R SparseMultivariatePolynomial(Fraction Integer,OrderedVariableList
--R [a,b])) has been added to workspace.
--R
--R Type: Void
--E 15

-- Reconstruct the polynomial term by term
--S 16 of 24
newpoly(t:H):H == reduce(+,map(newterm,listOfTerms(t)))
--R
--R Function declaration newpoly : XDistributedPolynomial(
--R OrderedVariableList [x,y],SparseMultivariatePolynomial(Fraction
--R Integer,OrderedVariableList [a,b])) -> XDistributedPolynomial(
--R OrderedVariableList [x,y],SparseMultivariatePolynomial(Fraction
--R Integer,OrderedVariableList [a,b])) has been added to workspace.
--R
--R Type: Void

```

--E 16

-- Example calcuations

--S 17 of 24

p1:(x::V+y::V)\$H^2

--R

--R

--RDaly Bug

--R Category, domain or package constructor ^ is not available.

--E 17

--S 18 of 24

newpoly(p1)

--R

--R Compiling function newpoly with type XDistributedPolynomial(

--R OrderedVariableList [x,y],SparseMultivariatePolynomial(Fraction

--R Integer,OrderedVariableList [a,b])) -> XDistributedPolynomial(

--R OrderedVariableList [x,y],SparseMultivariatePolynomial(Fraction

--R Integer,OrderedVariableList [a,b]))

--R There are no library operations named subs

--R Use HyperDoc Browse or issue

--R )what op subs

--R to learn if there is any operation containing " subs " in its

--R name.

--R Cannot find a definition or applicable library operation named subs

--R with argument type(s)

--RRecord(k: OrderedFreeMonoid OrderedVariableList [x,y],c: SparseMultivariatePolynomial(Fra

--R Variable k

--R

--R Perhaps you should use "@" to indicate the required return type,

--R or "\$" to specify which version of the function you need.

--R AXIOM will attempt to step through and interpret the code.

--R Compiling function newterm with type Record(k: OrderedFreeMonoid

--R OrderedVariableList [x,y],c: SparseMultivariatePolynomial(

--R Fraction Integer,OrderedVariableList [a,b])) ->

--R XDistributedPolynomial(OrderedVariableList [x,y],

--R SparseMultivariatePolynomial(Fraction Integer,OrderedVariableList

--R [a,b]))

--R There are no library operations named subs

--R Use HyperDoc Browse or issue

--R )what op subs

--R to learn if there is any operation containing " subs " in its

--R name.

--R

--RDaly Bug

--R Cannot find a definition or applicable library operation named subs

--R with argument type(s)

--RRecord(k: OrderedFreeMonoid OrderedVariableList [x,y],c: SparseMultivariatePolynomial(Fra

--R Variable k

```

--R
--R      Perhaps you should use "@" to indicate the required return type,
--R      or "$" to specify which version of the function you need.
--E 18

--S 19 of 24
p2:=(x::V+y::V)$H^3
--R
--R
--R      3      2      2      2      2      3
--R      (17)  y  + y x + y x y + y x  + x y  + x y x + x y + x
--RType: XDistributedPolynomial(OrderedVariableList [x,y],SparseMultivariatePolynomial(Fraction Integer,
--E 19

--S 20 of 24
pNew:=newpoly(p2)
--R
--R      There are no library operations named subs
--R      Use HyperDoc Browse or issue
--R      )what op subs
--R      to learn if there is any operation containing " subs " in its
--R      name.
--R      Cannot find a definition or applicable library operation named subs
--R      with argument type(s)
--RRecord(k: OrderedFreeMonoid OrderedVariableList [x,y],c: SparseMultivariatePolynomial(Fraction Integer,
--R      Variable k
--R
--R      Perhaps you should use "@" to indicate the required return type,
--R      or "$" to specify which version of the function you need.
--R      AXIOM will attempt to step through and interpret the code.
--R      Compiling function newterm with type Record(k: OrderedFreeMonoid
--R      OrderedVariableList [x,y],c: SparseMultivariatePolynomial(
--R      Fraction Integer,OrderedVariableList [a,b])) ->
--R      XDistributedPolynomial(OrderedVariableList [x,y],
--R      SparseMultivariatePolynomial(Fraction Integer,OrderedVariableList
--R      [a,b]))
--R      There are no library operations named subs
--R      Use HyperDoc Browse or issue
--R      )what op subs
--R      to learn if there is any operation containing " subs " in its
--R      name.
--R
--RDaly Bug
--R      Cannot find a definition or applicable library operation named subs
--R      with argument type(s)
--RRecord(k: OrderedFreeMonoid OrderedVariableList [x,y],c: SparseMultivariatePolynomial(Fraction Integer,
--R      Variable k
--R
--R      Perhaps you should use "@" to indicate the required return type,
--R      or "$" to specify which version of the function you need.

```

--E 20

-- But the rules should be applied more than once

--S 21 of 24

while pNew ~= p2 repeat

  p2 := pNew

  pNew := newpoly(p2)

--R

--R There are no library operations named subs

--R Use HyperDoc Browse or issue

--R                                    )what op subs

--R to learn if there is any operation containing " subs " in its

--R name.

--R Cannot find a definition or applicable library operation named subs

--R with argument type(s)

--RRecord(k: OrderedFreeMonoid OrderedVariableList [x,y],c: SparseMultivariatePolynomial(Fra

--R                                    Variable k

--R

--R Perhaps you should use "@" to indicate the required return type,

--R or "\$" to specify which version of the function you need.

--R AXIOM will attempt to step through and interpret the code.

--R Compiling function newterm with type Record(k: OrderedFreeMonoid

--R   OrderedVariableList [x,y],c: SparseMultivariatePolynomial(

--R   Fraction Integer,OrderedVariableList [a,b])) ->

--R   XDistributedPolynomial(OrderedVariableList [x,y],

--R   SparseMultivariatePolynomial(Fraction Integer,OrderedVariableList

--R   [a,b]))

--R There are no library operations named subs

--R Use HyperDoc Browse or issue

--R                                    )what op subs

--R to learn if there is any operation containing " subs " in its

--R name.

--R

--RDaly Bug

--R Cannot find a definition or applicable library operation named subs

--R with argument type(s)

--RRecord(k: OrderedFreeMonoid OrderedVariableList [x,y],c: SparseMultivariatePolynomial(Fra

--R                                    Variable k

--R

--R Perhaps you should use "@" to indicate the required return type,

--R or "\$" to specify which version of the function you need.

--E 21

--S 22 of 24

pNew

--R

--R

--R (18) pNew

--R

Type: Variable pNew

--E 22

```

--S 23 of 24
reduce(p:H):H ==
  p2 := newpoly(p)
  p3 := newpoly(p2)
  while p3 ~= p2 repeat
    p2 := p3
    p3 := newpoly(p2)
  p3
--R
--R   Function declaration reduce : XDistributedPolynomial(
--R     OrderedVariableList [x,y],SparseMultivariatePolynomial(Fraction
--R     Integer,OrderedVariableList [a,b])) -> XDistributedPolynomial(
--R     OrderedVariableList [x,y],SparseMultivariatePolynomial(Fraction
--R     Integer,OrderedVariableList [a,b])) has been added to workspace.
--R   Compiled code for newpoly has been cleared.
--R
--R                                          Type: Void
--E 23

--S 24 of 24
reduce(p2)
--R
--R   Compiling function newpoly with type XDistributedPolynomial(
--R     OrderedVariableList [x,y],SparseMultivariatePolynomial(Fraction
--R     Integer,OrderedVariableList [a,b])) -> XDistributedPolynomial(
--R     OrderedVariableList [x,y],SparseMultivariatePolynomial(Fraction
--R     Integer,OrderedVariableList [a,b]))
--R   Compiling function reduce with type XDistributedPolynomial(
--R     OrderedVariableList [x,y],SparseMultivariatePolynomial(Fraction
--R     Integer,OrderedVariableList [a,b])) -> XDistributedPolynomial(
--R     OrderedVariableList [x,y],SparseMultivariatePolynomial(Fraction
--R     Integer,OrderedVariableList [a,b]))
--R   There are no library operations named subs
--R   Use HyperDoc Browse or issue
--R
--R                                     )what op subs
--R   to learn if there is any operation containing " subs " in its
--R   name.
--R   Cannot find a definition or applicable library operation named subs
--R   with argument type(s)
--RRecord(k: OrderedFreeMonoid OrderedVariableList [x,y],c: SparseMultivariatePolynomial(Fraction Integer
--R                                     Variable k
--R
--R   Perhaps you should use "@" to indicate the required return type,
--R   or "$" to specify which version of the function you need.
--R   AXIOM will attempt to step through and interpret the code.
--R   Compiling function newterm with type Record(k: OrderedFreeMonoid
--R     OrderedVariableList [x,y],c: SparseMultivariatePolynomial(
--R     Fraction Integer,OrderedVariableList [a,b])) ->
--R     XDistributedPolynomial(OrderedVariableList [x,y],
--R     SparseMultivariatePolynomial(Fraction Integer,OrderedVariableList

```



```

--R      [a,b]))
--R      There are no library operations named subs
--R      Use HyperDoc Browse or issue
--R      )what op subs
--R      to learn if there is any operation containing " subs " in its
--R      name.
--R
--RDaly Bug
--R      Cannot find a definition or applicable library operation named subs
--R      with argument type(s)
--RRecord(k: OrderedFreeMonoid OrderedVariableList [x,y],c: SparseMultivariatePolynomial(Fra
--R      Variable k
--R
--R      Perhaps you should use "@" to indicate the required return type,
--R      or "$" to specify which version of the function you need.
--E 24

```

```

)spool
)lisp (bye)

```

---

— OrderedFreeMonoid.help —

```

=====
OrderedFreeMonoid examples
=====

```

```
m1:=(x*y*y*z)$OFMONOID(Symbol)
```

```
m2:=(x*y)$OFMONOID(Symbol)
```

```
lquo(m1,m2)
```

```
m3:=(y*y)$OFMONOID(Symbol)
```

```
div(m1,m2)
```

```
div(m1,m3)
```

```
m4:=(y^3)$OFMONOID(Symbol)
```

```
div(m1,m4)
```

Build the non-commutative algebra  $h=k[x,y]$  and then make computations in  $h$  using some predefined rules for  $x$  and  $y$ . For example, giving

```

x*y*x=y*x*y
x*x=a*x+b

```

```
y*y=a*y+b
```

where  $a$  and  $b$  are generic elements of  $k$ .

Then reduce the polynomials in  $x$  and  $y$  according to the previous rules. That is, given

```
(x+y)^2  ( = x^2+x*y+y*x+y^2)
```

should reduce to

```
a*(x+y)+2*b*x*y+y*x
```

We can reduce the clutter of the work by defining macros for the types of the domains we need to create. So first we create those macros.

We create generic elements of  $k$ . First we define a macro for the domain of ordered variables (OVAR is OrderedVariableList)

```
C ==> OVAR [a,b]
```

Next we define a macro for the commutative field domain  $k = Q[a,b]$  where  $Q$  is Fraction(Integer) and SMP is SparseMultivariatePolynomials

```
K ==> SMP(FRAC INT,C)
```

Now we need some non-commutative variables so we create a macro for that.

```
V ==> OVAR [x,y]
```

And now we need to define the non-commutative algebra  $k=k[x,y]$ . We use the domain XDistributedPolynomial (XDPOLY) as a macro.

```
H ==> XDPOLY(V,K)
```

The non-commutative variables are in an Ordered Free Monoid

```
M ==> OFMONOID V
```

We have three rules to apply. So we create a function that examines one term. Substitution rules are applied to words from the monoid over the variables and return polynomials. If any rule matches we construct the substitution, create a new monomial term and return it.

```
subs(w:M):H ==
-- x*y*x = y*x*y
n1:=lquo(w,(x::V*y::V*x::V)$M)$M
n1 case "failed" => monom(w,1)$H
-- x*x = a*x+b
n2:=lquo(w,(x::V^2)$M)$M
```

```

n2 case "failed" => monom(w,1)$H
-- y*y = a*y+b
n3:lquo(w,(y::V^2)$M)$M
n3 case "failed" => monom(w,1)$H
monom(n3,1)$H * (a::K*y::V+b::K)$M * monom(n3,1)$H
monom(n2,1)$H * (a::K*x::V+b::K)$H * monom(n2,1)$H
monom(n1,1)$H * (y::V*x::V*y::V)$H * monom(n1,1)$H

```

We apply these rules to a term, remembering the coefficient

```
newterm(x:Record(k:M,c:K)):H == x.c*subs(x,k)
```

And now we create a function to reconstruct the polynomial term by term.

```
newpoly(t:H):H == reduce(+,map(newterm,listOfTerms(t)))
```

For example,

```

p1:(x::V+y::V)$H^2

newpoly(p1)

p2:=(x::V+y::V)$H^3

pNew:=newpoly(p2)

```

But the rules should be applied more than once so we create a function to iterate the rules until nothing changes.

```

reduce(p:H):H ==
  p2 := newpoly(p)
  p3 := newpoly(p2)
  while p3 ~= p2 repeat
    p2 := p3
    p3 := newpoly(p2)
  p3

```

And we can see it work

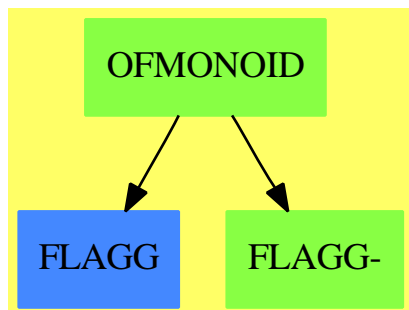
```
reduce(p2)
```

See Also:

```
o )show OrderedFreeMonoid
```

---

## 16.14.1 OrderedFreeMonoid (OFMONOID)

**Exports:**

|           |              |         |        |          |
|-----------|--------------|---------|--------|----------|
| 1         | coerce       | factors | first  | hash     |
| helf      | hcrf         | latex   | length | lexico   |
| lquo      | max          | min     | mirror | nthExpon |
| nthFactor | one?         | overlap | recip  | rest     |
| retract   | retractIfCan | rquo    | sample | size     |
| varList   | ?*?          | ?**?    | ?<?    | ?<=?     |
| ?=?       | ?>?          | ?>=?    | ?^?    | ?~=?     |
| ?div?     |              |         |        |          |

— domain OFMONOID OrderedFreeMonoid —

```

)abbrev domain OFMONOID OrderedFreeMonoid
++ Author: Michel Petitot petitot@lifl.fr
++ Date Created: 91
++ Date Last Updated: 7 Juillet 92
++ Fix History: compilation v 2.1 le 13 dec 98
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The free monoid on a set \spad{S} is the monoid of finite products of
++ the form \spad{reduce(*,[si ** ni])} where the si's are in S, and the ni's
++ are non-negative integers. The multiplication is not commutative.
++ For two elements \spad{x} and \spad{y} the relation \spad{x < y}
++ holds if either \spad{length(x) < length(y)} holds or if these lengths
++ are equal and if \spad{x} is smaller than \spad{y} w.r.t. the
++ lexicographical ordering induced by \spad{S}.
++ This domain inherits implementation from \spadtype{FreeMonoid}.

```

```

OrderedFreeMonoid(S: OrderedSet): OFMcategory == OFMdefinition where
  NNI ==> NonNegativeInteger

```

```

REC ==> Record(gen:S, exp:NNI)
RESULT ==> Union(Record(lm:Union(%, "failed"), rm:Union(%, "failed")), "failed")

OFMcategory == Join(OrderedMonoid, RetractableTo S) with
"*": (S, %) -> %
  ++ \spad{s*x} returns the product of \spad{x} by \spad{s} on the left.
  ++
  ++X m1:=(x*y*y*z)$OFMONOID(Symbol)
  ++X x*m1
"*": (% , S) -> %
  ++ \spad{x*s} returns the product of \spad{x} by \spad{s} on the right.
  ++
  ++X m1:=(y**3)$OFMONOID(Symbol)
  ++X m1*x
"**": (S, NNI) -> %
  ++ \spad{s**n} returns the product of \spad{s} by itself \spad{n} times.
  ++
  ++X m1:=(y**3)$OFMONOID(Symbol)
first: % -> S
  ++ \spad{first(x)} returns the first letter of \spad{x}.
  ++
  ++X m1:=(x*y*y*z)$OFMONOID(Symbol)
  ++X first m1
rest: % -> %
  ++ \spad{rest(x)} returns \spad{x} except the first letter.
  ++
  ++X m1:=(x*y*y*z)$OFMONOID(Symbol)
  ++X rest m1
mirror: % -> %
  ++ \spad{mirror(x)} returns the reversed word of \spad{x}.
  ++
  ++X m1:=(x*y*y*z)$OFMONOID(Symbol)
  ++X mirror m1
lexico: (% , %) -> Boolean
  ++ \spad{lexico(x,y)} returns \spad{true}
  ++ iff \spad{x} is smaller than \spad{y}
  ++ w.r.t. the pure lexicographical ordering induced by \spad{S}.
  ++
  ++X m1:=(x*y*y*z)$OFMONOID(Symbol)
  ++X m2:=(x*y)$OFMONOID(Symbol)
  ++X lexico(m1,m2)
  ++X lexico(m2,m1)
hclf: (% , %) -> %
  ++ \spad{hclf(x, y)} returns the highest common left factor
  ++ of \spad{x} and \spad{y},
  ++ that is the largest \spad{d} such that \spad{x = d a}
  ++ and \spad{y = d b}.
  ++
  ++X m1:=(x*y*z)$OFMONOID(Symbol)
  ++X m2:=(x*y)$OFMONOID(Symbol)

```

```

++X hclrf(m1,m2)
hcrf:  (% , %) -> %
++ \spad{hcrf(x, y)} returns the highest common right
++ factor of \spad{x} and \spad{y},
++ that is the largest \spad{d} such that \spad{x = a d}
++ and \spad{y = b d}.
++
++X m1:=(x*y*z)$OFMONOID(Symbol)
++X m2:=(y*z)$OFMONOID(Symbol)
++X hcrf(m1,m2)
lquo:  (% , %) -> Union(% , "failed")
++ \spad{lquo(x, y)} returns the exact left quotient of \spad{x}
++ by \spad{y} that is \spad{q} such that \spad{x = y * q},
++ "failed" if \spad{x} is not of the form \spad{y * q}.
++
++X m1:=(x*y*y*z)$OFMONOID(Symbol)
++X m2:=(x*y)$OFMONOID(Symbol)
++X lquo(m1,m2)
rquo:  (% , %) -> Union(% , "failed")
++ \spad{rquo(x, y)} returns the exact right quotient of \spad{x}
++ by \spad{y} that is \spad{q} such that \spad{x = q * y},
++ "failed" if \spad{x} is not of the form \spad{q * y}.
++
++X m1:=(q*y^3)$OFMONOID(Symbol)
++X m2:=(y^2)$OFMONOID(Symbol)
++X lquo(m1,m2)
lquo:  (% , S) -> Union(% , "failed")
++ \spad{lquo(x, s)} returns the exact left quotient of \spad{x}
++ by \spad{s}.
++
++X m1:=(x*y*y*z)$OFMONOID(Symbol)
++X lquo(m1,x)
rquo:  (% , S) -> Union(% , "failed")
++ \spad{rquo(x, s)} returns the exact right quotient
++ of \spad{x} by \spad{s}.
++
++X m1:=(x*y)$OFMONOID(Symbol)
++X div(m1,y)
divide: (% , %) -> RESULT
++ \spad{divide(x,y)} returns the left and right exact quotients of
++ \spad{x} by \spad{y}, that is \spad{[l,r]} such that \spad{x = l*y*r}.
++ "failed" is returned iff \spad{x} is not of the form \spad{l * y * r}.
++
++X m1:=(x*y*y*z)$OFMONOID(Symbol)
++X m2:=(x*y)$OFMONOID(Symbol)
++X divide(m1,m2)
overlap: (% , %) -> Record(lm: % , mm: % , rm: %)
++ \spad{overlap(x, y)} returns \spad{[l, m, r]} such that
++ \spad{x = l * m} and \spad{y = m * r} hold and such that
++ \spad{l} and \spad{r} have no overlap,

```

```

++ that is \spad{overlap(l, r) = [1, 1, r]}.
++
++X m1:=(x*y*y*z)$OFMONOID(Symbol)
++X m2:=(x*y)$OFMONOID(Symbol)
++X overlap(m1,m2)
size: % -> NNI
++ \spad{size(x)} returns the number of monomials in \spad{x}.
++
++X m1:=(x*y*y*z)$OFMONOID(Symbol)
++X size(m1,2)
nthExpon: (% , Integer) -> NNI
++ \spad{nthExpon(x, n)} returns the exponent of the
++ \spad{n-th} monomial of \spad{x}.
++
++X m1:=(x*y*y*z)$OFMONOID(Symbol)
++X nthExpon(m1,2)
nthFactor: (% , Integer) -> S
++ \spad{nthFactor(x, n)} returns the factor of the \spad{n-th}
++ monomial of \spad{x}.
++
++X m1:=(x*y*y*z)$OFMONOID(Symbol)
++X nthFactor(m1,2)
factors: % -> List REC
++ \spad{factors(a1\^e1,...,an\^en)} returns
++ \spad{[[a1, e1],...,[an, en]]}.
++
++X m1:=(x*y*y*z)$OFMONOID(Symbol)
++X factors m1
length: % -> NNI
++ \spad{length(x)} returns the length of \spad{x}.
++
++X m1:=(x*y*y*z)$OFMONOID(Symbol)
++X length m1
varList: % -> List S
++ \spad{varList(x)} returns the list of variables of \spad{x}.
++
++X m1:=(x*y*y*z)$OFMONOID(Symbol)
++X varList m1

OFMdefinition == FreeMonoid(S) add
Rep := ListMonoidOps(S, NNI, 1)

-- definitions
lquo(w:%, l:S) ==
x: List REC := listOfMonoms(w)$Rep
null x      => "failed"
fx: REC := first x
fx.gen ^ = 1 => "failed"
fx.exp = 1   => makeMulti rest(x)
makeMulti [[fx.gen, (fx.exp - 1)::NNI ]$REC, :rest x]

```

```

rquo(w:%, l:S) ==
  u:% := reverse w
  (r := lquo (u,l)) case "failed" => "failed"
  reverse_! (r::%)

divide(left:%,right:%) ==
  a:=lquo(left,right)
  b:=rquo(left,right)
  [a,b]

length x == reduce("+", [f.exp for f in listOfMonoms x], 0)

varList x ==
  le: List S := [t.gen for t in listOfMonoms x]
  sort_! removeDuplicates(le)

first w ==
  x: List REC := listOfMonoms w
  null x => error "empty word !!!"
  x.first.gen

rest w ==
  x: List REC := listOfMonoms w
  null x => error "empty word !!!"
  fx: REC := first x
  fx.exp = 1 => makeMulti rest x
  makeMulti [[fx.gen, (fx.exp - 1)::NNI] $REC, :rest x]

lexico(a,b) ==      -- ordre lexicographique
  la := listOfMonoms a
  lb := listOfMonoms b
  while (not null la) and (not null lb) repeat
    la.first.gen > lb.first.gen => return false
    la.first.gen < lb.first.gen => return true
    if la.first.exp = lb.first.exp then
      la:=rest la
      lb:=rest lb
    else if la.first.exp > lb.first.exp then
      la:=concat([la.first.gen,
        (la.first.exp - lb.first.exp)::NNI], rest lb)
      lb:=rest lb
    else
      lb:=concat([lb.first.gen,
        (lb.first.exp-la.first.exp)::NNI], rest la)
      la:=rest la
  empty? la and not empty? lb

a < b ==      -- ordre lexicographique par longueur

```





```

--S 3 of 5
size()$Z
--R
--R
--R (3)  3
--R
--R                                          Type: NonNegativeInteger
--E 3

--S 4 of 5
lv:=index(i::PI)$Z for i in 1..size()$Z]
--R
--R   Compiling function G1408 with type Integer -> Boolean
--R   Compiling function G1572 with type NonNegativeInteger -> Boolean
--R
--R (4)  [x,a,z]
--R
--R                                          Type: List OrderedVariableList [x,a,z]
--E 4

--S 5 of 5
sorted?(>,lv)
--R
--R
--R (5)  true
--R
--R                                          Type: Boolean
--E 5
)spool
)lisp (bye)

```

---

— OrderedVariableList.help —

```

=====
OrderedVariableList examples
=====

```

The domain OrderedVariableList provides symbols which are restricted to a particular list and have a definite ordering. Those two features are specified by a List Symbol object that is the argument to the domain.

This is a sample ordering of three symbols.

```

ls:List Symbol:=['x','a','z]
[x,a,z]

```

Type: List Symbol

Let's build the domain

```
Z:=OVAR ls
OrderedVariableList [x,a,z]
Type: Domain
```

How many variables does it have?

```
size()$Z
3
Type: NonNegativeInteger
```

They are (in the imposed order)

```
lv:=[index(i::PI)$Z for i in 1..size()$Z]
[x,a,z]
Type: List OrderedVariableList [x,a,z]
```

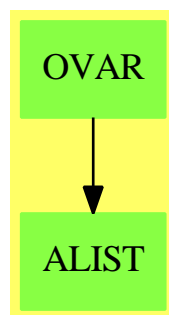
Check that the ordering is right

```
sorted?(>,lv)
true
Type: Boolean
```

See Also:

o )show OrderedVariableList

### 16.15.1 OrderedVariableList (OVAR)



#### Exports:

|          |         |      |        |       |
|----------|---------|------|--------|-------|
| coerce   | convert | hash | index  | latex |
| lookup   | max     | min  | random | size  |
| variable | ?~=?    | ?<?  | ?<=?   | ?=?   |
| ?>?      | ?>=?    |      |        |       |

— domain OVAR OrderedVariableList —

```

)abbrev domain OVAR OrderedVariableList
++ Author: Mark Botch
++ Description:
++ This domain implements ordered variables

OrderedVariableList(VariableList:List Symbol):
  Join(OrderedFinite, ConvertibleTo Symbol, ConvertibleTo InputForm,
    ConvertibleTo Pattern Float, ConvertibleTo Pattern Integer) with
    variable: Symbol -> Union(%, "failed")
    ++ variable(s) returns a member of the variable set or failed

== add
VariableList := removeDuplicates VariableList
Rep := PositiveInteger
s1,s2:%
convert(s1):Symbol == VariableList.((s1::Rep)::PositiveInteger)
coerce(s1):OutputForm == (convert(s1)@Symbol)::OutputForm
convert(s1):InputForm == convert(convert(s1)@Symbol)
convert(s1):Pattern(Integer) == convert(convert(s1)@Symbol)
convert(s1):Pattern(Float) == convert(convert(s1)@Symbol)
index i == i::%
lookup j == j :: Rep
size () == #VariableList
variable(exp:Symbol) ==
  for i in 1.. for exp2 in VariableList repeat
    if exp=exp2 then return i::PositiveInteger::%
  "failed"
s1 < s2 == s2 <$Rep s1
s1 = s2 == s1 =$Rep s2
latex(x:%):String == latex(convert(x)@Symbol)

```

—

— OVAR.dotabb —

```

"OVAR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=OVAR"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"OVAR" -> "ALIST"

```

—

## 16.16 domain ODPOL OrderlyDifferentialPolynomial

## — OrderlyDifferentialPolynomial.input —

```

)set break resume
)sys rm -f OrderlyDifferentialPolynomial.output
)spool OrderlyDifferentialPolynomial.output
)set message test on
)set message auto off
)clear all
--S 1 of 36
dpol:= ODPOL(FRAC INT)
--R
--R
--R (1) OrderlyDifferentialPolynomial Fraction Integer
--R
--R                                         Type: Domain
--E 1

--S 2 of 36
w := makeVariable('w)$dpol
--R
--R
--R (2) theMap(DPOLCAT-;makeVariable;AM;17!0,0)
--R Type: (NonNegativeInteger -> OrderlyDifferentialPolynomial Fraction Integer)
--E 2

--S 3 of 36
z := makeVariable('z)$dpol
--R
--R
--R (3) theMap(DPOLCAT-;makeVariable;AM;17!0,0)
--R Type: (NonNegativeInteger -> OrderlyDifferentialPolynomial Fraction Integer)
--E 3

--S 4 of 36
w.5
--R
--R
--R (4) w
--R      5
--R
--R                                         Type: OrderlyDifferentialPolynomial Fraction Integer
--E 4

--S 5 of 36
w 0
--R
--R
--R (5) w
--R
--R                                         Type: OrderlyDifferentialPolynomial Fraction Integer
--E 5

--S 6 of 36

```

```

[z.i for i in 1..5]
--R
--R
--R (6) [z ,z ,z ,z ,z ]
--R      1 2 3 4 5
--R                                     Type: List OrderlyDifferentialPolynomial Fraction Integer
--E 6

--S 7 of 36
f:= w.4 - w.1 * w.1 * z.3
--R
--R
--R      2
--R (7) w  - w  z
--R      4    1 3
--R                                     Type: OrderlyDifferentialPolynomial Fraction Integer
--E 7

--S 8 of 36
g:=(z.1)**3 * (z.2)**2 - w.2
--R
--R
--R      3 2
--R (8) z  z  - w
--R      1 2    2
--R                                     Type: OrderlyDifferentialPolynomial Fraction Integer
--E 8

--S 9 of 36
D(f)
--R
--R
--R      2
--R (9) w  - w  z  - 2w w z
--R      5    1 4    1 2 3
--R                                     Type: OrderlyDifferentialPolynomial Fraction Integer
--E 9

--S 10 of 36
D(f,4)
--R
--R
--R (10)
--R      2                                2
--R      w  - w  z  - 8w w z  + (- 12w w  - 12w )z  - 2w z w
--R      8    1 7    1 2 6    1 3    2 5    1 3 5
--R  +
--R      2
--R      (- 8w w  - 24w w )z  - 8w z w  - 6w  z
--R      1 4    2 3 4    2 3 4    3 3

```

[illegible]





```

--S 22 of 36
eval(g,['w::Symbol],[f])
--R
--R
--R
--R      2      2      3  2
--R      (22)  - w + w z + 4w w z + (2w w + 2w )z + z z
--R             6      1  5      1 2 4      1 3      2  3      1  2
--R
--R                                         Type: OrderlyDifferentialPolynomial Fraction Integer
--E 22

--S 23 of 36
eval(g,variables(w.0),[f])
--R
--R
--R
--R      3  2
--R      (23)  z z - w
--R             1  2      2
--R
--R                                         Type: OrderlyDifferentialPolynomial Fraction Integer
--E 23

--S 24 of 36
monomials(g)
--R
--R
--R
--R      3  2
--R      (24)  [z z , - w ]
--R             1  2      2
--R
--R                                         Type: List OrderlyDifferentialPolynomial Fraction Integer
--E 24

--S 25 of 36
variables(g)
--R
--R
--R
--R      (25)  [z ,w ,z ]
--R             2  2  1
--R
--R                                         Type: List OrderlyDifferentialVariable Symbol
--E 25

--S 26 of 36
gcd(f,g)
--R
--R
--R
--R      (26)  1
--R
--R                                         Type: OrderlyDifferentialPolynomial Fraction Integer
--E 26

--S 27 of 36
groebner([f,g])
--R

```

```

--R
--R      2      3  2
--R      (27) [w - w z , z z - w ]
--R      4      1  3  1  2      2
--R                                     Type: List OrderlyDifferentialPolynomial Fraction Integer
--E 27

--S 28 of 36
lg:=leader(g)
--R
--R
--R      (28) z
--R      2
--R                                     Type: OrderlyDifferentialVariable Symbol
--E 28

--S 29 of 36
sg:=separant(g)
--R
--R
--R      3
--R      (29) 2z z
--R      1  2
--R                                     Type: OrderlyDifferentialPolynomial Fraction Integer
--E 29

--S 30 of 36
ig:=initial(g)
--R
--R
--R      3
--R      (30) z
--R      1
--R                                     Type: OrderlyDifferentialPolynomial Fraction Integer
--E 30

--S 31 of 36
g1 := D g
--R
--R
--R      3      2  3
--R      (31) 2z z z - w + 3z z
--R      1  2  3      3      1  2
--R                                     Type: OrderlyDifferentialPolynomial Fraction Integer
--E 31

--S 32 of 36
lg1:= leader g1
--R
--R

```

```

--R (32) z
--R      3
--R
--R                                         Type: OrderlyDifferentialVariable Symbol
--E 32

--S 33 of 36
pdf:=D(f, lg1)
--R
--R
--R      2
--R (33) - w
--R      1
--R
--R                                         Type: OrderlyDifferentialPolynomial Fraction Integer
--E 33

--S 34 of 36
prf:=sg * f- pdf * g1
--R
--R
--R      3      2      2 2 3
--R (34) 2z z w - w w + 3w z z
--R      1 2 4      1 3      1 1 2
--R
--R                                         Type: OrderlyDifferentialPolynomial Fraction Integer
--E 34

--S 35 of 36
lcf:=leadingCoefficient univariate(prf, lg)
--R
--R
--R      2 2
--R (35) 3w z
--R      1 1
--R
--R                                         Type: OrderlyDifferentialPolynomial Fraction Integer
--E 35

--S 36 of 36
ig * prf - lcf * g * lg
--R
--R
--R      6      2 3      2 2
--R (36) 2z z w - w z w + 3w z w z
--R      1 2 4      1 1 3      1 1 2 2
--R
--R                                         Type: OrderlyDifferentialPolynomial Fraction Integer
--E 36
)spool
)lisp (bye)

```

---

## — OrderlyDifferentialPolynomial.help —

```
=====
OrderlyDifferentialPolynomial examples
=====
```

Many systems of differential equations may be transformed to equivalent systems of ordinary differential equations where the equations are expressed polynomially in terms of the unknown functions. In Axiom, the domain constructors `OrderlyDifferentialPolynomial` (abbreviated `ODPOL`) and `SequentialDifferentialPolynomial` (abbreviation `SDPOL`) implement two domains of ordinary differential polynomials over any differential ring. In the simplest case, this differential ring is usually either the ring of integers, or the field of rational numbers. However, Axiom can handle ordinary differential polynomials over a field of rational functions in a single indeterminate.

The two domains `ODPOL` and `SDPOL` are almost identical, the only difference being the choice of a different ranking, which is an ordering of the derivatives of the indeterminates. The first domain uses an orderly ranking, that is, derivatives of higher order are ranked higher, and derivatives of the same order are ranked alphabetically. The second domain uses a sequential ranking, where derivatives are ordered first alphabetically by the differential indeterminates, and then by order. A more general domain constructor, `DifferentialSparseMultivariatePolynomial` (abbreviation `DSMP`) allows both a user-provided list of differential indeterminates as well as a user-defined ranking. We shall illustrate `ODPOL(FRAC INT)`, which constructs a domain of ordinary differential polynomials in an arbitrary number of differential indeterminates with rational numbers as coefficients.

```
dpol:= ODPOL(FRAC INT)
      OrderlyDifferentialPolynomial Fraction Integer
      Type: Domain
```

A differential indeterminate `w` may be viewed as an infinite sequence of algebraic indeterminates, which are the derivatives of `w`. To facilitate referencing these, Axiom provides the operation `makeVariable` to convert an element of type `Symbol` to a map from the natural numbers to the differential polynomial ring.

```
w := makeVariable('w)$dpol
    theMap(DPOLCAT-;makeVariable;AM;17!0,0)
Type: (NonNegativeInteger -> OrderlyDifferentialPolynomial Fraction Integer)

z := makeVariable('z)$dpol
    theMap(DPOLCAT-;makeVariable;AM;17!0,0)
Type: (NonNegativeInteger -> OrderlyDifferentialPolynomial Fraction Integer)
```

The fifth derivative of `w` can be obtained by applying the map `w` to the

number 5. Note that the order of differentiation is given as a subscript (except when the order is 0).

$$\frac{w.5}{w^5}$$

Type: OrderlyDifferentialPolynomial Fraction Integer

$$\frac{w^0}{w}$$

Type: OrderlyDifferentialPolynomial Fraction Integer

The first five derivatives of  $z$  can be generated by a list.

$$[z.i \text{ for } i \text{ in } 1..5]$$

$$[z_1, z_2, z_3, z_4, z_5]$$

Type: List OrderlyDifferentialPolynomial Fraction Integer

The usual arithmetic can be used to form a differential polynomial from the derivatives.

$$f := w.4 - w.1^2 * w.1 * z.3$$

$$\frac{w^4 - w^2 z^3}{4 \cdot 1 \cdot 3}$$

Type: OrderlyDifferentialPolynomial Fraction Integer

$$g := (z.1)^3 * (z.2)^2 - w.2$$

$$z_1^3 z_2^2 - w$$

$$\frac{z_1^3 z_2^2 - w}{1 \cdot 2 \cdot 2}$$

Type: OrderlyDifferentialPolynomial Fraction Integer

The operation  $D$  computes the derivative of any differential polynomial.

$$D(f)$$

$$\frac{w^2 - w z^2 - 2w w z}{5 \cdot 1 \cdot 4 \cdot 1 \cdot 2 \cdot 3}$$

Type: OrderlyDifferentialPolynomial Fraction Integer

The same operation can compute higher derivatives, like the fourth derivative.

$$D(f, 4)$$

$$\frac{w^2 - w z^2 - 8w w z + (-12w w - 12w^2)z - 2w z w}{8 \cdot 1 \cdot 7 \cdot 1 \cdot 2 \cdot 6 \cdot 1 \cdot 3 \cdot 2 \cdot 5 \cdot 1 \cdot 3 \cdot 5}$$

$$+$$

$$\frac{2}{2}$$

$$\begin{array}{ccccccc} (- & 8w & w & - & 24w & w & )z & - & 8w & z & w & - & 6w & z \\ & 1 & 4 & & 2 & 3 & 4 & & 2 & 3 & 4 & & 3 & 3 \end{array}$$

Type: OrderlyDifferentialPolynomial Fraction Integer

The operation `makeVariable` creates a map to facilitate referencing the derivatives of `f`, similar to the map `w`.

```
df:=makeVariable(f)$dpol
  theMap(DPOLCAT-;makeVariable;AM;17!0,0)
Type: (NonNegativeInteger -> OrderlyDifferentialPolynomial Fraction Integer)
```

The fourth derivative of `f` may be referenced easily.

```
df.4
```

$$\begin{array}{ccccccc} & & 2 & & & & 2 \\ w & - & w & z & - & 8w & w & z & + & (- & 12w & w & - & 12w & )z & - & 2w & z & w \\ & 8 & 1 & 7 & & 1 & 2 & 6 & & 1 & 3 & & 2 & 5 & & 1 & 3 & 5 \\ + & & & & & & & & & & & & & & & & & & 2 \\ & & & & & & & & & (- & 8w & w & - & 24w & w & )z & - & 8w & z & w & - & 6w & z \\ & & & & & & & & & 1 & 4 & & 2 & 3 & 4 & & 2 & 3 & 4 & & 3 & 3 \end{array}$$

Type: OrderlyDifferentialPolynomial Fraction Integer

The operation `order` returns the order of a differential polynomial, or the order in a specified differential indeterminate.

```
order(g)
2
Type: PositiveInteger
```

```
order(g, 'w)
2
Type: PositiveInteger
```

The operation `differentialVariables` returns a list of differential indeterminates occurring in a differential polynomial.

```
differentialVariables(g)
[z,w]
Type: List Symbol
```

The operation `degree` returns the degree, or the degree in the differential indeterminate specified.

```
degree(g)
2 3
z z
2 1
Type: IndexedExponents OrderlyDifferentialVariable Symbol
```

```
degree(g, 'w)
1
```

Type: PositiveInteger

The operation `weights` returns a list of weights of differential monomials appearing in differential polynomial, or a list of weights in a specified differential indeterminate.

```
weights(g)
[7,2]
```

Type: List NonNegativeInteger

```
weights(g,'w)
[2]
```

Type: List NonNegativeInteger

The operation `weight` returns the maximum weight of all differential monomials appearing in the differential polynomial.

```
weight(g)
7
```

Type: PositiveInteger

A differential polynomial is isobaric if the weights of all differential monomials appearing in it are equal.

```
isobaric?(g)
false
```

Type: Boolean

To substitute differentially, use `eval`. Note that we must coerce `'w` to `Symbol`, since in `ODPOL`, differential indeterminates belong to the domain `Symbol`. Compare this result to the next, which substitutes algebraically (no substitution is done since `w.0` does not appear in `g`).

```
eval(g,['w::Symbol],[f])
```

$$- \frac{w^2}{6} + \frac{w^2 z}{1} + \frac{4w^2 z^2}{5} + \frac{4w^2 z^2}{12} + \frac{(2w^2 w + 2w^2)z}{13} + \frac{2z^3}{3} + \frac{z^2}{1} + \frac{z^2}{2}$$

Type: OrderlyDifferentialPolynomial Fraction Integer

```
eval(g,variables(w.0),[f])
```

$$\frac{z^3}{1} - \frac{z^2}{2} - \frac{w^2}{2}$$

Type: OrderlyDifferentialPolynomial Fraction Integer

Since `OrderlyDifferentialPolynomial` belongs to `PolynomialCategory`, all the operations defined in the latter category, or in packages for the latter category, are available.

```

monomials(g)
      3 2
      [z z , - w ]
      1 2 2
Type: List OrderlyDifferentialPolynomial Fraction Integer

```

```

variables(g)
      [z ,w ,z ]
      2 2 1
Type: List OrderlyDifferentialVariable Symbol

```

```

gcd(f,g)
1
Type: OrderlyDifferentialPolynomial Fraction Integer

```

```

groebner([f,g])
      2 3 2
      [w - w z ,z z - w ]
      4 1 3 1 2 2
Type: List OrderlyDifferentialPolynomial Fraction Integer

```

The next three operations are essential for elimination procedures in differential polynomial rings. The operation leader returns the leader of a differential polynomial, which is the highest ranked derivative of the differential indeterminates that occurs.

```

lg:=leader(g)
      z
      2
Type: OrderlyDifferentialVariable Symbol

```

The operation separant returns the separant of a differential polynomial, which is the partial derivative with respect to the leader.

```

sg:=separant(g)
      3
      2z z
      1 2
Type: OrderlyDifferentialPolynomial Fraction Integer

```

The operation initial returns the initial, which is the leading coefficient when the given differential polynomial is expressed as a polynomial in the leader.

```

ig:=initial(g)
      3
      z
      1
Type: OrderlyDifferentialPolynomial Fraction Integer

```



Using these three operations, it is possible to reduce  $f$  modulo the differential ideal generated by  $g$ . The general scheme is to first reduce the order, then reduce the degree in the leader. First, eliminate  $z^3$  using the derivative of  $g$ .

```
g1 := D g
      3          2 3
2z  z z  - w  + 3z  z
 1 2 3    3    1 2
Type: OrderlyDifferentialPolynomial Fraction Integer
```

Find its leader.

```
lg1:= leader g1
      z
      3
Type: OrderlyDifferentialVariable Symbol
```

Differentiate  $f$  partially with respect to this leader.

```
pdf:=D(f, lg1)
      2
      - w
      1
Type: OrderlyDifferentialPolynomial Fraction Integer
```

Compute the partial remainder of  $f$  with respect to  $g$ .

```
prf:=sg * f- pdf * g1
      3          2          2 2 3
2z  z w  - w  w  + 3w  z z
 1 2 4    1 3    1 1 2
Type: OrderlyDifferentialPolynomial Fraction Integer
```

Note that high powers of  $lg$  still appear in  $prf$ . Compute the leading coefficient of  $prf$  as a polynomial in the leader of  $g$ .

```
lcf:=leadingCoefficient univariate(prf, lg)
      2 2
3w  z
 1 1
Type: OrderlyDifferentialPolynomial Fraction Integer
```

Finally, continue eliminating the high powers of  $lg$  appearing in  $prf$  to obtain the (pseudo) remainder of  $f$  modulo  $g$  and its derivatives.

```
ig * prf - lcf * g * lg
      6          2 3          2 2
2z  z w  - w  z w  + 3w  z w z
```

```

1 2 4    1 1 3    1 1 2 2
Type: OrderlyDifferentialPolynomial Fraction Integer

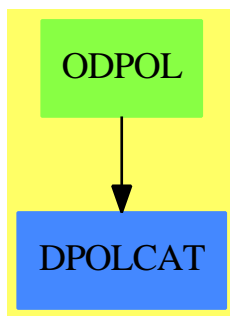
```

See Also:

o )show OrderlyDifferentialPolynomial

---

### 16.16.1 OrderlyDifferentialPolynomial (ODPOL)



See

- ⇒ “OrderlyDifferentialVariable” (ODVAR) 16.17.1 on page 1816
- ⇒ “SequentialDifferentialVariable” (SDVAR) 20.7.1 on page 2348
- ⇒ “DifferentialSparseMultivariatePolynomial” (DSMP) 5.8.1 on page 526
- ⇒ “SequentialDifferentialPolynomial” (SDPOL) 20.6.1 on page 2345

**Exports:**

|                               |                       |                            |
|-------------------------------|-----------------------|----------------------------|
| 0                             | 1                     | associates?                |
| binomThmExpt                  | characteristic        | charthRoot                 |
| coefficient                   | coefficients          | coerce                     |
| conditionP                    | content               | D                          |
| degree                        | differentialVariables | differentiate              |
| discriminant                  | eval                  | exquo                      |
| factor                        | factorPolynomial      | factorSquareFreePolynomial |
| gcd                           | gcdPolynomial         | ground                     |
| ground?                       | hash                  | initial                    |
| isExpt                        | isobaric?             | isPlus                     |
| isTimes                       | latex                 | lcm                        |
| leader                        | leadingCoefficient    | leadingMonomial            |
| mainVariable                  | map                   | mapExponents               |
| max                           | min                   | minimumDegree              |
| monicDivide                   | monomial              | monomial?                  |
| monomials                     | multivariate          | numberOfMonomials          |
| one?                          | order                 | patternMatch               |
| popopo!                       | prime?                | primitiveMonomials         |
| primitivePart                 | recip                 | reducedSystem              |
| reductum                      | resultant             | retract                    |
| retractIfCan                  | sample                | separant                   |
| solveLinearPolynomialEquation | squareFree            | squareFreePart             |
| squareFreePolynomial          | subtractIfCan         | totalDegree                |
| totalDegree                   | unit?                 | unitCanonical              |
| unitNormal                    | univariate            | univariate                 |
| variables                     | weight                | weights                    |
| zero?                         | ?*?                   | ?**?                       |
| ?+?                           | ?-?                   | -?                         |
| ?=?                           | ?^?                   | ?~=?                       |
| ?/?                           | ?<?                   | ?<=?                       |
| ?>?                           | ?>=?                  |                            |

— domain ODPOL OrderlyDifferentialPolynomial —

```

)abbrev domain ODPOL OrderlyDifferentialPolynomial
++ Author: William Sit
++ Date Created: 24 September, 1991
++ Date Last Updated: 7 February, 1992
++ Basic Operations:DifferentialPolynomialCategory
++ Related Constructors: DifferentialSparseMultivariatePolynomial
++ See Also:
++ AMS Classifications:12H05
++ Keywords: differential indeterminates, ranking, differential polynomials,
++           order, weight, leader, separant, initial, isobaric
++ References:Kolchin, E.R. "Differential Algebra and Algebraic Groups"
++           (Academic Press, 1973).
++ Description:

```

```

++ \spadtype{OrderlyDifferentialPolynomial} implements
++ an ordinary differential polynomial ring in arbitrary number
++ of differential indeterminates, with coefficients in a
++ ring. The ranking on the differential indeterminate is orderly.
++ This is analogous to the domain \spadtype{Polynomial}.

OrderlyDifferentialPolynomial(R):
  Exports == Implementation where
  R: Ring
  S ==> Symbol
  V ==> OrderlyDifferentialVariable S
  E ==> IndexedExponents(V)
  SMP ==> SparseMultivariatePolynomial(R, S)
  Exports ==> Join(DifferentialPolynomialCategory(R,S,V,E),
    RetractableTo SMP)

Implementation ==> DifferentialSparseMultivariatePolynomial(R,S,V)

```

---

— ODPOL.dotabb —

```

"ODPOL" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ODPOL"]
"DPOLCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=DPOLCAT"]
"ODPOL" -> "DPOLCAT"

```

## 16.17 domain ODVAR OrderlyDifferentialVariable

---

— OrderlyDifferentialVariable.input —

```

)set break resume
)sys rm -f OrderlyDifferentialVariable.output
)spool OrderlyDifferentialVariable.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OrderlyDifferentialVariable
--R OrderlyDifferentialVariable S: OrderedSet is a domain constructor
--R Abbreviation for OrderlyDifferentialVariable is ODVAR
--R This constructor is not exposed in this frame.

```

```

--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ODVAR
--R
--R----- Operations -----
--R ?<? : (% ,%) -> Boolean          ?<=? : (% ,%) -> Boolean
--R ?=? : (% ,%) -> Boolean          ?>? : (% ,%) -> Boolean
--R ?>=? : (% ,%) -> Boolean        coerce : S -> %
--R coerce : % -> OutputForm        differentiate : % -> %
--R hash : % -> SingleInteger       latex : % -> String
--R max : (% ,%) -> %               min : (% ,%) -> %
--R order : % -> NonNegativeInteger retract : % -> S
--R variable : % -> S              weight : % -> NonNegativeInteger
--R ?~=? : (% ,%) -> Boolean
--R differentiate : (% ,NonNegativeInteger) -> %
--R makeVariable : (S,NonNegativeInteger) -> %
--R retractIfCan : % -> Union(S,"failed")
--R
--E 1

)spool
)lisp (bye)

```

---

— OrderlyDifferentialVariable.help —

```

=====
OrderlyDifferentialVariable examples
=====

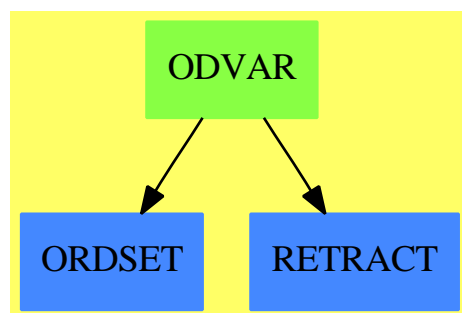
```

```

See Also:
o )show OrderlyDifferentialVariable

```

### 16.17.1 OrderlyDifferentialVariable (ODVAR)



See

- ⇒ “SequentialDifferentialVariable” (SDVAR) 20.7.1 on page 2348
- ⇒ “DifferentialSparseMultivariatePolynomial” (DSMP) 5.8.1 on page 526
- ⇒ “OrderlyDifferentialPolynomial” (ODPOL) 16.16.1 on page 1813
- ⇒ “SequentialDifferentialPolynomial” (SDPOL) 20.6.1 on page 2345

#### Exports:

|          |               |       |         |              |
|----------|---------------|-------|---------|--------------|
| coerce   | differentiate | hash  | latex   | makeVariable |
| max      | min           | order | retract | retractIfCan |
| variable | weight        | ?~=?  | ?<?     | ?<=?         |
| ?=?      | ?>?           | ?>=?  |         |              |

— domain ODVAR OrderlyDifferentialVariable —

```

)abbrev domain ODVAR OrderlyDifferentialVariable
++ Author: William Sit
++ Date Created: 19 July 1990
++ Date Last Updated: 13 September 1991
++ Basic Operations: differentiate, order, variable, <
++ Related Domains: OrderedVariableList,
++ SequentialDifferentialVariable.
++ See Also: DifferentialVariableCategory
++ AMS Classifications: 12H05
++ Keywords: differential indeterminates, orderly ranking.
++ References: Kolchin, E.R. "Differential Algebra and Algebraic Groups"
++ (Academic Press, 1973).
++ Description:
++ \spadtype{OrderlyDifferentialVariable} adds a commonly used orderly
++ ranking to the set of derivatives of an ordered list of differential
++ indeterminates. An orderly ranking is a ranking \spadfun{<} of the
++ derivatives with the property that for two derivatives u and v,
++ u \spadfun{<} v if the \spadfun{order} of u is less than that of v.
++ This domain belongs to \spadtype{DifferentialVariableCategory}. It
++ defines \spadfun{weight} to be just \spadfun{order}, and it
++ defines an orderly ranking \spadfun{<} on derivatives u via the
++ lexicographic order on the pair
++ (\spadfun{order}(u), \spadfun{variable}(u)).

OrderlyDifferentialVariable(S:OrderedSet):DifferentialVariableCategory(S)
== add
Rep := Record(var:S, ord:NonNegativeInteger)
makeVariable(s,n) == [s, n]
variable v == v.var
order v == v.ord

```

---

— ODVAR.dotabb —

```
"ODVAR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ODVAR"]
"ORDSET" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ORDSET"]
"RETRACT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=RETRACT"]
"ODVAR" -> "ORDSET"
"ODVAR" -> "RETRACT"
```

## 16.18 domain ODR OrdinaryDifferentialRing

— OrdinaryDifferentialRing.input —

```
)set break resume
)sys rm -f OrdinaryDifferentialRing.output
)spool OrdinaryDifferentialRing.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OrdinaryDifferentialRing
--R OrdinaryDifferentialRing(Kernels: SetCategory,R: PartialDifferentialRing Kernels,var: Ke
--R Abbreviation for OrdinaryDifferentialRing is ODR
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ODR
--R
--R----- Operations -----
--R ?? : (% ,%) -> %
--R ?? : (Integer,% ) -> %
--R ??? : (% ,PositiveInteger) -> %
--R ?-? : (% ,%) -> %
--R ?? : (% ,%) -> Boolean
--R D : (% ,NonNegativeInteger) -> %
--R 0 : () -> %
--R coerce : % -> R
--R coerce : Integer -> %
--R differentiate : % -> %
--R inv : % -> % if R has FIELD
--R one? : % -> Boolean
--R sample : () -> %
--R ~=? : (% ,%) -> Boolean
--R ?? : (% ,Fraction Integer) -> % if R has FIELD
--R ?? : (Fraction Integer,% ) -> % if R has FIELD
--R ?? : (NonNegativeInteger,% ) -> %
--R ?? : (% ,%) -> %
--R ?? : (PositiveInteger,% ) -> %
--R ?? : (% ,%) -> %
--R -? : % -> %
--R D : % -> %
--R 1 : () -> %
--R ?? : (% ,PositiveInteger) -> %
--R coerce : R -> %
--R coerce : % -> OutputForm
--R hash : % -> SingleInteger
--R latex : % -> String
--R recip : % -> Union(% ,"failed")
--R zero? : % -> Boolean
```

```

--R ??? : (%,Integer) -> % if R has FIELD
--R ??? : (%,NonNegativeInteger) -> %
--R ?/? : (%,% ) -> % if R has FIELD
--R ?? : (%,Integer) -> % if R has FIELD
--R ?? : (%,NonNegativeInteger) -> %
--R associates? : (%,% ) -> Boolean if R has FIELD
--R characteristic : () -> NonNegativeInteger
--R coerce : % -> % if R has FIELD
--R coerce : Fraction Integer -> % if R has FIELD
--R differentiate : (%,NonNegativeInteger) -> %
--R divide : (%,% ) -> Record(quotient: %,remainder: %) if R has FIELD
--R euclideanSize : % -> NonNegativeInteger if R has FIELD
--R expressIdealMember : (List %,%) -> Union(List %,"failed") if R has FIELD
--R exquo : (%,% ) -> Union(%,"failed") if R has FIELD
--R extendedEuclidean : (%,% ) -> Record(coef1: %,coef2: %,generator: %) if R has FIELD
--R extendedEuclidean : (%,%,% ) -> Union(Record(coef1: %,coef2: %),"failed") if R has FIELD
--R factor : % -> Factored % if R has FIELD
--R gcd : (%,% ) -> % if R has FIELD
--R gcd : List % -> % if R has FIELD
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolynomial %
--R lcm : (%,% ) -> % if R has FIELD
--R lcm : List % -> % if R has FIELD
--R multiEuclidean : (List %,%) -> Union(List %,"failed") if R has FIELD
--R prime? : % -> Boolean if R has FIELD
--R principalIdeal : List % -> Record(coef: List %,generator: %) if R has FIELD
--R ?quo? : (%,% ) -> % if R has FIELD
--R ?rem? : (%,% ) -> % if R has FIELD
--R sizeLess? : (%,% ) -> Boolean if R has FIELD
--R squareFree : % -> Factored % if R has FIELD
--R squareFreePart : % -> % if R has FIELD
--R subtractIfCan : (%,% ) -> Union(%,"failed")
--R unit? : % -> Boolean if R has FIELD
--R unitCanonical : % -> % if R has FIELD
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has FIELD
--R
--E 1

)spool
)lisp (bye)

```

---

— OrdinaryDifferentialRing.help —

```

=====
OrdinaryDifferentialRing examples
=====

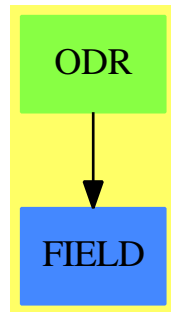
```

See Also:



```
o )show OrdinaryDifferentialRing
```

### 16.18.1 OrdinaryDifferentialRing (ODR)



See

⇒ “OppositeMonogenicLinearOperator” (OMLO) 16.11.1 on page 1768

⇒ “DirectProductModule” (DPMO) 5.11.1 on page 542

⇒ “DirectProductMatrixModule” (DPMM) 5.10.1 on page 538

#### Exports:

|               |                    |                |                   |
|---------------|--------------------|----------------|-------------------|
| 0             | 1                  | associates?    | characteristic    |
| coerce        | D                  | differentiate  | divide            |
| euclideanSize | expressIdealMember | exquo          | extendedEuclidean |
| factor        | gcd                | gcdPolynomial  | hash              |
| inv           | latex              | lcm            | multiEuclidean    |
| one?          | prime?             | principalIdeal | recip             |
| sample        | sizeLess?          | squareFree     | squareFreePart    |
| subtractIfCan | unit?              | unitCanonical  | unitNormal        |
| zero?         | ?*?                | ?**?           | ?+?               |
| ?-?           | -?                 | ?=?            | ?^?               |
| ?~=?          | ?/?                | ?quo?          | ?rem?             |

— domain ODR OrdinaryDifferentialRing —

```
)abbrev domain ODR OrdinaryDifferentialRing
```

```
++ Author: Stephen M. Watt
```

```
++ Date Created: 1986
```

```
++ Date Last Updated: June 3, 1991
```

```
++ Basic Operations:
```

```
++ Related Domains:
```

```
++ Also See:
```

```
++ AMS Classifications:
```

```
++ Keywords: differential ring
```

```

++ Examples:
++ References:
++ Description:
++ This constructor produces an ordinary differential ring from
++ a partial differential ring by specifying a variable.

OrdinaryDifferentialRing(Kernels,R,var): DRcategory == DRcapsule where
  Kernels:SetCategory
  R: PartialDifferentialRing(Kernels)
  var : Kernels
  DRcategory == Join(BiModule($,$), DifferentialRing) with
    if R has Field then Field
      coerce: R -> $
        ++ coerce(r) views r as a value in the ordinary differential ring.
      coerce: $ -> R
        ++ coerce(p) views p as a value in the partial differential ring.
  DRcapsule == R add
    n: Integer
    Rep := R
    coerce(u:R):$ == u::Rep::$
    coerce(p:$):R == p::Rep::R
    differentiate p == differentiate(p, var)

    if R has Field then
      p / q == ((p::R) /$R (q::R))::$
      p ** n == ((p::R) **$R n)::R
      inv(p) == (inv(p::R)$R)::R

  -----

  — ODR.dotabb —

"ODR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ODR"]
"FIELD" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FIELD"]
"ODR" -> "FIELD"

  -----

```

## 16.19 domain OWP OrdinaryWeightedPolynomials

— OrdinaryWeightedPolynomials.input —

```

)set break resume
)sys rm -f OrdinaryWeightedPolynomials.output

```

```

)spool OrdinaryWeightedPolynomials.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OrdinaryWeightedPolynomials
--R OrdinaryWeightedPolynomials(R: Ring,v1: List Symbol,w1: List NonNegativeInteger,wtlevel:
--R Abbreviation for OrdinaryWeightedPolynomials is OWP
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for OWP
--R
--R----- Operations -----
--R ??? : (%,%) -> %               ??? : (Integer,%) -> %
--R ??? : (PositiveInteger,%) -> %   ??? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %               ?-? : (%,%) -> %
--R -? : % -> %                   ?=? : (%,%) -> Boolean
--R 1 : () -> %                   0 : () -> %
--R ?? : (%,PositiveInteger) -> %   coerce : Polynomial R -> %
--R coerce : % -> Polynomial R       coerce : Integer -> %
--R coerce : % -> OutputForm         hash : % -> SingleInteger
--R latex : % -> String              one? : % -> Boolean
--R recip : % -> Union(%, "failed")  sample : () -> %
--R zero? : % -> Boolean              ?~=? : (%,%) -> Boolean
--R ??? : (%,R) -> % if R has COMRING
--R ??? : (R,%) -> % if R has COMRING
--R ??? : (NonNegativeInteger,%) -> %
--R ??? : (%,NonNegativeInteger) -> %
--R ?/? : (%,%) -> Union(%, "failed") if R has FIELD
--R ?? : (%,NonNegativeInteger) -> %
--R changeWeightLevel : NonNegativeInteger -> Void
--R characteristic : () -> NonNegativeInteger
--R coerce : R -> % if R has COMRING
--R subtractIfCan : (%,%) -> Union(%, "failed")
--R
--E 1

)spool
)lisp (bye)

```

---

— OrdinaryWeightedPolynomials.help —

```

=====
OrdinaryWeightedPolynomials examples
=====

```

See Also:



```

Ring with
  if R has CommutativeRing then Algebra(R)
  coerce: $ -> Polynomial(R)
    ++ coerce(p) converts back into a Polynomial(R), ignoring weights
  coerce: Polynomial(R) -> $
    ++ coerce(p) coerces a Polynomial(R) into Weighted form,
    ++ applying weights and ignoring terms
  if R has Field then "/": ($,$) -> Union($,"failed")
    ++ x/y division (only works if minimum weight
    ++ of divisor is zero, and if R is a Field)
  changeWeightLevel: NonNegativeInteger -> Void
    ++ changeWeightLevel(n) This changes the weight level to the
    ++ new value given:
    ++ NB: previously calculated terms are not affected
== WeightedPolynomials(R,Symbol,IndexedExponents(Symbol),
                        Polynomial(R),
                        vl,wl,wlevel)

```

---

— OWP.dotabb —

```

"OWP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=OWP"]
"FIELD" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FIELD"]
"OWP" -> "FIELD"

```

## 16.20 domain OSI OrdSetInts

---

— OrdSetInts.input —

```

)set break resume
)sys rm -f OrdSetInts.output
)spool OrdSetInts.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OrdSetInts
--R OrdSetInts is a domain constructor
--R Abbreviation for OrdSetInts is OSI
--R This constructor is exposed in this frame.

```

```

--R Issue )edit bookvol10.3.pamphlet to see algebra source code for OSI
--R
--R----- Operations -----
--R ?<? : (%,% ) -> Boolean      ?<=? : (%,% ) -> Boolean
--R ?=? : (%,% ) -> Boolean      ?>? : (%,% ) -> Boolean
--R ?>=? : (%,% ) -> Boolean     coerce : Integer -> %
--R coerce : % -> OutputForm     hash : % -> SingleInteger
--R latex : % -> String          max : (%,% ) -> %
--R min : (%,% ) -> %           value : % -> Integer
--R ?~=? : (%,% ) -> Boolean
--R
--E 1

)spool
)lisp (bye)

```

\_\_\_\_\_

— OrdSetInts.help —

```

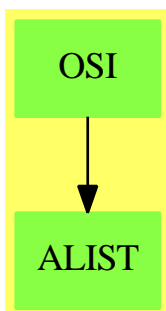
=====
OrdSetInts examples
=====

```

See Also:  
o )show OrdSetInts

\_\_\_\_\_

### 16.20.1 OrdSetInts (OSI)



See

⇒ “Commutator” (COMM) 4.7.1 on page 395  
 ⇒ “FreeNilpotentLie” (FNLA) 7.33.1 on page 993

**Exports:**

```

coerce  hash    latex  max    min
value   ?~=?    ?<?    ?<=?  ?=?
?>?     ?>=?

```

— domain OSI OrdSetInts —

```

)abbrev domain OSI OrdSetInts
++ Author: Larry Lambe
++ Date created: 14 August 1988
++ Date Last Updated: 11 March 1991
++ Description:
++ A domain used in order to take the free R-module on the
++ Integers I. This is actually the forgetful functor from OrderedRings
++ to OrderedSets applied to I

```

```

OrdSetInts: Export == Implement where

```

```

I ==> Integer
L ==> List
O ==> OutputForm

```

```

Export == OrderedSet with

```

```

  coerce : Integer -> %
    ++ coerce(i) returns the element corresponding to i
  value   : % -> I
    ++ value(x) returns the integer associated with x

```

```

Implement == add

```

```

  Rep := Integer
  x,y: %

```

```

  x = y == x =$Rep y
  x < y == x <$Rep y

```

```

  coerce(i:Integer):% == i

```

```

  value(x) == x:Rep

```

```

  coerce(x):O ==
    sub(e::Symbol::O, coerce(x)$Rep)$O

```

—————

— OSI.dotabb —

```

"OSI" [color="#88FF44",href="bookvol10.3.pdf#nameddest=OSI"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"OSI" -> "ALIST"

```

## 16.21 domain OUTPUTFORM OutputForm

— OutputForm.input —

```
)set break resume
)sys rm -f OutputForm.output
)spool OutputForm.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OutputForm
--R OutputForm is a domain constructor
--R Abbreviation for OutputForm is OUTPUTFORM
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for OUTPUTFORM
--R
--R----- Operations -----
--R ?? : (% , %) -> %
--R ?+? : (% , %) -> %
--R ?-? : (% , %) -> %
--R ?<? : (% , %) -> %
--R ?=? : (% , %) -> %
--R ?>? : (% , %) -> %
--R ?SEGMENT : % -> %
--R ?^=? : (% , %) -> %
--R assign : (% , %) -> %
--R blankSeparate : List % -> %
--R brace : List % -> %
--R bracket : List % -> %
--R center : % -> %
--R coerce : % -> OutputForm
--R ?div? : (% , %) -> %
--R ?.? : (% , List %) -> %
--R exquo : (% , %) -> %
--R hconcat : List % -> %
--R height : () -> Integer
--R hspace : Integer -> %
--R infix : (% , List %) -> %
--R int : (% , % , %) -> %
--R int : % -> %
--R latex : % -> String
--R ***? : (% , %) -> %
--R -? : % -> %
--R ?/? : (% , %) -> %
--R ?<=? : (% , %) -> %
--R ?=? : (% , %) -> Boolean
--R ?>=? : (% , %) -> %
--R ?..? : (% , %) -> %
--R ?and? : (% , %) -> %
--R binomial : (% , %) -> %
--R box : % -> %
--R brace : % -> %
--R bracket : % -> %
--R center : (% , Integer) -> %
--R commaSeparate : List % -> %
--R dot : % -> %
--R empty : () -> %
--R hash : % -> SingleInteger
--R hconcat : (% , %) -> %
--R height : % -> Integer
--R infix : (% , % , %) -> %
--R infix? : % -> Boolean
--R int : (% , %) -> %
--R label : (% , %) -> %
--R left : % -> %
```



```

--R left : (%,Integer) -> %
--R message : String -> %
--R not? : % -> %
--R outputForm : DoubleFloat -> %
--R outputForm : Symbol -> %
--R over : (%,%) -> %
--R overlabel : (%,%) -> %
--R paren : % -> %
--R postfix : (%,%) -> %
--R presub : (%,%) -> %
--R prime : % -> %
--R prod : (%,%,%) -> %
--R prod : % -> %
--R quote : % -> %
--R ?rem? : (%,%) -> %
--R right : (%,Integer) -> %
--R root : % -> %
--R scripts : (%,List %) -> %
--R slash : (%,%) -> %
--R sub : (%,%) -> %
--R sum : (%,%,%) -> %
--R sum : % -> %
--R superHeight : % -> Integer
--R vconcat : List % -> %
--R vspace : Integer -> %
--R width : % -> Integer
--R ?~=?: (%,%) -> Boolean
--R differentiate : (%,NonNegativeInteger) -> %
--R dot : (%,NonNegativeInteger) -> %
--R prime : (%,NonNegativeInteger) -> %
--R
--E 1

)spool
)lisp (bye)

```

---

— OutputForm.help —

```

=====
OutputForm examples
=====

```

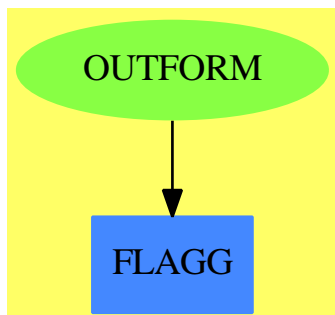
```

See Also:
o )show OutputForm

```

---

## 16.21.1 OutputForm (OUTPUTFORM)

**Exports:**

|              |                   |               |               |               |
|--------------|-------------------|---------------|---------------|---------------|
| assign       | binomial          | blankSeparate | box           | brace         |
| bracket      | center            | coerce        | commaSeparate | differentiate |
| dot          | empty             | exquo         | hash          | hconcat       |
| height       | hspace            | infix         | infix?        | int           |
| label        | latex             | left          | matrix        | message       |
| messagePrint | not?              | outputForm    | over          | overbar       |
| overlabel    | paren             | pile          | postfix       | prefix        |
| presub       | presuper          | prime         | print         | prod          |
| quote        | rarrow            | right         | root          | rspace        |
| scripts      | semicolonSeparate | slash         | string        | sub           |
| subHeight    | sum               | super         | superHeight   | supersub      |
| vconcat      | vspace            | width         | zag           | ?*?           |
| ?**?         | ?+?               | -?            | ?-?           | ?/?           |
| ?<?          | ?<=?              | ?=?           | ?>?           | ?>=?          |
| ?SEGMENT     | ?..?              | ?^=?          | ?and?         | ?div?         |
| ?..?         | ?or?              | ?quo?         | ?rem?         | ?~=?          |

— domain OUTPUTFORM OutputForm —

```

)abbrev domain OUTPUTFORM OutputForm
++ Keywords: output, I/O, expression
++ SMW March/88
++ Description:
++ This domain is used to create and manipulate mathematical expressions
++ for output. It is intended to provide an insulating layer between
++ the expression rendering software (e.g.FORTRAN, TeX, or Script) and
++ the output coercions in the various domains.

```

```

OutputForm(): SetCategory with
  --% Printing
  print : $ -> Void
  ++ print(u) prints the form u.
  message: String -> $

```

```

    ++ message(s) creates an form with no string quotes
    ++ from string s.
messagePrint: String -> Void
    ++ messagePrint(s) prints s without string quotes. Note:
    ++ \spad{messagePrint(s)} is equivalent to \spad{print message(s)}.
--% Creation of atomic forms
outputForm: Integer -> $
    ++ outputForm(n) creates an form for integer n.
outputForm: Symbol -> $
    ++ outputForm(s) creates an form for symbol s.
outputForm: String -> $
    ++ outputForm(s) creates an form for string s.
outputForm: DoubleFloat -> $
    ++ outputForm(sf) creates an form for small float sf.
empty : () -> $
    ++ empty() creates an empty form.

--% Sizings
width: $ -> Integer
    ++ width(f) returns the width of form f (an integer).
height: $ -> Integer
    ++ height(f) returns the height of form f (an integer).
width: -> Integer
    ++ width() returns the width of the display area (an integer).
height: -> Integer
    ++ height() returns the height of the display area (an integer).
subHeight: $ -> Integer
    ++ subHeight(f) returns the height of form f below the base line.
superHeight: $ -> Integer
    ++ superHeight(f) returns the height of form f above the base line.
--% Space manipulations
hspace: Integer -> $ ++ hspace(n) creates white space of width n.
vspace: Integer -> $ ++ vspace(n) creates white space of height n.
rspace: (Integer,Integer) -> $
    ++ rspace(n,m) creates rectangular white space, n wide by m high.
--% Area adjustments
left: ($,Integer) -> $
    ++ left(f,n) left-justifies form f within space of width n.
right: ($,Integer) -> $
    ++ right(f,n) right-justifies form f within space of width n.
center: ($,Integer) -> $
    ++ center(f,n) centers form f within space of width n.
left: $ -> $
    ++ left(f) left-justifies form f in total space.
right: $ -> $
    ++ right(f) right-justifies form f in total space.
center: $ -> $
    ++ center(f) centers form f in total space.

--% Area manipulations

```

```

hconcat: ($,$) -> $
    ++ hconcat(f,g) horizontally concatenate forms f and g.
vconcat: ($,$) -> $
    ++ vconcat(f,g) vertically concatenates forms f and g.
hconcat: List $ -> $
    ++ hconcat(u) horizontally concatenates all forms in list u.
vconcat: List $ -> $
    ++ vconcat(u) vertically concatenates all forms in list u.

--% Application formers
prefix: ($, List $) -> $
    ++ prefix(f,l) creates a form depicting the n-ary prefix
    ++ application of f to a tuple of arguments given by list l.
infix: ($, List $) -> $
    ++ infix(f,l) creates a form depicting the n-ary application
    ++ of infix operation f to a tuple of arguments l.
infix: ($, $, $) -> $
    ++ infix(op, a, b) creates a form which prints as: a op b.
postfix: ($, $) -> $
    ++ postfix(op, a) creates a form which prints as: a op.
infix?: $ -> Boolean
    ++ infix?(op) returns true if op is an infix operator,
    ++ and false otherwise.
elt: ($, List $) -> $
    ++ elt(op,l) creates a form for application of op
    ++ to list of arguments l.

--% Special forms
string: $ -> $
    ++ string(f) creates f with string quotes.
label: ($, $) -> $
    ++ label(n,f) gives form f an equation label n.
box: $ -> $
    ++ box(f) encloses f in a box.
matrix: List List $ -> $
    ++ matrix(llf) makes llf (a list of lists of forms) into
    ++ a form which displays as a matrix.
zag: ($, $) -> $
    ++ zag(f,g) creates a form for the continued fraction form for f over g.
root: $ -> $
    ++ root(f) creates a form for the square root of form f.
root: ($, $) -> $
    ++ root(f,n) creates a form for the nth root of form f.
over: ($, $) -> $
    ++ over(f,g) creates a form for the vertical fraction of f over g.
slash: ($, $) -> $
    ++ slash(f,g) creates a form for the horizontal fraction of f over g.
assign: ($, $) -> $
    ++ assign(f,g) creates a form for the assignment \spad{f := g}.
rarrow: ($, $) -> $

```

```

++ rarrow(f,g) creates a form for the mapping \spad{f -> g}.
differentiate: ($, NonNegativeInteger) -> $
++ differentiate(f,n) creates a form for the nth derivative of f,
++ e.g. \spad{f'}, \spad{f''}, \spad{f'''},
++ "f super \spad{iv}".
binomial: ($, $) -> $
++ binomial(n,m) creates a form for the binomial coefficient of n and m.

--% Scripts
sub:      ($, $) -> $
++ sub(f,n) creates a form for f subscripted by n.
super:    ($, $) -> $
++ super(f,n) creates a form for f superscripted by n.
presub:   ($, $) -> $
++ presub(f,n) creates a form for f presubscripted by n.
presuper: ($, $) -> $
++ presuper(f,n) creates a form for f presuperscripted by n.
scripts:  ($, List $) -> $
++ \spad{scripts(f, [sub, super, presuper, presub])}
++ creates a form for f with scripts on all 4 corners.
supersub: ($, List $) -> $
++ supersub(a,[sub1,super1,sub2,super2,...])
++ creates a form with each subscript aligned
++ under each superscript.

--% Diacritical marks
quote:    $ -> $
++ quote(f) creates the form f with a prefix quote.
dot:      $ -> $
++ dot(f) creates the form with a one dot overhead.
dot:      ($, NonNegativeInteger) -> $
++ dot(f,n) creates the form f with n dots overhead.
prime:    $ -> $
++ prime(f) creates the form f followed by a suffix prime (single quote).
prime:    ($, NonNegativeInteger) -> $
++ prime(f,n) creates the form f followed by n primes.
overbar:  $ -> $
++ overbar(f) creates the form f with an overbar.
overlabel: ($, $) -> $
++ overlabel(x,f) creates the form f with "x overbar" over the top.

--% Plexes
sum:      ($) -> $
++ sum(expr) creates the form prefixing expr by a capital sigma.
sum:      ($, $) -> $
++ sum(expr,lowerlimit) creates the form prefixing expr by
++ a capital sigma with a lowerlimit.
sum:      ($, $, $) -> $
++ sum(expr,lowerlimit,upperlimit) creates the form prefixing expr by
++ a capital sigma with both a lowerlimit and upperlimit.

```

```

prod:    ($)      -> $
++ prod(expr) creates the form prefixing expr by a capital pi.
prod:    ($, $)   -> $
++ prod(expr,lowerlimit) creates the form prefixing expr by
++ a capital pi with a lowerlimit.
prod:    ($, $, $) -> $
++ prod(expr,lowerlimit,upperlimit) creates the form prefixing expr by
++ a capital pi with both a lowerlimit and upperlimit.
int:     ($)      -> $
++ int(expr) creates the form prefixing expr with an integral sign.
int:     ($, $)   -> $
++ int(expr,lowerlimit) creates the form prefixing expr by an
++ integral sign with a lowerlimit.
int:     ($, $, $) -> $
++ int(expr,lowerlimit,upperlimit) creates the form prefixing expr by
++ an integral sign with both a lowerlimit and upperlimit.

--% Matchfix forms
brace:   $ -> $
++ brace(f) creates the form enclosing f in braces (curly brackets).
brace:   List $ -> $
++ brace(lf) creates the form separating the elements of lf
++ by commas and encloses the result in curly brackets.
bracket: $ -> $
++ bracket(f) creates the form enclosing f in square brackets.
bracket: List $ -> $
++ bracket(lf) creates the form separating the elements of lf
++ by commas and encloses the result in square brackets.
paren:   $ -> $
++ paren(f) creates the form enclosing f in parentheses.
paren:   List $ -> $
++ paren(lf) creates the form separating the elements of lf
++ by commas and encloses the result in parentheses.

--% Separators for aggregates
pile:    List $ -> $
++ pile(l) creates the form consisting of the elements of l which
++ displays as a pile, i.e. the elements begin on a new line and
++ are indented right to the same margin.

commaSeparate: List $ -> $
++ commaSeparate(l) creates the form separating the elements of l
++ by commas.
semicolonSeparate: List $ -> $
++ semicolonSeparate(l) creates the form separating the elements of l
++ by semicolons.
blankSeparate: List $ -> $
++ blankSeparate(l) creates the form separating the elements of l
++ by blanks.
--% Specific applications

```

```

"=:      ($, $) -> $
++ f = g creates the equivalent infix form.
"^=:     ($, $) -> $
++ f ^= g creates the equivalent infix form.
"<=:     ($, $) -> $
++ f < g creates the equivalent infix form.
">=:     ($, $) -> $
++ f > g creates the equivalent infix form.
"<=:     ($, $) -> $
++ f <= g creates the equivalent infix form.
">=:     ($, $) -> $
++ f >= g creates the equivalent infix form.
"+=:     ($, $) -> $
++ f + g creates the equivalent infix form.
"-=:     ($, $) -> $
++ f - g creates the equivalent infix form.
"-=:     ($)   -> $
++ - f creates the equivalent prefix form.
"*=:     ($, $) -> $
++ f * g creates the equivalent infix form.
"/=:     ($, $) -> $
++ f / g creates the equivalent infix form.
"**=:    ($, $) -> $
++ f ** g creates the equivalent infix form.
"div=:   ($, $) -> $
++ f div g creates the equivalent infix form.
"rem=:   ($, $) -> $
++ f rem g creates the equivalent infix form.
"quo=:   ($, $) -> $
++ f quo g creates the equivalent infix form.
"exquo=: ($, $) -> $
++ exquo(f,g) creates the equivalent infix form.
"and=:   ($, $) -> $
++ f and g creates the equivalent infix form.
"or=:    ($, $) -> $
++ f or g creates the equivalent infix form.
"not=:   ($)   -> $
++ not f creates the equivalent prefix form.
SEGMENT: ($,$) -> $
++ SEGMENT(x,y) creates the infix form: \spad{x..y}.
SEGMENT: ($)   -> $
++ SEGMENT(x) creates the prefix form: \spad{x..}.

== add
import NumberFormats

-- Todo:
--   program forms, greek letters
--   infix, prefix, postfix, matchfix support in OUT BOOT
--   labove rabove, corresponding overs.

```

```

--    better super script, overmark, undermark
--    bug in product, paren blankSeparate []
--    uniformize integrals, products, etc as plexes.

cons ==> CONS$Lisp
car  ==> CAR$Lisp
cdr  ==> CDR$Lisp

Rep := List $

a, b: $
l: List $
s: String
e: Symbol
n: Integer
nn: NonNegativeInteger

sform:    String -> $
eform:    Symbol -> $
iform:    Integer -> $

print x          == mathprint(x)$Lisp
message s        == (empty? s => empty(); s pretend $)
messagePrint s   == print message s
(a:$ = b:$):Boolean == EQUAL(a, b)$Lisp
(a:$ = b:$):$     == [sform "=", a, b]
coerce(a):OutputForm == a pretend OutputForm
outputForm n     == n pretend $
outputForm e     == e pretend $
outputForm(f:DoubleFloat) == f pretend $
sform s          == s pretend $
eform e          == e pretend $
iform n          == n pretend $

outputForm s ==
    sform concat(quote())$Character, concat(s, quote())$Character))

width(a) == outformWidth(a)$Lisp
height(a) == height(a)$Lisp
subHeight(a) == subspan(a)$Lisp
superHeight(a) == superspan(a)$Lisp
height() == 20
width() == 66

center(a,w) == hconcat(hspace((w - width(a)) quo 2),a)
left(a,w) == hconcat(a,hspace((w - width(a))))
right(a,w) == hconcat(hspace(w - width(a)),a)
center(a) == center(a,width())
left(a) == left(a,width())
right(a) == right(a,width())

```



```

vspace(n) ==
  n = 0 => empty()
  vconcat(sform " ",vspace(n - 1))

hspace(n) ==
  n = 0 => empty()
  sform(fillerSpaces(n)$Lisp)

rspace(n, m) ==
  n = 0 or m = 0 => empty()
  vconcat(hspace n, rspace(n, m - 1))

matrix ll ==
  lv:$ := [LIST2VEC$Lisp l for l in ll]
  CONS(eform MATRIX, LIST2VEC$Lisp lv)$Lisp

pile l          == cons(eform SC, l)
commaSeparate l == cons(eform AGGLST, l)
semicolonSeparate l == cons(eform AGGSET, l)
blankSeparate l ==
  c:=eform CONCATB
  l1:$:=[]
  for u in reverse l repeat
    if EQCAR(u,c)$Lisp
      then l1:=[:cdr u,:l1]
      else l1:=[:u,:l1]
  cons(c, l1)

brace a        == [eform BRACE, a]
brace l        == brace commaSeparate l
bracket a      == [eform BRACKET, a]
bracket l      == bracket commaSeparate l
paren a        == [eform PAREN, a]
paren l        == paren commaSeparate l

sub (a,b) == [eform SUB, a, b]
super (a, b) == [eform SUPERSUB,a,sform " ",b]
presub(a,b) == [eform SUPERSUB,a,sform " ",sform " ",sform " ",b]
presuper(a, b) == [eform SUPERSUB,a,sform " ",sform " ",b]
scripts (a, l) ==
  null l => a
  null rest l => sub(a, first l)
  cons(eform SUPERSUB, cons(a, l))
supersub(a, l) ==
  if odd? (#l) then l := append(l, [empty()])
  cons(eform ALTSUPERSUB, cons(a, l))

hconcat(a,b) == [eform CONCAT, a, b]
hconcat l == cons(eform CONCAT, l)

```

```

vconcat(a,b) == [eform VCONCAT, a, b]
vconcat l    == cons(eform VCONCAT, l)

a ^= b      == [sform "^=", a, b]
a < b       == [sform "<", a, b]
a > b       == [sform ">", a, b]
a <= b      == [sform "<=", a, b]
a >= b      == [sform ">=", a, b]

a + b       == [sform "+", a, b]
a - b       == [sform "-", a, b]
- a         == [sform "-", a]
a * b       == [sform "*", a, b]
a / b       == [sform "/", a, b]
a ** b      == [sform "**", a, b]
a div b     == [sform "div", a, b]
a rem b     == [sform "rem", a, b]
a quo b     == [sform "quo", a, b]
a exquo b   == [sform "exquo", a, b]
a and b     == [sform "and", a, b]
a or b      == [sform "or", a, b]
not a       == [sform "not", a]
SEGMENT(a,b)== [eform SEGMENT, a, b]
SEGMENT(a) == [eform SEGMENT, a]
binomial(a,b)==[eform BINOMIAL, a, b]

empty() == [eform NOTHING]

infix? a ==
  e:$ :=
    IDENTP$Lisp a => a
    STRINGP$Lisp a => INTERN$Lisp a
    return false
  if GET(e,QUOTE(INFIXOP$Lisp)$Lisp)$Lisp then true else false

elt(a, l) ==
  cons(a, l)
prefix(a,l) ==
  not infix? a => cons(a, l)
  hconcat(a, paren commaSeparate l)
infix(a, l) ==
  null l => empty()
  null rest l => first l
  infix? a => cons(a, l)
  hconcat [first l, a, infix(a, rest l)]
infix(a,b,c) ==
  infix? a => [a, b, c]
  hconcat [b, a, c]
postfix(a, b) ==
  hconcat(b, a)

```

```

string a == [eform STRING, a]
quote a == [eform QUOTE, a]
overbar a == [eform OVERBAR, a]
dot a == super(a, sform ".")
prime a == super(a, sform ",")
dot(a,nn) == (s := new(nn, char "."); super(a, sform s))
prime(a,nn) == (s := new(nn, char ","); super(a, sform s))

overlabel(a,b) == [eform OVERLABEL, a, b]
box a == [eform BOX, a]
zag(a,b) == [eform ZAG, a, b]
root a == [eform ROOT, a]
root(a,b) == [eform ROOT, a, b]
over(a,b) == [eform OVER, a, b]
slash(a,b) == [eform SLASH, a, b]
assign(a,b) == [eform LET, a, b]

label(a,b) == [eform EQUATNUM, a, b]
rarrow(a,b) == [eform TAG, a, b]
differentiate(a, nn) ==
  zero? nn => a
  nn < 4 => prime(a, nn)
  r := FormatRoman(nn::PositiveInteger)
  s := lowerCase(r::String)
  super(a, paren sform s)

sum(a) == [eform SIGMA, empty(), a]
sum(a,b) == [eform SIGMA, b, a]
sum(a,b,c) == [eform SIGMA2, b, c, a]
prod(a) == [eform PI, empty(), a]
prod(a,b) == [eform PI, b, a]
prod(a,b,c) == [eform PI2, b, c, a]
int(a) == [eform INTSIGN, empty(), empty(), a]
int(a,b) == [eform INTSIGN, b, empty(), a]
int(a,b,c) == [eform INTSIGN, b, c, a]

```

---

— **OUTFORM.dotabb** —

```

"OUTFORM" [color="#88FF44",href="bookvol10.3.pdf#nameddest=OUTFORM",
           shape=ellipse]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"OUTFORM" -> "FLAGG"

```

---

## Chapter 17

# Chapter P

### 17.1 domain PADIC PAdicInteger

— PAdicInteger.input —

```
)set break resume
)sys rm -f PAdicInteger.output
)spool PAdicInteger.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PAdicInteger
--R PAdicInteger p: Integer is a domain constructor
--R Abbreviation for PAdicInteger is PADIC
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PADIC
--R
--R----- Operations -----
--R ??? : (%,%) -> %                ??? : (Integer,%) -> %
--R ??? : (PositiveInteger,%) -> %  ??? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                ?-? : (%,%) -> %
--R -? : % -> %                     ?=? : (%,%) -> Boolean
--R 1 : () -> %                     0 : () -> %
--R ?? : (%,PositiveInteger) -> %  associates? : (%,%) -> Boolean
--R coerce : % -> %                 coerce : Integer -> %
--R coerce : % -> OutputForm         complete : % -> %
--R digits : % -> Stream Integer     extend : (%,Integer) -> %
--R gcd : List % -> %                gcd : (%,%) -> %
--R hash : % -> SingleInteger        latex : % -> String
--R lcm : List % -> %                lcm : (%,%) -> %
```

```

--R moduloP : % -> Integer
--R one? : % -> Boolean
--R ?quo? : (% , %) -> %
--R recip : % -> Union(%, "failed")
--R sample : () -> %
--R sqrt : (% , Integer) -> %
--R unitCanonical : % -> %
--R ?~=?: (% , %) -> Boolean
--R ?*?: (NonNegativeInteger, %) -> %
--R ?**?: (% , NonNegativeInteger) -> %
--R ?^?: (% , NonNegativeInteger) -> %
--R approximate : (% , Integer) -> Integer
--R characteristic : () -> NonNegativeInteger
--R divide : (% , %) -> Record(quotient: %, remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %, %) -> Union(List %, "failed")
--R exquo : (% , %) -> Union(%, "failed")
--R extendedEuclidean : (% , %, %) -> Union(Record(coef1: %, coef2: %), "failed")
--R extendedEuclidean : (% , %) -> Record(coef1: %, coef2: %, generator: %)
--R gcdPolynomial : (SparseUnivariatePolynomial %, SparseUnivariatePolynomial %) -> SparseUni
--R multiEuclidean : (List %, %) -> Union(List %, "failed")
--R principalIdeal : List % -> Record(coef: List %, generator: %)
--R root : (SparseUnivariatePolynomial Integer, Integer) -> %
--R subtractIfCan : (% , %) -> Union(%, "failed")
--R unitNormal : % -> Record(unit: %, canonical: %, associate: %)
--R
--E 1

)spool
)lisp (bye)

```

---

— PAdicInteger.help —

```

=====
PAdicInteger examples
=====

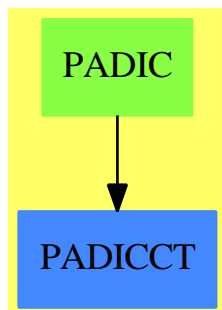
```

```

See Also:
o )show PAdicInteger

```

## 17.1.1 PAdicInteger (PADIC)



See

- ⇒ “InnerPAdicInteger” (IPADIC) 10.24.1 on page 1258
- ⇒ “BalancedPAdicInteger” (BPADIC) 3.2.1 on page 240
- ⇒ “PAdicRationalConstructor” (PADICRC) 17.3.1 on page 1850
- ⇒ “PAdicRational” (PADICRAT) 17.2.1 on page 1845
- ⇒ “BalancedPAdicRational” (BPADICRT) 3.3.1 on page 244

**Exports:**

|                |                   |                    |               |
|----------------|-------------------|--------------------|---------------|
| 0              | 1                 | approximate        | associates?   |
| characteristic | coerce            | complete           | digits        |
| divide         | euclideanSize     | expressIdealMember | exquo         |
| extend         | extendedEuclidean | gcd                | gcdPolynomial |
| hash           | latex             | lcm                | lcm           |
| moduloP        | modulus           | multiEuclidean     | one?          |
| order          | principalIdeal    | quotientByP        | recip         |
| root           | sample            | sizeLess?          | sqrt          |
| subtractIfCan  | unit?             | unitCanonical      | unitNormal    |
| zero?          | ?*?               | ?**?               | ?+?           |
| ?-?            | -?                | ?=?                | ?^?           |
| ?~=?           | ?quo?             | ?rem?              |               |

— domain PADIC PAdicInteger —

```

)abbrev domain PADIC PAdicInteger
++ Author: Clifton J. Williamson
++ Date Created: 20 August 1989
++ Date Last Updated: 15 May 1990
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Keywords: p-adic, completion
++ Examples:
++ References:

```

```

++ Description:
++ Stream-based implementation of  $\mathbb{Z}_p$ : p-adic numbers are represented as
++  $\sum_{i=0..} a[i] \cdot p^i$ , where the  $a[i]$  lie in  $0, 1, \dots, (p - 1)$ .

PAdicInteger(p:Integer) == InnerPAdicInteger(p,true$Boolean)

```

---

— PADIC.dotabb —

```

"PADIC" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PADIC"]
"PADICCT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PADICCT"]
"PADIC" -> "PADICCT"

```

---

## 17.2 domain PADICRAT PAdicRational

— PAdicRational.input —

```

)set break resume
)sys rm -f PAdicRational.output
)spool PAdicRational.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PAdicRational
--R PAdicRational p: Integer is a domain constructor
--R Abbreviation for PAdicRational is PADICRAT
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PADICRAT
--R
--R----- Operations -----
--R ?? : (%,PAdicInteger p) -> %          ??? : (PAdicInteger p,%) -> %
--R ?? : (Fraction Integer,%) -> %        ??? : (%,Fraction Integer) -> %
--R ?? : (%,%) -> %                       ??? : (Integer,%) -> %
--R ?? : (PositiveInteger,%) -> %          ??? : (%,Integer) -> %
--R ??? : (%,PositiveInteger) -> %         ??+ : (%,%) -> %
--R ?-? : (%,%) -> %                       -? : % -> %
--R ?/? : (%,%) -> %                       ?? : (%,%) -> Boolean
--R 1 : () -> %                             0 : () -> %
--R ?? : (%,Integer) -> %                   ?? : (%,PositiveInteger) -> %

```

```

--R associates? : (%,%) -> Boolean
--R coerce : Fraction Integer -> %
--R coerce : Integer -> %
--R denom : % -> PAdicInteger p
--R factor : % -> Factored %
--R gcd : (%,%) -> %
--R inv : % -> %
--R lcm : List % -> %
--R numer : % -> PAdicInteger p
--R one? : % -> Boolean
--R ?quo? : (%,%) -> %
--R ?rem? : (%,%) -> %
--R removeZeroes : % -> %
--R sample : () -> %
--R squareFree : % -> Factored %
--R unit? : % -> Boolean
--R zero? : % -> Boolean
--R ?? : (NonNegativeInteger,%) -> %
--R ??? : (%,NonNegativeInteger) -> %
--R ?/? : (PAdicInteger p,PAdicInteger p) -> %
--R ?<? : (%,%) -> Boolean if PAdicInteger p has ORDSET
--R ?<=? : (%,%) -> Boolean if PAdicInteger p has ORDSET
--R ?>? : (%,%) -> Boolean if PAdicInteger p has ORDSET
--R ?>=? : (%,%) -> Boolean if PAdicInteger p has ORDSET
--R D : (%,(PAdicInteger p -> PAdicInteger p)) -> %
--R D : (%,(PAdicInteger p -> PAdicInteger p),NonNegativeInteger) -> %
--R D : (%,List Symbol,List NonNegativeInteger) -> % if PAdicInteger p has PDRING SYMBOL
--R D : (%,Symbol,NonNegativeInteger) -> % if PAdicInteger p has PDRING SYMBOL
--R D : (%,List Symbol) -> % if PAdicInteger p has PDRING SYMBOL
--R D : (%,Symbol) -> % if PAdicInteger p has PDRING SYMBOL
--R D : (%,NonNegativeInteger) -> % if PAdicInteger p has DIFRING
--R D : % -> % if PAdicInteger p has DIFRING
--R ?? : (%,NonNegativeInteger) -> %
--R abs : % -> % if PAdicInteger p has OINTDOM
--R approximate : (%,Integer) -> Fraction Integer
--R ceiling : % -> PAdicInteger p if PAdicInteger p has INS
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%, "failed") if $ has CHARNZ and PAdicInteger p has PFECAT or PAdicInteger p
--R coerce : Symbol -> % if PAdicInteger p has RETRACT SYMBOL
--R conditionP : Matrix % -> Union(Vector %, "failed") if $ has CHARNZ and PAdicInteger p has PFECAT
--R continuedFraction : % -> ContinuedFraction Fraction Integer
--R convert : % -> DoubleFloat if PAdicInteger p has REAL
--R convert : % -> Float if PAdicInteger p has REAL
--R convert : % -> InputForm if PAdicInteger p has KONVERT INFORM
--R convert : % -> Pattern Float if PAdicInteger p has KONVERT PATTERN FLOAT
--R convert : % -> Pattern Integer if PAdicInteger p has KONVERT PATTERN INT
--R differentiate : (%,(PAdicInteger p -> PAdicInteger p)) -> %
--R differentiate : (%,(PAdicInteger p -> PAdicInteger p),NonNegativeInteger) -> %
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if PAdicInteger p has PDRING SYMBOL
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if PAdicInteger p has PDRING SYMBOL
--R coerce : PAdicInteger p -> %
--R coerce : % -> %
--R coerce : % -> OutputForm
--R denominator : % -> %
--R gcd : List % -> %
--R hash : % -> SingleInteger
--R latex : % -> String
--R lcm : (%,%) -> %
--R numerator : % -> %
--R prime? : % -> Boolean
--R recip : % -> Union(%, "failed")
--R removeZeroes : (Integer,%) -> %
--R retract : % -> PAdicInteger p
--R sizeLess? : (%,%) -> Boolean
--R squareFreePart : % -> %
--R unitCanonical : % -> %
--R ~=? : (%,%) -> Boolean

```



```

--R differentiate : (%,List Symbol) -> % if PAdicInteger p has PDRING SYMBOL
--R differentiate : (%,Symbol) -> % if PAdicInteger p has PDRING SYMBOL
--R differentiate : (%,NonNegativeInteger) -> % if PAdicInteger p has DIFRING
--R differentiate : % -> % if PAdicInteger p has DIFRING
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R ?.? : (%,PAdicInteger p) -> % if PAdicInteger p has ELTAB(PADIC p,PADIC p)
--R euclideanSize : % -> NonNegativeInteger
--R eval : (%,Symbol,PAdicInteger p) -> % if PAdicInteger p has IEVALAB(SYMBOL,PADIC p)
--R eval : (%,List Symbol,List PAdicInteger p) -> % if PAdicInteger p has IEVALAB(SYMBOL,PADIC p)
--R eval : (%,List Equation PAdicInteger p) -> % if PAdicInteger p has EVALAB PADIC p
--R eval : (%,Equation PAdicInteger p) -> % if PAdicInteger p has EVALAB PADIC p
--R eval : (%,PAdicInteger p,PAdicInteger p) -> % if PAdicInteger p has EVALAB PADIC p
--R eval : (%,List PAdicInteger p,List PAdicInteger p) -> % if PAdicInteger p has EVALAB PADIC p
--R expressIdealMember : (List %,%) -> Union(List %,"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial %
--R factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial %
--R floor : % -> PAdicInteger p if PAdicInteger p has INS
--R fractionPart : % -> % if PAdicInteger p has EUCDOM
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolynomial %
--R init : () -> % if PAdicInteger p has STEP
--R map : ((PAdicInteger p -> PAdicInteger p),%) -> %
--R max : (%,%) -> % if PAdicInteger p has ORDSET
--R min : (%,%) -> % if PAdicInteger p has ORDSET
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R negative? : % -> Boolean if PAdicInteger p has OINTDOM
--R nextItem : % -> Union(%,"failed") if PAdicInteger p has STEP
--R patternMatch : (%,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchResult(Float,%)
--R patternMatch : (%,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMatchResult(Integer,%)
--R positive? : % -> Boolean if PAdicInteger p has OINTDOM
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R random : () -> % if PAdicInteger p has INS
--R reducedSystem : Matrix % -> Matrix PAdicInteger p
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix PAdicInteger p,vec: Vector PAdicInteger p)
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) if PAdicInteger p has LINEXP INT
--R reducedSystem : Matrix % -> Matrix Integer if PAdicInteger p has LINEXP INT
--R retract : % -> Integer if PAdicInteger p has RETRACT INT
--R retract : % -> Fraction Integer if PAdicInteger p has RETRACT INT
--R retract : % -> Symbol if PAdicInteger p has RETRACT SYMBOL
--R retractIfCan : % -> Union(Integer,"failed") if PAdicInteger p has RETRACT INT
--R retractIfCan : % -> Union(Fraction Integer,"failed") if PAdicInteger p has RETRACT INT
--R retractIfCan : % -> Union(Symbol,"failed") if PAdicInteger p has RETRACT SYMBOL
--R retractIfCan : % -> Union(PAdicInteger p,"failed")
--R sign : % -> Integer if PAdicInteger p has OINTDOM
--R solveLinearPolynomialEquation : (List SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolynomial %
--R squareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)

```

```
--R wholePart : % -> PAdicInteger p if PAdicInteger p has EUCDOM
--R
--E 1
```

```
)spool
)lisp (bye)
```

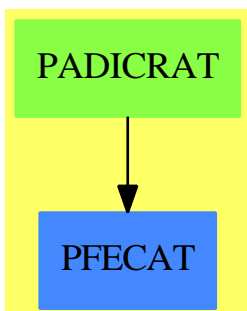
---

— PAdicRational.help —

```
=====
PAdicRational examples
=====
```

```
See Also:
o )show PAdicRational
```

### 17.2.1 PAdicRational (PADICRAT)



See

- ⇒ “InnerPAdicInteger” (IPADIC) 10.24.1 on page 1258
- ⇒ “PAdicInteger” (PADIC) 17.1.1 on page 1841
- ⇒ “BalancedPAdicInteger” (BPADIC) 3.2.1 on page 240
- ⇒ “PAdicRationalConstructor” (PADICRC) 17.3.1 on page 1850
- ⇒ “BalancedPAdicRational” (BPADICRT) 3.3.1 on page 244

**Exports:**

|                            |                    |                               |
|----------------------------|--------------------|-------------------------------|
| 0                          | 1                  | abs                           |
| approximate                | associates?        | ceiling                       |
| characteristic             | charthRoot         | coerce                        |
| conditionP                 | continuedFraction  | convert                       |
| D                          | denom              | denominator                   |
| differentiate              | divide             | euclideanSize                 |
| eval                       | expressIdealMember | exquo                         |
| extendedEuclidean          | factor             | factorPolynomial              |
| factorSquareFreePolynomial | floor              | fractionPart                  |
| gcd                        | gcdPolynomial      | hash                          |
| init                       | inv                | latex                         |
| lcm                        | map                | max                           |
| min                        | multiEuclidean     | negative?                     |
| nextItem                   | numer              | numerator                     |
| one?                       | patternMatch       | positive?                     |
| prime?                     | principalIdeal     | random                        |
| recip                      | reducedSystem      | removeZeroes                  |
| retract                    | retractIfCan       | sample                        |
| sign                       | sizeLess?          | solveLinearPolynomialEquation |
| squareFree                 | squareFreePart     | squareFreePolynomial          |
| subtractIfCan              | unit?              | unitCanonical                 |
| unitNormal                 | wholePart          | zero?                         |
| ?*?                        | ?**?               | ?+?                           |
| ?-?                        | -?                 | ?/?                           |
| ?=?                        | ?^?                | ?~=?                          |
| ?<?                        | ?<=?               | ?>?                           |
| ?>=?                       | ?..?               | ?quo?                         |
| ?rem?                      |                    |                               |

— domain PADICRAT PAdicRational —

```

)abbrev domain PADICRAT PAdicRational
++ Author: Clifton J. Williamson
++ Date Created: 15 May 1990
++ Date Last Updated: 15 May 1990
++ Keywords: p-adic, complementation
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: p-adic, completion
++ Examples:
++ References:
++ Description:
++ Stream-based implementation of Qp: numbers are represented as
++ sum(i = k.., a[i] * p^i) where the a[i] lie in 0,1,...,(p - 1).

PAdicRational(p:Integer) == PAdicRationalConstructor(p,PAdicInteger p)

```

---

— PADICRAT.dotabb —

```
"PADICRAT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PADICRAT"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"PADICRAT" -> "PFECAT"
```

---

## 17.3 domain PADICRC PAdicRationalConstructor

— PAdicRationalConstructor.input —

```
)set break resume
)sys rm -f PAdicRationalConstructor.output
)spool PAdicRationalConstructor.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PAdicRationalConstructor
--R PAdicRationalConstructor(p: Integer,PADIC: PAdicIntegerCategory p) is a domain constructor
--R Abbreviation for PAdicRationalConstructor is PADICRC
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PADICRC
--R
--R----- Operations -----
--R ??? : (% ,PADIC) -> %
--R ??? : (Fraction Integer,% ) -> %
--R ??? : (% ,%) -> %
--R ??? : (PositiveInteger,% ) -> %
--R ??? : (% ,PositiveInteger) -> %
--R ?-? : (% ,%) -> %
--R ?/? : (PADIC,PADIC) -> %
--R ?? =? : (% ,%) -> Boolean
--R 1 : () -> %
--R ??^? : (% ,Integer) -> %
--R associates? : (% ,%) -> Boolean
--R coerce : Fraction Integer -> %
--R coerce : Integer -> %
--R denom : % -> PADIC
--R ??? : (PADIC,% ) -> %
--R ??? : (% ,Fraction Integer) -> %
--R ??? : (Integer,% ) -> %
--R ??? : (% ,Integer) -> %
--R ?+? : (% ,%) -> %
--R -? : % -> %
--R ?/? : (% ,%) -> %
--R D : (% ,(PADIC -> PADIC)) -> %
--R 0 : () -> %
--R ??^? : (% ,PositiveInteger) -> %
--R coerce : PADIC -> %
--R coerce : % -> %
--R coerce : % -> OutputForm
--R denominator : % -> %
```

```

--R factor : % -> Factored %
--R gcd : (% , %) -> %
--R inv : % -> %
--R lcm : List % -> %
--R map : ((PADIC -> PADIC) , %) -> %
--R numerator : % -> %
--R prime? : % -> Boolean
--R recip : % -> Union(%, "failed")
--R removeZeroes : (Integer , %) -> %
--R retract : % -> PADIC
--R sizeLess? : (% , %) -> Boolean
--R squareFreePart : % -> %
--R unitCanonical : % -> %
--R ?~=? : (% , %) -> Boolean
--R ?? : (NonNegativeInteger , %) -> %
--R ***? : (% , NonNegativeInteger) -> %
--R ?<? : (% , %) -> Boolean if PADIC has ORDSET
--R ?<=? : (% , %) -> Boolean if PADIC has ORDSET
--R ?>? : (% , %) -> Boolean if PADIC has ORDSET
--R ?>=? : (% , %) -> Boolean if PADIC has ORDSET
--R D : (% , (PADIC -> PADIC) , NonNegativeInteger) -> %
--R D : (% , List Symbol , List NonNegativeInteger) -> % if PADIC has PDRING SYMBOL
--R D : (% , Symbol , NonNegativeInteger) -> % if PADIC has PDRING SYMBOL
--R D : (% , List Symbol) -> % if PADIC has PDRING SYMBOL
--R D : (% , Symbol) -> % if PADIC has PDRING SYMBOL
--R D : (% , NonNegativeInteger) -> % if PADIC has DIFRING
--R D : % -> % if PADIC has DIFRING
--R ?^? : (% , NonNegativeInteger) -> %
--R abs : % -> % if PADIC has OINTDOM
--R approximate : (% , Integer) -> Fraction Integer
--R ceiling : % -> PADIC if PADIC has INS
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%, "failed") if $ has CHARNZ and PADIC has PFECAT or PADIC has CH
--R coerce : Symbol -> % if PADIC has RETRACT SYMBOL
--R conditionP : Matrix % -> Union(Vector %, "failed") if $ has CHARNZ and PADIC has PFECAT
--R continuedFraction : % -> ContinuedFraction Fraction Integer
--R convert : % -> DoubleFloat if PADIC has REAL
--R convert : % -> Float if PADIC has REAL
--R convert : % -> InputForm if PADIC has KONVERT INFORM
--R convert : % -> Pattern Float if PADIC has KONVERT PATTERN FLOAT
--R convert : % -> Pattern Integer if PADIC has KONVERT PATTERN INT
--R differentiate : (% , (PADIC -> PADIC)) -> %
--R differentiate : (% , (PADIC -> PADIC) , NonNegativeInteger) -> %
--R differentiate : (% , List Symbol , List NonNegativeInteger) -> % if PADIC has PDRING SYMBOL
--R differentiate : (% , Symbol , NonNegativeInteger) -> % if PADIC has PDRING SYMBOL
--R differentiate : (% , List Symbol) -> % if PADIC has PDRING SYMBOL
--R differentiate : (% , Symbol) -> % if PADIC has PDRING SYMBOL
--R differentiate : (% , NonNegativeInteger) -> % if PADIC has DIFRING
--R differentiate : % -> % if PADIC has DIFRING
--R divide : (% , %) -> Record(quotient: % , remainder: %)

```

```

--R ?.? : (% , PADIC) -> % if PADIC has ELTAB(PADIC, PADIC)
--R euclideanSize : % -> NonNegativeInteger
--R eval : (% , Symbol, PADIC) -> % if PADIC has IEVALAB(SYMBOL, PADIC)
--R eval : (% , List Symbol, List PADIC) -> % if PADIC has IEVALAB(SYMBOL, PADIC)
--R eval : (% , List Equation PADIC) -> % if PADIC has EVALAB PADIC
--R eval : (% , Equation PADIC) -> % if PADIC has EVALAB PADIC
--R eval : (% , PADIC, PADIC) -> % if PADIC has EVALAB PADIC
--R eval : (% , List PADIC, List PADIC) -> % if PADIC has EVALAB PADIC
--R expressIdealMember : (List % , %) -> Union(List % , "failed")
--R exquo : (% , %) -> Union(% , "failed")
--R extendedEuclidean : (% , % , %) -> Union(Record(coef1: % , coef2: % ) , "failed")
--R extendedEuclidean : (% , %) -> Record(coef1: % , coef2: % , generator: %)
--R factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % if PADIC has
--R factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % if
--R floor : % -> PADIC if PADIC has INS
--R fractionPart : % -> % if PADIC has EUCDOM
--R gcdPolynomial : (SparseUnivariatePolynomial % , SparseUnivariatePolynomial %) -> SparseUnivariatePolynomial %
--R init : () -> % if PADIC has STEP
--R max : (% , %) -> % if PADIC has ORDSET
--R min : (% , %) -> % if PADIC has ORDSET
--R multiEuclidean : (List % , %) -> Union(List % , "failed")
--R negative? : % -> Boolean if PADIC has OINTDOM
--R nextItem : % -> Union(% , "failed") if PADIC has STEP
--R patternMatch : (% , Pattern Float, PatternMatchResult(Float, %)) -> PatternMatchResult(Float, %) if PADIC has
--R patternMatch : (% , Pattern Integer, PatternMatchResult(Integer, %)) -> PatternMatchResult(Integer, %) if
--R positive? : % -> Boolean if PADIC has OINTDOM
--R principalIdeal : List % -> Record(coef: List % , generator: %)
--R random : () -> % if PADIC has INS
--R reducedSystem : Matrix % -> Matrix PADIC
--R reducedSystem : (Matrix % , Vector %) -> Record(mat: Matrix PADIC, vec: Vector PADIC)
--R reducedSystem : (Matrix % , Vector %) -> Record(mat: Matrix Integer, vec: Vector Integer) if PADIC has
--R reducedSystem : Matrix % -> Matrix Integer if PADIC has LINEXP INT
--R retract : % -> Integer if PADIC has RETRACT INT
--R retract : % -> Fraction Integer if PADIC has RETRACT INT
--R retract : % -> Symbol if PADIC has RETRACT SYMBOL
--R retractIfCan : % -> Union(Integer, "failed") if PADIC has RETRACT INT
--R retractIfCan : % -> Union(Fraction Integer, "failed") if PADIC has RETRACT INT
--R retractIfCan : % -> Union(Symbol, "failed") if PADIC has RETRACT SYMBOL
--R retractIfCan : % -> Union(PADIC, "failed")
--R sign : % -> Integer if PADIC has OINTDOM
--R solveLinearPolynomialEquation : (List SparseUnivariatePolynomial % , SparseUnivariatePolynomial %) ->
--R squareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % if PADIC has
--R subtractIfCan : (% , %) -> Union(% , "failed")
--R unitNormal : % -> Record(unit: % , canonical: % , associate: %)
--R wholePart : % -> PADIC if PADIC has EUCDOM
--R
--E 1

)spool
)lisp (bye)

```

---

— **PAdicRationalConstructor.help** —

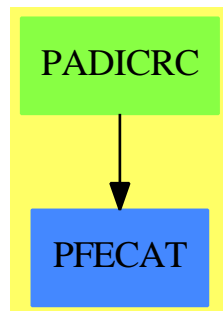
```
=====
PAdicRationalConstructor examples
=====
```

See Also:

```
o )show PAdicRationalConstructor
```

---

### 17.3.1 PAdicRationalConstructor (PADICRC)



See

- ⇒ “InnerPAdicInteger” (IPADIC) 10.24.1 on page 1258
- ⇒ “PAdicInteger” (PADIC) 17.1.1 on page 1841
- ⇒ “BalancedPAdicInteger” (BPADIC) 3.2.1 on page 240
- ⇒ “PAdicRational” (PADICRAT) 17.2.1 on page 1845
- ⇒ “BalancedPAdicRational” (BPADICRT) 3.3.1 on page 244

**Exports:**

|                               |                    |                  |
|-------------------------------|--------------------|------------------|
| 0                             | 1                  | abs              |
| approximate                   | associates?        | ceiling          |
| characteristic                | charthRoot         | coerce           |
| conditionP                    | continuedFraction  | convert          |
| D                             | denom              | denominator      |
| differentiate                 | divide             | euclideanSize    |
| eval                          | expressIdealMember | exquo            |
| extendedEuclidean             | factor             | factorPolynomial |
| factorSquareFreePolynomial    | floor              | fractionPart     |
| gcd                           | gcdPolynomial      | gcd              |
| hash                          | init               | inv              |
| latex                         | lcm                | map              |
| max                           | min                | multiEuclidean   |
| negative?                     | nextItem           | numer            |
| numerator                     | one?               | patternMatch     |
| positive?                     | prime?             | principalIdeal   |
| random                        | recip              | reducedSystem    |
| removeZeroes                  | retract            | retractIfCan     |
| sample                        | sign               | sizeLess?        |
| solveLinearPolynomialEquation | squareFree         | squareFreePart   |
| squareFreePolynomial          | subtractIfCan      | unit?            |
| unitCanonical                 | unitNormal         | wholePart        |
| zero?                         | ?*?                | ?**?             |
| ?+?                           | ?-?                | -?               |
| ?/?                           | ?=?                | ?^?              |
| ?~=?                          | ?<?                | ?<=?             |
| ?>?                           | ?>=?               | ?..?             |
| ?quo?                         | ?rem?              |                  |

— domain PADICRC PAdicRationalConstructor —

```

)abbrev domain PADICRC PAdicRationalConstructor
++ Author: Clifton J. Williamson
++ Date Created: 10 May 1990
++ Date Last Updated: 10 May 1990
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Keywords: p-adic, completion
++ Examples:
++ References:
++ Description:
++ This is the category of stream-based representations of  $\mathbb{Q}_p$ .

PAdicRationalConstructor(p,PADIC): Exports == Implementation where
  p      : Integer

```



```

PADIC :   PAdicIntegerCategory p
CF      ==> ContinuedFraction
I       ==> Integer
NNI     ==> NonNegativeInteger
OUT     ==> OutputForm
L       ==> List
RN      ==> Fraction Integer
ST      ==> Stream

Exports ==> QuotientFieldCategory(PADIC) with
approximate: (% , I) -> RN
  ++ \spad{approximate(x,n)} returns a rational number y such that
  ++ \spad{y = x (mod p^n)}.
continuedFraction: % -> CF RN
  ++ \spad{continuedFraction(x)} converts the p-adic rational number x
  ++ to a continued fraction.
removeZeroes: % -> %
  ++ \spad{removeZeroes(x)} removes leading zeroes from the
  ++ representation of the p-adic rational \spad{x}.
  ++ A p-adic rational is represented by (1) an exponent and
  ++ (2) a p-adic integer which may have leading zero digits.
  ++ When the p-adic integer has a leading zero digit, a 'leading zero'
  ++ is removed from the p-adic rational as follows:
  ++ the number is rewritten by increasing the exponent by 1 and
  ++ dividing the p-adic integer by p.
  ++ Note: \spad{removeZeroes(f)} removes all leading zeroes from f.
removeZeroes: (I,% ) -> %
  ++ \spad{removeZeroes(n,x)} removes up to n leading zeroes from
  ++ the p-adic rational \spad{x}.

Implementation ==> add

PEXPR := p :: OUT

--% representation

Rep := Record(expon:I,pint:PADIC)

getExpon: % -> I
getZp    : % -> PADIC
makeQp   : (I,PADIC) -> %

getExpon x    == x.expon
getZp x       == x.pint
makeQp(r,int) == [r,int]

--% creation

0 == makeQp(0,0)
1 == makeQp(0,1)

```

```

coerce(x:I)      == x :: PADIC :: %
coerce(r:RN)     == (numer(r) :: %)/(denom(r) :: %)
coerce(x:PADIC) == makeQp(0,x)

--% normalizations

removeZeroes x ==
  empty? digits(xx := getZp x) => 0
  zero? moduloP xx =>
    removeZeroes makeQp(getExpon x + 1,quotientByP xx)
  x

removeZeroes(n,x) ==
  n <= 0 => x
  empty? digits(xx := getZp x) => 0
  zero? moduloP xx =>
    removeZeroes(n - 1,makeQp(getExpon x + 1,quotientByP xx))
  x

--% arithmetic

x = y ==
  EQ(x,y)$Lisp => true
  n := getExpon(x) - getExpon(y)
  n >= 0 =>
    (p**(n :: NNI) * getZp(x)) = getZp(y)
    (p**((-n) :: NNI) * getZp(y)) = getZp(x)

x + y ==
  n := getExpon(x) - getExpon(y)
  n >= 0 =>
    makeQp(getExpon y,getZp(y) + p**(n :: NNI) * getZp(x))
    makeQp(getExpon x,getZp(x) + p**((-n) :: NNI) * getZp(y))

-x == makeQp(getExpon x,-getZp(x))

x - y ==
  n := getExpon(x) - getExpon(y)
  n >= 0 =>
    makeQp(getExpon y,p**(n :: NNI) * getZp(x) - getZp(y))
    makeQp(getExpon x,getZp(x) - p**((-n) :: NNI) * getZp(y))

n:I * x:% == makeQp(getExpon x,n * getZp x)
x:% * y:% == makeQp(getExpon x + getExpon y,getZp x * getZp y)

x:% ** n:I ==
  zero? n => 1
  positive? n => expt(x,n :: PositiveInteger)$RepeatedSquaring(%)
  inv expt(x,(-n) :: PositiveInteger)$RepeatedSquaring(%)

```

```

recip x ==
  x := removeZeroes(1000,x)
  zero? moduloP(xx := getZp x) => "failed"
  (inv := recip xx) case "failed" => "failed"
  makeQp(- getExpon x,inv :: PADIC)

inv x ==
  (inv := recip x) case "failed" => error "inv: no inverse"
  inv :: %

x:% / y:% == x * inv y
x:PADIC / y:PADIC == (x :: %) / (y :: %)
x:PADIC * y:% == makeQp(getExpon y,x * getZp y)

approximate(x,n) ==
  k := getExpon x
  (p :: RN) ** k * approximate(getZp x,n - k)

cfStream: % -> Stream RN
cfStream x == delay
-- zero? x => empty()
  invx := inv x; x0 := approximate(invx,1)
  concat(x0,cfStream(invx - (x0 :: %)))

continuedFraction x ==
  x0 := approximate(x,1)
  reducedContinuedFraction(x0,cfStream(x - (x0 :: %)))

termOutput:(I,I) -> OUT
termOutput(k,c) ==
  k = 0 => c :: OUT
  mon := (k = 1 => PEXPR; PEXPR ** (k :: OUT))
  c = 1 => mon
  c = -1 => -mon
  (c :: OUT) * mon

showAll?:() -> Boolean
-- check a global Lisp variable
showAll?() == true

coerce(x:%):OUT ==
  x := removeZeroes(_$streamCount$Lisp,x)
  m := getExpon x; zp := getZp x
  uu := digits zp
  l : L OUT := empty()
  empty? uu => 0 :: OUT
  n : NNI ; count : NNI := _$streamCount$Lisp
  for n in 0..count while not empty? uu repeat
    if frst(uu) ^= 0 then

```

```

      l := concat(termOutput((n :: I) + m,first(uu)),l)
      uu := rst uu
    if showAll?() then
      for n in (count + 1).. while explicitEntries? uu and _
        not eq?(uu,rst uu) repeat
        if first(uu) ^= 0 then
          l := concat(termOutput((n::I) + m,first(uu)),l)
          uu := rst uu
      l :=
        explicitlyEmpty? uu => l
        eq?(uu,rst uu) and first uu = 0 => l
        concat(prefix("0" :: OUT,[PEXPR ** ((n :: I) + m) :: OUT]),l)
    empty? l => 0 :: OUT
    reduce("+",reverse_! l)

```

---

— PADICRC.dotabb —

```

"PADICRC" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PADICRC"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"PADICRC" -> "PFECAT"

```

---

## 17.4 domain PALETTE Palette

— Palette.input —

```

)set break resume
)sys rm -f Palette.output
)spool Palette.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Palette
--R Palette is a domain constructor
--R Abbreviation for Palette is PALETTE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PALETTE
--R
--R----- Operations -----

```

```

--R ?=? : (% ,%) -> Boolean
--R coerce : Color -> %
--R dark : Color -> %
--R hash : % -> SingleInteger
--R latex : % -> String
--R pastel : Color -> %
--R ?~=? : (% ,%) -> Boolean
--R
--R
--E 1

)spool
)lisp (bye)

```

---

— Palette.help —

=====

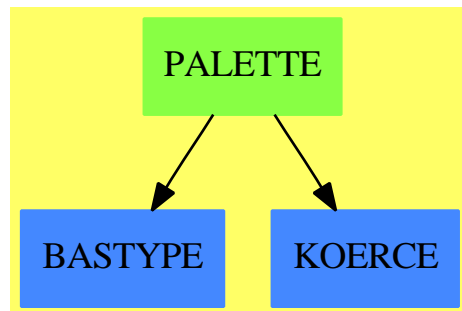
Palette examples

=====

See Also:

- o )show Palette

### 17.4.1 Palette (PALETTE)



See

⇒ “Color” (COLOR) 4.6.1 on page 392

**Exports:**

|        |        |       |        |       |
|--------|--------|-------|--------|-------|
| bright | coerce | dark  | dim    | hash  |
| hue    | latex  | light | pastel | shade |
| ?~=?   | ?=?    |       |        |       |

## — domain PALETTE Palette —

```

)abbrev domain PALETTE Palette
++ Author: Jim Wen
++ Date Created: May 10th 1989
++ Date Last Updated: Jan 19th 1990
++ Basic Operations: dark, dim, bright, pastel, light, hue, shade, coerce
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords: dim,bright,pastel,coerce
++ References:
++ Description:
++ This domain describes four groups of color shades (palettes).

Palette(): Exports == Implementation where
  I      ==> Integer
  C      ==> Color
  SHADE  ==> ["Dark","Dim","Bright","Pastel","Light"]

Exports ==> SetCategory with
  dark   : C -> %
    ++ dark(c) sets the shade of the indicated hue of c to it's lowest value.
  dim    : C -> %
    ++ dim(c) sets the shade of a hue, c,  above dark, but below bright.
  bright : C -> %
    ++ bright(c) sets the shade of a hue, c, above dim, but below pastel.
  pastel : C -> %
    ++ pastel(c) sets the shade of a hue, c,  above bright, but below light.
  light  : C -> %
    ++ light(c) sets the shade of a hue, c,  to it's highest value.
  hue    : % -> C
    ++ hue(p) returns the hue field of the indicated palette p.
  shade  : % -> I
    ++ shade(p) returns the shade index of the indicated palette p.
  coerce : C -> %
    ++ coerce(c) sets the average shade for the palette to that of the
    ++ indicated color c.

Implementation ==> add
  Rep := Record(shadeField:I, hueField:C)

  dark   c == [1,c]
  dim    c == [2,c]
  bright c == [3,c]
  pastel c == [4,c]
  light  c == [5,c]
  hue    p == p.hueField
  shade  p == p.shadeField

```

```

sample() == bright(sample())
coerce(c:Color):% == bright c
coerce(p:%):OutputForm ==
  hconcat ["[",coerce(p.hueField),"] from the ",_
          SHADE.(p.shadeField)," palette"]

```

---

— PALETTE.dotabb —

```

"PALETTE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PALETTE"]
"BASTYPE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=BASTYPE"]
"KOERCE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=KOERCE"]
"PALETTE" -> "BASTYPE"
"PALETTE" -> "KOERCE"

```

---

## 17.5 domain PARPCURV ParametricPlaneCurve

— ParametricPlaneCurve.input —

```

)set break resume
)sys rm -f ParametricPlaneCurve.output
)spool ParametricPlaneCurve.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ParametricPlaneCurve
--R ParametricPlaneCurve ComponentFunction: Type is a domain constructor
--R Abbreviation for ParametricPlaneCurve is PARPCURV
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PARPCURV
--R
--R----- Operations -----
--R coordinate : (% ,NonNegativeInteger) -> ComponentFunction
--R curve : (ComponentFunction,ComponentFunction) -> %
--R
--E 1

)spool
)lisp (bye)

```

---

— ParametricPlaneCurve.help —

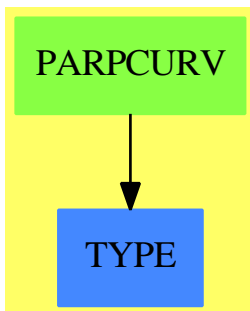
```
=====
ParametricPlaneCurve examples
=====
```

See Also:

```
o )show ParametricPlaneCurve
```

---

### 17.5.1 ParametricPlaneCurve (PARPCURV)



See

⇒ “ParametricSpaceCurve” (PARSCURV) 17.6.1 on page 1861

⇒ “ParametricSurface” (PARSURF) 17.7.1 on page 1864

**Exports:**

coordinate    curve

---

— domain PARPCURV ParametricPlaneCurve —

```
)abbrev domain PARPCURV ParametricPlaneCurve
++ Author: Clifton J. Williamson
++ Date Created: 24 May 1990
++ Date Last Updated: 24 May 1990
++ Basic Operations: curve, coordinate
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords: parametric curve, graphics
++ References:
```



```

++ Description:
++ ParametricPlaneCurve is used for plotting parametric plane
++ curves in the affine plane.

ParametricPlaneCurve(ComponentFunction): Exports == Implementation where
  ComponentFunction : Type
  NNI                ==> NonNegativeInteger

Exports ==> with
  curve: (ComponentFunction,ComponentFunction) -> %
    ++ curve(c1,c2) creates a plane curve from 2 component functions \spad{c1}
    ++ and \spad{c2}.
  coordinate: (% ,NNI) -> ComponentFunction
    ++ coordinate(c,i) returns a coordinate function for c using 1-based
    ++ indexing according to i. This indicates what the function for the
    ++ coordinate component i of the plane curve is.

Implementation ==> add

Rep := Record(xCoord:ComponentFunction,yCoord:ComponentFunction)

curve(x,y) == [x,y]
coordinate(c,n) ==
  n = 1 => c.xCoord
  n = 2 => c.yCoord
  error "coordinate: index out of bounds"

-----

— PARPCURV.dotabb —

"PARPCURV" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PARPCURV"]
"TYPE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=TYPE"]
"PARPCURV" -> "TYPE"

-----

```

## 17.6 domain PARSCURV ParametricSpaceCurve

— ParametricSpaceCurve.input —

```

)set break resume
)sys rm -f ParametricSpaceCurve.output
)spool ParametricSpaceCurve.output

```

```

)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ParametricSpaceCurve
--R ParametricSpaceCurve ComponentFunction: Type is a domain constructor
--R Abbreviation for ParametricSpaceCurve is PARSCURV
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PARSCURV
--R
--R----- Operations -----
--R coordinate : (% ,NonNegativeInteger) -> ComponentFunction
--R curve : (ComponentFunction,ComponentFunction,ComponentFunction) -> %
--R
--E 1

)spool
)lisp (bye)

```

---

### — ParametricSpaceCurve.help —

```

=====
ParametricSpaceCurve examples
=====

```

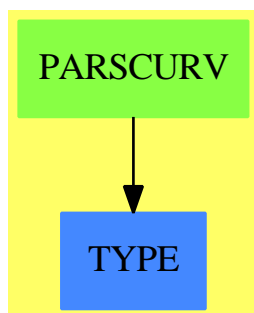
```

See Also:
o )show ParametricSpaceCurve

```

---

#### 17.6.1 ParametricSpaceCurve (PARSCURV)



See

⇒ “ParametricPlaneCurve” (PARPCURV) 17.5.1 on page 1859

⇒ “ParametricSurface” (PARSURF) 17.7.1 on page 1864

### Exports:

coordinate curve

### — domain PARSCURV ParametricSpaceCurve —

```
)abbrev domain PARSCURV ParametricSpaceCurve
++ Author: Clifton J. Williamson
++ Date Created: 24 May 1990
++ Date Last Updated: 24 May 1990
++ Basic Operations: curve, coordinate
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords: parametric curve, graphics
++ References:
++ Description:
++ ParametricSpaceCurve is used for plotting parametric space
++ curves in affine 3-space.

ParametricSpaceCurve(ComponentFunction): Exports == Implementation where
  ComponentFunction : Type
  NNI                ==> NonNegativeInteger

Exports ==> with
  curve: (ComponentFunction,ComponentFunction,ComponentFunction) -> %
    ++ curve(c1,c2,c3) creates a space curve from 3 component functions
    ++ \spad{c1}, \spad{c2}, and \spad{c3}.
  coordinate: (% ,NNI) -> ComponentFunction
    ++ coordinate(c,i) returns a coordinate function of c using 1-based
    ++ indexing according to i. This indicates what the function for the
    ++ coordinate component, i, of the space curve is.

Implementation ==> add

Rep := Record(xCoord:ComponentFunction,_
              yCoord:ComponentFunction,_
              zCoord:ComponentFunction)

curve(x,y,z) == [x,y,z]
coordinate(c,n) ==
  n = 1 => c.xCoord
  n = 2 => c.yCoord
  n = 3 => c.zCoord
  error "coordinate: index out of bounds"
```

---



---

— PARSCURV.dotabb —

```
"PARSCURV" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PARSCURV"]
"TYPE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=TYPE"]
"PARSCURV" -> "TYPE"
```

---

## 17.7 domain PARSURF ParametricSurface

---

— ParametricSurface.input —

```
)set break resume
)sys rm -f ParametricSurface.output
)spool ParametricSurface.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ParametricSurface
--R ParametricSurface ComponentFunction: Type is a domain constructor
--R Abbreviation for ParametricSurface is PARSURF
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PARSURF
--R
--R----- Operations -----
--R coordinate : (%,NonNegativeInteger) -> ComponentFunction
--R surface : (ComponentFunction,ComponentFunction,ComponentFunction) -> %
--R
--E 1

)spool
)lisp (bye)
```

---



---

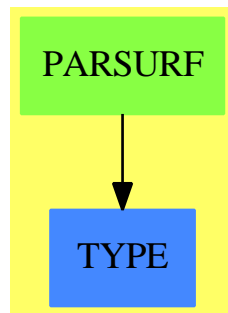
— ParametricSurface.help —

```
=====
ParametricSurface examples
=====
```

See Also:

o )show ParametricSurface

### 17.7.1 ParametricSurface (PARSURF)



See

⇒ “ParametricPlaneCurve” (PARPCURV) 17.5.1 on page 1859

⇒ “ParametricSpaceCurve” (PARSCURV) 17.6.1 on page 1861

**Exports:**

coordinate surface

— domain PARSURF ParametricSurface —

```

)abbrev domain PARSURF ParametricSurface
++ Author: Clifton J. Williamson
++ Date Created: 24 May 1990
++ Date Last Updated: 24 May 1990
++ Basic Operations: surface, coordinate
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords: parametric surface, graphics
++ References:
++ Description:
++ ParametricSurface is used for plotting parametric surfaces in
++ affine 3-space.

ParametricSurface(ComponentFunction): Exports == Implementation where
  ComponentFunction : Type
  NNI                ==> NonNegativeInteger
  
```

```
Exports ==> with
  surface: (ComponentFunction,ComponentFunction,ComponentFunction) -> %
    ++ surface(c1,c2,c3) creates a surface from 3 parametric component
    ++ functions \spad{c1}, \spad{c2}, and \spad{c3}.
  coordinate: (% ,NNI) -> ComponentFunction
    ++ coordinate(s,i) returns a coordinate function of s using 1-based
    ++ indexing according to i. This indicates what the function for the
    ++ coordinate component, i, of the surface is.
```

```
Implementation ==> add
```

```
Rep := Record(xCoord:ComponentFunction,_
              yCoord:ComponentFunction,_
              zCoord:ComponentFunction)
```

```
surface(x,y,z) == [x,y,z]
coordinate(c,n) ==
  n = 1 => c.xCoord
  n = 2 => c.yCoord
  n = 3 => c.zCoord
  error "coordinate: index out of bounds"
```

---

— PARSURF.dotabb —

```
"PARSURF" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PARSURF"]
"TYPE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=TYPE"]
"PARSURF" -> "TYPE"
```

---

## 17.8 domain PFR PartialFraction

— PartialFraction.input —

```
)set break resume
)sys rm -f PartialFraction.output
)spool PartialFraction.output
)set message test on
)set message auto off
)clear all
```

```
--S 1 of 22
```

[illegible]

```

--S 6 of 22
partialFraction(1,- 13 + 14 * %i)
--R
--R
--R      1      4
--R  (6)  - ---- + ----
--R      1 + 2%i  3 + 8%i
--R
--R                                          Type: PartialFraction Complex Integer
--E 6

--S 7 of 22
% :: Fraction Complex Integer
--R
--R
--R      %i
--R  (7)  - ----
--R      14 + 13%i
--R
--R                                          Type: Fraction Complex Integer
--E 7

--S 8 of 22
u : FR UP(x, FRAC INT) := reduce(*,[primeFactor(x+i,i) for i in 1..4])
--R
--R
--R      2      3      4
--R  (8)  (x + 1)(x + 2)(x + 3)(x + 4)
--R
--R                                          Type: Factored UnivariatePolynomial(x,Fraction Integer)
--E 8

--S 9 of 22
partialFraction(1,u)
--R
--R
--R  (9)
--R      1      1      7      17 2      139  607 3      10115 2      391      44179
--R      ---  - x + --  - -- x - 12x - ---  --- x + ----- x + --- x + -----
--R      648  4      16      8      8      324      432      4      324
--R      ----- + ----- + ----- + -----
--R      x + 1      2      3      4
--R      (x + 2)      (x + 3)      (x + 4)
--R
--R                                          Type: PartialFraction UnivariatePolynomial(x,Fraction Integer)
--E 9

--S 10 of 22
pAdicFraction %
--R
--R
--R  (10)
--R      1      1      1      17      3      1      607      403

```



```

--R      ---      -      --      --      -      -      ---      ---
--R      648      4      16      8      4      2      324      432
--R      ----- + ----- - ----- - ----- + ----- - ----- + ----- + -----
--R      x + 1    x + 2      2    x + 3      2      3      x + 4      2
--R      (x + 2)      (x + 3)      (x + 3)      (x + 4)
--R      +
--R      13      1
--R      --      --
--R      36      12
--R      ----- + -----
--R      3      4
--R      (x + 4)      (x + 4)
--R      Type: PartialFraction UnivariatePolynomial(x,Fraction Integer)
--E 10

```

```

--S 11 of 22
fraction:=Fraction(Polynomial(Integer))
--R
--R
--R      (11) Fraction Polynomial Integer
--R
--R      Type: Domain
--E 11

```

```

--S 12 of 22
up:=UnivariatePolynomial(y,fraction)
--R
--R
--R      (12) UnivariatePolynomial(y,Fraction Polynomial Integer)
--R
--R      Type: Domain
--E 12

```

```

--S 13 of 22
pfup:=PartialFraction(up)
--R
--R
--R      (13) PartialFraction UnivariatePolynomial(y,Fraction Polynomial Integer)
--R
--R      Type: Domain
--E 13

```

```

--S 14 of 22
a:=x+1/(y+1)
--R
--R
--R      x y + x + 1
--R      (14) -----
--R      y + 1
--R
--R      Type: Fraction Polynomial Integer
--E 14

```

```

--S 15 of 22

```

```

b:=partialFraction(a,y)$PartialFractionPackage(Integer)
--R
--R
--R      1
--R  (15)  x + ----
--R      y + 1
--R      Type: PartialFraction UnivariatePolynomial(y,Fraction Polynomial Integer)
--E 15

--S 16 of 22
c:=b::pfup
--R
--R
--R      1
--R  (16)  x + ----
--R      y + 1
--R      Type: PartialFraction UnivariatePolynomial(y,Fraction Polynomial Integer)
--E 16

--S 17 of 22
cw:=(wholePart c)::Expression(Integer)
--R
--R
--R  (17)  x
--R
--R                                          Type: Expression Integer
--E 17

--S 18 of 22
m:=numberOfFractionalTerms(c)
--R
--R
--R  (18)  1
--R
--R                                          Type: PositiveInteger
--E 18

--S 19 of 22
crList:=[nthFractionalTerm(c,i) for i in 1..m]
--R
--R
--R      1
--R  (19)  [-----]
--R      y + 1
--R      Type: List PartialFraction UnivariatePolynomial(y,Fraction Polynomial Integer)
--E 19

--S 20 of 22
cc:=reduce(+,crList)
--R
--R
--R      1

```

```

--R (20) -----
--R      y + 1
--R      Type: PartialFraction UnivariatePolynomial(y,Fraction Polynomial Integer)
--E 20

--S 21 of 22
ccx:=cc::(Fraction(up))::(Expression(Integer))
--R
--R
--R      1
--R (21) -----
--R      y + 1
--R
--R                                          Type: Expression Integer
--E 21

--S 22 of 22
sin(cw)*cos(ccx)+sin(ccx)*cos(cw)
--R
--R
--R      1      1
--R (22) cos(-----)sin(x) + cos(x)sin(-----)
--R      y + 1      y + 1
--R
--R                                          Type: Expression Integer
--E 22

)spool
)lisp (bye)

```

---

— PartialFraction.help —

=====  
PartialFraction examples  
=====

A partial fraction is a decomposition of a quotient into a sum of quotients where the denominators of the summands are powers of primes. Most people first encounter partial fractions when they are learning integral calculus. For a technical discussion of partial fractions, see, for example, Lang's Algebra. For example, the rational number  $1/6$  is decomposed into  $1/2-1/3$ . You can compute partial fractions of quotients of objects from domains belonging to the category EuclideanDomain. For example, Integer, Complex Integer, and UnivariatePolynomial(x, Fraction Integer) all belong to EuclideanDomain. In the examples following, we demonstrate how to decompose quotients of each of these kinds of object into partial fractions.

It is necessary that we know how to factor the denominator when we

want to compute a partial fraction. Although the interpreter can often do this automatically, it may be necessary for you to include a call to `factor`. In these examples, it is not necessary to factor the denominators explicitly.

The main operation for computing partial fractions is called `partialFraction` and we use this to compute a decomposition of  $1/10!$ . The first argument to `partialFraction` is the numerator of the quotient and the second argument is the factored denominator.

```
partialFraction(1,factorial 10)
159 23 12 1
--- - -- - -- + -
 8 4 2 7
2 3 5
Type: PartialFraction Integer
```

Since the denominators are powers of primes, it may be possible to expand the numerators further with respect to those primes. Use the operation `padicFraction` to do this.

```
f := padicFraction(%)
1 1 1 1 1 1 2 1 2 2 2 1
- + -- + -- + -- + -- + -- - -- - -- - -- - -- + -
2 4 5 6 7 8 2 3 4 5 2 7
2 2 2 2 2 3 3 3 5
Type: PartialFraction Integer
```

The operation `compactFraction` returns an expanded fraction into the usual form. The compacted version is used internally for computational efficiency.

```
compactFraction(f)
159 23 12 1
--- - -- - -- + -
 8 4 2 7
2 3 5
Type: PartialFraction Integer
```

You can add, subtract, multiply and divide partial fractions. In addition, you can extract the parts of the decomposition. `numberOfFractionalTerms` computes the number of terms in the fractional part. This does not include the whole part of the fraction, which you get by calling `wholePart`. In this example, the whole part is just 0.

```
numberOfFractionalTerms(f)
12
Type: PositiveInteger
```

The operation `nthFractionalTerm` returns the individual terms in the decomposition. Notice that the object returned is a partial fraction

itself. `firstNumer` and `firstDenom` extract the numerator and denominator of the first term of the fraction.

```
nthFractionalTerm(f,3)
  1
  --
  5
  2
                                     Type: PartialFraction Integer
```

Given two gaussian integers, you can decompose their quotient into a partial fraction.

```
partialFraction(1,- 13 + 14 * %i)
  1      4
  - ---- + ----
  1 + 2%i 3 + 8%i
                                     Type: PartialFraction Complex Integer
```

To convert back to a quotient, simply use a conversion.

```
% :: Fraction Complex Integer
      %i
  - ----
  14 + 13%i
                                     Type: Fraction Complex Integer
```

To conclude this section, we compute the decomposition of

$$\frac{1}{(x+1)(x+2)(x+3)(x+4)}$$

The polynomials in this object have type `UnivariatePolynomial(x, Fraction Integer)`.

We use the `primeFactor` operation to create the denominator in factored form directly.

```
u : FR UP(x, FRAC INT) := reduce(*,[primeFactor(x+i,i) for i in 1..4])
      2      3      4
  (x + 1)(x + 2) (x + 3) (x + 4)
                                     Type: Factored UnivariatePolynomial(x,Fraction Integer)
```

These are the compact and expanded partial fractions for the quotient.

```
partialFraction(1,u)
  1      1      7      17 2      139 607 3      10115 2      391      44179
```

$$\frac{\frac{1}{648}x + \frac{1}{4} - \frac{1}{16}}{x+1} + \frac{\frac{1}{8}x^2 - \frac{12x}{8}}{(x+2)^2} + \frac{\frac{1}{8}x^3 - \frac{12x}{8}}{(x+3)^3} + \frac{\frac{1}{324}x^4 + \frac{1}{432}x^3 + \frac{1}{4}x^2 + \frac{1}{324}}{(x+4)^4}$$

Type: PartialFraction UnivariatePolynomial(x,Fraction Integer)

padicFraction %

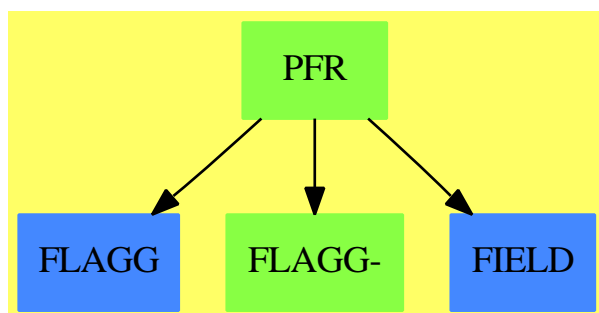
$$\frac{1}{648}x + \frac{1}{4} - \frac{1}{16} + \frac{17}{8}x^2 + \frac{3}{4}x^3 - \frac{1}{2}x^4 + \frac{607}{324}x^3 + \frac{403}{432}x^2 + \frac{13}{36}x + \frac{1}{12}$$

Type: PartialFraction UnivariatePolynomial(x,Fraction Integer)

See Also:

- o )help Factored
- o )help Complex
- o )help FullPartialFractionExpansionXmpPage
- o )show PartialFraction

### 17.8.1 PartialFraction (PFR)



Exports:

|                    |                 |                   |                         |
|--------------------|-----------------|-------------------|-------------------------|
| 0                  | 1               | associates?       | characteristic          |
| coerce             | compactFraction | divide            | euclideanSize           |
| expressIdealMember | exquo           | extendedEuclidean | extendedEuclidean       |
| factor             | firstDenom      | firstNumer        | gcd                     |
| gcdPolynomial      | hash            | inv               | latex                   |
| lcm                | multiEuclidean  | nthFractionalTerm | numberOfFractionalTerms |
| one?               | padicallyExpand | padicFraction     | partialFraction         |
| prime?             | principalIdeal  | recip             | sample                  |
| sizeLess?          | squareFree      | squareFreePart    | subtractIfCan           |
| unit?              | unitCanonical   | unitNormal        | wholePart               |
| zero?              | ?*?             | ?**?              | ?+?                     |
| ?-?                | -?              | ?/?               | ?=?                     |
| ?^?                | ?~=?            | ?quo?             | ?rem?                   |

— domain PFR PartialFraction —

```

)abbrev domain PFR PartialFraction
++ Author: Robert S. Sutor
++ Date Created: 1986
++ Change History:
++   05/20/91 BMT Converted to the new library
++ Basic Operations: (Field), (Algebra),
++   coerce, compactFraction, firstDenom, firstNumer,
++   nthFractionalTerm, numberOfFractionalTerms, padicallyExpand,
++   padicFraction, partialFraction, wholePart
++ Related Constructors:
++ Also See: ContinuedFraction
++ AMS Classifications:
++ Keywords: partial fraction, factorization, euclidean domain
++ References:
++ Description:
++ The domain \spadtype{PartialFraction} implements partial fractions
++ over a euclidean domain \spad{R}. This requirement on the
++ argument domain allows us to normalize the fractions. Of
++ particular interest are the 2 forms for these fractions. The
++ ‘compact’ form has only one fractional term per prime in the
++ denominator, while the ‘p-adic’ form expands each numerator
++ p-adically via the prime p in the denominator. For computational
++ efficiency, the compact form is used, though the p-adic form may
++ be gotten by calling the function padicFraction}. For a
++ general euclidean domain, it is not known how to factor the
++ denominator. Thus the function partialFraction takes as its
++ second argument an element of \spadtype{Factored(R)}.

PartialFraction(R: EuclideanDomain): Cat == Capsule where
  FRR ==> Factored R
  SUPR ==> SparseUnivariatePolynomial R

  Cat == Join(Field, Algebra R) with

```

```

coerce: % -> Fraction R
++ coerce(p) sums up the components of the partial fraction and
++ returns a single fraction.
++
++X a:=(13/74)::PFR(INT)
++X a::FRAC(INT)

coerce: Fraction FRR -> %
++ coerce(f) takes a fraction with numerator and denominator in
++ factored form and creates a partial fraction. It is
++ necessary for the parts to be factored because it is not
++ known in general how to factor elements of \spad{R} and
++ this is needed to decompose into partial fractions.
++
++X (13/74)::PFR(INT)

compactFraction: % -> %
++ compactFraction(p) normalizes the partial fraction \spad{p}
++ to the compact representation. In this form, the partial
++ fraction has only one fractional term per prime in the
++ denominator.
++
++X a:=partialFraction(1,factorial 10)
++X b:=padicFraction(a)
++X compactFraction(b)

firstDenom: % -> FRR
++ firstDenom(p) extracts the denominator of the first fractional
++ term. This returns 1 if there is no fractional part (use
++ wholePart from PartialFraction to get the whole part).
++
++X a:=partialFraction(1,factorial 10)
++X firstDenom(a)

firstNumerator: % -> R
++ firstNumerator(p) extracts the numerator of the first fractional
++ term. This returns 0 if there is no fractional part (use
++ wholePart from PartialFraction to get the whole part).
++
++X a:=partialFraction(1,factorial 10)
++X firstNumerator(a)

nthFractionalTerm: (%,Integer) -> %
++ nthFractionalTerm(p,n) extracts the nth fractional term from
++ the partial fraction \spad{p}. This returns 0 if the index
++ \spad{n} is out of range.
++
++X a:=partialFraction(1,factorial 10)
++X b:=padicFraction(a)
++X nthFractionalTerm(b,3)

```



```

numberOfFractionalTerms: % -> Integer
++ numberOfFractionalTerms(p) computes the number of fractional
++ terms in \spad{p}. This returns 0 if there is no fractional
++ part.
++
++X a:=partialFraction(1,factorial 10)
++X b:=padicFraction(a)
++X numberOfFractionalTerms(b)

padicallyExpand: (R,R) -> SUPR
++ padicallyExpand(p,x) is a utility function that expands
++ the second argument \spad{x} 'p-adically' in
++ the first.

padicFraction: % -> %
++ padicFraction(q) expands the fraction p-adically in the primes
++ \spad{p} in the denominator of \spad{q}. For example,
++ \spad{padicFraction(3/(2**2))} = 1/2 + 1/(2**2)}.
++ Use compactFraction from PartialFraction to
++ return to compact form.
++
++X a:=partialFraction(1,factorial 10)
++X padicFraction(a)

partialFraction: (R, FRR) -> %
++ partialFraction(num,denom) is the main function for
++ constructing partial fractions. The second argument is the
++ denominator and should be factored.
++
++X partialFraction(1,factorial 10)

wholePart: % -> R
++ wholePart(p) extracts the whole part of the partial fraction
++ \spad{p}.
++
++X a:=(74/13)::PFR(INT)
++X wholePart(a)

Capsule == add

-- some constructor assignments and macros

Ex      ==> OutputForm
fTerm   ==> Record(num: R, den: FRR)           -- den should have
                                                -- unit = 1 and only
                                                -- 1 factor

LfTerm  ==> List Record(num: R, den: FRR)
QR      ==> Record(quotient: R, remainder: R)

```

```

Rep      := Record(whole:R, fract: LfTerm)

-- private function signatures

copypf: % -> %
LessThan: (fTerm, fTerm) -> Boolean
multiplyFracTerms: (fTerm, fTerm) -> %
normalizeFracTerm: fTerm -> %
partialFractionNormalized: (R, FRR) -> %

-- declarations

a,b: %
n: Integer
r: R

-- private function definitions

copypf(a: %): % == [a.whole, copy a.fract]$$%

LessThan(s: fTerm, t: fTerm) ==
  -- have to wait until FR has < operation
  if (GGREATERP(s.den, t.den)$Lisp : Boolean) then false
  else true

multiplyFracTerms(s : fTerm, t : fTerm) ==
  nthFactor(s.den, 1) = nthFactor(t.den, 1) =>
    normalizeFracTerm([s.num * t.num, s.den * t.den]$fTerm) : Rep
  i : Union(Record(coef1: R, coef2: R), "failed")
  coefs : Record(coef1: R, coef2: R)
  i := extendedEuclidean(expand t.den, expand s.den, s.num * t.num)
  i case "failed" => error "PartialFraction: not in ideal"
  coefs := (i :: Record(coef1: R, coef2: R))
  c : % := copypf 0$$%
  d : %
  if coefs.coef2 ^= 0$R then
    c := normalizeFracTerm ([coefs.coef2, t.den]$fTerm)
  if coefs.coef1 ^= 0$R then
    d := normalizeFracTerm ([coefs.coef1, s.den]$fTerm)
    c.whole := c.whole + d.whole
    not (null d.fract) => c.fract := append(d.fract, c.fract)
  c

normalizeFracTerm(s : fTerm) ==
  -- makes sure num is "less than" den, whole may be non-zero
  qr : QR := divide(s.num, (expand s.den))
  qr.remainder = 0$R => [qr.quotient, nil()$LfTerm]
  -- now verify num and den are coprime
  f : R := nthFactor(s.den, 1)
  nexpon : Integer := nthExponent(s.den, 1)

```

```

expon : Integer := 0
q : QR := divide(qr.remainder, f)
while (q.remainder = 0$R) and (expon < nexpon) repeat
  expon := expon + 1
  qr.remainder := q.quotient
  q := divide(qr.remainder, f)
expon = 0 => [qr.quotient, [[qr.remainder, s.den]$fTerm]$LfTerm]
expon = nexpon => (qr.quotient + qr.remainder) :: %
[qr.quotient, [[qr.remainder, nilFactor(f, nexpon-expon)]$fTerm]$LfTerm]

partialFractionNormalized(nm: R, dn : FRR) ==
-- assume unit dn = 1
nm = 0$R => 0$%
dn = 1$FRR => nm :: %
qr : QR := divide(nm, expand dn)
c : % := [0$R, [[qr.remainder,
  nilFactor(nthFactor(dn,1), nthExponent(dn,1))$fTerm]$LfTerm]
d : %
for i in 2..numberOfFactors(dn) repeat
  d :=
    [0$R, [[1$R, nilFactor(nthFactor(dn,i), nthExponent(dn,i))$fTerm]$LfTerm]
  c := c * d
(qr.quotient :: %) + c

-- public function definitions

padicFraction(a : %) ==
  b : % := compactFraction a
  null b.fract => b
  l : LfTerm := nil
  s : fTerm
  f : R
  e,d: Integer
  for s in b.fract repeat
    e := nthExponent(s.den,1)
    e = 1 => l := cons(s,l)
    f := nthFactor(s.den,1)
    d := degree(sp := padicallyExpand(f,s.num))
    while (sp ^= 0$SUPR) repeat
      l := cons([leadingCoefficient sp, nilFactor(f,e-d)]$fTerm, l)
      d := degree(sp := reductum sp)
  [b.whole, sort(LessThan,l)]$%

compactFraction(a : %) ==
-- only one power for each distinct denom will remain
2 > # a.fract => a
af : LfTerm := reverse a.fract
bf : LfTerm := nil
bw : R := a.whole
b : %

```

```

s : fTerm := [(first af).num,(first af).den]$fTerm
f : R := nthFactor(s.den,1)
e : Integer := nthExponent(s.den,1)
t : fTerm
for t in rest af repeat
  f = nthFactor(t.den,1) =>
    s.num := s.num + (t.num *
      (f **$R ((e - nthExponent(t.den,1)) : NonNegativeInteger)))
  b := normalizeFracTerm s
  bw := bw + b.whole
  if not (null b.fract) then bf := cons(first b.fract,bf)
  s := [t.num, t.den]$fTerm
  f := nthFactor(s.den,1)
  e := nthExponent(s.den,1)
b := normalizeFracTerm s
[bw + b.whole,append(b.fract,bf)]$%

0 == [0$R, nil()$LfTerm]
1 == [1$R, nil()$LfTerm]
characteristic() == characteristic()$R

coerce(r): % == [r, nil()$LfTerm]
coerce(n): % == [(n :: R), nil()$LfTerm]
coerce(a): Fraction R ==
  q : Fraction R := (a.whole :: Fraction R)
  s : fTerm
  for s in a.fract repeat
    q := q + (s.num / (expand s.den))
  q
coerce(q: Fraction FRR): % ==
  u : R := (recip unit denom q):: R
  r1 : R := u * expand numer q
  partialFractionNormalized(r1, u * denom q)

a exquo b ==
  b = 0$% => "failed"
  b = 1$% => a
  br : Fraction R := inv (b :: Fraction R)
  a * partialFraction(numer br,(denom br) :: FRR)
recip a == (1$% exquo a)

firstDenom a ==      -- denominator of 1st fractional term
  null a.fract => 1$FRR
  (first a.fract).den
firstNumer a ==      -- numerator of 1st fractional term
  null a.fract => 0$R
  (first a.fract).num
numberOfFractionalTerms a == # a.fract
nthFractionalTerm(a,n) ==
  l : LfTerm := a.fract

```

```

(n < 1) or (n > # 1) => 0$%
[0$R,[1..n]$LfTerm]$%
wholePart a == a.whole

partialFraction(nm: R, dn : FRR) ==
  nm = 0$R => 0$%
  -- move inv unit of den to numerator
  u : R := unit dn
  u := (recip u) :: R
  partialFractionNormalized(u * nm,u * dn)

padicallyExpand(p : R, r : R) ==
  -- expands r as a sum of powers of p, with coefficients
  -- r = HornerEval(padicallyExpand(p,r),p)
  qr : QR := divide(r, p)
  qr.quotient = 0$R => qr.remainder :: SUPR
  (qr.remainder :: SUPR) + monomial(1$R,1$NonNegativeInteger)$SUPR *
    padicallyExpand(p,qr.quotient)

a = b ==
  a.whole ^= b.whole => false -- must verify this
  (null a.fract) =>
    null b.fract => a.whole = b.whole
    false
  null b.fract => false
  -- oh, no! following is temporary
  (a :: Fraction R) = (b :: Fraction R)

- a ==
  s: fTerm
  l: LfTerm := nil
  for s in reverse a.fract repeat l := cons([- s.num,s.den]$fTerm,l)
  [- a.whole,l]

r * a ==
  r = 0$R => 0$%
  r = 1$R => a
  b : % := (r * a.whole) :: %
  c : %
  s : fTerm
  for s in reverse a.fract repeat
    c := normalizeFracTerm [r * s.num, s.den]$fTerm
    b.whole := b.whole + c.whole
    not (null c.fract) => b.fract := append(c.fract, b.fract)
  b

n * a == (n :: R) * a

a + b ==
  compactFraction

```

```

    [a.whole + b.whole,
     sort(LessThan,append(a.fract,copy b.fract))]]$%

a * b ==
null a.fract => a.whole * b
null b.fract => b.whole * a
af : % := [0$R, a.fract]$% -- a - a.whole
c : % := (a.whole * b) + (b.whole * af)
s,t : fTerm
for s in a.fract repeat
  for t in b.fract repeat
    c := c + multiplyFracTerms(s,t)
c

coerce(a): Ex ==
null a.fract => a.whole :: Ex
s : fTerm
l : List Ex
if a.whole = 0 then l := nil else l := [a.whole :: Ex]
for s in a.fract repeat
  s.den = 1$FRR => l := cons(s.num :: Ex, l)
  l := cons(s.num :: Ex / s.den :: Ex, l)
# l = 1 => first l
reduce("+", reverse l)

```

---

— PFR.dotabb —

```

"PFR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PFR"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"FLAGG-" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FLAGG"]
"FIELD" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FIELD"]
"PFR" -> "FIELD"
"PFR" -> "FLAGG-"
"PFR" -> "FLAGG"

```

---

## 17.9 domain PARTITION Partition

— Partition.input —

```
)set break resume
```

```

)sys rm -f Partition.output
)spool Partition.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Partition
--R Partition is a domain constructor
--R Abbreviation for Partition is PRITITION
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PRITITION
--R
--R----- Operations -----
--R ?? : (PositiveInteger,%) -> %      ?? : (%,%) -> %
--R ?<? : (%,%) -> Boolean             ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean             ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean            0 : () -> %
--R coerce : % -> List Integer         coerce : % -> OutputForm
--R conjugate : % -> %                 convert : % -> List Integer
--R hash : % -> SingleInteger          latex : % -> String
--R max : (%,%) -> %                  min : (%,%) -> %
--R partition : List Integer -> %      pdct : % -> Integer
--R sample : () -> %                  zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (NonNegativeInteger,%) -> %
--R powers : List Integer -> List List Integer
--R subtractIfCan : (%,%) -> Union(%, "failed")
--R
--E 1

)spool
)lisp (bye)

```

---

— Partition.help —

```

=====
Partition examples
=====

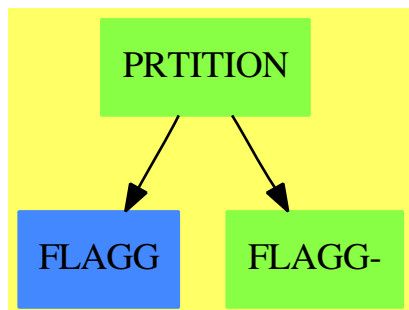
```

```

See Also:
o )show Partition

```

## 17.9.1 Partition (PRTITION)



See

⇒ “SymmetricPolynomial” (SYMPOLY) 20.39.1 on page 2613

**Exports:**

|        |        |               |           |      |
|--------|--------|---------------|-----------|------|
| 0      | coerce | conjugate     | convert   | hash |
| latex  | max    | min           | partition | pdct |
| powers | sample | subtractIfCan | zero?     | ?~=? |
| ?*?    | ?+?    | ?<?           | ?<=?      | ?=?  |
| ?>?    | ?>=?   |               |           |      |

— domain PRTITION Partition —

```

)abbrev domain PRTITION Partition
++ Author: William H. Burge
++ Date Created: 29 October 1987
++ Date Last Updated: 23 Sept 1991
++ Keywords:
++ Examples:
++ References:
++ Description:
++ Domain for partitions of positive integers
++ Partition is an OrderedCancellationAbelianMonoid which is used
++ as the basis for symmetric polynomial representation of the
++ sums of powers in SymmetricPolynomial. Thus, \spad{(5 2 2 1)} will
++ represent \spad{s5 * s2**2 * s1}.

```

Partition: Exports == Implementation where

```

L ==> List
I ==> Integer
OUT ==> OutputForm
NNI ==> NonNegativeInteger
UN ==> Union(%,"failed")

```

```

Exports ==> Join(OrderedCancellationAbelianMonoid,
                  ConvertibleTo List Integer) with
partition: L I -> %

```



```

    ++ partition(li) converts a list of integers li to a partition
powers: L I -> L L I
    ++ powers(li) returns a list of 2-element lists. For each 2-element
    ++ list, the first element is an entry of li and the second
    ++ element is the multiplicity with which the first element
    ++ occurs in li. There is a 2-element list for each value
    ++ occurring in l.
pdct: % -> I
    ++ \spad{pdct(a1**n1 a2**n2 ...)} returns
    ++ \spad{n1! * a1**n1 * n2! * a2**n2 * ...}.
    ++ This function is used in the package \spadtype{CycleIndicators}.
conjugate: % -> %
    ++ conjugate(p) returns the conjugate partition of a partition p
coerce:% -> List Integer
    ++ coerce(p) coerces a partition into a list of integers

Implementation ==> add

import PartitionsAndPermutations

Rep := List Integer
0 == nil()

coerce (s:%) == s pretend List Integer
convert x == copy(x pretend L I)

partition list == sort((i1:Integer,i2:Integer):Boolean +-> i2 < i1,list)

x < y ==
    empty? x => not empty? y
    empty? y => false
    first x = first y => rest x < rest y
    first x < first y

x = y ==
    EQUAL(x,y)$Lisp
--     empty? x => empty? y
--     empty? y => false
--     first x = first y => rest x = rest y
--     false

x + y ==
    empty? x => y
    empty? y => x
    first x > first y => concat(first x,rest(x) + y)
    concat(first y,x + rest(y))
n:NNI * x:% == (zero? n => 0; x + (subtractIfCan(n,1) :: NNI) * x)

dp: (I,%) -> %
dp(i,x) ==

```

```

empty? x => 0
first x = i => rest x
concat(first x,dp(i,rest x))

remv: (I,%) -> UN
remv(i,x) == (member?(i,x) => dp(i,x); "failed")

subtractIfCan(x, y) ==
  empty? x =>
    empty? y => 0
    "failed"
  empty? y => x
  (aa := remv(first y,x)) case "failed" => "failed"
  subtractIfCan((aa :: %), rest y)

li1 : L I --!! 'bite' won't compile without this
bite: (I,L I) -> L I
bite(i,li) ==
  empty? li => concat(0,nil())
  first li = i =>
    li1 := bite(i,rest li)
    concat(first(li1) + 1,rest li1)
  concat(0,li)

li : L I --!! 'powers' won't compile without this
powers l ==
  empty? l => nil()
  li := bite(first l,rest l)
  concat([first l,first(li) + 1],powers(rest li))

conjugate x == conjugate(x pretend Rep)$PartitionsAndPermutations

mkterm: (I,I) -> OUT
mkterm(i1,i2) ==
  i2 = 1 => (i1 :: OUT) ** (" " :: OUT)
  (i1 :: OUT) ** (i2 :: OUT)

mkexp1: L L I -> L OUT
mkexp1 lli ==
  empty? lli => nil()
  li := first lli
  empty?(rest lli) and second(li) = 1 =>
    concat(first(li) :: OUT,nil())
  concat(mkterm(first li,second li),mkexp1(rest lli))

coerce(x:%):OUT ==
  empty? (x pretend Rep) => coerce(x pretend Rep)$Rep
  paren(reduce("*",mkexp1(powers(x pretend Rep))))

pdct x ==

```

```
*/[factorial(second a) * (first(a) ** (second(a) pretend NNI))
   for a in powers(x pretend Rep)]
```

---

— PRTITION.dotabb —

```
"PRTITION" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PRTITION"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"FLAGG-" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FLAGG"]
"PRTITION" -> "FLAGG-"
"PRTITION" -> "FLAGG"
```

---

## 17.10 domain PATTERN Pattern

— Pattern.input —

```
)set break resume
)sys rm -f Pattern.output
)spool Pattern.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Pattern
--R Pattern R: SetCategory is a domain constructor
--R Abbreviation for Pattern is PATTERN
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PATTERN
--R
--R----- Operations -----
--R ?? : (%,% ) -> %               ??? : (%,% ) -> %
--R +? : (%,% ) -> %               ?/? : (%,% ) -> %
--R ?? : (%,% ) -> Boolean         1 : () -> %
--R 0 : () -> %                   addBadValue : (% ,Any) -> %
--R coerce : Symbol -> %          coerce : R -> %
--R coerce : % -> OutputForm      constant? : % -> Boolean
--R convert : List % -> %         copy : % -> %
--R depth : % -> NonNegativeInteger generic? : % -> Boolean
--R getBadValues : % -> List Any  hasPredicate? : % -> Boolean
--R hasTopPredicate? : % -> Boolean hash : % -> SingleInteger
```

```

--R inR? : % -> Boolean                latex : % -> String
--R multiple? : % -> Boolean            optional? : % -> Boolean
--R predicates : % -> List Any          quoted? : % -> Boolean
--R resetBadValues : % -> %            retract : % -> Symbol
--R retract : % -> R                   symbol? : % -> Boolean
--R variables : % -> List %            ?~=? : (%,% ) -> Boolean
--R ?**? : (% ,NonNegativeInteger) -> %
--R elt : (BasicOperator,List %) -> %
--R isExpt : % -> Union(Record(val: %,exponent: NonNegativeInteger),"failed")
--R isList : % -> Union(List %,"failed")
--R isOp : % -> Union(Record(op: BasicOperator,arg: List %),"failed")
--R isOp : (% ,BasicOperator) -> Union(List %,"failed")
--R isPlus : % -> Union(List %,"failed")
--R isPower : % -> Union(Record(val: %,exponent: %),"failed")
--R isQuotient : % -> Union(Record(num: %,den: %),"failed")
--R isTimes : % -> Union(List %,"failed")
--R optpair : List % -> Union(List %,"failed")
--R patternVariable : (Symbol,Boolean,Boolean,Boolean) -> %
--R retractIfCan : % -> Union(Symbol,"failed")
--R retractIfCan : % -> Union(R,"failed")
--R setPredicates : (% ,List Any) -> %
--R setTopPredicate : (% ,List Symbol,Any) -> %
--R topPredicate : % -> Record(var: List Symbol,pred: Any)
--R withPredicates : (% ,List Any) -> %
--R
--E 1

)spool
)lisp (bye)

```

---

— Pattern.help —

=====

Pattern examples

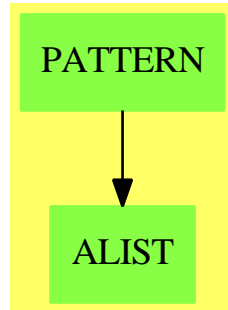
=====

See Also:

o )show Pattern

---

## 17.10.1 Pattern (PATTERN)

**Exports:**

|                |               |                  |                |                 |
|----------------|---------------|------------------|----------------|-----------------|
| 0              | 1             | addBadValue      | coerce         | constant?       |
| convert        | copy          | depth            | elt            | generic?        |
| getBadValues   | hasPredicate? | hasTopPredicate? | hash           | inR?            |
| isExpt         | isList        | isOp             | isOp           | isPlus          |
| isPower        | isQuotient    | isTimes          | latex          | multiple?       |
| optional?      | optpair       | patternVariable  | predicates     | quoted?         |
| resetBadValues | retract       | retractIfCan     | setPredicates  | setTopPredicate |
| symbol?        | topPredicate  | variables        | withPredicates | ?*              |
| ***?           | ?+?           | ?/?              | ?=?            | ?~=?            |

— domain PATTERN Pattern —

```

)abbrev domain PATTERN Pattern
++ Author: Manuel Bronstein
++ Date Created: 10 Nov 1988
++ Date Last Updated: 20 June 1991
++ Keywords: pattern, matching.
++ Description:
++ Patterns for use by the pattern matcher.
-- Not exposed.
-- Patterns are optimized for quick answers to structural questions.

```

```

Pattern(R:SetCategory): Exports == Implementation where
  B ==> Boolean
  SI ==> SingleInteger
  Z ==> Integer
  SY ==> Symbol
  O ==> OutputForm
  BOP ==> BasicOperator
  QOT ==> Record(num:%, den:%)
  REC ==> Record(val:%, exponent:NonNegativeInteger)
  RSY ==> Record(tag:SI, val: SY, pred:List Any, bad:List Any)
  KER ==> Record(tag:SI, op:BOP, arg:List %)
  PAT ==> Union(ret:R, ker: KER, exp:REC, qot: QOT, sym:RSY)

```

```

-- the following MUST be the name of the formal exponentiation operator
POWER ==> "%power"::Symbol

-- the 4 SYM_ constants must be disting powers of 2 (bitwise arithmetic)
SYM_GENERIC ==> 1::SI
SYM_MULTIPLE ==> 2::SI
SYM_OPTIONAL ==> 4::SI

PAT_PLUS ==> 1::SI
PAT_TIMES ==> 2::SI
PAT_LIST ==> 3::SI
PAT_ZERO ==> 4::SI
PAT_ONE ==> 5::SI
PAT_EXPT ==> 6::SI

Exports ==> Join(SetCategory, RetractableTo R, RetractableTo SY) with
0 : constant -> % ++ 0
1 : constant -> % ++ 1
isPlus : % -> Union(List %, "failed")
++ isPlus(p) returns \spad{[a1,...,an]} if \spad{n > 1}
++ and \spad{p = a1 + ... + an},
++ and "failed" otherwise.
isTimes : % -> Union(List %, "failed")
++ isTimes(p) returns \spad{[a1,...,an]} if \spad{n > 1} and
++ \spad{p = a1 * ... * an}, and
++ "failed" otherwise.
isOp : (%, BOP) -> Union(List %, "failed")
++ isOp(p, op) returns \spad{[a1,...,an]} if \spad{p = op(a1,...,an)}, and
++ "failed" otherwise.
isOp : % -> Union(Record(op:BOP, arg:List %), "failed")
++ isOp(p) returns \spad{[op, [a1,...,an]]} if
++ \spad{p = op(a1,...,an)}, and
++ "failed" otherwise;
isExpt : % -> Union(REC, "failed")
++ isExpt(p) returns \spad{[q, n]} if \spad{n > 0} and \spad{p = q ** n},
++ and "failed" otherwise.
isQuotient : % -> Union(QOT, "failed")
++ isQuotient(p) returns \spad{[a, b]} if \spad{p = a / b}, and
++ "failed" otherwise.
isList : % -> Union(List %, "failed")
++ isList(p) returns \spad{[a1,...,an]} if \spad{p = [a1,...,an]},
++ "failed" otherwise;
isPower : % -> Union(Record(val:%, exponent:%), "failed")
++ isPower(p) returns \spad{[a, b]} if \spad{p = a ** b}, and
++ "failed" otherwise.
elt : (BOP, List %) -> %
++ \spad{elt(op, [a1,...,an])} returns \spad{op(a1,...,an)}.
"+" : (%, %) -> %
++ \spad{a + b} returns the pattern \spad{a + b}.

```

```

"%"      : (% , %) -> %
++ \spad{a * b} returns the pattern \spad{a * b}.
"%"      : (% , NonNegativeInteger) -> %
++ \spad{a ** n} returns the pattern \spad{a ** n}.
"%"      : (% , %) -> %
++ \spad{a ** b} returns the pattern \spad{a ** b}.
"/"      : (% , %) -> %
++ \spad{a / b} returns the pattern \spad{a / b}.
depth    : % -> NonNegativeInteger
++ depth(p) returns the nesting level of p.
convert   : List % -> %
++ \spad{convert([a1,...,an])} returns the pattern \spad{[a1,...,an]}.
copy      : % -> %
++ copy(p) returns a recursive copy of p.
inR?     : % -> B
++ inR?(p) tests if p is an atom (i.e. an element of R).
quoted?   : % -> B
++ quoted?(p) tests if p is of the form 's for a symbol s.
symbol?   : % -> B
++ symbol?(p) tests if p is a symbol.
constant? : % -> B
++ constant?(p) tests if p contains no matching variables.
generic?  : % -> B
++ generic?(p) tests if p is a single matching variable.
multiple? : % -> B
++ multiple?(p) tests if p is a single matching variable
++ allowing list matching or multiple term matching in a
++ sum or product.
optional? : % -> B
++ optional?(p) tests if p is a single matching variable
++ which can match an identity.
hasPredicate?: % -> B
++ hasPredicate?(p) tests if p has predicates attached to it.
predicates : % -> List Any
++ predicates(p) returns \spad{[p1,...,pn]} such that the predicate
++ attached to p is p1 and ... and pn.
setPredicates: (% , List Any) -> %
++ \spad{setPredicates(p, [p1,...,pn])} attaches the predicate
++ p1 and ... and pn to p.
withPredicates:(% , List Any) -> %
++ \spad{withPredicates(p, [p1,...,pn])} makes a copy of p and attaches
++ the predicate p1 and ... and pn to the copy, which is
++ returned.
patternVariable: (SY, B, B, B) -> %
++ patternVariable(x, c?, o?, m?) creates a pattern variable x,
++ which is constant if \spad{c? = true}, optional if \spad{o? = true},
++ and multiple if \spad{m? = true}.
setTopPredicate: (% , List SY, Any) -> %
++ \spad{setTopPredicate(x, [a1,...,an], f)} returns x with
++ the top-level predicate set to \spad{f(a1,...,an)}.

```

```

topPredicate: % -> Record(var:List SY, pred:Any)
  ++ topPredicate(x) returns \spad{[[a1,...,an], f]} where the top-level
  ++ predicate of x is \spad{f(a1,...,an)}.
  ++ Note: n is 0 if x has no top-level
  ++ predicate.
hasTopPredicate?: % -> B
  ++ hasTopPredicate?(p) tests if p has a top-level predicate.
resetBadValues: % -> %
  ++ resetBadValues(p) initializes the list of "bad values" for p
  ++ to \spad{[]}.
  ++ Note: p is not allowed to match any of its "bad values".
addBadValue: (% , Any) -> %
  ++ addBadValue(p, v) adds v to the list of "bad values" for p.
  ++ Note: p is not allowed to match any of its "bad values".
getBadValues: % -> List Any
  ++ getBadValues(p) returns the list of "bad values" for p.
  ++ Note: p is not allowed to match any of its "bad values".
variables: % -> List %
  ++ variables(p) returns the list of matching variables
  ++ appearing in p.
optpair: List % -> Union(List %, "failed")
  ++ optpair(l) returns l has the form \spad{[a, b]} and
  ++ a is optional, and
  ++ "failed" otherwise;

Implementation ==> add
Rep := Record(cons?: B, pat: PAT, lev: NonNegativeInteger,
              topvar: List SY, toppred: Any)

dummy:BOP := operator(new()$Symbol)
nopred    := coerce(0$Integer)$AnyFunctions1(Integer)

mkPat      : (B, PAT, NonNegativeInteger) -> %
mkrsy      : (SY, B, B, B) -> RSY
SYM20      : RSY -> 0
PAT20      : PAT -> 0
patcopy    : PAT -> PAT
bitSet?    : (SI , SI) -> B
pateq?     : (PAT, PAT) -> B
LPAT20     : ((0, 0) -> 0, List %) -> 0
taggedElt  : (SI, List %) -> %
isTaggedOp : (% , SI) -> Union(List %, "failed")
incmax     : List % -> NonNegativeInteger

coerce(r:R):% == mkPat(true, [r], 0)
mkPat(c, p, l) == [c, p, l, empty(), nopred]
hasTopPredicate? x == not empty?(x.topvar)
topPredicate x == [x.topvar, x.toppred]
setTopPredicate(x, l, f) == (x.topvar := l; x.toppred := f; x)
constant? p == p.cons?

```



```

depth p          == p.lev
inR? p           == p.pat case ret
symbol? p        == p.pat case sym
isPlus p         == isTaggedOp(p, PAT_PLUS)
isTimes p        == isTaggedOp(p, PAT_TIMES)
isList p         == isTaggedOp(p, PAT_LIST)
isExpt p         == (p.pat case exp => p.pat.exp; "failed")
isQuotient p     == (p.pat case qot => p.pat.qot; "failed")
hasPredicate? p == not empty? predicates p
quoted? p        == symbol? p and zero?(p.pat.sym.tag)
generic? p       == symbol? p and bitSet?(p.pat.sym.tag, SYM_GENERIC)
multiple? p      == symbol? p and bitSet?(p.pat.sym.tag, SYM_MULTIPLE)
optional? p      == symbol? p and bitSet?(p.pat.sym.tag, SYM_OPTIONAL)
bitSet?(a, b)    == And(a, b) ^= 0
coerce(p:%):0    == PAT20(p.pat)
p1:% ** p2:%     == taggedElt(PAT_EXPT, [p1, p2])
LPAT20(f, l)     == reduce(f, [x::0 for x in l])$List(0)
retract(p:%):R   == (inR? p => p.pat.ret; error "Not retractable")
convert(l:List %):% == taggedElt(PAT_LIST, l)
retractIfCan(p:%):Union(R,"failed") == (inR? p => p.pat.ret; "failed")
withPredicates(p, l) == setPredicates(copy p, l)
coerce(sy:SY):%  == patternVariable(sy, false, false, false)
copy p           == [constant? p, patcopy(p.pat), p.lev, p.topvar, p.toppred]

-- returns [a, b] if #l = 2 and optional? a, "failed" otherwise
optpair l ==
  empty? rest rest l =>
    b := first rest l
    optional?(a := first l) => l
    optional? b => reverse l
    "failed"
    "failed"

incmax l ==
  1 + reduce("max", [p.lev for p in l], 0)$List(NonNegativeInteger)

p1 = p2 ==
  (p1.cons? = p2.cons?) and (p1.lev = p2.lev) and
  (p1.topvar = p2.topvar) and
  ((EQ(p1.toppred, p2.toppred)$Lisp) pretend B) and
  pateq?(p1.pat, p2.pat)

isPower p ==
  (u := isTaggedOp(p, PAT_EXPT)) case "failed" => "failed"
  [first(u::List(%)), second(u::List(%))]

taggedElt(n, l) ==
  mkPat(every?(constant?, l), [[n, dummy, l]$KER], incmax l)

elt(o, l) ==

```

```

is?(o, POWER) and #1 = 2 => first(1) ** last(1)
mkPat(every?(constant?, 1), [[0, o, 1]$KER], incmax 1)

isOp p ==
  (p.pat case ker) and zero?(p.pat.ker.tag) =>
    [p.pat.ker.op, p.pat.ker.arg]
  "failed"

isTaggedOp(p,t) ==
  (p.pat case ker) and (p.pat.ker.tag = t) => p.pat.ker.arg
  "failed"

if R has Monoid then
  1 == 1::R::%
else
  1 == taggedElt(PAT_ONE, empty())

if R has AbelianMonoid then
  0 == 0::R::%
else
  0 == taggedElt(PAT_ZERO, empty())

p:% ** n:NonNegativeInteger ==
  p = 0 and n > 0 => 0
  p = 1 or zero? n => 1
--  one? n => p
  (n = 1) => p
  mkPat(constant? p, [[p, n]$REC], 1 + (p.lev))

p1 / p2 ==
  p2 = 1 => p1
  mkPat(constant? p1 and constant? p2, [[p1, p2]$QOT],
        1 + max(p1.lev, p2.lev))

p1 + p2 ==
  p1 = 0 => p2
  p2 = 0 => p1
  (u1 := isPlus p1) case List(%) =>
    (u2 := isPlus p2) case List(%) =>
      taggedElt(PAT_PLUS, concat(u1::List %, u2::List %))
      taggedElt(PAT_PLUS, concat(u1::List %, p2))
  (u2 := isPlus p2) case List(%) =>
    taggedElt(PAT_PLUS, concat(p1, u2::List %))
    taggedElt(PAT_PLUS, [p1, p2])

p1 * p2 ==
  p1 = 0 or p2 = 0 => 0
  p1 = 1 => p2
  p2 = 1 => p1
  (u1 := isTimes p1) case List(%) =>

```

```

(u2 := isTimes p2) case List(%) =>
  taggedElt(PAT_TIMES, concat(u1::List %, u2::List %))
  taggedElt(PAT_TIMES, concat(u1::List %, p2))
(u2 := isTimes p2) case List(%) =>
  taggedElt(PAT_TIMES, concat(p1, u2::List %))
taggedElt(PAT_TIMES, [p1, p2])

isOp(p, o) ==
  (p.pat case ker) and zero?(p.pat.ker.tag) and (p.pat.ker.op = o) =>
    p.pat.ker.arg
  "failed"

predicates p ==
  symbol? p => p.pat.sym.pred
  empty()

setPredicates(p, l) ==
  generic? p => (p.pat.sym.pred := l; p)
  error "Can only attach predicates to generic symbol"

resetBadValues p ==
  generic? p => (p.pat.sym.bad := empty()$List(Any); p)
  error "Can only attach bad values to generic symbol"

addBadValue(p, a) ==
  generic? p =>
    if not member?(a, p.pat.sym.bad) then
      p.pat.sym.bad := concat(a, p.pat.sym.bad)
    p
  error "Can only attach bad values to generic symbol"

getBadValues p ==
  generic? p => p.pat.sym.bad
  error "Not a generic symbol"

SYM20 p ==
  sy := (p.val)::0
  empty?(p.pred) => sy
  paren infix(" | "::0, sy,
    reduce("and",[sub("f"::0, i::0) for i in 1..#(p.pred)])$List(0))

variables p ==
  constant? p => empty()
  generic? p => [p]
  q := p.pat
  q case ret => empty()
  q case exp => variables(q.exp.val)
  q case qot => concat_!(variables(q.qot.num), variables(q.qot.den))
  q case ker => concat [variables r for r in q.ker.arg]
  empty()

```

```

PAT20 p ==
  p case ret => (p.ret)::0
  p case sym => SYM20(p.sym)
  p case exp => (p.exp.val)::0 ** (p.exp.exponent)::0
  p case qot => (p.qot.num)::0 / (p.qot.den)::0
  p.ker.tag = PAT_PLUS => LPAT20("+", p.ker.arg)
  p.ker.tag = PAT_TIMES => LPAT20("*", p.ker.arg)
  p.ker.tag = PAT_LIST => (p.ker.arg)::0
  p.ker.tag = PAT_ZERO => 0::Integer::0
  p.ker.tag = PAT_ONE => 1::Integer::0
  l := [x::0 for x in p.ker.arg]$List(0)
  (u:=display(p.ker.op)) case "failed" =>prefix(name(p.ker.op)::0,l)
  (u::(List 0 -> 0)) 1

patcopy p ==
  p case ret => [p.ret]
  p case sym =>
    [[p.sym.tag, p.sym.val, copy(p.sym.pred), copy(p.sym.bad)]$RSY]
  p case ker=>[[p.ker.tag,p.ker.op,[copy x for x in p.ker.arg]]$KER]
  p case qot => [[copy(p.qot.num), copy(p.qot.den)]$QOT]
  [[copy(p.exp.val), p.exp.exponent]$REC]

pateq?(p1, p2) ==
  p1 case ret => (p2 case ret) and (p1.ret = p2.ret)
  p1 case qot =>
    (p2 case qot) and (p1.qot.num = p2.qot.num)
    and (p1.qot.den = p2.qot.den)
  p1 case sym =>
    (p2 case sym) and (p1.sym.val = p2.sym.val)
    and {p1.sym.pred} =$Set(Any) {p2.sym.pred}
    and {p1.sym.bad} =$Set(Any) {p2.sym.bad}
  p1 case ker =>
    (p2 case ker) and (p1.ker.tag = p2.ker.tag)
    and (p1.ker.op = p2.ker.op) and (p1.ker.arg = p2.ker.arg)
  (p2 case exp) and (p1.exp.exponent = p2.exp.exponent)
  and (p1.exp.val = p2.exp.val)

retractIfCan(p:%):Union(SY, "failed") ==
  symbol? p => p.pat.sym.val
  "failed"

mkrsy(t, c?, o?, m?) ==
  c? => [0, t, empty(), empty()]
  mlt := (m? => SYM_MULTIPLE; 0)
  opt := (o? => SYM_OPTIONAL; 0)
  [Or(Or(SYM_GENERIC, mlt), opt), t, empty(), empty()]

patternVariable(sy, c?, o?, m?) ==
  rsy := mkrsy(sy, c?, o?, m?)

```

```
mkPat(zero?(rsy.tag), [rsy], 0)
```

---

— PATTERN.dotabb —

```
"PATTERN" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PATTERN"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"PATTERN" -> "ALIST"
```

---

## 17.11 domain PATLRES PatternMatchListResult

— PatternMatchListResult.input —

```
)set break resume
)sys rm -f PatternMatchListResult.output
)spool PatternMatchListResult.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PatternMatchListResult
--R PatternMatchListResult(R: SetCategory,S: SetCategory,L: ListAggregate S) is a domain co
--R Abbreviation for PatternMatchListResult is PATLRES
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PATLRES
--R
--R----- Operations -----
--R ?? : (%,% ) -> Boolean               coerce : % -> OutputForm
--R failed : () -> %                     failed? : % -> Boolean
--R hash : % -> SingleInteger           latex : % -> String
--R new : () -> %                       ?~=? : (%,% ) -> Boolean
--R atoms : % -> PatternMatchResult(R,S)
--R lists : % -> PatternMatchResult(R,L)
--R makeResult : (PatternMatchResult(R,S),PatternMatchResult(R,L)) -> %
--R
--E 1

)spool
)lisp (bye)
```

---



---

— PatternMatchListResult.help —

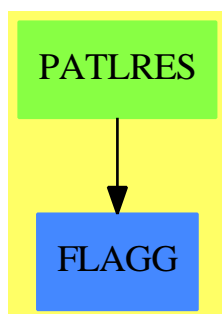
---

```
=====
PatternMatchListResult examples
=====
```

See Also:  
o )show PatternMatchListResult

---

### 17.11.1 PatternMatchListResult (PATLRES)



See

⇒ “PatternMatchResult” (PATRES) 17.12.1 on page 1900

#### Exports:

```
atoms  coerce  failed      failed?  hash
latex  lists   makeResult  new      ~=?
?~=?
```

---

— domain PATLRES PatternMatchListResult —

---

```
)abbrev domain PATLRES PatternMatchListResult
++ Author: Manuel Bronstein
++ Date Created: 4 Dec 1989
++ Date Last Updated: 4 Dec 1989
++ Keywords: pattern, matching, list.
++ Description:
++ A PatternMatchListResult is an object internally returned by the
++ pattern matcher when matching on lists.
++ It is either a failed match, or a pair of PatternMatchResult,
++ one for atoms (elements of the list), and one for lists.
```

```

-- not exported

PatternMatchListResult(R:SetCategory, S:SetCategory, L>ListAggregate S):
  SetCategory with
    failed? : % -> Boolean
    ++ failed?(r) tests if r is a failed match.
    failed : () -> %
    ++ failed() returns a failed match.
    new : () -> %
    ++ new() returns a new empty match result.
    makeResult: (PatternMatchResult(R,S), PatternMatchResult(R,L)) -> %
    ++ makeResult(r1,r2) makes the combined result [r1,r2].
    atoms : % -> PatternMatchResult(R, S)
    ++ atoms(r) returns the list of matches that match atoms
    ++ (elements of the lists).
    lists : % -> PatternMatchResult(R, L)
    ++ lists(r) returns the list of matches that match lists.
== add
Rep := Record(a:PatternMatchResult(R, S), l:PatternMatchResult(R, L))

new() == [new(), new()]
atoms r == r.a
lists r == r.l
failed() == [failed(), failed()]
failed? r == failed?(atoms r)
x = y == (atoms x = atoms y) and (lists x = lists y)

makeResult(r1, r2) ==
  failed? r1 or failed? r2 => failed()
  [r1, r2]

coerce(r:%):OutputForm ==
  failed? r => atoms(r)::OutputForm
  RecordPrint(r, Rep)$Lisp

```

---

— PATLRES.dotabb —

```

"PATLRES" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PATLRES"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"PATLRES" -> "FLAGG"

```

---

## 17.12 domain PATRES PatternMatchResult

### — PatternMatchResult.input —

```

)set break resume
)sys rm -f PatternMatchResult.output
)spool PatternMatchResult.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PatternMatchResult
--R PatternMatchResult(R: SetCategory,S: SetCategory) is a domain constructor
--R Abbreviation for PatternMatchResult is PATRES
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PATRES
--R
--R----- Operations -----
--R ==? : (%,% ) -> Boolean          addMatch : (Pattern R,S,%) -> %
--R coerce : % -> OutputForm        failed : () -> %
--R failed? : % -> Boolean          hash : % -> SingleInteger
--R latex : % -> String             new : () -> %
--R union : (%,% ) -> %             ~=? : (%,% ) -> Boolean
--R addMatchRestricted : (Pattern R,S,%,S) -> %
--R construct : List Record(key: Symbol,entry: S) -> %
--R destruct : % -> List Record(key: Symbol,entry: S)
--R getMatch : (Pattern R,%) -> Union(S,"failed")
--R insertMatch : (Pattern R,S,%) -> %
--R satisfy? : (%,(Pattern R) -> Union(Boolean,"failed"))
--R
--E 1

)spool
)lisp (bye)

```

### — PatternMatchResult.help —

```

=====
PatternMatchResult examples
=====

```

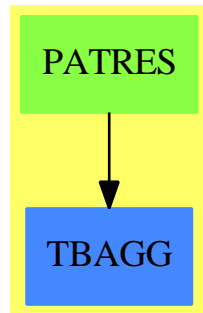
```

See Also:
o )show PatternMatchResult

```



### 17.12.1 PatternMatchResult (PATRES)



See

⇒ “PatternMatchListResult” (PATLRES) 17.11.1 on page 1897

#### Exports:

|          |                    |          |           |             |
|----------|--------------------|----------|-----------|-------------|
| addMatch | addMatchRestricted | coerce   | construct | destruct    |
| failed   | failed?            | getMatch | hash      | insertMatch |
| latex    | new                | satisfy? | union     | ?=?         |
| ?~=?     |                    |          |           |             |

— domain PATRES PatternMatchResult —

```

)abbrev domain PATRES PatternMatchResult
++ Author: Manuel Bronstein
++ Date Created: 28 Nov 1989
++ Date Last Updated: 5 Jul 1990
++ Keywords: pattern, matching.
++ Description:
++ A PatternMatchResult is an object internally returned by the
++ pattern matcher; It is either a failed match, or a list of
++ matches of the form (var, expr) meaning that the variable var
++ matches the expression expr.
-- not exported

PatternMatchResult(R:SetCategory, S:SetCategory): SetCategory with
  failed?          : % -> Boolean
    ++ failed?(r) tests if r is a failed match.
  failed          : () -> %
    ++ failed() returns a failed match.
  new             : () -> %
    ++ new() returns a new empty match result.
  union           : (% , %) -> %
    ++ union(a, b) makes the set-union of two match results.
  getMatch        : (Pattern R, %) -> Union(S, "failed")
  
```

```

    ++ getMatch(var, r) returns the expression that var matches
    ++ in the result r, and "failed" if var is not matched in r.
addMatch      : (Pattern R, S, %) -> %
    ++ addMatch(var, expr, r) adds the match (var, expr) in r,
    ++ provided that expr satisfies the predicates attached to var,
    ++ and that var is not matched to another expression already.
insertMatch   : (Pattern R, S, %) -> %
    ++ insertMatch(var, expr, r) adds the match (var, expr) in r,
    ++ without checking predicates or previous matches for var.
addMatchRestricted: (Pattern R, S, %, S) -> %
    ++ addMatchRestricted(var, expr, r, val) adds the match
    ++ (var, expr) in r,
    ++ provided that expr satisfies the predicates attached to var,
    ++ that var is not matched to another expression already,
    ++ and that either var is an optional pattern variable or that
    ++ expr is not equal to val (usually an identity).
destruct      : % -> List Record(key:Symbol, entry:S)
    ++ destruct(r) returns the list of matches (var, expr) in r.
    ++ Error: if r is a failed match.
construct     : List Record(key:Symbol, entry:S) -> %
    ++ construct([v1,e1],...,[vn,en]) returns the match result
    ++ containing the matches (v1,e1),...,(vn,en).
satisfy?     : (% , Pattern R) -> Union(Boolean, "failed")
    ++ satisfy?(r, p) returns true if the matches satisfy the
    ++ top-level predicate of p, false if they don't, and "failed"
    ++ if not enough variables of p are matched in r to decide.

== add
LR ==> AssociationList(Symbol, S)

import PatternFunctions1(R, S)

Rep := Union(LR, "failed")

new()          == empty()
failed()       == "failed"
failed? x      == x case "failed"
insertMatch(p, x, l) == concat([retract p, x], l::LR)
construct l    == construct(l)$LR
destruct l     == entries(l::LR)$LR

-- returns "failed" if not all the variables of the pred. are matched
satisfy?(r, p) ==
    failed? r => false
    lr := r::LR
    lv := [if (u := search(v, lr)) case "failed" then return "failed"
           else u::S for v in topPredicate(p).var]$List(S)
    satisfy?(lv, p)

union(x, y) ==

```

```

failed? x or failed? y => failed()
removeDuplicates concat(x::LR, y::LR)

x = y ==
  failed? x => failed? y
  failed? y => false
  x::LR ==LR y::LR

coerce(x:%):OutputForm ==
  failed? x => "Does not match":OutputForm
  destruct(x)::OutputForm

addMatchRestricted(p, x, l, ident) ==
  (not optional? p) and (x = ident) => failed()
  addMatch(p, x, l)

addMatch(p, x, l) ==
  failed?(l) or not(satisfy?(x, p)) => failed()
  al := l::LR
  sy := retract(p)@Symbol
  (r := search(sy, al)) case "failed" => insertMatch(p, x, l)
  r::S = x => l
  failed()

getMatch(p, l) ==
  failed? l => "failed"
  search(retract(p)@Symbol, l::LR)

```

---

— PATRES.dotabb —

```

"PATRES" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PATRES"]
"TBAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=TBAGG"]
"PATRES" -> "TBAGG"

```

---

## 17.13 domain PENDTREE PendantTree

— PendantTree.input —

```

)set break resume
)sys rm -f PendantTree.output

```

```

)spool PendantTree.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PendantTree
--R PendantTree S: SetCategory is a domain constructor
--R Abbreviation for PendantTree is PENDTREE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PENDTREE
--R
--R----- Operations -----
--R children : % -> List %               coerce : % -> Tree S
--R copy : % -> %                       cyclic? : % -> Boolean
--R distance : (%,% ) -> Integer         ?.right : (% ,right) -> %
--R ?.left : (% ,left) -> %             ?.value : (% ,value) -> S
--R empty : () -> %                     empty? : % -> Boolean
--R eq? : (%,% ) -> Boolean             leaf? : % -> Boolean
--R leaves : % -> List S               left : % -> %
--R map : ((S -> S),%) -> %            nodes : % -> List %
--R ptree : (%,% ) -> %               ptree : S -> %
--R right : % -> %                    sample : () -> %
--R value : % -> S
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ==? : (%,% ) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R child? : (%,% ) -> Boolean if S has SETCAT
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,% ) -> NonNegativeInteger if $ has finiteAggregate and S has SETCAT
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (% ,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (% ,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (% ,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (% ,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (% ,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,% ) -> Boolean if $ has finiteAggregate and S has SETCAT
--R members : % -> List S if $ has finiteAggregate
--R more? : (% ,NonNegativeInteger) -> Boolean
--R node? : (%,% ) -> Boolean if S has SETCAT
--R parts : % -> List S if $ has finiteAggregate
--R setchildren! : (% ,List %) -> % if $ has shallowlyMutable
--R setelt : (% ,right,%) -> % if $ has shallowlyMutable
--R setelt : (% ,left,%) -> % if $ has shallowlyMutable
--R setelt : (% ,value,S) -> S if $ has shallowlyMutable
--R setleft! : (%,% ) -> % if $ has shallowlyMutable

```

```

--R setright! : (%,% ) -> % if $ has shallowlyMutable
--R setvalue! : (% ,S) -> S if $ has shallowlyMutable
--R size? : (% ,NonNegativeInteger) -> Boolean
--R ?~=? : (% ,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)

```

---

— **PendantTree.help** —

```

=====
PendantTree examples
=====

```

```

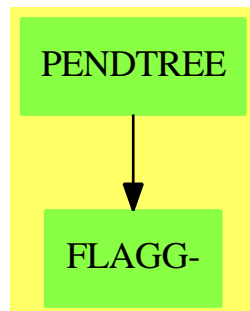
See Also:
o )show PendantTree

```

---

A `PendantTree(S)` is either a leaf? and is an `S` or has a left and a right both `PendantTree(S)`'s

### 17.13.1 PendantTree (PENDTREE)



See

- ⇒ “Tree” (TREE) 21.10.1 on page 2699
- ⇒ “BinaryTree” (BTREE) 3.11.1 on page 292
- ⇒ “BinarySearchTree” (BSTREE) 3.9.1 on page 285
- ⇒ “BinaryTournament” (BTourn) 3.10.1 on page 289
- ⇒ “BalancedBinaryTree” (BBTREE) 3.1.1 on page 234

**Exports:**

|              |         |          |           |           |
|--------------|---------|----------|-----------|-----------|
| any?         | child?  | children | coerce    | copy      |
| count        | cyclic? | distance | empty     | empty?    |
| eq?          | eval    | every?   | hash      | latex     |
| leaf?        | leaves  | left     | less?     | map       |
| map!         | member? | members  | more?     | node?     |
| nodes        | parts   | ptree    | right     | sample    |
| setchildren! | setelt  | setleft! | setright! | setvalue! |
| size?        | value   | #?       | ?=?       | ?~=?      |
| ?right       | ?left   | ?value   |           |           |

— domain PENDTREE PendantTree —

```

)abbrev domain PENDTREE PendantTree
++ Author: Mark Botch
++ Description:
++ This domain has no description

PendantTree(S: SetCategory): T == C where
  T == BinaryRecursiveAggregate(S) with
    ptree : S->%
      ++ ptree(s) is a leaf? pendant tree
      ++
      ++X t1:=ptree([1,2,3])

    ptree:(%, %)->%
      ++ ptree(x,y) is not documented
      ++
      ++X t1:=ptree([1,2,3])
      ++X ptree(t1,ptree([1,2,3]))

    coerce:%->Tree S
      ++ coerce(x) is not documented
      ++
      ++X t1:=ptree([1,2,3])
      ++X t2:=ptree(t1,ptree([1,2,3]))
      ++X t2::Tree List PositiveInteger

C == add
  Rep := Tree S
  import Tree S
  coerce (t:%):Tree S == t pretend Tree S
  ptree(n) == tree(n,[])$Rep pretend %
  ptree(l,r) == tree(value(r:Rep)$Rep,cons(l,children(r:Rep)$Rep)):%
  leaf? t == empty?(children(t)$Rep)
  t1=t2 == (t1:Rep) = (t2:Rep)
  left b ==
    leaf? b => error "ptree:no left"
    first(children(b)$Rep)

```

---

---

— Permutation.input —

[illegible]

[illegible]



```
)lisp (bye)
```

---

— **Permutation.help** —

```
=====
Permutation Examples
=====
```

We represent a permutation as two lists of equal length representing preimages and images of moved points. I.e., fixed points do not occur in either of these lists. This enables us to compute the set of fixed points and the set of moved points easily.

```
p := coercePreimagesImages([[1,2,3],[1,2,3]])
1
                                Type: Permutation PositiveInteger

movedPoints p
{}
                                Type: Set PositiveInteger

even? p
true
                                Type: Boolean

p := coercePreimagesImages([[0,1,2,3],[3,0,2,1]])$PERM ZMOD 4
(1 0 3)
                                Type: Permutation IntegerMod 4

fixedPoints p
{2}
                                Type: Set IntegerMod 4

q := coercePreimagesImages([[0,1,2,3],[1,0]])$PERM ZMOD 4
(1 0)
                                Type: Permutation IntegerMod 4

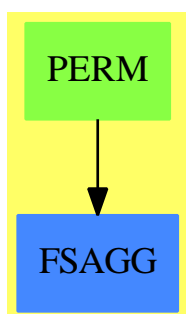
fixedPoints(p*q)
{2,0}
                                Type: Set IntegerMod 4

even?(p*q)
false
                                Type: Boolean
```

See Also:

o )show Permutation

## 17.14.1 Permutation (PERM)

**Exports:**

|                   |                       |                    |
|-------------------|-----------------------|--------------------|
| 1                 | coerce                | coerceImages       |
| coerceListOfPairs | coercePreimagesImages | commutator         |
| conjugate         | cycle                 | cyclePartition     |
| cycles            | degree                | eval               |
| even?             | fixedPoints           | hash               |
| inv               | latex                 | listRepresentation |
| max               | min                   | movedPoints        |
| numberOfCycles    | odd?                  | one?               |
| orbit             | order                 | recip              |
| sample            | sign                  | sort               |
| ?*?               | ?**?                  | ?/?                |
| ?<?               | ?=?                   | ?^?                |
| ?..?              | ?~=?                  | ?<=?               |
| ?>?               | ?>=?                  |                    |

— domain PERM Permutation —

```

)abbrev domain PERM Permutation
++ Authors: Johannes Grabmeier, Holger Gollan, Martin Rubey
++ Date Created: 19 May 1989
++ Date Last Updated: 2 June 2006
++ Basic Operations: _, degree, movedPoints, cyclePartition, order,
++   numberOfCycles, sign, even?, odd?
++ Related Constructors: PermutationGroup, PermutationGroupExamples
++ Also See: RepresentationTheoryPackage1
++ AMS Classifications:
++ Keywords:
++ Reference: G. James/A. Kerber: The Representation Theory of the Symmetric
++   Group. Encycl. of Math. and its Appl., Vol. 16., Cambridge
  
```

```

++ Description:
++ Permutation(S) implements the group of all bijections
++ on a set S, which move only a finite number of points.
++ A permutation is considered as a map from S into S. In particular
++ multiplication is defined as composition of maps:\br
++ pi1 * pi2 = pi1 o pi2.\br
++ The internal representation of permutations are two lists
++ of equal length representing preimages and images.

Permutation(S:SetCategory): public == private where

B      ==> Boolean
PI     ==> PositiveInteger
I      ==> Integer
L      ==> List
NNI    ==> NonNegativeInteger
V      ==> Vector
PT     ==> Partition
OUTFORM ==> OutputForm
RECCYPE ==> Record(cycl: L L S, permut: %)
RECPRI ==> Record(preimage: L S, image : L S)

public ==> PermutationCategory S with

listRepresentation: %          -> RECPRI
++ listRepresentation(p) produces a representation rep of
++ the permutation p as a list of preimages and images, i.e
++ p maps (rep.preimage).k to (rep.image).k for all
++ indices k. Elements of \spad{S} not in (rep.preimage).k
++ are fixed points, and these are the only fixed points of the
++ permutation.
coercePreimagesImages : List List S -> %
++ coercePreimagesImages(lls) coerces the representation lls
++ of a permutation as a list of preimages and images to a permutation.
++ We assume that both preimage and image do not contain repetitions.
++
++X p := coercePreimagesImages([[1,2,3],[1,2,3]])
++X q := coercePreimagesImages([[0,1,2,3],[3,0,2,1]])$PERM ZMOD 4
coerce      : List List S -> %
++ coerce(lls) coerces a list of cycles lls to a
++ permutation, each cycle being a list with no
++ repetitions, is coerced to the permutation, which maps
++ ls.i to ls.i+1, indices modulo the length of the list,
++ then these permutations are multiplied.
++ Error: if repetitions occur in one cycle.
coerce      : List S -> %
++ coerce(ls) coerces a cycle ls, i.e. a list with not
++ repetitions to a permutation, which maps ls.i to
++ ls.i+1, indices modulo the length of the list.
++ Error: if repetitions occur.

```

```

coerceListOfPairs : List List S      -> %
  ++ coerceListOfPairs(lis) coerces a list of pairs lis to a
  ++ permutation.
  ++ Error: if not consistent, i.e. the set of the first elements
  ++ coincides with the set of second elements.
--coerce          : %                  -> OUTFORM
  ++ coerce(p) generates output of the permutation p with domain
  ++ OutputForm.
degree            : %                  -> NonNegativeInteger
  ++ degree(p) returns the number of points moved by the
  ++ permutation p.
movedPoints       : %                  -> Set S
  ++ movedPoints(p) returns the set of points moved by the permutation p.
  ++
  ++X p := coercePreimagesImages([[1,2,3],[1,2,3]])
  ++X movedPoints p
cyclePartition    : %                  -> Partition
  ++ cyclePartition(p) returns the cycle structure of a permutation
  ++ p including cycles of length 1 only if S is finite.
order             : %                  -> NonNegativeInteger
  ++ order(p) returns the order of a permutation p as a group element.
numberOfCycles    : %                  -> NonNegativeInteger
  ++ numberOfCycles(p) returns the number of non-trivial cycles of
  ++ the permutation p.
sign              : %                  -> Integer
  ++ sign(p) returns the signum of the permutation p, +1 or -1.
even?             : %                  -> Boolean
  ++ even?(p) returns true if and only if p is an even permutation,
  ++ i.e. sign(p) is 1.
  ++
  ++X p := coercePreimagesImages([[1,2,3],[1,2,3]])
  ++X even? p
odd?              : %                  -> Boolean
  ++ odd?(p) returns true if and only if p is an odd permutation
  ++ i.e. sign(p) is -1.
sort              : L %                -> L %
  ++ sort(lp) sorts a list of permutations lp according to
  ++ cycle structure first according to length of cycles,
  ++ second, if S has \spadtype{Finite} or S has
  ++ \spadtype{OrderedSet} according to lexicographical order of
  ++ entries in cycles of equal length.
if S has Finite then
  fixedPoints     : %                  -> Set S
    ++ fixedPoints(p) returns the points fixed by the permutation p.
    ++X p := coercePreimagesImages([[0,1,2,3],[3,0,2,1]])$PERM ZMOD 4
    ++X fixedPoints p
if S has IntegerNumberSystem or S has Finite then
coerceImages      : L S                -> %
  ++ coerceImages(ls) coerces the list ls to a permutation
  ++ whose image is given by ls and the preimage is fixed

```

```

++ to be [1,...,n].
++ Note: {coerceImages(ls)=coercePreimagesImages([1,...,n],ls)}.
++ We assume that both preimage and image do not contain repetitions.

private ==> add

-- representation of the object:

Rep := V L S

-- import of domains and packages

import OutputForm
import Vector List S

-- variables

p,q      : %
exp      : I

-- local functions first, signatures:

smaller? : (S,S) -> B
rotateCycle: L S -> L S
coerceCycle: L L S -> %
smallerCycle?: (L S, L S) -> B
shorterCycle?: (L S, L S) -> B
permord: (RECCYPE,RECCYPE) -> B
coerceToCycle: (% ,B) -> L L S
duplicates?: L S -> B

smaller?(a:S, b:S): B ==
  S has OrderedSet => a <$S b
  S has Finite      => lookup a < lookup b
  false

rotateCycle(cyc: L S): L S ==
  -- smallest element is put in first place
  -- doesn't change cycle if underlying set
  -- is not ordered or not finite.
  min:S := first cyc
  minpos:I := 1          -- 1 = minIndex cyc
  for i in 2..maxIndex cyc repeat
    if smaller?(cyc.i,min) then
      min := cyc.i
      minpos := i
--  one? minpos => cyc
  (minpos = 1) => cyc
  concat(last(cyc,((#cyc-minpos+1)::NNI)),first(cyc,(minpos-1)::NNI))

```

```

coerceCycle(lls : L L S): % ==
  perm : % := 1
  for lists in reverse lls repeat
    perm := cycle lists * perm
  perm

smallerCycle?(cyca: L S, cycb: L S): B ==
  #cyca ^= #cycb =>
    #cyca < #cycb
  for i in cyca for j in cycb repeat
    i ^= j => return smaller?(i, j)
  false

shorterCycle?(cyca: L S, cycb: L S): B ==
  #cyca < #cycb

permord(pa: RECCYPE, pb : RECCYPE): B ==
  for i in pa.cycl for j in pb.cycl repeat
    i ^= j => return smallerCycle?(i, j)
  #pa.cycl < #pb.cycl

coerceToCycle(p: %, doSorting?: B): L L S ==
  preim := p.1
  im := p.2
  cycles := nil()$(L L S)
  while not null preim repeat
    -- start next cycle
    firstEltInCycle: S := first preim
    nextCycle : L S := list firstEltInCycle
    preim := rest preim
    nextEltInCycle := first im
    im := rest im
    while nextEltInCycle ^= firstEltInCycle repeat
      nextCycle := cons(nextEltInCycle, nextCycle)
      i := position(nextEltInCycle, preim)
      preim := delete(preim,i)
      nextEltInCycle := im.i
      im := delete(im,i)
    nextCycle := reverse nextCycle
    -- check on 1-cycles, we don't list these
    if not null rest nextCycle then
      if doSorting? and (S has OrderedSet or S has Finite) then
        -- put smallest element in cycle first:
        nextCycle := rotateCycle nextCycle
      cycles := cons(nextCycle, cycles)
  not doSorting? => cycles
  -- sort cycles
  S has OrderedSet or S has Finite =>
    sort(smallerCycle?,cycles)$(L L S)
  sort(shorterCycle?,cycles)$(L L S)

```

```

duplicates? (ls : L S ): B ==
  x := copy ls
  while not null x repeat
    member? (first x ,rest x) => return true
    x := rest x
  false

-- now the exported functions

listRepresentation p ==
  s : RECPRI := [p.1,p.2]

coercePreimagesImages preImageAndImage ==
  preImage: List S := []
  image: List S := []
  for i in preImageAndImage.1
    for pi in preImageAndImage.2 repeat
      if i ~= pi then
        preImage := cons(i, preImage)
        image := cons(pi, image)

  [preImage, image]

movedPoints p == construct p.1

degree p == #movedPoints p

p = q ==
  #(preimp := p.1) ^= #(preimpq := q.1) => false
  for i in 1..maxIndex preimp repeat
    pos := position(preimp.i, preimpq)
    pos = 0 => return false
    (p.2).i ^= (q.2).pos => return false
  true

orbit(p ,el) ==
  -- start with a 1-element list:
  out : Set S := brace list el
  el2 := eval(p, el)
  while el2 ^= el repeat
    -- be carefull: insert adds one element
    -- as side effect to out
    insert_!(el2, out)
    el2 := eval(p, el2)
  out

cyclePartition p ==
  partition([#c for c in coerceToCycle(p, false)])$Partition

```

```

order p ==
  ord: I := lcm removeDuplicates convert cyclePartition p
  ord::NNI

sign(p) ==
  even? p => 1
  - 1

even?(p) == even?((#(p.1) - numberOfCycles p)
  -- see the book of James and Kerber on symmetric groups
  -- for this formula.

odd?(p) == odd?((#(p.1) - numberOfCycles p)

pa < pb ==
  pacyc:= coerceToCycle(pa,true)
  pbcyc:= coerceToCycle(pb,true)
  for i in pacyc for j in pbcyc repeat
    i ^= j => return smallerCycle? ( i, j )
  maxIndex pacyc < maxIndex pbcyc

coerce(lls : L L S): % == coerceCycle lls

coerce(ls : L S): % == cycle ls

sort(inList : L %): L % ==
  not (S has OrderedSet or S has Finite) => inList
  ownList: L RECCYPE := nil()$(L RECCYPE)
  for sigma in inList repeat
    ownList :=
      cons([coerceToCycle(sigma,true),sigma]::RECCYPE, ownList)
  ownList := sort(permod, ownList)$(L RECCYPE)
  outList := nil()$(L %)
  for rec in ownList repeat
    outList := cons(rec.permut, outList)
  reverse outList

coerce (p: %): OUTFORM ==
  cycles: L L S := coerceToCycle(p,true)
  outfmL : L OUTFORM := nil()
  for cycle in cycles repeat
    outcycL: L OUTFORM := nil()
    for elt in cycle repeat
      outcycL := cons(elt :: OUTFORM, outcycL)
    outfmL := cons(paren blankSeparate reverse outcycL, outfmL)
  -- The identity element will be output as 1:
  null outfmL => outputForm(1@Integer)
  -- represent a single cycle in the form (a b c d)
  -- and not in the form ((a b c d)):
  null rest outfmL => first outfmL

```



```

hconcat reverse outfml

cycles(vs ) == coerceCycle vs

cycle(ls) ==
  #ls < 2 => 1
  duplicates? ls => error "cycle: the input contains duplicates"
  [ls, append(rest ls, list first ls)]

coerceListOfPairs(loP) ==
  preim := nil()$(L S)
  im := nil()$(L S)
  for pair in loP repeat
    if first pair ^= second pair then
      preim := cons(first pair, preim)
      im := cons(second pair, im)
  duplicates?(preim) or duplicates?(im) or brace(preim)$(Set S) _
    ^= brace(im)$(Set S) =>
    error "coerceListOfPairs: the input cannot be interpreted as a permutation"
  [preim, im]

q * p ==
  -- use vectors for efficiency??
  preimOfp : V S := construct p.1
  imOfp : V S := construct p.2
  preimOfq := q.1
  imOfq := q.2
  preimOfqp := nil()$(L S)
  imOfqp := nil()$(L S)
  -- 1 = minIndex preimOfp
  for i in 1..(maxIndex preimOfp) repeat
    -- find index of image of p.i in q if it exists
    j := position(imOfp.i, preimOfq)
    if j = 0 then
      -- it does not exist
      preimOfqp := cons(preimOfp.i, preimOfqp)
      imOfqp := cons(imOfp.i, imOfqp)
    else
      -- it exists
      el := imOfq.j
      -- if the composition fixes the element, we don't
      -- have to do anything
      if el ^= preimOfp.i then
        preimOfqp := cons(preimOfp.i, preimOfqp)
        imOfqp := cons(el, imOfqp)
      -- we drop the parts of q which have to do with p
      preimOfq := delete(preimOfq, j)
      imOfq := delete(imOfq, j)
  [append(preimOfqp, preimOfq), append(imOfqp, imOfq)]

```

```

1 == new(2,empty())$Rep

inv p == [p.2, p.1]

eval(p, el) ==
  pos := position(el, p.1)
  pos = 0 => el
  (p.2).pos

elt(p, el) == eval(p, el)

numberOfCycles p == #coerceToCycle(p, false)

if S has IntegerNumberSystem then

  coerceImages (image) ==
    preImage : L S := [i::S for i in 1..maxIndex image]
    coercePreimagesImages [preImage,image]

if S has Finite then

  coerceImages (image) ==
    preImage : L S := [index(i::PI)::S for i in 1..maxIndex image]
    coercePreimagesImages [preImage,image]

fixedPoints ( p ) == complement movedPoints p

cyclePartition p ==
  pt := partition([#c for c in coerceToCycle(p, false)])$Partition
  pt +$PT conjugate(partition([#fixedPoints(p)])$PT)$PT

```

---

— PERM.dotabb —

```

"PERM" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PERM"]
"FSAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FSAGG"]
"PERM" -> "FSAGG"

```

---

## 17.15 domain PERMGRP PermutationGroup

— PermutationGroup.input —

```

)set break resume
)sys rm -f PermutationGroup.output
)spool PermutationGroup.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PermutationGroup
--R PermutationGroup S: SetCategory is a domain constructor
--R Abbreviation for PermutationGroup is PERMGRP
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PERMGRP
--R
--R----- Operations -----
--R ?<? : (%,% ) -> Boolean          ?<=? : (%,% ) -> Boolean
--R ?=? : (%,% ) -> Boolean          base : % -> List S
--R coerce : List Permutation S -> %   coerce : % -> List Permutation S
--R coerce : % -> OutputForm           degree : % -> NonNegativeInteger
--R hash : % -> SingleInteger          latex : % -> String
--R movedPoints : % -> Set S           orbit : (% ,List S) -> Set List S
--R orbit : (% ,Set S) -> Set Set S    orbit : (% ,S) -> Set S
--R orbits : % -> Set Set S           order : % -> NonNegativeInteger
--R random : % -> Permutation S        ?~=? : (%,% ) -> Boolean
--R ?.? : (% ,NonNegativeInteger) -> Permutation S
--R generators : % -> List Permutation S
--R initializeGroupForWordProblem : (% ,Integer,Integer) -> Void
--R initializeGroupForWordProblem : % -> Void
--R member? : (Permutation S,% ) -> Boolean
--R permutationGroup : List Permutation S -> %
--R random : (% ,Integer) -> Permutation S
--R strongGenerators : % -> List Permutation S
--R wordInGenerators : (Permutation S,% ) -> List NonNegativeInteger
--R wordInStrongGenerators : (Permutation S,% ) -> List NonNegativeInteger
--R wordsForStrongGenerators : % -> List List NonNegativeInteger
--R
--E 1

)spool
)lisp (bye)

```

---

— PermutationGroup.help —

```

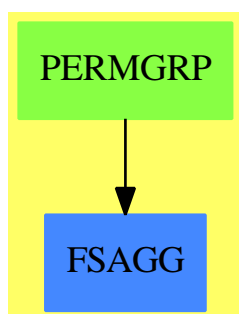
=====
PermutationGroup examples
=====

```

See Also:

o )show PermutationGroup

### 17.15.1 PermutationGroup (PERMGRP)



#### Exports:

|                          |                  |                               |
|--------------------------|------------------|-------------------------------|
| base                     | coerce           | degree                        |
| hash                     | latex            | movedPoints                   |
| orbit                    | orbits           | order                         |
| random                   | generators       | initializeGroupForWordProblem |
| member?                  | permutationGroup | random                        |
| strongGenerators         | wordInGenerators | wordInStrongGenerators        |
| wordsForStrongGenerators | ?~=?             | ??                            |
| ?<?                      | ?<=?             | ?=?                           |

— domain PERMGRP PermutationGroup —

```

)abbrev domain PERMGRP PermutationGroup
++ Authors: G. Schneider, H. Gollan, J. Grabmeier
++ Date Created: 13 February 1987
++ Date Last Updated: 24 May 1991
++ Basic Operations:
++ Related Constructors: PermutationGroupExamples, Permutation
++ Also See: RepresentationTheoryPackage1
++ AMS Classifications:
++ Keywords: permutation, permutation group, group operation, word problem
++ References:
++   C. Sims: Determining the conjugacy classes of a permutation group,
++   in Computers in Algebra and Number Theory, SIAM-AMS Proc., Vol. 4,
++   Amer. Math. Soc., Providence, R. I., 1971, pp. 191-195
++ Description:
++ PermutationGroup implements permutation groups acting
  
```

```

++ on a set S, i.e. all subgroups of the symmetric group of S,
++ represented as a list of permutations (generators). Note that
++ therefore the objects are not members of the \Language category
++ \spadtype{Group}.
++ Using the idea of base and strong generators by Sims,
++ basic routines and algorithms
++ are implemented so that the word problem for
++ permutation groups can be solved.
--++ Note: we plan to implement lattice operations on the subgroup
--++ lattice in a later release

```

```

PermutationGroup(S:SetCategory): public == private where

```

```

L      ==> List
PERM ==> Permutation
FSET ==> Set
I      ==> Integer
NNI    ==> NonNegativeInteger
V      ==> Vector
B      ==> Boolean
OUT    ==> OutputForm
SYM    ==> Symbol
REC    ==> Record ( orb : L NNI , svc : V I )
REC2   ==> Record(order:NNI,sgset:L V NNI,_,
                  gpbase:L NNI,orbs:L REC,mp:L S,wd:L L NNI)
REC3   ==> Record(elt:V NNI,lst:L NNI)
REC4   ==> Record(bool:B,lst:L NNI)

public ==> SetCategory with

coerce      : %          -> L PERM S
++ coerce(gp) returns the generators of the group gp.
generators  : %          -> L PERM S
++ generators(gp) returns the generators of the group gp.
elt         : (% ,NNI)   -> PERM S
++ elt(gp,i) returns the i-th generator of the group gp.
random      : (% ,I)     -> PERM S
++ random(gp,i) returns a random product of maximal i generators
++ of the group gp.
random      : %          -> PERM S
++ random(gp) returns a random product of maximal 20 generators
++ of the group gp.
++ Note: random(gp)=random(gp,20).
order       : %          -> NNI
++ order(gp) returns the order of the group gp.
degree      : %          -> NNI
++ degree(gp) returns the number of points moved by all permutations
++ of the group gp.
base        : %          -> L S
++ base(gp) returns a base for the group gp.

```

```

strongGenerators : %          -> L PERM S
++ strongGenerators(gp) returns strong generators for
++ the group gp.
wordsForStrongGenerators      : %          -> L L NNI
++ wordsForStrongGenerators(gp) returns the words for the strong
++ generators of the group gp in the original generators of
++ gp, represented by their indices in the list, given by
++ generators.
coerce                      : L PERM S -> %
++ coerce(ls) coerces a list of permutations ls to the group
++ generated by this list.
permutationGroup            : L PERM S -> %
++ permutationGroup(ls) coerces a list of permutations ls to
++ the group generated by this list.
orbit                       : (% ,S)      -> FSET S
++ orbit(gp,el) returns the orbit of the element el under the
++ group gp, i.e. the set of all points gained by applying
++ each group element to el.
orbits                      : %          -> FSET FSET S
++ orbits(gp) returns the orbits of the group gp, i.e.
++ it partitions the (finite) of all moved points.
orbit                       : (% ,FSET S)-> FSET FSET S
++ orbit(gp,els) returns the orbit of the unordered
++ set els under the group gp.
orbit                       : (% ,L S)    -> FSET L S
++ orbit(gp,ls) returns the orbit of the ordered
++ list ls under the group gp.
++ Note: return type is L L S temporarily because FSET L S has an error.
-- (GILT DAS NOCH?)
member?                     : (PERM S, %) -> B
++ member?(pp,gp) answers the question, whether the
++ permutation pp is in the group gp or not.
wordInStrongGenerators      : (PERM S, %) -> L NNI
++ wordInStrongGenerators(p,gp) returns the word for the
++ permutation p in the strong generators of the group gp,
++ represented by the indices of the list, given by strongGenerators.
wordInGenerators            : (PERM S, %) -> L NNI
++ wordInGenerators(p,gp) returns the word for the permutation p
++ in the original generators of the group gp,
++ represented by the indices of the list, given by generators.
movedPoints                 : %          -> FSET S
++ movedPoints(gp) returns the points moved by the group gp.
"<"                        : (% ,%)      -> B
++ gp1 < gp2 returns true if and only if gp1
++ is a proper subgroup of gp2.
"<="                       : (% ,%)      -> B
++ gp1 <= gp2 returns true if and only if gp1
++ is a subgroup of gp2.
++ Note: because of a bug in the parser you have to call this
++ function explicitly by gp1 <=$(PERMGRP S) gp2.

```

```

-- (GILT DAS NOCH?)
initializeGroupForWordProblem : %    -> Void
++ initializeGroupForWordProblem(gp) initializes the group gp
++ for the word problem.
++ Notes: it calls the other function of this name with parameters
++ 0 and 1: initializeGroupForWordProblem(gp,0,1).
++ Notes: (1) be careful: invoking this routine will destroy the
++ possibly information about your group (but will recompute it again)
++ (2) users need not call this function normally for the solution of
++ the word problem.
initializeGroupForWordProblem : (%,I,I) -> Void
++ initializeGroupForWordProblem(gp,m,n) initializes the group
++ gp for the word problem.
++ Notes: (1) with a small integer you get shorter words, but the
++ routine takes longer than the standard routine for longer words.
++ (2) be careful: invoking this routine will destroy the possibly stored
++ information about your group (but will recompute it again).
++ (3) users need not call this function normally for the solution of
++ the word problem.

private ==> add

-- representation of the object:

Rep := Record ( gens : L PERM S , information : REC2 )

-- import of domains and packages

import Permutation S
import OutputForm
import Symbol
import Void

--first the local variables

sgs                : L V NNI      := []
baseOfGroup        : L NNI        := []
sizeOfGroup        : NNI          := 1
degree             : NNI          := 0
gporb              : L REC        := []
out                : L L V NNI     := []
outword            : L L L NNI     := []
wordlist           : L L NNI      := []
basePoint          : NNI          := 0
newBasePoint       : B            := true
supp               : L S          := []
ord                : NNI          := 1
wordProblem        : B            := true

--local functions first, signatures:

```

```

shortenWord:(L NNI, %)->L NNI
times:(V NNI, V NNI)->V NNI
strip:(V NNI,REC,L V NNI,L L NNI)->REC3
orbitInternal:(%,L S )->L L S
inv: V NNI->V NNI
ranelt:(L V NNI,L L NNI, I)->REC3
testIdentity:V NNI->B
pointList: %->L S
orbitWithSvc:(L V NNI ,NNI )->REC
cosetRep:(NNI ,REC ,L V NNI )->REC3
bsgs1:(L V NNI,NNI,L L NNI,I,%,I)->NNI
computeOrbits: I->L NNI
reduceGenerators: I->Void
bsgs:(%, I, I)->NNI
initialize: %->FSET PERM S
knownGroup?: %->Void
subgroup:(%, %)->B
memberInternal:(PERM S, %, B)->REC4

--local functions first, implementations:

shortenWord ( lw : L NNI , gp : % ) : L NNI ==
  -- tries to shorten a word in the generators by removing identities
  gpgens : L PERM S := coerce gp
  orderList : L NNI := [ order gen for gen in gpgens ]
  newlw : L NNI := copy lw
  for i in 1.. maxIndex orderList repeat
    if orderList.i = 1 then
      while member?(i,newlw) repeat
        -- removing the trivial element
        pos := position(i,newlw)
        newlw := delete(newlw,pos)
  flag : B := true
  while flag repeat
    actualLength : NNI := (maxIndex newlw) pretend NNI
    pointer := actualLength
    test := newlw.pointer
    anzahl : NNI := 1
    flag := false
    while pointer > 1 repeat
      pointer := ( pointer - 1 )::NNI
      if newlw.pointer ^= test then
        -- don't get a trivial element, try next
        test := newlw.pointer
        anzahl := 1
      else
        anzahl := anzahl + 1
        if anzahl = orderList.test then
          -- we have an identity, so remove it

```



```

        for i in (pointer+anzahl)..actualLength repeat
            newlw.(i-anzahl) := newlw.i
        newlw := first(newlw, (actualLength - anzahl) :: NNI)
        flag := true
        pointer := 1
    newlw

times ( p : V NNI , q : V NNI ) : V NNI ==
-- internal multiplication of permutations
[ qelt(p,qelt(q,i)) for i in 1..degree ]

strip(element:V NNI,orbit:REC,group:L V NNI,words:L L NNI) : REC3 ==
-- strip an element into the stabilizer
actelt      := element
schreierVector := orbit.svc
point       := orbit.orb.1
outlist     := nil()$(L NNI)
entryLessZero : B := false
while ^entryLessZero repeat
    entry := schreierVector.(actelt.point)
    entryLessZero := (entry < 0)
    if ^entryLessZero then
        actelt := times(group.entry, actelt)
        if wordProblem then outlist := append ( words.(entry::NNI) , outlist )
[ actelt , reverse outlist ]

orbitInternal ( gp : % , startList : L S ) : L L S ==
orbitList : L L S := [ startList ]
pos : I := 1
while not zero? pos repeat
    gpset : L PERM S := gp.gens
    for gen in gpset repeat
        newList := nil()$(L S)
        workList := orbitList.pos
        for j in #workList..1 by -1 repeat
            newList := cons ( eval ( gen , workList.j ) , newList )
        if ^member?( newList , orbitList ) then
            orbitList := cons ( newList , orbitList )
            pos := pos + 1
    pos := pos - 1
reverse orbitList

inv ( p : V NNI ) : V NNI ==
-- internal inverse of a permutation
q : V NNI := new(degree,0)$(V NNI)
for i in 1..degree repeat q.(qelt(p,i)) := i
q

ranelt ( group : L V NNI , word : L L NNI , maxLoops : I ) : REC3 ==
-- generate a "random" element

```

```

numberOfGenerators := # group
randomInteger : I := 1 + (random()$Integer rem numberOfGenerators)
randomElement : V NNI := group.randomInteger
words := nil()$(L NNI)
if wordProblem then words := word.(randomInteger::NNI)
if maxLoops > 0 then
  numberOfLoops : I := 1 + (random()$Integer rem maxLoops)
else
  numberOfLoops : I := maxLoops
while numberOfLoops > 0 repeat
  randomInteger : I := 1 + (random()$Integer rem numberOfGenerators)
  randomElement := times ( group.randomInteger , randomElement )
  if wordProblem then words := append ( word.(randomInteger::NNI) , words)
  numberOfLoops := numberOfLoops - 1
[ randomElement , words ]

testIdentity ( p : V NNI ) : B ==
-- internal test for identity
for i in 1..degree repeat qelt(p,i) ^= i => return false
true

pointList(group : %) : L S ==
support : FSET S := brace() -- empty set !!
for perm in group.gens repeat
  support := union(support, movedPoints perm)
parts support

orbitWithSvc ( group : L V NNI , point : NNI ) : REC ==
-- compute orbit with Schreier vector, "-2" means not in the orbit,
-- "-1" means starting point, the PI correspond to generators
newGroup := nil()$(L V NNI)
for el in group repeat
  newGroup := cons ( inv el , newGroup )
newGroup := reverse newGroup
orbit : L NNI := [ point ]
schreierVector : V I := new ( degree , -2 )
schreierVector.point := -1
position : I := 1
while not zero? position repeat
  for i in 1..#newGroup repeat
    newPoint := orbit.position
    newPoint := newGroup.i.newPoint
    if ^ member? ( newPoint , orbit ) then
      orbit := cons ( newPoint , orbit )
      position := position + 1
      schreierVector.newPoint := i
  position := position - 1
[ reverse orbit , schreierVector ]

cosetRep ( point : NNI , o : REC , group : L V NNI ) : REC3 ==

```

```

ppt          := point
xelt : V NNI := [ n for n in 1..degree ]
word        := nil()$(L NNI)
oorb        := o.orb
osvc        := o.svc
while degree > 0 repeat
  p := osvc.ppt
  p < 0 => return [ xelt , word ]
  x  := group.p
  xelt := times ( x , xelt )
  if wordProblem then word := append ( wordlist.p , word )
  ppt := x.ppt

bsgs1 (group:L V NNI,number1:NNI,words:L L NNI,maxLoops:I,gp:%,diff:I)_
: NNI ==
-- try to get a good approximation for the strong generators and base
for i in number1..degree repeat
  ort := orbitWithSvc ( group , i )
  k    := ort.orb
  k1   := # k
  if k1 ^= 1 then leave
gpsgs := nil()$(L V NNI)
words2 := nil()$(L L NNI)
gplength : NNI := #group
for jj in 1..gplength repeat if (group.jj).i ^= i then leave
for k in 1..gplength repeat
  el2 := group.k
  if el2.i ^= i then
    gpsgs := cons ( el2 , gpsgs )
    if wordProblem then words2 := cons ( words.k , words2 )
  else
    gpsgs := cons ( times ( group.jj , el2 ) , gpsgs )
    if wordProblem _
      then words2 := cons ( append ( words.jj , words.k ) , words2 )
group2 := nil()$(L V NNI)
words3 := nil()$(L L NNI)
j : I := 15
while j > 0 repeat
  -- find generators for the stabilizer
  ran := ranelt ( group , words , maxLoops )
  str := strip ( ran.el , ort , group , words )
  el2 := str.el
  if ^ testIdentity el2 then
    if ^ member?(el2,group2) then
      group2 := cons ( el2 , group2 )
      if wordProblem then
        help : L NNI := append ( reverse str.lst , ran.lst )
        help      := shortenWord ( help , gp )
        words3     := cons ( help , words3 )
      j := j - 2

```

```

    j := j - 1
-- this is for word length control
if wordProblem then maxLoops := maxLoops - diff
if ( null group2 ) or ( maxLoops < 0 ) then
    sizeOfGroup := k1
    baseOfGroup := [ i ]
    out := [ gpsgs ]
    outword := [ words2 ]
    return sizeOfGroup
k2 := bsgs1 ( group2 , i + 1 , words3 , maxLoops , gp , diff )
sizeOfGroup := k1 * k2
out := append ( out , [ gpsgs ] )
outword := append ( outword , [ words2 ] )
baseOfGroup := cons ( i , baseOfGroup )
sizeOfGroup

computeOrbits ( kkk : I ) : L NNI ==
-- compute the orbits for the stabilizers
sgs := nil()
orbitLength := nil()$(L NNI)
gporb := nil()
for i in 1..#baseOfGroup repeat
    sgs := append ( sgs , out.i )
    pt := #baseOfGroup - i + 1
    obs := orbitWithSvc ( sgs , baseOfGroup.pt )
    orbitLength := cons ( #obs.orb , orbitLength )
    gporb := cons ( obs , gporb )
gporb := reverse gporb
reverse orbitLength

reduceGenerators ( kkk : I ) : Void ==
-- try to reduce number of strong generators
orbitLength := computeOrbits ( kkk )
sgs := nil()
wordlist := nil()
for i in 1..(kkk-1) repeat
    sgs := append ( sgs , out.i )
    if wordProblem then wordlist := append ( wordlist , outword.i )
removedGenerator := false
baseLength : NNI := #baseOfGroup
for nnn in kkk..(baseLength-1) repeat
    sgs := append ( sgs , out.nnn )
    if wordProblem then wordlist := append ( wordlist , outword.nnn )
    pt := baseLength - nnn + 1
    obs := orbitWithSvc ( sgs , baseOfGroup.pt )
    i := 1
    while not ( i > # out.nnn ) repeat
        pos := position ( out.nnn.i , sgs )
        sgs2 := delete(sgs, pos)
        obs2 := orbitWithSvc ( sgs2 , baseOfGroup.pt )

```

```

if # obs2.orb = orbitLength.nnn then
  test := true
  for j in (nnn+1)..(baseLength-1) repeat
    pt2 := baseLength - j + 1
    sgs2 := append ( sgs2 , out.j )
    obs2 := orbitWithSvc ( sgs2 , baseOfGroup.pt2 )
    if # obs2.orb ^= orbitLength.j then
      test := false
      leave
  if test then
    removedGenerator := true
    sgs := delete (sgs, pos)
    if wordProblem then wordlist := delete(wordlist, pos)
    out.nnn := delete (out.nnn, i)
    if wordProblem then _
      outword.nnn := delete(outword.nnn, i )
  else
    i := i + 1
  else
    i := i + 1
if removedGenerator then orbitLength := computeOrbits ( kkk )
void()

bsgs ( group : % ,maxLoops : I , diff : I ) : NNI ==
-- the MOST IMPORTANT part of the package
supp := pointList group
degree := # supp
if degree = 0 then
  sizeOfGroup := 1
  sgs := [ [ 0 ] ]
  baseOfGroup := nil()
  gporb := nil()
  return sizeOfGroup
newGroup := nil()$(L V NNI)
gp : L PERM S := group.gens
words := nil()$(L L NNI)
for ggg in 1..#gp repeat
  q := new(degree,0)$(V NNI)
  for i in 1..degree repeat
    newEl := eval ( gp.ggg , supp.i )
    pos2 := position ( newEl , supp )
    q.i := pos2 pretend NNI
  newGroup := cons ( q , newGroup )
  if wordProblem then words := cons(list ggg, words)
if maxLoops < 1 then
-- try to get the (approximate) base length
if zero? ( # ((group.information).gpbases) ) then
  wordProblem := false
  k := bsgs1 ( newGroup , 1 , words , 20 , group , 0 )

```

```

        wordProblem := true
        maxLoops    := (# baseOfGroup) - 1
    else
        maxLoops    := (# ((group.information).gpbase)) - 1
    k      := bsgs1 ( newGroup , 1 , words , maxLoops , group , diff )
    kkk : I := 1
    newGroup := reverse newGroup
    noAnswer : B := true
    while noAnswer repeat
        reduceGenerators kkk
-- *** Here is former "bsgs2" *** --
-- test whether we have a base and a strong generating set
    sgs := nil()
    wordlist := nil()
    for i in 1..(kkk-1) repeat
        sgs := append ( sgs , out.i )
        if wordProblem then wordlist := append ( wordlist , outword.i )
    noresult : B := true
    for i in kkk..#baseOfGroup while noresult repeat
        sgs := append ( sgs , out.i )
        if wordProblem then wordlist := append ( wordlist , outword.i )
    gporbi := gporb.i
    for pt in gporbi.orb while noresult repeat
        ppp := cosetRep ( pt , gporbi , sgs )
        y1  := inv ppp.elc
        word3 := ppp.lst
        for jjj in 1..#sgs while noresult repeat
            word := nil()$(L NNI)
            z := times ( sgs.jjj , y1 )
            if wordProblem then word := append ( wordlist.jjj , word )
            ppp := cosetRep ( (sgs.jjj).pt , gporbi , sgs )
            z := times ( ppp.elc , z )
            if wordProblem then word := append ( ppp.lst , word )
        newBasePoint := false
        for j in (i-1)..1 by -1 while noresult repeat
            s := gporb.j.svc
            p := gporb.j.orb.1
            while ( degree > 0 ) and noresult repeat
                entry := s.(z.p)
                if entry < 0 then
                    if entry = -1 then leave
                    basePoint := j::NNI
                    noresult := false
            else
                ee := sgs.entry
                z := times ( ee , z )
                if wordProblem then word := append ( wordlist.entry , word )
        if noresult then
            basePoint := 1
            newBasePoint := true

```

```

        noresult := testIdentity z
noAnswer := not (testIdentity z)
if noAnswer then
    -- we have missed something
    word2 := nil()$(L NNI)
    if wordProblem then
        for wd in word3 repeat
            ttt := newGroup.wd
            while not (testIdentity ttt) repeat
                word2 := cons ( wd , word2 )
                ttt := times ( ttt , newGroup.wd )
            word := append ( word , word2 )
            word := shortenWord ( word , group )
    if newBasePoint then
        for i in 1..degree repeat
            if z.i ^= i then
                baseOfGroup := append ( baseOfGroup , [ i ] )
                leave
        out := cons (list z, out )
        if wordProblem then outword := cons (list word , outword )
    else
        out.basePoint := cons ( z , out.basePoint )
        if wordProblem then outword.basePoint := cons(word ,outword.basePoint )
    kkk := basePoint
sizeOfGroup := 1
for j in 1..#baseOfGroup repeat
    sizeOfGroup := sizeOfGroup * # gporb.j.orb
sizeOfGroup

initialize ( group : % ) : FSET PERM S ==
group2 := brace()$(FSET PERM S)
gp : L PERM S := group.gens
for gen in gp repeat
    if degree gen > 0 then insert_!(gen, group2)
group2

knownGroup? (gp : %) : Void ==
-- do we know the group already?
result := gp.information
if result.order = 0 then
    wordProblem      := false
    ord              := bsgs ( gp , 20 , 0 )
    result           := [ ord , sgs , baseOfGroup , gporb , supp , [] ]
    gp.information   := result
else
    ord              := result.order
    sgs              := result.sgset
    baseOfGroup      := result.gpbase
    gporb            := result.orbs

```

```

    supp      := result.mp
    wordlist   := result.wd
void

subgroup ( gp1 : % , gp2 : % ) : B ==
  gpset1 := initialize gp1
  gpset2 := initialize gp2
  empty? difference (gpset1, gpset2) => true
  for el in parts gpset1 repeat
    not member? (el, gp2) => return false
  true

memberInternal ( p : PERM S , gp : % , flag : B ) : REC4 ==
  -- internal membership testing
  supp      := pointList gp
  outlist := nil()$(L NNI)
  mP : L S := parts movedPoints p
  for x in mP repeat
    not member? (x, supp) => return [ false , nil()$(L NNI) ]
  if flag then
    member? ( p , gp.gens ) => return [ true , nil()$(L NNI) ]
    knownGroup? gp
  else
    result := gp.information
    if #(result.wd) = 0 then
      initializeGroupForWordProblem gp
    else
      ord      := result.order
      sgs      := result.sgset
      baseOfGroup := result.gpbases
      gporb    := result.orbs
      supp     := result.mp
      wordlist  := result.wd
    degree := # supp
    pp := new(degree,0)$(V NNI)
    for i in 1..degree repeat
      el := eval ( p , supp.i )
      pos := position ( el , supp )
      pp.i := pos::NNI
    words := nil()$(L L NNI)
    if wordProblem then
      for i in 1..#sgs repeat
        lw : L NNI := [ (#sgs - i + 1)::NNI ]
        words := cons ( lw , words )
    for i in #baseOfGroup..1 by -1 repeat
      str := strip ( pp , gporb.i , sgs , words )
      pp := str.elc
      if wordProblem then outlist := append ( outlist , str.lst )
  [ testIdentity pp , reverse outlist ]

```



```

--now the exported functions

coerce ( gp : % ) : L PERM S == gp.gens
generators ( gp : % ) : L PERM S == gp.gens

strongGenerators ( group ) ==
  knownGroup? group
  degree := # supp
  strongGens := nil()$(L PERM S)
  for i in sgs repeat
    pairs := nil()$(L L S)
    for j in 1..degree repeat
      pairs := cons ( [ supp.j , supp.(i.j) ] , pairs )
    strongGens := cons ( coerceListOfPairs pairs , strongGens )
  reverse strongGens

elt ( gp , i ) == (gp.gens).i

movedPoints ( gp ) == brace pointList gp

random ( group , maximalNumberOfFactors ) ==
  maximalNumberOfFactors < 1 => 1$(PERM S)
  gp : L PERM S := group.gens
  numberOfGenerators := # gp
  randomInteger : I := 1 + (random())$Integer rem numberOfGenerators
  randomElement := gp.randomInteger
  numberOfLoops : I := 1 + (random())$Integer rem maximalNumberOfFactors
  while numberOfLoops > 0 repeat
    randomInteger : I := 1 + (random())$Integer rem numberOfGenerators
    randomElement := gp.randomInteger * randomElement
    numberOfLoops := numberOfLoops - 1
  randomElement

random ( group ) == random ( group , 20 )

order ( group ) ==
  knownGroup? group
  ord

degree ( group ) == # pointList group

base ( group ) ==
  knownGroup? group
  groupBase := nil()$(L S)
  for i in baseOfGroup repeat
    groupBase := cons ( supp.i , groupBase )
  reverse groupBase

wordsForStrongGenerators ( group ) ==
  knownGroup? group

```

```

wordlist

coerce ( gp : L PERM S ) : % ==
  result : REC2 := [ 0 , [] , [] , [] , [] , [] ]
  group      := [ gp , result ]

permutationGroup ( gp : L PERM S ) : % ==
  result : REC2 := [ 0 , [] , [] , [] , [] , [] ]
  group      := [ gp , result ]

coerce(group: %) : OUT ==
  outList := nil()$(L OUT)
  gp : L PERM S := group.gens
  for i in (maxIndex gp)..1 by -1 repeat
    outList := cons(coerce gp.i, outList)
  postfix(outputForm(">":SYM),postfix(commaSeparate outList,outputForm("<":SYM)))

orbit ( gp : % , el : S ) : FSET S ==
  elList : L S := [ el ]
  outList      := orbitInternal ( gp , elList )
  outSet       := brace()$(FSET S)
  for i in 1..#outList repeat
    insert_! ( outList.i.1 , outSet )
  outSet

orbits ( gp ) ==
  spp      := movedPoints gp
  orbits := nil()$(L FSET S)
  while cardinality spp > 0 repeat
    el      := extract_! spp
    orbitSet := orbit ( gp , el )
    orbits   := cons ( orbitSet , orbits )
    spp      := difference ( spp , orbitSet )
  brace orbits

member? (p, gp) ==
  wordProblem := false
  mi := memberInternal ( p , gp , true )
  mi.bool

wordInStrongGenerators (p, gp) ==
  mi := memberInternal ( inv p , gp , false )
  not mi.bool => error "p is not an element of gp"
  mi.lst

wordInGenerators (p, gp) ==
  lll : L NNI := wordInStrongGenerators (p, gp)
  outlist := nil()$(L NNI)
  for wd in lll repeat
    outlist := append ( outlist , wordlist.wd )

```

```

shortenWord ( outlist , gp )

gp1 < gp2 ==
  not empty? difference ( movedPoints gp1 , movedPoints gp2 ) => false
  not subgroup ( gp1 , gp2 ) => false
  order gp1 = order gp2 => false
  true

gp1 <= gp2 ==
  not empty? difference ( movedPoints gp1 , movedPoints gp2 ) => false
  subgroup ( gp1 , gp2 )

gp1 = gp2 ==
  movedPoints gp1 ^= movedPoints gp2 => false
  if #(gp1.gens) <= #(gp2.gens) then
    not subgroup ( gp1 , gp2 ) => return false
  else
    not subgroup ( gp2 , gp1 ) => return false
  order gp1 = order gp2 => true
  false

orbit ( gp : % , startSet : FSET S ) : FSET FSET S ==
  startList : L S := parts startSet
  outList      := orbitInternal ( gp , startList )
  outSet       := brace()$(FSET FSET S)
  for i in 1..#outList repeat
    newSet : FSET S := brace outList.i
    insert_! ( newSet , outSet )
  outSet

orbit ( gp : % , startList : L S ) : FSET L S ==
  brace orbitInternal(gp, startList)

initializeGroupForWordProblem ( gp , maxLoops , diff ) ==
  wordProblem      := true
  ord              := bsgs ( gp , maxLoops , diff )
  gp.information := [ ord , sgs , baseOfGroup , gporb , supp , wordlist ]
  void

initializeGroupForWordProblem ( gp ) == initializeGroupForWordProblem ( gp , 0 , 1 )

```

---

— PERMGRP.dotabb —

```

"PERMGRP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PERMGRP"]
"FSAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FSAGG"]
"PERMGRP" -> "FSAGG"

```

## 17.16 domain HACKPI Pi

— Pi.input —

```
)set break resume
)sys rm -f Pi.output
)spool Pi.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Pi
--R Pi is a domain constructor
--R Abbreviation for Pi is HACKPI
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for HACKPI
--R
--R----- Operations -----
--R ??? : (Fraction Integer,%) -> %      ??? : (%,Fraction Integer) -> %
--R ??? : (%,%) -> %                    ??? : (Integer,%) -> %
--R ??? : (PositiveInteger,%) -> %      ??? : (%,Integer) -> %
--R ??? : (%,PositiveInteger) -> %     ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                   -? : % -> %
--R ?/? : (%,%) -> %                   ?? : (%,%) -> Boolean
--R 1 : () -> %                        0 : () -> %
--R ?? : (%,Integer) -> %              ?? : (%,PositiveInteger) -> %
--R associates? : (%,%) -> Boolean     coerce : % -> Float
--R coerce : % -> DoubleFloat           coerce : Fraction Integer -> %
--R coerce : % -> %                     coerce : Integer -> %
--R coerce : % -> OutputForm            convert : % -> InputForm
--R convert : % -> DoubleFloat           convert : % -> Float
--R factor : % -> Factored %            gcd : List % -> %
--R gcd : (%,%) -> %                   hash : % -> SingleInteger
--R inv : % -> %                        latex : % -> String
--R lcm : List % -> %                  lcm : (%,%) -> %
--R one? : % -> Boolean                 pi : () -> %
--R prime? : % -> Boolean               ?quo? : (%,%) -> %
--R recip : % -> Union(%, "failed")     ?rem? : (%,%) -> %
--R retract : % -> Fraction Integer     retract : % -> Integer
--R sample : () -> %                   sizeLess? : (%,%) -> Boolean
--R squareFree : % -> Factored %        squareFreePart : % -> %
--R unit? : % -> Boolean                unitCanonical : % -> %
```

```

--R zero? : % -> Boolean          ?~=? : (%,% ) -> Boolean
--R ?? : (NonNegativeInteger,% ) -> %
--R ??? : (% ,NonNegativeInteger) -> %
--R ?? : (% ,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R convert : % -> Fraction SparseUnivariatePolynomial Integer
--R divide : (% ,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List % ,%) -> Union(List % ,"failed")
--R exquo : (% ,%) -> Union(% ,"failed")
--R extendedEuclidean : (% ,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (% ,%) -> Record(coef1: %,coef2: %,generator: %)
--R gcdPolynomial : (SparseUnivariatePolynomial % ,SparseUnivariatePolynomial % ) -> SparseUni
--R multiEuclidean : (List % ,%) -> Union(List % ,"failed")
--R principalIdeal : List % -> Record(coef: List % ,generator: %)
--R retractIfCan : % -> Union(Fraction Integer,"failed")
--R retractIfCan : % -> Union(Integer,"failed")
--R subtractIfCan : (% ,%) -> Union(% ,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)

```

---

— Pi.help —

```

=====
Pi examples
=====

```

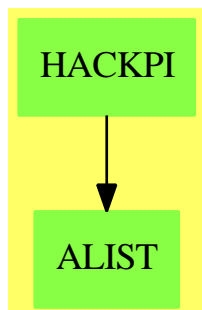
```

See Also:
o )show Pi

```

---

## 17.16.1 Pi (HACKPI)



See

⇒ “Expression” (EXPR) 6.6.1 on page 691

**Exports:**

|                   |                   |               |                    |                |
|-------------------|-------------------|---------------|--------------------|----------------|
| 0                 | 1                 | associates?   | characteristic     | coerce         |
| convert           | divide            | euclideanSize | expressIdealMember | exquo          |
| extendedEuclidean | extendedEuclidean | factor        | gcd                | gcdPolynomial  |
| hash              | inv               | latex         | lcm                | multiEuclidean |
| one?              | pi                | prime?        | principalIdeal     | recip          |
| retract           | retractIfCan      | retractIfCan  | sample             | sizeLess?      |
| squareFree        | squareFreePart    | subtractIfCan | unit?              | unitCanonical  |
| unitNormal        | zero?             | ?*?           | ?**?               | ?+?            |
| ?-?               | -?                | ?/?           | ?=?                | ?^?            |
| ?~=?              | ?quo?             | ?rem?         |                    |                |

— domain HACKPI Pi —

```

)abbrev domain HACKPI Pi
++ Author: Manuel Bronstein
++ Date Created: 21 Feb 1990
++ Date Last Updated: 12 Mai 1992
++ Description:
++ Symbolic fractions in %pi with integer coefficients;
++ The point for using Pi as the default domain for those fractions
++ is that Pi is coercible to the float types, and not Expression.

Pi(): Exports == Implementation where
  PZ ==> Polynomial Integer
  UP ==> SparseUnivariatePolynomial Integer
  RF ==> Fraction UP

Exports ==> Join(Field, CharacteristicZero, RetractableTo Integer,
  RetractableTo Fraction Integer, RealConstant,
  CoercibleTo DoubleFloat, CoercibleTo Float,
  ConvertibleTo RF, ConvertibleTo InputForm) with

```

```

pi: () -> % ++ pi() returns the symbolic %pi.
Implementation ==> RF add
Rep := RF

sympi := "%pi"::Symbol

p2sf: UP -> DoubleFloat
p2f : UP -> Float
p2o : UP -> OutputForm
p2i : UP -> InputForm
p2p:  UP -> PZ

pi()                == (monomial(1, 1)$UP :: RF) pretend %
convert(x:):RF      == x pretend RF
convert(x:):Float   == x::Float
convert(x:):DoubleFloat == x::DoubleFloat
coerce(x:):DoubleFloat == p2sf( numer x) / p2sf(denom x)
coerce(x:):Float     == p2f( numer x) / p2f(denom x)
p2o p                == outputForm(p, sympi::OutputForm)
p2i p                == convert p2p p

p2p p ==
  ans:PZ := 0
  while p ^= 0 repeat
    ans := ans + monomial(leadingCoefficient(p)::PZ, sympi, degree p)
    p    := reductum p
  ans

coerce(x:):OutputForm ==
  (r := retractIfCan(x)@Union(UP, "failed")) case UP => p2o(r::UP)
  p2o( numer x) / p2o(denom x)

convert(x:):InputForm ==
  (r := retractIfCan(x)@Union(UP, "failed")) case UP => p2i(r::UP)
  p2i( numer x) / p2i(denom x)

p2sf p ==
  map((x:Integer):DoubleFloat+->x::DoubleFloat, p)_
  $SparseUnivariatePolynomialFunctions2(Integer, DoubleFloat)
  (pi())$DoubleFloat

p2f p ==
  map((x:Integer):Float+->x::Float,p)_
  $SparseUnivariatePolynomialFunctions2(Integer, Float)
  (pi())$Float

```

---

— HACKPI.dotabb —

```
"HACKPI" [color="#88FF44",href="bookvol10.3.pdf#nameddest=HACKPI"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"HACKPI" -> "ALIST"
```

## 17.17 domain ACLOT PlaneAlgebraicCurvePlot

— PlaneAlgebraicCurvePlot.input —

```
)set break resume
)sys rm -f PlaneAlgebraicCurvePlot.output
)spool PlaneAlgebraicCurvePlot.output
)set message test on
)set message auto off
)clear all

--S 1 of 5
sketch:=makeSketch(x+y,x,y,-1/2..1/2,-1/2..1/2)$ACLOT
--R
--R (1) ACLOT
--R
--R      1      1      1      1
--R      y + x = 0,  - - <= x <= -,  - - <= y <= -
--R      2      2      2      2
--R      [0.5,- 0.5]
--R      [- 0.5,0.5]
--R
--R                                          Type: PlaneAlgebraicCurvePlot
--E 1

--S 2 of 5
refined:=refine(sketch,0.1)
--R
--R (2) ACLOT
--R
--R      1      1      1      1
--R      y + x = 0,  - - <= x <= -,  - - <= y <= -
--R      2      2      2      2
--R      [0.5,- 0.5]
--R      [0.49600000000000083,- 0.49600000000000083]
--R      [0.49200000000000083,- 0.49200000000000083]
--R      [0.48800000000000082,- 0.48800000000000082]
--R      [0.48400000000000082,- 0.48400000000000082]
--R      [0.48000000000000081,- 0.48000000000000081]
--R      [0.47600000000000081,- 0.47600000000000081]
--R      [0.47200000000000081,- 0.47200000000000081]
```



```
--R      [0.46800000000000008,- 0.46800000000000008]
--R      [0.46400000000000008,- 0.46400000000000008]
--R      [0.46000000000000008,- 0.46000000000000008]
--R      [0.456000000000000079,- 0.456000000000000079]
--R      [0.452000000000000079,- 0.452000000000000079]
--R      [0.448000000000000079,- 0.448000000000000079]
--R      [0.444000000000000078,- 0.444000000000000078]
--R      [0.440000000000000078,- 0.440000000000000078]
--R      [0.436000000000000078,- 0.436000000000000078]
--R      [0.432000000000000077,- 0.432000000000000077]
--R      [0.428000000000000077,- 0.428000000000000077]
--R      [0.424000000000000077,- 0.424000000000000077]
--R      [0.420000000000000076,- 0.420000000000000076]
--R      [0.416000000000000076,- 0.416000000000000076]
--R      [0.412000000000000075,- 0.412000000000000075]
--R      [0.408000000000000075,- 0.408000000000000075]
--R      [0.404000000000000075,- 0.404000000000000075]
--R      [0.400000000000000074,- 0.400000000000000074]
--R      [0.396000000000000074,- 0.396000000000000074]
--R      [0.392000000000000074,- 0.392000000000000074]
--R      [0.388000000000000073,- 0.388000000000000073]
--R      [0.384000000000000073,- 0.384000000000000073]
--R      [0.380000000000000073,- 0.380000000000000073]
--R      [0.376000000000000072,- 0.376000000000000072]
--R      [0.372000000000000072,- 0.372000000000000072]
--R      [0.368000000000000072,- 0.368000000000000072]
--R      [0.364000000000000071,- 0.364000000000000071]
--R      [0.360000000000000071,- 0.360000000000000071]
--R      [0.35600000000000007,- 0.35600000000000007]
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--R      [- 0.5,0.5]
--R
--R                                          Type: PlaneAlgebraicCurvePlot
--E 2

--S 3 of 5
listBranches(sketch)
--R
--R      (3)  [[0.5,- 0.5],[- 0.5,0.5]]

```

```

--R                                                    Type: List List Point DoubleFloat
--E 3

--S 4 of 5
listBranches(refined)
--R
--R (4)
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--R      [- 0.1439999999999968, 0.1439999999999968] ,
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--R      [- 0.1559999999999969, 0.1559999999999969] ,
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--R      [- 0.2319999999999976, 0.2319999999999976] ,
--R      [- 0.2359999999999977, 0.2359999999999977] ,
--R      [- 0.2399999999999977, 0.2399999999999977] ,
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--R      [- 0.2559999999999978, 0.2559999999999978] ,
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--R      [- 0.2679999999999979, 0.2679999999999979] ,
```

```
--R      [- 0.271999999999998,0.271999999999998],
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--R      [- 0.4359999999999994,0.4359999999999994],
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--R      [- 0.4559999999999996,0.4559999999999996],
--R      [- 0.4599999999999996,0.4599999999999996],
--R      [- 0.4639999999999997,0.4639999999999997],
--R      [- 0.4679999999999997,0.4679999999999997],
```

```

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--R      [- 0.4799999999999998,0.4799999999999998],
--R      [- 0.4839999999999999,0.4839999999999999],
--R      [- 0.4879999999999999,0.4879999999999999],
--R      [- 0.4919999999999999,0.4919999999999999], [- 0.496,0.496],
--R      [- 0.5,0.5]]
--R      ]
--R
--R                                          Type: List List Point DoubleFloat
--E 4

--S 5 of 5
)show ACPLLOT
--R PlaneAlgebraicCurvePlot is a domain constructor
--R Abbreviation for PlaneAlgebraicCurvePlot is ACPLLOT
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ACPLLOT
--R
--R----- Operations -----
--R coerce : % -> OutputForm          refine : (% ,DoubleFloat) -> %
--R xRange : % -> Segment DoubleFloat  yRange : % -> Segment DoubleFloat
--R listBranches : % -> List List Point DoubleFloat
--R makeSketch : (Polynomial Integer,Symbol,Symbol,Segment Fraction Integer,Segment Fraction
--R
--E 5

)spool
)lisp (bye)

```

---

— PlaneAlgebraicCurvePlot.help —

=====

PlaneAlgebraicCurvePlot examples

=====

```
sketch:=makeSketch(x+y,x,y,-1/2..1/2,-1/2..1/2)$ACPLLOT
```

```

ACPLLOT
      1      1      1      1
y + x = 0,  - - <= x <= -, - - <= y <= -
      2      2      2      2
      [0.5,- 0.5]
      [- 0.5,0.5]

```

```
refined:=refine(sketch,0.1)
```

```

ACPLOT
      1      1      1      1
y + x = 0,  - - <= x <= -,  - - <= y <= -
      2      2      2      2
      [0.5,- 0.5]
[0.496000000000000083,- 0.496000000000000083]
[0.492000000000000083,- 0.492000000000000083]
[0.488000000000000082,- 0.488000000000000082]
[0.484000000000000082,- 0.484000000000000082]
...
[- 0.48399999999999999,0.48399999999999999]
[- 0.48799999999999999,0.48799999999999999]
[- 0.49199999999999999,0.49199999999999999]
      [- 0.496,0.496]
      [- 0.5,0.5]

listBranches(sketch)

[[[0.5,- 0.5],[- 0.5,0.5]]]

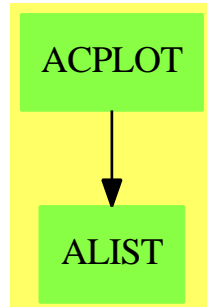
listBranches(refined)

[
  [[0.5,- 0.5], [0.496000000000000083,- 0.496000000000000083],
   [0.492000000000000083,- 0.492000000000000083],
   [0.488000000000000082,- 0.488000000000000082],
   ...
   [- 0.48399999999999999,0.48399999999999999],
   [- 0.48799999999999999,0.48799999999999999],
   [- 0.49199999999999999,0.49199999999999999], [- 0.496,0.496],

```

---

## 17.17.1 PlaneAlgebraicCurvePlot (ACPLOT)

**Exports:**

coerce listBranches makeSketch refine xRange yRange

— domain ACPLOT PlaneAlgebraicCurvePlot —

```

)abbrev domain ACPLOT PlaneAlgebraicCurvePlot
++ Author: Clifton J. Williamson and Timothy Daly
++ Date Created: Fall 1988
++ Date Last Updated: 27 April 1990
++ Keywords: algebraic curve, non-singular, plot
++ Examples:
++ References:
++ Description:
++ Plot a NON-SINGULAR plane algebraic curve  $p(x,y) = 0$ .

PlaneAlgebraicCurvePlot(): PlottablePlaneCurveCategory _
  with

  makeSketch:(Polynomial Integer,Symbol,Symbol,Segment Fraction Integer,_
    Segment Fraction Integer) -> %
    ++ makeSketch(p,x,y,a..b,c..d) creates an ACPLOT of the
    ++ curve \spad{p = 0} in the region  $a \leq x \leq b$ ,  $c \leq y \leq d$ .
    ++ More specifically, 'makeSketch' plots a non-singular algebraic curve
    ++ \spad{p = 0} in an rectangular region  $x_{\min} \leq x \leq x_{\max}$ ,
    ++  $y_{\min} \leq y \leq y_{\max}$ . The user inputs
    ++ \spad{makeSketch(p,x,y,xMin..xMax,yMin..yMax)}.
    ++ Here p is a polynomial in the variables x and y with
    ++ integer coefficients (p belongs to the domain
    ++ \spad{Polynomial Integer}). The case
    ++ where p is a polynomial in only one of the variables is
    ++ allowed. The variables x and y are input to specify the
    ++ the coordinate axes. The horizontal axis is the x-axis and
    ++ the vertical axis is the y-axis. The rational numbers
    ++ xMin,...,yMax specify the boundaries of the region in
    ++ which the curve is to be plotted.
  
```

```

++
++X makeSketch(x+y,x,y,-1/2..1/2,-1/2..1/2)$ACPLLOT

refine:(%,DoubleFloat) -> %
++ refine(p,x) is not documented
++
++X sketch:=makeSketch(x+y,x,y,-1/2..1/2,-1/2..1/2)$ACPLLOT
++X refined:=refine(sketch,0.1)

== add

import PointPackage DoubleFloat
import Plot
import RealSolvePackage

BoundaryPts ==> Record(left: List Point DoubleFloat,_
                      right: List Point DoubleFloat,_
                      bottom: List Point DoubleFloat,_
                      top: List Point DoubleFloat)

NewPtInfo ==> Record(newPt: Point DoubleFloat,_
                    type: String)

Corners ==> Record(minXVal: DoubleFloat,_
                  maxXVal: DoubleFloat,_
                  minYVal: DoubleFloat,_
                  maxYVal: DoubleFloat)

kinte ==> solve$RealSolvePackage()

rsolve ==> realSolve$RealSolvePackage()

singValBetween?:(DoubleFloat,DoubleFloat,List DoubleFloat) -> Boolean

segmentInfo:(DoubleFloat -> DoubleFloat,DoubleFloat,DoubleFloat,_
             List DoubleFloat,List DoubleFloat,List DoubleFloat,_
             DoubleFloat,DoubleFloat) -> _
Record(seg:Segment DoubleFloat,_
      left: DoubleFloat,_
      lowerVals: List DoubleFloat,_
      upperVals:List DoubleFloat)

swapCoords:Point DoubleFloat -> Point DoubleFloat

samePlottedPt?:(Point DoubleFloat,Point DoubleFloat) -> Boolean

findPtOnList:(Point DoubleFloat,List Point DoubleFloat) -> _
Union(Point DoubleFloat,"failed")

makeCorners:(DoubleFloat,DoubleFloat,DoubleFloat,DoubleFloat) -> Corners

```

```

getXMin: Corners -> DoubleFloat

getXMax: Corners -> DoubleFloat

getYMin: Corners -> DoubleFloat

getYMax: Corners -> DoubleFloat

SFPolyToUPoly:Polynomial DoubleFloat -> _
  SparseUnivariatePolynomial DoubleFloat

RNPolyToUPoly:Polynomial Fraction Integer -> _
  SparseUnivariatePolynomial Fraction Integer

coerceCoefsToSFs:Polynomial Integer -> Polynomial DoubleFloat

coerceCoefsToRNs:Polynomial Integer -> Polynomial Fraction Integer

RNtoSF:Fraction Integer -> DoubleFloat

RNtoNF:Fraction Integer -> Float

SFtoNF:DoubleFloat -> Float

listPtsOnHorizBdry:(Polynomial Fraction Integer,Symbol,Fraction Integer,_
  Float,Float) -> _
  List Point DoubleFloat

listPtsOnVertBdry:(Polynomial Fraction Integer,Symbol,Fraction Integer,_
  Float,Float) -> _
  List Point DoubleFloat

listPtsInRect:(List List Float,Float,Float,Float,Float) -> _
  List Point DoubleFloat

ptsSuchThat?:(List List Float,List Float -> Boolean) -> Boolean

inRect?:(List Float,Float,Float,Float,Float) -> Boolean

onHorzSeg?:(List Float,Float,Float,Float) -> Boolean

onVertSeg?:(List Float,Float,Float,Float) -> Boolean

newX:(List List Float,List List Float,Float,Float,Float,Float,Fraction Integer,_
  Fraction Integer) -> Fraction Integer

newY:(List List Float,List List Float,Float,Float,Float,_
  Fraction Integer,Fraction Integer) -> Fraction Integer

```

```

makeOneVarSketch:(Polynomial Integer,Symbol,Symbol,Fraction Integer,_
                  Fraction Integer,Fraction Integer,Fraction Integer,_
                  Symbol) -> %

makeLineSketch:(Polynomial Integer,Symbol,Symbol,Fraction Integer,_
                 Fraction Integer,Fraction Integer,Fraction Integer) -> %

makeRatFcnSketch:(Polynomial Integer,Symbol,Symbol,Fraction Integer,_
                  Fraction Integer,Fraction Integer,Fraction Integer,_
                  Symbol) -> %

makeGeneralSketch:(Polynomial Integer,Symbol,Symbol,Fraction Integer,_
                   Fraction Integer,Fraction Integer,Fraction Integer) -> %

traceBranches:(Polynomial DoubleFloat,Polynomial DoubleFloat,_
               Polynomial DoubleFloat,Symbol,Symbol,Corners,DoubleFloat,_
               DoubleFloat,PositiveInteger, List Point DoubleFloat,_
               BoundaryPts) -> List List Point DoubleFloat

dummyFirstPt:(Point DoubleFloat,Polynomial DoubleFloat,_
              Polynomial DoubleFloat,Symbol,Symbol,List Point DoubleFloat,_
              List Point DoubleFloat,List Point DoubleFloat,_
              List Point DoubleFloat) -> Point DoubleFloat

listPtsOnSegment:(Polynomial DoubleFloat,Polynomial DoubleFloat,_
                 Polynomial DoubleFloat,Symbol,Symbol,Point DoubleFloat,_
                 Point DoubleFloat,Corners, DoubleFloat,DoubleFloat,_
                 PositiveInteger,List Point DoubleFloat,_
                 List Point DoubleFloat) -> List List Point DoubleFloat

listPtsOnLoop:(Polynomial DoubleFloat,Polynomial DoubleFloat,_
              Polynomial DoubleFloat,Symbol,Symbol,Point DoubleFloat,_
              Corners, DoubleFloat,DoubleFloat,PositiveInteger,_
              List Point DoubleFloat,List Point DoubleFloat) -> _
              List List Point DoubleFloat

computeNextPt:(Polynomial DoubleFloat,Polynomial DoubleFloat,_
              Polynomial DoubleFloat,Symbol,Symbol,Point DoubleFloat,_
              Point DoubleFloat,Corners, DoubleFloat,DoubleFloat,_
              PositiveInteger,List Point DoubleFloat,_
              List Point DoubleFloat) -> NewPtInfo

newtonApprox:(SparseUnivariatePolynomial DoubleFloat, DoubleFloat, _
             DoubleFloat, PositiveInteger) -> Union(DoubleFloat, "failed")

--% representation

Rep := Record(poly      : Polynomial Integer,_
              xVar      : Symbol,_
              yVar      : Symbol,_

```



```

        minXVal : Fraction Integer, _
        maxXVal : Fraction Integer, _
        minYVal : Fraction Integer, _
        maxYVal : Fraction Integer, _
        bdryPts : BoundaryPts, _
        hTanPts : List Point DoubleFloat, _
        vTanPts : List Point DoubleFloat, _
        branches: List List Point DoubleFloat)

--% global constants

EPSILON : Float := .000001 -- precision to which realSolve finds roots
PLOTERR : DoubleFloat := float(1,-3,10)
-- maximum allowable difference in each coordinate when
-- determining if 2 plotted points are equal

--% global flags

NADA    : String := "nothing in particular"
BDRY    : String := "boundary point"
CRIT    : String := "critical point"
BOTTOM  : String := "bottom"
TOP     : String := "top"

--% hacks

NFtoSF: Float -> DoubleFloat
NFtoSF x == 0 + convert(x)$Float

--% points

makePt: (DoubleFloat, DoubleFloat) -> Point DoubleFloat
makePt(xx,yy) == point(1 : List DoubleFloat := [xx,yy])

swapCoords(pt) == makePt(yCoord pt, xCoord pt)

samePlottedPt?(p0,p1) ==
  -- determines if p1 lies in a square with side 2 PLOTERR
  -- centered at p0
  x0 := xCoord p0; y0 := yCoord p0
  x1 := xCoord p1; y1 := yCoord p1
  (abs(x1-x0) < PLOTERR) and (abs(y1-y0) < PLOTERR)

findPtOnList(pt, pointList) ==
  for point in pointList repeat
    samePlottedPt?(pt, point) => return point
  "failed"

--% corners

makeCorners(xMinSF, xMaxSF, yMinSF, yMaxSF) ==

```

```

[xMinSF,xMaxSF,yMinSF,yMaxSF]

getXMin(corners) == corners.minXVal
getXMax(corners) == corners.maxXVal
getYMin(corners) == corners.minYVal
getYMax(corners) == corners.maxYVal

--% coercions

SFPolyToUPoly(p) ==
-- 'p' is of type Polynomial, but has only one variable
zero? p => 0
monomial(leadingCoefficient p,totalDegree p) +
  SFPolyToUPoly(reductum p)

RNPolyToUPoly(p) ==
-- 'p' is of type Polynomial, but has only one variable
zero? p => 0
monomial(leadingCoefficient p,totalDegree p) +
  RNPolyToUPoly(reductum p)

coerceCoefsToSFs(p) ==
-- coefficients of 'p' are coerced to be DoubleFloat's
map(coerce,p)$PolynomialFunctions2(Integer,DoubleFloat)

coerceCoefsToRNs(p) ==
-- coefficients of 'p' are coerced to be DoubleFloat's
map(coerce,p)$PolynomialFunctions2(Integer,Fraction Integer)

RNtoSF(r) == coerce(r)@DoubleFloat
RNtoNF(r) == coerce(r)@Float
SFtoNF(x) == convert(x)@Float

--% computation of special points

listPtsOnHorizBdry(pRN,y,y0,xMinNF,xMaxNF) ==
-- strict inequality here: corners on vertical boundary
pointList : List Point DoubleFloat := nil()
ySF := RNtoSF(y0)
f := eval(pRN,y,y0)
roots : List Float := kinte(f,EPSILON)
for root in roots repeat
  if (xMinNF < root) and (root < xMaxNF) then
    pointList := cons(makePt(NFtoSF root, ySF), pointList)
pointList

listPtsOnVertBdry(pRN,x,x0,yMinNF,yMaxNF) ==
pointList : List Point DoubleFloat := nil()
xSF := RNtoSF(x0)
f := eval(pRN,x,x0)

```

```

roots : List Float := kinte(f,EPSILON)
for root in roots repeat
  if (yMinNF <= root) and (root <= yMaxNF) then
    pointList := cons(makePt(xSF, NFtoSF root), pointList)
pointList

listPtsInRect(points,xMin,xMax,yMin,yMax) ==
pointList : List Point DoubleFloat := nil()
for point in points repeat
  xx := first point; yy := second point
  if (xMin<=xx) and (xx<=xMax) and (yMin<=yy) and (yy<=yMax) then
    pointList := cons(makePt(NFtoSF xx,NFtoSF yy),pointList)
pointList

ptsSuchThat?(points,pred) ==
for point in points repeat
  if pred point then return true
false

inRect?(point,xMinNF,xMaxNF,yMinNF,yMaxNF) ==
xx := first point; yy := second point
xMinNF <= xx and xx <= xMaxNF and yMinNF <= yy and yy <= yMaxNF

onHorzSeg?(point,xMinNF,xMaxNF,yNF) ==
xx := first point; yy := second point
yy = yNF and xMinNF <= xx and xx <= xMaxNF

onVertSeg?(point,yMinNF,yMaxNF,xNF) ==
xx := first point; yy := second point
xx = xNF and yMinNF <= yy and yy <= yMaxNF

newX(vtanPts,singPts,yMinNF,yMaxNF,xNF,xRN,horizInc) ==
xNewNF := xNF + RNtoNF horizInc
xRtNF := max(xNF,xNewNF); xLftNF := min(xNF,xNewNF)
-- ptsSuchThat?(singPts,inRect?(#1,xLftNF,xRtNF,yMinNF,yMaxNF)) =>
foo : List Float -> Boolean := x +-> inRect?(x,xLftNF,xRtNF,yMinNF,yMaxNF)
ptsSuchThat?(singPts,foo) =>
  newX(vtanPts,singPts,yMinNF,yMaxNF,xNF,xRN,_
    horizInc/2:(Fraction Integer))
-- ptsSuchThat?(vtanPts,onVertSeg?(#1,yMinNF,yMaxNF,xNewNF)) =>
goo : List Float -> Boolean := x +-> onVertSeg?(x,yMinNF,yMaxNF,xNewNF)
ptsSuchThat?(vtanPts,goo) =>
  newX(vtanPts,singPts,yMinNF,yMaxNF,xNF,xRN,_
    horizInc/2:(Fraction Integer))
xRN + horizInc

newY(htanPts,singPts,xMinNF,xMaxNF,yNF,yRN,vertInc) ==
yNewNF := yNF + RNtoNF vertInc
yTopNF := max(yNF,yNewNF); yBotNF := min(yNF,yNewNF)
-- ptsSuchThat?(singPts,inRect?(#1,xMinNF,xMaxNF,yBotNF,yTopNF)) =>

```

```

foo : List Float -> Boolean := x +-> inRect?(x,xMinNF,xMaxNF,yBotNF,yTopNF)
ptsSuchThat?(singPts,foo) =>
  newY(htanPts,singPts,xMinNF,xMaxNF,yNF,yRN,_
    vertInc/2::(Fraction Integer))
-- ptsSuchThat?(htanPts,onHorzSeg?(#1,xMinNF,xMaxNF,yNewNF)) =>
goo : List Float -> Boolean := x +-> onHorzSeg?(x,xMinNF,xMaxNF,yNewNF)
ptsSuchThat?(htanPts,goo) =>
  newY(htanPts,singPts,xMinNF,xMaxNF,yNF,yRN,_
    vertInc/2::(Fraction Integer))
yRN + vertInc

--% creation of sketches

makeSketch(p,x,y,xRange,yRange) ==
  xMin := lo xRange; xMax := hi xRange
  yMin := lo yRange; yMax := hi yRange
  -- test input for consistency
  xMax <= xMin =>
    error "makeSketch: bad range for first variable"
  yMax <= yMin =>
    error "makeSketch: bad range for second variable"
  varList := variables p
  # varList > 2 =>
    error "makeSketch: polynomial in more than 2 variables"
  # varList = 0 =>
    error "makeSketch: constant polynomial"
  -- polynomial in 1 variable
  # varList = 1 =>
    (not member?(x,varList)) and (not member?(y,varList)) =>
      error "makeSketch: bad variables"
    makeOneVarSketch(p,x,y,xMin,xMax,yMin,yMax,first varList)
  -- polynomial in 2 variables
  (not member?(x,varList)) or (not member?(y,varList)) =>
    error "makeSketch: bad variables"
  totalDegree p = 1 =>
    makeLineSketch(p,x,y,xMin,xMax,yMin,yMax)
  -- polynomial is linear in one variable
  -- y is a rational function of x
  degree(p,y) = 1 =>
    makeRatFcnSketch(p,x,y,xMin,xMax,yMin,yMax,y)
  -- x is a rational function of y
  degree(p,x) = 1 =>
    makeRatFcnSketch(p,x,y,xMin,xMax,yMin,yMax,x)
  -- the general case
  makeGeneralSketch(p,x,y,xMin,xMax,yMin,yMax)

--% special cases

makeOneVarSketch(p,x,y,xMin,xMax,yMin,yMax,var) ==
  -- the case where 'p' is a polynomial in only one variable

```

```

-- the graph consists of horizontal or vertical lines
if var = x then
  minVal := RNtoNF xMin
  maxVal := RNtoNF xMax
else
  minVal := RNtoNF yMin
  maxVal := RNtoNF yMax
lf : List Point DoubleFloat := nil()
rt : List Point DoubleFloat := nil()
bt : List Point DoubleFloat := nil()
tp : List Point DoubleFloat := nil()
htans : List Point DoubleFloat := nil()
vtans : List Point DoubleFloat := nil()
bran : List List Point DoubleFloat := nil()
roots := kinte(p,EPSILON)
sketchRoots : List DoubleFloat := nil()
for root in roots repeat
  if (minVal <= root) and (root <= maxVal) then
    sketchRoots := cons(NFtoSF root,sketchRoots)
null sketchRoots =>
  [p,x,y,xMin,xMax,yMin,yMax,[lf,rt,bt,tp],htans,vtans,bran]
if var = x then
  yMinSF := RNtoSF yMin; yMaxSF := RNtoSF yMax
  for rootSF in sketchRoots repeat
    tp := cons(pt1 := makePt(rootSF,yMaxSF),tp)
    bt := cons(pt2 := makePt(rootSF,yMinSF),bt)
    branch : List Point DoubleFloat := [pt1,pt2]
    bran := cons(branch,bran)
else
  xMinSF := RNtoSF xMin; xMaxSF := RNtoSF xMax
  for rootSF in sketchRoots repeat
    rt := cons(pt1 := makePt(xMaxSF,rootSF),rt)
    lf := cons(pt2 := makePt(xMinSF,rootSF),lf)
    branch : List Point DoubleFloat := [pt1,pt2]
    bran := cons(branch,bran)
[p,x,y,xMin,xMax,yMin,yMax,[lf,rt,bt,tp],htans,vtans,bran]

makeLineSketch(p,x,y,xMin,xMax,yMin,yMax) ==
-- the case where  $p(x,y) = a x + b y + c$  with  $a \neq 0$ ,  $b \neq 0$ 
-- this is a line which is neither vertical nor horizontal
xMinSF := RNtoSF xMin; xMaxSF := RNtoSF xMax
yMinSF := RNtoSF yMin; yMaxSF := RNtoSF yMax
-- determine the coefficients a, b, and c
a := ground(coefficient(p,x,1)) :: DoubleFloat
b := ground(coefficient(p,y,1)) :: DoubleFloat
c := ground(coefficient(coefficient(p,x,0),y,0)) :: DoubleFloat
lf : List Point DoubleFloat := nil()
rt : List Point DoubleFloat := nil()
bt : List Point DoubleFloat := nil()
tp : List Point DoubleFloat := nil()

```

```

htans : List Point DoubleFloat := nil()
vtans : List Point DoubleFloat := nil()
branch : List Point DoubleFloat := nil()
bran : List List Point DoubleFloat := nil()
-- compute x coordinate of point on line with y = yMin
xBottom := (- b*yMinSF - c)/a
-- compute x coordinate of point on line with y = yMax
xTop := (- b*yMaxSF - c)/a
-- compute y coordinate of point on line with x = xMin
yLeft := (- a*xMinSF - c)/b
-- compute y coordinate of point on line with x = xMax
yRight := (- a*xMaxSF - c)/b
-- determine which of the above 4 points are in the region
-- to be plotted and list them as a branch
if (xMinSF < xBottom) and (xBottom < xMaxSF) then
  bt := cons(pt := makePt(xBottom,yMinSF),bt)
  branch := cons(pt,branch)
if (xMinSF < xTop) and (xTop < xMaxSF) then
  tp := cons(pt := makePt(xTop,yMaxSF),tp)
  branch := cons(pt,branch)
if (yMinSF <= yLeft) and (yLeft <= yMaxSF) then
  lf := cons(pt := makePt(xMinSF,yLeft),lf)
  branch := cons(pt,branch)
if (yMinSF <= yRight) and (yRight <= yMaxSF) then
  rt := cons(pt := makePt(xMaxSF,yRight),rt)
  branch := cons(pt,branch)
bran := cons(branch,bran)
[p,x,y,xMin,xMax,yMin,yMax,[lf,rt,bt,tp],htans,vtans,bran]

singValBetween?(xCurrent,xNext,xSingList) ==
  for xVal in xSingList repeat
    (xCurrent < xVal) and (xVal < xNext) => return true
  false

segmentInfo(f,lo,hi,botList,topList,singList,minSF,maxSF) ==
  repeat
    -- 'current' is the smallest element of 'topList' and 'botList'
    -- 'currentFrom' records the list from which it was taken
    if null topList then
      if null botList then
        return [segment(lo,hi),hi,nil(),nil()]
      else
        current := first botList
        botList := rest botList
        currentFrom := BOTTOM
    else
      if null botList then
        current := first topList
        topList := rest topList
        currentFrom := TOP

```

```

else
  bot := first botList
  top := first topList
  if bot < top then
    current := bot
    botList := rest botList
    currentFrom := BOTTOM
  else
    current := top
    topList := rest topList
    currentFrom := TOP
-- 'nxt' is the next smallest element of 'topList'
-- and 'botList'
-- 'nextFrom' records the list from which it was taken
if null topList then
  if null botList then
    return [segment(lo,hi),hi,nil(),nil()]
  else
    nxt := first botList
    botList := rest botList
    nextFrom := BOTTOM
else
  if null botList then
    nxt := first topList
    topList := rest topList
    nextFrom := TOP
  else
    bot := first botList
    top := first topList
    if bot < top then
      nxt := bot
      botList := rest botList
      nextFrom := BOTTOM
    else
      nxt := top
      topList := rest topList
      nextFrom := TOP
if currentFrom = nextFrom then
  if singValBetween?(current,nxt,singList) then
    return [segment(lo,current),nxt,botList,topList]
  else
    val := f((nxt - current)/2::DoubleFloat)
    if (val <= minSF) or (val >= maxSF) then
      return [segment(lo,current),nxt,botList,topList]
else
  if singValBetween?(current,nxt,singList) then
    return [segment(lo,current),nxt,botList,topList]

makeRatFcnSketch(p,x,y,xMin,xMax,yMin,yMax,depVar) ==
-- the case where p(x,y) is linear in x or y

```

```

-- Thus, one variable is a rational function of the other.
-- Therefore, we may use the 2-dimensional function plotting
-- package. The only problem is determining the intervals on
-- on which the function is to be plotted.
--!! corners: e.g. upper left corner is on graph with  $y' > 0$ 
factoredP := p :: (Factored Polynomial Integer)
numberOfFactors(factoredP) > 1 =>
    error "reducible polynomial" --!! sketch each factor
dpdx := differentiate(p,x)
dpdy := differentiate(p,y)
pRN := coerceCoefsToRNs p
xMinSF := RNtoSF xMin; xMaxSF := RNtoSF xMax
yMinSF := RNtoSF yMin; yMaxSF := RNtoSF yMax
xMinNF := RNtoNF xMin; xMaxNF := RNtoNF xMax
yMinNF := RNtoNF yMin; yMaxNF := RNtoNF yMax
-- 'p' is of degree 1 in the variable 'depVar'.
-- Thus, 'depVar' is a rational function of the other variable.
num := -coefficient(p,depVar,0)
den := coefficient(p,depVar,1)
numUPolySF := SFPolyToUPoly(coerceCoefsToSFs(num))
denUPolySF := SFPolyToUPoly(coerceCoefsToSFs(den))
-- this is the rational function
f : DoubleFloat -> DoubleFloat := s +-> elt(numUPolySF,s)/elt(denUPolySF,s)
-- values of the dependent and independent variables
if depVar = x then
    indVarMin := yMin; indVarMax := yMax
    indVarMinNF := yMinNF; indVarMaxNF := yMaxNF
    indVarMinSF := yMinSF; indVarMaxSF := yMaxSF
    depVarMin := xMin; depVarMax := xMax
    depVarMinSF := xMinSF; depVarMaxSF := xMaxSF
else
    indVarMin := xMin; indVarMax := xMax
    indVarMinNF := xMinNF; indVarMaxNF := xMaxNF
    indVarMinSF := xMinSF; indVarMaxSF := xMaxSF
    depVarMin := yMin; depVarMax := yMax
    depVarMinSF := yMinSF; depVarMaxSF := yMaxSF
-- Create lists of critical points.
htanPts := rsolve([p,dpdx],[x,y],EPSILON)
vtanPts := rsolve([p,dpdy],[x,y],EPSILON)
htans := listPtsInRect(htanPts,xMinNF,xMaxNF,yMinNF,yMaxNF)
vtans := listPtsInRect(vtanPts,xMinNF,xMaxNF,yMinNF,yMaxNF)
-- Create lists which will contain boundary points.
lf : List Point DoubleFloat := nil()
rt : List Point DoubleFloat := nil()
bt : List Point DoubleFloat := nil()
tp : List Point DoubleFloat := nil()
-- Determine values of the independent variable at the which
-- the rational function has a pole as well as the values of
-- the independent variable for which there is a point on the
-- upper or lower boundary.

```



```

singList : List DoubleFloat :=
  roots : List Float := kinte(den,EPSILON)
  outList : List DoubleFloat := nil()
  for root in roots repeat
    if (indVarMinNF < root) and (root < indVarMaxNF) then
      outList := cons(NFtoSF root,outList)
  sort((x,y) +-> x < y, outList)
topList : List DoubleFloat :=
  roots : List Float := kinte(eval(pRN,depVar,depVarMax),EPSILON)
  outList : List DoubleFloat := nil()
  for root in roots repeat
    if (indVarMinNF < root) and (root < indVarMaxNF) then
      outList := cons(NFtoSF root,outList)
  sort((x,y) +-> x < y, outList)
botList : List DoubleFloat :=
  roots : List Float := kinte(eval(pRN,depVar,depVarMin),EPSILON)
  outList : List DoubleFloat := nil()
  for root in roots repeat
    if (indVarMinNF < root) and (root < indVarMaxNF) then
      outList := cons(NFtoSF root,outList)
  sort((x,y) +-> x < y, outList)
-- We wish to determine if the graph has points on the 'left'
-- and 'right' boundaries, so we compute the value of the
-- rational function at the lefthand and righthand values of
-- the dependent variable. If the function has a singularity
-- on the left or right boundary, then 'leftVal' or 'rightVal'
-- is given a dummy value which will convince the program that
-- there is no point on the left or right boundary.
denUPolyRN := RNPolyToUPoly(coerceCoefsToRNs(den))
if elt(denUPolyRN,indVarMin) = 0$(Fraction Integer) then
  leftVal := depVarMinSF - (abs(depVarMinSF) + 1$DoubleFloat)
else
  leftVal := f(indVarMinSF)
if elt(denUPolyRN,indVarMax) = 0$(Fraction Integer) then
  rightVal := depVarMinSF - (abs(depVarMinSF) + 1$DoubleFloat)
else
  rightVal := f(indVarMaxSF)
-- Now put boundary points on the appropriate lists.
if depVar = x then
  if (xMinSF < leftVal) and (leftVal < xMaxSF) then
    bt := cons(makePt(leftVal,yMinSF),bt)
  if (xMinSF < rightVal) and (rightVal < xMaxSF) then
    tp := cons(makePt(rightVal,yMaxSF),tp)
  for val in botList repeat
    lf := cons(makePt(xMinSF,val),lf)
  for val in topList repeat
    rt := cons(makePt(xMaxSF,val),rt)
else
  if (yMinSF < leftVal) and (leftVal < yMaxSF) then
    lf := cons(makePt(xMinSF,leftVal),lf)

```

```

    if (yMinSF < rightVal) and (rightVal < yMaxSF) then
      rt := cons(makePt(xMaxSF, rightVal), rt)
    for val in botList repeat
      bt := cons(makePt(val, yMinSF), bt)
    for val in topList repeat
      tp := cons(makePt(val, yMaxSF), tp)
  bran : List List Point DoubleFloat := nil()
  -- Determine segments on which the rational function is to
  -- be plotted.
  if (depVarMinSF < leftVal) and (leftVal < depVarMaxSF) then
    lo := indVarMinSF
  else
    if null topList then
      if null botList then
        return [p, x, y, xMin, xMax, yMin, yMax, [lf, rt, bt, tp], _
              htans, vtans, bran]
      else
        lo := first botList
        botList := rest botList
    else
      if null botList then
        lo := first topList
        topList := rest topList
      else
        bot := first botList
        top := first topList
        if bot < top then
          lo := bot
          botList := rest botList
        else
          lo := top
          topList := rest topList
  hi := 0$DoubleFloat -- @#$%^&* compiler
  if (depVarMinSF < rightVal) and (rightVal < depVarMaxSF) then
    hi := indVarMaxSF
  else
    if null topList then
      if null botList then
        error "makeRatFcnSketch: plot domain"
      else
        hi := last botList
        botList := remove(hi, botList)
    else
      if null botList then
        hi := last topList
        topList := remove(hi, topList)
      else
        bot := last botList
        top := last topList
        if bot > top then

```

```

        hi := bot
        botList := remove(hi,botList)
    else
        hi := top
        topList := remove(hi,topList)
    if (depVar = x) then
        (minSF := xMinSF; maxSF := xMaxSF)
    else
        (minSF := yMinSF; maxSF := yMaxSF)
    segList : List Segment DoubleFloat := nil()
    repeat
        segInfo := segmentInfo(f,lo,hi,botList,topList,singList,_,
                               minSF,maxSF)
        segList := cons(segInfo.seg,segList)
        lo := segInfo.left
        botList := segInfo.lowerVals
        topList := segInfo.upperVals
        if lo = hi then break
    for segment in segList repeat
        RFPlot : Plot := plot(f,segment)
        curve := first(listBranches(RFPlot))
        if depVar = y then
            bran := cons(curve,bran)
        else
            bran := cons(map(swapCoords,curve),bran)
    [p,x,y,xMin,xMax,yMin,yMax,[lf,rt,bt,tp],htans,vtans,bran]

--% the general case

makeGeneralSketch(pol,x,y,xMin,xMax,yMin,yMax) ==
--!! corners of region should not be on curve
--!! enlarge region if necessary
factoredPol := pol :: (Factored Polynomial Integer)
numberOfFactors(factoredPol) > 1 =>
    error "reducible polynomial" --!! sketch each factor
p := nthFactor(factoredPol,1)
dpdx := differentiate(p,x); dpdy := differentiate(p,y)
xMinNF := RNtoNF xMin; xMaxNF := RNtoNF xMax
yMinNF := RNtoNF yMin; yMaxNF := RNtoNF yMax
-- compute singular points; error if singularities in region
singPts := rsolve([p,dpdx,dpdy],[x,y],EPSILON)
-- ptsSuchThat?(singPts,inRect?(#1,xMinNF,xMaxNF,yMinNF,yMaxNF)) =>
foo : List Float -> Boolean := s +-> inRect?(s,xMinNF,xMaxNF,yMinNF,yMaxNF)
ptsSuchThat?(singPts,foo) =>
    error "singular pts in region of sketch"
-- compute critical points
htanPts := rsolve([p,dpdx],[x,y],EPSILON)
vtanPts := rsolve([p,dpdy],[x,y],EPSILON)
critPts := append(htanPts,vtanPts)
-- if there are critical points on the boundary, then enlarge

```

```

-- the region, but be sure that the new region does not contain
-- any singular points
hInc : Fraction Integer := (1/20) * (xMax - xMin)
vInc : Fraction Integer := (1/20) * (yMax - yMin)
-- if ptsSuchThat?(critPts,onVertSeg?(#1,yMinNF,yMaxNF,xMinNF)) then
foo : List Float -> Boolean := s +-> onVertSeg?(s,yMinNF,yMaxNF,xMinNF)
if ptsSuchThat?(critPts,foo) then
  xMin := newX(critPts,singPts,yMinNF,yMaxNF,xMinNF,xMin,-hInc)
  xMinNF := RNtoNF xMin
-- if ptsSuchThat?(critPts,onVertSeg?(#1,yMinNF,yMaxNF,xMaxNF)) then
foo : List Float -> Boolean := s +-> onVertSeg?(s,yMinNF,yMaxNF,xMaxNF)
if ptsSuchThat?(critPts,foo) then
  xMax := newX(critPts,singPts,yMinNF,yMaxNF,xMaxNF,xMax,hInc)
  xMaxNF := RNtoNF xMax
-- if ptsSuchThat?(critPts,onHorzSeg?(#1,xMinNF,xMaxNF,yMinNF)) then
foo : List Float -> Boolean := s +-> onHorzSeg?(s,xMinNF,xMaxNF,yMinNF)
if ptsSuchThat?(critPts,foo) then
  yMin := newY(critPts,singPts,xMinNF,xMaxNF,yMinNF,yMin,-vInc)
  yMinNF := RNtoNF yMin
-- if ptsSuchThat?(critPts,onHorzSeg?(#1,xMinNF,xMaxNF,yMaxNF)) then
foo : List Float -> Boolean := s +-> onHorzSeg?(s,xMinNF,xMaxNF,yMaxNF)
if ptsSuchThat?(critPts,foo) then
  yMax := newY(critPts,singPts,xMinNF,xMaxNF,yMaxNF,yMax,vInc)
  yMaxNF := RNtoNF yMax
htans := listPtsInRect(htanPts,xMinNF,xMaxNF,yMinNF,yMaxNF)
vtans := listPtsInRect(vtanPts,xMinNF,xMaxNF,yMinNF,yMaxNF)
crits := append(htans,vtans)
-- conversions to DoubleFloats
xMinSF := RNtoSF xMin; xMaxSF := RNtoSF xMax
yMinSF := RNtoSF yMin; yMaxSF := RNtoSF yMax
corners := makeCorners(xMinSF,xMaxSF,yMinSF,yMaxSF)
pSF := coerceCoefsToSFs p
dpdxSF := coerceCoefsToSFs dpdx
dpdySF := coerceCoefsToSFs dpdy
delta := min((xMaxSF - xMinSF)/25,(yMaxSF - yMinSF)/25)
err := min(delta/100,PLOTERR/100)
bound : PositiveInteger := 10
-- compute points on the boundary
pRN := coerceCoefsToRNs(p)
lf : List Point DoubleFloat :=
  listPtsOnVertBdry(pRN,x,xMin,yMinNF,yMaxNF)
rt : List Point DoubleFloat :=
  listPtsOnVertBdry(pRN,x,xMax,yMinNF,yMaxNF)
bt : List Point DoubleFloat :=
  listPtsOnHorizBdry(pRN,y,yMin,xMinNF,xMaxNF)
tp : List Point DoubleFloat :=
  listPtsOnHorizBdry(pRN,y,yMax,xMinNF,xMaxNF)
bdPts : BoundaryPts := [lf,rt,bt,tp]
bran := traceBranches(pSF,dpdxSF,dpdySF,x,y,corners,delta,err,_
  bound,crits,bdPts)

```

```

[p,x,y,xMin,xMax,yMin,yMax,bdPts,htans,vtans,bran]

refine(plot,stepFraction) ==
  p := plot.poly; x := plot.xVar; y := plot.yVar
  dpdx := differentiate(p,x); dpdy := differentiate(p,y)
  pSF := coerceCoefsToSFs p
  dpdxSF := coerceCoefsToSFs dpdx
  dpdySF := coerceCoefsToSFs dpdy
  xMin := plot.minXVal; xMax := plot.maxXVal
  yMin := plot.minYVal; yMax := plot.maxYVal
  xMinSF := RNtoSF xMin; xMaxSF := RNtoSF xMax
  yMinSF := RNtoSF yMin; yMaxSF := RNtoSF yMax
  corners := makeCorners(xMinSF,xMaxSF,yMinSF,yMaxSF)
  pSF := coerceCoefsToSFs p
  dpdxSF := coerceCoefsToSFs dpdx
  dpdySF := coerceCoefsToSFs dpdy
  delta :=
    stepFraction * min((xMaxSF - xMinSF)/25,(yMaxSF - yMinSF)/25)
  err := min(delta/100,PLOTERR/100)
  bound : PositiveInteger := 10
  crits := append(plot.hTanPts,plot.vTanPts)
  bdPts := plot.bdryPts
  bran := traceBranches(pSF,dpdxSF,dpdySF,x,y,cornerRadius,delta,err,_
    bound,crits,bdPts)
  htans := plot.hTanPts; vtans := plot.vTanPts
  [p,x,y,xMin,xMax,yMin,yMax,bdPts,htans,vtans,bran]

traceBranches(pSF,dpdxSF,dpdySF,x,y,cornerRadius,delta,err,bound,_
  crits,bdPts) ==
  -- for boundary points, trace curve from boundary to boundary
  -- add the branch to the list of branches
  -- update list of boundary points by deleting first and last
  -- points on this branch
  -- update list of critical points by deleting any critical
  -- points which were plotted
  lf := bdPts.left; rt := bdPts.right
  tp := bdPts.top ; bt := bdPts.bottom
  bdry := append(append(lf,rt),append(bt,tp))
  bran : List List Point DoubleFloat := nil()
  while not null bdry repeat
    pt := first bdry
    p0 := dummyFirstPt(pt,dpdxSF,dpdySF,x,y,lf,rt,bt,tp)
    segInfo := listPtsOnSegment(pSF,dpdxSF,dpdySF,x,y,p0,pt,_
      corners,delta,err,bound,crits,bdry)
    bran := cons(first segInfo,bran)
    crits := second segInfo
    bdry := third segInfo
  -- trace loops beginning and ending with critical points
  -- add the branch to the list of branches
  -- update list of critical points by deleting any critical

```

```

-- points which were plotted
while not null crits repeat
  pt := first crits
  segInfo := listPtsOnLoop(pSF,dpdxSF,dpdySF,x,y,pt,_
                           corners,delta,err,bound,crits,bdry)
  bran := cons(first segInfo,bran)
  crits := second segInfo
bran

dummyFirstPt(p1,dpdxSF,dpdySF,x,y,lf,rt,bt,tp) ==
-- The function 'computeNextPt' requires 2 points, p0 and p1.
-- When computing the second point on a branch which starts
-- on the boundary, we use the boundary point as p1 and the
-- 'dummy' point returned by this function as p0.
x1 := xCoord p1; y1 := yCoord p1
zero := 0$DoubleFloat; one := 1$DoubleFloat
px := ground(eval(dpdxSF,[x,y],[x1,y1]))
py := ground(eval(dpdySF,[x,y],[x1,y1]))
if px * py < zero then      -- positive slope at p1
  member?(p1,lf) or member?(p1,bt) =>
    makePt(x1 - one,y1 - one)
    makePt(x1 + one,y1 + one)
else
  member?(p1,lf) or member?(p1,tp) =>
    makePt(x1 - one,y1 + one)
    makePt(x1 + one,y1 - one)

listPtsOnSegment(pSF,dpdxSF,dpdySF,x,y,p0,p1,corners,_
                 delta,err,bound,crits,bdry) ==
-- p1 is a boundary point; p0 is a 'dummy' point
bdry := remove(p1,bdry)
pointList : List Point DoubleFloat := [p1]
ptInfo := computeNextPt(pSF,dpdxSF,dpdySF,x,y,p0,p1,corners,_
                       delta,err,bound,crits,bdry)

p2 := ptInfo.newPt
ptInfo.type = BDRY =>
  bdry := remove(p2,bdry)
  pointList := cons(p2,pointList)
  [pointList,crits,bdry]
if ptInfo.type = CRIT then crits := remove(p2,crits)
pointList := cons(p2,pointList)
repeat
  pt0 := second pointList; pt1 := first pointList
  ptInfo := computeNextPt(pSF,dpdxSF,dpdySF,x,y,pt0,pt1,corners,_
                          delta,err,bound,crits,bdry)

  p2 := ptInfo.newPt
  ptInfo.type = BDRY =>
    bdry := remove(p2,bdry)
    pointList := cons(p2,pointList)

```

```

        return [pointList,crits,bdry]
    if ptInfo.type = CRIT then crits := remove(p2,crits)
    pointList := cons(p2,pointList)
--!! delete next line (compiler bug)
[pointList,crits,bdry]

listPtsOnLoop(pSF,dpdxSF,dpdySF,x,y,p1,cornerRadius,
              delta,err,bound,crits,bdry) ==
x1 := xCoord p1; y1 := yCoord p1
px := ground(eval(dpdxSF,[x,y],[x1,y1]))
py := ground(eval(dpdySF,[x,y],[x1,y1]))
p0 := makePt(x1 - 1$DoubleFloat,y1 - 1$DoubleFloat)
pointList : List Point DoubleFloat := [p1]
ptInfo := computeNextPt(pSF,dpdxSF,dpdySF,x,y,p0,p1,cornerRadius,
                        delta,err,bound,crits,bdry)

p2 := ptInfo.newPt
ptInfo.type = BDRY =>
    error "boundary reached while on loop"
if ptInfo.type = CRIT then
    p1 = p2 =>
        error "first and second points on loop are identical"
    crits := remove(p2,crits)
pointList := cons(p2,pointList)
repeat
    pt0 := second pointList; pt1 := first pointList
    ptInfo := computeNextPt(pSF,dpdxSF,dpdySF,x,y,pt0,pt1,cornerRadius,
                           delta,err,bound,crits,bdry)

    p2 := ptInfo.newPt
    ptInfo.type = BDRY =>
        error "boundary reached while on loop"
    if ptInfo.type = CRIT then
        crits := remove(p2,crits)
        p1 = p2 =>
            pointList := cons(p2,pointList)
            return [pointList,crits,bdry]
        pointList := cons(p2,pointList)
--!! delete next line (compiler bug)
[pointList,crits,bdry]

computeNextPt(pSF,dpdxSF,dpdySF,x,y,p0,p1,cornerRadius,
              delta,err,bound,crits,bdry) ==
-- p0=(x0,y0) and p1=(x1,y1) are the last two points on the curve.
-- The function computes the next point on the curve.
-- The function determines if the next point is a critical point
-- or a boundary point.
-- The function returns a record of the form
-- Record(newPt:Point DoubleFloat,type:String).
-- If the new point is a boundary point, then 'type' is
-- "boundary point" and 'newPt' is a boundary point to be

```

```

-- deleted from the list of boundary points yet to be plotted.
-- Similarly, if the new point is a critical point, then 'type' is
-- "critical point" and 'newPt' is a critical point to be
-- deleted from the list of critical points yet to be plotted.
-- If the new point is neither a critical point nor a boundary
-- point, then 'type' is "nothing in particular".
xMinSF := getXMin corners; xMaxSF := getXMax corners
yMinSF := getYMin corners; yMaxSF := getYMax corners
x0 := xCoord p0; y0 := yCoord p0
x1 := xCoord p1; y1 := yCoord p1
px := ground(eval(dpdxF, [x,y], [x1,y1]))
py := ground(eval(dpdxF, [x,y], [x1,y1]))
-- let m be the slope of the tangent line at p1
-- if |m| < 1, we will increment the x-coordinate by delta
-- (indicated by 'incVar = x'), find an approximate
-- y-coordinate using the tangent line, then find the actual
-- y-coordinate using a Newton iteration
if abs(py) > abs(px) then
  incVar0 := incVar := x
  deltaX := (if x1 > x0 then delta else -delta)
  x2Approx := x1 + deltaX
  y2Approx := y1 + (-px/py)*deltaX
-- if |m| >= 1, we interchange the roles of the x- and y-
-- coordinates
else
  incVar0 := incVar := y
  deltaY := (if y1 > y0 then delta else -delta)
  x2Approx := x1 + (-py/px)*deltaY
  y2Approx := y1 + deltaY
lookingFor := NADA
-- See if (x2Approx,y2Approx) is out of bounds.
-- If so, find where the line segment connecting (x1,y1) and
-- (x2Approx,y2Approx) intersects the boundary and use this
-- point as (x2Approx,y2Approx).
-- If the resulting point is on the left or right boundary,
-- we will now consider x as the 'incremented variable' and we
-- will compute the y-coordinate using a Newton iteration.
-- Similarly, if the point is on the top or bottom boundary,
-- we will consider y as the 'incremented variable' and we
-- will compute the x-coordinate using a Newton iteration.
if x2Approx >= xMaxSF then
  incVar := x
  lookingFor := BDRY
  x2Approx := xMaxSF
  y2Approx := y1 + (-px/py)*(x2Approx - x1)
else
  if x2Approx <= xMinSF then
    incVar := x
    lookingFor := BDRY
    x2Approx := xMinSF

```



```

    y2Approx := y1 + (-px/py)*(x2Approx - x1)
  if y2Approx >= yMaxSF then
    incVar := y
    lookingFor := BDRY
    y2Approx := yMaxSF
    x2Approx := x1 + (-py/px)*(y2Approx - y1)
  else
    if y2Approx <= yMinSF then
      incVar := y
      lookingFor := BDRY
      y2Approx := yMinSF
      x2Approx := x1 + (-py/px)*(y2Approx - y1)
    -- set xLo = min(x1,x2Approx), xHi = max(x1,x2Approx)
    -- set yLo = min(y1,y2Approx), yHi = max(y1,y2Approx)
  if x1 < x2Approx then
    xLo := x1
    xHi := x2Approx
  else
    xLo := x2Approx
    xHi := x1
  if y1 < y2Approx then
    yLo := y1
    yHi := y2Approx
  else
    yLo := y2Approx
    yHi := y1
  -- check for critical points (x*,y*) with x* between
  -- x1 and x2Approx or y* between y1 and y2Approx
  -- store values of x2Approx and y2Approx
  x2Approxx := x2Approx
  y2Approxx := y2Approx
  -- xPointList will contain all critical points (x*,y*)
  -- with x* between x1 and x2Approx
  xPointList : List Point DoubleFloat := nil()
  -- yPointList will contain all critical points (x*,y*)
  -- with y* between y1 and y2Approx
  yPointList : List Point DoubleFloat := nil()
  for pt in crits repeat
    xx := xCoord pt; yy := yCoord pt
    -- if x1 = x2Approx, then p1 is a point with horizontal
    -- tangent line
    -- in this case, we don't want critical points with
    -- x-coordinate x1
    if xx = x2Approx and not (xx = x1) then
      if min(abs(yy-yLo),abs(yy-yHi)) < delta then
        xPointList := cons(pt,xPointList)
    if ((xLo < xx) and (xx < xHi)) then
      if min(abs(yy-yLo),abs(yy-yHi)) < delta then
        xPointList := cons(pt,nil())
        x2Approx := xx

```

```

        if xx < x1 then xLo := xx else xHi := xx
-- if y1 = y2Approx, then p1 is a point with vertical
-- tangent line
-- in this case, we don't want critical points with
-- y-coordinate y1
if yy = y2Approx and not (yy = y1) then
    yPointList := cons(pt,yPointList)
if ((yLo < yy) and (yy < yHi)) then
    if min(abs(xx-xLo),abs(xx-xHi)) < delta then
        yPointList := cons(pt,nil())
        y2Approx := yy
        if yy < y1 then yLo := yy else yHi := yy
-- points in both xPointList and yPointList
if (not null xPointList) and (not null yPointList) then
    xPointList = yPointList =>
-- this implies that the lists have only one point
    incVar := incVar0
    if incVar = x then
        y2Approx := y1 + (-px/py)*(x2Approx - x1)
    else
        x2Approx := x1 + (-py/px)*(y2Approx - y1)
    lookingFor := CRIT          -- proceed
incVar0 = x =>
-- first try Newton iteration with 'y' as incremented variable
x2Temp := x1 + (-py/px)*(y2Approx - y1)
f := SFPolyToUPoly(eval(pSF,y,y2Approx))
x2New := newtonApprox(f,x2Temp,err,bound)
x2New case "failed" =>
    y2Approx := y1 + (-px/py)*(x2Approx - x1)
    incVar := x
    lookingFor := CRIT          -- proceed
y2Temp := y1 + (-px/py)*(x2Approx - x1)
f := SFPolyToUPoly(eval(pSF,x,x2Approx))
y2New := newtonApprox(f,y2Temp,err,bound)
y2New case "failed" =>
    return computeNextPt(pSF,dpdxSF,dpdySF,x,y,p0,p1,corners,_,
        abs((x2Approx-x1)/2),err,bound,crits,bdry)
pt1 := makePt(x2Approx,y2New :: DoubleFloat)
pt2 := makePt(x2New :: DoubleFloat,y2Approx)
critPt1 := findPtOnList(pt1,crits)
critPt2 := findPtOnList(pt2,crits)
(critPt1 case "failed") and (critPt2 case "failed") =>
    abs(x2Approx - x1) > abs(x2Temp - x1) =>
        return [pt1,NADA]
    return [pt2,NADA]
(critPt1 case "failed") =>
    return [critPt2::(Point DoubleFloat),CRIT]
(critPt2 case "failed") =>
    return [critPt1::(Point DoubleFloat),CRIT]
abs(x2Approx - x1) > abs(x2Temp - x1) =>

```

```

        return [critPt2::(Point DoubleFloat),CRIT]
    return [critPt1::(Point DoubleFloat),CRIT]
y2Temp := y1 + (-px/py)*(x2Approx - x1)
f := SFPolyToUPoly(eval(pSF,x,x2Approx))
y2New := newtonApprox(f,y2Temp,err,bound)
y2New case "failed" =>
    x2Approx := x1 + (-py/px)*(y2Approx - y1)
    incVar := y
    lookingFor := CRIT      -- proceed
x2Temp := x1 + (-py/px)*(y2Approx - y1)
f := SFPolyToUPoly(eval(pSF,y,y2Approx))
x2New := newtonApprox(f,x2Temp,err,bound)
x2New case "failed" =>
    return computeNextPt(pSF,dpdxSF,dpdySF,x,y,p0,p1,cornerRadius,
        abs((y2Approx-y1)/2),err,bound,crits,bdry)
pt1 := makePt(x2Approx,y2New :: DoubleFloat)
pt2 := makePt(x2New :: DoubleFloat,y2Approx)
critPt1 := findPtOnList(pt1,crits)
critPt2 := findPtOnList(pt2,crits)
(critPt1 case "failed") and (critPt2 case "failed") =>
    abs(y2Approx - y1) > abs(y2Temp - y1) =>
        return [pt2,NADA]
    return [pt1,NADA]
(critPt1 case "failed") =>
    return [critPt2::(Point DoubleFloat),CRIT]
(critPt2 case "failed") =>
    return [critPt1::(Point DoubleFloat),CRIT]
abs(y2Approx - y1) > abs(y2Temp - y1) =>
    return [critPt1::(Point DoubleFloat),CRIT]
return [critPt2::(Point DoubleFloat),CRIT]
if (not null xPointList) and (null yPointList) then
y2Approx := y1 + (-px/py)*(x2Approx - x1)
incVar0 = x =>
    incVar := x
    lookingFor := CRIT      -- proceed
f := SFPolyToUPoly(eval(pSF,x,x2Approx))
y2New := newtonApprox(f,y2Approx,err,bound)
y2New case "failed" =>
    x2Approx := x2Approxx
    y2Approx := y2Approxx      -- proceed
pt := makePt(x2Approx,y2New::DoubleFloat)
critPt := findPtOnList(pt,crits)
critPt case "failed" =>
    return [pt,NADA]
return [critPt :: (Point DoubleFloat),CRIT]
if (null xPointList) and (not null yPointList) then
x2Approx := x1 + (-py/px)*(y2Approx - y1)
incVar0 = y =>
    incVar := y
    lookingFor := CRIT      -- proceed

```

```

f := SFPolyToUPoly(eval(pSF,y,y2Approx))
x2New := newtonApprox(f,x2Approx,err,bound)
x2New case "failed" =>
  x2Approx := x2Approxx
  y2Approx := y2Approxx -- proceed
pt := makePt(x2New::DoubleFloat,y2Approx)
critPt := findPtOnList(pt,crits)
critPt case "failed" =>
  return [pt,NADA]
return [critPt :: (Point DoubleFloat),CRIT]
if incVar = x then
  x2 := x2Approx
  f := SFPolyToUPoly(eval(pSF,x,x2))
  y2New := newtonApprox(f,y2Approx,err,bound)
  y2New case "failed" =>
    return computeNextPt(pSF,dpdxSF,dpdySF,x,y,p0,p1,cornerRadius,
      abs((x2-x1)/2),err,bound,crits,bdry)
  y2 := y2New :: DoubleFloat
else
  y2 := y2Approx
  f := SFPolyToUPoly(eval(pSF,y,y2))
  x2New := newtonApprox(f,x2Approx,err,bound)
  x2New case "failed" =>
    return computeNextPt(pSF,dpdxSF,dpdySF,x,y,p0,p1,cornerRadius,
      abs((y2-y1)/2),err,bound,crits,bdry)
  x2 := x2New :: DoubleFloat
pt := makePt(x2,y2)
--!! check that 'pt' is not out of bounds
-- check if you've gotten a critical or boundary point
lookingFor = NADA =>
  [pt,lookingFor]
lookingFor = BDRY =>
  bdryPt := findPtOnList(pt,bdry)
  bdryPt case "failed" =>
    error "couldn't find boundary point"
  [bdryPt :: (Point DoubleFloat),BDRY]
critPt := findPtOnList(pt,crits)
critPt case "failed" =>
  [pt,NADA]
[critPt :: (Point DoubleFloat),CRIT]

--% Newton iterations

newtonApprox(f,a0,err,bound) ==
-- Newton iteration to approximate a root of the polynomial 'f'
-- using an initial approximation of 'a0'
-- Newton iteration terminates when consecutive approximations
-- are within 'err' of each other
-- returns "failed" if this has not been achieved after 'bound'
-- iterations

```

```

Df := differentiate f
oldApprox := a0
newApprox := a0 - elt(f,a0)/elt(Df,a0)
i : PositiveInteger := 1
while abs(newApprox - oldApprox) > err repeat
  i = bound => return "failed"
  oldApprox := newApprox
  newApprox := oldApprox - elt(f,oldApprox)/elt(Df,oldApprox)
  i := i+1
newApprox

--% graphics output

listBranches(acplot) == acplot.branches

--% terminal output

coerce(acplot:%) ==
  pp := acplot.poly :: OutputForm
  xx := acplot.xVar :: OutputForm
  yy := acplot.yVar :: OutputForm
  xLo := acplot.minXVal :: OutputForm
  xHi := acplot.maxXVal :: OutputForm
  yLo := acplot.minYVal :: OutputForm
  yHi := acplot.maxYVal :: OutputForm
  zip := message(" = 0")
  com := message(", ")
  les := message(" <= ")
  l : List OutputForm :=
    [pp,zip,com,xLo,les,xx,les,xHi,com,yLo,les,yy,les,yHi]
  f : List OutputForm := nil()
  for branch in acplot.branches repeat
    ll : List OutputForm := [p :: OutputForm for p in branch]
    f := cons(vconcat ll,f)
  ff := vconcat(hconcat l,vconcat f)
  vconcat(message "ACPLOT",ff)

```

---

— ACPLOT.dotabb —

```

"ACPLOT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ACPLOT"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"ACPLOT" -> "ALIST"

```

---

## 17.18 domain PLACES Places

— Places.input —

```
)set break resume
)sys rm -f Places.output
)spool Places.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Places
--R Places K: Field  is a domain constructor
--R Abbreviation for Places is PLACES
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PLACES
--R
--R----- Operations -----
--R ?? : (Integer,%) -> Divisor %      ?+? : (%,%) -> Divisor %
--R -? : % -> Divisor %                ?-? : (%,%) -> Divisor %
--R ?? : (%,%) -> Boolean              coerce : % -> OutputForm
--R create : Symbol -> %               create : List K -> %
--R degree : % -> PositiveInteger      ?.? : (%,Integer) -> K
--R foundPlaces : () -> List %         hash : % -> SingleInteger
--R itsALeaf! : % -> Void              latex : % -> String
--R leaf? : % -> Boolean               reduce : List % -> Divisor %
--R ?~=? : (%,%) -> Boolean
--R ?+? : (%,Divisor %) -> Divisor %
--R ?+? : (Divisor %,%) -> Divisor %
--R ?-? : (%,Divisor %) -> Divisor %
--R ?-? : (Divisor %,%) -> Divisor %
--R localParam : % -> List NeitherSparseOrDensePowerSeries K
--R setDegree! : (%,PositiveInteger) -> Void
--R setFoundPlacesToEmpty : () -> List %
--R setParam! : (%,List NeitherSparseOrDensePowerSeries K) -> Void
--R
--E 1

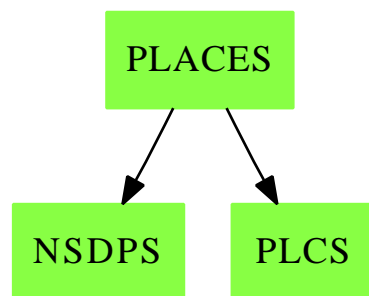
)spool
)lisp (bye)
```

— Places.help —

```
=====
Places examples
```

See Also:  
 o )show Places

### 17.18.1 Places (PLACES)



#### Exports:

|             |        |            |                       |           |
|-------------|--------|------------|-----------------------|-----------|
| -?          | ?*?    | ?+?        | ?-?                   | ??        |
| ?=?         | ?~=?   | coerce     | create                | degree    |
| foundPlaces | hash   | itsALeaf!  | latex                 | leaf?     |
| localParam  | reduce | setDegree! | setFoundPlacesToEmpty | setParam! |

— domain **PLACES** Places —

```

)abbrev domain PLACES Places
++ Author: Gaetan Hache
++ Date Created: 17 nov 1992
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
++ The following is part of the PAFF package
Places(K):Exports == Implementation where
  K:Field
  PCS    ==> NeitherSparseOrDensePowerSeries(K)

  Exports ==> PlacesCategory(K,PCS)

  Implementation ==> Plcs(K,PCS)
  
```

— PLACES.dotabb —

```
"PLACES" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PLACES"];
"NSDPS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=NSDPS"];
"PLCS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PLCS"]
"PLACES" -> "NSDPS"
"PLACES" -> "PLCS"
```

## 17.19 domain PLACESPS PlacesOverPseudoAlgebraic-ClosureOfFiniteField

— PlacesOverPseudoAlgebraicClosureOfFiniteField.input —

```
)set break resume
)sys rm -f PlacesOverPseudoAlgebraicClosureOfFiniteField.output
)spool PlacesOverPseudoAlgebraicClosureOfFiniteField.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PlacesOverPseudoAlgebraicClosureOfFiniteField
--R PlacesOverPseudoAlgebraicClosureOfFiniteField K: FiniteFieldCategory is a domain constructor
--R Abbreviation for PlacesOverPseudoAlgebraicClosureOfFiniteField is PLACESPS
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PLACESPS
--R
--R----- Operations -----
--R ?? : (Integer,%) -> Divisor %      ??? : (%,%) -> Divisor %
--R -? : % -> Divisor %                ?-? : (%,%) -> Divisor %
--R ?? : (%,%) -> Boolean              coerce : % -> OutputForm
--R create : Symbol -> %               degree : % -> PositiveInteger
--R foundPlaces : () -> List %         hash : % -> SingleInteger
--R itsALeaf! : % -> Void              latex : % -> String
--R leaf? : % -> Boolean               reduce : List % -> Divisor %
--R ?~=? : (%,%) -> Boolean
--R ??? : (%,Divisor %) -> Divisor %
--R ?+? : (Divisor %,%) -> Divisor %
--R ?-? : (%,Divisor %) -> Divisor %
--R ?-? : (Divisor %,%) -> Divisor %
--R create : List PseudoAlgebraicClosureOfFiniteField K -> %
--R ?.? : (%,Integer) -> PseudoAlgebraicClosureOfFiniteField K
```



```

--R localParam : % -> List NeitherSparseOrDensePowerSeries PseudoAlgebraicClosureOfFiniteField
--R setDegree! : (% ,PositiveInteger) -> Void
--R setFoundPlacesToEmpty : () -> List %
--R setParam! : (% ,List NeitherSparseOrDensePowerSeries PseudoAlgebraicClosureOfFiniteField) -> Void
--R
--E 1

)spool
)lisp (bye)

```

---

— PlacesOverPseudoAlgebraicClosureOfFiniteField.help —

```

=====
PlacesOverPseudoAlgebraicClosureOfFiniteField examples
=====

```

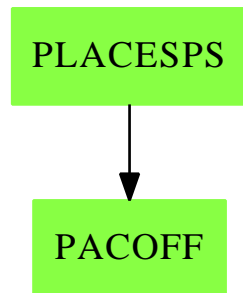
See Also:

```

o )show PlacesOverPseudoAlgebraicClosureOfFiniteField

```

### 17.19.1 PlacesOverPseudoAlgebraicClosureOfFiniteField (PLACESPS)



**Exports:**

|             |        |            |                       |           |
|-------------|--------|------------|-----------------------|-----------|
| -?          | ?*?    | ?+?        | ?-?                   | ?.?       |
| ?=?         | ?~=?   | coerce     | create                | degree    |
| foundPlaces | hash   | itsALeaf!  | latex                 | leaf?     |
| localParam  | reduce | setDegree! | setFoundPlacesToEmpty | setParam! |

— domain PLACESPS PlacesOverPseudoAlgebraicClosureOfFiniteField —

```
)abbrev domain PLACESPS PlacesOverPseudoAlgebraicClosureOfFiniteField
++ Author: Gaetan Hache
++ Date Created: 17 nov 1992
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
++ The following is part of the PAFF package
PlacesOverPseudoAlgebraicClosureOfFiniteField(K):Exports
== Implementation where

K:FiniteFieldCategory
KK ==> PseudoAlgebraicClosureOfFiniteField(K)
PCS ==> NeitherSparseOrDensePowerSeries(KK)

Exports ==> PlacesCategory(KK,PCS)

Implementation ==> Plcs(KK,PCS)
```

— PLACESPS.dotabb —

```
"PLACESPS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PLACESPS"];
"PACOFF" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PACOFF"]
"PLACESPS" -> "PACOFF"
```

## 17.20 domain PLCS Plcs

— Plcs.input —

```
)set break resume
)sys rm -f Plcs.output
)spool Plcs.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Plcs
--R Plcs(K: Field,PCS: LocalPowerSeriesCategory K) is a domain constructor
--R Abbreviation for Plcs is PLCS
```

```

--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PLCS
--R
--R----- Operations -----
--R ?? : (Integer,%) -> Divisor %      ?? : (%,%) -> Divisor %
--R -? : % -> Divisor %                ?-? : (%,%) -> Divisor %
--R ?=? : (%,%) -> Boolean             coerce : % -> OutputForm
--R create : Symbol -> %               create : List K -> %
--R degree : % -> PositiveInteger      ?? : (%,Integer) -> K
--R foundPlaces : () -> List %         hash : % -> SingleInteger
--R itsALeaf! : % -> Boolean           latex : % -> String
--R leaf? : % -> Boolean              localParam : % -> List PCS
--R reduce : List % -> Divisor %      setParam! : (%,List PCS) -> Void
--R ?~=? : (%,%) -> Boolean
--R ?+? : (%,Divisor %) -> Divisor %
--R ?+? : (Divisor %,%) -> Divisor %
--R ?-? : (%,Divisor %) -> Divisor %
--R ?-? : (Divisor %,%) -> Divisor %
--R setDegree! : (%,PositiveInteger) -> Void
--R setFoundPlacesToEmpty : () -> List %
--R
--E 1

)spool
)lisp (bye)

```

---

— Plcs.help —

```

=====
Plcs examples
=====

```

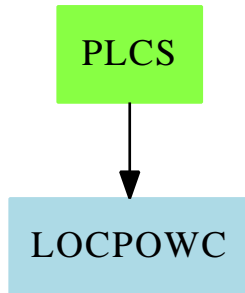
```

See Also:
o )show Plcs

```

---

## 17.20.1 Plcs (PLCS)



Exports:

— domain PLCS Plcs —

```

)abbrev domain PLCS Plcs
++ Author: Gaetan Hache
++ Date Created: 17 nov 1992
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
++ The following is part of the PAFF package
Plcs(K:Field,PCS:LocalPowerSeriesCategory(K)):Exports == Implementation where

  nameOfPlace ==> Union( List(K), Symbol )

  rec ==> Record(theName:nameOfPlace,_
                 locPar: List PCS,_
                 deg: PositiveInteger,_
                 isALeaf:Boolean,_
                 inName:Symbol,_
                 actualSet:Symbol)

  Exports ==> PlacesCategory(K,PCS)

  Implementation ==>   add

    Rep:= rec

    setOfPlacesName:Symbol:=new(ActualSetOfPlacesName)$Symbol

    a:% + b:% == (a:: Divisor(%)) +$Divisor(%) (b::Divisor(%))

    a:% - b:% == (a:: Divisor(%)) -$Divisor(%) (b::Divisor(%))

    n:Integer * b:% == n *$Divisor(%) (b :: Divisor(%))

```

```

reduce(lp)==
  lpd:List Divisor(%) := [p :: Divisor(%) for p in lp]
  reduce("+", lpd, 0$Divisor(%))

d:Divisor(%) + b:% == d + (b::Divisor(%))

a:% + d:Divisor(%) == (a::Divisor(%)) + d

d:Divisor(%) - b:% == d - (b::Divisor(%))

a:% - d:Divisor(%) == (a::Divisor(%)) - d

-a:% == - ( a::Divisor(%))

outName: nameOfPlace -> OutputForm

outName(pt)==
  pt case Symbol => pt :: OutputForm
  dd:OutputForm:= ":" :: OutputForm
  llout:List(OutputForm):=[ hconcat(dd, a::OutputForm) for a in rest pt]
  lout:= cons( (first pt)::OutputForm , llout)
  out:= hconcat lout
  bracket(out)

coerce(pt:%):OutputForm ==
  nn:OutputForm:= outName(pt.theName)
  ee:OutputForm:= degree(pt) :: OutputForm
  nn ** ee

a:% = b:% ==
  ^ (a.actualSet =$Symbol b.actualSet) =>
  a:String:=
    "From Places Domain: Cannot use old places with new places."
    " You have declared two different package PAFF or PAFFFF with the "
    "same arguments. This is not allowed because in that case the two "
    "packages used the same domain to represent the set of places. "
    "Two packages having the same arguments should be used in "
    "different frame"
  error a
  a.inName =$Symbol b.inName

elt(pl,n)==
  pt:= (pl :: Rep).theName
  pt case Symbol => _
    error "From Places domain : cannot return the coordinates of a leaf"
  elt(pt,n)$List(K)

leaf?(pl)==pl.isALeaf

```

```

itsALeaf_!(pl)==
  pl.isALeaf := true()
  void()

listOfFoundPlaces:List %:=[]

foundPlaces()==listOfFoundPlaces

setFoundPlacesToEmpty()==
  tmp:=copy listOfFoundPlaces
  listOfFoundPlaces:=[]
  setOfPlacesName:Symbol:=new(ActualSetOfPlacesName)$Symbol
  tmp

findInExistOnes: % -> %
findInExistOnes(pt)==
  ll:=listOfFoundPlaces
  found:Boolean:=false()
  fpl:%
  while ^found and ^empty?(ll) repeat
    fpl:= first ll
    -- remember: the "=" test is on done on the symbolic name
    found:= pt.theName = fpl.theName
    ll:=rest ll
  ^found =>
    listOfFoundPlaces:=cons(pt,listOfFoundPlaces)
    pt
  fpl

create(pt:List(K)):%==
  newName:=new(SIMPLE)$Symbol
  newPt: %:= [pt, [], 1, false(), newName, setOfPlacesName]$rec
  findInExistOnes(newPt)

create(pt:Symbol): %==
  newPt: %:= [pt, [], 1, false(), pt, setOfPlacesName]$rec
  findInExistOnes(newPt)

setDegree_!(pt,d)==
  pt.deg := d
  void()

setParam_!(pt,ls)==
  pt.locPar:=ls
  void()

localParam(pt)==pt.locPar

degree(pl)==pl.deg

```

---

— PLCS.dotabb —

```
"PLCS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PLCS"];
"LOCP0WC" [color=lightblue,href="bookvol10.2.pdf#nameddest=LOCP0WC"];
"PLCS" -> "LOCP0WC"
```

---

## 17.21 domain PLOT Plot

— Plot.input —

```
)set break resume
)sys rm -f Plot.output
)spool Plot.output
)set message test on
)set message auto off
)clear all
--S 1 of 2
fp:=(t:DFLOAT):DFLOAT +-> sin(t)
--R
--R (1) theMap(Closure)
--R
--R                                          Type: (DoubleFloat -> DoubleFloat)
--E 1

--S 2 of 2
plot(fp,-1.0..1.0)$PLOT
--R
--R
--R (2) PLOT(x = (- 1.)..1.    y = (- 0.8414709848078965)..0.8414709848078965)
--R      [- 1., - 0.8414709848078965]
--R      [- 0.9583333333333337, - 0.81823456433427133]
--R      [- 0.91666666666666674, - 0.79357780324894212]
--R      [- 0.87500000000000011, - 0.76754350223602708]
--R      [- 0.83333333333333348, - 0.74017685319603721]
--R      [- 0.79166666666666685, - 0.7115253607990657]
--R      [- 0.75000000000000022, - 0.68163876002333434]
--R      [- 0.70833333333333359, - 0.65056892982223602]
--R      [- 0.66666666666666696, - 0.61836980306973721]
--R      [- 0.62500000000000033, - 0.58509727294046243]
--R      [- 0.5833333333333337, - 0.55080909588697013]
```

```

--R      [- 0.54166666666666707,- 0.51556479138264011]
--R      [- 0.50000000000000044,- 0.47942553860420339]
--R      [- 0.45833333333333376,- 0.44245407023325911]
--R      [- 0.41666666666666707,- 0.40471456356112506]
--R      [- 0.37500000000000039,- 0.3662725290860479]
--R      [- 0.33333333333333337,- 0.3271946967961526]
--R      [- 0.29166666666666702,- 0.28754890033552849]
--R      [- 0.25000000000000033,- 0.24740395925452324]
--R      [- 0.20833333333333368,- 0.20682955954864138]
--R      [- 0.16666666666666702,- 0.16589613269341538]
--R      [- 0.12500000000000036,- 0.12467473338522805]
--R      [- 8.3333333333333703E-2,- 8.3236916200310623E-2]
--R      [- 4.16666666666667039E-2,- 4.1654611386019461E-2]
--R      [- 3.7470027081099033E-16,- 3.7470027081099033E-16]
--R      [4.166666666666629E-2,4.1654611386018711E-2]
--R      [8.3333333333332954E-2,8.3236916200309874E-2]
--R      [0.12499999999999961,0.1246747333852273]
--R      [0.16666666666666627,0.16589613269341463]
--R      [0.20833333333333293,0.20682955954864066]
--R      [0.24999999999999958,0.24740395925452252]
--R      [0.29166666666666624,0.28754890033552777]
--R      [0.33333333333333293,0.32719469679615187]
--R      [0.37499999999999961,0.36627252908604718]
--R      [0.41666666666666663,0.4047145635611244]
--R      [0.45833333333333298,0.44245407023325839]
--R      [0.49999999999999967,0.47942553860420273]
--R      [0.54166666666666663,0.51556479138263944]
--R      [0.58333333333333293,0.55080909588696947]
--R      [0.62499999999999956,0.58509727294046177]
--R      [0.66666666666666619,0.61836980306973666]
--R      [0.70833333333333282,0.65056892982223535]
--R      [0.74999999999999944,0.68163876002333379]
--R      [0.79166666666666607,0.71152536079906514]
--R      [0.8333333333333327,0.74017685319603665]
--R      [0.87499999999999933,0.76754350223602663]
--R      [0.91666666666666596,0.79357780324894167]
--R      [0.95833333333333259,0.81823456433427078]
--R      [1.,0.8414709848078965]
--R
--R
--E 2
)spool
)lisp (bye)

```

Type: Plot

— Plot.help —

```

=====
Plot examples

```



=====

The Plot (PLOT) domain supports plotting of functions defined over a real number system. Plot is limited to 2 dimensional plots.

The function plot: (F -> F,R) -> % plots the function f(x) on the interval a..b. So we need to define a function that maps from DoubleFloat to DoubleFloat:

```
fp:=(t:DFLOAT):DFLOAT +-> sin(t)
```

and then feed it to the plot function with a Segment DoubleFloat

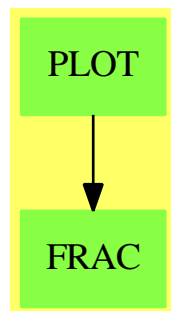
```
plot(fp,-1.0..1.0)$PLOT
```

See Also:

o )show Plot

—————

### 17.21.1 Plot (PLOT)



#### Exports:

|              |                     |                  |              |              |
|--------------|---------------------|------------------|--------------|--------------|
| adaptive?    | coerce              | debug            | listBranches | maxPoints    |
| minPoints    | numFunEvals         | parametric?      | plot         | plotPolar    |
| pointPlot    | refine              | screenResolution | setAdaptive  | setMaxPoints |
| setMinPoints | setScreenResolution | tRange           | xRange       | yRange       |
| zoom         |                     |                  |              |              |

— domain PLOT Plot —

)abbrev domain PLOT Plot

++ Author: Michael Monagan (revised by Clifton J. Williamson)

++ Date Created: Jan 1988

```

++ Date Last Updated: 30 Nov 1990 by Jonathan Steinbach
++ Basic Operations: plot, pointPlot, plotPolar, parametric?, zoom, refine,
++ tRange, minPoints, setMinPoints, maxPoints, screenResolution, adaptive?,
++ setAdaptive, numFunEvals, debug
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords: plot, function, parametric
++ References:
++ Description:
++ The Plot domain supports plotting of functions defined over a
++ real number system. A real number system is a model for the real
++ numbers and as such may be an approximation. For example
++ floating point numbers and infinite continued fractions.
++ The facilities at this point are limited to 2-dimensional plots
++ or either a single function or a parametric function.

```

Plot(): Exports == Implementation where

```

B ==> Boolean
F ==> DoubleFloat
I ==> Integer
L ==> List
N ==> NonNegativeInteger
OUT ==> OutputForm
P ==> Point F
RN ==> Fraction Integer
S ==> String
SEG ==> Segment
R ==> Segment F
C ==> Record(source: F -> P, ranges: L R, knots: L F, points: L P)

```

Exports ==> PlottablePlaneCurveCategory with

--% function plots

```

plot: (F -> F,R) -> %
++ plot(f,a..b) plots the function \spad{f(x)}
++ on the interval \spad{[a,b]}.
++
++X fp:=(t:DFLOAT):DFLOAT +-> sin(t)
++X plot(fp,-1.0..1.0)$PLOT

plot: (F -> F,R,R) -> %
++ plot(f,a..b,c..d) plots the function \spad{f(x)} on the interval
++ \spad{[a,b]}; y-range of \spad{[c,d]} is noted in Plot object.

```

--% multiple function plots

```

plot: (L(F -> F),R) -> %
++ plot([f1,...,fm],a..b) plots the functions \spad{y = f1(x)},...,

```

```

++ \spad{y = fm(x)} on the interval \spad{a..b}.
plot: (L(F -> F),R,R) -> %
++ plot([f1,...,fm],a..b,c..d) plots the functions \spad{y = f1(x)},...,
++ \spad{y = fm(x)} on the interval \spad{a..b}; y-range of \spad{[c,d]} is
++ noted in Plot object.

--% parametric plots

plot: (F -> F,F -> F,R) -> %
++ plot(f,g,a..b) plots the parametric curve \spad{x = f(t)},
++ \spad{y = g(t)} as t ranges over the interval \spad{[a,b]}.
plot: (F -> F,F -> F,R,R) -> %
++ plot(f,g,a..b,c..d,e..f) plots the parametric curve \spad{x = f(t)},
++ \spad{y = g(t)} as t ranges over the interval \spad{[a,b]}; x-range
++ of \spad{[c,d]} and y-range of \spad{[e,f]} are noted in Plot object.

--% parametric plots

pointPlot: (F -> P,R) -> %
++ pointPlot(t +-> (f(t),g(t)),a..b) plots the parametric curve
++ \spad{x = f(t)}, \spad{y = g(t)} as t ranges over the interval
++ \spad{[a,b]}.
pointPlot: (F -> P,R,R) -> %
++ pointPlot(t +-> (f(t),g(t)),a..b,c..d,e..f) plots the parametric
++ curve \spad{x = f(t)}, \spad{y = g(t)} as t ranges over the interval
++ \spad{[a,b]}; x-range of \spad{[c,d]} and y-range of \spad{[e,f]}
++ are noted in Plot object.

--% polar plots

plotPolar: (F -> F,R) -> %
++ plotPolar(f,a..b) plots the polar curve \spad{r = f(theta)} as
++ theta ranges over the interval \spad{[a,b]}; this is the same as
++ the parametric curve \spad{x = f(t)*cos(t)}, \spad{y = f(t)*sin(t)}.

plotPolar: (F -> F) -> %
++ plotPolar(f) plots the polar curve \spad{r = f(theta)} as theta
++ ranges over the interval \spad{[0,2*pi]}; this is the same as
++ the parametric curve \spad{x = f(t)*cos(t)}, \spad{y = f(t)*sin(t)}.

plot: (% ,R) -> %          -- change the range
++ plot(x,r) is not documented
parametric?: % -> B
++ parametric? determines whether it is a parametric plot?

zoom: (% ,R) -> %
++ zoom(x,r) is not documented
zoom: (% ,R,R) -> %
++ zoom(x,r,s) is not documented
refine: (% ,R) -> %

```

```

    ++ refine(x,r) is not documented
refine: % -> %
    ++ refine(p) performs a refinement on the plot p

tRange: % -> R
    ++ tRange(p) returns the range of the parameter in a parametric plot p

minPoints: () -> I
    ++ minPoints() returns the minimum number of points in a plot
setMinPoints: I -> I
    ++ setMinPoints(i) sets the minimum number of points in a plot to i
maxPoints: () -> I
    ++ maxPoints() returns the maximum number of points in a plot
setMaxPoints: I -> I
    ++ setMaxPoints(i) sets the maximum number of points in a plot to i
screenResolution: () -> I
    ++ screenResolution() returns the screen resolution
setScreenResolution: I -> I
    ++ setScreenResolution(i) sets the screen resolution to i
adaptive?: () -> B
    ++ adaptive?() determines whether plotting be done adaptively
setAdaptive: B -> B
    ++ setAdaptive(true) turns adaptive plotting on
    ++ \spad{setAdaptive(false)} turns adaptive plotting off
numFunEvals: () -> I
    ++ numFunEvals() returns the number of points computed
debug: B -> B
    ++ debug(true) turns debug mode on
    ++ \spad{debug(false)} turns debug mode off

Implementation ==> add
    import PointPackage(DoubleFloat)

--% local functions

checkRange      : R -> R
    -- checks that left-hand endpoint is less than right-hand endpoint
intersect       : (R,R) -> R
    -- intersection of two intervals
union           : (R,R) -> R
    -- union of two intervals
join            : (L C,I) -> R
parametricRange: % -> R
select          : (L P,P -> F,(F,F) -> F) -> F
rangeRefine     : (C,R) -> C
adaptivePlot    : (C,R,R,R,I) -> C
basicPlot       : (F -> P,R) -> C
basicRefine     : (C,R) -> C
pt              : (F,F) -> P
Fnan?           : F -> Boolean

```

```

Pnan?          : P -> Boolean

--% representation

Rep := Record( parametric: B, _
               display: L R, _
               bounds: L R, _
               axisLabels: L S, _
               functions: L C )

--% global constants

ADAPTIVE: B := true
MINPOINTS: I := 49
MAXPOINTS: I := 1000
NUMFUNVALS: I := 0
SCREENRES: I := 500
ANGLEBOUND: F := cos inv (4::F)
DEBUG: B := false

Fnan?(x) == x ~= x
Pnan?(x) == any?(Fnan?,x)

--% graphics output

listBranches plot ==
  outList : L L P := nil()
  for curve in plot.functions repeat
    -- curve is C
    newl:L P:=nil()
    for p in curve.points repeat
      if not Pnan? p then newl:=cons(p,newl)
      else if not empty? newl then
        outList := concat(newl:=reverse! newl,outList)
        newl:=nil()
      if not empty? newl then outList := concat(newl:=reverse! newl,outList)
--    print(outList::OutputForm)
  outList

checkRange r == (lo r > hi r => error "ranges cannot be negative"; r)
intersect(s,t) == checkRange (max(lo s,lo t) .. min(hi s,hi t))
union(s,t) == min(lo s,lo t) .. max(hi s,hi t)
join(l,i) ==
  rr := first l
  u : R :=
    i = 0 => first(rr.ranges)
    i = 1 => second(rr.ranges)
    third(rr.ranges)
  for r in rest l repeat
    i = 0 => u := union(u,first(r.ranges))

```

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    i = 1 => u := union(u,second(r.ranges))
    u := union(u,third(r.ranges))
  u
parametricRange r == first(r.bounds)

minPoints() == MINPOINTS
setMinPoints n ==
  if n < 3 then error "three points minimum required"
  if MAXPOINTS < n then MAXPOINTS := n
  MINPOINTS := n
maxPoints() == MAXPOINTS
setMaxPoints n ==
  if n < 3 then error "three points minimum required"
  if MINPOINTS > n then MINPOINTS := n
  MAXPOINTS := n
screenResolution() == SCREENRES
setScreenResolution n ==
  if n < 2 then error "buy a new terminal"
  SCREENRES := n
adaptive?() == ADAPTIVE
setAdaptive b == ADAPTIVE := b
parametric? p == p.parametric

numFunEvals() == NUMFUN EVALS
debug b == DEBUG := b

xRange plot == second plot.bounds
yRange plot == third plot.bounds
tRange plot == first plot.bounds

select(l,f,g) ==
  m := f first l
  if Fnan? m then m := 0
  for p in rest l repeat
    n := m
    m := g(m, f p)
    if Fnan? m then m := n
  m

rangeRefine(curve,nRange) ==
  checkRange nRange; l := lo nRange; h := hi nRange
  t := curve.knots; p := curve.points; f := curve.source
  while not null t and first t < l repeat
    (t := rest t; p := rest p)
  c: L F := nil(); q: L P := nil()
  while not null t and (first t) <= h repeat
    c := concat(first t,c); q := concat(first p,q)
    t := rest t; p := rest p
  if null c then return basicPlot(f,nRange)
  if first c < h then

```

```

c := concat(h,c)
q := concat(f h,q)
NUMFUNEVALS := NUMFUNEVALS + 1
t := c := reverse_! c; p := q := reverse_! q
s := (h-1)/(minPoints()::F-1)
if (first t) ^= 1 then
  t := c := concat(l,c)
  p := q := concat(f l,p)
  NUMFUNEVALS := NUMFUNEVALS + 1
while not null rest t repeat
  n := wholePart((second(t) - first(t))/s)
  d := (second(t) - first(t))/((n+1)::F)
  for i in 1..n repeat
    t.rest := concat(first(t) + d,rest t)
    p.rest := concat(f second t,rest p)
    NUMFUNEVALS := NUMFUNEVALS + 1
    t := rest t; p := rest p
  t := rest t
  p := rest p
xRange := select(q,xCoord,min) .. select(q,xCoord,max)
yRange := select(q,yCoord,min) .. select(q,yCoord,max)
[ f, [nRange,xRange,yRange], c, q]

adaptivePlot(curve,tRange,xRange,yRange,pixelfraction) ==
xDiff := hi xRange - lo xRange
yDiff := hi yRange - lo yRange
xDiff = 0 or yDiff = 0 => curve
l := lo tRange; h := hi tRange
(tDiff := h-l) = 0 => curve
--   if (EQL(yDiff, _$NaNvalue$Lisp)$Lisp) then yDiff := 1::F
t := curve.knots
#t < 3 => curve
p := curve.points; f := curve.source
minLength:F := 4::F/500::F
maxLength:F := 1::F/6::F
tLimit := tDiff/(pixelfraction*500)::F
while not null t and first t < 1 repeat (t := rest t; p := rest p)
#t < 3 => curve
headert := t; headerp := p

-- jitter the input points
--   while not null rest rest t repeat
--     t0 := second(t); t1 := third(t)
--     jitter := (random()$I) :: F
--     jitter := sin (jitter)
--     val := t0 + jitter * (t1-t0)/10::F
--     t.2 := val; p.2 := f val
--     t := rest t; p := rest p
--   t := headert; p := headerp

```

```

st := t; sp := p
todot : L L F := nil()
todop : L L P := nil()
while not null rest rest st repeat
  todot := concat_!(todot, st)
  todop := concat_!(todop, sp)
  st := rest st; sp := rest sp
st := headert; sp := headerp
todo1 := todot; todo2 := todop
n : I := 0
while not null todo1 repeat
  st := first(todo1)
  t0 := first(st); t1 := second(st); t2 := third(st)
  if t2 > h then leave
  t2 - t0 < tLimit =>
    todo1 := rest todo1
    todo2 := rest todo2
    if not null todo1 then (t := first(todo1); p := first(todo2))
  sp := first(todo2)
  x0 := xCoord first(sp); y0 := yCoord first(sp)
  x1 := xCoord second(sp); y1 := yCoord second(sp)
  x2 := xCoord third(sp); y2 := yCoord third(sp)
  a1 := (x1-x0)/xDiff; b1 := (y1-y0)/yDiff
  a2 := (x2-x1)/xDiff; b2 := (y2-y1)/yDiff
  s1 := sqrt(a1**2+b1**2); s2 := sqrt(a2**2+b2**2)
  dp := a1*a2+b1*b2

  s1 < maxLength and s2 < maxLength and _
  (s1 = 0::F or s2 = 0::F or
   s1 < minLength and s2 < minLength or _
   dp/s1/s2 > ANGLEBOUND) =>
    todo1 := rest todo1
    todo2 := rest todo2
    if not null todo1 then (t := first(todo1); p := first(todo2))
if n > MAXPOINTS then leave else n := n + 1
st := rest t
if not null rest rest st then
  tm := (t0+t1)/2::F
  tj := tm
  t.rest := concat(tj, rest t)
  p.rest := concat(f tj, rest p)
  todo1 := concat_!(todo1, t)
  todo2 := concat_!(todo2, p)
  t := rest t; p := rest p
  todo1 := concat_!(todo1, t)
  todo2 := concat_!(todo2, p)
  t := rest t; p := rest p
  todo1 := rest todo1; todo2 := rest todo2

  tm := (t1+t2)/2::F

```



```

    tj := tm
    t.rest := concat(tj, rest t)
    p.rest := concat(f tj, rest p)
    todo1 := concat_!(todo1, t)
    todo2 := concat_!(todo2, p)
    t := rest t; p := rest p
    todo1 := concat_!(todo1, t)
    todo2 := concat_!(todo2, p)
    todo1 := rest todo1
    todo2 := rest todo2
    if not null todo1 then (t := first(todo1); p := first(todo2))
  else
    tm := (t0+t1)/2::F
    tj := tm
    t.rest := concat(tj, rest t)
    p.rest := concat(f tj, rest p)
    todo1 := concat_!(todo1, t)
    todo2 := concat_!(todo2, p)
    t := rest t; p := rest p
    todo1 := concat_!(todo1, t)
    todo2 := concat_!(todo2, p)
    t := rest t; p := rest p

    tm := (t1+t2)/2::F
    tj := tm
    t.rest := concat(tj, rest t)
    p.rest := concat(f tj, rest p)
    todo1 := concat_!(todo1, t)
    todo2 := concat_!(todo2, p)
    todo1 := rest todo1
    todo2 := rest todo2
    if not null todo1 then (t := first(todo1); p := first(todo2))
  n > 0 =>
    NUMFUNEVALS := NUMFUNEVALS + n
    t := curve.knots; p := curve.points
    xRange := select(p,xCoord,min) .. select(p,xCoord,max)
    yRange := select(p,yCoord,min) .. select(p,yCoord,max)
    [ curve.source, [tRange,xRange,yRange], t, p ]
  curve

basicPlot(f,tRange) ==
  checkRange tRange
  l := lo tRange
  h := hi tRange
  t : L F := list l
  p : L P := list f l
  s := (h-l)/(minPoints()-1)::F
  for i in 2..minPoints()-1 repeat
    l := l+s
    t := concat(l,t)

```

```

    p := concat(f l,p)
    t := reverse_! concat(h,t)
    p := reverse_! concat(f h,p)
--    print(p::OutputForm)
    xRange : R := select(p,xCoord,min) .. select(p,xCoord,max)
    yRange : R := select(p,yCoord,min) .. select(p,yCoord,max)
    [ f, [tRange,xRange,yRange], t, p ]

zoom(p,xRange) ==
    [p.parametric, [xRange,third(p.display)], p.bounds, _
    p.axisLabels, p.functions]
zoom(p,xRange,yRange) ==
    [p.parametric, [xRange,yRange], p.bounds, _
    p.axisLabels, p.functions]

basicRefine(curve,nRange) ==
    tRange:R := first curve.ranges
    -- curve := copy$C curve -- Yet another compiler bug
    curve: C := [curve.source,curve.ranges,curve.knots,curve.points]
    t := curve.knots := copy curve.knots
    p := curve.points := copy curve.points
    l := lo nRange; h := hi nRange
    f := curve.source
    while not null rest t and first t < h repeat
        second(t) < 1 => (t := rest t; p := rest p)
        -- insert new point between t.0 and t.1
        tm : F := (first(t) + second(t))/2::F
--        if DEBUG then output$0 (tm::E)
        pm := f tm
        NUMFUNEVALS := NUMFUNEVALS + 1
        t.rest := concat(tm,rest t); t := rest rest t
        p.rest := concat(pm,rest p); p := rest rest p
    t := curve.knots; p := curve.points
    xRange := select(p,xCoord,min) .. select(p,xCoord,max)
    yRange := select(p,yCoord,min) .. select(p,yCoord,max)
    [ curve.source, [tRange,xRange,yRange], t, p ]

refine p == refine(p,parametricRange p)
refine(p,nRange) ==
    NUMFUNEVALS := 0
    tRange := parametricRange p
    nRange := intersect(tRange,nRange)
    curves: L C := [basicRefine(c,nRange) for c in p.functions]
    xRange := join(curves,1); yRange := join(curves,2)
    if adaptive? then
        tlimit := if parametric? p then 8 else 1
        curves := [adaptivePlot(c,nRange,xRange,yRange, _
            tlimit) for c in curves]
    xRange := join(curves,1); yRange := join(curves,2)
--    print(NUMFUNEVALS::OUT)

```

```

[p.parametric, p.display, [tRange,xRange,yRange], _
 p.axisLabels, curves ]

plot(p:%,tRange:R) ==
  -- re plot p on a new range making use of the points already
  -- computed if possible
  NUMFUNEVALS := 0
  curves: L C := [rangeRefine(c,tRange) for c in p.functions]
  xRange := join(curves,1); yRange := join(curves,2)
  if adaptive? then
    tlimit := if parametric? p then 8 else 1
    curves := [adaptivePlot(c,tRange,xRange,yRange,tlimit) for c in curves]
    xRange := join(curves,1); yRange := join(curves,2)
  -- print(NUMFUNEVALS::OUT)
  [ p.parametric, [xRange,yRange], [tRange,xRange,yRange],
    p.axisLabels, curves ]

pt(xx,yy) == point(1 : L F := [xx,yy])

myTrap: (F-> F, F) -> F
myTrap(ff:F-> F, f:F):F ==
  s := trapNumericErrors(ff(f))$Lisp :: Union(F, "failed")
  s case "failed" => _$NaNvalue$Lisp
  r:F:=s::F
  r > max()$F or r < min()$F => _$NaNvalue$Lisp
  r

plot(f:F -> F,xRange:R) ==
  p := basicPlot((u1:F):P +-> pt(u1,myTrap(f,u1)),xRange)
  r := p.ranges
  NUMFUNEVALS := minPoints()
  if adaptive? then
    p := adaptivePlot(p,first r,second r,third r,1)
    r := p.ranges
  [ false, rest r, r, nil(), [ p ] ]

plot(f:F -> F,xRange:R,yRange:R) ==
  p := plot(f,xRange)
  p.display := [xRange,checkRange yRange]
  p

plot(f:F -> F,g:F -> F,tRange:R) ==
  p := basicPlot((z1:F):P +-> pt(myTrap(f,z1),myTrap(g,z1)),tRange)
  r := p.ranges
  NUMFUNEVALS := minPoints()
  if adaptive? then
    p := adaptivePlot(p,first r,second r,third r,8)
    r := p.ranges
  [ true, rest r, r, nil(), [ p ] ]

```

```

plot(f:F -> F,g:F -> F,tRange:R,xRange:R,yRange:R) ==
  p := plot(f,g,tRange)
  p.display := [checkRange xRange,checkRange yRange]
  p

pointPlot(f:F -> P,tRange:R) ==
  p := basicPlot(f,tRange)
  r := p.ranges
  NUMFUNEVALS := minPoints()
  if adaptive? then
    p := adaptivePlot(p,first r,second r,third r,8)
    r := p.ranges
  [ true, rest r, r, nil(), [ p ] ]

pointPlot(f:F -> P,tRange:R,xRange:R,yRange:R) ==
  p := pointPlot(f,tRange)
  p.display := [checkRange xRange,checkRange yRange]
  p

plot(l:L(F -> F),xRange:R) ==
  if null l then error "empty list of functions"
  t: L C :=
    [ basicPlot((z1:F):P +-> pt(z1,myTrap(f,z1)),xRange) for f in l ]
  yRange := join(t,2)
  NUMFUNEVALS := # l * minPoints()
  if adaptive? then
    t := [adaptivePlot(p,xRange,xRange,yRange,1) _
          for f in l for p in t]
  yRange := join(t,2)
--  print(NUMFUNEVALS::OUT)
  [false, [xRange,yRange], [xRange,xRange,yRange], nil(), t ]

plot(l:L(F -> F),xRange:R,yRange:R) ==
  p := plot(l,xRange)
  p.display := [xRange,checkRange yRange]
  p

plotPolar(f,thetaRange) ==
  plot((u1:F):F +-> f(u1) * cos(u1),
       (v1:F):F +-> f(v1) * sin(v1),thetaRange)

plotPolar f == plotPolar(f,segment(0,2*pi()))

--% terminal output

coerce r ==
  spaces: OUT := coerce "  "
  xSymbol := "x = " :: OUT
  ySymbol := "y = " :: OUT
  tSymbol := "t = " :: OUT

```

```

plotSymbol := "PLOT" :: OUT
tRange := (parametricRange r) :: OUT
f : L OUT := nil()
for curve in r.functions repeat
  xRange := second(curve.ranges) :: OUT
  yRange := third(curve.ranges) :: OUT
  l : L OUT := [xSymbol,xRange,spaces,ySymbol,yRange]
  if parametric? r then
    l := concat_!([tSymbol,tRange,spaces],l)
  h : OUT := hconcat l
  l := [p::OUT for p in curve.points]
  f := concat(vconcat concat(h,l),f)
prefix("PLOT" :: OUT, reverse! f)

```

---

— PLOT.dotabb —

```

"PLOT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PLOT"]
"FRAC" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FRAC"]
"PLOT" -> "FRAC"

```

---

## 17.22 domain PLOT3D Plot3D

— Plot3D.input —

```

)set break resume
)sys rm -f Plot3D.output
)spool Plot3D.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Plot3D
--R Plot3D is a domain constructor
--R Abbreviation for Plot3D is PLOT3D
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PLOT3D
--R
--R----- Operations -----
--R adaptive3D? : () -> Boolean          coerce : % -> OutputForm

```

```

--R debug3D : Boolean -> Boolean          maxPoints3D : () -> Integer
--R minPoints3D : () -> Integer           numFunEvals3D : () -> Integer
--R refine : % -> %                       screenResolution3D : () -> Integer
--R setAdaptive3D : Boolean -> Boolean     tRange : % -> Segment DoubleFloat
--R xRange : % -> Segment DoubleFloat     yRange : % -> Segment DoubleFloat
--R zRange : % -> Segment DoubleFloat
--R listBranches : % -> List List Point DoubleFloat
--R plot : (% ,Segment DoubleFloat) -> %
--R plot : ((DoubleFloat -> DoubleFloat),(DoubleFloat -> DoubleFloat),(DoubleFloat -> DoubleFloat),(DoubleFloat -> DoubleFloat)) -> %
--R plot : ((DoubleFloat -> DoubleFloat),(DoubleFloat -> DoubleFloat),(DoubleFloat -> DoubleFloat),(DoubleFloat -> DoubleFloat)) -> %
--R pointPlot : ((DoubleFloat -> Point DoubleFloat),Segment DoubleFloat,Segment DoubleFloat,Segment DoubleFloat) -> %
--R pointPlot : ((DoubleFloat -> Point DoubleFloat),Segment DoubleFloat) -> %
--R refine : (% ,Segment DoubleFloat) -> %
--R setMaxPoints3D : Integer -> Integer
--R setMinPoints3D : Integer -> Integer
--R setScreenResolution3D : Integer -> Integer
--R tValues : % -> List List DoubleFloat
--R zoom : (% ,Segment DoubleFloat,Segment DoubleFloat,Segment DoubleFloat) -> %
--R
--E 1

)spool
)lisp (bye)

```

---

— Plot3D.help —

```

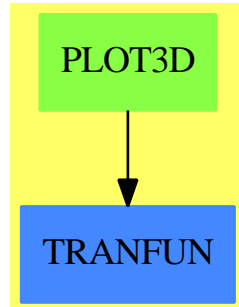
=====
Plot3D examples
=====

```

See Also:

- o )show Plot3D

## 17.22.1 Plot3D (PLOT3D)

**Exports:**

|                    |               |                |                |                       |
|--------------------|---------------|----------------|----------------|-----------------------|
| adaptive3D?        | coerce        | debug3D        | listBranches   | maxPoints3D           |
| minPoints3D        | numFunEvals3D | plot           | pointPlot      | refine                |
| screenResolution3D | setAdaptive3D | setMaxPoints3D | setMinPoints3D | setScreenResolution3D |
| tRange             | tValues       | xRange         | yRange         | zRange                |
| zoom               |               |                |                |                       |

— domain PLOT3D Plot3D —

```

)abbrev domain PLOT3D Plot3D
++ Author: Clifton J. Williamson based on code by Michael Monagan
++ Date Created: Jan 1989
++ Date Last Updated: 22 November 1990 (Jon Steinbach)
++ Basic Operations: pointPlot, plot, zoom, refine, tRange, tValues,
++ minPoints3D, setMinPoints3D, maxPoints3D, setMaxPoints3D,
++ screenResolution3D, setScreenResolution3D, adaptive3D?, setAdaptive3D,
++ numFunEvals3D, debug3D
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords: plot, parametric
++ References:
++ Description:
++ Plot3D supports parametric plots defined over a real
++ number system. A real number system is a model for the real
++ numbers and as such may be an approximation. For example,
++ floating point numbers and infinite continued fractions are
++ real number systems. The facilities at this point are limited
++ to 3-dimensional parametric plots.

```

Plot3D(): Exports == Implementation where

```

B ==> Boolean
F ==> DoubleFloat
I ==> Integer
L ==> List

```

```

N ==> NonNegativeInteger
OUT ==> OutputForm
P ==> Point F
S ==> String
R ==> Segment F
O ==> OutputPackage
C ==> Record(source: F -> P, ranges: L R, knots: L F, points: L P)

Exports ==> PlottableSpaceCurveCategory with

pointPlot: (F -> P,R) -> %
  ++ pointPlot(f,g,h,a..b) plots {/emx = f(t), y = g(t), z = h(t)} as
  ++ t ranges over {/em[a,b]}.
pointPlot: (F -> P,R,R,R,R) -> %
  ++ pointPlot(f,x,y,z,w) is not documented
plot: (F -> F,F -> F,F -> F,F -> F,R) -> %
  ++ plot(f,g,h,a..b) plots {/emx = f(t), y = g(t), z = h(t)} as
  ++ t ranges over {/em[a,b]}.
plot: (F -> F,F -> F,F -> F,F -> F,R,R,R,R) -> %
  ++ plot(f1,f2,f3,f4,x,y,z,w) is not documented

plot: (% ,R) -> % -- change the range
  ++ plot(x,r) is not documented
zoom: (% ,R,R,R) -> %
  ++ zoom(x,r,s,t) is not documented
refine: (% ,R) -> %
  ++ refine(x,r) is not documented
refine: % -> %
  ++ refine(x) is not documented

tRange: % -> R
  ++ tRange(p) returns the range of the parameter in a parametric plot p.
tValues: % -> L L F
  ++ tValues(p) returns a list of lists of the values of the parameter for
  ++ which a point is computed, one list for each curve in the plot p.

minPoints3D: () -> I
  ++ minPoints3D() returns the minimum number of points in a plot.
setMinPoints3D: I -> I
  ++ setMinPoints3D(i) sets the minimum number of points in a plot to i.
maxPoints3D: () -> I
  ++ maxPoints3D() returns the maximum number of points in a plot.
setMaxPoints3D: I -> I
  ++ setMaxPoints3D(i) sets the maximum number of points in a plot to i.
screenResolution3D: () -> I
  ++ screenResolution3D() returns the screen resolution for a 3d graph.
setScreenResolution3D: I -> I
  ++ setScreenResolution3D(i) sets the screen resolution for a 3d graph to i.
adaptive3D?: () -> B
  ++ adaptive3D?() determines whether plotting be done adaptively.

```



```

setAdaptive3D: B -> B
  ++ setAdaptive3D(true) turns adaptive plotting on;
  ++ setAdaptive3D(false) turns adaptive plotting off.
numFunEvals3D: () -> I
  ++ numFunEvals3D() returns the number of points computed.
debug3D: B -> B
  ++ debug3D(true) turns debug mode on;
  ++ debug3D(false) turns debug mode off.

Implementation ==> add
  import PointPackage(F)

--% local functions

fourth      : L R -> R
checkRange  : R -> R
  -- checks that left-hand endpoint is less than right-hand endpoint
intersect   : (R,R) -> R
  -- intersection of two intervals
union       : (R,R) -> R
  -- union of two intervals
join        : (L C,I) -> R
parametricRange: % -> R
-- setColor  : (P,F) -> F
select      : (L P,P -> F,(F,F) -> F) -> F
-- normalizeColor : (P,F,F) -> F
rangeRefine : (C,R) -> C
adaptivePlot : (C,R,R,R,R,I,I) -> C
basicPlot    : (F -> P,R) -> C
basicRefine  : (C,R) -> C
point        : (F,F,F,F) -> P

--% representation

Rep := Record( display: L R, _
               bounds: L R, _
               screenres: I, _
               axisLabels: L S, _
               functions: L C )

--% global constants

ADAPTIVE      : B := true
MINPOINTS     : I := 49
MAXPOINTS     : I := 1000
NUMFUNVALS    : I := 0
SCREENRES     : I := 500
ANGLEBOUND    : F := cos inv (4::F)
DEBUG         : B := false

```

```

point(xx,yy,zz,col) == point(l : L F := [xx,yy,zz,col])

fourth list == first rest rest rest list

checkRange r == (lo r > hi r => error "ranges cannot be negative"; r)
intersect(s,t) == checkRange (max(lo s,lo t) .. min(hi s,hi t))
union(s:R,t:R) == min(lo s,lo t) .. max(hi s,hi t)
join(l,i) ==
  rr := first l
  u : R :=
    i = 0 => first(rr.ranges)
    i = 1 => second(rr.ranges)
    i = 2 => third(rr.ranges)
    fourth(rr.ranges)
  for r in rest l repeat
    i = 0 => union(u,first(r.ranges))
    i = 1 => union(u,second(r.ranges))
    i = 2 => union(u,third(r.ranges))
    union(u,fourth(r.ranges))
  u
parametricRange r == first(r.bounds)

minPoints3D() == MINPOINTS
setMinPoints3D n ==
  if n < 3 then error "three points minimum required"
  if MAXPOINTS < n then MAXPOINTS := n
  MINPOINTS := n
maxPoints3D() == MAXPOINTS
setMaxPoints3D n ==
  if n < 3 then error "three points minimum required"
  if MINPOINTS > n then MINPOINTS := n
  MAXPOINTS := n
screenResolution3D() == SCREENRES
setScreenResolution3D n ==
  if n < 2 then error "buy a new terminal"
  SCREENRES := n
adaptive3D?() == ADAPTIVE
setAdaptive3D b == ADAPTIVE := b

numFunEvals3D() == NUMFUNEVALS
debug3D b == DEBUG := b

--      setColor(p,c) == p.colNum := c

xRange plot == second plot.bounds
yRange plot == third plot.bounds
zRange plot == fourth plot.bounds
tRange plot == first plot.bounds

tValues plot ==

```

```

outList : L L F := nil()
for curve in plot.functions repeat
  outList := concat(curve.knots,outList)
outList

select(l,f,g) ==
  m := f first l
  if (EQL(m, _$NaNvalue$Lisp)$Lisp) then m := 0
--   for p in rest l repeat m := g(m,fp)
  for p in rest l repeat
    fp : F := f p
    if (EQL(fp, _$NaNvalue$Lisp)$Lisp) then fp := 0
    m := g(m,fp)
  m

--   normalizeColor(p,lo,diff) ==
--   p.colNum := (p.colNum - lo)/diff

rangeRefine(curve,nRange) ==
  checkRange nRange; l := lo nRange; h := hi nRange
  t := curve.knots; p := curve.points; f := curve.source
  while not null t and first t < l repeat
    (t := rest t; p := rest p)
  c : L F := nil(); q : L P := nil()
  while not null t and first t <= h repeat
    c := concat(first t,c); q := concat(first p,q)
    t := rest t; p := rest p
  if null c then return basicPlot(f,nRange)
  if first c < h then
    c := concat(h,c); q := concat(f h,q)
    NUMFUNEVALS := NUMFUNEVALS + 1
  t := c := reverse_! c; p := q := reverse_! q
  s := (h-l)/(MINPOINTS::F-1)
  if (first t) ^= 1 then
    t := c := concat(l,c); p := q := concat(f l,p)
    NUMFUNEVALS := NUMFUNEVALS + 1
  while not null rest t repeat
    n := wholePart((second(t) - first(t))/s)
    d := (second(t) - first(t))/((n+1)::F)
    for i in 1..n repeat
      t.rest := concat(first(t) + d,rest t); t1 := second t
      p.rest := concat(f t1,rest p)
      NUMFUNEVALS := NUMFUNEVALS + 1
      t := rest t; p := rest p
    t := rest t
    p := rest p
  xRange := select(q,xCoord,min) .. select(q,xCoord,max)
  yRange := select(q,yCoord,min) .. select(q,yCoord,max)
  zRange := select(q,zCoord,min) .. select(q,zCoord,max)
--   colorLo := select(q,color,min); colorHi := select(q,color,max)

```

```

--      (diff := colorHi - colorLo) = 0 =>
--      error "all points are the same color"
--      map(normalizeColor(#1,colorLo,diff),q)$ListPackage1(P)
--      [f,[nRange,xRange,yRange,zRange],c,q]

adaptivePlot(curve,tRg,xRg,yRg,zRg,pixelfraction,resolution) ==
  xDiff := hi xRg - lo xRg
  yDiff := hi yRg - lo yRg
  zDiff := hi zRg - lo zRg
--      xDiff = 0 or yDiff = 0 or zDiff = 0 => curve--!! delete this?
  if xDiff = 0::F then xDiff := 1::F
  if yDiff = 0::F then yDiff := 1::F
  if zDiff = 0::F then zDiff := 1::F
  l := lo tRg; h := hi tRg
  (tDiff := h-l) = 0 => curve
  t := curve.knots
  #t < 3 => curve
  p := curve.points; f := curve.source
  minLength:F := 4::F/resolution::F
  maxLength := 1/4::F
  tLimit := tDiff/(pixelfraction*resolution)::F
  while not null t and first t < l repeat (t := rest t; p := rest p)
  #t < 3 => curve
  headert := t; headerp := p
  st := t; sp := p
  todot : L L F := nil()
  todop : L L P := nil()
  while not null rest rest st repeat
    todot := concat_!(todot, st)
    todop := concat_!(todop, sp)
    st := rest st; sp := rest sp
  st := headert; sp := headerp
  todo1 := todot; todo2 := todop
  n : I := 0

  while not null todo1 repeat
    st := first(todo1)
    t0 := first(st); t1 := second(st); t2 := third(st)
    if t2 > h then leave
    t2 - t0 < tLimit =>
      todo1 := rest todo1
      todo2 := rest todo2;
      if not null todo1 then (t := first(todo1); p := first(todo2))
    sp := first(todo2)
    x0 := xCoord first(sp); y0 := yCoord first(sp); z0 := zCoord first(sp)
    x1 := xCoord second(sp); y1 := yCoord second(sp); z1 := zCoord second(sp)
    x2 := xCoord third(sp); y2 := yCoord third(sp); z2 := zCoord third(sp)
    a1 := (x1-x0)/xDiff; b1 := (y1-y0)/yDiff; c1 := (z1-z0)/zDiff
    a2 := (x2-x1)/xDiff; b2 := (y2-y1)/yDiff; c2 := (z2-z1)/zDiff

```

```

s1 := sqrt(a1**2+b1**2+c1**2); s2 := sqrt(a2**2+b2**2+c2**2)
dp := a1*a2+b1*b2+c1*c2
s1 < maxLength and s2 < maxLength and _
  (s1 = 0 or s2 = 0 or
   s1 < minLength and s2 < minLength or _
   dp/s1/s2 > ANGLEBOUND) =>
  todo1 := rest todo1
  todo2 := rest todo2
  if not null todo1 then (t := first(todo1); p := first(todo2))
if n = MAXPOINTS then leave else n := n + 1
--if DEBUG then
  --r : L F := [minLength,maxLength,s1,s2,dp/s1/s2,ANGLEBOUND]
  --output(r::E)$0
st := rest t
if not null rest rest st then
  tm := (t0+t1)/2::F
  tj := tm
  t.rest := concat(tj,rest t)
  p.rest := concat(f tj, rest p)
  todo1 := concat_!(todo1, t)
  todo2 := concat_!(todo2, p)
  t := rest t; p := rest p
  todo1 := concat_!(todo1, t)
  todo2 := concat_!(todo2, p)
  t := rest t; p := rest p
  todo1 := rest todo1; todo2 := rest todo2

  tm := (t1+t2)/2::F
  tj := tm
  t.rest := concat(tj, rest t)
  p.rest := concat(f tj, rest p)
  todo1 := concat_!(todo1, t)
  todo2 := concat_!(todo2, p)
  t := rest t; p := rest p
  todo1 := concat_!(todo1, t)
  todo2 := concat_!(todo2, p)
  todo1 := rest todo1; todo2 := rest todo2
  if not null todo1 then (t := first(todo1); p := first(todo2))
else
  tm := (t0+t1)/2::F
  tj := tm
  t.rest := concat(tj,rest t)
  p.rest := concat(f tj, rest p)
  todo1 := concat_!(todo1, t)
  todo2 := concat_!(todo2, p)
  t := rest t; p := rest p
  todo1 := concat_!(todo1, t)
  todo2 := concat_!(todo2, p)
  t := rest t; p := rest p

```

```

    tm := (t1+t2)/2::F
    tj := tm
    t.rest := concat(tj, rest t)
    p.rest := concat(f tj, rest p)
    todo1 := concat_!(todo1, t)
    todo2 := concat_!(todo2, p)
    todo1 := rest todo1; todo2 := rest todo2
    if not null todo1 then (t := first(todo1); p := first(todo2))
if n > 0 then
    NUMFUNEVALS := NUMFUNEVALS + n
    t := curve.knots; p := curve.points
    xRg := select(p,xCoord,min) .. select(p,xCoord,max)
    yRg := select(p,yCoord,min) .. select(p,yCoord,max)
    zRg := select(p,zCoord,min) .. select(p,zCoord,max)
    [curve.source,[tRg,xRg,yRg,zRg],t,p]
else curve

basicPlot(f,tRange) ==
    checkRange tRange; l := lo tRange; h := hi tRange
    t : L F := list l; p : L P := list f l
    s := (h-l)/(MINPOINTS-1)::F
    for i in 2..MINPOINTS-1 repeat
        l := l+s; t := concat(l,t)
        p := concat(f l,p)
    t := reverse_! concat(h,t)
    p := reverse_! concat(f h,p)
    xRange : R := select(p,xCoord,min) .. select(p,xCoord,max)
    yRange : R := select(p,yCoord,min) .. select(p,yCoord,max)
    zRange : R := select(p,zCoord,min) .. select(p,zCoord,max)
    [f,[tRange,xRange,yRange,zRange],t,p]

zoom(p,xRange,yRange,zRange) ==
    [[xRange,yRange,zRange],p.bounds,
    p.screenres,p.axisLabels,p.functions]

basicRefine(curve,nRange) ==
    tRange:R := first curve.ranges
    -- curve := copy$C curve -- Yet another @#%~&* compiler bug
    curve: C := [curve.source,curve.ranges,curve.knots,curve.points]
    t := curve.knots := copy curve.knots
    p := curve.points := copy curve.points
    l := lo nRange; h := hi nRange
    f := curve.source
    while not null rest t and first(t) < h repeat
        second(t) < l => (t := rest t; p := rest p)
        -- insert new point between t.0 and t.1
        tm:F := (first(t) + second(t))/2::F
        -- if DEBUG then output$0 (tm::E)
        pm := f tm
        NUMFUNEVALS := NUMFUNEVALS + 1

```

```

    t.rest := concat(tm,rest t); t := rest rest t
    p.rest := concat(pm,rest p); p := rest rest p
    t := curve.knots; p := curve.points
    xRange := select(p,xCoord,min) .. select(p,xCoord,max)
    yRange := select(p,yCoord,min) .. select(p,yCoord,max)
    zRange := select(p,zCoord,min) .. select(p,zCoord,max)
    [curve.source,[tRange,xRange,yRange,zRange],t,p]

refine p == refine(p,parametricRange p)
refine(p,nRange) ==
    NUMFUNEVALS := 0
    tRange := parametricRange p
    nRange := intersect(tRange,nRange)
    curves: L C := [basicRefine(c,nRange) for c in p.functions]
    xRange := join(curves,1); yRange := join(curves,2)
    zRange := join(curves,3)
    scrres := p.screenres
    if adaptive3D? then
        tlimit := 8
        curves := [adaptivePlot(c,nRange,xRange,yRange,zRange, _
            tlimit,scrres := 2*scrres) for c in curves]
        xRange := join(curves,1); yRange := join(curves,2)
        zRange := join(curves,3)
    [p.display,[tRange,xRange,yRange,zRange], _
        scrres,p.axisLabels,curves]

plot(p:%,tRange:R) ==
    -- re plot p on a new range making use of the points already
    -- computed if possible
    NUMFUNEVALS := 0
    curves: L C := [rangeRefine(c,tRange) for c in p.functions]
    xRange := join(curves,1); yRange := join(curves,2)
    zRange := join(curves,3)
    if adaptive3D? then
        tlimit := 8
        curves := [adaptivePlot(c,tRange,xRange,yRange,zRange,tlimit, _
            p.screenres) for c in curves]
        xRange := join(curves,1); yRange := join(curves,2)
        zRange := join(curves,3)
    -- print(NUMFUNEVALS::OUT)
    [[xRange,yRange,zRange],[tRange,xRange,yRange,zRange],
        p.screenres,p.axisLabels,curves]

pointPlot(f:F -> P,tRange:R) ==
    p := basicPlot(f,tRange)
    r := p.ranges
    NUMFUNEVALS := MINPOINTS
    if adaptive3D? then
        p := adaptivePlot(p,first r,second r,third r,fourth r,8,SCREENRES)
    -- print(NUMFUNEVALS::OUT)

```

```

--      print(p::OUT)
      [ rest r, r, SCREENRES, nil(), [ p ] ]

pointPlot(f:F -> P,tRange:R,xRange:R,yRange:R,zRange:R) ==
  p := pointPlot(f,tRange)
  p.display:= [checkRange xRange,checkRange yRange,checkRange zRange]
  p

myTrap: (F-> F, F) -> F
myTrap(ff:F-> F, f:F):F ==
  s := trapNumericErrors(ff(f))$Lisp :: Union(F, "failed")
  if (s) case "failed" then
    r:F := _$NaNvalue$Lisp
  else
    r:F := s
  r

plot(f1:F -> F,f2:F -> F,f3:F -> F,col:F -> F,tRange:R) ==
  p := basicPlot(
    (z:F):P+>point(myTrap(f1,z),myTrap(f2,z),myTrap(f3,z),col(z)),tRange)
  r := p.ranges
  NUMFUNEVALS := MINPOINTS
  if adaptive3D? then
    p := adaptivePlot(p,first r,second r,third r,fourth r,8,SCREENRES)
--      print(NUMFUNEVALS::OUT)
      [ rest r, r, SCREENRES, nil(), [ p ] ]

plot(f1:F -> F,f2:F -> F,f3:F -> F,col:F -> F,_
      tRange:R,xRange:R,yRange:R,zRange:R) ==
  p := plot(f1,f2,f3,col,tRange)
  p.display:= [checkRange xRange,checkRange yRange,checkRange zRange]
  p

--% terminal output

coerce r ==
  spaces := " " :: OUT
  xSymbol := "x = " :: OUT; ySymbol := "y = " :: OUT
  zSymbol := "z = " :: OUT; tSymbol := "t = " :: OUT
  tRange := (parametricRange r) :: OUT
  f : L OUT := nil()
  for curve in r.functions repeat
    xRange := coerce curve.ranges.1
    yRange := coerce curve.ranges.2
    zRange := coerce curve.ranges.3
    l : L OUT := [xSymbol,xRange,spaces,ySymbol,yRange,_
      spaces,zSymbol,zRange]
    l := concat_!([tSymbol,tRange,spaces],l)
    h : OUT := hconcat l
    l := [p::OUT for p in curve.points]

```



```

      f := concat(vconcat concat(h,l),f)
      prefix("PLOT" :: OUT,reverse_! f)

-----% graphics output

listBranches plot ==
  outList : L L P := nil()
  for curve in plot.functions repeat
    outList := concat(curve.points,outList)
  outList

-----

— PLOT3D.dotabb —

"PLOT3D" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PLOT3D"]
"TRANFUN" [color="#4488FF",href="bookvol10.2.pdf#nameddest=TRANFUN"]
"PLOT3D" -> "TRANFUN"

-----

```

## 17.23 domain PBWLB PoincareBirkhoffWittLyndonBasis

```

— PoincareBirkhoffWittLyndonBasis.input —

)set break resume
)sys rm -f PoincareBirkhoffWittLyndonBasis.output
)spool PoincareBirkhoffWittLyndonBasis.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PoincareBirkhoffWittLyndonBasis
--R PoincareBirkhoffWittLyndonBasis VarSet: OrderedSet is a domain constructor
--R Abbreviation for PoincareBirkhoffWittLyndonBasis is PBWLB
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PBWLB
--R
--R----- Operations -----
--R ?<? : (%,%) -> Boolean          ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean          ?>? : (%,%) -> Boolean

```

```

--R ?>=? : (% , %) -> Boolean
--R coerce : VarSet -> %
--R coerce : % -> OutputForm
--R hash : % -> SingleInteger
--R length : % -> NonNegativeInteger
--R min : (% , %) -> %
--R retract : % -> LyndonWord VarSet
--R varList : % -> List VarSet
--R coerce : % -> OrderedFreeMonoid VarSet
--R listOfTerms : % -> List LyndonWord VarSet
--R retractIfCan : % -> Union(LyndonWord VarSet, "failed")
--R
--E 1

1 : () -> %
coerce : LyndonWord VarSet -> %
first : % -> LyndonWord VarSet
latex : % -> String
max : (% , %) -> %
rest : % -> %
retractable? : % -> Boolean
?~=? : (% , %) -> Boolean

)spool
)lisp (bye)

```

---

— PoincareBirkhoffWittLyndonBasis.help —

```

=====
PoincareBirkhoffWittLyndonBasis examples
=====

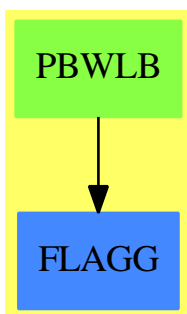
```

See Also:

- o )show PoincareBirkhoffWittLyndonBasis

---

### 17.23.1 PoincareBirkhoffWittLyndonBasis (PBWL B)



Exports:

|         |              |              |         |       |
|---------|--------------|--------------|---------|-------|
| 1       | coerce       | first        | hash    | latex |
| length  | listOfTerms  | max          | min     | rest  |
| retract | retractable? | retractIfCan | varList | ?~=?  |
| ?<?     | ?<=?         | ?=?          | ?>?     | ?>=?  |

— domain PBWLB PoincareBirkhoffWittLyndonBasis —

```

)abbrev domain PBWLB PoincareBirkhoffWittLyndonBasis
++ Author: Michel Petitot (petitot@lifl.fr).
++ Date Created: 91
++ Date Last Updated: 7 Juillet 92
++ Fix History: compilation v 2.1 le 13 dec 98
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This domain provides the internal representation
++ of polynomials in non-commutative variables written
++ over the Poincare-Birkhoff-Witt basis.
++ See the \spadtype{XPBWPolynomial} domain constructor.
++ See Free Lie Algebras by C. Reutenauer
++ (Oxford science publications).

PoincareBirkhoffWittLyndonBasis(VarSet: OrderedSet): Public == Private where
  WORD    ==> OrderedFreeMonoid(VarSet)
  LWORD    ==> LyndonWord(VarSet)
  LWORDS   ==> List(LWORD)
  PI       ==> PositiveInteger
  NNI      ==> NonNegativeInteger
  EX       ==> OutputForm

Public == Join(OrderedSet, RetractableTo LWORD) with
  1: constant -> %
      ++ \spad{1} returns the empty list.
  coerce      : $ -> WORD
      ++ \spad{coerce([l1]*[l2]*...[ln])} returns the word \spad{l1*l2*...*ln},
      ++ where \spad{[l_i]} is the bracketed form of the Lyndon word \spad{l_i}.
  coerce      : VarSet -> $
      ++ \spad{coerce(v)} return \spad{v}
  first       : $ -> LWORD
      ++ \spad{first([l1]*[l2]*...[ln])} returns the Lyndon word \spad{l1}.
  length      : $ -> NNI
      ++ \spad{length([l1]*[l2]*...[ln])} returns the length of the word \spad{l1*l2*...*ln}.
  listOfTerms : $ -> LWORDS
      ++ \spad{listOfTerms([l1]*[l2]*...[ln])} returns the list of words \spad{l1, l2, ...}.
  rest        : $ -> $
      ++ \spad{rest([l1]*[l2]*...[ln])} returns the list \spad{l2, ..., ln}.

```

```

retractable? : $ -> Boolean
  ++ \spad{retractable?([l1]*[l2]*...[ln])} returns true iff \spad{n} equals \spad{1}.
varList      : $ -> List VarSet
  ++ \spad{varList([l1]*[l2]*...[ln])} returns the list of
  ++ variables in the word \spad{l1*l2*...*ln}.

Private == add

-- Representation
Rep := LWORDS

-- Locales
recursif: ($,$) -> Boolean

-- Define
1 == nil

x = y == x =$Rep y

varList x ==
  null x => nil
  le: List VarSet := "setUnion"/ [varList$LWORD l for l in x]

first x == first(x)$Rep
rest x == rest(x)$Rep

coerce(v: VarSet):$ == [ v::LWORD ]
coerce(l: LWORD):$ == [l]
listOfTerms(x:$):LWORDS == x pretend LWORDS

coerce(x:$):WORD ==
  null x => 1
  x.first :: WORD *$WORD coerce(x.rest)

coerce(x:$):EX ==
  null x => outputForm(1$Integer)$EX
  reduce(_* , [l :: EX for l in x])$List(EX)

retractable? x ==
  null x => false
  null x.rest

retract x ==
  #x ^= 1 => error "cannot convert to Lyndon word"
  x.first

retractIfCan x ==
  retractable? x => x.first
  "failed"

```

```

length x ==
  n: Integer := +/[ length l for l in x]
  n::NNI

recursif(x, y) ==
  null y => false
  null x => true
  x.first = y.first => recursif(rest(x), rest(y))
  lexico(x.first, y.first)

x < y ==
  lx: NNI := length x; ly: NNI := length y
  lx = ly => recursif(x,y)
  lx < ly

```

---

— PBWLB.dotabb —

```

"PBWLB" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PBWLB"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"PBWLB" -> "FLAGG"

```

---

## 17.24 domain POINT Point

— Point.input —

```

)set break resume
)sys rm -f Point.output
)spool Point.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Point
--R Point R: Ring is a domain constructor
--R Abbreviation for Point is POINT
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for POINT
--R
--R----- Operations -----

```

```

--R ? : % -> % if R has ABELGRP
--R concat : (%,% ) -> %
--R concat : (% ,R) -> %
--R convert : List R -> %
--R cross : (%,% ) -> %
--R dimension : % -> PositiveInteger
--R elt : (% ,Integer,R) -> R
--R empty? : % -> Boolean
--R eq? : (%,% ) -> Boolean
--R index? : (Integer,% ) -> Boolean
--R insert : (% ,% ,Integer) -> %
--R map : ((R,R) -> R) ,% ,% ) -> %
--R new : (NonNegativeInteger,R) -> %
--R qelt : (% ,Integer) -> R
--R sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?? : (% ,R) -> % if R has MONOID
--R ?? : (R,% ) -> % if R has MONOID
--R ?? : (Integer,% ) -> % if R has ABELGRP
--R ?? : (% ,% ) -> % if R has ABELSG
--R ?? : (% ,% ) -> % if R has ABELGRP
--R ?<? : (% ,% ) -> Boolean if R has ORDSET
--R ?<=? : (% ,% ) -> Boolean if R has ORDSET
--R ?=? : (% ,% ) -> Boolean if R has SETCAT
--R ?>? : (% ,% ) -> Boolean if R has ORDSET
--R ?>=? : (% ,% ) -> Boolean if R has ORDSET
--R any? : ((R -> Boolean) ,% ) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if R has SETCAT
--R convert : % -> InputForm if R has KONVERT INFORM
--R copyInto! : (% ,% ,Integer) -> % if $ has shallowlyMutable
--R count : (R,% ) -> NonNegativeInteger if $ has finiteAggregate and R has SETCAT
--R count : ((R -> Boolean) ,% ) -> NonNegativeInteger if $ has finiteAggregate
--R delete : (% ,UniversalSegment Integer) -> %
--R dot : (% ,% ) -> R if R has RING
--R ?.? : (% ,UniversalSegment Integer) -> %
--R entry? : (R,% ) -> Boolean if $ has finiteAggregate and R has SETCAT
--R eval : (% ,List R,List R) -> % if R has EVALAB R and R has SETCAT
--R eval : (% ,R,R) -> % if R has EVALAB R and R has SETCAT
--R eval : (% ,Equation R) -> % if R has EVALAB R and R has SETCAT
--R eval : (% ,List Equation R) -> % if R has EVALAB R and R has SETCAT
--R every? : ((R -> Boolean) ,% ) -> Boolean if $ has finiteAggregate
--R fill! : (% ,R) -> % if $ has shallowlyMutable
--R find : ((R -> Boolean) ,% ) -> Union(R,"failed")
--R first : % -> R if Integer has ORDSET
--R hash : % -> SingleInteger if R has SETCAT
--R latex : % -> String if R has SETCAT
--R length : % -> R if R has RADCAT and R has RING
--R less? : (% ,NonNegativeInteger) -> Boolean
--R magnitude : % -> R if R has RADCAT and R has RING
--R map! : ((R -> R) ,% ) -> % if $ has shallowlyMutable
concat : List % -> %
concat : (R,% ) -> %
construct : List R -> %
copy : % -> %
delete : (% ,Integer) -> %
?.? : (% ,Integer) -> R
empty : () -> %
entries : % -> List R
extend : (% ,List R) -> %
indices : % -> List Integer
insert : (R,% ,Integer) -> %
map : ((R -> R) ,% ) -> %
point : List R -> %
reverse : % -> %

```

```

--R max : (%,%) -> % if R has ORDSET
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (R,%) -> Boolean if $ has finiteAggregate and R has SETCAT
--R members : % -> List R if $ has finiteAggregate
--R merge : (%,%) -> % if R has ORDSET
--R merge : (((R,R) -> Boolean),%,%) -> %
--R min : (%,%) -> % if R has ORDSET
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R outerProduct : (%,%) -> Matrix R if R has RING
--R parts : % -> List R if $ has finiteAggregate
--R position : (R,%,Integer) -> Integer if R has SETCAT
--R position : (R,%) -> Integer if R has SETCAT
--R position : ((R -> Boolean),%) -> Integer
--R qsetelt! : (%,Integer,R) -> R if $ has shallowlyMutable
--R reduce : (((R,R) -> R),%) -> R if $ has finiteAggregate
--R reduce : (((R,R) -> R),%,R) -> R if $ has finiteAggregate
--R reduce : (((R,R) -> R),%,R,R) -> R if $ has finiteAggregate and R has SETCAT
--R remove : ((R -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (R,%) -> % if $ has finiteAggregate and R has SETCAT
--R removeDuplicates : % -> % if $ has finiteAggregate and R has SETCAT
--R reverse! : % -> % if $ has shallowlyMutable
--R select : ((R -> Boolean),%) -> % if $ has finiteAggregate
--R setelt : (%,UniversalSegment Integer,R) -> R if $ has shallowlyMutable
--R setelt : (%,Integer,R) -> R if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : % -> % if R has ORDSET
--R sort : (((R,R) -> Boolean),%) -> %
--R sort! : % -> % if $ has shallowlyMutable and R has ORDSET
--R sort! : (((R,R) -> Boolean),%) -> % if $ has shallowlyMutable
--R sorted? : % -> Boolean if R has ORDSET
--R sorted? : (((R,R) -> Boolean),%) -> Boolean
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R zero : NonNegativeInteger -> % if R has ABELMON
--R ?~=? : (%,%) -> Boolean if R has SETCAT
--R
--E 1

)spool
)lisp (bye)

```

— Point.help —

```

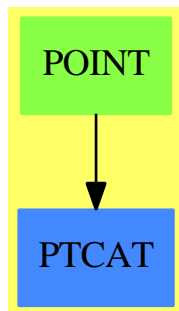
=====
Point examples
=====

```

See Also:

o )show Point

### 17.24.1 Point (POINT)



See

⇒ “SubSpaceComponentProperty” (COMPPROP) 20.34.1 on page 2583

⇒ “SubSpace” (SUBSPACE) 20.33.1 on page 2573

#### Exports:

|         |              |          |           |                  |
|---------|--------------|----------|-----------|------------------|
| any?    | coerce       | concat   | construct | convert          |
| copy    | copyInto!    | count    | count     | cross            |
| delete  | dimension    | dot      | elt       | empty            |
| empty?  | entries      | entry?   | eq?       | eval             |
| every?  | extend       | fill!    | find      | first            |
| hash    | index?       | indices  | insert    | insert           |
| latex   | length       | less?    | magnitude | map              |
| map!    | max          | maxIndex | member?   | members          |
| merge   | merge        | min      | minIndex  | more?            |
| new     | outerProduct | parts    | point     | position         |
| qelt    | qsetelt!     | reduce   | remove    | removeDuplicates |
| reverse | reverse!     | sample   | select    | setelt           |
| size?   | sort         | sort!    | sorted?   | swap!            |
| zero    | #?           | ?*?      | ?+?       | ?-?              |
| ?<?     | ?<=?         | ?=?      | ?>?       | ?>=?             |
| ?..?    | ?~=?         | -?       | ?..?      |                  |

— domain POINT Point —

)abbrev domain POINT Point

++ Author: Mark Botch

++ Description:



```

++ This domain implements points in coordinate space

Point(R:Ring) : Exports == Implementation where
  -- Domains for points, subspaces and properties of components in
  -- a subspace

Exports ==> PointCategory(R)

Implementation ==> Vector (R) add
  PI ==> PositiveInteger

point(l:List R):% ==
  pt := new(#l,R)
  for x in l for i in minIndex(pt).. repeat
    pt.i := x
  pt
dimension p == (# p)::PI -- Vector returns NonNegativeInteger...?
convert(l:List R):% == point(l)
cross(p0, p1) ==
  #p0 ^=3 or #p1^=3 => error "Arguments to cross must be three dimensional"
  point [p0.2 * p1.3 - p1.2 * p0.3, _
         p1.1 * p0.3 - p0.1 * p1.3, _
         p0.1 * p1.2 - p1.1 * p0.2]
extend(p,l) == concat(p,point l)

```

---

— POINT.dotabb —

```

"POINT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=POINT"]
"PTCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PTCAT"]
"POINT" -> "PTCAT"

```

## 17.25 domain POLY Polynomial

---

— Polynomial.input —

```

)set break resume
)sys rm -f Polynomial.output
)spool Polynomial.output
)set message test on
)set message auto off

```

```

--S 1 of 46
x + 1
--R
--R
--R (1)  $x + 1$ 
--R
--R                                         Type: Polynomial Integer
--E 1

```

```

--S 2 of 46
z - 2.3
--R
--R
--R (2)  $z - 2.3$ 
--R
--R                                         Type: Polynomial Float
--E 2

```

```

--S 3 of 46
y**2 - z + 3/4
--R
--R
--R (3)  $-z + y^2 + \frac{3}{4}$ 
--R
--R                                         Type: Polynomial Fraction Integer
--E 3

```

```

--S 4 of 46
y **2 + x*y + y
--R
--R
--R (4)  $y^2 + (x + 1)y$ 
--R
--R                                         Type: Polynomial Integer
--E 4

```

```

--S 5 of 46
% :: DMP([y,x],INT)
--R
--R
--R (5)  $y^2 + yx + y$ 
--R
--R                                         Type: DistributedMultivariatePolynomial([y,x],Integer)
--E 5

```

```

--S 6 of 46
p := (y-1)**2 * x * z
--R
--R
--R (6)  $(x y^2 - 2x y + x)z$ 

```

```
--R                                         Type: Polynomial Integer
--E 6
```

```
--S 7 of 46
q := (y-1) * x * (z+5)
--R
--R
--R      (7)  (x y - x)z + 5x y - 5x
--R
--R                                         Type: Polynomial Integer
--E 7
```

```
--S 8 of 46
factor(q)
--R
--R
--R      (8)  x(y - 1)(z + 5)
--R
--R                                         Type: Factored Polynomial Integer
--E 8
```

```
--S 9 of 46
p - q**2
--R
--R
--R      (9)
--R      2 2      2      2 2      2      2      2      2
--R      (- x y + 2x y - x )z + ((- 10x + x)y + (20x - 2x)y - 10x + x)z
--R      +
--R      2 2      2      2
--R      - 25x y + 50x y - 25x
--R
--R                                         Type: Polynomial Integer
--E 9
```

```
--S 10 of 46
gcd(p,q)
--R
--R
--R      (10)  x y - x
--R
--R                                         Type: Polynomial Integer
--E 10
```

```
--S 11 of 46
factor %
--R
--R
--R      (11)  x(y - 1)
--R
--R                                         Type: Factored Polynomial Integer
--E 11
```

```
--S 12 of 46
lcm(p,q)
```

```

--R
--R
--R      2      2      2
--R      (12)  (x y  - 2x y + x)z  + (5x y  - 10x y + 5x)z
--R                                                    Type: Polynomial Integer
--E 12

```

```

--S 13 of 46
content p
--R
--R
--R      (13)  1
--R                                                    Type: PositiveInteger
--E 13

```

```

--S 14 of 46
resultant(p,q,z)
--R
--R
--R      2 3      2 2      2      2
--R      (14)  5x y  - 15x y  + 15x y - 5x
--R                                                    Type: Polynomial Integer
--E 14

```

```

--S 15 of 46
resultant(p,q,x)
--R
--R
--R      (15)  0
--R                                                    Type: Polynomial Integer
--E 15

```

```

--S 16 of 46
mainVariable p
--R
--R
--R      (16)  z
--R                                                    Type: Union(Symbol,...)
--E 16

```

```

--S 17 of 46
mainVariable(1 :: POLY INT)
--R
--R
--R      (17)  "failed"
--R                                                    Type: Union("failed",...)
--E 17

```

```

--S 18 of 46
ground? p

```

```

--R
--R
--R (18) false
--R
--R                                         Type: Boolean
--E 18

--S 19 of 46
ground?(1 :: POLY INT)
--R
--R
--R (19) true
--R
--R                                         Type: Boolean
--E 19

--S 20 of 46
variables p
--R
--R
--R (20) [z,y,x]
--R
--R                                         Type: List Symbol
--E 20

--S 21 of 46
degree(p,x)
--R
--R
--R (21) 1
--R
--R                                         Type: PositiveInteger
--E 21

--S 22 of 46
degree(p,y)
--R
--R
--R (22) 2
--R
--R                                         Type: PositiveInteger
--E 22

--S 23 of 46
degree(p,z)
--R
--R
--R (23) 1
--R
--R                                         Type: PositiveInteger
--E 23

--S 24 of 46
degree(p,[x,y,z])
--R
--R

```

```

--R (24) [1,2,1]
--R
--R                                         Type: List NonNegativeInteger
--E 24

```

```

--S 25 of 46
minimumDegree(p,z)
--R
--R
--R (25) 1
--R
--R                                         Type: PositiveInteger
--E 25

```

```

--S 26 of 46
totalDegree p
--R
--R
--R (26) 4
--R
--R                                         Type: PositiveInteger
--E 26

```

```

--S 27 of 46
leadingMonomial p
--R
--R
--R          2
--R (27) x y z
--R
--R                                         Type: Polynomial Integer
--E 27

```

```

--S 28 of 46
reductum p
--R
--R
--R (28) (- 2x y + x)z
--R
--R                                         Type: Polynomial Integer
--E 28

```

```

--S 29 of 46
p - leadingMonomial p - reductum p
--R
--R
--R (29) 0
--R
--R                                         Type: Polynomial Integer
--E 29

```

```

--S 30 of 46
leadingCoefficient p
--R
--R
--R (30) 1

```

```
--R
--E 30
```

Type: PositiveInteger

```
--S 31 of 46
```

```
p
--R
--R
--R      2
--R (31) (x y  - 2x y + x)z
--R
--E 31
```

Type: Polynomial Integer

```
--S 32 of 46
```

```
eval(p,x,w)
--R
--R
--R      2
--R (32) (w y  - 2w y + w)z
--R
--E 32
```

Type: Polynomial Integer

```
--S 33 of 46
```

```
eval(p,x,1)
--R
--R
--R      2
--R (33) (y  - 2y + 1)z
--R
--E 33
```

Type: Polynomial Integer

```
--S 34 of 46
```

```
eval(p,x,y^2 - 1)
--R
--R
--R      4      3
--R (34) (y  - 2y  + 2y - 1)z
--R
--E 34
```

Type: Polynomial Integer

```
--S 35 of 46
```

```
D(p,x)
--R
--R
--R      2
--R (35) (y  - 2y + 1)z
--R
--E 35
```

Type: Polynomial Integer

```
--S 36 of 46
```

```
D(p,y)
```

[illegible]



--S 42 of 46

p/q

--R

--R

--R (y - 1)z

--R (42) -----

--R z + 5

--R

Type: Fraction Polynomial Integer

--E 42

--S 43 of 46

(2/3) \* x\*\*2 - y + 4/5

--R

--R

--R 2 2 4

--R (43) - y + - x + -

--R 3 5

--R

Type: Polynomial Fraction Integer

--E 43

--S 44 of 46

% :: FRAC POLY INT

--R

--R

--R 2

--R - 15y + 10x + 12

--R (44) -----

--R 15

--R

Type: Fraction Polynomial Integer

--E 44

--S 45 of 46

% :: POLY FRAC INT

--R

--R

--R 2 2 4

--R (45) - y + - x + -

--R 3 5

--R

Type: Polynomial Fraction Integer

--E 45

--S 46 of 46

map(numeric,%)

--R

--R

--R 2

--R (46) - 1.0 y + 0.6666666666 6666666667 x + 0.8

--R

Type: Polynomial Float

--E 46

```
)spool
)lisp (bye)
```

---

— Polynomial.help —

=====

Polynomial examples

=====

The domain constructor Polynomial (abbreviation: POLY) provides polynomials with an arbitrary number of unspecified variables.

It is used to create the default polynomial domains in Axiom. Here the coefficients are integers.

```
x + 1
x + 1
```

Type: Polynomial Integer

Here the coefficients have type Float.

```
z - 2.3
z - 2.3
```

Type: Polynomial Float

And here we have a polynomial in two variables with coefficients which have type Fraction Integer.

```
y**2 - z + 3/4
      2 3
- z + y + -
      4
```

Type: Polynomial Fraction Integer

The representation of objects of domains created by Polynomial is that of recursive univariate polynomials. The term univariate means "one variable". The term multivariate means "possibly more than one variable".

This recursive structure is sometimes obvious from the display of a polynomial.

```
y **2 + x*y + y
      2
y + (x + 1)y
```

Type: Polynomial Integer

In this example, you see that the polynomial is stored as a polynomial in y with coefficients that are polynomials in x with integer coefficients.

In fact, you really don't need to worry about the representation unless you are working on an advanced application where it is critical. The polynomial types created from `DistributedMultivariatePolynomial` and `NewDistributedMultivariatePolynomial` are stored and displayed in a non-recursive manner.

You see a "flat" display of the above polynomial by converting to one of those types.

```
% :: DMP([y,x],INT)
      2
      y  + y x + y
                                     Type: DistributedMultivariatePolynomial([y,x],Integer)
```

We will demonstrate many of the polynomial facilities by using two polynomials with integer coefficients.

By default, the interpreter expands polynomial expressions, even if they are written in a factored format.

```
p := (y-1)**2 * x * z
      2
      (x y  - 2x y + x)z
                                     Type: Polynomial Integer
```

See `Factored` to see how to create objects in factored form directly.

```
q := (y-1) * x * (z+5)
      (x y - x)z + 5x y - 5x
                                     Type: Polynomial Integer
```

The fully factored form can be recovered by using `factor`.

```
factor(q)
      x(y - 1)(z + 5)
                                     Type: Factored Polynomial Integer
```

This is the same name used for the operation to factor integers. Such reuse of names is called overloading and makes it much easier to think of solving problems in general ways. Axiom facilities for factoring polynomials created with `Polynomial` are currently restricted to the integer and rational number coefficient cases.

The standard arithmetic operations are available for polynomials.

```
p - q**2
      2 2      2      2 2      2      2      2      2
      (- x y  + 2x y - x )z  + ((- 10x  + x)y  + (20x  - 2x)y - 10x  + x)z
      +
      2 2      2      2
      (- x y  + 2x y - x )z  + ((- 10x  + x)y  + (20x  - 2x)y - 10x  + x)z
```

```

- 25x y + 50x y - 25x
Type: Polynomial Integer

```

The operation gcd is used to compute the greatest common divisor of two polynomials.

```

gcd(p,q)
x y - x
Type: Polynomial Integer

```

In the case of p and q, the gcd is obvious from their definitions. We factor the gcd to show this relationship better.

```

factor %
x(y - 1)
Type: Factored Polynomial Integer

```

The least common multiple is computed by using lcm.

```

lcm(p,q)
      2      2      2
(x y - 2x y + x)z + (5x y - 10x y + 5x)z
Type: Polynomial Integer

```

Use content to compute the greatest common divisor of the coefficients of the polynomial.

```

content p
1
Type: PositiveInteger

```

Many of the operations on polynomials require you to specify a variable. For example, resultant requires you to give the variable in which the polynomials should be expressed.

This computes the resultant of the values of p and q, considering them as polynomials in the variable z. They do not share a root when thought of as polynomials in z.

```

resultant(p,q,z)
      2 3      2 2      2      2
5x y - 15x y + 15x y - 5x
Type: Polynomial Integer

```

This value is 0 because as polynomials in x the polynomials have a common root.

```

resultant(p,q,x)
0
Type: Polynomial Integer

```

The data type used for the variables created by Polynomial is Symbol. As mentioned above, the representation used by Polynomial is recursive and so there is a main variable for nonconstant polynomials.

The operation `mainVariable` returns this variable. The return type is actually a union of Symbol and "failed".

```
mainVariable p
  z
                                Type: Union(Symbol,...)
```

The latter branch of the union is be used if the polynomial has no variables, that is, is a constant.

```
mainVariable(1 :: POLY INT)
  "failed"
                                Type: Union("failed",...)
```

You can also use the predicate `ground?` to test whether a polynomial is in fact a member of its ground ring.

```
ground? p
  false
                                Type: Boolean
```

```
ground?(1 :: POLY INT)
  true
                                Type: Boolean
```

The complete list of variables actually used in a particular polynomial is returned by `variables`. For constant polynomials, this list is empty.

```
variables p
  [z,y,x]
                                Type: List Symbol
```

The degree operation returns the degree of a polynomial in a specific variable.

```
degree(p,x)
  1
                                Type: PositiveInteger
```

```
degree(p,y)
  2
                                Type: PositiveInteger
```

```
degree(p,z)
  1
                                Type: PositiveInteger
```

If you give a list of variables for the second argument, a list of the degrees in those variables is returned.

```
degree(p,[x,y,z])
[1,2,1]
Type: List NonNegativeInteger
```

The minimum degree of a variable in a polynomial is computed using `minimumDegree`.

```
minimumDegree(p,z)
1
Type: PositiveInteger
```

The total degree of a polynomial is returned by `totalDegree`.

```
totalDegree p
4
Type: PositiveInteger
```

It is often convenient to think of a polynomial as a leading monomial plus the remaining terms.

```
leadingMonomial p
2
x y z
Type: Polynomial Integer
```

The `reductum` operation returns a polynomial consisting of the sum of the monomials after the first.

```
reductum p
(- 2x y + x)z
Type: Polynomial Integer
```

These have the obvious relationship that the original polynomial is equal to the leading monomial plus the reductum.

```
p - leadingMonomial p - reductum p
0
Type: Polynomial Integer
```

The value returned by `leadingMonomial` includes the coefficient of that term. This is extracted by using `leadingCoefficient` on the original polynomial.

```
leadingCoefficient p
1
Type: PositiveInteger
```

The operation `eval` is used to substitute a value for a variable in a polynomial.

```

P
      2
(x y  - 2x y + x)z
                                Type: Polynomial Integer

```

This value may be another variable, a constant or a polynomial.

```

eval(p,x,w)
      2
(w y  - 2w y + w)z
                                Type: Polynomial Integer

```

```

eval(p,x,1)
      2
(y  - 2y + 1)z
                                Type: Polynomial Integer

```

Actually, all the things being substituted are just polynomials, some more trivial than others.

```

eval(p,x,y^2 - 1)
      4      3
(y  - 2y  + 2y - 1)z
                                Type: Polynomial Integer

```

Derivatives are computed using the `D` operation.

```

D(p,x)
      2
(y  - 2y + 1)z
                                Type: Polynomial Integer

```

The first argument is the polynomial and the second is the variable.

```

D(p,y)
      2
(2x y - 2x)z
                                Type: Polynomial Integer

```

Even if the polynomial has only one variable, you must specify it.

```

D(p,z)
      2
x y  - 2x y + x
                                Type: Polynomial Integer

```

Integration of polynomials is similar and the `integrate` operation is used.

Integration requires that the coefficients support division. Axiom converts polynomials over the integers to polynomials over the rational numbers before integrating them.

```
integrate(p,y)
      1      3      2
      (- x y  - x y  + x y)z
      3
Type: Polynomial Fraction Integer
```

It is not possible, in general, to divide two polynomials. In our example using polynomials over the integers, the operation `monicDivide` divides a polynomial by a monic polynomial (that is, a polynomial with leading coefficient equal to 1). The result is a record of the quotient and remainder of the division.

You must specify the variable in which to express the polynomial.

```
qr := monicDivide(p,x+1,x)
      2      2
      [quotient= (y  - 2y + 1)z, remainder= (- y  + 2y - 1)z]
      Type: Record(quotient: Polynomial Integer, remainder: Polynomial Integer)
```

The selectors of the components of the record are `quotient` and `remainder`. Issue this to extract the remainder.

```
qr.remainder
      2
      (- y  + 2y - 1)z
Type: Polynomial Integer
```

Now that we can extract the components, we can demonstrate the relationship among them and the arguments to our original expression `qr := monicDivide(p,x+1,x)`.

```
p - ((x+1) * qr.quotient + qr.remainder)
0
Type: Polynomial Integer
```

If the `/` operator is used with polynomials, a fraction object is created. In this example, the result is an object of type `Fraction Polynomial Integer`.

```
p/q
      (y - 1)z
      -----
      z + 5
Type: Fraction Polynomial Integer
```

If you use rational numbers as polynomial coefficients, the



resulting object is of type Polynomial Fraction Integer.

$$\begin{array}{r} (2/3) * x^{**2} - y + 4/5 \\ \phantom{(2/3) * x^{**2} - y + 4/5} \phantom{2} \phantom{2} \phantom{4} \\ - y + - x + - \\ \phantom{- y + - x + -} \phantom{3} \phantom{5} \end{array}$$

Type: Polynomial Fraction Integer

This can be converted to a fraction of polynomials and back again, if required.

$$\begin{array}{r} \% :: \text{FRAC POLY INT} \\ \phantom{\% :: \text{FRAC POLY INT}} \phantom{2} \\ - 15y + 10x^2 + 12 \\ \hline \phantom{- 15y + 10x^2 + 12} 15 \end{array}$$

Type: Fraction Polynomial Integer

$$\begin{array}{r} \% :: \text{POLY FRAC INT} \\ \phantom{\% :: \text{POLY FRAC INT}} \phantom{2} \phantom{2} \phantom{4} \\ - y + - x + - \\ \phantom{- y + - x + -} \phantom{3} \phantom{5} \end{array}$$

Type: Polynomial Fraction Integer

To convert the coefficients to floating point, map the numeric operation on the coefficients of the polynomial.

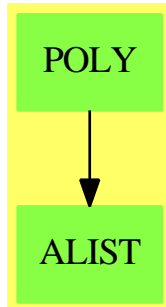
$$\begin{array}{r} \text{map(numeric,\%)} \\ - 1.0 y + 0.6666666666 6666666667 x^2 + 0.8 \end{array}$$

Type: Polynomial Float

See Also:

- o )help Factored
- o )help UnivariatePolynomial
- o )help MultivariatePolynomial
- o )help DistributedMultivariatePolynomial
- o )help NewDistributedMultivariatePolynomial
- o )show Polynomial

---

**17.25.1 Polynomial (POLY)**

**See**

- ⇒ “MultivariatePolynomial” (MPOLY) 14.16.1 on page 1645
- ⇒ “SparseMultivariatePolynomial” (SMP) 20.14.1 on page 2381
- ⇒ “IndexedExponents” (INDE) 10.9.1 on page 1183

**Exports:**

|                 |                   |                               |
|-----------------|-------------------|-------------------------------|
| 0               | 1                 | associates?                   |
| binomThmExpt    | characteristic    | charthRoot                    |
| coefficient     | coefficients      | coerce                        |
| conditionP      | content           | convert                       |
| D               | degree            | differentiate                 |
| discriminant    | eval              | exquo                         |
| factor          | factorPolynomial  | factorSquareFreePolynomial    |
| gcd             | gcdPolynomial     | ground                        |
| ground?         | hash              | integrate                     |
| isExpt          | isPlus            | isTimes                       |
| latex           | lcm               | leadingCoefficient            |
| leadingMonomial | mainVariable      | map                           |
| mapExponents    | max               | min                           |
| minimumDegree   | monicDivide       | monomial                      |
| monomial?       | monomials         | multivariate                  |
| one?            | numberOfMonomials | patternMatch                  |
| pomopo!         | prime?            | primitiveMonomials            |
| primitivePart   | recip             | reducedSystem                 |
| reductum        | resultant         | retract                       |
| retractIfCan    | sample            | solveLinearPolynomialEquation |
| squareFree      | squareFreePart    | squareFreePolynomial          |
| subtractIfCan   | totalDegree       | totalDegree                   |
| unit?           | unitCanonical     | unitNormal                    |
| univariate      | variables         | zero?                         |
| ?*?             | ?**?              | ?+?                           |
| ?-?             | -?                | ?=?                           |
| ?^?             | ?~=?              | ?/?                           |
| ?<?             | ?<=?              | ?>?                           |
| ?>=?            |                   |                               |

— domain **POLY Polynomial** —

```

)abbrev domain POLY Polynomial
++ Author: Dave Barton, Barry Trager
++ Date Created:
++ Date Last Updated:
++ Basic Functions: Ring, degree, eval, coefficient, monomial, differentiate,
++ resultant, gcd
++ Related Constructors: SparseMultivariatePolynomial, MultivariatePolynomial
++ Also See:
++ AMS Classifications:
++ Keywords: polynomial, multivariate
++ References:
++ Description:
++ This type is the basic representation of sparse recursive multivariate
++ polynomials whose variables are arbitrary symbols. The ordering
++ is alphabetic determined by the Symbol type.
++ The coefficient ring may be non commutative,

```

```

++ but the variables are assumed to commute.

Polynomial(R:Ring):
  PolynomialCategory(R, IndexedExponents Symbol, Symbol) with
  if R has Algebra Fraction Integer then
    integrate: (%, Symbol) -> %
    ++ integrate(p,x) computes the integral of \spad{p*dx}, i.e.
    ++ integrates the polynomial p with respect to the variable x.
  == SparseMultivariatePolynomial(R, Symbol) add

  import UserDefinedPartialOrdering(Symbol)

  coerce(p:%):OutputForm ==
    (r:= retractIfCan(p)@Union(R,"failed")) case R => r::R::OutputForm
    a :=
      userOrdered?() => largest variables p
      mainVariable(p)::Symbol
      outputForm(univariate(p, a), a::OutputForm)

  if R has Algebra Fraction Integer then
    integrate(p, x) == (integrate univariate(p, x)) (x::%)

  -----

  — POLY.dotabb —

  "POLY" [color="#88FF44",href="bookvol10.3.pdf#nameddest=POLY"]
  "ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
  "POLY" -> "ALIST"

  -----

```

## 17.26 domain IDEAL PolynomialIdeals

```

  -----

  — PolynomialIdeals.input —

)set break resume
)sys rm -f PolynomialIdeals.output
)spool PolynomialIdeals.output
)set message test on
)set message auto off
)clear all

--S 1 of 1

```

```

)show PolynomialIdeals
--R PolynomialIdeals(F: Field,Expon: OrderedAbelianMonoidSup,VarSet: OrderedSet,DPoly: Polynom
--R Abbreviation for PolynomialIdeals is IDEAL
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for IDEAL
--R
--R----- Operations -----
--R ?? : (% ,%) -> %
--R ==? : (% ,%) -> Boolean
--R coerce : % -> OutputForm
--R element? : (DPoly ,%) -> Boolean
--R groebner : % -> %
--R groebnerIdeal : List DPoly -> %
--R ideal : List DPoly -> %
--R inRadical? : (DPoly ,%) -> Boolean
--R intersect : (% ,%) -> %
--R leadingIdeal : % -> %
--R quotient : (% ,DPoly) -> %
--R saturate : (% ,DPoly) -> %
--R zeroDim? : % -> Boolean
--R ***? : (% ,NonNegativeInteger) -> %
--R backOldPos : Record(mval: Matrix F,invmmval: Matrix F,genIdeal: %) -> %
--R dimension : (% ,List VarSet) -> Integer
--R generalPosition : (% ,List VarSet) -> Record(mval: Matrix F,invmmval: Matrix F,genIdeal: %)
--R relationsIdeal : List DPoly -> SuchThat(List Polynomial F,List Equation Polynomial F) if
--R saturate : (% ,DPoly,List VarSet) -> %
--R zeroDim? : (% ,List VarSet) -> Boolean
--R
--E 1

)spool
)lisp (bye)

```

---

— PolynomialIdeals.help —

```

=====
PolynomialIdeals examples
=====

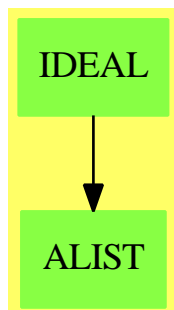
```

```

See Also:
o )show PolynomialIdeals

```

## 17.26.1 PolynomialIdeals (IDEAL)

**Exports:**

|              |          |            |                |                 |
|--------------|----------|------------|----------------|-----------------|
| backOldPos   | coerce   | dimension  | element?       | generalPosition |
| generators   | groebner | groebner?  | groebnerIdeal  | hash            |
| ideal        | in?      | inRadical? | intersect      | latex           |
| leadingIdeal | one?     | quotient   | relationsIdeal | saturate        |
| zero?        | zeroDim? | ?~=?       | ***?           | *?*             |
| ?+?          | ?=?      |            |                |                 |

— domain IDEAL PolynomialIdeals —

```

)abbrev domain IDEAL PolynomialIdeals
++ Author: P. Gianni
++ Date Created: summer 1986
++ Date Last Updated: September 1996
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References: GTZ
++ Description:
++ This domain represents polynomial ideals with coefficients in any
++ field and supports the basic ideal operations, including intersection
++ sum and quotient.
++ An ideal is represented by a list of polynomials (the generators of
++ the ideal) and a boolean that is true if the generators are a Groebner
++ basis.
++ The algorithms used are based on Groebner basis computations. The
++ ordering is determined by the datatype of the input polynomials.
++ Users may use refinements of total degree orderings.

```

```

PolynomialIdeals(F,Expon,VarSet,DPoly) : C == T
where
  F      : Field
  Expon  : OrderedAbelianMonoidSup

```

```

VarSet      : OrderedSet
DPoly       : PolynomialCategory(F,Expon,VarSet)

SUP         ==> SparseUnivariatePolynomial(DPoly)
NNI         ==> NonNegativeInteger
Z           ==> Integer
P           ==> Polynomial F
MF          ==> Matrix(F)
ST          ==> SuchThat(List P, List Equation P)

GenMPos     ==> Record(mval:MF,invmmval:MF,genIdeal:Ideal)
Ideal       ==> %

C == SetCategory with

"*"          : (Ideal,Ideal)      -> Ideal
++ I*J computes the product of the ideal I and J.
" **"       : (Ideal,NNI)        -> Ideal
++ I**n computes the nth power of the ideal I.
"+"         : (Ideal,Ideal)      -> Ideal
++ I+J computes the ideal generated by the union of I and J.
one?        : Ideal              -> Boolean
++ one?(I) tests whether the ideal I is the unit ideal,
++ i.e. contains 1.
zero?       : Ideal              -> Boolean
++ zero?(I) tests whether the ideal I is the zero ideal
element?    : (DPoly,Ideal)      -> Boolean
++ element?(f,I) tests whether the polynomial f belongs to
++ the ideal I.
in?         : (Ideal,Ideal)      -> Boolean
++ in?(I,J) tests if the ideal I is contained in the ideal J.
inRadical?  : (DPoly,Ideal)      -> Boolean
++ inRadical?(f,I) tests if some power of the polynomial f
++ belongs to the ideal I.
zeroDim?    : (Ideal,List VarSet) -> Boolean
++ zeroDim?(I,lvar) tests if the ideal I is zero dimensional, i.e.
++ all its associated primes are maximal,
++ in the ring \spad{F[lvar]}
zeroDim?    : Ideal              -> Boolean
++ zeroDim?(I) tests if the ideal I is zero dimensional, i.e.
++ all its associated primes are maximal,
++ in the ring \spad{F[lvar]},
++ where lvar are the variables appearing in I
intersect    : (Ideal,Ideal)      -> Ideal
++ intersect(I,J) computes the intersection of the ideals I and J.
intersect    : List(Ideal)        -> Ideal
++ intersect(LI) computes the intersection of the list of ideals LI.
quotient     : (Ideal,Ideal)      -> Ideal
++ quotient(I,J) computes the quotient of the ideals I and J,
++ \spad{(I:J)}.

```

```

quotient      : (Ideal,DPoly)      -> Ideal
++ quotient(I,f) computes the quotient of the ideal I by the principal
++ ideal generated by the polynomial f, \spad{(I:(f))}.
groebner      : Ideal              -> Ideal
++ groebner(I) returns a set of generators of I that are a
++ Groebner basis for I.
generalPosition : (Ideal,List VarSet) -> GenMPos
++ generalPosition(I,listvar) perform a random linear
++ transformation on the variables in listvar and returns
++ the transformed ideal along with the change of basis matrix.
backOldPos    : GenMPos            -> Ideal
++ backOldPos(genPos) takes the result
++ produced by generalPosition from PolynomialIdeals
++ and performs the inverse transformation, returning the original ideal
++ \spad{backOldPos(generalPosition(I,listvar))} = I.
dimension     : (Ideal,List VarSet) -> Z
++ dimension(I,lvar) gives the dimension of the ideal I,
++ in the ring \spad{F[lvar]}
dimension     : Ideal              -> Z
++ dimension(I) gives the dimension of the ideal I.
++ in the ring \spad{F[lvar]}, where lvar are the variables
++ appearing in I
leadingIdeal  : Ideal              -> Ideal
++ leadingIdeal(I) is the ideal generated by the
++ leading terms of the elements of the ideal I.
ideal        : List DPoly         -> Ideal
++ ideal(polyList) constructs the ideal generated by the list
++ of polynomials polyList.
groebnerIdeal : List DPoly         -> Ideal
++ groebnerIdeal(polyList) constructs the ideal generated by the list
++ of polynomials polyList which are assumed to be a Groebner
++ basis.
++ Note: this operation avoids a Groebner basis computation.
groebner?    : Ideal              -> Boolean
++ groebner?(I) tests if the generators of the ideal I are a
++ Groebner basis.
generators    : Ideal              -> List DPoly
++ generators(I) returns a list of generators for the ideal I.
coerce       : List DPoly         -> Ideal
++ coerce(polyList) converts the list of polynomials polyList
++ to an ideal.

saturate     : (Ideal,DPoly)      -> Ideal
++ saturate(I,f) is the saturation of the ideal I
++ with respect to the multiplicative
++ set generated by the polynomial f.
saturate     : (Ideal,DPoly,List VarSet) -> Ideal
++ saturate(I,f,lvar) is the saturation with respect to the prime
++ principal ideal which is generated by f in the polynomial ring
++ \spad{F[lvar]}.
```



```

if VarSet has ConvertibleTo Symbol then
  relationsIdeal : List DPoly      -> ST
  ++ relationsIdeal(polyList) returns the ideal of relations among the
  ++ polynomials in polyList.

T == add

--- Representation ---
Rep := Record(idl:List DPoly,isGr:Boolean)

----- Local Functions -----

contractGrob : newIdeal      -> Ideal
npoly        : DPoly         -> newPoly
oldpoly      : newPoly       -> Union(DPoly,"failed")
leadterm     : (DPoly,VarSet) -> DPoly
choosel      : (DPoly,DPoly) -> DPoly
isMonic?     : (DPoly,VarSet) -> Boolean
randomat     : List Z        -> Record(mM:MF,imM:MF)
monomDim     : (Ideal,List VarSet) -> NNI
variables    : Ideal         -> List VarSet
subset       : List VarSet   -> List List VarSet
makeleast    : (List VarSet,List VarSet) -> List VarSet

newExpon: OrderedAbelianMonoidSup
newExpon:= Product(NNI,Expon)
newPoly := PolynomialRing(F,newExpon)

import GaloisGroupFactorizer(SparseUnivariatePolynomial Z)
import GroebnerPackage(F,Expon,VarSet,DPoly)
import GroebnerPackage(F,newExpon,VarSet,newPoly)

newIdeal ==> List(newPoly)

npoly(f:DPoly) : newPoly ==
  f=0$DPoly => 0$newPoly
  monomial(leadingCoefficient f,makeprod(0,degree f))$newPoly +
  npoly(reductum f)

oldpoly(q:newPoly) : Union(DPoly,"failed") ==
  q=0$newPoly => 0$DPoly
  dq:newExpon:=degree q
  n:NNI:=selectfirst (dq)
  n^=0 => "failed"
  ((g:=oldpoly reductum q) case "failed") => "failed"
  monomial(leadingCoefficient q,selectsecond dq)$DPoly + (g::DPoly)

leadterm(f:DPoly,lvar:List VarSet) : DPoly ==
  empty?(lf:=variables f) or lf=lvar => f

```

```

    leadterm(leadingCoefficient univariate(f,lf.first),lvar)

choose1(f:DPoly,g:DPoly) : DPoly ==
  g=0 => f
  (f1:=f exquo g) case "failed" => f
  choose1(f1::DPoly,g)

contractGrob(I1:newIdeal) : Ideal ==
  J1:List(newPoly):=groebner(I1)
  while (oldpoly J1.first) case "failed" repeat J1:=J1.rest
  [[(oldpoly f)::DPoly for f in J1],true]

makeleast(fullVars: List VarSet,leastVars:List VarSet) : List VarSet ==
  n:= # leastVars
  #fullVars < n => error "wrong vars"
  n=0 => fullVars
  append([vv for vv in fullVars| ^member?(vv,leastVars)],leastVars)

isMonic?(f:DPoly,x:VarSet) : Boolean ==
  ground? leadingCoefficient univariate(f,x)

subset(lv : List VarSet) : List List VarSet ==
  #lv =1 => [lv,empty()]
  v:=lv.1
  ll:=subset(rest lv)
  l1:=[concat(v,set) for set in ll]
  concat(l1,ll)

monomDim(listm:Ideal,lv:List VarSet) : NNI ==
  monvar: List List VarSet := []
  for f in generators listm repeat
    mvset := variables f
    #mvset > 1 => monvar:=concat(mvset,monvar)
    lv:=delete(lv,position(mvset.1,lv))
  empty? lv => 0
  lsubset : List List VarSet := sort((a,b)+->#a > #b ,subset(lv))
  for subs in lsubset repeat
    ldif:List VarSet:= lv
    for mvset in monvar while ldif ^=[] repeat
      ldif:=setDifference(mvset,subs)
      if ^(empty? ldif) then return #subs
  0

--      Exported Functions      ----

---- is I = J ? ----
(I:Ideal = J:Ideal) == in?(I,J) and in?(J,I)

---- check if f is in I ----
element?(f:DPoly,I:Ideal) : Boolean ==

```

```

Id:=(groebner I).idl
empty? Id => f = 0
normalForm(f,Id) = 0

---- check if I is contained in J ----
in?(I:Ideal,J:Ideal):Boolean ==
  J:= groebner J
  empty?(I.idl) => true
  "and"/[element?(f,J) for f in I.idl ]

---- groebner base for an Ideal ----
groebner(I:Ideal) : Ideal ==
  I.isGr =>
    "or"/[^zero? f for f in I.idl] => I
    [empty(),true]
    [groebner I.idl ,true]

---- Intersection of two ideals ----
intersect(I:Ideal,J:Ideal) : Ideal ==
  empty?(Id:=I.idl) => I
  empty?(Jd:=J.idl) => J
  tp:newPoly := monomial(1,makeprod(1,0$Expon))$newPoly
  tp1:newPoly:= tp-1
  contractGrob(concat([tp*npoly f for f in Id],
    [tp1*npoly f for f in Jd]))

---- intersection for a list of ideals ----
intersect(lid:List(Ideal)) : Ideal == "intersect"/[l for l in lid]

---- quotient by an element ----
quotient(I:Ideal,f:DPoly) : Ideal ==
  --[[ (g exquo f)::DPoly for g in (intersect(I,[f]::%)).idl ],true]
  import GroebnerInternalPackage(F,Expon,VarSet,DPoly)
  [minGbasis [(g exquo f)::DPoly
    for g in (intersect(I,[f]::%)).idl ],true]

---- quotient of two ideals ----
quotient(I:Ideal,J:Ideal) : Ideal ==
  Jd1 := J.idl
  empty?(Jd1) => ideal [1]
  [("intersect"/[quotient(I,f) for f in Jd1 ]).idl ,true]

---- sum of two ideals ----
(I:Ideal + J:Ideal) : Ideal == [groebner(concat(I.idl ,J.idl )),true]

---- product of two ideals ----

```

```

(I:Ideal * J:Ideal):Ideal ==
  [groebner([f*g for f in I.idl ] for g in J.idl ]),true]

      ---- power of an ideal ----
(I:Ideal ** n:NNI) : Ideal ==
  n=0 => [[1$DPoly],true]
  (I * (I**(n-1):NNI))

      ---- saturation with respect to the multiplicative set f**n ----
saturate(I:Ideal,f:DPoly) : Ideal ==
  f=0 => error "f is zero"
  tp:newPoly := (monomial(1,makeprod(1,0$Expon))$newPoly * npoly f)-1
  contractGrob(concat(tp,[npoly g for g in I.idl ]))

      ---- saturation with respect to a prime principal ideal in lvar ---
saturate(I:Ideal,f:DPoly,lvar>List(VarSet)) : Ideal ==
  Id := I.idl
  fullVars := "setUnion"/[variables g for g in Id]
  newVars:=makeleast(fullVars,lvar)
  subVars := [monomial(1,vv,1) for vv in newVars]
  J>List DPoly:=groebner([eval(g,fullVars,subVars) for g in Id])
  ltJ:=leadterm(g,lvar) for g in J]
  s:DPoly:=_[choose1(ltg,f) for ltg in ltJ]
  fullPol:=monomial(1,vv,1) for vv in fullVars]
  [[eval(g,newVars,fullPol) for g in (saturate(J::%,s)).idl],true]

      ---- is the ideal zero dimensional? ----
      ---- in the ring F[lvar]? ----
zeroDim?(I:Ideal,lvar>List(VarSet)) : Boolean ==
  J:=(groebner I).idl
  empty? J => false
  J = [1] => false
  n:NNI := # lvar
  #J < n => false
  for f in J while ~empty?(lvar) repeat
    x:=(mainVariable f)::VarSet
    if isMonic?(f,x) then lvar:=delete(lvar,position(x,lvar))
  empty?(lvar)

      ---- is the ideal zero dimensional? ----
zeroDim?(I:Ideal):Boolean == zeroDim?(I,"setUnion"/[variables g for g in I.idl])

      ---- test if f is in the radical of I ----
inRadical?(f:DPoly,I:Ideal) : Boolean ==
  f=0$DPoly => true
  tp:newPoly :=(monomial(1,makeprod(1,0$Expon))$newPoly * npoly f)-1
  Id:=I.idl
  normalForm(1$newPoly,groebner concat(tp,[npoly g for g in Id])) = 0

      ---- dimension of an ideal ----

```

```

----- in the ring F[lvar] -----
dimension(I:Ideal,lvar>List VarSet) : Z ==
  I:=groebner I
  empty?(I.idl) => # lvar
  element?(1,I) => -1
  truelist:="setUnion"/[variables f for f in I.idl]
  "or"/[^member?(vv,lvar) for vv in truelist] => error "wrong variables"
  truelist:=setDifference(lvar,setDifference(lvar,truelist))
  ed:Z:=#lvar - #truelist
  leadid:=leadingIdeal(I)
  n1:Z:=monomDim(leadid,truelist)::Z
  ed+n1

dimension(I:Ideal) : Z == dimension(I,"setUnion"/[variables g for g in I.idl])

-- leading term ideal --
leadingIdeal(I : Ideal) : Ideal ==
  Idl:= (groebner I).idl
  [[(f-reductum f) for f in Idl],true]

----- ideal of relations among the fi -----
if VarSet has ConvertibleTo Symbol then

  monompol(df>List NNI,lcf:F,lv>List VarSet) : P ==
    g:P:=lcf::P
    for dd in df for v in lv repeat
      g:= monomial(g,convert v,dd)
    g

  relationsIdeal(listf : List DPoly): ST ==
    empty? listf => [empty(),empty()],$ST
    nf:=#listf
    lvint := "setUnion"/[variables g for g in listf]
    vl: List Symbol := [convert vv for vv in lvint]
    nvar:List Symbol:=[new() for i in 1..nf]
    VarSet1:=OrderedVariableList(concat(vl,nvar))
    lv1:=[variable(vv)$VarSet1::VarSet1 for vv in nvar]
    DirP:=DirectProduct(nf,NNI)
    nExponent:=Product(Expon,DirP)
    nPoly := PolynomialRing(F,nExponent)
    gp:=GroebnerPackage(F,nExponent,VarSet1,nPoly)
    lf:List nPoly :=[]
    lp:List P:=[]
    for f in listf for i in 1.. repeat
      vec2:Vector(NNI):=new(nf,0$NNI)
      vec2.i:=1
      g:nPoly:=0$nPoly
      pol:=0$P
      while f^=0 repeat
        df:=degree(f-reductum f,lvint)

```

```

    lcf:=leadingCoefficient f
    pol:=pol+monopol(df,lcf,lvint)
    g:=g+monomial(lcf,makeprod(degree f,0))$nPoly
    f:=reductum f
    lp:=concat(pol,lp)
    lf:=concat(monomial(1,makeprod(0,directProduct vec2))-g,lf)
    npol:List P :=[v::P for v in nvar]
    leq : List Equation P :=
      [p = pol for p in npol for pol in reverse lp ]
    lf:=(groebner lf)$gp
    while lf^=[] repeat
      q:=lf.first
      dq:nExponent:=degree q
      n:=selectfirst (dq)
      if n=0 then leave "done"
      lf:=lf.rest
    solsn:List P:=[]
    for q in lf repeat
      g:Polynomial F :=0
      while q^=0 repeat
        dq:=degree q
        lcq:=leadingCoefficient q
        q:=reductum q
        vdq:=(selectsecond dq):Vector NNI
        g:=g+ lcq*
          _*/[p**vdq.j for p in npol for j in 1..]
      solsn:=concat(g,solsn)
    [solsn,leq]$ST

coerce(Id:List DPoly) : Ideal == [Id,false]

coerce(I:Ideal) : OutputForm ==
  Idl := I.idl
  empty? Idl => [0$DPoly] :: OutputForm
  Idl :: OutputForm

ideal(Id:List DPoly) :Ideal == [[f for f in Id|f^=0],false]

groebnerIdeal(Id:List DPoly) : Ideal == [Id,true]

generators(I:Ideal) : List DPoly == I.idl

groebner?(I:Ideal) : Boolean == I.isGr

one?(I:Ideal) : Boolean == element?(1, I)

zero?(I:Ideal) : Boolean == empty? (groebner I).idl

```

---

— IDEAL.dotabb —

```
"IDEAL" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IDEAL"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"IDEAL" -> "ALIST"
```

—

## 17.27 domain PR PolynomialRing

— PolynomialRing.input —

```
)set break resume
)sys rm -f PolynomialRing.output
)spool PolynomialRing.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PolynomialRing
--R PolynomialRing(R: Ring,E: OrderedAbelianMonoid) is a domain constructor
--R Abbreviation for PolynomialRing is PR
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PR
--R
--R----- Operations -----
--R ???: (R,%) -> %               ??? : (%,R) -> %
--R ??? : (%,%) -> %             ??? : (Integer,%) -> %
--R ??? : (PositiveInteger,%) -> % ??? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %             ?-? : (%,%) -> %
--R -? : % -> %                  ?? : (%,%) -> Boolean
--R 1 : () -> %                  0 : () -> %
--R ^? : (%,PositiveInteger) -> % coefficient : (%,E) -> R
--R coefficients : % -> List R    coerce : R -> %
--R coerce : Integer -> %         coerce : % -> OutputForm
--R degree : % -> E               ground : % -> R
--R ground? : % -> Boolean        hash : % -> SingleInteger
--R latex : % -> String           leadingCoefficient : % -> R
--R leadingMonomial : % -> %      map : ((R -> R),%) -> %
--R mapExponents : ((E -> E),%) -> % minimumDegree : % -> E
--R monomial : (R,E) -> %         monomial? : % -> Boolean
--R one? : % -> Boolean           pomopo! : (%,R,E,%) -> %
--R recip : % -> Union(%, "failed") reductum : % -> %
--R retract : % -> R              sample : () -> %
```

```

--R zero? : % -> Boolean          ?~=? : (%,% ) -> Boolean
--R ?? : (% ,Fraction Integer) -> % if R has ALGEBRA FRAC INT
--R ?? : (Fraction Integer,% ) -> % if R has ALGEBRA FRAC INT
--R ?? : (NonNegativeInteger,% ) -> %
--R ***? : (% ,NonNegativeInteger) -> %
--R ?/? : (% ,R) -> % if R has FIELD
--R ?? : (% ,NonNegativeInteger) -> %
--R associates? : (% ,%) -> Boolean if R has INTDOM
--R binomThmExpt : (% ,% ,NonNegativeInteger) -> % if R has COMRING
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(% ,"failed") if R has CHARNZ
--R coerce : Fraction Integer -> % if R has ALGEBRA FRAC INT or R has RETRACT FRAC INT
--R coerce : % -> % if R has INTDOM
--R content : % -> R if R has GCDDOM
--R exquo : (% ,R) -> Union(% ,"failed") if R has INTDOM
--R exquo : (% ,%) -> Union(% ,"failed") if R has INTDOM
--R fmeq : (% ,E,R,% ) -> % if E has CABMON and R has INTDOM
--R numberOfMonomials : % -> NonNegativeInteger
--R primitivePart : % -> % if R has GCDDOM
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retract : % -> Integer if R has RETRACT INT
--R retractIfCan : % -> Union(R ,"failed")
--R retractIfCan : % -> Union(Fraction Integer ,"failed") if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(Integer ,"failed") if R has RETRACT INT
--R subtractIfCan : (% ,%) -> Union(% ,"failed")
--R unit? : % -> Boolean if R has INTDOM
--R unitCanonical : % -> % if R has INTDOM
--R unitNormal : % -> Record(unit: % ,canonical: % ,associate: % ) if R has INTDOM
--R
--E 1

```

```

)spool
)lisp (bye)

```

— PolynomialRing.help —

```

=====
PolynomialRing examples
=====

```

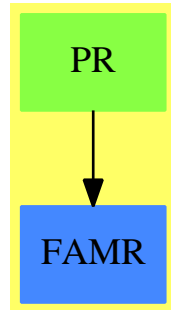
```

See Also:
o )show PolynomialRing

```



### 17.27.1 PolynomialRing (PR)



See

⇒ “FreeModule” (FM) 7.30.1 on page 980

⇒ “SparseUnivariatePolynomial” (SUP) 20.18.1 on page 2425

⇒ “UnivariatePolynomial” (UP) 22.4.1 on page 2784

#### Exports:

|                   |               |               |                    |                 |
|-------------------|---------------|---------------|--------------------|-----------------|
| 0                 | 1             | associates?   | binomThmExpt       | characteristic  |
| charthRoot        | coerce        | coefficient   | coefficients       | content         |
| degree            | exquo         | exquo         | fmech              | ground          |
| ground?           | hash          | latex         | leadingCoefficient | leadingMonomial |
| map               | mapExponents  | minimumDegree | monomial           | monomial?       |
| numberOfMonomials | one?          | popopo!       | primitivePart      | recip           |
| reductum          | retract       | retractIfCan  | sample             | subtractIfCan   |
| unit?             | unitCanonical | unitNormal    | zero?              | ?*?             |
| ?**?              | ?+?           | ?-?           | -?                 | ?=?             |
| ?^?               | ?~=?          | ?/?           |                    |                 |

— domain PR PolynomialRing —

```

)abbrev domain PR PolynomialRing
++ Author: Dave Barton, James Davenport, Barry Trager
++ Date Created:
++ Date Last Updated: 14.08.2000. Improved exponentiation [MMM/TTT]
++ Basic Functions: Ring, degree, coefficient, monomial, reductum
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This domain represents generalized polynomials with coefficients
++ (from a not necessarily commutative ring), and terms
++ indexed by their exponents (from an arbitrary ordered abelian monoid).
++ This type is used, for example,
++ by the \spadtype{DistributedMultivariatePolynomial} domain where
  
```

```

++ the exponent domain is a direct product of non negative integers.

PolynomialRing(R:Ring,E:OrderedAbelianMonoid): T == C
where
  T == FiniteAbelianMonoidRing(R,E) with
    --assertions
      if R has IntegralDomain and E has CancellationAbelianMonoid then
        fme cg: (% ,E,R,%) -> %
        ++ fme cg(p1,e,r,p2) finds x : p1 - r * x**e * p2
      if R has canonicalUnitNormal then canonicalUnitNormal
        ++ canonicalUnitNormal guarantees that the function
        ++ unitCanonical returns the same representative for all
        ++ associates of any particular element.

C == FreeModule(R,E) add
  --representations
    Term:= Record(k:E,c:R)
    Rep:= List Term

  --declarations
    x,y,p,p1,p2: %
    n: Integer
    nn: NonNegativeInteger
    np: PositiveInteger
    e: E
    r: R
  --local operations
    1 == [[0$E,1$R]]
    characteristic == characteristic$R
    numberOfMonomials x == (# x)$Rep
    degree p == if null p then 0 else p.first.k
    minimumDegree p == if null p then 0 else (last p).k
    leadingCoefficient p == if null p then 0$R else p.first.c
    leadingMonomial p == if null p then 0 else [p.first]
    reductum p == if null p then p else p.rest
    retractIfCan(p:%):Union(R,"failed") ==
      null p => 0$R
      not null p.rest => "failed"
      zero?(p.first.k) => p.first.c
      "failed"
    coefficient(p,e) ==
      for tm in p repeat
        tm.k=e => return tm.c
        tm.k < e => return 0$R
      0$R
    recip(p) ==
      null p => "failed"
      p.first.k > 0$E => "failed"
      (u:=recip(p.first.c)) case "failed" => "failed"

```

```

(u::R)::%

coerce(r) == if zero? r then 0$% else [[0$E,r]]
coerce(n) == (n::R)::%

ground?(p): Boolean == empty? p or (empty? rest p and zero? degree p)

qsetrest!: (Rep, Rep) -> Rep
qsetrest!(l: Rep, e: Rep): Rep == RPLACD(l, e)$Lisp

times!: (R, %) -> %
times: (R, E, %) -> %

entireRing? := R has EntireRing

times!(r: R, x: %): % ==
  res, endcell, newend, xx: Rep
  if entireRing? then
    for tx in x repeat tx.c := r*tx.c
  else
    xx := x
    res := empty()
    while not empty? xx repeat
      tx := first xx
      tx.c := r * tx.c
      if zero? tx.c then
        xx := rest xx
      else
        newend := xx
        xx := rest xx
        if empty? res then
          res := newend
          endcell := res
        else
          qsetrest!(endcell, newend)
          endcell := newend
    res;

--- term * polynomial
termTimes: (R, E, Term) -> Term
termTimes(r: R, e: E, tx:Term): Term == [e+tx.k, r*tx.c]
times(tco: R, tex: E, rx: %): % ==
  if entireRing? then
    map(x1+>termTimes(tco, tex, x1), rx::Rep)
  else
    [[tex + tx.k, r] for tx in rx::Rep | not zero? (r := tco * tx.c)]

-- local addm!

```

```

addm!: (Rep, R, E, Rep) -> Rep
-- p1 + coef*x^E * p2
-- 'spare' (commented out) is for storage efficiency (not so good for
-- performance though.
addm!(p1:Rep, coef:R, exp: E, p2:Rep): Rep ==
--local res, newend, last: Rep
res, newcell, endcell: Rep
spare: List Rep
res      := empty()
endcell := empty()
--spare  := empty()
while not empty? p1 and not empty? p2 repeat
    tx := first p1
    ty := first p2
    exy := exp + ty.k
    newcell := empty();
    if tx.k = exy then
        newcoef := tx.c + coef * ty.c
        if not zero? newcoef then
            tx.c := newcoef
            newcell := p1
        --else
        --      spare := cons(p1, spare)
        p1 := rest p1
        p2 := rest p2
    else if tx.k > exy then
        newcell := p1
        p1 := rest p1
    else
        newcoef := coef * ty.c
        if not entireRing? and zero? newcoef then
            newcell := empty()
        --else if empty? spare then
        --      ttt := [exy, newcoef]
        --      newcell := cons(ttt, empty())
        --else
        --      newcell := first spare
        --      spare := rest spare
        --      ttt := first newcell
        --      ttt.k := exy
        --      ttt.c := newcoef
        else
            ttt := [exy, newcoef]
            newcell := cons(ttt, empty())
        p2 := rest p2
    if not empty? newcell then
        if empty? res then
            res := newcell
            endcell := res
        else

```

```

                                qsetrest!(endcell, newcell)
                                endcell := newcell
    if not empty? p1 then -- then end is const * p1
        newcell := p1
    else -- then end is (coef, exp) * p2
        newcell := times(coef, exp, p2)
    empty? res => newcell
    qsetrest!(endcell, newcell)
    res
pomopo! (p1, r, e, p2) == addm!(p1, r, e, p2)
p1 * p2 ==
    xx := p1::Rep
    empty? xx => p1
    yy := p2::Rep
    empty? yy => p2
    zero? first(xx).k => first(xx).c * p2
    zero? first(yy).k => p1 * first(yy).c
    --if #xx > #yy then
    --    (xx, yy) := (yy, xx)
    --    (p1, p2) := (p2, p1)
    xx := reverse xx
    res : Rep := empty()
    for tx in xx repeat res:=addm!(res,tx.c,tx.k,yy)
    res

-- if R has EntireRing then
--     p1 * p2 ==
--         null p1 => 0
--         null p2 => 0
--         zero?(p1.first.k) => p1.first.c * p2
--         one? p2 => p1
--         +/[[t1.k+t2.k,t1.c*t2.c]$Term for t2 in p2]
--             for t1 in reverse(p1)]
--         -- This 'reverse' is an efficiency improvement:
--         -- reduces both time and space [Abbott/Bradford/Davenport]
--     else
--         p1 * p2 ==
--             null p1 => 0
--             null p2 => 0
--             zero?(p1.first.k) => p1.first.c * p2
--             one? p2 => p1
--             +/[[t1.k+t2.k,r]$Term for t2 in p2 | (r:=t1.c*t2.c) ^= 0]
--                 for t1 in reverse(p1)]
--             -- This 'reverse' is an efficiency improvement:
--             -- reduces both time and space [Abbott/Bradford/Davenport]
if R has CommutativeRing then
    p ** np == p ** (np pretend NonNegativeInteger)
    p ^ np == p ** (np pretend NonNegativeInteger)
    p ^ nn == p ** nn

```

```

p ** nn ==
  null p => 0
  zero? nn => 1
--   one? nn => p
  (nn = 1) => p
  empty? p.rest =>
    zero?(cc:=p.first.c ** nn) => 0
    [[nn * p.first.k, cc]]
  binomThmExpt([p.first], p.rest, nn)

if R has Field then
  unitNormal(p) ==
    null p or (lcf:=p.first.c) = 1 => [1,p,1]
    a := inv lcf
    [lcf::%, [[p.first.k,1],:(a * p.rest)], a::%]
  unitCanonical(p) ==
    null p or (lcf:=p.first.c) = 1 => p
    a := inv lcf
    [[p.first.k,1],:(a * p.rest)]
else if R has IntegralDomain then
  unitNormal(p) ==
    null p or p.first.c = 1 => [1,p,1]
    (u,cf,a):=unitNormal(p.first.c)
    [u::%, [[p.first.k,cf],:(a * p.rest)], a::%]
  unitCanonical(p) ==
    null p or p.first.c = 1 => p
    (u,cf,a):=unitNormal(p.first.c)
    [[p.first.k,cf],:(a * p.rest)]
if R has IntegralDomain then
  associates?(p1,p2) ==
    null p1 => null p2
    null p2 => false
    p1.first.k = p2.first.k and
    associates?(p1.first.c,p2.first.c) and
    ((p2.first.c exquo p1.first.c)::R * p1.rest = p2.rest)
p exquo r ==
  [(if (a:= tm.c exquo r) case "failed"
    then return "failed" else [tm.k,a])
   for tm in p] :: Union(%, "failed")
if E has CancellationAbelianMonoid then
  fmecg(p1::%,e:E,r:R,p2::%)::% ==      -- p1 - r * X**e * p2
  rout::%:= []
  r:= - r
  for tm in p2 repeat
    e2:= e + tm.k
    c2:= r * tm.c
    c2 = 0 => "next term"
  while not null p1 and p1.first.k > e2 repeat
    (rout:=[p1.first,:rout]; p1:=p1.rest) --use PUSH and POP?

```

```

      null p1 or p1.first.k < e2 => rout:=[[e2,c2],:rout]
      if (u:=p1.first.c + c2) ^= 0 then rout:=[[e2, u],:rout]
      p1:=p1.rest
      NRECONC(rout,p1)$Lisp
if R has approximate then
  p1 exquo p2 ==
    null p2 => error "Division by 0"
    p2 = 1 => p1
    p1=p2 => 1
  --(p1.lastElt.c exquo p2.lastElt.c) case "failed" => "failed"
  rout:= []@List(Term)
  while not null p1 repeat
    (a:= p1.first.c exquo p2.first.c)
    a case "failed" => return "failed"
    ee:= subtractIfCan(p1.first.k, p2.first.k)
    ee case "failed" => return "failed"
    p1:= fmech(p1.rest, ee, a, p2.rest)
    rout:= [[ee,a], :rout]
    null p1 => reverse(rout)::%    -- nreverse?
    "failed"
else -- R not approximate
  p1 exquo p2 ==
    null p2 => error "Division by 0"
    p2 = 1 => p1
  --(p1.lastElt.c exquo p2.lastElt.c) case "failed" => "failed"
  rout:= []@List(Term)
  while not null p1 repeat
    (a:= p1.first.c exquo p2.first.c)
    a case "failed" => return "failed"
    ee:= subtractIfCan(p1.first.k, p2.first.k)
    ee case "failed" => return "failed"
    p1:= fmech(p1.rest, ee, a, p2.rest)
    rout:= [[ee,a], :rout]
    null p1 => reverse(rout)::%    -- nreverse?
    "failed"
if R has Field then
  x/r == inv(r)*x

```

---

— PR.dotabb —

"PR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PR"]  
 "FAMR" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FAMR"]  
 "PR" -> "FAMR"

---

## 17.28 domain PI PositiveInteger

### — PositiveInteger.input —

```

)set break resume
)sys rm -f PositiveInteger.output
)spool PositiveInteger.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PositiveInteger
--R PositiveInteger is a domain constructor
--R Abbreviation for PositiveInteger is PI
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PI
--R
--R----- Operations -----
--R ?? : (%,%) -> %           ??? : (PositiveInteger,%) -> %
--R ??? : (%,PositiveInteger) -> %   ??+ : (%,%) -> %
--R ?<? : (%,%) -> Boolean         ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean         ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean       1 : () -> %
--R ??? : (%,PositiveInteger) -> %   coerce : % -> OutputForm
--R gcd : (%,%) -> %               hash : % -> SingleInteger
--R latex : % -> String           max : (%,%) -> %
--R min : (%,%) -> %              one? : % -> Boolean
--R recip : % -> Union(%, "failed")  sample : () -> %
--R ?~=? : (%,%) -> Boolean
--R ??? : (%,NonNegativeInteger) -> %
--R ??^? : (%,NonNegativeInteger) -> %
--R
--E 1

)spool
)lisp (bye)

```

### — PositiveInteger.help —

```

=====
PositiveInteger examples
=====

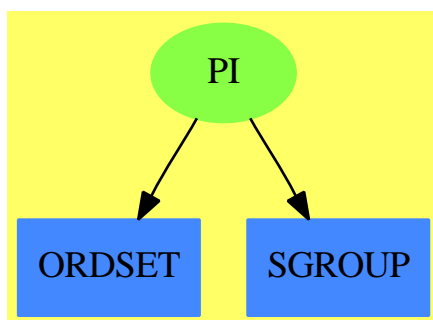
```

See Also:

o )show PositiveInteger



### 17.28.1 PositiveInteger (PI)



See

⇒ “Integer” (INT) 10.30.1 on page 1325

⇒ “NonNegativeInteger” (NNI) 15.5.1 on page 1702

⇒ “RomanNumeral” (ROMAN) 19.12.1 on page 2286

#### Exports:

|     |        |      |       |        |
|-----|--------|------|-------|--------|
| 1   | coerce | gcd  | hash  | latex  |
| max | min    | one? | recip | sample |
| ?^? | ?~=?   | ?**? | ?*?   | ?+?    |
| ?<? | ?<=?   | ?=?  | ?>?   | ?>=?   |

— domain PI PositiveInteger —

```

)abbrev domain PI PositiveInteger
++ Author: Mark Botch
++ Date Created:
++ Change History:
++ Basic Operations:
++ Related Constructors:
++ Keywords: positive integer
++ Description:
++ \spadtype{PositiveInteger} provides functions for
++ positive integers.

PositiveInteger: Join(AbelianSemiGroup,OrderedSet,Monoid) with
  gcd: (%,% ) -> %
    ++ gcd(a,b) computes the greatest common divisor of two
    ++ positive integers \spad{a} and b.
  commutative("*")
    ++ commutative("*") means multiplication is commutative : x*y = y*x
  
```

```

== SubDomain(NonNegativeInteger,#1 > 0) add
  x:%
  y:%

```

---

— PI.dotabb —

```

"PI" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PI",shape=ellipse]
"ORDSET" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ORDSET"]
"SGROUP" [color="#4488FF",href="bookvol10.2.pdf#nameddest=SGROUP"]
"PI" -> "ORDSET"
"PI" -> "SGROUP"

```

---

## 17.29 domain PF PrimeField

— PrimeField.input —

```

)set break resume
)sys rm -f PrimeField.output
)spool PrimeField.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PrimeField
--R PrimeField p: PositiveInteger is a domain constructor
--R Abbreviation for PrimeField is PF
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PF
--R
--R----- Operations -----
--R ?? : (Fraction Integer,%) -> %      ?? : (%,Fraction Integer) -> %
--R ?? : (%,%) -> %                    ?? : (Integer,%) -> %
--R ?? : (PositiveInteger,%) -> %      ***? : (%,Integer) -> %
--R ***? : (%,PositiveInteger) -> %    ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                  -? : % -> %
--R ?/? : (%,%) -> %                  ?=? : (%,%) -> Boolean
--R D : % -> %                        D : (%,NonNegativeInteger) -> %
--R 1 : () -> %                       0 : () -> %
--R ?? : (%,Integer) -> %             ?? : (%,PositiveInteger) -> %

```

```

--R algebraic? : % -> Boolean
--R basis : () -> Vector %
--R coerce : Fraction Integer -> %
--R coerce : Integer -> %
--R convert : % -> Integer
--R createPrimitiveElement : () -> %
--R differentiate : % -> %
--R factor : % -> Factored %
--R gcd : (%,%) -> %
--R inGroundField? : % -> Boolean
--R init : () -> %
--R latex : % -> String
--R lcm : (%,%) -> %
--R norm : % -> %
--R order : % -> PositiveInteger
--R primeFrobenius : % -> %
--R primitiveElement : () -> %
--R random : () -> %
--R ?rem? : (%,%) -> %
--R retract : % -> %
--R size : () -> NonNegativeInteger
--R squareFree : % -> Factored %
--R trace : % -> %
--R unit? : % -> Boolean
--R zero? : % -> Boolean
--R ?? : (NonNegativeInteger,%) -> %
--R ***? : (%,NonNegativeInteger) -> %
--R Frobenius : % -> % if $ has FINITE
--R Frobenius : (%,NonNegativeInteger) -> % if $ has FINITE
--R ?? : (%,NonNegativeInteger) -> %
--R basis : PositiveInteger -> Vector %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%, "failed")
--R conditionP : Matrix % -> Union(Vector %, "failed")
--R coordinates : Vector % -> Matrix %
--R createNormalElement : () -> % if $ has FINITE
--R definingPolynomial : () -> SparseUnivariatePolynomial %
--R degree : % -> OnePointCompletion PositiveInteger
--R differentiate : (%,NonNegativeInteger) -> %
--R discreteLog : % -> NonNegativeInteger
--R discreteLog : (%,%) -> Union(NonNegativeInteger, "failed")
--R divide : (%,%) -> Record(quotient: %, remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %,%) -> Union(List %, "failed")
--R exquo : (%,%) -> Union(%, "failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %), "failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R extensionDegree : () -> OnePointCompletion PositiveInteger
--R extensionDegree : () -> PositiveInteger
--R factorsOfCyclicGroupSize : () -> List Record(factor: Integer,exponent: Integer)
--R associates? : (%,%) -> Boolean
--R charthRoot : % -> %
--R coerce : % -> %
--R coerce : % -> OutputForm
--R coordinates : % -> Vector %
--R degree : % -> PositiveInteger
--R dimension : () -> CardinalNumber
--R gcd : List % -> %
--R hash : % -> SingleInteger
--R index : PositiveInteger -> %
--R inv : % -> %
--R lcm : List % -> %
--R lookup : % -> PositiveInteger
--R one? : % -> Boolean
--R prime? : % -> Boolean
--R primitive? : % -> Boolean
--R ?quo? : (%,%) -> %
--R recip : % -> Union(%, "failed")
--R represents : Vector % -> %
--R sample : () -> %
--R sizeLess? : (%,%) -> Boolean
--R squareFreePart : % -> %
--R transcendental? : % -> Boolean
--R unitCanonical : % -> %
--R ?~=? : (%,%) -> Boolean

```

```

--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolynomial %
--R generator : () -> % if $ has FINITE
--R linearAssociatedExp : (%,SparseUnivariatePolynomial %) -> % if $ has FINITE
--R linearAssociatedLog : % -> SparseUnivariatePolynomial % if $ has FINITE
--R linearAssociatedLog : (%,%) -> Union(SparseUnivariatePolynomial %,"failed") if $ has FINITE
--R linearAssociatedOrder : % -> SparseUnivariatePolynomial % if $ has FINITE
--R minimalPolynomial : % -> SparseUnivariatePolynomial %
--R minimalPolynomial : (%,PositiveInteger) -> SparseUnivariatePolynomial % if $ has FINITE
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R nextItem : % -> Union(%,"failed")
--R norm : (%,PositiveInteger) -> % if $ has FINITE
--R normal? : % -> Boolean if $ has FINITE
--R normalElement : () -> % if $ has FINITE
--R order : % -> OnePointCompletion PositiveInteger
--R primeFrobenius : (%,NonNegativeInteger) -> %
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R representationType : () -> Union("prime",polynomial,normal,cyclic)
--R retractIfCan : % -> Union(%,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger,NonNegativeInteger)
--R trace : (%,PositiveInteger) -> % if $ has FINITE
--R transcendenceDegree : () -> NonNegativeInteger
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

```

```

)spool
)lisp (bye)

```

---

— PrimeField.help —

```

=====
PrimeField examples
=====

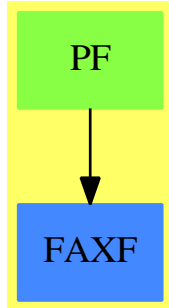
```

```

See Also:
o )show PrimeField

```

---

**17.29.1 PrimeField (PF)**

**See**

⇒ “InnerPrimeField” (IPF) 10.25.1 on page 1267

**Exports:**

|                           |                          |                     |
|---------------------------|--------------------------|---------------------|
| 0                         | 1                        | algebraic?          |
| associates?               | basis                    | characteristic      |
| charthRoot                | conditionP               | coordinates         |
| coerce                    | convert                  | coordinates         |
| createPrimitiveElement    | createNormalElement      | D                   |
| definingPolynomial        | degree                   | differentiate       |
| dimension                 | discreteLog              | discreteLog         |
| divide                    | euclideanSize            | expressIdealMember  |
| exquo                     | extendedEuclidean        | extensionDegree     |
| factor                    | factorsOfCyclicGroupSize | Frobenius           |
| gcd                       | gcdPolynomial            | generator           |
| hash                      | inGroundField?           | index               |
| init                      | inv                      | latex               |
| lcm                       | linearAssociatedExp      | linearAssociatedLog |
| linearAssociatedOrder     | lookup                   | minimalPolynomial   |
| multiEuclidean            | nextItem                 | norm                |
| normal?                   | normalElement            | one?                |
| order                     | prime?                   | primeFrobenius      |
| primitive?                | primitiveElement         | principalIdeal      |
| random                    | recip                    | representationType  |
| represents                | retract                  | retractIfCan        |
| sample                    | size                     | sizeLess?           |
| squareFree                | squareFreePart           | subtractIfCan       |
| tableForDiscreteLogarithm | trace                    | transcendenceDegree |
| transcendent?             | unit?                    | unitCanonical       |
| unitNormal                | zero?                    | ?*?                 |
| ？**?                      | ?+?                      | ?-?                 |
| -?                        | ?/?                      | ?=?                 |
| ?^?                       | ?~=?                     | ?quo?               |
| ?rem?                     |                          |                     |

— domain PF PrimeField —

```

)abbrev domain PF PrimeField
++ Authors: N.N.,
++ Date Created: November 1990, 26.03.1991
++ Date Last Updated: 31 March 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords: prime characteristic, prime field, finite field
++ References:
++ R.Lidl, H.Niederreiter: Finite Field, Encycoldia of Mathematics and
++ Its Applications, Vol. 20, Cambridge Univ. Press, 1983, ISBN 0 521 30240 4
++ Description:
++ PrimeField(p) implements the field with p elements if p is a prime number.

```

```

++ Error: if p is not prime.
++ Note: this domain does not check that argument is a prime.

PrimeField(p:PositiveInteger): Exp == Impl where
  Exp ==> Join(FiniteFieldCategory,FiniteAlgebraicExtensionField($),_
    ConvertibleTo(Integer))
  Impl ==> InnerPrimeField(p) add
    if not prime?(p)$IntegerPrimesPackage(Integer) then
      error "Argument to prime field must be a prime"

```

---

— PF.dotabb —

```

"PF" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PF"]
"FAXF" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FAXF"]
"PF" -> "FAXF"

```

---

## 17.30 domain PRIMARR PrimitiveArray

— PrimitiveArray.input —

```

)set break resume
)sys rm -f PrimitiveArray.output
)spool PrimitiveArray.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PrimitiveArray
--R PrimitiveArray S: Type is a domain constructor
--R Abbreviation for PrimitiveArray is PRIMARR
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PRIMARR
--R
--R----- Operations -----
--R concat : List % -> %                concat : (%,% ) -> %
--R concat : (S,% ) -> %                concat : (% ,S) -> %
--R construct : List S -> %              copy : % -> %
--R delete : (% ,Integer) -> %           ?.? : (% ,Integer) -> S
--R elt : (% ,Integer,S) -> S            empty : () -> %

```

```

--R empty? : % -> Boolean
--R eq? : (%,% ) -> Boolean
--R indices : % -> List Integer
--R insert : (S,% ,Integer) -> %
--R map : ((S -> S),%) -> %
--R qelt : (% ,Integer) -> S
--R sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?<? : (%,% ) -> Boolean if S has ORDSET
--R ?<=? : (%,% ) -> Boolean if S has ORDSET
--R ==? : (%,% ) -> Boolean if S has SETCAT
--R ?>? : (%,% ) -> Boolean if S has ORDSET
--R ?>=? : (%,% ) -> Boolean if S has ORDSET
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R convert : % -> InputForm if S has KONVERT INFORM
--R copyInto! : (%,% ,Integer) -> % if $ has shallowlyMutable
--R count : (S,% ) -> NonNegativeInteger if $ has finiteAggregate and S has SETCAT
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R delete : (% ,UniversalSegment Integer) -> %
--R ?.? : (% ,UniversalSegment Integer) -> %
--R entry? : (S,% ) -> Boolean if $ has finiteAggregate and S has SETCAT
--R eval : (% ,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (% ,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (% ,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (% ,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (% ,S) -> % if $ has shallowlyMutable
--R find : ((S -> Boolean),%) -> Union(S,"failed")
--R first : % -> S if Integer has ORDSET
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (% ,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R max : (%,% ) -> % if S has ORDSET
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (S,% ) -> Boolean if $ has finiteAggregate and S has SETCAT
--R members : % -> List S if $ has finiteAggregate
--R merge : (%,% ) -> % if S has ORDSET
--R merge : (((S,S) -> Boolean),%,%) -> %
--R min : (%,% ) -> % if S has ORDSET
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (% ,NonNegativeInteger) -> Boolean
--R parts : % -> List S if $ has finiteAggregate
--R position : (S,% ,Integer) -> Integer if S has SETCAT
--R position : (S,% ) -> Integer if S has SETCAT
--R position : ((S -> Boolean),%) -> Integer
--R qsetelt! : (% ,Integer,S) -> S if $ has shallowlyMutable
--R reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate

```



```

--R reduce : (((S,S) -> S),%,S,S) -> S if $ has finiteAggregate and S has SETCAT
--R remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (S,%) -> % if $ has finiteAggregate and S has SETCAT
--R removeDuplicates : % -> % if $ has finiteAggregate and S has SETCAT
--R reverse! : % -> % if $ has shallowlyMutable
--R select : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R setelt : (%,UniversalSegment Integer,S) -> S if $ has shallowlyMutable
--R setelt : (%,Integer,S) -> S if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : % -> % if S has ORDSET
--R sort : (((S,S) -> Boolean),%) -> %
--R sort! : % -> % if $ has shallowlyMutable and S has ORDSET
--R sort! : (((S,S) -> Boolean),%) -> % if $ has shallowlyMutable
--R sorted? : % -> Boolean if S has ORDSET
--R sorted? : (((S,S) -> Boolean),%) -> Boolean
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R ~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

```

```

)spool
)lisp (bye)

```

---

— PrimitiveArray.help —

```

=====
PrimitiveArray examples
=====

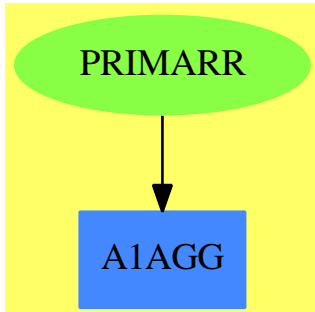
```

```

See Also:
o )show PrimitiveArray

```

## 17.30.1 PrimitiveArray (PRIMARR)



See

- ⇒ “Tuple” (TUPLE) 21.12.1 on page 2711
- ⇒ “IndexedFlexibleArray” (IFARRAY) 10.10.1 on page 1187
- ⇒ “FlexibleArray” (FARRAY) 7.14.1 on page 853
- ⇒ “IndexedOneDimensionalArray” (IARRAY1) 10.13.1 on page 1208
- ⇒ “OneDimensionalArray” (ARRAY1) 16.3.1 on page 1736

**Exports:**

|          |           |         |           |                  |
|----------|-----------|---------|-----------|------------------|
| any?     | coerce    | concat  | construct | convert          |
| copy     | copyInto! | count   | delete    | entry?           |
| elt      | empty     | empty?  | entries   | eq?              |
| eval     | every?    | fill!   | find      | first            |
| hash     | index?    | indices | insert    | insert           |
| latex    | less?     | map     | map!      | max              |
| maxIndex | member?   | members | merge     | min              |
| minIndex | more?     | new     | parts     | position         |
| qelt     | qsetelt!  | reduce  | remove    | removeDuplicates |
| reverse  | reverse!  | sample  | select    | setelt           |
| size?    | sort      | sort!   | sorted?   | swap!            |
| #?       | ??        | ?~=?    | ?<?       | ?<=?             |
| ?=?      | ?>?       | ?>=?    |           |                  |

— domain PRIMARR PrimitiveArray —

```

)abbrev domain PRIMARR PrimitiveArray
++ Author: Mark Botch
++ Description:
++ This provides a fast array type with no bound checking on elt's.
++ Minimum index is 0 in this type, cannot be changed
  
```

```

PrimitiveArray(S:Type): OneDimensionalArrayAggregate S == add
  Qmax ==> QVMAXINDEX$Lisp
  Qsize ==> QVSIZE$Lisp
--  Qelt ==> QVELT$Lisp
--  Qsetelt ==> QSETVELT$Lisp
  
```

---

---

— Product.input —

[illegible]

```

--S 2 of 6
f(3)
--R
--R
--R (2)  9
--R
--R                                         Type: PositiveInteger
--E 2

--S 3 of 6
g:=(x:INT):INT +-> x^3
--R
--R
--R (3)  theMap(Closure)
--R
--R                                         Type: (Integer -> Integer)
--E 3

--S 4 of 6
g(3)
--R
--R
--R (4)  27
--R
--R                                         Type: PositiveInteger
--E 4

--S 5 of 6
h(x:INT):Product(INT,INT) == makeprod(f x, g x)
--R
--R Function declaration h : Integer -> Product(Integer,Integer) has
--R been added to workspace.
--R
--R                                         Type: Void
--E 5

--S 6 of 6
h(3)
--R
--R Compiling function h with type Integer -> Product(Integer,Integer)
--R
--R (6)  (9,27)
--R
--R                                         Type: Product(Integer,Integer)
--E 6

)spool
)lisp (bye)

```

---

— Product.help —

=====

## Product examples

=====

The direct product of two functions over the same domain is another function over that domain whose co-domain is the product of their co-domains.

So we can define two functions,  $f$  and  $g$ , that go from  $\text{INT} \rightarrow \text{INT}$

```
f:=(x:INT):INT +-> 3*x
g:=(x:INT):INT +-> x^3
```

so

```
f(3) is 9
g(3) is 27
```

and we can construct the direct product of those functions  $h=f,g$

```
h(x:INT):Product(INT,INT) == makeprod(f x, g x)
```

so

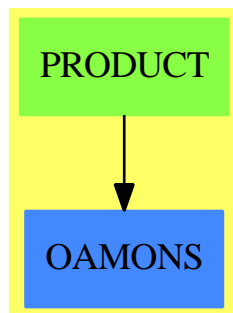
```
h(3) is (9,27)
```

See Also:

```
o )show Product
```

---

## 17.31.1 Product (PRODUCT)



Exports:

|               |        |             |              |           |
|---------------|--------|-------------|--------------|-----------|
| 0             | 1      | coerce      | commutator   | conjugate |
| hash          | index  | inv         | latex        | lookup    |
| makeprod      | max    | min         | one?         | random    |
| recip         | sample | selectfirst | selectsecond | size      |
| subtractIfCan | sup    | zero?       | ?=?          | ?~=?      |
| ?*?           | ?**?   | ?+?         | ?-?          | ?-?       |
| ?/?           | ?<?    | ?<=?        | ?>?          | ?>=?      |
| ?^?           |        |             |              |           |

— domain **PRODUCT** Product —

```

)abbrev domain PRODUCT Product
++ Author: Mark Botch
++ Description:
++ This domain implements cartesian product

Product (A:SetCategory,B:SetCategory) : C == T
where
  C == SetCategory with
    if A has Finite and B has Finite then Finite
    if A has Monoid and B has Monoid then Monoid
    if A has AbelianMonoid and B has AbelianMonoid then AbelianMonoid
    if A has CancellationAbelianMonoid and
      B has CancellationAbelianMonoid then CancellationAbelianMonoid
    if A has Group and B has Group then Group
    if A has AbelianGroup and B has AbelianGroup then AbelianGroup
    if A has OrderedAbelianMonoidSup and B has OrderedAbelianMonoidSup
      then OrderedAbelianMonoidSup
    if A has OrderedSet and B has OrderedSet then OrderedSet

  makeprod      : (A,B) -> %
  ++ makeprod(a,b) computes the product of two functions
  ++
  ++X f:=(x:INT):INT --> 3*x
  ++X g:=(x:INT):INT --> x^3
  ++X h(x:INT):Product(INT,INT) == makeprod(f x, g x)
  ++X h(3)
  selectfirst   : %   -> A
  ++ selectfirst(x) is not documented
  selectsecond  : %   -> B
  ++ selectsecond(x) is not documented

T == add

--representations
  Rep := Record(acomp:A,bcomp:B)

--declarations
  x,y: %

```

```

i: NonNegativeInteger
p: NonNegativeInteger
a: A
b: B
d: Integer

--define
coerce(x):OutputForm == paren [(x.accomp)::OutputForm,
                                (x.bcomp)::OutputForm]

x=y ==
  x.accomp = y.accomp => x.bcomp = y.bcomp
  false
makeprod(a:A,b:B) :% == [a,b]

selectfirst(x:%) : A == x.accomp
selectsecond (x:%) : B == x.bcomp

if A has Monoid and B has Monoid then
  1 == [1$A,1$B]
  x * y == [x.accomp * y.accomp,x.bcomp * y.bcomp]
  x ** p == [x.accomp ** p ,x.bcomp ** p]

if A has Finite and B has Finite then
  size == size$A () * size$B ()

if A has Group and B has Group then
  inv(x) == [inv(x.accomp),inv(x.bcomp)]

if A has AbelianMonoid and B has AbelianMonoid then
  0 == [0$A,0$B]

  x + y == [x.accomp + y.accomp,x.bcomp + y.bcomp]

  c:NonNegativeInteger * x == [c * x.accomp,c*x.bcomp]

if A has CancellationAbelianMonoid and
B has CancellationAbelianMonoid then
  subtractIfCan(x, y) : Union(%, "failed") ==
    (na:= subtractIfCan(x.accomp, y.accomp)) case "failed" => "failed"
    (nb:= subtractIfCan(x.bcomp, y.bcomp)) case "failed" => "failed"
    [na::A,nb::B]

if A has AbelianGroup and B has AbelianGroup then
  - x == [- x.accomp,-x.bcomp]
  (x - y):% == [x.accomp - y.accomp,x.bcomp - y.bcomp]
  d * x == [d * x.accomp,d * x.bcomp]

if A has OrderedAbelianMonoidSup and B has OrderedAbelianMonoidSup then
  sup(x,y) == [sup(x.accomp,y.accomp),sup(x.bcomp,y.bcomp)]

```

```

if A has OrderedSet and B has OrderedSet then
  x < y ==
    xa:= x.accomp ; ya:= y.accomp
    xa < ya => true
    xb:= x.bcomp ; yb:= y.bcomp
    xa = ya => (xb < yb)
    false

--  coerce(x:%):Symbol ==
--  PrintableForm()
--  formList([x.accomp::Expression,x.bcomp::Expression])$PrintableForm

```

---

— PRODUCT.dotabb —

```

"PRODUCT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PRODUCT"]
"OAMONS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=OAMONS"]
"PRODUCT" -> "OAMONS"

```

---

## 17.32 domain PROJPL ProjectivePlane

— ProjectivePlane.input —

```

)set break resume
)sys rm -f ProjectivePlane.output
)spool ProjectivePlane.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ProjectivePlane
--R ProjectivePlane K: Field is a domain constructor
--R Abbreviation for ProjectivePlane is PROJPL
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PROJPL
--R
--R----- Operations -----
--R ?? : (%,% ) -> Boolean          coerce : List K -> %
--R coerce : % -> List K          coerce : % -> OutputForm

```



```

--R conjugate : % -> %
--R degree : % -> PositiveInteger
--R hash : % -> SingleInteger
--R homogenize : (%,Integer) -> %
--R lastNonNull : % -> Integer
--R list : % -> List K
--R pointValue : % -> List K
--R rational? : % -> Boolean
--R ?~=? : (%,% ) -> Boolean
--R conjugate : (% ,NonNegativeInteger) -> %
--R orbit : (% ,NonNegativeInteger) -> List %
--R rational? : (% ,NonNegativeInteger) -> Boolean
--R removeConjugate : List % -> List %
--R removeConjugate : (List % ,NonNegativeInteger) -> List %
--R
--E 1

)spool
)lisp (bye)

```

---

— ProjectivePlane.help —

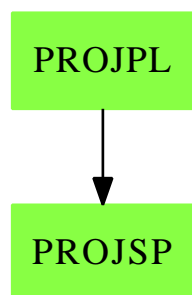
```

=====
ProjectivePlane examples
=====

See Also:
o )show ProjectivePlane

```

### 17.32.1 ProjectivePlane (PROJPL)



### 17.33. DOMAIN PROJPLPS PROJECTIVEPLANEOVERPSEUDOALGEBRAICCLOSUREOFFINITEFIELD

#### Exports:

|                 |           |                 |            |            |
|-----------------|-----------|-----------------|------------|------------|
| ??              | ?=?       | ?~=?            | coerce     | conjugate  |
| definingField   | degree    | hash            | homogenize | lastNonNul |
| lastNonNull     | latex     | list            | orbit      | pointValue |
| projectivePoint | rational? | removeConjugate | setelt     |            |

— domain PROJPL ProjectivePlane —

```
)abbrev domain PROJPL ProjectivePlane
++ Author: Gaetan Hache
++ Date Created: 17 nov 1992
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
++ This is part of the PAFF package, related to projective space.
ProjectivePlane(K):Exports == Implementation where
  K:Field

Exports ==> ProjectiveSpaceCategory(K)

Implementation ==> ProjectiveSpace(3,K)
```

\_\_\_\_\_

— PROJPL.dotabb —

```
"PROJPL" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PROJPL"];
"PROJSP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PROJSP"];
"PROJPL" -> "PROJSP"
```

\_\_\_\_\_

### 17.33 domain PROJPLPS ProjectivePlaneOverPseudoAlgebraicClosureOfFiniteField

— ProjectivePlaneOverPseudoAlgebraicClosureOfFiniteField.input —

```
)set break resume
)sys rm -f ProjectivePlaneOverPseudoAlgebraicClosureOfFiniteField.output
)spool ProjectivePlaneOverPseudoAlgebraicClosureOfFiniteField.output
)set message test on
)set message auto off
```

```

)clear all

--S 1 of 1
)show ProjectivePlaneOverPseudoAlgebraicClosureOfFiniteField
--R ProjectivePlaneOverPseudoAlgebraicClosureOfFiniteField K: FiniteFieldCategory is a domain
--R Abbreviation for ProjectivePlaneOverPseudoAlgebraicClosureOfFiniteField is PROJPLPS
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PROJPLPS
--R
--R----- Operations -----
--R ?=? : (%,% ) -> Boolean          coerce : % -> OutputForm
--R conjugate : % -> %                degree : % -> PositiveInteger
--R hash : % -> SingleInteger         homogenize : % -> %
--R homogenize : (%,Integer) -> %     lastNonNul : % -> Integer
--R lastNonNull : % -> Integer        latex : % -> String
--R orbit : % -> List %              rational? : % -> Boolean
--R ?~=? : (%,% ) -> Boolean
--R coerce : List PseudoAlgebraicClosureOfFiniteField K -> %
--R coerce : % -> List PseudoAlgebraicClosureOfFiniteField K
--R conjugate : (% ,NonNegativeInteger) -> %
--R definingField : % -> PseudoAlgebraicClosureOfFiniteField K
--R ?.? : (% ,Integer) -> PseudoAlgebraicClosureOfFiniteField K
--R list : % -> List PseudoAlgebraicClosureOfFiniteField K
--R orbit : (% ,NonNegativeInteger) -> List %
--R pointValue : % -> List PseudoAlgebraicClosureOfFiniteField K
--R projectivePoint : List PseudoAlgebraicClosureOfFiniteField K -> %
--R rational? : (% ,NonNegativeInteger) -> Boolean
--R removeConjugate : List % -> List %
--R removeConjugate : (List % ,NonNegativeInteger) -> List %
--R setelt : (% ,Integer,PseudoAlgebraicClosureOfFiniteField K) -> PseudoAlgebraicClosureOfFiniteField K
--R
--E 1

)spool
)lisp (bye)

```

— ProjectivePlaneOverPseudoAlgebraicClosureOfFiniteField.help —

```

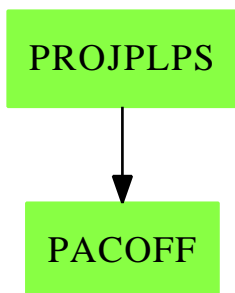
=====
ProjectivePlaneOverPseudoAlgebraicClosureOfFiniteField examples
=====

```

See Also:

```
o )show ProjectivePlaneOverPseudoAlgebraicClosureOfFiniteField
```

### 17.33.1 ProjectivePlaneOverPseudoAlgebraicClosureOfFiniteField (PROJPLPS)



#### Exports:

|               |                 |           |                 |            |
|---------------|-----------------|-----------|-----------------|------------|
| ??            | ?=?             | ?~=?      | coerce          | conjugate  |
| definingField | degree          | hash      | homogenize      | lastNonNul |
| lastNonNull   | latex           | list      | orbit           | orbit      |
| pointValue    | projectivePoint | rational? | removeConjugate | setelt     |

— domain PROJPLPS ProjectivePlaneOverPseudoAlgebraicClosureOfFiniteField —

```

)abbrev domain PROJPLPS ProjectivePlaneOverPseudoAlgebraicClosureOfFiniteField
++ Author: Gaetan Hache
++ Date Created: 17 nov 1992
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
++ This is part of the PAFF package, related to projective space.
ProjectivePlaneOverPseudoAlgebraicClosureOfFiniteField(K):Exp == Impl where
  K:FiniteFieldCategory

  KK  ==> PseudoAlgebraicClosureOfFiniteField(K)

  Exp ==> ProjectiveSpaceCategory(KK)

  Impl ==> ProjectivePlane(KK)
  
```

— PROJPLPS.dotabb —

```
"PROJPLPS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PROJPLPS"];
"PACOFF" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PACOFF"];
"PROJPLPS" -> "PACOFF"
```

## 17.34 domain PROJSP ProjectiveSpace

— ProjectiveSpace.input —

```
)set break resume
)sys rm -f ProjectiveSpace.output
)spool ProjectiveSpace.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ProjectiveSpace
--R ProjectiveSpace(dim: NonNegativeInteger,K: Field) is a domain constructor
--R Abbreviation for ProjectiveSpace is PROJSP
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PROJSP
--R
--R----- Operations -----
--R ?=? : (%,%) -> Boolean          coerce : List K -> %
--R coerce : % -> List K           coerce : % -> OutputForm
--R conjugate : % -> %             definingField : % -> K
--R degree : % -> PositiveInteger  ?.? : (%,Integer) -> K
--R hash : % -> SingleInteger       homogenize : % -> %
--R homogenize : (%,Integer) -> %   lastNonNul : % -> Integer
--R lastNonNull : % -> Integer      latex : % -> String
--R list : % -> List K             orbit : % -> List %
--R pointValue : % -> List K       projectivePoint : List K -> %
--R rational? : % -> Boolean       setelt : (%,Integer,K) -> K
--R ?~=? : (%,%) -> Boolean
--R conjugate : (%,NonNegativeInteger) -> %
--R orbit : (%,NonNegativeInteger) -> List %
--R rational? : (%,NonNegativeInteger) -> Boolean
--R removeConjugate : List % -> List %
--R removeConjugate : (List %,NonNegativeInteger) -> List %
--R
--E 1

)spool
)lisp (bye)
```

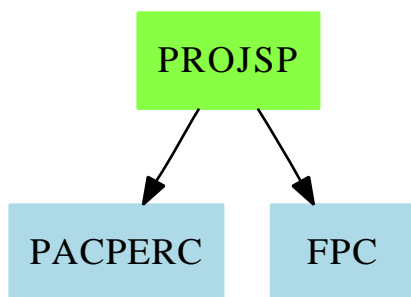
— ProjectiveSpace.help —

```
=====
ProjectiveSpace examples
=====
```

See Also:

```
o )show ProjectiveSpace
```

### 17.34.1 ProjectiveSpace (PROJSP)



#### Exports:

|               |                 |           |                 |            |
|---------------|-----------------|-----------|-----------------|------------|
| ?.?           | ?=?             | ?~=?      | coerce          | conjugate  |
| definingField | degree          | hash      | homogenize      | lastNonNul |
| lastNonNull   | latex           | list      | orbit           | orbit      |
| pointValue    | projectivePoint | rational? | removeConjugate | setelt     |

— domain PROJSP ProjectiveSpace —

```
)abbrev domain PROJSP ProjectiveSpace
++ Author: Gaetan Hache
++ Date Created: 17 nov 1992
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
++ This is part of the PAFF package, related to projective space.
ProjectiveSpace(dim,K):Exports == Implementation where
```

```
NNI ==> NonNegativeInteger
```

```
LIST ==> List
```

```

dim:NNI
K:Field

Exports ==> ProjectiveSpaceCategory(K)

Implementation ==> List(K) add

Rep:= List(K)

coerce(pt:%):OutputForm ==
  dd:OutputForm:= ":" :: OutputForm
  llout:List(OutputForm):=[ hconcat(dd, a::OutputForm) for a in rest pt]
  lout:= cons( (first pt)::OutputForm , llout)
  out:= hconcat lout
  oo:=paren(out)
  ee:OutputForm:= degree(pt) :: OutputForm
  oo**ee

definingField(pt)==
  K has PseudoAlgebraicClosureOfPerfectFieldCategory => _
    maxTower(pt pretend Rep)
  1$K

degree(pt)==
  K has PseudoAlgebraicClosureOfPerfectFieldCategory => _
    extDegree definingField pt
  1

coerce(pt:%):List(K) == pt pretend Rep

projectivePoint(pt:LIST(K))==
  pt :: %

list(ptt)==
  ptt pretend Rep

pointValue(ptt)==
  ptt pretend Rep

conjugate(p,e)==
  lp:Rep:=p
  pc:List(K):=[c**e for c in lp]
  projectivePoint(pc)

homogenize(ptt,nV)==
  if K has Field then
    pt:=list(ptt)$%
    zero?(pt.nV) => error "Impossible to homogenize this point"
    divPt:=pt.nV

```

```

      ((a/divPt) for a in pt])
    else
      ptt

rational?(p,n)== p=conjugate(p,n)

rational?(p)==rational?(p,characteristic()$K)

removeConjugate(l)==removeConjugate(l,characteristic()$K)

removeConjugate(l:LIST(%),n:NNI):LIST(%)==
  if K has FiniteFieldCategory then
    allconj:LIST(%):=empty()
    conjrem:LIST(%):=empty()
    for p in l repeat
      if ~member?(p,allconj) then
        conjrem:=cons(p,conjrem)
        allconj:=concat(allconj,orbit(p,n))
    conjrem
  else
    error "The field is not finite"

conjugate(p)==conjugate(p,characteristic()$K)

orbit(p)==orbit(p,characteristic()$K)

orbit(p,e)==
  if K has FiniteFieldCategory then
    l:LIST(%) := [p]
    np: % := conjugate(p,e)
    flag: ~ (np=p) :: Boolean
    while flag repeat
      l:=concat(np,l)
      np:=conjugate(np,e)
      flag:=not (np=p) :: Boolean
    l
  else
    error "Cannot compute the conjugate"

aa:% = bb:% ==
  ah:=homogenize(aa)
  bh:=homogenize(bb)
  ah =$Rep bh

coerce(pt:LIST(K))==
  ^(dim=#pt) => error "Le point n'a pas la bonne dimension"
  reduce("and",[zero?(a) for a in pt]) => _
    error "Ce n'est pas un point projectif"
  ptt: % := pt
  homogenize ptt

```



```

homogenize(ptt)==
  homogenize(ptt,lastNonNull(ptt))

nonZero?: K -> Boolean
nonZero?(a)==
  not(zero?(a))

lastNonNull(ptt)==
  pt:=ptt pretend Rep
  (dim pretend Integer)+1-
    (position("nonZero?",(reverse(pt)$LIST(K)))$LIST(K))

lastNonNul(pt)==lastNonNull(pt)

```

---

— PROJSP.dotabb —

```

"PROJSP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PROJSP"];
"PACPERC" [color=lightblue,href="bookvol10.2.pdf#nameddest=PACPERC"];
"FPC" [color=lightblue,href="bookvol10.2.pdf#nameddest=FPC"];
"PROJSP" -> "PACPERC"
"PROJSP" -> "FPC"

```

---

### 17.35 domain PACEXT PseudoAlgebraicClosureOfAlgExtOfRationalNumber

— PseudoAlgebraicClosureOfAlgExtOfRationalNumber.input —

```

)set break resume
)sys rm -f PseudoAlgebraicClosureOfAlgExtOfRationalNumber.output
)spool PseudoAlgebraicClosureOfAlgExtOfRationalNumber.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PseudoAlgebraicClosureOfAlgExtOfRationalNumber
--E 1

```

```
)spool
)lisp (bye)
```

\_\_\_\_\_

— PseudoAlgebraicClosureOfAlgExtOfRationalNumber.help —

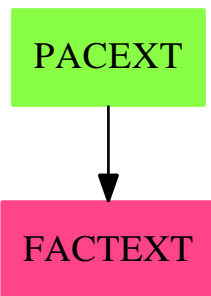
```
=====
PseudoAlgebraicClosureOfAlgExtOfRationalNumber examples
=====
```

See Also:

```
o )show PseudoAlgebraicClosureOfAlgExtOfRationalNumber
```

\_\_\_\_\_

### 17.35.1 PseudoAlgebraicClosureOfAlgExtOfRationalNumber (PACEXT)



Exports:

— domain PACEXT PseudoAlgebraicClosureOfAlgExtOfRational-  
Number —

```
)abbrev domain PACEXT PseudoAlgebraicClosureOfAlgExtOfRationalNumber
-- PseudoAlgebraicClosureOfAlgExtOfRationalNumber
++ Authors: Gaetan Hache
++ Date Created: jan 1998
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
++ This domain implement dynamic extension over the
++ PseudoAlgebraicClosureOfRationalNumber.
```

```

++ A tower extension T of the ground field K is any sequence of field
++ extension (T : K_0, K_1, ..., K_i...,K_n) where K_0 = K
++ and for i =1,2,...,n, K_i is an extension of K_{i-1} of degree > 1
++ and defined by an irreducible polynomial p(Z) in K_{i-1}.
++ Two towers (T_1: K_01, K_11,...,K_i1,...,K_n1) and
++ (T_2: K_02, K_12,...,K_i2,...,K_n2)
++ are said to be related if T_1 <= T_2 (or T_1 >= T_2),
++ that is if K_i1 = K_i2 for i=1,2,...,n1
++ (or i=1,2,...,n2). Any algebraic operations defined for several elements
++ are only defined if all of the concerned elements are coming from
++ a set of related tower extensions.
PseudoAlgebraicClosureOfAlgExtOfRationalNumber(downLevel:K):Exp == Impl where
  K      ==> PseudoAlgebraicClosureOfRationalNumber
  INT     ==> Integer
  NNI     ==> NonNegativeInteger
  SUP     ==> SparseUnivariatePolynomial
  BOOLEAN ==> Boolean
  PI      ==> PositiveInteger
  FACTRN  ==> FactorisationOverPseudoAlgebraicClosureOfAlgExtOfRationalNumber

recRep ==> Record(recEl:SUP(%),_
                  recTower:SUP(%),_
                  recDeg:PI,_
                  recPrevTower:%,_
                  recName:Symbol)

Exp == PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory with

  fullOutput: % -> OutputForm

  retractToGrn: % -> K

Impl == add
  Rep := Union(recRep,K)

  -- signature of local function
  replaceRecEl: (% ,SUP(%)) -> %

  down: % -> %

  retractPol( pol:SUP(%)) :SUP(K)==
    zero? pol => 0$SUP(K)
    lc := leadingCoefficient pol
    d := degree pol
    rlc := retractToGrn( lc )
    monomial( rlc , d )$SUP(K) + retractPol( reductum pol )

  retractToGrn(aa)==
    aa case K => aa
    a:=(aa pretend recRep)

```

```

el:= a.recEl
t:= a.recTower
d:= a.recDeg * extDegree downLevel
pt:= a.recPrevTower
n:= a.recName
newElement(retractPol el, retractPol t, d, retractToGrn pt, n)$K

newElement(pol,subF,inName) ==
  -- pol is an irreducible polynomial over the field extension
  -- given by subF.
  -- The output of this function is a root of pol.
dp:=degree pol
listCoef: List % := coefficients pol
a1:% := inv first listCoef
b1:% := last listCoef
rr:% := b1*a1
one?(dp) =>
  one?(#listCoef) => 0
  - rr
ground?(pol) => error "Cannot create a new element with a constant"
d:PI := (dp pretend PI) * extDegree(subF)
[monomial(1$,1)$SUP(%),pol,d,subF,inName] :: Rep

coerce(a:Integer):%== (a :: K)

down(a:%) ==
  a case K => a
  aa:=(a pretend recRep)
  el1:= aa.recEl
  ^ground?(el1)$SUP(%) => a
  gel:%:=ground(el1)
  down(gel)

n:INT * a:% ==
  one?(n) => a
  zero?(a) or zero?(n) => 0
  (n < 0) => -((-n)*a)
  mm:PositiveInteger:=(n pretend PositiveInteger)
  double(mm,a)$RepeatedDoubling(%)

replaceRecEl(a,el)==
  a case K => a
  aa:=copy a
  aa.recEl := el
  aa

localTower :% := downLevel

lift(a) ==
  a case K => monomial(a,0)

```

```

(a pretend recRep).recEl

lift(a,b)==
  extDegree a > extDegree b => _
    error "Cannot lift something at lower level !!!!!"
  extDegree a < extDegree b => monomial(a,0)$SUP(%)
  lift a

reduce(a)==
  localTower case K =>
    coefficient(a,0)
  ar:= a rem (localTower pretend recRep).recTower
  replaceRecEl(localTower,ar)

maxTower(la)==
  --return an element from the list la which is in the largest
  --extension of the ground field
  --PRECONDITION: all elements in same tower, else no meaning?
  m:="max"/[extDegree(a)$% for a in la]
  first [b for b in la | extDegree(b)=m]

ground?(a)== a case K

vectorise(a,lev)==
  da:=extDegree a
  dlev:=extDegree lev
  dlev < da => _
    error "Cannot vectorise at a lower level than the element to vectorise"
  lev case K => [a]
  pa:SUP(%)
  na:%
  ~(da = dlev) =>
    pa:= monomial(a,0)$SUP(%)
    na:= replaceRecEl(lev,pa)
    vectorise(na,lev)$%

prevLev:=previousTower(lev)
a case K => error "At this point a is not suppose to be in K"
aEl:=(a pretend recRep).recEl
daEl:=degree definingPolynomial(a)$%
lv:=[vectorise(c,prevLev)$% for c in entries(vectorise(aEl,daEl)$SUP(%))]
concat lv

retractIfCan(a:):Union(K,"failed")==
  a case K => a
  "failed"

retractIfCan(a:):Union(Integer,"failed")==
  a case K => retractIfCan(a)$K
  "failed"

```

```

setTower!(a) ==
  if a case K then
    localTower := downLevel
  else
    localTower:=a
  void()

definingPolynomial == definingPolynomial(localTower)

a:% + b:% ==
  (a case K) and (b case K) => a +$K b
  extDegree(a) > extDegree(b) => b + a
  res1:SUP(%)
  res2:%
  if extDegree(a) = extDegree(b) then
    res1:= b.recEl +$SUP(%) a.recEl
    res2:= replaceRecEl(b,res1)
  else
    res1:= b.recEl +$SUP(%) monomial(a,0)$SUP(%)
    res2:= replaceRecEl(b,res1)
  down(res2)

a:% * b:% ==
  (a case K) and (b case K) => a *$K b
  extDegree(a) > extDegree(b) => b * a
  res1:SUP(%)
  res2:%
  if extDegree(a) = extDegree(b) then
    res1:= b.recEl *$SUP(%) a.recEl rem b.recTower
    res2:= replaceRecEl(b,res1)
  else
    res1:= b.recEl *$SUP(%) monomial(a,0)$SUP(%)
    res2:= replaceRecEl(b,res1)
  down(res2)

distinguishedRootsOf(polyZero,ee) ==
  setTower!(ee)
  zero?(polyZero) => error "to lazy to give you all the roots of 0 !!!"
  factorf: Factored SUP % := factor(polyZero,localTower)$FACTRN(%)
  listFact:List SUP % := [pol.fctr for pol in factorList(factorf)]
  listOfZeros:List(%):=empty()
  for p in listFact repeat
    root:=newElement(p, new(E::Symbol)$Symbol)
    listOfZeros:List(%):=concat([ root ], listOfZeros)
  listOfZeros

1 == 1$K

0 == 0$K

```

```

newElement(poll:SUP(%),inName:Symbol)==
  newElement(poll,localTower,inName)$%

--Field operations
inv(a)==
  a case K => inv(a)$K
  aRecEl:= (a pretend recRep).recEl
  aDefPoly:= (a pretend recRep).recTower
  aInv := extendedEuclidean( aRecEl , aDefPoly, 1 )
  aInv case "failed" => error "PACOFF : division by zero"
  -- On doit retourner un Record representant l'inverse de a.
  -- Ce Record est exactement le mme que celui de a sauf
  -- qu'il faut remplacer le polynme du selecteur recEl
  -- par le polynme representant l'inverse de a :
  -- C'est ce que fait la fonction replaceRecEl.
  replaceRecEl( a , aInv.coef1 )

a:% / b:% == a * inv(b)

a:K * b:%==
  (a :: %) * b

b:% * a:K == a*b

a:% - b:% ==
  a + (-b)

a:% * b:Fraction(Integer) ==
  bn:=numer b
  bd:=denom b
  ebn:%:= bn * 1$%
  ebd:%:= bd * 1$%
  a * ebn * inv(ebd)

-a:% ==
  a case K => -$K a
  [-$SUP(%) (a pretend recRep).recEl,_
   (a pretend recRep).recTower,_
   (a pretend recRep).recDeg,_
   (a pretend recRep).recPrevTower,_
   (a pretend recRep).recName ]

bb:% = aa:% ==
  b:=down bb
  a:=down aa
  ^( extDegree(b) =$NNI extDegree(a) ) => false
  (b case K) => ( (retract a)@K =$K (retract b)@K )
  rda := a :: recRep
  rdb := b :: recRep

```

```

not (rda.recTower = $SUP(%) rdb.recTower) => false
rdb.recEl = $SUP(%) rda.recEl

zero?(a:%) ==
  da:=down a -- just to be sure !!!
  ~(da case K) => false
  zero?(da)$K

one?(a:%) ==
  da:= down a -- just to be sure !!!
  ~(da case K) => false
  one?(da)$K

coerce(a:K):% == a

coerce(a:%):OutputForm ==
  a case K => ((retract a)@K) ::OutputForm
  outputForm((a pretend recRep).recEl,
    ((a pretend recRep).recName)::OutputForm) $SUP(%)

fullOutput(a:%):OutputForm==
  a case K => ((retract a)@K) ::OutputForm
  (a pretend recRep)::OutputForm

definingPolynomial(a:%): SUP % ==
  a case K => monomial(1,1)$SUP(%)
  (a pretend recRep).recTower

extDegree(a:%): PI ==
  a case K => 1
  (a pretend recRep).recDeg

previousTower(a:~):% ==
  a case K => error "No previous extension for ground field element"
  (a pretend recRep).recPrevTower

name(a:~):Symbol ==
  a case K => error "No name for ground field element"
  (a pretend recRep).recName

characteristic == characteristic()$K

```

---

— PACEXT.dotabb —

"PACEXT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PACEXT"];  
 "FACTEXT" [color="#FF4488",href="bookvol10.4.pdf#nameddest=FACTEXT"]



"PACEXT" -> "FACTEXT"

## 17.36 domain PACOFF PseudoAlgebraicClosureOfFiniteField

— PseudoAlgebraicClosureOfFiniteField.input —

```
)set break resume
)sys rm -f PseudoAlgebraicClosureOfFiniteField.output
)spool PseudoAlgebraicClosureOfFiniteField.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PseudoAlgebraicClosureOfFiniteField
--R PseudoAlgebraicClosureOfFiniteField K: FiniteFieldCategory is a domain constructor
--R Abbreviation for PseudoAlgebraicClosureOfFiniteField is PACOFF
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PACOFF
--R
--R----- Operations -----
--R ?? : (%,K) -> %                ?? : (K,%) -> %
--R ?? : (Fraction Integer,%) -> %  ?? : (%,Fraction Integer) -> %
--R ?? : (%,%) -> %                ?? : (Integer,%) -> %
--R ?? : (PositiveInteger,%) -> %   ??? : (%,Integer) -> %
--R ??? : (%,PositiveInteger) -> %  ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                -? : % -> %
--R ?/? : (%,K) -> %                ?/? : (%,%) -> %
--R ?=? : (%,%) -> Boolean           D : % -> %
--R D : (%,NonNegativeInteger) -> %  1 : () -> %
--R 0 : () -> %                     ?? : (%,Integer) -> %
--R ?? : (%,PositiveInteger) -> %    algebraic? : % -> Boolean
--R associates? : (%,%) -> Boolean    charthRoot : % -> %
--R coerce : K -> %                  coerce : Fraction Integer -> %
--R coerce : % -> %                  coerce : Integer -> %
--R coerce : % -> OutputForm          conjugate : % -> %
--R createPrimitiveElement : () -> %  differentiate : % -> %
--R dimension : () -> CardinalNumber  extDegree : % -> PositiveInteger
--R factor : % -> Factored %          fullOutput : % -> OutputForm
--R gcd : List % -> %                 gcd : (%,%) -> %
--R ground? : % -> Boolean            hash : % -> SingleInteger
--R inGroundField? : % -> Boolean      index : PositiveInteger -> %
```

```

--R init : () -> %
--R latex : % -> String
--R lcm : (% , %) -> %
--R maxTower : List % -> %
--R order : % -> PositiveInteger
--R prime? : % -> Boolean
--R primitive? : % -> Boolean
--R ?quo? : (% , %) -> %
--R recip : % -> Union(%, "failed")
--R retract : % -> K
--R setTower! : % -> Void
--R sizeLess? : (% , %) -> Boolean
--R squareFreePart : % -> %
--R unit? : % -> Boolean
--R vectorise : (% , %) -> Vector %
--R ?~=?: (% , %) -> Boolean
--R ?*?: (NonNegativeInteger, %) -> %
--R ?**?: (% , NonNegativeInteger) -> %
--R Frobenius : % -> % if K has FINITE
--R Frobenius : (% , NonNegativeInteger) -> % if K has FINITE
--R ?^?: (% , NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%, "failed")
--R conditionP : Matrix % -> Union(Vector %, "failed")
--R definingPolynomial : () -> SparseUnivariatePolynomial %
--R definingPolynomial : % -> SparseUnivariatePolynomial %
--R degree : % -> OnePointCompletion PositiveInteger
--R differentiate : (% , NonNegativeInteger) -> %
--R discreteLog : % -> NonNegativeInteger
--R discreteLog : (% , %) -> Union(NonNegativeInteger, "failed")
--R distinguishedRootsOf : (SparseUnivariatePolynomial %, %) -> List %
--R divide : (% , %) -> Record(quotient: %, remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %, %) -> Union(List %, "failed")
--R exquo : (% , %) -> Union(%, "failed")
--R extendedEuclidean : (% , %, %) -> Union(Record(coef1: %, coef2: %), "failed")
--R extendedEuclidean : (% , %) -> Record(coef1: %, coef2: %, generator: %)
--R extensionDegree : () -> OnePointCompletion PositiveInteger
--R factorsOfCyclicGroupSize : () -> List Record(factor: Integer, exponent: Integer)
--R gcdPolynomial : (SparseUnivariatePolynomial %, SparseUnivariatePolynomial %) -> SparseUnivariatePolynomial %
--R lift : % -> SparseUnivariatePolynomial %
--R lift : (% , %) -> SparseUnivariatePolynomial %
--R multiEuclidean : (List %, %) -> Union(List %, "failed")
--R newElement : (SparseUnivariatePolynomial %, %, Symbol) -> %
--R newElement : (SparseUnivariatePolynomial %, Symbol) -> %
--R nextItem : % -> Union(%, "failed")
--R order : % -> OnePointCompletion PositiveInteger
--R primeFrobenius : (% , NonNegativeInteger) -> %
--R principalIdeal : List % -> Record(coef: List %, generator: %)
--R reduce : SparseUnivariatePolynomial % -> %
--R inv : % -> %
--R lcm : List % -> %
--R lookup : % -> PositiveInteger
--R one? : % -> Boolean
--R previousTower : % -> %
--R primeFrobenius : % -> %
--R primitiveElement : () -> %
--R random : () -> %
--R ?rem?: (% , %) -> %
--R sample : () -> %
--R size : () -> NonNegativeInteger
--R squareFree : % -> Factored %
--R transcendent? : % -> Boolean
--R unitCanonical : % -> %
--R zero? : % -> Boolean

```

```

--R representationType : () -> Union("prime",polynomial,normal,cyclic)
--R retractIfCan : % -> Union(K,"failed")
--R subtractIfCan : (%,%) -> Union(%, "failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger,NonNegativeInteger)
--R transcendenceDegree : () -> NonNegativeInteger
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)

```

---

— PseudoAlgebraicClosureOfFiniteField.help —

```

=====
PseudoAlgebraicClosureOfFiniteField examples
=====

```

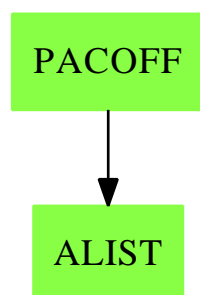
```

See Also:
o )show PseudoAlgebraicClosureOfFiniteField

```

---

### 17.36.1 PseudoAlgebraicClosureOfFiniteField (PACOFF)



Exports:

|                          |                           |                     |
|--------------------------|---------------------------|---------------------|
| 0                        | 1                         | -?                  |
| ?**?                     | ?*?                       | ?+?                 |
| ?-?                      | ?/?                       | ?=?                 |
| ?^?                      | ?^=?                      | ?quo?               |
| ?rem?                    | D                         | algebraic?          |
| associates?              | characteristic            | charthRoot          |
| coerce                   | conditionP                | conjugate           |
| createPrimitiveElement   | definingPolynomial        | degree              |
| differentiate            | dimension                 | discreteLog         |
| distinguishedRootsOf     | divide                    | euclideanSize       |
| expressIdealMember       | exquo                     | extDegree           |
| extendedEuclidean        | extensionDegree           | factor              |
| factorsOfCyclicGroupSize | Frobenius                 | fullOutput          |
| gcd                      | gcdPolynomial             | ground?             |
| hash                     | inGroundField?            | index               |
| init                     | inv                       | latex               |
| lcm                      | lift                      | lookup              |
| maxTower                 | multiEuclidean            | newElement          |
| nextItem                 | one?                      | order               |
| previousTower            | prime?                    | primeFrobenius      |
| primitive?               | primitiveElement          | principalIdeal      |
| random                   | recip                     | reduce              |
| representationType       | retract                   | retractIfCan        |
| sample                   | setTower!                 | size                |
| sizeLess?                | squareFree                | squareFreePart      |
| subtractIfCan            | tableForDiscreteLogarithm | transcendenceDegree |
| transcendent?            | unit?                     | unitCanonical       |
| unitNormal               | vectorise                 | zero?               |

— domain PACOFF PseudoAlgebraicClosureOfFiniteField —

```
)abbrev domain PACOFF PseudoAlgebraicClosureOfFiniteField
++ Authors: Gaetan Hache
++ Date Created: june 1996
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
++ This domain implement dynamic extension using the simple notion of
++ tower extensions. A tower extension T of the ground field K is any
++ sequence of field extension (T : K_0, K_1, ..., K_i...,K_n) where K_0 = K
++ and for i =1,2,...,n, K_i is an extension of K_{i-1} of degree > 1 and
++ defined by an irreducible polynomial p(Z) in K_{i-1}.
++ Two towers (T_1: K_01, K_11,...,K_i1,...,K_n1)
++ and (T_2: K_02, K_12,...,K_i2,...,K_n2) are said to be related
++ if T_1 <= T_2 (or T_1 >= T_2), that is if K_i1 = K_i2 for i=1,2,...,n1
++ (or i=1,2,...,n2). Any algebraic operations defined for several elements
++ are only defined if all of the concerned elements are coming from
++ a set of related tower extensions.
PseudoAlgebraicClosureOfFiniteField(K):Exports == Implementation where
```

```

K:FiniteFieldCategory

INT      ==> Integer
NNI      ==> NonNegativeInteger
SUP      ==> SparseUnivariatePolynomial
BOOLEAN  ==> Boolean
PI       ==> PositiveInteger
FFFACTSE ==> FiniteFieldFactorizationWithSizeParseBySideEffect

recRep ==> Record(recEl:SUP(%),_
                  recTower:SUP(%),_
                  recDeg:PI,_
                  recPrevTower:%,_
                  recName:Symbol)

Exports == Join(PseudoAlgebraicClosureOfFiniteFieldCategory,_
               ExtensionField(K)) with

fullOutput: % -> OutputForm

Implementation == add
Rep := Union(recRep,K)

-- signature of local function
replaceRecEl: (% ,SUP(%)) -> %
down: % -> %
localRandom: % -> %
repPolynomial : % -> SUP(%)

replaceRecEl(a,el)==
  a case K => a
  aa:=copy a
  aa.recEl := el
  aa

-- local variable
localTower :% := 1$K

localSize :NNI := size()$K
-- implemetation of exported function

degree(a)==
  da:PositiveInteger:= extDegree a
  coerce(da@PositiveInteger)$OnePointCompletion(PositiveInteger)

repPolynomial(a)==
  a case K => error "Is in ground field"
  (a pretend recRep).recEl

inv(a)==

```

```

a case K => inv(a)$K
aRecEl:= repPolynomial a
aDefPoly:= definingPolynomial a
aInv := extendedEuclidean( aRecEl , aDefPoly, 1 )
aInv case "failed" => error "PACOFF : division by zero"
down replaceRecEl( a , aInv.coef1 )

a:% ** n:PositiveInteger ==
zero?(a) => 0
expt( a , n )$RepeatedSquaring(%)

a:% ** n:NonNegativeInteger ==
zero?(a) and zero?(n) => error " --- 0^0 not defined "
zero?(n) => 1$%
a ** ( n pretend PositiveInteger )

a:% ** n:Integer ==
n < 0 => inv( a ** ( (-n) pretend PositiveInteger ) )
a ** ( n pretend NonNegativeInteger )

unitNormal(a)==
zero? a => [1,0,1]
[a,1,inv a]

ground?(a)== a case K

vectorise(a,lev)==
da:=extDegree a
dlev:=extDegree lev
dlev < da => _
error "Cannot vectorise at a lower level than the element to vectorise"
lev case K => [a]
pa:SUP(%)
na:%
^(da = dlev) =>
pa:= monomial(a,0)$SUP(%)
na:= replaceRecEl(lev,pa)
vectorise(na,lev)$%
prevLev:=previousTower(lev)
a case K => _
error "At this point a is not suppose to be in K, big error"
aEl:=(a pretend recRep).recEl
daEl:=degree(definingPolynomial a)$SUP(%)
lv:=[vectorise(c,prevLev)$% for c in entries(vectorise(aEl,daEl)$SUP(%))]
concat lv

size == localSize

setTower!(a) ==
localTower:=a

```

```

localSize:=(size())$K)**extDegree(a)
void()

localRandom(a) ==
  --return a random element at the extension of a
  a case K => random()$K
  subF:=previousTower(a)
  d:=degree(a.recTower)-1
  pol:=reduce("+",[monomial(localRandom(subF),i)$SUP(%) for i in 0..d])
  down replaceRecEl(a,pol)

a:% + b:% ==
  (a case K) and (b case K) => a +$K b
  extDegree(a) > extDegree(b) => b + a
  res1:SUP(%)
  res2:%
  if extDegree(a) = extDegree(b) then
    res1:= b.recEl +$SUP(%) a.recEl
    res2:= replaceRecEl(b,res1)
  else
    res1:= b.recEl +$SUP(%) monomial(a,0)$SUP(%)
    res2:= replaceRecEl(b,res1)
  down(res2)

a:% * b:% ==
  (a case K) and (b case K) => a *$K b
  extDegree(a) > extDegree(b) => b * a
  res1:SUP(%)
  res2:%
  if extDegree(a) = extDegree(b) then
    res1:= b.recEl *$SUP(%) a.recEl rem b.recTower
    res2:= replaceRecEl(b,res1)
  else
    res1:= b.recEl *$SUP(%) monomial(a,0)$SUP(%)
    res2:= replaceRecEl(b,res1)
  down(res2)

distinguishedRootsOf(polyZero,ee) ==
  setTower!(ee)
  zero?(polyZero) => error "to lazy to give you all the roots of 0 !!!"
  factorf: Factored SUP % := factor(polyZero)$FFFACTSE(%,SUP(%)
  listFact:List SUP % := [pol.fctr for pol in factorList(factorf)]
  listOfZeros:List(%) := empty()
  for p in listFact repeat
    root:=newElement(p, new(D::Symbol)$Symbol)
    listOfZeros:List(%) := concat([ root ], listOfZeros)
  listOfZeros

random==
  localRandom(localTower)

```

```

extDegOfGrdField:PI :=
  i: PI := 1
  while characteristic()$K ** i < size()$K repeat
    i:= i + 1
  i

charthRoot(a : %): % ==
  --return a**(1/characteristic )
  a case K => charthRoot(retract a)$K
  b:NNI := extDegree(a) * extDegOfGrdField
  j := subtractIfCan(b,1)
  if (j case "failed") then b:= 0
  else b:= j
  c:= (characteristic()$K) ** b
  a**c

conjugate(a)==
  a ** size()$K

1 == 1$K

0 == 0$K

newElement(pol:SUP(%),subF:%,inName:Symbol): % ==
  -- pol is an irreducible polynomial over the field extension
  -- given by subF.
  -- The output of this function is a root of pol.
  dp:=degree pol
  one?(dp) =>
    listCoef:=coefficients(pol)
    one?(#listCoef) => 0
    - last(listCoef) / first(listCoef)
  ground?(pol) => error "Cannot create a new element with a constant"
  d:PI := (dp pretend PI) * extDegree(subF)
  [monomial(1$%,1),pol,d,subF,inName] :: Rep

newElement(poll:SUP(%),inName:Symbol)==
  newElement(poll,localTower,inName)

maxTower(la)==
  --return an element from the list la which is in the largest
  --extension of the ground field
  --PRECONDITION: all elements in same tower, else no meaning?
  m:=reduce("max",[extDegree(a) for a in la])
  first [b for b in la | extDegree(b)=m]

--Field operations

a:% / b:% == a * inv(b)

```



```

a:K * b:%==
(a :: %) * b

b:% * a:K == a*b

a:% - b:% ==
a + (-b)

a:% * b:Fraction(Integer) ==
bn:=numer b
bd:=denom b
ebn:%:= bn * 1$%
ebd:%:= bd * 1$%
a * ebn * inv(ebd)

-a:% ==
a case K => -$K a
[-$SUP(%) (a pretend recRep).recEl,_
(a pretend recRep).recTower,_
(a pretend recRep).recDeg,_
(a pretend recRep).recPrevTower,_
(a pretend recRep).recName ]

n:INT * a:% ==
one?(n) => a
zero?(a) or zero?(n) => 0
(n < 0) => -((-n)*a)
mm:PositiveInteger:=(n pretend PositiveInteger)
double(mm,a)$RepeatedDoubling(%)

bb:% = aa:% ==
b:=down bb
a:=down aa
^( extDegree(b) =$NNI extDegree(a) ) => false
(b case K) => ( (retract a) =$K (retract b) )
rda := a :: recRep
rdb := b :: recRep
not (rda.recTower =$SUP(%) rdb.recTower) => false
rdb.recEl =$SUP(%) rda.recEl

zero?(a:%) ==
da:=down a -- just to be sure !!!
^(da case K) => false
zero?(da)$K

one?(a:%) ==
da:= down a -- just to be sure !!!
^(da case K) => false
one?(da)$K

```

```

--Coerce Functions

coerce(a:K) == a

retractIfCan(a)==
  a case K => a
  "failed"

coerce(a:%):OutputForm ==
  a case K => (retract a)::OutputForm
  outputForm((a pretend recRep).recEl, _
    ((a pretend recRep).recName)::OutputForm) $SUP(%)

fullOutput(a:%):OutputForm==
  a case K => (retract a)::OutputForm
  (a pretend recRep)::OutputForm

definingPolynomial(a:%): SUP % ==
  a case K => 1
  (a pretend recRep).recTower

extDegree(a:%): PI ==
  a case K => 1
  (a pretend recRep).recDeg

previousTower(a:%):% ==
  a case K => error "No previous extension for ground field element"
  (a pretend recRep).recPrevTower

name(a:%):Symbol ==
  a case K => error "No name for ground field element"
  (a pretend recRep).recName

-- function related to the ground field

lookup(a:%)==
  aa:=down a
  ^(aa case K) => _
    error "From NonGlobalDynamicExtensionOfFiniteField fnc Lookup: Cannot take i-dex"
  lookup(retract aa)$K

index(i)==(index(i)$K)

fromPrimeField? == characteristic()$K = size()$K

representationType == representationType()$K

characteristic == characteristic()$K

```

```

-- implementation of local functions

down(a:%) ==
  a case K => a
  aa:=(a pretend recRep)
  el1 := aa.recEl
  ^ground?(el1) => a
  gel:%=ground(el1)
  down(gel)

-----

— PACOFF.dotabb —

"PACOFF" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PACOFF"]
"ALIST"  [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"PACOFF" -> "ALIST"

```

## 17.37 domain PACRAT PseudoAlgebraicClosureOfRationalNumber

```

— PseudoAlgebraicClosureOfRationalNumber.input —

)set break resume
)sys rm -f PseudoAlgebraicClosureOfRationalNumber.output
)spool PseudoAlgebraicClosureOfRationalNumber.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PseudoAlgebraicClosureOfRationalNumber
--R PseudoAlgebraicClosureOfRationalNumber is a domain constructor
--R Abbreviation for PseudoAlgebraicClosureOfRationalNumber is PACRAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for PACRAT
--R
--R----- Operations -----
--R ??? : (%,Fraction Integer) -> %      ??? : (Fraction Integer,%) -> %
--R ??? : (Fraction Integer,%) -> %      ??? : (%,Fraction Integer) -> %
--R ??? : (%,%) -> %                    ??? : (Integer,%) -> %

```

```

--R ??? : (PositiveInteger,%) -> %
--R ??? : (%,PositiveInteger) -> %
--R ?-? : (%,%) -> %
--R ?/? : (%,Fraction Integer) -> %
--R ==? : (%,%) -> Boolean
--R 0 : () -> %
--R ?? : (%,PositiveInteger) -> %
--R associates? : (%,%) -> Boolean
--R coerce : Fraction Integer -> %
--R coerce : Fraction Integer -> %
--R coerce : Integer -> %
--R conjugate : % -> %
--R extDegree : % -> PositiveInteger
--R fullOutput : % -> OutputForm
--R gcd : (%,%) -> %
--R hash : % -> SingleInteger
--R inv : % -> %
--R lcm : List % -> %
--R maxTower : List % -> %
--R previousTower : % -> %
--R ?quo? : (%,%) -> %
--R ?rem? : (%,%) -> %
--R retract : % -> Fraction Integer
--R sample : () -> %
--R sizeLess? : (%,%) -> Boolean
--R squareFreePart : % -> %
--R unit? : % -> Boolean
--R vectorise : (%,%) -> Vector %
--R ~=? : (%,%) -> Boolean
--R ?? : (NonNegativeInteger,%) -> %
--R ??? : (%,NonNegativeInteger) -> %
--R Frobenius : % -> % if Fraction Integer has FINITE
--R Frobenius : (%,NonNegativeInteger) -> % if Fraction Integer has FINITE
--R ?? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%, "failed") if Fraction Integer has CHARNZ or Fraction Integer has FINITE
--R definingPolynomial : % -> SparseUnivariatePolynomial %
--R definingPolynomial : () -> SparseUnivariatePolynomial %
--R degree : % -> OnePointCompletion PositiveInteger
--R discreteLog : (%,%) -> Union(NonNegativeInteger, "failed") if Fraction Integer has CHARNZ or Fraction Integer has FINITE
--R distinguishedRootsOf : (SparseUnivariatePolynomial %,%) -> List %
--R divide : (%,%) -> Record(quotient: %, remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List %,%) -> Union(List %, "failed")
--R exquo : (%,%) -> Union(%, "failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %, coef2: %), "failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %, coef2: %, generator: %)
--R extensionDegree : () -> OnePointCompletion PositiveInteger
--R gcdPolynomial : (SparseUnivariatePolynomial %, SparseUnivariatePolynomial %) -> SparseUnivariatePolynomial %
--R lift : (%,%) -> SparseUnivariatePolynomial %

***? : (%,Integer) -> %
+? : (%,%) -> %
-? : % -> %
?/? : (%,%) -> %
1 : () -> %
?? : (%,Integer) -> %
algebraic? : % -> Boolean
coerce : Fraction Integer -> %
coerce : Integer -> %
coerce : % -> %
coerce : % -> OutputForm
dimension : () -> CardinalNumber
factor : % -> Factored %
gcd : List % -> %
ground? : % -> Boolean
inGroundField? : % -> Boolean
latex : % -> String
lcm : (%,%) -> %
one? : % -> Boolean
prime? : % -> Boolean
recip : % -> Union(%, "failed")
retract : % -> Fraction Integer
retract : % -> Integer
setTower! : % -> Void
squareFree : % -> Factored %
transcendent? : % -> Boolean
unitCanonical : % -> %
zero? : % -> Boolean

```

```

--R lift : % -> SparseUnivariatePolynomial %
--R multiEuclidean : (List %,%) -> Union(List %,"failed")
--R newElement : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %,PositiveInteger,%
--R newElement : (SparseUnivariatePolynomial %,Symbol) -> %
--R newElement : (SparseUnivariatePolynomial %,%,Symbol) -> %
--R order : % -> OnePointCompletion PositiveInteger if Fraction Integer has CHARNZ or Fraction
--R primeFrobenius : % -> % if Fraction Integer has CHARNZ or Fraction Integer has FINITE
--R primeFrobenius : (%,NonNegativeInteger) -> % if Fraction Integer has CHARNZ or Fraction
--R principalIdeal : List % -> Record(coef: List %,generator: %)
--R reduce : SparseUnivariatePolynomial % -> %
--R retractIfCan : % -> Union(Fraction Integer,"failed")
--R retractIfCan : % -> Union(Fraction Integer,"failed")
--R retractIfCan : % -> Union(Integer,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R transcendenceDegree : () -> NonNegativeInteger
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)

```

---

— PseudoAlgebraicClosureOfRationalNumber.help —

```

=====
PseudoAlgebraicClosureOfRationalNumber examples
=====

```

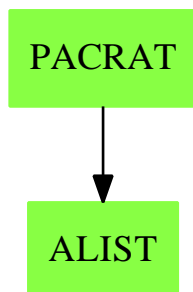
See Also:

```

o )show PseudoAlgebraicClosureOfRationalNumber

```

## 17.37.1 PseudoAlgebraicClosureOfRationalNumber (PACRAT)



## Exports:

|                |                   |                     |                      |      |
|----------------|-------------------|---------------------|----------------------|------|
| 0              | 0                 | 1                   | -?                   | ?**? |
| ?*?            | ?+?               | ?-?                 | ?/?                  |      |
| ?=?            | ?^?               | ?~=?                | ?quo?                |      |
| ?rem?          | algebraic?        | associates?         | characteristic       |      |
| charthRoot     | coerce            | conjugate           | definingPolynomial   |      |
| degree         | dimension         | discreteLog         | distinguishedRootsOf |      |
| divide         | euclideanSize     | expressIdealMember  | exquo                |      |
| extDegree      | extendedEuclidean | extensionDegree     | factor               |      |
| Frobenius      | fullOutput        | gcd                 | gcdPolynomial        |      |
| ground?        | hash              | inGroundField?      | inv                  |      |
| latex          | lcm               | lift                | maxTower             |      |
| multiEuclidean | newElement        | one?                | order                |      |
| previousTower  | prime?            | primeFrobenius      | principalIdeal       |      |
| recip          | reduce            | retract             | retractIfCan         |      |
| sample         | setTower!         | sizeLess?           | squareFree           |      |
| squareFreePart | subtractIfCan     | transcendenceDegree | transcendent?        |      |
| unit?          | unitCanonical     | unitNormal          | vectorise            |      |
| zero?          |                   |                     |                      |      |

— domain PACRAT PseudoAlgebraicClosureOfRationalNumber —

```

)abbrev domain PACRAT PseudoAlgebraicClosureOfRationalNumber
++ Authors: Gaetan Hache
++ Date Created: feb 1997
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
++ This domain implements dynamic extension using the simple notion of
++ tower extensions. A tower extension T of the ground field K is any
++ sequence of field extension (T : K_0, K_1, ..., K_i...,K_n) where K_0 = K
++ and for i =1,2,...,n, K_i is an extension of K_{i-1} of degree > 1 and
++ defined by an irreducible polynomial p(Z) in K_{i-1}.
++ Two towers (T_1: K_01, K_11,...,K_i1,...,K_n1) and
++ (T_2: K_02, K_12,...,K_i2,...,K_n2) are said to be related if T_1 <= T_2

```

```

++ (or T_1 >= T_2), that is if K_i1 = K_i2 for i=1,2,...,n1
++ (or i=1,2,...,n2). Any algebraic operations defined for several elements
++ are only defined if all of the concerned elements are coming from
++ a set of related tour extensions.
PseudoAlgebraicClosureOfRationalNumber:Exports == Implementation where

INT      ==> Integer
K        ==> Fraction Integer
NNI      ==> NonNegativeInteger
SUP      ==> SparseUnivariatePolynomial
BOOLEAN ==> Boolean
PI       ==> PositiveInteger
FACTRN   ==> FactorisationOverPseudoAlgebraicClosureOfRationalNumber

recRep   ==> Record(recEl:SUP(%),_
                    recTower:SUP(%),_
                    recDeg:PI,_
                    recPrevTower:%,_
                    recName:Symbol)

Exports == PseudoAlgebraicClosureOfRationalNumberCategory with

fullOutput: % -> OutputForm

newElement: (SUP(%), SUP(%), PI, %, Symbol) -> %

Implementation == add
Rep := Union(recRep,K)

-- signature of local function
replaceRecEl: (%,SUP(%)) -> %
down: % -> %

down(a:%) ==
  a case K => a
  aa:=(a pretend recRep)
  ele1 := aa.recEl
  ^ground?(ele1)$SUP(%) => a
  gel:%=ground(ele1)
  down(gel)

coerce(a:Integer):%== (a :: K)

n:INT * a:% ==
  one?(n) => a
  zero?(a) or zero?(n) => 0
  (n < 0) => -((-n)*a)
mm:PositiveInteger:=(n pretend PositiveInteger)
double(mm,a)$RepeatedDoubling(%)

```

```

replaceRecEl(a,el)==
  a case K => a
  aa:=copy a
  aa.recEl := el
  aa

-- local variable
localTower :% := 1$K

-- implemetation of exported function

lift(a) ==
  a case K => monomial(a,0)
  (a pretend recRep).recEl

lift(a,b)==
  extDegree a > extDegree b => _
    error "Cannot lift something at lower level !!!!!"
  extDegree a < extDegree b => monomial(a,0)$SUP(%)
  lift a

reduce(a)==
  localTower case K =>
    coefficient(a,0)
  ar:= a rem (localTower pretend recRep).recTower
  replaceRecEl(localTower,ar)

maxTower(la)==
  --return an element from the list la which is in the largest
  --extension of the ground field
  --PRECONDITION: all elements in same tower, else no meaning?
  m:="max"/[extDegree(a)$% for a in la]
  first [b for b in la | extDegree(b)=m]

ground?(a)== a case K

vectorise(a,lev)==
  da:=extDegree a
  dlev:=extDegree lev
  dlev < da => _
    error "Cannot vectorise at a lower level than the element to vectorise"
  lev case K => [a]
  pa:SUP(%)
  na:%
  ~(da = dlev) =>
    pa:= monomial(a,0)$SUP(%)
    na:= replaceRecEl(lev,pa)
    vectorise(na,lev)$%
  prevLev:=previousTower(lev)
  a case K => error "At this point a is not suppose to be in K"

```



```

aEl:=(a pretend recRep).recEl
daEl:=degree definingPolynomial(a)$%
lv:=[vectorise(c,prevLev)$% for c in entries(vectorise(aEl,daEl)$SUP(%))]
concat lv

setTower!(a) ==
  localTower:=a
  void()

definingPolynomial == definingPolynomial(localTower)

a:% + b:% ==
  (a case K) and (b case K) => a +$K b
  extDegree(a) > extDegree(b) => b + a
  res1:SUP(%)
  res2:%
  if extDegree(a) = extDegree(b) then
    res1:= b.recEl +$SUP(%) a.recEl
    res2:= replaceRecEl(b,res1)
  else
    res1:= b.recEl +$SUP(%) monomial(a,0)$SUP(%)
    res2:= replaceRecEl(b,res1)
  down(res2)

a:% * b:% ==
  (a case K) and (b case K) => a *$K b
  extDegree(a) > extDegree(b) => b * a
  res1:SUP(%)
  res2:%
  if extDegree(a) = extDegree(b) then
    res1:= b.recEl *$SUP(%) a.recEl rem b.recTower
    res2:= replaceRecEl(b,res1)
  else
    res1:= b.recEl *$SUP(%) monomial(a,0)$SUP(%)
    res2:= replaceRecEl(b,res1)
  down(res2)

distinguishedRootsOf(polyZero,ee) ==
  setTower!(ee)
  zero?(polyZero) => error "to lazy to give you all the roots of 0 !!!"
  factorf: Factored SUP % := factor(polyZero,ee)$FACTRN(%)
  listFact:List SUP % := [pol.fctr for pol in factorList(factorf)]
  listOfZeros:List(%) := empty()
  for p in listFact repeat
    root:=newElement(p, new(D::Symbol)$Symbol)
    listOfZeros:List(%) := concat([ root ], listOfZeros)
  listOfZeros

1 == 1$K

```

```

0 == 0$K

newElement(pol:SUP(%),subF:%,inName:Symbol): % ==
  -- pol is an irreducible polynomial over the field extension
  -- given by subF.
  -- The output of this function is a root of pol.
dp:=degree pol
one?(dp) =>
  listCoef:=coefficients(pol)
  one?(#listCoef) => 0
  - last(listCoef) / first(listCoef)
ground?(pol) => error "Cannot create a new element with a constant"
d:PI := (dp pretend PI) * extDegree(subF)
[monomial(1$%,1),pol,d,subF,inName] :: Rep

newElement(poll:SUP(%),inName:Symbol)==
  newElement(poll,localTower,inName)

newElement(elPol:SUP(%),pol:SUP(%),d:PI,subF:%,inName:Symbol): % ==
  [elPol, pol,d,subF,inName] :: Rep

--Field operations
inv(a)==
  a case K => inv(a)$K
  aRecEl:= (a pretend recRep).recEl
  aDefPoly:= (a pretend recRep).recTower
  aInv := extendedEuclidean( aRecEl , aDefPoly, 1 )
  aInv case "failed" => error "PACOFF : division by zero"
  -- On doit retourner un Record representant l'inverse de a.
  -- Ce Record est exactement le mme que celui de a sauf
  -- qu'il faut remplacer le polynme du selecteur recEl
  -- par le polynme representant l'inverse de a :
  -- C'est ce que fait la fonction replaceRecEl.
  replaceRecEl( a , aInv.coef1 )

a:% / b:% == a * inv(b)

a:K * b:%==
  (a :: %) * b

b:% * a:K == a*b

a:% - b:% ==
  a + (-b)

a:% * b:Fraction(Integer) ==
  bn:=numer b
  bd:=denom b
  ebn%:= bn * 1$%
  ebd%:= bd * 1$%

```

```

a * ebn * inv(ebd)

-a:% ==
  a case K => -$K a
  [-$SUP(%) (a pretend recRep).recEl,_
   (a pretend recRep).recTower,_
   (a pretend recRep).recDeg,_
   (a pretend recRep).recPrevTower,_
   (a pretend recRep).recName ]

bb:% = aa:% ==
  b:=down bb
  a:=down aa
  ^( extDegree(b) =$NNI extDegree(a) ) => false
  (b case K) => ( (retract a)@K =$K (retract b)@K )
  rda := a :: recRep
  rdb := b :: recRep
  not (rda.recTower =$SUP(%) rdb.recTower) => false
  rdb.recEl =$SUP(%) rda.recEl

zero?(a:%) ==
  da:=down a -- just to be sure !!!
  ~(da case K) => false
  zero?(da)$K

one?(a:%) ==
  da:= down a -- just to be sure !!!
  ~(da case K) => false
  one?(da)$K

--Coerce Functions

coerce(a:K):% == a

retractIfCan(a:%):Union(Integer,"failed")==
  a case K => retractIfCan(a)$K
  "failed"

retractIfCan(a:%):Union(K,"failed")==
  a case K => a
  "failed"

coerce(a:%):OutputForm ==
  a case K => ((retract a)@K) ::OutputForm
  outputForm((a pretend recRep).recEl,_
              ((a pretend recRep).recName)::OutputForm) $SUP(%)

fullOutput(a:%):OutputForm==
  a case K => ((retract a)@K) ::OutputForm
  (a pretend recRep)::OutputForm

```

17.37. DOMAIN PACRAT PSEUDOALGEBRAIC CLOSURE OF RATIONAL NUMBER 2111

```

definingPolynomial(a:%): SUP % ==
  a case K => monomial(1,1)$SUP(%)
  (a pretend recRep).recTower

extDegree(a:%): PI ==
  a case K => 1
  (a pretend recRep).recDeg

previousTower(a:%):% ==
  a case K => error "No previous extension for ground field element"
  (a pretend recRep).recPrevTower

name(a:%):Symbol ==
  a case K => error "No name for ground field element"
  (a pretend recRep).recName

-- function related to the ground field

characteristic == characteristic()$K

```

---

— PACRAT.dotabb —

```

"PACRAT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=PRODUCT"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"PACRAT" -> "ALIST"

```

---



## Chapter 18

# Chapter Q

### 18.1 domain QFORM QuadraticForm

— QuadraticForm.input —

```
)set break resume
)sys rm -f QuadraticForm.output
)spool QuadraticForm.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show QuadraticForm
--R QuadraticForm(n: PositiveInteger,K: Field) is a domain constructor
--R Abbreviation for QuadraticForm is QFORM
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for QFORM
--R
--R----- Operations -----
--R ??? : (Integer,%) -> %          ??? : (PositiveInteger,%) -> %
--R ?+? : (%,%) -> %              ?-? : (%,%) -> %
--R -? : % -> %                  ?=? : (%,%) -> Boolean
--R 0 : () -> %                  coerce : % -> OutputForm
--R ?.? : (%,DirectProduct(n,K)) -> K  hash : % -> SingleInteger
--R latex : % -> String           matrix : % -> SquareMatrix(n,K)
--R sample : () -> %             zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ??? : (NonNegativeInteger,%) -> %
--R quadraticForm : SquareMatrix(n,K) -> %
--R subtractIfCan : (%,%) -> Union(%, "failed")
--R
```

```
--E 1
```

```
)spool
)lisp (bye)
```

```
-----
```

```
— QuadraticForm.help —
```

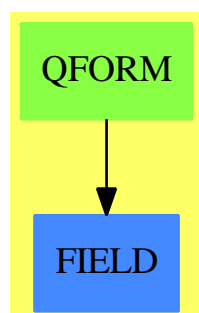
```
=====
QuadraticForm examples
=====
```

```
See Also:
```

```
o )show QuadraticForm
```

```
-----
```

### 18.1.1 QuadraticForm (QFORM)



See

⇒ “CliffordAlgebra” (CLIF) 4.5.12 on page 386

**Exports:**

|               |        |               |       |        |
|---------------|--------|---------------|-------|--------|
| 0             | coerce | hash          | latex | matrix |
| quadraticForm | sample | subtractIfCan | zero? | ?~=?   |
| ?*?           | ?.?    | ?+?           | ?-?   | -?     |
| ?=?           |        |               |       |        |

```
— domain QFORM QuadraticForm —
```

```
)abbrev domain QFORM QuadraticForm
++ Author: Stephen M. Watt
++ Date Created: August 1988
++ Date Last Updated: May 17, 1991
```

```

++ Basic Operations: quadraticForm, elt
++ Related Domains: Matrix, SquareMatrix
++ Also See:
++ AMS Classifications:
++ Keywords: quadratic form
++ Examples:
++ References:
++
++ Description:
++ This domain provides modest support for quadratic forms.

```

```

QuadraticForm(n, K): T == Impl where

```

```

  n: PositiveInteger
  K: Field
  SM ==> SquareMatrix
  V ==> DirectProduct

```

```

  T ==> AbelianGroup with
    quadraticForm: SM(n, K) -> %
      ++ quadraticForm(m) creates a quadratic form from a symmetric,
      ++ square matrix m.
    matrix: % -> SM(n, K)
      ++ matrix(qf) creates a square matrix from the quadratic form qf.
    elt: (% , V(n, K)) -> K
      ++ elt(qf,v) evaluates the quadratic form qf on the vector v,
      ++ producing a scalar.

```

```

Impl ==> SM(n,K) add
  Rep := SM(n,K)

```

```

  quadraticForm m ==
    not symmetric? m =>
      error "quadraticForm requires a symmetric matrix"
    m::%
  matrix q == q pretend SM(n,K)
  elt(q,v) == dot(v, (matrix q * v))

```

---

— QFORM.dotabb —

```

"QFORM" [color="#88FF44",href="bookvol10.3.pdf#nameddest=QFORM"]
"FIELD" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FIELD"]
"QFORM" -> "FIELD"

```

---



## 18.2 domain QALGSET QuasiAlgebraicSet

— QuasiAlgebraicSet.input —

```
)set break resume
)sys rm -f QuasiAlgebraicSet.output
)spool QuasiAlgebraicSet.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show QuasiAlgebraicSet
--R QuasiAlgebraicSet(R: GcdDomain,Var: OrderedSet,Expon: OrderedAbelianMonoidSup,Dpoly: Poly)
--R Abbreviation for QuasiAlgebraicSet is QALGSET
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for QALGSET
--R
--R----- Operations -----
--R ?? : (%,% ) -> Boolean          coerce : % -> OutputForm
--R definingInequation : % -> Dpoly    empty : () -> %
--R empty? : % -> Boolean              hash : % -> SingleInteger
--R idealSimplify : % -> %             latex : % -> String
--R ?~=? : (%,% ) -> Boolean
--R definingEquations : % -> List Dpoly
--R quasiAlgebraicSet : (List Dpoly,Dpoly) -> %
--R setStatus : (% ,Union(Boolean,"failed")) -> %
--R simplify : % -> % if R has CHARZ and R has EUCDOM
--R status : % -> Union(Boolean,"failed")
--R
--E 1

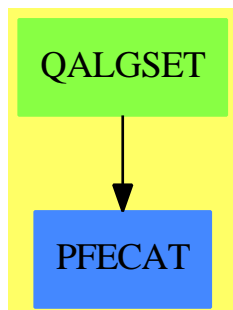
)spool
)lisp (bye)
```

— QuasiAlgebraicSet.help —

```
=====
QuasiAlgebraicSet examples
=====
```

```
See Also:
o )show QuasiAlgebraicSet
```

## 18.2.1 QuasiAlgebraicSet (QALGSET)

**Exports:**

|                   |                   |                    |        |
|-------------------|-------------------|--------------------|--------|
| coerce            | definingEquations | definingInequation | empty  |
| empty?            | hash              | idealSimplify      | latex  |
| quasiAlgebraicSet | setStatus         | simplify           | status |
| ?=?               | ?~=?              |                    |        |

— domain QALGSET QuasiAlgebraicSet —

```

)abbrev domain QALGSET QuasiAlgebraicSet
++ Author: William Sit
++ Date Created: March 13, 1992
++ Date Last Updated: June 12, 1992
++ Basic Operations:
++ Related Constructors: GroebnerPackage
++ See Also: QuasiAlgebraicSet2
++ AMS Classifications:
++ Keywords: Zariski closed sets, quasi-algebraic sets
++ References: William Sit, "An Algorithm for Parametric Linear Systems"
++ J. Sym. Comp., April, 1992
++ Description:
++ \spadtype{QuasiAlgebraicSet} constructs a domain representing
++ quasi-algebraic sets, which is the intersection of a Zariski
++ closed set, defined as the common zeros of a given list of
++ polynomials (the defining polynomials for equations), and a principal
++ Zariski open set, defined as the complement of the common
++ zeros of a polynomial f (the defining polynomial for the inequation).
++ This domain provides simplification of a user-given representation
++ using groebner basis computations.
++ There are two simplification routines: the first function
++ \spadfun{idealSimplify} uses groebner
++ basis of ideals alone, while the second, \spadfun{simplify} uses both
++ groebner basis and factorization. The resulting defining equations L
++ always form a groebner basis, and the resulting defining
++ inequation f is always reduced. The function \spadfun{simplify} may
++ be applied several times if desired. A third simplification

```

```

++ routine \spadfun{radicalSimplify} is provided in
++ \spadtype{QuasiAlgebraicSet2} for comparison study only,
++ as it is inefficient compared to the other two, as well as is
++ restricted to only certain coefficient domains. For detail analysis
++ and a comparison of the three methods, please consult the reference
++ cited.
++
++ A polynomial function q defined on the quasi-algebraic set
++ is equivalent to its reduced form with respect to L. While
++ this may be obtained using the usual normal form
++ algorithm, there is no canonical form for q.
++
++ The ordering in groebner basis computation is determined by
++ the data type of the input polynomials. If it is possible
++ we suggest to use refinements of total degree orderings.

QuasiAlgebraicSet(R, Var,Expon,Dpoly) : C == T
where
  R      : GcdDomain
  Expon   : OrderedAbelianMonoidSup
  Var     : OrderedSet
  Dpoly   : PolynomialCategory(R,Expon,Var)
  NNI     ==> NonNegativeInteger
  newExpon ==> Product(NNI,Expon)
  newPoly ==> PolynomialRing(R,newExpon)
  Ex      ==> OutputForm
  mrf     ==> MultivariateFactorize(Var,Expon,R,Dpoly)
  Status  ==> Union(Boolean,"failed") -- empty or not, or don't know

C == Join(SetCategory, CoercibleTo OutputForm) with
--- should be Object instead of SetCategory, bug in LIST Object ---
--- equality is not implemented ---
empty: () -> $
++ empty() returns the empty quasi-algebraic set
quasiAlgebraicSet: (List Dpoly, Dpoly) -> $
++ quasiAlgebraicSet(pl,q) returns the quasi-algebraic set
++ with defining equations  $p = 0$  for  $p$  belonging to the list pl, and
++ defining inequation  $q \neq 0$ .
status: $ -> Status
++ status(s) returns true if the quasi-algebraic set is empty,
++ false if it is not, and "failed" if not yet known
setStatus: ($, Status) -> $
++ setStatus(s,t) returns the same representation for s, but
++ asserts the following: if t is true, then s is empty,
++ if t is false, then s is non-empty, and if t = "failed",
++ then no assertion is made (that is, "don't know").
++ Note: for internal use only, with care.
empty?      : $ -> Boolean
++ empty?(s) returns
++ true if the quasialgebraic set s has no points,

```

```

    ++ and false otherwise.
definingEquations: $ -> List Dpoly
    ++ definingEquations(s) returns a list of defining polynomials
    ++ for equations, that is, for the Zariski closed part of s.
definingInequation: $ -> Dpoly
    ++ definingInequation(s) returns a single defining polynomial for the
    ++ inequation, that is, the Zariski open part of s.
idealSimplify:$ -> $
    ++ idealSimplify(s) returns a different and presumably simpler
    ++ representation of s with the defining polynomials for the
    ++ equations
    ++ forming a groebner basis, and the defining polynomial for the
    ++ inequation reduced with respect to the basis, using Buchberger's
    ++ algorithm.
if (R has EuclideanDomain) and (R has CharacteristicZero) then
  simplify:$ -> $
    ++ simplify(s) returns a different and presumably simpler
    ++ representation of s with the defining polynomials for the
    ++ equations
    ++ forming a groebner basis, and the defining polynomial for the
    ++ inequation reduced with respect to the basis, using a heuristic
    ++ algorithm based on factoring.
T == add
Rep := Record(status:Status,zero:List Dpoly, nzero:Dpoly)
x:$

import GroebnerPackage(R,Expon,Var,Dpoly)
import GroebnerPackage(R,newExpon,Var,newPoly)
import GroebnerInternalPackage(R,Expon,Var,Dpoly)

----- Local Functions -----

minset   : List List Dpoly -> List List Dpoly
overset? : (List Dpoly, List List Dpoly) -> Boolean
npoly    : Dpoly           -> newPoly
oldpoly  : newPoly         -> Union(Dpoly,"failed")

if (R has EuclideanDomain) and (R has CharacteristicZero) then
  factorset (y:Dpoly):List Dpoly ==
    ground? y => []
    [j.factor for j in factors factor$mrf y]

simplify x ==
  if x.status case "failed" then
    x:=quasiAlgebraicSet(zro:=groebner x.zero, redPol(x.nzero,zro))
    (pnzero:=x.nzero)=0 => empty()
    nzro:=factorset pnzero
    mset:=minset [factorset p for p in x.zero]
    mset:=[setDifference(s,nzro) for s in mset]

```

```

      zro:=groebner [*/s for s in mset]
      member? (1$Dpoly, zro) => empty()
      [x.status, zro, primitivePart redPol(*/nzro, zro)]

npoly(f:Dpoly) : newPoly ==
  zero? f => 0
  monomial(leadingCoefficient f,makeprod(0,degree f))$newPoly +
    npoly(reductum f)

oldpoly(q:newPoly) : Union(Dpoly,"failed") ==
  q=0$newPoly => 0$Dpoly
  dq:newExpon:=degree q
  n:NNI:=selectfirst (dq)
  n^=0 => "failed"
  ((g:=oldpoly reductum q) case "failed") => "failed"
  monomial(leadingCoefficient q,selectsecond dq)$Dpoly + (g::Dpoly)

coerce x ==
  x.status = true => "Empty":Ex
  bracket [[hconcat(f::Ex, " = 0":Ex) for f in x.zero ]::Ex,
    hconcat( x.nzero::Ex, " != 0":Ex)]

empty? x ==
  if x.status case "failed" then x:=idealSimplify x
  x.status :: Boolean

empty() == [true::Status, [1$Dpoly], 0$Dpoly]
status x == x.status
setStatus(x,t) == [t,x.zero,x.nzero]
definingEquations x == x.zero
definingInequation x == x.nzero
quasiAlgebraicSet(z0,n0) == ["failed", z0, n0]

idealSimplify x ==
  x.status case Boolean => x
  z0:= x.zero
  n0:= x.nzero
  empty? z0 => [false, z0, n0]
  member? (1$Dpoly, z0) => empty()
  tp:newPoly:=(monomial(1,makeprod(1,0$Expon))$newPoly * npoly n0)-1
  ngb:=groebner concat(tp, [npoly g for g in z0])
  member? (1$newPoly, ngb) => empty()
  gb:List Dpoly:=nil
  while not empty? ngb repeat
    if ((f:=oldpoly ngb.first) case Dpoly) then gb:=concat(f, gb)
    ngb:=ngb.rest
  [false::Status, gb, primitivePart redPol(n0, gb)]

minset lset ==

```

```

empty? lset => lset
[s for s in lset | ^(overiset?(s,lset))]

overiset?(p,qlist) ==
empty? qlist => false
or/[(brace$(Set Dpoly) q) <$(Set Dpoly) (brace$(Set Dpoly) p) for q in qlist]

```

---

— QALGSET.dotabb —

```

"QALGSET" [color="#88FF44",href="bookvol10.3.pdf#nameddest=QALGSET"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"QALGSET" -> "PFECAT"

```

---

## 18.3 domain QUAT Quaternion

— Quaternion.input —

```

)set break resume
)sys rm -f Quaternion.output
)spool Quaternion.output
)set message test on
)set message auto off
)clear all
--S 1 of 13
q := quatern(2/11,-8,3/4,1)
--R
--R
--R      2      3
--R  (1)  -- - 8i + - j + k
--R      11      4
--R
--R                                          Type: Quaternion Fraction Integer
--E 1

--S 2 of 13
[real q, imagI q, imagJ q, imagK q]
--R
--R
--R      2      3
--R  (2)  [--,- 8,-,1]
--R      11      4

```

Type: Quaternion Integer

```

--S 8 of 13
j:=quatern(0,0,1,0)
--R
--R
--R (8)  j
--R
--R                                          Type: Quaternion Integer
--E 8

--S 9 of 13
k:=quatern(0,0,0,1)
--R
--R
--R (9)  k
--R
--R                                          Type: Quaternion Integer
--E 9

--S 10 of 13
[i*i, j*j, k*k, i*j, j*k, k*i, q*i]
--R
--R
--R
--R (10)  [- 1,- 1,- 1,k,i,j,8 + -- i + j - - k]
--R                                     2      3
--R                                     11     4
--R
--R                                          Type: List Quaternion Fraction Integer
--E 10

--S 11 of 13
norm q
--R
--R
--R
--R (11)  126993
--R      -----
--R      1936
--R
--R                                          Type: Fraction Integer
--E 11

--S 12 of 13
conjugate q
--R
--R
--R
--R (12)  -- + 8i - - j - k
--R          2      3
--R          11     4
--R
--R                                          Type: Quaternion Fraction Integer
--E 12

--S 13 of 13
q * %
--R

```



```

--R
--R      126993
--R  (13) -----
--R      1936
--R
--R                                          Type: Quaternion Fraction Integer
--E 13
)spool
)lisp (bye)

```

---

— Quaternion.help —

=====

Quaternion examples

=====

The domain constructor Quaternion implements quaternions over commutative rings.

The basic operation for creating quaternions is `quatern`. This is a quaternion over the rational numbers.

```

q := quatern(2/11,-8,3/4,1)
      2      3
-- - 8i + - j + k
     11     4

```

Type: Quaternion Fraction Integer

The four arguments are the real part, the `i` imaginary part, the `j` imaginary part, and the `k` imaginary part, respectively.

```

[real q, imagI q, imagJ q, imagK q]
      2      3
[--, - 8, -, 1]
     11     4

```

Type: List Fraction Integer

Because `q` is over the rationals (and nonzero), you can invert it.

```

inv q
      352      15488      484      1936
----- + ----- i - ----- j - ----- k
126993  126993      42331  126993

```

Type: Quaternion Fraction Integer

The usual arithmetic (ring) operations are available

`q^6`

[illegible]

In general, multiplication is not commutative.

```

q * r - r * q
      2495      817
      ---- i - 1418j - --- k
        18        18
Type: Quaternion Fraction Integer

```

There are no predefined constants for the imaginary i, j, and k parts, but you can easily define them.

```
i:=quatern(0,1,0,0)
i
Type: Quaternion Integer

j:=quatern(0,0,1,0)
j
Type: Quaternion Integer

k:=quatern(0,0,0,1)
k
Type: Quaternion Integer
```

These satisfy the normal identities.

```
[i*i, j*j, k*k, i*j, j*k, k*i, q*i]
      2      3
[- 1,- 1,- 1,k,i,j,8 + -- i + j - - k]
      11      4
Type: List Quaternion Fraction Integer
```

The norm is the quaternion times its conjugate.

```

norm q
126993
-----
1936
Type: Fraction Integer

```

```
conjugate q
      2      3
    -- + 8i - - j - k
     11      4
```

Type: Quaternion Fraction Integer

```
q * %
126993
-----
 1936
```

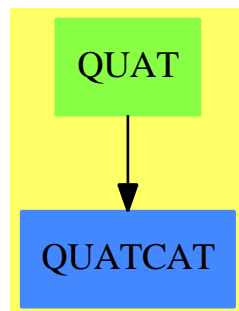
Type: Quaternion Fraction Integer

See Also:

- o )help Octonion
- o )help Complex
- o )help CliffordAlgebra
- o )show Quaternion

---

### 18.3.1 Quaternion (QUAT)



#### Exports:

|               |           |               |                |               |
|---------------|-----------|---------------|----------------|---------------|
| 0             | 1         | abs           | characteristic | charthRoot    |
| coerce        | conjugate | convert       | D              | differentiate |
| eval          | hash      | imagI         | imagJ          | imagK         |
| inv           | latex     | map           | max            | min           |
| norm          | one?      | quatern       | rational       | rational?     |
| rationalIfCan | real      | recip         | reducedSystem  | retract       |
| retractIfCan  | sample    | subtractIfCan | zero?          | ?*?           |
| ?**?          | ?+?       | ?-?           | -?             | ?=?           |
| ?^?           | ?~=?      | ?<?           | ?<=?           | ?>?           |
| ?>=?          | ?..?      |               |                |               |

## — domain QUAT Quaternion —

```

)abbrev domain QUAT Quaternion
++ Author: Robert S. Sutor
++ Date Created: 23 May 1990
++ Change History:
++   10 September 1990
++ Basic Operations: (Algebra)
++   abs, conjugate, imagI, imagJ, imagK, norm, quatern, rational,
++   rational?, real
++ Related Constructors: QuaternionCategoryFunctions2
++ Also See: QuaternionCategory, DivisionRing
++ AMS Classifications: 11R52
++ Keywords: quaternions, division ring, algebra
++ Description:
++ \spadtype{Quaternion} implements quaternions over a
++ commutative ring. The main constructor function is \spadfun{quatern}
++ which takes 4 arguments: the real part, the i imaginary part, the j
++ imaginary part and the k imaginary part.

```

```

Quaternion(R:CommutativeRing): QuaternionCategory(R) == add
  Rep := Record(r:R,i:R,j:R,k:R)

```

```

0 == [0,0,0,0]
1 == [1,0,0,0]

```

```

a,b,c,d : R
x,y : $

```

```

real x == x.r
imagI x == x.i
imagJ x == x.j
imagK x == x.k

```

```

quatern(a,b,c,d) == [a,b,c,d]

```

```

x * y == [x.r*y.r-x.i*y.i-x.j*y.j-x.k*y.k,
          x.r*y.i+x.i*y.r+x.j*y.k-x.k*y.j,
          x.r*y.j+x.j*y.r+x.k*y.i-x.i*y.k,
          x.r*y.k+x.k*y.r+x.i*y.j-x.j*y.i]

```

## — QUAT.dotabb —

```

"QUAT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=QUAT"]
"QUATCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=QUATCAT"]

```

```
"QUAT" -> "QUATCAT"
```

---

## 18.4 domain QEQUAT QueryEquation

— QueryEquation.input —

```
)set break resume
)sys rm -f QueryEquation.output
)spool QueryEquation.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show QueryEquation
--R QueryEquation is a domain constructor
--R Abbreviation for QueryEquation is QEQUAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for QEQUAT
--R
--R----- Operations -----
--R coerce : % -> OutputForm          equation : (Symbol,String) -> %
--R value : % -> String                variable : % -> Symbol
--R
--E 1

)spool
)lisp (bye)
```

---

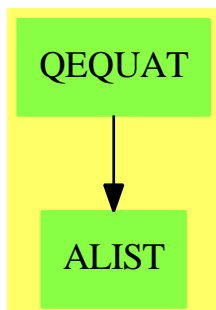
— QueryEquation.help —

```
=====
QueryEquation examples
=====
```

```
See Also:
o )show QueryEquation
```

---

## 18.4.1 QueryEquation (QEUAT)



See

- ⇒ “DataList” (DLIST) 5.2.1 on page 445
- ⇒ “IndexCard” (ICARD) 10.2.1 on page 1159
- ⇒ “Database” (DBASE) 5.1.1 on page 440

**Exports:**

coerce equation value variable

— domain QEUAT QueryEquation —

```

)abbrev domain QEUAT QueryEquation
++ Author: Mark Botch
++ Description:
++ This domain implements simple database queries

QueryEquation(): Exports == Implementation where
  Exports == CoercibleTo(OutputForm) with
    equation: (Symbol,String) -> %
      ++ equation(s,"a") creates a new equation.
    variable: % -> Symbol
      ++ variable(q) returns the variable (i.e. left hand side) of \axiom{q}.
    value: % -> String
      ++ value(q) returns the value (i.e. right hand side) of \axiom{q}.
  Implementation == add
    Rep := Record(var:Symbol, val:String)
    coerce(u) == coerce(u.var)$Symbol = coerce(u.val)$String
    equation(x,s) == [x,s]
    variable q == q.var
    value q == q.val
  
```

— QEUAT.dotabb —

```
"QEQUAT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=QEQUAT"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"QEQUAT" -> "ALIST"
```

## 18.5 domain QUEUE Queue

— Queue.input —

```
)set break resume
)sys rm -f Queue.output
)spool Queue.output
)set message test on
)set message auto off
)clear all

--S 1 of 46
a:Queue INT:= queue [1,2,3,4,5]
--R
--R
--R (1) [1,2,3,4,5]
--R
--R                                          Type: Queue Integer
--E 1

--S 2 of 46
dequeue! a
--R
--R
--R (2) 1
--R
--R                                          Type: PositiveInteger
--E 2

--S 3 of 46
a
--R
--R
--R (3) [2,3,4,5]
--R
--R                                          Type: Queue Integer
--E 3

--S 4 of 46
extract! a
--R
--R
--R (4) 2
```

[illegible]



```
--S 11 of 46
empty? a
--R
--R
--R (11) false
--R
--E 11
```

Type: Boolean

```
--S 12 of 46
front a
--R
--R
--R (12) 3
--R
--E 12
```

Type: PositiveInteger

```
--S 13 of 46
back a
--R
--R
--R (13) 8
--R
--E 13
```

Type: PositiveInteger

```
--S 14 of 46
rotate! a
--R
--R
--R (14) [4,5,9,8,3]
--R
--E 14
```

Type: Queue Integer

```
--S 15 of 46
#a
--R
--R
--R (15) 5
--R
--E 15
```

Type: PositiveInteger

```
--S 16 of 46
length a
--R
--R
--R (16) 5
--R
--E 16
```

Type: PositiveInteger

```
--S 17 of 46
```

```

less?(a,9)
--R
--R
--R (17) true
--R
--R                                          Type: Boolean
--E 17

--S 18 of 46
more?(a,9)
--R
--R
--R (18) false
--R
--R                                          Type: Boolean
--E 18

--S 19 of 46
size?(a,#a)
--R
--R
--R (19) true
--R
--R                                          Type: Boolean
--E 19

--S 20 of 46
size?(a,9)
--R
--R
--R (20) false
--R
--R                                          Type: Boolean
--E 20

--S 21 of 46
parts a
--R
--R
--R (21) [4,5,9,8,3]
--R
--R                                          Type: List Integer
--E 21

--S 22 of 46
bag([1,2,3,4,5])$Queue(INT)
--R
--R
--R (22) [1,2,3,4,5]
--R
--R                                          Type: Queue Integer
--E 22

--S 23 of 46
b:=empty()$(Queue INT)
--R

```

```

--R
--R (23) []
--R
--R                                         Type: Queue Integer
--E 23

--S 24 of 46
empty? b
--R
--R
--R (24) true
--R
--R                                         Type: Boolean
--E 24

--S 25 of 46
sample()$Queue(INT)
--R
--R
--R (25) []
--R
--R                                         Type: Queue Integer
--E 25

--S 26 of 46
c:=copy a
--R
--R
--R (26) [4,5,9,8,3]
--R
--R                                         Type: Queue Integer
--E 26

--S 27 of 46
eq?(a,c)
--R
--R
--R (27) false
--R
--R                                         Type: Boolean
--E 27

--S 28 of 46
eq?(a,a)
--R
--R
--R (28) true
--R
--R                                         Type: Boolean
--E 28

--S 29 of 46
(a=c)@Boolean
--R
--R
--R (29) true

```

```
--R
--E 29
Type: Boolean

--S 30 of 46
(a=a)@Boolean
--R
--R
--R (30) true
--R
--R
--E 30
Type: Boolean

--S 31 of 46
a~c
--R
--R
--R (31) false
--R
--R
--E 31
Type: Boolean

--S 32 of 46
any?(x+-(x=4),a)
--R
--R
--R (32) true
--R
--R
--E 32
Type: Boolean

--S 33 of 46
any?(x+-(x=11),a)
--R
--R
--R (33) false
--R
--R
--E 33
Type: Boolean

--S 34 of 46
every?(x+-(x=11),a)
--R
--R
--R (34) false
--R
--R
--E 34
Type: Boolean

--S 35 of 46
count(4,a)
--R
--R
--R (35) 1
--R
--R
--E 35
Type: PositiveInteger
```

```

--S 36 of 46
count(x+-->(x>2),a)
--R
--R
--R (36)  5
--R
--R                                          Type: PositiveInteger
--E 36

--S 37 of 46
map(x+-->x+10,a)
--R
--R
--R (37)  [14,15,19,18,13]
--R
--R                                          Type: Queue Integer
--E 37

--S 38 of 46
a
--R
--R
--R (38)  [4,5,9,8,3]
--R
--R                                          Type: Queue Integer
--E 38

--S 39 of 46
map!(x+-->x+10,a)
--R
--R
--R (39)  [14,15,19,18,13]
--R
--R                                          Type: Queue Integer
--E 39

--S 40 of 46
a
--R
--R
--R (40)  [14,15,19,18,13]
--R
--R                                          Type: Queue Integer
--E 40

--S 41 of 46
members a
--R
--R
--R (41)  [14,15,19,18,13]
--R
--R                                          Type: List Integer
--E 41

--S 42 of 46

```

```

member?(14,a)
--R
--R
--R (42) true
--R
--R                                          Type: Boolean
--E 42

--S 43 of 46
coerce a
--R
--R
--R (43) [14,15,19,18,13]
--R
--R                                          Type: OutputForm
--E 43

--S 44 of 46
hash a
--R
--R
--R (44) 4999531
--R
--R                                          Type: SingleInteger
--E 44

--S 45 of 46
latex a
--R
--R
--R (45) "\mbox{\bf Unimplemented}"
--R
--R                                          Type: String
--E 45

--S 46 of 46
)show Queue
--R
--R Queue S: SetCategory is a domain constructor
--R Abbreviation for Queue is QUEUE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for QUEUE
--R
--R----- Operations -----
--R back : % -> S                      bag : List S -> %
--R copy : % -> %                      dequeue! : % -> S
--R empty : () -> %                    empty? : % -> Boolean
--R enqueue! : (S,%) -> S              eq? : (%,%) -> Boolean
--R extract! : % -> S                  front : % -> S
--R insert! : (S,%) -> %               inspect : % -> S
--R length : % -> NonNegativeInteger  map : ((S -> S),%) -> %
--R queue : List S -> %               rotate! : % -> %
--R sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate

```

```

--R ?=? : (%,% ) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,% ) -> NonNegativeInteger if $ has finiteAggregate and S has SETCAT
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (% ,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (% ,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (% ,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (% ,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (% ,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,% ) -> Boolean if $ has finiteAggregate and S has SETCAT
--R members : % -> List S if $ has finiteAggregate
--R more? : (% ,NonNegativeInteger) -> Boolean
--R parts : % -> List S if $ has finiteAggregate
--R size? : (% ,NonNegativeInteger) -> Boolean
--R ?~=? : (%,% ) -> Boolean if S has SETCAT
--R
--E 46

```

```

)spool
)lisp (bye)

```

---

— Queue.help —

```

=====
Queue examples
=====

```

A Queue object is represented as a list ordered by first-in, first-out. It operates like a line of people, where the "next" person is the one at the front of the line.

Here we create an queue of integers from a list. Notice that the order in the list is the order in the queue.

```

a:Queue INT:= queue [1,2,3,4,5]
[1,2,3,4,5]

```

We can remove the top of the queue using dequeue!:

```

dequeue! a

```

1

Notice that the use of `dequeue!` is destructive (destructive operations in Axiom usually end with `!` to indicate that the underlying data structure is changed).

```
a
  [2,3,4,5]
```

The `extract!` operation is another name for the `pop!` operation and has the same effect. This operation treats the queue as a `BagAggregate`:

```
extract! a
2
```

and you can see that it also has destructively modified the queue:

```
a
  [3,4,5]
```

Next we use `enqueue!` to add a new element to the end of the queue:

```
push!(9,a)
9
```

Again, the `push!` operation is destructive so the queue is changed:

```
a
  [3,4,5,9]
```

Another name for `enqueue!` is `insert!`, which treats the queue as a `BagAggregate`:

```
insert!(8,a)
[3,4,5,9,8]
```

and it modifies the queue:

```
a
  [3,4,5,9,8]
```

The `inspect` function returns the top of the queue without modification, viewed as a `BagAggregate`:

```
inspect a
8
```

The `empty?` operation returns true only if there are no element on the queue, otherwise it returns false:



```
empty? a
false
```

The front operation returns the front of the queue without modification:

```
front a
3
```

The back operation returns the back of the queue without modification:

```
back a
8
```

The rotate! operation moves the item at the front of the queue to the back of the queue:

```
rotate! a
[4,5,9,8,3]
```

The # (length) operation:

```
#a
5
```

The length operation does the same thing:

```
length a
5
```

The less? predicate will compare the queue length to an integer:

```
less?(a,9)
true
```

The more? predicate will compare the queue length to an integer:

```
more?(a,9)
false
```

The size? operation will compare the queue length to an integer:

```
size?(a,#a)
true
```

and since the last computation must always be true we try:

```
size?(a,9)
false
```

The parts function will return the queue as a list of its elements:

```
parts a
  [8,9,3,4,5]
```

If we have a BagAggregate of elements we can use it to construct a queue:

```
bag([1,2,3,4,5])$Queue(INT)
  [1,2,3,4,5]
```

The empty function will construct an empty queue of a given type:

```
b:=empty()$(Queue INT)
  []
```

and the empty? predicate allows us to find out if a queue is empty:

```
empty? b
  true
```

The sample function returns a sample, empty queue:

```
sample()$Queue(INT)
  []
```

We can copy a queue and it does not share storage so subsequent modifications of the original queue will not affect the copy:

```
c:=copy a
  [4,5,9,8,3]
```

The eq? function is only true if the lists are the same reference, so even though c is a copy of a, they are not the same:

```
eq?(a,c)
  false
```

However, a clearly shares a reference with itself:

```
eq?(a,a)
  true
```

But we can compare a and c for equality:

```
(a=c)@Boolean
  true
```

and clearly a is equal to itself:

```
(a=a)@Boolean
  true
```

and since a and c are equal, they are clearly NOT not-equal:

```
a~=c
false
```

We can use the any? function to see if a predicate is true for any element:

```
any?(x+>(x=4),a)
true
```

or false for every element:

```
any?(x+>(x=11),a)
false
```

We can use the every? function to check every element satisfies a predicate:

```
every?(x+>(x=11),a)
false
```

We can count the elements that are equal to an argument of this type:

```
count(4,a)
1
```

or we can count against a boolean function:

```
count(x+>(x>2),a)
5
```

You can also map a function over every element, returning a new queue:

```
map(x+>x+10,a)
[14,15,19,18,13]
```

Notice that the original queue is unchanged:

```
a
[4,5,9,8,3]
```

You can use map! to map a function over every element and change the original queue since map! is destructive:

```
map!(x+>x+10,a)
[14,15,19,18,13]
```

o

Notice that the original queue has been changed:

```
a
```

```
[14,15,19,18,13]
```

The member function can also get the element of the queue as a list:

```
members a
[18,19,13,14,15]
```

and using member? we can test if the queue holds a given element:

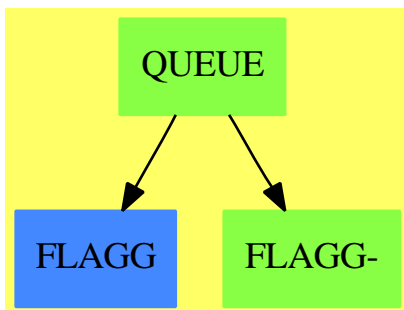
```
member?(14,a)
true
```

See Also:

- o )show Stack
- o )show ArrayStack
- o )show Queue
- o )show Dequeue
- o )show Heap
- o )show BagAggregate

---

### 18.5.1 Queue (QUEUE)



See

- ⇒ “Stack” (STACK) 20.28.1 on page 2521
- ⇒ “ArrayStack” (ASTACK) 2.10.1 on page 65
- ⇒ “Dequeue” (DEQUEUE) 5.5.1 on page 497
- ⇒ “Heap” (HEAP) 9.2.1 on page 1100

**Exports:**

|       |          |         |          |          |
|-------|----------|---------|----------|----------|
| any?  | back     | bag     | coerce   | copy     |
| count | dequeue! | empty   | empty?   | enqueue! |
| eq?   | eval     | every?  | extract! | front    |
| hash  | insert!  | inspect | latex    | length   |
| less? | map      | map!    | member?  | members  |
| more? | parts    | queue   | rotate!  | sample   |
| size? | #?       | ?=?     | ?~=?     |          |

— domain QUEUE Queue —

```

)abbrev domain QUEUE Queue
++ Author: Michael Monagan and Stephen Watt
++ Date Created: June 86 and July 87
++ Date Last Updated: Feb 92
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ Linked List implementation of a Queue
--% Dequeue and Heap data types

Queue(S:SetCategory): QueueAggregate S with
  queue: List S -> %
  ++ queue([x,y,...,z]) creates a queue with first (top)
  ++ element x, second element y,...,and last (bottom) element z.
  ++
  ++E e:Queue INT:= queue [1,2,3,4,5]

-- Inherited Signatures repeated for examples documentation

dequeue_! : % -> S
++
++X a:Queue INT:= queue [1,2,3,4,5]
++X dequeue! a
++X a
extract_! : % -> S
++
++X a:Queue INT:= queue [1,2,3,4,5]
++X extract! a
++X a
enqueue_! : (S,%) -> S
++
++X a:Queue INT:= queue [1,2,3,4,5]
++X enqueue! (9,a)
++X a
insert_! : (S,%) -> %

```

```

++
++X a:Queue INT:= queue [1,2,3,4,5]
++X insert! (8,a)
++X a
inspect : % -> S
++
++X a:Queue INT:= queue [1,2,3,4,5]
++X inspect a
front : % -> S
++
++X a:Queue INT:= queue [1,2,3,4,5]
++X front a
back : % -> S
++
++X a:Queue INT:= queue [1,2,3,4,5]
++X back a
rotate_! : % -> %
++
++X a:Queue INT:= queue [1,2,3,4,5]
++X rotate! a
length : % -> NonNegativeInteger
++
++X a:Queue INT:= queue [1,2,3,4,5]
++X length a
less? : (%,NonNegativeInteger) -> Boolean
++
++X a:Queue INT:= queue [1,2,3,4,5]
++X less?(a,9)
more? : (%,NonNegativeInteger) -> Boolean
++
++X a:Queue INT:= queue [1,2,3,4,5]
++X more?(a,9)
size? : (%,NonNegativeInteger) -> Boolean
++
++X a:Queue INT:= queue [1,2,3,4,5]
++X size?(a,5)
bag : List S -> %
++
++X bag([1,2,3,4,5])$Queue(INT)
empty? : % -> Boolean
++
++X a:Queue INT:= queue [1,2,3,4,5]
++X empty? a
empty : () -> %
++
++X b:=empty()$(Queue INT)
sample : () -> %
++
++X sample()$(Queue(INT))
copy : % -> %

```

```

++
++X a:Queue INT:= queue [1,2,3,4,5]
++X copy a
eq? : (%,%) -> Boolean
++
++X a:Queue INT:= queue [1,2,3,4,5]
++X b:=copy a
++X eq?(a,b)
map : ((S -> S),%) -> %
++
++X a:Queue INT:= queue [1,2,3,4,5]
++X map(x+>x+10,a)
++X a
if $ has shallowlyMutable then
map! : ((S -> S),%) -> %
++
++X a:Queue INT:= queue [1,2,3,4,5]
++X map!(x+>x+10,a)
++X a
if S has SetCategory then
latex : % -> String
++
++X a:Queue INT:= queue [1,2,3,4,5]
++X latex a
hash : % -> SingleInteger
++
++X a:Queue INT:= queue [1,2,3,4,5]
++X hash a
coerce : % -> OutputForm
++
++X a:Queue INT:= queue [1,2,3,4,5]
++X coerce a
"=" : (%,%) -> Boolean
++
++X a:Queue INT:= queue [1,2,3,4,5]
++X b:Queue INT:= queue [1,2,3,4,5]
++X (a=b)@Boolean
"~=" : (%,%) -> Boolean
++
++X a:Queue INT:= queue [1,2,3,4,5]
++X b:=copy a
++X (a~=b)
if % has finiteAggregate then
every? : ((S -> Boolean),%) -> Boolean
++
++X a:Queue INT:= queue [1,2,3,4,5]
++X every?(x+>(x=4),a)
any? : ((S -> Boolean),%) -> Boolean
++
++X a:Queue INT:= queue [1,2,3,4,5]

```

```

++X any?(x+>(x=4),a)
count : ((S -> Boolean),%) -> NonNegativeInteger
++
++X a:Queue INT:= queue [1,2,3,4,5]
++X count(x+>(x>2),a)
_# : % -> NonNegativeInteger
++
++X a:Queue INT:= queue [1,2,3,4,5]
++X #a
parts : % -> List S
++
++X a:Queue INT:= queue [1,2,3,4,5]
++X parts a
members : % -> List S
++
++X a:Queue INT:= queue [1,2,3,4,5]
++X members a
if % has finiteAggregate and S has SetCategory then
member? : (S,%) -> Boolean
++
++X a:Queue INT:= queue [1,2,3,4,5]
++X member?(3,a)
count : (S,%) -> NonNegativeInteger
++
++X a:Queue INT:= queue [1,2,3,4,5]
++X count(4,a)

== Stack S add
Rep := Reference List S
lastTail==> LAST$Lisp
enqueue_!(e,q) ==
  if null deref q then setref(q, list e)
  else lastTail.(deref q).rest := list e
  e
insert_!(e,q) == (enqueue_!(e,q);q)
dequeue_! q ==
  empty? q => error "empty queue"
  e := first deref q
  setref(q,rest deref q)
  e
extract_! q == dequeue_! q
rotate_! q == if empty? q then q else (enqueue_!(dequeue_! q,q); q)
length q == # deref q
front q == if empty? q then error "empty queue" else first deref q
inspect q == front q
back q == if empty? q then error "empty queue" else last deref q
queue q == ref copy q

```

---



## — QUEUE.dotabb —

```
"QUEUE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=QUEUE"]  
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]  
"FLAGG-" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FLAGG"]  
"QUEUE" -> "FLAGG"  
"QUEUE" -> "FLAGG-"
```

---

## Chapter 19

# Chapter R

### 19.1 domain RADFF RadicalFunctionField

— RadicalFunctionField.input —

```
)set break resume
)sys rm -f RadicalFunctionField.output
)spool RadicalFunctionField.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show RadicalFunctionField
--R RadicalFunctionField(F: UniqueFactorizationDomain,UP: UnivariatePolynomialCategory F,UPUP: UnivariatePolynomialCategory F)
--R Abbreviation for RadicalFunctionField is RADFF
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for RADFF
--R
--R----- Operations -----
--R ??? : (Fraction UP,%) -> %          ??? : (%,Fraction UP) -> %
--R ??? : (%,%) -> %                  ??? : (Integer,%) -> %
--R ??? : (PositiveInteger,%) -> %      ??? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                  ?-? : (%,%) -> %
--R -? : % -> %                      ?? : (%,%) -> Boolean
--R 1 : () -> %                      0 : () -> %
--R ?? : (%,PositiveInteger) -> %      basis : () -> Vector %
--R branchPoint? : UP -> Boolean        branchPoint? : F -> Boolean
--R coerce : Fraction UP -> %           coerce : Integer -> %
--R coerce : % -> OutputForm           convert : UPUP -> %
--R convert : % -> UPUP                 convert : Vector Fraction UP -> %
--R convert : % -> Vector Fraction UP   definingPolynomial : () -> UPUP
```

```

--R discriminant : () -> Fraction UP      elt : (% ,F,F) -> F
--R generator : () -> %                   genus : () -> NonNegativeInteger
--R hash : % -> SingleInteger              integral? : (% ,UP) -> Boolean
--R integral? : (% ,F) -> Boolean           integral? : % -> Boolean
--R integralBasis : () -> Vector %         latex : % -> String
--R lift : % -> UPUP                       norm : % -> Fraction UP
--R one? : % -> Boolean                    primitivePart : % -> %
--R ramified? : UP -> Boolean               ramified? : F -> Boolean
--R rank : () -> PositiveInteger            rationalPoint? : (F,F) -> Boolean
--R recip : % -> Union(%,"failed")         reduce : UPUP -> %
--R represents : (Vector UP,UP) -> %       retract : % -> Fraction UP
--R sample : () -> %                       singular? : UP -> Boolean
--R singular? : F -> Boolean                trace : % -> Fraction UP
--R zero? : % -> Boolean                    ~=?: (% ,%) -> Boolean

--R ?? : (% ,Fraction Integer) -> % if Fraction UP has FIELD
--R ?? : (Fraction Integer,% ) -> % if Fraction UP has FIELD
--R ?? : (NonNegativeInteger,% ) -> %
--R ??? : (% ,Integer) -> % if Fraction UP has FIELD
--R ??? : (% ,NonNegativeInteger) -> %
--R ?/? : (% ,%) -> % if Fraction UP has FIELD
--R D : % -> % if Fraction UP has DIFRING and Fraction UP has FIELD or Fraction UP has FFIELDC
--R D : (% ,NonNegativeInteger) -> % if Fraction UP has DIFRING and Fraction UP has FIELD or FFIELDC
--R D : (% ,Symbol) -> % if Fraction UP has FIELD and Fraction UP has PDRING SYMBOL
--R D : (% ,List Symbol) -> % if Fraction UP has FIELD and Fraction UP has PDRING SYMBOL
--R D : (% ,Symbol,NonNegativeInteger) -> % if Fraction UP has FIELD and Fraction UP has PDRING SYMBOL
--R D : (% ,List Symbol,List NonNegativeInteger) -> % if Fraction UP has FIELD and Fraction UP has PDRING SYMBOL
--R D : (% ,(Fraction UP -> Fraction UP)) -> % if Fraction UP has FIELD
--R D : (% ,(Fraction UP -> Fraction UP),NonNegativeInteger) -> % if Fraction UP has FIELD
--R ?? : (% ,Integer) -> % if Fraction UP has FIELD
--R ?? : (% ,NonNegativeInteger) -> %
--R absolutelyIrreducible? : () -> Boolean
--R algSplitSimple : (% ,(UP -> UP)) -> Record(num: % ,den: UP,derivden: UP,gd: UP)
--R associates? : (% ,%) -> Boolean if Fraction UP has FIELD
--R branchPointAtInfinity? : () -> Boolean
--R characteristic : () -> NonNegativeInteger
--R characteristicPolynomial : % -> UPUP
--R charthRoot : % -> Union(%,"failed") if Fraction UP has CHARNZ
--R charthRoot : % -> % if Fraction UP has FFIELDC
--R coerce : % -> % if Fraction UP has FIELD
--R coerce : Fraction Integer -> % if Fraction UP has FIELD or Fraction UP has RETRACT FRAC
--R complementaryBasis : Vector % -> Vector %
--R conditionP : Matrix % -> Union(Vector %,"failed") if Fraction UP has FFIELDC
--R coordinates : Vector % -> Matrix Fraction UP
--R coordinates : % -> Vector Fraction UP
--R coordinates : (Vector % ,Vector %) -> Matrix Fraction UP
--R coordinates : (% ,Vector %) -> Vector Fraction UP
--R createPrimitiveElement : () -> % if Fraction UP has FFIELDC
--R derivationCoordinates : (Vector % ,(Fraction UP -> Fraction UP)) -> Matrix Fraction UP if
--R differentiate : % -> % if Fraction UP has DIFRING and Fraction UP has FIELD or Fraction UP has FFIELDC
--R differentiate : (% ,NonNegativeInteger) -> % if Fraction UP has DIFRING and Fraction UP has FFIELDC

```

```

--R differentiate : (% , Symbol) -> % if Fraction UP has FIELD and Fraction UP has PDRING SYMBOL
--R differentiate : (% , List Symbol) -> % if Fraction UP has FIELD and Fraction UP has PDRING SYMBOL
--R differentiate : (% , Symbol , NonNegativeInteger) -> % if Fraction UP has FIELD and Fraction UP has PDRING SYMBOL
--R differentiate : (% , List Symbol , List NonNegativeInteger) -> % if Fraction UP has FIELD and Fraction UP has PDRING SYMBOL
--R differentiate : (% , (UP -> UP)) -> %
--R differentiate : (% , (Fraction UP -> Fraction UP)) -> % if Fraction UP has FIELD
--R differentiate : (% , (Fraction UP -> Fraction UP) , NonNegativeInteger) -> % if Fraction UP has FIELD
--R discreteLog : (% , %) -> Union(NonNegativeInteger , "failed") if Fraction UP has FFIELDC
--R discreteLog : % -> NonNegativeInteger if Fraction UP has FFIELDC
--R discriminant : Vector % -> Fraction UP
--R divide : (% , %) -> Record(quotient: % , remainder: %) if Fraction UP has FIELD
--R elliptic : () -> Union(UP , "failed")
--R euclideanSize : % -> NonNegativeInteger if Fraction UP has FIELD
--R expressIdealMember : (List % , %) -> Union(List % , "failed") if Fraction UP has FIELD
--R exquo : (% , %) -> Union(% , "failed") if Fraction UP has FIELD
--R extendedEuclidean : (% , %) -> Record(coef1: % , coef2: % , generator: %) if Fraction UP has FIELD
--R extendedEuclidean : (% , % , %) -> Union(Record(coef1: % , coef2: % , "failed") if Fraction UP has FIELD
--R factor : % -> Factored % if Fraction UP has FIELD
--R factorsOfCyclicGroupSize : () -> List Record(factor: Integer , exponent: Integer) if Fraction UP has FIELD
--R gcd : (% , %) -> % if Fraction UP has FIELD
--R gcd : List % -> % if Fraction UP has FIELD
--R gcdPolynomial : (SparseUnivariatePolynomial % , SparseUnivariatePolynomial %) -> SparseUnivariatePolynomial %
--R hyperelliptic : () -> Union(UP , "failed")
--R index : PositiveInteger -> % if Fraction UP has FINITE
--R init : () -> % if Fraction UP has FFIELDC
--R integralAtInfinity? : % -> Boolean
--R integralBasisAtInfinity : () -> Vector %
--R integralCoordinates : % -> Record(num: Vector UP , den: UP)
--R integralDerivationMatrix : (UP -> UP) -> Record(num: Matrix UP , den: UP)
--R integralMatrix : () -> Matrix Fraction UP
--R integralMatrixAtInfinity : () -> Matrix Fraction UP
--R integralRepresents : (Vector UP , UP) -> %
--R inv : % -> % if Fraction UP has FIELD
--R inverseIntegralMatrix : () -> Matrix Fraction UP
--R inverseIntegralMatrixAtInfinity : () -> Matrix Fraction UP
--R lcm : (% , %) -> % if Fraction UP has FIELD
--R lcm : List % -> % if Fraction UP has FIELD
--R lookup : % -> PositiveInteger if Fraction UP has FINITE
--R minimalPolynomial : % -> UPUP if Fraction UP has FIELD
--R multiEuclidean : (List % , %) -> Union(List % , "failed") if Fraction UP has FIELD
--R nextItem : % -> Union(% , "failed") if Fraction UP has FFIELDC
--R nonSingularModel : Symbol -> List Polynomial F if F has FIELD
--R normalizeAtInfinity : Vector % -> Vector %
--R numberOfComponents : () -> NonNegativeInteger
--R order : % -> OnePointCompletion PositiveInteger if Fraction UP has FFIELDC
--R order : % -> PositiveInteger if Fraction UP has FFIELDC
--R prime? : % -> Boolean if Fraction UP has FIELD
--R primeFrobenius : % -> % if Fraction UP has FFIELDC
--R primeFrobenius : (% , NonNegativeInteger) -> % if Fraction UP has FFIELDC
--R primitive? : % -> Boolean if Fraction UP has FFIELDC

```

```

--R primitiveElement : () -> % if Fraction UP has FFIELDC
--R principalIdeal : List % -> Record(coef: List %,generator: %) if Fraction UP has FIELD
--R ?quo? : (%,% ) -> % if Fraction UP has FIELD
--R ramifiedAtInfinity? : () -> Boolean
--R random : () -> % if Fraction UP has FINITE
--R rationalPoints : () -> List List F if F has FINITE
--R reduce : Fraction UPUP -> Union(%, "failed") if Fraction UP has FIELD
--R reduceBasisAtInfinity : Vector % -> Vector %
--R reducedSystem : Matrix % -> Matrix Fraction UP
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Fraction UP,vec: Vector Fraction UP)
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) if Fraction UP has FIELD
--R reducedSystem : Matrix % -> Matrix Integer if Fraction UP has LINEXP INT
--R regularRepresentation : % -> Matrix Fraction UP
--R regularRepresentation : (% ,Vector %) -> Matrix Fraction UP
--R ?rem? : (%,% ) -> % if Fraction UP has FIELD
--R representationType : () -> Union("prime",polynomial,normal,cyclic) if Fraction UP has FIELD
--R represents : Vector Fraction UP -> %
--R represents : (Vector Fraction UP,Vector %) -> %
--R retract : % -> Fraction Integer if Fraction UP has RETRACT FRAC INT
--R retract : % -> Integer if Fraction UP has RETRACT INT
--R retractIfCan : % -> Union(Fraction UP, "failed")
--R retractIfCan : % -> Union(Fraction Integer, "failed") if Fraction UP has RETRACT FRAC INT
--R retractIfCan : % -> Union(Integer, "failed") if Fraction UP has RETRACT INT
--R singularAtInfinity? : () -> Boolean
--R size : () -> NonNegativeInteger if Fraction UP has FINITE
--R sizeLess? : (%,% ) -> Boolean if Fraction UP has FIELD
--R squareFree : % -> Factored % if Fraction UP has FIELD
--R squareFreePart : % -> % if Fraction UP has FIELD
--R subtractIfCan : (%,% ) -> Union(%, "failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger,NonNegativeInteger) if Fraction UP has FIELD
--R traceMatrix : () -> Matrix Fraction UP
--R traceMatrix : Vector % -> Matrix Fraction UP
--R unit? : % -> Boolean if Fraction UP has FIELD
--R unitCanonical : % -> % if Fraction UP has FIELD
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if Fraction UP has FIELD
--R yCoordinates : % -> Record(num: Vector UP,den: UP)
--R
--E 1

```

```

)spool
)lisp (bye)

```

---

— RadicalFunctionField.help —

```

=====
RadicalFunctionField examples
=====

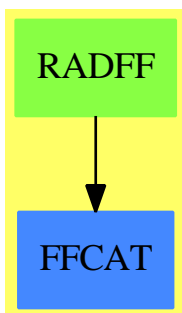
```

See Also:

o )show RadicalFunctionField

---

### 19.1.1 RadicalFunctionField (RADFF)



See

⇒ “AlgebraicFunctionField” (ALGFF) 2.5.1 on page 27

**Exports:**

|                                 |                          |                           |
|---------------------------------|--------------------------|---------------------------|
| 0                               | 1                        | absolutelyIrreducible?    |
| algSplitSimple                  | associates?              | basis                     |
| branchPoint?                    | branchPointAtInfinity?   | characteristic            |
| characteristicPolynomial        | charthRoot               | coerce                    |
| complementaryBasis              | conditionP               | convert                   |
| coordinates                     | createPrimitiveElement   | D                         |
| derivationCoordinates           | definingPolynomial       | differentiate             |
| discreteLog                     | discriminant             | divide                    |
| elliptic                        | elt                      | euclideanSize             |
| expressIdealMember              | exquo                    | extendedEuclidean         |
| factor                          | factorsOfCyclicGroupSize | gcd                       |
| gcdPolynomial                   | generator                | genus                     |
| hash                            | hyperelliptic            | index                     |
| init                            | integral?                | integralAtInfinity?       |
| integralBasis                   | integralBasisAtInfinity  | integralCoordinates       |
| integralDerivationMatrix        | integralMatrix           | integralMatrixAtInfinity  |
| integralRepresents              | inv                      | inverseIntegralMatrix     |
| inverseIntegralMatrixAtInfinity | latex                    | lcm                       |
| lift                            | lookup                   | minimalPolynomial         |
| multiEuclidean                  | nextItem                 | nonSingularModel          |
| norm                            | normalizeAtInfinity      | numberOfComponents        |
| one?                            | order                    | prime?                    |
| primeFrobenius                  | primitive?               | primitiveElement          |
| primitivePart                   | principalIdeal           | ramified?                 |
| ramifiedAtInfinity?             | random                   | rank                      |
| rationalPoint?                  | rationalPoints           | recip                     |
| reduce                          | reduce                   | reduceBasisAtInfinity     |
| reducedSystem                   | regularRepresentation    | representationType        |
| represents                      | retract                  | retractIfCan              |
| sample                          | singular?                | singularAtInfinity?       |
| size                            | sizeLess?                | squareFree                |
| squareFreePart                  | subtractIfCan            | tableForDiscreteLogarithm |
| trace                           | traceMatrix              | unit?                     |
| unitCanonical                   | unitNormal               | yCoordinates              |
| zero?                           | ?*?                      | ?**?                      |
| ?+?                             | ?-?                      | -?                        |
| ?=?                             | ?^?                      | ?~=?                      |
| ?/?                             | ?quo?                    | ?rem?                     |

— domain RADFF RadicalFunctionField —

```

)abbrev domain RADFF RadicalFunctionField
++ Author: Manuel Bronstein
++ Date Created: 1987
++ Date Last Updated: 27 July 1993
++ Keywords: algebraic, curve, radical, function, field.
++ Examples: )r RADFF INPUT

```

```

++ Description:
++ Function field defined by  $y^n = f(x)$ ;

RadicalFunctionField(F, UP, UPUP, radicnd, n): Exports == Impl where
  F      : UniqueFactorizationDomain
  UP     : UnivariatePolynomialCategory F
  UPUP   : UnivariatePolynomialCategory Fraction UP
  radicnd : Fraction UP
  n      : NonNegativeInteger

  N ==> NonNegativeInteger
  Z ==> Integer
  RF ==> Fraction UP
  QF ==> Fraction UPUP
  UP2 ==> SparseUnivariatePolynomial UP
  REC ==> Record(factor:UP, exponent:Z)
  MOD ==> monomial(1, n)$UPUP - radicnd::UPUP
  INIT ==> if (deref brandNew?) then startUp false

Exports ==> FunctionFieldCategory(F, UP, UPUP)

Impl ==> SimpleAlgebraicExtension(RF, UPUP, MOD) add
  import ChangeOfVariable(F, UP, UPUP)
  import InnerCommonDenominator(UP, RF, Vector UP, Vector RF)
  import UnivariatePolynomialCategoryFunctions2(RF, UPUP, UP, UP2)

  diag      : Vector RF -> Vector $
  startUp   : Boolean -> Void
  fullVector : (Factored UP, N) -> PrimitiveArray UP
  iBasis     : (UP, N) -> Vector UP
  infyBasis  : (RF, N) -> Vector RF
  basisvec   : () -> Vector RF
  char0StartUp: () -> Void
  charPStartUp: () -> Void
  getInfBasis : () -> Void
  radcand    : () -> UP
  charPintbas : (UPUP, RF, Vector RF, Vector RF) -> Void

  brandNew?:Reference(Boolean) := ref true
  discPoly:Reference(RF) := ref(0$RF)
  newrad:Reference(UP) := ref(0$UP)
  n1 := (n - 1)::N
  modulus := MOD
  ibasis:Vector(RF) := new(n, 0)
  invibasis:Vector(RF) := new(n, 0)
  infbasis:Vector(RF) := new(n, 0)
  invinfbasis:Vector(RF) := new(n, 0)
  mini := minIndex ibasis

discriminant() == (INIT; discPoly())

```



```

radcand() == (INIT; newrad())
integralBasis() == (INIT; diag ibasis)
integralBasisAtInfinity() == (INIT; diag infbasis)
basisvec() == (INIT; ibasis)
integralMatrix() == diagonalMatrix basisvec()
integralMatrixAtInfinity() == (INIT; diagonalMatrix infbasis)
inverseIntegralMatrix() == (INIT; diagonalMatrix invibasis)
inverseIntegralMatrixAtInfinity() == (INIT; diagonalMatrix invinfbasis)
definingPolynomial() == modulus
ramified?(point:F) == zero?(radcand() point)
branchPointAtInfinity?() == (degree(radcand()) exquo n) case "failed"
elliptic() == (n = 2 and degree(radcand()) = 3 => radcand(); "failed")
hyperelliptic() == (n=2 and odd? degree(radcand()) => radcand(); "failed")
diag v == [reduce monomial(qelt(v,i+mini), i) for i in 0..n1]

integralRepresents(v, d) ==
  ib := basisvec()
  represents
    [qelt(ib, i) * (qelt(v, i) /$RF d) for i in mini .. maxIndex ib]

integralCoordinates f ==
  v := coordinates f
  ib := basisvec()
  splitDenominator
    [qelt(v,i) / qelt(ib,i) for i in mini .. maxIndex ib]$Vector(RF)

integralDerivationMatrix d ==
  dlogp := differentiate(radicnd, d) / (n * radicnd)
  v := basisvec()
  cd := splitDenominator(
    [(i - mini) * dlogp + differentiate(qelt(v, i), d) / qelt(v, i)
     for i in mini..maxIndex v]$Vector(RF))
  [diagonalMatrix(cd.num), cd.den]

-- return (d0,...,d(n-1)) s.t. (1/d0, y/d1,...,y**(n-1)/d(n-1))
-- is an integral basis for the curve y**d = p
-- requires that p has no factor of multiplicity >= d
iBasis(p, d) ==
  pl := fullVector(squareFree p, d)
  d1 := (d - 1)::N
  [*/[pl.j ** ((i * j) quo d) for j in 0..d1] for i in 0..d1]

-- returns a vector [a0,a1,...,a_{m-1}] of length m such that
-- p = a0^0 a1^1 ... a_{m-1}^{m-1}
fullVector(p, m) ==
  ans:PrimitiveArray(UP) := new(m, 0)
  ans.0 := unit p
  l := factors p
  for i in 1..maxIndex ans repeat
    ans.i :=

```

```

(u := find(s+->s.exponent = i, 1)) case "failed" => 1
(u::REC).factor
ans

-- return (f0,...,f(n-1)) s.t. (f0, y f1,..., y**(n-1) f(n-1))
-- is a local integral basis at infinity for the curve y**d = p
infyBasis(p, m) ==
  rt := rootPoly(p(x := inv(monomial(1, 1)$UP :: RF)), m)
  m ^= rt.exponent =>
    error "Curve not irreducible after change of variable 0 -> infinity"
  a := (rt.coef) x
  b:RF := 1
  v := iBasis(rt.radicand, m)
  w:Vector(RF) := new(m, 0)
  for i in mini..maxIndex v repeat
    qsetelt_!(w, i, b / (qelt(v, i)::RF) x)
    b := b * a
  w

charPintbas(p, c, v, w) ==
  degree(p) ^= n => error "charPintbas: should not happen"
  q:UP2 := map(s+->retract(s)@UP, p)
  ib := integralBasis()$FunctionFieldIntegralBasis(UP, UP2,
    SimpleAlgebraicExtension(UP, UP2, q))

  not diagonal?(ib.basis)=>
    error "charPintbas: integral basis not diagonal"
  a:RF := 1
  for i in minRowIndex(ib.basis) .. maxRowIndex(ib.basis)
    for j in minColIndex(ib.basis) .. maxColIndex(ib.basis)
      for k in mini .. maxIndex v repeat
        qsetelt_!(v, k, (qelt(ib.basis, i, j) / ib.basisDen) * a)
        qsetelt_!(w, k, qelt(ib.basisInv, i, j) * inv a)
      a := a * c
  void

charPStartUp() ==
  r := mkIntegral modulus
  charPintbas(r.poly, r.coef, ibasis, invibasis)
  x := inv(monomial(1, 1)$UP :: RF)
  invmod := monomial(1, n)$UPUP - (radicnd x)::UPUP
  r := mkIntegral invmod
  charPintbas(r.poly, (r.coef) x, infbasis, invinfbasis)

startUp b ==
  brandNew?() := b
  if zero?(p := characteristic()$F) or p > n then char0StartUp()
    else charPStartUp()
  dsc:RF := ((-1)$Z ** ((n*$N n1) quo 2::N) * (n::Z)**n)$Z *
    radicnd ** n1 *
    */[qelt(ibasis, i) ** 2 for i in mini..maxIndex ibasis]

```

```

discPoly() := primitivePart(numer dsc) / denom(dsc)
void

char0StartUp() ==
  rp      := rootPoly(radicnd, n)
  rp.exponent ^= n =>
    error "RadicalFunctionField: curve is not irreducible"
  newrad() := rp.radicand
  ib       := iBasis(newrad(), n)
  infb     := infthyBasis(radicnd, n)
  invden:RF := 1
  for i in mini..maxIndex ib repeat
    qsetelt_!(invibasis, i, a := qelt(ib, i) * invden)
    qsetelt_!(ibasis, i, inv a)
    invden := invden / rp.coef      -- always equals 1/rp.coef**(i-mini)
    qsetelt_!(infbasis, i, a := qelt(infb, i))
    qsetelt_!(invinfbasis, i, inv a)
  void

ramified?(p:UP) ==
  (r := retractIfCan(p)@Union(F, "failed")) case F =>
    singular?(r::F)
  (radcand() exquo p) case UP

singular?(p:UP) ==
  (r := retractIfCan(p)@Union(F, "failed")) case F =>
    singular?(r::F)
  (radcand() exquo(p**2)) case UP

branchPoint?(p:UP) ==
  (r := retractIfCan(p)@Union(F, "failed")) case F =>
    branchPoint?(r::F)
  ((q := (radcand() exquo p)) case UP) and
  ((q::UP exquo p) case "failed")

singular?(point:F) ==
  zero?(radcand() point) and
  zero?(((radcand() exquo (monomial(1,1)$UP-point::UP))::UP) point)

branchPoint?(point:F) ==
  zero?(radcand() point) and not
  zero?(((radcand() exquo (monomial(1,1)$UP-point::UP))::UP) point)

```

---

— RADFF.dotabb —

"RADFF" [color="#88FF44",href="bookvol10.3.pdf#nameddest=RADFF"]

"FFCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FFCAT"]  
 "RADFF" -> "FFCAT"

## 19.2 domain RADIX RadixExpansion

— RadixExpansion.input —

```
)set break resume
)sys rm -f RadixExpansion.output
)spool RadixExpansion.output
)set message test on
)set message auto off
)clear all
--S 1 of 17
111::RadixExpansion(5)
--R
--R
--R (1) 421
--R
--R                                          Type: RadixExpansion 5
--E 1

--S 2 of 17
(5/24)::RadixExpansion(2)
--R
--R
--R --
--R (2) 0.00110
--R
--R                                          Type: RadixExpansion 2
--E 2

--S 3 of 17
(5/24)::RadixExpansion(3)
--R
--R
--R --
--R (3) 0.012
--R
--R                                          Type: RadixExpansion 3
--E 3

--S 4 of 17
(5/24)::RadixExpansion(8)
--R
--R
--R --
```

```

--R (4) 0.152
--R
--E 4
Type: RadixExpansion 8

--S 5 of 17
(5/24)::RadixExpansion(10)
--R
--R
--R
--R (5) 0.2083
--R
--E 5
Type: RadixExpansion 10

--S 6 of 17
(5/24)::RadixExpansion(12)
--R
--R
--R
--R (6) 0.26
--R
--E 6
Type: RadixExpansion 12

--S 7 of 17
(5/24)::RadixExpansion(16)
--R
--R
--R
--R (7) 0.35
--R
--E 7
Type: RadixExpansion 16

--S 8 of 17
(5/24)::RadixExpansion(36)
--R
--R
--R
--R (8) 0.7I
--R
--E 8
Type: RadixExpansion 36

--S 9 of 17
(5/24)::RadixExpansion(38)
--R
--R
--R
--R (9) 0 . 7 34 31 25 12
--R
--E 9
Type: RadixExpansion 38

--S 10 of 17
a := (76543/210)::RadixExpansion(8)
--R

```

```

--R
--R      ----
--R      (10)  554.37307
--R
--R                                          Type: RadixExpansion 8
--E 10

--S 11 of 17
w := wholeRagits a
--R
--R
--R      (11)  [5,5,4]
--R
--R                                          Type: List Integer
--E 11

--S 12 of 17
f0 := prefixRagits a
--R
--R
--R      (12)  [3]
--R
--R                                          Type: List Integer
--E 12

--S 13 of 17
f1 := cycleRagits a
--R
--R
--R      (13)  [7,3,0,7]
--R
--R                                          Type: List Integer
--E 13

--S 14 of 17
u:RadixExpansion(8):=wholeRadix(w)+fractRadix(f0,f1)
--R
--R
--R      ----
--R      (14)  554.37307
--R
--R                                          Type: RadixExpansion 8
--E 14

--S 15 of 17
v: RadixExpansion(12) := fractRadix([1,2,3,11], [0])
--R
--R
--R      -
--R      (15)  0.123B0
--R
--R                                          Type: RadixExpansion 12
--E 15

--S 16 of 17
fractRagits(u)

```

```

--R
--R
--R      -----
--R      (16)  [3,7,3,0,7,7]
--R
--R                                          Type: Stream Integer
--E 16

--S 17 of 17
a :: Fraction(Integer)
--R
--R
--R      76543
--R      (17)  -----
--R      210
--R
--R                                          Type: Fraction Integer
--E 17
)spool
)lisp (bye)

```

---

— RadixExpansion.help —

=====

RadixExpansion examples

=====

It possible to expand numbers in general bases.

Here we expand 111 in base 5. This means

$$10^2 + 10^1 + 10^0 = 4 * 5^2 + 2 * 5^1 + 5^0$$

```

111::RadixExpansion(5)
421

```

Type: RadixExpansion 5

You can expand fractions to form repeating expansions.

```

(5/24)::RadixExpansion(2)

```

```

0.00110

```

Type: RadixExpansion 2

```

(5/24)::RadixExpansion(3)

```

```

0.012

```

Type: RadixExpansion 3

```

(5/24)::RadixExpansion(8)

```

$$0.\overline{152}$$

Type: RadixExpansion 8

(5/24)::RadixExpansion(10)

$$0.\overline{2083}$$

Type: RadixExpansion 10

For bases from 11 to 36 the letters A through Z are used.

(5/24)::RadixExpansion(12)

$$0.\overline{26}$$

Type: RadixExpansion 12

(5/24)::RadixExpansion(16)

$$0.\overline{35}$$

Type: RadixExpansion 16

(5/24)::RadixExpansion(36)

$$0.\overline{7I}$$

Type: RadixExpansion 36

For bases greater than 36, the ragits are separated by blanks.

(5/24)::RadixExpansion(38)

$$0.\overline{7\ 34\ 31\ 25\ 12}$$

Type: RadixExpansion 38

The RadixExpansion type provides operations to obtain the individual ragits. Here is a rational number in base 8.

a := (76543/210)::RadixExpansion(8)

$$554.\overline{37307}$$

Type: RadixExpansion 8

The operation wholeRagits returns a list of the ragits for the integral part of the number.

```
w := wholeRagits a
[5,5,4]
```

Type: List Integer

The operations prefixRagits and cycleRagits return lists of the initial and repeating ragits in the fractional part of the number.

f0 := prefixRagits a



[3]

Type: List Integer

```
f1 := cycleRagits a
      [7,3,0,7]
```

Type: List Integer

You can construct any radix expansion by giving the whole, prefix and cycle parts. The declaration is necessary to let Axiom know the base of the ragits.

```
u:RadixExpansion(8):=wholeRadix(w)+fractRadix(f0,f1)
```

```
-----
554.37307
```

Type: RadixExpansion 8

If there is no repeating part, then the list [0] should be used.

```
v: RadixExpansion(12) := fractRadix([1,2,3,11], [0])
```

```
-----
0.123B0
```

Type: RadixExpansion 12

If you are not interested in the repeating nature of the expansion, an infinite stream of ragits can be obtained using fractRagits.

```
fractRagits(u)
```

```
-----
[3,7,3,0,7,7]
```

Type: Stream Integer

Of course, it's possible to recover the fraction representation:

```
a :: Fraction(Integer)
```

```
76543
```

```
-----
```

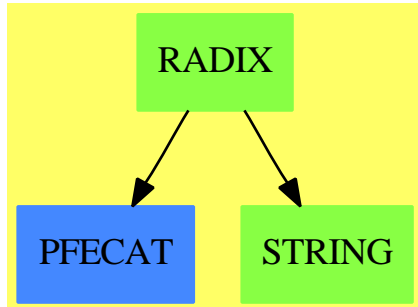
```
210
```

Type: Fraction Integer

See Also:

- o )help DecimalExpansion
- o )help BinaryExpansion
- o )help HexadecimalExpansion
- o )show RadixExpansion

---

**19.2.1 RadixExpansion (RADIX)**

**See**

⇒ “BinaryExpansion” (BINARY) 3.7.1 on page 274

⇒ “DecimalExpansion” (DECIMAL) 5.3.1 on page 451

⇒ “HexadecimalExpansion” (HEXADEC) 9.3.1 on page 1108

**Exports:**

|                               |                  |                            |
|-------------------------------|------------------|----------------------------|
| 0                             | 1                | abs                        |
| associates?                   | ceiling          | characteristic             |
| charthRoot                    | coerce           | conditionP                 |
| convert                       | cycleRagits      | D                          |
| denom                         | denominator      | differentiate              |
| divide                        | euclideanSize    | eval                       |
| expressIdealMember            | exquo            | extendedEuclidean          |
| factor                        | factorPolynomial | factorSquareFreePolynomial |
| floor                         | fractRadix       | fractRagits                |
| fractionPart                  | gcd              | gcdPolynomial              |
| hash                          | init             | inv                        |
| latex                         | lcm              | map                        |
| max                           | min              | multiEuclidean             |
| negative?                     | nextItem         | numer                      |
| numerator                     | one?             | patternMatch               |
| positive?                     | prefixRagits     | prime?                     |
| principalIdeal                | random           | recip                      |
| reducedSystem                 | retract          | retractIfCan               |
| sample                        | sign             | sizeLess?                  |
| solveLinearPolynomialEquation | squareFree       | squareFreePart             |
| squareFreePolynomial          | subtractIfCan    | unit?                      |
| unitCanonical                 | unitNormal       | wholePart                  |
| wholeRadix                    | wholeRagits      | zero?                      |
| ?.?                           | ?*?              | ?**?                       |
| ?+?                           | ?-?              | -?                         |
| ?/?                           | ?=?              | ?^?                        |
| ?~=?                          | ?<?              | ?<=?                       |
| ?>?                           | ?>=?             | ?quo?                      |
| ?rem?                         |                  |                            |

— domain RADIX RadixExpansion —

```

)abbrev domain RADIX RadixExpansion
++ Author: Stephen M. Watt
++ Date Created: October 1986
++ Date Last Updated: May 15, 1991
++ Basic Operations: wholeRadix, fractRadix, wholeRagits, fractRagits
++ Related Domains: BinaryExpansion, DecimalExpansion, HexadecimalExpansion,
++   RadixUtilities
++ Also See:
++ AMS Classifications:
++ Keywords: radix, base, repeating decimal
++ Examples:
++ References:
++ Description:
++ This domain allows rational numbers to be presented as repeating
++ decimal expansions or more generally as repeating expansions in any base.

```

```

RadixExpansion(bb): Exports == Implementation where
  bb : Integer
  I ==> Integer
  NNI ==> NonNegativeInteger
  OUT ==> OutputForm
  RN ==> Fraction Integer
  ST ==> Stream Integer
  QuoRem ==> Record(quotient: Integer, remainder: Integer)

Exports ==> QuotientFieldCategory(Integer) with
  coerce: % -> Fraction Integer
  ++ coerce(rx) converts a radix expansion to a rational number.
  fractionPart: % -> Fraction Integer
  ++ fractionPart(rx) returns the fractional part of a radix expansion.
  wholeRagits: % -> List Integer
  ++ wholeRagits(rx) returns the ragits of the integer part
  ++ of a radix expansion.
  fractRagits: % -> Stream Integer
  ++ fractRagits(rx) returns the ragits of the fractional part
  ++ of a radix expansion.
  prefixRagits: % -> List Integer
  ++ prefixRagits(rx) returns the non-cyclic part of the ragits
  ++ of the fractional part of a radix expansion.
  ++ For example, if \spad{x = 3/28 = 0.10 714285 714285 ...},
  ++ then \spad{prefixRagits(x)}=[1,0]}.
  cycleRagits: % -> List Integer
  ++ cycleRagits(rx) returns the cyclic part of the ragits of the
  ++ fractional part of a radix expansion.
  ++ For example, if \spad{x = 3/28 = 0.10 714285 714285 ...},
  ++ then \spad{cycleRagits(x)} = [7,1,4,2,8,5]}.
  wholeRadix: List Integer -> %
  ++ wholeRadix(l) creates an integral radix expansion from a list
  ++ of ragits.
  ++ For example, \spad{wholeRadix([1,3,4])} will return \spad{134}.
  fractRadix: (List Integer, List Integer) -> %
  ++ fractRadix(pre,cyc) creates a fractional radix expansion
  ++ from a list of prefix ragits and a list of cyclic ragits.
  ++ e.g., \spad{fractRadix([1],[6])} will return \spad{0.16666666...}.

Implementation ==> add
  -- The efficiency of arithmetic operations is poor.
  -- Could use a lazy eval where either rational rep
  -- or list of ragit rep (the current) or both are kept
  -- as demanded.

  bb < 2 => error "Radix base must be at least 2"
  Rep := Record(sgn: Integer, int: List Integer,
    pfx: List Integer, cyc: List Integer)

  q: RN

```

```

qr:      QuoRem
a,b:     %
n:       I

radixInt:  (I, I)      -> List I
radixFrac: (I, I, I) -> Record(pfx: List I, cyc: List I)
checkRagits: List I   -> Boolean

-- Arithmetic operations
characteristic() == 0
differentiate a == 0

0      == [1, nil(), nil(), nil()]
1      == [1, [1], nil(), nil()]
- a    == (a = 0 => 0; [-a.sgn, a.int, a.pfx, a.cyc])
a + b  == (a::RN + b::RN)::%
a - b  == (a::RN - b::RN)@RN::%
n * a  == (n      * a::RN)::%
a * b  == (a::RN * b::RN)::%
a / b  == (a::RN / b::RN)::%
(i:I) / (j:I) == (i/j)@RN :: %
a < b  == a::RN < b::RN
a = b  == a.sgn = b.sgn and a.int = b.int and
        a.pfx = b.pfx and a.cyc = b.cyc
numer a == numer(a::RN)
denom a == denom(a::RN)

-- Algebraic coercions
coerce(a):RN == (wholePart a) :: RN + fractionPart a
coerce(n):%  == n :: RN :: %
coerce(q):%  ==
  s := 1; if q < 0 then (s := -1; q := -q)
  qr := divide(numer q,denom q)
  whole := radixInt (qr.quotient,bb)
  fractn := radixFrac(qr.remainder,denom q,bb)
  cycle := (fractn.cyc = [0] => nil(); fractn.cyc)
  [s,whole,fractn.pfx,cycle]

retractIfCan(a):Union(RN,"failed") == a::RN
retractIfCan(a):Union(I,"failed") ==
  empty?(a.pfx) and empty?(a.cyc) => wholePart a
  "failed"

-- Exported constructor/destructors
ceiling a == ceiling(a::RN)
floor a == floor(a::RN)

wholePart a ==
  n0 := 0
  for r in a.int repeat n0 := bb*n0 + r

```

```

    a.sgn*n0
fractionPart a ==
  n0 := 0
  for r in a.pfx repeat n0 := bb*n0 + r
  null a.cyc =>
    a.sgn*n0/bb**((#a.pfx)::NNI)
  n1 := n0
  for r in a.cyc repeat n1 := bb*n1 + r
  n := n1 - n0
  d := (bb**((#a.cyc)::NNI) - 1) * bb**((#a.pfx)::NNI)
  a.sgn*n/d

wholeRagits a == a.int
fractRagits a == concat(construct(a.pfx)@ST, repeating a.cyc)
prefixRagits a == a.pfx
cycleRagits a == a.cyc

wholeRadix li ==
  checkRagits li
  [1, li, nil(), nil()]
fractRadix(lpfx, lcyc) ==
  checkRagits lpfx; checkRagits lcyc
  [1, nil(), lpfx, lcyc]

-- Output

ALPHAS : String := "ABCDEFGHIJKLMNOPQRSTUVWXYZ"

intToExpr(i:I): OUT ==
  -- computes a digit for bases between 11 and 36
  i < 10 => i :: OUT
  elt(ALPHAS, (i-10) + minIndex(ALPHAS)) :: OUT

exprgroup(le: List OUT): OUT ==
  empty? le      => error "exprgroup needs non-null list"
  empty? rest le => first le
  abs bb <= 36 => hconcat le
  blankSeparate le

intgroup(li: List I): OUT ==
  empty? li      => error "intgroup needs non-null list"
  empty? rest li => intToExpr first(li)
  abs bb <= 10 => hconcat [i :: OUT for i in li]
  abs bb <= 36 => hconcat [intToExpr(i) for i in li]
  blankSeparate [i :: OUT for i in li]

overBar(li: List I): OUT == overbar intgroup li

coerce(a): OUT ==
  le : List OUT := nil()

```

```

    if not null a.cyc then le := concat(overBar a.cyc,le)
    if not null a.pfx then le := concat(intgroup a.pfx,le)
    if not null le then le := concat(".", :: OUT,le)
    if not null a.int then le := concat(intgroup a.int,le)
    else le := concat(0 :: OUT,le)
    rex := exprgroup le
    if a.sgn < 0 then -rex else rex

-- Construction utilities
checkRagits li ==
  for i in li repeat if i < 0 or i >= bb then
    error "Each ragit (digit) must be between 0 and base-1"
  true

radixInt(n,bas) ==
  rits: List I := nil()
  while abs n ^= 0 repeat
    qr := divide(n,bas)
    n := qr.quotient
    rits := concat(qr.remainder,rits)
  rits

radixFrac(num,den,bas) ==
  -- Rits is the sequence of quotient/remainder pairs
  -- in calculating the radix expansion of the rational number.
  -- We wish to find p and c such that
  --   rits.i are distinct for 0<=i<=p+c-1
  --   rits.i = rits.(i+p) for i>p
  -- I.e. p is the length of the non-periodic prefix and c is
  -- the length of the cycle.

  -- Compute p and c using Floyd's algorithm.
  -- 1. Find smallest n s.t. rits.n = rits.(2*n)
  qr := divide(bas * num, den)
  i : I := 0
  qr1i := qr2i := qr
  rits: List QuoRem := [qr]
  until qr1i = qr2i repeat
    qr1i := divide(bas * qr1i.remainder,den)
    qrt := divide(bas * qr2i.remainder,den)
    qr2i := divide(bas * qrt.remainder,den)
    rits := concat(qr2i, concat(qrt, rits))
    i := i + 1
  rits := reverse_! rits
  n := i
  -- 2. Find p = first i such that rits.i = rits.(i+n)
  ritsi := rits
  ritsn := rits; for i in 1..n repeat ritsn := rest ritsn
  i := 0
  while first(ritsi) ^= first(ritsn) repeat

```

```

    ritsi := rest ritsi
    ritsn := rest ritsn
    i      := i + 1
p := i
-- 3. Find c = first i such that rits.p = rits.(p+i)
ritsn := rits; for i in 1..n repeat ritsn := rest ritsn
rn     := first ritsn
cfound:= false
c : I := 0
for i in 1..p while not cfound repeat
    ritsn := rest ritsn
    if rn = first(ritsn) then
        c := i
        cfound := true
if not cfound then c := n
-- 4. Now produce the lists of ragits.
ritspfx: List I := nil()
ritscyc: List I := nil()
for i in 1..p repeat
    ritspfx := concat(first(rits).quotient, ritspfx)
    rits     := rest rits
for i in 1..c repeat
    ritscyc := concat(first(rits).quotient, ritscyc)
    rits     := rest rits
[reverse_! ritspfx, reverse_! ritscyc]

```

---

— RADIX.dotabb —

```

"RADIX" [color="#88FF44",href="bookvol10.3.pdf#nameddest=RADIX"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"RADIX" -> "PFECAT"
"RADIX" -> "STRING"

```

---

## 19.3 domain RECLOS RealClosure

The domain constructore **RealClosure** by Renaud Rioboo (University of Paris 6, France) provides the real closure of an ordered field. The implementation is based on interval arithmetic. Moreover, the design of this constructor and its related packages allows an easy use of other codings for real algebraic numbers. ordered field



The RealClosure domain is the end-user code, it provides usual arithmetics with real algebraic numbers, along with the functionalities of a real closed field. It also provides functions to approximate a real algebraic number by an element of the base field. This approximation may either be absolute (approximate) or relative (realtivApprox).

### CAVEATS

Since real algebraic expressions are stored as depending on "real roots" which are managed like variables, there is an ordering on these. This ordering is dynamical in the sense that any new algebraic takes precedence over older ones. In particular every creation function raises a new "real root". This has the effect that when you type something like  $\sqrt{2} + \sqrt{2}$  you have two new variables which happen to be equal. To avoid this name the expression such as in `s2 := sqrt(2) ; s2 + s2`

Also note that computing times depend strongly on the ordering you implicitly provide. Please provide algebraics in the order which most natural to you.

### LIMITATIONS

The file `reclos.input` show some basic use of the package. This package uses algorithms which are published in [1] and [2] which are based on field arithmetics, in particular for polynomial gcd related algorithms. This can be quite slow for high degree polynomials and subresultants methods usually work best. Beta versions of the package try to use these techniques in a better way and work significantly faster. These are mostly based on unpublished algorithms and cannot be distributed. Please contact the author if you have a particular problem to solve or want to use these versions.

Be aware that approximations behave as post-processing and that all computations are done exactly. They can thus be quite time consuming when depending on several "real roots".

### — RealClosure.input —

```
)set break resume
)sys rm -f RealClosure.output
)spool RealClosure.output
)set message test on
)set message auto off
)clear all
--S 1 of 67
Ran := RECLOS(FRAC INT)
--R
--R
--R (1) RealClosure Fraction Integer
--R
--R                                          Type: Domain
--E 1

--S 2 of 67
fourSquares(a:Ran,b:Ran,c:Ran,d:Ran):Ran==sqrt(a)+sqrt(b)-sqrt(c)-sqrt(d)
--R
--R Function declaration fourSquares : (RealClosure Fraction Integer,
--R      RealClosure Fraction Integer,RealClosure Fraction Integer,
```

```

--R      RealClosure Fraction Integer) -> RealClosure Fraction Integer has
--R      been added to workspace.
--R
--R                                          Type: Void
--E 2

--S 3 of 67
squareDiff1 := fourSquares(73,548,60,586)
--R
--R      Compiling function fourSquares with type (RealClosure Fraction
--R      Integer,RealClosure Fraction Integer,RealClosure Fraction Integer
--R      ,RealClosure Fraction Integer) -> RealClosure Fraction Integer
--R
--R      +---+ +---+ +---+ +---+
--R      (3) - \|586 - \|60 + \|548 + \|73
--R
--R                                          Type: RealClosure Fraction Integer
--E 3

--S 4 of 67
recip(squareDiff1)
--R
--R
--R      (4)
--R      +---+ +---+ +---+ +---+ +---+ +---+
--R      ((54602\|548 + 149602\|73 )\|60 + 49502\|73 \|548 + 9900895)\|586
--R      +
--R      +---+ +---+ +---+ +---+ +---+
--R      (154702\|73 \|548 + 30941947)\|60 + 10238421\|548 + 28051871\|73
--R
--R                                          Type: Union(RealClosure Fraction Integer,...)
--E 4

--S 5 of 67
sign(squareDiff1)
--R
--R
--R      (5) 1
--R
--R                                          Type: PositiveInteger
--E 5

--S 6 of 67
squareDiff2 := fourSquares(165,778,86,990)
--R
--R
--R      +---+ +---+ +---+ +---+
--R      (6) - \|990 - \|86 + \|778 + \|165
--R
--R                                          Type: RealClosure Fraction Integer
--E 6

--S 7 of 67
recip(squareDiff2)
--R

```

```

--R      (7)
--R      +----+      +----+ +---+      +----+ +----+
--R      ((556778\|778 + 1209010\|165 )\|86 + 401966\|165 \|778 + 144019431)
--R      *
--R      +----+
--R      \|990
--R      +
--R      +----+ +----+      +---+      +----+      +----+
--R      (1363822\|165 \|778 + 488640503)\|86 + 162460913\|778 + 352774119\|165
--R      Type: Union(RealClosure Fraction Integer,...)
--E 7

--S 8 of 67
sign(squareDiff2)
--R
--R
--R      (8)  1
--R
--R      Type: PositiveInteger
--E 8

--S 9 of 67
squareDiff3 := fourSquares(217,708,226,692)
--R
--R
--R      +----+      +----+      +----+      +----+
--R      (9)  - \|692 - \|226 + \|708 + \|217
--R
--R      Type: RealClosure Fraction Integer
--E 9

--S 10 of 67
recip(squareDiff3)
--R
--R
--R      (10)
--R      +----+      +----+ +---+      +----+ +----+      +----+
--R      ((- 34102\|708 - 61598\|217 )\|226 - 34802\|217 \|708 - 13641141)\|692
--R      +
--R      +----+ +----+      +----+      +----+      +----+
--R      (- 60898\|217 \|708 - 23869841)\|226 - 13486123\|708 - 24359809\|217
--R      Type: Union(RealClosure Fraction Integer,...)
--E 10

--S 11 of 67
sign(squareDiff3)
--R
--R
--R      (11)  - 1
--R
--R      Type: Integer
--E 11

```

```

--S 12 of 67
squareDiff4 := fourSquares(155,836,162,820)
--R
--R
--R      +---+ +---+ +---+ +---+
--R      (12) - \|820 - \|162 + \|836 + \|155
--R                                                    Type: RealClosure Fraction Integer
--E 12

--S 13 of 67
recip(squareDiff4)
--R
--R
--R      (13)
--R      +---+ +---+ +---+ +---+ +---+ +---+
--R      ((- 37078\|836 - 86110\|155 )\|162 - 37906\|155 \|836 - 13645107)\|820
--R      +
--R      +---+ +---+ +---+ +---+ +---+ +---+
--R      (- 85282\|155 \|836 - 30699151)\|162 - 13513901\|836 - 31384703\|155
--R                                                    Type: Union(RealClosure Fraction Integer,...)
--E 13

--S 14 of 67
sign(squareDiff4)
--R
--R
--R      (14) - 1
--R                                                    Type: Integer
--E 14

--S 15 of 67
squareDiff5 := fourSquares(591,772,552,818)
--R
--R
--R      +---+ +---+ +---+ +---+
--R      (15) - \|818 - \|552 + \|772 + \|591
--R                                                    Type: RealClosure Fraction Integer
--E 15

--S 16 of 67
recip(squareDiff5)
--R
--R
--R      (16)
--R      +---+ +---+ +---+ +---+ +---+ +---+
--R      ((70922\|772 + 81058\|591 )\|552 + 68542\|591 \|772 + 46297673)\|818
--R      +
--R      +---+ +---+ +---+ +---+ +---+ +---+
--R      (83438\|591 \|772 + 56359389)\|552 + 47657051\|772 + 54468081\|591

```

```

--R                                     Type: Union(RealClosure Fraction Integer,...)
--E 16

--S 17 of 67
sign(squareDiff5)
--R
--R
--R (17)  1
--R
--R                                     Type: PositiveInteger
--E 17

--S 18 of 67
squareDiff6 := fourSquares(434,1053,412,1088)
--R
--R
--R          +-----+   +-----+   +-----+   +-----+
--R (18)  - \|1088  - \|412  + \|1053  + \|434
--R
--R                                     Type: RealClosure Fraction Integer
--E 18

--S 19 of 67
recip(squareDiff6)
--R
--R
--R (19)
--R          +-----+   +-----+   +-----+   +-----+   +-----+
--R ((115442\|1053  + 179818\|434 )\|412  + 112478\|434 \|1053  + 76037291)
--R *
--R          +-----+
--R          \|1088
--R +
--R          +-----+ +-----+   +-----+   +-----+   +-----+
--R (182782\|434 \|1053  + 123564147)\|412  + 77290639\|1053  + 120391609\|434
--R
--R                                     Type: Union(RealClosure Fraction Integer,...)
--E 19

--S 20 of 67
sign(squareDiff6)
--R
--R
--R (20)  1
--R
--R                                     Type: PositiveInteger
--E 20

--S 21 of 67
squareDiff7 := fourSquares(514,1049,446,1152)
--R
--R
--R          +-----+   +-----+   +-----+   +-----+
--R (21)  - \|1152  - \|446  + \|1049  + \|514

```

```

--R                                                    Type: RealClosure Fraction Integer
--E 21

--S 22 of 67
recip(squareDiff7)
--R
--R
--R (22)
--R      +-----+      +----+ +----+      +----+ +-----+
--R      ((349522\|1049 + 499322\|514 )\|446 + 325582\|514 \|1049 + 239072537)
--R      *
--R      +-----+
--R      \|1152
--R      +
--R      +----+ +-----+      +----+      +-----+      +----+
--R      (523262\|514 \|1049 + 384227549)\|446 + 250534873\|1049 + 357910443\|514
--R                                                    Type: Union(RealClosure Fraction Integer,...)
--E 22

--S 23 of 67
sign(squareDiff7)
--R
--R
--R (23)  1
--R
--R                                                    Type: PositiveInteger
--E 23

--S 24 of 67
squareDiff8 := fourSquares(190,1751,208,1698)
--R
--R
--R      +-----+      +----+      +-----+      +----+
--R      (24)  - \|1698 - \|208 + \|1751 + \|190
--R                                                    Type: RealClosure Fraction Integer
--E 24

--S 25 of 67
recip(squareDiff8)
--R
--R
--R (25)
--R      +-----+      +----+ +----+      +----+ +-----+
--R      (- 214702\|1751 - 651782\|190 )\|208 - 224642\|190 \|1751
--R      +
--R      - 129571901
--R      *
--R      +-----+
--R      \|1698
--R      +
--R      +----+ +-----+      +----+      +-----+

```

```

--R      (- 641842\|190 \|1751 - 370209881)\|208 - 127595865\|1751
--R      +
--R      +----+
--R      - 387349387\|190
--R
--R                                          Type: Union(RealClosure Fraction Integer,...)
--E 25

--S 26 of 67
sign(squareDiff8)
--R
--R
--R      (26)  - 1
--R
--R                                          Type: Integer
--E 26

--S 27 of 67
relativeApprox(squareDiff8,10**(-3))::Float
--R
--R
--R      (27)  - 0.2340527771 5937700123 E -10
--R
--R                                          Type: Float
--E 27

--S 28 of 67
l := allRootsOf((x**2-2)**2-2)$Ran
--R
--R
--R      (28)  [%A33,%A34,%A35,%A36]
--R
--R                                          Type: List RealClosure Fraction Integer
--E 28

--S 29 of 67
removeDuplicates map(mainDefiningPolynomial,l)
--R
--R
--R      4      2
--R      (29)  [ ? - 4 ? + 2 ]
--RType: List Union(SparseUnivariatePolynomial RealClosure Fraction Integer,"failed")
--E 29

--S 30 of 67
map(mainCharacterization,l)
--R
--R
--R      (30)  [[- 2,- 1[,- 1,0[, [0,1[, [1,2[
--RType: List Union(RightOpenIntervalRootCharacterization(RealClosure Fraction Integer,Spars
--E 30

--S 31 of 67
[reduce(+,l),reduce(*,l)-2]

```

Type: Boolean



```

--S 37 of 67
s3 := sqrt(3)$Ran
--R
--R
--R      +-+
--R  (37)  \|3
--R
--R                                          Type: RealClosure Fraction Integer
--E 37

--S 38 of 67
s7:= sqrt(7)$Ran
--R
--R
--R      +-+
--R  (38)  \|7
--R
--R                                          Type: RealClosure Fraction Integer
--E 38

--S 39 of 67
e1 := sqrt(2*s7-3*s3,3)
--R
--R
--R      +-----+
--R      3|  +-+  +-+
--R  (39)  \|2\|7  - 3\|3
--R
--R                                          Type: RealClosure Fraction Integer
--E 39

--S 40 of 67
e2 := sqrt(2*s7+3*s3,3)
--R
--R
--R      +-----+
--R      3|  +-+  +-+
--R  (40)  \|2\|7  + 3\|3
--R
--R                                          Type: RealClosure Fraction Integer
--E 40

--S 41 of 67
e2-e1-s3
--R
--R
--R  (41)  0
--R
--R                                          Type: RealClosure Fraction Integer
--E 41

--S 42 of 67
pol : UP(x,Ran) := x**4+(7/3)*x**2+30*x-(100/3)
--R

```

```

--R
--R      4  7  2      100
--R  (42)  x  + - x  + 30x - ---
--R      3      3
--R                                     Type: UnivariatePolynomial(x,RealClosure Fraction Integer)
--E 42

--S 43 of 67
r1 := sqrt(7633)$Ran
--R
--R
--R      +-----+
--R  (43)  \ | 7633
--R
--R                                     Type: RealClosure Fraction Integer
--E 43

--S 44 of 67
alpha := sqrt(5*r1-436,3)/3
--R
--R
--R      +-----+
--R      1 3|  +-----+
--R  (44)  - \ | 5 \ | 7633  - 436
--R      3
--R
--R                                     Type: RealClosure Fraction Integer
--E 44

--S 45 of 67
beta := -sqrt(5*r1+436,3)/3
--R
--R
--R      +-----+
--R      1 3|  +-----+
--R  (45)  - - \ | 5 \ | 7633  + 436
--R      3
--R
--R                                     Type: RealClosure Fraction Integer
--E 45

--S 46 of 67
pol.(alpha+beta-1/3)
--R
--R
--R  (46)  0
--R
--R                                     Type: RealClosure Fraction Integer
--E 46

--S 47 of 67
qol : UP(x,Ran) := x**5+10*x**3+20*x+22
--R
--R

```

```

--R      5      3
--R (47) x  + 10x  + 20x + 22
--R                                     Type: UnivariatePolynomial(x,RealClosure Fraction Integer)
--E 47

--S 48 of 67
r2 := sqrt(153)$Ran
--R
--R
--R      +---+
--R (48) \|153
--R
--R                                     Type: RealClosure Fraction Integer
--E 48

--S 49 of 67
alpha2 := sqrt(r2-11,5)
--R
--R
--R      +-----+
--R      5| +---+
--R (49) \|\|153 - 11
--R
--R                                     Type: RealClosure Fraction Integer
--E 49

--S 50 of 67
beta2 := -sqrt(r2+11,5)
--R
--R
--R      +-----+
--R      5| +---+
--R (50) - \|\|153 + 11
--R
--R                                     Type: RealClosure Fraction Integer
--E 50

--S 51 of 67
qol(alpha2+beta2)
--R
--R
--R (51) 0
--R
--R                                     Type: RealClosure Fraction Integer
--E 51

--S 52 of 67
dst1:=sqrt(9+4*s2)=1+2*s2
--R
--R
--R      +-----+
--R      | +-+      +-+
--R (52) \|4\|2 + 9 = 2\|2 + 1
--R
--R                                     Type: Equation RealClosure Fraction Integer

```

```

--E 52

--S 53 of 67
dst1::Boolean
--R
--R
--R (53) true
--R
--R                                          Type: Boolean
--E 53

--S 54 of 67
s6:Ran:=sqrt 6
--R
--R
--R      +-+
--R (54) \|6
--R
--R                                          Type: RealClosure Fraction Integer
--E 54

--S 55 of 67
dst2:=sqrt(5+2*s6)+sqrt(5-2*s6) = 2*s3
--R
--R
--R      +-----+      +-----+
--R      |      +-+      |      +-+      +-+
--R (55) \|- 2\|6  + 5  + \|2\|6  + 5 = 2\|3
--R
--R                                          Type: Equation RealClosure Fraction Integer
--E 55

--S 56 of 67
dst2::Boolean
--R
--R
--R (56) true
--R
--R                                          Type: Boolean
--E 56

--S 57 of 67
s29:Ran:=sqrt 29
--R
--R
--R      +---+
--R (57) \|29
--R
--R                                          Type: RealClosure Fraction Integer
--E 57

--S 58 of 67
dst4:=sqrt(16-2*s29+2*sqrt(55-10*s29)) = sqrt(22+2*s5)-sqrt(11+2*s29)+s5
--R
--R

```

```

--R (58)
--R +-----+
--R | +-----+ +-----+ +-----+
--R | | +--+ +--+ | +--+ | +--+ +--+
--R \|2\|- 10\|29 + 55 - 2\|29 + 16 = - \|2\|29 + 11 + \|2\|5 + 22 + \|5
--R                                         Type: Equation RealClosure Fraction Integer
--E 58

--S 59 of 67
dst4::Boolean
--R
--R
--R (59) true
--R
--R                                         Type: Boolean
--E 59

--S 60 of 67
dst6:=sqrt((112+70*s2)+(46+34*s2)*s5) = (5+4*s2)+(3+s2)*s5
--R
--R
--R +-----+
--R | +--+ +--+ +--+ +--+ +--+ +--+ +--+
--R (60) \|(34\|2 + 46)\|5 + 70\|2 + 112 = (\|2 + 3)\|5 + 4\|2 + 5
--R                                         Type: Equation RealClosure Fraction Integer
--E 60

--S 61 of 67
dst6::Boolean
--R
--R
--R (61) true
--R
--R                                         Type: Boolean
--E 61

--S 62 of 67
f3:Ran:=sqrt(3,5)
--R
--R
--R 5+--+
--R (62) \|3
--R
--R                                         Type: RealClosure Fraction Integer
--E 62

--S 63 of 67
f25:Ran:=sqrt(1/25,5)
--R
--R
--R +--+
--R | 1
--R (63) 5|--

```

```

--R      \|25
--R
--R                                          Type: RealClosure Fraction Integer
--E 63

--S 64 of 67
f32:Ran:=sqrt(32/5,5)
--R
--R
--R      +---+
--R      |32
--R  (64) 5|--
--R      \| 5
--R
--R                                          Type: RealClosure Fraction Integer
--E 64

--S 65 of 67
f27:Ran:=sqrt(27/5,5)
--R
--R
--R      +---+
--R      |27
--R  (65) 5|--
--R      \| 5
--R
--R                                          Type: RealClosure Fraction Integer
--E 65

--S 66 of 67
dst5:=sqrt((f32-f27,3)) = f25*(1+f3-f3**2)
--R
--R
--R      +-----+
--R      |  +---+  +---+
--R      |  |27  |32      5+--+  5+--+      +---+
--R  (66) 3|-- 5|-- + 5|-- = (- \|3  + \|3  + 1) 5|--
--R      \|  \| 5  \| 5      \|25
--R
--R                                          Type: Equation RealClosure Fraction Integer
--E 66

--S 67 of 67
dst5::Boolean
--R
--R
--R  (67) true
--R
--R                                          Type: Boolean
--E 67
)spool
)lisp (bye)

```

---

## — RealClosure.help —

```
=====
RealClosure examples
=====
```

The Real Closure 1.0 package provided by Renaud Rioboo consists of different packages, categories and domains :

The package `RealPolynomialUtilitiesPackage` which needs a `Field F` and a `UnivariatePolynomialCategory` domain with coefficients in `F`. It computes some simple functions such as Sturm and Sylvester sequences `sturmSequence`, `sylvesterSequence`.

The category `RealRootCharacterizationCategory` provides abstract functions to work with "real roots" of univariate polynomials. These resemble variables with some functionality needed to compute important operations.

The category `RealClosedField` provides common operations available over real closed fields. These include finding all the roots of a univariate polynomial, taking square (and higher) roots, ...

The domain `RightOpenIntervalRootCharacterization` is the main code that provides the functionality of `RealRootCharacterizationCategory` for the case of archimedean fields. Abstract roots are encoded with a left closed right open interval containing the root together with a defining polynomial for the root.

The `RealClosure` domain is the end-user code. It provides usual arithmetic with real algebraic numbers, along with the functionality of a real closed field. It also provides functions to approximate a real algebraic number by an element of the base field. This approximation may either be absolute, approximate or relative (`relativeApprox`).

```
=====
CAVEATS
=====
```

Since real algebraic expressions are stored as depending on "real roots" which are managed like variables, there is an ordering on these. This ordering is dynamical in the sense that any new algebraic takes precedence over older ones. In particular every creation function raises a new "real root". This has the effect that when you type something like `sqrt(2) + sqrt(2)` you have two new variables which happen to be equal. To avoid this name the expression such as in `s2 := sqrt(2) ; s2 + s2`

Also note that computing times depend strongly on the ordering you

implicitly provide. Please provide algebraics in the order which seems most natural to you.

```
=====
LIMITATIONS
=====
```

This packages uses algorithms which are published in [1] and [2] which are based on field arithmetics, in particular for polynomial gcd related algorithms. This can be quite slow for high degree polynomials and subresultants methods usually work best. Beta versions of the package try to use these techniques in a better way and work significantly faster. These are mostly based on unpublished algorithms and cannot be distributed. Please contact the author if you have a particular problem to solve or want to use these versions.

Be aware that approximations behave as post-processing and that all computations are done exactly. They can thus be quite time consuming when depending on several ‘‘real roots’’.

```
=====
REFERENCES
=====
```

- [1] R. Rioboo : Real Algebraic Closure of an ordered Field : Implementation in Axiom.  
In proceedings of the ISSAC’92 Conference, Berkeley 1992 pp. 206-215.
- [2] Z. Ligatsikas, R. Rioboo, M. F. Roy : Generic computation of the real closure of an ordered field.  
In Mathematics and Computers in Simulation Volume 42, Issue 4-6, November 1996.

```
=====
EXAMPLES
=====
```

We shall work with the real closure of the ordered field of rational numbers.

```
Ran := RECLOS(FRAC INT)
      RealClosure Fraction Integer
                        Type: Domain
```

Some simple signs for square roots, these correspond to an extension of degree 16 of the rational numbers. Examples provided by J. Abbot.

```
fourSquares(a:Ran,b:Ran,c:Ran,d:Ran):Ran==sqrt(a)+sqrt(b)-sqrt(c)-sqrt(d)
                        Type: Void
```



These produce values very close to zero.

```

squareDiff1 := fourSquares(73,548,60,586)
      +---+   +---+   +---+   +---+
      - \|586 - \|60 + \|548 + \|73
                                         Type: RealClosure Fraction Integer

recip(squareDiff1)
      +---+   +---+   +---+   +---+   +---+
      ((54602\|548 + 149602\|73 )\|60 + 49502\|73 \|548 + 9900895)\|586
+
      +---+ +---+   +---+   +---+   +---+
      (154702\|73 \|548 + 30941947)\|60 + 10238421\|548 + 28051871\|73
                                         Type: Union(RealClosure Fraction Integer,...)

sign(squareDiff1)
1
                                         Type: PositiveInteger

squareDiff2 := fourSquares(165,778,86,990)
      +---+   +---+   +---+   +---+
      - \|990 - \|86 + \|778 + \|165
                                         Type: RealClosure Fraction Integer

recip(squareDiff2)
      +---+   +---+   +---+   +---+   +---+
      ((556778\|778 + 1209010\|165 )\|86 + 401966\|165 \|778 + 144019431)
*
      +---+
      \|990
+
      +---+ +---+   +---+   +---+   +---+
      (1363822\|165 \|778 + 488640503)\|86 + 162460913\|778 + 352774119\|165
                                         Type: Union(RealClosure Fraction Integer,...)

sign(squareDiff2)
1
                                         Type: PositiveInteger

squareDiff3 := fourSquares(217,708,226,692)
      +---+   +---+   +---+   +---+
      - \|692 - \|226 + \|708 + \|217
                                         Type: RealClosure Fraction Integer

recip(squareDiff3)
      +---+   +---+   +---+   +---+   +---+
      ((- 34102\|708 - 61598\|217 )\|226 - 34802\|217 \|708 - 13641141)\|692
+
      +---+ +---+   +---+   +---+   +---+

```

```

      (- 60898\|217 \|708 - 23869841)\|226 - 13486123\|708 - 24359809\|217
      Type: Union(RealClosure Fraction Integer,...)

sign(squareDiff3)
- 1
      Type: Integer

squareDiff4 := fourSquares(155,836,162,820)
      +----+ +----+ +----+ +----+
      - \|820 - \|162 + \|836 + \|155
      Type: RealClosure Fraction Integer

recip(squareDiff4)
      +----+ +----+ +----+ +----+ +----+ +----+
      ((- 37078\|836 - 86110\|155 )\|162 - 37906\|155 \|836 - 13645107)\|820
      +
      +----+ +----+ +----+ +----+ +----+
      (- 85282\|155 \|836 - 30699151)\|162 - 13513901\|836 - 31384703\|155
      Type: Union(RealClosure Fraction Integer,...)

sign(squareDiff4)
- 1
      Type: Integer

squareDiff5 := fourSquares(591,772,552,818)
      +----+ +----+ +----+ +----+
      - \|818 - \|552 + \|772 + \|591
      Type: RealClosure Fraction Integer

recip(squareDiff5)
      +----+ +----+ +----+ +----+ +----+ +----+
      ((70922\|772 + 81058\|591 )\|552 + 68542\|591 \|772 + 46297673)\|818
      +
      +----+ +----+ +----+ +----+ +----+
      (83438\|591 \|772 + 56359389)\|552 + 47657051\|772 + 54468081\|591
      Type: Union(RealClosure Fraction Integer,...)

sign(squareDiff5)
1
      Type: PositiveInteger

squareDiff6 := fourSquares(434,1053,412,1088)
      +----+ +----+ +----+ +----+
      - \|1088 - \|412 + \|1053 + \|434
      Type: RealClosure Fraction Integer

recip(squareDiff6)
      +----+ +----+ +----+ +----+
      ((115442\|1053 + 179818\|434 )\|412 + 112478\|434 \|1053 + 76037291)
      *

```

```

      +-----+
      \|1088
+
      +-----+ +-----+          +-----+          +-----+          +-----+
      (182782\|434 \|1053 + 123564147)\|412 + 77290639\|1053 + 120391609\|434
      Type: Union(RealClosure Fraction Integer,...)

sign(squareDiff6)
1
      Type: PositiveInteger

squareDiff7 := fourSquares(514,1049,446,1152)
      +-----+ +-----+ +-----+ +-----+
      - \|1152 - \|446 + \|1049 + \|514
      Type: RealClosure Fraction Integer

recip(squareDiff7)
      +-----+          +-----+ +-----+          +-----+ +-----+
      ((349522\|1049 + 499322\|514 )\|446 + 325582\|514 \|1049 + 239072537)
      *
      +-----+
      \|1152
+
      +-----+ +-----+          +-----+          +-----+          +-----+
      (523262\|514 \|1049 + 384227549)\|446 + 250534873\|1049 + 357910443\|514
      Type: Union(RealClosure Fraction Integer,...)

sign(squareDiff7)
1
      Type: PositiveInteger

squareDiff8 := fourSquares(190,1751,208,1698)
      +-----+ +-----+ +-----+ +-----+
      - \|1698 - \|208 + \|1751 + \|190
      Type: RealClosure Fraction Integer

recip(squareDiff8)
      +-----+          +-----+ +-----+          +-----+ +-----+
      (- 214702\|1751 - 651782\|190 )\|208 - 224642\|190 \|1751
      +
      - 129571901
      *
      +-----+
      \|1698
+
      +-----+ +-----+          +-----+          +-----+
      (- 641842\|190 \|1751 - 370209881)\|208 - 127595865\|1751
+
      +-----+
      - 387349387\|190

```

```
Type: Union(RealClosure Fraction Integer,...)
```

```
sign(squareDiff8)
- 1
```

```
Type: Integer
```

This should give three digits of precision

```
relativeApprox(squareDiff8,10**(-3))::Float
- 0.2340527771 5937700123 E -10
```

```
Type: Float
```

The sum of these 4 roots is 0

```
l := allRootsOf((x**2-2)**2-2)$Ran
[%A33,%A34,%A35,%A36]
```

```
Type: List RealClosure Fraction Integer
```

Check that they are all roots of the same polynomial

```
removeDuplicates map(mainDefiningPolynomial,l)
```

```
4      2
[? - 4? + 2]
```

```
Type: List Union(
    SparseUnivariatePolynomial RealClosure Fraction Integer,
    "failed")
```

We can see at a glance that they are separate roots

```
map(mainCharacterization,l)
[[- 2,- 1],[ - 1,0],[0,1],[1,2[]]
```

```
Type: List Union(
    RightOpenIntervalRootCharacterization(
        RealClosure Fraction Integer,
        SparseUnivariatePolynomial RealClosure Fraction Integer),
    "failed")
```

Check the sum and product

```
[reduce(+,l),reduce(*,l)-2]
[0,0]
```

```
Type: List RealClosure Fraction Integer
```

A more complicated test that involve an extension of degree 256.  
This is a way of checking nested radical identities.

```
(s2, s5, s10) := (sqrt(2)$Ran, sqrt(5)$Ran, sqrt(10)$Ran)
+---+
\|10
```

```
Type: RealClosure Fraction Integer
```

```
eq1::Boolean
  true
                                     Type: Boolean
```

```
eq2::Boolean
true
Type: Boolean
```

```
s3 := sqrt(3)$Ran
++
\|3
Type: RealClosure Fraction Integer
```

```
s7:= sqrt(7)$Ran
++
\|7
Type: RealClosure Fraction Integer
```

```
e1 := sqrt(2*s7-3*s3,3)
+-----+
3|  +-+    +-+
\2\|7  - 3\3
Type: RealClosure Fraction Integer
```

```
e2 := sqrt(2*s7+3*s3,3)
+-----+
3| +-+ +-+
\2\17 + 3\13
Type: RealClosure Fraction Integer
```

```
e2-e1-s3
0
Type: RealClosure Fraction Integer
```

A quartic polynomial

```
pol : UP(x,Ran) := x**4+(7/3)*x**2+30*x-(100/3)
      4   7   2       100
      x  + - x  + 30x - ---
          3         3
Type: UnivariatePolynomial(x,RealClosure Fraction Integer)
```

Add some cubic roots

```
r1 := sqrt(7633)$Ran
      +-----+
      \|7633
Type: RealClosure Fraction Integer
```

```
alpha := sqrt(5*r1-436,3)/3
      +-----+
      1 3| +-----+
      - \|5\|7633 - 436
      3
Type: RealClosure Fraction Integer
```

```
beta := -sqrt(5*r1+436,3)/3
      +-----+
      1 3| +-----+
      - - \|5\|7633 + 436
      3
Type: RealClosure Fraction Integer
```

this should be null

```
pol.(alpha+beta-1/3)
      0
Type: RealClosure Fraction Integer
```

A quintic polynomial

```
qol : UP(x,Ran) := x**5+10*x**3+20*x+22
      5       3
      x  + 10x  + 20x + 22
Type: UnivariatePolynomial(x,RealClosure Fraction Integer)
```

Add some cubic roots

```
r2 := sqrt(153)$Ran
      +-----+
      \|153
Type: RealClosure Fraction Integer
```

```
alpha2 := sqrt(r2-11,5)
+-----+
5| +---+
\\|153  - 11
Type: RealClosure Fraction Integer
```

```
beta2 := -sqrt(r2+11,5)
+-----+
5| +---+
- \\|153  + 11
Type: RealClosure Fraction Integer
```

this should be null

```
qol(alpha2+beta2)
0
Type: RealClosure Fraction Integer
```

Finally, some examples from the book Computer Algebra by Davenport, Siret and Tournier (page 77). The last one is due to Ramanujan.

```
dst1:=sqrt(9+4*s2)=1+2*s2
+-----+
| +-+ +-+
\\4\\2  + 9 = 2\\2  + 1
Type: Equation RealClosure Fraction Integer
```

```
dst1::Boolean
true
Type: Boolean
```

```
s6:Ran:=sqrt 6
+-+
\\6
Type: RealClosure Fraction Integer
```

```
dst2:=sqrt(5+2*s6)+sqrt(5-2*s6) = 2*s3
+-----+ +-----+
| +-+ | +-+ +-+
\\- 2\\6  + 5 + \\2\\6  + 5 = 2\\3
Type: Equation RealClosure Fraction Integer
```

```
dst2::Boolean
true
Type: Boolean
```

```
s29:Ran:=sqrt 29
+---+
\\29
Type: RealClosure Fraction Integer
```

```

dst4:=sqrt(16-2*s29+2*sqrt(55-10*s29)) = sqrt(22+2*s5)-sqrt(11+2*s29)+s5
+-----+
| +-----+ +-----+ +-----+
| | +--+ +--+ | +--+ | +--+ +--+
\|2\|- 10\|29 + 55 - 2\|29 + 16 = - \|2\|29 + 11 + \|2\|5 + 22 + \|5
Type: Equation RealClosure Fraction Integer

```

```

dst4::Boolean
true
Type: Boolean

```

```

dst6:=sqrt((112+70*s2)+(46+34*s2)*s5) = (5+4*s2)+(3+s2)*s5
+-----+
| +--+ +--+ +--+ +--+ +--+ +--+
\|(34\|2 + 46)\|5 + 70\|2 + 112 = (\|2 + 3)\|5 + 4\|2 + 5
Type: Equation RealClosure Fraction Integer

```

```

dst6::Boolean
true
Type: Boolean

```

```

f3:Ran:=sqrt(3,5)
5+--+
\|3
Type: RealClosure Fraction Integer

```

```

f25:Ran:=sqrt(1/25,5)
+--+
| 1
5|--
\|25
Type: RealClosure Fraction Integer

```

```

f32:Ran:=sqrt(32/5,5)
+--+
|32
5|--
\| 5
Type: RealClosure Fraction Integer

```

```

f27:Ran:=sqrt(27/5,5)
+--+
|27
5|--
\| 5
Type: RealClosure Fraction Integer

```

```

dst5:=sqrt((f32-f27,3)) = f25*(1+f3-f3**2)
+-----+

```



```

      |  +---+  +---+      +---+
      |  |27   |32      5+--+2  5+--+  | 1
3|- 5|--- + 5|--- = (- \|3   + \|3   + 1) 5|---
\|  \| 5   \| 5               \|25
                                     Type: Equation RealClosure Fraction Integer

```

```

dst5::Boolean
true
                                     Type: Boolean

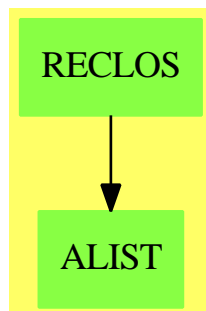
```

See Also:

- o )help RightOpenIntervalRootCharacterization
- o )help RealClosedField
- o )help RealRootCharacterizationCategory
- o )help UnivariatePolynomialCategory
- o )help Field
- o )help RealPolynomialUtilitiesPackage
- o )show RealClosure

---

### 19.3.1 RealClosure (RECLOS)



See

⇒ “RightOpenIntervalRootCharacterization” (ROIRC) 19.11.1 on page 2270

**Exports:**

|               |                      |                        |                    |
|---------------|----------------------|------------------------|--------------------|
| 0             | 1                    | abs                    | algebraicOf        |
| allRootsOf    | approximate          | associates?            | characteristic     |
| coerce        | divide               | euclideanSize          | expressIdealMember |
| exquo         | extendedEuclidean    | factor                 | gcd                |
| gcdPolynomial | hash                 | inv                    | latex              |
| lcm           | mainCharacterization | mainDefiningPolynomial | mainForm           |
| mainValue     | max                  | multiEuclidean         | min                |
| negative?     | nthRoot              | one?                   | positive?          |
| prime?        | principalIdeal       | recip                  | relativeApprox     |
| rename        | rename!              | retract                | retractIfCan       |
| rootOf        | sample               | sign                   | sizeLess?          |
| sqrt          | squareFree           | squareFreePart         | subtractIfCan      |
| unit?         | unitCanonical        | unitNormal             | zero?              |
| ?*?           | ?**?                 | ?+?                    | ?-?                |
| -?            | ?/?                  | ?<?                    | ?<=?               |
| ?=?           | ?>?                  | ?>=?                   | ?^?                |
| ?~=?          | ?quo?                | ?rem?                  |                    |

— domain RECLOS RealClosure —

```
)abbrev domain RECLOS RealClosure
++ Author: Renaud Rioboo
++ Date Created: summer 1988
++ Date Last Updated: January 2004
++ Basic Functions: provides computations in an ordered real closure
++ Related Constructors: RightOpenIntervalRootCharacterization
++ Also See:
++ AMS Classifications:
++ Keywords: Real Algebraic Numbers
++ References:
++ Description:
++ This domain implements the real closure of an ordered field.
++ Note:
++ The code here is generic i.e. it does not depend of the way the operations
++ are done. The two macros PME and SEG should be passed as functorial
++ arguments to the domain. It does not help much to write a category
++ since non trivial methods cannot be placed there either.
++
```

```
RealClosure(TheField): PUB == PRIV where
```

```

    TheField    : Join(OrderedRing, Field, RealConstant)

--   ThePolS      : UnivariatePolynomialCategory($)
--   PME          ==> ThePolS
--   TheCharDom   : RealRootCharacterizationCategory($, ThePolS )
--   SEG          ==> TheCharDom
-- this does not work yet
```

```

E      ==> OutputForm
Z      ==> Integer
SE     ==> Symbol
B      ==> Boolean
SUP    ==> SparseUnivariatePolynomial($)
N      ==> PositiveInteger
RN     ==> Fraction Z
LF     ==> ListFunctions2($,N)
PME    ==> SparseUnivariatePolynomial($)
SEG    ==> RightOpenIntervalRootCharacterization($,PME)

PUB == Join(RealClosedField,
            FullyRetractableTo TheField,
            Algebra TheField) with

algebraicOf : (SEG,E) -> $
++ \axiom{algebraicOf(char)} is the external number

mainCharacterization : $ -> Union(SEG,"failed")
++ \axiom{mainCharacterization(x)} is the main algebraic
++ quantity of \axiom{x} (\axiom{SEG})

relativeApprox : ($,$) -> RN
++ \axiom{relativeApprox(n,p)} gives a relative
++ approximation of \axiom{n}
++ that has precision \axiom{p}

PRIV == add

-- local functions

lessAlgebraic : $ -> $
newElementIfneeded : (SEG,E) -> $

-- Representation

Rec := Record(seg: SEG, val:PME, outForm:E, order:N)
Rep := Union(TheField,Rec)

-- global (mutable) variables

orderOfCreation : N := 1$N
-- it is internally used to sort the algebraic levels

instanceName : Symbol := new()$Symbol
-- this used to print the results, thus different instanciations
-- use different names

-- now the code

```

```

relativeApprox(nbe,prec) ==
  nbe case TheField => retract(nbe)
  appr := relativeApprox(nbe.val, nbe.seg, prec)
  -- now appr has the good exact precision but is $
  relativeApprox(appr,prec)

approximate(nbe,prec) ==
  abs(nbe) < prec => 0
  nbe case TheField => retract(nbe)
  appr := approximate(nbe.val, nbe.seg, prec)
  -- now appr has the good exact precision but is $
  approximate(appr,prec)

newElementIfneeded(s,o) ==
  p := definingPolynomial(s)
  degree(p) = 1 =>
    - coefficient(p,0) / leadingCoefficient(p)
  res := [s, monomial(1,1), o, orderOfCreation ]$Rec
  orderOfCreation := orderOfCreation + 1
  res :: $

algebraicOf(s,o) ==
  pol := definingPolynomial(s)
  degree(pol) = 1 =>
    -coefficient(pol,0) / leadingCoefficient(pol)
  res := [s, monomial(1,1), o, orderOfCreation ]$Rec
  orderOfCreation := orderOfCreation + 1
  res :: $

rename!(x,o) ==
  x.outForm := o
  x

rename(x,o) ==
  [x.seg, x.val, o, x.order]$Rec

rootOf(pol,n) ==
  degree(pol) = 0 => "failed"
  degree(pol) = 1 =>
    if n=1
    then
      -coefficient(pol,0) / leadingCoefficient(pol)
    else
      "failed"
  r := rootOf(pol,n)$SEG
  r case "failed" => "failed"
  o := hconcat(instanceName :: E , orderOfCreation :: E)$E
  algebraicOf(r,o)

```

```

allRootsOf(pol:SUP):List($)==
degree(pol)=0 => []
degree(pol)=1 => [-coefficient(pol,0) / leadingCoefficient(pol)]
liste := allRootsOf(pol)$SEG
res : List $ := []
for term in liste repeat
  o := hconcat(instanceName :: E , orderOfCreation :: E)$E
  res := cons(algebraicOf(term,o), res)
reverse! res

coerce(x:$):$ ==
x case TheField => x
[x.seg,x.val rem$PME definingPolynomial(x.seg),x.outForm,x.order]$Rec

positive?(x) ==
x case TheField => positive?(x)$TheField
positive?(x.val,x.seg)$SEG

negative?(x) ==
x case TheField => negative?(x)$TheField
negative?(x.val,x.seg)$SEG

abs(x) == sign(x)*x

sign(x) ==
x case TheField => sign(x)$TheField
sign(x.val,x.seg)$SEG

x < y == positive?(y-x)

x = y == zero?(x-y)

mainCharacterization(x) ==
x case TheField => "failed"
x.seg

mainDefiningPolynomial(x) ==
x case TheField => "failed"
definingPolynomial x.seg

mainForm(x) ==
x case TheField => "failed"
x.outForm

mainValue(x) ==
x case TheField => "failed"
x.val

coerce(x:$):E ==
x case TheField => x::TheField :: E

```

```

xx:$ := coerce(x)
outputForm(univariate(xx.val),x.outForm)$SUP

inv(x) ==
  (res:= recip x) case "failed" => error "Division by 0"
  res :: $

recip(x) ==
  x case TheField =>
    if ((r := recip(x)$TheField) case TheField)
    then r::$
    else "failed"
  if ((r := recip(x.val,x.seg)$SEG) case "failed")
  then "failed"
  else lessAlgebraic([x.seg,r::PME,x.outForm,x.order]$Rec)

(n:Z * x:$):$ ==
  x case TheField => n *$TheField x
  zero?(n) => 0
  one?(n) => x
  [x.seg,map(z+>n*z, x.val),x.outForm,x.order]$Rec

(rn:TheField * x:$):$ ==
  x case TheField => rn *$TheField x
  zero?(rn) => 0
  one?(rn) => x
  [x.seg,map(z+>rn*z, x.val),x.outForm,x.order]$Rec

(x:$ * y:$):$ ==
  (x case TheField) and (y case TheField) => x *$TheField y
  (x case TheField) => x::$TheField * y
  -- x is no longer TheField
  (y case TheField) => y::$TheField * x
  -- now both are algebraic
  y.order > x.order =>
    [y.seg,map(z+>x*z, y.val),y.outForm,y.order]$Rec
  x.order > y.order =>
    [x.seg,map(z+>z*y, x.val),x.outForm,x.order]$Rec
  -- now x.exp = y.exp
  -- we will multiply the polynomials and then reduce
  -- however we need to call lessAlgebraic
  lessAlgebraic([x.seg,
    (x.val * y.val) rem definingPolynomial(x.seg),
    x.outForm,
    x.order]$Rec)

nonNull(rep:Rec):$ ==
  degree(rep.val)=0 => leadingCoefficient(rep.val)
  numberOfMonomials(rep.val) = 1 => rep

```

```

zero?(rep.val,rep.seg)$SEG => 0
rep

--      zero?(x) ==
--      x case TheField => zero?(x)$TheField
--      zero?(x.val,x.seg)$SEG

zero?(x) ==
  x case TheField => zero?(x)$TheField
  false

x + y ==
  (x case TheField) and (y case TheField) => x +$TheField y
  (x case TheField) =>
    if zero?(x)
    then
      y
    else
      nonNull([y.seg,x::PME+(y.val),y.outForm,y.order]$Rec)
      -- x is no longer TheField
  (y case TheField) =>
    if zero?(y)
    then
      x
    else
      nonNull([x.seg,(x.val)+y::PME,x.outForm,x.order]$Rec)
      -- now both are algebraic
  y.order > x.order =>
    nonNull([y.seg,x::PME+y.val,y.outForm,y.order]$Rec)
  x.order > y.order =>
    nonNull([x.seg,(x.val)+y::PME,x.outForm,x.order]$Rec)
    -- now x.exp = y.exp
    -- we simply add polynomials (since degree cannot increase)
    -- however we need to call lessAlgebraic
  nonNull([x.seg,x.val + y.val,x.outForm,x.order])

-x ==
  x case TheField => -$TheField (x::TheField)
  [x.seg,-$PME x.val,x.outForm,x.order]$Rec

retractIfCan(x:$):Union(TheField,"failed") ==
  x case TheField => x
  o := x.order
  res := lessAlgebraic x
  res case TheField => res
  o = res.order => "failed"
  retractIfCan res

```

```

retract(x:$):TheField ==
  x case TheField => x
  o := x.order
  res := lessAlgebraic x
  res case TheField => res
  o = res.order => error "Can't retract"
  retract res

lessAlgebraic(x) ==
  x case TheField => x
  degree(x.val) = 0 => leadingCoefficient(x.val)
  def := definingPolynomial(x.seg)
  degree(def) = 1 =>
    x.val.(- coefficient(def,0) / leadingCoefficient(def))
  x

0 == (0$TheField) :: $

1 == (1$TheField) :: $

coerce(rn:TheField):$ == rn :: $

```

---

— RECLOS.dotabb —

```

"RECLOS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=RECLOS"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"RECLOS" -> "ALIST"

```

---

## 19.4 domain RMATRIX RectangularMatrix

— RectangularMatrix.input —

```

)set break resume
)sys rm -f RectangularMatrix.output
)spool RectangularMatrix.output
)set message test on
)set message auto off
)clear all

```



```

--S 1 of 1
)show RectangularMatrix
--R RectangularMatrix(m: NonNegativeInteger,n: NonNegativeInteger,R: Ring) is a domain constructor
--R Abbreviation for RectangularMatrix is RMATRIX
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for RMATRIX
--R
--R----- Operations -----
--R ?? : (%,R) -> %               ?? : (R,%) -> %
--R ?? : (Integer,%) -> %         ?? : (PositiveInteger,%) -> %
--R ?? : (%,%) -> %              ?-? : (%,%) -> %
--R -? : % -> %                   ?=? : (%,%) -> Boolean
--R 0 : () -> %                  antisymmetric? : % -> Boolean
--R coerce : % -> Matrix R       coerce : % -> OutputForm
--R copy : % -> %                 diagonal? : % -> Boolean
--R elt : (%,Integer,Integer,R) -> R elt : (%,Integer,Integer) -> R
--R empty : () -> %              empty? : % -> Boolean
--R eq? : (%,%) -> Boolean        hash : % -> SingleInteger
--R latex : % -> String          listOfLists : % -> List List R
--R map : ((R,R) -> R),%,%) -> % map : ((R -> R),%) -> %
--R matrix : List List R -> %    maxColIndex : % -> Integer
--R maxRowIndex : % -> Integer   minColIndex : % -> Integer
--R minRowIndex : % -> Integer   ncols : % -> NonNegativeInteger
--R nrows : % -> NonNegativeInteger qelt : (%,Integer,Integer) -> R
--R rectangularMatrix : Matrix R -> % sample : () -> %
--R square? : % -> Boolean       symmetric? : % -> Boolean
--R zero? : % -> Boolean         ?~=? : (%,%) -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?? : (NonNegativeInteger,%) -> %
--R ?/? : (%,R) -> % if R has FIELD
--R any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R column : (%,Integer) -> DirectProduct(m,R)
--R convert : % -> InputForm if R has KONVERT INFORM
--R count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R count : (R,%) -> NonNegativeInteger if $ has finiteAggregate and R has SETCAT
--R dimension : () -> CardinalNumber if R has FIELD
--R eval : (%,List Equation R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,Equation R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,R,R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,List R,List R) -> % if R has EVALAB R and R has SETCAT
--R every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R exquo : (%,R) -> Union(%, "failed") if R has INTDOM
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((R -> R),%) -> % if $ has shallowlyMutable
--R member? : (R,%) -> Boolean if $ has finiteAggregate and R has SETCAT
--R members : % -> List R if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R nullSpace : % -> List DirectProduct(m,R) if R has INTDOM
--R nullity : % -> NonNegativeInteger if R has INTDOM
--R parts : % -> List R if $ has finiteAggregate

```

```

--R rank : % -> NonNegativeInteger if R has INTDOM
--R row : (%,Integer) -> DirectProduct(n,R)
--R rowEchelon : % -> % if R has EUCDOM
--R size? : (%NonNegativeInteger) -> Boolean
--R subtractIfCan : (%,% ) -> Union(%, "failed")
--R
--E 1

```

```

)spool
)lisp (bye)

```

---

— RectangularMatrix.help —

```

=====
RectangularMatrix examples
=====

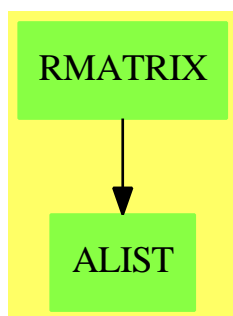
```

```

See Also:
o )show RectangularMatrix

```

### 19.4.1 RectangularMatrix (RMATRIX)



See

- ⇒ “IndexedMatrix” (IMATRIX) 10.12.1 on page 1204
- ⇒ “Matrix” (MATRIX) 14.7.1 on page 1586
- ⇒ “SquareMatrix” (SQMATRIX) 20.27.1 on page 2505

**Exports:**

|             |                |         |                   |               |
|-------------|----------------|---------|-------------------|---------------|
| 0           | antisymmetric? | any?    | coerce            | column        |
| convert     | copy           | count   | diagonal?         | dimension     |
| elt         | empty          | empty?  | eq?               | eval          |
| every?      | exquo          | hash    | latex             | less?         |
| listOfLists | map            | map!    | matrix            | maxColIndex   |
| maxRowIndex | member?        | members | minColIndex       | minRowIndex   |
| more?       | ncols          | nrows   | nullSpace         | nullity       |
| parts       | qelt           | rank    | rectangularMatrix | row           |
| rowEchelon  | sample         | size?   | square?           | subtractIfCan |
| symmetric?  | zero?          | #?      | ?~=?              | ?*?           |
| ?/?         | ?+?            | ?-?     | -?                | ?=?           |

— domain RMATRIX RectangularMatrix —

```

)abbrev domain RMATRIX RectangularMatrix
++ Author: Grabmeier, Gschnitzer, Williamson
++ Date Created: 1987
++ Date Last Updated: July 1990
++ Basic Operations:
++ Related Domains: IndexedMatrix, Matrix, SquareMatrix
++ Also See:
++ AMS Classifications:
++ Keywords: matrix, linear algebra
++ Examples:
++ References:
++ Description:
++ \spadtype{RectangularMatrix} is a matrix domain where the number of rows
++ and the number of columns are parameters of the domain.

RectangularMatrix(m,n,R): Exports == Implementation where
  m,n : NonNegativeInteger
  R   : Ring
  Row ==> DirectProduct(n,R)
  Col ==> DirectProduct(m,R)
  Exports ==> Join(RectangularMatrixCategory(m,n,R,Row,Col),_
    CoercibleTo Matrix R) with

  if R has Field then VectorSpace R

  if R has ConvertibleTo InputForm then ConvertibleTo InputForm

rectangularMatrix: Matrix R -> $
  ++ \spad{rectangularMatrix(m)} converts a matrix of type \spadtype{Matrix}
  ++ to a matrix of type \spad{RectangularMatrix}.
coerce: $ -> Matrix R
  ++ \spad{coerce(m)} converts a matrix of type \spadtype{RectangularMatrix}
  ++ to a matrix of type \spad{Matrix}.

Implementation ==> Matrix R add

```

```

minr ==> minRowIndex
maxr ==> maxRowIndex
minc ==> minColIndex
maxc ==> maxColIndex
mini ==> minIndex
maxi ==> maxIndex

ZERO := new(m,n,0)$Matrix(R) pretend $
0     == ZERO

coerce(x:$):OutputForm == coerce(x pretend Matrix R)$Matrix(R)

matrix(l: List List R) ==
  -- error check: this is a top level function
  #l ^= m => error "matrix: wrong number of rows"
  for ll in l repeat
    #ll ^= n => error "matrix: wrong number of columns"
  ans : Matrix R := new(m,n,0)
  for i in minr(ans)..maxr(ans) for ll in l repeat
    for j in minc(ans)..maxc(ans) for r in ll repeat
      qsetelt_!(ans,i,j,r)
  ans pretend $

row(x,i)    == directProduct row(x pretend Matrix(R),i)
column(x,j) == directProduct column(x pretend Matrix(R),j)

coerce(x:$):Matrix(R) == copy(x pretend Matrix(R))

rectangularMatrix x ==
  (nrows(x) ^= m) or (ncols(x) ^= n) =>
    error "rectangularMatrix: matrix of bad dimensions"
  copy(x) pretend $

if R has EuclideanDomain then

  rowEchelon x == rowEchelon(x pretend Matrix(R)) pretend $

if R has IntegralDomain then

  rank x    == rank(x pretend Matrix(R))
  nullity x == nullity(x pretend Matrix(R))
  nullSpace x ==
    [directProduct c for c in nullSpace(x pretend Matrix(R))]]

if R has Field then

  dimension() == (m * n) :: CardinalNumber

if R has ConvertibleTo InputForm then
  convert(x:$):InputForm ==

```

```
convert [convert("rectangularMatrix":Symbol)@InputForm,
         convert(x:Matrix(R)))]$List(InputForm)
```

---

— RMATRIX.dotabb —

```
"RMATRIX" [color="#88FF44",href="bookvol10.3.pdf#nameddest=RMATRIX"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"RMATRIX" -> "ALIST"
```

---

## 19.5 domain REF Reference

— Reference.input —

```
)set break resume
)sys rm -f Reference.output
)spool Reference.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Reference
--R Reference S: Type is a domain constructor
--R Abbreviation for Reference is REF
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for REF
--R
--R----- Operations -----
--R ?=? : (%,% ) -> Boolean          deref : % -> S
--R elt : % -> S                    ref : S -> %
--R setelt : (% ,S) -> S            setref : (% ,S) -> S
--R coerce : % -> OutputForm if S has SETCAT
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R ?~=? : (%,% ) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

---



---

— **Reference.help** —

---

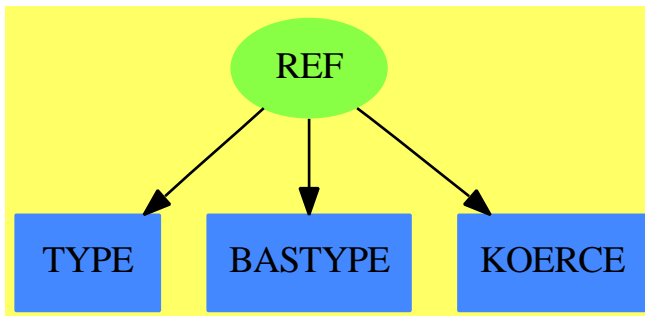
```
=====
Reference examples
=====
```

See Also:

o )show Reference

---

### 19.5.1 Reference (REF)



See

⇒ “Boolean” (BOOLEAN) 3.15.1 on page 304

⇒ “IndexedBits” (IBITS) 10.3.1 on page 1165

⇒ “Bits” (BITS) 3.12.1 on page 297

#### Exports:

|        |        |        |      |       |
|--------|--------|--------|------|-------|
| coerce | deref  | elt    | hash | latex |
| ref    | setelt | setref | ?=?  | ?~=?  |

---

— **domain REF Reference** —

---

```
)abbrev domain REF Reference
++ Author: Stephen M. Watt
++ Date Created:
++ Change History:
++ Basic Operations: deref, elt, ref, setelt, setref, =
++ Related Constructors:
++ Keywords: reference
++ Description:
++ \spadtype{Reference} is for making a changeable instance
```

++ of something.

```

Reference(S:Type): Type with
  ref   : S -> %
  ++ ref(n) creates a pointer (reference) to the object n.
  elt   : % -> S
  ++ elt(n) returns the object n.
  setelt: (% , S) -> S
  ++ setelt(n,m) changes the value of the object n to m.
  -- alternates for when bugs don't allow the above
  deref : % -> S
  ++ deref(n) is equivalent to \spad{elt(n)}.
  setref: (% , S) -> S
  ++ setref(n,m) same as \spad{setelt(n,m)}.
  _=    : (% , %) -> Boolean
  ++ a=b tests if \spad{a} and b are equal.
  if S has SetCategory then SetCategory

== add
  Rep := Record(value: S)

  p = q      == EQ(p, q)$Lisp
  ref v      == [v]
  elt p      == p.value
  setelt(p, v) == p.value := v
  deref p    == p.value
  setref(p, v) == p.value := v

  if S has SetCategory then
    coerce p ==
      prefix(message("ref"@String), [p.value::OutputForm])

  -----

  — REF.dotabb —

"REF" [color="#88FF44",href="bookvol10.3.pdf#nameddest=REF",shape=ellipse]
"TYPE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=TYPE"]
"BASTYPE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=BASTYPE"]
"KOERCE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=KOERCE"]
"REF" -> "TYPE"
"REF" -> "BASTYPE"
"REF" -> "KOERCE"

  -----

```

— RegularChain.input —

```

)set break resume
)sys rm -f RegularChain.output
)spool RegularChain.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show RegularChain
--R RegularChain(R: GcdDomain,ls: List Symbol) is a domain constructor
--R Abbreviation for RegularChain is RGCHAIN
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for RGCHAIN
--R
--R----- Operations -----
--R ==? : (% ,%) -> Boolean               coerce : % -> OutputForm
--R collectQuasiMonic : % -> %             copy : % -> %
--R degree : % -> NonNegativeInteger       empty : () -> %
--R empty? : % -> Boolean                  eq? : (% ,%) -> Boolean
--R hash : % -> SingleInteger              headReduced? : % -> Boolean
--R infRittWu? : (% ,%) -> Boolean          initiallyReduced? : % -> Boolean
--R latex : % -> String                    normalized? : % -> Boolean
--R purelyAlgebraic? : % -> Boolean         rest : % -> Union(%, "failed")
--R sample : () -> %                       stronglyReduced? : % -> Boolean
--R trivialIdeal? : % -> Boolean            ~=? : (% ,%) -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R algebraic? : (OrderedVariableList ls, %) -> Boolean
--R algebraicCoefficients? : (NewSparseMultivariatePolynomial(R, OrderedVariableList ls), %) -> Boolean
--R algebraicVariables : % -> List OrderedVariableList ls
--R any? : ((NewSparseMultivariatePolynomial(R, OrderedVariableList ls) -> Boolean), %) -> Boolean if $ has finiteAggregate
--R augment : (List NewSparseMultivariatePolynomial(R, OrderedVariableList ls), List %) -> List %
--R augment : (List NewSparseMultivariatePolynomial(R, OrderedVariableList ls), %) -> List %
--R augment : (NewSparseMultivariatePolynomial(R, OrderedVariableList ls), List %) -> List %
--R augment : (NewSparseMultivariatePolynomial(R, OrderedVariableList ls), %) -> List %
--R autoReduced? : (% , ((NewSparseMultivariatePolynomial(R, OrderedVariableList ls), List NewSparseMultivariatePolynomial(R, OrderedVariableList ls))) -> Boolean) -> Boolean
--R basicSet : (List NewSparseMultivariatePolynomial(R, OrderedVariableList ls), (NewSparseMultivariatePolynomial(R, OrderedVariableList ls))) -> List %
--R basicSet : (List NewSparseMultivariatePolynomial(R, OrderedVariableList ls), ((NewSparseMultivariatePolynomial(R, OrderedVariableList ls))) -> List %
--R coHeight : % -> NonNegativeInteger if OrderedVariableList ls has FINITE
--R coerce : % -> List NewSparseMultivariatePolynomial(R, OrderedVariableList ls)
--R collect : (% , OrderedVariableList ls) -> %
--R collectUnder : (% , OrderedVariableList ls) -> %
--R collectUpper : (% , OrderedVariableList ls) -> %
--R construct : List NewSparseMultivariatePolynomial(R, OrderedVariableList ls) -> %
--R convert : % -> InputForm if NewSparseMultivariatePolynomial(R, OrderedVariableList ls) has KONVERT

```



```

--R count : ((NewSparseMultivariatePolynomial(R,OrderedVariableList ls) -> Boolean),%) -> NonNegativeInteger
--R count : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> NonNegativeInteger
--R eval : (%,List Equation NewSparseMultivariatePolynomial(R,OrderedVariableList ls)) -> %
--R eval : (%,Equation NewSparseMultivariatePolynomial(R,OrderedVariableList ls)) -> % if NewSparseMultivariatePolynomial(R,OrderedVariableList ls) is an equation
--R eval : (%,NewSparseMultivariatePolynomial(R,OrderedVariableList ls),NewSparseMultivariatePolynomial(R,OrderedVariableList ls)) -> %
--R eval : (%,List NewSparseMultivariatePolynomial(R,OrderedVariableList ls),List NewSparseMultivariatePolynomial(R,OrderedVariableList ls)) -> %
--R every? : ((NewSparseMultivariatePolynomial(R,OrderedVariableList ls) -> Boolean),%) -> Boolean
--R extend : (List NewSparseMultivariatePolynomial(R,OrderedVariableList ls),List %) -> List NewSparseMultivariatePolynomial(R,OrderedVariableList ls)
--R extend : (List NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> List %
--R extend : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),List %) -> List %
--R extend : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> List %
--R extend : (%,NewSparseMultivariatePolynomial(R,OrderedVariableList ls)) -> %
--R extendIfCan : (%,NewSparseMultivariatePolynomial(R,OrderedVariableList ls)) -> Union(%,NewSparseMultivariatePolynomial(R,OrderedVariableList ls))
--R find : ((NewSparseMultivariatePolynomial(R,OrderedVariableList ls) -> Boolean),%) -> Union(%,NewSparseMultivariatePolynomial(R,OrderedVariableList ls))
--R first : % -> Union(NewSparseMultivariatePolynomial(R,OrderedVariableList ls),"failed")
--R headReduce : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> NewSparseMultivariatePolynomial(R,OrderedVariableList ls)
--R headReduced? : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> Boolean
--R headRemainder : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> Record(close: NewSparseMultivariatePolynomial(R,OrderedVariableList ls),remainder: NewSparseMultivariatePolynomial(R,OrderedVariableList ls))
--R initiallyReduce : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> NewSparseMultivariatePolynomial(R,OrderedVariableList ls)
--R initiallyReduced? : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> Boolean
--R initials : % -> List NewSparseMultivariatePolynomial(R,OrderedVariableList ls)
--R internalAugment : (List NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> List NewSparseMultivariatePolynomial(R,OrderedVariableList ls)
--R internalAugment : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> %
--R intersect : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),List %) -> List %
--R intersect : (List NewSparseMultivariatePolynomial(R,OrderedVariableList ls),List %) -> List %
--R intersect : (List NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> List %
--R intersect : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> List %
--R invertible? : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> Boolean
--R invertible? : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> List Record(close: NewSparseMultivariatePolynomial(R,OrderedVariableList ls),remainder: NewSparseMultivariatePolynomial(R,OrderedVariableList ls))
--R invertibleElseSplit? : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> Union(%,NewSparseMultivariatePolynomial(R,OrderedVariableList ls))
--R invertibleSet : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> List %
--R last : % -> Union(NewSparseMultivariatePolynomial(R,OrderedVariableList ls),"failed")
--R lastSubResultant : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),NewSparseMultivariatePolynomial(R,OrderedVariableList ls)) -> NewSparseMultivariatePolynomial(R,OrderedVariableList ls)
--R lastSubResultantElseSplit : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),NewSparseMultivariatePolynomial(R,OrderedVariableList ls)) -> Union(%,NewSparseMultivariatePolynomial(R,OrderedVariableList ls))
--R less? : (%,NonNegativeInteger) -> Boolean
--R mainVariable? : (OrderedVariableList ls,%) -> Boolean
--R mainVariables : % -> List OrderedVariableList ls
--R map : ((NewSparseMultivariatePolynomial(R,OrderedVariableList ls) -> NewSparseMultivariatePolynomial(R,OrderedVariableList ls)),List %) -> List NewSparseMultivariatePolynomial(R,OrderedVariableList ls)
--R map! : ((NewSparseMultivariatePolynomial(R,OrderedVariableList ls) -> NewSparseMultivariatePolynomial(R,OrderedVariableList ls)),List %) -> List NewSparseMultivariatePolynomial(R,OrderedVariableList ls)
--R member? : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> Boolean if % is in the list
--R members : % -> List NewSparseMultivariatePolynomial(R,OrderedVariableList ls) if % has finitely many members
--R more? : (%,NonNegativeInteger) -> Boolean
--R mvar : % -> OrderedVariableList ls
--R normalized? : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> Boolean
--R parts : % -> List NewSparseMultivariatePolynomial(R,OrderedVariableList ls) if % has finitely many parts
--R purelyAlgebraic? : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> Boolean
--R purelyAlgebraicLeadingMonomial? : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> NewSparseMultivariatePolynomial(R,OrderedVariableList ls)
--R purelyTranscendental? : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%) -> Boolean
--R quasiComponent : % -> Record(close: List NewSparseMultivariatePolynomial(R,OrderedVariableList ls),remainder: NewSparseMultivariatePolynomial(R,OrderedVariableList ls))
--R reduce : (NewSparseMultivariatePolynomial(R,OrderedVariableList ls),%,(NewSparseMultivariatePolynomial(R,OrderedVariableList ls),NewSparseMultivariatePolynomial(R,OrderedVariableList ls))) -> NewSparseMultivariatePolynomial(R,OrderedVariableList ls)

```

```
)spool
)lisp (bye)
```

---

RegularChain examples

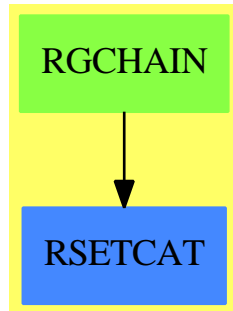
---

See Also:

- o )show RegularChain

---

### 19.6.1 RegularChain (RGCHAIN)



Exports:

|                                 |                                   |
|---------------------------------|-----------------------------------|
| algebraic?                      | algebraicCoefficients?            |
| algebraicVariables              | any?                              |
| augment                         | autoReduced?                      |
| basicSet                        | coHeight                          |
| coerce                          | collect                           |
| collectQuasiMonic               | collectUnder                      |
| collectUpper                    | construct                         |
| convert                         | copy                              |
| count                           | degree                            |
| empty                           | empty?                            |
| eq?                             | eval                              |
| every?                          | extend                            |
| extendIfCan                     | find                              |
| first                           | hash                              |
| headReduce                      | headReduced?                      |
| headRemainder                   | infRittWu?                        |
| initiallyReduce                 | initiallyReduced?                 |
| initials                        | internalAugment                   |
| intersect                       | invertible?                       |
| invertibleElseSplit?            | invertibleSet                     |
| last                            | lastSubResultant                  |
| lastSubResultantElseSplit       | latex                             |
| less?                           | mainVariable?                     |
| mainVariables                   | map                               |
| map!                            | member?                           |
| members                         | more?                             |
| mvar                            | normalized?                       |
| parts                           | purelyAlgebraic?                  |
| purelyAlgebraicLeadingMonomial? | purelyTranscendental?             |
| quasiComponent                  | reduce                            |
| reduceByQuasiMonic              | reduced?                          |
| remainder                       | remove                            |
| removeDuplicates                | removeZero                        |
| rest                            | retract                           |
| retractIfCan                    | rewriteIdealWithHeadRemainder     |
| rewriteIdealWithRemainder       | rewriteSetWithReduction           |
| roughBase?                      | roughEqualIdeals?                 |
| roughSubIdeal?                  | roughUnitIdeal?                   |
| sample                          | select                            |
| size?                           | sort                              |
| squareFreePart                  | stronglyReduce                    |
| stronglyReduced?                | triangular?                       |
| trivialIdeal?                   | variables                         |
| zeroSetSplit                    | zeroSetSplitIntoTriangularSystems |
| #?                              | ?~=?                              |
| ?=?                             |                                   |

## — domain RGCHAIN RegularChain —

```

)abbrev domain RGCHAIN RegularChain
++ Author: Marc Moreno Maza
++ Date Created: 01/1999
++ Date Last Updated: 23/01/1999
++ Description:
++ A domain for regular chains (i.e. regular triangular sets) over
++ a Gcd-Domain and with a fix list of variables.
++ This is just a front-end for the \spadtype{RegularTriangularSet}
++ domain constructor.

RegularChain(R,ls): Exports == Implementation where
  R : GcdDomain
  ls: List Symbol
  V ==> OrderedVariableList ls
  E ==> IndexedExponents V
  P ==> NewSparseMultivariatePolynomial(R,V)
  TS ==> RegularTriangularSet(R,E,V,P)

Exports == RegularTriangularSetCategory(R,E,V,P) with
  zeroSetSplit: (List P, Boolean, Boolean) -> List $
    ++ \spad{zeroSetSplit(lp,clos?,info?)} returns a list \spad{lts} of
    ++ regular chains such that the union of the closures of their regular
    ++ zero sets equals the affine variety associated with \spad{lp}.
    ++ Moreover, if \spad{clos?} is \spad{false} then the union of the
    ++ regular zero set of the \spad{ts} (for \spad{ts} in \spad{lts})
    ++ equals this variety.
    ++ If \spad{info?} is \spad{true} then some information is
    ++ displayed during the computations. See
    ++ zeroSetSplit from RegularTriangularSet.

Implementation == RegularTriangularSet(R,E,V,P)

```

## — RGCHAIN.dotabb —

```

"RGCHAIN" [color="#88FF44",href="bookvol10.3.pdf#nameddest=RGCHAIN"]
"RSETCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=RSETCAT"]
"RGCHAIN" -> "RSETCAT"

```

Several domain constructors implement regular triangular sets (or regular chains). Among them **RegularTriangularSet** and **SquareFreeRegularTriangularSet**. They also implement an algorithm by Marc Moreno Maza for computing triangular decompositions of polynomial systems. This method is refined in the package **LazardSetSolvingPackage** in order to produce decompositions by means of Lazard triangular sets.

[illegible]



```

--S 11 of 34
p1 := x ** 31 - x ** 6 - x - y
--R
--R
--R      31      6
--R (11)  x  - x  - x - y
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 11

--S 12 of 34
p2 := x ** 8 - z
--R
--R
--R      8
--R (12)  x  - z
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 12

--S 13 of 34
p3 := x ** 10 - t
--R
--R
--R      10
--R (13)  x  - t
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 13

--S 14 of 34
lp := [p1, p2, p3]
--R
--R
--R      31      6      8      10
--R (14)  [x  - x  - x - y, x  - z, x  - t]
--R Type: List NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 14

--S 15 of 34
zeroSetSplit(lp)$T
--R
--R
--R      5      4      2      3      8      5      3      2      4      2
--R (15)  [{z  - t , t z y  + 2z y - t  + 2t  + t  - t , (t  - t)x - t y - z}]
--R Type: List RegularTriangularSet(Integer,IndexedExponents OrderedVariableList [x,y,z,t],OrderedVariableList [x,y,z,t])
--E 15

--S 16 of 34
lts := zeroSetSplit(lp,false)$T
--R
--R
--R (16)

```



```

--R      5      4      2      3      8      5      3      2      4      2
--R      [{z - t , t z y + 2z y - t + 2t + t - t , (t - t)x - t y - z },
--R      3      5      2      3      2
--R      {t - 1, z - t, t z y + 2z y + 1, z x - t}, {t, z, y, x}]
--RType: List RegularTriangularSet(Integer, IndexedExponents OrderedVariableList [x, y, z, t], Or
--E 16

--S 17 of 34
[coHeight(ts) for ts in lts]
--R
--R
--R      (17)  [1, 0, 0]
--R
--R                                          Type: List NonNegativeInteger
--E 17

--S 18 of 34
f1 := y**2*z+2*x*y*t-2*x-z
--R
--R
--R      2
--R      (18)  (2t y - 2)x + z y - z
--R Type: NewSparseMultivariatePolynomial(Integer, OrderedVariableList [x, y, z, t])
--E 18

--S 19 of 34
f2:=-x**3*z+ 4*x*y**2*z+4*x**2*y*t+2*y**3*t+4*x**2-10*y**2+4*x*z-10*y*t+2
--R
--R
--R      3      2      2      3      2
--R      (19)  - z x + (4t y + 4)x + (4z y + 4z)x + 2t y - 10y - 10t y + 2
--R Type: NewSparseMultivariatePolynomial(Integer, OrderedVariableList [x, y, z, t])
--E 19

--S 20 of 34
f3 := 2*y*z*t+x*t**2-x-2*z
--R
--R
--R      2
--R      (20)  (t - 1)x + 2t z y - 2z
--R Type: NewSparseMultivariatePolynomial(Integer, OrderedVariableList [x, y, z, t])
--E 20

--S 21 of 34
f4:=-x*z**3+4*y*z**2*t+4*x*z*t**2+2*y*t**3+4*x*z+4*z**2-10*y*t- 10*t**2+2
--R
--R
--R      3      2      2      3      2      2
--R      (21)  (- z + (4t + 4)z)x + (4t z + 2t - 10t)y + 4z - 10t + 2
--R Type: NewSparseMultivariatePolynomial(Integer, OrderedVariableList [x, y, z, t])
--E 21

```

```

--S 22 of 34
lf := [f1, f2, f3, f4]
--R
--R
--R (22)
--R

$$\begin{aligned} & [(2t^2y - 2)x + z^2y^2 - z^3, \\ & -z^3x^3 + (4t^2y + 4)x^2 + (4z^2y + 4z)x + 2t^3y^3 - 10y^2 - 10t^2y + 2, \\ & (t^2 - 1)x + 2tz^2y - 2z, \\ & (-z^3 + (4t^2 + 4)z)x + (4t^2z^2 + 2t^3 - 10t)y + 4z^2 - 10t^2 + 2] \end{aligned}$$

--RType: List NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 22

--S 23 of 34
zeroSetSplit(lf)$T
--R
--R
--R (23)
--R

$$\begin{aligned} & [{t^2 - 1, z^8 - 16z^6 + 256z^2 - 256, t^3y - 1, (z^3 - 8z)x - 8z^2 + 16}, \\ & {3t^2 + 1, z^2 - 7t^2 - 1, y + t, x + z}, \\ & {t^8 - 10t^6 + 10t^2 - 1, z, (t^3 - 5t)y - 5t^2 + 1, x}, \\ & {t^2 + 3, z^2 - 4, y + t, x - z}] \end{aligned}$$

--RType: List RegularTriangularSet(Integer,IndexedExponents OrderedVariableList [x,y,z,t],OrderedVariableList)
--E 23

--S 24 of 34
lts2 := zeroSetSplit(lf,false)$T
--R
--R
--R (24)
--R

$$\begin{aligned} & [{t^8 - 10t^6 + 10t^2 - 1, z, (t^3 - 5t)y - 5t^2 + 1, x}, \\ & {t^2 - 1, z^8 - 16z^6 + 256z^2 - 256, t^3y - 1, (z^3 - 8z)x - 8z^2 + 16}, \\ & {3t^2 + 1, z^2 - 7t^2 - 1, y + t, x + z}, {t^2 + 3, z^2 - 4, y + t, x - z}] \end{aligned}$$

--RType: List RegularTriangularSet(Integer,IndexedExponents OrderedVariableList [x,y,z,t],OrderedVariableList)
--E 24

--S 25 of 34
[coHeight(ts) for ts in lts2]
--R

```

```

--R
--R (25) [0,0,0,0]
--R
--R                                         Type: List NonNegativeInteger
--E 25

--S 26 of 34
degrees := [degree(ts) for ts in lts2]
--R
--R
--R (26) [8,16,4,4]
--R
--R                                         Type: List NonNegativeInteger
--E 26

--S 27 of 34
reduce(+,degrees)
--R
--R
--R (27) 32
--R
--R                                         Type: PositiveInteger
--E 27

--S 28 of 34
u : R := 2
--R
--R
--R (28) 2
--R
--R                                         Type: Integer
--E 28

--S 29 of 34
q1 := 2*(u-1)**2+ 2*(x-z*x+z**2)+ y**2*(x-1)**2- 2*u*x+ 2*y*t*(1-x)*(x-z)+_
      2*u*z*t*(t-y)+ u**2*t**2*(1-2*z)+ 2*u*t**2*(z-x)+ 2*u*t*y*(z-1)+_
      2*u*z*x*(y+1)+ (u**2-2*u)*z**2*t**2+ 2*u**2*z**2+ 4*u*(1-u)*z+_
      t**2*(z-x)**2
--R
--R
--R (29)
--R      2      2 2      2      2      2      2      2
--R      (y - 2t y + t )x + (- 2y + ((2t + 4)z + 2t)y + (- 2t + 2)z - 4t - 2)x
--R      +
--R      2      2      2      2
--R      y + (- 2t z - 4t)y + (t + 10)z - 8z + 4t + 2
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 29

--S 30 of 34
q2 := t*(2*z+1)*(x-z)+ y*(z+2)*(1-x)+ u*(u-2)*t+ u*(1-2*u)*z*t+_
      u*y*(x+u-z*x-1)+ u*(u+1)*z**2*t
--R
--R

```

```

--R
--R      2
--R (30) (- 3z y + 2t z + t)x + (z + 4)y + 4t z - 7t z
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 30

--S 31 of 34
q3 := -u**2*(z-1)**2+ 2*z*(z-x)-2*(x-1)
--R
--R
--R      2
--R (31) (- 2z - 2)x - 2z + 8z - 2
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 31

--S 32 of 34
q4 := u**2+4*(z-x**2)+3*y**2*(x-1)**2- 3*t**2*(z-x)**2+_
      3*u**2*t**2*(z-1)**2+u**2*z*(z-2)+6*u*t*y*(z+x+z*x-1)
--R
--R
--R (32)
--R      2      2      2      2      2      2      2
--R (3y - 3t - 4)x + (- 6y + (12t z + 12t)y + 6t z)x + 3y + (12t z - 12t)y
--R +
--R      2      2      2      2
--R (9t + 4)z + (- 24t - 4)z + 12t + 4
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 32

--S 33 of 34
lq := [q1, q2, q3, q4]
--R
--R
--R (33)
--R [
--R      2      2      2
--R (y - 2t y + t )x
--R +
--R      2      2      2      2      2      2
--R (- 2y + ((2t + 4)z + 2t)y + (- 2t + 2)z - 4t - 2)x + y
--R +
--R      2      2      2
--R (- 2t z - 4t)y + (t + 10)z - 8z + 4t + 2
--R ,
--R      2      2
--R (- 3z y + 2t z + t)x + (z + 4)y + 4t z - 7t z, (- 2z - 2)x - 2z + 8z - 2,
--R
--R      2      2      2      2      2      2
--R (3y - 3t - 4)x + (- 6y + (12t z + 12t)y + 6t z)x + 3y
--R +
--R      2      2      2      2

```

23

```
--R      26604210869491302385515265737052082361668474181372891857784t
--R      +
--R      22
--R      443104378424686086067294899528296664238693556855017735265295t
--R      +
--R      21
--R      279078393286701234679141342358988327155321305829547090310242t
--R      +
--R      20
--R      3390276361413232465107617176615543054620626391823613392185226t
--R      +
--R      19
--R      941478179503540575554198645220352803719793196473813837434129t
--R      +
--R      18
--R      11547855194679475242211696749673949352585747674184320988144390t
--R      +
--R      17
--R      1343609566765597789881701656699413216467215660333356417241432t
--R      +
--R      16
--R      23233813868147873503933551617175640859899102987800663566699334t
--R      +
--R      15
--R      869574020537672336950845440508790740850931336484983573386433t
--R      +
--R      14
--R      31561554305876934875419461486969926554241750065103460820476969t
--R      +
--R      13
--R      1271400990287717487442065952547731879554823889855386072264931t
--R      +
--R      12
--R      31945089913863736044802526964079540198337049550503295825160523t
--R      +
--R      11
--R      3738735704288144509871371560232845884439102270778010470931960t
--R      +
--R      10
--R      25293997512391412026144601435771131587561905532992045692885927t
--R      +
--R      9
--R      5210239009846067123469262799870052773410471135950175008046524t
--R      +
--R      8
--R      15083887986930297166259870568608270427403187606238713491129188t
--R      +
--R      7
--R      3522087234692930126383686270775779553481769125670839075109000t
--R      +
```

```

--R                                                                 6
--R      6079945200395681013086533792568886491101244247440034969288588t
--R      +
--R                                                                 5
--R      1090634852433900888199913756247986023196987723469934933603680t
--R      +
--R                                                                 4
--R      1405819430871907102294432537538335402102838994019667487458352t
--R      +
--R                                                                 3
--R      88071527950320450072536671265507748878347828884933605202432t
--R      +
--R                                                                 2
--R      135882489433640933229781177155977768016065765482378657129440t
--R      +
--R      - 13957283442882262230559894607400314082516690749975646520320t
--R      +
--R      334637692973189299277258325709308472592117112855749713920
--R      *
--R      z
--R      +
--R                                                                 23
--R      8567175484043952879756725964506833932149637101090521164936t
--R      +
--R                                                                 22
--R      149792392864201791845708374032728942498797519251667250945721t
--R      +
--R                                                                 21
--R      77258371783645822157410861582159764138123003074190374021550t
--R      +
--R                                                                 20
--R      1108862254126854214498918940708612211184560556764334742191654t
--R      +
--R                                                                 19
--R      213250494460678865219774480106826053783815789621501732672327t
--R      +
--R                                                                 18
--R      3668929075160666195729177894178343514501987898410131431699882t
--R      +
--R                                                                 17
--R      171388906471001872879490124368748236314765459039567820048872t
--R      +
--R                                                                 16
--R      7192430746914602166660233477331022483144921771645523139658986t
--R      +
--R                                                                 15
--R      - 128798674689690072812879965633090291959663143108437362453385t
--R      +
--R                                                                 14
--R      9553010858341425909306423132921134040856028790803526430270671t

```

```

--R      +
--R                                                    13
--R      - 13296096245675492874538687646300437824658458709144441096603t
--R      +
--R                                                    12
--R      9475806805814145326383085518325333106881690568644274964864413t
--R      +
--R                                                    11
--R      803234687925133458861659855664084927606298794799856265539336t
--R      +
--R                                                    10
--R      7338202759292865165994622349207516400662174302614595173333825t
--R      +
--R                                                    9
--R      1308004628480367351164369613111971668880538855640917200187108t
--R      +
--R                                                    8
--R      4268059455741255498880229598973705747098216067697754352634748t
--R      +
--R                                                    7
--R      892893526858514095791318775904093300103045601514470613580600t
--R      +
--R                                                    6
--R      1679152575460683956631925852181341501981598137465328797013652t
--R      +
--R                                                    5
--R      269757415767922980378967154143357835544113158280591408043936t
--R      +
--R                                                    4
--R      380951527864657529033580829801282724081345372680202920198224t
--R      +
--R                                                    3
--R      19785545294228495032998826937601341132725035339452913286656t
--R      +
--R                                                    2
--R      36477412057384782942366635303396637763303928174935079178528t
--R      +
--R      - 3722212879279038648713080422224976273210890229485838670848t
--R      +
--R      89079724853114348361230634484013862024728599906874105856
--R      ,
--R      3      2      3      2
--R      (3z - 11z + 8z + 4)y + 2t z + 4t z - 5t z - t,
--R      2
--R      (z + 1)x + z - 4z + 1}
--R      ]
--RType: List RegularTriangularSet(Integer,IndexedExponents OrderedVariableList [x,y,z,t],OrderedVariableList)
--E 34
)spool
)lisp (bye)

```



---

— RegularTriangularSet.help —

```
=====
RegularTriangularSet examples
=====
```

The RegularTriangularSet domain constructor implements regular triangular sets. These particular triangular sets were introduced by M. Kalkbrener (1991) in his PhD Thesis under the name regular chains. Regular chains and their related concepts are presented in the paper "On the Theories of Triangular sets" By P. Aubry, D. Lazard and M. Moreno Maza (to appear in the Journal of Symbolic Computation). The RegularTriangularSet constructor also provides a new method (by the third author) for solving polynomial system by means of regular chains. This method has two ways of solving. One has the same specifications as Kalkbrener's algorithm (1991) and the other is closer to Lazard's method (Discr. App. Math, 1991). Moreover, this new method removes redundant component from the decompositions when this is not too expensive. This is always the case with square-free regular chains. So if you want to obtain decompositions without redundant components just use the SquareFreeRegularTriangularSet domain constructor or the LazardSetSolvingPackage package constructor. See also the LexTriangularPackage and ZeroDimensionalSolvePackage for the case of algebraic systems with a finite number of (complex) solutions.

One of the main features of regular triangular sets is that they naturally define towers of simple extensions of a field. This allows to perform with multivariate polynomials the same kind of operations as one can do in an EuclideanDomain.

The RegularTriangularSet constructor takes four arguments. The first one, R, is the coefficient ring of the polynomials; it must belong to the category GcdDomain. The second one, E, is the exponent monoid of the polynomials; it must belong to the category OrderedAbelianMonoidSup. The third one, V, is the ordered set of variables; it must belong to the category OrderedSet. The last one is the polynomial ring; it must belong to the category RecursivePolynomialCategory(R,E,V). The abbreviation for RegularTriangularSet is REGSET. See also the constructor RegularChain which only takes two arguments, the coefficient ring and the ordered set of variables; in that case, polynomials are necessarily built with the NewSparseMultivariatePolynomial domain constructor.

We shall explain now how to use the constructor REGSET and how to read the decomposition of a polynomial system by means of regular sets.

Let us give some examples. We start with an easy one (Donati-Traverso) in order to understand the two ways of solving

polynomial systems provided by the REGSET constructor.

Define the coefficient ring.

```
R := Integer
Integer
Type: Domain
```

Define the list of variables,

```
ls : List Symbol := [x,y,z,t]
[x,y,z,t]
Type: List Symbol
```

and make it an ordered set;

```
V := OVAR(ls)
OrderedVariableList [x,y,z,t]
Type: Domain
```

then define the exponent monoid.

```
E := IndexedExponents V
IndexedExponents OrderedVariableList [x,y,z,t]
Type: Domain
```

Define the polynomial ring.

```
P := NSMP(R, V)
NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
Type: Domain
```

Let the variables be polynomial.

```
x: P := 'x
x
Type: NewSparseMultivariatePolynomial(Integer,
OrderedVariableList [x,y,z,t])
y: P := 'y
y
Type: NewSparseMultivariatePolynomial(Integer,
OrderedVariableList [x,y,z,t])
z: P := 'z
z
Type: NewSparseMultivariatePolynomial(Integer,
OrderedVariableList [x,y,z,t])
t: P := 't
t
```

```

Type: NewSparseMultivariatePolynomial(Integer,
      OrderedVariableList [x,y,z,t])

```

Now call the RegularTriangularSet domain constructor.

```

T := REGSET(R,E,V,P)
RegularTriangularSet(Integer,IndexedExponents OrderedVariableList [x,y,z,t],0
rderedVariableList [x,y,z,t],NewSparseMultivariatePolynomial(Integer,OrderedV
ariableList [x,y,z,t]))
Type: Domain

```

Define a polynomial system.

```

p1 := x ** 31 - x ** 6 - x - y
      31      6
      x  - x  - x - y
Type: NewSparseMultivariatePolynomial(Integer,
      OrderedVariableList [x,y,z,t])

p2 := x ** 8 - z
      8
      x  - z
Type: NewSparseMultivariatePolynomial(Integer,
      OrderedVariableList [x,y,z,t])

p3 := x ** 10 - t
      10
      x  - t
Type: NewSparseMultivariatePolynomial(Integer,
      OrderedVariableList [x,y,z,t])

lp := [p1, p2, p3]
      31      6      8      10
      [x  - x  - x - y, x  - z, x  - t]
Type: List NewSparseMultivariatePolynomial(Integer,
      OrderedVariableList [x,y,z,t])

```

First of all, let us solve this system in the sense of Kalkbrener.

```

zeroSetSplit(lp)$T
      5      4      2      3      8      5      3      2      4      2
      [{z  - t , t z y + 2z y - t + 2t + t - t , (t - t)x - t y - z }]
Type: List RegularTriangularSet(Integer,
      IndexedExponents OrderedVariableList [x,y,z,t],
      OrderedVariableList [x,y,z,t],
      NewSparseMultivariatePolynomial(Integer,
      OrderedVariableList [x,y,z,t]))

```

And now in the sense of Lazard (or Wu and other authors).

```

lts := zeroSetSplit(lp,false)$T
      5      4      2      3      8      5      3      2      4      2
[[z  - t  ,t z y  + 2z y - t  + 2t  + t  - t  ,(t  - t)x - t y - z ],
      3      5      2      3      2
{t  - 1,z  - t,t z y  + 2z y + 1,z x  - t}, {t,z,y,x}]
Type: List RegularTriangularSet(Integer,
      IndexedExponents OrderedVariableList [x,y,z,t],
      OrderedVariableList [x,y,z,t],
      NewSparseMultivariatePolynomial(Integer,
      OrderedVariableList [x,y,z,t]))

```

We can see that the first decomposition is a subset of the second.  
So how can both be correct ?

Recall first that polynomials from a domain of the category  
RecursivePolynomialCategory are regarded as univariate polynomials in  
their main variable. For instance the second polynomial in the first  
set of each decomposition has main variable y and its initial  
(i.e. its leading coefficient w.r.t. its main variable) is t z.

Now let us explain how to read the second decomposition. Note that  
the non-constant initials of the first set are  $t^4-t$  and  $t z$ . Then  
the solutions described by this first set are the common zeros of  
its polynomials that do not cancel the polynomials  $t^4-t$  and  $ty z$ .  
Now the solutions of the input system lp satisfying these equations  
are described by the second and the third sets of the decomposition.  
Thus, in some sense, they can be considered as degenerated solutions.  
The solutions given by the first set are called the generic points of  
the system; they give the general form of the solutions. The first  
decomposition only provides these generic points. This latter  
decomposition is useful when they are many degenerated solutions  
(which is sometimes hard to compute) and when one is only interested  
in general informations, like the dimension of the input system.

We can get the dimensions of each component of a decomposition as follows.

```

[coHeight(ts) for ts in lts]
[1,0,0]
Type: List NonNegativeInteger

```

Thus the first set has dimension one. Indeed t can take any value,  
except 0 or any third root of 1, whereas z is completely determined  
from t, y is given by z and t, and finally x is given by the other  
three variables. In the second and the third sets of the second  
decomposition the four variables are completely determined and thus  
these sets have dimension zero.

We give now the precise specifications of each decomposition. This  
assume some mathematical knowledge. However, for the non-expert user,  
the above explanations will be sufficient to understand the other

features of the RSEGSET constructor.

The input system  $lp$  is decomposed in the sense of Kalkbrener as finitely many regular sets  $T_1, \dots, T_s$  such that the radical ideal generated by  $lp$  is the intersection of the radicals of the saturated ideals of  $T_1, \dots, T_s$ . In other words, the affine variety associated with  $lp$  is the union of the closures (w.r.t. Zarisky topology) of the regular-zeros sets of  $T_1, \dots, T_s$ .

N. B. The prime ideals associated with the radical of the saturated ideal of a regular triangular set have all the same dimension; moreover these prime ideals can be given by characteristic sets with the same main variables. Thus a decomposition in the sense of Kalkbrener is unmixed dimensional. Then it can be viewed as a lazy decomposition into prime ideals (some of these prime ideals being merged into unmixed dimensional ideals).

Now we explain the other way of solving by means of regular triangular sets. The input system  $lp$  is decomposed in the sense of Lazard as finitely many regular triangular sets  $T_1, \dots, T_s$  such that the affine variety associated with  $lp$  is the union of the regular-zeros sets of  $T_1, \dots, T_s$ . Thus a decomposition in the sense of Lazard is also a decomposition in the sense of Kalkbrener; the converse is false as we have seen before.

When the input system has a finite number of solutions, both ways of solving provide similar decompositions as we shall see with this second example (Caprasse).

Define a polynomial system.

```
f1 := y**2*z+2*x*y*t-2*x-z
      2
(2t y - 2)x + z y - z
      Type: NewSparseMultivariatePolynomial(Integer,
      OrderedVariableList [x,y,z,t])

f2:=-x**3*z+ 4*x*y**2*z+4*x**2*y*t+2*y**3*t+4*x**2-10*y**2+4*x*z-10*y*t+2
      3      2      2      3      2
- z x + (4t y + 4)x + (4z y + 4z)x + 2t y - 10y - 10t y + 2
      Type: NewSparseMultivariatePolynomial(Integer,
      OrderedVariableList [x,y,z,t])

f3 := 2*y*z*t+x*t**2-x-2*z
      2
(t - 1)x + 2t z y - 2z
      Type: NewSparseMultivariatePolynomial(Integer,
      OrderedVariableList [x,y,z,t])

f4:=-x*z**3+4*y*z**2*t+4*x*z*t**2+2*y*t**3+4*x*z+4*z**2-10*y*t- 10*t**2+2
```

```

      3      2      2      3      2      2
      (- z  + (4t  + 4)z)x + (4t z  + 2t  - 10t)y + 4z  - 10t  + 2
      Type: NewSparseMultivariatePolynomial(Integer,
      OrderedVariableList [x,y,z,t])

lf := [f1, f2, f3, f4]
      2
      [(2t y - 2)x + z y  - z,
      3      2      2      3      2
      - z x  + (4t y + 4)x  + (4z y  + 4z)x + 2t y  - 10y  - 10t y + 2,
      2
      (t  - 1)x + 2t z y - 2z,
      3      2      2      3      2      2
      (- z  + (4t  + 4)z)x + (4t z  + 2t  - 10t)y + 4z  - 10t  + 2]
      Type: List NewSparseMultivariatePolynomial(Integer,
      OrderedVariableList [x,y,z,t])

```

First of all, let us solve this system in the sense of Kalkbrener.

```

zeroSetSplit(lf)$T
      2      8      6      2      3      2
      [{t  - 1, z  - 16z  + 256z  - 256, t y - 1, (z  - 8z)x - 8z  + 16},
      2      2      2
      {3t  + 1, z  - 7t  - 1, y + t, x + z},
      8      6      2      3      2
      {t  - 10t  + 10t  - 1, z, (t  - 5t)y - 5t  + 1, x},
      2      2
      {t  + 3, z  - 4, y + t, x - z}]
      Type: List RegularTriangularSet(Integer,
      IndexedExponents OrderedVariableList [x,y,z,t],
      OrderedVariableList [x,y,z,t],
      NewSparseMultivariatePolynomial(Integer,
      OrderedVariableList [x,y,z,t]))

```

And now in the sense of Lazard (or Wu and other authors).

```

lts2 := zeroSetSplit(lf,false)$T
      8      6      2      3      2
      [{t  - 10t  + 10t  - 1, z, (t  - 5t)y - 5t  + 1, x},
      2      8      6      2      3      2
      {t  - 1, z  - 16z  + 256z  - 256, t y - 1, (z  - 8z)x - 8z  + 16},
      2      2      2      2      2
      {3t  + 1, z  - 7t  - 1, y + t, x + z}, {t  + 3, z  - 4, y + t, x - z}]
      Type: List RegularTriangularSet(Integer,
      IndexedExponents OrderedVariableList [x,y,z,t],
      OrderedVariableList [x,y,z,t],
      NewSparseMultivariatePolynomial(Integer,
      OrderedVariableList [x,y,z,t]))

```

Up to the ordering of the components, both decompositions are identical.

Let us check that each component has a finite number of solutions.

```
[coHeight(ts) for ts in lts2]
[0,0,0,0]
Type: List NonNegativeInteger
```

Let us count the degrees of each component,

```
degrees := [degree(ts) for ts in lts2]
[8,16,4,4]
Type: List NonNegativeInteger
```

and compute their sum.

```
reduce(+,degrees)
32
Type: PositiveInteger
```

We study now the options of the `zeroSetSplit` operation. As we have seen yet, there is an optional second argument which is a boolean value. If this value is true (this is the default) then the decomposition is computed in the sense of Kalkbrener, otherwise it is computed in the sense of Lazard.

There is a second boolean optional argument that can be used (in that case the first optional argument must be present). This second option allows you to get some information during the computations.

Therefore, we need to understand a little what is going on during the computations. An important feature of the algorithm is that the intermediate computations are managed in some sense like the processes of a Unix system. Indeed, each intermediate computation may generate other intermediate computations and the management of all these computations is a crucial task for the efficiency. Thus any intermediate computation may be suspended, killed or resumed, depending on algebraic considerations that determine priorities for these processes. The goal is of course to go as fast as possible towards the final decomposition which means to avoid as much as possible unnecessary computations.

To follow the computations, one needs to set to true the second argument. Then a lot of numbers and letters are displayed. Between a [ and a ] one has the state of the processes at a given time. Just after [ one can see the number of processes. Then each process is represented by two numbers between < and >. A process consists of a list of polynomial `ps` and a triangular set `ts`; its goal is to compute the common zeros of `ps` that belong to the regular-zeros set of `ts`. After the processes, the number between pipes gives the total number of polynomials in all the sets `ps`. Finally, the number between braces gives the number of components of a decomposition that are already

computed. This number may decrease.

Let us take a third example (Czapor-Geddes-Wang) to see how this information is displayed.

Define a polynomial system.

```

u : R := 2
2
Type: Integer

q1 := 2*(u-1)**2+ 2*(x-z*x+z**2)+ y**2*(x-1)**2- 2*u*x+ 2*y*t*(1-x)*(x-z)+_
2*u*z*t*(t-y)+ u**2*t**2*(1-2*z)+ 2*u*t**2*(z-x)+ 2*u*t*y*(z-1)+_
2*u*z*x*(y+1)+ (u**2-2*u)*z**2*t**2+ 2*u**2*z**2+ 4*u*(1-u)*z+_
t**2*(z-x)**2}
2 2 2 2 2 2 2 2 2 2
(y - 2t y + t )x + (- 2y + ((2t + 4)z + 2t)y + (- 2t + 2)z - 4t - 2)x
+
2 2 2 2 2 2
y + (- 2t z - 4t)y + (t + 10)z - 8z + 4t + 2
Type: NewSparseMultivariatePolynomial(Integer,
OrderedVariableList [x,y,z,t])

q2 := t*(2*z+1)*(x-z)+ y*(z+2)*(1-x)+ u*(u-2)*t+ u*(1-2*u)*z*t+_
u*y*(x+u-z*x-1)+ u*(u+1)*z**2*t}
(- 3z y + 2t z + t)x + (z + 4)y + 4t z - 7t z
Type: NewSparseMultivariatePolynomial(Integer,
OrderedVariableList [x,y,z,t])

q3 := -u**2*(z-1)**2+ 2*z*(z-x)-2*(x-1)
(- 2z - 2)x - 2z + 8z - 2
Type: NewSparseMultivariatePolynomial(Integer,
OrderedVariableList [x,y,z,t])

q4 := u**2+4*(z-x**2)+3*y**2*(x-1)**2- 3*t**2*(z-x)**2+_
3*u**2*t**2*(z-1)**2+u**2*z*(z-2)+6*u*t*y*(z+x+z*x-1)}
2 2 2 2 2 2 2 2 2 2
(3y - 3t - 4)x + (- 6y + (12t z + 12t)y + 6t z)x + 3y + (12t z - 12t)y
+
2 2 2 2
(9t + 4)z + (- 24t - 4)z + 12t + 4
Type: NewSparseMultivariatePolynomial(Integer,
OrderedVariableList [x,y,z,t])

lq := [q1, q2, q3, q4]
[
2 2 2 2
(y - 2t y + t )x
+
2 2 2 2

```



```

      2      2      2
      (- 2y + ((2t + 4)z + 2t)y + (- 2t + 2)z - 4t - 2)x + y
+
      2      2      2
      (- 2t z - 4t)y + (t + 10)z - 8z + 4t + 2
,
      2      2      2
      (- 3z y + 2t z + t)x + (z + 4)y + 4t z - 7t z, (- 2z - 2)x - 2z + 8z - 2,

      2      2      2      2      2      2
      (3y - 3t - 4)x + (- 6y + (12t z + 12t)y + 6t z)x + 3y
+
      2      2      2      2
      (12t z - 12t)y + (9t + 4)z + (- 24t - 4)z + 12t + 4
]
Type: List NewSparseMultivariatePolynomial(Integer,
      OrderedVariableList [x,y,z,t])

```

Let us try the information option. N.B. The timing should be between 1 and 10 minutes, depending on your machine.

```

zeroSetSplit(lq,true,true)$T
[1 <4,0> -> |4|; {0}]W[2 <5,0>,<3,1> -> |8|; {0}]
[2 <4,1>,<3,1> -> |7|; {0}]
[1 <3,1> -> |3|; {0}]G
[2 <4,1>,<4,1> -> |8|; {0}]W
[3 <5,1>,<4,1>,<3,2> -> |12|; {0}]GI
[3 <4,2>,<4,1>,<3,2> -> |11|; {0}]Gww
[3 <4,1>,<3,2>,<5,2> -> |12|; {0}]
[3 <3,2>,<3,2>,<5,2> -> |11|; {0}]GIwWWWw
[4 <3,2>,<4,2>,<5,2>,<2,3> -> |14|; {0}]
[4 <2,2>,<4,2>,<5,2>,<2,3> -> |13|; {0}]Gwww
[5 <3,2>,<3,2>,<4,2>,<5,2>,<2,3> -> |17|; {0}]Gwwwww
[8 <3,2>,<4,2>,<4,2>,<4,2>,<4,2>,<4,2>,<5,2>,<2,3> -> |30|; {0}]Gwwwww
[8 <4,2>,<4,2>,<4,2>,<4,2>,<4,2>,<4,2>,<5,2>,<2,3> -> |31|; {0}]
[8 <3,3>,<4,2>,<4,2>,<4,2>,<4,2>,<4,2>,<5,2>,<2,3> -> |30|; {0}]
[8 <2,3>,<4,2>,<4,2>,<4,2>,<4,2>,<4,2>,<5,2>,<2,3> -> |29|; {0}]
[8 <1,3>,<4,2>,<4,2>,<4,2>,<4,2>,<4,2>,<5,2>,<2,3> -> |28|; {0}]
[7 <4,2>,<4,2>,<4,2>,<4,2>,<4,2>,<5,2>,<2,3> -> |27|; {0}]
[6 <4,2>,<4,2>,<4,2>,<4,2>,<5,2>,<2,3> -> |23|; {0}]
[5 <4,2>,<4,2>,<4,2>,<5,2>,<2,3> -> |19|; {0}]GIGIwww
[6 <5,2>,<4,2>,<4,2>,<5,2>,<3,3>,<2,3> -> |23|; {0}]
[6 <4,3>,<4,2>,<4,2>,<5,2>,<3,3>,<2,3> -> |22|; {0}]GIGI
[6 <3,4>,<4,2>,<4,2>,<5,2>,<3,3>,<2,3> -> |21|; {0}]
[6 <2,4>,<4,2>,<4,2>,<5,2>,<3,3>,<2,3> -> |20|; {0}]GGG
[5 <4,2>,<4,2>,<5,2>,<3,3>,<2,3> -> |18|; {0}]GIGIwwwW
[6 <5,2>,<4,2>,<5,2>,<3,3>,<3,3>,<2,3> -> |22|; {0}]
[6 <4,3>,<4,2>,<5,2>,<3,3>,<3,3>,<2,3> -> |21|; {0}]
GIwWWWWWWWWWWWWWWWWWW
[8 <4,2>,<5,2>,<3,3>,<3,3>,<4,3>,<2,3>,<3,4>,<3,4> -> |27|; {0}]
[8 <3,3>,<5,2>,<3,3>,<3,3>,<4,3>,<2,3>,<3,4>,<3,4> -> |26|; {0}]

```

[illegible]

[illegible]

```

[8 <3,3>,<4,3>,<2,3>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4> -> |24|; {1}]W
[8 <2,4>,<4,3>,<2,3>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4> -> |23|; {1}]
[8 <1,4>,<4,3>,<2,3>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4> -> |22|; {1}]
[7 <4,3>,<2,3>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4> -> |21|; {1}]w
[7 <3,4>,<2,3>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4> -> |20|; {1}]
[7 <2,4>,<2,3>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4> -> |19|; {1}]
[7 <1,4>,<2,3>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4> -> |18|; {1}]
[6 <2,3>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4> -> |17|; {1}]GGwwwww
[7 <3,3>,<3,3>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4> -> |21|; {1}]GIW
[7 <2,4>,<3,3>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4> -> |20|; {1}]GG
[6 <3,3>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4> -> |18|; {1}]Gwwwww
[7 <4,3>,<4,3>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4> -> |23|; {1}]GIW
[7 <3,4>,<4,3>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4> -> |22|; {1}]
[6 <4,3>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4> -> |19|; {1}]GIW
[6 <3,4>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4> -> |18|; {1}]GGW
[6 <2,4>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4> -> |17|; {1}]
[6 <1,4>,<3,3>,<3,3>,<3,3>,<3,4>,<3,4> -> |16|; {1}]GGG
[5 <3,3>,<3,3>,<3,3>,<3,4>,<3,4> -> |15|; {1}]GIW
[5 <2,4>,<3,3>,<3,3>,<3,4>,<3,4> -> |14|; {1}]GG
[4 <3,3>,<3,3>,<3,4>,<3,4> -> |12|; {1}]
[3 <3,3>,<3,4>,<3,4> -> |9|; {1}]W
[3 <2,4>,<3,4>,<3,4> -> |8|; {1}]
[3 <1,4>,<3,4>,<3,4> -> |7|; {1}]G
[2 <3,4>,<3,4> -> |6|; {1}]G
[1 <3,4> -> |3|; {1}]
[1 <2,4> -> |2|; {1}]
[1 <1,4> -> |1|; {1}]
*** QCMACK Statistics ***
Table      size: 36
Entries reused: 255

*** REGSETGCD: Gcd Statistics ***
Table      size: 125
Entries reused: 0

*** REGSETGCD: Inv Set Statistics ***
Table      size: 30
Entries reused: 0

[
{
          24          23          22
960725655771966t  + 386820897948702t  + 8906817198608181t
+
          21          20          19
2704966893949428t  + 37304033340228264t  + 7924782817170207t
+
          18          17          16
93126799040354990t  + 13101273653130910t  + 156146250424711858t
+

```

$$\begin{aligned}
& \begin{array}{r} 15 \\ 16626490957259119t \end{array} + \begin{array}{r} 14 \\ 190699288479805763t \end{array} + \begin{array}{r} 13 \\ 24339173367625275t \end{array} \\
+ & \begin{array}{r} 12 \\ 180532313014960135t \end{array} + \begin{array}{r} 11 \\ 35288089030975378t \end{array} + \begin{array}{r} 10 \\ 135054975747656285t \end{array} \\
+ & \begin{array}{r} 9 \\ 34733736952488540t \end{array} + \begin{array}{r} 8 \\ 75947600354493972t \end{array} + \begin{array}{r} 7 \\ 19772555692457088t \end{array} \\
+ & \begin{array}{r} 6 \\ 28871558573755428t \end{array} + \begin{array}{r} 5 \\ 5576152439081664t \end{array} + \begin{array}{r} 4 \\ 6321711820352976t \end{array} \\
+ & \begin{array}{r} 3 \\ 438314209312320t \end{array} + \begin{array}{r} 2 \\ 581105748367008t \end{array} - 60254467992576t + 1449115951104 \\
& ,
\end{aligned}$$

$$\begin{aligned}
& \begin{array}{r} 23 \\ 26604210869491302385515265737052082361668474181372891857784t \end{array} \\
+ & \begin{array}{r} 22 \\ 443104378424686086067294899528296664238693556855017735265295t \end{array} \\
+ & \begin{array}{r} 21 \\ 279078393286701234679141342358988327155321305829547090310242t \end{array} \\
+ & \begin{array}{r} 20 \\ 3390276361413232465107617176615543054620626391823613392185226t \end{array} \\
+ & \begin{array}{r} 19 \\ 941478179503540575554198645220352803719793196473813837434129t \end{array} \\
+ & \begin{array}{r} 18 \\ 11547855194679475242211696749673949352585747674184320988144390t \end{array} \\
+ & \begin{array}{r} 17 \\ 1343609566765597789881701656699413216467215660333356417241432t \end{array} \\
+ & \begin{array}{r} 16 \\ 23233813868147873503933551617175640859899102987800663566699334t \end{array} \\
+ & \begin{array}{r} 15 \\ 869574020537672336950845440508790740850931336484983573386433t \end{array} \\
+ & \begin{array}{r} 14 \\ 31561554305876934875419461486969926554241750065103460820476969t \end{array} \\
+ & \begin{array}{r} 13 \\ 1271400990287717487442065952547731879554823889855386072264931t \end{array} \\
& +
\end{aligned}$$

12

```

31945089913863736044802526964079540198337049550503295825160523t
+
11
3738735704288144509871371560232845884439102270778010470931960t
+
10
25293997512391412026144601435771131587561905532992045692885927t
+
9
5210239009846067123469262799870052773410471135950175008046524t
+
8
15083887986930297166259870568608270427403187606238713491129188t
+
7
3522087234692930126383686270775779553481769125670839075109000t
+
6
6079945200395681013086533792568886491101244247440034969288588t
+
5
1090634852433900888199913756247986023196987723469934933603680t
+
4
1405819430871907102294432537538335402102838994019667487458352t
+
3
88071527950320450072536671265507748878347828884933605202432t
+
2
135882489433640933229781177155977768016065765482378657129440t
+
- 13957283442882262230559894607400314082516690749975646520320t
+
334637692973189299277258325709308472592117112855749713920
*
z
+
23
8567175484043952879756725964506833932149637101090521164936t
+
22
149792392864201791845708374032728942498797519251667250945721t
+
21
77258371783645822157410861582159764138123003074190374021550t
+
20
1108862254126854214498918940708612211184560556764334742191654t
+

```

$$\begin{aligned}
& 213250494460678865219774480106826053783815789621501732672327t \\
& + \\
& 3668929075160666195729177894178343514501987898410131431699882t \\
& + \\
& 171388906471001872879490124368748236314765459039567820048872t \\
& + \\
& 7192430746914602166660233477331022483144921771645523139658986t \\
& + \\
& - 128798674689690072812879965633090291959663143108437362453385t \\
& + \\
& 9553010858341425909306423132921134040856028790803526430270671t \\
& + \\
& - 13296096245675492874538687646300437824658458709144441096603t \\
& + \\
& 9475806805814145326383085518325333106881690568644274964864413t \\
& + \\
& 803234687925133458861659855664084927606298794799856265539336t \\
& + \\
& 7338202759292865165994622349207516400662174302614595173333825t \\
& + \\
& 1308004628480367351164369613111971668880538855640917200187108t \\
& + \\
& 4268059455741255498880229598973705747098216067697754352634748t \\
& + \\
& 892893526858514095791318775904093300103045601514470613580600t \\
& + \\
& 1679152575460683956631925852181341501981598137465328797013652t \\
& + \\
& 269757415767922980378967154143357835544113158280591408043936t \\
& + \\
& 380951527864657529033580829801282724081345372680202920198224t \\
& + \\
& 19785545294228495032998826937601341132725035339452913286656t
\end{aligned}$$

```

+
                                     2
36477412057384782942366635303396637763303928174935079178528t
+
- 3722212879279038648713080422224976273210890229485838670848t
+
89079724853114348361230634484013862024728599906874105856
,
      3      2      3      2
(3z  - 11z  + 8z + 4)y + 2t z  + 4t z  - 5t z - t,
      2
(z + 1)x + z  - 4z + 1}
]
Type: List RegularTriangularSet(Integer,
      IndexedExponents OrderedVariableList [x,y,z,t],
      OrderedVariableList [x,y,z,t],
      NewSparseMultivariatePolynomial(Integer,
      OrderedVariableList [x,y,z,t]))

```

Between a sequence of processes, thus between a ] and a [ you can see capital letters W, G, I and lower case letters i, w. Each time a capital letter appears a non-trivial computation has be performed and its result is put in a hash-table. Each time a lower case letter appears a needed result has been found in an hash-table. The use of these hash-tables generally speed up the computations. However, on very large systems, it may happen that these hash-tables become too big to be handle by your AXIOM configuration. Then in these exceptional cases, you may prefer getting a result (even if it takes a long time) than getting nothing. Hence you need to know how to prevent the RSEGET constructor from using these hash-tables. In that case you will be using the zeroSetSplit with five arguments. The first one is the input system lp as above. The second one is a boolean value hash? which is true iff you want to use hash-tables. The third one is boolean value clos? which is true iff you want to solve your system in the sense of Kalkbrener, the other way remaining that of Lazard. The fourth argument is boolean value info? which is true iff you want to display information during the computations. The last one is boolean value prep? which is true iff you want to use some heuristics that are performed on the input system before starting the real algorithm. The value of this flag is true when you are using zeroSetSplit with less than five arguments. Note that there is no available signature for zeroSetSplit with four arguments.

We finish this section by some remarks about both ways of solving, in the sense of Kalkbrener or in the sense of Lazard. For problems with a finite number of solutions, there are theoretically equivalent and the resulting decompositions are identical, up to the ordering of the components. However, when solving in the sense of Lazard, the algorithm behaves differently. In that case, it becomes more incremental than in the sense of Kalkbrener. That means the



polynomials of the input system are considered one after another whereas in the sense of Kalkbrener the input system is treated more globally.

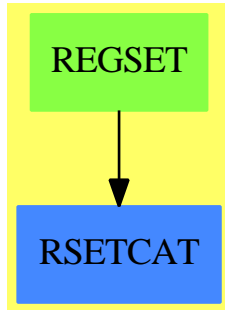
This makes an important difference in positive dimension. Indeed when solving in the sense of Kalkbrener, the Primeidealkettensatz of Krull is used. That means any regular triangular containing more polynomials than the input system can be deleted. This is not possible when solving in the sense of Lazard. This explains why Kalkbrener's decompositions usually contain less components than those of Lazard. However, it may happen with some examples that the incremental process (that cannot be used when solving in the sense of Kalkbrener) provide a more efficient way of solving than the global one even if the Primeidealkettensatz is used. Thus just try both, with the various options, before concluding that you cannot solve your favorite system with `zeroSetSplit`. There exist more options at the development level that are not currently available in this public version.

See Also:

- o `)help GcdDomain`
- o `)help OrderedAbelianMonoidSup`
- o `)help OrderedSet`
- o `)help RecursivePolynomialCategory`
- o `)help RegularChain`
- o `)help NewSparseMultivariatePolynomial`
- o `)help ZeroDimensionalSolvePackage`
- o `)help LexTriangularPackage`
- o `)help LazardSetSolvingPackage`
- o `)help SquareFreeRegularTriangularSet`
- o `)show RegularTriangularSet`

---

### 19.7.1 RegularTriangularSet (REGSET)



**Exports:**

|                                 |                                   |
|---------------------------------|-----------------------------------|
| algebraic?                      | algebraicCoefficients?            |
| algebraicVariables              | any?                              |
| augment                         | autoReduced?                      |
| basicSet                        | coerce                            |
| coHeight                        | collect                           |
| collectQuasiMonic               | collectUnder                      |
| collectUpper                    | construct                         |
| convert                         | copy                              |
| count                           | degree                            |
| empty                           | empty?                            |
| eq?                             | eval                              |
| every?                          | extend                            |
| extendIfCan                     | find                              |
| first                           | hash                              |
| headRemainder                   | headReduce                        |
| headReduced?                    | infRittWu?                        |
| initiallyReduce                 | initiallyReduced?                 |
| initials                        | internalAugment                   |
| internalZeroSetSplit            | intersect                         |
| invertible?                     | invertibleElseSplit?              |
| invertibleSet                   | last                              |
| lastSubResultant                | lastSubResultantElseSplit         |
| latex                           | less?                             |
| mainVariable?                   | mainVariables                     |
| map                             | map!                              |
| member?                         | members                           |
| more?                           | mvar                              |
| normalized?                     | parts                             |
| preprocess                      | purelyAlgebraic?                  |
| purelyAlgebraicLeadingMonomial? | purelyTranscendental?             |
| quasiComponent                  | reduce                            |
| reduced?                        | reduceByQuasiMonic                |
| remainder                       | remove                            |
| removeDuplicates                | removeZero                        |
| rest                            | retract                           |
| retractIfCan                    | rewriteIdealWithHeadRemainder     |
| rewriteIdealWithRemainder       | rewriteSetWithReduction           |
| roughBase?                      | roughEqualIdeals?                 |
| roughSubIdeal?                  | roughUnitIdeal?                   |
| sample                          | select                            |
| size?                           | sort                              |
| squareFreePart                  | stronglyReduce                    |
| stronglyReduced?                | triangular?                       |
| trivialIdeal?                   | variables                         |
| zeroSetSplit                    | zeroSetSplitIntoTriangularSystems |
| #?                              | ?~=?                              |
| ?=?                             |                                   |

## — domain REGSET RegularTriangularSet —

```

)abbrev domain REGSET RegularTriangularSet
++ Author: Marc Moreno Maza
++ Date Created: 08/25/1998
++ Date Last Updated: 16/12/1998
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References :
++ [1] M. MORENO MAZA "A new algorithm for computing triangular
++      decomposition of algebraic varieties" NAG Tech. Rep. 4/98.
++ Description:
++ This domain provides an implementation of regular chains.
++ Moreover, the operation zeroSetSplit is an implementation of a new
++ algorithm for solving polynomial systems by means of regular chains.

```

```

RegularTriangularSet(R,E,V,P) : Exports == Implementation where

```

```

R : GcdDomain
E : OrderedAbelianMonoidSup
V : OrderedSet
P : RecursivePolynomialCategory(R,E,V)
N ==> NonNegativeInteger
Z ==> Integer
B ==> Boolean
LP ==> List P
PtoP ==> P -> P
PS ==> GeneralPolynomialSet(R,E,V,P)
PWT ==> Record(val : P, tower : $)
BWT ==> Record(val : Boolean, tower : $)
LpWT ==> Record(val : (List P), tower : $)
Split ==> List $
iprintpack ==> InternalPrintPackage()
polsetpack ==> PolynomialSetUtilitiesPackage(R,E,V,P)
quasicomppack ==> QuasiComponentPackage(R,E,V,P,$)
regsetgcdpack ==> RegularTriangularSetGcdPackage(R,E,V,P,$)
regsetdecomppack ==> RegularSetDecompositionPackage(R,E,V,P,$)

```

```

Exports == RegularTriangularSetCategory(R,E,V,P) with

```

```

internalAugment: (P,$,B,B,B,B) -> List $
++ \axiom{internalAugment(p,ts,b1,b2,b3,b4,b5)}
++ is an internal subroutine, exported only for developement.
zeroSetSplit: (LP, B, B) -> Split
++ \axiom{zeroSetSplit(lp,clos?,info?)} has the same specifications as
++ zeroSetSplit from RegularTriangularSetCategory.

```

```

    ++ Moreover, if \axiom{clos?} then solves in the sense of the Zariski closure
    ++ else solves in the sense of the regular zeros. If \axiom{info?} then
    ++ do print messages during the computations.
zeroSetSplit: (LP, B, B, B, B) -> Split
    ++ \axiom{zeroSetSplit(lp,b1,b2.b3,b4)}
    ++ is an internal subroutine, exported only for developement.
internalZeroSetSplit: (LP, B, B, B) -> Split
    ++ \axiom{internalZeroSetSplit(lp,b1,b2,b3)}
    ++ is an internal subroutine, exported only for developement.
pre_process: (LP, B, B) -> Record(val: LP, towers: Split)
    ++ \axiom{pre_process(lp,b1,b2)}
    ++ is an internal subroutine, exported only for developement.

Implementation == add

Rep ==> LP

rep(s:$):Rep == s pretend Rep
per(l:Rep):$ == l pretend $

copy ts ==
    per(copy(rep(ts))$LP)
empty() ==
    per([])
empty?(ts:$) ==
    empty?(rep(ts))
parts ts ==
    rep(ts)
members ts ==
    rep(ts)
map (f : PtoP, ts : $) : $ ==
    construct(map(f,rep(ts))$LP)$
map! (f : PtoP, ts : $) : $ ==
    construct(map!(f,rep(ts))$LP)$
member? (p,ts) ==
    member?(p,rep(ts))$LP
unitIdealIfCan() ==
    "failed"::Union($,"failed")
roughUnitIdeal? ts ==
    false
coerce(ts:$) : OutputForm ==
    lp : List(P) := reverse(rep(ts))
    brace([p::OutputForm for p in lp]$List(OutputForm))$OutputForm
mvar ts ==
    empty? ts => error "mvar$REGSET: #1 is empty"
    mvar(first(rep(ts)))$P
first ts ==
    empty? ts => "failed"::Union(P,"failed")
    first(rep(ts))::Union(P,"failed")
last ts ==

```

```

    empty? ts => "failed"::Union(P,"failed")
    last(rep(ts))::Union(P,"failed")
rest ts ==
    empty? ts => "failed"::Union($,"failed")
    per(rest(rep(ts)))::Union($,"failed")
coerce(ts:$) : (List P) ==
    rep(ts)

collectUpper (ts,v) ==
    empty? ts => ts
    lp := rep(ts)
    newlp : Rep := []
    while (not empty? lp) and (mvar(first(lp)) > v) repeat
        newlp := cons(first(lp),newlp)
        lp := rest lp
    per(reverse(newlp))

collectUnder (ts,v) ==
    empty? ts => ts
    lp := rep(ts)
    while (not empty? lp) and (mvar(first(lp)) >= v) repeat
        lp := rest lp
    per(lp)

construct(lp:List(P)) ==
    ts : $ := per([])
    empty? lp => ts
    lp := sort(infRittWu?,lp)
    while not empty? lp repeat
        eif := extendIfCan(ts,first(lp))
        not (eif case $) =>
            error"in construct : List P -> $ from REGSET : bad #1"
        ts := eif::$
        lp := rest lp
    ts

extendIfCan(ts:$,p:P) ==
    ground? p => "failed"::Union($,"failed")
    empty? ts =>
        p := primitivePart p
        (per([p]))::Union($,"failed")
    not (mvar(ts) < mvar(p)) => "failed"::Union($,"failed")
    invertible?(init(p),ts)@Boolean =>
        (per(cons(p,rep(ts))))::Union($,"failed")
    "failed"::Union($,"failed")

removeZero(p:P, ts:$): P ==
    (ground? p) or (empty? ts) => p
    v := mvar(p)
    ts_v_- := collectUnder(ts,v)

```

```

if algebraic?(v,ts)
then
  q := lazyPrem(p,select(ts,v)::P)
  zero? q => return q
  zero? removeZero(q,ts_v_-) => return 0
empty? ts_v_- => p
q: P := 0
while positive? degree(p,v) repeat
  q := removeZero(init(p),ts_v_-) * mainMonomial(p) + q
  p := tail(p)
q + removeZero(p,ts_v_-)

internalAugment(p:P,ts:$): $ ==
-- ASSUME that adding p to ts DOES NOT require any split
ground? p => error "in internalAugment$REGSET: ground? #1"
first(internalAugment(p,ts,false,false,false,false,false))

internalAugment(lp:List(P),ts:$): $ ==
-- ASSUME that adding p to ts DOES NOT require any split
empty? lp => ts
internalAugment(rest lp, internalAugment(first lp, ts))

internalAugment(p:P,ts:$,rem?:B,red?:B,prim?:B,sqfr?:B,extend?:B): Split ==
-- ASSUME p is not a constant
-- ASSUME mvar(p) is not algebraic w.r.t. ts
-- ASSUME init(p) invertible modulo ts
-- if rem? then REDUCE p by remainder
-- if prim? then REPLACE p by its main primitive part
-- if sqfr? then FACTORIZE SQUARE FREE p over R
-- if extend? DO NOT ASSUME every pol in ts_v_+ is invertible modulo ts
v := mvar(p)
ts_v_- := collectUnder(ts,v)
ts_v_+ := collectUpper(ts,v)
if rem? then p := remainder(p,ts_v_-).polnum
-- if rem? then p := reduceByQuasiMonic(p,ts_v_-)
if red? then p := removeZero(p,ts_v_-)
if prim? then p := mainPrimitivePart p
if sqfr?
then
  lsfp := squareFreeFactors(p)$polsetpack
  lts: Split := [per(cons(f,rep(ts_v_-))) for f in lsfp]
else
  lts: Split := [per(cons(p,rep(ts_v_-)))]
extend? => extend(members(ts_v_+),lts)
[per(concat(rep(ts_v_+),rep(us))) for us in lts]

augment(p:P,ts:$): List $ ==
ground? p => error "in augment$REGSET: ground? #1"
algebraic?(mvar(p),ts) => error "in augment$REGSET: bad #1"
-- ASSUME init(p) invertible modulo ts

```

```

-- DOES NOT ASSUME anything else.
-- THUS reduction, mainPrimitivePart and squareFree are NEEDED
internalAugment(p,ts,true,true,true,true)

extend(p:P,ts:$): List $ ==
  ground? p => error "in extend$REGSET: ground? #1"
  v := mvar(p)
  not (mvar(ts) < mvar(p)) => error "in extend$REGSET: bad #1"
  lts: List($):= []
  split: List($):= invertibleSet(init(p),ts)
  for us in split repeat
    lts := concat(augment(p,us),lts)
  lts

invertible?(p:P,ts:$): Boolean ==
  toseInvertible?(p,ts)$regsetgcdpack

invertible?(p:P,ts:$): List BWT ==
  toseInvertible?(p,ts)$regsetgcdpack

invertibleSet(p:P,ts:$): Split ==
  toseInvertibleSet(p,ts)$regsetgcdpack

lastSubResultant(p1:P,p2:P,ts:$): List PWT ==
  toseLastSubResultant(p1,p2,ts)$regsetgcdpack

squareFreePart(p:P, ts: $): List PWT ==
  toseSquareFreePart(p,ts)$regsetgcdpack

intersect(p:P, ts: $): List($):= decompose([p], [ts], false, false)$regsetdecomppack

intersect(lp: LP, lts: List($)): List($):= decompose(lp, lts, false, false)$regsetdecomppack
  -- SOLVE in the regular zero sense
  -- and DO NOT PRINT info

decompose(p:P, ts: $): List($):= decompose([p], [ts], true, false)$regsetdecomppack

decompose(lp: LP, lts: List($)): List($):= decompose(lp, lts, true, false)$regsetdecomppack
  -- SOLVE in the closure sense
  -- and DO NOT PRINT info

zeroSetSplit(lp:List(P)) == zeroSetSplit(lp,true,false)
  -- by default SOLVE in the closure sense
  -- and DO NOT PRINT info

zeroSetSplit(lp:List(P), clos?: B) == zeroSetSplit(lp,clos?, false)
  -- DO NOT PRINT info

zeroSetSplit(lp:List(P), clos?: B, info?: B) ==
  -- if clos? then SOLVE in the closure sense

```



```

-- if info? then PRINT info
-- by default USE hash-tables
-- and PREPROCESS the input system
zeroSetSplit(lp,true,clos?,info?,true)

zeroSetSplit(lp>List(P),hash?:B,clos?:B,info?:B,prep?:B) ==
-- if hash? then USE hash-tables
-- if info? then PRINT information
-- if clos? then SOLVE in the closure sense
-- if prep? then PREPROCESS the input system
if hash?
then
  s1, s2, s3, dom1, dom2, dom3: String
  e: String := empty()$String
  if info? then (s1,s2,s3) := ("w","g","i") else (s1,s2,s3) := (e,e,e)
  if info?
  then
    (dom1, dom2, dom3) := ("QCMACK", "REGSETGCD: Gcd", "REGSETGCD: Inv Set")
  else
    (dom1, dom2, dom3) := (e,e,e)
  startTable!(s1,"W",dom1)$quasicomppack
  startTableGcd!(s2,"G",dom2)$regsetgcdpack
  startTableInvSet!(s3,"I",dom3)$regsetgcdpack
  lts := internalZeroSetSplit(lp,clos?,info?,prep?)
if hash?
then
  stopTable!()$quasicomppack
  stopTableGcd!()$regsetgcdpack
  stopTableInvSet!()$regsetgcdpack
lts

internalZeroSetSplit(lp:LP,clos?:B,info?:B,prep?:B) ==
-- if info? then PRINT information
-- if clos? then SOLVE in the closure sense
-- if prep? then PREPROCESS the input system
if prep?
then
  pp := pre_process(lp,clos?,info?)
  lp := pp.val
  lts := pp.towers
else
  ts: $ := [[]]
  lts := [ts]
lp := remove(zero?, lp)
any?(ground?, lp) => []
empty? lp => lts
empty? lts => lts
lp := sort(infRittWu?,lp)
clos? => decompose(lp,lts, clos?, info?)$regsetdecomppack
-- IN DIM > 0 with clos? the following is false ...

```

```

    for p in lp repeat
        lts := decompose([p],lts, clos?, info?)$regsetdecomppack
    lts

largeSystem?(lp:LP): Boolean ==
    -- Gonnet and Gerdt and not Wu-Wang.2
    #lp > 16 => true
    #lp < 13 => false
    lts: List($) := []
    (#lp :: Z - numberOfVariables(lp,lts)$regsetdecomppack :: Z) > 3

smallSystem?(lp:LP): Boolean ==
    -- neural, Vermeer, Liu, and not f-633 and not Hairer-2
    #lp < 5

mediumSystem?(lp:LP): Boolean ==
    -- f-633 and not Hairer-2
    lts: List($) := []
    (numberOfVariables(lp,lts)$regsetdecomppack :: Z - #lp :: Z) < 2

--    lin?(p:P):Boolean == ground?(init(p)) and one?(mdeg(p))
lin?(p:P):Boolean == ground?(init(p)) and (mdeg(p) = 1)

pre_process(lp:LP,clos?:B,info?:B): Record(val: LP, towers: Split) ==
    -- if info? then PRINT information
    -- if clos? then SOLVE in the closure sense
    ts: $ := [[]];
    lts: Split := [ts]
    empty? lp => [lp,lts]
    lp1: List P := []
    lp2: List P := []
    for p in lp repeat
        ground? (tail p) => lp1 := cons(p, lp1)
        lp2 := cons(p, lp2)
    lts: Split := decompose(lp1,[ts],clos?,info?)$regsetdecomppack
    probablyZeroDim?(lp)$polsetpack =>
        largeSystem?(lp) => return [lp2,lts]
    if #lp > 7
    then
        -- Butcher (8,8) + Wu-Wang.2 (13,16)
        lp2 := crushedSet(lp2)$polsetpack
        lp2 := remove(zero?,lp2)
        any?(ground?,lp2) => return [lp2, lts]
        lp3 := [p for p in lp2 | lin?(p)]
        lp4 := [p for p in lp2 | not lin?(p)]
        if clos?
        then
            lts := decompose(lp4,lts, clos?, info?)$regsetdecomppack
        else
            lp4 := sort(infRittWu?,lp4)

```

```

        for p in lp4 repeat
            lts := decompose([p],lts, clos?, info?)$regsetdecomppack
        lp2 := lp3
    else
        lp2 := crushedSet(lp2)$polsetpack
        lp2 := remove(zero?,lp2)
        any?(ground?,lp2) => return [lp2, lts]
    if clos?
    then
        lts := decompose(lp2,lts, clos?, info?)$regsetdecomppack
    else
        lp2 := sort(infRittWu?,lp2)
        for p in lp2 repeat
            lts := decompose([p],lts, clos?, info?)$regsetdecomppack
        lp2 := []
        return [lp2,lts]
    smallSystem?(lp) => [lp2,lts]
    mediumSystem?(lp) => [crushedSet(lp2)$polsetpack,lts]
    lp3 := [p for p in lp2 | lin?(p)]
    lp4 := [p for p in lp2 | not lin?(p)]
    if clos?
    then
        lts := decompose(lp4,lts, clos?, info?)$regsetdecomppack
    else
        lp4 := sort(infRittWu?,lp4)
        for p in lp4 repeat
            lts := decompose([p],lts, clos?, info?)$regsetdecomppack
    if clos?
    then
        lts := decompose(lp3,lts, clos?, info?)$regsetdecomppack
    else
        lp3 := sort(infRittWu?,lp3)
        for p in lp3 repeat
            lts := decompose([p],lts, clos?, info?)$regsetdecomppack
    lp2 := []
    return [lp2,lts]

```

---

— REGSET.dotabb —

```

"REGSET" [color="#88FF44",href="bookvol10.3.pdf#nameddest=REGSET"]
"RSETCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=RSETCAT"]
"REGSET" -> "RSETCAT"

```

---

## 19.8 domain RESRING ResidueRing

— ResidueRing.input —

```
)set break resume
)sys rm -f ResidueRing.output
)spool ResidueRing.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ResidueRing
--R ResidueRing(F: Field,Expon: OrderedAbelianMonoidSup,VarSet: OrderedSet,FPol: PolynomialCategory(F,Ex
--R Abbreviation for ResidueRing is RESRING
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for RESRING
--R
--R----- Operations -----
--R ??? : (F,% ) -> %          ??? : (F,% ) -> %
--R ??? : (%,% ) -> %          ??? : (Integer,% ) -> %
--R ??? : (PositiveInteger,% ) -> %    ??? : (% ,PositiveInteger) -> %
--R ?+? : (%,% ) -> %          ?-? : (%,% ) -> %
--R -? : % -> %                ?=? : (%,% ) -> Boolean
--R 1 : () -> %                0 : () -> %
--R ??? : (% ,PositiveInteger) -> %    coerce : FPol -> %
--R coerce : F -> %              coerce : Integer -> %
--R coerce : % -> OutputForm        hash : % -> SingleInteger
--R latex : % -> String            lift : % -> FPol
--R one? : % -> Boolean            recip : % -> Union(% ,"failed")
--R reduce : FPol -> %            sample : () -> %
--R zero? : % -> Boolean          ?~=? : (%,% ) -> Boolean
--R ??? : (NonNegativeInteger,% ) -> %
--R ??? : (% ,NonNegativeInteger) -> %
--R ?^? : (% ,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R subtractIfCan : (%,% ) -> Union(% ,"failed")
--R
--E 1

)spool
)lisp (bye)
```

— ResidueRing.help —

=====

ResidueRing examples

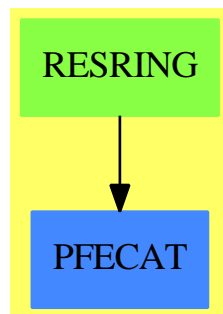
=====

See Also:

o )show ResidueRing

—

### 19.8.1 ResidueRing (RESRING)



#### Exports:

|        |               |                |        |        |
|--------|---------------|----------------|--------|--------|
| 0      | 1             | characteristic | coerce | hash   |
| latex  | lift          | one?           | recip  | reduce |
| sample | subtractIfCan | zero?          | ?~=?   | ?*?    |
| ?**?   | ?^?           | ?+?            | ?-?    | -?     |
| ?=?    |               |                |        |        |

— domain RESRING ResidueRing —

```

)abbrev domain RESRING ResidueRing
++ Author: P.Gianni
++ Date Created: December 1992
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ ResidueRing is the quotient of a polynomial ring by an ideal.
++ The ideal is given as a list of generators. The elements of the domain
++ are equivalence classes expressed in terms of reduced elements
  
```

```

ResidueRing(F,Expon,VarSet,FPol,LFPol) : Dom == Body
  
```

```

where
  F      : Field
  Expon  : OrderedAbelianMonoidSup
  VarSet : OrderedSet
  FPol   : PolynomialCategory(F, Expon, VarSet)
  LFPol  : List FPol

  Dom == Join(CommutativeRing, Algebra F) with
    reduce : FPol -> $
    ++ reduce(f) produces the equivalence class of f in the residue ring
    coerce : FPol -> $
    ++ coerce(f) produces the equivalence class of f in the residue ring
    lift   : $ -> FPol
    ++ lift(x) return the canonical representative of the equivalence class x
  Body == add
    --representation
    Rep:= FPol
    import GroebnerPackage(F,Expon,VarSet,FPol)
    relations:= groebner(LFPol)
    relations = [1] => error "the residue ring is the zero ring"
    --declarations
    x,y: $
    --definitions
    0 == 0$Rep
    1 == 1$Rep
    reduce(f : FPol) : $ == normalForm(f,relations)
    coerce(f : FPol) : $ == normalForm(f,relations)
    lift x == x :: Rep :: FPol
    x + y == x +$Rep y
    -x == -$Rep x
    x*y == normalForm(lift(x *$Rep y),relations)
    (n : Integer) * x == n *$Rep x
    (a : F) * x == a *$Rep x
    x = y == x =$Rep y
    characteristic() == characteristic()$F
    coerce(x) : OutputForm == coerce(x)$Rep

```

---

— RESRING.dotabb —

```

"RESRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=RESRING"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"RESRING" -> "PFECAT"

```

---

## 19.9 domain RESULT Result

— Result.input —

```

)set break resume
)sys rm -f Result.output
)spool Result.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Result
--R Result is a domain constructor
--R Abbreviation for Result is RESULT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for RESULT
--R
--R----- Operations -----
--R copy : % -> % dictionary : () -> %
--R elt : (% , Symbol, Any) -> Any ?.? : (% , Symbol) -> Any
--R empty : () -> % empty? : % -> Boolean
--R entries : % -> List Any eq? : (% , %) -> Boolean
--R index? : (Symbol, %) -> Boolean indices : % -> List Symbol
--R key? : (Symbol, %) -> Boolean keys : % -> List Symbol
--R map : ((Any -> Any), %) -> % qelt : (% , Symbol) -> Any
--R sample : () -> % setelt : (% , Symbol, Any) -> Any
--R table : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?? : (% , %) -> Boolean if Any has SETCAT or Record(key: Symbol, entry: Any) has SETCAT
--R any? : ((Any -> Boolean), %) -> Boolean if $ has finiteAggregate
--R any? : ((Record(key: Symbol, entry: Any) -> Boolean), %) -> Boolean if $ has finiteAggregate
--R bag : List Record(key: Symbol, entry: Any) -> %
--R coerce : % -> OutputForm if Any has SETCAT or Record(key: Symbol, entry: Any) has SETCAT
--R construct : List Record(key: Symbol, entry: Any) -> %
--R convert : % -> InputForm if Record(key: Symbol, entry: Any) has KONVERT INFORM
--R count : ((Any -> Boolean), %) -> NonNegativeInteger if $ has finiteAggregate
--R count : (Any, %) -> NonNegativeInteger if $ has finiteAggregate and Any has SETCAT
--R count : (Record(key: Symbol, entry: Any), %) -> NonNegativeInteger if $ has finiteAggregate
--R count : ((Record(key: Symbol, entry: Any) -> Boolean), %) -> NonNegativeInteger if $ has finiteAggregate
--R dictionary : List Record(key: Symbol, entry: Any) -> %
--R entry? : (Any, %) -> Boolean if $ has finiteAggregate and Any has SETCAT
--R eval : (% , List Equation Any) -> % if Any has EVALAB ANY and Any has SETCAT
--R eval : (% , Equation Any) -> % if Any has EVALAB ANY and Any has SETCAT
--R eval : (% , Any, Any) -> % if Any has EVALAB ANY and Any has SETCAT
--R eval : (% , List Any, List Any) -> % if Any has EVALAB ANY and Any has SETCAT
--R eval : (% , List Record(key: Symbol, entry: Any), List Record(key: Symbol, entry: Any)) -> % if Record(
--R eval : (% , Record(key: Symbol, entry: Any), Record(key: Symbol, entry: Any)) -> % if Record(

```

```

--R eval : (% ,Equation Record(key: Symbol,entry: Any)) -> % if Record(key: Symbol,entry: Any) has EVALAB
--R eval : (% ,List Equation Record(key: Symbol,entry: Any)) -> % if Record(key: Symbol,entry: Any) has E
--R every? : ((Any -> Boolean),%) -> Boolean if $ has finiteAggregate
--R every? : ((Record(key: Symbol,entry: Any) -> Boolean),%) -> Boolean if $ has finiteAggregate
--R extract! : % -> Record(key: Symbol,entry: Any)
--R fill! : (% ,Any) -> % if $ has shallowlyMutable
--R find : ((Record(key: Symbol,entry: Any) -> Boolean),%) -> Union(Record(key: Symbol,entry: Any),"fail
--R first : % -> Any if Symbol has ORDSET
--R hash : % -> SingleInteger if Any has SETCAT or Record(key: Symbol,entry: Any) has SETCAT
--R insert! : (Record(key: Symbol,entry: Any),%) -> %
--R inspect : % -> Record(key: Symbol,entry: Any)
--R latex : % -> String if Any has SETCAT or Record(key: Symbol,entry: Any) has SETCAT
--R less? : (% ,NonNegativeInteger) -> Boolean
--R map : (((Any,Any) -> Any),%,%) -> %
--R map : ((Record(key: Symbol,entry: Any) -> Record(key: Symbol,entry: Any)),%) -> %
--R map! : ((Any -> Any),%) -> % if $ has shallowlyMutable
--R map! : ((Record(key: Symbol,entry: Any) -> Record(key: Symbol,entry: Any)),%) -> % if $ has shallow
--R maxIndex : % -> Symbol if Symbol has ORDSET
--R member? : (Any,%) -> Boolean if $ has finiteAggregate and Any has SETCAT
--R member? : (Record(key: Symbol,entry: Any),%) -> Boolean if $ has finiteAggregate and Record(key: Sym
--R members : % -> List Any if $ has finiteAggregate
--R members : % -> List Record(key: Symbol,entry: Any) if $ has finiteAggregate
--R minIndex : % -> Symbol if Symbol has ORDSET
--R more? : (% ,NonNegativeInteger) -> Boolean
--R parts : % -> List Any if $ has finiteAggregate
--R parts : % -> List Record(key: Symbol,entry: Any) if $ has finiteAggregate
--R qsetelt! : (% ,Symbol,Any) -> Any if $ has shallowlyMutable
--R reduce : (((Record(key: Symbol,entry: Any),Record(key: Symbol,entry: Any)) -> Record(key: Symbol,ent
--R reduce : (((Record(key: Symbol,entry: Any),Record(key: Symbol,entry: Any)) -> Record(key: Symbol,ent
--R reduce : (((Record(key: Symbol,entry: Any),Record(key: Symbol,entry: Any)) -> Record(key: Symbol,ent
--R remove : ((Record(key: Symbol,entry: Any) -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (Record(key: Symbol,entry: Any),%) -> % if $ has finiteAggregate and Record(key: Symbol,ent
--R remove! : (Symbol,%) -> Union(Any,"failed")
--R remove! : ((Record(key: Symbol,entry: Any) -> Boolean),%) -> % if $ has finiteAggregate
--R remove! : (Record(key: Symbol,entry: Any),%) -> % if $ has finiteAggregate
--R removeDuplicates : % -> % if $ has finiteAggregate and Record(key: Symbol,entry: Any) has SETCAT
--R search : (Symbol,%) -> Union(Any,"failed")
--R select : ((Record(key: Symbol,entry: Any) -> Boolean),%) -> % if $ has finiteAggregate
--R select! : ((Record(key: Symbol,entry: Any) -> Boolean),%) -> % if $ has finiteAggregate
--R showArrayValues : Boolean -> Boolean
--R showScalarValues : Boolean -> Boolean
--R size? : (% ,NonNegativeInteger) -> Boolean
--R swap! : (% ,Symbol,Symbol) -> Void if $ has shallowlyMutable
--R table : List Record(key: Symbol,entry: Any) -> %
--R ~=? : (% ,%) -> Boolean if Any has SETCAT or Record(key: Symbol,entry: Any) has SETCAT
--R
--E 1

)spool
)lisp (bye)

```



---

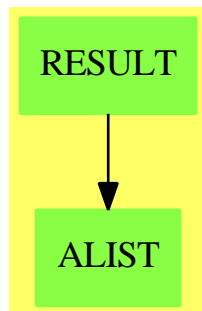
— **Result.help** —

```
=====
Result examples
=====
```

```
See Also:
o )show Result
```

---

### 19.9.1 Result (RESULT)



**See**

- ⇒ “FortranCode” (FC) 7.16.1 on page 898
- ⇒ “FortranProgram” (FORTRAN) 7.18.1 on page 923
- ⇒ “ThreeDimensionalMatrix” (M3D) 21.7.1 on page 2661
- ⇒ “SimpleFortranProgram” (SFORT) 20.11.1 on page 2364
- ⇒ “Switch” (SWITCH) 20.36.1 on page 2588
- ⇒ “FortranTemplate” (FTEM) 7.20.1 on page 934
- ⇒ “FortranExpression” (FEXPR) 7.17.1 on page 914

**Exports:**

|        |          |                  |                 |                  |
|--------|----------|------------------|-----------------|------------------|
| any?   | bag      | coerce           | construct       | convert          |
| copy   | count    | dictionary       | entry?          | elt              |
| empty  | empty?   | entries          | eq?             | eval             |
| every? | extract! | fill!            | find            | first            |
| hash   | index?   | indices          | insert!         | inspect          |
| key?   | keys     | latex            | less?           | map              |
| map!   | maxIndex | member?          | members         | minIndex         |
| more?  | parts    | qelt             | qsetelt!        | reduce           |
| remove | remove!  | removeDuplicates | sample          | search           |
| select | select!  | setelt           | showArrayValues | showScalarValues |
| size?  | swap!    | table            | #?              | ?=?              |
| ?~=?   | ?..?     |                  |                 |                  |

— domain RESULT Result —

```

)abbrev domain RESULT Result
++ Author: Didier Pinchon and Mike Dewar
++ Date Created: 8 April 1994
++ Date Last Updated: 28 June 1994
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ A domain used to return the results from a call to the NAG
++ Library. It prints as a list of names and types, though the user may
++ choose to display values automatically if he or she wishes.

Result():Exports==Implementation where

0 ==> OutputForm

Exports ==> TableAggregate(Symbol,Any) with
  showScalarValues : Boolean -> Boolean
    ++ showScalarValues(true) forces the values of scalar components to be
    ++ displayed rather than just their types.
  showArrayValues : Boolean -> Boolean
    ++ showArrayValues(true) forces the values of array components to be
    ++ displayed rather than just their types.
  finiteAggregate

Implementation ==> Table(Symbol,Any) add

-- Constant
colon := ": " :: Symbol::0

```

```

elide := "...":Symbol::0

-- Flags
showScalarValuesFlag : Boolean := false
showArrayValuesFlag  : Boolean := false

cleanUpDomainForm(d:SExpression):0 ==
  not list? d => d::0
  #d=1 => (car d)::0
  -- If the car is an atom then we have a domain constructor, if not
  -- then we have some kind of value. Since we often can't print these
  -- **eters we just elide them.
  not atom? car d => elide
  prefix((car d)::0,[cleanUpDomainForm(u) for u in destruct cdr(d)]$List(0))

display(v:Any,d:SExpression):0 ==
  not list? d => error "Domain form is non-list"
  #d=1 =>
    showScalarValuesFlag => objectOf v
    cleanUpDomainForm d
    car(d) = convert("Complex":Symbol)@SExpression =>
      showScalarValuesFlag => objectOf v
      cleanUpDomainForm d
    showArrayValuesFlag => objectOf v
    cleanUpDomainForm d

makeEntry(k:Symbol,v:Any):0 ==
  hconcat [k::0,colon,display(v,dom v)]

coerce(r:%):0 ==
  bracket [makeEntry(key,r.key) for key in reverse! keys(r)]

showArrayValues(b:Boolean):Boolean == showArrayValuesFlag := b
showScalarValues(b:Boolean):Boolean == showScalarValuesFlag := b

```

---

— RESULT.dotabb —

```

"RESULT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=RESULT"]
"ALIST"  [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"RESULT" -> "ALIST"

```

---

## 19.10 domain RULE RewriteRule

— RewriteRule.input —

```
)set break resume
)sys rm -f RewriteRule.output
)spool RewriteRule.output
)set message test on
)set message auto off
)clear all

--S 1 of 4
)show RewriteRule
--R RewriteRule(Base: SetCategory,R: Join(Ring,PatternMatchable Base,OrderedSet,ConvertibleTo Pattern Ba
--R Abbreviation for RewriteRule is RULE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for RULE
--R
--R----- Operations -----
--R ==? : (%,% ) -> Boolean          coerce : Equation F -> %
--R coerce : % -> OutputForm          elt : (% ,F,PositiveInteger) -> F
--R ?.? : (% ,F) -> F                hash : % -> SingleInteger
--R latex : % -> String              lhs : % -> F
--R pattern : % -> Pattern Base      retract : % -> Equation F
--R rhs : % -> F                     rule : (F,F,List Symbol) -> %
--R rule : (F,F) -> %                ~=? : (% ,%) -> Boolean
--R quotedOperators : % -> List Symbol
--R retractIfCan : % -> Union(Equation F,"failed")
--R suchThat : (% ,List Symbol,(List F -> Boolean)) -> %
--R
--E 1

--S 2 of 4
logrule := rule log(x) + log(y) == log(x*y)
--R
--R (1) log(y) + log(x) + %B == log(x y) + %B
--R                                     Type: RewriteRule(Integer,Integer,Expression Integer)
--E 2

--S 3 of 4
f := log(sin(x)) + log(x)
--R
--R (2) log(sin(x)) + log(x)
--R                                     Type: Expression Integer
--E 3

--S 4 of 4
logrule f
```

```

--R
--R (3) log(x sin(x))
--R
--R                                         Type: Expression Integer
--E 4

)spool
)lisp (bye)

```

---

— RewriteRule.help —

=====

RewriteRule examples

=====

For example:

```

logrule := rule log(x) + log(y) == log(x*y)
          log(y) + log(x) + %C == log(x y) + %C

f := log(sin(x)) + log(x)
   log(sin(x)) + log(x)

logrule f
   log(x sin(x))

```

Note that you cannot write the simple form of a rule as:

```
rule1 := rule a*x+b == b*x+a
```

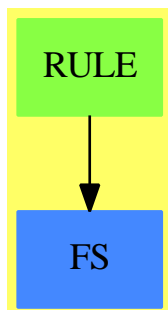
as this causes an infinite loop. The pattern is properly matched but the result also properly matches and the pattern is applied again.

See Also:

- o )show ApplyRules
- o )show RewriteRule

---

## 19.10.1 RewriteRule (RULE)



See

⇒ “Ruleset” (RULESET) 19.15.1 on page 2303

**Exports:**

|         |                 |         |              |     |
|---------|-----------------|---------|--------------|-----|
| coerce  | elt             | hash    | latex        | lhs |
| pattern | quotedOperators | retract | retractIfCan | rhs |
| rule    | suchThat        | ?.?     | ?~=?         | ?=? |

— domain **RULE RewriteRule** —

```
)abbrev domain RULE RewriteRule
++ Author: Manuel Bronstein
++ Date Created: 24 Oct 1988
++ Date Last Updated: 26 October 1993
++ Keywords: pattern, matching, rule.
++ Description:
++ Rules for the pattern matcher
```

```
RewriteRule(Base, R, F): Exports == Implementation where
  Base   : SetCategory
  R       : Join(Ring, PatternMatchable Base, OrderedSet,
                ConvertibleTo Pattern Base)
  F       : Join(FunctionSpace R, PatternMatchable Base,
                ConvertibleTo Pattern Base)
```

```
P      ==> Pattern Base
```

```
Exports ==>
Join(SetCategory, Eltable(F, F), RetractableTo Equation F) with
  rule   : (F, F) -> $
    ++ rule(f, g) creates the rewrite rule: \spad{f == eval(g, g is f)},
    ++ with left-hand side f and right-hand side g.
    ++
    ++X logrule := rule log(x) + log(y) == log(x*y)
    ++X f := log(sin(x)) + log(x)
    ++X logrule f
```

```

rule      : (F, F, List Symbol) -> $
  ++ rule(f, g, [f1,...,fn]) creates the rewrite rule
  ++ \spad{f == eval(eval(g, g is f), [f1,...,fn])},
  ++ that is a rule with left-hand side f and right-hand side g;
  ++ The symbols f1,...,fn are the operators that are considered
  ++ quoted, that is they are not evaluated during any rewrite,
  ++ but just applied formally to their arguments.
suchThat: ($, List Symbol, List F -> Boolean) -> $
  ++ suchThat(r, [a1,...,an], f) returns the rewrite rule r with
  ++ the predicate \spad{f(a1,...,an)} attached to it.
pattern : $ -> P
  ++ pattern(r) returns the pattern corresponding to
  ++ the left hand side of the rule r.
lhs      : $ -> F
  ++ lhs(r) returns the left hand side of the rule r.
rhs      : $ -> F
  ++ rhs(r) returns the right hand side of the rule r.
elt      : ($, F, PositiveInteger) -> F
  ++ elt(r,f,n) or r(f, n) applies the rule r to f at most n times.
quotedOperators: $ -> List Symbol
  ++ quotedOperators(r) returns the list of operators
  ++ on the right hand side of r that are considered
  ++ quoted, that is they are not evaluated during any rewrite,
  ++ but just applied formally to their arguments.

Implementation ==> add
import ApplyRules(Base, R, F)
import PatternFunctions1(Base, F)
import FunctionSpaceAssertions(R, F)

Rep := Record(pat: P, lft: F, rgt: F, qot: List Symbol)

mkRule      : (P, F, F, List Symbol) -> $
transformLhs: P -> Record(plus: F, times: F)
bad?        : Union(List P, "failed") -> Boolean
appear?     : (P, List P) -> Boolean
opt         : F -> P
F2Symbol    : F -> F

pattern x          == x.pat
lhs x              == x.lft
rhs x              == x.rgt
quotedOperators x == x.qot
mkRule(pt, p, s, l) == [pt, p, s, l]
coerce(eq:Equation F):$ == rule(lhs eq, rhs eq, empty())
rule(l, r)         == rule(l, r, empty())
elt(r:$, s:F) == applyRules([r pretend RewriteRule(Base, R, F)], s)

suchThat(x, l, f) ==
  mkRule(suchThat(pattern x,l,f), lhs x, rhs x, quotedOperators x)

```

```

x = y ==
  (lhs x = lhs y) and (rhs x = rhs y) and
    (quotedOperators x = quotedOperators y)

elt(r:$, s:F, n:PositiveInteger) ==
  applyRules([r pretend RewriteRule(Base, R, F)], s, n)

-- remove the extra properties from the constant symbols in f
F2Symbol f ==
  l := select_!(z+>symbolIfCan z case Symbol, tower f)$List(Kernel F)
  eval(f, l, [symbolIfCan(k)::Symbol::F for k in l])

retractIfCan r ==
  constant? pattern r =>
    (u:= retractIfCan(lhs r)@Union(Kernel F,"failed")) case "failed"
      => "failed"
    F2Symbol(u::Kernel(F)::F) = rhs r
  "failed"

rule(p, s, l) ==
  lh := transformLhs(pt := convert(p)@P)
  mkRule(opt(lh.times) * (opt(lh.plus) + pt),
    lh.times * (lh.plus + p), lh.times * (lh.plus + s), l)

opt f ==
  retractIfCan(f)@Union(R, "failed") case R => convert f
  convert optional f

-- appear?(x, [p1,...,pn]) is true if x appears as a variable in
-- a composite pattern pi.
appear?(x, l) ==
  for p in l | p ^= x repeat
    member?(x, variables p) => return true
  false

-- a sum/product p1 @ ... @ pn is "bad" if it will not match
-- a sum/product p1 @ ... @ pn @ p(n+1)
-- in which case one should transform p1 @ ... @ pn to
-- p1 @ ... @ ?p(n+1) which does not change its meaning.
-- examples of "bad" combinations
--   sin(x) @ sin(y)      sin(x) @ x
-- examples of "good" combinations
--   sin(x) @ y
bad? u ==
  u case List(P) =>
    for x in u::List(P) repeat
      generic? x and not appear?(x, u::List(P)) => return false
    true
  false

```



```

transformLhs p ==
  bad? isPlus p  => [new()$Symbol :: F, 1]
  bad? isTimes p => [0, new()$Symbol :: F]
  [0, 1]

coerce(x:$):OutputForm ==
  infix(" == " :: Symbol :: OutputForm,
        lhs(x)::OutputForm, rhs(x)::OutputForm)

```

---

— RULE.dotabb —

```

"RULE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=RULE"]
"FS"   [color="#4488FF",href="bookvol10.2.pdf#nameddest=FS"]
"RULE" -> "FS"

```

---

## 19.11 domain ROIRC RightOpenIntervalRootCharacterization

The domain `RightOpenIntervalRootCharacterization` is the main code that provides the functionalities of `RealRootCharacterizationCategory` for the case of archimedean fields. Abstract roots are encoded with a left closed right open interval containing the root together with a defining polynomial for the root.

### CAVEATS

Since real algebraic expressions are stored as depending on "real roots" which are managed like variables, there is an ordering on these. This ordering is dynamical in the sense that any new algebraic takes precedence over older ones. In particular every creation function raises a new "real root". This has the effect that when you type something like  $\sqrt{2} + \sqrt{2}$  you have two new variables which happen to be equal. To avoid this name the expression such as in `s2 := sqrt(2) ; s2 + s2`

Also note that computing times depend strongly on the ordering you implicitly provide. Please provide algebraics in the order which most natural to you.

### LIMITATIONS

The file `reclos.input` show some basic use of the package. This package uses algorithms which are published in [1] and [2] which are based on field arithmetics, in particular for polynomial gcd related algorithms. This can be quite slow for high degree polynomials and subresultants methods usually work best. Beta versions of the package try to use these techniques in a

better way and work significantly faster. These are mostly based on unpublished algorithms and cannot be distributed. Please contact the author if you have a particular problem to solve or want to use these versions.

Be aware that approximations behave as post-processing and that all computations are done exactly. They can thus be quite time consuming when depending on several "real roots".

— RightOpenIntervalRootCharacterization.input —

```
)set break resume
)sys rm -f RightOpenIntervalRootCharacterization.output
)spool RightOpenIntervalRootCharacterization.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show RightOpenIntervalRootCharacterization
--R RightOpenIntervalRootCharacterization(TheField: Join(OrderedRing,Field),ThePolDom: UnivariatePolynom
--R Abbreviation for RightOpenIntervalRootCharacterization is ROIRC
--R This constructor is exposed in this frame.
--R Issue )edit NIL to see algebra source code for ROIRC
--R
--R----- Operations -----
--R ?=? : (%,% ) -> Boolean          allRootsOf : ThePolDom -> List %
--R coerce : % -> OutputForm          hash : % -> SingleInteger
--R latex : % -> String                left : % -> TheField
--R middle : % -> TheField             refine : % -> %
--R right : % -> TheField              sign : (ThePolDom,%) -> Integer
--R size : % -> TheField               zero? : (ThePolDom,%) -> Boolean
--R ?~=? : (%,% ) -> Boolean
--R approximate : (ThePolDom,%,TheField) -> TheField
--R definingPolynomial : % -> ThePolDom
--R mightHaveRoots : (ThePolDom,%) -> Boolean
--R negative? : (ThePolDom,%) -> Boolean
--R positive? : (ThePolDom,%) -> Boolean
--R recip : (ThePolDom,%) -> Union(ThePolDom,"failed")
--R relativeApprox : (ThePolDom,%,TheField) -> TheField
--R rootOf : (ThePolDom,PositiveInteger) -> Union(%, "failed")
--R
--E 1

)spool
)lisp (bye)
```

— RightOpenIntervalRootCharacterization.help —

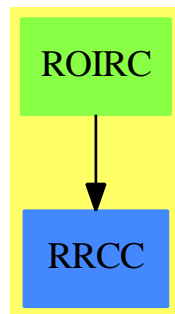
```
=====
RightOpenIntervalRootCharacterization examples
=====
```

See Also:

```
o )show RightOpenIntervalRootCharacterization
```

—

### 19.11.1 RightOpenIntervalRootCharacterization (ROIRC)



See

⇒ “RealClosure” (RECLOS) 19.3.1 on page 2196

#### Exports:

|            |             |        |                    |           |
|------------|-------------|--------|--------------------|-----------|
| allRootsOf | approximate | coerce | definingPolynomial | hash      |
| latex      | left        | middle | mightHaveRoots     | negative? |
| positive?  | recip       | refine | relativeApprox     | right     |
| rootOf     | sign        | size   | zero?              | ?=?       |
| ?~=?       |             |        |                    |           |

— domain ROIRC RightOpenIntervalRootCharacterization —

```
)abbrev domain ROIRC RightOpenIntervalRootCharacterization
++ Author: Renaud Rioboo
++ Date Created: summer 1992
++ Date Last Updated: January 2004
++ Basic Functions: provides computations with real roots of olynomials
++ Related Constructors: RealRootCharacterizationCategory, RealClosure
++ Also See:
++ AMS Classifications:
++ Keywords: Real Algebraic Numbers
++ References:
++ Description:
++ \axiomType{RightOpenIntervalRootCharacterization} provides work with
```

```
++ interval root coding.
```

```
RightOpenIntervalRootCharacterization(TheField,ThePolDom) : PUB == PRIV where
```

```
TheField : Join(OrderedRing,Field)
```

```
ThePolDom : UnivariatePolynomialCategory(TheField)
```

```
Z      ==> Integer
```

```
P      ==> ThePolDom
```

```
N      ==> NonNegativeInteger
```

```
B      ==> Boolean
```

```
UTIL   ==> RealPolynomialUtilitiesPackage(TheField,ThePolDom)
```

```
RRCC    ==> RealRootCharacterizationCategory
```

```
O ==> OutputForm
```

```
TwoPoints ==> Record(low:TheField , high:TheField)
```

```
PUB == RealRootCharacterizationCategory(TheField, ThePolDom) with
```

```
left   :      $      -> TheField
```

```
++ \axiom{left(rootChar)} is the left bound of the isolating
```

```
++ interval
```

```
right  :      $      -> TheField
```

```
++ \axiom{right(rootChar)} is the right bound of the isolating
```

```
++ interval
```

```
size   :      $      -> TheField
```

```
++ The size of the isolating interval
```

```
middle :      $      -> TheField
```

```
++ \axiom{middle(rootChar)} is the middle of the isolating
```

```
++ interval
```

```
refine :      $      -> $
```

```
++ \axiom{refine(rootChar)} shrinks isolating interval around
```

```
++ \axiom{rootChar}
```

```
mightHaveRoots : (P,$) -> B
```

```
++ \axiom{mightHaveRoots(p,r)} is false if \axiom{p.r} is not 0
```

```
relativeApprox : (P,$,TheField) -> TheField
```

```
++ \axiom{relativeApprox(exp,c,p) = a} is relatively close to exp
```

```
++ as a polynomial in c ip to precision p
```

```
PRIV == add
```

```
-- local functions
```

```
makeChar:      (TheField,TheField,ThePolDom) -> $
```

```
refine! :      $      -> $
```

```
sturmIsolate : (List(P), TheField, TheField,N,N) -> List TwoPoints
```

```
isolate :      List(P)      -> List TwoPoints
```

```

    rootBound :                               P                -> TheField
--  varStar :                               P                -> N
    linearRecip :                             ( P , $)        -> Union(P, "failed")
    linearZero? :                             (TheField,$)    -> B
    linearSign :                              (P,$)           -> Z
    sturmNthRoot : (List(P), TheField, TheField,N,N,N) -> Union(TwoPoints,"failed")
    addOne :                                   P               -> P
    minus :                                   P               -> P
    translate :                               (P,TheField)    -> P
    dilate :                                  (P,TheField)    -> P
    invert :                                  P               -> P
    evalOne :                                 P               -> TheField
    hasVarsl :                               List(TheField)   -> B
    hasVars :                                P               -> B

-- Representation

Rep := Record(low:TheField,high:TheField,defPol:ThePolDom)

-- and now the code !

size(rootCode) ==
    rootCode.high - rootCode.low

relativeApprox(pval,rootCode,prec) ==
    -- beurk !
    dPol := rootCode.defPol
    degree(dPol) = 1 =>
        c := -coefficient(dPol,0)/leadingCoefficient(dPol)
        pval.c
    pval := pval rem dPol
    degree(pval) = 0 => leadingCoefficient(pval)
    zero?(pval,rootCode) => 0
    while mightHaveRoots(pval,rootCode) repeat
        rootCode := refine(rootCode)
    dpval := differentiate(pval)
    degree(dpval) = 0 =>
        l := left(rootCode)
        r := right(rootCode)
        a := pval.l
        b := pval.r
        while ( abs(2*(a-b)/(a+b)) > prec ) repeat
            rootCode := refine(rootCode)
            l := left(rootCode)
            r := right(rootCode)
            a := pval.l
            b := pval.r
        (a+b)/(2::TheField)
    zero?(dpval,rootCode) =>

```

```

    relativeApprox(pval,
                    [left(rootCode),
                     right(rootCode),
                     gcd(dpval, rootCode.defPol)]$Rep,
                    prec)
while mightHaveRoots(dpval, rootCode) repeat
    rootCode := refine(rootCode)
l := left(rootCode)
r := right(rootCode)
a := pval.l
b := pval.r
while ( abs(2*(a-b)/(a+b)) > prec ) repeat
    rootCode := refine(rootCode)
    l := left(rootCode)
    r := right(rootCode)
    a := pval.l
    b := pval.r
(a+b)/(2::TheField)

approximate(pval, rootCode, prec) ==
-- glurp
dPol := rootCode.defPol
degree(dPol) = 1 =>
    c := -coefficient(dPol, 0)/leadingCoefficient(dPol)
    pval.c
pval := pval rem dPol
degree(pval) = 0 => leadingCoefficient(pval)
dpval := differentiate(pval)
degree(dpval) = 0 =>
    l := left(rootCode)
    r := right(rootCode)
    while ( abs((a := pval.l) - (b := pval.r)) > prec ) repeat
        rootCode := refine(rootCode)
        l := left(rootCode)
        r := right(rootCode)
    (a+b)/(2::TheField)
zero?(dpval, rootCode) =>
    approximate(pval,
                [left(rootCode),
                 right(rootCode),
                 gcd(dpval, rootCode.defPol)]$Rep,
                prec)
while mightHaveRoots(dpval, rootCode) repeat
    rootCode := refine(rootCode)
l := left(rootCode)
r := right(rootCode)
while ( abs((a := pval.l) - (b := pval.r)) > prec ) repeat
    rootCode := refine(rootCode)
    l := left(rootCode)
    r := right(rootCode)

```

```

(a+b)/(2::TheField)

addOne(p) == p.(monomial(1,1)+(1::P))

minus(p) == p.(monomial(-1,1))

translate(p,a) == p.(monomial(1,1)+(a::P))

dilate(p,a) == p.(monomial(a,1))

evalOne(p) == "+" / coefficients(p)

invert(p) ==
  d := degree(p)
  mapExponents(z +-> (d-z)::N, p)

rootBound(p) ==
  res : TheField := 1
  raw :TheField := 1+boundOfCauchy(p)$UTIL
  while (res < raw) repeat
    res := 2*(res)
  res

sturmNthRoot(lp,l,r,vl,vr,n) ==
  nv := (vl - vr)::N
  nv < n => "failed"
  ((nv = 1) and (n = 1)) => [l,r]
  int := (l+r)/(2::TheField)
  lt:List(TheField):=[]
  for t in lp repeat
    lt := cons(t.int , lt)
  vi := sturmVariationsOf(reverse! lt)$UTIL
  o :Z := n - vl + vi
  if o > 0
  then
    sturmNthRoot(lp,int,r,vi,vr,o::N)
  else
    sturmNthRoot(lp,l,int,vl,vi,n)

sturmIsolate(lp,l,r,vl,vr) ==
  r <= l => error "ROIRC: sturmIsolate: bad bounds"
  n := (vl - vr)::N
  zero?(n) => []
  one?(n) => [[l,r]]
  int := (l+r)/(2::TheField)
  vi := sturmVariationsOf( [t.int for t in lp ] )$UTIL
  append(sturmIsolate(lp,l,int,vl,vi),sturmIsolate(lp,int,r,vi,vr))

isolate(lp) ==

```

```

b := rootBound(first(lp))
l1,l2 : List(TheField)
(l1,l2) := ([], [])
for t in reverse(lp) repeat
  if odd?(degree(t))
  then
    (l1,l2) := (cons(-leadingCoefficient(t),l1),
                cons(leadingCoefficient(t),l2))
  else
    (l1,l2) := (cons(leadingCoefficient(t),l1),
                cons(leadingCoefficient(t),l2))
 SturmIsolate(lp,
              -b,
              b,
              sturmVariationsOf(l1)$UTIL,
              sturmVariationsOf(l2)$UTIL)

rootOf(pol,n) ==
ls := sturmSequence(pol)$UTIL
pol := unitCanonical(first(ls)) -- this one is SqFR
degree(pol) = 0 => "failed"
numberOfMonomials(pol) = 1 => ([0,1,monomial(1,1)]$Rep)::F
b := rootBound(pol)
l1,l2 : List(TheField)
(l1,l2) := ([], [])
for t in reverse(ls) repeat
  if odd?(degree(t))
  then
    (l1,l2) := (cons(leadingCoefficient(t),l1),
                cons(-leadingCoefficient(t),l2))
  else
    (l1,l2) := (cons(leadingCoefficient(t),l1),
                cons(leadingCoefficient(t),l2))
res := sturmNthRoot(ls,
                  -b,
                  b,
                  sturmVariationsOf(l2)$UTIL,
                  sturmVariationsOf(l1)$UTIL,
                  n)
res case "failed" => "failed"
makeChar(res.low,res.high,pol)

allRootsOf(pol) ==
ls := sturmSequence(unitCanonical pol)$UTIL
pol := unitCanonical(first(ls)) -- this one is SqFR
degree(pol) = 0 => []
numberOfMonomials(pol) = 1 => [[0,1,monomial(1,1)]$Rep]
[ makeChar(term.low,term.high,pol) for term in isolate(ls) ]

```



```

hasVarsl(l:List(TheField)) ==
  null(l) => false
  f := sign(first(l))
  for term in rest(l) repeat
    if f*term < 0 then return(true)
  false

hasVars(p:P) ==
  zero?(p) => error "ROIRC: hasVars: null polynomial"
  zero?(coefficient(p,0)) => true
  hasVarsl(coefficients(p))

mightHaveRoots(p,rootChar) ==
  a := rootChar.low
  q := translate(p,a)
  not(hasVars(q)) => false
--   varStar(q) = 0 => false
  a := (rootChar.high) - a
  q := dilate(q,a)
  sign(coefficient(q,0))*sign(evalOne(q)) <= 0 => true
  q := minus(addOne(q))
  not(hasVars(q)) => false
--   varStar(q) = 0 => false
  q := invert(q)
  hasVars(addOne(q))
--   ^(varStar(addOne(q)) = 0)

coerce(rootChar:$):0 ==
  commaSeparate([ hconcat("[" :: 0 , (rootChar.low)::0),
                  hconcat((rootChar.high)::0,"[" :: 0 ) ])

c1 = c2 ==
  mM := max(c1.low,c2.low)
  Mm := min(c1.high,c2.high)
  mM >= Mm => false
  rr : ThePolDom := gcd(c1.defPol,c2.defPol)
  degree(rr) = 0 => false
  sign(rr.mM) * sign(rr.Mm) <= 0

makeChar(left,right,pol) ==
-- The following lines of code, which check for a possible error,
-- cause major performance problems and were removed by Renaud Rioboo,
-- the original author. They were originally inserted for debugging.
--   right <= left => error "ROIRC: makeChar: Bad interval"
--   (pol.left * pol.right) > 0 => error "ROIRC: makeChar: Bad pol"
  res :$ := [left,right,leadingMonomial(pol)+reductum(pol)]$Rep -- safe copy
  while zero?(pol.(res.high)) repeat refine!(res)
  while (res.high * res.low < 0 ) repeat refine!(res)
  zero?(pol.(res.low)) => [res.low,res.high,monomial(1,1)-(res.low)::P]

```

```

res

definingPolynomial(rootChar) == rootChar.defPol

linearRecip(toTest,rootChar) ==
  c := - inv(leadingCoefficient(toTest)) * coefficient(toTest,0)
  r := recip(rootChar.defPol.c)
  if (r case "failed")
  then
    if (c - rootChar.low) * (c - rootChar.high) <= 0
    then
      "failed"
    else
      newPol := (rootChar.defPol exquo toTest)::P
      ((1$ThePolDom - inv(newPol.c)*newPol) exquo toTest)::P
  else
    ((1$ThePolDom - (r::TheField)*rootChar.defPol) exquo toTest)::P

recip(toTest,rootChar) ==
  degree(toTest) = 0 or degree(rootChar.defPol) <= degree(toTest) =>
    error "IRC: recip: Not reduced"
  degree(rootChar.defPol) = 1 =>
    error "IRC: recip: Linear Defining Polynomial"
  degree(toTest) = 1 =>
    linearRecip(toTest, rootChar)
  d := extendedEuclidean((rootChar.defPol),toTest)
  (degree(d.generator) = 0 ) =>
    d.coef2
  d.generator := unitCanonical(d.generator)
  (d.generator.(rootChar.low) *
   d.generator.(rootChar.high)<= 0) => "failed"
  newPol := (rootChar.defPol exquo (d.generator))::P
  degree(newPol) = 1 =>
    c := - inv(leadingCoefficient(newPol)) * coefficient(newPol,0)
    inv(toTest.c)::P
  degree(toTest) = 1 =>
    c := - coefficient(toTest,0)/ leadingCoefficient(toTest)
    ((1$ThePolDom - inv(newPol.c))*newPol) exquo toTest)::P
  d := extendedEuclidean(newPol,toTest)
  d.coef2

linearSign(toTest,rootChar) ==
  c := - inv(leadingCoefficient(toTest)) * coefficient(toTest,0)
  ev := sign(rootChar.defPol.c)
  if zero?(ev)
  then
    if (c - rootChar.low) * (c - rootChar.high) <= 0
    then
      0
    else

```

```

        sign(toTest.(rootChar.high))
    else
        if (ev*sign(rootChar.defPol.(rootChar.high)) <= 0 )
        then
            sign(toTest.(rootChar.high))
        else
            sign(toTest.(rootChar.low))

sign(toTest,rootChar) ==
degree(toTest) = 0 or degree(rootChar.defPol) <= degree(toTest) =>
    error "IRC: sign: Not reduced"
degree(rootChar.defPol) = 1 =>
    error "IRC: sign: Linear Defining Polynomial"
degree(toTest) = 1 =>
    linearSign(toTest, rootChar)
s := sign(leadingCoefficient(toTest))
toTest := monomial(1,degree(toTest))+
    inv(leadingCoefficient(toTest))*reductum(toTest)
delta := gcd(toTest,rootChar.defPol)
newChar := [rootChar.low,rootChar.high,rootChar.defPol]$Rep
if degree(delta) > 0
then
    if sign(delta.(rootChar.low) * delta.(rootChar.high)) <= 0
    then
        return(0)
    else
        newChar.defPol := (newChar.defPol exquo delta) :: P
        toTest := toTest rem (newChar.defPol)
        degree(toTest) = 0 => s * sign(leadingCoefficient(toTest))
        degree(toTest) = 1 => s * linearSign(toTest, newChar)
        while mightHaveRoots(toTest,newChar) repeat
            newChar := refine(newChar)
            s*sign(toTest.(newChar.low))

linearZero?(c,rootChar) ==
    zero?((rootChar.defPol).c) and
    (c - rootChar.low) * (c - rootChar.high) <= 0

zero?(toTest,rootChar) ==
degree(toTest) = 0 or degree(rootChar.defPol) <= degree(toTest) =>
    error "IRC: zero?: Not reduced"
degree(rootChar.defPol) = 1 =>
    error "IRC: zero?: Linear Defining Polynomial"
degree(toTest) = 1 =>
    linearZero?(- inv(leadingCoefficient(toTest)) * coefficient(toTest,0),
        rootChar)
toTest := monomial(1,degree(toTest))+
    inv(leadingCoefficient(toTest))*reductum(toTest)
delta := gcd(toTest,rootChar.defPol)
degree(delta) = 0 => false

```

```

sign(delta.(rootChar.low) * delta.(rootChar.high)) <= 0

refine!(rootChar) ==
  -- this is not a safe function, it can work with badly created object
  -- we do not assume (rootChar.defPol).(rootChar.high) <> 0
  int := middle(rootChar)
  s1 := sign((rootChar.defPol).(rootChar.low))
  zero?(s1) =>
    rootChar.high := int
    rootChar.defPol := monomial(1,1) - (rootChar.low)::P
    rootChar
  s2 := sign((rootChar.defPol).int)
  zero?(s2) =>
    rootChar.low := int
    rootChar.defPol := monomial(1,1) - int::P
    rootChar
  if (s1*s2 < 0)
  then
    rootChar.high := int
  else
    rootChar.low := int
  rootChar

refine(rootChar) ==
  -- we assume (rootChar.defPol).(rootChar.high) <> 0
  int := middle(rootChar)
  s := (rootChar.defPol).int * (rootChar.defPol).(rootChar.high)
  zero?(s) => [int, rootChar.high, monomial(1,1)-int::P]
  if s < 0
  then
    [int, rootChar.high, rootChar.defPol]
  else
    [rootChar.low, int, rootChar.defPol]

left(rootChar) == rootChar.low

right(rootChar) == rootChar.high

middle(rootChar) == (rootChar.low + rootChar.high)/(2::TheField)

-- varStar(p) == -- if 0 no roots in [0,:infy[
--   res : N := 0
--   lsg := sign(coefficient(p,0))
--   l := [ sign(i) for i in reverse!(coefficients(p))]
--   for sg in l repeat
--     if (sg ^= lsg) then res := res + 1
--     lsg := sg
--   res

```

```
"ROIRC" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ROIRC"]
"RRCC" [color="#4488FF",href="bookvol10.2.pdf#nameddest=RRCC"]
"ROIRC" -> "RRCC"
```

— RomanNumeral.input —

[illegible]

```

--S 4 of 15
x : UTS(ROMAN,'x,0) := x
--R
--R
--R (4) x
--R
--R                                     Type: UnivariateTaylorSeries(RomanNumeral,x,0)
--E 4

--S 5 of 15
recip(1 - x - x**2)
--R
--R
--R (5)
--R          2      3      4      5      6      7      8
--R      I + x + II x + III x + V x + VIII x + XIII x + XXI x + XXXIV x
--R      +
--R          9      10      11
--R      LV x + LXXXIX x + O(x )
--R
--R                                     Type: Union(UnivariateTaylorSeries(RomanNumeral,x,0),...)
--E 5

--S 6 of 15
m : MATRIX FRAC ROMAN
--R
--R
--R                                     Type: Void
--E 6

--S 7 of 15
m := matrix [ [1/(i + j) for i in 1..3] for j in 1..3]
--R
--R
--R      + I      I      I+
--R      |-- --- --|
--R      |II  III IV|
--R      |      |
--R      | I      I      I |
--R (7) |-- -- - |
--R      |III IV  V |
--R      |      |
--R      | I      I      I|
--R      |-- - --|
--R      +IV  V  VI+
--R
--R                                     Type: Matrix Fraction RomanNumeral
--E 7

--S 8 of 15
inverse m
--R
--R

```

[illegible]

```

--S 14 of 15
a * b
--R
--R
--R (14) MMMMMDCCLXXXVI
--R
--R                                          Type: RomanNumeral
--E 14

--S 15 of 15
b rem a
--R
--R
--R (15) IX
--R
--R                                          Type: RomanNumeral
--E 15
)spool
)lisp (bye)

```

---

— RomanNumeral.help —

=====

RomanNumeral Examples

=====

The Roman numeral package was added to Axiom in MCMLXXXVI for use in denoting higher order derivatives.

For example, let f be a symbolic operator.

```

f := operator 'f
f

```

Type: BasicOperator

This is the seventh derivative of f with respect to x.

```

D(f x,x,7)
(vii)
f      (x)

```

Type: Expression Integer

You can have integers printed as Roman numerals by declaring variables to be of type RomanNumeral (abbreviation ROMAN).

```

a := roman(1978 - 1965)
XIII

```

Type: RomanNumeral

This package now has a small but devoted group of followers that claim



this domain has shown its efficacy in many other contexts. They claim that Roman numerals are every bit as useful as ordinary integers.

In a sense, they are correct, because Roman numerals form a ring and you can therefore construct polynomials with Roman numeral coefficients, matrices over Roman numerals, etc..

```
x : UTS(ROMAN,'x,0) := x
x
Type: UnivariateTaylorSeries(RomanNumeral,x,0)
```

Was Fibonacci Italian or ROMAN?

```
recip(1 - x - x**2)
      2      3      4      5      6      7      8
I + x + II x + III x + V x + VIII x + XIII x + XXI x + XXXIV x
+
      9      10      11
LV x + LXXXIX x + 0(x )
Type: Union(UnivariateTaylorSeries(RomanNumeral,x,0),...)
```

You can also construct fractions with Roman numeral numerators and denominators, as this matrix Hilberticus illustrates.

```
m : MATRIX FRAC ROMAN
Type: Void

m := matrix [ [1/(i + j) for i in 1..3] for j in 1..3]
+ I      I      I+
|--  ---  --|
|II   III  IV|
|      |      |
| I    I    I |
|---  --  - |
|III  IV  V |
|      |      |
| I    I    I |
|--  -  --|
+IV    V    VI+
Type: Matrix Fraction RomanNumeral
```

Note that the inverse of the matrix has integral ROMAN entries.

```
inverse m
+LXXII  - CCXL  CLXXX +
|      |      |
|- CCXL  CM    - DCCXX|
|      |      |
+CLXXX  - DCCXX  DC    +
Type: Union(Matrix Fraction RomanNumeral,...)
```

Unfortunately, the spoil-sports say that the fun stops when the numbers get big---mostly because the Romans didn't establish conventions about representing very large numbers.

```
y := factorial 10
3628800
Type: PositiveInteger
```

You work it out!

```
roman y
((((I))))((((I))))((((I)))) ((I))((I))((I))((I))((I))((I))((I)) ((I)) (
(I)) MMMMMMMDCCC
Type: RomanNumeral
```

Issue the system command `)show RomanNumeral` to display the full list of operations defined by `RomanNumeral`.

```
a := roman(78)
LXXVIII
Type: RomanNumeral
```

```
b := roman(87)
LXXXVII
Type: RomanNumeral
```

```
a + b
CLXV
Type: RomanNumeral
```

```
a * b
MMMMMDCCLXXXVI
Type: RomanNumeral
```

```
b rem a
IX
Type: RomanNumeral
```

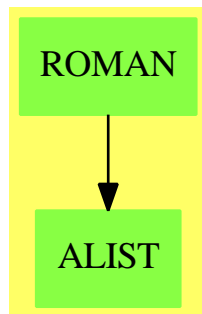
See Also:

- o )help Integer
- o )help Complex
- o )help Factored
- o )help Records
- o )help Fraction
- o )help RadixExpansion
- o )help HexadecimalExpansion
- o )help BinaryExpansion
- o )help DecimalExpansion

```
o )help IntegerNumberTheoryFunctions
o )show RomanNumeral
```

---

### 19.12.1 RomanNumeral (ROMAN)



**See**

⇒ “Integer” (INT) 10.30.1 on page 1325  
⇒ “NonNegativeInteger” (NNI) 15.5.1 on page 1702  
⇒ “PositiveInteger” (PI) 17.28.1 on page 2060

**Exports:**

|                   |                   |                    |                |
|-------------------|-------------------|--------------------|----------------|
| 0                 | 1                 | abs                | addmod         |
| associates?       | base              | binomial           | bit?           |
| characteristic    | coerce            | convert            | copy           |
| D                 | dec               | differentiate      | divide         |
| euclideanSize     | even?             | expressIdealMember | exquo          |
| extendedEuclidean | extendedEuclidean | factor             | factorial      |
| gcd               | gcdPolynomial     | hash               | inc            |
| init              | invmod            | latex              | lcm            |
| length            | mask              | max                | min            |
| mulmod            | multiEuclidean    | negative?          | nextItem       |
| odd?              | one?              | patternMatch       | permutation    |
| positive?         | positiveRemainder | powmod             | prime?         |
| principalIdeal    | random            | rational           | rational?      |
| rationalIfCan     | recip             | reducedSystem      | retract        |
| retractIfCan      | roman             | sample             | shift          |
| sign              | sizeLess?         | squareFree         | squareFreePart |
| submod            | subtractIfCan     | symmetricRemainder | unit?          |
| unitCanonical     | unitNormal        | zero?              | ?*?            |
| ?**?              | ?+?               | ?-?                | -?             |
| ?<?               | ?<=?              | ?=?                | ?>?            |
| ?>=?              | ?^?               | ?~=?               | ?quo?          |
| ?rem?             |                   |                    |                |

— domain ROMAN RomanNumeral —

```
)abbrev domain ROMAN RomanNumeral
++ Author: Mark Botch
++ Date Created:
++ Change History:
++ Related Constructors:
++ Keywords: roman numerals
++ Description:
++ \spadtype{RomanNumeral} provides functions for converting
++ integers to roman numerals.
```

```
RomanNumeral(): IntegerNumberSystem with
  canonical
    ++ mathematical equality is data structure equality.
  canonicalsClosed
    ++ two positives multiply to give positive.
  noetherian
    ++ ascending chain condition on ideals.
  convert: Symbol -> %
    ++ convert(n) creates a roman numeral for symbol n.
  roman : Symbol -> %
    ++ roman(n) creates a roman numeral for symbol n.
  roman : Integer -> %
    ++ roman(n) creates a roman numeral for n.
```

```

== Integer add
import NumberFormats()

roman(n:Integer) == n::%
roman(sy:Symbol) == convert sy
convert(sy:Symbol):% == ScanRoman(string sy)::%

coerce(r:%):OutputForm ==
  n := convert(r)@Integer
  -- okay, we stretch it
  zero? n => n::OutputForm
  negative? n => -((-r)::OutputForm)
  FormatRoman(n::PositiveInteger)::Symbol::OutputForm

-----

— ROMAN.dotabb —

"ROMAN" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ROMAN"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"ROMAN" -> "ALIST"

-----

```

### 19.13 domain ROUTINE RoutinesTable

```

— RoutinesTable.input —

)set break resume
)sys rm -f RoutinesTable.output
)spool RoutinesTable.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show RoutinesTable
--R RoutinesTable is a domain constructor
--R Abbreviation for RoutinesTable is ROUTINE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ROUTINE
--R
--R----- Operations -----

```

```

--R concat : (%,%) -> %
--R deleteRoutine! : (%,Symbol) -> %
--R elt : (%,Symbol,Any) -> Any
--R empty : () -> %
--R entries : % -> List Any
--R getMeasure : (%,Symbol) -> Float
--R indices : % -> List Symbol
--R keys : % -> List Symbol
--R qelt : (%,Symbol) -> Any
--R sample : () -> %
--R selectNonFiniteRoutines : % -> %
--R selectPDERoutines : % -> %
--R showTheRoutinesTable : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?? : (%,%) -> Boolean if Any has SETCAT or Record(key: Symbol,entry: Any) has SETCAT
--R any? : ((Any -> Boolean),%) -> Boolean if $ has finiteAggregate
--R any? : ((Record(key: Symbol,entry: Any) -> Boolean),%) -> Boolean if $ has finiteAggregate
--R bag : List Record(key: Symbol,entry: Any) -> %
--R changeMeasure : (%,Symbol,Float) -> %
--R changeThreshold : (%,Symbol,Float) -> %
--R coerce : % -> OutputForm if Any has SETCAT or Record(key: Symbol,entry: Any) has SETCAT
--R construct : List Record(key: Symbol,entry: Any) -> %
--R convert : % -> InputForm if Record(key: Symbol,entry: Any) has KONVERT INFORM
--R count : ((Any -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R count : (Any,%) -> NonNegativeInteger if $ has finiteAggregate and Any has SETCAT
--R count : (Record(key: Symbol,entry: Any),%) -> NonNegativeInteger if $ has finiteAggregate and Record
--R count : ((Record(key: Symbol,entry: Any) -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R dictionary : List Record(key: Symbol,entry: Any) -> %
--R entry? : (Any,%) -> Boolean if $ has finiteAggregate and Any has SETCAT
--R eval : (%,List Equation Any) -> % if Any has EVALAB ANY and Any has SETCAT
--R eval : (%,Equation Any) -> % if Any has EVALAB ANY and Any has SETCAT
--R eval : (%,Any,Any) -> % if Any has EVALAB ANY and Any has SETCAT
--R eval : (%,List Any,List Any) -> % if Any has EVALAB ANY and Any has SETCAT
--R eval : (%,List Record(key: Symbol,entry: Any),List Record(key: Symbol,entry: Any)) -> % if Record(ke
--R eval : (%,Record(key: Symbol,entry: Any),Record(key: Symbol,entry: Any)) -> % if Record(key: Symbol,
--R eval : (%,Equation Record(key: Symbol,entry: Any)) -> % if Record(key: Symbol,entry: Any) has EVALAB
--R eval : (%,List Equation Record(key: Symbol,entry: Any)) -> % if Record(key: Symbol,entry: Any) has E
--R every? : ((Any -> Boolean),%) -> Boolean if $ has finiteAggregate
--R every? : ((Record(key: Symbol,entry: Any) -> Boolean),%) -> Boolean if $ has finiteAggregate
--R extract! : % -> Record(key: Symbol,entry: Any)
--R fill! : (%,Any) -> % if $ has shallowlyMutable
--R find : ((Record(key: Symbol,entry: Any) -> Boolean),%) -> Union(Record(key: Symbol,entry: Any),"fail
--R first : % -> Any if Symbol has ORDSET
--R getExplanations : (%,String) -> List String
--R hash : % -> SingleInteger if Any has SETCAT or Record(key: Symbol,entry: Any) has SETCAT
--R insert! : (Record(key: Symbol,entry: Any),%) -> %
--R inspect : % -> Record(key: Symbol,entry: Any)
--R latex : % -> String if Any has SETCAT or Record(key: Symbol,entry: Any) has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map : (((Any,Any) -> Any),%,%) -> %
--R copy : % -> %
--R dictionary : () -> %
--R ?.? : (%,Symbol) -> Any
--R empty? : % -> Boolean
--R eq? : (%,%) -> Boolean
--R index? : (Symbol,%) -> Boolean
--R key? : (Symbol,%) -> Boolean
--R map : ((Any -> Any),%) -> %
--R routines : () -> %
--R selectFiniteRoutines : % -> %
--R selectODEIVPRoutines : % -> %
--R setelt : (%,Symbol,Any) -> Any
--R table : () -> %

```

```

--R map : ((Record(key: Symbol,entry: Any) -> Record(key: Symbol,entry: Any)),%) -> %
--R map! : ((Any -> Any),%) -> % if $ has shallowlyMutable
--R map! : ((Record(key: Symbol,entry: Any) -> Record(key: Symbol,entry: Any)),%) -> % if $ has shallowlyMutable
--R maxIndex : % -> Symbol if Symbol has ORDSET
--R member? : (Any,%) -> Boolean if $ has finiteAggregate and Any has SETCAT
--R member? : (Record(key: Symbol,entry: Any),%) -> Boolean if $ has finiteAggregate and Record(key: Symbol,entry: Any) has SETCAT
--R members : % -> List Any if $ has finiteAggregate
--R members : % -> List Record(key: Symbol,entry: Any) if $ has finiteAggregate
--R minIndex : % -> Symbol if Symbol has ORDSET
--R more? : (% , NonNegativeInteger) -> Boolean
--R parts : % -> List Any if $ has finiteAggregate
--R parts : % -> List Record(key: Symbol,entry: Any) if $ has finiteAggregate
--R qsetelt! : (% , Symbol, Any) -> Any if $ has shallowlyMutable
--R recoverAfterFail : (% , String, Integer) -> Union(String, "failed")
--R reduce : (((Record(key: Symbol,entry: Any), Record(key: Symbol,entry: Any)) -> Record(key: Symbol,entry: Any)),%) -> Record(key: Symbol,entry: Any)
--R reduce : (((Record(key: Symbol,entry: Any), Record(key: Symbol,entry: Any)) -> Record(key: Symbol,entry: Any)),%) -> Record(key: Symbol,entry: Any)
--R reduce : (((Record(key: Symbol,entry: Any), Record(key: Symbol,entry: Any)) -> Record(key: Symbol,entry: Any)),%) -> Record(key: Symbol,entry: Any)
--R remove : ((Record(key: Symbol,entry: Any) -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (Record(key: Symbol,entry: Any),%) -> % if $ has finiteAggregate and Record(key: Symbol,entry: Any) has SETCAT
--R remove! : (Symbol,%) -> Union(Any, "failed")
--R remove! : ((Record(key: Symbol,entry: Any) -> Boolean),%) -> % if $ has finiteAggregate
--R remove! : (Record(key: Symbol,entry: Any),%) -> % if $ has finiteAggregate
--R removeDuplicates : % -> % if $ has finiteAggregate and Record(key: Symbol,entry: Any) has SETCAT
--R search : (Symbol,%) -> Union(Any, "failed")
--R select : ((Record(key: Symbol,entry: Any) -> Boolean),%) -> % if $ has finiteAggregate
--R select! : ((Record(key: Symbol,entry: Any) -> Boolean),%) -> % if $ has finiteAggregate
--R selectIntegrationRoutines : % -> %
--R selectMultiDimensionalRoutines : % -> %
--R selectOptimizationRoutines : % -> %
--R selectSumOfSquaresRoutines : % -> %
--R size? : (% , NonNegativeInteger) -> Boolean
--R swap! : (% , Symbol, Symbol) -> Void if $ has shallowlyMutable
--R table : List Record(key: Symbol,entry: Any) -> %
--R ~=? : (% , %) -> Boolean if Any has SETCAT or Record(key: Symbol,entry: Any) has SETCAT
--R
--E 1

)spool
)lisp (bye)

```

---

— RoutinesTable.help —

```

=====
RoutinesTable examples
=====

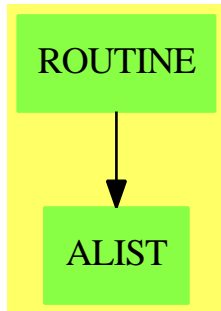
```

See Also:

o )show RoutinesTable

—————▶

### 19.13.1 RoutinesTable (ROUTINE)



**Exports:**



|                                |                         |
|--------------------------------|-------------------------|
| any?                           | bag                     |
| changeMeasure                  | changeThreshold         |
| coerce                         | concat                  |
| construct                      | convert                 |
| copy                           | count                   |
| deleteRoutine!                 | dictionary              |
| elt                            | empty                   |
| empty?                         | entries                 |
| entry?                         | eq?                     |
| eval                           | every?                  |
| extract!                       | fill!                   |
| find                           | first                   |
| getExplanations                | getMeasure              |
| hash                           | index?                  |
| indices                        | insert!                 |
| inspect                        | key?                    |
| keys                           | latex                   |
| less?                          | map                     |
| map!                           | maxIndex                |
| member?                        | members                 |
| minIndex                       | more?                   |
| parts                          | qelt                    |
| qsetelt!                       | recoverAfterFail        |
| reduce                         | remove                  |
| remove!                        | removeDuplicates        |
| routines                       | sample                  |
| search                         | select                  |
| select!                        | selectFiniteRoutines    |
| selectIntegrationRoutines      | selectNonFiniteRoutines |
| selectMultiDimensionalRoutines | selectODEIVPRoutines    |
| selectOptimizationRoutines     | selectPDERoutines       |
| selectSumOfSquaresRoutines     | setelt                  |
| showTheRoutinesTable           | size?                   |
| swap!                          | table                   |
| #?                             | ?=?                     |
| ?~=?                           | ?..?                    |

— domain ROUTINE RoutinesTable —

```

)abbrev domain ROUTINE RoutinesTable
++ Author: Brian Dupee
++ Date Created: August 1994
++ Date Last Updated: December 1997
++ Basic Operations: routines, getMeasure
++ Related Constructors: TableAggregate(Symbol,Any)
++ Description:
++ \axiomType{RoutinesTable} implements a database and associated tuning

```

```

++ mechanisms for a set of known NAG routines

RoutinesTable(): E == I where
  F ==> Float
  ST ==> String
  LST ==> List String
  Rec ==> Record(key:Symbol,entry:Any)
  RList ==> List(Record(key:Symbol,entry:Any))
  IFL ==> List(Record(iffail:Integer,instruction:ST))
  Entry ==> Record(chapter:ST, type:ST, domainName: ST,
                    defaultMin:F, measure:F, failList:IFL, explList:LST)

E ==> TableAggregate(Symbol,Any) with

  concat:(%,%) -> %
    ++ concat(x,y) merges two tables x and y
  routines:() -> %
    ++ routines() initialises a database of known NAG routines
  selectIntegrationRoutines:% -> %
    ++ selectIntegrationRoutines(R) chooses only those routines from
    ++ the database which are for integration
  selectOptimizationRoutines:% -> %
    ++ selectOptimizationRoutines(R) chooses only those routines from
    ++ the database which are for integration
  selectPDERoutines:% -> %
    ++ selectPDERoutines(R) chooses only those routines from the
    ++ database which are for the solution of PDE's
  selectODEIVPRoutines:% -> %
    ++ selectODEIVPRoutines(R) chooses only those routines from the
    ++ database which are for the solution of ODE's
  selectFiniteRoutines:% -> %
    ++ selectFiniteRoutines(R) chooses only those routines from the
    ++ database which are designed for use with finite expressions
  selectSumOfSquaresRoutines:% -> %
    ++ selectSumOfSquaresRoutines(R) chooses only those routines from the
    ++ database which are designed for use with sums of squares
  selectNonFiniteRoutines:% -> %
    ++ selectNonFiniteRoutines(R) chooses only those routines from the
    ++ database which are designed for use with non-finite expressions.
  selectMultiDimensionalRoutines:% -> %
    ++ selectMultiDimensionalRoutines(R) chooses only those routines from
    ++ the database which are designed for use with multi-dimensional
    ++ expressions
  changeThreshold:(%,Symbol,F) -> %
    ++ changeThreshold(R,s,newValue) changes the value below which,
    ++ given a NAG routine generating a higher measure, the routines will
    ++ make no attempt to generate a measure.
  changeMeasure:(%,Symbol,F) -> %
    ++ changeMeasure(R,s,newValue) changes the maximum value for a
    ++ measure of the given NAG routine.

```

```

getMeasure:(%,Symbol) -> F
  ++ getMeasure(R,s) gets the current value of the maximum measure for
  ++ the given NAG routine.
getExplanations:(%,ST) -> LST
  ++ getExplanations(R,s) gets the explanations of the output parameters for
  ++ the given NAG routine.
deleteRoutine!:(%,Symbol) -> %
  ++ deleteRoutine!(R,s) destructively deletes the given routine from
  ++ the current database of NAG routines
showTheRoutinesTable:() -> %
  ++ showTheRoutinesTable() returns the current table of NAG routines.
recoverAfterFail:(%,ST,Integer) -> Union(ST,"failed")
  ++ recoverAfterFail(routs,routineName,ifailValue) acts on the
  ++ instructions given by the ifail list
finiteAggregate

```

```

I ==> Result add

```

```

Rep := Result
import Rep

```

```

theRoutinesTable:% := routines()

```

```

showTheRoutinesTable():% == theRoutinesTable

```

```

integrationRoutine?(r:Record(key:Symbol,entry:Any)):Boolean ==
  (a := retractIfCan(r.entry)$AnyFunctions1(Entry)) case Entry =>
    elt(a,chapter) = "Integration"
  false

```

```

selectIntegrationRoutines(R:%):% == select(integrationRoutine?,R)

```

```

optimizationRoutine?(r:Record(key:Symbol,entry:Any)):Boolean ==
  (a := retractIfCan(r.entry)$AnyFunctions1(Entry)) case Entry =>
    elt(a,chapter) = "Optimization"
  false

```

```

selectOptimizationRoutines(R:%):% == select(optimizationRoutine?,R)

```

```

PDERoutine?(r:Record(key:Symbol,entry:Any)):Boolean ==
  (a := retractIfCan(r.entry)$AnyFunctions1(Entry)) case Entry =>
    elt(a,chapter) = "PDE"
  false

```

```

selectPDERoutines(R:%):% == select(PDERoutine?,R)

```

```

ODERoutine?(r:Record(key:Symbol,entry:Any)):Boolean ==
  (a := retractIfCan(r.entry)$AnyFunctions1(Entry)) case Entry =>
    elt(a,chapter) = "ODE"
  false

```

```

selectODEIVPRoutines(R:%):% == select(ODERoutine?,R)

sumOfSquaresRoutine?(r:Record(key:Symbol,entry:Any)):Boolean ==
  (a := retractIfCan(r.entry)$AnyFunctions1(Entry)) case Entry =>
    elt(a,type) = "SS"
  false

selectSumOfSquaresRoutines(R:%):% == select(sumOfSquaresRoutine?,R)

finiteRoutine?(r:Record(key:Symbol,entry:Any)):Boolean ==
  (a := retractIfCan(r.entry)$AnyFunctions1(Entry)) case Entry =>
    elt(a,type) = "One-dimensional finite"
  false

selectFiniteRoutines(R:%):% == select(finiteRoutine?,R)

infiniteRoutine?(r:Record(key:Symbol,entry:Any)):Boolean ==
  (a := retractIfCan(r.entry)$AnyFunctions1(Entry)) case Entry =>
    elt(a,type) = "One-dimensional infinite"
  false

semiInfiniteRoutine?(r:Record(key:Symbol,entry:Any)):Boolean ==
  (a := retractIfCan(r.entry)$AnyFunctions1(Entry)) case Entry =>
    elt(a,type) = "One-dimensional semi-infinite"
  false

nonFiniteRoutine?(r:Record(key:Symbol,entry:Any)):Boolean ==
  (semiInfiniteRoutine?(r) or infiniteRoutine?(r))

selectNonFiniteRoutines(R:%):% == select(nonFiniteRoutine?,R)

multiDimensionalRoutine?(r:Record(key:Symbol,entry:Any)):Boolean ==
  (a := retractIfCan(r.entry)$AnyFunctions1(Entry)) case Entry =>
    elt(a,type) = "Multi-dimensional"
  false

selectMultiDimensionalRoutines(R:%):% == select(multiDimensionalRoutine?,R)

concat(a:%,b:%):% ==
  membersOfa := (members(a)@List(Record(key:Symbol,entry:Any)))
  membersOfb := (members(b)@List(Record(key:Symbol,entry:Any)))
  allMembers:=
    concat(membersOfa,membersOfb)$List(Record(key:Symbol,entry:Any))
  construct(allMembers)

changeThreshold(R:%,s:Symbol,newValue:F):% ==
  (a := search(s,R)) case Any =>
    e := retract(a)$AnyFunctions1(Entry)
    e.defaultMin := newValue

```

```

    a := coerce(e)$AnyFunctions1(Entry)
    insert!([s,a],R)
    error("changeThreshold","Cannot find routine of that name")$ErrorFunctions

changeMeasure(R:%,s:Symbol,newValue:F):% ==
(a := search(s,R)) case Any =>
    e := retract(a)$AnyFunctions1(Entry)
    e.measure := newValue
    a := coerce(e)$AnyFunctions1(Entry)
    insert!([s,a],R)
    error("changeMeasure","Cannot find routine of that name")$ErrorFunctions

getMeasure(R:%,s:Symbol):F ==
(a := search(s,R)) case Any =>
    e := retract(a)$AnyFunctions1(Entry)
    e.measure
    error("getMeasure","Cannot find routine of that name")$ErrorFunctions

deleteRoutine!(R:%,s:Symbol):% ==
(a := search(s,R)) case Any =>
    e:Record(key:Symbol,entry:Any) := [s,a]
    remove!(e,R)
    error("deleteRoutine!","Cannot find routine of that name")$ErrorFunctions

routines():% ==
f := "One-dimensional finite"
s := "One-dimensional semi-infinite"
i := "One-dimensional infinite"
m := "Multi-dimensional"
int := "Integration"
ode := "ODE"
pde := "PDE"
opt := "Optimization"
d01ajfExplList:LST := ["result: Calculated value of the integral",
    "iw: iw(1) contains the actual number of sub-intervals used",
    "w: contains the end-points of the sub-intervals used along with",
    "abserr: the estimate of the absolute error of the result",
    "ifail: the error warning parameter",
    "method: details of the method used and measures of accuracy",
    "attributes: a list of the attributes pertaining to the result"]
d01asfExplList:LST := ["result: Calculated value of the integral",
    "iw: iw(1) contains the actual number of sub-intervals used",
    "lst: contains the actual number of sub-intervals used",
    "erlst: contains the error estimates over the sub-intervals",
    "rslst: contains the integral contributions of the sub-intervals",
    "ierlst: contains the error flags corresponding to the sub-intervals",
    "abserr: the estimate of the absolute error of the result",
    "ifail: the error warning parameter",
    "method: details of the method used and measures of accuracy",
    "attributes: a list of the attributes pertaining to the result"]

```

```

d01fcfExplList:LST := ["result: Calculated value of the integral",
    "acc: the estimate of the relative error of the result",
    "minpts: the number of integrand evaluations",
    "ifail: the error warning parameter",
    "method: details of the method used and measures of all methods",
    "attributes: a list of the attributes pertaining to the integrand"]
d01transExplList:LST := ["result: Calculated value of the integral",
    "abserr: the estimate of the absolute error of the result",
    "method: details of the method and transformation used and measures",
    "d01***AnnaTypeAnswer: the individual results from the routines",
    "attributes: a list of the attributes pertaining to the integrand"]
d02bhfExplList:LST := ["x: the value of x at the end of the calculation",
    "y: the computed values of Y[1\].Y[n\] at x",
    "tol: the (possible) estimate of the error; this is not guaranteed",
    "ifail: the error warning parameter",
    "method: details of the method used and measures of all methods",
    "intensityFunctions: a list of the attributes and values pertaining"]
d02bbfExplList:LST := concat(["result: the computed values of the solution at the required points"]
d03eefExplList:LST := ["See the NAG On-line Documentation for D03EEF/D03EDF",
    "u: the computed solution u[i][j] is returned in u(i+(j-1)*ngx),for i"]
e04fdfExplList:LST := ["x: the position of the minimum",
    "objf: the value of the objective function at x",
    "ifail: the error warning parameter",
    "method: details of the method used and measures of all methods",
    "attributes: a list of the attributes pertaining to the function"]
e04dggExplList:LST := concat(e04fdfExplList,
    ["objgrd: the values of the derivatives at x",
    "iter: the number of iterations performed"])$LST
e04jafExplList:LST := concat(e04fdfExplList,
    ["bu: the values of the upper bounds used",
    "bl: the values of the lower bounds used"])$LST
e04ucfExplList:LST := concat(e04dggExplList,
    ["istate: the status of every constraint at x",
    "clamda: the QP multipliers for the last QP sub-problem",
    "For other output parameters see the NAG On-line Documentation"])
e04mbfExplList:LST := concat(e04fdfExplList,
    ["istate: the status of every constraint at x",
    "clamda: the Lagrange multipliers for each constraint"])$LST
d01ajfIfail:IFL := [[1,"incrFunEvals"], [2,"delete"], [3,"delete"], [4,"delete"],
    [5,"delete"], [6,"delete"]]
d01akfIfail:IFL := [[1,"incrFunEvals"], [2,"delete"], [3,"delete"], [4,"delete"]]
d01alfIfail:IFL := [[1,"incrFunEvals"], [2,"delete"], [3,"delete"], [4,"delete"],
    [5,"delete"], [6,"delete"], [7,"delete"]]
d01amfIfail:IFL := [[1,"incrFunEvals"], [2,"delete"], [3,"delete"], [4,"delete"],
    [5,"delete"], [6,"delete"]]
d01anfIfail:IFL := [[1,"incrFunEvals"], [2,"delete"], [3,"delete"], [4,"delete"],
    [5,"delete"], [6,"delete"], [7,"delete"]]
d01apfIfail:IFL :=
    [[1,"incrFunEvals"], [2,"delete"], [3,"delete"], [4,"delete"], [5,"delete"]]
d01aqfIfail:IFL :=

```

```

[[1,"incrFunEvals"], [2,"delete"], [3,"delete"], [4,"delete"], [5,"delete"]]
d01asfIfail:IFL := [[1,"incrFunEvals"], [2,"delete"], [3,"delete"], [4,"delete"],
    [5,"delete"], [6,"delete"], [7,"delete"], [8,"delete"], [9,"delete"]]
d01fcfIfail:IFL := [[1,"delete"], [2,"incrFunEvals"], [3,"delete"]]
d01gbfIfail:IFL := [[1,"delete"], [2,"incrFunEvals"]]
d02bbfIfail:IFL :=
    [[1,"delete"], [2,"decrease tolerance"], [3,"increase tolerance"],
    [4,"delete"], [5,"delete"], [6,"delete"], [7,"delete"]]
d02bhfIfail:IFL :=
    [[1,"delete"], [2,"decrease tolerance"], [3,"increase tolerance"],
    [4,"no action"], [5,"delete"], [6,"delete"], [7,"delete"]]
d02cjfIfail:IFL :=
    [[1,"delete"], [2,"decrease tolerance"], [3,"increase tolerance"],
    [4,"delete"], [5,"delete"], [6,"no action"], [7,"delete"]]
d02ejfIfail:IFL :=
    [[1,"delete"], [2,"decrease tolerance"], [3,"increase tolerance"],
    [4,"delete"], [5,"delete"], [6,"no action"], [7,"delete"], [8,"delete"],
    [9,"delete"]]
e04dgmIfail:IFL := [[3,"delete"], [4,"no action"], [6,"delete"],
    [7,"delete"], [8,"delete"], [9,"delete"]]
e04fdfIfail:IFL :=
    [[1,"delete"], [2,"delete"], [3,"delete"], [4,"delete"],
    [5,"no action"], [6,"no action"], [7,"delete"], [8,"delete"]]
e04gcfIfail:IFL := [[1,"delete"], [2,"delete"], [3,"delete"], [4,"delete"],
    [5,"no action"], [6,"no action"], [7,"delete"], [8,"delete"], [9,"delete"]]
e04jafIfail:IFL := [[1,"delete"], [2,"delete"], [3,"delete"], [4,"delete"],
    [5,"no action"], [6,"no action"], [7,"delete"], [8,"delete"], [9,"delete"]]
e04mbfIfail:IFL :=
    [[1,"delete"], [2,"delete"], [3,"delete"], [4,"delete"], [5,"delete"]]
e04nafIfail:IFL :=
    [[1,"delete"], [2,"delete"], [3,"delete"], [4,"delete"], [5,"delete"],
    [6,"delete"], [7,"delete"], [8,"delete"], [9,"delete"]]
e04ucfIfail:IFL := [[1,"delete"], [2,"delete"], [3,"delete"], [4,"delete"],
    [5,"delete"], [6,"delete"], [7,"delete"], [8,"delete"], [9,"delete"]]
d01ajfEntry:Entry := [int, f, "d01ajfAnnaType",0.4,0.4,d01ajfIfail,d01ajfExplList]
d01akfEntry:Entry := [int, f, "d01akfAnnaType",0.6,1.0,d01akfIfail,d01ajfExplList]
d01alfEntry:Entry := [int, f, "d01alfAnnaType",0.6,0.6,d01alfIfail,d01ajfExplList]
d01amfEntry:Entry := [int, i, "d01amfAnnaType",0.5,0.5,d01amfIfail,d01ajfExplList]
d01anfEntry:Entry := [int, f, "d01anfAnnaType",0.6,0.9,d01anfIfail,d01ajfExplList]
d01apfEntry:Entry := [int, f, "d01apfAnnaType",0.7,0.7,d01apfIfail,d01ajfExplList]
d01aqfEntry:Entry := [int, f, "d01aqfAnnaType",0.6,0.7,d01aqfIfail,d01ajfExplList]
d01asfEntry:Entry := [int, s, "d01asfAnnaType",0.6,0.9,d01asfIfail,d01asfExplList]
d01transEntry:Entry:= [int, i, "d01TransformFunctionType",0.6,0.9,[],d01transExplList]
d01gbfEntry:Entry := [int, m, "d01gbfAnnaType",0.6,0.6,d01gbfIfail,d01fcfExplList]
d01fcfEntry:Entry := [int, m, "d01fcfAnnaType",0.5,0.5,d01fcfIfail,d01fcfExplList]
d02bbfEntry:Entry := [ode, "IVP", "d02bbfAnnaType",0.7,0.5,d02bbfIfail,d02bbfExplList]
d02bhfEntry:Entry := [ode, "IVP", "d02bhfAnnaType",0.7,0.49,d02bhfIfail,d02bhfExplList]
d02cjfEntry:Entry := [ode, "IVP", "d02cjfAnnaType",0.7,0.5,d02cjfIfail,d02bbfExplList]
d02ejfEntry:Entry := [ode, "IVP", "d02ejfAnnaType",0.7,0.5,d02ejfIfail,d02bbfExplList]
d03eefEntry:Entry := [pde, "2", "d03eefAnnaType",0.6,0.5,[],d03eefExplList]

```

```

--d03fafEntry:Entry := [pde, "3", "d03fafAnnaType",0.6,0.5,[],[]]
e04dggEntry:Entry := [opt, "CGA", "e04dggAnnaType",0.4,0.4,e04dggIfail,e04dggExplList]
e04fdfEntry:Entry := [opt, "SS", "e04fdfAnnaType",0.7,0.7,e04fdfIfail,e04fdfExplList]
e04gcfEntry:Entry := [opt, "SS", "e04gcfAnnaType",0.8,0.8,e04gcfIfail,e04gcfExplList]
e04jafEntry:Entry := [opt, "QNA", "e04jafAnnaType",0.5,0.5,e04jafIfail,e04jafExplList]
e04mbfEntry:Entry := [opt, "LP", "e04mbfAnnaType",0.7,0.7,e04mbfIfail,e04mbfExplList]
e04nafEntry:Entry := [opt, "QP", "e04nafAnnaType",0.7,0.7,e04nafIfail,e04nafExplList]
e04ucfEntry:Entry := [opt, "SQP", "e04ucfAnnaType",0.6,0.6,e04ucfIfail,e04ucfExplList]
rl:RList :=
  [{"d01apf" :: Symbol, coerce(d01apfEntry)$AnyFunctions1(Entry)],_
  [{"d01aqf" :: Symbol, coerce(d01aqfEntry)$AnyFunctions1(Entry)],_
  [{"d01alf" :: Symbol, coerce(d01alfEntry)$AnyFunctions1(Entry)],_
  [{"d01anf" :: Symbol, coerce(d01anfEntry)$AnyFunctions1(Entry)],_
  [{"d01akf" :: Symbol, coerce(d01akfEntry)$AnyFunctions1(Entry)],_
  [{"d01ajf" :: Symbol, coerce(d01ajfEntry)$AnyFunctions1(Entry)],_
  [{"d01asf" :: Symbol, coerce(d01asfEntry)$AnyFunctions1(Entry)],_
  [{"d01amf" :: Symbol, coerce(d01amfEntry)$AnyFunctions1(Entry)],_
  [{"d01transform" :: Symbol, coerce(d01transEntry)$AnyFunctions1(Entry)],_
  [{"d01gbf" :: Symbol, coerce(d01gbfEntry)$AnyFunctions1(Entry)],_
  [{"d01fcf" :: Symbol, coerce(d01fcfEntry)$AnyFunctions1(Entry)],_
  [{"d02bbf" :: Symbol, coerce(d02bbfEntry)$AnyFunctions1(Entry)],_
  [{"d02bhf" :: Symbol, coerce(d02bhfEntry)$AnyFunctions1(Entry)],_
  [{"d02cjf" :: Symbol, coerce(d02cjfEntry)$AnyFunctions1(Entry)],_
  [{"d02ejf" :: Symbol, coerce(d02ejfEntry)$AnyFunctions1(Entry)],_
  [{"d03eef" :: Symbol, coerce(d03eefEntry)$AnyFunctions1(Entry)],_
  --["d03faf" :: Symbol, coerce(d03fafEntry)$AnyFunctions1(Entry)],_
  [{"e04dgg" :: Symbol, coerce(e04dggEntry)$AnyFunctions1(Entry)],_
  [{"e04fdf" :: Symbol, coerce(e04fdfEntry)$AnyFunctions1(Entry)],_
  [{"e04gcf" :: Symbol, coerce(e04gcfEntry)$AnyFunctions1(Entry)],_
  [{"e04jaf" :: Symbol, coerce(e04jafEntry)$AnyFunctions1(Entry)],_
  [{"e04mbf" :: Symbol, coerce(e04mbfEntry)$AnyFunctions1(Entry)],_
  [{"e04naf" :: Symbol, coerce(e04nafEntry)$AnyFunctions1(Entry)],_
  [{"e04ucf" :: Symbol, coerce(e04ucfEntry)$AnyFunctions1(Entry)}]
construct(rl)

getIFL(s:Symbol,l:%):Union(IFL,"failed") ==
  o := search(s,l)$%
  o case "failed" => "failed"
  e := retractIfCan(o)$AnyFunctions1(Entry)
  e case "failed" => "failed"
  e.failList

getInstruction(l:IFL,ifailValue:Integer):Union(ST,"failed") ==
  output := empty()$ST
  for i in 1..#l repeat
    if ((l.i).ifail=ifailValue)@Boolean then
      output := (l.i).instruction
  empty?(output)$ST => "failed"
  output

```



```

recoverAfterFail(routs:%,routineName:ST,
                  ifailValue:Integer):Union(ST,"failed") ==
  name := routineName :: Symbol
  failedList := getIFL(name,routs)
  failedList case "failed" => "failed"
  empty? failedList => "failed"
  instr := getInstruction(failedList,ifailValue)
  instr case "failed" => concat(routineName," failed")$ST
  (instr = "delete")@Boolean =>
    deleteRoutine!(routs,name)
    concat(routineName," failed - trying alternatives")$ST
  instr

getExplanations(R:%,routineName:ST):LST ==
  name := routineName :: Symbol
  (a := search(name,R)) case Any =>
    e := retract(a)$AnyFunctions1(Entry)
    e.explList
  empty()$LST

```

---

— ROUTINE.dotabb —

```

"ROUTINE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ROUTINE"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"ROUTINE" -> "ALIST"

```

---

## 19.14 domain RULECOLD RuleCalled

— RuleCalled.input —

```

)set break resume
)sys rm -f RuleCalled.output
)spool RuleCalled.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show RuleCalled
--R RuleCalled f: Symbol is a domain constructor

```

```

--R Abbreviation for RuleCalled is RULECOLD
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for RULECOLD
--R
--R----- Operations -----
--R ?=? : (% ,%) -> Boolean          coerce : % -> OutputForm
--R hash : % -> SingleInteger        latex : % -> String
--R name : % -> Symbol              ?~=? : (% ,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)

```

---

### — RuleCalled.help —

```

=====
RuleCalled examples
=====

```

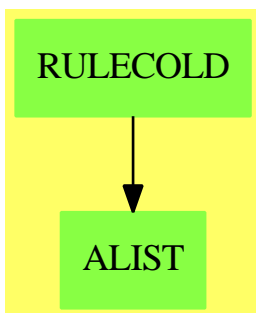
```

See Also:
o )show RuleCalled

```

---

#### 19.14.1 RuleCalled (RULECOLD)



**Exports:**  
 coerce hash latex name ?=? ?~=?

— domain RULECOLD RuleCalled —

```

)abbrev domain RULECOLD RuleCalled
++ Author: Mark Botch
++ Description:
++ This domain implements named rules

RuleCalled(f:Symbol): SetCategory with
  name: % -> Symbol
  ++ name(x) returns the symbol

== add
  name r              == f
  coerce(r:%):OutputForm == f::OutputForm
  x = y                == true
  latex(x:%):String    == latex f

```

---

— RULECOLD.dotabb —

```

"RULECOLD" [color="#88FF44",href="bookvol10.3.pdf#nameddest=RULECOLD"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"RULECOLD" -> "ALIST"

```

---

## 19.15 domain RULESET Ruleset

— Ruleset.input —

```

)set break resume
)sys rm -f Ruleset.output
)spool Ruleset.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Ruleset
--R Ruleset(Base: SetCategory,R: Join(Ring,PatternMatchable Base,OrderedSet,ConvertibleTo Pat
--R Abbreviation for Ruleset is RULESET
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for RULESET
--R
--R----- Operations -----
--R ?=? : (%,% ) -> Boolean          coerce : % -> OutputForm

```

```

--R elt : (% , F , PositiveInteger) -> F      ?..? : (% , F) -> F
--R hash : % -> SingleInteger                  latex : % -> String
--R ?~=? : (% , %) -> Boolean
--R rules : % -> List RewriteRule(Base,R,F)
--R ruleset : List RewriteRule(Base,R,F) -> %
--R
--E 1

)spool
)lisp (bye)

```

— Ruleset.help —

```

=====
Ruleset examples
=====

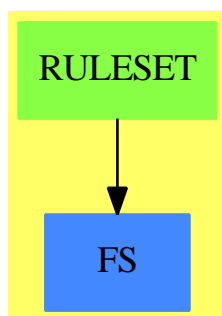
```

```

See Also:
o )show Ruleset

```

### 19.15.1 Ruleset (RULESET)



See

⇒ “RewriteRule” (RULE) 19.10.1 on page 2265

#### Exports:

```

coerce  elt    hash  latex  rules
ruleset  ?=?   ?~=?  ?..?

```

— domain RULESET Ruleset —

```

)abbrev domain RULESET Ruleset
++ Author: Manuel Bronstein
++ Date Created: 20 Mar 1990
++ Date Last Updated: 29 Jun 1990
++ Keywords: pattern, matching, rule.
++ Description:
++ Sets of rules for the pattern matcher.
++ A ruleset is a set of pattern matching rules grouped together.

Ruleset(Base, R, F): Exports == Implementation where
  Base      : SetCategory
  R          : Join(Ring, PatternMatchable Base, OrderedSet,
                    ConvertibleTo Pattern Base)
  F          : Join(FunctionSpace R, PatternMatchable Base,
                    ConvertibleTo Pattern Base)

RR ==> RewriteRule(Base, R, F)

Exports ==> Join(SetCategory, Eltable(F, F)) with
  ruleset: List RR -> $
    ++ ruleset([r1,...,rn]) creates the rule set \spad{{r1,...,rn}}.
  rules   : $ -> List RR
    ++ rules(r) returns the rules contained in r.
  elt     : ($, F, PositiveInteger) -> F
    ++ elt(r,f,n) or r(f, n) applies all the rules of r to f at most n times.

Implementation ==> add
  import ApplyRules(Base, R, F)

  Rep := Set RR

  ruleset l                               == {l}$Rep
  coerce(x:$):OutputForm                  == coerce(x)$Rep
  x = y                                    == x =$Rep y
  elt(x:$, f:F)                           == applyRules(rules x, f)
  elt(r:$, s:F, n:PositiveInteger) == applyRules(rules r, s, n)
  rules x                                  == parts(x)$Rep

-----

— RULESET.dotabb —

"RULESET" [color="#88FF44",href="bookvol10.3.pdf#nameddest=RULESET"]
"FS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FS"]
"RULESET" -> "FS"

-----

```

## Chapter 20

# Chapter S

### 20.1 domain FORMULA ScriptFormulaFormat

— ScriptFormulaFormat.input —

```
)set break resume
)sys rm -f ScriptFormulaFormat.output
)spool ScriptFormulaFormat.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ScriptFormulaFormat
--R ScriptFormulaFormat is a domain constructor
--R Abbreviation for ScriptFormulaFormat is FORMULA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for FORMULA
--R
--R----- Operations -----
--R ?=? : (%,%) -> Boolean          coerce : OutputForm -> %
--R coerce : % -> OutputForm        display : % -> Void
--R display : (%,Integer) -> Void    epilogue : % -> List String
--R formula : % -> List String        hash : % -> SingleInteger
--R latex : % -> String              new : () -> %
--R prologue : % -> List String       ?~=? : (%,%) -> Boolean
--R convert : (OutputForm,Integer) -> %
--R setEpilogue! : (%,List String) -> List String
--R setFormula! : (%,List String) -> List String
--R setPrologue! : (%,List String) -> List String
--R
--E 1
```

```
)spool
)lisp (bye)
```

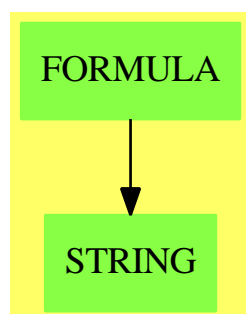
---

— ScriptFormulaFormat.help —

```
=====
ScriptFormulaFormat examples
=====
```

```
See Also:
o )show ScriptFormulaFormat
```

### 20.1.1 ScriptFormulaFormat (FORMULA)



#### Exports:

|             |              |          |         |              |
|-------------|--------------|----------|---------|--------------|
| coerce      | display      | epilogue | formula | hash         |
| latex       | new          | prologue | convert | setEpilogue! |
| setFormula! | setPrologue! | ?=?      | ?~=?    |              |

---

— domain FORMULA ScriptFormulaFormat —

```
)abbrev domain FORMULA ScriptFormulaFormat
++ Author: Robert S. Sutor
++ Date Created: 1987 through 1990
++ Change History:
++ Basic Operations: coerce, convert, display, epilogue,
++   formula, new, prologue, setEpilogue!, setFormula!, setPrologue!
++ Related Constructors: ScriptFormulaFormat1
++ Also See: TexFormat
```

```

++ AMS Classifications:
++ Keywords: output, format, SCRIPT, BookMaster, formula
++ References:
++   SCRIPT Mathematical Formula Formatter User's Guide, SH20-6453,
++   IBM Corporation, Publishing Systems Information Development,
++   Dept. G68, P.O. Box 1900, Boulder, Colorado, USA 80301-9191.
++ Description:
++ \spadtype{ScriptFormulaFormat} provides a coercion from
++ \spadtype{OutputForm} to IBM SCRIPT/VS Mathematical Formula Format.
++ The basic SCRIPT formula format object consists of three parts: a
++ prologue, a formula part and an epilogue. The functions
++ \spadfun{prologue}, \spadfun{formula} and \spadfun{epilogue}
++ extract these parts, respectively. The central parts of the expression
++ go into the formula part. The other parts can be set
++ (\spadfun{setPrologue!}, \spadfun{setEpilogue!}) so that contain the
++ appropriate tags for printing. For example, the prologue and
++ epilogue might simply contain ":df." and ":edf." so that the
++ formula section will be printed in display math mode.

ScriptFormulaFormat(): public == private where
  E      ==> OutputForm
  I      ==> Integer
  L      ==> List
  S      ==> String

public == SetCategory with
  coerce:  E -> %
    ++ coerce(o) changes o in the standard output format to
    ++ SCRIPT formula format.
  convert: (E,I) -> %
    ++ convert(o,step) changes o in standard output format to
    ++ SCRIPT formula format and also adds the given step number.
    ++ This is useful if you want to create equations with given numbers
    ++ or have the equation numbers correspond to the interpreter step
    ++ numbers.
  display: (% , I) -> Void
    ++ display(t,width) outputs the formatted code t so that each
    ++ line has length less than or equal to \spadvar{width}.
  display: % -> Void
    ++ display(t) outputs the formatted code t so that each
    ++ line has length less than or equal to the value set by
    ++ the system command \spadsyscom{set output length}.
  epilogue: % -> L S
    ++ epilogue(t) extracts the epilogue section of a formatted object t.
  formula:  % -> L S
    ++ formula(t) extracts the formula section of a formatted object t.
  new:      () -> %
    ++ new() create a new, empty object. Use \spadfun{setPrologue!},
    ++ \spadfun{setFormula!} and \spadfun{setEpilogue!} to set the
    ++ various components of this object.

```



```

prologue: % -> L S
  ++ prologue(t) extracts the prologue section of a formatted object t.
setEpilogue!: (% , L S) -> L S
  ++ setEpilogue!(t,strings) sets the epilogue section of a
  ++ formatted object t to strings.
setFormula!: (% , L S) -> L S
  ++ setFormula!(t,strings) sets the formula section of a
  ++ formatted object t to strings.
setPrologue!: (% , L S) -> L S
  ++ setPrologue!(t,strings) sets the prologue section of a
  ++ formatted object t to strings.

private == add
import OutputForm
import Character
import Integer
import List OutputForm
import List String

Rep := Record(prolog : L S, formula : L S, epilog : L S)

-- local variables declarations and definitions

expr: E
prec,opPrec: I
str: S
blank      : S := " @@ "

maxPrec    : I    := 1000000
minPrec    : I    := 0

splitChars : S    := " <>[](){}+*=-,% "

unaryOps    : L S := ["-", "^"]$(L S)
unaryPrecs  : L I := [700,260]$(L I)

-- the precedence of / in the following is relatively low because
-- the bar obviates the need for parentheses.
binaryOps   : L S := ["+>", "|", "**", "/", "<", ">", "=", "OVER"]$(L S)
binaryPrecs : L I := [0,0,900, 700,400,400,400, 700]$(L I)

naryOps     : L S := ["-", "+", "*", blank, ",", ";", " ", "ROW", "",
  " habove ", " here ", " labove "]$(L S)
naryPrecs   : L I := [700,700,800, 800,110,110, 0, 0, 0,
  0, 0, 0]$(L I)
-- naryNGOps : L S := ["ROW", " here "]$(L S)
naryNGOps   : L S := nil$(L S)

plexOps     : L S := ["SIGMA", "PI", "INTSIGN", "INDEFINTEGRAL"]$(L S)
plexPrecs   : L I := [ 700, 800, 700, 700]$(L I)

```

```

specialOps      : L S := ["MATRIX","BRACKET","BRACE","CONCATB",    -
                          "AGGLST","CONCAT","OVERBAR","ROOT","SUB",  -
                          "SUPERSUB","ZAG","AGGSET","SC","PAREN"]

-- the next two lists provide translations for some strings for
-- which the formula formatter provides special variables.

specialStrings : L S :=
  ["5","..."]
specialStringsInFormula : L S :=
  [" alpha "," ellipsis "]

-- local function signatures

addBraces:      S -> S
addBrackets:    S -> S
group:          S -> S
formatBinary:   (S,L E, I) -> S
formatFunction: (S,L E, I) -> S
formatMatrix:   L E -> S
formatNary:     (S,L E, I) -> S
formatNaryNoGroup: (S,L E, I) -> S
formatNullary:  S -> S
formatPlex:     (S,L E, I) -> S
formatSpecial:  (S,L E, I) -> S
formatUnary:    (S,  E, I) -> S
formatFormula:  (E,I) -> S
parenthesize:   S -> S
precondition:   E -> E
postcondition:  S -> S
splitLong:      (S,I) -> L S
splitLong1:     (S,I) -> L S
stringify:      E -> S

-- public function definitions

new() : % == [[".eq set blank @",":df." ]$(L S),
  [""]$(L S), [":edf." ]$(L S)]$Rep

coerce(expr : E): % ==
  f : % := new()$%
  f.formula := [postcondition
    formatFormula(precondition expr, minPrec)]$(L S)
  f

convert(expr : E, stepNum : I): % ==
  f : % := new()$%
  f.formula := concat(["<leqno lparen ",string(stepNum)$S,
    " rparen>"], [postcondition

```

```

        formatFormula(precondition expr, minPrec)]$(L S))
    f

display(f : %, len : I) ==
    s,t : S
    for s in f.prolog repeat sayFORMULA(s)$Lisp
    for s in f.formula repeat
        for t in splitLong(s, len) repeat sayFORMULA(t)$Lisp
    for s in f.epilog repeat sayFORMULA(s)$Lisp
    void()$Void

display(f : %) ==
    display(f, _$LINELENGTH$Lisp pretend I)

prologue(f : %) == f.prolog
formula(f : %) == f.formula
epilogue(f : %) == f.epilog

setPrologue!(f : %, l : L S) == f.prolog := l
setFormula!(f : %, l : L S) == f.formula := l
setEpilogue!(f : %, l : L S) == f.epilog := l

coerce(f : %): E ==
    s,t : S
    l : L S := nil
    for s in f.prolog repeat l := concat(s,l)
    for s in f.formula repeat
        for t in splitLong(s, (_$LINELENGTH$Lisp pretend Integer) - 4) repeat
            l := concat(t,l)
    for s in f.epilog repeat l := concat(s,l)
    (reverse l) :: E

-- local function definitions

postcondition(str: S): S ==
    len : I := #str
    len < 4 => str
    plus : Character := char "+"
    minus: Character := char "-"
    for i in 1..(len-1) repeat
        if (str.i =$Character plus) and (str.(i+1) =$Character minus)
            then setelt(str,i,char " ")$S
    str

stringify expr == object2String(expr)$Lisp pretend S

splitLong(str : S, len : I): L S ==
    -- this blocks into lines
    if len < 20 then len := _$LINELENGTH$Lisp
    splitLong1(str, len)

```

```

splitLong1(str : S, len : I) ==
  l : List S := nil
  s : S := ""
  ls : I := 0
  ss : S
  lss : I
  for ss in split(str,char " ") repeat
    lss := #ss
    if ls + lss > len then
      l := concat(s,l)$List(S)
      s := ""
      ls := 0
    lss > len => l := concat(ss,l)$List(S)
    ls := ls + lss + 1
    s := concat(s,concat(ss," ")$S)$S
  if ls > 0 then l := concat(s,l)$List(S)
  reverse l

group str ==
  concat ["<",str,">"]

addBraces str ==
  concat ["left lbrace ",str," right rbrace"]

addBrackets str ==
  concat ["left lb ",str," right rb"]

parenthesize str ==
  concat ["left lparen ",str," right rparen"]

precondition expr ==
  outputTran(expr)$Lisp

formatSpecial(op : S, args : L E, prec : I) : S ==
  op = "AGGLST" =>
    formatNary(",",args,prec)
  op = "AGGSET" =>
    formatNary(";",args,prec)
  op = "CONCATB" =>
    formatNary(" ",args,prec)
  op = "CONCAT" =>
    formatNary("",args,prec)
  op = "BRACKET" =>
    group addBrackets formatFormula(first args, minPrec)
  op = "BRACE" =>
    group addBraces formatFormula(first args, minPrec)
  op = "PAREN" =>
    group parenthesize formatFormula(first args, minPrec)
  op = "OVERBAR" =>

```

```

    null args => ""
    group concat [formatFormula(first args, minPrec), " bar"]
op = "ROOT" =>
    null args => ""
    tmp : S := formatFormula(first args, minPrec)
    null rest args => group concat ["sqrt ", tmp]
    group concat ["midsup adjust(u 1.5 r 9) ",
        formatFormula(first rest args, minPrec), " sqrt ", tmp]
op = "SC" =>
    formatNary(" labove ", args, prec)
op = "SUB" =>
    group concat [formatFormula(first args, minPrec), " sub ",
        formatSpecial("AGGLST", rest args, minPrec)]
op = "SUPERSUB" =>
    -- variable name
    form : List S := [formatFormula(first args, minPrec)]
    -- subscripts
    args := rest args
    null args => concat form
    tmp : S := formatFormula(first args, minPrec)
    if tmp ^= "" then form := append(form, [" sub ", tmp])$(List S)
    -- superscripts
    args := rest args
    null args => group concat form
    tmp : S := formatFormula(first args, minPrec)
    if tmp ^= "" then form := append(form, [" sup ", tmp])$(List S)
    -- presuperscripts
    args := rest args
    null args => group concat form
    tmp : S := formatFormula(first args, minPrec)
    if tmp ^= "" then form := append(form, [" presup ", tmp])$(List S)
    -- presubscripts
    args := rest args
    null args => group concat form
    tmp : S := formatFormula(first args, minPrec)
    if tmp ^= "" then form := append(form, [" presub ", tmp])$(List S)
    group concat form
op = "MATRIX" => formatMatrix rest args
-- op = "ZAG" =>
--     concat ["\zag{", formatFormula(first args, minPrec), "}{" ,
--         formatFormula(first rest args, minPrec), "}"]
--     concat ["not done yet for ", op]

formatPlex(op : S, args : L E, prec : I) : S ==
    hold : S
    p : I := position(op, plexOps)
    p < 1 => error "unknown Script Formula Formatter unary op"
    opPrec := plexPrecs.p
    n : I := #args
    (n ^= 2) and (n ^= 3) => error "wrong number of arguments for plex"

```

```

s : S :=
  op = "SIGMA"    => "sum"
  op = "PI"       => "product"
  op = "INTSIGN"  => "integral"
  op = "INDEFINTEGRAL" => "integral"
  "???"
hold := formatFormula(first args,minPrec)
args := rest args
if op ^= "INDEFINTEGRAL" then
  if hold ^= "" then
    s := concat [s," from",group concat ["\displaystyle ",hold]]
  if not null rest args then
    hold := formatFormula(first args,minPrec)
    if hold ^= "" then
      s := concat [s," to",group concat ["\displaystyle ",hold]]
    args := rest args
    s := concat [s," ",formatFormula(first args,minPrec)]
  else
    hold := group concat [hold," ",formatFormula(first args,minPrec)]
    s := concat [s," ",hold]
  if opPrec < prec then s := parenthesize s
  group s

formatMatrix(args : L E) : S ==
  -- format for args is [[ROW ...],[ROW ...],[ROW ...]]
  group addBrackets formatNary(" habove ",args,minPrec)

formatFunction(op : S, args : L E, prec : I) : S ==
  group concat [op, " ", parenthesize formatNary(" ",args,minPrec)]

formatNullary(op : S) ==
  op = "NOTHING" => ""
  group concat [op,"()"]

formatUnary(op : S, arg : E, prec : I) ==
  p : I := position(op,unaryOps)
  p < 1 => error "unknown Script Formula Formatter unary op"
  opPrec := unaryPrecs.p
  s : S := concat [op,formatFormula(arg,opPrec)]
  opPrec < prec => group parenthesize s
  op = "-" => s
  group s

formatBinary(op : S, args : L E, prec : I) : S ==
  p : I := position(op,binaryOps)
  p < 1 => error "unknown Script Formula Formatter binary op"
  op :=
    op = "**"    => " sup "
    op = "/"    => " over "
    op = "OVER" => " over "

```

```

    op
    opPrec := binaryPrecs.p
    s : S := formatFormula(first args, opPrec)
    s := concat [s,op,formatFormula(first rest args, opPrec)]
    group
    op = " over " => s
    opPrec < prec => parenthesize s
    s

formatNary(op : S, args : L E, prec : I) : S ==
    group formatNaryNoGroup(op, args, prec)

formatNaryNoGroup(op : S, args : L E, prec : I) : S ==
    null args => ""
    p : I := position(op,naryOps)
    p < 1 => error "unknown Script Formula Formatter nary op"
    op :=
        op = ","      => ", @@ "
        op = ";"      => "; @@ "
        op = "*"      => blank
        op = " "      => blank
        op = "ROW"    => " here "
    op
    l : L S := nil
    opPrec := naryPrecs.p
    for a in args repeat
        l := concat(op,concat(formatFormula(a,opPrec),l)$L(S))$L(S)
    s : S := concat reverse rest l
    opPrec < prec => parenthesize s
    s

formatFormula(expr,prec) ==
    i : Integer
    ATOM(expr)$Lisp pretend Boolean =>
        str := stringify expr
        INTEGERP(expr)$Lisp =>
            i := expr : Integer
            if (i < 0) or (i > 9) then group str else str
        (i := position(str,specialStrings)) > 0 =>
            specialStringsInFormula.i
        str
    l : L E := (expr pretend L E)
    null l => blank
    op : S := stringify first l
    args : L E := rest l
    nargs : I := #args

-- special cases
member?(op, specialOps) => formatSpecial(op,args,prec)
member?(op, plexOps)    => formatPlex(op,args,prec)

```

```

-- nullary case
0 = nargs => formatNullary op

-- unary case
(1 = nargs) and member?(op, unaryOps) =>
  formatUnary(op, first args, prec)

-- binary case
(2 = nargs) and member?(op, binaryOps) =>
  formatBinary(op, args, prec)

-- nary case
member?(op,naryNGOps) => formatNaryNoGroup(op,args, prec)
member?(op,naryOps) => formatNary(op,args, prec)
op := formatFormula(first 1,minPrec)
formatFunction(op,args,prec)

```

---

— FORMULA.dotabb —

```

"FORMULA" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FORMULA"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"FORMULA" -> "STRING"

```

---

## 20.2 domain SEG Segment

— Segment.input —

```

)set break resume
)sys rm -f Segment.output
)spool Segment.output
)set message test on
)set message auto off
)clear all
--S 1 of 10
s := 3..10
--R
--R
--R (1) 3..10
--R

```

Type: Segment PositiveInteger



```
--E 1
```

```
--S 2 of 10
```

```
lo s
```

```
--R
```

```
--R
```

```
--R (2) 3
```

```
--R
```

Type: PositiveInteger

```
--E 2
```

```
--S 3 of 10
```

```
hi s
```

```
--R
```

```
--R
```

```
--R (3) 10
```

```
--R
```

Type: PositiveInteger

```
--E 3
```

```
--S 4 of 10
```

```
t := 10..3 by -2
```

```
--R
```

```
--R
```

```
--R (4) 10..3 by - 2
```

```
--R
```

Type: Segment PositiveInteger

```
--E 4
```

```
--S 5 of 10
```

```
incr s
```

```
--R
```

```
--R
```

```
--R (5) 1
```

```
--R
```

Type: PositiveInteger

```
--E 5
```

```
--S 6 of 10
```

```
incr t
```

```
--R
```

```
--R
```

```
--R (6) - 2
```

```
--R
```

Type: Integer

```
--E 6
```

```
--S 7 of 10
```

```
l := [1..3, 5, 9, 15..11 by -1]
```

```
--R
```

```
--R
```

```
--R (7) [1..3,5..5,9..9,15..11 by - 1]
```

```
--R
```

Type: List Segment PositiveInteger

```
--E 7
```



These names are used even though the end points might belong to an unordered set.

```
hi s
  10
```

Type: PositiveInteger

In addition to the end points, each segment has an integer "increment". An increment can be specified using the "by" construct.

```
t := 10..3 by -2
    10..3 by - 2
```

Type: Segment PositiveInteger

This part can be obtained using the incr function.

```
incr s
  1
```

Type: PositiveInteger

Unless otherwise specified, the increment is 1.

```
incr t
  - 2
```

Type: Integer

A single value can be converted to a segment with equal end points. This happens if segments and single values are mixed in a list.

```
l := [1..3, 5, 9, 15..11 by -1]
     [1..3,5..5,9..9,15..11 by - 1]
```

Type: List Segment PositiveInteger

If the underlying type is an ordered ring, it is possible to perform additional operations. The expand operation creates a list of points in a segment.

```
expand s
  [3,4,5,6,7,8,9,10]
```

Type: List Integer

If  $k > 0$ , then `expand(l..h by k)` creates the list `[l, l+k, ..., lN]` where  $lN \leq h < lN+k$ . If  $k < 0$ , then  $lN \geq h > lN+k$ .

```
expand t
  [10,8,6,4]
```

Type: List Integer

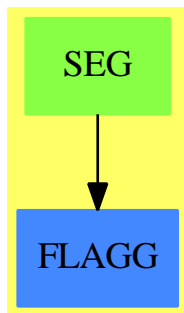
It is also possible to expand a list of segments. This is equivalent to appending lists obtained by expanding each segment individually.

```
expand 1
  [1,2,3,5,9,15,14,13,12,11]
                                     Type: List Integer
```

See Also:

- o )help UniversalSegment
- o )help SegmentBinding
- o )show Segment

### 20.2.1 Segment (SEG)



See

⇒ “SegmentBinding” (SEGBIND) 20.3.1 on page 2324  
 ⇒ “UniversalSegment” (UNISEG) 22.11.1 on page 2853

#### Exports:

|       |        |         |        |      |
|-------|--------|---------|--------|------|
| BY    | coerce | convert | expand | hash |
| hi    | high   | incr    | latex  | lo   |
| low   | map    | segment | ?=?    | ?~=? |
| ?...? |        |         |        |      |

— domain SEG Segment —

```
)abbrev domain SEG Segment
++ Author: Stephen M. Watt
++ Date Created: December 1986
++ Date Last Updated: June 3, 1991
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
```

```

++ Keywords: range, segment
++ Examples:
++ References:
++ Description:
++ This type is used to specify a range of values from type \spad{S}.

Segment(S:Type): SegmentCategory(S) with
  if S has SetCategory then SetCategory
  if S has OrderedRing then SegmentExpansionCategory(S, List S)
== add

Rep := Record(low: S, high: S, incr: Integer)

a..b == [a,b,1]
lo s == s.low
low s == s.low
hi s == s.high
high s == s.high
incr s == s.incr
segment(a,b) == [a,b,1]
BY(s, r) == [lo s, hi s, r]

if S has SetCategory then
  (s1:%) = (s2:%) ==
    s1.low = s2.low and s1.high=s2.high and s1.incr = s2.incr

coerce(s:%):OutputForm ==
  seg := SEGMENT(s.low::OutputForm, s.high::OutputForm)
  s.incr = 1 => seg
  infix(" by "::OutputForm, seg, s.incr::OutputForm)

convert a == [a,a,1]

if S has OrderedRing then
  expand(ls: List %):List S ==
    lr := nil()$List(S)
    for s in ls repeat
      l := lo s
      h := hi s
      inc := (incr s)::S
      zero? inc => error "Cannot expand a segment with an increment of zero"
      if inc > 0 then
        while l <= h repeat
          lr := concat(l, lr)
          l := l + inc
      else
        while l >= h repeat
          lr := concat(l, lr)
          l := l + inc
    reverse_! lr

```

```

expand(s : %) == expand([s]$List(%))$%
map(f : S->S, s : %): List S ==
  lr := nil()$List(S)
  l := lo s
  h := hi s
  inc := (incr s)::S
  if inc > 0 then
    while l <= h repeat
      lr := concat(f l, lr)
      l := l + inc
  else
    while l >= h repeat
      lr := concat(f l, lr)
      l := l + inc
  reverse_! lr

```

---

— SEG.dotabb —

```

"SEG" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SEG"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"SEG" -> "FLAGG"

```

---

## 20.3 domain SEGBIND SegmentBinding

— SegmentBinding.input —

```

)set break resume
)sys rm -f SegmentBinding.output
)spool SegmentBinding.output
)set message test on
)set message auto off
)clear all
--S 1 of 5
x = a..b
--R
--R
--R (1) x= a..b
--R
--E 1

```

Type: SegmentBinding Symbol

```

--S 2 of 5
sum(i**2, i = 0..n)
--R
--R
--R      3      2
--R      2n  + 3n  + n
--R      (2)  -----
--R              6
--R
--R                                          Type: Fraction Polynomial Integer
--E 2

--S 3 of 5
sb := y = 1/2..3/2
--R
--R
--R      1      3
--R      (-) .. (-)
--R      2      2
--R
--R                                          Type: SegmentBinding Fraction Integer
--E 3

--S 4 of 5
variable(sb)
--R
--R
--R      (4)  y
--R
--R                                          Type: Symbol
--E 4

--S 5 of 5
segment(sb)
--R
--R
--R      1      3
--R      (-) .. (-)
--R      2      2
--R
--R                                          Type: Segment Fraction Integer
--E 5
)spool
)lisp (bye)

```

---

— SegmentBinding.help —

```

=====
SegmentBinding examples
=====

```

The SegmentBinding type is used to indicate a range for a named symbol.

First give the symbol, then an = and finally a segment of values.

```
x = a..b
x= a..b
```

Type: SegmentBinding Symbol

This is used to provide a convenient syntax for arguments to certain operations.

```
sum(i**2, i = 0..n)
  3      2
  2n  + 3n  + n
  -----
      6
```

Type: Fraction Polynomial Integer

```
draw(x**2, x = -2..2)
TwoDimensionalViewport: "x*x"
```

Type: TwoDimensionalViewport

The left-hand side must be of type Symbol but the right-hand side can be a segment over any type.

```
sb := y = 1/2..3/2
      1      3
y= (-)..(-)
      2      2
```

Type: SegmentBinding Fraction Integer

The left- and right-hand sides can be obtained using the variable and segment operations.

```
variable(sb)
y
```

Type: Symbol

```
segment(sb)
  1      3
  (-)..(-)
  2      2
```

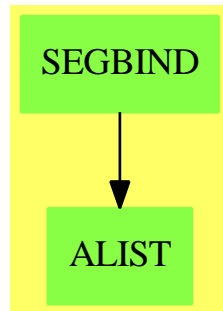
Type: Segment Fraction Integer

See Also:

- o )help Segment
- o )help UniversalSegment
- o )show SegmentBinding



### 20.3.1 SegmentBinding (SEGBIND)



See

⇒ “Segment” (SEG) 20.2.1 on page 2319

⇒ “UniversalSegment” (UNISEG) 22.11.1 on page 2853

#### Exports:

coerce    equation    hash    latex    segment  
variable    ?=?            ?~=?

— domain SEGBIND SegmentBinding —

```

)abbrev domain SEGBIND SegmentBinding
++ Author: Mark Botch
++ Date Created:
++ Date Last Updated: June 4, 1991
++ Basic Operations:
++ Related Domains: Equation, Segment, Symbol
++ Also See:
++ AMS Classifications:
++ Keywords: equation
++ Examples:
++ References:
++ Description:
++ This domain is used to provide the function argument syntax \spad{v=a..b}.
++ This is used, for example, by the top-level \spadfun{draw} functions.

SegmentBinding(S:Type): Type with
  equation: (Symbol, Segment S) -> %
    ++ equation(v,a..b) creates a segment binding value with variable
    ++ \spad{v} and segment \spad{a..b}. Note that the interpreter parses
  
```

```

++ \spad{v=a..b} to this form.
variable: % -> Symbol
++ variable(segb) returns the variable from the left hand side of
++ the \spadtype{SegmentBinding}. For example, if \spad{segb} is
++ \spad{v=a..b}, then \spad{variable(segb)} returns \spad{v}.
segment : % -> Segment S
++ segment(segb) returns the segment from the right hand side of
++ the \spadtype{SegmentBinding}. For example, if \spad{segb} is
++ \spad{v=a..b}, then \spad{segment(segb)} returns \spad{a..b}.

if S has SetCategory then SetCategory
== add
Rep := Record(var:Symbol, seg:Segment S)
equation(x,s) == [x, s]
variable b    == b.var
segment b     == b.seg

if S has SetCategory then

b1 = b2      == variable b1 = variable b2 and segment b1 = segment b2

coerce(b:%):OutputForm ==
variable(b)::OutputForm = segment(b)::OutputForm

-----

— SEGBIND.dotabb —

"SEGBIND" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SEGBIND"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"SEGBIND" -> "ALIST"

-----

```

## 20.4 domain SET Set

```

— Set.input —

)set break resume
)sys rm -f Set.output
)spool Set.output
)set message test on
)set message auto off
)clear all

```

```

--S 1 of 20
s := set [x**2-1, y**2-1, z**2-1]
--R
--R
--R      2      2      2
--R (1) {x  - 1, y  - 1, z  - 1}
--R
--R                                          Type: Set Polynomial Integer
--E 1

--S 2 of 20
t := set [x**i - i+1 for i in 2..10 | prime? i]
--R
--R
--R      2      3      5      7
--R (2) {x  - 1, x  - 2, x  - 4, x  - 6}
--R
--R                                          Type: Set Polynomial Integer
--E 2

--S 3 of 20
i := intersect(s,t)
--R
--R
--R      2
--R (3) {x  - 1}
--R
--R                                          Type: Set Polynomial Integer
--E 3

--S 4 of 20
u := union(s,t)
--R
--R
--R      2      3      5      7      2      2
--R (4) {x  - 1, x  - 2, x  - 4, x  - 6, y  - 1, z  - 1}
--R
--R                                          Type: Set Polynomial Integer
--E 4

--S 5 of 20
difference(s,t)
--R
--R
--R      2      2
--R (5) {y  - 1, z  - 1}
--R
--R                                          Type: Set Polynomial Integer
--E 5

--S 6 of 20
symmetricDifference(s,t)
--R
--R
--R      3      5      7      2      2

```

[illegible]

--E 12

--S 13 of 20

a := set [i\*\*2 for i in 1..5]

--R

--R

--R (13) {1,4,9,16,25}

--R

Type: Set PositiveInteger

--E 13

--S 14 of 20

insert!(32, a)

--R

--R

--R (14) {1,4,9,16,25,32}

--R

Type: Set PositiveInteger

--E 14

--S 15 of 20

remove!(25, a)

--R

--R

--R (15) {1,4,9,16,32}

--R

Type: Set PositiveInteger

--E 15

--S 16 of 20

a

--R

--R

--R (16) {1,4,9,16,32}

--R

Type: Set PositiveInteger

--E 16

--S 17 of 20

b := b0 := set [i\*\*2 for i in 1..5]

--R

--R

--R (17) {1,4,9,16,25}

--R

Type: Set PositiveInteger

--E 17

--S 18 of 20

b := union(b, {32})

--R

--R

--R (18) {1,4,9,16,25,32}

--R

Type: Set PositiveInteger

--E 18

```

--S 19 of 20
b := difference(b, {25})
--R
--R
--R (19) {1,4,9,16,32}
--R
--R                                         Type: Set PositiveInteger
--E 19

--S 20 of 20
b0
--R
--R
--R (20) {1,4,9,16,25}
--R
--R                                         Type: Set PositiveInteger
--E 20
)spool
)lisp (bye)

```

— Set.help —

=====

Set examples

=====

The Set domain allows one to represent explicit finite sets of values. These are similar to lists, but duplicate elements are not allowed.

Sets can be created by giving a fixed set of values ...

```

s := set [x**2-1, y**2-1, z**2-1]
      2      2      2
      {x  - 1, y  - 1, z  - 1}
                                         Type: Set Polynomial Integer

```

or by using a collect form, just as for lists. In either case, the set is formed from a finite collection of values.

```

t := set [x**i - i+1 for i in 2..10 | prime? i]
      2      3      5      7
      {x  - 1, x  - 2, x  - 4, x  - 6}
                                         Type: Set Polynomial Integer

```

The basic operations on sets are intersect, union, difference, and symmetricDifference.

```

i := intersect(s,t)
      2

```

```
{x2 - 1}
Type: Set Polynomial Integer
```

```
u := union(s,t)
      2      3      5      7      2      2
{x2 - 1, x3 - 2, x5 - 4, x7 - 6, y2 - 1, z2 - 1}
Type: Set Polynomial Integer
```

The set difference(s,t) contains those members of s which are not in t.

```
difference(s,t)
      2      2
{y2 - 1, z2 - 1}
Type: Set Polynomial Integer
```

The set symmetricDifference(s,t) contains those elements which are in s or t but not in both.

```
symmetricDifference(s,t)
      3      5      7      2      2
{x3 - 2, x5 - 4, x7 - 6, y2 - 1, z2 - 1}
Type: Set Polynomial Integer
```

Set membership is tested using the member? operation.

```
member?(y, s)
false
Type: Boolean
```

```
member?((y+1)*(y-1), s)
true
Type: Boolean
```

The subset? function determines whether one set is a subset of another.

```
subset?(i, s)
true
Type: Boolean
```

```
subset?(u, s)
false
Type: Boolean
```

When the base type is finite, the absolute complement of a set is defined. This finds the set of all multiplicative generators of PrimeField 11---the integers mod 11.

```
gs := set [g for i in 1..11 | primitive?(g := i::PF 11)]
{2,6,7,8}
Type: Set PrimeField 11
```

The following values are not generators.

```
complement gs
{1,3,4,5,9,10,0}
Type: Set PrimeField 11
```

Often the members of a set are computed individually; in addition, values can be inserted or removed from a set over the course of a computation.

There are two ways to do this:

```
a := set [i**2 for i in 1..5]
{1,4,9,16,25}
Type: Set PositiveInteger
```

One is to view a set as a data structure and to apply updating operations.

```
insert!(32, a)
{1,4,9,16,25,32}
Type: Set PositiveInteger
```

```
remove!(25, a)
{1,4,9,16,32}
Type: Set PositiveInteger
```

```
a
{1,4,9,16,32}
Type: Set PositiveInteger
```

The other way is to view a set as a mathematical entity and to create new sets from old.

```
b := b0 := set [i**2 for i in 1..5]
{1,4,9,16,25}
Type: Set PositiveInteger
```

```
b := union(b, {32})
{1,4,9,16,25,32}
Type: Set PositiveInteger
```

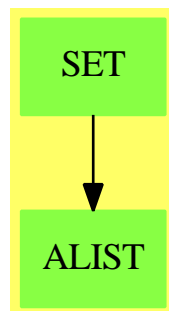
```
b := difference(b, {25})
{1,4,9,16,32}
Type: Set PositiveInteger
```

```
b0
{1,4,9,16,25}
Type: Set PositiveInteger
```



See Also:  
 o )help List  
 o )show Set

### 20.4.1 Set (SET)



#### Exports:

|            |            |                     |             |                  |
|------------|------------|---------------------|-------------|------------------|
| any?       | bag        | brace               | cardinality | coerce           |
| complement | construct  | convert             | copy        | count            |
| dictionary | difference | empty               | empty?      | eq?              |
| eval       | every?     | extract!            | find        | hash             |
| index      | insert!    | inspect             | intersect   | latex            |
| less?      | lookup     | map                 | map!        | max              |
| member?    | members    | min                 | more?       | parts            |
| random     | reduce     | remove              | remove!     | removeDuplicates |
| sample     | select     | select!             | set         | size             |
| size?      | subset?    | symmetricDifference | union       | universe         |
| #?         | ?~=?       | ?<?                 | ?=?         |                  |

— domain SET Set —

```

)abbrev domain SET Set
++ Author: Michael Monagan; revised by Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: May 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
  
```

```

++ Description:
++ A set over a domain D models the usual mathematical notion of a finite set
++ of elements from D.
++ Sets are unordered collections of distinct elements
++ (that is, order and duplication does not matter).
++ The notation \spad{set [a,b,c]} can be used to create
++ a set and the usual operations such as union and intersection are available
++ to form new sets.
++ In our implementation, \Language{} maintains the entries in
++ sorted order. Specifically, the parts function returns the entries
++ as a list in ascending order and
++ the extract operation returns the maximum entry.
++ Given two sets s and t where \spad{#s = m} and \spad{#t = n},
++ the complexity of\br
++ \tab{5}\spad{s = t} is \spad{0(min(n,m))}\br
++ \tab{5}\spad{s < t} is \spad{0(max(n,m))}\br
++ \tab{5}\spad{union(s,t)}, \spad{intersect(s,t)}, \spad{minus(s,t)},\br
++ \tab{10} \spad{symmetricDifference(s,t)} is \spad{0(max(n,m))}\br
++ \tab{5}\spad{member(x,t)} is \spad{0(n log n)}\br
++ \tab{5}\spad{insert(x,t)} and \spad{remove(x,t)} is \spad{0(n)}

Set(S:SetCategory): FiniteSetAggregate S == add
  Rep := FlexibleArray(S)
  # s      == _#$Rep s
  brace()  == empty()
  set()     == empty()
  empty()  == empty()$Rep
  copy s    == copy(s)$Rep
  parts s   == parts(s)$Rep
  inspect s == (empty? s => error "Empty set"; s(maxIndex s))

  extract_! s ==
    x := inspect s
    delete_!(s, maxIndex s)
    x

  find(f, s) == find(f, s)$Rep

  map(f, s) == map_!(f,copy s)

  map_!(f,s) ==
    map_!(f,s)$Rep
    removeDuplicates_! s

  reduce(f, s) == reduce(f, s)$Rep

  reduce(f, s, x) == reduce(f, s, x)$Rep

  reduce(f, s, x, y) == reduce(f, s, x, y)$Rep

```

```

if S has ConvertibleTo InputForm then
  convert(x:%):InputForm ==
    convert [convert("set"::Symbol)@InputForm,
              convert(parts x)@InputForm]

if S has OrderedSet then
  s = t == s =$Rep t
  max s == inspect s
  min s == (empty? s => error "Empty set"; s(minIndex s))

construct l ==
  zero?(n := #l) => empty()
  a := new(n, first l)
  for i in minIndex(a).. for x in l repeat a.i := x
  removeDuplicates_! sort_! a

insert_!(x, s) ==
  n := inc maxIndex s
  k := minIndex s
  while k < n and x > s.k repeat k := inc k
  k < n and s.k = x => s
  insert_!(x, s, k)

member?(x, s) == -- binary search
  empty? s => false
  t := maxIndex s
  b := minIndex s
  while b < t repeat
    m := (b+t) quo 2
    if x > s.m then b := m+1 else t := m
  x = s.t

remove_!(x:S, s:%) ==
  n := inc maxIndex s
  k := minIndex s
  while k < n and x > s.k repeat k := inc k
  k < n and x = s.k => delete_!(s, k)
  s

-- the set operations are implemented as variations of merging
intersect(s, t) ==
  m := maxIndex s
  n := maxIndex t
  i := minIndex s
  j := minIndex t
  r := empty()
  while i <= m and j <= n repeat
    s.i = t.j => (concat_!(r, s.i); i := i+1; j := j+1)
    if s.i < t.j then i := i+1 else j := j+1
  r

```

```

difference(s:%, t:%) ==
  m := maxIndex s
  n := maxIndex t
  i := minIndex s
  j := minIndex t
  r := empty()
  while i <= m and j <= n repeat
    s.i = t.j => (i := i+1; j := j+1)
    s.i < t.j => (concat_!(r, s.i); i := i+1)
    j := j+1
  while i <= m repeat (concat_!(r, s.i); i := i+1)
  r

symmetricDifference(s, t) ==
  m := maxIndex s
  n := maxIndex t
  i := minIndex s
  j := minIndex t
  r := empty()
  while i <= m and j <= n repeat
    s.i < t.j => (concat_!(r, s.i); i := i+1)
    s.i > t.j => (concat_!(r, t.j); j := j+1)
    i := i+1; j := j+1
  while i <= m repeat (concat_!(r, s.i); i := i+1)
  while j <= n repeat (concat_!(r, t.j); j := j+1)
  r

subset?(s, t) ==
  m := maxIndex s
  n := maxIndex t
  m > n => false
  i := minIndex s
  j := minIndex t
  while i <= m and j <= n repeat
    s.i = t.j => (i := i+1; j := j+1)
    s.i > t.j => j := j+1
  return false
  i > m

union(s:%, t:%) ==
  m := maxIndex s
  n := maxIndex t
  i := minIndex s
  j := minIndex t
  r := empty()
  while i <= m and j <= n repeat
    s.i = t.j => (concat_!(r, s.i); i := i+1; j := j+1)
    s.i < t.j => (concat_!(r, s.i); i := i+1)
    (concat_!(r, t.j); j := j+1)

```

```

while i <= m repeat (concat_!(r, s.i); i := i+1)
while j <= n repeat (concat_!(r, t.j); j := j+1)
r

else
insert_!(x, s) ==
  for k in minIndex s .. maxIndex s repeat
    s.k = x => return s
  insert_!(x, s, inc maxIndex s)

remove_!(x:S, s:%) ==
  n := inc maxIndex s
  k := minIndex s
  while k < n repeat
    x = s.k => return delete_!(s, k)
    k := inc k
s

```

---

— SET.dotabb —

```

"SET" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SET"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"SET" -> "ALIST"

```

## 20.5 domain SETMN SetOfMIntegersInOneToN

---

— SetOfMIntegersInOneToN.input —

```

)set break resume
)sys rm -f SetOfMIntegersInOneToN.output
)spool SetOfMIntegersInOneToN.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SetOfMIntegersInOneToN
--R SetOfMIntegersInOneToN(m: PositiveInteger,n: PositiveInteger) is a domain constructor
--R Abbreviation for SetOfMIntegersInOneToN is SETMN
--R This constructor is not exposed in this frame.

```

```

--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SETMN
--R
--R----- Operations -----
--R ==? : (%,% ) -> Boolean          coerce : % -> OutputForm
--R enumerate : () -> Vector %      hash : % -> SingleInteger
--R index : PositiveInteger -> %    latex : % -> String
--R lookup : % -> PositiveInteger    random : () -> %
--R size : () -> NonNegativeInteger  ?~=? : (%,% ) -> Boolean
--R delta : (% ,PositiveInteger,PositiveInteger) -> NonNegativeInteger
--R elements : % -> List PositiveInteger
--R incrementKthElement : (% ,PositiveInteger) -> Union(%, "failed")
--R member? : (PositiveInteger,%) -> Boolean
--R replaceKthElement : (% ,PositiveInteger,PositiveInteger) -> Union(%, "failed")
--R setOfMinN : List PositiveInteger -> %
--R
--E 1

)spool
)lisp (bye)

```

---

— SetOfMinIntegersInOneToN.help —

```

=====
SetOfMinIntegersInOneToN examples
=====

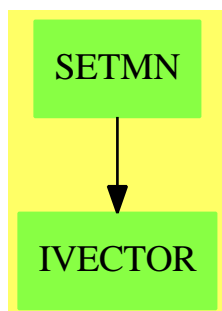
```

```

See Also:
o )show SetOfMinIntegersInOneToN

```

### 20.5.1 SetOfMinIntegersInOneToN (SETMN)



**Exports:**

|                     |                   |           |           |         |
|---------------------|-------------------|-----------|-----------|---------|
| coerce              | delta             | elements  | enumerate | hash    |
| incrementKthElement | index             | latex     | lookup    | member? |
| random              | replaceKthElement | setOfMinN | size      | ?=?     |
| ?~=?                |                   |           |           |         |

— domain SETMN SetOfMIntegersInOneToN —

```

)abbrev domain SETMN SetOfMIntegersInOneToN
++ Author: Manuel Bronstein
++ Date Created: 10 January 1994
++ Date Last Updated: 10 January 1994
++ Description:
++ \spadtype{SetOfMIntegersInOneToN} implements the subsets of M integers
++ in the interval \spad{1..n}

SetOfMIntegersInOneToN(m, n): Exports == Implementation where
  PI ==> PositiveInteger
  N  ==> NonNegativeInteger
  U  ==> Union(%, "failed")
  n,m: PI

Exports ==> Finite with
  incrementKthElement: (%, PI) -> U
    ++ incrementKthElement(S,k) increments the kth element of S,
    ++ and returns "failed" if the result is not a set of M integers
    ++ in \spad{1..n} any more.
  replaceKthElement: (%, PI, PI) -> U
    ++ replaceKthElement(S,k,p) replaces the kth element of S by p,
    ++ and returns "failed" if the result is not a set of M integers
    ++ in \spad{1..n} any more.
  elements: % -> List PI
    ++ elements(S) returns the list of the elements of S in increasing order.
  setOfMinN: List PI -> %
    ++ setOfMinN([a1,...,am]) returns the set {a1,...,am}.
    ++ Error if {a1,...,am} is not a set of M integers in \spad{1..n}.
  enumerate: () -> Vector %
    ++ enumerate() returns a vector of all the sets of M integers in
    ++ \spad{1..n}.
  member?: (PI, %) -> Boolean
    ++ member?(p, s) returns true is p is in s, false otherwise.
  delta: (%, PI, PI) -> N
    ++ delta(S,k,p) returns the number of elements of S which are strictly
    ++ between p and the kth element of S.

Implementation ==> add
  Rep := Record(bits:Bits, pos:N)

  reallyEnumerate: () -> Vector %
  enum: (N, N, PI) -> List Bits

```

```

all:Reference Vector % := ref empty()
sz:Reference N := ref 0

s1 = s2 == s1.bits == $Bits s2.bits
coerce(s:%):OutputForm == brace [i::OutputForm for i in elements s]
random() == index((1 + (random())$Integer rem size()))::PI
reallyEnumerate() == [[b, i] for b in enum(m, n, n) for i in 1..]
member?(p, s) == s.bits.p

enumerate() ==
  if empty? all() then all() := reallyEnumerate()
  all()

-- enumerates the sets of p integers in 1..q, returns them as sets in 1..n
-- must have p <= q
enum(p, q, n) ==
  zero? p or zero? q => empty()
  p = q =>
    b := new(n, false)$Bits
    for i in 1..p repeat b.i := true
    [b]
  q1 := (q - 1)::N
  l := enum((p - 1)::N, q1, n)
  if empty? l then l := [new(n, false)$Bits]
  for s in l repeat s.q := true
  concat_!(enum(p, q1, n), l)

size() ==
  if zero? sz() then
    sz() := binomial(n, m)$IntegerCombinatoricFunctions(Integer) :: N
  sz()

lookup s ==
  if empty? all() then all() := reallyEnumerate()
  if zero?(s.pos) then s.pos := position(s, all()) :: N
  s.pos :: PI

index p ==
  p > size() => error "index: argument too large"
  if empty? all() then all() := reallyEnumerate()
  all().p

setOfMinN l ==
  s := new(n, false)$Bits
  count:N := 0
  for i in l repeat
    count := count + 1
    count > m or zero? i or i > n or s.i =>
      error "setOfMinN: improper set of integers"

```



```

    s.i := true
    count < m => error "setOfMinN: improper set of integers"
    [s, 0]

elements s ==
  b := s.bits
  l:List PI := empty()
  found:N := 0
  i:PI := 1
  while found < m repeat
    if b.i then
      l := concat(i, l)
      found := found + 1
      i := i + 1
  reverse_! l

incrementKthElement(s, k) ==
  b := s.bits
  found:N := 0
  i:N := 1
  while found < k repeat
    if b.i then found := found + 1
    i := i + 1
  i > n or b.i => "failed"
  newb := copy b
  newb.i := true
  newb.((i-1):N) := false
  [newb, 0]

delta(s, k, p) ==
  b := s.bits
  count:N := found:N := 0
  i:PI := 1
  while found < k repeat
    if b.i then
      found := found + 1
      if i > p and found < k then count := count + 1
      i := i + 1
  count

replaceKthElement(s, k, p) ==
  b := s.bits
  found:N := 0
  i:PI := 1
  while found < k repeat
    if b.i then found := found + 1
    if found < k then i := i + 1
  b.p and i ^= p => "failed"
  newb := copy b
  newb.p := true

```

```
newb.i := false
[newb, (i = p => s.pos; 0)]
```

---

— SETMN.dotabb —

```
"SETMN" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SETMN"]
"IVECTOR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=IVECTOR"]
"SETMN" -> "IVECTOR"
```

---

## 20.6 domain SDPOL SequentialDifferentialPolynomial

— SequentialDifferentialPolynomial.input —

```
)set break resume
)sys rm -f SequentialDifferentialPolynomial.output
)spool SequentialDifferentialPolynomial.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SequentialDifferentialPolynomial
--R SequentialDifferentialPolynomial R: Ring is a domain constructor
--R Abbreviation for SequentialDifferentialPolynomial is SDPOL
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SDPOL
--R
--R----- Operations -----
--R ??? : (%,R) -> %               ??? : (R,%) -> %
--R ??? : (%,%) -> %               ??? : (Integer,%) -> %
--R ??? : (PositiveInteger,%) -> %  ??? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %               ?-? : (%,%) -> %
--R -? : % -> %                     ?? : (%,%) -> Boolean
--R D : (%,(R -> R)) -> %          D : % -> % if R has DIFRING
--R 1 : () -> %                     0 : () -> %
--R ?? : (%,PositiveInteger) -> %  coefficients : % -> List R
--R coerce : Symbol -> %           coerce : R -> %
--R coerce : Integer -> %          coerce : % -> OutputForm
--R eval : (%,List %,List %) -> %  eval : (%,%,%) -> %
--R eval : (%,Equation %) -> %    eval : (%,List Equation %) -> %
```

```

--R ground : % -> R
--R hash : % -> SingleInteger
--R isobaric? : % -> Boolean
--R leadingCoefficient : % -> R
--R map : ((R -> R),%) -> %
--R monomials : % -> List %
--R order : % -> NonNegativeInteger
--R recip : % -> Union(%, "failed")
--R retract : % -> Symbol
--R sample : () -> %
--R weight : % -> NonNegativeInteger
--R ~=? : (%,%) -> Boolean
--R ?? : (Fraction Integer,%) -> % if R has ALGEBRA FRAC INT
--R ?? : (%, Fraction Integer) -> % if R has ALGEBRA FRAC INT
--R ?? : (NonNegativeInteger,%) -> %
--R ***? : (%, NonNegativeInteger) -> %
--R ?/? : (%, R) -> % if R has FIELD
--R ?<? : (%,%) -> Boolean if R has ORDSET
--R ?<=? : (%,%) -> Boolean if R has ORDSET
--R ?>? : (%,%) -> Boolean if R has ORDSET
--R ?>=? : (%,%) -> Boolean if R has ORDSET
--R D : (%, (R -> R), NonNegativeInteger) -> %
--R D : (%, List Symbol, List NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (%, Symbol, NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (%, List Symbol) -> % if R has PDRING SYMBOL
--R D : (%, Symbol) -> % if R has PDRING SYMBOL
--R D : (%, NonNegativeInteger) -> % if R has DIFRING
--R D : (%, List SequentialDifferentialVariable Symbol, List NonNegativeInteger) -> %
--R D : (%, SequentialDifferentialVariable Symbol, NonNegativeInteger) -> %
--R D : (%, List SequentialDifferentialVariable Symbol) -> %
--R D : (%, SequentialDifferentialVariable Symbol) -> %
--R ?? : (%, NonNegativeInteger) -> %
--R associates? : (%,%) -> Boolean if R has INTDOM
--R binomThmExpt : (%,%, NonNegativeInteger) -> % if R has COMRING
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%, "failed") if $ has CHARNZ and R has PFECAT or R has CHARNZ
--R coefficient : (%, List SequentialDifferentialVariable Symbol, List NonNegativeInteger) -> %
--R coefficient : (%, SequentialDifferentialVariable Symbol, NonNegativeInteger) -> %
--R coefficient : (%, IndexedExponents SequentialDifferentialVariable Symbol) -> R
--R coerce : % -> % if R has INTDOM
--R coerce : Fraction Integer -> % if R has ALGEBRA FRAC INT or R has RETRACT FRAC INT
--R coerce : SparseMultivariatePolynomial(R, Symbol) -> %
--R coerce : SequentialDifferentialVariable Symbol -> %
--R conditionP : Matrix % -> Union(Vector %, "failed") if $ has CHARNZ and R has PFECAT
--R content : (%, SequentialDifferentialVariable Symbol) -> % if R has GCDDOM
--R content : % -> R if R has GCDDOM
--R convert : % -> InputForm if SequentialDifferentialVariable Symbol has KONVERT INFORM and
--R convert : % -> Pattern Integer if SequentialDifferentialVariable Symbol has KONVERT PATTI
--R convert : % -> Pattern Float if SequentialDifferentialVariable Symbol has KONVERT PATTERN
--R degree : (%, Symbol) -> NonNegativeInteger

```

```

--R degree : (% ,List SequentialDifferentialVariable Symbol) -> List NonNegativeInteger
--R degree : (% ,SequentialDifferentialVariable Symbol) -> NonNegativeInteger
--R degree : % -> IndexedExponents SequentialDifferentialVariable Symbol
--R differentialVariables : % -> List Symbol
--R differentiate : (% ,(R -> R)) -> %
--R differentiate : (% ,(R -> R),NonNegativeInteger) -> %
--R differentiate : (% ,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
--R differentiate : (% ,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
--R differentiate : (% ,List Symbol) -> % if R has PDRING SYMBOL
--R differentiate : (% ,Symbol) -> % if R has PDRING SYMBOL
--R differentiate : (% ,NonNegativeInteger) -> % if R has DIFRING
--R differentiate : % -> % if R has DIFRING
--R differentiate : (% ,List SequentialDifferentialVariable Symbol,List NonNegativeInteger) -> %
--R differentiate : (% ,SequentialDifferentialVariable Symbol,NonNegativeInteger) -> %
--R differentiate : (% ,List SequentialDifferentialVariable Symbol) -> %
--R differentiate : (% ,SequentialDifferentialVariable Symbol) -> %
--R discriminant : (% ,SequentialDifferentialVariable Symbol) -> % if R has COMRING
--R eval : (% ,List Symbol,List R) -> % if R has DIFRING
--R eval : (% ,Symbol,R) -> % if R has DIFRING
--R eval : (% ,List Symbol,List %) -> % if R has DIFRING
--R eval : (% ,Symbol,%) -> % if R has DIFRING
--R eval : (% ,List SequentialDifferentialVariable Symbol,List %) -> %
--R eval : (% ,SequentialDifferentialVariable Symbol,%) -> %
--R eval : (% ,List SequentialDifferentialVariable Symbol,List R) -> %
--R eval : (% ,SequentialDifferentialVariable Symbol,R) -> %
--R exquo : (% ,%) -> Union(% ,"failed") if R has INTDOM
--R exquo : (% ,R) -> Union(% ,"failed") if R has INTDOM
--R factor : % -> Factored % if R has PFECAT
--R factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % if R has PF
--R factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % i
--R gcd : (% ,%) -> % if R has GCDDOM
--R gcd : List % -> % if R has GCDDOM
--R gcdPolynomial : (SparseUnivariatePolynomial % ,SparseUnivariatePolynomial %) -> SparseUnivariatePolyn
--R isExpt : % -> Union(Record(var: SequentialDifferentialVariable Symbol,exponent: NonNegativeInteger),
--R isPlus : % -> Union(List % ,"failed")
--R isTimes : % -> Union(List % ,"failed")
--R lcm : (% ,%) -> % if R has GCDDOM
--R lcm : List % -> % if R has GCDDOM
--R leader : % -> SequentialDifferentialVariable Symbol
--R mainVariable : % -> Union(SequentialDifferentialVariable Symbol,"failed")
--R makeVariable : % -> (NonNegativeInteger -> %) if R has DIFRING
--R makeVariable : Symbol -> (NonNegativeInteger -> %)
--R mapExponents : ((IndexedExponents SequentialDifferentialVariable Symbol -> IndexedExponents Sequenti
--R max : (% ,%) -> % if R has ORDSET
--R min : (% ,%) -> % if R has ORDSET
--R minimumDegree : (% ,List SequentialDifferentialVariable Symbol) -> List NonNegativeInteger
--R minimumDegree : (% ,SequentialDifferentialVariable Symbol) -> NonNegativeInteger
--R minimumDegree : % -> IndexedExponents SequentialDifferentialVariable Symbol
--R monicDivide : (% ,%,SequentialDifferentialVariable Symbol) -> Record(quotient: %,remainder: %)
--R monomial : (% ,List SequentialDifferentialVariable Symbol,List NonNegativeInteger) -> %

```

```

--R monomial : (%,SequentialDifferentialVariable Symbol,NonNegativeInteger) -> %
--R monomial : (R,IndexedExponents SequentialDifferentialVariable Symbol) -> %
--R multivariate : (SparseUnivariatePolynomial %,SequentialDifferentialVariable Symbol) -> %
--R multivariate : (SparseUnivariatePolynomial R,SequentialDifferentialVariable Symbol) -> %
--R numberOfMonomials : % -> NonNegativeInteger
--R order : (%,Symbol) -> NonNegativeInteger
--R patternMatch : (%,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMatchResult(Integer,%)
--R patternMatch : (%,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchResult(Float,%)
--R pomopo! : (%,R,IndexedExponents SequentialDifferentialVariable Symbol,%) -> %
--R prime? : % -> Boolean if R has PFECAT
--R primitivePart : (%,SequentialDifferentialVariable Symbol) -> % if R has GCDDOM
--R primitivePart : % -> % if R has GCDDOM
--R reducedSystem : Matrix % -> Matrix R
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) if R has LINEXP INT
--R reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT
--R resultant : (%,%,SequentialDifferentialVariable Symbol) -> % if R has COMRING
--R retract : % -> SparseMultivariatePolynomial(R,Symbol)
--R retract : % -> SequentialDifferentialVariable Symbol
--R retract : % -> Integer if R has RETRACT INT
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(SparseMultivariatePolynomial(R,Symbol),"failed")
--R retractIfCan : % -> Union(Symbol,"failed")
--R retractIfCan : % -> Union(SequentialDifferentialVariable Symbol,"failed")
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(R,"failed")
--R solveLinearPolynomialEquation : (List SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> List SparseUnivariatePolynomial %
--R squareFree : % -> Factored % if R has GCDDOM
--R squareFreePart : % -> % if R has GCDDOM
--R squareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R totalDegree : (%,List SequentialDifferentialVariable Symbol) -> NonNegativeInteger
--R totalDegree : % -> NonNegativeInteger
--R unit? : % -> Boolean if R has INTDOM
--R unitCanonical : % -> % if R has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has INTDOM
--R univariate : % -> SparseUnivariatePolynomial R
--R univariate : (%,SequentialDifferentialVariable Symbol) -> SparseUnivariatePolynomial %
--R variables : % -> List SequentialDifferentialVariable Symbol
--R weight : (%,Symbol) -> NonNegativeInteger
--R weights : (%,Symbol) -> List NonNegativeInteger
--R weights : % -> List NonNegativeInteger
--R
--E 1

)spool
)lisp (bye)

```

---

## — SequentialDifferentialPolynomial.help —

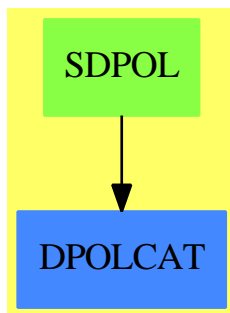
```
=====
SequentialDifferentialPolynomial examples
=====
```

See Also:

```
o )show SequentialDifferentialPolynomial
```

---

## 20.6.1 SequentialDifferentialPolynomial (SDPOL)



See

- ⇒ “OrderlyDifferentialVariable” (ODVAR) 16.17.1 on page 1816
- ⇒ “SequentialDifferentialVariable” (SDVAR) 20.7.1 on page 2348
- ⇒ “DifferentialSparseMultivariatePolynomial” (DSMP) 5.8.1 on page 526
- ⇒ “OrderlyDifferentialPolynomial” (ODPOL) 16.16.1 on page 1813

**Exports:**

|                            |                    |                               |
|----------------------------|--------------------|-------------------------------|
| 0                          | 1                  | associates?                   |
| binomThmExpt               | characteristic     | charthRoot                    |
| coefficient                | coefficients       | coerce                        |
| conditionP                 | content            | convert                       |
| D                          | degree             | differentialVariables         |
| differentiate              | discriminant       | eval                          |
| exquo                      | factor             | factorPolynomial              |
| factorSquareFreePolynomial | gcd                | gcdPolynomial                 |
| ground                     | ground?            | hash                          |
| initial                    | isExpt             | isobaric?                     |
| isPlus                     | isTimes            | latex                         |
| lcm                        | leader             | leadingCoefficient            |
| leadingMonomial            | mainVariable       | makeVariable                  |
| map                        | mapExponents       | max                           |
| min                        | minimumDegree      | monicDivide                   |
| monomial                   | monomial?          | monomials                     |
| multivariate               | numberOfMonomials  | one?                          |
| order                      | patternMatch       | pomopo!                       |
| prime?                     | primitiveMonomials | primitivePart                 |
| recip                      | reducedSystem      | reductum                      |
| resultant                  | retract            | retractIfCan                  |
| sample                     | separant           | solveLinearPolynomialEquation |
| squareFree                 | squareFreePart     | squareFreePolynomial          |
| subtractIfCan              | totalDegree        | unit?                         |
| unitCanonical              | unitNormal         | univariate                    |
| variables                  | weight             | weights                       |
| zero?                      | ?^?                | ?*?                           |
| ?**?                       | ?+?                | ?-?                           |
| -?                         | ?=?                | ?~=?                          |
| ?/?                        | ?<?                | ?<=?                          |
| ?>?                        | ?>=?               |                               |

— domain SDPOL SequentialDifferentialPolynomial —

```

)abbrev domain SDPOL SequentialDifferentialPolynomial
++ Author: William Sit
++ Date Created: 24 September, 1991
++ Date Last Updated: 7 February, 1992
++ Basic Operations:DifferentialPolynomialCategory
++ Related Constructors: DifferentialSparseMultivariatePolynomial
++ See Also:
++ AMS Classifications:12H05
++ Keywords: differential indeterminates, ranking, differential polynomials,
++          order, weight, leader, separant, initial, isobaric
++ References:Kolchin, E.R. "Differential Algebra and Algebraic Groups"
++          (Academic Press, 1973).
++ Description:

```

```

++ \spadtype{SequentialDifferentialPolynomial} implements
++ an ordinary differential polynomial ring in arbitrary number
++ of differential indeterminates, with coefficients in a
++ ring. The ranking on the differential indeterminate is sequential.

```

```

SequentialDifferentialPolynomial(R):
  Exports == Implementation where
  R: Ring
  S ==> Symbol
  V ==> SequentialDifferentialVariable S
  E ==> IndexedExponents(V)
  SMP ==> SparseMultivariatePolynomial(R, S)
  Exports ==> Join(DifferentialPolynomialCategory(R,S,V,E),
    RetractableTo SMP)

Implementation ==> DifferentialSparseMultivariatePolynomial(R,S,V)

```

---

— SDPOL.dotabb —

```

"SDPOL" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SDPOL"]
"DPOLCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=DPOLCAT"]
"SDPOL" -> "DPOLCAT"

```

## 20.7 domain SDVAR SequentialDifferentialVariable

— SequentialDifferentialVariable.input —

```

)set break resume
)sys rm -f SequentialDifferentialVariable.output
)spool SequentialDifferentialVariable.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SequentialDifferentialVariable
--R SequentialDifferentialVariable S: OrderedSet is a domain constructor
--R Abbreviation for SequentialDifferentialVariable is SDVAR
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SDVAR

```



```

--R
--R----- Operations -----
--R ?<? : (%,% ) -> Boolean          ?<=? : (%,% ) -> Boolean
--R ?=? : (%,% ) -> Boolean          ?>? : (%,% ) -> Boolean
--R ?>=? : (%,% ) -> Boolean        coerce : S -> %
--R coerce : % -> OutputForm        differentiate : % -> %
--R hash : % -> SingleInteger        latex : % -> String
--R max : (%,% ) -> %                min : (%,% ) -> %
--R order : % -> NonNegativeInteger  retract : % -> S
--R variable : % -> S                weight : % -> NonNegativeInteger
--R ?~=? : (%,% ) -> Boolean
--R differentiate : (% ,NonNegativeInteger) -> %
--R makeVariable : (S,NonNegativeInteger) -> %
--R retractIfCan : % -> Union(S,"failed")
--R
--E 1

)spool
)lisp (bye)

```

— SequentialDifferentialVariable.help —

```

=====
SequentialDifferentialVariable examples
=====

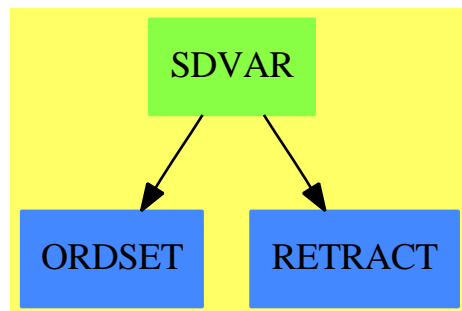
```

```

See Also:
o )show SequentialDifferentialVariable

```

### 20.7.1 SequentialDifferentialVariable (SDVAR)



See

- ⇒ “OrderlyDifferentialVariable” (ODVAR) 16.17.1 on page 1816
- ⇒ “DifferentialSparseMultivariatePolynomial” (DSMP) 5.8.1 on page 526
- ⇒ “OrderlyDifferentialPolynomial” (ODPOL) 16.16.1 on page 1813
- ⇒ “SequentialDifferentialPolynomial” (SDPOL) 20.6.1 on page 2345

**Exports:**

|          |               |       |         |              |
|----------|---------------|-------|---------|--------------|
| coerce   | differentiate | hash  | latex   | makeVariable |
| max      | min           | order | retract | retractIfCan |
| variable | weight        | ?~=?  | ?<?     | ?<=?         |
| ?=?      | ?>?           | ?>=?  |         |              |

— domain SDVAR SequentialDifferentialVariable —

```

)abbrev domain SDVAR SequentialDifferentialVariable
++ Author: William Sit
++ Date Created: 19 July 1990
++ Date Last Updated: 13 September 1991
++ Basic Operations: differentiate, order, variable, <
++ Related Domains: OrderedVariableList,
++                      OrderlyDifferentialVariable.
++ See Also: DifferentialVariableCategory
++ AMS Classifications: 12H05
++ Keywords: differential indeterminates, sequential ranking.
++ References: Kolchin, E.R. "Differential Algebra and Algebraic Groups"
++             (Academic Press, 1973).
++ Description:
++ \spadtype{OrderlyDifferentialVariable} adds a commonly used sequential
++ ranking to the set of derivatives of an ordered list of differential
++ indeterminates. A sequential ranking is a ranking \spadfun{<} of the
++ derivatives with the property that for any derivative v,
++ there are only a finite number of derivatives u with u \spadfun{<} v.
++ This domain belongs to \spadtype{DifferentialVariableCategory}. It
++ defines \spadfun{weight} to be just \spadfun{order}, and it
++ defines a sequential ranking \spadfun{<} on derivatives u by the
++ lexicographic order on the pair
++ (\spadfun{variable}(u), \spadfun{order}(u)).

SequentialDifferentialVariable(S:OrderedSet):DifferentialVariableCategory(S)
== add
  Rep := Record(var:S, ord:NonNegativeInteger)
  makeVariable(s,n) == [s, n]
  variable v      == v.var
  order v         == v.ord
  v < u ==
    variable v = variable u => order v < order u
    variable v < variable u

```

— SDVAR.dotabb —

```
"SDVAR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SDVAR"]
"ORDSET" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ORDSET"]
"RETRACT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=RETRACT"]
"SDVAR" -> "ORDSET"
"SDVAR" -> "RETRACT"
```

## 20.8 domain SEX SExpression

— SExpression.input —

```
)set break resume
)sys rm -f SExpression.output
)spool SExpression.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SExpression
--R SExpression is a domain constructor
--R Abbreviation for SExpression is SEX
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SEX
--R
--R----- Operations -----
--R #? : % -> Integer           ?? : (%,%) -> Boolean
--R atom? : % -> Boolean       car : % -> %
--R cdr : % -> %               coerce : % -> OutputForm
--R convert : OutputForm -> %   convert : DoubleFloat -> %
--R convert : Integer -> %      convert : Symbol -> %
--R convert : String -> %       convert : List % -> %
--R destruct : % -> List %      ?? : (%,List Integer) -> %
--R ?.? : (%,Integer) -> %      eq : (%,%) -> Boolean
--R expr : % -> OutputForm      float : % -> DoubleFloat
--R float? : % -> Boolean       hash : % -> SingleInteger
--R integer : % -> Integer      integer? : % -> Boolean
--R latex : % -> String         list? : % -> Boolean
--R null? : % -> Boolean        pair? : % -> Boolean
--R string : % -> String        string? : % -> Boolean
--R symbol : % -> Symbol        symbol? : % -> Boolean
--R ~=? : (%,%) -> Boolean
```

```
--R
--E 1

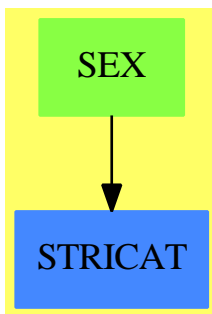
)spool
)lisp (bye)
```

— SExpression.help —

```
=====
SExpression examples
=====
```

```
See Also:
o )show SExpression
```

### 20.8.1 SExpression (SEX)



See

⇒ “SExpressionOf” (SEXOF) 20.9.1 on page 2353

#### Exports:

|          |         |          |         |         |
|----------|---------|----------|---------|---------|
| atom?    | car     | cdr      | coerce  | convert |
| destruct | eq      | expr     | float   | float?  |
| hash     | integer | integer? | latex   | list?   |
| null?    | pair?   | string   | string? | symbol  |
| symbol?  | #?      | ?.?      | ?=?     | ?~=?    |

— domain SEX SExpression —

```
)abbrev domain SEX SExpression
++ Author: S.M.Watt
```

```

++ Date Created: July 1987
++ Date Last Modified: 23 May 1991
++ Description:
++ This domain allows the manipulation of the usual Lisp values;

SExpression()
  == SExpressionOf(String, Symbol, Integer, DoubleFloat, OutputForm)

```

---

— SEX.dotabb —

```

"SEX" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SEX"]
"STRICAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=STRICAT"]
"SEX" -> "STRICAT"

```

---

## 20.9 domain SEXOF SExpressionOf

— SExpressionOf.input —

```

)set break resume
)sys rm -f SExpressionOf.output
)spool SExpressionOf.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SExpressionOf
--R SExpressionOf(Str: SetCategory,Sym: SetCategory,Int: SetCategory,Flt: SetCategory,Expr: S
--R Abbreviation for SExpressionOf is SEXOF
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SEXOF
--R
--R----- Operations -----
--R #? : % -> Integer          ?? : (%,%) -> Boolean
--R atom? : % -> Boolean      car : % -> %
--R cdr : % -> %              coerce : % -> OutputForm
--R convert : Expr -> %       convert : Flt -> %
--R convert : Int -> %        convert : Sym -> %
--R convert : Str -> %        convert : List % -> %
--R destruct : % -> List %    ?? : (%,List Integer) -> %

```

```

--R ?.? : (%,Integer) -> %
--R expr : % -> Expr
--R float? : % -> Boolean
--R integer : % -> Int
--R latex : % -> String
--R null? : % -> Boolean
--R string : % -> Str
--R symbol : % -> Sym
--R ?~=? : (%,% ) -> Boolean
--R
--E 1

)spool
)lisp (bye)

```

---

— SExpressionOf.help —

=====

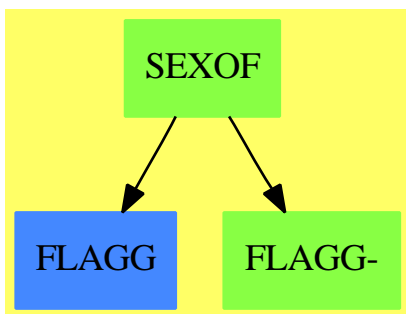
SExpressionOf examples

=====

See Also:

- o )show SExpressionOf

### 20.9.1 SExpressionOf (SEXOF)



See

⇒ “SExpression” (SEX) 20.8.1 on page 2351

**Exports:**

|          |         |          |         |         |
|----------|---------|----------|---------|---------|
| atom?    | car     | cdr      | coerce  | convert |
| destruct | eq      | expr     | float   | float?  |
| hash     | integer | integer? | latex   | list?   |
| null?    | pair?   | string   | string? | symbol  |
| symbol?  | #?      | ??       | ?=?     | ?~=?    |

— domain SEXOF SExpressionOf —

```

)abbrev domain SEXOF SExpressionOf
++ Author: S.M.Watt
++ Date Created: July 1987
++ Date Last Modified: 23 May 1991
++ Description:
++ This domain allows the manipulation of Lisp values over
++ arbitrary atomic types.
-- Allows the names of the atomic types to be chosen.
-- *** Warning *** Although the parameters are declared only to be Sets,
-- *** Warning *** they must have the appropriate representations.

SExpressionOf(Str, Sym, Int, Flt, Expr): Decl == Body where
  Str, Sym, Int, Flt, Expr: SetCategory

Decl ==> SExpressionCategory(Str, Sym, Int, Flt, Expr)

Body ==> add
  Rep := Expr

  dotex:OutputForm := INTERN(".")$Lisp

  coerce(b:%):OutputForm ==
    null? b => paren empty()
    atom? b => coerce(b)$Rep
    r := b
    while not atom? r repeat r := cdr r
    l1 := [b1::OutputForm for b1 in (l := destruct b)]
    not null? r =>
      paren blankSeparate concat_!(l1, [dotex, r::OutputForm])
    #l = 2 and (first(l1) = QUOTE)@Boolean => quote first rest l1
    paren blankSeparate l1

  b1 = b2      == EQUAL(b1,b2)$Lisp
  eq(b1, b2)   == EQ(b1,b2)$Lisp

  null? b      == NULL(b)$Lisp
  atom? b      == ATOM(b)$Lisp
  pair? b      == CONSP(b)$Lisp

  list?  b     == CONSP(b)$Lisp or NULL(b)$Lisp
  string? b    == STRINGP(b)$Lisp
  symbol? b    == IDENTP(b)$Lisp

```

```

integer? b == INTEGERP(b)$Lisp
float?  b  == RNUMP(b)$Lisp

destruct b == (list? b    => b pretend List %; error "Non-list")
string b == (STRINGP(b)$Lisp=> b pretend Str;error "Non-string")
symbol b == (IDENTP(b)$Lisp => b pretend Sym;error "Non-symbol")
float  b == (RNUMP(b)$Lisp => b pretend Flt;error "Non-float")
integer b == (INTEGERP(b)$Lisp => b pretend Int;error "Non-integer")
expr   b == b pretend Expr

convert(l: List %) == l pretend %
convert(st: Str)   == st pretend %
convert(sy: Sym)   == sy pretend %
convert(n: Int)    == n  pretend %
convert(f: Flt)    == f  pretend %
convert(e: Expr)   == e  pretend %

car b          == CAR(b)$Lisp
cdr b          == CDR(b)$Lisp
# b           == LENGTH(b)$Lisp
elt(b:%, i:Integer) == destruct(b).i
elt(b:%, li:List Integer) ==
  for i in li repeat b := destruct(b).i
  b

```

---

— SEXOF.dotabb —

```

"SEXOF" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SEXOF"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"FLAGG-" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FLAGG"]
"SEXOF" -> "FLAGG"
"SEXOF" -> "FLAGG-"

```

---

## 20.10 domain SAE SimpleAlgebraicExtension

— SimpleAlgebraicExtension.input —

```

)set break resume
)sys rm -f SimpleAlgebraicExtension.output
)spool SimpleAlgebraicExtension.output

```



```

)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SimpleAlgebraicExtension
--R SimpleAlgebraicExtension(R: CommutativeRing,UP: UnivariatePolynomialCategory R,M: UP) i
--R Abbreviation for SimpleAlgebraicExtension is SAE
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SAE
--R
--R----- Operations -----
--R ?? : (R,%) -> %          ?? : (%,R) -> %
--R ?? : (%,%) -> %          ?? : (Integer,%) -> %
--R ?? : (PositiveInteger,%) -> %    ??? : (%,PositiveInteger) -> %
--R +? : (%,%) -> %          -? : (%,%) -> %
--R -? : % -> %              ?? : (%,%) -> Boolean
--R 1 : () -> %              0 : () -> %
--R ^? : (%,PositiveInteger) -> %    basis : () -> Vector %
--R coerce : R -> %              coerce : Integer -> %
--R coerce : % -> OutputForm        convert : UP -> %
--R convert : % -> UP              convert : Vector R -> %
--R convert : % -> Vector R        coordinates : % -> Vector R
--R definingPolynomial : () -> UP    discriminant : () -> R
--R discriminant : Vector % -> R      generator : () -> %
--R hash : % -> SingleInteger        inv : % -> % if R has FIELD
--R latex : % -> String              lift : % -> UP
--R norm : % -> R                    one? : % -> Boolean
--R rank : () -> PositiveInteger      recip : % -> Union(%, "failed")
--R reduce : UP -> %                  represents : Vector R -> %
--R retract : % -> R                  sample : () -> %
--R trace : % -> R                    traceMatrix : () -> Matrix R
--R zero? : % -> Boolean              ~=? : (%,%) -> Boolean
--R ?? : (%,Fraction Integer) -> % if R has FIELD
--R ?? : (Fraction Integer,%) -> % if R has FIELD
--R ?? : (NonNegativeInteger,%) -> %
--R ??? : (%,Integer) -> % if R has FIELD
--R ??? : (%,NonNegativeInteger) -> %
--R ?/? : (%,%) -> % if R has FIELD
--R D : (%,(R -> R)) -> % if R has FIELD
--R D : (%,(R -> R),NonNegativeInteger) -> % if R has FIELD
--R D : (%,List Symbol,List NonNegativeInteger) -> % if R has FIELD and R has PDRING SYMBOL
--R D : (%,Symbol,NonNegativeInteger) -> % if R has FIELD and R has PDRING SYMBOL
--R D : (%,List Symbol) -> % if R has FIELD and R has PDRING SYMBOL
--R D : (%,Symbol) -> % if R has FIELD and R has PDRING SYMBOL
--R D : (%,NonNegativeInteger) -> % if R has DIFRING and R has FIELD or R has FFIELDC
--R D : % -> % if R has DIFRING and R has FIELD or R has FFIELDC
--R ^? : (%,Integer) -> % if R has FIELD
--R ^? : (%,NonNegativeInteger) -> %
--R associates? : (%,%) -> Boolean if R has FIELD

```

```

--R characteristic : () -> NonNegativeInteger
--R characteristicPolynomial : % -> UP
--R charthRoot : % -> Union(%, "failed") if R has CHARNZ
--R charthRoot : % -> % if R has FFIELDC
--R coerce : Fraction Integer -> % if R has FIELD or R has RETRACT FRAC INT
--R coerce : % -> % if R has FIELD
--R conditionP : Matrix % -> Union(Vector %, "failed") if R has FFIELDC
--R coordinates : Vector % -> Matrix R
--R coordinates : (Vector %, Vector %) -> Matrix R
--R coordinates : (%, Vector %) -> Vector R
--R createPrimitiveElement : () -> % if R has FFIELDC
--R derivationCoordinates : (Vector %, (R -> R)) -> Matrix R if R has FIELD
--R differentiate : (%, (R -> R)) -> % if R has FIELD
--R differentiate : (%, (R -> R), NonNegativeInteger) -> % if R has FIELD
--R differentiate : (%, List Symbol, List NonNegativeInteger) -> % if R has FIELD and R has PDRING SYMBOL
--R differentiate : (%, Symbol, NonNegativeInteger) -> % if R has FIELD and R has PDRING SYMBOL
--R differentiate : (%, List Symbol) -> % if R has FIELD and R has PDRING SYMBOL
--R differentiate : (%, Symbol) -> % if R has FIELD and R has PDRING SYMBOL
--R differentiate : (%, NonNegativeInteger) -> % if R has DIFRING and R has FIELD or R has FFIELDC
--R differentiate : % -> % if R has DIFRING and R has FIELD or R has FFIELDC
--R discreteLog : (%, %) -> Union(NonNegativeInteger, "failed") if R has FFIELDC
--R discreteLog : % -> NonNegativeInteger if R has FFIELDC
--R divide : (%, %) -> Record(quotient: %, remainder: %) if R has FIELD
--R euclideanSize : % -> NonNegativeInteger if R has FIELD
--R expressIdealMember : (List %, %) -> Union(List %, "failed") if R has FIELD
--R exquo : (%, %) -> Union(%, "failed") if R has FIELD
--R extendedEuclidean : (%, %) -> Record(coef1: %, coef2: %, generator: %) if R has FIELD
--R extendedEuclidean : (%, %, %) -> Union(Record(coef1: %, coef2: %), "failed") if R has FIELD
--R factor : % -> Factored % if R has FIELD
--R factorsOfCyclicGroupSize : () -> List Record(factor: Integer, exponent: Integer) if R has FFIELDC
--R gcd : (%, %) -> % if R has FIELD
--R gcd : List % -> % if R has FIELD
--R gcdPolynomial : (SparseUnivariatePolynomial %, SparseUnivariatePolynomial %) -> SparseUnivariatePolynomial %
--R index : PositiveInteger -> % if R has FINITE
--R init : () -> % if R has FFIELDC
--R lcm : (%, %) -> % if R has FIELD
--R lcm : List % -> % if R has FIELD
--R lookup : % -> PositiveInteger if R has FINITE
--R minimalPolynomial : % -> UP if R has FIELD
--R multiEuclidean : (List %, %) -> Union(List %, "failed") if R has FIELD
--R nextItem : % -> Union(%, "failed") if R has FFIELDC
--R order : % -> OnePointCompletion PositiveInteger if R has FFIELDC
--R order : % -> PositiveInteger if R has FFIELDC
--R prime? : % -> Boolean if R has FIELD
--R primeFrobenius : % -> % if R has FFIELDC
--R primeFrobenius : (%, NonNegativeInteger) -> % if R has FFIELDC
--R primitive? : % -> Boolean if R has FFIELDC
--R primitiveElement : () -> % if R has FFIELDC
--R principalIdeal : List % -> Record(coef: List %, generator: %) if R has FIELD
--R ?quo? : (%, %) -> % if R has FIELD

```

```

--R random : () -> % if R has FINITE
--R reduce : Fraction UP -> Union(%, "failed") if R has FIELD
--R reducedSystem : Matrix % -> Matrix R
--R reducedSystem : (Matrix %, Vector %) -> Record(mat: Matrix R, vec: Vector R)
--R reducedSystem : (Matrix %, Vector %) -> Record(mat: Matrix Integer, vec: Vector Integer) i
--R reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT
--R regularRepresentation : % -> Matrix R
--R regularRepresentation : (%, Vector %) -> Matrix R
--R ?rem? : (%, %) -> % if R has FIELD
--R representationType : () -> Union("prime", polynomial, normal, cyclic) if R has FFIELDC
--R represents : (Vector R, Vector %) -> %
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retract : % -> Integer if R has RETRACT INT
--R retractIfCan : % -> Union(R, "failed")
--R retractIfCan : % -> Union(Fraction Integer, "failed") if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(Integer, "failed") if R has RETRACT INT
--R size : () -> NonNegativeInteger if R has FINITE
--R sizeLess? : (%, %) -> Boolean if R has FIELD
--R squareFree : % -> Factored % if R has FIELD
--R squareFreePart : % -> % if R has FIELD
--R subtractIfCan : (%, %) -> Union(%, "failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger, NonNegativeInteger) if R has
--R traceMatrix : Vector % -> Matrix R
--R unit? : % -> Boolean if R has FIELD
--R unitCanonical : % -> % if R has FIELD
--R unitNormal : % -> Record(unit: %, canonical: %, associate: %) if R has FIELD
--R
--E 1

)spool
)lisp (bye)

```

---

— SimpleAlgebraicExtension.help —

```

=====
SimpleAlgebraicExtension examples
=====

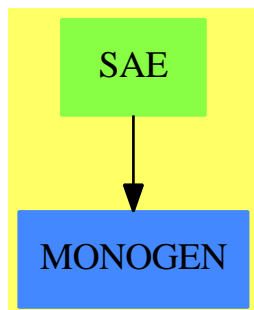
```

```

See Also:
o )show SimpleAlgebraicExtension

```

## 20.10.1 SimpleAlgebraicExtension (SAE)

**Exports:**

|                           |                       |                          |
|---------------------------|-----------------------|--------------------------|
| 0                         | 1                     | associates?              |
| basis                     | characteristic        | characteristicPolynomial |
| charthRoot                | coerce                | conditionP               |
| convert                   | coordinates           | createPrimitiveElement   |
| D                         | definingPolynomial    | derivationCoordinates    |
| differentiate             | discreteLog           | discriminant             |
| divide                    | euclideanSize         | expressIdealMember       |
| exquo                     | extendedEuclidean     | factor                   |
| factorsOfCyclicGroupSize  | gcd                   | gcdPolynomial            |
| generator                 | hash                  | index                    |
| init                      | inv                   | latex                    |
| lcm                       | lift                  | lookup                   |
| minimalPolynomial         | multiEuclidean        | nextItem                 |
| norm                      | one?                  | order                    |
| prime?                    | primeFrobenius        | primitive?               |
| primitiveElement          | principalIdeal        | random                   |
| rank                      | recip                 | reduce                   |
| reducedSystem             | regularRepresentation | representationType       |
| represents                | retract               | retractIfCan             |
| sample                    | size                  | sizeLess?                |
| squareFree                | squareFreePart        | subtractIfCan            |
| tableForDiscreteLogarithm | trace                 | traceMatrix              |
| unit?                     | unitCanonical         | unitNormal               |
| zero?                     | ?*?                   | ?**?                     |
| ?+?                       | ?-?                   | ~?                       |
| ?=?                       | ?^?                   | ?~=?                     |
| ?/?                       | ?quo?                 | ?rem?                    |

— domain SAE SimpleAlgebraicExtension —

```

)abbrev domain SAE SimpleAlgebraicExtension
++ Author: Barry Trager, Manuel Bronstein, Clifton Williamson

```

```

++ Date Created: 1986
++ Date Last Updated: 9 May 1994
++ Keywords: ring, algebraic, extension
++ Description:
++ Algebraic extension of a ring by a single polynomial.
++ Domain which represents simple algebraic extensions of arbitrary
++ rings. The first argument to the domain, R, is the underlying ring,
++ the second argument is a domain of univariate polynomials over K,
++ while the last argument specifies the defining minimal polynomial.
++ The elements of the domain are canonically represented as polynomials
++ of degree less than that of the minimal polynomial with coefficients
++ in R. The second argument is both the type of the third argument and
++ the underlying representation used by \spadtype{SAE} itself.

SimpleAlgebraicExtension(R:CommutativeRing,
  UP:UnivariatePolynomialCategory R, M:UP): MonogenicAlgebra(R, UP) == add
  --sqFr(pb): FactorS(Poly) from UnivPolySquareFree(Poly)

  --degree(M) > 0 and M must be monic if R is not a field.
  if (r := recip leadingCoefficient M) case "failed" then
    error "Modulus cannot be made monic"

  Rep := UP
  x,y :$
  c: R

  mkDisc : Boolean -> Void
  mkDiscMat: Boolean -> Void

  M := r::R * M
  d := degree M
  d1 := subtractIfCan(d,1)::NonNegativeInteger
  discmat:Matrix(R) := zero(d, d)
  nodiscmat?:Reference(Boolean) := ref true
  disc:Reference(R) := ref 0
  nodisc?:Reference(Boolean) := ref true
  basis := [monomial(1, i)$Rep for i in 0..d1]$Vector(Rep)

  if R has Finite then
    size == size$R ** d
    random == represents([random()$R for i in 0..d1])
  0 == 0$Rep
  1 == 1$Rep
  c * x == c *$Rep x
  n:Integer * x == n *$Rep x
  coerce(n:Integer):$ == coerce(n)$Rep
  coerce(c) == monomial(c,0)$Rep
  coerce(x):OutputForm == coerce(x)$Rep
  lift(x) == x pretend Rep
  reduce(p:UP):$ == (monicDivide(p,M)$Rep).remainder
  x = y == x =$Rep y

```

```

x + y == x + $Rep y
- x == -$Rep x
x * y == reduce((x * $Rep y) pretend UP)
coordinates(x) == [coefficient(lift(x),i) for i in 0..d1]
represents(vect) == +/[monomial(vect.(i+1),i) for i in 0..d1]
definingPolynomial() == M
characteristic() == characteristic()$R
rank() == d::PositiveInteger
basis() == copy(basis@Vector(Rep) pretend Vector($))
--!! I inserted 'copy' in the definition of 'basis' -- cjlw 7/19/91

if R has Field then
  minimalPolynomial x == squareFreePart characteristicPolynomial x

if R has Field then
  coordinates(x:$,bas: Vector $) ==
    (m := inverse transpose coordinates bas) case "failed" =>
      error "coordinates: second argument must be a basis"
    (m :: Matrix R) * coordinates(x)

else if R has IntegralDomain then
  coordinates(x:$,bas: Vector $) ==
    -- we work over the quotient field of R to invert a matrix
    qf := Fraction R
    imatqf := InnerMatrixQuotientFieldFunctions(R,Vector R,Vector R,
      Matrix R,qf,Vector qf,Vector qf,Matrix qf)
    mat := transpose coordinates bas
    (m := inverse(mat)$imatqf) case "failed" =>
      error "coordinates: second argument must be a basis"
    coordsQF: Vector qf :=
      map(y +-> y::qf,coordinates x)$VectorFunctions2(R,qf)
    -- here are the coordinates as elements of the quotient field:
    vecQF := (m :: Matrix qf) * coordsQF
    vec : Vector R := new(d,0)
    for i in 1..d repeat
      xi := qelt(vecQF,i)
      denom(xi) = 1 => qsetelt_!(vec,i,numer xi)
      error "coordinates: coordinates are not integral over ground ring"
    vec

reducedSystem(m:Matrix $):Matrix(R) ==
  reducedSystem(map(lift, m)$MatrixCategoryFunctions2($, Vector $,
    Vector $, Matrix $, UP, Vector UP, Vector UP, Matrix UP))

reducedSystem(m:Matrix $, v:Vector $):Record(mat:Matrix R,vec:Vector R) ==
  reducedSystem(map(lift, m)$MatrixCategoryFunctions2($, Vector $,
    Vector $, Matrix $, UP, Vector UP, Vector UP, Matrix UP),
    map(lift, v)$VectorFunctions2($, UP))

discriminant() ==

```

```

    if nodisc?() then mkDisc false
    disc()

mkDisc b ==
    nodisc?() := b
    disc() := discriminant M
    void

traceMatrix() ==
    if nodiscmat?() then mkDiscMat false
    discmat

mkDiscMat b ==
    nodiscmat?() := b
    mr := minRowIndex discmat; mc := minColIndex discmat
    for i in 0..d1 repeat
        for j in 0..d1 repeat
            qsetelt_!(discmat,mr + i,mc + j,trace reduce monomial(1,i + j))
    void

trace x ==          --this could be coded perhaps more efficiently
    xn := x;  ans := coefficient(lift xn, 0)
    for n in 1..d1 repeat
        (xn := generator() * xn;  ans := coefficient(lift xn, n) + ans)
    ans

if R has Finite then
    index k ==
        i:Integer := k rem size()
        p:Integer := size()$R
        ans:$ := 0
        for j in 0.. while i > 0 repeat
            h := i rem p
            -- index(p) = 0$R
            if h ^= 0 then
                -- here was a bug: "index" instead of
                -- "coerce", otherwise it wouldn't work for
                -- Rings R where "coerce: I-> R" is not surjective
                a := index(h :: PositiveInteger)$R
                ans := ans + reduce monomial(a, j)
            i := i quo p
        ans
lookup(z : $) : PositiveInteger ==
    -- z = index lookup z, n = lookup index n
    -- the answer is merely the Horner evaluation of the
    -- representation with the size of R (as integers).
    zero?(z) => size()$$ pretend PositiveInteger
    p : Integer := size()$R
    co : Integer := lookup(leadingCoefficient z)$R
    n : NonNegativeInteger := degree(z)

```

```

while not zero?(z := reductum z) repeat
  co := co * p ** ((n - (n := degree z)) pretend
    NonNegativeInteger) + lookup(leadingCoefficient z)$R
  n = 0 => co pretend PositiveInteger
  (co * p ** n) pretend PositiveInteger

--
-- KA:=BasicPolynomialFunctions(Poly)
-- minPoly(x) ==
--   ffe:= SqFr(resultant(M::KA, KA.var - lift(x)::KA)).fs.first
--   ffe.flag = "SQFR" => ffe.f
--   mdeg:= (degree(ffe.f) // K.characteristic)::Integer
--   mat:= Zero()::Matrix<mdeg+1,deg+mdeg+1>(K)
--   xi:=L.1; setelt(mat,1,1,K.1); setelt(mat,1,(deg+1),K.1)
--   for i in 1..mdeg repeat
--     xi:= x * xi; xp:= lift(xi)
--     while xp ^= KA.0 repeat
--       setelt(mat,(mdeg+1),(degree(xp)+1),LeadingCoef(xp))
--       xp:=reductum(xp)
--     setelt(mat,(mdeg+1),(deg+i+1),K.1)
--     EchelonLastRow(mat)
--     if and/(elt(mat,(i+1),j) = K.0 for j in 1..deg)
--       then return unitNormal(+/(elt(mat,(i+1),(deg+j+1))*(B::KA)**j
--         for j in 0..i)).a
--   ffe.f

```

---

— SAE.dotabb —

```

"SAE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SAE"]
"MONOGEN" [color="#4488FF",href="bookvol10.2.pdf#nameddest=MONOGEN"]
"SAE" -> "MONOGEN"

```

---

## 20.11 domain SFORT SimpleFortranProgram

---

— SimpleFortranProgram.input —

```

)set break resume
)sys rm -f SimpleFortranProgram.output
)spool SimpleFortranProgram.output
)set message test on

```



```

)set message auto off
)clear all

--S 1 of 1
)show SimpleFortranProgram
--R SimpleFortranProgram(R: OrderedSet,FS: FunctionSpace R) is a domain constructor
--R Abbreviation for SimpleFortranProgram is SFORT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SFORT
--R
--R----- Operations -----
--R coerce : % -> OutputForm          outputAsFortran : % -> Void
--R fortran : (Symbol,FortranScalarType,FS) -> %
--R
--E 1

)spool
)lisp (bye)

```

---

— SimpleFortranProgram.help —

```

=====
SimpleFortranProgram examples
=====

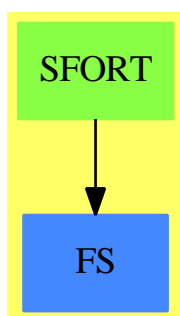
```

```

See Also:
o )show SimpleFortranProgram

```

### 20.11.1 SimpleFortranProgram (SFORT)



See

⇒ “Result” (RESULT) 19.9.1 on page 2260  
 ⇒ “FortranCode” (FC) 7.16.1 on page 898  
 ⇒ “FortranProgram” (FORTRAN) 7.18.1 on page 923  
 ⇒ “ThreeDimensionalMatrix” (M3D) 21.7.1 on page 2661  
 ⇒ “Switch” (SWITCH) 20.36.1 on page 2588  
 ⇒ “FortranTemplate” (FTEM) 7.20.1 on page 934  
 ⇒ “FortranExpression” (FEXPR) 7.17.1 on page 914

**Exports:**

coerce    fortran    outputAsFortran

## — domain SFORT SimpleFortranProgram —

```

)abbrev domain SFORT SimpleFortranProgram
-- Because of a bug in the compiler:
)bo $noSubsumption:=true

++ Author: Mike Dewar
++ Date Created: November 1992
++ Date Last Updated:
++ Basic Operations:
++ Related Constructors: FortranType, FortranCode, Switch
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ \axiomType{SimpleFortranProgram(f,type)} provides a simple model of some
++ FORTRAN subprograms, making it possible to coerce objects of various
++ domains into a FORTRAN subprogram called \axiom{f}.
++ These can then be translated into legal FORTRAN code.

SimpleFortranProgram(R,FS): Exports == Implementation where
  R  : OrderedSet
  FS : FunctionSpace(R)

  FST ==> FortranScalarType

Exports ==> FortranProgramCategory with
  fortran : (Symbol,FST,FS) -> $
  ++fortran(fname,ftype,body) builds an object of type
  ++\axiomType{FortranProgramCategory}. The three arguments specify
  ++the name, the type and the body of the program.

Implementation ==> add

  Rep := Record(name : Symbol, type : FST, body : FS )

  fortran(fname, ftype, res) ==

```

```

construct(fname,ftype,res)$Rep

nameOf(u:$):Symbol == u . name

typeOf(u:$):Union(FST,"void") == u . type

bodyOf(u:$):FS == u . body

argumentsOf(u:$):List Symbol == variables(bodyOf u)$FS

coerce(u:$):OutputForm ==
  coerce(nameOf u)$Symbol

outputAsFortran(u:$):Void ==
  ftype := (checkType(typeOf(u)::OutputForm)$Lisp)::OutputForm
  fname := nameOf(u)::OutputForm
  args := argumentsOf(u)
  nargs:=args::OutputForm
  val := bodyOf(u)::OutputForm
  fortFormatHead(ftype,fname,nargs)$Lisp
  fortFormatTypes(ftype,args)$Lisp
  dispfortexp1$Lisp ["="::OutputForm, fname, val]@List(OutputForm)
  dispfortexp1$Lisp "RETURN"::OutputForm
  dispfortexp1$Lisp "END"::OutputForm
  void()$Void

```

---

— SFORT.dotabb —

```

"SFORT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SFORT"]
"FS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FS"]
"SFORT" -> "FS"

```

## 20.12 domain SINT SingleInteger

The definition of **one?** has been rewritten as it relies on calling **ONEP** which is a function specific to Codemist Common Lisp but is not defined in Common Lisp.

---

— SingleInteger.input —

```

)set break resume

```

```
)sys rm -f SingleInteger.output
)spool SingleInteger.output
)set message test on
)set message auto off
)clear all
--S 1 of 11
min()$SingleInteger
--R
--R
--R (1) - 2147483648
--R
--R                                          Type: SingleInteger
--E 1

--S 2 of 11
max()$SingleInteger
--R
--R
--R (2) 2147483647
--R
--R                                          Type: SingleInteger
--E 2

--S 3 of 11
a := 1234 :: SingleInteger
--R
--R
--R (3) 1234
--R
--R                                          Type: SingleInteger
--E 3

--S 4 of 11
b := 124$SingleInteger
--R
--R
--R (4) 124
--R
--R                                          Type: SingleInteger
--E 4

--S 5 of 11
gcd(a,b)
--R
--R
--R (5) 2
--R
--R                                          Type: SingleInteger
--E 5

--S 6 of 11
lcm(a,b)
--R
--R
--R (6) 76508
```

```

--R                                                    Type: SingleInteger
--E 6

--S 7 of 11
mulmod(5,6,13)$SingleInteger
--R
--R
--R      (7)  4
--R
--R                                                    Type: SingleInteger
--E 7

--S 8 of 11
positiveRemainder(37,13)$SingleInteger
--R
--R
--R      (8)  11
--R
--R                                                    Type: SingleInteger
--E 8

--S 9 of 11
And(3,4)$SingleInteger
--R
--R
--R      (9)  0
--R
--R                                                    Type: SingleInteger
--E 9

--S 10 of 11
shift(1,4)$SingleInteger
--R
--R
--R      (10) 16
--R
--R                                                    Type: SingleInteger
--E 10

--S 11 of 11
shift(31,-1)$SingleInteger
--R
--R
--R      (11) 15
--R
--R                                                    Type: SingleInteger
--E 11
)spool
)lisp (bye)

```

```
=====
SingleInteger examples
=====
```

The SingleInteger domain is intended to provide support in Axiom for machine integer arithmetic. It is generally much faster than (bignum) Integer arithmetic but suffers from a limited range of values. Since Axiom can be implemented on top of various dialects of Lisp, the actual representation of small integers may not correspond exactly to the host machines integer representation.

You can discover the minimum and maximum values in your implementation by using min and max.

```
min()$SingleInteger
- 2147483648
                                     Type: SingleInteger
```

```
max()$SingleInteger
2147483647
                                     Type: SingleInteger
```

To avoid confusion with Integer, which is the default type for integers, you usually need to work with declared variables.

```
a := 1234 :: SingleInteger
1234
                                     Type: SingleInteger
```

or use package calling

```
b := 124$SingleInteger
124
                                     Type: SingleInteger
```

You can add, multiply and subtract SingleInteger objects, and ask for the greatest common divisor (gcd).

```
gcd(a,b)
2
                                     Type: SingleInteger
```

The least common multiple (lcm) is also available.

```
lcm(a,b)
76508
                                     Type: SingleInteger
```

Operations mulmod, addmod, submod, and invmod are similar - they provide arithmetic modulo a given small integer.

Here is  $5 * 6 \bmod 13$ .

```
mulmod(5,6,13)$SingleInteger
```

```
4
```

Type: SingleInteger

To reduce a small integer modulo a prime, use `positiveRemainder`.

```
positiveRemainder(37,13)$SingleInteger
```

```
11
```

Type: SingleInteger

Operations `And`, `Or`, `xor`, and `Not` provide bit level operations on small integers.

```
And(3,4)$SingleInteger
```

```
0
```

Type: SingleInteger

Use `shift(int,numToShift)` to shift bits, where `i` is shifted left if `numToShift` is positive, right if negative.

```
shift(1,4)$SingleInteger
```

```
16
```

Type: SingleInteger

```
shift(31,-1)$SingleInteger
```

```
15
```

Type: SingleInteger

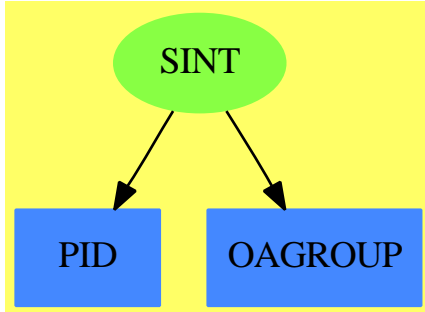
Many other operations are available for small integers, including many of those provided for `Integer`.

See Also:

- o `)help Integer`
- o `)show SingleInteger`

---

## 20.12.1 SingleInteger (SINT)

**Exports:**

|                |                    |                   |                    |
|----------------|--------------------|-------------------|--------------------|
| 0              | 1                  | abs               | addmod             |
| And            | associates?        | base              | binomial           |
| bit?           | characteristic     | coerce            | convert            |
| copy           | D                  | dec               | differentiate      |
| divide         | euclideanSize      | even?             | expressIdealMember |
| exquo          | extendedEuclidean  | factor            | factorial          |
| gcd            | gcdPolynomial      | hash              | inc                |
| init           | invmod             | latex             | lcm                |
| length         | mask               | max               | min                |
| mulmod         | multiEuclidean     | negative?         | nextItem           |
| Not            | not?               | odd?              | OMwrite            |
| one?           | Or                 | patternMatch      | permutation        |
| principalIdeal | positive?          | positiveRemainder | powmod             |
| prime?         | random             | rational          | rationalIfCan      |
| rational?      | recip              | reducedSystem     | retract            |
| retractIfCan   | sample             | shift             | sign               |
| sizeLess?      | squareFree         | squareFreePart    | subtractIfCan      |
| submod         | symmetricRemainder | unit?             | unitCanonical      |
| unitNormal     | xor                | zero?             | ?*?                |
| ?**?           | ?+?                | ?-?               | -?                 |
| ?/\?           | ?<?                | ?<=?              | ?=?                |
| ?>?            | ?>=?               | ?\/?              | ?^?                |
| ?              | ?~=?               | ?quo?             | ?rem?              |

— domain SINT SingleInteger —

```
)abbrev domain SINT SingleInteger
```

```
-- following patch needed to deal with *(I,%) -> %
-- affects behavior of SourceLevelSubset
--)bo $noSubsets := true
-- No longer - JHD !! still needed 5/3/91 BMT
```



```

++ Author: Michael Monagan
++ Date Created:
++ January 1988
++ Change History:
++ Basic Operations: max, min,
++ not, and, or, xor, Not, And, Or
++ Related Constructors:
++ Keywords: single integer
++ Description:
++ SingleInteger is intended to support machine integer arithmetic.

-- MAXINT, BASE (machine integer constants)
-- MODULUS, MULTIPLIER (random number generator constants)

-- Lisp dependencies
-- EQ, ABSVAL, TIMES, INTEGER-LENGTH, HASHEQ, REMAINDER
-- QSLESSP, QSGREATERP, QSADD1, QSSUB1, QSMINUS, QSPLUS, QSDIFFERENCE
-- QSTIMES, QSREMAINDER, QSODDP, QSZEROP, QSMAX, QSMIN, QSNOT, QSAND
-- QSOR, QSXOR, QSLEFTSHIFT, QSADDMOD, QSDIFMOD, QSMULTMOD

SingleInteger(): Join(IntegerNumberSystem,Logic,OpenMath) with
canonical
    ++ \spad{canonical} means that mathematical equality is
    ++ implied by data structure equality.
canonicalsClosed
    ++ \spad{canonicalClosed} means two positives multiply to
    ++ give positive.
noetherian
    ++ \spad{noetherian} all ideals are finitely generated
    ++ (in fact principal).

max      : () -> %
    ++ max() returns the largest single integer.
min      : () -> %
    ++ min() returns the smallest single integer.

-- bit operations
"not":    % -> %
    ++ not(n) returns the bit-by-bit logical not of the single integer n.
"~" :    % -> %
    ++ ~ n returns the bit-by-bit logical not of the single integer n.
"/\" : (% , %) -> %
    ++ n /\ m returns the bit-by-bit logical and of
    ++ the single integers n and m.
"\\" : (% , %) -> %
    ++ n \/ m returns the bit-by-bit logical or of
    ++ the single integers n and m.

```

```

"xor": (% , %) -> %
  ++ xor(n,m) returns the bit-by-bit logical xor of
  ++ the single integers n and m.
Not : % -> %
  ++ Not(n) returns the bit-by-bit logical not of the single integer n.
And : (% , %) -> %
  ++ And(n,m) returns the bit-by-bit logical and of
  ++ the single integers n and m.
Or : (% , %) -> %
  ++ Or(n,m) returns the bit-by-bit logical or of
  ++ the single integers n and m.

== add

seed : % := 1$Lisp -- for random()
MAXINT ==> MOST_POSITIVE_FIXNUM$Lisp
MININT ==> MOST_NEGATIVE_FIXNUM$Lisp
BASE ==> 67108864$Lisp -- 2**26
MULTIPLIER ==> 314159269$Lisp -- from Knuth's table
MODULUS ==> 2147483647$Lisp -- 2**31-1

writeOMSingleInt(dev: OpenMathDevice, x: %): Void ==
  if x < 0 then
    OmputApp(dev)
    OmputSymbol(dev, "arith1", "unary_minus")
    OmputInteger(dev, convert(-x))
    OmputEndApp(dev)
  else
    OmputInteger(dev, convert(x))

OMwrite(x: %): String ==
  s: String := ""
  sp := OM_STRINGTOSTRINGPTR(s)$Lisp
  dev: OpenMathDevice := OMOpenString(sp pretend String, OMencodingXML)
  OmputObject(dev)
  writeOMSingleInt(dev, x)
  OmputEndObject(dev)
  OMclose(dev)
  s := OM_STRINGPTRTOSTRING(sp)$Lisp pretend String
  s

OMwrite(x: %, wholeObj: Boolean): String ==
  s: String := ""
  sp := OM_STRINGTOSTRINGPTR(s)$Lisp
  dev: OpenMathDevice := OMOpenString(sp pretend String, OMencodingXML)
  if wholeObj then
    OmputObject(dev)
  writeOMSingleInt(dev, x)
  if wholeObj then
    OmputEndObject(dev)

```

```

OMclose(dev)
s := OM_STRINGPTRTOSTRING(sp)$Lisp pretend String
s

OMwrite(dev: OpenMathDevice, x: %): Void ==
  OMputObject(dev)
  writeOMSingleInt(dev, x)
  OMputEndObject(dev)

OMwrite(dev: OpenMathDevice, x: %, wholeObj: Boolean): Void ==
  if wholeObj then
    OMputObject(dev)
    writeOMSingleInt(dev, x)
  if wholeObj then
    OMputEndObject(dev)

reducedSystem m      == m pretend Matrix(Integer)
coerce(x):OutputForm == (convert(x)@Integer)::OutputForm
convert(x:%):Integer == x pretend Integer
i:Integer * y:%      == i::% * y
0                    == 0$Lisp
1                    == 1$Lisp
base()              == 2$Lisp
max()               == MAXINT
min()               == MININT
x = y               == EQL(x,y)$Lisp
~ x                 == LOGNOT(x)$Lisp
not(x)              == LOGNOT(x)$Lisp
_/_\ (x,y)          == LOGAND(x,y)$Lisp
_/_/(x,y)           == LOGIOR(x,y)$Lisp
Not(x)              == LOGNOT(x)$Lisp
And(x,y)            == LOGAND(x,y)$Lisp
Or(x,y)             == LOGIOR(x,y)$Lisp
xor(x,y)            == LOGXOR(x,y)$Lisp
x < y               == QSLESSP(x,y)$Lisp
inc x               == QSADD1(x)$Lisp
dec x               == QSSUB1(x)$Lisp
- x                 == QSMINUS(x)$Lisp
x + y               == QSPLUS(x,y)$Lisp
x:% - y:%           == QSDIFFERENCE(x,y)$Lisp
x:% * y:%           == QSTIMES(x,y)$Lisp
x:% ** n:NonNegativeInteger == ((EXPT(x, n)$Lisp) pretend Integer)::%
x quo y            == QSQUOTIENT(x,y)$Lisp
x rem y            == QSREMAINDER(x,y)$Lisp
divide(x, y)       == CONS(QSQUOTIENT(x,y)$Lisp, QSREMAINDER(x,y)$Lisp)$Lisp
gcd(x,y)           == GCD(x,y)$Lisp
abs(x)             == QSABSVAL(x)$Lisp
odd?(x)            == QSODDP(x)$Lisp
zero?(x)           == QSZEROP(x)$Lisp
-- one?(x)         == ONEP(x)$Lisp

```

```

one?(x)    == x = 1
max(x,y)   == QSMAX(x,y)$Lisp
min(x,y)   == QSMIN(x,y)$Lisp
hash(x)    == SXHASH(x)$Lisp
length(x)  == INTEGER_-LENGTH(x)$Lisp
shift(x,n) == QSLEFTSHIFT(x,n)$Lisp
mulmod(a,b,p) == QSMULTMOD(a,b,p)$Lisp
addmod(a,b,p) == QSADDMOD(a,b,p)$Lisp
submod(a,b,p) == QSDIFMOD(a,b,p)$Lisp
negative?(x) == QSMINUSP$Lisp x

reducedSystem(m, v) ==
  [m pretend Matrix(Integer), v pretend Vector(Integer)]

positiveRemainder(x,n) ==
  r := QSREMAINDER(x,n)$Lisp
  QSMINUSP(r)$Lisp =>
    QSMINUSP(n)$Lisp => QSDIFFERENCE(x, n)$Lisp
    QSPPLUS(r, n)$Lisp
  r

coerce(x:Integer):% ==
  (x <= max pretend Integer) and (x >= min pretend Integer) =>
    x pretend %
  error "integer too large to represent in a machine word"

random() ==
  seed := REMAINDER(TIMES(MULTIPLIER,seed)$Lisp,MODULUS)$Lisp
  REMAINDER(seed,BASE)$Lisp

random(n) == RANDOM(n)$Lisp

UCA ==> Record(unit:%,canonical:%,associate:%)
unitNormal x ==
  x < 0 => [-1,-x,-1]$UCA
  [1,x,1]$UCA

)bo $noSubsets := false

-----

— SINT.dotabb —

"SINT" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SINT",shape=ellipse]
"PID"  [color="#4488FF",href="bookvol10.2.pdf#nameddest=PID"]
"OAGROUP" [color="#4488FF",href="bookvol10.2.pdf#nameddest=OAGROUP"]
"SINT" -> "PID"

```

"SINT" -> "OAGROUP"

## 20.13 domain SAOS SingletonAsOrderedSet

— SingletonAsOrderedSet.input —

```
)set break resume
)sys rm -f SingletonAsOrderedSet.output
)spool SingletonAsOrderedSet.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SingletonAsOrderedSet
--R SingletonAsOrderedSet is a domain constructor
--R Abbreviation for SingletonAsOrderedSet is SAOS
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SAOS
--R
--R----- Operations -----
--R ?<? : (%,% ) -> Boolean          ?<=? : (%,% ) -> Boolean
--R ?=? : (%,% ) -> Boolean          ?>? : (%,% ) -> Boolean
--R ?>=? : (%,% ) -> Boolean         coerce : % -> OutputForm
--R convert : % -> Symbol            create : () -> %
--R hash : % -> SingleInteger         latex : % -> String
--R max : (%,% ) -> %                min : (%,% ) -> %
--R ?~=? : (%,% ) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

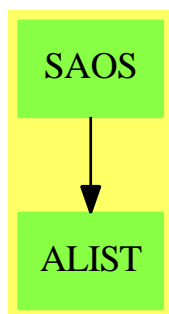
— SingletonAsOrderedSet.help —

```
=====
SingletonAsOrderedSet examples
=====
```

See Also:

```
o )show SingletonAsOrderedSet
```

### 20.13.1 SingletonAsOrderedSet (SAOS)



#### Exports:

```
coerce  convert  create  hash  latex
max      min      ?~=?   ?<?   ?<=?
?=?      ?>?      ?>=?
```

— domain SAOS SingletonAsOrderedSet —

```
)abbrev domain SAOS SingletonAsOrderedSet
++ Author: Mark Botch
++ Description:
++ This trivial domain lets us build Univariate Polynomials
++ in an anonymous variable

SingletonAsOrderedSet(): OrderedSet with
    create:() -> %
    convert:% -> Symbol

== add
create() == "?" pretend %
a<b == false -- only one element
coerce(a) == outputForm "?" -- CJW doesn't like this: change ?
a=b == true -- only one element
min(a,b) == a -- only one element
max(a,b) == a -- only one element
convert a == coerce("?")
```

— SAOS.dotabb —

```
"SAOS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SAOS"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"SAOS" -> "ALIST"
```

## 20.14 domain SMP SparseMultivariatePolynomial

— SparseMultivariatePolynomial.input —

```
)set break resume
)sys rm -f SparseMultivariatePolynomial.output
)spool SparseMultivariatePolynomial.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SparseMultivariatePolynomial
--R SparseMultivariatePolynomial(R: Ring,VarSet: OrderedSet) is a domain constructor
--R Abbreviation for SparseMultivariatePolynomial is SMP
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SMP
--R
--R----- Operations -----
--R ?? : (%,R) -> %          ?? : (R,%) -> %
--R ?? : (%,%) -> %          ?? : (Integer,%) -> %
--R ?? : (PositiveInteger,%) -> %    ??? : (%,PositiveInteger) -> %
--R ?? : (%,%) -> %          ?-? : (%,%) -> %
--R -? : % -> %              ?=? : (%,%) -> Boolean
--R D : (%,List VarSet) -> %        D : (%,VarSet) -> %
--R 1 : () -> %                0 : () -> %
--R ?? : (%,PositiveInteger) -> %    coefficients : % -> List R
--R coerce : VarSet -> %            coerce : R -> %
--R coerce : Integer -> %           coerce : % -> OutputForm
--R differentiate : (%,VarSet) -> %    eval : (%,VarSet,%) -> %
--R eval : (%,VarSet,R) -> %          eval : (%,List %,List %) -> %
--R eval : (%,%,%) -> %              eval : (%,Equation %) -> %
--R eval : (%,List Equation %) -> %    ground : % -> R
--R ground? : % -> Boolean           hash : % -> SingleInteger
--R latex : % -> String              leadingCoefficient : % -> R
--R leadingMonomial : % -> %          map : ((R -> R),%) -> %
--R monomial? : % -> Boolean          monomials : % -> List %
--R one? : % -> Boolean              primitiveMonomials : % -> List %
--R recip : % -> Union(%, "failed")    reductum : % -> %
--R retract : % -> VarSet            retract : % -> R
```

```

--R sample : () -> %                               variables : % -> List VarSet
--R zero? : % -> Boolean                             ~=? : (%,% ) -> Boolean
--R ?? : (Fraction Integer,% ) -> % if R has ALGEBRA FRAC INT
--R ?? : (% ,Fraction Integer) -> % if R has ALGEBRA FRAC INT
--R ?? : (NonNegativeInteger,% ) -> %
--R ***? : (% ,NonNegativeInteger) -> %
--R ?/? : (% ,R) -> % if R has FIELD
--R ?<? : (% ,%) -> Boolean if R has ORDSET
--R ?<=? : (% ,%) -> Boolean if R has ORDSET
--R ?>? : (% ,%) -> Boolean if R has ORDSET
--R ?>=? : (% ,%) -> Boolean if R has ORDSET
--R D : (% ,List VarSet,List NonNegativeInteger) -> %
--R D : (% ,VarSet,NonNegativeInteger) -> %
--R ?? : (% ,NonNegativeInteger) -> %
--R associates? : (% ,%) -> Boolean if R has INTDOM
--R binomThmExpt : (% ,%,NonNegativeInteger) -> % if R has COMRING
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(% ,"failed") if $ has CHARNZ and R has PFECAT or R has CHARNZ
--R coefficient : (% ,List VarSet,List NonNegativeInteger) -> %
--R coefficient : (% ,VarSet,NonNegativeInteger) -> %
--R coefficient : (% ,IndexedExponents VarSet) -> R
--R coerce : Fraction Integer -> % if R has ALGEBRA FRAC INT or R has RETRACT FRAC INT
--R coerce : % -> % if R has INTDOM
--R conditionP : Matrix % -> Union(Vector % ,"failed") if $ has CHARNZ and R has PFECAT
--R content : (% ,VarSet) -> % if R has GCDDOM
--R content : % -> R if R has GCDDOM
--R convert : % -> InputForm if R has KONVERT INFORM and VarSet has KONVERT INFORM
--R convert : % -> Pattern Integer if R has KONVERT PATTERN INT and VarSet has KONVERT PATTERN INT
--R convert : % -> Pattern Float if R has KONVERT PATTERN FLOAT and VarSet has KONVERT PATTERN FLOAT
--R degree : (% ,List VarSet) -> List NonNegativeInteger
--R degree : (% ,VarSet) -> NonNegativeInteger
--R degree : % -> IndexedExponents VarSet
--R differentiate : (% ,List VarSet,List NonNegativeInteger) -> %
--R differentiate : (% ,VarSet,NonNegativeInteger) -> %
--R differentiate : (% ,List VarSet) -> %
--R discriminant : (% ,VarSet) -> % if R has COMRING
--R eval : (% ,List VarSet,List %) -> %
--R eval : (% ,List VarSet,List R) -> %
--R exquo : (% ,%) -> Union(% ,"failed") if R has INTDOM
--R exquo : (% ,R) -> Union(% ,"failed") if R has INTDOM
--R factor : % -> Factored % if R has PFECAT
--R factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % if R has PF
--R factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % i
--R gcd : (% ,%) -> % if R has GCDDOM
--R gcd : List % -> % if R has GCDDOM
--R gcdPolynomial : (SparseUnivariatePolynomial % ,SparseUnivariatePolynomial %) -> SparseUnivariatePolyn
--R isExpt : % -> Union(Record(var: VarSet,exponent: NonNegativeInteger),"failed")
--R isPlus : % -> Union(List % ,"failed")
--R isTimes : % -> Union(List % ,"failed")
--R lcm : (% ,%) -> % if R has GCDDOM

```



```

--R lcm : List % -> % if R has GCDDOM
--R mainVariable : % -> Union(VarSet,"failed")
--R mapExponents : ((IndexedExponents VarSet -> IndexedExponents VarSet),%) -> %
--R max : (%,% ) -> % if R has ORDSET
--R min : (%,% ) -> % if R has ORDSET
--R minimumDegree : (% ,List VarSet) -> List NonNegativeInteger
--R minimumDegree : (% ,VarSet) -> NonNegativeInteger
--R minimumDegree : % -> IndexedExponents VarSet
--R monicDivide : (% ,%,VarSet) -> Record(quotient: %,remainder: %)
--R monomial : (% ,List VarSet,List NonNegativeInteger) -> %
--R monomial : (% ,VarSet,NonNegativeInteger) -> %
--R monomial : (R,IndexedExponents VarSet) -> %
--R multivariate : (SparseUnivariatePolynomial %,VarSet) -> %
--R multivariate : (SparseUnivariatePolynomial R,VarSet) -> %
--R numberOfMonomials : % -> NonNegativeInteger
--R patternMatch : (% ,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMatchResult(Integer,% )
--R patternMatch : (% ,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchResult(Float,% )
--R pomopo! : (% ,R,IndexedExponents VarSet,% ) -> %
--R prime? : % -> Boolean if R has PFECAT
--R primitivePart : (% ,VarSet) -> % if R has GCDDOM
--R primitivePart : % -> % if R has GCDDOM
--R reducedSystem : Matrix % -> Matrix R
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) if R has LINEXP INT
--R reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT
--R resultant : (% ,%,VarSet) -> % if R has COMRING
--R retract : % -> Integer if R has RETRACT INT
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(VarSet,"failed")
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(R,"failed")
--R solveLinearPolynomialEquation : (List SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> List SparseUnivariatePolynomial %
--R squareFree : % -> Factored % if R has GCDDOM
--R squareFreePart : % -> % if R has GCDDOM
--R squareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial %
--R subtractIfCan : (% ,%) -> Union(%,"failed")
--R totalDegree : (% ,List VarSet) -> NonNegativeInteger
--R totalDegree : % -> NonNegativeInteger
--R unit? : % -> Boolean if R has INTDOM
--R unitCanonical : % -> % if R has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has INTDOM
--R univariate : % -> SparseUnivariatePolynomial R
--R univariate : (% ,VarSet) -> SparseUnivariatePolynomial %
--R
--E 1

)spool
)lisp (bye)

```

---



---

— SparseMultivariatePolynomial.help —

---

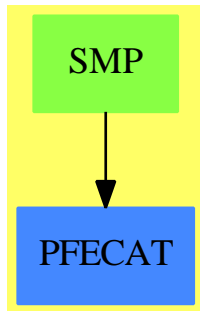
```
=====
SparseMultivariatePolynomial examples
=====
```

See Also:

o )show SparseMultivariatePolynomial

---

### 20.14.1 SparseMultivariatePolynomial (SMP)



See

⇒ “Polynomial” (POLY) 17.25.1 on page 2037

⇒ “MultivariatePolynomial” (MPOLY) 14.16.1 on page 1645

⇒ “IndexedExponents” (INDE) 10.9.1 on page 1183

**Exports:**

|                |                               |                            |
|----------------|-------------------------------|----------------------------|
| 0              | 1                             | associates?                |
| binomThmExpt   | characteristic                | charthRoot                 |
| coefficient    | coefficients                  | coerce                     |
| conditionP     | content                       | convert                    |
| D              | degree                        | differentiate              |
| discriminant   | eval                          | exquo                      |
| factor         | factorPolynomial              | factorSquareFreePolynomial |
| gcd            | gcdPolynomial                 | ground                     |
| ground?        | hash                          | latex                      |
| isExpt         | isPlus                        | isTimes                    |
| lcm            | leadingCoefficient            | leadingMonomial            |
| mainVariable   | map                           | mapExponents               |
| max            | min                           | minimumDegree              |
| monicDivide    | monomial                      | monomial?                  |
| monomials      | multivariate                  | numberOfMonomials          |
| one?           | patternMatch                  | pomopo!                    |
| prime?         | primitivePart                 | primitiveMonomials         |
| recip          | reducedSystem                 | reductum                   |
| resultant      | retract                       | retractIfCan               |
| sample         | solveLinearPolynomialEquation | squareFree                 |
| squareFreePart | squareFreePolynomial          | subtractIfCan              |
| totalDegree    | unit?                         | unitCanonical              |
| unitNormal     | univariate                    | variables                  |
| zero?          | ?*?                           | ?**?                       |
| ?+?            | ?-?                           | -?                         |
| ?=?            | ?^?                           | ?~=?                       |
| ?/?            | ?<?                           | ?<=?                       |
| ?>?            | ?>=?                          |                            |

— domain SMP SparseMultivariatePolynomial —

```
)abbrev domain SMP SparseMultivariatePolynomial
++ Author: Dave Barton, Barry Trager
++ Date Created:
++ Date Last Updated: 30 November 1994
++ Fix History:
++ 30 Nov 94: added gcdPolynomial for float-type coefficients
++ Basic Functions: Ring, degree, eval, coefficient, monomial, differentiate,
++ resultant, gcd
++ Related Constructors: Polynomial, MultivariatePolynomial
++ Also See:
++ AMS Classifications:
++ Keywords: polynomial, multivariate
++ References:
++ Description:
++ This type is the basic representation of sparse recursive multivariate
```

```

++ polynomials. It is parameterized by the coefficient ring and the
++ variable set which may be infinite. The variable ordering is determined
++ by the variable set parameter. The coefficient ring may be non-commutative,
++ but the variables are assumed to commute.

```

```

SparseMultivariatePolynomial(R: Ring, VarSet: OrderedSet): C == T where
  pgcd ==> PolynomialGcdPackage(IndexedExponents VarSet, VarSet, R, %)
  C == PolynomialCategory(R, IndexedExponents(VarSet), VarSet)
  SUP ==> SparseUnivariatePolynomial
  T == add
    --constants
    --D := F(%) replaced by next line until compiler support completed

    --representations
    D := SparseUnivariatePolynomial(%)
    VPoly := Record(v: VarSet, ts: D)
    Rep := Union(R, VPoly)

    --local function

    --declarations
    fn: R -> R
    n: Integer
    k: NonNegativeInteger
    kp: PositiveInteger
    k1: NonNegativeInteger
    c: R
    mvar: VarSet
    val : R
    var: VarSet
    up: D
    p, p1, p2, pval: %
    Lval : List(R)
    Lpval : List(%)
    Lvar : List(VarSet)

    --define
    0 == 0$R::%
    1 == 1$R::%

    zero? p == p case R and zero?(p)$R
  -- one? p == p case R and one?(p)$R
    one? p == p case R and ((p) = 1)$R
  -- a local function
    red(p: %): % ==
      p case R => 0
      if ground?(reductum p.ts) then
        leadingCoefficient(reductum p.ts) else [p.v, reductum p.ts]$VPoly

```

```

numberOfMonomials(p): NonNegativeInteger ==
  p case R =>
    zero?(p)$R => 0
    1
  +/[numberOfMonomials q for q in coefficients(p.ts)]

coerce(mvar):% == [mvar,monomial(1,1)$D]$VPoly

monomial? p ==
  p case R => true
  sup : D := p.ts
  1 ^= numberOfMonomials(sup) => false
  monomial? leadingCoefficient(sup)$D

-- local
moreThanOneVariable?: % -> Boolean

moreThanOneVariable? p ==
  p case R => false
  q:=p.ts
  any?(x1+-->not ground? x1 ,coefficients q) => true
  false

-- if we already know we use this (slightly) faster function
univariateKnown: % -> SparseUnivariatePolynomial R

univariateKnown p ==
  p case R => (leadingCoefficient p) :: SparseUnivariatePolynomial(R)
  monomial( leadingCoefficient p,degree p.ts)+ univariateKnown(red p)

univariate p ==
  p case R =>(leadingCoefficient p) :: SparseUnivariatePolynomial(R)
  moreThanOneVariable? p => error "not univariate"
  monomial( leadingCoefficient p,degree p.ts)+ univariate(red p)

multivariate (u:SparseUnivariatePolynomial(R),var:VarSet) ==
  ground? u => (leadingCoefficient u) ::%
  [var,monomial(leadingCoefficient u,degree u)$D]$VPoly +
  multivariate(reductum u,var)

univariate(p:%,mvar:VarSet):SparseUnivariatePolynomial(%) ==
  p case R or mvar>p.v => monomial(p,0)$D
  pt:=p.ts
  mvar=p.v => pt
  monomial(1,p.v,degree pt)*univariate(leadingCoefficient pt,mvar)+
  univariate(red p,mvar)

-- a local functions, used in next definition
unlikeUnivReconstruct(u:SparseUnivariatePolynomial(%),mvar:VarSet):% ==

```

```

zero? (d:=degree u) => coefficient(u,0)
monomial(leadingCoefficient u,mvar,d)+
  unlikeUnivReconstruct(reductum u,mvar)

multivariate(u: SparseUnivariatePolynomial(%),mvar: VarSet):% ==
  ground? u => coefficient(u,0)
  uu:=u
  while not zero? uu repeat
    cc:=leadingCoefficient uu
    cc case R or mvar > cc.v => uu:=reductum uu
    return unlikeUnivReconstruct(u,mvar)
  [mvar,u]$VPoly

ground?(p:%): Boolean ==
  p case R => true
  false

--      const p ==
--      p case R => p
--      error "the polynomial is not a constant"

monomial(p,mvar,k1) ==
  zero? k1 or zero? p => p
  p case R or mvar>p.v => [mvar,monomial(p,k1)$D]$VPoly
  p*[mvar,monomial(1,k1)$D]$VPoly

monomial(c:R,e: IndexedExponents(VarSet)):% ==
  zero? e => (c::%)
  monomial(1,leadingSupport e, leadingCoefficient e) *
    monomial(c,reductum e)

coefficient(p:%, e: IndexedExponents(VarSet)) : R ==
  zero? e =>
    p case R => p::R
    coefficient(coefficient(p.ts,0),e)
  p case R => 0
  ve := leadingSupport e
  vp := p.v
  ve < vp =>
    coefficient(coefficient(p.ts,0),e)
  ve > vp => 0
  coefficient(coefficient(p.ts,leadingCoefficient e),reductum e)

--      coerce(e: IndexedExponents(VarSet)) : % ==
--      e = 0 => 1
--      monomial(1,leadingSupport e, leadingCoefficient e) *
--      (reductum e)::%

--      retract(p:%): IndexedExponents(VarSet) ==
--      q: Union(IndexedExponents(VarSet),"failed"):=retractIfCan p

```

```

--      q :: IndexedExponents(VarSet)

--      retractIfCan(p:%):Union(IndexedExponents(VarSet),"failed") ==
--      p = 0 => degree p
--      reductum(p)=0 and leadingCoefficient(p)=1 => degree p
--      "failed"

coerce(n) == n::R::%
coerce(c) == c::%
characteristic == characteristic$R

recip(p) ==
  p case R => (uu:=recip(p::R);uu case "failed" => "failed"; uu::%)
  "failed"

- p ==
  p case R => -$R p
  [p.v, - p.ts]$VPoly
n * p ==
  p case R => n * p::R
  mvar:=p.v
  up:=n*p.ts
  if ground? up then leadingCoefficient(up) else [mvar,up]$VPoly
c * p ==
  c = 1 => p
  p case R => c * p::R
  mvar:=p.v
  up:=c*p.ts
  if ground? up then leadingCoefficient(up) else [mvar,up]$VPoly
p1 + p2 ==
  p1 case R and p2 case R => p1 +$R p2
  p1 case R => [p2.v, p1::D + p2.ts]$VPoly
  p2 case R => [p1.v, p1.ts + p2::D]$VPoly
  p1.v = p2.v =>
    mvar:=p1.v
    up:=p1.ts+p2.ts
    if ground? up then leadingCoefficient(up) else [mvar,up]$VPoly
  p1.v < p2.v =>
    [p2.v, p1::D + p2.ts]$VPoly
    [p1.v, p1.ts + p2::D]$VPoly

p1 - p2 ==
  p1 case R and p2 case R => p1 -$R p2
  p1 case R => [p2.v, p1::D - p2.ts]$VPoly
  p2 case R => [p1.v, p1.ts - p2::D]$VPoly
  p1.v = p2.v =>
    mvar:=p1.v
    up:=p1.ts-p2.ts
    if ground? up then leadingCoefficient(up) else [mvar,up]$VPoly
  p1.v < p2.v =>

```

```

      [p2.v, p1::D - p2.ts]$VPoly
      [p1.v, p1.ts - p2::D]$VPoly

p1 = p2 ==
  p1 case R =>
    p2 case R => p1 =$R p2
    false
  p2 case R => false
  p1.v = p2.v => p1.ts = p2.ts
  false

p1 * p2 ==
  p1 case R => p1::R * p2
  p2 case R =>
    mvar:=p1.v
    up:=p1.ts*p2
    if ground? up then leadingCoefficient(up) else [mvar,up]$VPoly
  p1.v = p2.v =>
    mvar:=p1.v
    up:=p1.ts*p2.ts
    if ground? up then leadingCoefficient(up) else [mvar,up]$VPoly
  p1.v > p2.v =>
    mvar:=p1.v
    up:=p1.ts*p2
    if ground? up then leadingCoefficient(up) else [mvar,up]$VPoly
    --- p1.v < p2.v
  mvar:=p2.v
  up:=p1*p2.ts
  if ground? up then leadingCoefficient(up) else [mvar,up]$VPoly

p ^ kp == p ** (kp pretend NonNegativeInteger)
p ** kp == p ** (kp pretend NonNegativeInteger )
p ^ k == p ** k
p ** k ==
  p case R => p::R ** k
  -- univariate special case
  not moreThanOneVariable? p =>
    multivariate( (univariateKnown p) ** k , p.v)
  mvar:=p.v
  up:=p.ts ** k
  if ground? up then leadingCoefficient(up) else [mvar,up]$VPoly

if R has IntegralDomain then
  UnitCorrAssoc ==> Record(unit:%,canonical:%,associate:%)
  unitNormal(p) ==
    u,c,a:R
    p case R =>
      (u,c,a):= unitNormal(p::R)$R
      [u::%,c::%,a::%]$UnitCorrAssoc
      (u,c,a):= unitNormal(leadingCoefficient(p))$R

```



```

      [u::%, (a*p)::%, a::%]$UnitCorrAssoc
unitCanonical(p) ==
  p case R => unitCanonical(p::R)$R
  (u,c,a):= unitNormal(leadingCoefficient(p))$R
  a*p
unit? p ==
  p case R => unit?(p::R)$R
  false
associates?(p1,p2) ==
  p1 case R => p2 case R and associates?(p1,p2)$R
  p2 case VPoly and p1.v = p2.v and associates?(p1.ts,p2.ts)

if R has approximate then
  p1 exquo p2 ==
    p1 case R and p2 case R =>
      a:= (p1::R exquo p2::R)
      if a case "failed" then "failed" else a::%
    zero? p1 => p1
    one? p2 => p1
    (p2 = 1) => p1
    p1 case R or p2 case VPoly and p1.v < p2.v => "failed"
    p2 case R or p1.v > p2.v =>
      a:= (p1.ts exquo p2::D)
      a case "failed" => "failed"
      [p1.v,a]$VPoly::%
    -- The next test is useful in the case that R has inexact
    -- arithmetic (in particular when it is Interval(...)).
    -- In the case where the test succeeds, empirical evidence
    -- suggests that it can speed up the computation several times,
    -- but in other cases where there are a lot of variables
    -- and p1 and p2 differ only in the low order terms (e.g. p1=p2+1)
    -- it slows exquo down by about 15-20%.
    p1 = p2 => 1
    a:= p1.ts exquo p2.ts
    a case "failed" => "failed"
    mvar:=p1.v
    up:SUP %:=a
    if ground? (up) then
      leadingCoefficient(up) else [mvar,up]$VPoly::%
else
  p1 exquo p2 ==
    p1 case R and p2 case R =>
      a:= (p1::R exquo p2::R)
      if a case "failed" then "failed" else a::%
    zero? p1 => p1
    one? p2 => p1
    (p2 = 1) => p1
    p1 case R or p2 case VPoly and p1.v < p2.v => "failed"
    p2 case R or p1.v > p2.v =>
      a:= (p1.ts exquo p2::D)

```

```

        a case "failed" => "failed"
        [p1.v,a]$VPoly::%
a:= p1.ts exquo p2.ts
a case "failed" => "failed"
mvar:=p1.v
up:SUP %:=a
if ground? up then leadingCoefficient(up) else [mvar,up]$VPoly::%

map(fn,p) ==
  p case R => fn(p)
  mvar:=p.v
  up:=map(x1+>->map(fn,x1),p.ts)
  if ground? up then leadingCoefficient(up) else [mvar,up]$VPoly

if R has Field then
  (p : %) / (r : R) == inv(r) * p

if R has GcdDomain then
  content(p) ==
    p case R => p
    c :R :=0
    up:=p.ts
--    while not(zero? up) and not(one? c) repeat
    while not(zero? up) and not(c = 1) repeat
      c:=gcd(c,content leadingCoefficient(up))
      up := reductum up
    c

if R has EuclideanDomain and
  R has CharacteristicZero and
  not(R has FloatingPointSystem) then

  content(p,mvar) ==
    p case R => p
    gcd(coefficients univariate(p,mvar))$pgcd

  gcd(p1,p2) == gcd(p1,p2)$pgcd

  gcd(lp>List %) == gcd(lp)$pgcd

  gcdPolynomial(a:SUP $,b:SUP $):SUP $ == gcd(a,b)$pgcd

else if R has GcdDomain then
  content(p,mvar) ==
    p case R => p
    content univariate(p,mvar)

  gcd(p1,p2) ==
    p1 case R =>
      p2 case R => gcd(p1,p2)$R::%

```

```

    zero? p1 => p2
    gcd(p1, content(p2.ts))
p2 case R =>
    zero? p2 => p1
    gcd(p2, content(p1.ts))
p1.v < p2.v => gcd(p1, content(p2.ts))
p1.v > p2.v => gcd(content(p1.ts), p2)
mvar:=p1.v
up:=gcd(p1.ts, p2.ts)
if ground? up then leadingCoefficient(up) else [mvar,up]$VPoly

if R has FloatingPointSystem then
-- eventually need a better notion of gcd's over floats
-- this essentially computes the gcds of the monomial contents
gcdPolynomial(a:SUP $,b:SUP $):SUP $ ==
    ground? (a) =>
        zero? a => b
        gcd(leadingCoefficient a, content b)::SUP $
    ground?(b) =>
        zero? b => a
        gcd(leadingCoefficient b, content a)::SUP $
    conta := content a
    mona:SUP $ := monomial(conta, minimumDegree a)
    if mona ^= 1 then
        a := (a exquo mona)::SUP $
    contb := content b
    monb:SUP $ := monomial(contb, minimumDegree b)
    if monb ^= 1 then
        b := (b exquo monb)::SUP $
    mong:SUP $ := monomial(gcd(conta, contb),
        min(degree mona, degree monb))
    degree(a) >= degree b =>
        not((a exquo b) case "failed") =>
            mong * b
        mong
    not((b exquo a) case "failed") => mong * a
    mong

coerce(p):OutputForm ==
    p case R => (p::R)::OutputForm
    outputForm(p.ts,p.v::OutputForm)

coefficients p ==
    p case R => list(p :: R)$List(R)
    "append"/[coefficients(p1)$% for p1 in coefficients(p.ts)]

retract(p:%):R ==
    p case R => p :: R
    error "cannot retract nonconstant polynomial"

```

```

retractIfCan(p:%):Union(R, "failed") ==
  p case R => p::R
  "failed"

--      leadingCoefficientRecursive(p:%):% ==
--      p case R => p
--      leadingCoefficient p.ts

mymerge:(List VarSet,List VarSet) ->List VarSet
mymerge(l:List VarSet,m:List VarSet):List VarSet ==
  empty? l => m
  empty? m => l
  first l = first m =>
    empty? rest l =>
      setrest!(l,rest m)
      l
    empty? rest m => l
    setrest!(l, mymerge(rest l, rest m))
    l
  first l > first m =>
    empty? rest l =>
      setrest!(l,m)
      l
    setrest!(l, mymerge(rest l, m))
    l
  empty? rest m =>
    setrest!(m,l)
    m
  setrest!(m,mymerge(l,rest m))
  m

variables p ==
  p case R => empty()
  lv:List VarSet:=empty()
  q := p.ts
  while not zero? q repeat
    lv:=mymerge(lv,variables leadingCoefficient q)
    q := reductum q
  cons(p.v,lv)

mainVariable p ==
  p case R => "failed"
  p.v

eval(p,mvar,pval) == univariate(p,mvar)(pval)
eval(p,mvar,val) == univariate(p,mvar)(val)

evalSortedVarlist(p,Lvar,Lpval):% ==
  p case R => p
  empty? Lvar or empty? Lpval => p

```

```

mvar := Lvar.first
mvar > p.v => evalSortedVarlist(p,Lvar.rest,Lpval.rest)
pval := Lpval.first
pts := map(x1+>=>evalSortedVarlist(x1,Lvar,Lpval),p.ts)
mvar=p.v =>
  pval case R => pts (pval::R)
  pts pval
multivariate(pts,p.v)

eval(p,Lvar,Lpval) ==
  empty? rest Lvar => evalSortedVarlist(p,Lvar,Lpval)
  sorted?((x1,x2) +> x1 > x2, Lvar) => evalSortedVarlist(p,Lvar,Lpval)
  nlvar := sort((x1,x2) +> x1 > x2,Lvar)
  nlpval :=
    Lvar = nlvar => Lpval
    nlpval := [Lpval.position(mvar,Lvar) for mvar in nlvar]
  evalSortedVarlist(p,nlvar,nlpval)

eval(p,Lvar,Lval) ==
  eval(p,Lvar,[val::% for val in Lval]$(List %)) -- kill?

degree(p,mvar) ==
  p case R => 0
  mvar= p.v => degree p.ts
  mvar > p.v => 0 -- might as well take advantage of the order
  max(degree(leadingCoefficient p.ts,mvar),degree(red p,mvar))

degree(p,Lvar) == [degree(p,mvar) for mvar in Lvar]

degree p ==
  p case R => 0
  degree(leadingCoefficient(p.ts)) + monomial(degree(p.ts), p.v)

minimumDegree p ==
  p case R => 0
  md := minimumDegree p.ts
  minimumDegree(coefficient(p.ts,md)) + monomial(md, p.v)

minimumDegree(p,mvar) ==
  p case R => 0
  mvar = p.v => minimumDegree p.ts
  md:=minimumDegree(leadingCoefficient p.ts,mvar)
  zero? (p1:=red p) => md
  min(md,minimumDegree(p1,mvar))

minimumDegree(p,Lvar) ==
  [minimumDegree(p,mvar) for mvar in Lvar]

totalDegree(p, Lvar) ==
  ground? p => 0

```

```

null setIntersection(Lvar, variables p) => 0
u := univariate(p, mv := mainVariable(p)::VarSet)
weight:NonNegativeInteger := (member?(mv,Lvar) => 1; 0)
tdeg:NonNegativeInteger := 0
while u ^= 0 repeat
    termdeg:NonNegativeInteger := weight*degree u +
        totalDegree(leadingCoefficient u, Lvar)
    tdeg := max(tdeg, termdeg)
    u := reductum u
tdeg

if R has CommutativeRing then
    differentiate(p,mvar) ==
        p case R => 0
        mvar=p.v =>
            up:=differentiate p.ts
            if ground? up then leadingCoefficient(up) else [mvar,up]$VPoly
            up:=map(x1 +-> differentiate(x1,mvar),p.ts)
            if ground? up then leadingCoefficient(up) else [p.v,up]$VPoly

    leadingCoefficient(p) ==
        p case R => p
        leadingCoefficient(leadingCoefficient(p.ts))

--    trailingCoef(p) ==
--        p case R => p
--        coef(p.ts,0) case R => coef(p.ts,0)
--        trailingCoef(red p)
--    TrailingCoef(p) == trailingCoef(p)

    leadingMonomial p ==
        p case R => p
        monomial(leadingMonomial leadingCoefficient(p.ts),
            p.v, degree(p.ts))

    reductum(p) ==
        p case R => 0
        p - leadingMonomial p

--
--    if R is Integer then
--        pgcd := PolynomialGcdPackage(%,VarSet)
--        gcd(p1,p2) ==
--            gcd(p1,p2)$pgcd
--
--    else if R is RationalNumber then
--        gcd(p1,p2) ==
--            mrat:= MRationalFactorize(VarSet,%)
--            gcd(p1,p2)$mrat
--

```

```

--      else gcd(p1,p2) ==
--      p1 case R =>
--      p2 case R => gcd(p1,p2)$R::%
--      p1 = 0 => p2
--      gcd(p1, content(p2.ts))
--      p2 case R =>
--      p2 = 0 => p1
--      gcd(p2, content(p1.ts))
--      p1.v < p2.v => gcd(p1, content(p2.ts))
--      p1.v > p2.v => gcd(content(p1.ts), p2)
--      PSimp(p1.v, gcd(p1.ts, p2.ts))

```

---

— SMP.dotabb —

```

"SMP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SMP"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"SMP" -> "PFECAT"

```

---

## 20.15 domain SMTS SparseMultivariateTaylorSeries

— SparseMultivariateTaylorSeries.input —

```

)set break resume
)sys rm -f SparseMultivariateTaylorSeries.output
)spool SparseMultivariateTaylorSeries.output
)set message test on
)set message auto off
)clear all

--S 1 of 10
xts:=x::TaylorSeries Fraction Integer
--R
--R
--R (1) x
--R
--R                                          Type: TaylorSeries Fraction Integer
--E 1

--S 2 of 10
yts:=y::TaylorSeries Fraction Integer
--R

```

```

--R
--R (2) y
--R
--R                                     Type: TaylorSeries Fraction Integer
--E 2

--S 3 of 10
zts:=z::TaylorSeries Fraction Integer
--R
--R
--R (3) z
--R
--R                                     Type: TaylorSeries Fraction Integer
--E 3

--S 4 of 10
t1:=sin(xts)
--R
--R
--R
--R      1 3      1 5      1 7      1 9
--R (4)  x - - x + --- x - ---- x + ----- x + 0(11)
--R      6      120     5040    362880
--R
--R                                     Type: TaylorSeries Fraction Integer
--E 4

--S 5 of 10
coefficient(t1,3)
--R
--R
--R      1 3
--R (5)  - - x
--R      6
--R
--R                                     Type: Polynomial Fraction Integer
--E 5

--S 6 of 10
coefficient(t1,monomial(3,x)$IndexedExponents Symbol)
--R
--R
--R      1
--R (6)  - -
--R      6
--R
--R                                     Type: Fraction Integer
--E 6

--S 7 of 10
t2:=sin(xts + yts)
--R
--R
--R (7)
--R
--R      1 3      1 2      1 2      1 3
--R (y + x) + (- - y - - x y - - x y - - x )

```



```

--R          6      2      2      6
--R      +
--R      1 5      1 4      1 2 3      1 3 2      1 4      1 5
--R      (--- y + -- x y + -- x y + -- x y + -- x y + --- x )
--R      120      24      12      12      24      120
--R      +
--R      PAREN
--R      1 7      1 6      1 2 5      1 3 4      1 4 3      1 5 2
--R      - ---- y - --- x y - --- x y - --- x y - --- x y - --- x y
--R      5040      720      240      144      144      240
--R      +
--R      1 6      1 7
--R      - --- x y - ---- x
--R      720      5040
--R      +
--R      PAREN
--R      1 9      1 8      1 2 7      1 3 6      1 4 5
--R      ----- y + ----- x y + ----- x y + ----- x y + ----- x y
--R      362880      40320      10080      4320      2880
--R      +
--R      1 5 4      1 6 3      1 7 2      1 8      1 9
--R      ---- x y + ---- x y + ---- x y + ---- x y + ----- x
--R      2880      4320      10080      40320      362880
--R      +
--R      0(11)
--R
--R                                          Type: TaylorSeries Fraction Integer
--E 7

--S 8 of 10
coefficient(t2,3)
--R
--R
--R      1 3      1 2      1 2      1 3
--R      (8) - - y - - x y - - x y - - x
--R      6      2      2      6
--R
--R                                          Type: Polynomial Fraction Integer
--E 8

--S 9 of 10
coefficient(t2,monomial(3,x)$IndexedExponents Symbol)
--R
--R
--R      1
--R      (9) - -
--R      6
--R
--R                                          Type: Fraction Integer
--E 9

--S 10 of 10
polynomial(t2,5)

```

```

--R
--R
--R (10)
--R      1 5 1 4 1 2 1 3 1 3 1 2 1 4 1 2
--R      --- y + -- x y + (-- x - -)y + (-- x - - x)y + (-- x - - x + 1)y
--R      120 24 12 6 12 2 24 2
--R +
--R      1 5 1 3
--R      --- x - - x + x
--R      120 6
--R
--R                                          Type: Polynomial Fraction Integer
--E 10

```

```

)spool
)lisp (bye)

```

---

— SparseMultivariateTaylorSeries.help —

```

=====
SparseMultivariateTaylorSeries examples
=====

```

Assume we have three variables which get expressed as sparse multivariate taylor series.

```

xts:=x::TaylorSeries Fraction Integer
yts:=y::TaylorSeries Fraction Integer
zts:=z::TaylorSeries Fraction Integer

```

These will cause traditional routines to expand in series form:

```

t1:=sin(xts)

```

$$x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{5040}x^7 + \frac{1}{362880}x^9 + O(11)$$

We can ask for a specific coefficient, in this case, the coefficient of the third power.

```

coefficient(t1,3)

```

$$-\frac{1}{6}x^3$$

And we can get that coefficient, expressed as a monomial.

```
coefficient(t1,monomial(3,x)$IndexedExponents Symbol)
```

$$-\frac{1}{6}$$

In a multivariate version we get a polynomial in x and y

```
t2:=sin(xts + yts)
```

$$\begin{aligned}
 & (y + x) + \left( -\frac{1}{6}y^3 - \frac{1}{2}xy^2 - \frac{1}{2}x^2y - \frac{1}{6}x^3 \right) \\
 & + \left( \frac{1}{120}y^5 + \frac{1}{24}xy^4 + \frac{1}{12}x^2y^3 + \frac{1}{12}x^3y^2 + \frac{1}{24}x^4y + \frac{1}{120}x^5 \right) \\
 & + \text{PAREN} \\
 & \quad -\frac{1}{5040}y^7 - \frac{1}{720}xy^6 - \frac{1}{240}x^2y^5 - \frac{1}{144}x^3y^4 - \frac{1}{144}x^4y^3 - \frac{1}{240}x^5y^2 \\
 & \quad + \frac{1}{720}x^6y - \frac{1}{5040}x^7 \\
 & + \text{PAREN} \\
 & \quad \frac{1}{362880}y^9 + \frac{1}{40320}xy^8 + \frac{1}{10080}x^2y^7 + \frac{1}{4320}x^3y^6 + \frac{1}{2880}x^4y^5 \\
 & \quad + \frac{1}{2880}x^5y^4 + \frac{1}{4320}x^6y^3 + \frac{1}{10080}x^7y^2 + \frac{1}{40320}x^8y + \frac{1}{362880}x^9 \\
 & + 0(11)
 \end{aligned}$$

We can ask for the third coefficient which is

```
coefficient(t2,3)
```

$$-\frac{1}{6}y^3 - \frac{1}{2}xy^2 - \frac{1}{2}x^2y - \frac{1}{6}x^3$$

And we can ask for the third coefficient of that coefficient in x

```
coefficient(t2,monomial(3,x)$IndexedExponents Symbol)
```

$$-\frac{1}{6}$$

And we can convert that result to a polynomial

```
polynomial(t2,5)
```

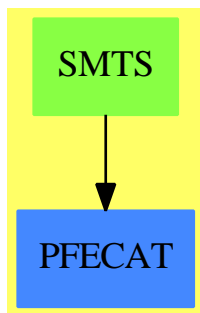
$$\begin{aligned} & \frac{1}{120}y^5 + \frac{1}{24}x^4y + \left(\frac{1}{12}x^2 - \frac{1}{6}\right)y^3 + \left(\frac{1}{12}x^3 - \frac{1}{2}x\right)y^2 + \left(\frac{1}{24}x^4 - \frac{1}{2}x^2 + 1\right)y \\ & + \frac{1}{120}x^5 - \frac{1}{6}x^3 + x \end{aligned}$$

See Also:

- o )show SparseMultivariateTaylorSeries
- o )display op coefficient

---

### 20.15.1 SparseMultivariateTaylorSeries (SMTS)



See

⇒ “TaylorSeries” (TS) 21.3.1 on page 2628

**Exports:**

|                |            |               |                    |                 |
|----------------|------------|---------------|--------------------|-----------------|
| 0              | 1          | acos          | acosh              | acot            |
| acoth          | acsc       | acsch         | asec               | asech           |
| asin           | asinh      | associates?   | atan               | atanh           |
| characteristic | charthRoot | coefficient   | coerce             | complete        |
| cos            | cosh       | cot           | coth               | csc             |
| csch           | csubst     | D             | degree             | differentiate   |
| eval           | exp        | exquo         | extend             | fintegrate      |
| hash           | integrate  | latex         | leadingCoefficient | leadingMonomial |
| log            | map        | monomial      | monomial?          | nthRoot         |
| one?           | order      | pi            | pole?              | polynomial      |
| recip          | reductum   | sample        | sec                | sech            |
| sin            | sinh       | sqrt          | subtractIfCan      | tan             |
| tanh           | unit?      | unitCanonical | unitNormal         | variables       |
| zero?          | ?*?        | ?**?          | ?+?                | ?-?             |
| -?             | ?=?        | ?^?           | ?~=?               |                 |

## — domain SMTS SparseMultivariateTaylorSeries —

```

)abbrev domain SMTS SparseMultivariateTaylorSeries
++ Authors: William Burge, Stephen Watt, Clifton Williamson
++ Date Created: 15 August 1988
++ Date Last Updated: 18 May 1991
++ Basic Operations:
++ Related Domains:
++ Also See: UnivariateTaylorSeries
++ AMS Classifications:
++ Keywords: multivariate, Taylor, series
++ Examples:
++ References:
++ Description:
++ This domain provides multivariate Taylor series with variables
++ from an arbitrary ordered set. A Taylor series is represented
++ by a stream of polynomials from the polynomial domain SMP.
++ The nth element of the stream is a form of degree n. SMTS is an
++ internal domain.

```

```

SparseMultivariateTaylorSeries(Coef,Var,SMP):_
Exports == Implementation where
  Coef : Ring
  Var  : OrderedSet
  SMP  : PolynomialCategory(Coef,IndexedExponents Var,Var)
  I    ==> Integer
  L    ==> List
  NNI  ==> NonNegativeInteger
  OUT  ==> OutputForm
  PS   ==> InnerTaylorSeries SMP
  RN   ==> Fraction Integer
  ST   ==> Stream

```

```

StS ==> Stream SMP
STT ==> StreamTaylorSeriesOperations SMP
STF ==> StreamTranscendentalFunctions SMP
ST2 ==> StreamFunctions2
ST3 ==> StreamFunctions3

Exports ==> MultivariateTaylorSeriesCategory(Coef,Var) with
  coefficient: (%,NNI) -> SMP
    ++ \spad{coefficient(s, n)} gives the terms of total degree n.
    ++
    ++X xts:=x::TaylorSeries Fraction Integer
    ++X t1:=sin(xts)
    ++X coefficient(t1,3)

  coerce: Var -> %
    ++ \spad{coerce(var)} converts a variable to a Taylor series
  coerce: SMP -> %
    ++ \spad{coerce(poly)} regroups the terms by total degree and forms
    ++ a series.
  "*" : (SMP,%) -> %
    ++ \spad{smp*ts} multiplies a TaylorSeries by a monomial SMP.
  csubst: (L Var, L StS) -> (SMP -> StS)
    ++ \spad{csubst(a,b)} is for internal use only

if Coef has Algebra Fraction Integer then
  integrate: (%,Var,Coef) -> %
    ++ \spad{integrate(s,v,c)} is the integral of s with respect
    ++ to v and having c as the constant of integration.
  fintegrate: (() -> %,Var,Coef) -> %
    ++ \spad{fintegrate(f,v,c)} is the integral of \spad{f()} with respect
    ++ to v and having c as the constant of integration.
    ++ The evaluation of \spad{f()} is delayed.

Implementation ==> PS add

Rep := StS -- Below we use the fact that Rep of PS is Stream SMP.
extend(x,n) == extend(x,n + 1)$Rep
complete x == complete(x)$Rep

evalstream:(%,L Var,L SMP) -> StS
evalstream(s:%,lv:(L Var),lsmp:(L SMP)):(ST SMP) ==
  scan(0,_+$SMP,
    map((z1:SMP):SMP+>eval(z1,lv,lsmp),s pretend StS))$ST2(SMP,SMP)

addvariable:(Var,InnerTaylorSeries Coef) -> %
addvariable(v,s) ==
  ints := integers(0)$STT pretend ST NNI
  map((n1:NNI,c2:Coef):SMP+>monomial(c2 :: SMP,v,n1)$SMP,
    ints,s pretend ST Coef)$ST3(NNI,Coef,SMP)

```

```

-- We can extract a polynomial giving the terms of given total degree
coefficient(s,n) == elt(s,n + 1)$Rep -- 1-based indexing for streams

-- Here we have to take into account that we reduce the degree of each
-- term of the stream by a constant
coefficient(s:%,lv>List Var,ln>List NNI):% ==
  map ((z1:SMP):SMP +-> coefficient(z1,lv,ln),rest(s,reduce(_+,ln)))

-- the coefficient of a particular monomial:
coefficient(s:%,m:IndexedExponents Var):Coef ==
  n:=leadingCoefficient(mon:=m)
  while not zero?(mon:=reductum mon) repeat
    n:=n+leadingCoefficient mon
  coefficient(coefficient(s,n),m)

--% creation of series

coerce(r:Coef) == monom(r::SMP,0)$STT
smp:SMP * p:% == (((smp) * (p pretend Rep))$STT)pretend %
r:Coef * p:% == (((r::SMP) * (p pretend Rep))$STT)pretend %
p:% * r:Coef == (((r::SMP) * (p pretend Rep))$STT)pretend %
mts(p:SMP):% ==
  (uv := mainVariable p) case "failed" => monom(p,0)$STT
  v := uv :: Var
  s : % := 0
  up := univariate(p,v)
  while not zero? up repeat
    s := s + monomial(1,v,degree up) * mts(leadingCoefficient up)
    up := reductum up
  s

coerce(p:SMP) == mts p
coerce(v:Var) == v :: SMP :: %

monomial(r:%,v:Var,n:NNI) ==
  r * monom(monomial(1,v,n)$SMP,n)$STT

--% evaluation

substvar: (SMP,L Var,L %) -> %
substvar(p,vl,q) ==
  null vl => monom(p,0)$STT
  (uv := mainVariable p) case "failed" => monom(p,0)$STT
  v := uv :: Var
  v = first vl =>
    s : % := 0
    up := univariate(p,v)
    while not zero? up repeat
      c := leadingCoefficient up
      s := s + first q ** degree up * substvar(c,rest vl,rest q)

```

```

    up := reductum up
  s
  substvar(p,rest vl,rest q)

sortmfirst:(SMP,L Var,L %) -> %
sortmfirst(p,vl,q) ==
  nlv : L Var := sort((v1:Var,v2:Var):Boolean +-> v1 > v2,vl)
  nq : L % := [q position$(L Var) (i,vl) for i in nlv]
  substvar(p,nlv,nq)

csubst(vl,q) == (p1:SMP):StS+>sortmfirst(p1,vl,q pretend L(%)) pretend StS

restCheck(s:StS):StS ==
  -- checks that stream is null or first element is 0
  -- returns empty() or rest of stream
  empty? s => s
  not zero? first s =>
    error "eval: constant coefficient should be 0"
  rst s

eval(s:%,v:L Var,q:L %) ==
  #v ^= #q =>
    error "eval: number of variables should equal number of values"
  nq : L StS := [restCheck(i pretend StS) for i in q]
  adddiag(map(csubst(v,nq),s pretend StS)$ST2(SMP,StS))$STT pretend %

substmmts(v:Var,p:SMP,q:%):% ==
  up := univariate(p,v)
  ss : % := 0
  while not zero? up repeat
    d:=degree up
    c:SMP:=leadingCoefficient up
    ss := ss + c* q ** d
    up := reductum up
  ss

subststream(v:Var,p:SMP,q:StS):StS==
  substmmts(v,p,q pretend %) pretend StS

comp1:(Var,StS,StS) -> StS
comp1(v,r,t)==
  adddiag(map((p1:SMP):StS +-> subststream(v,p1,t),r)$ST2(SMP,StS))$STT

comp(v:Var,s:StS,t:StS):StS == delay
  empty? s => s
  f := first s; r : StS := rst s;
  empty? r => s
  empty? t => concat(f,comp1(v,r,empty())$StS)
  not zero? first t =>
    error "eval: constant coefficient should be zero"

```



```

concat(f,comp1(v,r,rst t))

eval(s:%,v:Var,t:%) == comp(v,s pretend StS,t pretend StS)

--% differentiation and integration

differentiate(s:%,v:Var):% ==
  empty? s => 0
  map((z1:SMP):SMP +-> differentiate(z1,v),rst s)

if Coef has Algebra Fraction Integer then

stream(x:%):Rep == x pretend Rep

(x:%) ** (r:RN) == powern(r,stream x)$STT
(r:RN) * (x:%) ==
  map((z1:SMP):SMP +-> r*z1,stream x)$ST2(SMP,SMP) pretend %
(x:%) * (r:RN) ==
  map((z1:SMP):SMP +-> z1*r,stream x)$ST2(SMP,SMP) pretend %

exp x == exp(stream x)$STF
log x == log(stream x)$STF

sin x == sin(stream x)$STF
cos x == cos(stream x)$STF
tan x == tan(stream x)$STF
cot x == cot(stream x)$STF
sec x == sec(stream x)$STF
csc x == csc(stream x)$STF

asin x == asin(stream x)$STF
acos x == acos(stream x)$STF
atan x == atan(stream x)$STF
acot x == acot(stream x)$STF
asec x == asec(stream x)$STF
acsc x == acsc(stream x)$STF

sinh x == sinh(stream x)$STF
cosh x == cosh(stream x)$STF
tanh x == tanh(stream x)$STF
coth x == coth(stream x)$STF
sech x == sech(stream x)$STF
csch x == csch(stream x)$STF

asinh x == asinh(stream x)$STF
acosh x == acosh(stream x)$STF
atanh x == atanh(stream x)$STF
acoth x == acoth(stream x)$STF
asech x == asech(stream x)$STF
acsch x == acsch(stream x)$STF

```

```

intsmp(v:Var,p: SMP): SMP ==
  up := univariate(p,v)
  ss : SMP := 0
  while not zero? up repeat
    d := degree up
    c := leadingCoefficient up
    ss := ss + inv((d+1) :: RN) * monomial(c,v,d+1)$SMP
    up := reductum up
  ss

fintegrate(f,v,r) ==
  concat(r::SMP,delay map((z1:SMP):SMP +-> intsmv(v,z1),f() pretend StS))
integrate(s,v,r) ==
  concat(r::SMP,map((z1:SMP):SMP +-> intsmv(v,z1),s pretend StS))

-- If there is more than one term of the same order, group them.
tout(p:SMP):OUT ==
  pe := p :: OUT
  monomial? p => pe
  paren pe

showAll?: () -> Boolean
-- check a global Lisp variable
showAll?() == true

coerce(s:%):OUT ==
  uu := s pretend Stream(SMP)
  empty? uu => (0$SMP) :: OUT
  n : NNI; count : NNI := _$streamCount$Lisp
  l : List OUT := empty()
  for n in 0..count while not empty? uu repeat
    if frst(uu) ^= 0 then l := concat(tout frst uu,l)
    uu := rst uu
  if showAll?() then
    for n in n.. while explicitEntries? uu and _
      not eq?(uu,rst uu) repeat
        if frst(uu) ^= 0 then l := concat(tout frst uu,l)
        uu := rst uu
  l :=
    explicitlyEmpty? uu => l
    eq?(uu,rst uu) and frst uu = 0 => l
    concat(prefix("0" :: OUT,[n :: OUT]),l)
  empty? l => (0$SMP) :: OUT
  reduce("+",reverse_! l)
if Coef has Field then
  stream(x:%):Rep == x pretend Rep
  SF2==> StreamFunctions2
  p:% / r:Coef ==
    (map((z1:SMP):SMP +-> z1/$SMP r,stream p)$SF2(SMP,SMP)) pretend %

```



```

--S 4 of 7
t.3
--R
--R
--R (4) "Number three"
--R
--R                                         Type: String
--E 4

--S 5 of 7
t.2
--R
--R
--R (5) "Try again!"
--R
--R                                         Type: String
--E 5

--S 6 of 7
keys t
--R
--R
--R (6) [4,3]
--R
--R                                         Type: List Integer
--E 6

--S 7 of 7
entries t
--R
--R
--R (7) ["Number four","Number three"]
--R
--R                                         Type: List String
--E 7
)spool
)lisp (bye)

```

---

— SparseTable.help —

```

=====
SparseTable examples
=====

```

The SparseTable domain provides a general purpose table type with default entries.

Here we create a table to save strings under integer keys. The value "Try again!" is returned if no other value has been stored for a key.

```
t: SparseTable(Integer, String, "Try again!") := table()
table()
Type: SparseTable(Integer,String,Try again!)
```

Entries can be stored in the table.

```
t.3 := "Number three"
"Number three"
Type: String
```

```
t.4 := "Number four"
"Number four"
Type: String
```

These values can be retrieved as usual, but if a look up fails the default entry will be returned.

```
t.3
"Number three"
Type: String
```

```
t.2
"Try again!"
Type: String
```

To see which values are explicitly stored, the keys and entries functions can be used.

```
keys t
[4,3]
Type: List Integer
```

```
entries t
["Number four","Number three"]
Type: List String
```

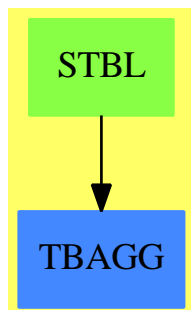
If a specific table representation is required, the `GeneralSparseTable` constructor should be used. The domain `SparseTable(K, E, dflt)` is equivalent to `GeneralSparseTable(K,E,Table(K,E), dflt)`.

See Also:

- o )help Table
- o )help GeneralSparseTable
- o )show SparseTable

---

### 20.16.1 SparseTable (STBL)



See

- ⇒ “HashTable” (HASHTBL) 9.1.1 on page 1085
- ⇒ “InnerTable” (INTABL) 10.27.1 on page 1299
- ⇒ “Table” (TABLE) 21.1.1 on page 2621
- ⇒ “EqTable” (EQTBL) 6.2.1 on page 667
- ⇒ “StringTable” (STRTBL) 20.32.1 on page 2569
- ⇒ “GeneralSparseTable” (GSTBL) 8.5.1 on page 1044

#### Exports:

|        |          |                  |           |          |
|--------|----------|------------------|-----------|----------|
| any?   | bag      | coerce           | construct | convert  |
| copy   | count    | dictionary       | elt       | empty    |
| empty? | entries  | entry?           | eq?       | eval     |
| every? | extract! | fill!            | find      | first    |
| hash   | index?   | indices          | insert!   | inspect  |
| key?   | keys     | latex            | less?     | map      |
| map!   | maxIndex | member?          | members   | minIndex |
| more?  | parts    | qelt             | qsetelt!  | reduce   |
| remove | remove!  | removeDuplicates | sample    | search   |
| setelt | select   | select!          | size?     | swap!    |
| table  | #?       | ?=?              | ?~=?      | ?..?     |

— domain STBL SparseTable —

```

)abbrev domain STBL SparseTable
++ Author: Stephen M. Watt
++ Date Created: 1986
++ Date Last Updated: June 21, 1991
++ Basic Operations:
++ Related Domains: Table
++ Also See:
++ AMS Classifications:
++ Keywords: equation
++ Examples:
++ References:
++ Description:
  
```

```

++ A sparse table has a default entry, which is returned if no other
++ value has been explicitly stored for a key.

```

```

SparseTable(Key:SetCategory, Ent:SetCategory, dent:Ent) ==
    GeneralSparseTable(Key, Ent, Table(Key, Ent), dent)

```

---

— STBL.dotabb —

```

"STBL" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STBL"]
"TBAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=TBAGG"]
"STBL" -> "TBAGG"

```

---

## 20.17 domain SULS SparseUnivariateLaurentSeries

— SparseUnivariateLaurentSeries.input —

```

)set break resume
)sys rm -f SparseUnivariateLaurentSeries.output
)spool SparseUnivariateLaurentSeries.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SparseUnivariateLaurentSeries
--R SparseUnivariateLaurentSeries(Coef: Ring,var: Symbol,cen: Coef) is a domain constructor
--R Abbreviation for SparseUnivariateLaurentSeries is SULS
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SULS
--R
--R----- Operations -----
--R ?? : (Coef,%) -> %           ?? : (%,Coef) -> %
--R ?? : (%,%) -> %           ?? : (Integer,%) -> %
--R ?? : (PositiveInteger,%) -> %   ??? : (%,PositiveInteger) -> %
--R ?? : (%,%) -> %           ?-? : (%,%) -> %
--R -? : % -> %               ?? : (%,%) -> Boolean
--R 1 : () -> %               0 : () -> %
--R ?? : (%,PositiveInteger) -> %   center : % -> Coef
--R coefficient : (%,Integer) -> Coef   coerce : Variable var -> %
--R coerce : Integer -> %           coerce : % -> OutputForm

```

```

--R complete : % -> %
--R ?..? : (%,Integer) -> Coef
--R hash : % -> SingleInteger
--R leadingCoefficient : % -> Coef
--R map : ((Coef -> Coef),%) -> %
--R monomial? : % -> Boolean
--R order : (%,Integer) -> Integer
--R pole? : % -> Boolean
--R reductum : % -> %
--R removeZeroes : % -> %
--R truncate : (%,Integer) -> %
--R zero? : % -> Boolean
--R ?? : (%Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
--R ?? : (SparseUnivariateTaylorSeries(Coef,var,cen),%) -> % if Coef has FIELD
--R ?? : (%SparseUnivariateTaylorSeries(Coef,var,cen)) -> % if Coef has FIELD
--R ?? : (NonNegativeInteger,%) -> %
--R ?? : (%Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?? : (%,) -> % if Coef has ALGEBRA FRAC INT
--R ?? : (%Integer) -> % if Coef has FIELD
--R ?? : (%NonNegativeInteger) -> %
--R ?/? : (SparseUnivariateTaylorSeries(Coef,var,cen),SparseUnivariateTaylorSeries(Coef,var,cen)) -> % if Coef has FIELD
--R ?/? : (%,) -> % if Coef has FIELD
--R ?/? : (%Coef) -> % if Coef has FIELD
--R ?<? : (%,) -> Boolean if SparseUnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef has FIELD
--R ?<=? : (%,) -> Boolean if SparseUnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef has FIELD
--R ?>? : (%,) -> Boolean if SparseUnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef has FIELD
--R ?>=? : (%,) -> Boolean if SparseUnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef has FIELD
--R D : (%Symbol) -> % if SparseUnivariateTaylorSeries(Coef,var,cen) has PDRING SYMBOL and Coef has FIELD
--R D : (%List Symbol) -> % if SparseUnivariateTaylorSeries(Coef,var,cen) has PDRING SYMBOL and Coef has FIELD
--R D : (%Symbol,NonNegativeInteger) -> % if SparseUnivariateTaylorSeries(Coef,var,cen) has PDRING SYMBOL and Coef has FIELD
--R D : (%List Symbol,List NonNegativeInteger) -> % if SparseUnivariateTaylorSeries(Coef,var,cen) has PDRING SYMBOL and Coef has FIELD
--R D : % -> % if SparseUnivariateTaylorSeries(Coef,var,cen) has DIFRING and Coef has FIELD or Coef has FIELD
--R D : (%NonNegativeInteger) -> % if SparseUnivariateTaylorSeries(Coef,var,cen) has DIFRING and Coef has FIELD
--R D : (%(SparseUnivariateTaylorSeries(Coef,var,cen) -> SparseUnivariateTaylorSeries(Coef,var,cen)),NonNegativeInteger) -> % if SparseUnivariateTaylorSeries(Coef,var,cen) has DIFRING and Coef has FIELD
--R D : (%(SparseUnivariateTaylorSeries(Coef,var,cen) -> SparseUnivariateTaylorSeries(Coef,var,cen)),NonNegativeInteger) -> % if SparseUnivariateTaylorSeries(Coef,var,cen) has DIFRING and Coef has FIELD
--R ?^? : (%Integer) -> % if Coef has FIELD
--R ?^? : (%NonNegativeInteger) -> %
--R abs : % -> % if SparseUnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef has FIELD
--R acos : % -> % if Coef has ALGEBRA FRAC INT
--R acosh : % -> % if Coef has ALGEBRA FRAC INT
--R acot : % -> % if Coef has ALGEBRA FRAC INT
--R acoth : % -> % if Coef has ALGEBRA FRAC INT
--R acsc : % -> % if Coef has ALGEBRA FRAC INT
--R acsch : % -> % if Coef has ALGEBRA FRAC INT
--R approximate : (%Integer) -> Coef if Coef has **: (Coef,Integer) -> Coef and Coef has coerce: Symbol
--R asec : % -> % if Coef has ALGEBRA FRAC INT
--R asech : % -> % if Coef has ALGEBRA FRAC INT
--R asin : % -> % if Coef has ALGEBRA FRAC INT
--R asinh : % -> % if Coef has ALGEBRA FRAC INT

```



```

--R associates? : (% , %) -> Boolean if SparseUnivariateTaylorSeries(Coef,var,cen) has OINTDOM
--R atan : % -> % if Coef has ALGEBRA FRAC INT
--R atanh : % -> % if Coef has ALGEBRA FRAC INT
--R ceiling : % -> SparseUnivariateTaylorSeries(Coef,var,cen) if SparseUnivariateTaylorSeries
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if $ has CHARNZ and SparseUnivariateTaylorSeries(Coe
--R coerce : Fraction Integer -> % if SparseUnivariateTaylorSeries(Coef,var,cen) has RETRACT
--R coerce : % -> % if SparseUnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef has F
--R coerce : Symbol -> % if SparseUnivariateTaylorSeries(Coef,var,cen) has RETRACT SYMBOL and
--R coerce : SparseUnivariateTaylorSeries(Coef,var,cen) -> %
--R coerce : Coef -> % if Coef has COMRING
--R conditionP : Matrix % -> Union(Vector %,"failed") if $ has CHARNZ and SparseUnivariateTay
--R convert : % -> Pattern Integer if SparseUnivariateTaylorSeries(Coef,var,cen) has KONVERT
--R convert : % -> Pattern Float if SparseUnivariateTaylorSeries(Coef,var,cen) has KONVERT P
--R convert : % -> DoubleFloat if SparseUnivariateTaylorSeries(Coef,var,cen) has REAL and Coe
--R convert : % -> Float if SparseUnivariateTaylorSeries(Coef,var,cen) has REAL and Coef has
--R convert : % -> InputForm if SparseUnivariateTaylorSeries(Coef,var,cen) has KONVERT INFOR
--R cos : % -> % if Coef has ALGEBRA FRAC INT
--R cosh : % -> % if Coef has ALGEBRA FRAC INT
--R cot : % -> % if Coef has ALGEBRA FRAC INT
--R coth : % -> % if Coef has ALGEBRA FRAC INT
--R csc : % -> % if Coef has ALGEBRA FRAC INT
--R csch : % -> % if Coef has ALGEBRA FRAC INT
--R denom : % -> SparseUnivariateTaylorSeries(Coef,var,cen) if Coef has FIELD
--R denominator : % -> % if Coef has FIELD
--R differentiate : (% , Symbol) -> % if SparseUnivariateTaylorSeries(Coef,var,cen) has PDRING
--R differentiate : (% , List Symbol) -> % if SparseUnivariateTaylorSeries(Coef,var,cen) has P
--R differentiate : (% , Symbol, NonNegativeInteger) -> % if SparseUnivariateTaylorSeries(Coef,
--R differentiate : (% , List Symbol, List NonNegativeInteger) -> % if SparseUnivariateTaylorSer
--R differentiate : % -> % if SparseUnivariateTaylorSeries(Coef,var,cen) has DIFRING and Coe
--R differentiate : (% , NonNegativeInteger) -> % if SparseUnivariateTaylorSeries(Coef,var,cen)
--R differentiate : (% , Variable var) -> %
--R differentiate : (% , (SparseUnivariateTaylorSeries(Coef,var,cen) -> SparseUnivariateTaylors
--R differentiate : (% , (SparseUnivariateTaylorSeries(Coef,var,cen) -> SparseUnivariateTaylors
--R divide : (% , %) -> Record(quotient: %, remainder: %) if Coef has FIELD
--R ?.? : (% , SparseUnivariateTaylorSeries(Coef,var,cen)) -> % if SparseUnivariateTaylorSeries
--R ?.? : (% , %) -> % if Integer has SGROUP
--R euclideanSize : % -> NonNegativeInteger if Coef has FIELD
--R eval : (% , List SparseUnivariateTaylorSeries(Coef,var,cen), List SparseUnivariateTaylorSer
--R eval : (% , SparseUnivariateTaylorSeries(Coef,var,cen), SparseUnivariateTaylorSeries(Coef, v
--R eval : (% , Equation SparseUnivariateTaylorSeries(Coef,var,cen)) -> % if SparseUnivariateT
--R eval : (% , List Equation SparseUnivariateTaylorSeries(Coef,var,cen)) -> % if SparseUnivar
--R eval : (% , List Symbol, List SparseUnivariateTaylorSeries(Coef,var,cen)) -> % if SparseUni
--R eval : (% , Symbol, SparseUnivariateTaylorSeries(Coef,var,cen)) -> % if SparseUnivariateTay
--R eval : (% , Coef) -> Stream Coef if Coef has **: (Coef, Integer) -> Coef
--R exp : % -> % if Coef has ALGEBRA FRAC INT
--R expressIdealMember : (List % , %) -> Union(List % , "failed") if Coef has FIELD
--R exquo : (% , %) -> Union(% , "failed") if SparseUnivariateTaylorSeries(Coef,var,cen) has OINT
--R extendedEuclidean : (% , %) -> Record(coef1: %, coef2: %, generator: %) if Coef has FIELD
--R extendedEuclidean : (% , % , %) -> Union(Record(coef1: %, coef2: %), "failed") if Coef has FIE

```

```

--R factor : % -> Factored % if Coef has FIELD
--R factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % if SparseUn
--R factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % i
--R floor : % -> SparseUnivariateTaylorSeries(Coef,var,cen) if SparseUnivariateTaylorSeries(Coef,var,cen)
--R fractionPart : % -> % if SparseUnivariateTaylorSeries(Coef,var,cen) has EUCDOM and Coef has FIELD
--R gcd : (%,% ) -> % if Coef has FIELD
--R gcd : List % -> % if Coef has FIELD
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolym
--R init : () -> % if SparseUnivariateTaylorSeries(Coef,var,cen) has STEP and Coef has FIELD
--R integrate : (% ,Variable var) -> % if Coef has ALGEBRA FRAC INT
--R integrate : (% ,Symbol) -> % if Coef has integrate: (Coef,Symbol) -> Coef and Coef has variables: Coe
--R integrate : % -> % if Coef has ALGEBRA FRAC INT
--R inv : % -> % if Coef has FIELD
--R laurent : (Integer,SparseUnivariateTaylorSeries(Coef,var,cen)) -> %
--R lcm : (%,% ) -> % if Coef has FIELD
--R lcm : List % -> % if Coef has FIELD
--R log : % -> % if Coef has ALGEBRA FRAC INT
--R map : ((SparseUnivariateTaylorSeries(Coef,var,cen) -> SparseUnivariateTaylorSeries(Coef,var,cen)),%)
--R max : (%,% ) -> % if SparseUnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef has FIELD or Spa
--R min : (%,% ) -> % if SparseUnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef has FIELD or Spa
--R monomial : (% ,List SingletonAsOrderedSet,List Integer) -> %
--R monomial : (% ,SingletonAsOrderedSet,Integer) -> %
--R multiEuclidean : (List % ,%) -> Union(List % ,"failed") if Coef has FIELD
--R multiplyCoefficients : ((Integer -> Coef),%) -> %
--R multiplyExponents : (% ,PositiveInteger) -> %
--R negative? : % -> Boolean if SparseUnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef has FIEL
--R nextItem : % -> Union(% ,"failed") if SparseUnivariateTaylorSeries(Coef,var,cen) has STEP and Coef ha
--R nthRoot : (% ,Integer) -> % if Coef has ALGEBRA FRAC INT
--R numer : % -> SparseUnivariateTaylorSeries(Coef,var,cen) if Coef has FIELD
--R numerator : % -> % if Coef has FIELD
--R patternMatch : (% ,Pattern Float,PatternMatchResult(Float,% )) -> PatternMatchResult(Float,% ) if Spars
--R patternMatch : (% ,Pattern Integer,PatternMatchResult(Integer,% )) -> PatternMatchResult(Integer,% ) if
--R pi : () -> % if Coef has ALGEBRA FRAC INT
--R positive? : % -> Boolean if SparseUnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef has FIEL
--R prime? : % -> Boolean if Coef has FIELD
--R principalIdeal : List % -> Record(coef: List % ,generator: %) if Coef has FIELD
--R ?quo? : (% ,%) -> % if Coef has FIELD
--R random : () -> % if SparseUnivariateTaylorSeries(Coef,var,cen) has INS and Coef has FIELD
--R rationalFunction : (% ,Integer,Integer) -> Fraction Polynomial Coef if Coef has INTDOM
--R rationalFunction : (% ,Integer) -> Fraction Polynomial Coef if Coef has INTDOM
--R reducedSystem : Matrix % -> Matrix Integer if SparseUnivariateTaylorSeries(Coef,var,cen) has LINEXP
--R reducedSystem : (Matrix % ,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) if SparseUniv
--R reducedSystem : (Matrix % ,Vector %) -> Record(mat: Matrix SparseUnivariateTaylorSeries(Coef,var,cen)
--R reducedSystem : Matrix % -> Matrix SparseUnivariateTaylorSeries(Coef,var,cen) if Coef has FIELD
--R ?rem? : (% ,%) -> % if Coef has FIELD
--R retract : % -> Integer if SparseUnivariateTaylorSeries(Coef,var,cen) has RETRACT INT and Coef has FI
--R retract : % -> Fraction Integer if SparseUnivariateTaylorSeries(Coef,var,cen) has RETRACT INT and Co
--R retract : % -> Symbol if SparseUnivariateTaylorSeries(Coef,var,cen) has RETRACT SYMBOL and Coef has
--R retract : % -> SparseUnivariateTaylorSeries(Coef,var,cen)
--R retractIfCan : % -> Union(Integer,"failed") if SparseUnivariateTaylorSeries(Coef,var,cen) has RETRAC

```

```

--R retractIfCan : % -> Union(Fraction Integer,"failed") if SparseUnivariateTaylorSeries(Coe
--R retractIfCan : % -> Union(Symbol,"failed") if SparseUnivariateTaylorSeries(Coef,var,cen)
--R retractIfCan : % -> Union(SparseUnivariateTaylorSeries(Coef,var,cen),"failed")
--R sec : % -> % if Coef has ALGEBRA FRAC INT
--R sech : % -> % if Coef has ALGEBRA FRAC INT
--R series : Stream Record(k: Integer,c: Coef) -> %
--R sign : % -> Integer if SparseUnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef h
--R sin : % -> % if Coef has ALGEBRA FRAC INT
--R sinh : % -> % if Coef has ALGEBRA FRAC INT
--R sizeLess? : (%,%) -> Boolean if Coef has FIELD
--R solveLinearPolynomialEquation : (List SparseUnivariatePolynomial %,SparseUnivariatePolyn
--R sqrt : % -> % if Coef has ALGEBRA FRAC INT
--R squareFree : % -> Factored % if Coef has FIELD
--R squareFreePart : % -> % if Coef has FIELD
--R squareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomi
--R subtractIfCan : (%,%) -> Union(%, "failed")
--R tan : % -> % if Coef has ALGEBRA FRAC INT
--R tanh : % -> % if Coef has ALGEBRA FRAC INT
--R taylor : % -> SparseUnivariateTaylorSeries(Coef,var,cen)
--R taylorIfCan : % -> Union(SparseUnivariateTaylorSeries(Coef,var,cen),"failed")
--R taylorRep : % -> SparseUnivariateTaylorSeries(Coef,var,cen)
--R terms : % -> Stream Record(k: Integer,c: Coef)
--R truncate : (%,Integer,Integer) -> %
--R unit? : % -> Boolean if SparseUnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef I
--R unitCanonical : % -> % if SparseUnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coe
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if SparseUnivariateTaylorSer
--R variables : % -> List SingletonAsOrderedSet
--R wholePart : % -> SparseUnivariateTaylorSeries(Coef,var,cen) if SparseUnivariateTaylorSer
--R
--E 1

)spool
)lisp (bye)

```

---

— SparseUnivariateLaurentSeries.help —

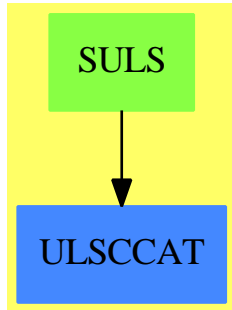
```

=====
SparseUnivariateLaurentSeries examples
=====

```

See Also:

- o )show SparseUnivariateLaurentSeries

**20.17.1 SparseUnivariateLaurentSeries (SULS)**

**Exports:**

|                               |                      |                            |
|-------------------------------|----------------------|----------------------------|
| 0                             | 1                    | abs                        |
| acos                          | acosh                | acot                       |
| acoth                         | acsc                 | acsch                      |
| approximate                   | asec                 | asech                      |
| asin                          | asinh                | associates?                |
| atan                          | atanh                | ceiling                    |
| characteristic                | charthRoot           | center                     |
| coefficient                   | coerce               | complete                   |
| conditionP                    | convert              | cos                        |
| cosh                          | cot                  | coth                       |
| csc                           | csch                 | D                          |
| degree                        | denom                | denominator                |
| differentiate                 | divide               | euclideanSize              |
| eval                          | exp                  | expressIdealMember         |
| exquo                         | extend               | extendedEuclidean          |
| factor                        | factorPolynomial     | factorSquareFreePolynomial |
| floor                         | fractionPart         | gcd                        |
| gcdPolynomial                 | hash                 | init                       |
| integrate                     | inv                  | latex                      |
| laurent                       | lcm                  | leadingCoefficient         |
| leadingMonomial               | log                  | map                        |
| max                           | min                  | monomial                   |
| monomial?                     | multiEuclidean       | multiplyCoefficients       |
| multiplyExponents             | negative?            | nextItem                   |
| nthRoot                       | numer                | numerator                  |
| one?                          | order                | patternMatch               |
| pi                            | pole?                | positive?                  |
| prime?                        | principalIdeal       | random                     |
| rationalFunction              | recip                | reducedSystem              |
| reductum                      | removeZeroes         | retract                    |
| retractIfCan                  | sample               | sec                        |
| sech                          | series               | sign                       |
| sin                           | sinh                 | sizeLess?                  |
| solveLinearPolynomialEquation | sqrt                 | squareFree                 |
| squareFreePart                | squareFreePolynomial | subtractIfCan              |
| tan                           | tanh                 | taylor                     |
| taylorIfCan                   | taylorRep            | terms                      |
| truncate                      | unit?                | unitCanonical              |
| unitNormal                    | variable             | variables                  |
| wholePart                     | zero?                | ?*?                        |
| **?                           | ?+?                  | ?-?                        |
| -?                            | ?=?                  | ?^?                        |
| ?.?                           | ?~=?                 | ?/?                        |
| ?<?                           | ?<=?                 | ?>?                        |
| ?>=?                          | ?^?                  | ?quo?                      |
| ?rem?                         |                      |                            |

## — domain SULS SparseUnivariateLaurentSeries —

```

)abbrev domain SULS SparseUnivariateLaurentSeries
++ Author: Clifton J. Williamson
++ Date Created: 11 November 1994
++ Date Last Updated: 10 March 1995
++ Basic Operations:
++ Related Domains: InnerSparseUnivariatePowerSeries,
++   SparseUnivariateTaylorSeries, SparseUnivariatePuisseuxSeries
++ Also See:
++ AMS Classifications:
++ Keywords: sparse, series
++ Examples:
++ References:
++ Description:
++ Sparse Laurent series in one variable
++ \spadtype{SparseUnivariateLaurentSeries} is a domain representing Laurent
++ series in one variable with coefficients in an arbitrary ring. The
++ parameters of the type specify the coefficient ring, the power series
++ variable, and the center of the power series expansion. For example,
++ \spad{SparseUnivariateLaurentSeries(Integer,x,3)} represents Laurent
++ series in \spad{(x - 3)} with integer coefficients.

SparseUnivariateLaurentSeries(Coef,var,cen): Exports == Implementation where
  Coef : Ring
  var  : Symbol
  cen  : Coef
  I    ==> Integer
  NNI  ==> NonNegativeInteger
  OUT  ==> OutputForm
  P    ==> Polynomial Coef
  RF   ==> Fraction Polynomial Coef
  RN   ==> Fraction Integer
  S    ==> String
  SUTS ==> SparseUnivariateTaylorSeries(Coef,var,cen)
  EFULS ==> ElementaryFunctionsUnivariateLaurentSeries(Coef,SUTS,%)

Exports ==> UnivariateLaurentSeriesConstructorCategory(Coef,SUTS) with
  coerce: Variable(var) -> %
    ++ \spad{coerce(var)} converts the series variable \spad{var} into a
    ++ Laurent series.
  differentiate: (%,Variable(var)) -> %
    ++ \spad{differentiate(f(x),x)} returns the derivative of
    ++ \spad{f(x)} with respect to \spad{x}.
  if Coef has Algebra Fraction Integer then
    integrate: (%,Variable(var)) -> %
      ++ \spad{integrate(f(x))} returns an anti-derivative of the power
      ++ series \spad{f(x)} with constant coefficient 0.
      ++ We may integrate a series when we can divide coefficients

```

```

    ++ by integers.

Implementation ==> InnerSparseUnivariatePowerSeries(Coef) add

Rep := InnerSparseUnivariatePowerSeries(Coef)

variable x == var
center   x == cen

coerce(v: Variable(var)) ==
  zero? cen => monomial(1,1)
  monomial(1,1) + monomial(cen,0)

pole? x == negative? order(x,0)

--% operations with Taylor series

coerce(uts:SUTS) == uts pretend %

taylorIfCan uls ==
  pole? uls => "failed"
  uls pretend SUTS

taylor uls ==
  (uts := taylorIfCan uls) case "failed" =>
    error "taylor: Laurent series has a pole"
  uts :: SUTS

retractIfCan(x:%):Union(SUTS,"failed") == taylorIfCan x

laurent(n,uts) == monomial(1,n) * (uts :: %)

removeZeroes uls    == uls
removeZeroes(n,uls) == uls

taylorRep uls == taylor(monomial(1,-order(uls,0)) * uls)
degree uls    == order(uls,0)

numer uls == taylorRep uls
denom uls == monomial(1,(-order(uls,0)) :: NNI)$SUTS

(uts:SUTS) * (uls:%) == (uts :: %) * uls
(uls:%) * (uts:SUTS) == uls * (uts :: %)

if Coef has Field then
  (uts1:SUTS) / (uts2:SUTS) == (uts1 :: %) / (uts2 :: %)

recip(uls) == iExquo(1,uls,false)

if Coef has IntegralDomain then

```

```

uls1 exquo uls2 == iExquo(uls1,uls2,false)

if Coef has Field then
  uls1:% / uls2:% ==
    (q := uls1 exquo uls2) case "failed" =>
      error "quotient cannot be computed"
    q :: %

differentiate(uls:%,v:Variable(var)) == differentiate uls

elt(uls1:%,uls2:%) ==
  order(uls2,1) < 1 =>
    error "elt: second argument must have positive order"
  negative?(ord := order(uls1,0)) =>
    (recipr := recip uls2) case "failed" =>
      error "elt: second argument not invertible"
    uls3 := uls1 * monomial(1,-ord)
    iCompose(uls3,uls2) * (recipr :: %) ** ((-ord) :: NNI)
    iCompose(uls1,uls2)

if Coef has IntegralDomain then
  rationalFunction(uls,n) ==
    zero?(e := order(uls,0)) =>
      negative? n => 0
      polynomial(taylor uls,n :: NNI) :: RF
    negative?(m := n - e) => 0
    poly := polynomial(taylor(monomial(1,-e) * uls),m :: NNI) :: RF
    v := variable(uls) :: RF; c := center(uls) :: P :: RF
    poly / (v - c) ** ((-e) :: NNI)

rationalFunction(uls,n1,n2) == rationalFunction(truncate(uls,n1,n2),n2)

if Coef has Algebra Fraction Integer then

  integrate uls ==
    zero? coefficient(uls,-1) =>
      error "integrate: series has term of order -1"
    integrate(uls)$Rep

  integrate(uls:%,v:Variable(var)) == integrate uls

(uls1:%) ** (uls2:%) == exp(log(uls1) * uls2)

exp uls == exp(uls)$EFULS
log uls == log(uls)$EFULS
sin uls == sin(uls)$EFULS
cos uls == cos(uls)$EFULS
tan uls == tan(uls)$EFULS
cot uls == cot(uls)$EFULS
sec uls == sec(uls)$EFULS

```



```

csc uls == csc(uls)$EFULS
asin uls == asin(uls)$EFULS
acos uls == acos(uls)$EFULS
atan uls == atan(uls)$EFULS
acot uls == acot(uls)$EFULS
asec uls == asec(uls)$EFULS
acsc uls == acsc(uls)$EFULS
sinh uls == sinh(uls)$EFULS
cosh uls == cosh(uls)$EFULS
tanh uls == tanh(uls)$EFULS
coth uls == coth(uls)$EFULS
sech uls == sech(uls)$EFULS
csch uls == csch(uls)$EFULS
asinh uls == asinh(uls)$EFULS
acosh uls == acosh(uls)$EFULS
atanh uls == atanh(uls)$EFULS
acoth uls == acoth(uls)$EFULS
asech uls == asech(uls)$EFULS
acsch uls == acsch(uls)$EFULS

if Coef has CommutativeRing then

  (uls:%) ** (r:RN) == cRationalPower(uls,r)

else

  (uls:%) ** (r:RN) ==
    negative?(ord0 := order(uls,0)) =>
      order := ord0 :: I
      (n := order exquo denom(r)) case "failed" =>
        error "**: rational power does not exist"
      uts := retract(uls * monomial(1,-order))@SUTS
      utsPow := (uts ** r) :: %
      monomial(1,(n :: I) * numer(r)) * utsPow
      uts := retract(uls)@SUTS
      (uts ** r) :: %

--% OutputForms

coerce(uls:%): OUT ==
  st := getStream uls
  if not(explicitlyEmpty? st or explicitEntries? st) _
    and (nx := retractIfCan(elt getRef uls))@Union(I,"failed") case I then
    count : NNI := _$streamCount$Lisp
    degr := min(count,(nx :: I) + count + 1)
    extend(uls,degr)
  seriesToOutputForm(st,getRef uls,variable uls,center uls,1)

```

---

```
"SULS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SULS"]
"ULSCCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ULSCCAT"]
"SULS" -> "ULSCCAT"
```

— SparseUnivariatePolynomial.input —

```

)set break resume
)sys rm -f SparseUnivariatePolynomial.output
)spool SparseUnivariatePolynomial.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SparseUnivariatePolynomial
--R SparseUnivariatePolynomial R: Ring is a domain constructor
--R Abbreviation for SparseUnivariatePolynomial is SUP
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SUP
--R
--R----- Operations -----
--R ?? : (%,R) -> %           ?? : (R,%) -> %
--R ?? : (%,%) -> %           ?? : (Integer,%) -> %
--R ?? : (PositiveInteger,%) -> %   ??? : (%,PositiveInteger) -> %
--R ?+ : (%,%) -> %           ?- : (%,%) -> %
--R -? : % -> %               ?=: (%,%) -> Boolean
--R D : (%,(R -> R)) -> %       D : % -> %
--R D : (%,NonNegativeInteger) -> %  1 : () -> %
--R 0 : () -> %               ?^? : (%,PositiveInteger) -> %
--R coefficients : % -> List R     coerce : R -> %
--R coerce : Integer -> %         coerce : % -> OutputForm
--R degree : % -> NonNegativeInteger  differentiate : % -> %
--R ?.? : (%,%) -> %           ?.? : (%,R) -> R
--R eval : (%,List %,List %) -> %   eval : (%,%,%) -> %
--R eval : (%,Equation %) -> %     eval : (%,List Equation %) -> %
--R ground : % -> R             ground? : % -> Boolean
--R hash : % -> SingleInteger     init : () -> % if R has STEP
--R latex : % -> String          leadingCoefficient : % -> R
--R leadingMonomial : % -> %      map : ((R -> R),%) -> %

```

```

--R monomial? : % -> Boolean
--R one? : % -> Boolean
--R pseudoRemainder : (%,% ) -> %
--R reductum : % -> %
--R sample : () -> %
--R ?~=?: (%,% ) -> Boolean
--R ?*?: (Fraction Integer,% ) -> % if R has ALGEBRA FRAC INT
--R ?*?: (% ,Fraction Integer) -> % if R has ALGEBRA FRAC INT
--R ?*?: (NonNegativeInteger,% ) -> %
--R ?**?: (% ,NonNegativeInteger) -> %
--R ?/? : (% ,R) -> % if R has FIELD
--R ?<?: (% ,%) -> Boolean if R has ORDSET
--R ?<=?: (% ,%) -> Boolean if R has ORDSET
--R ?>?: (% ,%) -> Boolean if R has ORDSET
--R ?>=?: (% ,%) -> Boolean if R has ORDSET
--R D : (% ,(R -> R),NonNegativeInteger) -> %
--R D : (% ,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (% ,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (% ,List Symbol) -> % if R has PDRING SYMBOL
--R D : (% ,Symbol) -> % if R has PDRING SYMBOL
--R D : (% ,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R D : (% ,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R D : (% ,List SingletonAsOrderedSet) -> %
--R D : (% ,SingletonAsOrderedSet) -> %
--R ???: (% ,NonNegativeInteger) -> %
--R associates? : (% ,%) -> Boolean if R has INTDOM
--R binomThmExpt : (% ,%,NonNegativeInteger) -> % if R has COMRING
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(% ,"failed") if $ has CHARNZ and R has PFECAT or R has CHARNZ
--R coefficient : (% ,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R coefficient : (% ,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R coefficient : (% ,NonNegativeInteger) -> R
--R coerce : % -> % if R has INTDOM
--R coerce : Fraction Integer -> % if R has ALGEBRA FRAC INT or R has RETRACT FRAC INT
--R coerce : SingletonAsOrderedSet -> %
--R composite : (Fraction % ,%) -> Union(Fraction % ,"failed") if R has INTDOM
--R composite : (% ,%) -> Union(% ,"failed") if R has INTDOM
--R conditionP : Matrix % -> Union(Vector % ,"failed") if $ has CHARNZ and R has PFECAT
--R content : (% ,SingletonAsOrderedSet) -> % if R has GCDDOM
--R content : % -> R if R has GCDDOM
--R convert : % -> InputForm if SingletonAsOrderedSet has KONVERT INFORM and R has KONVERT I
--R convert : % -> Pattern Integer if SingletonAsOrderedSet has KONVERT PATTERN INT and R has
--R convert : % -> Pattern Float if SingletonAsOrderedSet has KONVERT PATTERN FLOAT and R has
--R degree : (% ,List SingletonAsOrderedSet) -> List NonNegativeInteger
--R degree : (% ,SingletonAsOrderedSet) -> NonNegativeInteger
--R differentiate : (% ,(R -> R),%) -> %
--R differentiate : (% ,(R -> R)) -> %
--R differentiate : (% ,(R -> R),NonNegativeInteger) -> %
--R differentiate : (% ,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
--R differentiate : (% ,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL

```

```

--R differentiate : (% ,List Symbol) -> % if R has PDRING SYMBOL
--R differentiate : (% ,Symbol) -> % if R has PDRING SYMBOL
--R differentiate : (% ,NonNegativeInteger) -> %
--R differentiate : (% ,List SingletonAsOrderedSet ,List NonNegativeInteger) -> %
--R differentiate : (% ,SingletonAsOrderedSet ,NonNegativeInteger) -> %
--R differentiate : (% ,List SingletonAsOrderedSet) -> %
--R differentiate : (% ,SingletonAsOrderedSet) -> %
--R discriminant : % -> R if R has COMRING
--R discriminant : (% ,SingletonAsOrderedSet) -> % if R has COMRING
--R divide : (% ,%) -> Record(quotient: % ,remainder: %) if R has FIELD
--R divideExponents : (% ,NonNegativeInteger) -> Union(% , "failed")
--R ?.? : (% ,Fraction %) -> Fraction % if R has INTDOM
--R elt : (Fraction % ,R) -> R if R has FIELD
--R elt : (Fraction % ,Fraction %) -> Fraction % if R has INTDOM
--R euclideanSize : % -> NonNegativeInteger if R has FIELD
--R eval : (% ,List SingletonAsOrderedSet ,List %) -> %
--R eval : (% ,SingletonAsOrderedSet ,%) -> %
--R eval : (% ,List SingletonAsOrderedSet ,List R) -> %
--R eval : (% ,SingletonAsOrderedSet ,R) -> %
--R expressIdealMember : (List % ,%) -> Union(List % , "failed") if R has FIELD
--R exquo : (% ,%) -> Union(% , "failed") if R has INTDOM
--R exquo : (% ,R) -> Union(% , "failed") if R has INTDOM
--R extendedEuclidean : (% ,%) -> Record(coef1: % ,coef2: % ,generator: %) if R has FIELD
--R extendedEuclidean : (% ,% ,%) -> Union(Record(coef1: % ,coef2: % ) , "failed") if R has FIELD
--R factor : % -> Factored % if R has PFECAT
--R factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % if R has PF
--R factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % i
--R fmecg : (% ,NonNegativeInteger ,R ,%) -> %
--R gcd : (% ,%) -> % if R has GCDDOM
--R gcd : List % -> % if R has GCDDOM
--R gcdPolynomial : (SparseUnivariatePolynomial % ,SparseUnivariatePolynomial %) -> SparseUnivariatePolym
--R integrate : % -> % if R has ALGEBRA FRAC INT
--R isExpt : % -> Union(Record(var: SingletonAsOrderedSet ,exponent: NonNegativeInteger) , "failed")
--R isPlus : % -> Union(List % , "failed")
--R isTimes : % -> Union(List % , "failed")
--R karatsubaDivide : (% ,NonNegativeInteger) -> Record(quotient: % ,remainder: %)
--R lcm : (% ,%) -> % if R has GCDDOM
--R lcm : List % -> % if R has GCDDOM
--R mainVariable : % -> Union(SingletonAsOrderedSet , "failed")
--R makeSUP : % -> SparseUnivariatePolynomial R
--R mapExponents : ((NonNegativeInteger -> NonNegativeInteger) ,%) -> %
--R max : (% ,%) -> % if R has ORDSET
--R min : (% ,%) -> % if R has ORDSET
--R minimumDegree : (% ,List SingletonAsOrderedSet) -> List NonNegativeInteger
--R minimumDegree : (% ,SingletonAsOrderedSet) -> NonNegativeInteger
--R minimumDegree : % -> NonNegativeInteger
--R monicDivide : (% ,%) -> Record(quotient: % ,remainder: %)
--R monicDivide : (% ,% ,SingletonAsOrderedSet) -> Record(quotient: % ,remainder: %)
--R monomial : (% ,List SingletonAsOrderedSet ,List NonNegativeInteger) -> %
--R monomial : (% ,SingletonAsOrderedSet ,NonNegativeInteger) -> %

```

```

--R monomial : (R,NonNegativeInteger) -> %
--R multiEuclidean : (List %,%) -> Union(List %,"failed") if R has FIELD
--R multiplyExponents : (%,NonNegativeInteger) -> %
--R multivariate : (SparseUnivariatePolynomial %,SingletonAsOrderedSet) -> %
--R multivariate : (SparseUnivariatePolynomial R,SingletonAsOrderedSet) -> %
--R nextItem : % -> Union(%,"failed") if R has STEP
--R numberOfMonomials : % -> NonNegativeInteger
--R order : (%,%) -> NonNegativeInteger if R has INTDOM
--R outputForm : (%,OutputForm) -> OutputForm
--R patternMatch : (%,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMatchResult(Integer,%)
--R patternMatch : (%,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchResult(Float,%)
--R pomopo! : (%,R,NonNegativeInteger,%)-> %
--R prime? : % -> Boolean if R has PFECAT
--R primitivePart : (%,SingletonAsOrderedSet) -> % if R has GCDDOM
--R primitivePart : % -> % if R has GCDDOM
--R principalIdeal : List % -> Record(coef: List %,generator: %) if R has FIELD
--R pseudoDivide : (%,%) -> Record(coef: R,quotient: %,remainder: %) if R has INTDOM
--R pseudoQuotient : (%,%) -> % if R has INTDOM
--R ?quo? : (%,%) -> % if R has FIELD
--R reducedSystem : Matrix % -> Matrix R
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) if R has INTDOM
--R reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT
--R ?rem? : (%,%) -> % if R has FIELD
--R resultant : (%,%) -> R if R has COMRING
--R resultant : (%,%,SingletonAsOrderedSet) -> % if R has COMRING
--R retract : % -> SingletonAsOrderedSet
--R retract : % -> Integer if R has RETRACT INT
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(SingletonAsOrderedSet,"failed")
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(R,"failed")
--R separate : (%,%) -> Record(primePart: %,commonPart: %) if R has GCDDOM
--R shiftLeft : (%,NonNegativeInteger) -> %
--R shiftRight : (%,NonNegativeInteger) -> %
--R sizeLess? : (%,%) -> Boolean if R has FIELD
--R solveLinearPolynomialEquation : (List SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> %
--R squareFree : % -> Factored % if R has GCDDOM
--R squareFreePart : % -> % if R has GCDDOM
--R squareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial %
--R subResultantGcd : (%,%) -> % if R has INTDOM
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R totalDegree : (%,List SingletonAsOrderedSet) -> NonNegativeInteger
--R totalDegree : % -> NonNegativeInteger
--R unit? : % -> Boolean if R has INTDOM
--R unitCanonical : % -> % if R has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has INTDOM
--R univariate : % -> SparseUnivariatePolynomial R
--R univariate : (%,SingletonAsOrderedSet) -> SparseUnivariatePolynomial %

```

```

--R unmakeSUP : SparseUnivariatePolynomial R -> %
--R variables : % -> List SingletonAsOrderedSet
--R vectorise : (% , NonNegativeInteger) -> Vector R
--R
--E 1

```

```

)spool
)lisp (bye)

```

---

— SparseUnivariatePolynomial.help —

```

=====
SparseUnivariatePolynomial examples
=====

```

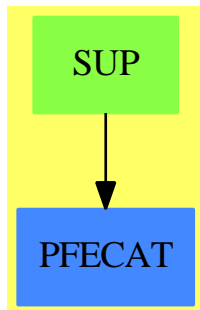
```

See Also:
o )show SparseUnivariatePolynomial

```

---

### 20.18.1 SparseUnivariatePolynomial (SUP)



See

- ⇒ “FreeModule” (FM) 7.30.1 on page 980
- ⇒ “PolynomialRing” (PR) 17.27.1 on page 2052
- ⇒ “UnivariatePolynomial” (UP) 22.4.1 on page 2784

**Exports:**

|                      |                               |
|----------------------|-------------------------------|
| 0                    | 1                             |
| associates?          | binomThmExpt                  |
| characteristic       | charthRoot                    |
| coefficient          | coefficients                  |
| coerce               | composite                     |
| conditionP           | content                       |
| convert              | D                             |
| degree               | differentiate                 |
| discriminant         | divide                        |
| divideExponents      | elt                           |
| euclideanSize        | eval                          |
| expressIdealMember   | exquo                         |
| extendedEuclidean    | factor                        |
| factorPolynomial     | factorSquareFreePolynomial    |
| fmeqg                | gcd                           |
| gcdPolynomial        | ground                        |
| ground?              | hash                          |
| init                 | integrate                     |
| isExpt               | isPlus                        |
| isTimes              | karatsubaDivide               |
| latex                | lcm                           |
| leadingCoefficient   | leadingMonomial               |
| mainVariable         | makeSUP                       |
| map                  | mapExponents                  |
| max                  | min                           |
| minimumDegree        | monicDivide                   |
| monomial             | monomial?                     |
| monomials            | multiEuclidean                |
| multiplyExponents    | multivariate                  |
| nextItem             | numberOfMonomials             |
| one?                 | order                         |
| outputForm           | patternMatch                  |
| popopo!              | prime?                        |
| primitiveMonomials   | primitivePart                 |
| principalIdeal       | pseudoDivide                  |
| pseudoQuotient       | pseudoRemainder               |
| recip                | reducedSystem                 |
| reductum             | resultant                     |
| retract              | retractIfCan                  |
| sample               | separate                      |
| shiftLeft            | shiftRight                    |
| sizeLess?            | solveLinearPolynomialEquation |
| squareFree           | squareFreePart                |
| squareFreePolynomial | subResultantGcd               |
| subtractIfCan        | totalDegree                   |
| totalDegree          | unit?                         |
| unitCanonical        | unitNormal                    |
| univariate           | univariate                    |
| unmakeSUP            | variables                     |
| vectorise            | zero?                         |
| ?*?                  | ?**?                          |
| ?+?                  | ?-?                           |
| -?                   | ?=?                           |
| ?^?                  | ?..?                          |
| ?~=?                 | ?/?                           |
| ?<?                  | ?<=?                          |
| ?>?                  | ?>=?                          |
| ?quo?                | ?rem?                         |

## — domain SUP SparseUnivariatePolynomial —

```

)abbrev domain SUP SparseUnivariatePolynomial
++ Author: Dave Barton, Barry Trager
++ Date Created:
++ Date Last Updated:
++ Basic Functions: Ring, monomial, coefficient, reductum, differentiate,
++ elt, map, resultant, discriminant
++ Related Constructors: UnivariatePolynomial, Polynomial
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This domain represents univariate polynomials over arbitrary
++ (not necessarily commutative) coefficient rings. The variable is
++ unspecified so that the variable displays as \spad{?} on output.
++ If it is necessary to specify the variable name, use type \spadtype{UnivariatePolynomial}.
++ The representation is sparse
++ in the sense that only non-zero terms are represented.
++ Note that if the coefficient ring is a field, this domain forms a euclidean domain.

SparseUnivariatePolynomial(R:Ring): UnivariatePolynomialCategory(R) with
  outputForm : (%,OutputForm) -> OutputForm
    ++ outputForm(p,var) converts the SparseUnivariatePolynomial p to
    ++ an output form (see \spadtype{OutputForm}) printed as a polynomial in the
    ++ output form variable.
  fmcg: (%,NonNegativeInteger,R,%) -> %
    ++ fmcg(p1,e,r,p2) finds  $x : p1 - r * x**e * p2$ 
  == PolynomialRing(R,NonNegativeInteger)
add
--representations
Term := Record(k:NonNegativeInteger,c:R)
Rep := List Term
p:%
n:NonNegativeInteger
np: PositiveInteger
FP ==> SparseUnivariatePolynomial %
pp,qq: FP
lpp:List FP

-- for karatsuba
kBound: NonNegativeInteger := 63
upmp := UnivariatePolynomialMultiplicationPackage(R,%)

if R has FieldOfPrimeCharacteristic then
  p ** np == p ** (np pretend NonNegativeInteger)
  p ^ np == p ** (np pretend NonNegativeInteger)

```



```

p ^ n == p ** n
p ** n ==
  null p => 0
  zero? n => 1
  one? n => p
  (n = 1) => p
  empty? p.rest =>
    zero?(cc:=p.first.c ** n) => 0
    [[n * p.first.k, cc]]
  -- not worth doing special trick if characteristic is too small
  if characteristic()$R < 3 then return expt(p,n pretend PositiveInteger)$Repeated
  y:=1
  -- break up exponent in qn * characteristic + rn
  -- exponentiating by the characteristic is fast
  rec := divide(n, characteristic()$R)
  qn:= rec.quotient
  rn:= rec.remainder
  repeat
    if rn = 1 then y := y * p
    if rn > 1 then y:= y * binomThmExpt([p.first], p.rest, rn)
    zero? qn => return y
    -- raise to the characteristic power
    p:= [[t.k * characteristic()$R , primeFrobenius(t.c)$R ]$Term for t in p]
    rec := divide(qn, characteristic()$R)
    qn:= rec.quotient
    rn:= rec.remainder
  y

zero?(p): Boolean == empty?(p)
-- one?(p): Boolean == not empty? p and (empty? rest p and zero? first(p).k and one? first(p).c)
one?(p): Boolean == not empty? p and (empty? rest p and zero? first(p).k and (first(p).c = 1))
ground?(p): Boolean == empty? p or (empty? rest p and zero? first(p).k)
multiplyExponents(p,n) == [ [u.k*n,u.c] for u in p]
divideExponents(p,n) ==
  null p => p
  m:= (p.first.k :: Integer exquo n::Integer)
  m case "failed" => "failed"
  u:= divideExponents(p.rest,n)
  u case "failed" => "failed"
  [[m::Integer::NonNegativeInteger,p.first.c],:u]
karatsubaDivide(p, n) ==
  zero? n => [p, 0]
  lowp: Rep := p
  highp: Rep := []
  repeat
    if empty? lowp then break
    t := first lowp
    if t.k < n then break

```

```

    lowp := rest lowp
    highp := cons([subtractIfCan(t.k,n)::NonNegativeInteger,t.c]$Term,highp)
  [ reverse highp, lowp]
shiftRight(p, n) ==
  [[subtractIfCan(t.k,n)::NonNegativeInteger,t.c]$Term for t in p]
shiftLeft(p, n) ==
  [[t.k + n,t.c]$Term for t in p]
pomopo!(p1,r,e,p2) ==
  rout:=%:= []
  for tm in p2 repeat
    e2:= e + tm.k
    c2:= r * tm.c
    c2 = 0 => "next term"
    while not null p1 and p1.first.k > e2 repeat
      (rout:=[p1.first,:rout]; p1:=p1.rest) --use PUSH and POP?
    null p1 or p1.first.k < e2 => rout:=[[e2,c2],:rout]
    if (u:=p1.first.c + c2) ^= 0 then rout:=[[e2, u],:rout]
    p1:=p1.rest
  NRECONC(rout,p1)$Lisp

-- implementation using karatsuba algorithm conditionally
--
-- p1 * p2 ==
--   xx := p1::Rep
--   empty? xx => p1
--   yy := p2::Rep
--   empty? yy => p2
--   zero? first(xx).k => first(xx).c * p2
--   zero? first(yy).k => p1 * first(yy).c
--   (first(xx).k > kBound) and (first(yy).k > kBound) and (#xx > kBound) and (#yy > kBound) =>
--     karatsubaOnce(p1,p2)$upmp
--   xx := reverse xx
--   res : Rep := empty()
--   for tx in xx repeat res:= rep pomopo!( res,tx.c,tx.k,p2)
--   res

univariate(p:%) == p pretend SparseUnivariatePolynomial(R)
multivariate(sup: SparseUnivariatePolynomial(R),v: SingletonAsOrderedSet) ==
  sup pretend %
univariate(p:%,v: SingletonAsOrderedSet) ==
  zero? p => 0
  monomial(leadingCoefficient(p)::%,degree p) +
  univariate(reductum p,v)
multivariate(supp: SparseUnivariatePolynomial(%),v: SingletonAsOrderedSet) ==
  zero? supp => 0
  lc:=leadingCoefficient supp
  degree lc > 0 => error "bad form polynomial"
  monomial(leadingCoefficient lc,degree supp) +
  multivariate(reductum supp,v)

```

```

if R has FiniteFieldCategory and R has PolynomialFactorizationExplicit then
  RXY ==> SparseUnivariatePolynomial SparseUnivariatePolynomial R
  squareFreePolynomial pp ==
    squareFree(pp)$UnivariatePolynomialSquareFree(%,FP)
  factorPolynomial pp ==
    (generalTwoFactor(pp pretend RXY)$TwoFactorize(R))
    pretend Factored SparseUnivariatePolynomial %
  factorSquareFreePolynomial pp ==
    (generalTwoFactor(pp pretend RXY)$TwoFactorize(R))
    pretend Factored SparseUnivariatePolynomial %
  gcdPolynomial(pp,qq) == gcd(pp,qq)$FP
  factor p == factor(p)$DistinctDegreeFactorize(R,%)
  solveLinearPolynomialEquation(lpp,pp) ==
    solveLinearPolynomialEquation(lpp, pp)$FiniteFieldSolveLinearPolynomialEquation(R,%,F)
else if R has PolynomialFactorizationExplicit then
  import PolynomialFactorizationByRecursionUnivariate(R,%)
  solveLinearPolynomialEquation(lpp,pp)==
    solveLinearPolynomialEquationByRecursion(lpp,pp)
  factorPolynomial(pp) ==
    factorByRecursion(pp)
  factorSquareFreePolynomial(pp) ==
    factorSquareFreeByRecursion(pp)

if R has IntegralDomain then
  if R has approximate then
    p1 exquo p2 ==
      null p2 => error "Division by 0"
      p2 = 1 => p1
      p1=p2 => 1
    --(p1.lastElt.c exquo p2.lastElt.c) case "failed" => "failed"
    rout:= []@List(Term)
    while not null p1 repeat
      (a:= p1.first.c exquo p2.first.c)
      a case "failed" => return "failed"
      ee:= subtractIfCan(p1.first.k, p2.first.k)
      ee case "failed" => return "failed"
      p1:= fmecg(p1.rest, ee, a, p2.rest)
      rout:= [[ee,a], :rout]
    null p1 => reverse(rout)::% -- nreverse?
    "failed"
  else -- R not approximate
    p1 exquo p2 ==
      null p2 => error "Division by 0"
      p2 = 1 => p1
    --(p1.lastElt.c exquo p2.lastElt.c) case "failed" => "failed"
    rout:= []@List(Term)
    while not null p1 repeat
      (a:= p1.first.c exquo p2.first.c)
      a case "failed" => return "failed"
      ee:= subtractIfCan(p1.first.k, p2.first.k)

```

```

        ee case "failed" => return "failed"
        p1:= fmecg(p1.rest, ee, a, p2.rest)
        rout:= [[ee,a], :rout]
        null p1 => reverse(rout)::%    -- nreverse?
        "failed"
    fmecg(p1,e,r,p2) ==          -- p1 - r * x**e * p2
        rout:%= []
        r:= - r
        for tm in p2 repeat
            e2:= e + tm.k
            c2:= r * tm.c
            c2 = 0 => "next term"
            while not null p1 and p1.first.k > e2 repeat
                (rout:=[p1.first,:rout]; p1:=p1.rest) --use PUSH and POP?
            null p1 or p1.first.k < e2 => rout:=[[e2,c2],:rout]
            if (u:=p1.first.c + c2) ^= 0 then rout:=[[e2, u],:rout]
            p1:=p1.rest
        NRECONC(rout,p1)$Lisp
    pseudoRemainder(p1,p2) ==
        null p2 => error "PseudoDivision by Zero"
        null p1 => 0
        co:=p2.first.c;
        e:=p2.first.k;
        p2:=p2.rest;
        e1:=max(p1.first.k:Integer-e+1,0):NonNegativeInteger
        while not null p1 repeat
            if (u:=subtractIfCan(p1.first.k,e)) case "failed" then leave
            p1:=fmecg(co * p1.rest, u, p1.first.c, p2)
            e1:= (e1 - 1):NonNegativeInteger
        e1 = 0 => p1
        co ** e1 * p1
    toOutput(t1:Term,v:OutputForm):OutputForm ==
        t1.k = 0 => t1.c :: OutputForm
        if t1.k = 1
            then mon:= v
            else mon := v ** t1.k::OutputForm
        t1.c = 1 => mon
        t1.c = -1 and
            ((t1.c :: OutputForm) = (-1$Integer)::OutputForm)@Boolean => - mon
        t1.c::OutputForm * mon
    outputForm(p:%,v:OutputForm) ==
        l: List(OutputForm)
        l:=[toOutput(t,v) for t in p]
        null l => (0$Integer)::OutputForm -- else FreeModule 0 problems
        reduce("+",l)

    coerce(p:%)::OutputForm == outputForm(p, "?"::OutputForm)
    elt(p:%,val:R) ==
        null p => 0$R
        co:=p.first.c

```

```

n:=p.first.k
for tm in p.rest repeat
  co:= co * val ** (n - (n:=tm.k)):NonNegativeInteger + tm.c
n = 0 => co
co * val ** n
elt(p:%,val:%) ==
  null p => 0$%
  coef:% := p.first.c :: %
  n:=p.first.k
  for tm in p.rest repeat
    coef:= coef * val ** (n-(n:=tm.k)):NonNegativeInteger+(tm.c::%)
  n = 0 => coef
  coef * val ** n

monicDivide(p1:%,p2:%) ==
  null p2 => error "monicDivide: division by 0"
  leadingCoefficient p2 ^= 1 => error "Divisor Not Monic"
  p2 = 1 => [p1,0]
  null p1 => [0,0]
  degree p1 < (n:=degree p2) => [0,p1]
  rout:Rep := []
  p2 := p2.rest
  while not null p1 repeat
    (u:=subtractIfCan(p1.first.k, n)) case "failed" => leave
    rout:=[u, p1.first.c], :rout]
    p1:=fmecg(p1.rest, rout.first.k, rout.first.c, p2)
  [reverse_!(rout),p1]

if R has IntegralDomain then
  discriminant(p) == discriminant(p)$PseudoRemainderSequence(R,%)
-- discriminant(p) ==
--   null p or zero?(p.first.k) => error "cannot take discriminant of constants"
--   dp:=differentiate p
--   corr:= p.first.c ** ((degree p - 1 - degree dp)::NonNegativeInteger)
--   (-1)**((p.first.k*(p.first.k-1)) quo 2):NonNegativeInteger
--   * (corr * resultant(p,dp) exquo p.first.c)::R

subResultantGcd(p1,p2) == subResultantGcd(p1,p2)$PseudoRemainderSequence(R,%)
-- subResultantGcd(p1,p2) == --args # 0, non-coef, prim, ans not prim
-- --see algorithm 1 (p. 4) of Brown's latest (unpublished) paper
-- if p1.first.k < p2.first.k then (p1,p2):=(p2,p1)
-- p:=pseudoRemainder(p1,p2)
-- co:=1$R;
-- e:= (p1.first.k - p2.first.k):NonNegativeInteger
-- while not null p and p.first.k ^= 0 repeat
--   p1:=p2; p2:=p; p:=pseudoRemainder(p1,p2)
--   null p or p.first.k = 0 => "enuf"
--   co:=(p1.first.c ** e exquo co ** max(0, (e-1))::NonNegativeInteger)::R
--   e:= (p1.first.k - p2.first.k):NonNegativeInteger; c1:=co**e
--   p:=[tm.k,((tm.c exquo p1.first.c)::R exquo c1)::R] for tm in p]

```

```

--      if null p then p2 else 1$%

resultant(p1,p2) == resultant(p1,p2)$PseudoRemainderSequence(R,%)
--      resultant(p1,p2) ==      --SubResultant PRS Algorithm
--      null p1 or null p2 => 0$R
--      0 = degree(p1) => ((first p1).c)**degree(p2)
--      0 = degree(p2) => ((first p2).c)**degree(p1)
--      if p1.first.k < p2.first.k then
--          (if odd?(p1.first.k) then p1:=-p1; (p1,p2):=(p2,p1))
--      p:=pseudoRemainder(p1,p2)
--      co:=1$R; e:=(p1.first.k-p2.first.k):NonNegativeInteger
--      while not null p repeat
--          if not odd?(e) then p:=-p
--          p1:=p2; p2:=p; p:=pseudoRemainder(p1,p2)
--          co:=(p1.first.c ** e exquo co ** max(e:Integer-1,0):NonNegativeInteger)::R
--          e:= (p1.first.k - p2.first.k):NonNegativeInteger; c1:=co**e
--          p:=(p exquo ((leadingCoefficient p1) * c1))::%
--          degree p2 > 0 => 0$R
--          (p2.first.c**e exquo co**((e-1)::NonNegativeInteger))::R
if R has GcdDomain then
    content(p) == if null p then 0$R else "gcd"/[tm.c for tm in p]
    --make CONTENT more efficient?

primitivePart(p) ==
    null p => p
    ct :=content(p)
    unitCanonical((p exquo ct)::%)
    -- exquo present since % is now an IntegralDomain

gcd(p1,p2) ==
    gcdPolynomial(p1 pretend SparseUnivariatePolynomial R,
        p2 pretend SparseUnivariatePolynomial R) pretend %

if R has Field then
    divide( p1, p2) ==
        zero? p2 => error "Division by 0"
--        one? p2 => [p1,0]
        (p2 = 1) => [p1,0]
        ct:=inv(p2.first.c)
        n:=p2.first.k
        p2:=p2.rest
        rout:=empty()$List(Term)
        while p1 ^= 0 repeat
            (u:=subtractIfCan(p1.first.k, n)) case "failed" => leave
            rout:=[[u, ct * p1.first.c], :rout]
            p1:=fmecg(p1.rest, rout.first.k, rout.first.c, p2)
        [reverse_!(rout),p1]

p / co == inv(co) * p

```

---

— SUP.dotabb —

```
"SUP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SUP"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"SUP" -> "PFECAT"
```

---

## 20.19 domain SUEXPR SparseUnivariatePolynomial-Expressions

This domain is a hack, in some sense. What I'd really like to do - automatically - is to provide all operations supported by the coefficient domain, as long as the polynomials can be retracted to that domain, i.e., as long as they are just constants. I don't see another way to do this, unfortunately.

— SparseUnivariatePolynomialExpressions.input —

```
)set break resume
)sys rm -f SparseUnivariatePolynomialExpressions.output
)spool SparseUnivariatePolynomialExpressions.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SparseUnivariatePolynomialExpressions
--R SparseUnivariatePolynomialExpressions R: Ring is a domain constructor
--R Abbreviation for SparseUnivariatePolynomialExpressions is SUEXPR
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SUEXPR
--R
--R----- Operations -----
--R ?? : (%,R) -> %           ?? : (R,%) -> %
--R ?? : (%,%) -> %           ?? : (Integer,%) -> %
--R ?? : (PositiveInteger,%) -> %   ??? : (%,PositiveInteger) -> %
--R ?? : (%,%) -> %           ?-? : (%,%) -> %
--R -? : % -> %               ?? : (%,%) -> Boolean
--R D : (%,(R -> R)) -> %       D : % -> %
--R D : (%,NonNegativeInteger) -> %   1 : () -> %
--R 0 : () -> %                 ?? : (%,PositiveInteger) -> %
--R coefficients : % -> List R       coerce : R -> %
--R coerce : Integer -> %           coerce : % -> OutputForm
```

```

--R degree : % -> NonNegativeInteger      differentiate : % -> %
--R ?.? : (%,% ) -> %                      ?.? : (% ,R) -> R
--R eval : (% ,List % ,List % ) -> %       eval : (% ,% ,% ) -> %
--R eval : (% ,Equation % ) -> %          eval : (% ,List Equation % ) -> %
--R ground : % -> R                        ground? : % -> Boolean
--R hash : % -> SingleInteger              init : () -> % if R has STEP
--R latex : % -> String                    leadingCoefficient : % -> R
--R leadingMonomial : % -> %               map : ((R -> R),%) -> %
--R monomial? : % -> Boolean               monomials : % -> List %
--R one? : % -> Boolean                    pi : () -> % if R has TRANFUN
--R primitiveMonomials : % -> List %       pseudoRemainder : (% ,%) -> %
--R recip : % -> Union(% , "failed")       reductum : % -> %
--R retract : % -> R                      sample : () -> %
--R zero? : % -> Boolean                   ?~=? : (% ,%) -> Boolean
--R ?? : (Fraction Integer ,%) -> % if R has ALGEBRA FRAC INT
--R ?? : (% ,Fraction Integer) -> % if R has ALGEBRA FRAC INT
--R ?? : (NonNegativeInteger ,%) -> %
--R ??? : (% ,%) -> % if R has TRANFUN
--R ??? : (% ,NonNegativeInteger) -> %
--R ?/? : (% ,R) -> % if R has FIELD
--R ?<? : (% ,%) -> Boolean if R has ORDSET
--R ?<=? : (% ,%) -> Boolean if R has ORDSET
--R ?>? : (% ,%) -> Boolean if R has ORDSET
--R ?>=? : (% ,%) -> Boolean if R has ORDSET
--R D : (% ,(R -> R) ,NonNegativeInteger) -> %
--R D : (% ,List Symbol ,List NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (% ,Symbol ,NonNegativeInteger) -> % if R has PDRING SYMBOL
--R D : (% ,List Symbol) -> % if R has PDRING SYMBOL
--R D : (% ,Symbol) -> % if R has PDRING SYMBOL
--R D : (% ,List SingletonAsOrderedSet ,List NonNegativeInteger) -> %
--R D : (% ,SingletonAsOrderedSet ,NonNegativeInteger) -> %
--R D : (% ,List SingletonAsOrderedSet) -> %
--R D : (% ,SingletonAsOrderedSet) -> %
--R ?? : (% ,NonNegativeInteger) -> %
--R acos : % -> % if R has TRANFUN
--R acosh : % -> % if R has TRANFUN
--R acot : % -> % if R has TRANFUN
--R acoth : % -> % if R has TRANFUN
--R acsc : % -> % if R has TRANFUN
--R acsch : % -> % if R has TRANFUN
--R asec : % -> % if R has TRANFUN
--R asech : % -> % if R has TRANFUN
--R asin : % -> % if R has TRANFUN
--R asinh : % -> % if R has TRANFUN
--R associates? : (% ,%) -> Boolean if R has INTDOM
--R atan : % -> % if R has TRANFUN
--R atanh : % -> % if R has TRANFUN
--R binomThmExpt : (% ,% ,NonNegativeInteger) -> % if R has COMRING
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(% , "failed") if $ has CHARNZ and R has PFECAT or R has CHARNZ

```



```

--R coefficient : (% ,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R coefficient : (% ,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R coefficient : (% ,NonNegativeInteger) -> R
--R coerce : % -> % if R has INTDOM
--R coerce : Fraction Integer -> % if R has ALGEBRA FRAC INT or R has RETRACT FRAC INT
--R coerce : SingletonAsOrderedSet -> %
--R composite : (Fraction %,%) -> Union(Fraction %,"failed") if R has INTDOM
--R composite : (% ,%) -> Union(%,"failed") if R has INTDOM
--R conditionP : Matrix % -> Union(Vector %,"failed") if $ has CHARNZ and R has PFECAT
--R content : (% ,SingletonAsOrderedSet) -> % if R has GCDDOM
--R content : % -> R if R has GCDDOM
--R convert : % -> InputForm if SingletonAsOrderedSet has KONVERT INFORM and R has KONVERT I
--R convert : % -> Pattern Integer if SingletonAsOrderedSet has KONVERT PATTERN INT and R has
--R convert : % -> Pattern Float if SingletonAsOrderedSet has KONVERT PATTERN FLOAT and R has
--R cos : % -> % if R has TRANFUN
--R cosh : % -> % if R has TRANFUN
--R cot : % -> % if R has TRANFUN
--R coth : % -> % if R has TRANFUN
--R csc : % -> % if R has TRANFUN
--R csch : % -> % if R has TRANFUN
--R degree : (% ,List SingletonAsOrderedSet) -> List NonNegativeInteger
--R degree : (% ,SingletonAsOrderedSet) -> NonNegativeInteger
--R differentiate : (% ,(R -> R),%) -> %
--R differentiate : (% ,(R -> R)) -> %
--R differentiate : (% ,(R -> R),NonNegativeInteger) -> %
--R differentiate : (% ,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
--R differentiate : (% ,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
--R differentiate : (% ,List Symbol) -> % if R has PDRING SYMBOL
--R differentiate : (% ,Symbol) -> % if R has PDRING SYMBOL
--R differentiate : (% ,NonNegativeInteger) -> %
--R differentiate : (% ,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R differentiate : (% ,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R differentiate : (% ,List SingletonAsOrderedSet) -> %
--R differentiate : (% ,SingletonAsOrderedSet) -> %
--R discriminant : % -> R if R has COMRING
--R discriminant : (% ,SingletonAsOrderedSet) -> % if R has COMRING
--R divide : (% ,%) -> Record(quotient: %,remainder: %) if R has FIELD
--R divideExponents : (% ,NonNegativeInteger) -> Union(%,"failed")
--R ?.? : (% ,Fraction %) -> Fraction % if R has INTDOM
--R elt : (Fraction %,R) -> R if R has FIELD
--R elt : (Fraction %,Fraction %) -> Fraction % if R has INTDOM
--R euclideanSize : % -> NonNegativeInteger if R has FIELD
--R eval : (% ,List SingletonAsOrderedSet,List %) -> %
--R eval : (% ,SingletonAsOrderedSet,%) -> %
--R eval : (% ,List SingletonAsOrderedSet,List R) -> %
--R eval : (% ,SingletonAsOrderedSet,R) -> %
--R exp : % -> % if R has TRANFUN
--R expressIdealMember : (List %,%) -> Union(List %,"failed") if R has FIELD
--R exquo : (% ,%) -> Union(%,"failed") if R has INTDOM
--R exquo : (% ,R) -> Union(%,"failed") if R has INTDOM

```

```

--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if R has FIELD
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if R has FIELD
--R factor : % -> Factored % if R has PFECAT
--R factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % if R has PF
--R factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % i
--R gcd : (%,%) -> % if R has GCDDOM
--R gcd : List % -> % if R has GCDDOM
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolym
--R integrate : % -> % if R has ALGEBRA FRAC INT
--R isExpt : % -> Union(Record(var: SingletonAsOrderedSet,exponent: NonNegativeInteger),"failed")
--R isPlus : % -> Union(List %,"failed")
--R isTimes : % -> Union(List %,"failed")
--R karatsubaDivide : (%,NonNegativeInteger) -> Record(quotient: %,remainder: %)
--R lcm : (%,%) -> % if R has GCDDOM
--R lcm : List % -> % if R has GCDDOM
--R log : % -> % if R has TRANFUN
--R mainVariable : % -> Union(SingletonAsOrderedSet,"failed")
--R makeSUP : % -> SparseUnivariatePolynomial R
--R mapExponents : ((NonNegativeInteger -> NonNegativeInteger),%) -> %
--R max : (%,%) -> % if R has ORDSET
--R min : (%,%) -> % if R has ORDSET
--R minimumDegree : (%,List SingletonAsOrderedSet) -> List NonNegativeInteger
--R minimumDegree : (%,SingletonAsOrderedSet) -> NonNegativeInteger
--R minimumDegree : % -> NonNegativeInteger
--R monicDivide : (%,%) -> Record(quotient: %,remainder: %)
--R monicDivide : (%,%,SingletonAsOrderedSet) -> Record(quotient: %,remainder: %)
--R monomial : (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R monomial : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R monomial : (R,NonNegativeInteger) -> %
--R multiEuclidean : (List %,%) -> Union(List %,"failed") if R has FIELD
--R multiplyExponents : (%,NonNegativeInteger) -> %
--R multivariate : (SparseUnivariatePolynomial %,SingletonAsOrderedSet) -> %
--R multivariate : (SparseUnivariatePolynomial R,SingletonAsOrderedSet) -> %
--R nextItem : % -> Union(%,"failed") if R has STEP
--R numberOfMonomials : % -> NonNegativeInteger
--R order : (%,%) -> NonNegativeInteger if R has INTDOM
--R patternMatch : (%,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMatchResult(Integer,%) if
--R patternMatch : (%,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchResult(Float,%) if Singl
--R pomopo! : (%,R,NonNegativeInteger,%) -> %
--R prime? : % -> Boolean if R has PFECAT
--R primitivePart : (%,SingletonAsOrderedSet) -> % if R has GCDDOM
--R primitivePart : % -> % if R has GCDDOM
--R principalIdeal : List % -> Record(coef: List %,generator: %) if R has FIELD
--R pseudoDivide : (%,%) -> Record(coef: R,quotient: %,remainder: %) if R has INTDOM
--R pseudoQuotient : (%,%) -> % if R has INTDOM
--R ?quo? : (%,%) -> % if R has FIELD
--R reducedSystem : Matrix % -> Matrix R
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) if R has LINE
--R reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT

```

```

--R ?rem? : (% , %) -> % if R has FIELD
--R resultant : (% , %) -> R if R has COMRING
--R resultant : (% , %, SingletonAsOrderedSet) -> % if R has COMRING
--R retract : % -> SingletonAsOrderedSet
--R retract : % -> Integer if R has RETRACT INT
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(SingletonAsOrderedSet, "failed")
--R retractIfCan : % -> Union(Integer, "failed") if R has RETRACT INT
--R retractIfCan : % -> Union(Fraction Integer, "failed") if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(R, "failed")
--R sec : % -> % if R has TRANFUN
--R sech : % -> % if R has TRANFUN
--R separate : (% , %) -> Record(primePart: %, commonPart: %) if R has GCDDOM
--R shiftLeft : (% , NonNegativeInteger) -> %
--R shiftRight : (% , NonNegativeInteger) -> %
--R sin : % -> % if R has TRANFUN
--R sinh : % -> % if R has TRANFUN
--R sizeLess? : (% , %) -> Boolean if R has FIELD
--R solveLinearPolynomialEquation : (List SparseUnivariatePolynomial %, SparseUnivariatePolynomial %) -> %
--R squareFree : % -> Factored % if R has GCDDOM
--R squareFreePart : % -> % if R has GCDDOM
--R squareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial %
--R subResultantGcd : (% , %) -> % if R has INTDOM
--R subtractIfCan : (% , %) -> Union(%, "failed")
--R tan : % -> % if R has TRANFUN
--R tanh : % -> % if R has TRANFUN
--R totalDegree : (% , List SingletonAsOrderedSet) -> NonNegativeInteger
--R totalDegree : % -> NonNegativeInteger
--R unit? : % -> Boolean if R has INTDOM
--R unitCanonical : % -> % if R has INTDOM
--R unitNormal : % -> Record(unit: %, canonical: %, associate: %) if R has INTDOM
--R univariate : % -> SparseUnivariatePolynomial R
--R univariate : (% , SingletonAsOrderedSet) -> SparseUnivariatePolynomial %
--R unmakeSUP : SparseUnivariatePolynomial R -> %
--R variables : % -> List SingletonAsOrderedSet
--R vectorise : (% , NonNegativeInteger) -> Vector R
--R
--E 1

)spool
)lisp (bye)

```

---

— SparseUnivariatePolynomialExpressions.help —

```

=====
SparseUnivariatePolynomialExpressions examples
=====

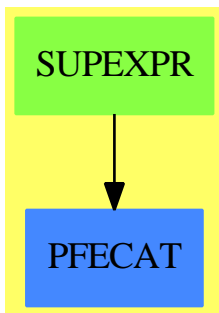
```

See Also:

o )show SparseUnivariatePolynomialExpressions

\_\_\_\_\_

### 20.19.1 SparseUnivariatePolynomialExpressions (SUEXPR)



**Exports:**

|                    |                               |                    |
|--------------------|-------------------------------|--------------------|
| 0                  | 1                             | acos               |
| acosh              | acot                          | acoth              |
| acsc               | acsch                         | asec               |
| asech              | asin                          | asinh              |
| associates?        | atan                          | atanh              |
| binomThmExpt       | characteristic                | charthRoot         |
| coefficient        | coefficients                  | coerce             |
| composite          | conditionP                    | content            |
| convert            | cos                           | cosh               |
| cot                | coth                          | csc                |
| csch               | D                             | degree             |
| differentiate      | discriminant                  | divide             |
| divideExponents    | elt                           | euclideanSize      |
| eval               | exp                           | expressIdealMember |
| exquo              | extendedEuclidean             | factor             |
| factorPolynomial   | factorSquareFreePolynomial    | gcd                |
| gcdPolynomial      | ground                        | ground?            |
| hash               | init                          | integrate          |
| isExpt             | isPlus                        | isTimes            |
| karatsubaDivide    | latex                         | lcm                |
| leadingCoefficient | leadingMonomial               | log                |
| mainVariable       | makeSUP                       | map                |
| mapExponents       | max                           | min                |
| minimumDegree      | monicDivide                   | monomial           |
| monomial?          | monomials                     | multiEuclidean     |
| multiplyExponents  | multivariate                  | nextItem           |
| numberOfMonomials  | one?                          | order              |
| patternMatch       | pi                            | pomopo!            |
| prime?             | primitiveMonomials            | primitivePart      |
| principalIdeal     | pseudoDivide                  | pseudoQuotient     |
| pseudoRemainder    | recip                         | reducedSystem      |
| reductum           | resultant                     | retract            |
| retractIfCan       | sample                        | sec                |
| sech               | separate                      | shiftLeft          |
| shiftRight         | sin                           | sinh               |
| sizeLess?          | solveLinearPolynomialEquation | squareFree         |
| squareFreePart     | squareFreePolynomial          | subResultantGcd    |
| subtractIfCan      | tan                           | tanh               |
| totalDegree        | unit?                         | unitCanonical      |
| unitNormal         | univariate                    | unmakeSUP          |
| variables          | vectorise                     | zero?              |
| ?*?                | ?**?                          | ?+?                |
| ?-?                | -?                            | ?=?                |
| ?^?                | ?..?                          | ?~=?               |
| ?/?                | ?<?                           | ?<=?               |
| ?>?                | ?>=?                          | ?quo?              |
| ?rem?              |                               |                    |

## — domain SUEXPR SparseUnivariatePolynomialExpressions —

```

)abbrev domain SUEXPR SparseUnivariatePolynomialExpressions
++ Author: Mark Botch
++ Description:
++ This domain has no description

SparseUnivariatePolynomialExpressions(R: Ring): Exports == Implementation where

  Exports == UnivariatePolynomialCategory R with

    if R has TranscendentalFunctionCategory
    then TranscendentalFunctionCategory

  Implementation == SparseUnivariatePolynomial R add

    if R has TranscendentalFunctionCategory then
      log(p: %): % ==
        ground? p => coerce log ground p
        output(hconcat("log p for p= ", p::OutputForm))$OutputPackage
        error "SUPTRAFUN: log only defined for elements of the coefficient ring"

      exp(p: %): % ==
        ground? p => coerce exp ground p
        output(hconcat("exp p for p= ", p::OutputForm))$OutputPackage
        error "SUPTRAFUN: exp only defined for elements of the coefficient ring"

      sin(p: %): % ==
        ground? p => coerce sin ground p
        output(hconcat("sin p for p= ", p::OutputForm))$OutputPackage
        error "SUPTRAFUN: sin only defined for elements of the coefficient ring"

      asin(p: %): % ==
        ground? p => coerce asin ground p
        output(hconcat("asin p for p= ", p::OutputForm))$OutputPackage
        error "SUPTRAFUN: asin only defined for elements of the coefficient ring"

      cos(p: %): % ==
        ground? p => coerce cos ground p
        output(hconcat("cos p for p= ", p::OutputForm))$OutputPackage
        error "SUPTRAFUN: cos only defined for elements of the coefficient ring"

      acos(p: %): % ==
        ground? p => coerce acos ground p
        output(hconcat("acos p for p= ", p::OutputForm))$OutputPackage
        error "SUPTRAFUN: acos only defined for elements of the coefficient ring"

```

---

## — SUEXPR.dotabb —

"SUEXPR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SUEXPR"]

```
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"SUPEXPR" -> "PFECAT"
```

## 20.20 domain SUPXS SparseUnivariatePuisseuxSeries

— SparseUnivariatePuisseuxSeries.input —

```
)set break resume
)sys rm -f SparseUnivariatePuisseuxSeries.output
)spool SparseUnivariatePuisseuxSeries.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SparseUnivariatePuisseuxSeries
--R SparseUnivariatePuisseuxSeries(Coef: Ring,var: Symbol,cen: Coef) is a domain constructor
--R Abbreviation for SparseUnivariatePuisseuxSeries is SUPXS
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SUPXS
--R
--R----- Operations -----
--R ?? : (Coef,%) -> %           ?? : (% ,Coef) -> %
--R ?? : (% ,%) -> %           ?? : (Integer,%) -> %
--R ?? : (PositiveInteger,%) -> %   ??? : (% ,PositiveInteger) -> %
--R ?+? : (% ,%) -> %           ?-? : (% ,%) -> %
--R -? : % -> %                 ?? : (% ,%) -> Boolean
--R 1 : () -> %                 0 : () -> %
--R ?? : (% ,PositiveInteger) -> %   center : % -> Coef
--R coerce : Variable var -> %       coerce : Integer -> %
--R coerce : % -> OutputForm         complete : % -> %
--R degree : % -> Fraction Integer   hash : % -> SingleInteger
--R latex : % -> String              leadingCoefficient : % -> Coef
--R leadingMonomial : % -> %         map : ((Coef -> Coef),%) -> %
--R monomial? : % -> Boolean         one? : % -> Boolean
--R order : % -> Fraction Integer    pole? : % -> Boolean
--R recip : % -> Union(%,"failed")   reductum : % -> %
--R sample : () -> %                variable : % -> Symbol
--R zero? : % -> Boolean             ?~=? : (% ,%) -> Boolean
--R ?? : (% ,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
--R ?? : (NonNegativeInteger,%) -> %
--R ??? : (% ,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ??? : (% ,%) -> % if Coef has ALGEBRA FRAC INT
```

```

--R ??? : (%,Integer) -> % if Coef has FIELD
--R ??? : (%NonNegativeInteger) -> %
--R ?/? : (%,% ) -> % if Coef has FIELD
--R ?/? : (%Coef) -> % if Coef has FIELD
--R D : % -> % if Coef has *: (Fraction Integer,Coef) -> Coef
--R D : (%NonNegativeInteger) -> % if Coef has *: (Fraction Integer,Coef) -> Coef
--R D : (%Symbol) -> % if Coef has *: (Fraction Integer,Coef) -> Coef and Coef has PDRING SYMBOL
--R D : (%List Symbol) -> % if Coef has *: (Fraction Integer,Coef) -> Coef and Coef has PDRING SYMBOL
--R D : (%Symbol,NonNegativeInteger) -> % if Coef has *: (Fraction Integer,Coef) -> Coef and Coef has P
--R D : (%List Symbol,List NonNegativeInteger) -> % if Coef has *: (Fraction Integer,Coef) -> Coef and
--R ?? : (%Integer) -> % if Coef has FIELD
--R ?? : (%NonNegativeInteger) -> %
--R acos : % -> % if Coef has ALGEBRA FRAC INT
--R acosh : % -> % if Coef has ALGEBRA FRAC INT
--R acot : % -> % if Coef has ALGEBRA FRAC INT
--R acoth : % -> % if Coef has ALGEBRA FRAC INT
--R acsc : % -> % if Coef has ALGEBRA FRAC INT
--R acsch : % -> % if Coef has ALGEBRA FRAC INT
--R approximate : (%Fraction Integer) -> Coef if Coef has **: (Coef,Fraction Integer) -> Coef and Coef
--R asec : % -> % if Coef has ALGEBRA FRAC INT
--R asech : % -> % if Coef has ALGEBRA FRAC INT
--R asin : % -> % if Coef has ALGEBRA FRAC INT
--R asinh : % -> % if Coef has ALGEBRA FRAC INT
--R associates? : (%,% ) -> Boolean if Coef has INTDOM
--R atan : % -> % if Coef has ALGEBRA FRAC INT
--R atanh : % -> % if Coef has ALGEBRA FRAC INT
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
--R coefficient : (%Fraction Integer) -> Coef
--R coerce : % -> % if Coef has INTDOM
--R coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
--R coerce : SparseUnivariateTaylorSeries(Coef,var,cen) -> %
--R coerce : SparseUnivariateLaurentSeries(Coef,var,cen) -> %
--R coerce : Coef -> % if Coef has COMRING
--R cos : % -> % if Coef has ALGEBRA FRAC INT
--R cosh : % -> % if Coef has ALGEBRA FRAC INT
--R cot : % -> % if Coef has ALGEBRA FRAC INT
--R coth : % -> % if Coef has ALGEBRA FRAC INT
--R csc : % -> % if Coef has ALGEBRA FRAC INT
--R csch : % -> % if Coef has ALGEBRA FRAC INT
--R differentiate : (%Variable var) -> %
--R differentiate : % -> % if Coef has *: (Fraction Integer,Coef) -> Coef
--R differentiate : (%NonNegativeInteger) -> % if Coef has *: (Fraction Integer,Coef) -> Coef
--R differentiate : (%Symbol) -> % if Coef has *: (Fraction Integer,Coef) -> Coef and Coef has PDRING S
--R differentiate : (%List Symbol) -> % if Coef has *: (Fraction Integer,Coef) -> Coef and Coef has PDR
--R differentiate : (%Symbol,NonNegativeInteger) -> % if Coef has *: (Fraction Integer,Coef) -> Coef an
--R differentiate : (%List Symbol,List NonNegativeInteger) -> % if Coef has *: (Fraction Integer,Coef)
--R divide : (%,% ) -> Record(quotient: %,remainder: %) if Coef has FIELD
--R ?.? : (%,% ) -> % if Fraction Integer has SGROUP
--R ?.? : (%Fraction Integer) -> Coef

```



```

--R euclideanSize : % -> NonNegativeInteger if Coef has FIELD
--R eval : (% , Coef) -> Stream Coef if Coef has **: (Coef, Fraction Integer) -> Coef
--R exp : % -> % if Coef has ALGEBRA FRAC INT
--R expressIdealMember : (List % , %) -> Union(List % , "failed") if Coef has FIELD
--R exquo : (% , %) -> Union(% , "failed") if Coef has INTDOM
--R extend : (% , Fraction Integer) -> %
--R extendedEuclidean : (% , %) -> Record(coef1: % , coef2: % , generator: %) if Coef has FIELD
--R extendedEuclidean : (% , % , %) -> Union(Record(coef1: % , coef2: % ) , "failed") if Coef has FIELD
--R factor : % -> Factored % if Coef has FIELD
--R gcd : (% , %) -> % if Coef has FIELD
--R gcd : List % -> % if Coef has FIELD
--R gcdPolynomial : (SparseUnivariatePolynomial % , SparseUnivariatePolynomial %) -> SparseUnivariatePolynomial %
--R integrate : (% , Variable var) -> % if Coef has ALGEBRA FRAC INT
--R integrate : (% , Symbol) -> % if Coef has integrate: (Coef, Symbol) -> Coef and Coef has var
--R integrate : % -> % if Coef has ALGEBRA FRAC INT
--R inv : % -> % if Coef has FIELD
--R laurent : % -> SparseUnivariateLaurentSeries(Coef, var, cen)
--R laurentIfCan : % -> Union(SparseUnivariateLaurentSeries(Coef, var, cen) , "failed")
--R laurentRep : % -> SparseUnivariateLaurentSeries(Coef, var, cen)
--R lcm : (% , %) -> % if Coef has FIELD
--R lcm : List % -> % if Coef has FIELD
--R log : % -> % if Coef has ALGEBRA FRAC INT
--R monomial : (% , List SingletonAsOrderedSet, List Fraction Integer) -> %
--R monomial : (% , SingletonAsOrderedSet, Fraction Integer) -> %
--R monomial : (Coef, Fraction Integer) -> %
--R multiEuclidean : (List % , %) -> Union(List % , "failed") if Coef has FIELD
--R multiplyExponents : (% , Fraction Integer) -> %
--R multiplyExponents : (% , PositiveInteger) -> %
--R nthRoot : (% , Integer) -> % if Coef has ALGEBRA FRAC INT
--R order : (% , Fraction Integer) -> Fraction Integer
--R pi : () -> % if Coef has ALGEBRA FRAC INT
--R prime? : % -> Boolean if Coef has FIELD
--R principalIdeal : List % -> Record(coef: List % , generator: %) if Coef has FIELD
--R puioux : (Fraction Integer, SparseUnivariateLaurentSeries(Coef, var, cen)) -> %
--R ?quo? : (% , %) -> % if Coef has FIELD
--R rationalPower : % -> Fraction Integer
--R ?rem? : (% , %) -> % if Coef has FIELD
--R retract : % -> SparseUnivariateTaylorSeries(Coef, var, cen)
--R retract : % -> SparseUnivariateLaurentSeries(Coef, var, cen)
--R retractIfCan : % -> Union(SparseUnivariateTaylorSeries(Coef, var, cen) , "failed")
--R retractIfCan : % -> Union(SparseUnivariateLaurentSeries(Coef, var, cen) , "failed")
--R sec : % -> % if Coef has ALGEBRA FRAC INT
--R sech : % -> % if Coef has ALGEBRA FRAC INT
--R series : (NonNegativeInteger, Stream Record(k: Fraction Integer, c: Coef)) -> %
--R sin : % -> % if Coef has ALGEBRA FRAC INT
--R sinh : % -> % if Coef has ALGEBRA FRAC INT
--R sizeLess? : (% , %) -> Boolean if Coef has FIELD
--R sqrt : % -> % if Coef has ALGEBRA FRAC INT
--R squareFree : % -> Factored % if Coef has FIELD
--R squareFreePart : % -> % if Coef has FIELD

```

```

--R subtractIfCan : (%,% ) -> Union(%, "failed")
--R tan : % -> % if Coef has ALGEBRA FRAC INT
--R tanh : % -> % if Coef has ALGEBRA FRAC INT
--R terms : % -> Stream Record(k: Fraction Integer, c: Coef)
--R truncate : (% , Fraction Integer, Fraction Integer) -> %
--R truncate : (% , Fraction Integer) -> %
--R unit? : % -> Boolean if Coef has INTDOM
--R unitCanonical : % -> % if Coef has INTDOM
--R unitNormal : % -> Record(unit: %, canonical: %, associate: %) if Coef has INTDOM
--R variables : % -> List SingletonAsOrderedSet
--R
--E 1

```

```

)spool
)lisp (bye)

```

---

### — SparseUnivariatePuisseuxSeries.help —

```

=====
SparseUnivariatePuisseuxSeries examples
=====

```

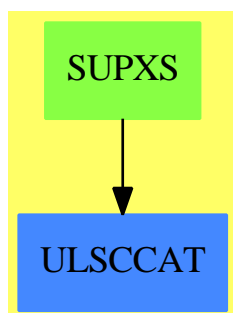
```

See Also:
o )show SparseUnivariatePuisseuxSeries

```

---

## 20.20.1 SparseUnivariatePuisseuxSeries (SUPXS)



**Exports:**

|                |                   |                    |                    |
|----------------|-------------------|--------------------|--------------------|
| 0              | 1                 | acos               | acosh              |
| acot           | acoth             | acsc               | acsch              |
| approximate    | asec              | asech              | asin               |
| asinh          | associates?       | atan               | atanh              |
| characteristic | charthRoot        | center             | coefficient        |
| coerce         | complete          | cos                | cosh               |
| cot            | coth              | csc                | csch               |
| D              | degree            | differentiate      | divide             |
| euclideanSize  | eval              | exp                | expressIdealMember |
| exquo          | extend            | extendedEuclidean  | factor             |
| gcd            | gcdPolynomial     | hash               | integrate          |
| inv            | latex             | laurent            | laurentIfCan       |
| laurentRep     | lcm               | leadingCoefficient | leadingMonomial    |
| log            | map               | monomial           | monomial?          |
| multiEuclidean | multiplyExponents | nthRoot            | one?               |
| order          | pi                | pole?              | prime?             |
| principalIdeal | puiseux           | rationalPower      | recip              |
| reductum       | retract           | retractIfCan       | sample             |
| sec            | sech              | series             | sin                |
| sinh           | sizeLess?         | sqrt               | squareFree         |
| squareFreePart | subtractIfCan     | tan                | tanh               |
| terms          | truncate          | unit?              | unitCanonical      |
| unitNormal     | variable          | variables          | zero?              |
| ??             | ?*?               | ?**?               | ?+?                |
| ?-?            | -?                | ?=?                | ?^?                |
| ?~=?           | ?/?               | ?quo?              | ?rem?              |

— domain SUPXS SparseUnivariatePuisseuxSeries —

```

)abbrev domain SUPXS SparseUnivariatePuisseuxSeries
++ Author: Clifton J. Williamson
++ Date Created: 11 November 1994
++ Date Last Updated: 28 February 1995
++ Basic Operations:
++ Related Domains: InnerSparseUnivariatePowerSeries,
++   SparseUnivariateTaylorSeries, SparseUnivariateLaurentSeries
++ Also See:
++ AMS Classifications:
++ Keywords: sparse, series
++ Examples:
++ References:
++ Description:
++ Sparse Puiseux series in one variable
++ \spadtype{SparseUnivariatePuisseuxSeries} is a domain representing Puiseux
++ series in one variable with coefficients in an arbitrary ring. The
++ parameters of the type specify the coefficient ring, the power series
++ variable, and the center of the power series expansion. For example,

```

```

++ \spad{SparseUnivariatePuisseuxSeries(Integer,x,3)} represents Puiseux
++ series in \spad{(x - 3)} with \spadtype{Integer} coefficients.

SparseUnivariatePuisseuxSeries(Coef,var,cen): Exports == Implementation where
  Coef : Ring
  var  : Symbol
  cen  : Coef
  I    ==> Integer
  NNI  ==> NonNegativeInteger
  OUT  ==> OutputForm
  RN   ==> Fraction Integer
  SUTS ==> SparseUnivariateTaylorSeries(Coef,var,cen)
  SULS ==> SparseUnivariateLaurentSeries(Coef,var,cen)
  SUPS ==> InnerSparseUnivariatePowerSeries(Coef)

Exports ==> Join(UnivariatePuisseuxSeriesConstructorCategory(Coef,SULS),_
  RetractableTo SUTS) with
  coerce: Variable(var) -> %
    ++ coerce(var) converts the series variable \spad{var} into a
    ++ Puiseux series.
  differentiate: (% ,Variable(var)) -> %
    ++ \spad{differentiate(f(x),x)} returns the derivative of
    ++ \spad{f(x)} with respect to \spad{x}.
  if Coef has Algebra Fraction Integer then
    integrate: (% ,Variable(var)) -> %
      ++ \spad{integrate(f(x))} returns an anti-derivative of the power
      ++ series \spad{f(x)} with constant coefficient 0.
      ++ We may integrate a series when we can divide coefficients
      ++ by integers.

Implementation ==> UnivariatePuisseuxSeriesConstructor(Coef,SULS) add

  Rep := Record(expon:RN,lSeries:SULS)

  getExpon: % -> RN
  getExpon pxs == pxs.expon

  variable x == var
  center   x == cen

  coerce(v: Variable(var)) ==
    zero? cen => monomial(1,1)
    monomial(1,1) + monomial(cen,0)

  coerce(uts:SUTS) == uts :: SULS :: %

  retractIfCan(upxs:%):Union(SUTS,"failed") ==
    (uls := retractIfCan(upxs)@Union(SULS,"failed")) case "failed" =>
      "failed"
    retractIfCan(uls :: SULS)@Union(SUTS,"failed")

```

```

if Coef has "*": (Fraction Integer, Coef) -> Coef then
  differentiate(upxs:%,v:Variable(var)) == differentiate upxs

if Coef has Algebra Fraction Integer then
  integrate(upxs:%,v:Variable(var)) == integrate upxs

--% OutputForms

coerce(x:%): OUT ==
  sups : SUPS := laurentRep(x) pretend SUPS
  st := getStream sups; refer := getRef sups
  if not(explicitlyEmpty? st or explicitEntries? st) _
    and (nx := retractIfCan(elt refer)@Union(I,"failed")) case I then
    count : NNI := _$streamCount$Lisp
    degr := min(count,(nx :: I) + count + 1)
    extend(sups,degr)
  seriesToOutputForm(st,refer,variable x,center x,rationalPower x)



---


— SUPXS.dotabb —

"SUPXS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SUPXS"]
"ULSCCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ULSCCAT"]
"SUPXS" -> "ULSCCAT"

```

---

## 20.21 domain ORESUP SparseUnivariateSkewPolynomial

```

— SparseUnivariateSkewPolynomial.input —

)set break resume
)sys rm -f SparseUnivariateSkewPolynomial.output
)spool SparseUnivariateSkewPolynomial.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SparseUnivariateSkewPolynomial

```

```

--R SparseUnivariateSkewPolynomial(R: Ring, sigma: Automorphism R, delta: (R -> R)) is a domain construct
--R Abbreviation for SparseUnivariateSkewPolynomial is ORESUP
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ORESUP
--R
--R----- Operations -----
--R ?? : (R,%) -> %          ?? : (% ,R) -> %
--R ?? : (% ,%) -> %          ?? : (Integer,%) -> %
--R ?? : (PositiveInteger,%) -> %    ??? : (% ,PositiveInteger) -> %
--R ?+? : (% ,%) -> %          ?-? : (% ,%) -> %
--R -? : % -> %              ?=? : (% ,%) -> Boolean
--R 1 : () -> %              0 : () -> %
--R ?? : (% ,PositiveInteger) -> %    apply : (% ,R,R) -> R
--R coefficients : % -> List R        coerce : R -> %
--R coerce : Integer -> %            coerce : % -> OutputForm
--R degree : % -> NonNegativeInteger  hash : % -> SingleInteger
--R latex : % -> String              leadingCoefficient : % -> R
--R one? : % -> Boolean              recip : % -> Union(% ,"failed")
--R reductum : % -> %              retract : % -> R
--R sample : () -> %              zero? : % -> Boolean
--R ~=? : (% ,%) -> Boolean
--R ?? : (NonNegativeInteger,%) -> %
--R ??? : (% ,NonNegativeInteger) -> %
--R ?? : (% ,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R coefficient : (% ,NonNegativeInteger) -> R
--R coerce : Fraction Integer -> % if R has RETRACT FRAC INT
--R content : % -> R if R has GCDDOM
--R exquo : (% ,R) -> Union(% ,"failed") if R has INTDOM
--R leftDivide : (% ,%) -> Record(quotient: %,remainder: %) if R has FIELD
--R leftExactQuotient : (% ,%) -> Union(% ,"failed") if R has FIELD
--R leftExtendedGcd : (% ,%) -> Record(coef1: %,coef2: %,generator: %) if R has FIELD
--R leftGcd : (% ,%) -> % if R has FIELD
--R leftLcm : (% ,%) -> % if R has FIELD
--R leftQuotient : (% ,%) -> % if R has FIELD
--R leftRemainder : (% ,%) -> % if R has FIELD
--R minimumDegree : % -> NonNegativeInteger
--R monicLeftDivide : (% ,%) -> Record(quotient: %,remainder: %) if R has INTDOM
--R monicRightDivide : (% ,%) -> Record(quotient: %,remainder: %) if R has INTDOM
--R monomial : (R,NonNegativeInteger) -> %
--R outputForm : (% ,OutputForm) -> OutputForm
--R primitivePart : % -> % if R has GCDDOM
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retract : % -> Integer if R has RETRACT INT
--R retractIfCan : % -> Union(R ,"failed")
--R retractIfCan : % -> Union(Fraction Integer ,"failed") if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(Integer ,"failed") if R has RETRACT INT
--R rightDivide : (% ,%) -> Record(quotient: %,remainder: %) if R has FIELD
--R rightExactQuotient : (% ,%) -> Union(% ,"failed") if R has FIELD
--R rightExtendedGcd : (% ,%) -> Record(coef1: %,coef2: %,generator: %) if R has FIELD

```

```

--R rightGcd : (%,%) -> % if R has FIELD
--R rightLcm : (%,%) -> % if R has FIELD
--R rightQuotient : (%,%) -> % if R has FIELD
--R rightRemainder : (%,%) -> % if R has FIELD
--R subtractIfCan : (%,%) -> Union(%, "failed")
--R
--E 1

```

```

)spool
)lisp (bye)

```

— SparseUnivariateSkewPolynomial.help —

```

=====
SparseUnivariateSkewPolynomial examples
=====

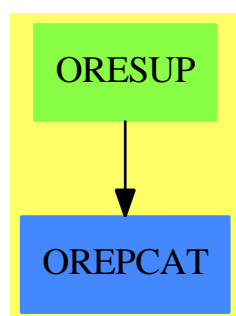
```

```

See Also:
o )show SparseUnivariateSkewPolynomial

```

### 20.21.1 SparseUnivariateSkewPolynomial (ORESUP)



See

- ⇒ “Automorphism” (AUTOMOR) 2.44.1 on page 228
- ⇒ “UnivariateSkewPolynomial” (OREUP) 22.8.1 on page 2829

**Exports:**

|                 |                 |                    |                  |                   |
|-----------------|-----------------|--------------------|------------------|-------------------|
| 0               | 1               | apply              | characteristic   | coefficient       |
| coefficients    | coerce          | content            | degree           | exquo             |
| hash            | latex           | leadingCoefficient | leftDivide       | leftExactQuotient |
| leftExtendedGcd | leftGcd         | leftLcm            | leftQuotient     | leftRemainder     |
| minimumDegree   | monicLeftDivide | monicRightDivide   | monomial         | one?              |
| outputForm      | primitivePart   | recip              | reductum         | retract           |
| retractIfCan    | rightDivide     | rightExactQuotient | rightExtendedGcd | rightGcd          |
| rightLcm        | rightQuotient   | rightRemainder     | sample           | subtractIfCan     |
| zero?           | ?*?             | ***?               | ?+?              | ?-?               |
| -?              | ?=?             | ?^?                | ?^=?             |                   |

— domain ORESUP SparseUnivariateSkewPolynomial —

```
)abbrev domain ORESUP SparseUnivariateSkewPolynomial
++ Author: Manuel Bronstein
++ Date Created: 19 October 1993
++ Date Last Updated: 1 February 1994
++ Description:
++ This is the domain of sparse univariate skew polynomials over an Ore
++ coefficient field.
++ The multiplication is given by \spad{x a = \sigma(a) x + \delta a}.

SparseUnivariateSkewPolynomial(R:Ring, sigma:Automorphism R, delta: R -> R):
  UnivariateSkewPolynomialCategory R with
    outputForm: (% , OutputForm) -> OutputForm
      ++ outputForm(p, x) returns the output form of p using x for the
      ++ otherwise anonymous variable.
  == SparseUnivariatePolynomial R add
  import UnivariateSkewPolynomialCategoryOps(R, %)

  x:% * y:%      == times(x, y, sigma, delta)
  apply(p, c, r) == apply(p, c, r, sigma, delta)

  if R has IntegralDomain then
    monicLeftDivide(a, b) == monicLeftDivide(a, b, sigma)
    monicRightDivide(a, b) == monicRightDivide(a, b, sigma)

  if R has Field then
    leftDivide(a, b) == leftDivide(a, b, sigma)
    rightDivide(a, b) == rightDivide(a, b, sigma)
```

—————

— ORESUP.dotabb —

"ORESUP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ORESUP"]



```
"OREPCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=OREPCAT"]
"ORESUP" -> "OREPCAT"
```

## 20.22 domain SUTS SparseUnivariateTaylorSeries

— SparseUnivariateTaylorSeries.input —

```
)set break resume
)sys rm -f SparseUnivariateTaylorSeries.output
)spool SparseUnivariateTaylorSeries.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SparseUnivariateTaylorSeries
--R SparseUnivariateTaylorSeries(Coef: Ring,var: Symbol,cen: Coef) is a domain constructor
--R Abbreviation for SparseUnivariateTaylorSeries is SUTS
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SUTS
--R
--R----- Operations -----
--R ??? : (Coef,%) -> %           ??? : (%,Coef) -> %
--R ??? : (%,%) -> %             ??? : (Integer,%) -> %
--R ??? : (PositiveInteger,%) -> %   ??? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %             ?-? : (%,%) -> %
--R -? : % -> %                  ?? : (%,%) -> Boolean
--R 1 : () -> %                  0 : () -> %
--R ?? : (%,PositiveInteger) -> %   center : % -> Coef
--R coefficients : % -> Stream Coef   coerce : Variable var -> %
--R coerce : Integer -> %             coerce : % -> OutputForm
--R complete : % -> %                 degree : % -> NonNegativeInteger
--R hash : % -> SingleInteger          latex : % -> String
--R leadingCoefficient : % -> Coef     leadingMonomial : % -> %
--R map : ((Coef -> Coef),%) -> %      monomial? : % -> Boolean
--R one? : % -> Boolean                 order : % -> NonNegativeInteger
--R pole? : % -> Boolean                quoByVar : % -> %
--R recip : % -> Union(%, "failed")    reductum : % -> %
--R sample : () -> %                  series : Stream Coef -> %
--R variable : % -> Symbol              zero? : % -> Boolean
--R ~=? : (%,%) -> Boolean
--R ??? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ??? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
--R ??? : (NonNegativeInteger,%) -> %
```

```

--R ??? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ??? : (%,%) -> % if Coef has ALGEBRA FRAC INT
--R ??? : (%,Coef) -> % if Coef has FIELD
--R ??? : (%,NonNegativeInteger) -> %
--R ?/? : (%,Coef) -> % if Coef has FIELD
--R D : % -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef
--R D : (%,NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef
--R D : (%,Symbol) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef and Coef has PDRING SYMBOL
--R D : (%,List Symbol) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef and Coef has PDRING SYMBOL
--R D : (%,Symbol,NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef and Coef has
--R D : (%,List Symbol,List NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef and
--R ?? : (%,NonNegativeInteger) -> %
--R acos : % -> % if Coef has ALGEBRA FRAC INT
--R acosh : % -> % if Coef has ALGEBRA FRAC INT
--R acot : % -> % if Coef has ALGEBRA FRAC INT
--R acoth : % -> % if Coef has ALGEBRA FRAC INT
--R acsc : % -> % if Coef has ALGEBRA FRAC INT
--R acsch : % -> % if Coef has ALGEBRA FRAC INT
--R approximate : (%,NonNegativeInteger) -> Coef if Coef has **: (Coef,NonNegativeInteger) -> Coef and C
--R asec : % -> % if Coef has ALGEBRA FRAC INT
--R asech : % -> % if Coef has ALGEBRA FRAC INT
--R asin : % -> % if Coef has ALGEBRA FRAC INT
--R asinh : % -> % if Coef has ALGEBRA FRAC INT
--R associates? : (%,%) -> Boolean if Coef has INTDOM
--R atan : % -> % if Coef has ALGEBRA FRAC INT
--R atanh : % -> % if Coef has ALGEBRA FRAC INT
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%, "failed") if Coef has CHARNZ
--R coefficient : (%,NonNegativeInteger) -> Coef
--R coerce : UnivariatePolynomial(var,Coef) -> %
--R coerce : Coef -> % if Coef has COMRING
--R coerce : % -> % if Coef has INTDOM
--R coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
--R cos : % -> % if Coef has ALGEBRA FRAC INT
--R cosh : % -> % if Coef has ALGEBRA FRAC INT
--R cot : % -> % if Coef has ALGEBRA FRAC INT
--R coth : % -> % if Coef has ALGEBRA FRAC INT
--R csc : % -> % if Coef has ALGEBRA FRAC INT
--R csch : % -> % if Coef has ALGEBRA FRAC INT
--R differentiate : (%,Variable var) -> %
--R differentiate : % -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef
--R differentiate : (%,NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef
--R differentiate : (%,Symbol) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef and Coef has PDRING
--R differentiate : (%,List Symbol) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef and Coef has P
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger,Coef
--R ?.? : (%,%) -> % if NonNegativeInteger has SGROUP
--R ?.? : (%,NonNegativeInteger) -> Coef
--R eval : (%,Coef) -> Stream Coef if Coef has **: (Coef,NonNegativeInteger) -> Coef
--R exp : % -> % if Coef has ALGEBRA FRAC INT

```

```

--R exquo : (%,%) -> Union(%, "failed") if Coef has INTDOM
--R extend : (%, NonNegativeInteger) -> %
--R integrate : (%, Variable var) -> % if Coef has ALGEBRA FRAC INT
--R integrate : (%, Symbol) -> % if Coef has integrate: (Coef, Symbol) -> Coef and Coef has va
--R integrate : % -> % if Coef has ALGEBRA FRAC INT
--R log : % -> % if Coef has ALGEBRA FRAC INT
--R monomial : (%, List SingletonAsOrderedSet, List NonNegativeInteger) -> %
--R monomial : (%, SingletonAsOrderedSet, NonNegativeInteger) -> %
--R monomial : (Coef, NonNegativeInteger) -> %
--R multiplyCoefficients : ((Integer -> Coef), %) -> %
--R multiplyExponents : (%, PositiveInteger) -> %
--R nthRoot : (%, Integer) -> % if Coef has ALGEBRA FRAC INT
--R order : (%, NonNegativeInteger) -> NonNegativeInteger
--R pi : () -> % if Coef has ALGEBRA FRAC INT
--R polynomial : (%, NonNegativeInteger, NonNegativeInteger) -> Polynomial Coef
--R polynomial : (%, NonNegativeInteger) -> Polynomial Coef
--R sec : % -> % if Coef has ALGEBRA FRAC INT
--R sech : % -> % if Coef has ALGEBRA FRAC INT
--R series : Stream Record(k: NonNegativeInteger, c: Coef) -> %
--R sin : % -> % if Coef has ALGEBRA FRAC INT
--R sinh : % -> % if Coef has ALGEBRA FRAC INT
--R sqrt : % -> % if Coef has ALGEBRA FRAC INT
--R subtractIfCan : (%, %) -> Union(%, "failed")
--R tan : % -> % if Coef has ALGEBRA FRAC INT
--R tanh : % -> % if Coef has ALGEBRA FRAC INT
--R terms : % -> Stream Record(k: NonNegativeInteger, c: Coef)
--R truncate : (%, NonNegativeInteger, NonNegativeInteger) -> %
--R truncate : (%, NonNegativeInteger) -> %
--R unit? : % -> Boolean if Coef has INTDOM
--R unitCanonical : % -> % if Coef has INTDOM
--R unitNormal : % -> Record(unit: %, canonical: %, associate: %) if Coef has INTDOM
--R univariatePolynomial : (%, NonNegativeInteger) -> UnivariatePolynomial(var, Coef)
--R variables : % -> List SingletonAsOrderedSet
--R
--E 1

)spool
)lisp (bye)

```

---

— SparseUnivariateTaylorSeries.help —

```

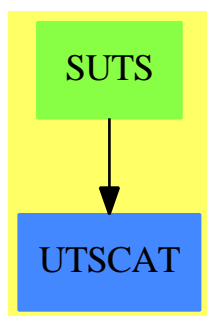
=====
SparseUnivariateTaylorSeries examples
=====

```

See Also:

o )show SparseUnivariateTaylorSeries

## 20.22.1 SparseUnivariateTaylorSeries (SUTS)

**Exports:**

|                      |                   |                |                      |                    |
|----------------------|-------------------|----------------|----------------------|--------------------|
| 0                    | 1                 | acos           | acosh                | acot               |
| acoth                | acsc              | acsch          | approximate          | asec               |
| asech                | asin              | asinh          | associates?          | atan               |
| atanh                | center            | characteristic | charthRoot           | coefficient        |
| coefficients         | coerce            | complete       | cos                  | cosh               |
| cot                  | coth              | csc            | csch                 | D                  |
| degree               | differentiate     | eval           | exp                  | exquo              |
| extend               | hash              | integrate      | latex                | leadingCoefficient |
| leadingMonomial      | log               | map            | monomial             | monomial?          |
| multiplyCoefficients | multiplyExponents | nthRoot        | one?                 | order              |
| pole?                | pi                | polynomial     | polynomial           | quoByVar           |
| recip                | reductum          | sample         | sec                  | sech               |
| series               | sin               | sinh           | sqrt                 | subtractIfCan      |
| tan                  | tanh              | terms          | truncate             | truncate           |
| unit?                | unitCanonical     | unitNormal     | univariatePolynomial | variable           |
| variables            | zero?             | ?*?            | ?**?                 | ?+?                |
| ?-?                  | -?                | ?=?            | ?^?                  | ?~=?               |
| ?/?                  | ?..?              |                |                      |                    |

— domain SUTS SparseUnivariateTaylorSeries —

```

)abbrev domain SUTS SparseUnivariateTaylorSeries
++ Author: Clifton J. Williamson
++ Date Created: 16 February 1990
++ Date Last Updated: 10 March 1995
++ Basic Operations:
++ Related Domains: InnerSparseUnivariatePowerSeries,

```

```

++ SparseUnivariateLaurentSeries, SparseUnivariatePuisseuxSeries
++ Also See:
++ AMS Classifications:
++ Keywords: Taylor series, sparse power series
++ Examples:
++ References:
++ Description:
++ Sparse Taylor series in one variable
++ \spadtype{SparseUnivariateTaylorSeries} is a domain representing Taylor
++ series in one variable with coefficients in an arbitrary ring. The
++ parameters of the type specify the coefficient ring, the power series
++ variable, and the center of the power series expansion. For example,
++ \spadtype{SparseUnivariateTaylorSeries}(Integer,x,3) represents Taylor
++ series in \spad{(x - 3)} with \spadtype{Integer} coefficients.

```

SparseUnivariateTaylorSeries(Coef,var,cen): Exports == Implementation where

```

Coef : Ring
var  : Symbol
cen  : Coef
COM ==> OrderedCompletion Integer
I    ==> Integer
L    ==> List
NNI  ==> NonNegativeInteger
OUT  ==> OutputForm
P    ==> Polynomial Coef
REF  ==> Reference OrderedCompletion Integer
RN   ==> Fraction Integer
Term ==> Record(k:Integer,c:Coef)
SG   ==> String
ST   ==> Stream Term
UP   ==> UnivariatePolynomial(var,Coef)

```

Exports ==> UnivariateTaylorSeriesCategory(Coef) with

```

coerce: UP -> %
  ++\spad{coerce(p)} converts a univariate polynomial p in the variable
  ++\spad{var} to a univariate Taylor series in \spad{var}.
univariatePolynomial: (%,NNI) -> UP
  ++\spad{univariatePolynomial(f,k)} returns a univariate polynomial
  ++ consisting of the sum of all terms of f of degree \spad{<= k}.
coerce: Variable(var) -> %
  ++\spad{coerce(var)} converts the series variable \spad{var} into a
  ++ Taylor series.
differentiate: (%,Variable(var)) -> %
  ++ \spad{differentiate(f(x),x)} computes the derivative of
  ++ \spad{f(x)} with respect to \spad{x}.
if Coef has Algebra Fraction Integer then
  integrate: (%,Variable(var)) -> %
    ++ \spad{integrate(f(x),x)} returns an anti-derivative of the power
    ++ series \spad{f(x)} with constant coefficient 0.
    ++ We may integrate a series when we can divide coefficients

```

```

    ++ by integers.

Implementation ==> InnerSparseUnivariatePowerSeries(Coef) add
import REF

Rep := InnerSparseUnivariatePowerSeries(Coef)

makeTerm: (Integer,Coef) -> Term
makeTerm(exp,coef) == [exp,coef]
getCoef: Term -> Coef
getCoef term == term.c
getExpon: Term -> Integer
getExpon term == term.k

monomial(coef,expon) == monomial(coef,expon)$Rep
extend(x,n) == extend(x,n)$Rep

0 == monomial(0,0)$Rep
1 == monomial(1,0)$Rep

recip uts == iExquo(1,uts,true)

if Coef has IntegralDomain then
  uts1 exquo uts2 == iExquo(uts1,uts2,true)

quoByVar uts == taylorQuoByVar(uts)$Rep

differentiate(x:%,v:Variable(var)) == differentiate x

--% Creation and destruction of series

coerce(v: Variable(var)) ==
  zero? cen => monomial(1,1)
  monomial(1,1) + monomial(cen,0)

coerce(p:UP) ==
  zero? p => 0
  if not zero? cen then p := p(monomial(1,1)$UP + monomial(cen,0)$UP)
  st : ST := empty()
  while not zero? p repeat
    st := concat(makeTerm(degree p,leadingCoefficient p),st)
    p := reductum p
  makeSeries(ref plusInfinity(),st)

univariatePolynomial(x,n) ==
  extend(x,n); st := getStream x
  ans : UP := 0; oldDeg : I := 0;
  mon := monomial(1,1)$UP - monomial(center x,0)$UP; monPow : UP := 1
  while explicitEntries? st repeat
    (xExpon := getExpon(xTerm := frst st)) > n => return ans

```

```

    pow := (xExpon - oldDeg) :: NNI; oldDeg := xExpon
    monPow := monPow * mon ** pow
    ans := ans + getCoef(xTerm) * monPow
    st := rst st
  ans

polynomial(x,n) ==
  extend(x,n); st := getStream x
  ans : P := 0; oldDeg : I := 0;
  mon := (var :: P) - (center(x) :: P); monPow : P := 1
  while explicitEntries? st repeat
    (xExpon := getExpon(xTerm := first st)) > n => return ans
    pow := (xExpon - oldDeg) :: NNI; oldDeg := xExpon
    monPow := monPow * mon ** pow
    ans := ans + getCoef(xTerm) * monPow
    st := rst st
  ans

polynomial(x,n1,n2) == polynomial(truncate(x,n1,n2),n2)

truncate(x,n)      == truncate(x,n)$Rep
truncate(x,n1,n2) == truncate(x,n1,n2)$Rep

iCoefficients: (ST,REF,I) -> Stream Coef
iCoefficients(x,refer,n) == delay
  -- when this function is called, we are computing the nth order
  -- coefficient of the series
  explicitlyEmpty? x => empty()
  -- if terms up to order n have not been computed,
  -- apply lazy evaluation
  nn := n :: COM
  while (nx := elt refer) < nn repeat lazyEvaluate x
  -- must have nx >= n
  explicitEntries? x =>
    xCoef := getCoef(xTerm := first x); xExpon := getExpon xTerm
    xExpon = n => concat(xCoef,iCoefficients(rst x,refer,n + 1))
    -- must have nx > n
    concat(0,iCoefficients(x,refer,n + 1))
    concat(0,iCoefficients(x,refer,n + 1))

coefficients uts ==
  refer := getRef uts; x := getStream uts
  iCoefficients(x,refer,0)

terms uts == terms(uts)$Rep pretend Stream Record(k::NNI,c::Coef)

iSeries: (Stream Coef,I,REF) -> ST
iSeries(st,n,refer) == delay
  -- when this function is called, we are creating the nth order
  -- term of a series

```

```

empty? st => (setelt(refer,plusInfinity()); empty())
setelt(refer,n :: COM)
zero? (coef := frst st) => iSeries(rst st,n + 1,refer)
concat(makeTerm(n,coef),iSeries(rst st,n + 1,refer))

series(st:Stream Coef) ==
  refer := ref(-1)
  makeSeries(refer,iSeries(st,0,refer))

nniToI: Stream Record(k>NNI,c:Coef) -> ST
nniToI st ==
  empty? st => empty()
  term : Term := [(frst st).k,(frst st).c]
  concat(term,nniToI rst st)

series(st:Stream Record(k>NNI,c:Coef)) == series(nniToI st)$Rep

--% Values

variable x == var
center   x == cen

coefficient(x,n) == coefficient(x,n)$Rep
elt(x:%,n:NonNegativeInteger) == coefficient(x,n)

pole? x == false

order x      == (order(x)$Rep) :: NNI
order(x,n) == (order(x,n)$Rep) :: NNI

--% Composition

elt(uts1:%,uts2:%) ==
  zero? uts2 => coefficient(uts1,0) :: %
  not zero? coefficient(uts2,0) =>
    error "elt: second argument must have positive order"
    iCompose(uts1,uts2)

--% Integration

if Coef has Algebra Fraction Integer then

  integrate(x:%,v:Variable(var)) == integrate x

--% Transcendental functions

(uts1:%) ** (uts2:%) == exp(log(uts1) * uts2)

if Coef has CommutativeRing then

```



```

(uts:%) ** (r:RN) == cRationalPower(uts,r)

exp uts == cExp uts
log uts == cLog uts

sin uts == cSin uts
cos uts == cCos uts
tan uts == cTan uts
cot uts == cCot uts
sec uts == cSec uts
csc uts == cCsc uts

asin uts == cAsin uts
acos uts == cAcos uts
atan uts == cAtan uts
acot uts == cAcot uts
asec uts == cAsec uts
acsc uts == cAcsc uts

sinh uts == cSinh uts
cosh uts == cCosh uts
tanh uts == cTanh uts
coth uts == cCoth uts
sech uts == cSech uts
csch uts == cCsch uts

asinh uts == cAsinh uts
acosh uts == cAcosh uts
atanh uts == cAtanh uts
acoth uts == cAcoth uts
asech uts == cAsech uts
acsch uts == cAcsch uts

else

ZERO      : SG := "series must have constant coefficient zero"
ONE       : SG := "series must have constant coefficient one"
NPOWERS   : SG := "series expansion has terms of negative degree"

(uts:%) ** (r:RN) ==
--      not one? coefficient(uts,0) =>
      not (coefficient(uts,0) = 1) =>
        error "**: constant coefficient must be one"
      onePlusX : % := monomial(1,0) + monomial(1,1)
      ratPow := cPower(uts,r :: Coef)
      iCompose(ratPow,uts - 1)

exp uts ==
      zero? coefficient(uts,0) =>
        expx := cExp monomial(1,1)

```

```

        iCompose(expx,uts)
        error concat("exp: ",ZERO)

log uts ==
--      one? coefficient(uts,0) =>
      (coefficient(uts,0) = 1) =>
        log1PlusX := cLog(monomial(1,0) + monomial(1,1))
        iCompose(log1PlusX,uts - 1)
        error concat("log: ",ONE)

sin uts ==
      zero? coefficient(uts,0) =>
        sinx := cSin monomial(1,1)
        iCompose(sinx,uts)
        error concat("sin: ",ZERO)

cos uts ==
      zero? coefficient(uts,0) =>
        cosx := cCos monomial(1,1)
        iCompose(cosx,uts)
        error concat("cos: ",ZERO)

tan uts ==
      zero? coefficient(uts,0) =>
        tanx := cTan monomial(1,1)
        iCompose(tanx,uts)
        error concat("tan: ",ZERO)

cot uts ==
      zero? uts => error "cot: cot(0) is undefined"
      zero? coefficient(uts,0) => error concat("cot: ",NPOWERS)
      error concat("cot: ",ZERO)

sec uts ==
      zero? coefficient(uts,0) =>
        secx := cSec monomial(1,1)
        iCompose(secx,uts)
        error concat("sec: ",ZERO)

csc uts ==
      zero? uts => error "csc: csc(0) is undefined"
      zero? coefficient(uts,0) => error concat("csc: ",NPOWERS)
      error concat("csc: ",ZERO)

asin uts ==
      zero? coefficient(uts,0) =>
        asinx := cAsin monomial(1,1)
        iCompose(asinx,uts)
        error concat("asin: ",ZERO)

```

```

atan uts ==
  zero? coefficient(uts,0) =>
    atanx := cAtan monomial(1,1)
    iCompose(atanx,uts)
    error concat("atan: ",ZERO)

acos z == error "acos: acos undefined on this coefficient domain"
acot z == error "acot: acot undefined on this coefficient domain"
asec z == error "asec: asec undefined on this coefficient domain"
acsc z == error "acsc: acsc undefined on this coefficient domain"

sinh uts ==
  zero? coefficient(uts,0) =>
    sinhx := cSinh monomial(1,1)
    iCompose(sinhx,uts)
    error concat("sinh: ",ZERO)

cosh uts ==
  zero? coefficient(uts,0) =>
    coshx := cCosh monomial(1,1)
    iCompose(coshx,uts)
    error concat("cosh: ",ZERO)

tanh uts ==
  zero? coefficient(uts,0) =>
    tanhx := cTanh monomial(1,1)
    iCompose(tanhx,uts)
    error concat("tanh: ",ZERO)

coth uts ==
  zero? uts => error "coth: coth(0) is undefined"
  zero? coefficient(uts,0) => error concat("coth: ",NPOWERS)
  error concat("coth: ",ZERO)

sech uts ==
  zero? coefficient(uts,0) =>
    sechx := cSech monomial(1,1)
    iCompose(sechx,uts)
    error concat("sech: ",ZERO)

csch uts ==
  zero? uts => error "csch: csch(0) is undefined"
  zero? coefficient(uts,0) => error concat("csch: ",NPOWERS)
  error concat("csch: ",ZERO)

asinh uts ==
  zero? coefficient(uts,0) =>
    asinhx := cAsinh monomial(1,1)
    iCompose(asinhx,uts)
    error concat("asinh: ",ZERO)

```

```

    atanh uts ==
      zero? coefficient(uts,0) =>
        atanhx := cAtanh monomial(1,1)
        iCompose(atanhx,uts)
        error concat("atanh: ",ZERO)

    acosh uts == error "acosh: acosh undefined on this coefficient domain"
    acoth uts == error "acoth: acoth undefined on this coefficient domain"
    asech uts == error "asech: asech undefined on this coefficient domain"
    acsch uts == error "acsch: acsch undefined on this coefficient domain"

    if Coef has Field then
      if Coef has Algebra Fraction Integer then

        (uts:%) ** (r:Coef) ==
--      not one? coefficient(uts,1) =>
        not (coefficient(uts,1) = 1) =>
          error "**: constant coefficient should be 1"
          cPower(uts,r)

--% OutputForms

    coerce(x:): OUT ==
      count : NNI := _$streamCount$Lisp
      extend(x,count)
      seriesToOutputForm(getStream x,getRef x,variable x,center x,1)

    -----

    — SUTS.dotabb —

    "SUTS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SUTS"]
    "UTSCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=UTSCAT"]
    "SUTS" -> "UTSCAT"

    -----

```

## 20.23 domain SHDP SplitHomogeneousDirectProduct

— SplitHomogeneousDirectProduct.input —

```

)set break resume
)sys rm -f SplitHomogeneousDirectProduct.output

```

```

)spool SplitHomogeneousDirectProduct.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SplitHomogeneousDirectProduct
--R SplitHomogeneousDirectProduct(dimtot: NonNegativeInteger,dim1: NonNegativeInteger,S: Ord
--R Abbreviation for SplitHomogeneousDirectProduct is SHDP
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SHDP
--R
--R----- Operations -----
--R -? : % -> % if S has RING          1 : () -> % if S has MONOID
--R 0 : () -> % if S has CABMON        coerce : % -> Vector S
--R copy : % -> %                      directProduct : Vector S -> %
--R ?.? : (%,Integer) -> S             elt : (%,Integer,S) -> S
--R empty : () -> %                   empty? : % -> Boolean
--R entries : % -> List S              eq? : (%,% ) -> Boolean
--R index? : (Integer,%) -> Boolean    indices : % -> List Integer
--R map : ((S -> S),%) -> %           qelt : (%,Integer) -> S
--R sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?? : (PositiveInteger,%) -> % if S has ABELSG
--R ?? : (NonNegativeInteger,%) -> % if S has CABMON
--R ?? : (S,%) -> % if S has RING
--R ?? : (% ,S) -> % if S has RING
--R ?? : (% ,%) -> % if S has MONOID
--R ?? : (Integer,%) -> % if S has RING
--R ??? : (% ,PositiveInteger) -> % if S has MONOID
--R ??? : (% ,NonNegativeInteger) -> % if S has MONOID
--R ?+? : (% ,%) -> % if S has ABELSG
--R ?-? : (% ,%) -> % if S has RING
--R ?/? : (% ,S) -> % if S has FIELD
--R ?<? : (% ,%) -> Boolean if S has OAMONS or S has ORDRING
--R ?<=? : (% ,%) -> Boolean if S has OAMONS or S has ORDRING
--R ?=? : (% ,%) -> Boolean if S has SETCAT
--R ?>? : (% ,%) -> Boolean if S has OAMONS or S has ORDRING
--R ?>=? : (% ,%) -> Boolean if S has OAMONS or S has ORDRING
--R D : (% ,(S -> S)) -> % if S has RING
--R D : (% ,(S -> S),NonNegativeInteger) -> % if S has RING
--R D : (% ,List Symbol,List NonNegativeInteger) -> % if S has PDRING SYMBOL and S has RING
--R D : (% ,Symbol,NonNegativeInteger) -> % if S has PDRING SYMBOL and S has RING
--R D : (% ,List Symbol) -> % if S has PDRING SYMBOL and S has RING
--R D : (% ,Symbol) -> % if S has PDRING SYMBOL and S has RING
--R D : (% ,NonNegativeInteger) -> % if S has DIFRING and S has RING
--R D : % -> % if S has DIFRING and S has RING
--R ?? : (% ,PositiveInteger) -> % if S has MONOID
--R ?? : (% ,NonNegativeInteger) -> % if S has MONOID
--R abs : % -> % if S has ORDRING

```

```

--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R characteristic : () -> NonNegativeInteger if S has RING
--R coerce : S -> % if S has SETCAT
--R coerce : Fraction Integer -> % if S has RETRACT FRAC INT and S has SETCAT
--R coerce : Integer -> % if S has RETRACT INT and S has SETCAT or S has RING
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if $ has finiteAggregate and S has SETCAT
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R differentiate : (%,(S -> S)) -> % if S has RING
--R differentiate : (%,(S -> S),NonNegativeInteger) -> % if S has RING
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if S has PDRING SYMBOL and S has RING
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if S has PDRING SYMBOL and S has RING
--R differentiate : (%,List Symbol) -> % if S has PDRING SYMBOL and S has RING
--R differentiate : (%,Symbol) -> % if S has PDRING SYMBOL and S has RING
--R differentiate : (%,NonNegativeInteger) -> % if S has DIFRING and S has RING
--R differentiate : % -> % if S has DIFRING and S has RING
--R dimension : () -> CardinalNumber if S has FIELD
--R dot : (%,%) -> S if S has RING
--R entry? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,S) -> % if $ has shallowlyMutable
--R first : % -> S if Integer has ORDSET
--R hash : % -> SingleInteger if S has SETCAT
--R index : PositiveInteger -> % if S has FINITE
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R lookup : % -> PositiveInteger if S has FINITE
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R max : (%,%) -> % if S has OAMONS or S has ORDRING
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R members : % -> List S if $ has finiteAggregate
--R min : (%,%) -> % if S has OAMONS or S has ORDRING
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R negative? : % -> Boolean if S has ORDRING
--R one? : % -> Boolean if S has MONOID
--R parts : % -> List S if $ has finiteAggregate
--R positive? : % -> Boolean if S has ORDRING
--R qsetelt! : (%,Integer,S) -> S if $ has shallowlyMutable
--R random : () -> % if S has FINITE
--R recip : % -> Union(%, "failed") if S has MONOID
--R reducedSystem : Matrix % -> Matrix S if S has RING
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix S,vec: Vector S) if S has RING
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) if S has LINE
--R reducedSystem : Matrix % -> Matrix Integer if S has LINEXP INT and S has RING

```

```

--R retract : % -> S if S has SETCAT
--R retract : % -> Fraction Integer if S has RETRACT FRAC INT and S has SETCAT
--R retract : % -> Integer if S has RETRACT INT and S has SETCAT
--R retractIfCan : % -> Union(S,"failed") if S has SETCAT
--R retractIfCan : % -> Union(Fraction Integer,"failed") if S has RETRACT FRAC INT and S has
--R retractIfCan : % -> Union(Integer,"failed") if S has RETRACT INT and S has SETCAT
--R setelt : (%,Integer,S) -> S if $ has shallowlyMutable
--R sign : % -> Integer if S has ORDRING
--R size : () -> NonNegativeInteger if S has FINITE
--R size? : (%,NonNegativeInteger) -> Boolean
--R subtractIfCan : (%,%) -> Union(%, "failed") if S has CABMON
--R sup : (%,%) -> % if S has OAMONS
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R unitVector : PositiveInteger -> % if S has RING
--R zero? : % -> Boolean if S has CABMON
--R ~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)

```

---

— SplitHomogeneousDirectProduct.help —

```

=====
SplitHomogeneousDirectProduct examples
=====

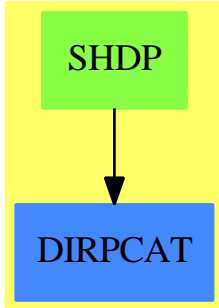
```

```

See Also:
o )show SplitHomogeneousDirectProduct

```

### 20.23.1 SplitHomogeneousDirectProduct (SHDP)



See

⇒ “OrderedDirectProduct” (ODP) 16.13.1 on page 1778

⇒ “HomogeneousDirectProduct” (HDP) 9.9.1 on page 1138

#### Exports:

|               |               |               |            |                |
|---------------|---------------|---------------|------------|----------------|
| 0             | 1             | abs           | any?       | characteristic |
| coerce        | copy          | count         | D          | differentiate  |
| dimension     | directProduct | dot           | elt        | empty          |
| empty?        | entries       | entry?        | eq?        | eval           |
| every?        | fill!         | first         | hash       | index          |
| index?        | indices       | latex         | less?      | lookup         |
| map           | map!          | max           | maxIndex   | member?        |
| members       | min           | minIndex      | more?      | negative?      |
| one?          | parts         | positive?     | qelt       | qsetelt!       |
| random        | recip         | reducedSystem | retract    | retractIfCan   |
| sample        | setelt        | sign          | size       | size?          |
| subtractIfCan | sup           | swap!         | unitVector | zero?          |
| #?            | ?*?           | ?**?          | ?+?        | ?-?            |
| ?/?           | ?<?           | ?<=?          | ?=?        | ?>?            |
| ?>=?          | ?^?           | ?~=?          | -?         | ?.?            |

— domain SHDP SplitHomogeneousDirectProduct —

```

)abbrev domain SHDP SplitHomogeneousDirectProduct
++ Author: Mark Botch
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors: Vector, DirectProduct
++ Also See: OrderedDirectProduct, HomogeneousDirectProduct
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This type represents the finite direct or cartesian product of an

```



```

++ underlying ordered component type. The vectors are ordered as if
++ they were split into two blocks. The dim1 parameter specifies the
++ length of the first block. The ordering is lexicographic between
++ the blocks but acts like \spadtype{HomogeneousDirectProduct}
++ within each block. This type is a suitable third argument for
++ \spadtype{GeneralDistributedMultivariatePolynomial}.

```

```

SplitHomogeneousDirectProduct(dimtot,dim1,S) : T == C where
  NNI ==> NonNegativeInteger
  dim1,dimtot : NNI
  S           : OrderedAbelianMonoidSup

  T == DirectProductCategory(dimtot,S)
  C == DirectProduct(dimtot,S) add
  Rep:=Vector(S)
  lessThanRlex(v1:%,v2:%,low:NNI,high:NNI):Boolean ==
-- reverse lexicographical ordering
  n1:S:=0
  n2:S:=0
  for i in low..high repeat
    n1:= n1+qelt(v1,i)
    n2:=n2+qelt(v2,i)
  n1<n2 => true
  n2<n1 => false
  for i in reverse(low..high) repeat
    if qelt(v2,i) < qelt(v1,i) then return true
    if qelt(v1,i) < qelt(v2,i) then return false
  false

(v1:% < v2:%):Boolean ==
  lessThanRlex(v1,v2,1,dim1) => true
  for i in 1..dim1 repeat
    if qelt(v1,i) ^= qelt(v2,i) then return false
  lessThanRlex(v1,v2,dim1+1,dimtot)

```

---

— SHDP.dotabb —

```

"SHDP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SHDP"]
"DIRPCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=DIRPCAT"]
"SHDP" -> "DIRPCAT"

```

---

## 20.24 domain SPLNODE SplittingNode

### — SplittingNode.input —

```

)set break resume
)sys rm -f SplittingNode.output
)spool SplittingNode.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SplittingNode
--R SplittingNode(V: Join(SetCategory,Aggregate),C: Join(SetCategory,Aggregate)) is a domain constructor
--R Abbreviation for SplittingNode is SPLNODE
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SPLNODE
--R
--R----- Operations -----
--R ?? : (%,% ) -> Boolean          coerce : % -> OutputForm
--R condition : % -> C              construct : (V,List C) -> List %
--R construct : (V,C) -> %          construct : (V,C,Boolean) -> %
--R copy : % -> %                  empty : () -> %
--R empty? : % -> Boolean          hash : % -> SingleInteger
--R latex : % -> String            setCondition! : (% ,C) -> %
--R setEmpty! : % -> %             setStatus! : (% ,Boolean) -> %
--R setValue! : (% ,V) -> %        status : % -> Boolean
--R value : % -> V                 ~=? : (% ,%) -> Boolean
--R construct : List Record(val: V,tower: C) -> List %
--R construct : Record(val: V,tower: C) -> %
--R infLex? : (% ,% ,((V,V) -> Boolean),((C,C) -> Boolean)) -> Boolean
--R subNode? : (% ,% ,((C,C) -> Boolean)) -> Boolean
--R
--E 1

)spool
)lisp (bye)

```

### — SplittingNode.help —

```

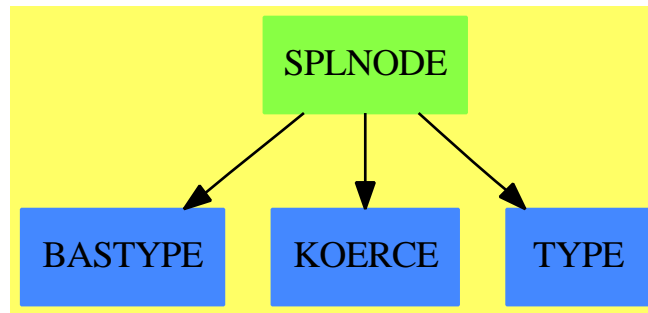
=====
SplittingNode examples
=====

```

See Also:

```
o )show SplittingNode
```

### 20.24.1 SplittingNode (SPLNODE)



See

⇒ “SplittingNode” (SPLNODE) 20.24.1 on page 2470

#### Exports:

|           |            |           |        |               |
|-----------|------------|-----------|--------|---------------|
| coerce    | condition  | construct | copy   | empty         |
| empty?    | hash       | infLex?   | latex  | setCondition! |
| setEmpty! | setStatus! | setValue! | status | subNode?      |
| value     | ?=?        | ?~=?      |        |               |

— domain SPLNODE SplittingNode —

```

)abbrev domain SPLNODE SplittingNode
++ Author: Marc Moereno Maza
++ Date Created: 07/05/1996
++ Date Last Updated: 07/19/1996
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ References:
++ Description:
++ This domain exports a modest implementation for the
++ vertices of splitting trees. These vertices are called
++ here splitting nodes. Every of these nodes store 3 informations.
++ The first one is its value, that is the current expression
++ to evaluate. The second one is its condition, that is the
++ hypothesis under which the value has to be evaluated.
++ The last one is its status, that is a boolean flag
++ which is true iff the value is the result of its
  
```

```

++ evaluation under its condition. Two splitting vertices
++ are equal iff they have the same values and the same
++ conditions (so their status do not matter).

```

SplittingNode(V,C) : Exports == Implementation where

```

V:Join(SetCategory,Aggregate)
C:Join(SetCategory,Aggregate)
Z ==> Integer
B ==> Boolean
O ==> OutputForm
VT ==> Record(val:V, tower:C)
VTB ==> Record(val:V, tower:C, flag:B)

```

Exports == SetCategory with

```

empty : () -> %
  ++ \axiom{empty()} returns the same as
  ++ \axiom{[empty()$V,empty()$C,false]$%}
empty? : % -> B
  ++ \axiom{empty?(n)} returns true iff the node n is \axiom{empty()$%}.
value : % -> V
  ++ \axiom{value(n)} returns the value of the node n.
condition : % -> C
  ++ \axiom{condition(n)} returns the condition of the node n.
status : % -> B
  ++ \axiom{status(n)} returns the status of the node n.
construct : (V,C,B) -> %
  ++ \axiom{construct(v,t,b)} returns the non-empty node with
  ++ value v, condition t and flag b
construct : (V,C) -> %
  ++ \axiom{construct(v,t)} returns the same as
  ++ \axiom{construct(v,t,false)}
construct : VT -> %
  ++ \axiom{construct(vt)} returns the same as
  ++ \axiom{construct(vt.val,vt.tower)}
construct : List VT -> List %
  ++ \axiom{construct(lvt)} returns the same as
  ++ \axiom{[construct(vt.val,vt.tower) for vt in lvt]}
construct : (V, List C) -> List %
  ++ \axiom{construct(v,lt)} returns the same as
  ++ \axiom{[construct(v,t) for t in lt]}
copy : % -> %
  ++ \axiom{copy(n)} returns a copy of n.
setValue! : (% ,V) -> %
  ++ \axiom{setValue!(n,v)} returns n whose value
  ++ has been replaced by v if it is not
  ++ empty, else an error is produced.
setCondition! : (% ,C) -> %
  ++ \axiom{setCondition!(n,t)} returns n whose condition

```

```

    ++ has been replaced by t if it is not
    ++ empty, else an error is produced.
setStatus!: (% ,B) -> %
    ++ \axiom{setStatus!(n,b)} returns n whose status
    ++ has been replaced by b if it is not
    ++ empty, else an error is produced.
setEmpty! : % -> %
    ++ \axiom{setEmpty!(n)} replaces n by \axiom{empty()}$.
infLex? : (% ,%, (V,V) -> B, (C,C) -> B) -> B
    ++ \axiom{infLex?(n1,n2,o1,o2)} returns true iff
    ++ \axiom{o1(value(n1),value(n2))} or
    ++ \axiom{value(n1) = value(n2)} and
    ++ \axiom{o2(condition(n1),condition(n2))}.
subNode? : (% ,%, (C,C) -> B) -> B
    ++ \axiom{subNode?(n1,n2,o2)} returns true iff
    ++ \axiom{value(n1) = value(n2)} and
    ++ \axiom{o2(condition(n1),condition(n2))}

Implementation == add

Rep ==> VTB

rep(n:%):Rep == n pretend Rep
per(r:Rep):% == r pretend %

empty() == per [empty()$V,empty()$C,false]$Rep
empty?(n:%) == empty?((rep n).val)$V and empty?((rep n).tower)$C
value(n:%) == (rep n).val
condition(n:%) == (rep n).tower
status(n:%) == (rep n).flag
construct(v:V,t:C,b:B) == per [v,t,b]$Rep
construct(v:V,t:C) == [v,t,false]$%
construct(vt:VT) == [vt.val,vt.tower]$%
construct(lvt:List VT) == [[vt]$% for vt in lvt]
construct(v:V,lt: List C) == [[v,t]$% for t in lt]
copy(n:%) == per copy rep n
setValue!(n:%,v:V) ==
    (rep n).val := v
    n
setCondition!(n:%,t:C) ==
    (rep n).tower := t
    n
setStatus!(n:%,b:B) ==
    (rep n).flag := b
    n
setEmpty!(n:%) ==
    (rep n).val := empty()$V
    (rep n).tower := empty()$C
    n
infLex?(n1,n2,o1,o2) ==

```

```

    o1((rep n1).val,(rep n2).val) => true
    (rep n1).val = (rep n2).val =>
        o2((rep n1).tower,(rep n2).tower)
    false
subNode?(n1,n2,o2) ==
    (rep n1).val = (rep n2).val =>
        o2((rep n1).tower,(rep n2).tower)
    false
-- sample() == empty()
n1:% = n2:% ==
    (rep n1).val ~= (rep n2).val => false
    (rep n1).tower = (rep n2).tower
n1:% ~= n2:% ==
    (rep n1).val = (rep n2).val => false
    (rep n1).tower ~= (rep n2).tower
coerce(n:%):0 ==
    l1,l2,l3,l : List 0
    l1 := [message("value == "), ((rep n).val)::0]
    o1 : 0 := blankSeparate l1
    l2 := [message(" tower == "), ((rep n).tower)::0]
    o2 : 0 := blankSeparate l2
    if ((rep n).flag)
    then
        o3 := message(" closed == true")
    else
        o3 := message(" closed == false")
    l := [o1,o2,o3]
    bracket commaSeparate l

```

---

— SPLNODE.dotabb —

```

"SPLNODE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SPLNODE"]
"BASTYPE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=BASTYPE"]
"KOERCE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=KOERCE"]
"TYPE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=TYPE"]
"SPLNODE" -> "BASTYPE"
"SPLNODE" -> "KOERCE"
"SPLNODE" -> "TYPE"

```

---

## 20.25 domain SPLTREE SplittingTree

— SplittingTree.input —

```

)set break resume
)sys rm -f SplittingTree.output
)spool SplittingTree.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SplittingTree
--R SplittingTree(V: Join(SetCategory,Aggregate),C: Join(SetCategory,Aggregate)) is a domain
--R Abbreviation for SplittingTree is SPLTREE
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SPLTREE
--R
--R----- Operations -----
--R children : % -> List %
--R construct : (V,C,V,List C) -> %
--R copy : % -> %
--R distance : (%,% ) -> Integer
--R empty? : % -> Boolean
--R leaf? : % -> Boolean
--R sample : () -> %
--R value : % -> SplittingNode(V,C)
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?? : (%,% ) -> Boolean if SplittingNode(V,C) has SETCAT
--R any? : ((SplittingNode(V,C) -> Boolean),%) -> Boolean if $ has finiteAggregate
--R child? : (%,% ) -> Boolean if SplittingNode(V,C) has SETCAT
--R coerce : % -> OutputForm if SplittingNode(V,C) has SETCAT
--R construct : (V,C,List SplittingNode(V,C)) -> %
--R construct : SplittingNode(V,C) -> %
--R count : (SplittingNode(V,C),%) -> NonNegativeInteger if $ has finiteAggregate and Splitt
--R count : ((SplittingNode(V,C) -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R ?.value : (% ,value) -> SplittingNode(V,C)
--R eval : (% ,List SplittingNode(V,C),List SplittingNode(V,C)) -> % if SplittingNode(V,C) has
--R eval : (% ,SplittingNode(V,C),SplittingNode(V,C)) -> % if SplittingNode(V,C) has EVALAB S
--R eval : (% ,Equation SplittingNode(V,C)) -> % if SplittingNode(V,C) has EVALAB SPLNODE(V,C)
--R eval : (% ,List Equation SplittingNode(V,C)) -> % if SplittingNode(V,C) has EVALAB SPLNOD
--R every? : ((SplittingNode(V,C) -> Boolean),%) -> Boolean if $ has finiteAggregate
--R extractSplittingLeaf : % -> Union(%,"failed")
--R hash : % -> SingleInteger if SplittingNode(V,C) has SETCAT
--R latex : % -> String if SplittingNode(V,C) has SETCAT
--R leaves : % -> List SplittingNode(V,C)
--R less? : (% ,NonNegativeInteger) -> Boolean
--R map : ((SplittingNode(V,C) -> SplittingNode(V,C)),%) -> %

```

```

--R map! : ((SplittingNode(V,C) -> SplittingNode(V,C)),%) -> % if $ has shallowlyMutable
--R member? : (SplittingNode(V,C),%) -> Boolean if $ has finiteAggregate and SplittingNode(V,C) has SETCAT
--R members : % -> List SplittingNode(V,C) if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R node? : (%,%) -> Boolean if SplittingNode(V,C) has SETCAT
--R nodeOf? : (SplittingNode(V,C),%) -> Boolean
--R parts : % -> List SplittingNode(V,C) if $ has finiteAggregate
--R remove : (SplittingNode(V,C),%) -> %
--R remove! : (SplittingNode(V,C),%) -> %
--R result : % -> List Record(val: V,tower: C)
--R setchildren! : (%,List %) -> % if $ has shallowlyMutable
--R setelt : (%,value,SplittingNode(V,C)) -> SplittingNode(V,C) if $ has shallowlyMutable
--R setvalue! : (%,SplittingNode(V,C)) -> SplittingNode(V,C) if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R splitNodeOf! : (%,%,List SplittingNode(V,C),((C,C) -> Boolean)) -> %
--R splitNodeOf! : (%,%,List SplittingNode(V,C)) -> %
--R subNodeOf? : (SplittingNode(V,C),%,((C,C) -> Boolean)) -> Boolean
--R ?~=? : (%,%) -> Boolean if SplittingNode(V,C) has SETCAT
--R
--E 1

)spool
)lisp (bye)

```

---

— SplittingTree.help —

```

=====
SplittingTree examples
=====

```

```

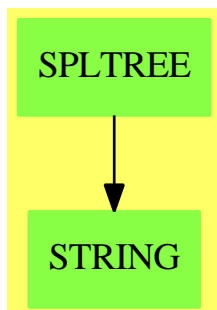
See Also:
o )show SplittingTree

```

---



### 20.25.1 SplittingTree (SPLTREE)



See

⇒ “SplittingTree” (SPLTREE) 20.25.1 on page 2476

#### Exports:

|            |               |              |                      |
|------------|---------------|--------------|----------------------|
| any?       | child?        | children     | coerce               |
| conditions | construct     | copy         | count                |
| cyclic?    | distance      | empty        | empty?               |
| eq?        | eval          | every?       | extractSplittingLeaf |
| hash       | latex         | leaf?        | leaves               |
| less?      | map           | map!         | member?              |
| members    | more?         | node?        | nodeOf?              |
| nodes      | parts         | remove       | remove!              |
| result     | sample        | setchildren! | setelt               |
| setvalue!  | size?         | splitNodeOf! | splitNodeOf!         |
| subNodeOf? | updateStatus! | value        | #?                   |
| ?=?        | ?.value       | ?~=?         |                      |

— domain SPLTREE SplittingTree —

```

)abbrev domain SPLTREE SplittingTree
++ Author: Marc Moereno Maza
++ Date Created: 07/05/1996
++ Date Last Updated: 07/19/1996
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++      M. MORENO MAZA "Calculs de pgcd au-dessus des tours
++      d'extensions simples et resolution des systemes d'equations
++      algebriques" These, Universite P.etM. Curie, Paris, 1997.
++ Description:
++ This domain exports a modest implementation of splitting
++ trees. Splitting trees are needed when the
  
```

```

++ evaluation of some quantity under some hypothesis
++ requires to split the hypothesis into sub-cases.
++ For instance by adding some new hypothesis on one
++ hand and its negation on another hand. The computations
++ are terminated is a splitting tree \axiom{a} when
++ \axiom{status(value(a))} is \axiom{true}. Thus,
++ if for the splitting tree \axiom{a} the flag
++ \axiom{status(value(a))} is \axiom{true}, then
++ \axiom{status(value(d))} is \axiom{true} for any
++ subtree \axiom{d} of \axiom{a}. This property
++ of splitting trees is called the termination
++ condition. If no vertex in a splitting tree \axiom{a}
++ is equal to another, \axiom{a} is said to satisfy
++ the no-duplicates condition. The splitting
++ tree \axiom{a} will satisfy this condition
++ if nodes are added to \axiom{a} by mean of
++ \axiom{splitNodeOf!} and if \axiom{construct}
++ is only used to create the root of \axiom{a}
++ with no children.

```

SplittingTree(V,C) : Exports == Implementation where

```

V:Join(SetCategory,Aggregate)
C:Join(SetCategory,Aggregate)
B ==> Boolean
O ==> OutputForm
NNI ==> NonNegativeInteger
VT ==> Record(val:V, tower:C)
VTB ==> Record(val:V, tower:C, flag:B)
S ==> SplittingNode(V,C)
A ==> Record(root:S,subTrees:List(%))

Exports == RecursiveAggregate(S) with
  shallowlyMutable
  finiteAggregate
  extractSplittingLeaf : % -> Union(%, "failed")
    ++ \axiom{extractSplittingLeaf(a)} returns the left
    ++ most leaf (as a tree) whose status is false
    ++ if any, else "failed" is returned.
  updateStatus! : % -> %
    ++ \axiom{updateStatus!(a)} returns a where the status
    ++ of the vertices are updated to satisfy
    ++ the "termination condition".
  construct : S -> %
    ++ \axiom{construct(s)} creates a splitting tree
    ++ with value (i.e. root vertex) given by
    ++ \axiom{s} and no children. Thus, if the
    ++ status of \axiom{s} is false, \axiom{[s]}
    ++ represents the starting point of the
    ++ evaluation \axiom{value(s)} under the

```

```

    ++ hypothesis \axiom{condition(s)}.
construct : (V,C, List %) -> %
    ++ \axiom{construct(v,t,la)} creates a splitting tree
    ++ with value (i.e. root vertex) given by
    ++ \axiom{[v,t]$S} and with \axiom{la} as
    ++ children list.
construct : (V,C,List S) -> %
    ++ \axiom{construct(v,t,ls)} creates a splitting tree
    ++ with value (i.e. root vertex) given by
    ++ \axiom{[v,t]$S} and with children list given by
    ++ \axiom{[[s]$% for s in ls]}.
construct : (V,C,V,List C) -> %
    ++ \axiom{construct(v1,t,v2,lt)} creates a splitting tree
    ++ with value (i.e. root vertex) given by
    ++ \axiom{[v,t]$S} and with children list given by
    ++ \axiom{[[[v,t]$S]$% for s in ls]}.
conditions : % -> List C
    ++ \axiom{conditions(a)} returns the list of the conditions
    ++ of the leaves of a
result : % -> List VT
    ++ \axiom{result(a)} where \axiom{ls} is the leaves list of \axiom{a}
    ++ returns \axiom{[[value(s),condition(s)]$VT for s in ls]}
    ++ if the computations are terminated in \axiom{a} else
    ++ an error is produced.
nodeOf? : (S,%) -> B
    ++ \axiom{nodeOf?(s,a)} returns true iff some node of \axiom{a}
    ++ is equal to \axiom{s}
subNodeOf? : (S,%,(C,C) -> B) -> B
    ++ \axiom{subNodeOf?(s,a,sub?) } returns true iff for some node
    ++ \axiom{n} in \axiom{a} we have \axiom{s = n} or
    ++ \axiom{status(n)} and \axiom{subNode?(s,n,sub?)}.
remove : (S,%) -> %
    ++ \axiom{remove(s,a)} returns the splitting tree obtained
    ++ from a by removing every sub-tree \axiom{b} such
    ++ that \axiom{value(b)} and \axiom{s} have the same
    ++ value, condition and status.
remove! : (S,%) -> %
    ++ \axiom{remove!(s,a)} replaces a by remove(s,a)
splitNodeOf! : (%,%,List(S)) -> %
    ++ \axiom{splitNodeOf!(l,a,ls)} returns \axiom{a} where the children
    ++ list of \axiom{l} has been set to
    ++ \axiom{[[s]$% for s in ls | not nodeOf?(s,a)]}.
    ++ Thus, if \axiom{l} is not a node of \axiom{a}, this
    ++ latter splitting tree is unchanged.
splitNodeOf! : (%,%,List(S),(C,C) -> B) -> %
    ++ \axiom{splitNodeOf!(l,a,ls,sub?) } returns \axiom{a} where the children
    ++ list of \axiom{l} has been set to
    ++ \axiom{[[s]$% for s in ls | not subNodeOf?(s,a,sub?)]].
    ++ Thus, if \axiom{l} is not a node of \axiom{a}, this
    ++ latter splitting tree is unchanged.

```

```

Implementation == add

Rep ==> A

rep(n:%):Rep == n pretend Rep
per(r:Rep):% == r pretend %

construct(s:S) ==
  per [s,[]]$A
construct(v:V,t:C,la:List(%)) ==
  per [[v,t]$S,la]$A
construct(v:V,t:C,ls:List(S)) ==
  per [[v,t]$S,[[s]$% for s in ls]]$A
construct(v1:V,t:C,v2:V,lt:List(C)) ==
  [v1,t,([v2,lt]$S)@(List S)]$%

empty?(a:%) == empty?((rep a).root) and empty?((rep a).subTrees)
empty() == [empty()$S]$%

remove(s:S,a:%) ==
  empty? a => a
  (s = value(a)) and (status(s) = status(value(a))) => empty()$%
  la := children(a)
  lb : List % := []
  while (not empty? la) repeat
    lb := cons(remove(s,first la), lb)
    la := rest la
  lb := reverse remove(empty?,lb)
  [value(value(a)),condition(value(a)),lb]$%

remove!(s:S,a:%) ==
  empty? a => a
  (s = value(a)) and (status(s) = status(value(a))) =>
    (rep a).root := empty()$S
    (rep a).subTrees := []
    a
  la := children(a)
  lb : List % := []
  while (not empty? la) repeat
    lb := cons(remove!(s,first la), lb)
    la := rest la
  lb := reverse remove(empty()$%,lb)
  setchildren!(a,lb)

value(a:%) ==
  (rep a).root
children(a:%) ==
  (rep a).subTrees

```

```

leaf?(a:%) ==
  empty? a => false
  empty? (rep a).subTrees
setchildren!(a:%,la:List(%)) ==
  (rep a).subTrees := la
  a
setvalue!(a:%,s:S) ==
  (rep a).root := s
  s
cyclic?(a:%) == false
map(foo:(S -> S),a:%) ==
  empty? a => a
  b : % := [foo(value(a))]%
  leaf? a => b
  setchildren!(b,[map(foo,c) for c in children(a)])
map!(foo:(S -> S),a:%) ==
  empty? a => a
  setvalue!(a,foo(value(a)))
  leaf? a => a
  setchildren!(a,[map!(foo,c) for c in children(a)])
copy(a:%) ==
  map(copy,a)
eq?(a1:%,a2:%) ==
  error"in eq? from SPLTREE : la vache qui rit est-elle folle?"
nodes(a:%) ==
  empty? a => []
  leaf? a => [a]
  cons(a,concat([nodes(c) for c in children(a)]))
leaves(a:%) ==
  empty? a => []
  leaf? a => [value(a)]
  concat([leaves(c) for c in children(a)])
members(a:%) ==
  empty? a => []
  leaf? a => [value(a)]
  cons(value(a),concat([members(c) for c in children(a)]))
#(a:%) ==
  empty? a => 0$NNI
  leaf? a => 1$NNI
  reduce("+",[#c for c in children(a)],1$NNI)$(List NNI)
a1:% = a2:% ==
  empty? a1 => empty? a2
  empty? a2 => false
  leaf? a1 =>
    not leaf? a2 => false
    value(a1) == value(a2)
  leaf? a2 => false
  value(a1) ~= value(a2) => false
  children(a1) = children(a2)
-- sample() == [sample()$S]%

```

```

localCoerce(a:%,k:NNI):O ==
  s : String
  if k = 1 then s := "*" else s := "-> "
  for i in 2..k repeat s := concat("-",s)$String
  ro : O := left(hconcat(message(s)$O,value(a)::O)$O)$O
  leaf? a => ro
  lo : List O := [localCoerce(c,k+1) for c in children(a)]
  lo := cons(ro,lo)
  vconcat(lo)$O
coerce(a:%):O ==
  empty? a => vconcat(message(" ")$O,message("* []")$O)
  vconcat(message(" ")$O,localCoerce(a,1))

extractSplittingLeaf(a:%) ==
  empty? a => "failed"::Union(%, "failed")
  status(value(a))$S => "failed"::Union(%, "failed")
  la := children(a)
  empty? la => a
  while (not empty? la) repeat
    esl := extractSplittingLeaf(first la)
    (esl case %) => return(esl)
    la := rest la
  "failed"::Union(%, "failed")

updateStatus!(a:%) ==
  la := children(a)
  (empty? la) or (status(value(a))$S) => a
  done := true
  while (not empty? la) and done repeat
    done := done and status(value(updateStatus! first la))
    la := rest la
  setStatus!(value(a),done)$S
  a

result(a:%) ==
  empty? a => []
  not status(value(a))$S =>
    error"in result from SLPTREE : mad cow!"
  ls : List S := leaves(a)
  [[value(s),condition(s)]$VT for s in ls]

conditions(a:%) ==
  empty? a => []
  ls : List S := leaves(a)
  [condition(s) for s in ls]

nodeOf?(s:S,a:%) ==
  empty? a => false
  s = $S value(a) => true
  la := children(a)

```

```

while (not empty? la) and (not nodeOf?(s,first la)) repeat
  la := rest la
not empty? la

subNodeOf?(s:S,a:%,sub?:((C,C) -> B)) ==
  empty? a => false
  -- s = $$ value(a) => true
  status(value(a)$%)$S and subNode?(s,value(a),sub?)$S => true
  la := children(a)
  while (not empty? la) and (not subNodeOf?(s,first la,sub?)) repeat
    la := rest la
  not empty? la

splitNodeOf!(l:%,a:%,ls:List(S)) ==
  ln := removeDuplicates ls
  la : List % := []
  while not empty? ln repeat
    if not nodeOf?(first ln,a)
      then
        la := cons([first ln]$%, la)
        ln := rest ln
  la := reverse la
  setchildren!(l,la)$%
  if empty? la then (rep l).root := [empty()$V,empty()$C,true]$S
  updateStatus!(a)

splitNodeOf!(l:%,a:%,ls:List(S),sub?:((C,C) -> B)) ==
  ln := removeDuplicates ls
  la : List % := []
  while not empty? ln repeat
    if not subNodeOf?(first ln,a,sub?)
      then
        la := cons([first ln]$%, la)
        ln := rest ln
  la := reverse la
  setchildren!(l,la)$%
  if empty? la then (rep l).root := [empty()$V,empty()$C,true]$S
  updateStatus!(a)

```

---

— SPLTREE.dotabb —

```

"SPLTREE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SPLTREE"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"SPLTREE" -> "STRING"

```

---

— SquareFreeRegularTriangularSet.input —

```

)set break resume
)sys rm -f SquareFreeRegularTriangularSet.output
)spool SquareFreeRegularTriangularSet.output
)set message test on
)set message auto off
)clear all
--S 1 of 23
R := Integer
--R
--R
--R (1) Integer
--R
--R                                         Type: Domain
--E 1

--S 2 of 23
ls : List Symbol := [x,y,z,t]
--R
--R
--R (2) [x,y,z,t]
--R
--R                                         Type: List Symbol
--E 2

--S 3 of 23
V := OVAR(ls)
--R
--R
--R (3) OrderedVariableList [x,y,z,t]
--R
--R                                         Type: Domain
--E 3

--S 4 of 23
E := IndexedExponents V
--R
--R
--R (4) IndexedExponents OrderedVariableList [x,y,z,t]
--R
--R                                         Type: Domain
--E 4

--S 5 of 23
P := NSMP(R, V)
--R
--R
--R (5) NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--R
--R                                         Type: Domain

```



--E 5

--S 6 of 23

x: P := 'x

--R

--R

--R (6) x

--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])

--E 6

--S 7 of 23

y: P := 'y

--R

--R

--R (7) y

--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])

--E 7

--S 8 of 23

z: P := 'z

--R

--R

--R (8) z

--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])

--E 8

--S 9 of 23

t: P := 't

--R

--R

--R (9) t

--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])

--E 9

--S 10 of 23

ST := SREGSET(R,E,V,P)

--R

--R

--R (10)

--R SquareFreeRegularTriangularSet(Integer,IndexedExponents OrderedVariableList [

--R x,y,z,t],OrderedVariableList [x,y,z,t],NewSparseMultivariatePolynomial(Intege

--R r,OrderedVariableList [x,y,z,t]))

--R

Type: Domain

--E 10

--S 11 of 23

p1 := x \*\* 31 - x \*\* 6 - x - y

--R

--R

--R 31 6

```
--R (11)  $x^2 - x - x - y$ 
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 11
```

```
--S 12 of 23
p2 := x ** 8 - z
--R
--R
--R      8
--R (12)  $x^8 - z$ 
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 12
```

```
--S 13 of 23
p3 := x ** 10 - t
--R
--R
--R      10
--R (13)  $x^{10} - t$ 
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 13
```

```
--S 14 of 23
lp := [p1, p2, p3]
--R
--R
--R      31      6      8      10
--R (14)  $[x^{31} - x^6 - x^8 - y, x^8 - z, x^{10} - t]$ 
--RType: List NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 14
```

```
--S 15 of 23
zeroSetSplit(lp)$ST
--R
--R
--R      5      4      2      3      8      5      3      2      4      2
--R (15)  $[{z^5 - t^4, t^2 z y^2 + 2z y^2 - t^3 + 2t^5 + t^3 - t^2, (t^8 - t^5)x - t^2 y - z^2}]$ 
--RType: List SquareFreeRegularTriangularSet(Integer,IndexedExponents OrderedVariableList [x,y,z,t],Order)
--E 15
```

```
--S 16 of 23
zeroSetSplit(lp,false)$ST
--R
--R
--R      5      4      2      3      8      5      3      2      4      2
--R (16)  $[{z^5 - t^4, t^2 z y^2 + 2z y^2 - t^3 + 2t^5 + t^3 - t^2, (t^8 - t^5)x - t^2 y - z^2},$ 
--R      3      5      2      2
--R       ${t^3 - 1, z^5 - t^2, t^2 y + z^2, z^2 x^2 - t^2}, {t,z,y,x}]$ 
--RType: List SquareFreeRegularTriangularSet(Integer,IndexedExponents OrderedVariableList [x,y,z,t],Order)
```

--E 16

--S 17 of 23

T := REGSET(R,E,V,P)

--R

--R

--R (17)

--R RegularTriangularSet(Integer,IndexedExponents OrderedVariableList [x,y,z,t],0

--R rderedVariableList [x,y,z,t],NewSparseMultivariatePolynomial(Integer,OrderedV

--R ariableList [x,y,z,t]))

--R

Type: Domain

--E 17

--S 18 of 23

lts := zeroSetSplit(lp,false)\$T

--R

--R

--R (18)

--R 
$$\{z^5 - t^4, t^2 z y + 2z^2 y - t^3 + 2t^5 + t^3 - t^2, (t^2 - t)x - t^2 y - z^2\},$$

--R 
$$\{t^3 - 1, z^5 - t^2, t^2 z y + 2z^2 y + 1, z^3 x - t^2\}, \{t, z, y, x\}$$

--RType: List RegularTriangularSet(Integer,IndexedExponents OrderedVariableList [x,y,z,t],OrderedV

--E 18

--S 19 of 23

ts := lts.2

--R

--R

--R 
$$\{t^3 - 1, z^5 - t^2, t^2 z y + 2z^2 y + 1, z^3 x - t^2\}$$

--RType: RegularTriangularSet(Integer,IndexedExponents OrderedVariableList [x,y,z,t],OrderedV

--E 19

--S 20 of 23

pol := select(ts,'y')\$T

--R

--R

--R 
$$t^2 z y + 2z^2 y + 1$$

--RType: Union(NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t]),...)

--E 20

--S 21 of 23

tower := collectUnder(ts,'y')\$T

--R

--R

--R 
$$\{t^3 - 1, z^5 - t^2\}$$

--RType: RegularTriangularSet(Integer,IndexedExponents OrderedVariableList [x,y,z,t],OrderedV

```

--E 21

--S 22 of 23
pack := RegularTriangularSetGcdPackage(R,E,V,P,T)
--R
--R
--R (22)
--R RegularTriangularSetGcdPackage(Integer,IndexedExponents OrderedVariableList [
--R x,y,z,t],OrderedVariableList [x,y,z,t],NewSparseMultivariatePolynomial(Integer
--R r,OrderedVariableList [x,y,z,t]),RegularTriangularSet(Integer,IndexedExponent
--R s OrderedVariableList [x,y,z,t],OrderedVariableList [x,y,z,t],NewSparseMultiv
--R ariatePolynomial(Integer,OrderedVariableList [x,y,z,t]))
--R
--R                                         Type: Domain
--E 22

--S 23 of 23
toseSquareFreePart(pol,tower)$pack
--R
--R
--R
--R (23) [[val= t2 y + z3, tower= {t3 - 1, z5 - t}]]
--RType: List Record(val: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t]),tower:
--E 23
)spool
)lisp (bye)

```

---

— SquareFreeRegularTriangularSet.help —

```

=====
SquareFreeRegularTriangularSet examples
=====

```

The SquareFreeRegularTriangularSet domain constructor implements square-free regular triangular sets. See the RegularTriangularSet domain constructor for general regular triangular sets. Let  $T$  be a regular triangular set consisting of polynomials  $t_1, \dots, t_m$  ordered by increasing main variables. The regular triangular set  $T$  is square-free if  $T$  is empty or if  $t_1, \dots, t_{m-1}$  is square-free and if the polynomial  $t_m$  is square-free as a univariate polynomial with coefficients in the tower of simple extensions associated with  $t_1, \dots, t_{m-1}$ .

The main interest of square-free regular triangular sets is that their associated towers of simple extensions are product of fields. Consequently, the saturated ideal of a square-free regular triangular set is radical. This property simplifies some of the operations related to regular triangular sets. However, building square-free regular triangular sets is generally more expensive than building

general regular triangular sets.

As the `RegularTriangularSet` domain constructor, the `SquareFreeRegularTriangularSet` domain constructor also implements a method for solving polynomial systems by means of regular triangular sets. This is in fact the same method with some adaptations to take into account the fact that the computed regular chains are square-free. Note that it is also possible to pass from a decomposition into general regular triangular sets to a decomposition into square-free regular triangular sets. This conversion is used internally by the `LazardSetSolvingPackage` package constructor.

N.B. When solving polynomial systems with the `SquareFreeRegularTriangularSet` domain constructor or the `LazardSetSolvingPackage` package constructor, decompositions have no redundant components. See also `LexTriangularPackage` and `ZeroDimensionalSolvePackage` for the case of algebraic systems with a finite number of (complex) solutions.

We shall explain now how to use the constructor `SquareFreeRegularTriangularSet`.

This constructor takes four arguments. The first one, `R`, is the coefficient ring of the polynomials; it must belong to the category `GcdDomain`. The second one, `E`, is the exponent monoid of the polynomials; it must belong to the category `OrderedAbelianMonoidSup`. The third one, `V`, is the ordered set of variables; it must belong to the category `OrderedSet`. The last one is the polynomial ring; it must belong to the category `RecursivePolynomialCategory(R,E,V)`. The abbreviation for `SquareFreeRegularTriangularSet` is `SREGSET`.

Note that the way of understanding triangular decompositions is detailed in the example of the `RegularTriangularSet` constructor.

Let us illustrate the use of this constructor with one example (Donati-Traverso). Define the coefficient ring.

```
R := Integer
Integer
Type: Domain
```

Define the list of variables,

```
ls : List Symbol := [x,y,z,t]
[x,y,z,t]
Type: List Symbol
```

and make it an ordered set;

```
V := OVAR(ls)
OrderedVariableList [x,y,z,t]
```

Type: Domain

then define the exponent monoid.

```
E := IndexedExponents V
   IndexedExponents OrderedVariableList [x,y,z,t]
                                     Type: Domain
```

Define the polynomial ring.

```
P := NSMP(R, V)
   NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
                                     Type: Domain
```

Let the variables be polynomial.

```
x: P := 'x
   x
                                     Type: NewSparseMultivariatePolynomial(Integer,
                                     OrderedVariableList [x,y,z,t])
```

```
y: P := 'y
   y
                                     Type: NewSparseMultivariatePolynomial(Integer,
                                     OrderedVariableList [x,y,z,t])
```

```
z: P := 'z
   z
                                     Type: NewSparseMultivariatePolynomial(Integer,
                                     OrderedVariableList [x,y,z,t])
```

```
t: P := 't
   t
                                     Type: NewSparseMultivariatePolynomial(Integer,
                                     OrderedVariableList [x,y,z,t])
```

Now call the SquareFreeRegularTriangularSet domain constructor.

```
ST := SREGSET(R,E,V,P)
SquareFreeRegularTriangularSet(Integer,IndexedExponents OrderedVariableList [
x,y,z,t],OrderedVariableList [x,y,z,t],NewSparseMultivariatePolynomial(Intege
r,OrderedVariableList [x,y,z,t]))
                                     Type: Domain
```

Define a polynomial system.

```
p1 := x ** 31 - x ** 6 - x - y
      31      6
      x  - x  - x - y
                                     Type: NewSparseMultivariatePolynomial(Integer,
```

```

OrderedVariableList [x,y,z,t])

p2 := x ** 8 - z
      8
      x - z
      Type: NewSparseMultivariatePolynomial(Integer,
      OrderedVariableList [x,y,z,t])

p3 := x ** 10 - t
      10
      x - t
      Type: NewSparseMultivariatePolynomial(Integer,
      OrderedVariableList [x,y,z,t])

lp := [p1, p2, p3]
      31      6      8      10
      [x - x - x - y, x - z, x - t]
      Type: List NewSparseMultivariatePolynomial(Integer,
      OrderedVariableList [x,y,z,t])

```

First of all, let us solve this system in the sense of Kalkbrener.

```

zeroSetSplit(lp)$ST
      5      4      2      3      8      5      3      2      4      2
      [{z - t , t z y + 2z y - t + 2t + t - t , (t - t)x - t y - z }]
      Type: List SquareFreeRegularTriangularSet(Integer,
      IndexedExponents OrderedVariableList [x,y,z,t],
      OrderedVariableList [x,y,z,t],
      NewSparseMultivariatePolynomial(Integer,
      OrderedVariableList [x,y,z,t]))

```

And now in the sense of Lazard (or Wu and other authors).

```

zeroSetSplit(lp,false)$ST
      5      4      2      3      8      5      3      2      4      2
      [{z - t , t z y + 2z y - t + 2t + t - t , (t - t)x - t y - z },
      3      5      2      2
      {t - 1, z - t, t y + z , z x - t}, {t,z,y,x}]
      Type: List SquareFreeRegularTriangularSet(Integer,
      IndexedExponents OrderedVariableList [x,y,z,t],
      OrderedVariableList [x,y,z,t],
      NewSparseMultivariatePolynomial(Integer,
      OrderedVariableList [x,y,z,t]))

```

Now to see the difference with the RegularTriangularSet domain constructor, we define:

```

T := REGSET(R,E,V,P)
RegularTriangularSet(Integer,IndexedExponents OrderedVariableList [x,y,z,t],O
rderedVariableList [x,y,z,t],NewSparseMultivariatePolynomial(Integer,OrderedV

```

```
variableList [x,y,z,t]))
```

```
Type: Domain
```

and compute:

```
lts := zeroSetSplit(lp,false)$T
      5      4      2      3      8      5      3      2      4      2
      [z  - t ,t z y  + 2z y - t  + 2t  + t  - t ,(t  - t)x - t y - z ],
      3      5      2      3      2
      {t  - 1,z  - t,t z y  + 2z y + 1,z x  - t}, {t,z,y,x}]
      Type: List RegularTriangularSet(Integer,
      IndexedExponents OrderedVariableList [x,y,z,t],
      OrderedVariableList [x,y,z,t],
      NewSparseMultivariatePolynomial(Integer,
      OrderedVariableList [x,y,z,t]))
```

If you look at the second set in both decompositions in the sense of Lazard, you will see that the polynomial with main variable  $y$  is not the same.

Let us understand what has happened.

We define:

```
ts := lts.2
      3      5      2      3      2
(19) {t  - 1,z  - t,t z y  + 2z y + 1,z x  - t}
      Type: RegularTriangularSet(Integer,
      IndexedExponents OrderedVariableList [x,y,z,t],
      OrderedVariableList [x,y,z,t],
      NewSparseMultivariatePolynomial(Integer,
      OrderedVariableList [x,y,z,t]))

pol := select(ts,'y)$T
      2      3
      t z y  + 2z y + 1
      Type: Union(NewSparseMultivariatePolynomial(Integer,
      OrderedVariableList [x,y,z,t]),...)

tower := collectUnder(ts,'y)$T
      3      5
      {t  - 1,z  - t}
      Type: RegularTriangularSet(Integer,
      IndexedExponents OrderedVariableList [x,y,z,t],
      OrderedVariableList [x,y,z,t],
      NewSparseMultivariatePolynomial(Integer,
      OrderedVariableList [x,y,z,t]))

pack := RegularTriangularSetGcdPackage(R,E,V,P,T)
RegularTriangularSetGcdPackage(Integer,IndexedExponents OrderedVariableList [
x,y,z,t],OrderedVariableList [x,y,z,t],NewSparseMultivariatePolynomial(Intege
```



```

r,OrderedVariableList [x,y,z,t]),RegularTriangularSet(Integer,IndexedExponent
s OrderedVariableList [x,y,z,t],OrderedVariableList [x,y,z,t],NewSparseMultiv
ariatePolynomial(Integer,OrderedVariableList [x,y,z,t]))
Type: Domain

```

Then we compute:

```

toseSquareFreePart(pol,tower)$pack
      2      3      5
[[val= t y + z ,tower= {t - 1,z - t}]]
Type: List Record(val: NewSparseMultivariatePolynomial(Integer,
OrderedVariableList [x,y,z,t]),
tower: RegularTriangularSet(Integer,
IndexedExponents OrderedVariableList [x,y,z,t],
OrderedVariableList [x,y,z,t],
NewSparseMultivariatePolynomial(Integer,
OrderedVariableList [x,y,z,t]))

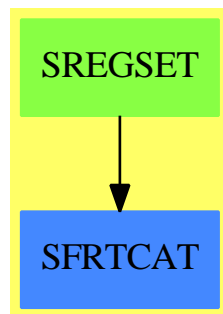
```

See Also:

- o )help GcdDomain
- o )help OrderedAbelianMonoidSup
- o )help OrderedSet
- o )help RecursivePolynomialCategory
- o )help ZeroDimensionalSolvePackage
- o )help LexTriangularPackage
- o )help LazardSetSolvingPackage
- o )help RegularTriangularSet
- o )show SquareFreeRegularTriangularSet

—

### 20.26.1 SquareFreeRegularTriangularSet (SREGSET)



**Exports:**

|                                 |                                   |
|---------------------------------|-----------------------------------|
| algebraic?                      | algebraicCoefficients?            |
| algebraicVariables              | any?                              |
| augment                         | autoReduced?                      |
| basicSet                        | coerce                            |
| coHeight                        | collect                           |
| collectQuasiMonic               | collectUnder                      |
| collectUpper                    | convert                           |
| construct                       | copy                              |
| count                           | degree                            |
| empty                           | empty?                            |
| eq?                             | eval                              |
| every?                          | extend                            |
| extendIfCan                     | find                              |
| first                           | hash                              |
| headReduce                      | headReduced?                      |
| headRemainder                   | infRittWu?                        |
| initiallyReduce                 | initiallyReduced?                 |
| initials                        | internalAugment                   |
| internalZeroSetSplit            | intersect                         |
| invertible?                     | invertibleSet                     |
| invertibleElseSplit?            | last                              |
| lastSubResultant                | lastSubResultantElseSplit         |
| less?                           | latex                             |
| mainVariable?                   | mainVariables                     |
| map                             | map!                              |
| member?                         | members                           |
| more?                           | mvar                              |
| normalized?                     | parts                             |
| preprocess                      | purelyAlgebraic?                  |
| purelyAlgebraicLeadingMonomial? | purelyTranscendental?             |
| quasiComponent                  | reduce                            |
| reduceByQuasiMonic              | reduced?                          |
| remainder                       | remove                            |
| removeDuplicates                | removeZero                        |
| rest                            | retract                           |
| retractIfCan                    | rewriteIdealWithHeadRemainder     |
| rewriteIdealWithRemainder       | rewriteSetWithReduction           |
| roughBase?                      | roughEqualIdeals?                 |
| roughSubIdeal?                  | roughUnitIdeal?                   |
| sample                          | select                            |
| size?                           | sort                              |
| squareFreePart                  | stronglyReduce                    |
| stronglyReduced?                | triangular?                       |
| trivialIdeal?                   | variables                         |
| zeroSetSplit                    | zeroSetSplitIntoTriangularSystems |
| #?                              | ?=?                               |
| ?~=?                            |                                   |

## — domain SREGSET SquareFreeRegularTriangularSet —

```

)abbrev domain SREGSET SquareFreeRegularTriangularSet
++ Author: Marc Moreno Maza
++ Date Created: 08/25/1998
++ Date Last Updated: 16/12/1998
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References :
++ [1] M. MORENO MAZA "A new algorithm for computing triangular
++      decomposition of algebraic varieties" NAG Tech. Rep. 4/98.
++ Description:
++ This domain provides an implementation of square-free regular chains.
++ Moreover, the operation zeroSetSplit
++ is an implementation of a new algorithm for solving polynomial systems by
++ means of regular chains.

SquareFreeRegularTriangularSet(R,E,V,P) : Exports == Implementation where

  R : GcdDomain
  E : OrderedAbelianMonoidSup
  V : OrderedSet
  P : RecursivePolynomialCategory(R,E,V)
  N ==> NonNegativeInteger
  Z ==> Integer
  B ==> Boolean
  LP ==> List P
  PtoP ==> P -> P
  PS ==> GeneralPolynomialSet(R,E,V,P)
  PWT ==> Record(val : P, tower : $)
  BWT ==> Record(val : Boolean, tower : $)
  LpWT ==> Record(val : (List P), tower : $)
  Split ==> List $
  iprintpack ==> InternalPrintPackage()
  polsetpack ==> PolynomialSetUtilitiesPackage(R,E,V,P)
  quasicomppack ==> SquareFreeQuasiComponentPackage(R,E,V,P,$)
  regsetgcdpack ==> SquareFreeRegularTriangularSetGcdPackage(R,E,V,P,$)
  regsetdecomppack ==> SquareFreeRegularSetDecompositionPackage(R,E,V,P,$)

Exports == SquareFreeRegularTriangularSetCategory(R,E,V,P) with

  internalAugment: (P,$,B,B,B,B,B) -> List $
  ++ \axiom{internalAugment(p,ts,b1,b2,b3,b4,b5)}
  ++ is an internal subroutine, exported only for developement.
  zeroSetSplit: (LP, B, B) -> Split
  ++ \axiom{zeroSetSplit(lp,clos?,info?)} has the same specifications as

```

```

++ zeroSetSplit from RegularTriangularSetCategory
++ from \spadtype{RegularTriangularSetCategory}
++ Moreover, if clos? then solves in the sense of the Zariski closure
++ else solves in the sense of the regular zeros. If \axiom{info?} then
++ do print messages during the computations.
zeroSetSplit: (LP, B, B, B, B) -> Split
++ \axiom{zeroSetSplit(lp,b1,b2.b3,b4)}
++ is an internal subroutine, exported only for developement.
internalZeroSetSplit: (LP, B, B, B) -> Split
++ \axiom{internalZeroSetSplit(lp,b1,b2,b3)}
++ is an internal subroutine, exported only for developement.
pre_process: (LP, B, B) -> Record(val: LP, towers: Split)
++ \axiom{pre_process(lp,b1,b2)}
++ is an internal subroutine, exported only for developement.

```

Implementation == add

```

Rep ==> LP

rep(s:$):Rep == s pretend Rep
per(l:Rep):$ == l pretend $

copy ts ==
  per(copy(rep(ts))$LP)
empty() ==
  per([])
empty?(ts:$) ==
  empty?(rep(ts))
parts ts ==
  rep(ts)
members ts ==
  rep(ts)
map (f : PtoP, ts : $) : $ ==
  construct(map(f,rep(ts))$LP)$
map! (f : PtoP, ts : $) : $ ==
  construct(map!(f,rep(ts))$LP)$
member? (p,ts) ==
  member?(p,rep(ts))$LP
unitIdealIfCan() ==
  "failed"::Union($,"failed")
roughUnitIdeal? ts ==
  false
coerce(ts:$) : OutputForm ==
  lp : List(P) := reverse(rep(ts))
  brace([p::OutputForm for p in lp]$List(OutputForm))$OutputForm
mvar ts ==
  empty? ts => error "mvar$SREGSET: #1 is empty"
  mvar(first(rep(ts)))$P
first ts ==
  empty? ts => "failed"::Union(P,"failed")

```

```

    first(rep(ts))::Union(P,"failed")
last ts ==
    empty? ts => "failed"::Union(P,"failed")
    last(rep(ts))::Union(P,"failed")
rest ts ==
    empty? ts => "failed"::Union($,"failed")
    per(rest(rep(ts))::Union($,"failed")
coerce(ts:$) : (List P) ==
    rep(ts)

collectUpper (ts,v) ==
    empty? ts => ts
    lp := rep(ts)
    newlp : Rep := []
    while (not empty? lp) and (mvar(first(lp)) > v) repeat
        newlp := cons(first(lp),newlp)
        lp := rest lp
    per(reverse(newlp))

collectUnder (ts,v) ==
    empty? ts => ts
    lp := rep(ts)
    while (not empty? lp) and (mvar(first(lp)) >= v) repeat
        lp := rest lp
    per(lp)

construct(lp:List(P)) ==
    ts : $ := per([])
    empty? lp => ts
    lp := sort(infRittWu?,lp)
    while not empty? lp repeat
        eif := extendIfCan(ts,first(lp))
        not (eif case $) =>
            error"in construct : List P -> $ from SREGSET : bad #1"
        ts := eif::$
        lp := rest lp
    ts

extendIfCan(ts:$,p:P) ==
    ground? p => "failed"::Union($,"failed")
    empty? ts =>
        p := squareFreePart primitivePart p
        (per([p]))::Union($,"failed")
    not (mvar(ts) < mvar(p)) => "failed"::Union($,"failed")
    invertible?(init(p),ts)@Boolean =>
        lts: Split := augment(p,ts)
        #lts ~= 1 => "failed"::Union($,"failed")
        (first lts)::Union($,"failed")
    "failed"::Union($,"failed")

```

```

removeZero(p:P, ts:$): P ==
  (ground? p) or (empty? ts) => p
  v := mvar(p)
  ts_v_- := collectUnder(ts,v)
  if algebraic?(v,ts)
  then
    q := lazyPrem(p,select(ts,v)::P)
    zero? q => return q
    zero? removeZero(q,ts_v_-) => return 0
  empty? ts_v_- => p
  q: P := 0
  while positive? degree(p,v) repeat
    q := removeZero(init(p),ts_v_-) * mainMonomial(p) + q
    p := tail(p)
  q + removeZero(p,ts_v_-)

internalAugment(p:P,ts:$): $ ==
  -- ASSUME that adding p to ts DOES NOT require any split
  ground? p => error "in internalAugment$$SREGSET: ground? #1"
  first(internalAugment(p,ts,false,false,false,false,false))

internalAugment(lp:List(P),ts:$): $ ==
  -- ASSUME that adding p to ts DOES NOT require any split
  empty? lp => ts
  internalAugment(rest lp, internalAugment(first lp, ts))

internalAugment(p:P,ts:$,rem?:B,red?:B,prim?:B,sqfr?:B,extend?:B): Split ==
  -- ASSUME p is not a constant
  -- ASSUME mvar(p) is not algebraic w.r.t. ts
  -- ASSUME init(p) invertible modulo ts
  -- if rem? then REDUCE p by remainder
  -- if prim? then REPLACE p by its main primitive part
  -- if sqfr? then FACTORIZE SQUARE FREE p over R
  -- if extend? DO NOT ASSUME every pol in ts_v_+ is invertible modulo ts
  v := mvar(p)
  ts_v_- := collectUnder(ts,v)
  ts_v_+ := collectUpper(ts,v)
  if rem? then p := remainder(p,ts_v_-).polnum
  -- if rem? then p := reduceByQuasiMonic(p,ts_v_-)
  if red? then p := removeZero(p,ts_v_-)
  if prim? then p := mainPrimitivePart p
  lts: Split
  if sqfr?
  then
    lts: Split := []
    lsfp := squareFreeFactors(p)$polsetpack
    for f in lsfp repeat
      (ground? f) or (mvar(f) < v) => "leave"
      lpwt := squareFreePart(f,ts_v_-)
      for pwt in lpwt repeat

```

```

      sfp := pwt.val; us := pwt.tower
      lts := cons( per(cons(pwt.val, rep(pwt.tower))), lts)
    else
      lts: Split := [per(cons(p,rep(ts_v-)))]
    extend? => extend(members(ts_v+),lts)
    [per(concat(rep(ts_v+),rep(us))) for us in lts]

augment(p:P,ts:$): List $ ==
  ground? p => error "in augment$SREGSET: ground? #1"
  algebraic?(mvar(p),ts) => error "in augment$SREGSET: bad #1"
  -- ASSUME init(p) invertible modulo ts
  -- DOES NOT ASSUME anything else.
  -- THUS reduction, mainPrimitivePart and squareFree are NEEDED
  internalAugment(p,ts,true,true,true,true)

extend(p:P,ts:$): List $ ==
  ground? p => error "in extend$SREGSET: ground? #1"
  v := mvar(p)
  not (mvar(ts) < mvar(p)) => error "in extend$SREGSET: bad #1"
  split: List($):= invertibleSet(init(p),ts)
  lts: List($):= []
  for us in split repeat
    lts := concat(augment(p,us),lts)
  lts

invertible?(p:P,ts:$): Boolean ==
  stoseInvertible?(p,ts)$regsetgcdpack

invertible?(p:P,ts:$): List BWT ==
  stoseInvertible?_sqfreg(p,ts)$regsetgcdpack

invertibleSet(p:P,ts:$): Split ==
  stoseInvertibleSet_sqfreg(p,ts)$regsetgcdpack

lastSubResultant(p1:P,p2:P,ts:$): List PWT ==
  stoseLastSubResultant(p1,p2,ts)$regsetgcdpack

squareFreePart(p:P, ts: $): List PWT ==
  stoseSquareFreePart(p,ts)$regsetgcdpack

intersect(p:P, ts: $): List($):= decompose([p], [ts], false, false)$regsetdecomppack

intersect(lp: LP, lts: List($)): List($):= decompose(lp, lts, false, false)$regsetdecomppack
  -- SOLVE in the regular zero sense
  -- and DO NOT PRINT info

decompose(p:P, ts: $): List($):= decompose([p], [ts], true, false)$regsetdecomppack

decompose(lp: LP, lts: List($)): List($):= decompose(lp, lts, true, false)$regsetdecomppack
  -- SOLVE in the closure sense

```

```

-- and DO NOT PRINT info

zeroSetSplit(lp:List(P)) == zeroSetSplit(lp,true,false)
  -- by default SOLVE in the closure sense
  -- and DO NOT PRINT info

zeroSetSplit(lp:List(P), clos?: B) == zeroSetSplit(lp,clos?, false)

zeroSetSplit(lp:List(P), clos?: B, info?: B) ==
  -- if clos? then SOLVE in the closure sense
  -- if info? then PRINT info
  -- by default USE hash-tables
  -- and PREPROCESS the input system
  zeroSetSplit(lp,true,clos?,info?,true)

zeroSetSplit(lp:List(P),hash?:B,clos?:B,info?:B,prep?:B) ==
  -- if hash? then USE hash-tables
  -- if info? then PRINT information
  -- if clos? then SOLVE in the closure sense
  -- if prep? then PREPROCESS the input system
  if hash?
  then
    s1, s2, s3, dom1, dom2, dom3: String
    e: String := empty()$String
    if info? then (s1,s2,s3) := ("w","g","i") else (s1,s2,s3) := (e,e,e)
    if info?
    then
      (dom1, dom2, dom3) := ("QCMPPACK", "REGSETGCD: Gcd", "REGSETGCD: Inv Set")
    else
      (dom1, dom2, dom3) := (e,e,e)
    startTable!(s1,"W",dom1)$quasicomppack
    startTableGcd!(s2,"G",dom2)$regsetgcdpack
    startTableInvSet!(s3,"I",dom3)$regsetgcdpack
  lts := internalZeroSetSplit(lp,clos?,info?,prep?)
  if hash?
  then
    stopTable!()$quasicomppack
    stopTableGcd!()$regsetgcdpack
    stopTableInvSet!()$regsetgcdpack
  lts

internalZeroSetSplit(lp:LP,clos?:B,info?:B,prep?:B) ==
  -- if info? then PRINT information
  -- if clos? then SOLVE in the closure sense
  -- if prep? then PREPROCESS the input system
  if prep?
  then
    pp := pre_process(lp,clos?,info?)
    lp := pp.val
    lts := pp.towers

```



```

    else
      ts: $ := [[]]
      lts := [ts]
    lp := remove(zero?, lp)
    any?(ground?, lp) => []
    empty? lp => lts
    empty? lts => lts
    lp := sort(infRittWu?,lp)
    clos? => decompose(lp,lts, clos?, info?)$regsetdecomppack
    -- IN DIM > 0 with clos? the following is not false ...
    for p in lp repeat
      lts := decompose([p],lts, clos?, info?)$regsetdecomppack
    lts

largeSystem?(lp:LP): Boolean ==
  -- Gonnet and Gerdt and not Wu-Wang.2
  #lp > 16 => true
  #lp < 13 => false
  lts: List($):= []
  (#lp :: Z - numberOfVariables(lp,lts)$regsetdecomppack :: Z) > 3

smallSystem?(lp:LP): Boolean ==
  -- neural, Vermeer, Liu, and not f-633 and not Hairer-2
  #lp < 5

mediumSystem?(lp:LP): Boolean ==
  -- f-633 and not Hairer-2
  lts: List($):= []
  (numberOfVariables(lp,lts)$regsetdecomppack :: Z - #lp :: Z) < 2

--   lin?(p:P):Boolean == ground?(init(p)) and one?(mdeg(p))
lin?(p:P):Boolean == ground?(init(p)) and (mdeg(p) = 1)

pre_process(lp:LP,clos?:B,info?:B): Record(val: LP, towers: Split) ==
  -- if info? then PRINT information
  -- if clos? then SOLVE in the closure sense
  ts: $ := [[]];
  lts: Split := [ts]
  empty? lp => [lp,lts]
  lp1: List P := []
  lp2: List P := []
  for p in lp repeat
    ground? (tail p) => lp1 := cons(p, lp1)
    lp2 := cons(p, lp2)
  lts: Split := decompose(lp1,[ts],clos?,info?)$regsetdecomppack
  probablyZeroDim?(lp)$polsetpack =>
    largeSystem?(lp) => return [lp2,lts]
  if #lp > 7
  then
    -- Butcher (8,8) + Wu-Wang.2 (13,16)

```

```

lp2 := crushedSet(lp2)$polsetpack
lp2 := remove(zero?,lp2)
any?(ground?,lp2) => return [lp2, lts]
lp3 := [p for p in lp2 | lin?(p)]
lp4 := [p for p in lp2 | not lin?(p)]
if clos?
then
  lts := decompose(lp4,lts, clos?, info?)$regsetdecomppack
else
  lp4 := sort(infRittWu?,lp4)
  for p in lp4 repeat
    lts := decompose([p],lts, clos?, info?)$regsetdecomppack
  lp2 := lp3
else
  lp2 := crushedSet(lp2)$polsetpack
  lp2 := remove(zero?,lp2)
  any?(ground?,lp2) => return [lp2, lts]
if clos?
then
  lts := decompose(lp2,lts, clos?, info?)$regsetdecomppack
else
  lp2 := sort(infRittWu?,lp2)
  for p in lp2 repeat
    lts := decompose([p],lts, clos?, info?)$regsetdecomppack
lp2 := []
return [lp2,lts]
smallSystem?(lp) => [lp2,lts]
mediumSystem?(lp) => [crushedSet(lp2)$polsetpack,lts]
lp3 := [p for p in lp2 | lin?(p)]
lp4 := [p for p in lp2 | not lin?(p)]
if clos?
then
  lts := decompose(lp4,lts, clos?, info?)$regsetdecomppack
else
  lp4 := sort(infRittWu?,lp4)
  for p in lp4 repeat
    lts := decompose([p],lts, clos?, info?)$regsetdecomppack
if clos?
then
  lts := decompose(lp3,lts, clos?, info?)$regsetdecomppack
else
  lp3 := sort(infRittWu?,lp3)
  for p in lp3 repeat
    lts := decompose([p],lts, clos?, info?)$regsetdecomppack
lp2 := []
return [lp2,lts]

```

---

— SREGSET.dotabb —

```
"SREGSET" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SREGSET"]
"SFRTCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=SFRTCAT"]
"SREGSET" -> "SFRTCAT"
```

—————

## 20.27 domain SQMATRIX SquareMatrix

— SquareMatrix.input —

```
)set break resume
)sys rm -f SquareMatrix.output
)spool SquareMatrix.output
)set message test on
)set message auto off
)clear all
--S 1 of 6
)set expose add constructor SquareMatrix
--R
--I SquareMatrix is now explicitly exposed in frame frame0
--E 1

--S 2 of 6
m := squareMatrix [ [1,-%i],[%i,4] ]
--R
--R
--R      +1   - %i+
--R  (1)  |      |
--R      +%i   4  +
--R
--R                                          Type: SquareMatrix(2,Complex Integer)
--E 2

--S 3 of 6
m*m - m
--R
--R
--R      + 1   - 4%i+
--R  (2)  |      |
--R      +4%i   13  +
--R
--R                                          Type: SquareMatrix(2,Complex Integer)
--E 3

--S 4 of 6
```

```

mm := squareMatrix [ [m, 1], [1-m, m**2] ]
--R
--R
--R      ++1  - %i+      +1  0+  +
--R      ||      |      |  |  |
--R      |+%i  4  +      +0  1+  |
--R  (3) |      |      |
--R      |+ 0      %i + + 2  - 5%i+|
--R      ||      |  |      ||
--R      ++- %i  - 3+ +5%i  17  ++
--R                                     Type: SquareMatrix(2,SquareMatrix(2,Complex Integer))
--E 4

--S 5 of 6
p := (x + m)**2
--R
--R
--R      2  + 2  - 2%i+      + 2  - 5%i+
--R  (4) x  + |      |x + |      |
--R      +2%i  8  +      +5%i  17  +
--R                                     Type: Polynomial SquareMatrix(2,Complex Integer)
--E 5

--S 6 of 6
p::SquareMatrix(2, ?)
--R
--R
--R      + 2
--R      |x  + 2x + 2  - 2%i x - 5%i|
--R  (5) |      |
--R      |      2      |
--R      +2%i x + 5%i  x  + 8x + 17 +
--R                                     Type: SquareMatrix(2,Polynomial Complex Integer)
--E 6
)spool
)lisp (bye)

```

---

— SquareMatrix.help —

=====

SquareMatrix examples

=====

The top level matrix type in Axiom is Matrix, which provides basic arithmetic and linear algebra functions. However, since the matrices can be of any size it is not true that any pair can be added or multiplied. Thus Matrix has little algebraic structure.

Sometimes you want to use matrices as coefficients for polynomials or in other algebraic contexts. In this case, `SquareMatrix` should be used. The domain `SquareMatrix(n,R)` gives the ring of  $n$  by  $n$  square matrices over  $R$ .

Since `SquareMatrix` is not normally exposed at the top level, you must expose it before it can be used.

```
)set expose add constructor SquareMatrix
```

Once `SQMATRIX` has been exposed, values can be created using the `squareMatrix` function.

```
m := squareMatrix [ [1,-%i],[%i,4] ]
+1 - %i+
|      |
+%i 4 +
Type: SquareMatrix(2,Complex Integer)
```

The usual arithmetic operations are available.

```
m*m - m
+ 1 - 4%i+
|      |
+4%i 13 +
Type: SquareMatrix(2,Complex Integer)
```

Square matrices can be used where ring elements are required. For example, here is a matrix with matrix entries.

```
mm := squareMatrix [ [m, 1], [1-m, m**2] ]
++1 - %i+ +1 0+ +
||      | |      |
|+%i 4 + +0 1+ |
|      |      |
|+ 0 %i + + 2 - 5%i+|
||      | |      ||
++- %i - 3+ +5%i 17 ++
Type: SquareMatrix(2,SquareMatrix(2,Complex Integer))
```

Or you can construct a polynomial with square matrix coefficients.

```
p := (x + m)**2
2 + 2 - 2%i+ + 2 - 5%i+
x + |      |x + |      |
+2%i 8 + +5%i 17 +
Type: Polynomial SquareMatrix(2,Complex Integer)
```

This value can be converted to a square matrix with polynomial coefficients.

```

p::SquareMatrix(2, ?)
+ 2 +
|x  + 2x + 2  - 2%i x - 5%i|
|                               |
|                               |
+2%i x + 5%i  x  + 8x + 17 +
                                     Type: SquareMatrix(2,Polynomial Complex Integer)

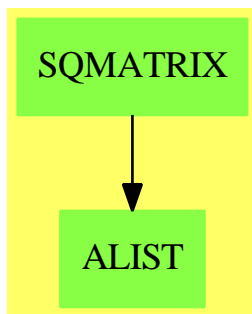
```

See Also:

- o )help Matrix
- o )show SquareMatrix

—————→

### 20.27.1 SquareMatrix (SQMATRIX)



See

- ⇒ “IndexedMatrix” (IMATRIX) 10.12.1 on page 1204
- ⇒ “Matrix” (MATRIX) 14.7.1 on page 1586
- ⇒ “RectangularMatrix” (RMATRIX) 19.4.1 on page 2205

**Exports:**

|                 |               |                |              |                |
|-----------------|---------------|----------------|--------------|----------------|
| 0               | 1             | antisymmetric? | any?         | characteristic |
| coerce          | column        | convert        | copy         | count          |
| D               | determinant   | diagonal       | diagonal?    | diagonalMatrix |
| diagonalProduct | differentiate | elt            | empty        | empty?         |
| eq?             | eval          | every?         | exquo        | hash           |
| inverse         | latex         | less?          | listOfLists  | map            |
| map!            | matrix        | maxColIndex    | maxRowIndex  | member?        |
| members         | minColIndex   | minordet       | minRowIndex  | more?          |
| ncols           | nrows         | nullSpace      | nullity      | one?           |
| parts           | qelt          | rank           | recip        | reducedSystem  |
| retract         | retractIfCan  | row            | rowEchelon   | sample         |
| scalarMatrix    | size?         | square?        | squareMatrix | subtractIfCan  |
| symmetric?      | trace         | transpose      | zero?        | #?             |
| ?*?             | ?**?          | ?+?            | ?-?          | -?             |
| ?=?             | ?^?           | ?~=?           | ?/?          |                |

## — domain SQMATRIX SquareMatrix —

```

)abbrev domain SQMATRIX SquareMatrix
++ Author: Grabmeier, Gschnitzer, Williamson
++ Date Created: 1987
++ Date Last Updated: July 1990
++ Basic Operations:
++ Related Domains: IndexedMatrix, Matrix, RectangularMatrix
++ Also See:
++ AMS Classifications:
++ Keywords: matrix, linear algebra
++ Examples:
++ References:
++ Description:
++ \spadtype{SquareMatrix} is a matrix domain of square matrices, where the
++ number of rows (= number of columns) is a parameter of the type.

```

```

SquareMatrix(ndim,R): Exports == Implementation where
  ndim : NonNegativeInteger
  R      : Ring
  Row ==> DirectProduct(ndim,R)
  Col ==> DirectProduct(ndim,R)
  MATLIN ==> MatrixLinearAlgebraFunctions(R,Row,Col,$)

```

```

Exports ==> Join(SquareMatrixCategory(ndim,R,Row,Col),_
  CoercibleTo Matrix R) with

```

```

transpose: $ -> $
++ \spad{transpose(m)} returns the transpose of the matrix m.
squareMatrix: Matrix R -> $
++ \spad{squareMatrix(m)} converts a matrix of type \spadtype{Matrix}
++ to a matrix of type \spadtype{SquareMatrix}.

```

```

coerce: $ -> Matrix R
  ++ \spad{coerce(m)} converts a matrix of type \spadtype{SquareMatrix}
  ++ to a matrix of type \spadtype{Matrix}.
-- symdecomp : $ -> Record(sym:$,antisym:$)
--   ++ \spad{symdecomp(m)} decomposes the matrix m as a sum of a symmetric
--   ++ matrix \spad{m1} and an antisymmetric matrix \spad{m2}. The object
--   ++ returned is the Record \spad{[m1,m2]}
-- if R has commutative("*") then
--   minorsVect: -> Vector(Union(R,"uncomputed")) --range: 1..2**n-1
--   ++ \spad{minorsVect(m)} returns a vector of the minors of the matrix m
if R has commutative("*") then central
  ++ the elements of the Ring R, viewed as diagonal matrices, commute
  ++ with all matrices and, indeed, are the only matrices which commute
  ++ with all matrices.
if R has commutative("*") and R has unitsKnown then unitsKnown
  ++ the invertible matrices are simply the matrices whose determinants
  ++ are units in the Ring R.
if R has ConvertibleTo InputForm then ConvertibleTo InputForm

Implementation ==> Matrix R add
minr ==> minRowIndex
maxr ==> maxRowIndex
minc ==> minColIndex
maxc ==> maxColIndex
mini ==> minIndex
maxi ==> maxIndex

ZERO := scalarMatrix 0
0    == ZERO
ONE  := scalarMatrix 1
1    == ONE

characteristic() == characteristic()$R

matrix(l: List List R) ==
  -- error check: this is a top level function
  #l ^= ndim => error "matrix: wrong number of rows"
  for ll in l repeat
    #ll ^= ndim => error "matrix: wrong number of columns"
  ans : Matrix R := new(ndim,ndim,0)
  for i in minr(ans)..maxr(ans) for ll in l repeat
    for j in minc(ans)..maxc(ans) for r in ll repeat
      qsetelt_!(ans,i,j,r)
  ans pretend $

row(x,i)    == directProduct row(x pretend Matrix(R),i)
column(x,j) == directProduct column(x pretend Matrix(R),j)
coerce(x:$):OutputForm == coerce(x pretend Matrix R)$Matrix(R)

scalarMatrix r == scalarMatrix(ndim,r)$Matrix(R) pretend $

```



```

diagonalMatrix l ==
  #l ~= ndim =>
    error "diagonalMatrix: wrong number of entries in list"
  diagonalMatrix(l)$Matrix(R) pretend $

coerce(x:$):Matrix(R) == copy(x pretend Matrix(R))

squareMatrix x ==
  (nrows(x) ~= ndim) or (ncols(x) ~= ndim) =>
    error "squareMatrix: matrix of bad dimensions"
  copy(x) pretend $

x:$ * v:Col ==
  directProduct((x pretend Matrix(R)) * (v :: Vector(R)))

v:Row * x:$ ==
  directProduct((v :: Vector(R)) * (x pretend Matrix(R)))

x:$ ** n:NonNegativeInteger ==
  ((x pretend Matrix(R)) ** n) pretend $

if R has commutative("*") then

  determinant x == determinant(x pretend Matrix(R))
  minordet x    == minordet(x pretend Matrix(R))

if R has EuclideanDomain then

  rowEchelon x == rowEchelon(x pretend Matrix(R)) pretend $

if R has IntegralDomain then

  rank x      == rank(x pretend Matrix(R))
  nullity x   == nullity(x pretend Matrix(R))
  nullSpace x ==
    [directProduct c for c in nullSpace(x pretend Matrix(R))]

if R has Field then

  dimension() == (m * n) :: CardinalNumber

  inverse x ==
    (u := inverse(x pretend Matrix(R))) case "failed" => "failed"
    (u :: Matrix(R)) pretend $

x:$ ** n:Integer ==
  ((x pretend Matrix(R)) ** n) pretend $

recip x == inverse x

```

```

if R has ConvertibleTo InputForm then
  convert(x:$):InputForm ==
    convert [convert("squareMatrix":Symbol)@InputForm,
             convert(x:Matrix(R))]]$List(InputForm)

```

---

— SQMATRIX.dotabb —

```

"SQMATRIX" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SQMATRIX"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"SQMATRIX" -> "ALIST"

```

---

## 20.28 domain STACK Stack

— Stack.input —

```

)set break resume
)sys rm -f Stack.output
)spool Stack.output
)set message test on
)set message auto off
)clear all

--S 1 of 44
a:Stack INT:= stack [1,2,3,4,5]
--R
--R (1) [1,2,3,4,5]
--R
--R                                          Type: Stack Integer
--E 1

--S 2 of 44
pop! a
--R
--R (2) 1
--R
--R                                          Type: PositiveInteger
--E 2

--S 3 of 44
a

```

```

--R
--R (3) [2,3,4,5]
--R
--R                                          Type: Stack Integer
--E 3

--S 4 of 44
extract! a
--R
--R (4) 2
--R
--R                                          Type: PositiveInteger
--E 4

--S 5 of 44
a
--R
--R (5) [3,4,5]
--R
--R                                          Type: Stack Integer
--E 5

--S 6 of 44
push!(9,a)
--R
--R (6) 9
--R
--R                                          Type: PositiveInteger
--E 6

--S 7 of 44
a
--R
--R (7) [9,3,4,5]
--R
--R                                          Type: Stack Integer
--E 7

--S 8 of 44
insert!(8,a)
--R
--R (8) [8,9,3,4,5]
--R
--R                                          Type: Stack Integer
--E 8

--S 9 of 44
a
--R
--R (9) [8,9,3,4,5]
--R
--R                                          Type: Stack Integer
--E 9

--S 10 of 44
inspect a
--R

```

```
--R (10) 8
--R                                          Type: PositiveInteger
--E 10

--S 11 of 44
empty? a
--R
--R (11) false
--R                                          Type: Boolean
--E 11

--S 12 of 44
top a
--R
--R (12) 8
--R                                          Type: PositiveInteger
--E 12

--S 13 of 44
depth a
--R
--R (13) 5
--R                                          Type: PositiveInteger
--E 13

--S 14 of 44
#a
--R
--R (14) 5
--R                                          Type: PositiveInteger
--E 14

--S 15 of 44
less?(a,9)
--R
--R (15) true
--R                                          Type: Boolean
--E 15

--S 16 of 44
more?(a,9)
--R
--R (16) false
--R                                          Type: Boolean
--E 16

--S 17 of 44
size?(a,#a)
--R
--R (17) true
```



--E 24

--S 25 of 44

eq?(a,c)

--R

--R (25) false

--R

Type: Boolean

--E 25

--S 26 of 44

eq?(a,a)

--R

--R (26) true

--R

Type: Boolean

--E 26

--S 27 of 44

(a=c)@Boolean

--R

--R (27) true

--R

Type: Boolean

--E 27

--S 28 of 44

(a=a)@Boolean

--R

--R (28) true

--R

Type: Boolean

--E 28

--S 29 of 44

a~c

--R

--R (29) false

--R

Type: Boolean

--E 29

--S 30 of 44

any?(x+->(x=4),a)

--R

--R (30) true

--R

Type: Boolean

--E 30

--S 31 of 44

any?(x+->(x=11),a)

--R

--R (31) false

--R

Type: Boolean

--E 31

```
--S 32 of 44
every?(x+-(x=11),a)
--R
--R (32) false
--R
--R Type: Boolean
--E 32

--S 33 of 44
count(4,a)
--R
--R (33) 1
--R
--R Type: PositiveInteger
--E 33

--S 34 of 44
count(x+-(x>2),a)
--R
--R (34) 5
--R
--R Type: PositiveInteger
--E 34

--S 35 of 44
map(x+-(x+10),a)
--R
--R (35) [18,19,13,14,15]
--R
--R Type: Stack Integer
--E 35

--S 36 of 44
a
--R
--R (36) [8,9,3,4,5]
--R
--R Type: Stack Integer
--E 36

--S 37 of 44
map!(x+-(x+10),a)
--R
--R (37) [18,19,13,14,15]
--R
--R Type: Stack Integer
--E 37

--S 38 of 44
a
--R
--R (38) [18,19,13,14,15]
--R
--R Type: Stack Integer
--E 38
```

```

--S 39 of 44
members a
--R
--R (39) [18,19,13,14,15]
--R
--R                                          Type: List Integer
--E 39

--S 40 of 44
member?(14,a)
--R
--R (40) true
--R
--R                                          Type: Boolean
--E 40

--S 41 of 44
coerce a
--R
--R
--R (41) [18,19,13,14,15]
--R
--R                                          Type: OutputForm
--E 41

--S 42 of 44
hash a
--R
--R
--R (42) 4999539
--R
--R                                          Type: SingleInteger
--E 42

--S 43 of 44
latex a
--R
--R
--R (43) "\mbox{\bf Unimplemented}"
--R
--R                                          Type: String
--E 43

--S 44 of 44
)show Stack
--R Stack S: SetCategory is a domain constructor
--R Abbreviation for Stack is STACK
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for STACK
--R
--R----- Operations -----
--R bag : List S -> %                copy : % -> %
--R depth : % -> NonNegativeInteger  empty : () -> %
--R empty? : % -> Boolean            eq? : (%,% ) -> Boolean
--R extract! : % -> S                insert! : (S,% ) -> %

```



```

--R inspect : % -> S                                map : ((S -> S),%) -> %
--R pop! : % -> S                                    push! : (S,%) -> S
--R sample : () -> %                                stack : List S -> %
--R top : % -> S
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if $ has finiteAggregate and S has SETCAT
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R members : % -> List S if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List S if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R ~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 44
)spool
)lisp (bye)

```

---

— Stack.help —

=====

Stack examples

=====

A Stack object is represented as a list ordered by last-in, first-out. It operates like a pile of books, where the "next" book is the one on the top of the pile.

Here we create a stack of integers from a list. Notice that the order in the list is the order in the stack.

```

a:Stack INT:= stack [1,2,3,4,5]
[1,2,3,4,5]

```

We can remove the top of the stack using pop!:

```
pop! a
1
```

Notice that the use of pop! is destructive (destructive operations in Axiom usually end with ! to indicate that the underlying data structure is changed).

```
a
[2,3,4,5]
```

The extract! operation is another name for the pop! operation and has the same effect. This operation treats the stack as a BagAggregate:

```
extract! a
2
```

and you can see that it also has destructively modified the stack:

```
a
[3,4,5]
```

Next we push a new element on top of the stack:

```
push!(9,a)
9
```

Again, the push! operation is destructive so the stack is changed:

```
a
[9,2,3,4,5]
```

Another name for push! is insert!, which treats the stack as a BagAggregate:

```
insert!(8,a)
[8,9,3,4,5]
```

and it modifies the stack:

```
a
[8,9,3,4,5]
```

The inspect function returns the top of the stack without modification, viewed as a BagAggregate:

```
inspect a
8
```

The empty? operation returns true only if there are no element on the

stack, otherwise it returns false:

```
empty? a
false
```

The top operation returns the top of stack without modification, viewed as a Stack:

```
top a
8
```

The depth operation returns the number of elements on the stack:

```
depth a
5
```

which is the same as the # (length) operation:

```
#a
5
```

The less? predicate will compare the stack length to an integer:

```
less?(a,9)
true
```

The more? predicate will compare the stack length to an integer:

```
more?(a,9)
false
```

The size? operation will compare the stack length to an integer:

```
size?(a,#a)
true
```

and since the last computation must always be true we try:

```
size?(a,9)
false
```

The parts function will return the stack as a list of its elements:

```
parts a
[8,9,3,4,5]
```

If we have a BagAggregate of elements we can use it to construct a stack. Notice that the elements are pushed in reverse order:

```
bag([1,2,3,4,5])$Stack(INT)
```

```
[5,4,3,2,1]
```

The empty function will construct an empty stack of a given type:

```
b:=empty()$(Stack INT)
[]
```

and the empty? predicate allows us to find out if a stack is empty:

```
empty? b
true
```

The sample function returns a sample, empty stack:

```
sample()$Stack(INT)
[]
```

We can copy a stack and it does not share storage so subsequent modifications of the original stack will not affect the copy:

```
c:=copy a
[8,9,3,4,5]
```

The eq? function is only true if the lists are the same reference, so even though c is a copy of a, they are not the same:

```
eq?(a,c)
false
```

However, a clearly shares a reference with itself:

```
eq?(a,a)
true
```

But we can compare a and c for equality:

```
(a=c)@Boolean
true
```

and clearly a is equal to itself:

```
(a=a)@Boolean
true
```

and since a and c are equal, they are clearly NOT not-equal:

```
a~=c
false
```

We can use the any? function to see if a predicate is true for any element:

```
any?(x+>(x=4),a)
true
```

or false for every element:

```
any?(x+>(x=11),a)
false
```

We can use the `every?` function to check every element satisfies a predicate:

```
every?(x+>(x=11),a)
false
```

We can count the elements that are equal to an argument of this type:

```
count(4,a)
1
```

or we can count against a boolean function:

```
count(x+>(x>2),a)
5
```

You can also map a function over every element, returning a new stack:

```
map(x+>x+10,a)
[18,19,13,14,15]
```

Notice that the original stack is unchanged:

```
a
[8,9,3,4,5]
```

You can use `map!` to map a function over every element and change the original stack since `map!` is destructive:

```
map!(x+>x+10,a)
[18,19,13,14,15]
```

Notice that the original stack has been changed:

```
a
[18,19,13,14,15]
```

The `member` function can also get the element of the stack as a list:

```
members a
[18,19,13,14,15]
```

and using `member?` we can test if the stack holds a given element:

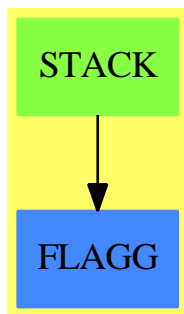
```
member?(14,a)
true
```

See Also:

- o `)show Stack`
- o `)show ArrayStack`
- o `)show Queue`
- o `)show Dequeue`
- o `)show Heap`
- o `)show BagAggregate`

—————→

### 20.28.1 Stack (STACK)



See

- ⇒ “ArrayStack” (ASTACK) 2.10.1 on page 65
- ⇒ “Queue” (QUEUE) 18.5.1 on page 2143
- ⇒ “Dequeue” (DEQUEUE) 5.5.1 on page 497
- ⇒ “Heap” (HEAP) 9.2.1 on page 1100

#### Exports:

|                      |                       |                     |                      |                      |
|----------------------|-----------------------|---------------------|----------------------|----------------------|
| <code>any?</code>    | <code>bag</code>      | <code>coerce</code> | <code>copy</code>    | <code>count</code>   |
| <code>depth</code>   | <code>empty</code>    | <code>empty?</code> | <code>eq?</code>     | <code>eval</code>    |
| <code>every?</code>  | <code>extract!</code> | <code>hash</code>   | <code>insert!</code> | <code>inspect</code> |
| <code>latex</code>   | <code>less?</code>    | <code>map</code>    | <code>map!</code>    | <code>member?</code> |
| <code>members</code> | <code>more?</code>    | <code>parts</code>  | <code>pop!</code>    | <code>push!</code>   |
| <code>sample</code>  | <code>size?</code>    | <code>stack</code>  | <code>top</code>     | <code>#?</code>      |
| <code>?=?</code>     | <code>?~=?</code>     |                     |                      |                      |

— domain STACK Stack —

```

)abbrev domain STACK Stack
++ Author: Michael Monagan, Stephen Watt, Timothy Daly
++ Date Created: June 86 and July 87
++ Date Last Updated: Feb 09
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ Linked List implementation of a Stack
--% Dequeue and Heap data types

Stack(S:SetCategory): StackAggregate S with
  stack: List S -> %
    ++ stack([x,y,...,z]) creates a stack with first (top)
    ++ element x, second element y,...,and last element z.
    ++
    ++X a:Stack INT:= stack [1,2,3,4,5]

-- Inherited Signatures repeated for examples documentation

pop_! : % -> S
++
++X a:Stack INT:= stack [1,2,3,4,5]
++X pop! a
++X a
extract_! : % -> S
++
++X a:Stack INT:= stack [1,2,3,4,5]
++X extract! a
++X a
push_! : (S,%) -> S
++
++X a:Stack INT:= stack [1,2,3,4,5]
++X push!(9,a)
++X a
insert_! : (S,%) -> %
++
++X a:Stack INT:= stack [1,2,3,4,5]
++X insert!(8,a)
++X a
inspect : % -> S
++
++X a:Stack INT:= stack [1,2,3,4,5]
++X inspect a
top : % -> S
++

```

```

++X a:Stack INT:= stack [1,2,3,4,5]
++X top a
depth : % -> NonNegativeInteger
++
++X a:Stack INT:= stack [1,2,3,4,5]
++X depth a
less? : (% ,NonNegativeInteger) -> Boolean
++
++X a:Stack INT:= stack [1,2,3,4,5]
++X less?(a,9)
more? : (% ,NonNegativeInteger) -> Boolean
++
++X a:Stack INT:= stack [1,2,3,4,5]
++X more?(a,9)
size? : (% ,NonNegativeInteger) -> Boolean
++
++X a:Stack INT:= stack [1,2,3,4,5]
++X size?(a,5)
bag : List S -> %
++
++X bag([1,2,3,4,5])$Stack(INT)
empty? : % -> Boolean
++
++X a:Stack INT:= stack [1,2,3,4,5]
++X empty? a
empty : () -> %
++
++X b:=empty()$(Stack INT)
sample : () -> %
++
++X sample()$Stack(INT)
copy : % -> %
++
++X a:Stack INT:= stack [1,2,3,4,5]
++X copy a
eq? : (% ,%) -> Boolean
++
++X a:Stack INT:= stack [1,2,3,4,5]
++X b:=copy a
++X eq?(a,b)
map : ((S -> S),%) -> %
++
++X a:Stack INT:= stack [1,2,3,4,5]
++X map(x+>x+10,a)
++X a
if $ has shallowlyMutable then
map! : ((S -> S),%) -> %
++
++X a:Stack INT:= stack [1,2,3,4,5]
++X map!(x+>x+10,a)

```



```

++X a
if S has SetCategory then
  latex : % -> String
  ++
  ++X a:Stack INT:= stack [1,2,3,4,5]
  ++X latex a
hash : % -> SingleInteger
++
++X a:Stack INT:= stack [1,2,3,4,5]
++X hash a
coerce : % -> OutputForm
++
++X a:Stack INT:= stack [1,2,3,4,5]
++X coerce a
"=": (%,% ) -> Boolean
++
++X a:Stack INT:= stack [1,2,3,4,5]
++X b:Stack INT:= stack [1,2,3,4,5]
++X (a=b)@Boolean
"~=" : (%,% ) -> Boolean
++
++X a:Stack INT:= stack [1,2,3,4,5]
++X b:=copy a
++X (a~b)
if % has finiteAggregate then
  every? : ((S -> Boolean),%) -> Boolean
  ++
  ++X a:Stack INT:= stack [1,2,3,4,5]
  ++X every?(x+>(x=4),a)
  any? : ((S -> Boolean),%) -> Boolean
  ++
  ++X a:Stack INT:= stack [1,2,3,4,5]
  ++X any?(x+>(x=4),a)
  count : ((S -> Boolean),%) -> NonNegativeInteger
  ++
  ++X a:Stack INT:= stack [1,2,3,4,5]
  ++X count(x+>(x>2),a)
  _# : % -> NonNegativeInteger
  ++
  ++X a:Stack INT:= stack [1,2,3,4,5]
  ++X #a
  parts : % -> List S
  ++
  ++X a:Stack INT:= stack [1,2,3,4,5]
  ++X parts a
  members : % -> List S
  ++
  ++X a:Stack INT:= stack [1,2,3,4,5]
  ++X members a
if % has finiteAggregate and S has SetCategory then

```

```

member? : (S,%) -> Boolean
++
++X a:Stack INT:= stack [1,2,3,4,5]
++X member?(3,a)
count : (S,%) -> NonNegativeInteger
++
++X a:Stack INT:= stack [1,2,3,4,5]
++X count(4,a)

== add
Rep := Reference List S
s = t == deref s = deref t
coerce(d:%): OutputForm == bracket [e::OutputForm for e in deref d]
copy s == ref copy deref s
depth s == # deref s
# s == depth s
pop_! (s:%):S ==
  empty? s => error "empty stack"
  e := first deref s
  setref(s,rest deref s)
  e
extract_! (s:%):S == pop_! s
top (s:%):S ==
  empty? s => error "empty stack"
  first deref s
inspect s == top s
push_!(e,s) == (setref(s,cons(e,deref s));e)
insert_!(e:S,s:%):% == (push_!(e,s);s)
empty() == ref nil()$List(S)
empty? s == null deref s
stack s == ref copy s
parts s == copy deref s
map(f,s) == ref map(f,deref s)
map!(f,s) == ref map!(f,deref s)

```

---

— STACK.dotabb —

```

"STACK" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STACK"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"STACK" -> "FLAGG"

```

---

## 20.29 domain SD StochasticDifferential

— StochasticDifferential.input —

```

)set break resume
)sys rm -f StochasticDifferential.output
)spool StochasticDifferential.output
)set message test on
)set message auto off
)clear all

--S 1 of 12
dt := introduce!(t,dt)
--R
--R
--R (1) dt
--R
--R                                         Type: Union(BasicStochasticDifferential,...)
--E 1

--S 2 of 12
dX := introduce!(X,dX)
--R
--R
--R (2) dX
--R
--R                                         Type: Union(BasicStochasticDifferential,...)
--E 2

--S 3 of 12
dY := introduce!(Y,dY)
--R
--R
--R (3) dY
--R
--R                                         Type: Union(BasicStochasticDifferential,...)
--E 3

--S 4 of 12
copyBSD()
--R
--R
--R (4) [dX,dY,dt]
--R
--R                                         Type: List BasicStochasticDifferential
--E 4

--S 5 of 12
copyIto()
--R
--R
--R (5) table(t= dt,Y= dY,X= dX)

```

```
--R                                     Type: Table(Symbol,BasicStochasticDifferential)
--E 5
```

```
--S 6 of 12
copyQuadVar()          -- display of multiplication table
--R
--R
--R (6) table()
--R Type: Table(StochasticDifferential Integer,StochasticDifferential Integer)
--E 6
```

```
--S 7 of 12
statusIto()
--R
--R
--R      +B S D      : dX dY dt+
--R      |
--R      |drift      : ? ? ? |
--R      |
--R      | *
--R (7) |
--R      | dX      : ? ? ? |
--R      |
--R      | dY      : ? ? ? |
--R      |
--R      + dt      : ? ? ? +
--R
--R                                     Type: OutputForm
--E 7
```

```
--S 8 of 12
copyDrift()          -- display of drift list
--R
--R
--R (8) table()
--R Type: Table(StochasticDifferential Integer,StochasticDifferential Integer)
--E 8
```

```
--S 9 of 12
nbsd := #copyBSD()
--R
--R
--R (9) 3
--R
--R                                     Type: PositiveInteger
--E 9
```

```
--S 10 of 12
ItoMultArray:ARRAY2(SD INT) :=new(nbsd,nbsd,0$SD(INT))
--R
--R
--R      +0 0 0+
```

```

--R      |      |
--R  (10) |0  0  0|
--R      |      |
--R      +0  0  0+
--R                                         Type: TwoDimensionalArray StochasticDifferential Integer
--E 10

--S 11 of 12
ItoMultArray
--R
--R
--R      +0  0  0+
--R      |      |
--R  (11) |0  0  0|
--R      |      |
--R      +0  0  0+
--R                                         Type: TwoDimensionalArray StochasticDifferential Integer
--E 11

--S 12 of 12
statusIto()
--R
--R
--R      +B S D      :  dX  dY  dt+
--R      |          |
--R      |drift      :  ?   ?   ? |
--R      |          |
--R      |   *          |
--R  (12) |          |
--R      |  dX      :  ?   ?   ? |
--R      |          |
--R      |  dY      :  ?   ?   ? |
--R      |          |
--R      +  dt      :  ?   ?   ? +
--R
--R                                         Type: OutputForm
--E 12

)spool
)lisp (bye)

```

---

— StochasticDifferential.help —

=====

StochasticDifferential examples

=====

A basic implementation of StochasticDifferential(R) using the  
associated domain BasicStochasticDifferential in the underlying

representation as sparse multivariate polynomials. The domain is a module over Expression(R), and is a ring without identity (AXIOM term is "Rng"). Note that separate instances, for example using R=Integer and R=Float, have different hidden structure (multiplication and drift tables).

```
dt := introduce!(t,dt)
dt

dX := introduce!(X,dX)
dX

dY := introduce!(Y,dY)
dY

copyBSD()
[dX,dY,dt]

copyIto()
table(t= dt,Y= dY,X= dX)

copyQuadVar()          -- display of multiplication table
table()

statusIto()
+B S D   :  dX  dY  dt+
|
|drift   :  ?   ?   ? |
|
|  *      |
|
|  dX     :  ?   ?   ? |
|
|  dY     :  ?   ?   ? |
|
+  dt     :  ?   ?   ? +

copyDrift()          -- display of drift list
table()

nbsd := #copyBSD()
3

ItoMultArray:ARRAY2(SD INT) :=new(nbsd,nbsd,0$SD(INT))
+0  0  0+
|
|0  0  0|
|
+0  0  0+
```

```
ItoMultArray
+0 0 0+
|
|0 0 0|
|
+0 0 0+

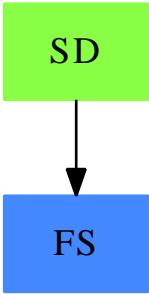
statusIto()
+B S D : dX dY dt+
|
|drift : ? ? ? |
|
| *
|
| dX : ? ? ? |
|
| dY : ? ? ? |
|
+ dt : ? ? ? +
```

See Also:

- o )help BasicStochasticDifferential
- o )show StochasticDifferential



20.29.1 StochasticDifferential (SD)



See

⇐ “BasicStochasticDifferential” (BSD) 3.6.1 on page 268

**Exports:**

|               |               |           |              |
|---------------|---------------|-----------|--------------|
| 0             | coerce        | copyDrift | copyQuadVar  |
| drift         | hash          | latex     | sample       |
| statusIto     | uncorrelated? | zero?     | alterDrift!  |
| alterQuadVar! | coefficient   | coerce    | equation     |
| freeOf?       | listSD        | retract   | retractIfCan |
| subtractIfCan | ?*?           | ***?      | ?+?          |
| ?-?           | -?            | ?/?       | ?=?          |
| ?^?           | ?~=?          |           |              |

— domain SD StochasticDifferential —

```

)abbrev domain SD StochasticDifferential
++ Author: Wilfrid S. Kendall
++ Last Last Updated: July 26, 1999
++ Related Domains: BasicStochasticDifferential
++ AMS Classifications:
++ Keywords: stochastic differential, semimartingale.
++ References: Ito (1975), Kendall (1991a,b; 1993a,b).
++ Description:
++ A basic implementation of StochasticDifferential(R) using the
++ associated domain BasicStochasticDifferential in the underlying
++ representation as sparse multivariate polynomials. The domain is
++ a module over Expression(R), and is a ring without identity
++ (AXIOM term is "Rng"). Note that separate instances, for example
++ using R=Integer and R=Float, have different hidden structure
++ (multiplication and drift tables).

```

```

StochasticDifferential(R:Join(OrderedSet, IntegralDomain)):

```

```

Category == Implementation where
ER ==> Expression(R)
PR ==> Polynomial(R)
FR ==> Fraction(PR)
BSD ==> BasicStochasticDifferential
PI ==> PositiveInteger
NNI ==> NonNegativeInteger
OF ==> OutputForm
Category ==> Join(Rng, Module(ER)) with

```

```

RetractableTo(BSD)

```

```

alterQuadVar!:(BSD,BSD,%) -> Union(%, "failed")
++ alterQuadVar! adds multiplication formula for a
++ pair of stochastic differentials to a private table.
++ Failure occurs if
++ (a) either of first or second arguments is not basic
++ (b) third argument is not exactly of first degree

```

```

alterDrift!:(BSD,%) -> Union(%, "failed")

```



```

++ alterDrift! adds drift formula for a
++ stochastic differential to a private table.
++ Failure occurs if
++ (a) first arguments is not basic
++ (b) second argument is not exactly of first degree

drift:% -> %
++ drift(dx) returns the drift of \axiom{dx}

freeOf?:(% ,BSD) -> Boolean
++ freeOf?(sd,dX) checks whether \axiom{dX} occurs in
++ \axiom{sd} as a module element

coefficient:(% ,BSD) -> ER
++ coefficient(sd,dX) returns the coefficient of \axiom{dX}
++ in the stochastic differential \axiom{sd}

listSD:(%) -> List BSD
++ listSD(dx) returns a list of all \axiom{BSD} involved
++ in the generation of \axiom{dx} as a module element

equation:(% ,R) -> Union(Equation %,"failed")
++ equation(dx,0) allows RHS of Equation % to be zero
equation:(R,% ) -> Union(Equation %,"failed")
++ equation(0,dx) allows LHS of Equation % to be zero

copyDrift:() -> Table(% ,%)
++ copyDrift returns private table of drifts
++ of basic stochastic differentials for inspection

copyQuadVar:() -> Table(% ,%)
++ copyQuadVar returns private multiplication table
++ of basic stochastic differentials for inspection

"/"      : (% , ER) -> %
++ dx/y divides the stochastic differential dx
++ by the previsible function y.

"*)"      : (% , PI) -> %
++ dx**n is dx multiplied by itself n times.

"^"      : (% , PI) -> %
++ dx^n is dx multiplied by itself n times.

statusIto:() -> OF
++ statusIto() displays the current state of \axiom{setBSD},
++ \axiom{tableDrift}, and \axiom{tableQuadVar}. Question
++ marks are printed instead of undefined entries
++
++X dt:=introduce!(t,dt)

```

```

++X dX:=introduce!(X,dX)
++X dY:=introduce!(Y,dY)
++X copyBSD()
++X copyIto()
++X copyhQuadVar()
++X statusIto()

uncorrelated?: (%,% ) -> Boolean
++ uncorrelated?(dx,dy) checks whether its two arguments
++ have zero quadratic co-variation.
uncorrelated?: (List %,List %) -> Boolean
++ uncorrelated?(l1,l2) checks whether its two arguments
++ are lists of stochastic differentials of zero inter-list
++ quadratic co-variation.
uncorrelated?: (List List %) -> Boolean
++ uncorrelated?(ll) checks whether its argument is a list
++ of lists of stochastic differentials of zero inter-list
++ quadratic co-variation.

Implementation ==> SparseMultivariatePolynomial(ER,BSD) add
Rep:=SparseMultivariatePolynomial(ER,BSD)

(v:% / s:ER):% == inv(s) * v

tableQuadVar:Table(%,% ) := table()
tableDrift:Table(%,% ) := table()

alterQuadVar!(da:BSD,db:BSD,dXdY:%):Union(%,"failed") ==
-- next two lines for security only!
1 < totalDegree(dXdY) => "failed"
0 ~= coefficient(dXdY,degree(1)$Rep) => "failed"
not(0::% = (dXdY*dXdY)::%) => "failed"
setelt(tableQuadVar,((da:Rep)*(db:Rep))$Rep,dXdY)$Table(%,% )
-- We have to take care here to avoid a bad
-- recursion on \axiom{*(%,%)->%}

alterDrift!(da:BSD,dx:%):Union(%,"failed") ==
1 < totalDegree(dx) => "failed"
0 ~= coefficient(dx,degree(1)$Rep) => "failed"
not(0::% = (dx*dx)::%) => "failed"
setelt(tableDrift,da:Rep,dx)$Table(%,% )

multSDOrError(dm:%):% ==
c := leadingCoefficient dm
(dmm := search(dm/c,tableQuadVar))
case "failed" =>
print hconcat(message("ERROR IN ")$OF,(dm/c)::OF)
error "Above product of sd's is not defined"
c*dmm

```

```

(dx:% * dy:%) : % ==
1 < totalDegree(dx) =>
  print hconcat(message("ERROR IN ")$OF,dx::OF)
  error "bad sd in lhs of sd product"
1 < totalDegree(dy) =>
  print hconcat(message("ERROR IN ")$OF,dy::OF)
  error "bad sd in rhs of sd product"
reduce("+",map(multSDOrError,monomials((dx*dy)$Rep)),0)
-- We have to take care here to avoid a bad
-- recursion on \axiom{*(%,%)->%}

(dx:% ** n:PI) : % ==
n = 1 => dx
n = 2 => dx*dx
n > 2 => 0::%

(dx:% ^ n:PI) : % == dx**n

driftSDOrError(dm:%):% ==
c := leadingCoefficient dm
(dmm := search(dm/c,tableDrift))
case "failed" =>
  print hconcat(message("ERROR IN ")$OF,(dm/c)::OF)
  error "drift of sd is not defined"
c*dmm

drift(dx:%):% ==
reduce("+",map(driftSDOrError,monomials(dx)),0)

freeOf?(sd,dX) == (0 = coefficient(sd,dX,1))

coefficient(sd:%,dX:BSD):ER ==
retract(coefficient(sd,dX,1))@ER

listSD(sd) ==
[retract(dX)@BSD for dX in primitiveMonomials(sd)]

equation(dx:%,zero:R):Union(Equation %,"failed") ==
not(0 = zero) => "failed"
equation(dx,0::%)
equation(zero:R,dx:%):Union(Equation %,"failed") ==
not(0 = zero) => "failed"
equation(0::%,dx)

copyDrift() == tableDrift
copyQuadVar() == tableQuadVar

xDrift(dx:BSD):OF ==
(xdx := search(dx::Rep,tableDrift)) case "failed" => "?":OF
xdx::OF

```

```

xQV(dx:BSD,dy:BSD):OF ==
  (xdxdy := search((dx::% * dy::%)$Rep,tableQuadVar))
  case "failed" => "?"::Symbol::OF
  xdxdy::OF

statusIto():OF ==
  bsd := copyBSD()$BSD
  bsdo := [dx::OF for dx in bsd]
  blank:= ":"::Symbol::OF
  colon:= ":"::Symbol::OF
  bsdh := "B S D " ::Symbol::OF
  dfth := "drift " ::Symbol::OF
  qvh := "*"::Symbol::OF
  head := append([bsdh,colon],bsdo)
  drift:= append([dfth,colon],[xDrift dx for dx in bsd])
  space:= append([qvh ,blank],[blank for dx in bsd])
  qv := [append([dy::OF,colon],[xQV(dx,dy) for dx in bsd])
        for dy in bsd]
  matrix(append([head,drift,space],qv))$OF

uncorrelated?(dx:%,dy:%): Boolean == (0::% = dx*dy)

uncorrelated?(l1:List %,l2:List %): Boolean ==
  reduce("and", [
    reduce("and",[uncorrelated?(dx,dy) for dy in l2],true)
    for dx in l1 ],true)

uncorrelated1?(l1:List %,l1:List List %): Boolean ==
  reduce("and",[uncorrelated?(l1,l2) for l2 in l1],true)

uncorrelated?(l1:List List %): Boolean ==
  (0$Integer = # l1) => true
  (1 = # l1) => true
  uncorrelated1?(first l1,rest l1) and uncorrelated?(rest l1)

```

---

— SD.dotabb —

```

"SD" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SD"]
"FS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FS"]
"SD" -> "FS"

```

---

## 20.30 domain STREAM Stream

— Stream.input —

```

)set break resume
)sys rm -f Stream.output
)spool Stream.output
)set message test on
)set message auto off
)clear all
--S 1 of 12
ints := [i for i in 0..]
--R
--R
--R (1) [0,1,2,3,4,5,6,7,8,9,...]
--R
--R                                          Type: Stream NonNegativeInteger
--E 1

--S 2 of 12
f : List INT -> List INT
--R
--R
--R                                          Type: Void
--E 2

--S 3 of 12
f x == [x.1 + x.2, x.1]
--R
--R
--R                                          Type: Void
--E 3

--S 4 of 12
fibs := [i.2 for i in [generate(f,[1,1])]]
--R
--R Compiling function f with type List Integer -> List Integer
--R
--R (4) [1,1,2,3,5,8,13,21,34,55,...]
--R
--R                                          Type: Stream Integer
--E 4

--S 5 of 12
[i for i in ints | odd? i]
--R
--R
--R (5) [1,3,5,7,9,11,13,15,17,19,...]
--R
--R                                          Type: Stream NonNegativeInteger
--E 5

--S 6 of 12

```

```

odds := [2*i+1 for i in ints]
--R
--R
--R (6) [1,3,5,7,9,11,13,15,17,19,...]
--R                                         Type: Stream NonNegativeInteger
--E 6

--S 7 of 12
scan(0,+,odds)
--R
--R
--R (7) [1,4,9,16,25,36,49,64,81,100,...]
--R                                         Type: Stream NonNegativeInteger
--E 7

--S 8 of 12
[i*j for i in ints for j in odds]
--R
--R
--R (8) [0,3,10,21,36,55,78,105,136,171,...]
--R                                         Type: Stream NonNegativeInteger
--E 8

--S 9 of 12
map(*,ints,odds)
--R
--R
--R (9) [0,3,10,21,36,55,78,105,136,171,...]
--R                                         Type: Stream NonNegativeInteger
--E 9

--S 10 of 12
first ints
--R
--R
--R (10) 0
--R                                         Type: NonNegativeInteger
--E 10

--S 11 of 12
rest ints
--R
--R
--R (11) [1,2,3,4,5,6,7,8,9,10,...]
--R                                         Type: Stream NonNegativeInteger
--E 11

--S 12 of 12
fibs 20
--R

```

```

--R
--R (12) 6765
--R
--R                                         Type: PositiveInteger
--E 12
)spool
)lisp (bye)

```

---

— Stream.help —

=====

Stream examples

=====

A Stream object is represented as a list whose last element contains the wherewithal to create the next element, should it ever be required.

Let ints be the infinite stream of non-negative integers.

```

ints := [i for i in 0..]
      [0,1,2,3,4,5,6,7,8,9,...]
                                         Type: Stream NonNegativeInteger

```

By default, ten stream elements are calculated. This number may be changed to something else by the system command

```

)set streams calculate

```

More generally, you can construct a stream by specifying its initial value and a function which, when given an element, creates the next element.

```

f : List INT -> List INT
                                         Type: Void

f x == [x.1 + x.2, x.1]
                                         Type: Void

fibs := [i.2 for i in [generate(f,[1,1])]]
      [1,1,2,3,5,8,13,21,34,55,...]
                                         Type: Stream Integer

```

You can create the stream of odd non-negative integers by either filtering them from the integers, or by evaluating an expression for each integer.

```

[i for i in ints | odd? i]
      [1,3,5,7,9,11,13,15,17,19,...]
                                         Type: Stream NonNegativeInteger

odds := [2*i+1 for i in ints]

```

```
[1,3,5,7,9,11,13,15,17,19,...]
```

```
Type: Stream NonNegativeInteger
```

You can accumulate the initial segments of a stream using the scan operation.

```
scan(0,+,odds)
```

```
[1,4,9,16,25,36,49,64,81,100,...]
```

```
Type: Stream NonNegativeInteger
```

The corresponding elements of two or more streams can be combined in this way.

```
[i*j for i in ints for j in odds]
```

```
[0,3,10,21,36,55,78,105,136,171,...]
```

```
Type: Stream NonNegativeInteger
```

```
map(*,ints,odds)
```

```
[0,3,10,21,36,55,78,105,136,171,...]
```

```
Type: Stream NonNegativeInteger
```

Many operations similar to those applicable to lists are available for streams.

```
first ints
```

```
0
```

```
Type: NonNegativeInteger
```

```
rest ints
```

```
[1,2,3,4,5,6,7,8,9,10,...]
```

```
Type: Stream NonNegativeInteger
```

```
fibs 20
```

```
6765
```

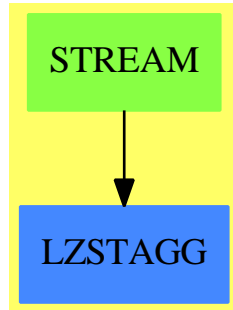
```
Type: PositiveInteger
```

See Also:

- o )help StreamFunctions1
- o )help StreamFunctions2
- o )help StreamFunctions3
- o )show Stream

---



**20.30.1 Stream (STREAM)**

**Exports:**

|                   |                  |                         |
|-------------------|------------------|-------------------------|
| any?              | child?           | children                |
| coerce            | complete         | concat                  |
| concat!           | cons             | construct               |
| convert           | copy             | count                   |
| cycleEntry        | cycleLength      | cycleSplit!             |
| cycleTail         | cyclic?          | delay                   |
| delete            | distance         | elt                     |
| empty             | empty?           | entries                 |
| entry?            | eq?              | eval                    |
| eval              | eval             | eval                    |
| every?            | explicitEntries? | explicitlyEmpty?        |
| explicitlyFinite? | extend           | fill!                   |
| filterUntil       | filterWhile      | find                    |
| findCycle         | first            | frst                    |
| generate          | hash             | index?                  |
| indices           | insert           | insert                  |
| last              | latex            | lazy?                   |
| lazyEvaluate      | leaf?            | leaves                  |
| less?             | map              | map!                    |
| maxIndex          | member?          | members                 |
| minIndex          | more?            | new                     |
| nodes             | node?            | numberOfComputedEntries |
| output            | parts            | possiblyInfinite?       |
| qelt              | qsetelt!         | reduce                  |
| remove            | removeDuplicates | repeating               |
| repeating?        | rest             | rst                     |
| sample            | second           | select                  |
| setchildren!      | setelt           | setfirst!               |
| setlast!          | setrest!         | setvalue!               |
| showAll?          | showAllElements  | size?                   |
| split!            | swap!            | tail                    |
| third             | value            | #?                      |
| ?=?               | ?~=?             | ?..?                    |
| ?.last            | ?.rest           | ?.first                 |
| ?.value           |                  |                         |

— domain **STREAM Stream** —

```

)abbrev domain STREAM Stream
++ Implementation of streams via lazy evaluation
++ Authors: Burge, Watt; updated by Clifton J. Williamson
++ Date Created: July 1986
++ Date Last Updated: 30 March 1990
++ Keywords: stream, infinite list, infinite sequence
++ Examples:
++ References:

```

```

++ Description:
++ A stream is an implementation of an infinite sequence using
++ a list of terms that have been computed and a function closure
++ to compute additional terms when needed.

Stream(S): Exports == Implementation where
-- problems:
-- 1) dealing with functions which basically want a finite structure
-- 2) 'map' doesn't deal with cycles very well

S : Type
B ==> Boolean
OUT ==> OutputForm
I ==> Integer
L ==> List
NNI ==> NonNegativeInteger
U ==> UniversalSegment I

Exports ==> LazyStreamAggregate(S) with
shallowlyMutable
++ one may destructively alter a stream by assigning new
++ values to its entries.

coerce: L S -> %
++ coerce(l) converts a list l to a stream.
++
++X m:=[1,2,3,4,5,6,7,8,9,10,11,12]
++X coerce(m)@Stream(Integer)
++X m::Stream(Integer)

repeating: L S -> %
++ repeating(l) is a repeating stream whose period is the list l.
++
++X m:=repeating([-1,0,1,2,3])

if S has SetCategory then
repeating?: (L S,% ) -> B
++ repeating?(l,s) returns true if a stream s is periodic
++ with period l, and false otherwise.
++
++X m:=[1,2,3]
++X n:=repeating(m)
++X repeating?(m,n)

findCycle: (NNI,% ) -> Record(cycle?: B, prefix: NNI, period: NNI)
++ findCycle(n,st) determines if st is periodic within n.
++
++X m:=[1,2,3]
++X n:=repeating(m)
++X findCycle(3,n)

```

```

++X findCycle(2,n)

delay: (() -> %) -> %
++ delay(f) creates a stream with a lazy evaluation defined by
++ function f.
++ Caution: This function can only be called in compiled code.
cons: (S,%) -> %
++ cons(a,s) returns a stream whose \spad{first} is \spad{a}
++ and whose \spad{rest} is s.
++ Note: \spad{cons(a,s) = concat(a,s)}.
++
++X m:=[1,2,3]
++X n:=repeating(m)
++X cons(4,n)

if S has SetCategory then
output: (I, %) -> Void
++ output(n,st) computes and displays the first n entries
++ of st.
++
++X m:=[1,2,3]
++X n:=repeating(m)
++X output(5,n)

showAllElements: % -> OUT
++ showAllElements(s) creates an output form which displays all
++ computed elements.
++
++X m:=[1,2,3,4,5,6,7,8,9,10,11,12]
++X n:=m::Stream(PositiveInteger)
++X showAllElements n

showAll?: () -> B
++ showAll?() returns true if all computed entries of streams
++ will be displayed.
--!! this should be a function of one argument
setrest_!: (% ,I,%) -> %
++ setrest!(x,n,y) sets rest(x,n) to y. The function will expand
++ cycles if necessary.
++
++X p:=[i for i in 1..]
++X q:=[i for i in 9..]
++X setrest!(p,4,q)
++X p

generate: (() -> S) -> %
++ generate(f) creates an infinite stream all of whose elements are
++ equal to \spad{f()}.
++ Note: \spad{generate(f) = [f(),f(),f(),...]}.
```

```

++X f():Integer == 1
++X generate(f)

generate: (S -> S,S) -> %
++ generate(f,x) creates an infinite stream whose first element is
++ x and whose nth element (\spad{n > 1}) is f applied to the previous
++ element. Note: \spad{generate(f,x) = [x,f(x),f(f(x)),...]}.
++
++X f(x:Integer):Integer == x+10
++X generate(f,10)

filterWhile: (S -> Boolean,%) -> %
++ filterWhile(p,s) returns \spad{[x0,x1,...,x(n-1)]} where
++ \spad{s = [x0,x1,x2,...]} and
++ n is the smallest index such that \spad{p(xn) = false}.
++
++X m:=[i for i in 1..]
++X f(x:PositiveInteger):Boolean == x < 5
++X filterWhile(f,m)

filterUntil: (S -> Boolean,%) -> %
++ filterUntil(p,s) returns \spad{[x0,x1,...,x(n)]} where
++ \spad{s = [x0,x1,x2,...]} and
++ n is the smallest index such that \spad{p(xn) = true}.
++
++X m:=[i for i in 1..]
++X f(x:PositiveInteger):Boolean == x < 5
++X filterUntil(f,m)

-- if S has SetCategory then
-- map: ((S,S) -> S,%,%,S) -> %
-- ++ map(f,x,y,a) is equivalent to map(f,x,y)
-- ++ If z = map(f,x,y,a), then z = map(f,x,y) except if
-- ++ x.n = a and rest(rest(x,n)) = rest(x,n) in which case
-- ++ rest(z,n) = rest(y,n) or if y.m = a and rest(rest(y,m)) =
-- ++ rest(y,m) in which case rest(z,n) = rest(x,n).
-- ++ Think of the case where f(xi,yi) = xi + yi and a = 0.

Implementation ==> add
MIN ==> 1 -- minimal stream index; see also the defaults in LZSTAGG
x:%

import CyclicStreamTools(S,%)

--% representation

-- This description of the rep is not quite true.
-- The Rep is a pair of one of three forms:
-- [value: S, rest: %]
-- [nullstream: Magic, NIL ]

```

```

--      [nonnullstream: Magic, fun: () -> %]
-- Could use a record of unions if we could guarantee no tags.

NullStream:   S := _$NullStream$Lisp   pretend S
NonNullStream: S := _$NonNullStream$Lisp pretend S

Rep := Record(firstElt: S, restOfStream: %)

explicitlyEmpty? x == EQ(first x, NullStream)$Lisp
lazy? x           == EQ(first x, NonNullStream)$Lisp

--% signatures of local functions

setfirst_!      : (%,S) -> S
setrst_!        : (%,%) -> %
setToNil_!      : % -> %
setrestt_!      : (%,I,%) -> %
lazyEval        : % -> %
expand_!        : (%,I) -> %

--% functions to access or change record fields without lazy evaluation

first x == x.firstElt
rst x == x.restOfStream

setfirst_!(x,s) == x.firstElt := s
setrst_!(x,y) == x.restOfStream := y

setToNil_! x ==
-- destructively changes x to a null stream
  setfirst_!(x, NullStream); setrst_!(x, NIL$Lisp)
  x

--% SETCAT functions

if S has SetCategory then

  getm          : (%,L OUT,I) -> L OUT
  streamCountCoerce : % -> OUT
  listm         : (%,L OUT,I) -> L OUT

  getm(x,le,n) ==
    explicitlyEmpty? x => le
    lazy? x =>
      n > 0 =>
        empty? x => le
        getm(rst x, concat(first(x) :: OUT, le), n - 1)
        concat(message("..."), le)
    eq?(x, rst x) => concat(overbar(first(x) :: OUT), le)
    n > 0 => getm(rst x, concat(first(x) :: OUT, le), n - 1)

```

```

concat(message("..."),le)

streamCountCoerce x ==
-- this will not necessarily display all stream elements
-- which have been computed
count := _$streamCount$Lisp
-- compute count elements
y := x
for i in 1..count while not empty? y repeat y := rst y
fc := findCycle(count,x)
not fc.cycle? => bracket reverse_! getm(x,empty(),count)
le : L OUT := empty()
for i in 1..fc.prefix repeat
  le := concat(first(x) :: OUT,le)
  x := rest x
pp : OUT :=
  fc.period = 1 => overbar(frst(x) :: OUT)
  pl : L OUT := empty()
  for i in 1..fc.period repeat
    pl := concat(frst(x) :: OUT,pl)
    x := rest x
  overbar commaSeparate reverse_! pl
bracket reverse_! concat(pp,le)

listm(x,le,n) ==
  explicitlyEmpty? x => le
  lazy? x =>
    n > 0 =>
      empty? x => le
      listm(rst x, concat(first(x) :: OUT,le),n-1)
      concat(message("..."),le)
      listm(rst x,concat(frst(x) :: OUT,le),n-1)

showAllElements x ==
-- this will display all stream elements which have been computed
-- and will display at least n elements with n = streamCount$Lisp
extend(x,_$streamCount$Lisp)
cycElt := cycleElt x
cycElt case "failed" =>
  le := listm(x,empty(),_$streamCount$Lisp)
  bracket reverse_! le
cycEnt := computeCycleEntry(x,cycElt :: %)
le : L OUT := empty()
while not eq?(x,cycEnt) repeat
  le := concat(frst(x) :: OUT,le)
  x := rst x
len := computeCycleLength(cycElt :: %)
pp : OUT :=
  len = 1 => overbar(frst(x) :: OUT)
  pl : L OUT := []

```

```

        for i in 1..len repeat
            pl := concat(first(x) :: OUT,pl)
            x := rst x
        overbar commaSeparate reverse_! pl
    bracket reverse_! concat(pp,le)

showAll?() ==
    NULL(_$streamsShowAll$Lisp)$Lisp => false
    true

coerce(x):OUT ==
    showAll?() => showAllElements x
    streamCountCoerce x

--% AGG functions

lazyCopy:% -> %
lazyCopy x == delay
    empty? x => empty()
    concat(first x, copy rst x)

copy x ==
    cycElt := cycleElt x
    cycElt case "failed" => lazyCopy x
    ce := cycElt :: %
    len := computeCycleLength(ce)
    e := computeCycleEntry(x,ce)
    d := distance(x,e)
    cycle := complete first(e,len)
    setrst_!(tail cycle,cycle)
    d = 0 => cycle
    head := complete first(x,d::NNI)
    setrst_!(tail head,cycle)
    head

--% CNAGG functions

construct l ==
    -- copied from defaults to avoid loading defaults
    empty? l => empty()
    concat(first l, construct rest l)

--% ELTAGG functions

elt(x:%,n:I) ==
    -- copied from defaults to avoid loading defaults
    n < MIN or empty? x => error "elt: no such element"
    n = MIN => first x
    elt(rst x,n - 1)

```



```

seteltt: (%, I, S) -> S
seteltt(x, n, s) ==
  n = MIN => setfirst_!(x, s)
  seteltt(rst x, n - 1, s)

setelt(x, n: I, s: S) ==
  n < MIN or empty? x => error "setelt: no such element"
  x := expand_!(x, n - MIN + 1)
  seteltt(x, n, s)

--% IXAGG functions

removee: ((S -> Boolean), %) -> %
removee(p, x) == delay
  empty? x => empty()
  p(first x) => remove(p, rst x)
  concat(first x, remove(p, rst x))

remove(p, x) ==
  explicitlyEmpty? x => empty()
  eq?(x, rst x) =>
    p(first x) => empty()
    x
  removee(p, x)

selectt: ((S -> Boolean), %) -> %
selectt(p, x) == delay
  empty? x => empty()
  not p(first x) => select(p, rst x)
  concat(first x, select(p, rst x))

select(p, x) ==
  explicitlyEmpty? x => empty()
  eq?(x, rst x) =>
    p(first x) => x
    empty()
  selectt(p, x)

map(f, x) ==
  map(f, x pretend Stream(S))$StreamFunctions2(S, S) pretend %

map(g, x, y) ==
  xs := x pretend Stream(S); ys := y pretend Stream(S)
  map(g, xs, ys)$StreamFunctions3(S, S, S) pretend %

fill_!(x, s) ==
  setfirst_!(x, s)
  setrst_!(x, x)

map_!(f, x) ==

```

```

-- too many problems with map_! on a lazy stream, so
-- in this case, an error message is returned
cyclic? x =>
  tail := cycleTail x ; y := x
  until y = tail repeat
    setfirst_!(y,f first y)
  y := rst y
x
explicitlyFinite? x =>
  y := x
  while not empty? y repeat
    setfirst_!(y,f first y)
  y := rst y
x
error "map!: stream with lazy evaluation"

swap_!(x,m,n) ==
  (not index?(m,x)) or (not index?(n,x)) =>
    error "swap!: no such elements"
  x := expand_!(x,max(m,n) - MIN + 1)
  xm := elt(x,m); xn := elt(x,n)
  setelt(x,m,xn); setelt(x,n,xm)
x

--% LNAGG functions

concat(x:%,s:S) == delay
  empty? x => concat(s,empty())
  concat(first x,concat(rst x,s))

concat(x:%,y:%) == delay
  empty? x => copy y
  concat(first x,concat(rst x, y))

concat l == delay
  empty? l => empty()
  empty?(x := first l) => concat rest l
  concat(first x,concat(rst x,concat rest l))

setelt(x,seg:U,s:S) ==
  low := lo seg
  hasHi seg =>
    high := hi seg
    high < low => s
    (not index?(low,x)) or (not index?(high,x)) =>
      error "setelt: index out of range"
    x := expand_!(x,high - MIN + 1)
    y := rest(x,(low - MIN) :: NNI)
    for i in 0..(high-low) repeat
      setfirst_!(y,s)

```

```

        y := rst y
      s
    not index?(low,x) => error "setelt: index out of range"
    x := rest(x,(low - MIN) :: NNI)
    setrst_!(x,x)
    setfirst_!(x,s)

--% RCAGG functions

empty() == [NullStream, NIL$Lisp]

lazyEval x == (rst(x):(()-> %)) ()

lazyEvaluate x ==
  st := lazyEval x
  setfirst_!(x, first st)
  setrst_!(x,if EQ(rst st,st)$Lisp then x else rst st)
  x

-- empty? is the only function that explicitly causes evaluation
-- of a stream element
empty? x ==
  while lazy? x repeat
    st := lazyEval x
    setfirst_!(x, first st)
    setrst_!(x,if EQ(rst st,st)$Lisp then x else rst st)
  explicitlyEmpty? x

--setvalue(x,s) == setfirst_!(x,s)

--setchildren(x,l) ==
  --empty? l => error "setchildren: empty list of children"
  --not(empty? rest l) => error "setchildren: wrong number of children"
  --setrest_!(x,first l)

--% URAGG functions

first(x,n) == delay
-- former name: take
  n = 0 or empty? x => empty()
  (concat(first x, first(rst x,(n-1) :: NNI)))

concat(s:S,x:%) == [s,x]
cons(s,x) == concat(s,x)

cycleSplit_! x ==
  cycElt := cycleElt x
  cycElt case "failed" =>
    error "cycleSplit_!: non-cyclic stream"
  y := computeCycleEntry(x,cycElt :: %)

```

```

eq?(x,y) => (setToNil_! x; return y)
z := rst x
repeat
  eq?(y,z) => (setrest_!(x,empty()); return y)
  x := z ; z := rst z

expand_!(x,n) ==
-- expands cycles (if necessary) so that the first n
-- elements of x will not be part of a cycle
n < 1 => x
y := x
for i in 1..n while not empty? y repeat y := rst y
cycElt := cycleElt x
cycElt case "failed" => x
e := computeCycleEntry(x,cycElt :: %)
d : I := distance(x,e)
d >= n => x
if d = 0 then
  -- roll the cycle 1 entry
  d := 1
  t := cycleTail e
  if eq?(t,e) then
    t := concat(first t,empty())
    e := setrst_!(t,t)
    setrst_!(x,e)
  else
    setrst_!(t,concat(first e,rst e))
    e := rst e
nLessD := (n-d) :: NNI
y := complete first(e,nLessD)
e := rest(e,nLessD)
setrst_!(tail y,e)
setrst_!(rest(x,(d-1) :: NNI),y)
x

first x ==
empty? x => error "Can't take the first of an empty stream."
rst x

concat_!(x:%,y:%) ==
empty? x => y
setrst_!(tail x,y)

concat_!(x:%,s:S) ==
concat_!(x,concat(s,empty()))

setfirst_!(x,s) == setelt(x,0,s)
setelt(x,"first",s) == setfirst_!(x,s)
setrest_!(x,y) ==
empty? x => error "setrest!: empty stream"

```

```

    setrst_!(x,y)
    setelt(x,"rest",y) == setrest_!(x,y)

    setlast_!(x,s) ==
      empty? x => error "setlast!: empty stream"
      setfirst_!(tail x, s)
    setelt(x,"last",s) == setlast_!(x,s)

    split_!(x,n) ==
      n < MIN => error "split!: index out of range"
      n = MIN =>
        y : % := empty()
        setfirst_!(y,first x)
        setrst_!(y,rst x)
        setToNil_! x
      y
      x := expand_!(x,n - MIN)
      x := rest(x,(n - MIN - 1) :: NNI)
      y := rest x
      setrst_!(x,empty())
      y

--% STREAM functions

coerce(l: L S) == construct l

repeating l ==
  empty? l =>
    error "Need a non-null list to make a repeating stream."
  x0 : % := x := construct l
  while not empty? rst x repeat x := rst x
  setrst_!(x,x0)

if S has SetCategory then

  repeating?(l, x) ==
    empty? l =>
      error "Need a non-empty? list to make a repeating stream."
    empty? rest l =>
      not empty? x and first x = first l and x = rst x
    x0 := x
    for s in l repeat
      empty? x or s ^!= first x => return false
      x := rst x
    eq?(x,x0)

  findCycle(n, x) ==
    hd := x
    -- Determine whether periodic within n.
    tl := rest(x, n)

```

```

explicitlyEmpty? tl => [false, 0, 0]
i := 0; while not eq?(x,tl) repeat (x := rst x; i := i + 1)
i = n => [false, 0, 0]
-- Find period. Now x=tl, so step over and find it again.
x := rst x; per := 1
while not eq?(x,tl) repeat (x := rst x; per := per + 1)
-- Find non-periodic part.
x := hd; xp := rest(hd, per); npp := 0
while not eq?(x,xp) repeat (x := rst x; xp := rst xp; npp := npp+1)
[true, npp, per]

delay(fs:()->%) == [NonNullStream, fs pretend %]

--      explicitlyEmpty? x == markedNull? x

explicitEntries? x ==
  not explicitlyEmpty? x and not lazy? x

numberOfComputedEntries x ==
  explicitEntries? x => numberOfComputedEntries(rst x) + 1
  0

if S has SetCategory then

  output(n,x) ==
    (not(n>0))or empty? x => void()
    mathPrint(first(x)::OUT)$Lisp
    output(n-1, rst x)

  setrestt_!(x,n,y) ==
    n = 0 => setrst_!(x,y)
    setrestt_!(rst x,n-1,y)

  setrest_!(x,n,y) ==
    n < 0 or empty? x => error "setrest!: no such rest"
    x := expand_!(x,n+1)
    setrestt_!(x,n,y)

  generate f    == delay concat(f(), generate f)
  gen:(S -> S,S) -> %
  gen(f,s) == delay(ss:=f s; concat(ss, gen(f,ss)))
  generate(f,s)==concat(s,gen(f,s))

  concat(x:%,y:%) ==delay
    empty? x => y
    concat(first x,concat(rst x,y))

  swhilee:(S -> Boolean,%) -> %
  swhilee(p,x) == delay
    empty? x      => empty()

```

```

    not p(first x) => empty()
    concat(first x,filterWhile(p,rst x))
filterWhile(p,x)==
    explicitlyEmpty? x => empty()
    eq?(x,rst x) =>
        p(first x) => x
        empty()
    swhilee(p,x)

suntill: (S -> Boolean,%) -> %
suntill(p,x) == delay
    empty? x => empty()
    p(first x) => concat(first x,empty())
    concat(first x, filterUntil(p, rst x))

filterUntil(p,x)==
    explicitlyEmpty? x => empty()
    eq?(x,rst x) =>
        p(first x) => concat(first x,empty())
        x
    suntill(p,x)

-- if S has SetCategory then
-- mapp: ((S,S) -> S,%,%,S) -> %
-- mapp(f,x,y,a) == delay
--     empty? x or empty? y => empty()
--     concat(f(first x,first y), map(f,rst x,rst y,a))
-- map(f,x,y,a) ==
--     explicitlyEmpty? x => empty()
--     eq?(x,rst x) =>
--         frst x=a => y
--         map(f(first x,#1),y)
--     explicitlyEmpty? y => empty()
--     eq?(y,rst y) =>
--         frst y=a => x
--         p(f(#1,first y),x)
-- mapp(f,x,y,a)

```

---

— STREAM.dotabb —

```

"STREAM" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STREAM"]
"LZSTAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=LZSTAGG"]
"STREAM" -> "LZSTAGG"

```

---

— String.input —

[illegible]





```
--S 12 of 35
concat("hello ", "goodbye")
--R
--R
--R (12) "hello goodbye"
--R
--R                                          Type: String
--E 12

--S 13 of 35
"This " "is " "several " "strings " "concatenated."
--R
--R
--R (13) "This is several strings concatenated."
--R
--R                                          Type: String
--E 13

--S 14 of 35
hello(1..5)
--R
--R
--R (14) "Hello"
--R
--R                                          Type: String
--E 14

--S 15 of 35
hello(8..)
--R
--R
--R (15) "I'm AXIOM!"
--R
--R                                          Type: String
--E 15

--S 16 of 35
split(hello, char " ")
--R
--R
--R (16) ["Hello,", "I'm", "AXIOM!"]
--R
--R                                          Type: List String
--E 16

--S 17 of 35
other := complement alphanumeric();
--R
--R
--R                                          Type: CharacterClass
--E 17

--S 18 of 35
split(said saw, other)
--R
--R
```

```

--R (18) ["alpha","omega"]
--R
--E 18
Type: List String

--S 19 of 35
trim("## ++ relax ++ ##", char "#")
--R
--R
--R (19) " ++ relax ++ "
--R
--E 19
Type: String

--S 20 of 35
trim("## ++ relax ++ ##", other)
--R
--R
--R (20) "relax"
--R
--E 20
Type: String

--S 21 of 35
leftTrim("## ++ relax ++ ##", other)
--R
--R
--R (21) "relax ++ ##"
--R
--E 21
Type: String

--S 22 of 35
rightTrim("## ++ relax ++ ##", other)
--R
--R
--R (22) "## ++ relax"
--R
--E 22
Type: String

--S 23 of 35
upperCase hello
--R
--R
--R (23) "HELLO, I'M AXIOM!"
--R
--E 23
Type: String

--S 24 of 35
lowerCase hello
--R
--R
--R (24) "hello, i'm axiom!"
--R
--E 24
Type: String

```



```

--S 31 of 35
n := position("nd", "underground", 1)
--R
--R
--R (31) 2
--R
--R                                         Type: PositiveInteger
--E 31

--S 32 of 35
n := position("nd", "underground", n+1)
--R
--R
--R (32) 10
--R
--R                                         Type: PositiveInteger
--E 32

--S 33 of 35
n := position("nd", "underground", n+1)
--R
--R
--R (33) 0
--R
--R                                         Type: NonNegativeInteger
--E 33

--S 34 of 35
position(char "d", "underground", 1)
--R
--R
--R (34) 3
--R
--R                                         Type: PositiveInteger
--E 34

--S 35 of 35
position(hexDigit(), "underground", 1)
--R
--R
--R (35) 3
--R
--R                                         Type: PositiveInteger
--E 35
)spool
)lisp (bye)

```

---

— String.help —

```

=====
String examples
=====

```

The type `String` provides character strings. Character strings provide all the operations for a one-dimensional array of characters, plus additional operations for manipulating text.

String values can be created using double quotes.

```
hello := "Hello, I'm AXIOM!"
      "Hello, I'm AXIOM!"
                                     Type: String
```

Note, however, that double quotes and underscores must be preceded by an extra underscore.

```
said := "Jane said, \"Look!\"_"
      "Jane said, \"Look!\"_"
                                     Type: String
```

```
saw := "She saw exactly one underscore: \"_\"_"
      "She saw exactly one underscore: \"_\"_"
                                     Type: String
```

It is also possible to use `new` to create a string of any size filled with a given character. Since there are many new functions it is necessary to indicate the desired type.

```
gasp: String := new(32, char "x")
      "xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx"
                                     Type: String
```

The length of a string is given by `#`.

```
#gasp
32
                                     Type: PositiveInteger
```

Indexing operations allow characters to be extracted or replaced in strings. For any string `s`, indices lie in the range `1..#s`.

```
hello.2
e
                                     Type: Character
```

Indexing is really just the application of a string to a subscript, so any application syntax works.

```
hello 2
e
                                     Type: Character
```

```
hello(2)
e
```

Type: Character

If it is important not to modify a given string, it should be copied before any updating operations are used.

```
hullo := copy hello
"Hello, I'm AXIOM!"
```

Type: String

```
hullo.2 := char "u"; [hello, hullo]
["Hello, I'm AXIOM!", "Hullo, I'm AXIOM!"]
```

Type: List String

Operations are provided to split and join strings. The concat operation allows several strings to be joined together.

```
sawdust := concat ["alpha", "---", "omega"]
"alpha---omega"
```

Type: String

There is a version of concat that works with two strings.

```
concat("hello ", "goodbye")
"hello goodbye"
```

Type: String

Juxtaposition can also be used to concatenate strings.

```
"This " "is " "several " "strings " "concatenated."
"This is several strings concatenated."
```

Type: String

Substrings are obtained by giving an index range.

```
hello(1..5)
"Hello"
```

Type: String

```
hello(8..)
"I'm AXIOM!"
```

Type: String

A string can be split into several substrings by giving a separation character or character class.

```
split(hello, char " ")
["Hello, ", "I'm ", "AXIOM!"]
```

Type: List String

```
other := complement alphanumeric();
                                Type: CharacterClass
```

```
split(said saw, other)
["alpha", "omega"]
                                Type: List String
```

Unwanted characters can be trimmed from the beginning or end of a string using the operations `trim`, `leftTrim` and `rightTrim`.

```
trim("## ++ relax ++ ##", char "#")
" ++ relax ++ "
                                Type: String
```

Each of these functions takes a string and a second argument to specify the characters to be discarded.

```
trim("## ++ relax ++ ##", other)
"relax"
                                Type: String
```

The second argument can be given either as a single character or as a character class.

```
leftTrim("## ++ relax ++ ##", other)
"relax ++ ##"
                                Type: String
```

```
rightTrim("## ++ relax ++ ##", other)
"## ++ relax"
                                Type: String
```

Strings can be changed to upper case or lower case using the operations `upperCase` and `lowerCase`.

```
upperCase hello
"HELLO, I'M AXIOM!"
                                Type: String
```

The versions with the exclamation mark change the original string, while the others produce a copy.

```
lowerCase hello
"hello, i'm axiom!"
                                Type: String
```

Some basic string matching is provided. The function `prefix?` tests whether one string is an initial prefix of another.



```

prefix?("He", "Hello")
  true
                                Type: Boolean

```

```

prefix?("Her", "Hello")
  false
                                Type: Boolean

```

A similar function, `suffix?`, tests for suffixes.

```

suffix?("", "Hello")
  true
                                Type: Boolean

```

```

suffix?("LO", "Hello")
  false
                                Type: Boolean

```

The function `substring?` tests for a substring given a starting position.

```

substring?("ll", "Hello", 3)
  true
                                Type: Boolean

```

```

substring?("ll", "Hello", 4)
  false
                                Type: Boolean

```

A number of position functions locate things in strings. If the first argument to `position` is a string, then `position(s,t,i)` finds the location of `s` as a substring of `t` starting the search at position `i`.

```

n := position("nd", "underground", 1)
  2
                                Type: PositiveInteger

```

```

n := position("nd", "underground", n+1)
  10
                                Type: PositiveInteger

```

If `s` is not found, then 0 is returned (`minIndex(s)-1` in `IndexedString`).

```

n := position("nd", "underground", n+1)
  0
                                Type: NonNegativeInteger

```

To search for a specific character or a member of a character class, a different first argument is used.

```

position(char "d", "underground", 1)

```

3

Type: PositiveInteger

```
position(hexDigit(), "underground", 1)
```

3

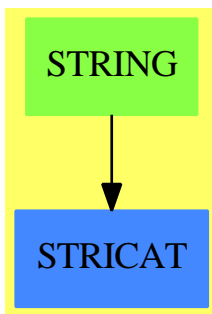
Type: PositiveInteger

See Also:

- o )help Character
- o )help CharacterClass
- o )show String

—————

### 20.31.1 String (STRING)



See

- ⇒ “Character” (CHAR) 4.3.1 on page 357
- ⇒ “CharacterClass” (CCLASS) 4.4.1 on page 365
- ⇒ “IndexedString” (ISTRING) 10.14.1 on page 1214

**Exports:**

|          |           |           |            |                  |
|----------|-----------|-----------|------------|------------------|
| any?     | coerce    | concat    | construct  | convert          |
| copy     | copyInto! | count     | delete     | elt              |
| empty    | empty?    | entries   | entry?     | eq?              |
| eval     | every?    | fill!     | find       | first            |
| hash     | index?    | indices   | insert     | latex            |
| leftTrim | less?     | lowerCase | lowerCase! | map              |
| map!     | match     | match?    | max        | maxIndex         |
| member?  | members   | merge     | min        | minIndex         |
| more?    | new       | OMwrite   | parts      | position         |
| prefix?  | qelt      | qsetelt!  | reduce     | removeDuplicates |
| replace  | reverse   | reverse!  | rightTrim  | sample           |
| select   | setelt    | size?     | sort       | sort!            |
| sorted?  | split     | string    | substring? | suffix?          |
| swap!    | trim      | upperCase | upperCase! | #?               |
| ?=?      | ?.?       | ?~=?      | ?<?        | ?<=?             |
| ?>?      | ?>=?      |           |            |                  |

— domain **STRING** String —

```

)abbrev domain STRING String
++ Author: Mark Botch
++ Description:
++ This is the domain of character strings. Strings are 1 based.

String(): StringCategory == IndexedString(1) add
  string n == STRINGIMAGE(n)$Lisp

OMwrite(x: %): String ==
  s: String := ""
  sp := OM_-STRINGTOSTRINGPTR(s)$Lisp
  dev: OpenMathDevice := OMopenString(sp pretend String, OMencodingXML)
  OMputObject(dev)
  OMputString(dev, x pretend String)
  OMputEndObject(dev)
  OMclose(dev)
  s := OM_-STRINGPTRTOSTRING(sp)$Lisp pretend String
  s

OMwrite(x: %, wholeObj: Boolean): String ==
  s: String := ""
  sp := OM_-STRINGTOSTRINGPTR(s)$Lisp
  dev: OpenMathDevice := OMopenString(sp pretend String, OMencodingXML)
  if wholeObj then
    OMputObject(dev)
  OMputString(dev, x pretend String)
  if wholeObj then
    OMputEndObject(dev)
  OMclose(dev)

```

```

s := OM_-STRINGPTRTOSTRING(sp)$Lisp pretend String
s

OMwrite(dev: OpenMathDevice, x: %): Void ==
  OMputObject(dev)
  OMputString(dev, x pretend String)
  OMputEndObject(dev)

OMwrite(dev: OpenMathDevice, x: %, wholeObj: Boolean): Void ==
  if wholeObj then
    OMputObject(dev)
  OMputString(dev, x pretend String)
  if wholeObj then
    OMputEndObject(dev)

-----

— STRING.dotabb —

"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"STRICAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=STRICAT"]
"STRING" -> "STRICAT"

-----

```

## 20.32 domain STRTBL StringTable

```

----- StringTable.input -----

)set break resume
)sys rm -f StringTable.output
)spool StringTable.output
)set message test on
)set message auto off
)clear all
--S 1 of 3
t: StringTable(Integer) := table()
--R
--R
--R (1) table()
--R
--R                                         Type: StringTable Integer
--E 1

--S 2 of 3

```

```

for s in split("My name is Ian Watt.",char " ")
  repeat
    t.s := #s
--R
--R
--R                                          Type: Void
--E 2

--S 3 of 3
for key in keys t repeat output [key, t.key]
--R
--R      ["Watt.",5]
--R      ["Ian",3]
--R      ["is",2]
--R      ["name",4]
--R      ["My",2]
--R
--R                                          Type: Void
--E 3
)spool
)lisp (bye)

```

---

— StringTable.help —

```

=====
StringTable examples
=====

```

This domain provides a table type in which the keys are known to be strings so special techniques can be used. Other than performance, the type `StringTable(S)` should behave exactly the same way as `Table(String,S)`.

This creates a new table whose keys are strings.

```

t: StringTable(Integer) := table()
table()
                                Type: StringTable Integer

```

The value associated with each string key is the number of characters in the string.

```

for s in split("My name is Ian Watt.",char " ")
  repeat
    t.s := #s
                                Type: Void

for key in keys t repeat output [key, t.key]
["Watt.",5]
["Ian",3]

```

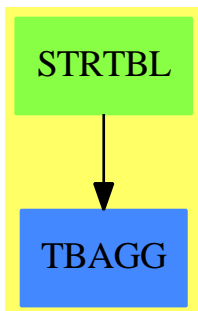
```
["is",2]
["name",4]
["My",2]
```

Type: Void

See Also:

- o )help Table
- o )show StringTable

### 20.32.1 StringTable (STRTBL)



See

- ⇒ “HashTable” (HASHTBL) 9.1.1 on page 1085
- ⇒ “InnerTable” (INTABL) 10.27.1 on page 1299
- ⇒ “Table” (TABLE) 21.1.1 on page 2621
- ⇒ “EqTable” (EQTBL) 6.2.1 on page 667
- ⇒ “GeneralSparseTable” (GSTBL) 8.5.1 on page 1044
- ⇒ “SparseTable” (STBL) 20.16.1 on page 2409

#### Exports:

|        |          |                  |           |          |
|--------|----------|------------------|-----------|----------|
| any?   | bag      | coerce           | construct | convert  |
| copy   | count    | dictionary       | elt       | empty    |
| empty? | entries  | entry?           | eq?       | eval     |
| every? | extract! | fill!            | find      | first    |
| hash   | index?   | indices          | insert!   | inspect  |
| key?   | keys     | latex            | less?     | map      |
| map!   | maxIndex | member?          | members   | minIndex |
| more?  | parts    | qelt             | qsetelt!  | reduce   |
| remove | remove!  | removeDuplicates | sample    | search   |
| select | select!  | setelt           | size?     | swap!    |
| table  | #?       | ?=?              | ?~=?      | ?..?     |

---

— domain STRTBL StringTable —

```

)abbrev domain STRTBL StringTable
++ Author: Stephen M. Watt
++ Date Created:
++ Date Last Updated: June 21, 1991
++ Basic Operations:
++ Related Domains: Table
++ Also See:
++ AMS Classifications:
++ Keywords: equation
++ Examples:
++ References:
++ Description:
++ This domain provides tables where the keys are strings.
++ A specialized hash function for strings is used.

StringTable(Entry: SetCategory) ==
  HashTable(String, Entry, "CVEC")

```

---

— STRTBL.dotabb —

```

"STRTBL" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRTBL"]
"TBAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=TBAGG"]
"STRTBL" -> "TBAGG"

```

---

### 20.33 domain SUBSPACE SubSpace

The first argument  $n$  is the dimension of the subSpace

The SubSpace domain is implemented as a tree. The root of the tree is the only node in which the field `dataList` - which points to a list of points over the ring,  $R$  - is defined. The children of the root are the top level components of the SubSpace (in 2D, these would be separate curves; in 3D, these would be separate surfaces).

The `pt` field is only defined in the leaves.

By way of example, consider a three dimensional subspace with two components - a three by three grid and a sphere. The internal representation of this subspace is a tree with a depth of three.

The root holds a list of all the points used in the subspace (so, if the grid and the sphere share points, the shared points would not be represented redundantly but would be referenced by index).

The root, in this case, has two children - the first points to the grid component and the second to the sphere component. The grid child has four children of its own - a 3x3 grid has 4 endpoints - and each of these point to a list of four points. To see it another way, the grid (child of the root) holds a list of line components which, when placed one above the next, forms a grid. Each of these line components is a list of points.

Points could be explicitly added to subspaces at any level. A path could be given as an argument to the `addPoint()` function. It is a list of `NonNegativeIntegers` and refers, in order, to the *n*-th child of the current node. For example,

```
addPoint(s,[2,3],p)
```

would add the point *p* to the subspace *s* by going to the second child of the root and then the third child of that node. If the path does extend to the full depth of the tree, nodes are automatically added so that the tree is of constant depth down any path. By not specifying the full path, new components could be added - e.g. for *s* from `SubSpace(3,Float)`

```
addPoint(s,[],p)
```

would create a new child to the root (a new component in *N*-space) and extend a path to a leaf of depth 3 that points to the data held in *p*. The subspace *s* would now have a new component which has one child which, in turn, has one child (the leaf). The new component is then a point.

#### — SubSpace.input —

```
)set break resume
)sys rm -f SubSpace.output
)spool SubSpace.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SubSpace
--R SubSpace(n: PositiveInteger,R: Ring) is a domain constructor
--R Abbreviation for SubSpace is SUBSPACE
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SUBSPACE
--R
--R----- Operations -----
--R ==? : (%,% ) -> Boolean          addPoint2 : (% ,Point R) -> %
--R birth : % -> %                  children : % -> List %
--R coerce : % -> OutputForm        deepCopy : % -> %
--R extractClosed : % -> Boolean     extractPoint : % -> Point R
```



```

--R hash : % -> SingleInteger          internal? : % -> Boolean
--R latex : % -> String                leaf? : % -> Boolean
--R level : % -> NonNegativeInteger    merge : List % -> %
--R merge : (%,%) -> %                 new : () -> %
--R parent : % -> %                    pointData : % -> List Point R
--R root? : % -> Boolean                separate : % -> List %
--R shallowCopy : % -> %                subspace : () -> %
--R ?~=? : (%,%) -> Boolean
--R addPoint : (%,Point R) -> NonNegativeInteger
--R addPoint : (%,List NonNegativeInteger,NonNegativeInteger) -> %
--R addPoint : (%,List NonNegativeInteger,Point R) -> %
--R addPointLast : (%,%,Point R,NonNegativeInteger) -> %
--R child : (%,NonNegativeInteger) -> %
--R closeComponent : (%,List NonNegativeInteger,Boolean) -> %
--R defineProperty : (%,List NonNegativeInteger,SubSpaceComponentProperty) -> %
--R extractIndex : % -> NonNegativeInteger
--R extractProperty : % -> SubSpaceComponentProperty
--R modifyPoint : (%,NonNegativeInteger,Point R) -> %
--R modifyPoint : (%,List NonNegativeInteger,NonNegativeInteger) -> %
--R modifyPoint : (%,List NonNegativeInteger,Point R) -> %
--R numberOfChildren : % -> NonNegativeInteger
--R traverse : (%,List NonNegativeInteger) -> %
--R
--E 1

```

```

)spool
)lisp (bye)

```

---

— SubSpace.help —

```

=====
SubSpace examples
=====

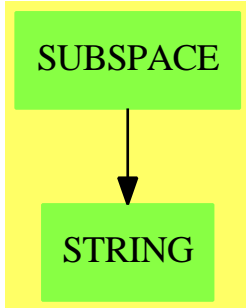
```

```

See Also:
o )show SubSpace

```

## 20.33.1 SubSpace (SUBSPACE)



See

⇒ “Point” (POINT) 17.24.1 on page 2019

⇒ “SubSpaceComponentProperty” (COMPPROP) 20.34.1 on page 2583

**Exports:**

|               |                |              |                  |                |
|---------------|----------------|--------------|------------------|----------------|
| addPoint      | addPointLast   | addPoint2    | birth            | child          |
| children      | closeComponent | coerce       | deepCopy         | defineProperty |
| extractClosed | extractIndex   | extractPoint | extractProperty  | hash           |
| internal?     | latex          | leaf?        | level            | merge          |
| merge         | modifyPoint    | new          | numberOfChildren | parent         |
| pointData     | root?          | separate     | shallowCopy      | subspace       |
| traverse      | ? =?           | ? =?         |                  |                |

— domain SUBSPACE SubSpace —

```
)abbrev domain SUBSPACE SubSpace
```

```
++ Author: Mark Botch
```

```
++ Description:
```

```
++ This domain is not documented
```

```
SubSpace(n:PI,R:Ring) : Exports == Implementation where
```

```
  I ==> Integer
```

```
  PI ==> PositiveInteger
```

```
  NNI ==> NonNegativeInteger
```

```
  L ==> List
```

```
  B ==> Boolean
```

```
  POINT ==> Point(R)
```

```
  PROP ==> SubSpaceComponentProperty()
```

```
  S ==> String
```

```
  O ==> OutputForm
```

```
  empty ==> nil -- macro to ease conversion to new aggcats.spad
```

```
Exports ==> SetCategory with
```

```
  leaf? : % -> B
```

```
    ++ leaf?(x) is not documented
```

```
  root? : % -> B
```

```

++ root?(x) is not documented
internal?      : % -> B
++ internal?(x) is not documented
new           : () -> %
++ new() is not documented
subspace      : () -> %
++ subspace() is not documented
birth         : % -> %      -- returns a pointer to the baby
++ birth(x) is not documented
child         : (%,NNI) -> %
++ child(x,n) is not documented
children      : % -> List %
++ children(x) is not documented
numberOfChildren: % -> NNI
++ numberOfChildren(x) is not documented
shallowCopy   : % -> %
++ shallowCopy(x) is not documented
deepCopy      : % -> %
++ deepCopy(x) is not documented
merge         : (%,%) -> %
++ merge(s1,s2) the subspaces s1 and s2 into a single subspace.
merge         : List % -> %
++ merge(ls) a list of subspaces, ls, into one subspace.
separate      : % -> List %
++ separate(s) makes each of the components of the \spadtype{SubSpace},
++ s, into a list of separate and distinct subspaces and returns
++ the list.
addPoint      : (%,List NNI,POINT) -> %
++ addPoint(s,li,p) adds the 4 dimensional point, p, to the 3
++ dimensional subspace, s. The list of non negative integers, li,
++ dictates the path to follow, or, to look at it another way,
++ points to the component in which the point is to be added. It's
++ length should range from 0 to \spad{n - 1} where n is the dimension
++ of the subspace. If the length is \spad{n - 1}, then a specific
++ lowest level component is being referenced. If it is less than
++ \spad{n - 1}, then some higher level component (0 indicates top
++ level component) is being referenced and a component of that level
++ with the desired point is created. The subspace s is returned
++ with the additional point.
addPoint2     : (%,POINT) -> %
++ addPoint2(s,p) adds the 4 dimensional point, p, to the 3
++ dimensional subspace, s.
++ The subspace s is returned with the additional point.
addPointLast  : (%,%,POINT, NNI) -> %
++ addPointLast(s,s2,li,p) adds the 4 dimensional point, p, to the 3
++ dimensional subspace, s. s2 point to the end of the subspace
++ s. n is the path in the s2 component.
++ The subspace s is returned with the additional point.
modifyPoint   : (%,List NNI,POINT) -> %
++ modifyPoint(s,li,p) replaces an existing point in the 3 dimensional

```

```

++ subspace, s, with the 4 dimensional point, p. The list of non
++ negative integers, li, dictates the path to follow, or, to look at
++ it another way, points to the component in which the existing point
++ is to be modified. An error message occurs if s is empty, otherwise
++ the subspace s is returned with the point modification.
addPoint      : (%,List NNI,NNI) -> %
++ addPoint(s,li,i) adds the 4 dimensional point indicated by the
++ index location, i, to the 3 dimensional subspace, s. The list of
++ non negative integers, li, dictates the path to follow, or, to
++ look at it another way, points to the component in which the point
++ is to be added. It's length should range from 0 to \spad{n - 1}
++ where n is the dimension of the subspace. If the length is
++ \spad{n - 1}, then a specific lowest level component is being
++ referenced. If it is less than \spad{n - 1}, then some higher
++ level component (0 indicates top level component) is being
++ referenced and a component of that level with the desired point
++ is created. The subspace s is returned with the additional point.
modifyPoint   : (%,List NNI,NNI) -> %
++ modifyPoint(s,li,i) replaces an existing point in the 3 dimensional
++ subspace, s, with the 4 dimensional point indicated by the index
++ location, i. The list of non negative integers, li, dictates
++ the path to follow, or, to look at it another way, points to the
++ component in which the existing point is to be modified. An error
++ message occurs if s is empty, otherwise the subspace s is returned
++ with the point modification.
addPoint      : (%,POINT) -> NNI
++ addPoint(s,p) adds the point, p, to the 3 dimensional subspace, s,
++ and returns the new total number of points in s.
modifyPoint   : (%,NNI,POINT) -> %
++ modifyPoint(s,ind,p) modifies the point referenced by the index
++ location, ind, by replacing it with the point, p in the 3 dimensional
++ subspace, s. An error message occurs if s is empty, otherwise the
++ subspace s is returned with the point modification.

closeComponent : (%,List NNI,B)      -> %
++ closeComponent(s,li,b) sets the property of the component in the
++ 3 dimensional subspace, s, to be closed if b is true, or open if
++ b is false. The list of non negative integers, li, dictates the
++ path to follow, or, to look at it another way, points to the
++ component whose closed property is to be set. The subspace, s,
++ is returned with the component property modification.
defineProperty : (%,List NNI,PROP) -> %
++ defineProperty(s,li,p) defines the component property in the
++ 3 dimensional subspace, s, to be that of p, where p is of the
++ domain \spadtype{SubSpaceComponentProperty}. The list of non
++ negative integers, li, dictates the path to follow, or, to look
++ at it another way, points to the component whose property is
++ being defined. The subspace, s, is returned with the component
++ property definition.
traverse      : (%,List NNI) -> %

```

```

++ traverse(s,li) follows the branch list of the 3 dimensional
++ subspace, s, along the path dictated by the list of non negative
++ integers, li, which points to the component which has been
++ traversed to. The subspace, s, is returned, where s is now
++ the subspace pointed to by li.
extractPoint : % -> POINT
++ extractPoint(s) returns the point which is given by the current
++ index location into the point data field of the 3 dimensional
++ subspace s.
extractIndex : % -> NNI
++ extractIndex(s) returns a non negative integer which is the current
++ index of the 3 dimensional subspace s.
extractClosed : % -> B
++ extractClosed(s) returns the \spadtype{Boolean} value of the closed
++ property for the indicated 3 dimensional subspace s. If the
++ property is closed, \spad{True} is returned, otherwise \spad{False}
++ is returned.
extractProperty : % -> PROP
++ extractProperty(s) returns the property of domain
++ \spadtype{SubSpaceComponentProperty} of the indicated 3 dimensional
++ subspace s.
level : % -> NNI
++ level(s) returns a non negative integer which is the current
++ level field of the indicated 3 dimensional subspace s.
parent : % -> %
++ parent(s) returns the subspace which is the parent of the indicated
++ 3 dimensional subspace s. If s is the top level subspace an error
++ message is returned.
pointData : % -> L POINT
++ pointData(s) returns the list of points from the point data field
++ of the 3 dimensional subspace s.

Implementation ==> add
import String()

Rep := Record(pt:POINT, index:NNI, property:PROP, _
              childrenField:List %, _
              lastChild: List %, _
              levelField:NNI, _
              pointDataField:L POINT, _
              lastPoint: L POINT, _
              noPoints: NNI, _
              noChildren: NNI, _
              parentField:List %) -- needn't be list but...base case?

TELLWATT : String := "Non-null list: Please inform Stephen Watt"

leaf? space == empty? children space
root? space == (space.levelField = 0$NNI)
internal? space == ^(root? space and leaf? space)

```

```

new() ==
  [point(empty())$POINT,0,new()$PROP,empty(),empty(),0,_
    empty(),empty(),0,0,empty()]]
subspace() == new()

birth momma ==
  baby := new()
  baby.levelField := momma.levelField+1
  baby.parentField := [momma]
  if not empty?(lastKid := momma.lastChild) then
    not empty? rest lastKid => error TELLWATT
  if empty? lastKid
  then
    momma.childrenField := [baby]
    momma.lastChild := momma.childrenField
    momma.noChildren := 1
  else
    setrest_!(lastKid,[baby])
    momma.lastChild := rest lastKid
    momma.noChildren := momma.noChildren + 1
  baby

child(space,num) ==
  space.childrenField.num

children space == space.childrenField
numberOfChildren space == space.noChildren

shallowCopy space ==
  node := new()
  node.pt      := space.pt
  node.index   := space.index
  node.property := copy(space.property)
  node.levelField := space.levelField
  node.parentField := nil()
  if root? space then
    node.pointDataField := copy(space.pointDataField)
    node.lastPoint := tail(node.pointDataField)
    node.noPoints := space.noPoints
  node

deepCopy space ==
  node := shallowCopy(space)
  leaf? space => node
  for c in children space repeat
    cc := deepCopy c
    cc.parentField := [node]
    node.childrenField := cons(cc,node.childrenField)
  node.childrenField := reverse_!(node.childrenField)

```

```

node.lastChild := tail node.childrenField
node

merge(s1,s2) ==
----- need to worry about reindexing s2 & parentField
n1 : Rep := deepCopy s1
n2 : Rep := deepCopy s2
n1.childrenField := append(children n1,children n2)
n1

merge listOfSpaces ==
----- need to worry about reindexing & parentField
empty? listOfSpaces => error "empty list passed as argument to merge"
-- notice that the properties of the first subspace on the
-- list are the ones that are inherited...hmmmm...
space := deepCopy first listOfSpaces
for s in rest listOfSpaces repeat
-- because of the initial deepCopy, above, everything is
-- deepCopied to be consistent...more hmmm...
space.childrenField := append(space.childrenField,[deepCopy c for c in s.childrenField])
space

separate space ==
----- need to worry about reindexing & parentField
spaceList := empty()
for s in space.childrenField repeat
  spc:=shallowCopy space
  spc.childrenField:=[deepCopy s]
  spaceList := cons(spc,spaceList)
spaceList

addPoint(space:%,path:List NNI,point:POINT) ==
if not empty?(lastPt := space.lastPoint) then
  not empty? rest lastPt => error TELLWATT
if empty? lastPt
then
  space.pointDataField := [point]
  space.lastPoint := space.pointDataField
else
  setrest_!(lastPt,[point])
  space.lastPoint := rest lastPt
space.noPoints := space.noPoints + 1
which := space.noPoints
node := space
depth : NNI := 0
for i in path repeat
  node := child(node,i)
  depth := depth + 1
for more in depth..(n-1) repeat
  node := birth node

```

```

node.pt := point      -- will be obsolete field
node.index := which
space

addPoint2(space:%,point:POINT) ==
  if not empty?(lastPt := space.lastPoint) then
    not empty? rest lastPt => error TELLWATT
  if empty? lastPt
  then
    space.pointDataField := [point]
    space.lastPoint := space.pointDataField
  else
    setrest_!(lastPt,[point])
    space.lastPoint := rest lastPt
  space.noPoints := space.noPoints + 1
  which := space.noPoints
  node := space
  depth : NNI := 0
  node := birth node
  first := node
  for more in 1..n-1 repeat
    node := birth node
  node.pt := point      -- will be obsolete field
  node.index := which
  first

addPointLast(space:%,node:%, point:POINT, depth:NNI) ==
  if not empty?(lastPt := space.lastPoint) then
    not empty? rest lastPt => error TELLWATT
  if empty? lastPt
  then
    space.pointDataField := [point]
    space.lastPoint := space.pointDataField
  else
    setrest_!(lastPt,[point])
    space.lastPoint := rest lastPt
  space.noPoints := space.noPoints + 1
  which := space.noPoints
  if depth = 2 then node := child(node, 2)
  for more in depth..(n-1) repeat
    node := birth node
  node.pt := point      -- will be obsolete field
  node.index := which
  node -- space

addPoint(space:%,path:List NNI,which:NNI) ==
  node := space
  depth : NNI := 0
  for i in path repeat
    node := child(node,i)

```



```

    depth := depth + 1
  for more in depth..(n-1) repeat
    node := birth node
    node.pt := space.pointDataField.which  -- will be obsolete field
    node.index := which
  space

addPoint(space:%,point:POINT) ==
  root? space =>
    if not empty?(lastPt := space.lastPoint) then
      not empty? rest lastPt => error TELLWATT
    if empty? lastPt
    then
      space.pointDataField := [point]
      space.lastPoint := space.pointDataField
    else
      setrest_!(lastPt,[point])
      space.lastPoint := rest lastPt
    space.noPoints := space.noPoints + 1
  error "You need to pass a top level SubSpace (level should be zero)"

modifyPoint(space:%,path:List NNI,point:POINT) ==
  if not empty?(lastPt := space.lastPoint) then
    not empty? rest lastPt => error TELLWATT
  if empty? lastPt
  then
    space.pointDataField := [point]
    space.lastPoint := space.pointDataField
  else
    setrest_!(lastPt,[point])
    space.lastPoint := rest lastPt
  space.noPoints := space.noPoints + 1
  which := space.noPoints
  node := space
  for i in path repeat
    node := child(node,i)
  node.pt := point  ----- will be obsolete field
  node.index := which
  space

modifyPoint(space:%,path:List NNI,which:NNI) ==
  node := space
  for i in path repeat
    node := child(node,i)
  node.pt := space.pointDataField.which ----- will be obsolete field
  node.index := which
  space

modifyPoint(space:%,which:NNI,point:POINT) ==
  root? space =>

```

```

    space.pointDataField.which := point
    space
    error "You need to pass a top level SubSpace (level should be zero)"

closeComponent(space,path,val) ==
  node := space
  for i in path repeat
    node := child(node,i)
  close(node.property,val)
  space

defineProperty(space,path,prop) ==
  node := space
  for i in path repeat
    node := child(node,i)
  node.property := prop
  space

traverse(space,path) ==
  for i in path repeat space := child(space,i)
  space

extractPoint space ==
  node := space
  while ^root? node repeat node := parent node
  (node.pointDataField).(space.index)
extractIndex space == space.index
extractClosed space == closed? space.property
extractProperty space == space.property

parent space ==
  empty? space.parentField => error "This is a top level SubSpace - it does not have a parent"
  first space.parentField
pointData space == space.pointDataField
level space == space.levelField
s1 = s2 ==
  ----- extra checks for list of point data
  (leaf? s1 and leaf? s2) =>
    (s1.pt = s2.pt) and (s1.property = s2.property) and (s1.levelField = s2.levelField)
  -- note that the ordering of children is important
  #s1.childrenField ^= #s2.childrenField => false
  and/[c1 = c2 for c1 in s1.childrenField for c2 in s2.childrenField]
  and (s1.property = s2.property) and (s1.levelField = s2.levelField)
coerce(space:):0 ==
  hconcat([n::0,"-Space with depth of "::0,
    (n - space.levelField)::0," and "::0,(s:=(#space.childrenField))::0,
    (s1 => " component "::0;" components "::0)])

```

---

— SUBSPACE.dotabb —

```
"SUBSPACE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SUBSPACE"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"SUBSPACE" -> "STRING"
```

—————

## 20.34 domain COMPPROP SubSpaceComponentProperty

— SubSpaceComponentProperty.input —

```
)set break resume
)sys rm -f SubSpaceComponentProperty.output
)spool SubSpaceComponentProperty.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SubSpaceComponentProperty
--R SubSpaceComponentProperty is a domain constructor
--R Abbreviation for SubSpaceComponentProperty is COMPPROP
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for COMPPROP
--R
--R----- Operations -----
--R ?=? : (%,% ) -> Boolean          close : (% ,Boolean) -> Boolean
--R closed? : % -> Boolean          coerce : % -> OutputForm
--R copy : % -> %                  hash : % -> SingleInteger
--R latex : % -> String            new : () -> %
--R solid : (% ,Boolean) -> Boolean solid? : % -> Boolean
--R ?~=? : (%,% ) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

—————

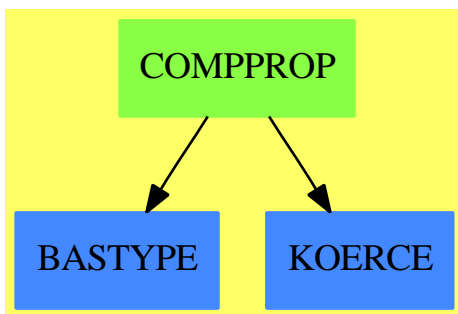
— SubSpaceComponentProperty.help —

```
=====
SubSpaceComponentProperty examples
=====
```

See Also:

```
o )show SubSpaceComponentProperty
```

### 20.34.1 SubSpaceComponentProperty (COMPPROP)



See

⇒ “Point” (POINT) 17.24.1 on page 2019

⇒ “SubSpace” (SUBSPACE) 20.33.1 on page 2573

**Exports:**

```
close  closed?  coerce  copy    hash
latex  new      solid   solid?  ~=?
```

— domain **COMPPROP** SubSpaceComponentProperty —

```
)abbrev domain COMPPROP SubSpaceComponentProperty
```

```
++ Author: Mark Botch
```

```
++ Description:
```

```
++ This domain implements some global properties of subspaces.
```

```
SubSpaceComponentProperty() : Exports == Implementation where
```

```

O ==> OutputForm
I  ==> Integer
PI ==> PositiveInteger
NNI ==> NonNegativeInteger
L  ==> List
B  ==> Boolean

```

```

Exports ==> SetCategory with
  new      : () -> %
    ++ new() is not documented
  closed?  : % -> B
    ++ closed?(x) is not documented
  solid?   : % -> B
    ++ solid?(x) is not documented
  close    : (%,B) -> B
    ++ close(x,b) is not documented
  solid    : (%,B) -> B
    ++ solid(x,b) is not documented
  copy     : % -> %
    ++ copy(x) is not documented

Implementation ==> add
  Rep := Record(closed:B, solid:B)
  closed? p == p.closed
  solid? p == p.solid
  close(p,b) == p.closed := b
  solid(p,b) == p.solid := b
  new() == [false,false]
  copy p ==
    annuderOne := new()
    close(annuderOne,closed? p)
    solid(annuderOne,solid? p)
    annuderOne
  coerce p ==
    hconcat(["Component is "::0,
      (closed? p => ":::0; "not ":::0),"closed, ":::0, _
      (solid? p => ":::0; "not ":::0),"solid":::0 ])

```

— COMPPROP.dotabb —

```

"COMPPROP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=COMPPROP"]
"BASTYPE"  [color="#4488FF",href="bookvol10.2.pdf#nameddest=BASTYPE"]
"KOERCE"   [color="#4488FF",href="bookvol10.2.pdf#nameddest=KOERCE"]
"COMPPROP" -> "BASTYPE"
"COMPPROP" -> "KOERCE"

```

## 20.35 domain SUCH SuchThat

## — SuchThat.input —

```

)set break resume
)sys rm -f SuchThat.output
)spool SuchThat.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SuchThat
--R SuchThat(S1: SetCategory,S2: SetCategory) is a domain constructor
--R Abbreviation for SuchThat is SUCH
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SUCH
--R
--R----- Operations -----
--R ?? : (%,% ) -> Boolean          coerce : % -> OutputForm
--R construct : (S1,S2) -> %        hash : % -> SingleInteger
--R latex : % -> String             lhs : % -> S1
--R rhs : % -> S2                  ?~=? : (%,% ) -> Boolean
--R
--E 1

)spool
)lisp (bye)

```

## — SuchThat.help —

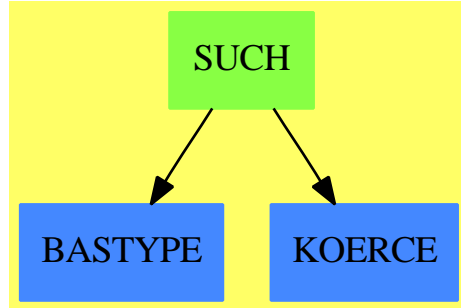
```

=====
SuchThat examples
=====

See Also:
o )show SuchThat

```

## 20.35.1 SuchThat (SUCH)

**Exports:**

```

coerce  construct  hash  latex  lhs
rhs      ~=?      ?~=?

```

— domain **SUCH** **SuchThat** —

```

)abbrev domain SUCH SuchThat
++ Author: Mark Botch
++ Description:
++ This domain implements "such that" forms

SuchThat(S1, S2): Cat == Capsule where
  E ==> OutputForm
  S1, S2: SetCategory

Cat == SetCategory with
  construct: (S1, S2) -> %
    ++ construct(s,t) makes a form s:t
  lhs: % -> S1
    ++ lhs(f) returns the left side of f
  rhs: % -> S2
    ++ rhs(f) returns the right side of f

Capsule == add
  Rep := Record(obj: S1, cond: S2)
  construct(o, c) == [o, c]$Record(obj: S1, cond: S2)
  lhs st == st.obj
  rhs st == st.cond
  coerce(w):E == infix("|"::E, w.obj::E, w.cond::E)

```

— —

— **SUCH.dotabb** —

```

"SUCH" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SUCH"]
"BASTYPE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=BASTYPE"]
"KOERCE" [color="#4488FF",href="bookvol10.2.pdf#nameddest=KOERCE"]
"SUCH" -> "BASTYPE"
"SUCH" -> "KOERCE"

```

---

## 20.36 domain SWITCH Switch

— Switch.input —

```

)set break resume
)sys rm -f Switch.output
)spool Switch.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Switch
--R Switch is a domain constructor
--R Abbreviation for Switch is SWITCH
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SWITCH
--R
--R----- Operations -----
--R NOT : % -> %               coerce : Symbol -> %
--R coerce : % -> OutputForm
--R AND : (Union(I: Expression Integer,F: Expression Float,CF: Expression Complex Float,switch: %),Union(
--R EQ : (Union(I: Expression Integer,F: Expression Float,CF: Expression Complex Float,switch: %),Union(
--R GE : (Union(I: Expression Integer,F: Expression Float,CF: Expression Complex Float,switch: %),Union(
--R GT : (Union(I: Expression Integer,F: Expression Float,CF: Expression Complex Float,switch: %),Union(
--R LE : (Union(I: Expression Integer,F: Expression Float,CF: Expression Complex Float,switch: %),Union(
--R LT : (Union(I: Expression Integer,F: Expression Float,CF: Expression Complex Float,switch: %),Union(
--R NOT : Union(I: Expression Integer,F: Expression Float,CF: Expression Complex Float,switch: %) -> %
--R OR : (Union(I: Expression Integer,F: Expression Float,CF: Expression Complex Float,switch: %),Union(
--R
--E 1

)spool
)lisp (bye)

```

---

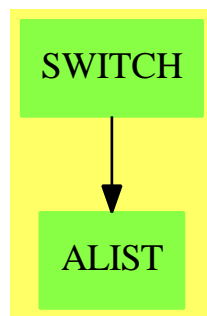
— Switch.help —



```
=====
Switch examples
=====
```

```
See Also:
o )show Switch
```

### 20.36.1 Switch (SWITCH)



See

```
⇒ “Result” (RESULT) 19.9.1 on page 2260
⇒ “FortranCode” (FC) 7.16.1 on page 898
⇒ “FortranProgram” (FORTRAN) 7.18.1 on page 923
⇒ “ThreeDimensionalMatrix” (M3D) 21.7.1 on page 2661
⇒ “SimpleFortranProgram” (SFORT) 20.11.1 on page 2364
⇒ “FortranTemplate” (FTEM) 7.20.1 on page 934
⇒ “FortranExpression” (FEXPR) 7.17.1 on page 914
```

#### Exports:

```
coerce  AND  EQ  GE  GT  LE  LT  NOT  OR
```

#### — domain SWITCH Switch —

```
)abbrev domain SWITCH Switch
-- Because of a bug in the compiler:
)bo $noSubsumption:=false

++ Author: Mike Dewar
++ Date Created: April 1991
++ Date Last Updated: March 1994 30.6.94 Added coercion from Symbol MCD
++ Basic Operations:
++ Related Constructors: FortranProgram, FortranCode, FortranTypes
++ Also See:
```

```

++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This domain builds representations of boolean expressions for use with
++ the \axiomType{FortranCode} domain.

Switch():public == private where
  EXPR ==> Union(I:Expression Integer,F:Expression Float,
                 CF:Expression Complex Float,switch:%)

public == CoercibleTo OutputForm with
  coerce : Symbol -> $
    ++ coerce(s) is not documented
  LT : (EXPR,EXPR) -> $
    ++ LT(x,y) returns the \axiomType{Switch} expression representing \spad{x<y}.
  GT : (EXPR,EXPR) -> $
    ++ GT(x,y) returns the \axiomType{Switch} expression representing \spad{x>y}.
  LE : (EXPR,EXPR) -> $
    ++ LE(x,y) returns the \axiomType{Switch} expression representing \spad{x<=y}.
  GE : (EXPR,EXPR) -> $
    ++ GE(x,y) returns the \axiomType{Switch} expression representing \spad{x>=y}.
  OR : (EXPR,EXPR) -> $
    ++ OR(x,y) returns the \axiomType{Switch} expression representing \spad{x or y}.
  EQ : (EXPR,EXPR) -> $
    ++ EQ(x,y) returns the \axiomType{Switch} expression representing \spad{x = y}.
  AND : (EXPR,EXPR) -> $
    ++ AND(x,y) returns the \axiomType{Switch} expression representing \spad{x and y}.
  NOT : EXPR -> $
    ++ NOT(x) returns the \axiomType{Switch} expression representing \spad{\~x}.
  NOT : $ -> $
    ++ NOT(x) returns the \axiomType{Switch} expression representing \spad{\~x}.

private == add
  Rep := Record(op:BasicOperator,rands:List EXPR)

  -- Public function definitions

  nullOp : BasicOperator := operator NULL

  coerce(s:%):OutputForm ==
    rat := (s . op)::OutputForm
    ran := [u::OutputForm for u in s.rands]
    (s . op) = nullOp => first ran
    #ran = 1 =>
      prefix(rat,ran)
    infix(rat,ran)

  coerce(s:Symbol):$ == [nullOp,[[s::Expression(Integer)]$EXPR]$List(EXPR)]$Rep

```

```

NOT(r:EXPR):% ==
  [operator("^":Symbol),[r]$List(EXPR)]$Rep

NOT(r:%):% ==
  [operator("^":Symbol),[[r]$EXPR]$List(EXPR)]$Rep

LT(r1:EXPR,r2:EXPR):% ==
  [operator("<":Symbol),[r1,r2]$List(EXPR)]$Rep

GT(r1:EXPR,r2:EXPR):% ==
  [operator(">":Symbol),[r1,r2]$List(EXPR)]$Rep

LE(r1:EXPR,r2:EXPR):% ==
  [operator("<=":Symbol),[r1,r2]$List(EXPR)]$Rep

GE(r1:EXPR,r2:EXPR):% ==
  [operator(">=":Symbol),[r1,r2]$List(EXPR)]$Rep

AND(r1:EXPR,r2:EXPR):% ==
  [operator("and":Symbol),[r1,r2]$List(EXPR)]$Rep

OR(r1:EXPR,r2:EXPR):% ==
  [operator("or":Symbol),[r1,r2]$List(EXPR)]$Rep

EQ(r1:EXPR,r2:EXPR):% ==
  [operator("EQ":Symbol),[r1,r2]$List(EXPR)]$Rep

```

---

— SWITCH.dotabb —

```

"SWITCH" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SWITCH"]
"ALIST"  [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"SWITCH" -> "ALIST"

```

---

## 20.37 domain SYMBOL Symbol

— Symbol.input —

```

)set break resume
)sys rm -f Symbol.output
)spool Symbol.output

```

```
)set message test on
)set message auto off
)clear all
--S 1 of 24
X: Symbol := 'x
--R
--R
--R (1) x
--R
--R Type: Symbol
--E 1

--S 2 of 24
XX: Symbol := x
--R
--R
--R (2) x
--R
--R Type: Symbol
--E 2

--S 3 of 24
A := 'a
--R
--R
--R (3) a
--R
--R Type: Variable a
--E 3

--S 4 of 24
B := b
--R
--R
--R (4) b
--R
--R Type: Variable b
--E 4

--S 5 of 24
x**2 + 1
--R
--R
--R 2
--R (5) x + 1
--R
--R Type: Polynomial Integer
--E 5

--S 6 of 24
"Hello":Symbol
--R
--R
--R (6) Hello
--R
--R Type: Symbol
```

--E 6

--S 7 of 24

new()\$Symbol

--R

--R

--R (7) %A

--R

Type: Symbol

--E 7

--S 8 of 24

new()\$Symbol

--R

--R

--R (8) %B

--R

Type: Symbol

--E 8

--S 9 of 24

new("xyz")\$Symbol

--R

--R

--R (9) %xyz0

--R

Type: Symbol

--E 9

--S 10 of 24

X[i,j]

--R

--R

--R (10) x

--R i,j

--R

Type: Symbol

--E 10

--S 11 of 24

U := subscript(u, [1,2,1,2])

--R

--R

--R (11) u

--R 1,2,1,2

--R

Type: Symbol

--E 11

--S 12 of 24

V := superscript(v, [n])

--R

--R

--R n

--R (12) v

```
--R
--E 12
```

Type: Symbol

```
--S 13 of 24
P := argscript(p, [t])
--R
--R
--R (13) p(t)
--R
--E 13
```

Type: Symbol

```
--S 14 of 24
scripted? U
--R
--R
--R (14) true
--R
--E 14
```

Type: Boolean

```
--S 15 of 24
scripted? X
--R
--R
--R (15) false
--R
--E 15
```

Type: Boolean

```
--S 16 of 24
string X
--R
--R
--R (16) "x"
--R
--E 16
```

Type: String

```
--S 17 of 24
name U
--R
--R
--R (17) u
--R
--E 17
```

Type: Symbol

```
--S 18 of 24
scripts U
--R
--R
--R (18) [sub= [1,2,1,2],sup= [],presup= [],presub= [],args= []]
--RType: Record(sub: List OutputForm,sup: List OutputForm,presup: List OutputForm,presub: List OutputForm,args: List OutputForm)
--E 18
```

--S 19 of 24

name X

--R

--R

--R (19) x

--R

Type: Symbol

--E 19

--S 20 of 24

scripts X

--R

--R

--R (20) [sub= [],sup= [],presup= [],presub= [],args= []]

--RType: Record(sub: List OutputForm,sup: List OutputForm,presup: List OutputForm,presub: Li

--E 20

--S 21 of 24

M := script(Mammoth, [ [i,j],[k,l],[0,1],[2],[u,v,w] ])

--R

--R

--R 0,1 k,l

--R (21) Mammoth (u,v,w)

--R 2 i,j

--R

Type: Symbol

--E 21

--S 22 of 24

scripts M

--R

--R

--R (22) [sub= [i,j],sup= [k,l],presup= [0,1],presub= [2],args= [u,v,w]]

--RType: Record(sub: List OutputForm,sup: List OutputForm,presup: List OutputForm,presub: Li

--E 22

--S 23 of 24

N := script(Nut, [ [i,j],[k,l],[0,1] ])

--R

--R

--R 0,1 k,l

--R (23) Nut

--R i,j

--R

Type: Symbol

--E 23

--S 24 of 24

scripts N

--R

--R

--R (24) [sub= [i,j],sup= [k,l],presup= [0,1],presub= [],args= []]

```
--RType: Record(sub: List OutputForm,sup: List OutputForm,presup: List OutputForm,presub: List OutputForm)
--E 24
)spool
)lisp (bye)
```

---

— Symbol.help —

=====

Symbol examples

=====

Symbols are one of the basic types manipulated by Axiom. The Symbol domain provides ways to create symbols of many varieties.

The simplest way to create a symbol is to "single quote" an identifier.

```
X: Symbol := 'x
x
Type: Symbol
```

This gives the symbol even if x has been assigned a value. If x has not been assigned a value, then it is possible to omit the quote.

```
XX: Symbol := x
x
Type: Symbol
```

Declarations must be used when working with symbols, because otherwise the interpreter tries to place values in a more specialized type Variable.

```
A := 'a
a
Type: Variable a
```

```
B := b
b
Type: Variable b
```

The normal way of entering polynomials uses this fact.

```
x**2 + 1
2
x + 1
Type: Polynomial Integer
```

Another convenient way to create symbols is to convert a string. This is useful when the name is to be constructed by a program.



```
"Hello"::Symbol
Hello
```

Type: Symbol

Sometimes it is necessary to generate new unique symbols, for example, to name constants of integration. The expression `new()` generates a symbol starting with %.

```
new()$Symbol
%A
```

Type: Symbol

Successive calls to `new` produce different symbols.

```
new()$Symbol
%B
```

Type: Symbol

The expression `new("s")` produces a symbol starting with %s.

```
new("xyz")$Symbol
%xyz0
```

Type: Symbol

A symbol can be adorned in various ways. The most basic thing is applying a symbol to a list of subscripts.

```
X[i,j]
  x
  i,j
```

Type: Symbol

Somewhat less pretty is to attach subscripts, superscripts or arguments.

```
U := subscript(u, [1,2,1,2])
  u
  1,2,1,2
```

Type: Symbol

```
V := superscript(v, [n])
  n
  v
```

Type: Symbol

```
P := argscript(p, [t])
  p(t)
```

Type: Symbol

It is possible to test whether a symbol has scripts using the `scripted?` test.

```

scripted? U
  true
                                     Type: Boolean

```

```

scripted? X
  false
                                     Type: Boolean

```

If a symbol is not scripted, then it may be converted to a string.

```

string X
  "x"
                                     Type: String

```

The basic parts can always be extracted using the name and scripts operations.

```

name U
  u
                                     Type: Symbol

```

```

scripts U
  [sub= [1,2,1,2],sup= [],presup= [],presub= [],args= []]
                                     Type: Record(sub: List OutputForm,
                                     sup: List OutputForm,
                                     presup: List OutputForm,
                                     presub: List OutputForm,
                                     args: List OutputForm)

```

```

name X
  x
                                     Type: Symbol

```

```

scripts X
  [sub= [],sup= [],presup= [],presub= [],args= []]
                                     Type: Record(sub: List OutputForm,
                                     sup: List OutputForm,
                                     presup: List OutputForm,
                                     presub: List OutputForm,
                                     args: List OutputForm)

```

The most general form is obtained using the script operation. This operation takes an argument which is a list containing, in this order, lists of subscripts, superscripts, presuperscripts, presubscripts and arguments to a symbol.

```

M := script(Mammoth, [ [i,j],[k,l],[0,1],[2],[u,v,w] ])
0,1      k,l
Mammoth  (u,v,w)
2      i,j

```

Type: Symbol

```
scripts M
  [sub= [i,j],sup= [k,l],presup= [0,1],presub= [2],args= [u,v,w]]
    Type: Record(sub: List OutputForm,
      sup: List OutputForm,
      presup: List OutputForm,
      presub: List OutputForm,
      args: List OutputForm)
```

If trailing lists of scripts are omitted, they are assumed to be empty.

```
N := script(Nut, [ [i,j],[k,l],[0,1] ])
      0,1    k,l
      Nut
      i,j
```

Type: Symbol

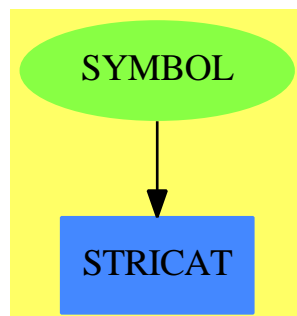
```
scripts N
  [sub= [i,j],sup= [k,l],presup= [0,1],presub= [],args= []]
    Type: Record(sub: List OutputForm,
      sup: List OutputForm,
      presup: List OutputForm,
      presub: List OutputForm,
      args: List OutputForm)
```

See Also:

o )show Symbol

—————

### 20.37.1 Symbol (SYMBOL)



**Exports:**

|           |              |          |           |             |
|-----------|--------------|----------|-----------|-------------|
| argscript | coerce       | convert  | hash      | latex       |
| list      | max          | min      | name      | new         |
| OMwrite   | patternMatch | resetNew | sample    | script      |
| scripts   | scripted?    | string   | subscript | superscript |
| ?~=?      | ?..?         | ?<?      | ?<=?      | ?=?         |
| ?>?       | ?>=?         |          |           |             |

— domain SYMBOL Symbol —

```

)abbrev domain SYMBOL Symbol
++ Author: Stephen Watt
++ Date Created: 1986
++ Date Last Updated: 7 Mar 1991, 29 Apr. 1994 (FDLL)
++ Keywords: symbol.
++ Description:
++ Basic and scripted symbols.

Symbol(): Exports == Implementation where
  L ==> List OutputForm
  Scripts ==> Record(sub:L,sup:L,presup:L,presub:L,args:L)

Exports ==> Join(OrderedSet, ConvertibleTo InputForm, OpenMath,
  ConvertibleTo Symbol,
  ConvertibleTo Pattern Integer, ConvertibleTo Pattern Float,
  PatternMatchable Integer, PatternMatchable Float) with
new: () -> %
  ++ new() returns a new symbol whose name starts with %.
new: % -> %
  ++ new(s) returns a new symbol whose name starts with %s.
resetNew: () -> Void
  ++ resetNew() resets the internal counters that new() and
  ++ new(s) use to return distinct symbols every time.
coerce: String -> %
  ++ coerce(s) converts the string s to a symbol.
name: % -> %
  ++ name(s) returns s without its scripts.
scripted?: % -> Boolean
  ++ scripted?(s) is true if s has been given any scripts.
scripts: % -> Scripts
  ++ scripts(s) returns all the scripts of s.
script: (% , List L) -> %
  ++ script(s, [a,b,c,d,e]) returns s with subscripts a,
  ++ superscripts b, pre-superscripts c, pre-subscripts d,
  ++ and argument-scripts e. Omitted components are taken to be empty.
  ++ For example, \spad{script(s, [a,b,c])} is equivalent to
  ++ \spad{script(s,[a,b,c,[],[]])}.
script: (% , Scripts) -> %
  ++ script(s, [a,b,c,d,e]) returns s with subscripts a,

```

```

    ++ superscripts b, pre-superscripts c, pre-subscripts d,
    ++ and argument-scripts e.
subscript: (% , L) -> %
    ++ subscript(s, [a1,...,an]) returns s
    ++ subscripted by \spad{[a1,...,an]}.
superscript: (% , L) -> %
    ++ superscript(s, [a1,...,an]) returns s
    ++ superscripted by \spad{[a1,...,an]}.
argscript: (% , L) -> %
    ++ argscript(s, [a1,...,an]) returns s
    ++ arg-scripted by \spad{[a1,...,an]}.
elt: (% , L) -> %
    ++ elt(s,[a1,...,an]) or s([a1,...,an]) returns s subscripted by \spad{[a1,...,an]}.
string: % -> String
    ++ string(s) converts the symbol s to a string.
    ++ Error: if the symbol is subscripted.
list: % -> List %
    ++ list(sy) takes a scripted symbol and produces a list
    ++ of the name followed by the scripts.
sample: constant -> %
    ++ sample() returns a sample of %

Implementation ==> add
count: Reference(Integer) := ref 0
xcount: AssociationList(% , Integer) := empty()
istrings:PrimitiveArray(String) :=
    construct ["0","1","2","3","4","5","6","7","8","9"]
-- the following 3 strings shall be of empty intersection
nums:String:="0123456789"
ALPHAS:String:="ABCDEFGHIJKLMNOPQRSTUVWXYZ"
alphas:String:="abcdefghijklmnopqrstuvwxyz"

writeOMSym(dev: OpenMathDevice, x: %): Void ==
    scripted? x =>
        error "Cannot convert a scripted symbol to OpenMath"
    OMputVariable(dev, x pretend Symbol)

OMwrite(x: %): String ==
    s: String := ""
    sp := OM_-STRINGTOSTRINGPTR(s)$Lisp
    dev: OpenMathDevice := OMopenString(sp pretend String, OMencodingXML)
    OMputObject(dev)
    writeOMSym(dev, x)
    OMputEndObject(dev)
    OMclose(dev)
    s := OM_-STRINGPTRTOSTRING(sp)$Lisp pretend String
    s

OMwrite(x: % , wholeObj: Boolean): String ==
    s: String := ""

```

```

sp := OM_-STRINGTOSTRINGPTR(s)$Lisp
dev: OpenMathDevice := OMopenString(sp pretend String, OMencodingXML)
if wholeObj then
  OMputObject(dev)
writeOMSym(dev, x)
if wholeObj then
  OMputEndObject(dev)
OMclose(dev)
s := OM_-STRINGPTRTOSTRING(sp)$Lisp pretend String
s

OMwrite(dev: OpenMathDevice, x: %): Void ==
  OMputObject(dev)
  writeOMSym(dev, x)
  OMputEndObject(dev)

OMwrite(dev: OpenMathDevice, x: %, wholeObj: Boolean): Void ==
  if wholeObj then
    OMputObject(dev)
  writeOMSym(dev, x)
  if wholeObj then
    OMputEndObject(dev)

hd:String      := "*"
lhd            := #hd
ord0           := ord char("0")$Character

istring : Integer -> String
symprefix : Scripts -> String
syscripts: Scripts -> L

convert(s:%):InputForm == convert(s pretend Symbol)$InputForm
convert(s:%):Symbol    == s pretend Symbol
coerce(s:String):%     == VALUES(INTERN(s)$Lisp)$Lisp
x = y                 == EQUAL(x,y)$Lisp
x < y                 == GGREATERP(y, x)$Lisp
coerce(x:%):OutputForm == outputForm(x pretend Symbol)
subscript(sy, lx)      == script(sy, [lx, nil, nil(), nil(), nil()])
elt(sy,lx)             == subscript(sy,lx)
superscript(sy, lx)    == script(sy,[nil(),lx, nil(), nil(), nil()])
argscript(sy, lx)      == script(sy,[nil(),nil(), nil(), nil(), lx])

patternMatch(x:%,p:Pattern Integer,l:PatternMatchResult(Integer,%))==
  (patternMatch(x pretend Symbol, p, l pretend
    PatternMatchResult(Integer, Symbol))$PatternMatchSymbol(Integer))
  pretend PatternMatchResult(Integer, %)

patternMatch(x:%, p:Pattern Float, l:PatternMatchResult(Float, %)) ==
  (patternMatch(x pretend Symbol, p, l pretend
    PatternMatchResult(Float, Symbol))$PatternMatchSymbol(Float))

```

```

pretend PatternMatchResult(Float, %)

convert(x: %):Pattern(Float) ==
  coerce(x pretend Symbol)$Pattern(Float)

convert(x: %):Pattern(Integer) ==
  coerce(x pretend Symbol)$Pattern(Integer)

symprefix sc ==
  ns: List Integer := [#sc.presub, #sc.presup, #sc.sup, #sc.sub]
  while #ns >= 2 and zero? first ns repeat ns := rest ns
  concat concat(concat(hd, istring(#sc.args)),
    [istring n for n in reverse_! ns])

syscripts sc ==
  all := sc.presub
  all := concat(sc.presup, all)
  all := concat(sc.sup, all)
  all := concat(sc.sub, all)
  concat(all, sc.args)

script(sy: %, ls: List L) ==
  sc: Scripts := [nil(), nil(), nil(), nil()]
  if not null ls then (sc.sub := first ls; ls := rest ls)
  if not null ls then (sc.sup := first ls; ls := rest ls)
  if not null ls then (sc.presup := first ls; ls := rest ls)
  if not null ls then (sc.presub := first ls; ls := rest ls)
  if not null ls then (sc.args := first ls; ls := rest ls)
  script(sy, sc)

script(sy: %, sc: Scripts) ==
  scripted? sy => error "Cannot add scripts to a scripted symbol"
  (concat(concat(symprefix sc, string name sy)::%::OutputForm,
    syscripts sc)) pretend %

string e ==
  not scripted? e => PNAME(e)$Lisp
  error "Cannot form string from non-atomic symbols."

-- Scripts ==> Record(sub:L,sup:L,presup:L,presub:L,args:L)
latex e ==
  s : String := (PNAME(name e)$Lisp) pretend String
  if #s > 1 and s.1 ^= char "\"" then
    s := concat("\mbox{\it ", concat(s, "}")$String)$String
  not scripted? e => s
  ss : Scripts := scripts e
  lo : List OutputForm := ss.sub
  sc : String
  if not empty? lo then
    sc := "__{"

```

```

while not empty? lo repeat
  sc := concat(sc, latex first lo)$String
  lo := rest lo
  if not empty? lo then sc := concat(sc, ", ")$String
sc := concat(sc, "}")$String
s := concat(s, sc)$String
lo := ss.sup
if not empty? lo then
  sc := "^{"
  while not empty? lo repeat
    sc := concat(sc, latex first lo)$String
    lo := rest lo
    if not empty? lo then sc := concat(sc, ", ")$String
  sc := concat(sc, "}")$String
  s := concat(s, sc)$String
lo := ss.presup
if not empty? lo then
  sc := "{}^{"
  while not empty? lo repeat
    sc := concat(sc, latex first lo)$String
    lo := rest lo
    if not empty? lo then sc := concat(sc, ", ")$String
  sc := concat(sc, "}")$String
  s := concat(sc, s)$String
lo := ss.presub
if not empty? lo then
  sc := "{}_{"
  while not empty? lo repeat
    sc := concat(sc, latex first lo)$String
    lo := rest lo
    if not empty? lo then sc := concat(sc, ", ")$String
  sc := concat(sc, "}")$String
  s := concat(sc, s)$String
lo := ss.args
if not empty? lo then
  sc := "\left( {"
  while not empty? lo repeat
    sc := concat(sc, latex first lo)$String
    lo := rest lo
    if not empty? lo then sc := concat(sc, ", ")$String
  sc := concat(sc, "} \right)")$String
  s := concat(s, sc)$String
s

anyRadix(n:Integer,s:String):String ==
ns:String:=""
repeat
  qr := divide(n,#s)
  n := qr.quotient
  ns := concat(s.(qr.remainder+minIndex s),ns)

```



```

    if zero?(n) then return ns

new() ==
  sym := anyRadix(count():Integer,ALPHAS)
  count() := count() + 1
  concat("%",sym):=%

new x ==
  n:Integer :=
    (u := search(x, xcount)) case "failed" => 0
    inc(u:Integer)
  xcount(x) := n
  xx :=
    not scripted? x => string x
    string name x
  xx := concat("%",xx)
  xx :=
    (position(xx.maxIndex(xx),nums)>=minIndex(nums)) =>
      concat(xx, anyRadix(n,alphas))
    concat(xx, anyRadix(n,nums))
  not scripted? x => xx:=%
  script(xx:=%,scripts x)

resetNew() ==
  count() := 0
  for k in keys xcount repeat remove_!(k, xcount)
  void

scripted? sy ==
  not ATOM(sy)$Lisp

name sy ==
  not scripted? sy => sy
  str := string first list sy
  for i in lhd+1..#str repeat
    not digit?(str.i) => return((str.(i..#str)):=%)
  error "Improper scripted symbol"

scripts sy ==
  not scripted? sy => [nil(), nil(), nil(), nil(), nil()]
  nscripts: List NonNegativeInteger := [0, 0, 0, 0, 0]
  lscripts: List L := [nil(), nil(), nil(), nil(), nil()]
  str := string first list sy
  nstr := #str
  m := minIndex nscripts
  for i in m.. for j in lhd+1..nstr while digit?(str.j) repeat
    nscripts.i := (ord(str.j) - ord0)::NonNegativeInteger
  -- Put the number of function scripts at the end.
  nscripts := concat(rest nscripts, first nscripts)
  allscripts := rest list sy

```

```

m := minIndex lscripts
for i in m.. for n in nscripts repeat
  #allscripts < n => error "Improper script count in symbol"
  lscripts.i := [a::OutputForm for a in first(allscripts, n)]
  allscripts := rest(allscripts, n)
[lscripts.m, lscripts.(m+1), lscripts.(m+2),
                                lscripts.(m+3), lscripts.(m+4)]

istring n ==
  n > 9 => error "Can have at most 9 scripts of each kind"
  istrings.(n + minIndex istrings)

list sy ==
  not scripted? sy =>
    error "Cannot convert a symbol to a list if it is not subscripted"
  sy pretend List(%)

sample() == "aSymbol"::%

```

---

— SYMBOL.dotabb —

```

"SYMBOL" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SYMBOL",
          shape=ellipse]
"STRICAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=STRICAT"]
"SYMBOL" -> "STRICAT"

```

---

## 20.38 domain SYMTAB SymbolTable

---

— SymbolTable.input —

```

)set break resume
)sys rm -f SymbolTable.output
)spool SymbolTable.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SymbolTable
--R SymbolTable is a domain constructor

```

```

--R Abbreviation for SymbolTable is SYMTAB
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SYMTAB
--R
--R----- Operations -----
--R coerce : % -> OutputForm          empty : () -> %
--R externalList : % -> List Symbol    newTypeLists : % -> SExpression
--R parametersOf : % -> List Symbol    printTypes : % -> Void
--R coerce : % -> Table(Symbol,FortranType)
--R declare! : (Symbol,FortranType,%) -> FortranType
--R declare! : (List Symbol,FortranType,%) -> FortranType
--R fortranTypeOf : (Symbol,%) -> FortranType
--R symbolTable : List Record(key: Symbol,entry: FortranType) -> %
--R typeList : (FortranScalarType,%) -> List Union(name: Symbol,bounds: List Union(S: Symbol
--R typeLists : % -> List List Union(name: Symbol,bounds: List Union(S: Symbol,P: Polynomial
--R
--E 1

)spool
)lisp (bye)

```

---

— SymbolTable.help —

```

=====
SymbolTable examples
=====

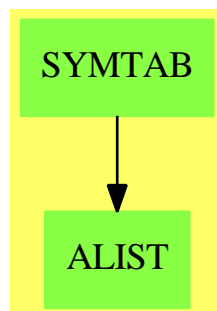
```

```

See Also:
o )show SymbolTable

```

### 20.38.1 SymbolTable (SYMTAB)



See

⇒ “FortranScalarType” (FST) 7.19.1 on page 929  
 ⇒ “FortranType” (FT) 7.21.1 on page 938  
 ⇒ “TheSymbolTable” (SYMS) 21.6.1 on page 2655

**Exports:**

|              |              |            |              |               |
|--------------|--------------|------------|--------------|---------------|
| coerce       | declare!     | empty      | externalList | fortranTypeOf |
| newTypeLists | parametersOf | printTypes | symbolTable  | typeList      |
| typeLists    |              |            |              |               |

— domain SYMTAB SymbolTable —

```
)abbrev domain SYMTAB SymbolTable
++ Author: Mike Dewar
++ Date Created: October 1992
++ Date Last Updated: 12 July 1994
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ Create and manipulate a symbol table for generated FORTRAN code

SymbolTable() : exports == implementation where

T ==> Union(S:Symbol,P:Polynomial Integer)
TL1 ==> List T
TU ==> Union(name:Symbol,bounds:TL1)
TL ==> List TU
SEX ==> SExpression
OFORM ==> OutputForm
L ==> List
FSTU ==> Union(fst:FortranScalarType,void:"void")

exports ==> CoercibleTo OutputForm with
  coerce : $ -> Table(Symbol,FortranType)
    ++ coerce(x) returns a table view of x
  empty : () -> $
    ++ empty() returns a new, empty symbol table
  declare! : (L Symbol,FortranType,$) -> FortranType
    ++ declare!(l,t,tab) creates new entrys in tab, declaring each of l
    ++ to be of type t
  declare! : (Symbol,FortranType,$) -> FortranType
    ++ declare!(u,t,tab) creates a new entry in tab, declaring u to be of
    ++ type t
  fortranTypeOf : (Symbol,$) -> FortranType
```

```

    ++ fortranTypeOf(u,tab) returns the type of u in tab
parametersOf: $ -> L Symbol
    ++ parametersOf(tab) returns a list of all the symbols declared in tab
typeList : (FortranScalarType,$) -> TL
    ++ typeList(t,tab) returns a list of all the objects of type t in tab
externalList : $ -> L Symbol
    ++ externalList(tab) returns a list of all the external symbols in tab
typeLists : $ -> L TL
    ++ typeLists(tab) returns a list of lists of types of objects in tab
newTypeLists : $ -> SEX
    ++ newTypeLists(x) is not documented
printTypes: $ -> Void
    ++ printTypes(tab) produces FORTRAN type declarations from tab, on the
    ++ current FORTRAN output stream
symbolTable: L Record(key:Symbol,entry:FortranType) -> $
    ++ symbolTable(l) creates a symbol table from the elements of l.

implementation ==> add

Rep := Table(Symbol,FortranType)

coerce(t:$):OFORM ==
    coerce(t)$Rep

coerce(t:$):Table(Symbol,FortranType) ==
    t pretend Table(Symbol,FortranType)

symbolTable(l:L Record(key:Symbol,entry:FortranType)):$ ==
    table(l)$Rep

empty():$ ==
    empty()$Rep

parametersOf(tab:$):L(Symbol) ==
    keys(tab)

declare!(name:Symbol,type:FortranType,tab:$):FortranType ==
    setelt(tab,name,type)$Rep
    type

declare!(names:L Symbol,type:FortranType,tab:$):FortranType ==
    for name in names repeat setelt(tab,name,type)$Rep
    type

fortranTypeOf(u:Symbol,tab:$):FortranType ==
    elt(tab,u)$Rep

externalList(tab:$):L(Symbol) ==
    [u for u in keys(tab) | external? fortranTypeOf(u,tab)]

```

```

typeList(type:FortranScalarType,tab:$):TL ==
  scalarList := []@TL
  arrayList  := []@TL
  for u in keys(tab)$Rep repeat
    uType : FortranType := fortranTypeOf(u,tab)
    sType : FSTU := scalarTypeOf(uType)
    if (sType case fst and (sType.fst)=type) then
      uDim : TL1 := [[v]$T for v in dimensionsOf(uType)]
      if empty? uDim then
        scalarList := cons([u]$TU,scalarList)
      else
        arrayList := cons([cons([u],uDim)$TL1]$TU,arrayList)
  -- Scalars come first in case they are integers which are later
  -- used as an array dimension.
  append(scalarList,arrayList)

typeList2(type:FortranScalarType,tab:$):TL ==
  t1 := []@TL
  symbolType : Symbol := coerce(type)$FortranScalarType
  for u in keys(tab)$Rep repeat
    uType : FortranType := fortranTypeOf(u,tab)
    sType : FSTU := scalarTypeOf(uType)
    if (sType case fst and (sType.fst)=type) then
      uDim : TL1 := [[v]$T for v in dimensionsOf(uType)]
      t1 := if empty? uDim then cons([u]$TU,t1)
            else cons([cons([u],uDim)$TL1]$TU,t1)
  empty? t1 => t1
  cons([symbolType]$TU,t1)

updateList(sType:SEX,name:SEX,lDims:SEX,t1:SEX):SEX ==
  l : SEX := ASSOC(sType,t1)$Lisp
  entry : SEX := if null?(lDims) then name else CONS(name,lDims)$Lisp
  null?(l) => CONS([sType,entry]$Lisp,t1)$Lisp
  RPLACD(l,CONS(entry,cdr l)$Lisp)$Lisp
  t1

newTypeLists(tab:$):SEX ==
  t1 := []$Lisp
  for u in keys(tab)$Rep repeat
    uType : FortranType := fortranTypeOf(u,tab)
    sType : FSTU := scalarTypeOf(uType)
    dims : L Polynomial Integer := dimensionsOf uType
    lDims : L SEX := [convert(convert(v)@InputForm)@SEX for v in dims]
    lType : SEX := if sType case void
      then convert(void::Symbol)@SEX
      else coerce(sType.fst)$FortranScalarType
    t1 := updateList(lType,convert(u)@SEX,convert(lDims)@SEX,t1)
  t1

typeLists(tab:$):L(TL) ==

```

```

fortranTypes := ["real"::FortranScalarType, _
  "double precision"::FortranScalarType, _
  "integer"::FortranScalarType, _
  "complex"::FortranScalarType, _
  "logical"::FortranScalarType, _
  "character"::FortranScalarType]@L(FortranScalarType)
t1 := []@L TL
for u in fortranTypes repeat
  types : TL := typeList2(u,tab)
  if (not null types) then
    t1 := cons(types,t1)$L TL
t1

oForm2(w:T):OFORM ==
  w case S => w.S::OFORM
  w case P => w.P::OFORM

oForm(v:TU):OFORM ==
  v case name => v.name::OFORM
  v case bounds =>
    ll : L OFORM := [oForm2(uu) for uu in v.bounds]
    ll :: OFORM

outForm(t:TL):L OFORM ==
  [oForm(u) for u in t]

printTypes(tab:$):Void ==
  -- It is important that INTEGER is the first element of this
  -- list since INTEGER symbols used in type declarations must
  -- be declared in advance.
  ft := ["integer"::FortranScalarType, _
    "real"::FortranScalarType, _
    "double precision"::FortranScalarType, _
    "complex"::FortranScalarType, _
    "logical"::FortranScalarType, _
    "character"::FortranScalarType]@L(FortranScalarType)
  for ty in ft repeat
    t1 : TL := typeList(ty,tab)
    ot1 : L OFORM := outForm(t1)
    fortFormatTypes(ty::OFORM,ot1)$Lisp
  e1 : L OFORM := [u::OFORM for u in externalList(tab)]
  fortFormatTypes("EXTERNAL"::OFORM,e1)$Lisp
  void()$Void

```

```
"SYMTAB" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SYMTAB"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"SYMTAB" -> "ALIST"
```

## 20.39 domain SYMPOLY SymmetricPolynomial

— SymmetricPolynomial.input —

```
)set break resume
)sys rm -f SymmetricPolynomial.output
)spool SymmetricPolynomial.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SymmetricPolynomial
--R SymmetricPolynomial R: Ring is a domain constructor
--R Abbreviation for SymmetricPolynomial is SYMPOLY
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SYMPOLY
--R
--R----- Operations -----
--R ?? : (R,%) -> %          ?? : (%,R) -> %
--R ?? : (%,%) -> %          ?? : (Integer,%) -> %
--R ?? : (PositiveInteger,%) -> %    ??? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %          ?-? : (%,%) -> %
--R -? : % -> %              ?=? : (%,%) -> Boolean
--R 1 : () -> %              0 : () -> %
--R ?? : (%,PositiveInteger) -> %    coefficient : (%,Partition) -> R
--R coefficients : % -> List R        coerce : R -> %
--R coerce : Integer -> %            coerce : % -> OutputForm
--R degree : % -> Partition          ground : % -> R
--R ground? : % -> Boolean          hash : % -> SingleInteger
--R latex : % -> String             leadingCoefficient : % -> R
--R leadingMonomial : % -> %         map : ((R -> R),%) -> %
--R minimumDegree : % -> Partition  monomial : (R,Partition) -> %
--R monomial? : % -> Boolean         one? : % -> Boolean
--R recip : % -> Union(%, "failed")  reductum : % -> %
--R retract : % -> R               sample : () -> %
--R zero? : % -> Boolean           ?~=? : (%,%) -> Boolean
--R ?? : (%,Fraction Integer) -> % if R has ALGEBRA FRAC INT
--R ?? : (Fraction Integer,%) -> % if R has ALGEBRA FRAC INT
--R ?? : (NonNegativeInteger,%) -> %
```



```

--R ***? : (% , NonNegativeInteger) -> %
--R ?/? : (% , R) -> % if R has FIELD
--R ?? : (% , NonNegativeInteger) -> %
--R associates? : (% , %) -> Boolean if R has INTDOM
--R binomThmExpt : (% , %, NonNegativeInteger) -> % if R has COMRING
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(% , "failed") if R has CHARNZ
--R coerce : Fraction Integer -> % if R has ALGEBRA FRAC INT or R has RETRACT FRAC INT
--R coerce : % -> % if R has INTDOM
--R content : % -> R if R has GCDDOM
--R exquo : (% , R) -> Union(% , "failed") if R has INTDOM
--R exquo : (% , %) -> Union(% , "failed") if R has INTDOM
--R fmeq : (% , Partition , R , %) -> % if Partition has CABMON and R has INTDOM
--R mapExponents : ((Partition -> Partition) , %) -> %
--R numberOfMonomials : % -> NonNegativeInteger
--R pomopo! : (% , R , Partition , %) -> %
--R primitivePart : % -> % if R has GCDDOM
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retract : % -> Integer if R has RETRACT INT
--R retractIfCan : % -> Union(R , "failed")
--R retractIfCan : % -> Union(Fraction Integer , "failed") if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(Integer , "failed") if R has RETRACT INT
--R subtractIfCan : (% , %) -> Union(% , "failed")
--R unit? : % -> Boolean if R has INTDOM
--R unitCanonical : % -> % if R has INTDOM
--R unitNormal : % -> Record(unit: % , canonical: % , associate: %) if R has INTDOM
--R
--E 1

)spool
)lisp (bye)

```

---

— SymmetricPolynomial.help —

```

=====
SymmetricPolynomial examples
=====

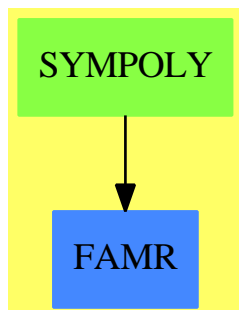
```

```

See Also:
o )show SymmetricPolynomial

```

## 20.39.1 SymmetricPolynomial (SYMPOLY)



See

⇒ “Partition” (PARTITION) 17.9.1 on page 1883

**Exports:**

|                 |               |                   |                    |
|-----------------|---------------|-------------------|--------------------|
| 0               | 1             | associates?       | binomThmExpt       |
| characteristic  | charthRoot    | coefficient       |                    |
| coefficients    | coerce        | content           | degree             |
| exquo           | exquo         | finecg            | ground             |
| ground?         | hash          | latex             | leadingCoefficient |
| leadingMonomial | map           | mapExponents      | minimumDegree      |
| monomial        | monomial?     | numberOfMonomials | one?               |
| pomopo!         | primitivePart | recip             | reductum           |
| retract         | retractIfCan  | sample            | subtractIfCan      |
| unit?           | unitCanonical | unitNormal        | zero?              |
| ?~=?            | ?**?          | ?/?               | ?^?                |
| ?*?             | ?+?           | ?-?               | -?                 |
| ?=?             |               |                   |                    |

— domain SYMPOLY SymmetricPolynomial —

```

)abbrev domain SYMPOLY SymmetricPolynomial
++ Author: Mark Botch
++ Description:
++ This domain implements symmetric polynomial

SymmetricPolynomial(R:Ring) == PolynomialRing(R,Partition) add
  Term:= Record(k:Partition,c:R)
  Rep:= List Term

-- override PR implementation because coeff. arithmetic too expensive (??)

if R has EntireRing then
  (p1:%) * (p2:%) ==
    null p1 => 0
    null p2 => 0
  
```

```

zero?(p1.first.k) => p1.first.c * p2
--
  one? p2 => p1
  (p2 = 1) => p1
  +/[[[t1.k+t2.k,t1.c*t2.c]$Term for t2 in p2]
    for t1 in reverse(p1)]
    -- This 'reverse' is an efficiency improvement:
    -- reduces both time and space [Abbott/Bradford/Davenport]
else
  (p1:%) * (p2:%) ==
  null p1 => 0
  null p2 => 0
  zero?(p1.first.k) => p1.first.c * p2
--
  one? p2 => p1
  (p2 = 1) => p1
  +/[[[t1.k+t2.k,r]$Term for t2 in p2 | (r:=t1.c*t2.c) ^= 0]
    for t1 in reverse(p1)]
    -- This 'reverse' is an efficiency improvement:
    -- reduces both time and space [Abbott/Bradford/Davenport]

```

---

— SYMPOLY.dotabb —

```

"SYMPOLY" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SYMPOLY"]
"FAMR" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FAMR"]
"SYMPOLY" -> "FAMR"

```

---

## Chapter 21

# Chapter T

### 21.1 domain TABLE Table

— Table.input —

```
)set break resume
)sys rm -f Table.output
)spool Table.output
)set message test on
)set message auto off
)clear all
--S 1 of 18
t: Table(Polynomial Integer, String) := table()
--R
--R
--R (1) table()
--R
--R                                          Type: Table(Polynomial Integer,String)
--E 1

--S 2 of 18
setelt(t, x**2 - 1, "Easy to factor")
--R
--R
--R (2) "Easy to factor"
--R
--R                                          Type: String
--E 2

--S 3 of 18
t(x**3 + 1) := "Harder to factor"
--R
--R
--R (3) "Harder to factor"
```



[illegible]

```

--S 16 of 18
#t
--R
--R
--R (16) 2
--R
--R                                         Type: PositiveInteger
--E 16

--S 17 of 18
members t
--R
--R
--R (17) ["The easiest to factor","Harder to factor"]
--R
--R                                         Type: List String
--E 17

--S 18 of 18
count(s: String +-> prefix?("Hard", s), t)
--R
--R
--R (18) 1
--R
--R                                         Type: PositiveInteger
--E 18
)spool
)lisp (bye)

```

---

— Table.help —

=====

Table examples

=====

The Table constructor provides a general structure for associative storage. This type provides hash tables in which data objects can be saved according to keys of any type. For a given table, specific types must be chosen for the keys and entries.

In this example the keys to the table are polynomials with integer coefficients. The entries in the table are strings.

```

t: Table(Polynomial Integer, String) := table()
table()
                                         Type: Table(Polynomial Integer,String)

```

To save an entry in the table, the setelt operation is used. This can be called directly, giving the table a key and an entry.

```
setelt(t, x**2 - 1, "Easy to factor")
    "Easy to factor"
                                Type: String
```

Alternatively, you can use assignment syntax.

```
t(x**3 + 1) := "Harder to factor"
    "Harder to factor"
                                Type: String
```

```
t(x) := "The easiest to factor"
    "The easiest to factor"
                                Type: String
```

Entries are retrieved from the table by calling the elt operation.

```
elt(t, x)
    "The easiest to factor"
                                Type: String
```

This operation is called when a table is "applied" to a key using this or the following syntax.

```
t.x
    "The easiest to factor"
                                Type: String
```

```
t x
    "The easiest to factor"
                                Type: String
```

Parentheses are used only for grouping. They are needed if the key is an infix expression.

```
t.(x**2 - 1)
    "Easy to factor"
                                Type: String
```

Note that the elt operation is used only when the key is known to be in the table, otherwise an error is generated.

```
t (x**3 + 1)
    "Harder to factor"
                                Type: String
```

You can get a list of all the keys to a table using the keys operation.

```
keys t
      3      2
    [x,x  + 1,x  - 1]
```



Type: List Polynomial Integer

If you wish to test whether a key is in a table, the search operation is used. This operation returns either an entry or "failed".

```
search(x, t)
"The easiest to factor"
Type: Union(String,...)
```

```
search(x**2, t)
"failed"
Type: Union("failed",...)
```

The return type is a union so the success of the search can be tested using case.

```
search(x**2, t) case "failed"
true
Type: Boolean
```

The remove operation is used to delete values from a table.

```
remove!(x**2-1, t)
"Easy to factor"
Type: Union(String,...)
```

If an entry exists under the key, then it is returned. Otherwise remove returns "failed".

```
remove!(x-1, t)
"failed"
Type: Union("failed",...)
```

The number of key-entry pairs can be found using the # operation.

```
#t
2
Type: PositiveInteger
```

Just as keys returns a list of keys to the table, a list of all the entries can be obtained using the members operation.

```
members t
(17) ["The easiest to factor","Harder to factor"]
Type: List String
```

A number of useful operations take functions and map them on to the table to compute the result. Here we count the entries which have "Hard" as a prefix.

```
count(s: String +-> prefix?("Hard", s), t)
1
```

Type: PositiveInteger

Other table types are provided to support various needs.

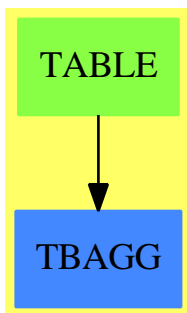
- o AssociationList gives a list with a table view. This allows new entries to be appended onto the front of the list to cover up old entries. This is useful when table entries need to be stacked or when frequent list traversals are required.
- o EqTable gives tables in which keys are considered equal only when they are in fact the same instance of a structure.
- o StringTable should be used when the keys are known to be strings.
- o SparseTable provides tables with default entries, so lookup never fails. The GeneralSparseTable constructor can be used to make any table type behave this way.
- o KeyedAccessFile allows values to be saved in a file, accessed as a table.

See Also:

- o )help AssociationList
- o )help EqTable
- o )help StringTable
- o )help SparseTable
- o )help GeneralSparseTable
- o )help KeyedAccessFile
- o )show Table

—————

### 21.1.1 Table (TABLE)



See

- ⇒ “HashTable” (HASHTBL) 9.1.1 on page 1085
- ⇒ “InnerTable” (INTABL) 10.27.1 on page 1299
- ⇒ “EqTable” (EQTBL) 6.2.1 on page 667

⇒ “StringTable” (STRTBL) 20.32.1 on page 2569  
 ⇒ “GeneralSparseTable” (GSTBL) 8.5.1 on page 1044  
 ⇒ “SparseTable” (STBL) 20.16.1 on page 2409

**Exports:**

|          |          |            |           |                  |
|----------|----------|------------|-----------|------------------|
| any?     | bag      | coerce     | construct | convert          |
| copy     | count    | dictionary | elt       | empty            |
| empty?   | entries  | entry?     | eq?       | eval             |
| eval     | every?   | extract!   | fill!     | find             |
| first    | hash     | index?     | indices   | insert!          |
| inspect  | key?     | keys       | latex     | less?            |
| map      | map      | map!       | maxIndex  | member?          |
| members  | minIndex | more?      | parts     | qelt             |
| qsetelt! | reduce   | remove     | remove!   | removeDuplicates |
| sample   | search   | select     | select!   | setelt           |
| size?    | swap!    | table      | #?        | ?=?              |
| ?~=?     | ?.?      |            |           |                  |

— domain TABLE Table —

```

)abbrev domain TABLE Table
++ Author: Stephen M. Watt, Barry Trager
++ Date Created: 1985
++ Date Last Updated: Sept 15, 1992
++ Basic Operations:
++ Related Domains: HashTable, EqTable, StringTable, AssociationList
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ This is the general purpose table type.
++ The keys are hashed to look up the entries.
++ This creates a \spadtype{HashTable} if equal for the Key
++ domain is consistent with Lisp EQUAL otherwise an
++ \spadtype{AssociationList}

Table(Key: SetCategory, Entry: SetCategory):Exports == Implementation where
  Exports ==> TableAggregate(Key, Entry) with
    finiteAggregate

Implementation ==> InnerTable(Key, Entry,
  if hashable(Key)$Lisp then HashTable(Key, Entry, "UEQUAL")
  else AssociationList(Key, Entry))

```

— — —

— TABLE.dotabb —

```
"TABLE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=TABLE"]
"TBAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=TBAGG"]
"TABLE" -> "TBAGG"
```

—

## 21.2 domain TABLEAU Tableau

— Tableau.input —

```
)set break resume
)sys rm -f Tableau.output
)spool Tableau.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Tableau
--R Tableau S: SetCategory is a domain constructor
--R Abbreviation for Tableau is TABLEAU
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for TABLEAU
--R
--R----- Operations -----
--R coerce : % -> OutputForm          listOfLists : % -> List List S
--R tableau : List List S -> %
--R
--E 1

)spool
)lisp (bye)
```

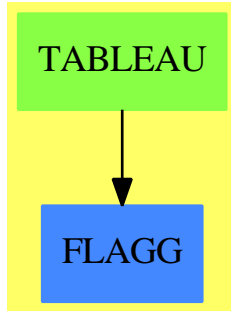
—

— Tableau.help —

```
=====
Tableau examples
=====
```

```
See Also:
o )show Tableau
```

### 21.2.1 Tableau (TABLEAU)



#### Exports:

coerce listOfLists tableau

— domain TABLEAU Tableau —

```

)abbrev domain TABLEAU Tableau
++ Author: William H. Burge
++ Date Created: 1987
++ Date Last Updated: 23 Sept 1991
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords: Young tableau
++ References:
++ Description:
++ The tableau domain is for printing Young tableaux, and
++ coercions to and from List List S where S is a set.

Tableau(S:SetCategory):Exports == Implementation where
  L ==> List
  I ==> Integer
  NNI ==> NonNegativeInteger
  OUT ==> OutputForm
  V ==> Vector
  fm==>formMatrix$PrintableForm()
  Exports ==> with
    tableau : L L S -> %
      ++ tableau(ll) converts a list of lists ll to a tableau.
    listOfLists : % -> L L S
      ++ listOfLists t converts a tableau t to a list of lists.

```

```

coerce : % -> OUT
  ++ coerce(t) converts a tableau t to an output form.
Implementation ==> add

Rep := L L S

tableau(lls:(L L S)) == lls pretend %
listOfLists(x:):(L L S) == x pretend (L L S)
makeupv : (NNI,L S) -> L OUT
makeupv(n,ls)==
  v:=new(n,message " ")(List OUT)
  for i in 1..#ls for s in ls repeat v.i:=box(s::OUT)
  v
maketab : L L S -> OUT
maketab lls ==
  ll : L OUT :=
    empty? lls => [[empty()]]
    sz:NNI:=# first lls
    [blankSeparate makeupv(sz,i) for i in lls]
  pile ll

coerce(x:):OUT == maketab listOfLists x

```

---

— TABLEAU.dotabb —

```

"TABLEAU" [color="#88FF44",href="bookvol10.3.pdf#nameddest=TABLEAU"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"TABLEAU" -> "FLAGG"

```

---

## 21.3 domain TS TaylorSeries

— TaylorSeries.input —

```

)set break resume
)sys rm -f TaylorSeries.output
)spool TaylorSeries.output
)set message test on
)set message auto off
)clear all

```

```

--S 1 of 1
)show TaylorSeries
--R TaylorSeries Coef: Ring is a domain constructor
--R Abbreviation for TaylorSeries is TS
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for TS
--R
--R----- Operations -----
--R ???: (% , Coef) -> %          ??? : (Coef, %) -> %
--R ???: (% , %) -> %            ??? : (Integer, %) -> %
--R ???: (PositiveInteger, %) -> %   ??? : (% , PositiveInteger) -> %
--R ?+?: (% , %) -> %             ?-?: (% , %) -> %
--R -?: % -> %                    ?=? : (% , %) -> Boolean
--R D : (% , List Symbol) -> %      D : (% , Symbol) -> %
--R 1 : () -> %                    0 : () -> %
--R ^?: (% , PositiveInteger) -> %   coerce : Polynomial Coef -> %
--R coerce : Symbol -> %             coerce : Integer -> %
--R coerce : % -> OutputForm         complete : % -> %
--R differentiate : (% , Symbol) -> % eval : (% , List %, List %) -> %
--R eval : (% , %, %) -> %           eval : (% , Equation %) -> %
--R eval : (% , List Equation %) -> % eval : (% , Symbol, %) -> %
--R hash : % -> SingleInteger        latex : % -> String
--R leadingCoefficient : % -> Coef    leadingMonomial : % -> %
--R map : ((Coef -> Coef), %) -> %    monomial?: % -> Boolean
--R one?: % -> Boolean                pole?: % -> Boolean
--R recip : % -> Union(%, "failed")   reductum : % -> %
--R sample : () -> %                  variables : % -> List Symbol
--R zero?: % -> Boolean                ?~=? : (% , %) -> Boolean
--R ???: (Fraction Integer, %) -> % if Coef has ALGEBRA FRAC INT
--R ???: (% , Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ???: (NonNegativeInteger, %) -> %
--R ????: (% , Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ????: (% , %) -> % if Coef has ALGEBRA FRAC INT
--R ????: (% , NonNegativeInteger) -> %
--R ?/? : (% , Coef) -> % if Coef has FIELD
--R D : (% , List Symbol, List NonNegativeInteger) -> %
--R D : (% , Symbol, NonNegativeInteger) -> %
--R ^?: (% , NonNegativeInteger) -> %
--R acos : % -> % if Coef has ALGEBRA FRAC INT
--R acosh : % -> % if Coef has ALGEBRA FRAC INT
--R acot : % -> % if Coef has ALGEBRA FRAC INT
--R acoth : % -> % if Coef has ALGEBRA FRAC INT
--R acsc : % -> % if Coef has ALGEBRA FRAC INT
--R acsch : % -> % if Coef has ALGEBRA FRAC INT
--R asec : % -> % if Coef has ALGEBRA FRAC INT
--R asech : % -> % if Coef has ALGEBRA FRAC INT
--R asin : % -> % if Coef has ALGEBRA FRAC INT
--R asinh : % -> % if Coef has ALGEBRA FRAC INT
--R associates?: (% , %) -> Boolean if Coef has INTDOM
--R atan : % -> % if Coef has ALGEBRA FRAC INT

```

```

--R atanh : % -> % if Coef has ALGEBRA FRAC INT
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%, "failed") if Coef has CHARNZ
--R coefficient : (%, NonNegativeInteger) -> Polynomial Coef
--R coefficient : (%, List Symbol, List NonNegativeInteger) -> %
--R coefficient : (%, Symbol, NonNegativeInteger) -> %
--R coefficient : (%, IndexedExponents Symbol) -> Coef
--R coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
--R coerce : % -> % if Coef has INTDOM
--R coerce : Coef -> % if Coef has COMRING
--R cos : % -> % if Coef has ALGEBRA FRAC INT
--R cosh : % -> % if Coef has ALGEBRA FRAC INT
--R cot : % -> % if Coef has ALGEBRA FRAC INT
--R coth : % -> % if Coef has ALGEBRA FRAC INT
--R csc : % -> % if Coef has ALGEBRA FRAC INT
--R csch : % -> % if Coef has ALGEBRA FRAC INT
--R degree : % -> IndexedExponents Symbol
--R differentiate : (%, List Symbol, List NonNegativeInteger) -> %
--R differentiate : (%, Symbol, NonNegativeInteger) -> %
--R differentiate : (%, List Symbol) -> %
--R eval : (%, List Symbol, List %) -> %
--R exp : % -> % if Coef has ALGEBRA FRAC INT
--R exquo : (%, %) -> Union(%, "failed") if Coef has INTDOM
--R extend : (%, NonNegativeInteger) -> %
--R fintegrate : (((() -> %), Symbol, Coef) -> % if Coef has ALGEBRA FRAC INT
--R integrate : (%, Symbol, Coef) -> % if Coef has ALGEBRA FRAC INT
--R integrate : (%, Symbol) -> % if Coef has ALGEBRA FRAC INT
--R log : % -> % if Coef has ALGEBRA FRAC INT
--R monomial : (%, List Symbol, List NonNegativeInteger) -> %
--R monomial : (%, Symbol, NonNegativeInteger) -> %
--R monomial : (Coef, IndexedExponents Symbol) -> %
--R monomial : (%, Symbol, IndexedExponents Symbol) -> %
--R monomial : (%, List Symbol, List IndexedExponents Symbol) -> %
--R nthRoot : (%, Integer) -> % if Coef has ALGEBRA FRAC INT
--R order : (%, Symbol, NonNegativeInteger) -> NonNegativeInteger
--R order : (%, Symbol) -> NonNegativeInteger
--R pi : () -> % if Coef has ALGEBRA FRAC INT
--R polynomial : (%, NonNegativeInteger, NonNegativeInteger) -> Polynomial Coef
--R polynomial : (%, NonNegativeInteger) -> Polynomial Coef
--R sec : % -> % if Coef has ALGEBRA FRAC INT
--R sech : % -> % if Coef has ALGEBRA FRAC INT
--R sin : % -> % if Coef has ALGEBRA FRAC INT
--R sinh : % -> % if Coef has ALGEBRA FRAC INT
--R sqrt : % -> % if Coef has ALGEBRA FRAC INT
--R subtractIfCan : (%, %) -> Union(%, "failed")
--R tan : % -> % if Coef has ALGEBRA FRAC INT
--R tanh : % -> % if Coef has ALGEBRA FRAC INT
--R unit? : % -> Boolean if Coef has INTDOM
--R unitCanonical : % -> % if Coef has INTDOM
--R unitNormal : % -> Record(unit: %, canonical: %, associate: %) if Coef has INTDOM

```



```
--R
--E 1

)spool
)lisp (bye)
```

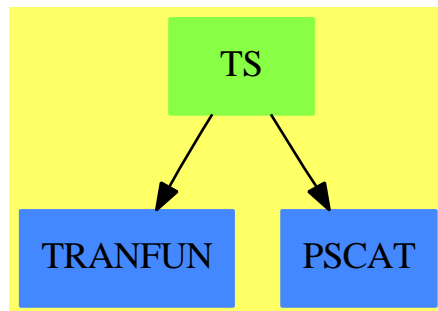
---

— TaylorSeries.help —

```
=====
TaylorSeries examples
=====
```

```
See Also:
o )show TaylorSeries
```

### 21.3.1 TaylorSeries (TS)



See

⇒ “SparseMultivariateTaylorSeries” (SMTS) 20.15.1 on page 2399

**Exports:**

|                |               |                    |                 |          |
|----------------|---------------|--------------------|-----------------|----------|
| 0              | 1             | acos               | acosh           | acot     |
| acoth          | acsc          | acsch              | asec            | asech    |
| asin           | asinh         | associates?        | atan            | atanh    |
| characteristic | charthRoot    | coefficient        | coerce          | complete |
| cos            | cosh          | cot                | coth            | csc      |
| csch           | D             | degree             | differentiate   | eval     |
| exp            | exquo         | extend             | fintegrate      | hash     |
| integrate      | latex         | leadingCoefficient | leadingMonomial | log      |
| map            | monomial      | monomial?          | nthRoot         | one?     |
| order          | pi            | pole?              | polynomial      | recip    |
| reductum       | sample        | sec                | sech            | sin      |
| sinh           | sqrt          | subtractIfCan      | tan             | tanh     |
| unit?          | unitCanonical | unitNormal         | variables       | zero?    |
| ?*?            | ?*?*          | ?+?                | ?-?             | -?       |
| ?=?            | ?^?           | ?^=?               | ?/?             |          |

— domain TS TaylorSeries —

```
)abbrev domain TS TaylorSeries
++ Authors: Burge, Watt, Williamson
++ Date Created: 15 August 1988
++ Date Last Updated: 18 May 1991
++ Basic Operations:
++ Related Domains: SparseMultivariateTaylorSeries
++ Also See: UnivariateTaylorSeries
++ AMS Classifications:
++ Keywords: multivariate, Taylor, series
++ Examples:
++ References:
++ Description:
++ \spadtype{TaylorSeries} is a general multivariate Taylor series domain
++ over the ring Coef and with variables of type Symbol.
```

```
TaylorSeries(Coef): Exports == Implementation where
```

```
Coef : Ring
L ==> List
NNI ==> NonNegativeInteger
SMP ==> Polynomial Coef
StS ==> Stream SMP
```

```
Exports ==> MultivariateTaylorSeriesCategory(Coef,Symbol) with
coefficient: (%,NNI) -> SMP
++\spad{coefficient(s, n)} gives the terms of total degree n.
coerce: Symbol -> %
++\spad{coerce(s)} converts a variable to a Taylor series
coerce: SMP -> %
++\spad{coerce(s)} regroups terms of s by total degree
++ and forms a series.
```

```

if Coef has Algebra Fraction Integer then
  integrate: (%,Symbol,Coef) -> %
    ++\spad{integrate(s,v,c)} is the integral of s with respect
    ++ to v and having c as the constant of integration.
  fintegrate: (() -> %,Symbol,Coef) -> %
    ++\spad{fintegrate(f,v,c)} is the integral of \spad{f()} with respect
    ++ to v and having c as the constant of integration.
    ++ The evaluation of \spad{f()} is delayed.

Implementation ==> SparseMultivariateTaylorSeries(Coef,Symbol,SMP) add
Rep := StS -- Below we use the fact that Rep of PS is Stream SMP.

polynomial(s,n) ==
  sum : SMP := 0
  for i in 0..n while not empty? s repeat
    sum := sum + frst s
    s:= rst s
  sum

```

---

— TS.dotabb —

```

"TS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=TS"]
"TRANFUN" [color="#4488FF",href="bookvol10.2.pdf#nameddest=TRANFUN"]
"PSCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PSCAT"]
"TS" -> "PSCAT"
"TS" -> "TRANFUN"

```

---

## 21.4 domain TEX TexFormat

### 21.4.1 product(product(i\*j,i=a..b),j=c..d) fix

The expression prints properly in ascii text but the tex output is incorrect. Originally the input

```
product(product(i*j,i=a..b),j=c..d)
```

prints as

$$(1) \quad PI2(j = c, d, PI2(i = a, b, i j))$$

but now says: The problem is in `[[src/algebra/tex.spad.pamphlet]]` in the list of constants. The code used to read

```
plexOps      : L S := ["SIGMA","SIGMA2","PI","INTSIGN","INDEFINTEGRAL"]$(L S)
plexPrecs    : L I := [ 700, 800,      700,      700]$(L I)
```

it now reads:

— **product**(**product**(**i\*j**,**i=a..b**),**j=c..d**) **fix** —

```
plexOps      : L S := ["SIGMA","SIGMA2","PI","PI2","INTSIGN","INDEFINTEGRAL"]$(L S)
plexPrecs    : L I := [ 700, 800,      700, 800 , 700,      700]$(L I)
```

in addition we need to add a line defining `[[PI2]]` in `[[formatPlex]]`:

— **define PI2** —

```
op = "PI2"      => "\prod"
```

— **TexFormat.input** —

```
)set break resume
)sys rm -f TexFormat.output
)spool TexFormat.output
)set message test on
)set message auto off
)clear all
```

```
--S 1 of 11
```

```
(1/2)::TEX
```

```
--R
```

```
--R
```

```
--R (1) [ "$$", "\frac{1}{2} ", "$$"]
```

```
--R
```

Type: TexFormat

```
--E 1
```

```
--S 2 of 11
```

```
(1/(x+5))::TEX
```

```
--R
```

```
--R
```

```
--R (2) [ "$$", "\frac{1}{x+5} ", "$$"]
```

```
--R
```

Type: TexFormat

```
--E 2
```

```
--S 3 of 11
```

```
((x+3)/(y-5))::TEX
```

```
--R
```

```
--R
```

```
--R (3) [ "$$", "\frac{x+3}{y-5} ", "$$"]
```

```
--R
```

Type: TexFormat

--E 3

```
--S 4 of 11
)set output fraction horizontal
--R
--E 4
```

```
--S 5 of 11
(1/2)::TEX
--R
--R
--R (4) ["$$", "SLASH ", "\left(", "{1, \: 2} ", "\right)", "$$"]
--R
--R                                          Type: TexFormat
--E 5
```

```
--S 6 of 11
(1/(x+5))::TEX
--R
--R
--R (5)
--R ["$$", "SLASH ", "\left(", "{1, \: {\left( x+5 ", "\right)}}", "\right)", "$$"]
--R
--R                                          Type: TexFormat
--E 6
```

```
--S 7 of 11
)set output mathml on
--R
--E 7
```

```
--S 8 of 11
1/2
--R
--R
--R (6) 1/2
--R<math xmlns="http://www.w3.org/1998/Math/MathML" mathsize="big" display="block">
--R<mrow><mn>1</mn><mo>/</mo><mn>2</mn></mrow>
--R</math>
--R
--R                                          Type: Fraction Integer
--E 8
```

```
--S 9 of 11
1/(x+5)
--R
--R
--R (7) 1/(x + 5)
--R<math xmlns="http://www.w3.org/1998/Math/MathML" mathsize="big" display="block">
--R<mrow><mn>1</mn><mo>/</mo><mrow><mo>(</mo><mi>x</mi><mo>+</mo><mn>5</mn><mo>)</mo></mrow>
--R</math>
--R
```

```
--R                                     Type: Fraction Polynomial Integer
--E 9
```

```
--S 10 of 11
(x+3)/(y-5)
```

```
--R
```

```
--R
```

```
--R      (8)  (x + 3)/(y - 5)
```

```
--R<math xmlns="http://www.w3.org/1998/Math/MathML" mathsize="big" display="block">
```

```
--R<mrow><mrow><mo></mo><mi>x</mi><mo>+</mo><mn>3</mn><mo></mo></mrow><mo></mo><mrow><mo></mo><mi>y</mi><mo>-</mo><mn>5</mn><mo></mo></mrow></math>
```

```
--R</math>
```

```
--R
```

```
                                     Type: Fraction Polynomial Integer
```

```
--E 10
```

```
--S 11 of 11
```

```
)show TexFormat
```

```
--R
```

```
--R TexFormat is a domain constructor
```

```
--R Abbreviation for TexFormat is TEX
```

```
--R This constructor is exposed in this frame.
```

```
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for TEX
```

```
--R
```

```
--R----- Operations -----
```

```
--R ?=? : (%,% ) -> Boolean          coerce : OutputForm -> %
--R coerce : % -> OutputForm          display : % -> Void
--R display : (%,Integer) -> Void      epilogue : % -> List String
--R hash : % -> SingleInteger          latex : % -> String
--R new : () -> %                      prologue : % -> List String
--R tex : % -> List String              ?~=? : (%,% ) -> Boolean
```

```
--R convert : (OutputForm,Integer,OutputForm) -> %
```

```
--R convert : (OutputForm,Integer) -> %
```

```
--R setEpilogue! : (%,List String) -> List String
```

```
--R setPrologue! : (%,List String) -> List String
```

```
--R setTex! : (%,List String) -> List String
```

```
--R
```

```
--E 11
```

```
)spool
```

```
)lisp (bye)
```

---

— TexFormat.help —

```
=====
TexFormat examples
=====
```

You can ask Axiom to show latex output. In particular, this can be used for complex output.

```
(1/2)::TEX
```

```
["$$","\frac{1}{2} ","$$"]
```

```
(1/(x+5))::TEX
```

```
["$$","\frac{1}{{x+5}} ","$$"]
```

```
((x+3)/(y-5))::TEX
```

```
["$$","\frac{{x+3}}{{y -5}} ","$$"]
```

We can change the fraction display so it is horizontal:

```
)set output fraction horizontal
```

```
(1/2)::TEX
```

```
["$$","SLASH ","\left(","{1, \: 2} ","\right)","$$"]
```

```
(1/(x+5))::TEX
```

```
["$$","SLASH ","\left(","{1, \: {x+5}} ","\right)","$$"]
```

```
((x+3)/(y-5))::TEX
```

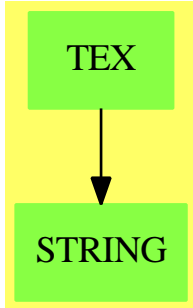
```
["$$","SLASH ","\left(","{{x+3}, \: {y -5}} ","\right)","$$"]
```

See Also:

- o )show TexFormat

---

## 21.4.2 TexFormat (TEX)

**Exports:**

|         |         |          |              |              |
|---------|---------|----------|--------------|--------------|
| coerce  | convert | display  | epilogue     | hash         |
| latex   | new     | prologue | setEpilogue! | setPrologue! |
| setTex! | tex     | convert  | ?=?          | ?~=?         |

— domain **TEX** **TexFormat** —

```

)abbrev domain TEX TexFormat
++ Author: Robert S. Sutor
++ Date Created: 1987 through 1992
++ Change History:
++   05/15/91 RSS Changed matrix formatting to use array environment.
++   06/27/91 RSS Fixed segments
++   08/12/91 RSS Removed some grouping for things, added newWithNum and
++             ungroup, improved line splitting
++   08/15/91 RSS Added mbox support for strings
++   10/15/91 RSS Handle \%\\% at beginning of string
++   01/22/92 RSS Use \[ and \] instead of $$ and $$$. Use
++             %AXIOM STEP NUMBER: instead of \leqno
++   02/27/92 RSS Escape dollar signs appearing in the input.
++   03/09/92 RSS Handle explicit blank appearing in the input.
++   11/28/93 JHD Added code for the VCONCAT and TAG operations.
++   06/27/95 RSS Change back to $$ and \leqno for Saturn
++ Basic Operations: coerce, convert, display, epilogue,
++   tex, new, prologue, setEpilogue!, setTex!, setPrologue!
++ Related Constructors: TexFormat1
++ Also See: ScriptFormulaFormat
++ AMS Classifications:
++ Keywords: TeX, LaTeX, output, format
++ References: \TeX{} is a trademark of the American Mathematical Society.
++ Description:
++ \spadtype{TexFormat} provides a coercion from \spadtype{OutputForm} to
++ \TeX{} format. The particular dialect of \TeX{} used is \LaTeX{}.
++ The basic object consists of three parts: a prologue, a
++ tex part and an epilogue. The functions \spadfun{prologue},

```



```

++ \spadfun{tex} and \spadfun{epilogue} extract these parts,
++ respectively. The main guts of the expression go into the tex part.
++ The other parts can be set (\spadfun{setPrologue!},
++ \spadfun{setEpilogue!}) so that contain the appropriate tags for
++ printing. For example, the prologue and epilogue might simply
++ contain ‘\verb+[+’ and ‘\verb+]\+’, respectively, so that
++ the TeX section will be printed in LaTeX display math mode.

TexFormat(): public == private where
E      ==> OutputForm
I      ==> Integer
L      ==> List
S      ==> String
US     ==> UniversalSegment(Integer)

public == SetCategory with
coerce: E -> $
  ++ coerce(o) changes o in the standard output format to TeX
  ++ format.
convert: (E,I) -> $
  ++ convert(o,step) changes o in standard output format to
  ++ TeX format and also adds the given step number. This is useful
  ++ if you want to create equations with given numbers or have the
  ++ equation numbers correspond to the interpreter step numbers.
convert: (E,I,E) -> $
  ++ convert(o,step,type) changes o in standard output format to
  ++ TeX format and also adds the given step number and type. This
  ++ is useful if you want to create equations with given numbers
  ++ or have the equation numbers correspond to the interpreter step
  ++ numbers.
display: ($, I) -> Void
  ++ display(t,width) outputs the TeX formatted code t so that each
  ++ line has length less than or equal to \spadvar{width}.
display: $ -> Void
  ++ display(t) outputs the TeX formatted code t so that each
  ++ line has length less than or equal to the value set by
  ++ the system command \spadsyscom{set output length}.
epilogue: $ -> L S
  ++ epilogue(t) extracts the epilogue section of a TeX form t.
tex:      $ -> L S
  ++ tex(t) extracts the TeX section of a TeX form t.
new:      () -> $
  ++ new() create a new, empty object. Use \spadfun{setPrologue!},
  ++ \spadfun{setTex!} and \spadfun{setEpilogue!} to set the various
  ++ components of this object.
prologue: $ -> L S
  ++ prologue(t) extracts the prologue section of a TeX form t.
setEpilogue!: ($, L S) -> L S
  ++ setEpilogue!(t,strings) sets the epilogue section of a TeX form t
  ++ to strings.

```

```

setTex!: ($, L S) -> L S
  ++ setTex!(t,strings) sets the TeX section of a TeX form t to strings.
setPrologue!: ($, L S) -> L S
  ++ setPrologue!(t,strings) sets the prologue section of a TeX form t
  ++ to strings.

private == add
import OutputForm
import Character
import Integer
import List OutputForm
import List String

Rep := Record(prolog : L S, TeX : L S, epilog : L S)

-- local variables declarations and definitions

expr: E
prec,opPrec: I
str: S
blank      : S := " \ "

maxPrec    : I := 1000000
minPrec    : I := 0

unaryOps    : L S := ["-", "^"]$(L S)
unaryPrecs  : L I := [700,260]$(L I)

-- the precedence of / in the following is relatively low because
-- the bar obviates the need for parentheses.
binaryOps   : L S := ["+>", "|", "***", "/", "<", ">", "=", "OVER"]$(L S)
binaryPrecs : L I := [0,0,900, 700,400,400,400, 700]$(L I)

naryOps      : L S := ["-", "+", "*", blank, ",", ";", " ", "ROW", "",
  " \cr ", "&", " \\\ " ]$(L S)
naryPrecs    : L I := [700,700,800, 800,110,110, 0, 0, 0,
  0, 0, 0]$(L I)
naryNGOps    : L S := ["ROW", "&"]$(L S)

\getchunk{product(product(i*j,i=a..b),j=c..d) fix}

specialOps   : L S := ["MATRIX", "BRACKET", "BRACE", "CONCATB", "VCONCAT", _
  "AGGLST", "CONCAT", "OVERBAR", "ROOT", "SUB", "TAG", _
  "SUPERSUB", "ZAG", "AGGSET", "SC", "PAREN", _
  "SEGMENT", "QUOTE", "theMap" ]

-- the next two lists provide translations for some strings for
-- which TeX provides special macros.

specialStrings : L S :=

```

```

["cos", "cot", "csc", "log", "sec", "sin", "tan",
 "cosh", "coth", "csch", "sech", "sinh", "tanh",
 "acos", "asin", "atan", "erf", "...", "$", "infinity"]
specialStringsInTeX : L S :=
["\cos", "\cot", "\csc", "\log", "\sec", "\sin", "\tan",
 "\cosh", "\coth", "\csch", "\sech", "\sinh", "\tanh",
 "\arccos", "\arcsin", "\arctan", "\erf", "\ldots", "\$", "\infty"]

-- local function signatures

addBraces:      S -> S
addBrackets:    S -> S
group:          S -> S
formatBinary:   (S, L E, I) -> S
formatFunction: (S, L E, I) -> S
formatMatrix:   L E -> S
formatNary:     (S, L E, I) -> S
formatNaryNoGroup: (S, L E, I) -> S
formatNullary:  S -> S
formatPlex:     (S, L E, I) -> S
formatSpecial:  (S, L E, I) -> S
formatUnary:    (S, E, I) -> S
formatTex:      (E, I) -> S
newWithNum:     I -> $
parenthesize:   S -> S
precondition:   E -> E
postcondition:  S -> S
splitLong:      (S, I) -> L S
splitLong1:     (S, I) -> L S
stringify:      E -> S
ungroup:        S -> S

-- public function definitions

new() : $ ==
--   [[["\["]$(L S), ["" ]$(L S), ["\"]$(L S)]$Rep
--     [["$"]$(L S), ["" ]$(L S), [$$$"]$(L S)]$Rep

newWithNum(stepNum: I) : $ ==
--   num : S := concat("%AXIOM STEP NUMBER: ", string(stepNum)$S)
--   [[["\["]$(L S), ["" ]$(L S), ["\", num]$(L S)]$Rep
--     num : S := concat(concat("\leqno(", string(stepNum)$S), ")")$S
--       [["$"]$(L S), ["" ]$(L S), [num, $$$"]$(L S)]$Rep

coerce(expr : E): $ ==
  f : $ := new()$$
  f.TeX := [postcondition
    formatTex(precondition expr, minPrec)]$(L S)
  f

```

```

convert(expr : E, stepNum : I): $ ==
  f : $ := newWithNum(stepNum)
  f.TeX := [postcondition
    formatTex(precondition expr, minPrec)]$(L S)
  f

display(f : $, len : I) ==
  s,t : S
  for s in f.prolog repeat sayTeX$Lisp s
  for s in f.TeX repeat
    for t in splitLong(s, len) repeat sayTeX$Lisp t
  for s in f.epilog repeat sayTeX$Lisp s
  void()$Void

display(f : $) ==
  display(f, _$LINELENGTH$Lisp pretend I)

prologue(f : $) == f.prolog
tex(f : $) == f.TeX
epilogue(f : $) == f.epilog

setPrologue!(f : $, l : L S) == f.prolog := l
setTex!(f : $, l : L S) == f.TeX := l
setEpilogue!(f : $, l : L S) == f.epilog := l

coerce(f : $): E ==
  s,t : S
  l : L S := nil
  for s in f.prolog repeat l := concat(s,l)
  for s in f.TeX repeat
    for t in splitLong(s, (_$LINELENGTH$Lisp pretend Integer) - 4) repeat
      l := concat(t,l)
  for s in f.epilog repeat l := concat(s,l)
  (reverse l) :: E

-- local function definitions

ungroup(str: S): S ==
  len : I := #str
  len < 2 => str
  lbrace : Character := char "{"
  rbrace : Character := char "}"
  -- drop leading and trailing braces
  if (str.1 = $Character lbrace) and (str.len = $Character rbrace) then
    u : US := segment(2,len-1)$US
    str := str.u
  str

postcondition(str: S): S ==
  str := ungroup str

```

```

len : I := #str
plus : Character := char "+"
minus: Character := char "-"
len < 4 => str
for i in 1..(len-1) repeat
  if (str.i = $Character plus) and (str.(i+1) = $Character minus)
    then setelt(str,i,char " ")$S
str

stringify expr == (object2String$Lisp expr) pretend S

lineConcat( line : S, lines: L S ) : L S ==
  length := #line

  if ( length > 0 ) then
    -- If the last character is a backslash then split at "\ ".
    -- Reinstate the blank.

    if (line.length = char "\" ) then line := concat(line, " ")

    -- Remark: for some reason, "%" at the beginning
    -- of a line has the "\" erased when printed

    if ( line.1 = char "%" ) then line := concat(" \", line)
    else if ( line.1 = char "\" ) and length > 1 and ( line.2 = char "%" ) then
      line := concat(" ", line)

    lines := concat(line,lines)$List(S)
  lines

splitLong(str : S, len : I): L S ==
  -- this blocks into lines
  if len < 20 then len := _$LINELENGTH$Lisp
  splitLong1(str, len)

splitLong1(str : S, len : I) ==
  -- We first build the list of lines backwards and then we
  -- reverse it.

  l : List S := nil
  s : S := ""
  ls : I := 0
  ss : S
  lss : I
  for ss in split(str,char " ") repeat
    -- have the newline macro end a line (even if it means the line
    -- is slightly too long)

    ss = "\\\" =>
      l := lineConcat( concat(s,ss), l )

```

```

s := ""
ls := 0

lss := #ss

-- place certain tokens on their own lines for clarity

ownLine : Boolean :=
  u : US := segment(1,4)$US
  (lss > 3) and ("\end" = ss.u) => true
  u := segment(1,5)$US
  (lss > 4) and ("\left" = ss.u) => true
  u := segment(1,6)$US
  (lss > 5) and ((("\right" = ss.u) or ("\begin" = ss.u)) => true
  false

if ownLine or (ls + lss > len) then
  if not empty? s then l := lineConcat( s, l )
  s := ""
  ls := 0

ownLine or lss > len => l := lineConcat( ss, l )

(lss = 1) and (ss.1 = char "\" ) =>
  ls := ls + lss + 2
  s := concat(s,concat(ss," ")$S)$S

ls := ls + lss + 1
s := concat(s,concat(ss," ")$S)$S

if ls > 0 then l := lineConcat( s, l )

reverse l

group str ==
  concat ["{",str,"}"]

addBraces str ==
  concat ["\left\{ ",str," \right\}"]

addBrackets str ==
  concat ["\left[ ",str," \right]"]

parenthesize str ==
  concat ["\left( ",str," \right)"]

precondition expr ==
  outputTran$Lisp expr

formatSpecial(op : S, args : L E, prec : I) : S ==

```

```

arg : E
prescript : Boolean := false
op = "theMap" => "\mbox{theMap(...)}"
op = "AGGLST" =>
  formatNary(" ",args,prec)
op = "AGGSET" =>
  formatNary(";",args,prec)
op = "TAG" =>
  group concat [formatTex(first args,prec),
    "\rightarrow",
    formatTex(second args,prec)]
op = "VCONCAT" =>
  group concat("\begin{array}{c}",
    concat(concat([concat(formatTex(u, minPrec),"\\")
      for u in args]::L S),
    "\end{array}"))
op = "CONCATB" =>
  formatNary(" ",args,prec)
op = "CONCAT" =>
  formatNary("",args,minPrec)
op = "QUOTE" =>
  group concat("{\tt '}",formatTex(first args, minPrec))
op = "BRACKET" =>
  group addBrackets ungroup formatTex(first args, minPrec)
op = "BRACE" =>
  group addBraces ungroup formatTex(first args, minPrec)
op = "PAREN" =>
  group parenthesize ungroup formatTex(first args, minPrec)
op = "OVERBAR" =>
  null args => ""
  group concat ["\overline ",formatTex(first args, minPrec)]
op = "ROOT" =>
  null args => ""
  tmp : S := group formatTex(first args, minPrec)
  null rest args => group concat ["\sqrt ",tmp]
  group concat
    ["\root ",group formatTex(first rest args, minPrec)," \of ",tmp]
op = "SEGMENT" =>
  tmp : S := concat [formatTex(first args, minPrec),".."]
  group
    null rest args => tmp
    concat [tmp,formatTex(first rest args, minPrec)]
op = "SUB" =>
  group concat [formatTex(first args, minPrec)," \sb ",
    formatSpecial("AGGLST",rest args,minPrec)]
op = "SUPERSUB" =>
  -- variable name
  form : List S := [formatTex(first args, minPrec)]
  -- subscripts
  args := rest args

```

```

null args => concat(form)$S
tmp : S := formatTex(first args, minPrec)
if (tmp ^="") and (tmp ^="{}") and (tmp ^=" ") then
  form := append(form,[" \sb ",group tmp])$(List S)
-- superscripts
args := rest args
null args => group concat(form)$S
tmp : S := formatTex(first args, minPrec)
if (tmp ^="") and (tmp ^="{}") and (tmp ^=" ") then
  form := append(form,[" \sp ",group tmp])$(List S)
-- presuperscripts
args := rest args
null args => group concat(form)$S
tmp : S := formatTex(first args, minPrec)
if (tmp ^="") and (tmp ^="{}") and (tmp ^=" ") then
  form := append([" \sp ",group tmp],form)$(List S)
  prescript := true
-- presubscripts
args := rest args
null args =>
  group concat
    prescript => cons("{",form)
    form
tmp : S := formatTex(first args, minPrec)
if (tmp ^="") and (tmp ^="{}") and (tmp ^=" ") then
  form := append([" \sb ",group tmp],form)$(List S)
  prescript := true
group concat
  prescript => cons("{",form)
  form
op = "SC" =>
  -- need to handle indentation someday
  null args => ""
  tmp := formatNaryNoGroup(" \\", args, minPrec)
  group concat ["\begin{array}{l} ",tmp," \end{array} "]
op = "MATRIX" => formatMatrix rest args
op = "ZAG" =>
  concat [" \zag{",formatTex(first args, minPrec),"}{",
    formatTex(first rest args,minPrec),"}"]
concat ["not done yet for ",op]

formatPlex(op : S, args : L E, prec : I) : S ==
  hold : S
  p : I := position(op,plexOps)
  p < 1 => error "unknown Tex unary op"
  opPrec := plexPrecs.p
  n : I := #args
  (n ^= 2) and (n ^= 3) => error "wrong number of arguments for plex"
  s : S :=
    op = "SIGMA"    => "\sum"

```



```

    op = "SIGMA2"    => "\sum"
    op = "PI"        => "\prod"
\getchunk{define PI2}
    op = "INTSIGN" => "\int"
    op = "INDEFINTEGRAL" => "\int"
    "???"
hold := formatTex(first args,minPrec)
args := rest args
if op ^="INDEFINTEGRAL" then
    if hold ^=" " then
        s := concat [s," \sb",group concat ["\displaystyle ",hold]]
    if not null rest args then
        hold := formatTex(first args,minPrec)
        if hold ^=" " then
            s := concat [s," \sp",group concat ["\displaystyle ",hold]]
        args := rest args
        s := concat [s," ",formatTex(first args,minPrec)]
    else
        hold := group concat [hold," ",formatTex(first args,minPrec)]
        s := concat [s," ",hold]
    if opPrec < prec then s := parenthesize s
    group s

formatMatrix(args : L E) : S ==
-- format for args is [[ROW ...],[ROW ...],[ROW ...]]
-- generate string for formatting columns (centered)
cols : S := "{"
for i in 2..#(first(args) pretend L E) repeat
    cols := concat(cols,"c")
cols := concat(cols,"}")
group addBrackets concat
["\begin{array}",cols,formatNaryNoGroup(" \\",args,minPrec),
" \end{array} "]

formatFunction(op : S, args : L E, prec : I) : S ==
    group concat [op, " ", parenthesize formatNary(" ",args,minPrec)]

formatNullary(op : S) ==
    op = "NOTHING" => ""
    group concat [op,"()"]

formatUnary(op : S, arg : E, prec : I) ==
    p : I := position(op,unaryOps)
    p < 1 => error "unknown Tex unary op"
    opPrec := unaryPrecs.p
    s : S := concat [op,formatTex(arg,opPrec)]
    opPrec < prec => group parenthesize s
    op = "-" => s
    group s

```

```

formatBinary(op : S, args : L E, prec : I) : S ==
  p : I := position(op,binaryOps)
  p < 1 => error "unknown Tex binary op"
  op :=
    op = "|"      => " \mid "
    op = "**"      => " \sp "
    op = "/"      => " \over "
    op = "OVER"   => " \over "
    op = "+->"    => " \mapsto "
    op
  opPrec := binaryPrecs.p
  s : S := formatTex(first args, opPrec)
  if op = " \over " then
    s := concat [" \frac{",s,"}{",formatTex(first rest args, opPrec),"}"]
  else if op = " \sp " then
    s := concat [s,"^",formatTex(first rest args, opPrec)]
  else
    s := concat [s,op,formatTex(first rest args, opPrec)]
  group
    op = " \over " => s
    opPrec < prec => parenthesize s
  s

formatNary(op : S, args : L E, prec : I) : S ==
  group formatNaryNoGroup(op, args, prec)

formatNaryNoGroup(op : S, args : L E, prec : I) : S ==
  null args => ""
  p : I := position(op,naryOps)
  p < 1 => error "unknown Tex nary op"
  op :=
    op = ","      => ", \: "
    op = ";"      => "; \: "
    op = "*"      => blank
    op = " "      => " \ "
    op = "ROW"    => " & "
    op
  l : L S := nil
  opPrec := naryPrecs.p
  for a in args repeat
    l := concat(op,concat(formatTex(a,opPrec),l)$L(S))$L(S)
  s : S := concat reverse rest l
  opPrec < prec => parenthesize s
  s

formatTex(expr,prec) ==
  i,len : Integer
  intSplitLen : Integer := 20
  ATOM(expr)$Lisp pretend Boolean =>
    str := stringify expr

```

```

len := #str
INTEGERP$Lisp expr =>
  i := expr pretend Integer
  if (i < 0) or (i > 9)
  then
    group
      nstr : String := ""
      -- insert some blanks into the string, if too long
      while ((len := #str) > intSplitLen) repeat
        nstr := concat [nstr, " ",
          elt(str,segment(1,intSplitLen)$US)]
        str := elt(str,segment(intSplitLen+1)$US)
      empty? nstr => str
      nstr :=
        empty? str => nstr
        concat [nstr, " ",str]
        elt(nstr,segment(2)$US)
      else str
  str = "%pi" => "\pi"
  str = "%e"  => "e"
  str = "%i"  => "i"
  len > 1 and str.1 = char "%" and str.2 = char "%" =>
    u : US := segment(3,len)$US
    concat(" %\%",str.u)
  len > 0 and str.1 = char "%" => concat(" \",str)
  len > 1 and digit? str.1 => group str -- should handle floats
  len > 0 and str.1 = char "_" =>
    concat(concat(" \mbox{\tt ",str),"} ")
  len = 1 and str.1 = char " " => "{\ "
  (i := position(str,specialStrings)) > 0 =>
    specialStringsInTeX.i
  (i := position(char " ",str)) > 0 =>
    -- We want to preserve spacing, so use a roman font.
    concat(concat(" \mbox{\rm ",str),"} ")
  str
l : L E := (expr pretend L E)
null l => blank
op : S := stringify first l
args : L E := rest l
nargs : I := #args

-- special cases
member?(op, specialOps) => formatSpecial(op,args,prec)
member?(op, plexOps)    => formatPlex(op,args,prec)

-- nullary case
0 = nargs => formatNullary op

-- unary case
(1 = nargs) and member?(op, unaryOps) =>

```

```

formatUnary(op, first args, prec)

-- binary case
(2 = nargs) and member?(op, binaryOps) =>
  formatBinary(op, args, prec)

-- nary case
member?(op,naryNGOps) => formatNaryNoGroup(op,args, prec)
member?(op,naryOps) => formatNary(op,args, prec)
op := formatTex(first 1,minPrec)
formatFunction(op,args,prec)

-----

— TEX.dotabb —

"TEX" [color="#88FF44",href="bookvol10.3.pdf#nameddest=TEX"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"TEX" -> "STRING"

-----

```

## 21.5 domain TEXTFILE TextFile

```

— TextFile.input —

)set break resume
)sys rm -f TextFile.output
)spool TextFile.output
)set message test on
)set message auto off
)clear all
--S 1 of 10
f1: TextFile := open("/etc/group", "input")
--R
--R
--R (1) "/etc/group"
--R
--R
--E 1

--S 2 of 10
f2: TextFile := open("MOTD", "output")
--R
--R

```

Type: TextFile

```

--R (2) "MOTD"
--R
--E 2
Type: TextFile

--S 3 of 10
l := readLine! f1
--R
--R
--I (3) "ROOT:x:0:"
--R
--E 3
Type: String

--S 4 of 10
writeLine!(f2, upperCase l)
--R
--R
--I (4) "ROOT:X:0:"
--R
--E 4
Type: String

--S 5 of 10
while not endOfFile? f1 repeat
  s := readLine! f1
  writeLine!(f2, upperCase s)
--R
--R
--E 5
Type: Void

--S 6 of 10
close! f1
--R
--R
--R (6) "/etc/group"
--R
--E 6
Type: TextFile

--S 7 of 10
write!(f2, "-The-")
--R
--R
--R (7) "-The-"
--R
--E 7
Type: String

--S 8 of 10
write!(f2, "-End-")
--R
--R
--R (8) "-End-"
--R
Type: String

```

```

--E 8

--S 9 of 10
writeLine! f2
--R
--R
--R (9) ""
--R
--R                                          Type: String
--E 9

--S 10 of 10
close! f2
--R
--R
--R (10) "MOTD"
--R
--R                                          Type: TextFile
--E 10
)system rm -f MOTD
)spool
)lisp (bye)

```

---

— TextFile.help —

=====

TextFile examples

=====

The domain TextFile allows Axiom to read and write character data and exchange text with other programs. This type behaves in Axiom much like a File of strings, with additional operations to cause new lines. We give an example of how to produce an upper case copy of a file.

This is the file from which we read the text.

```

f1: TextFile := open("/etc/group", "input")
"/etc/group"
                                     Type: TextFile

```

This is the file to which we write the text.

```

f2: TextFile := open("/tmp/MOTD", "output")
"MOTD"
                                     Type: TextFile

```

Entire lines are handled using the readLine and writeLine operations.

```

l := readLine! f1

```

```
"root:x:0:root"
```

Type: String

```
writeLine!(f2, upperCase 1)
"ROOT:X:0:ROOT"
```

Type: String

Use the `endOfFile?` operation to check if you have reached the end of the file.

```
while not endOfFile? f1 repeat
  s := readLine! f1
  writeLine!(f2, upperCase s)
```

Type: Void

The file `f1` is exhausted and should be closed.

```
close! f1
"/etc/group"
```

Type: TextFile

It is sometimes useful to write lines a bit at a time. The write operation allows this.

```
write!(f2, "-The-")
"-The-"
```

Type: String

```
write!(f2, "-End-")
"-End-"
```

Type: String

This ends the line. This is done in a machine-dependent manner.

```
writeLine! f2
""
```

Type: String

```
close! f2
"MOTD"
```

Type: TextFile

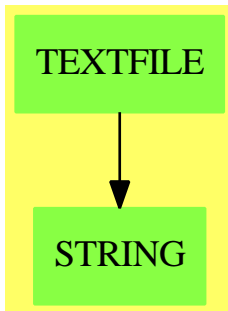
Finally, clean up.

```
)system rm /tmp/MOTD
```

See Also:

- o )help File
- o )help KeyedAccessFile
- o )help Library
- o )show TextFile

### 21.5.1 TextFile (TEXTFILE)



See

- ⇒ “File” (FILE) 7.2.1 on page 770
- ⇒ “BinaryFile” (BINFILE) 3.8.1 on page 277
- ⇒ “KeyedAccessFile” (KAFILE) 12.2.1 on page 1377
- ⇒ “Library” (LIB) 13.2.1 on page 1392

#### Exports:

|           |                |            |        |            |
|-----------|----------------|------------|--------|------------|
| close!    | coerce         | endOfFile? | hash   | iomode     |
| latex     | name           | open       | read!  | readIfCan! |
| readLine! | readLineIfCan! | reopen!    | write! | writeLine! |
| ?=?       | ?~=?           |            |        |            |

— domain TEXTFILE TextFile —

```

)abbrev domain TEXTFILE TextFile
++ Author: Stephen M. Watt
++ Date Created: 1985
++ Date Last Updated: June 4, 1991
++ Basic Operations: writeLine! readLine! readLineIfCan! readIfCan! endOfFile?
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This domain provides an implementation of text files. Text is stored
++ in these files using the native character set of the computer.

```

TextFile: Cat == Def where



```

StreamName ==> Union(FileName, "console")

Cat == FileCategory(FileName, String) with
writeLine_!: (%, String) -> String
  ++ writeLine!(f,s) writes the contents of the string s
  ++ and finishes the current line in the file f.
  ++ The value of s is returned.

writeLine_!: % -> String
  ++ writeLine!(f) finishes the current line in the file f.
  ++ An empty string is returned. The call \spad{writeLine!(f)} is
  ++ equivalent to \spad{writeLine!(f,"")}.

readLine_!: % -> String
  ++ readLine!(f) returns a string of the contents of a line from
  ++ the file f.

readLineIfCan_!: % -> Union(String, "failed")
  ++ readLineIfCan!(f) returns a string of the contents of a line from
  ++ file f, if possible. If f is not readable or if it is
  ++ positioned at the end of file, then \spad{"failed"} is returned.

readIfCan_!: % -> Union(String, "failed")
  ++ readIfCan!(f) returns a string of the contents of a line from
  ++ file f, if possible. If f is not readable or if it is
  ++ positioned at the end of file, then \spad{"failed"} is returned.

endOfFile?: % -> Boolean
  ++ endOfFile?(f) tests whether the file f is positioned after the
  ++ end of all text. If the file is open for output, then
  ++ this test is always true.

Def == File(String) add
FileState ==> SExpression

Rep := Record(fileName:  FileName,  _
               fileState: FileState, _
               fileIOmode: String)

read_! f      == readLine_! f
readIfCan_! f == readLineIfCan_! f

readLine_! f ==
  f.fileIOmode ^= "input" => error "File not in read state"
  s: String := read_-line(f.fileState)$Lisp
  PLACEP(s)$Lisp => error "End of file"
  s
readLineIfCan_! f ==
  f.fileIOmode ^= "input" => error "File not in read state"
  s: String := read_-line(f.fileState)$Lisp

```

```

PLACEP(s)$Lisp => "failed"
s
write_!(f, x) ==
  f.fileIOmode ^= "output" => error "File not in write state"
  PRINTEXP(x, f.fileState)$Lisp
  x
writeLine_! f ==
  f.fileIOmode ^= "output" => error "File not in write state"
  TERPRI(f.fileState)$Lisp
  ""
writeLine_!(f, x) ==
  f.fileIOmode ^= "output" => error "File not in write state"
  PRINTEXP(x, f.fileState)$Lisp
  TERPRI(f.fileState)$Lisp
  x
endOfFile? f ==
  f.fileIOmode = "output" => false
  (EOFP(f.fileState)$Lisp pretend Boolean) => true
  false

```

---

— TEXTFILE.dotabb —

```

"TEXTFILE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=TEXTFILE"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"TEXTFILE" -> "STRING"

```

---

## 21.6 domain SYMS TheSymbolTable

— TheSymbolTable.input —

```

)set break resume
)sys rm -f TheSymbolTable.output
)spool TheSymbolTable.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show TheSymbolTable
--R TheSymbolTable is a domain constructor

```

```

--R Abbreviation for TheSymbolTable is SYMS
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SYMS
--R
--R----- Operations -----
--R clearTheSymbolTable : () -> Void      coerce : % -> OutputForm
--R currentSubProgram : () -> Symbol      empty : () -> %
--R endSubProgram : () -> Symbol          newSubProgram : Symbol -> Void
--R printHeader : () -> Void              printHeader : Symbol -> Void
--R printHeader : (Symbol,%) -> Void      printTypes : Symbol -> Void
--R showTheSymbolTable : () -> %
--R argumentList! : List Symbol -> Void
--R argumentList! : (Symbol,List Symbol) -> Void
--R argumentList! : (Symbol,List Symbol,%) -> Void
--R argumentListOf : (Symbol,%) -> List Symbol
--R clearTheSymbolTable : Symbol -> Void
--R declare! : (Symbol,FortranType,Symbol) -> FortranType
--R declare! : (Symbol,FortranType) -> FortranType
--R declare! : (List Symbol,FortranType,Symbol,%) -> FortranType
--R declare! : (Symbol,FortranType,Symbol,%) -> FortranType
--R returnType! : Union(fst: FortranScalarType,void: void) -> Void
--R returnType! : (Symbol,Union(fst: FortranScalarType,void: void)) -> Void
--R returnType! : (Symbol,Union(fst: FortranScalarType,void: void),%) -> Void
--R returnTypeOf : (Symbol,%) -> Union(fst: FortranScalarType,void: void)
--R symbolTableOf : (Symbol,%) -> SymbolTable
--R
--E 1

)spool
)lisp (bye)

```

---

— TheSymbolTable.help —

```

=====
TheSymbolTable examples
=====

```

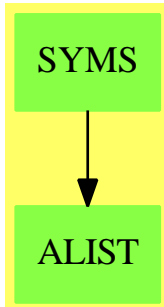
```

See Also:
o )show TheSymbolTable

```

---

### 21.6.1 TheSymbolTable (SYMS)



See

- ⇒ “FortranScalarType” (FST) 7.19.1 on page 929
- ⇒ “FortranType” (FT) 7.21.1 on page 938
- ⇒ “SymbolTable” (SYMTAB) 20.38.1 on page 2606

#### Exports:

|               |                |                     |                    |                   |
|---------------|----------------|---------------------|--------------------|-------------------|
| argumentList! | argumentListOf | clearTheSymbolTable | coerce             | currentSubProgram |
| declare!      | empty          | endSubProgram       | newSubProgram      | printHeader       |
| printTypes    | returnType!    | returnTypeOf        | showTheSymbolTable | symbolTableOf     |

— domain SYMS TheSymbolTable —

```

)abbrev domain SYMS TheSymbolTable
++ Author: Mike Dewar
++ Date Created:  October 1992
++ Date Last Updated:
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ Creates and manipulates one global symbol table for FORTRAN
++ code generation, containing details of types, dimensions, and argument
++ lists.

```

TheSymbolTable() : Exports == Implementation where

```

S      ==> Symbol
FST    ==> FortranScalarType
FSTU   ==> Union(fst:FST,void:"void")

```

```

Exports == CoercibleTo OutputForm with
  showTheSymbolTable : () -> $

```

```

    ++ showTheSymbolTable() returns the current symbol table.
clearTheSymbolTable : () -> Void
    ++ clearTheSymbolTable() clears the current symbol table.
clearTheSymbolTable : Symbol -> Void
    ++ clearTheSymbolTable(x) removes the symbol x from the table
declare! : (Symbol,FortranType,Symbol,$) -> FortranType
    ++ declare!(u,t,asp,tab) declares the parameter u of subprogram asp
    ++ to have type t in symbol table tab.
declare! : (List Symbol,FortranType,Symbol,$) -> FortranType
    ++ declare!(u,t,asp,tab) declares the parameters u of subprogram asp
    ++ to have type t in symbol table tab.
declare! : (Symbol,FortranType) -> FortranType
    ++ declare!(u,t) declares the parameter u to have type t in the
    ++ current level of the symbol table.
declare! : (Symbol,FortranType,Symbol) -> FortranType
    ++ declare!(u,t,asp) declares the parameter u to have type t in asp.
newSubProgram : Symbol -> Void
    ++ newSubProgram(f) asserts that from now on type declarations are part
    ++ of subprogram f.
currentSubProgram : () -> Symbol
    ++ currentSubProgram() returns the name of the current subprogram being
    ++ processed
endSubProgram : () -> Symbol
    ++ endSubProgram() asserts that we are no longer processing the current
    ++ subprogram.
argumentList! : (Symbol,List Symbol,$) -> Void
    ++ argumentList!(f,l,tab) declares that the argument list for subprogram f
    ++ in symbol table tab is l.
argumentList! : (Symbol,List Symbol) -> Void
    ++ argumentList!(f,l) declares that the argument list for subprogram f in
    ++ the global symbol table is l.
argumentList! : List Symbol -> Void
    ++ argumentList!(l) declares that the argument list for the current
    ++ subprogram in the global symbol table is l.
returnType! : (Symbol,FSTU,$) -> Void
    ++ returnType!(f,t,tab) declares that the return type of subprogram f in
    ++ symbol table tab is t.
returnType! : (Symbol,FSTU) -> Void
    ++ returnType!(f,t) declares that the return type of subprogram f in
    ++ the global symbol table is t.
returnType! : FSTU -> Void
    ++ returnType!(t) declares that the return type of the current subprogram
    ++ in the global symbol table is t.
printHeader : (Symbol,$) -> Void
    ++ printHeader(f,tab) produces the FORTRAN header for subprogram f in
    ++ symbol table tab on the current FORTRAN output stream.
printHeader : Symbol -> Void
    ++ printHeader(f) produces the FORTRAN header for subprogram f in
    ++ the global symbol table on the current FORTRAN output stream.
printHeader : () -> Void

```

```

    ++ printHeader() produces the FORTRAN header for the current subprogram in
    ++ the global symbol table on the current FORTRAN output stream.
printTypes: Symbol -> Void
    ++ printTypes(tab) produces FORTRAN type declarations from tab, on the
    ++ current FORTRAN output stream
empty : () -> $
    ++ empty() creates a new, empty symbol table.
returnTypeOf : (Symbol,$) -> FSTU
    ++ returnTypeOf(f,tab) returns the type of the object returned by f
argumentListOf : (Symbol,$) -> List(Symbol)
    ++ argumentListOf(f,tab) returns the argument list of f
symbolTableOf : (Symbol,$) -> SymbolTable
    ++ symbolTableOf(f,tab) returns the symbol table of f

Implementation == add

Entry : Domain := Record(symtab:SymbolTable, _
                        returnType:FSTU, _
                        argList:List Symbol)

Rep := Table(Symbol,Entry)

-- These are the global variables we want to update:
theSymbolTable : $ := empty()$Rep
currentSubProgramName : Symbol := MAIN

newEntry():Entry ==
    construct(empty()$SymbolTable,["void"]$FSTU,[],:List(Symbol))$Entry

checkIfEntryExists(name:Symbol,tab:$) : Void ==
    key?(name,tab) => void()$Void
    setelt(tab,name,newEntry())$Rep
    void()$Void

returnTypeOf(name:Symbol,tab:$):FSTU ==
    elt(elt(tab,name)$Rep,returnType)$Entry

argumentListOf(name:Symbol,tab:$):List(Symbol) ==
    elt(elt(tab,name)$Rep,argList)$Entry

symbolTableOf(name:Symbol,tab:$):SymbolTable ==
    elt(elt(tab,name)$Rep,symtab)$Entry

coerce(u:$):OutputForm ==
    coerce(u)$Rep

showTheSymbolTable():$ ==
    theSymbolTable

clearTheSymbolTable():Void ==

```

```

theSymbolTable := empty()$Rep
void()$Void

clearTheSymbolTable(u:Symbol):Void ==
  remove!(u,theSymbolTable)$Rep
  void()$Void

empty():$ ==
  empty()$Rep

currentSubProgram():Symbol ==
  currentSubProgramName

endSubProgram():Symbol ==
-- If we want to support more complex languages then we should keep
-- a list of subprograms / blocks - but for the moment lets stick with
-- Fortran.
  currentSubProgramName := MAIN

newSubProgram(u:Symbol):Void ==
  setelt(theSymbolTable,u,newEntry())$Rep
  currentSubProgramName := u
  void()$Void

argumentList!(u:Symbol,args>List Symbol,symbols:$):Void ==
  checkIfEntryExists(u,symbols)
  setelt(elt(symbols,u)$Rep,argList,args)$Entry

argumentList!(u:Symbol,args>List Symbol):Void ==
  argumentList!(u,args,theSymbolTable)

argumentList!(args>List Symbol):Void ==
  checkIfEntryExists(currentSubProgramName,theSymbolTable)
  setelt(elt(theSymbolTable,currentSubProgramName)$Rep, _
    argList,args)$Entry

returnType!(u:Symbol,type:FSTU,symbols:$):Void ==
  checkIfEntryExists(u,symbols)
  setelt(elt(symbols,u)$Rep,returnType,type)$Entry

returnType!(u:Symbol,type:FSTU):Void ==
  returnType!(u,type,theSymbolTable)

returnType!(type:FSTU ):Void ==
  checkIfEntryExists(currentSubProgramName,theSymbolTable)
  setelt(elt(theSymbolTable,currentSubProgramName)$Rep, _
    returnType,type)$Entry

declare!(u:Symbol,type:FortranType):FortranType ==
  declare!(u,type,currentSubProgramName,theSymbolTable)

```

```

declare!(u:Symbol,type:FortranType,asp:Symbol,symbols:):FortranType ==
  checkIfEntryExists(asp,symbols)
  declare!(u,type, elt(elt(symbols,asp)$Rep,symtab)$Entry)$SymbolTable

declare!(u>List Symbol,type:FortranType,asp:Symbol,syms:):FortranType ==
  checkIfEntryExists(asp,syms)
  declare!(u,type, elt(elt(syms,asp)$Rep,symtab)$Entry)$SymbolTable

declare!(u:Symbol,type:FortranType,asp:Symbol):FortranType ==
  checkIfEntryExists(asp,theSymbolTable)
  declare!(u,type,elt(elt(theSymbolTable,asp)$Rep,symtab)$Entry)$SymbolTable

printHeader(u:Symbol,symbols:):Void ==
  entry := elt(symbols,u)$Rep
  fortFormatHead(elt(entry,returnType)$Entry::OutputForm,u::OutputForm, _
    elt(entry,argList)$Entry::OutputForm)$Lisp
  printTypes(elt(entry,symtab)$Entry)$SymbolTable

printHeader(u:Symbol):Void ==
  printHeader(u,theSymbolTable)

printHeader():Void ==
  printHeader(currentSubProgramName,theSymbolTable)

printTypes(u:Symbol):Void ==
  printTypes(elt(elt(theSymbolTable,u)$Rep,symtab)$Entry)$SymbolTable

```

---

— SYMS.dotabb —

```

"SYMS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SYMS"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"SYMS" -> "ALIST"

```

---

## 21.7 domain M3D ThreeDimensionalMatrix

— ThreeDimensionalMatrix.input —

```

)set break resume
)sys rm -f ThreeDimensionalMatrix.output

```



```

)spool ThreeDimensionalMatrix.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ThreeDimensionalMatrix
--R ThreeDimensionalMatrix R: SetCategory is a domain constructor
--R Abbreviation for ThreeDimensionalMatrix is M3D
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for M3D
--R
--R----- Operations -----
--R construct : List List List R -> %      copy : % -> %
--R empty : () -> %                        empty? : % -> Boolean
--R eq? : (%,% ) -> Boolean                map : ((R -> R),%) -> %
--R sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,% ) -> Boolean if R has SETCAT
--R any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> PrimitiveArray PrimitiveArray PrimitiveArray R
--R coerce : PrimitiveArray PrimitiveArray PrimitiveArray R -> %
--R coerce : % -> OutputForm if R has SETCAT
--R count : (R,%) -> NonNegativeInteger if $ has finiteAggregate and R has SETCAT
--R count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R elt : (% ,NonNegativeInteger,NonNegativeInteger,NonNegativeInteger) -> R
--R eval : (% ,List R,List R) -> % if R has EVALAB R and R has SETCAT
--R eval : (% ,R,R) -> % if R has EVALAB R and R has SETCAT
--R eval : (% ,Equation R) -> % if R has EVALAB R and R has SETCAT
--R eval : (% ,List Equation R) -> % if R has EVALAB R and R has SETCAT
--R every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if R has SETCAT
--R identityMatrix : NonNegativeInteger -> % if R has RING
--R latex : % -> String if R has SETCAT
--R less? : (% ,NonNegativeInteger) -> Boolean
--R map! : ((R -> R),%) -> % if $ has shallowlyMutable
--R matrixConcat3D : (Symbol,%,%) -> %
--R matrixDimensions : % -> Vector NonNegativeInteger
--R member? : (R,%) -> Boolean if $ has finiteAggregate and R has SETCAT
--R members : % -> List R if $ has finiteAggregate
--R more? : (% ,NonNegativeInteger) -> Boolean
--R parts : % -> List R if $ has finiteAggregate
--R plus : (% ,%) -> % if R has RING
--R setelt! : (% ,NonNegativeInteger,NonNegativeInteger,NonNegativeInteger,R) -> R
--R size? : (% ,NonNegativeInteger) -> Boolean
--R zeroMatrix : (NonNegativeInteger,NonNegativeInteger,NonNegativeInteger) -> % if R has RING
--R ?=? : (% ,%) -> Boolean if R has SETCAT
--R
--E 1

```

```
)spool
)lisp (bye)
```

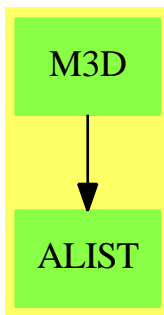
---

— **ThreeDimensionalMatrix.help** —

```
=====
ThreeDimensionalMatrix examples
=====
```

```
See Also:
o )show ThreeDimensionalMatrix
```

### 21.7.1 ThreeDimensionalMatrix (M3D)



**See**

```
⇒ “Result” (RESULT) 19.9.1 on page 2260
⇒ “FortranCode” (FC) 7.16.1 on page 898
⇒ “FortranProgram” (FORTRAN) 7.18.1 on page 923
⇒ “SimpleFortranProgram” (SFORT) 20.11.1 on page 2364
⇒ “Switch” (SWITCH) 20.36.1 on page 2588
⇒ “FortranTemplate” (FTEM) 7.20.1 on page 934
⇒ “FortranExpression” (FEXPR) 7.17.1 on page 914
```

**Exports:**

|         |        |                |                  |         |
|---------|--------|----------------|------------------|---------|
| any?    | coerce | construct      | copy             | count   |
| elt     | empty  | empty?         | eq?              | eval    |
| every?  | hash   | identityMatrix | latex            | less?   |
| map     | map!   | matrixConcat3D | matrixDimensions | member? |
| members | more?  | parts          | plus             | sample  |
| setelt! | size?  | zeroMatrix     | #?               | ?=?     |
| ?~=?    |        |                |                  |         |

## — domain M3D ThreeDimensionalMatrix —

```

)abbrev domain M3D ThreeDimensionalMatrix
++ Author: William Naylor
++ Date Created: 20 October 1993
++ Date Last Updated: 20 May 1994
++ BasicFunctions:
++ Related Constructors: Matrix
++ Also See: PrimitiveArray
++ AMS Classification:
++ Keywords:
++ References:
++ Description:
++ This domain represents three dimensional matrices over a general object type

ThreeDimensionalMatrix(R) : Exports == Implementation where

R : SetCategory
L ==> List
NNI ==> NonNegativeInteger
A1AGG ==> OneDimensionalArrayAggregate
ARRAY1 ==> OneDimensionalArray
PA ==> PrimitiveArray
INT ==> Integer
PI ==> PositiveInteger

Exports ==> HomogeneousAggregate(R) with

if R has Ring then
  zeroMatrix : (NNI,NNI,NNI) -> $
    ++ zeroMatrix(i,j,k) create a matrix with all zero terms
  identityMatrix : (NNI) -> $
    ++ identityMatrix(n) create an identity matrix
    ++ we note that this must be square
  plus : ($,$) -> $
    ++ plus(x,y) adds two matrices, term by term
    ++ we note that they must be the same size
  construct : (L L L R) -> $
    ++ construct(l1l1) creates a 3-D matrix from a List List List R l1l1
  elt : ($,NNI,NNI,NNI) -> R
    ++ elt(x,i,j,k) extract an element from the matrix x
  setelt! : ($,NNI,NNI,NNI,R) -> R
    ++ setelt!(x,i,j,k,s) (or x.i.j.k:=s) sets a specific element of the array to some va
  coerce : (PA PA PA R) -> $
    ++ coerce(p) moves from the representation type
    ++ (PrimitiveArray PrimitiveArray PrimitiveArray R)
    ++ to the domain
  coerce : $ -> (PA PA PA R)
    ++ coerce(x) moves from the domain to the representation type

```

```

matrixConcat3D : (Symbol,$,$) -> $
  ++ matrixConcat3D(s,x,y) concatenates two 3-D matrices along a specified axis
matrixDimensions : $ -> Vector NNI
  ++ matrixDimensions(x) returns the dimensions of a matrix

Implementation ==> (PA PA PA R) add

import (PA PA PA R)
import (PA PA R)
import (PA R)
import R

matrix1,matrix2,resultMatrix : $

-- function to concatenate two matrices
-- the first argument must be a symbol, which is either i,j or k
-- to specify the direction in which the concatenation is to take place
matrixConcat3D(dir : Symbol,mat1 : $,mat2 : $) : $ ==
  ^((dir = (i::Symbol)) or (dir = (j::Symbol)) or (dir = (k::Symbol)))_
  => error "the axis of concatenation must be i,j or k"
mat1Dim := matrixDimensions(mat1)
mat2Dim := matrixDimensions(mat2)
iDim1 := mat1Dim.1
jDim1 := mat1Dim.2
kDim1 := mat1Dim.3
iDim2 := mat2Dim.1
jDim2 := mat2Dim.2
kDim2 := mat2Dim.3
matRep1 : (PA PA PA R) := copy(mat1 :: (PA PA PA R))$(PA PA PA R)
matRep2 : (PA PA PA R) := copy(mat2 :: (PA PA PA R))$(PA PA PA R)
retVal : $

if (dir = (i::Symbol)) then
  -- j,k dimensions must agree
  if (^((jDim1 = jDim2) and (kDim1=kDim2)))
  then
    error "jxk do not agree"
  else
    retVal := (coerce(concat(matRep1,matRep2)$(PA PA PA R))$$)@$

if (dir = (j::Symbol)) then
  -- i,k dimensions must agree
  if (^((iDim1 = iDim2) and (kDim1=kDim2)))
  then
    error "ixk do not agree"
  else
    for i in 0..(iDim1-1) repeat
      setelt(matRep1,i,(concat(elt(matRep1,i)$(PA PA PA R)_
        ,elt(matRep2,i)$(PA PA PA R))$(PA PA R))@(PA PA R))$(PA PA PA R)
    retVal := (coerce(matRep1)$$)@$

```

```

if (dir = (k::Symbol)) then
  temp : (PA PA R)
  -- i,j dimensions must agree
  if (~((iDim1 = iDim2) and (jDim1=jDim2)))
  then
    error "ixj do not agree"
  else
    for i in 0..(iDim1-1) repeat
      temp := copy(elt(matRep1,i)$(PA PA PA R))$(PA PA R)
      for j in 0..(jDim1-1) repeat
        setelt(temp,j,concat(elt(elt(matRep1,i)$(PA PA PA R)_
          ,j)$(PA PA R),elt(elt(matRep2,i)$(PA PA PA R),j)$(PA PA R)_
            )$(PA R))$(PA PA R)
        setelt(matRep1,i,temp)$(PA PA PA R)
      retVal := (coerce(matRep1)$)$@$

retVal

matrixDimensions(mat : $) : Vector NNI ==
  matRep : (PA PA PA R) := mat :: (PA PA PA R)
  iDim : NNI := (#matRep)$(PA PA PA R)
  matRep2 : PA PA R := elt(matRep,0)$(PA PA PA R)
  jDim : NNI := (#matRep2)$(PA PA R)
  matRep3 : (PA R) := elt(matRep2,0)$(PA PA R)
  kDim : NNI := (#matRep3)$(PA R)
  retVal : Vector NNI := new(3,0)$(Vector NNI)
  retVal.1 := iDim
  retVal.2 := jDim
  retVal.3 := kDim
  retVal

coerce(matrixRep : (PA PA PA R)) : $ == matrixRep pretend $

coerce(mat : $) : (PA PA PA R) == mat pretend (PA PA PA R)

-- i,j,k must be with in the bounds of the matrix
elt(mat : $,i : NNI,j : NNI,k : NNI) : R ==
  matDims := matrixDimensions(mat)
  iLength := matDims.1
  jLength := matDims.2
  kLength := matDims.3
  ((i > iLength) or (j > jLength) or (k > kLength) or (i=0) or (j=0) or_
(k=0)) => error "coordinates must be within the bounds of the matrix"
  matrixRep : PA PA PA R := mat :: (PA PA PA R)
  elt(elt(elt(matrixRep,i-1)$(PA PA PA R),j-1)$(PA PA R),k-1)$(PA R)

setelt!(mat : $,i : NNI,j : NNI,k : NNI,val : R)_
: R ==
  matDims := matrixDimensions(mat)

```

```

iLength := matDims.1
jLength := matDims.2
kLength := matDims.3
((i > iLength) or (j > jLength) or (k > kLength) or (i=0) or (j=0) or_
(k=0)) => error "coordinates must be within the bounds of the matrix"
matrixRep : PA PA PA R := mat :: (PA PA PA R)
row2 : PA PA R := copy(elt(matrixRep,i-1)$(PA PA PA R))$(PA PA R)
row1 : PA R := copy(elt(row2,j-1)$(PA PA R))$(PA R)
setelt(row1,k-1,val)$(PA R)
setelt(row2,j-1,row1)$(PA PA R)
setelt(matrixRep,i-1,row2)$(PA PA PA R)
val

if R has Ring then
zeroMatrix(iLength:NNI,jLength:NNI,kLength:NNI) : $ ==
  (new(iLength,new(jLength,new(kLength,(0$R)))$(PA R))$(PA PA R))$(PA PA PA R)) :: $

identityMatrix(iLength:NNI) : $ ==
  retValueRep : PA PA PA R := zeroMatrix(iLength,iLength,iLength)$ $ :: (PA PA PA R)
  row1 : PA R
  row2 : PA PA R
  row1empty : PA R := new(iLength,0$R)$(PA R)
  row2empty : PA PA R := new(iLength,copy(row1empty)$(PA R))$(PA PA R)
  for count in 0..(iLength-1) repeat
    row1 := copy(row1empty)$(PA R)
    setelt(row1,count,1$R)$(PA R)
    row2 := copy(row2empty)$(PA PA R)
    setelt(row2,count,copy(row1)$(PA R))$(PA PA R)
    setelt(retValueRep,count,copy(row2)$(PA PA R))$(PA PA PA R)
  retValueRep :: $

plus(mat1 : $,mat2 : $) : $ ==

mat1Dims := matrixDimensions(mat1)
iLength1 := mat1Dims.1
jLength1 := mat1Dims.2
kLength1 := mat1Dims.3

mat2Dims := matrixDimensions(mat2)
iLength2 := mat2Dims.1
jLength2 := mat2Dims.2
kLength2 := mat2Dims.3

-- check that the dimensions are the same
(^ (iLength1 = iLength2) or ^ (jLength1 = jLength2) or ^ (kLength1 = kLength2))_
=> error "error the matrices are different sizes"

sum : R
row1 : (PA R) := new(kLength1,0$R)$(PA R)

```

```

row2 : (PA PA R) := new(jLength1,copy(row1$(PA R))$(PA PA R)
row3 : (PA PA PA R) := new(iLength1,copy(row2$(PA PA R))$(PA PA PA R)

for i in 1..iLength1 repeat
  for j in 1..jLength1 repeat
    for k in 1..kLength1 repeat
      sum := (elt(mat1,i,j,k)::R +$R_
              elt(mat2,i,j,k)::R)
      setelt(row1,k-1,sum)$(PA R)
      setelt(row2,j-1,copy(row1$(PA R))$(PA PA R)
      setelt(row3,i-1,copy(row2$(PA PA R))$(PA PA PA R)

resultMatrix := (row3 pretend $)

resultMatrix

construct(listRep : L L L R) : $ ==

(#listRep)$(L L L R) = 0 => error "empty list"
(#(listRep.1))$(L L R) = 0 => error "empty list"
(#((listRep.1).1))$(L R) = 0 => error "empty list"
iLength := (#listRep)$(L L L R)
jLength := (#(listRep.1))$(L L R)
kLength := (#((listRep.1).1))$(L R)

--first check that the matrix is in the correct form
for subList in listRep repeat
  ^((#subList)$(L L R) = jLength) => error_
"can not have an irregular shaped matrix"
  for subSubList in subList repeat
    ^((#(subSubList))$(L R) = kLength) => error_
"can not have an irregular shaped matrix"

row1 : (PA R) := new(kLength,((listRep.1).1).1)$(PA R)
row2 : (PA PA R) := new(jLength,copy(row1$(PA R))$(PA PA R)
row3 : (PA PA PA R) := new(iLength,copy(row2$(PA PA R))$(PA PA PA R)

for i in 1..iLength repeat
  for j in 1..jLength repeat
    for k in 1..kLength repeat

      element := elt(elt(elt(listRep,i)$(L L L R),j)$(L L R),k)$(L R)
      setelt(row1,k-1,element)$(PA R)
      setelt(row2,j-1,copy(row1$(PA R))$(PA PA R)
      setelt(row3,i-1,copy(row2$(PA PA R))$(PA PA PA R)

resultMatrix := (row3 pretend $)

resultMatrix

```

---



---

— M3D.dotabb —

```
"M3D" [color="#88FF44",href="bookvol10.3.pdf#nameddest=M3D"]
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"M3D" -> "ALIST"
```

---

## 21.8 domain VIEW3D ThreeDimensionalViewport

---

— ThreeDimensionalViewport.input —

```
)set break resume
)sys rm -f ThreeDimensionalViewport.output
)spool ThreeDimensionalViewport.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ThreeDimensionalViewport
--R ThreeDimensionalViewport is a domain constructor
--R Abbreviation for ThreeDimensionalViewport is VIEW3D
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for VIEW3D
--R
--R----- Operations -----
--R ?=? : (% ,%) -> Boolean
--R clipSurface : (% ,String) -> Void
--R coerce : % -> OutputForm
--R diagonals : (% ,String) -> Void
--R eyeDistance : (% ,Float) -> Void
--R hitherPlane : (% ,Float) -> Void
--R key : % -> Integer
--R makeViewport3D : % -> %
--R perspective : (% ,String) -> Void
--R rotate : (% ,Float,Float) -> Void
--R title : (% ,String) -> Void
--R viewDeltaXDefault : () -> Float
--R viewDeltaYDefault : () -> Float
--R viewPhiDefault : () -> Float
--R viewThetaDefault : () -> Float
--R viewZoomDefault : () -> Float
--R axes : (% ,String) -> Void
--R close : % -> Void
--R controlPanel : (% ,String) -> Void
--R drawStyle : (% ,String) -> Void
--R hash : % -> SingleInteger
--R intensity : (% ,Float) -> Void
--R latex : % -> String
--R options : % -> List DrawOption
--R reset : % -> Void
--R showRegion : (% ,String) -> Void
--R viewDeltaXDefault : Float -> Float
--R viewDeltaYDefault : Float -> Float
--R viewPhiDefault : Float -> Float
--R viewThetaDefault : Float -> Float
--R viewZoomDefault : Float -> Float
--R viewport3D : () -> %
```



```

--R write : (% ,String) -> String          zoom : (% ,Float) -> Void
--R ?~=? : (% ,%) -> Boolean
--R colorDef : (% ,Color,Color) -> Void
--R dimensions : (% ,NonNegativeInteger,NonNegativeInteger,PositiveInteger,PositiveInteger) -> %
--R lighting : (% ,Float,Float,Float) -> Void
--R makeViewport3D : (ThreeSpace DoubleFloat,List DrawOption) -> %
--R makeViewport3D : (ThreeSpace DoubleFloat,String) -> %
--R modifyPointData : (% ,NonNegativeInteger,Point DoubleFloat) -> Void
--R move : (% ,NonNegativeInteger,NonNegativeInteger) -> Void
--R options : (% ,List DrawOption) -> %
--R outlineRender : (% ,String) -> Void
--R resize : (% ,PositiveInteger,PositiveInteger) -> Void
--R rotate : (% ,Integer,Integer) -> Void
--R showClipRegion : (% ,String) -> Void
--R subspace : (% ,ThreeSpace DoubleFloat) -> %
--R subspace : % -> ThreeSpace DoubleFloat
--R translate : (% ,Float,Float) -> Void
--R viewpoint : (% ,Float,Float,Float) -> Void
--R viewpoint : (% ,Float,Float) -> Void
--R viewpoint : (% ,Integer,Integer,Float,Float,Float) -> Void
--R viewpoint : (% ,Record(theta: DoubleFloat,phi: DoubleFloat,scale: DoubleFloat,scaleX: DoubleFloat,scaleY: DoubleFloat,scaleZ: DoubleFloat)) -> %
--R viewpoint : % -> Record(theta: DoubleFloat,phi: DoubleFloat,scale: DoubleFloat,scaleX: DoubleFloat,scaleY: DoubleFloat,scaleZ: DoubleFloat)
--R viewpoint : (% ,Float,Float,Float,Float,Float) -> Void
--R write : (% ,String,List String) -> String
--R write : (% ,String,String) -> String
--R zoom : (% ,Float,Float,Float) -> Void
--R
--E 1

)spool
)lisp (bye)

```

---

### — ThreeDimensionalViewport.help —

```

=====
ThreeDimensionalViewport examples
=====

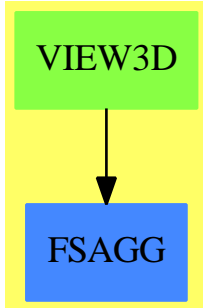
```

See Also:

- o )show ThreeDimensionalViewport

---

## 21.8.1 ThreeDimensionalViewport (VIEW3D)

**Exports:**

|                   |                   |                 |           |                  |
|-------------------|-------------------|-----------------|-----------|------------------|
| axes              | clipSurface       | colorDef        | close     | coerce           |
| controlPanel      | diagonals         | dimensions      | drawStyle | eyeDistance      |
| hash              | hitherPlane       | intensity       | key       | latex            |
| lighting          | makeViewport3D    | modifyPointData | move      | options          |
| outlineRender     | perspective       | reset           | resize    | rotate           |
| showClipRegion    | showRegion        | subspace        | title     | translate        |
| viewDeltaXDefault | viewDeltaYDefault | viewPhiDefault  | viewport  | viewThetaDefault |
| viewZoomDefault   | viewport3D        | write           | zoom      | ?=?              |
| ?~=?              |                   |                 |           |                  |

— domain VIEW3D ThreeDimensionalViewport —

```

)abbrev domain VIEW3D ThreeDimensionalViewport
++ Author: Jim Wen
++ Date Created: 28 April 1989
++ Date Last Updated: 2 November 1991, Jim Wen
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ ThreeDimensionalViewport creates viewports to display graphs

```

```

ThreeDimensionalViewport(): Exports == Implementation where

```

```

VIEW    ==> VIEWPORTSERVER$Lisp
sendI   ==> SOCK_-SEND_-INT
sendSF  ==> SOCK_-SEND_-FLOAT
sendSTR ==> SOCK_-SEND_-STRING
getI    ==> SOCK_-GET_-INT
getSF   ==> SOCK_-GET_-FLOAT

```

```

typeVIEW3D ==> 1$I

```

```

typeVIEWTube ==> 4

makeVIEW3D ==> (-1)$SingleInteger

I ==> Integer
PI ==> PositiveInteger
NNI ==> NonNegativeInteger
XY ==> Record( X:I, Y:I )
XYP ==> Record( X:PI, Y:PI )
XYNN ==> Record( X:NNI, Y:NNI )
SF ==> DoubleFloat
F ==> Float
L ==> List
Pt ==> ColoredThreeDimensionalPoint
SEG ==> Segment
S ==> String
E ==> OutputForm
PLOT3D ==> Plot3D
TUBE ==> TubePlot
V ==> Record( theta:SF, phi:SF, scale:SF, scaleX:SF, scaleY:SF, scaleZ:SF, deltaX:SF, de
H ==> Record( hueOffset:I, hueNumber:I)
FLAG ==> Record( showCP:I, style:I, axesOn:I, diagonalsOn:I, outlineRenderOn:I, showRegi
FR ==> Record( fn:Fn2, fc: FnU, xmin:SF, xmax:SF, ymin:SF, ymax:SF, xnum:I, ynum:I )
FParamR ==> Record( theTube:TUBE )
LR ==> Record( lightX:SF, lightY:SF, lightZ:SF, lightTheta:SF, lightPhi:SF , transluce
UFR ==> Union(FR,FParamR,"undefined")
PR ==> Record( perspectiveField:I, eyeDistance:SF, hitherPlane:SF)
VR ==> Record( clipXMin:SF, clipXMax:SF, clipYMin:SF, clipYMax:SF, clipZMin:SF, clipZMax:
C ==> Color()
B ==> Boolean
POINT ==> Point(SF)
SUBSPACE ==> SubSpace(3,SF)
SPACE3 ==> ThreeSpace(SF)
DROP ==> DrawOption
COORDSYS ==> CoordinateSystems(SF)

-- the below macros correspond to the ones in include/actions.h
ROTATE ==> 0$I -- rotate in actions.h
ZOOM ==> 1$I -- zoom in actions.h
TRANSLATE ==> 2 -- translate in actions.h
rendered ==> 3 -- render in actions.h
hideControl ==> 4
closeAll ==> 5
axesOnOff ==> 6
opaque ==> 7 -- opaqueMesh in action.h
contour ==> 24
RESET ==> 8
wireMesh ==> 9 -- transparent in actions.h
region3D ==> 12
smooth ==> 22

```

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diagOnOff      ==> 26
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noControl ==> 0$I

yes          ==> 1$I
no           ==> 0$I

EYED        ==> 500::SF -- see draw.h, should be the same(?) as clipOffset
HITHER      ==> (-250)::SF -- see process.h in view3D/ (not yet passed to viewman)

openTube    ==> 1$I
closedTube  ==> 0$I

fun2Var3D ==> "    Three Dimensional Viewport: Function of Two Variables"
para1Var3D ==> "    Three Dimensional Viewport: Parametric Curve of One Variable"
undef3D    ==> "    Three Dimensional Viewport: No function defined for this viewport yet"

Exports ==> SetCategory with
viewThetaDefault      : ()                                -> F
    ++ viewThetaDefault() returns the current default longitudinal
    ++ view angle in radians.
viewThetaDefault      : F                                -> F
    ++ viewThetaDefault(t) sets the current default longitudinal
    ++ view angle in radians to the value t and returns t.
viewPhiDefault        : ()                                -> F
    ++ viewPhiDefault() returns the current default latitudinal
    ++ view angle in radians.
viewPhiDefault        : F                                -> F

```

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++ viewPhiDefault(p) sets the current default latitudinal
++ view angle in radians to the value p and returns p.
viewZoomDefault      : ()                                -> F
++ viewZoomDefault() returns the current default graph scaling
++ value.
viewZoomDefault      : F                                -> F
++ viewZoomDefault(s) sets the current default graph scaling
++ value to s and returns s.
viewDeltaXDefault    : ()                                -> F
++ viewDeltaXDefault() returns the current default horizontal
++ offset from the center of the viewport window.
viewDeltaXDefault    : F                                -> F
++ viewDeltaXDefault(dx) sets the current default horizontal
++ offset from the center of the viewport window to be \spad{dx}
++ and returns \spad{dx}.
viewDeltaYDefault    : ()                                -> F
++ viewDeltaYDefault() returns the current default vertical
++ offset from the center of the viewport window.
viewDeltaYDefault    : F                                -> F
++ viewDeltaYDefault(dy) sets the current default vertical
++ offset from the center of the viewport window to be \spad{dy}
++ and returns \spad{dy}.
viewport3D           : ()                                -> %
++ viewport3D() returns an undefined three-dimensional viewport
++ of the domain \spadtype{ThreeDimensionalViewport} whose
++ contents are empty.
makeViewport3D       : %                                -> %
++ makeViewport3D(v) takes the given three-dimensional viewport,
++ v, of the domain \spadtype{ThreeDimensionalViewport} and
++ displays a viewport window on the screen which contains
++ the contents of v.
makeViewport3D       : (SPACE3,S)                       -> %
++ makeViewport3D(sp,s) takes the given space, \spad{sp} which is
++ of the domain \spadtype{ThreeSpace} and displays a viewport
++ window on the screen which contains the contents of \spad{sp},
++ and whose title is given by s.
makeViewport3D       : (SPACE3,L DROP)                   -> %
++ makeViewport3D(sp,lopt) takes the given space, \spad{sp} which is
++ of the domain \spadtype{ThreeSpace} and displays a viewport
++ window on the screen which contains the contents of \spad{sp},
++ and whose draw options are indicated by the list \spad{lopt}, which
++ is a list of options from the domain \spad{DrawOption}.
subspace             : %                                -> SPACE3
++ subspace(v) returns the contents of the viewport v, which is
++ of the domain \spadtype{ThreeDimensionalViewport}, as a subspace
++ of the domain \spad{ThreeSpace}.
subspace             : (%,SPACE3)                        -> %
++ subspace(v,sp) places the contents of the viewport v, which is
++ of the domain \spadtype{ThreeDimensionalViewport}, in the subspace
++ \spad{sp}, which is of the domain \spad{ThreeSpace}.

```

```

modifyPointData      : (%,NNI,POINT)                                -> Void
++ modifyPointData(v,ind,pt) takes the viewport, v, which is of the
++ domain \spadtype{ThreeDimensionalViewport}, and places the data
++ point, \spad{pt} into the list of points database of v at the index
++ location given by \spad{ind}.
options              : %                                           -> L DROP
++ options(v) takes the viewport, v, which is of the domain
++ \spadtype{ThreeDimensionalViewport} and returns a list of all
++ the draw options from the domain \spad{DrawOption} which are
++ being used by v.
options              : (%,L DROP)                                   -> %
++ options(v,lopt) takes the viewport, v, which is of the domain
++ \spadtype{ThreeDimensionalViewport} and sets the draw options
++ being used by v to those indicated in the list, \spad{lopt},
++ which is a list of options from the domain \spad{DrawOption}.
move                 : (%,NNI,NNI)                                  -> Void
++ move(v,x,y) displays the three-dimensional viewport, v, which
++ is of domain \spadtype{ThreeDimensionalViewport}, with the upper
++ left-hand corner of the viewport window at the screen
++ coordinate position x, y.
resize               : (%,PI,PI)                                    -> Void
++ resize(v,w,h) displays the three-dimensional viewport, v, which
++ is of domain \spadtype{ThreeDimensionalViewport}, with a width
++ of w and a height of h, keeping the upper left-hand corner
++ position unchanged.
title                : (%,S)                                        -> Void
++ title(v,s) changes the title which is shown in the three-dimensional
++ viewport window, v of domain \spadtype{ThreeDimensionalViewport}.
dimensions           : (%,NNI,NNI,PI,PI)                           -> Void
++ dimensions(v,x,y,width,height) sets the position of the
++ upper left-hand corner of the three-dimensional viewport, v,
++ which is of domain \spadtype{ThreeDimensionalViewport}, to
++ the window coordinate x, y, and sets the dimensions of the
++ window to that of \spad{width}, \spad{height}. The new
++ dimensions are not displayed until the function
++ \spadfun{makeViewport3D} is executed again for v.
viewpoint            : (%,F,F,F,F,F)                               -> Void
++ viewpoint(v,th,phi,s,dx,dy) sets the longitudinal view angle
++ to \spad{th} radians, the latitudinal view angle to \spad{phi}
++ radians, the scale factor to \spad{s}, the horizontal viewport
++ offset to \spad{dx}, and the vertical viewport offset to \spad{dy}
++ for the viewport v, which is of the domain
++ \spadtype{ThreeDimensionalViewport}. The new viewpoint position
++ is not displayed until the function \spadfun{makeViewport3D} is
++ executed again for v.
viewpoint            : (%) -> V
++ viewpoint(v) returns the current viewpoint setting of the given
++ viewport, v. This function is useful in the situation where the
++ user has created a viewport, proceeded to interact with it via
++ the control panel and desires to save the values of the viewpoint

```

```

++ as the default settings for another viewport to be created using
++ the system.
viewpoint : (%,V) -> Void
++ viewpoint(v,viewpt) sets the viewpoint for the viewport. The
++ viewport record consists of the latitudinal and longitudinal angles,
++ the zoom factor, the x,y and z scales, and the x and y displacements.
viewpoint      : (%,I,I,F,F,F) -> Void
++ viewpoint(v,th,phi,s,dx,dy) sets the longitudinal view angle
++ to \spad{th} degrees, the latitudinal view angle to \spad{phi}
++ degrees, the scale factor to \spad{s}, the horizontal viewport
++ offset to \spad{dx}, and the vertical viewport offset to \spad{dy}
++ for the viewport v, which is of the domain
++ \spadtype{ThreeDimensionalViewport}. The new viewpoint position
++ is not displayed until the function \spadfun{makeViewport3D} is
++ executed again for v.
viewpoint      : (%,F,F) -> Void
++ viewpoint(v,th,phi) sets the longitudinal view angle to \spad{th}
++ radians and the latitudinal view angle to \spad{phi} radians
++ for the viewport v, which is of the domain
++ \spadtype{ThreeDimensionalViewport}. The new viewpoint position
++ is not displayed until the function \spadfun{makeViewport3D} is
++ executed again for v.
viewpoint      : (%,F,F,F) -> Void
++ viewpoint(v,rotx,roty,rotz) sets the rotation about the x-axis
++ to be \spad{rotx} radians, sets the rotation about the y-axis
++ to be \spad{roty} radians, and sets the rotation about the z-axis
++ to be \spad{rotz} radians, for the viewport v, which is of the
++ domain \spadtype{ThreeDimensionalViewport} and displays v with
++ the new view position.
controlPanel    : (%,S) -> Void
++ controlPanel(v,s) displays the control panel of the given
++ three-dimensional viewport, v, which is of domain
++ \spadtype{ThreeDimensionalViewport}, if s is "on", or hides
++ the control panel if s is "off".
axes            : (%,S) -> Void
++ axes(v,s) displays the axes of the given three-dimensional
++ viewport, v, which is of domain \spadtype{ThreeDimensionalViewport},
++ if s is "on", or does not display the axes if s is "off".
diagonals       : (%,S) -> Void
++ diagonals(v,s) displays the diagonals of the polygon outline
++ showing a triangularized surface instead of a quadrilateral
++ surface outline, for the given three-dimensional viewport v
++ which is of domain \spadtype{ThreeDimensionalViewport}, if s is
++ "on", or does not display the diagonals if s is "off".
outlineRender   : (%,S) -> Void
++ outlineRender(v,s) displays the polygon outline showing either
++ triangularized surface or a quadrilateral surface outline depending
++ on the whether the \spadfun{diagonals} function has been set, for
++ the given three-dimensional viewport v which is of domain
++ \spadtype{ThreeDimensionalViewport}, if s is "on", or does not

```

```

++ display the polygon outline if s is "off".
drawStyle      : (%,S)                                -> Void
++ drawStyle(v,s) displays the surface for the given three-dimensional
++ viewport v which is of domain \spadtype{ThreeDimensionalViewport}
++ in the style of drawing indicated by s. If s is not a valid
++ drawing style the style is wireframe by default. Possible
++ styles are \spad{"shade"}, \spad{"solid"} or \spad{"opaque"},
++ \spad{"smooth"}, and \spad{"wireMesh"}.
rotate        : (%,F,F)                                -> Void
++ rotate(v,th,phi) rotates the graph to the longitudinal view angle
++ \spad{th} radians and the latitudinal view angle \spad{phi} radians
++ for the viewport v, which is of the domain
++ \spadtype{ThreeDimensionalViewport}.
rotate        : (%,I,I)                                -> Void
++ rotate(v,th,phi) rotates the graph to the longitudinal view angle
++ \spad{th} degrees and the latitudinal view angle \spad{phi} degrees
++ for the viewport v, which is of the domain
++ \spadtype{ThreeDimensionalViewport}. The new rotation position
++ is not displayed until the function \spadfun{makeViewport3D} is
++ executed again for v.
zoom          : (%,F)                                    -> Void
++ zoom(v,s) sets the graph scaling factor to s, for the viewport v,
++ which is of the domain \spadtype{ThreeDimensionalViewport}.
zoom          : (%,F,F,F)                                -> Void
++ zoom(v,sx,sy,sz) sets the graph scaling factors for the x-coordinate
++ axis to \spad{sx}, the y-coordinate axis to \spad{sy} and the
++ z-coordinate axis to \spad{sz} for the viewport v, which is of
++ the domain \spadtype{ThreeDimensionalViewport}.
translate     : (%,F,F)                                    -> Void
++ translate(v,dx,dy) sets the horizontal viewport offset to \spad{dx}
++ and the vertical viewport offset to \spad{dy}, for the viewport v,
++ which is of the domain \spadtype{ThreeDimensionalViewport}.
perspective   : (%,S)                                    -> Void
++ perspective(v,s) displays the graph in perspective if s is "on",
++ or does not display perspective if s is "off" for the given
++ three-dimensional viewport, v, which is of domain
++ \spadtype{ThreeDimensionalViewport}.
eyeDistance   : (%,F)                                    -> Void
++ eyeDistance(v,d) sets the distance of the observer from the center
++ of the graph to d, for the viewport v, which is of the domain
++ \spadtype{ThreeDimensionalViewport}.
hitherPlane   : (%,F)                                    -> Void
++ hitherPlane(v,h) sets the hither clipping plane of the graph to h,
++ for the viewport v, which is of the domain
++ \spadtype{ThreeDimensionalViewport}.
showRegion    : (%,S)                                    -> Void
++ showRegion(v,s) displays the bounding box of the given
++ three-dimensional viewport, v, which is of domain
++ \spadtype{ThreeDimensionalViewport}, if s is "on", or does not
++ display the box if s is "off".

```



```

showClipRegion      : (%,S)                                -> Void
++ showClipRegion(v,s) displays the clipping region of the given
++ three-dimensional viewport, v, which is of domain
++ \spadtype{ThreeDimensionalViewport}, if s is "on", or does not
++ display the region if s is "off".
clipSurface         : (%,S)                                -> Void
++ clipSurface(v,s) displays the graph with the specified
++ clipping region removed if s is "on", or displays the graph
++ without clipping implemented if s is "off", for the given
++ three-dimensional viewport, v, which is of domain
++ \spadtype{ThreeDimensionalViewport}.
lighting            : (%,F,F,F)                            -> Void
++ lighting(v,x,y,z) sets the position of the light source to
++ the coordinates x, y, and z and displays the graph for the given
++ three-dimensional viewport, v, which is of domain
++ \spadtype{ThreeDimensionalViewport}.
intensity           : (%,F)                                -> Void
++ intensity(v,i) sets the intensity of the light source to i, for
++ the given three-dimensional viewport, v, which is of domain
++ \spadtype{ThreeDimensionalViewport}.
reset               : %                                     -> Void
++ reset(v) sets the current state of the graph characteristics
++ of the given three-dimensional viewport, v, which is of domain
++ \spadtype{ThreeDimensionalViewport}, back to their initial settings.
colorDef            : (%,C,C)                              -> Void
++ colorDef(v,c1,c2) sets the range of colors along the colormap so
++ that the lower end of the colormap is defined by \spad{c1} and the
++ top end of the colormap is defined by \spad{c2}, for the given
++ three-dimensional viewport, v, which is of domain
++ \spadtype{ThreeDimensionalViewport}.
write               : (%,S)                                -> S
++ write(v,s) takes the given three-dimensional viewport, v, which
++ is of domain \spadtype{ThreeDimensionalViewport}, and creates
++ a directory indicated by s, which contains the graph data
++ file for v.
write               : (%,S,S)                              -> S
++ write(v,s,f) takes the given three-dimensional viewport, v, which
++ is of domain \spadtype{ThreeDimensionalViewport}, and creates
++ a directory indicated by s, which contains the graph data
++ file for v and an optional file type f.
write               : (%,S,L S)                            -> S
++ write(v,s,lf) takes the given three-dimensional viewport, v, which
++ is of domain \spadtype{ThreeDimensionalViewport}, and creates
++ a directory indicated by s, which contains the graph data
++ file for v and the optional file types indicated by the list lf.
close               : %                                     -> Void
++ close(v) closes the viewport window of the given
++ three-dimensional viewport, v, which is of domain
++ \spadtype{ThreeDimensionalViewport}, and terminates the
++ corresponding process ID.

```

```

key          : %                                -> I
++ key(v) returns the process ID number of the given three-dimensional
++ viewport, v, which is of domain \spadtype{ThreeDimensionalViewport}.
-- print     : %                                -> Void

Implementation ==> add
import Color()
import ViewDefaultsPackage()
import Plot3D()
import TubePlot()
import POINT
import PointPackage(SF)
import SubSpaceComponentProperty()
import SPACE3
import MeshCreationRoutinesForThreeDimensions()
import DrawOptionFunctions0
import COORDSYS
import Set(PositiveInteger)

Rep := Record (key:I, fun:I, _
               title:S, moveTo:XYNN, size:XYP, viewpoint:V, colors:H, flags:FLAG, _
               lighting:LR, perspective:PR, volume:VR, _
               space3D:SPACE3, _
               optionsField:L DROP)

degrees := pi()$F / 180.0
degreesSF := pi()$SF / 180
defaultTheta : Reference(SF) := ref(convert(pi()$F/4.0)$SF)
defaultPhi   : Reference(SF) := ref(convert(-pi()$F/4.0)$SF)
defaultZoom  : Reference(SF) := ref(convert(1.2)$SF)
defaultDeltaX : Reference(SF) := ref 0
defaultDeltaY : Reference(SF) := ref 0

--%Local Functions
checkViewport (viewport:%) : B ==
-- checks to see if this viewport still exists
-- by sending the key to the viewport manager and
-- waiting for its reply after it checks it against
-- the viewports in its list. a -1 means it doesn't
-- exist.
sendI(VIEW,viewport.key)$Lisp
i := getI(VIEW)$Lisp
(i < 0$I) =>
viewport.key := 0$I
error "This viewport has already been closed!"
true

arcsinTemp(x:SF):SF ==
-- the asin function doesn't exist in the SF domain currently

```

```

x >= 1 => (pi())$SF / 2) -- to avoid floating point error from SF (ie 1.0 -> 1.00001)
x <= -1 => 3 * pi())$SF / 2
convert(asin(convert(x)@Float)$Float)@SF

arctanTemp(x:SF):SF == convert(atan(convert(x)@Float)$Float)@SF

doOptions(v:Rep):Void ==
  v.title := title(v.optionsField,"AXIOM3D")
  st:S := style(v.optionsField,"wireMesh")
  if (st = "shade" or st = "render") then
    v.flags.style := rendered
  else if (st = "solid" or st = "opaque") then
    v.flags.style := opaque
  else if (st = "contour") then
    v.flags.style := contour
  else if (st = "smooth") then
    v.flags.style := smooth
  else v.flags.style := wireMesh
  v.viewpoint := viewpoint(v.optionsField,
    [deref defaultTheta,deref defaultPhi,deref defaultZoom, _
     1$SF,1$SF,1$SF,deref defaultDeltaX, deref defaultDeltaY])
  -- etc - 3D specific stuff...

--%Exported Functions : Default Settings
viewport3D() ==
  [0,typeVIEW3D,"AXIOM3D",[viewPosDefault().1,viewPosDefault().2], _
   [viewSizeDefault().1,viewSizeDefault().2], _
   [deref defaultTheta,deref defaultPhi,deref defaultZoom, _
    1$SF,1$SF,1$SF,deref defaultDeltaX, deref defaultDeltaY], [0,27], _
   [noControl,wireMesh,yes,no,no,no], [0$SF,0$SF,1$SF,0$SF,0$SF,1$SF], _
   [yes, EYED, HITHER], [0$SF,1$SF,0$SF,1$SF,0$SF,1$SF,no,yes], _
   create3Space()$SPACE3, [] ]

subspace viewport ==
  viewport.space3D

subspace(viewport,space) ==
  viewport.space3D := space
  viewport

options viewport ==
  viewport.optionsField

options(viewport,opts) ==
  viewport.optionsField := opts
  viewport

makeViewport3D(space:SPACE3,Title:S):% ==
  v := viewport3D()
  v.space3D := space

```

```

v.optionsField := [title(Title)]
makeViewport3D v

makeViewport3D(space:SPACE3,opts:L DROP):% ==
  v := viewport3D()
  v.space3D := space
  v.optionsField := opts
  makeViewport3D v

makeViewport3D viewport ==
  doOptions viewport --local function to extract and assign optional arguments for 3D viewports
  sayBrightly(["    Transmitting data...":E]$List(E))$Lisp
  transform := coord(viewport.optionsField,cartesian$COORDSYS)$DrawOptionFunctions0
  check(viewport.space3D)
  lpts := lp(viewport.space3D)
  lllipts := lllip(viewport.space3D)
  llprops := llprop(viewport.space3D)
  lprops := lprop(viewport.space3D)
  -- check for dimensionality of points
  -- if they are all 4D points, then everything is okay
  -- if they are all 3D points, then pad an extra constant
  -- coordinate for color
  -- if they have varying dimensionalities, give an error
  s := brace()$Set(PI)
  for pt in lpts repeat
    insert_!(dimension pt,s)
  #s > 1 => error "All points should have the same dimension"
  (n := first parts s) < 3 => error "Dimension of points should be greater than 2"
  sendI(VIEW,viewport.fun)$Lisp
  sendI(VIEW,makeVIEW3D)$Lisp
  sendSTR(VIEW,viewport.title)$Lisp
  sendSF(VIEW,viewport.viewpoint.deltaX)$Lisp
  sendSF(VIEW,viewport.viewpoint.deltaY)$Lisp
  sendSF(VIEW,viewport.viewpoint.scale)$Lisp
  sendSF(VIEW,viewport.viewpoint.scaleX)$Lisp
  sendSF(VIEW,viewport.viewpoint.scaleY)$Lisp
  sendSF(VIEW,viewport.viewpoint.scaleZ)$Lisp
  sendSF(VIEW,viewport.viewpoint.theta)$Lisp
  sendSF(VIEW,viewport.viewpoint.phi)$Lisp
  sendI(VIEW,viewport.moveTo.X)$Lisp
  sendI(VIEW,viewport.moveTo.Y)$Lisp
  sendI(VIEW,viewport.size.X)$Lisp
  sendI(VIEW,viewport.size.Y)$Lisp
  sendI(VIEW,viewport.flags.showCP)$Lisp
  sendI(VIEW,viewport.flags.style)$Lisp
  sendI(VIEW,viewport.flags.axesOn)$Lisp
  sendI(VIEW,viewport.flags.diagonalsOn)$Lisp
  sendI(VIEW,viewport.flags.outlineRenderOn)$Lisp
  sendI(VIEW,viewport.flags.showRegionField)$Lisp -- add to make3D.c
  sendI(VIEW,viewport.volume.clipRegionField)$Lisp -- add to make3D.c

```

```

sendI(VIEW,viewport.volume.clipSurfaceField)$Lisp -- add to make3D.c
sendI(VIEW,viewport.colors.hueOffset)$Lisp
sendI(VIEW,viewport.colors.hueNumber)$Lisp
sendSF(VIEW,viewport.lighting.lightX)$Lisp
sendSF(VIEW,viewport.lighting.lightY)$Lisp
sendSF(VIEW,viewport.lighting.lightZ)$Lisp
sendSF(VIEW,viewport.lighting.translucence)$Lisp
sendI(VIEW,viewport.perspective.perspectiveField)$Lisp
sendSF(VIEW,viewport.perspective.eyeDistance)$Lisp
-- new, crazy points domain stuff
-- first, send the point data list
sendI(VIEW,#lpts)$Lisp
for pt in lpts repeat
  aPoint := transform pt
  sendSF(VIEW,xCoord aPoint)$Lisp
  sendSF(VIEW,yCoord aPoint)$Lisp
  sendSF(VIEW,zCoord aPoint)$Lisp
  n = 3 => sendSF(VIEW,zCoord aPoint)$Lisp
  sendSF(VIEW,color aPoint)$Lisp -- change to c
  -- now, send the 3d subspace structure
sendI(VIEW,#lllpts)$Lisp
for allpts in lllpts for oneprop in lprops for onelprops in llprops repeat
  -- the following is false for f(x,y) and user-defined for [x(t),y(t),z(t)]
  -- this is temporary - until the generalized points stuff gets put in
  sendI(VIEW,(closed? oneprop => yes; no))$Lisp
  sendI(VIEW,(solid? oneprop => yes; no))$Lisp
  sendI(VIEW,#allpts)$Lisp
  for alipts in allpts for tinyprop in onelprops repeat
    -- the following is false for f(x,y) and true for [x(t),y(t),z(t)]
    -- this is temporary -- until the generalized points stuff gets put in
    sendI(VIEW,(closed? tinyprop => yes;no))$Lisp
    sendI(VIEW,(solid? tinyprop => yes;no))$Lisp
    sendI(VIEW,#alipts)$Lisp
    for oneIndexedPoint in alipts repeat
      sendI(VIEW,oneIndexedPoint)$Lisp
viewport.key := getI(VIEW)$Lisp
viewport
-- the key (now set to 0) should be what the viewport returns

viewThetaDefault == convert(defaultTheta())@F
viewThetaDefault t ==
  defaultTheta() := convert(t)@SF
  t
viewPhiDefault == convert(defaultPhi())@F
viewPhiDefault t ==
  defaultPhi() := convert(t)@SF
  t
viewZoomDefault == convert(defaultZoom())@F
viewZoomDefault t ==
  defaultZoom() := convert(t)@SF

```

```

t
viewDeltaXDefault == convert(defaultDeltaX())@F
viewDeltaXDefault t ==
  defaultDeltaX() := convert(t)@SF
t
viewDeltaYDefault == convert(defaultDeltaY())@F
viewDeltaYDefault t ==
  defaultDeltaY() := convert(t)@SF
t

--Exported Functions: Available features for 3D viewports
lighting(viewport,Xlight,Ylight,Zlight) ==
  viewport.lighting.lightX := convert(Xlight)@SF
  viewport.lighting.lightY := convert(Ylight)@SF
  viewport.lighting.lightZ := convert(Zlight)@SF
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW3D)$Lisp
    sendI(VIEW,lightDef)$Lisp
  checkViewport viewport =>
    sendSF(VIEW,viewport.lighting.lightX)$Lisp
    sendSF(VIEW,viewport.lighting.lightY)$Lisp
    sendSF(VIEW,viewport.lighting.lightZ)$Lisp
    getI(VIEW)$Lisp      -- acknowledge

axes (viewport,onOff) ==
  if onOff = "on" then viewport.flags.axesOn := yes
  else viewport.flags.axesOn := no
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW3D)$Lisp
    sendI(VIEW,axesOnOff)$Lisp
  checkViewport viewport =>
    sendI(VIEW,viewport.flags.axesOn)$Lisp
    getI(VIEW)$Lisp      -- acknowledge

diagonals (viewport,onOff) ==
  if onOff = "on" then viewport.flags.diagonalsOn := yes
  else viewport.flags.diagonalsOn := no
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW3D)$Lisp
    sendI(VIEW,diagOnOff)$Lisp
  checkViewport viewport =>
    sendI(VIEW,viewport.flags.diagonalsOn)$Lisp
    getI(VIEW)$Lisp      -- acknowledge

outlineRender (viewport,onOff) ==
  if onOff = "on" then viewport.flags.outlineRenderOn := yes
  else viewport.flags.outlineRenderOn := no
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW3D)$Lisp
    sendI(VIEW,outlineOnOff)$Lisp

```

```

checkViewport viewport =>
    sendI(VIEW,viewport.flags.outlineRenderOn)$Lisp
    getI(VIEW)$Lisp          -- acknowledge

controlPanel (viewport,onOff) ==
    if onOff = "on" then viewport.flags.showCP := yes
    else viewport.flags.showCP := no
    (key(viewport) ^= 0$I) =>
        sendI(VIEW,typeVIEW3D)$Lisp
        sendI(VIEW,hideControl)$Lisp
        checkViewport viewport =>
            sendI(VIEW,viewport.flags.showCP)$Lisp
            getI(VIEW)$Lisp          -- acknowledge

drawStyle (viewport,how) ==
    if (how = "shade") then          -- render
        viewport.flags.style := rendered
    else if (how = "solid") then      -- opaque
        viewport.flags.style := opaque
    else if (how = "contour") then    -- contour
        viewport.flags.style := contour
    else if (how = "smooth") then     -- smooth
        viewport.flags.style := smooth
    else viewport.flags.style := wireMesh
    (key(viewport) ^= 0$I) =>
        sendI(VIEW,typeVIEW3D)$Lisp
        sendI(VIEW,viewport.flags.style)$Lisp
        checkViewport viewport =>
            getI(VIEW)$Lisp          -- acknowledge

reset viewport ==
    (key(viewport) ^= 0$I) =>
        sendI(VIEW,typeVIEW3D)$Lisp
        sendI(VIEW,SPADBUTTONPRESS)$Lisp
        checkViewport viewport =>
            sendI(VIEW,RESET)$Lisp
            getI(VIEW)$Lisp          -- acknowledge

close viewport ==
    (key(viewport) ^= 0$I) =>
        sendI(VIEW,typeVIEW3D)$Lisp
        sendI(VIEW,closeAll)$Lisp
        checkViewport viewport =>
            getI(VIEW)$Lisp          -- acknowledge
        viewport.key := 0$I

viewpoint (viewport:):V ==
    (key(viewport) ^= 0$I) =>
        sendI(VIEW,typeVIEW3D)$Lisp
        sendI(VIEW,queryVIEWPOINT)$Lisp

```

```

checkViewport viewport =>
  deltaX_sf : SF := getSF(VIEW)$Lisp
  deltaY_sf : SF := getSF(VIEW)$Lisp
  scale_sf  : SF := getSF(VIEW)$Lisp
  scaleX_sf : SF := getSF(VIEW)$Lisp
  scaleY_sf : SF := getSF(VIEW)$Lisp
  scaleZ_sf : SF := getSF(VIEW)$Lisp
  theta_sf  : SF := getSF(VIEW)$Lisp
  phi_sf    : SF := getSF(VIEW)$Lisp
  getI(VIEW)$Lisp      -- acknowledge
  viewport.viewpoint :=
    [ theta_sf, phi_sf, scale_sf, scaleX_sf, scaleY_sf, scaleZ_sf,
      deltaX_sf, deltaY_sf ]
  viewport.viewpoint

viewpoint (viewport:%, viewpt:V):Void ==
  viewport.viewpoint := viewpt
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW3D)$Lisp
    sendI(VIEW,changeVIEWPOINT)$Lisp
    checkViewport viewport =>
      sendSF(VIEW,viewport.viewpoint.deltaX)$Lisp
      sendSF(VIEW,viewport.viewpoint.deltaY)$Lisp
      sendSF(VIEW,viewport.viewpoint.scale)$Lisp
      sendSF(VIEW,viewport.viewpoint.scaleX)$Lisp
      sendSF(VIEW,viewport.viewpoint.scaleY)$Lisp
      sendSF(VIEW,viewport.viewpoint.scaleZ)$Lisp
      sendSF(VIEW,viewport.viewpoint.theta)$Lisp
      sendSF(VIEW,viewport.viewpoint.phi)$Lisp
      getI(VIEW)$Lisp      -- acknowledge

viewpoint (viewport:%,Theta:F,Phi:F,Scale:F,DeltaX:F,DeltaY:F):Void ==
  viewport.viewpoint :=
    [convert(Theta)$SF,convert(Phi)$SF,convert(Scale)$SF,1$SF,1$SF,1$SF,convert(DeltaX)$SF,convert(DeltaY)$SF]

viewpoint (viewport:%,Theta:I,Phi:I,Scale:F,DeltaX:F,DeltaY:F):Void ==
  viewport.viewpoint := [convert(Theta)$SF * degreesSF,convert(Phi)$SF * degreesSF,
    convert(Scale)$SF,1$SF,1$SF,1$SF,convert(DeltaX)$SF,convert(DeltaY)$SF]

viewpoint (viewport:%,Theta:F,Phi:F):Void ==
  viewport.viewpoint.theta := convert(Theta)$SF * degreesSF
  viewport.viewpoint.phi   := convert(Phi)$SF * degreesSF

viewpoint (viewport:%,X:F,Y:F,Z:F):Void ==
  Theta : F
  Phi : F
  if (X=0$F) and (Y=0$F) then
    Theta := 0$F
    if (Z>=0$F) then

```



```

        Phi := 0$F
    else
        Phi := 180.0
    else
        Theta := asin(Y/(R := sqrt(X*X+Y*Y)))
        if (Z=0$F) then
            Phi := 90.0
        else
            Phi := atan(Z/R)
    rotate(viewport, Theta * degrees, Phi * degrees)

title (viewport,Title) ==
viewport.title := Title
(key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW3D)$Lisp
    sendI(VIEW,TITLE)$Lisp
    checkViewport viewport =>
        sendSTR(VIEW,Title)$Lisp
        getI(VIEW)$Lisp          -- acknowledge

colorDef (viewport,HueOffset,HueNumber) ==
viewport.colors := [h := (hue HueOffset),(hue HueNumber) - h]
(key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW3D)$Lisp
    sendI(VIEW,COLORDEF)$Lisp
    checkViewport viewport =>
        sendI(VIEW,hue HueOffset)$Lisp
        sendI(VIEW,hue HueNumber)$Lisp
        getI(VIEW)$Lisp          -- acknowledge

dimensions (viewport,ViewX,ViewY,ViewWidth,ViewHeight) ==
viewport.moveTo := [ViewX,ViewY]
viewport.size    := [ViewWidth,ViewHeight]

move(viewport,xLoc,yLoc) ==
viewport.moveTo := [xLoc,yLoc]
(key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW3D)$Lisp
    sendI(VIEW,MOVE)$Lisp
    checkViewport viewport =>
        sendI(VIEW,xLoc)$Lisp
        sendI(VIEW,yLoc)$Lisp
        getI(VIEW)$Lisp          -- acknowledge

resize(viewport,xSize,ySize) ==
viewport.size := [xSize,ySize]
(key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW3D)$Lisp
    sendI(VIEW,RESIZE)$Lisp
    checkViewport viewport =>

```

```

        sendI(VIEW,xSize)$Lisp
        sendI(VIEW,ySize)$Lisp
        getI(VIEW)$Lisp          -- acknowledge

coerce viewport ==
  (key(viewport) = 0$I) =>
    hconcat
      ["Closed or Undefined ThreeDimensionalViewport: "::E,
       (viewport.title)::E]
    hconcat ["ThreeDimensionalViewport: "::E, (viewport.title)::E]

key viewport == viewport.key

rotate(viewport:%,Theta:I,Phi:I) ==
  rotate(viewport,Theta::F * degrees,Phi::F * degrees)

rotate(viewport:%,Theta:F,Phi:F) ==
  viewport.viewpoint.theta := convert(Theta)@SF
  viewport.viewpoint.phi   := convert(Phi)@SF
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW3D)$Lisp
    sendI(VIEW,ROTATE)$Lisp
    checkViewport viewport =>
      sendSF(VIEW,viewport.viewpoint.theta)$Lisp
      sendSF(VIEW,viewport.viewpoint.phi)$Lisp
      getI(VIEW)$Lisp          -- acknowledge

zoom(viewport:%,Scale:F) ==
  viewport.viewpoint.scale := convert(Scale)@SF
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW3D)$Lisp
    sendI(VIEW,ZOOM)$Lisp
    checkViewport viewport =>
      sendSF(VIEW,viewport.viewpoint.scale)$Lisp
      getI(VIEW)$Lisp          -- acknowledge

zoom(viewport:%,ScaleX:F,ScaleY:F,ScaleZ:F) ==
  viewport.viewpoint.scaleX := convert(ScaleX)@SF
  viewport.viewpoint.scaleY := convert(ScaleY)@SF
  viewport.viewpoint.scaleZ := convert(ScaleZ)@SF
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW3D)$Lisp
    sendI(VIEW,zoomx)$Lisp
    checkViewport viewport =>
      sendSF(VIEW,viewport.viewpoint.scaleX)$Lisp
      sendSF(VIEW,viewport.viewpoint.scaleY)$Lisp
      sendSF(VIEW,viewport.viewpoint.scaleZ)$Lisp
      getI(VIEW)$Lisp          -- acknowledge

translate(viewport,DeltaX,DeltaY) ==

```

```

viewport.viewpoint.deltaX := convert(DeltaX)@SF
viewport.viewpoint.deltaY := convert(DeltaY)@SF
(key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW3D)$Lisp
    sendI(VIEW,TRANSLATE)$Lisp
    checkViewport viewport =>
        sendSF(VIEW,viewport.viewpoint.deltaX)$Lisp
        sendSF(VIEW,viewport.viewpoint.deltaY)$Lisp
        getI(VIEW)$Lisp          -- acknowledge

intensity(viewport,Amount) ==
    if (Amount < 0$F) or (Amount > 1$F) then
        error "The intensity must be a value between 0 and 1, inclusively."
    viewport.lighting.translucence := convert(Amount)@SF
(key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW3D)$Lisp
    sendI(VIEW,translucenceDef)$Lisp
    checkViewport viewport =>
        sendSF(VIEW,viewport.lighting.translucence)$Lisp
        getI(VIEW)$Lisp          -- acknowledge

write(viewport:%,Filename:S,aThingToWrite:S) ==
    write(viewport,Filename,[aThingToWrite])

write(viewport,Filename) ==
    write(viewport,Filename,viewWriteDefault())

write(viewport:%,Filename:S,thingsToWrite:L S) ==
    stringToSend : S := ""
(key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW3D)$Lisp
    sendI(VIEW,writeView)$Lisp
    checkViewport viewport =>
        sendSTR(VIEW,Filename)$Lisp
        m := minIndex(avail := viewWriteAvailable())
        for aTypeOfFile in thingsToWrite repeat
            if (writeTypeInt:= position(upperCase aTypeOfFile,avail)-m) < 0 then
                sayBrightly([" > ":E,(concat(aTypeOfFile, _
                    " is not a valid file type for writing a 3D viewport")):E]$List(E))$Lisp
            else
                sendI(VIEW,writeTypeInt+(1$I))$Lisp
        sendI(VIEW,0$I)$Lisp      -- no more types of things to write
        getI(VIEW)$Lisp          -- acknowledge
        Filename

perspective (viewport,onOff) ==
    if onOff = "on" then viewport.perspective.perspectiveField := yes
    else viewport.perspective.perspectiveField := no
(key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW3D)$Lisp

```

```

    sendI(VIEW,perspectiveOnOff)$Lisp
    checkViewport viewport =>
        sendI(VIEW,viewport.perspective.perspectiveField)$Lisp
        getI(VIEW)$Lisp          -- acknowledge

showRegion (viewport,onOff) ==
    if onOff = "on" then viewport.flags.showRegionField := yes
    else viewport.flags.showRegionField := no
    (key(viewport) ^= 0$I) =>
        sendI(VIEW,typeVIEW3D)$Lisp
        sendI(VIEW,region3D)$Lisp
        checkViewport viewport =>
            sendI(VIEW,viewport.flags.showRegionField)$Lisp
            getI(VIEW)$Lisp      -- acknowledge

showClipRegion (viewport,onOff) ==
    if onOff = "on" then viewport.volume.clipRegionField := yes
    else viewport.volume.clipRegionField := no
    (key(viewport) ^= 0$I) =>
        sendI(VIEW,typeVIEW3D)$Lisp
        sendI(VIEW,clipRegionOnOff)$Lisp
        checkViewport viewport =>
            sendI(VIEW,viewport.volume.clipRegionField)$Lisp
            getI(VIEW)$Lisp      -- acknowledge

clipSurface (viewport,onOff) ==
    if onOff = "on" then viewport.volume.clipSurfaceField := yes
    else viewport.volume.clipSurfaceField := no
    (key(viewport) ^= 0$I) =>
        sendI(VIEW,typeVIEW3D)$Lisp
        sendI(VIEW,clipSurfaceOnOff)$Lisp
        checkViewport viewport =>
            sendI(VIEW,viewport.volume.clipSurfaceField)$Lisp
            getI(VIEW)$Lisp      -- acknowledge

eyeDistance(viewport:%,EyeDistance:F) ==
    viewport.perspective.eyeDistance := convert(EyeDistance)@SF
    (key(viewport) ^= 0$I) =>
        sendI(VIEW,typeVIEW3D)$Lisp
        sendI(VIEW,eyeDistanceData)$Lisp
        checkViewport viewport =>
            sendSF(VIEW,viewport.perspective.eyeDistance)$Lisp
            getI(VIEW)$Lisp      -- acknowledge

hitherPlane(viewport:%,HitherPlane:F) ==
    viewport.perspective.hitherPlane := convert(HitherPlane)@SF
    (key(viewport) ^= 0$I) =>
        sendI(VIEW,typeVIEW3D)$Lisp
        sendI(VIEW,hitherPlaneData)$Lisp
        checkViewport viewport =>

```

```

        sendSF(VIEW,viewport.perspective.hitherPlane)$Lisp
        getI(VIEW)$Lisp          -- acknowledge

modifyPointData(viewport,anIndex,aPoint) ==
  (n := dimension aPoint) < 3 => error "The point should have dimension of at least 3"
  viewport.space3D := modifyPointData(viewport.space3D,anIndex,aPoint)
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW3D)$Lisp
    sendI(VIEW,modifyPOINT)$Lisp
    checkViewport viewport =>
      sendI(VIEW,anIndex)$Lisp
      sendSF(VIEW,xCoord aPoint)$Lisp
      sendSF(VIEW,yCoord aPoint)$Lisp
      sendSF(VIEW,zCoord aPoint)$Lisp
      if (n = 3) then sendSF(VIEW,convert(0.5)@SF)$Lisp
      else sendSF(VIEW,color aPoint)$Lisp
      getI(VIEW)$Lisp          -- acknowledge

--   print viewport ==
--   (key(viewport) ^= 0$I) =>
--   sendI(VIEW,typeVIEW3D)$Lisp
--   sendI(VIEW,printViewport)$Lisp
--   checkViewport viewport =>
--   getI(VIEW)$Lisp          -- acknowledge



---


— VIEW3D.dotabb —

"VIEW3D" [color="#88FF44",href="bookvol10.3.pdf#nameddest=VIEW3D"]
"FSAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FSAGG"]
"VIEW3D" -> "FSAGG"



---



```

## 21.9 domain SPACE3 ThreeSpace

— ThreeSpace.input —

```

)set break resume
)sys rm -f ThreeSpace.output
)spool ThreeSpace.output
)set message test on
)set message auto off

```

```

)clear all

--S 1 of 1
)show ThreeSpace
--R ThreeSpace R: Ring is a domain constructor
--R Abbreviation for ThreeSpace is SPACE3
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for SPACE3
--R
--R----- Operations -----
--R ?=? : (%,%) -> Boolean               check : % -> %
--R closedCurve : % -> List Point R      closedCurve : List Point R -> %
--R closedCurve? : % -> Boolean           coerce : % -> OutputForm
--R components : % -> List %              composite : List % -> %
--R composites : % -> List %              copy : % -> %
--R create3Space : SubSpace(3,R) -> %     create3Space : () -> %
--R curve : % -> List Point R             curve : List Point R -> %
--R curve : (%,List List R) -> %          curve : (%,List Point R) -> %
--R curve? : % -> Boolean                  hash : % -> SingleInteger
--R latex : % -> String                    lp : % -> List Point R
--R merge : (%,%) -> %                     merge : List % -> %
--R mesh : % -> List List Point R          mesh : List List Point R -> %
--R mesh? : % -> Boolean                   point : % -> Point R
--R point : Point R -> %                   point : (%,List R) -> %
--R point : (%,Point R) -> %               point? : % -> Boolean
--R polygon : % -> List Point R            polygon : List Point R -> %
--R polygon : (%,List List R) -> %         polygon : (%,List Point R) -> %
--R polygon? : % -> Boolean                 subspace : % -> SubSpace(3,R)
--R ?~=? : (%,%) -> Boolean
--R closedCurve : (%,List List R) -> %
--R closedCurve : (%,List Point R) -> %
--R enterPointData : (%,List Point R) -> NonNegativeInteger
--R lllip : % -> List List List NonNegativeInteger
--R lllp : % -> List List List Point R
--R llprop : % -> List List SubSpaceComponentProperty
--R lprop : % -> List SubSpaceComponentProperty
--R mesh : (List List Point R,Boolean,Boolean) -> %
--R mesh : (%,List List List R,Boolean,Boolean) -> %
--R mesh : (%,List List Point R,Boolean,Boolean) -> %
--R mesh : (%,List List List R,List SubSpaceComponentProperty,SubSpaceComponentProperty) -> %
--R mesh : (%,List List Point R,List SubSpaceComponentProperty,SubSpaceComponentProperty) -> %
--R modifyPointData : (%,NonNegativeInteger,Point R) -> %
--R numberOfComponents : % -> NonNegativeInteger
--R numberOfComposites : % -> NonNegativeInteger
--R objects : % -> Record(points: NonNegativeInteger,curves: NonNegativeInteger,polygons: NonNegativeInt
--R point : (%,NonNegativeInteger) -> %
--R
--E 1

)spool

```

```
)lisp (bye)
```

---

— **ThreeSpace.help** —

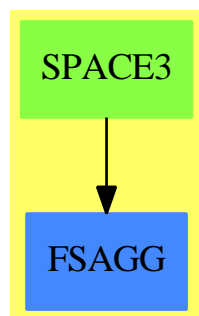
```
=====
ThreeSpace examples
=====
```

See Also:

```
o )show ThreeSpace
```

---

### 21.9.1 ThreeSpace (SPACE3)



#### Exports:

|           |                |                 |                    |            |
|-----------|----------------|-----------------|--------------------|------------|
| check     | closedCurve    | closedCurve?    | coerce             | components |
| composite | composites     | copy            | create3Space       | curve      |
| curve?    | enterPointData | hash            | latex              | lflip      |
| lflip     | lprop          | lprop           | lp                 | merge      |
| mesh      | mesh?          | modifyPointData | numberOfComponents | objects    |
| point     | point?         | polygon         | polygon?           | subspace   |
| ?=?       | ?~=?           |                 |                    |            |

— **domain SPACE3 ThreeSpace** —

```
)abbrev domain SPACE3 ThreeSpace
```

```
++ Author: Mark Botch
```

```
++ Date Created:
```

```
++ Date Last Updated:
```

```
++ Related Constructors:
```

```
++ Also See:
```

```

++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The domain ThreeSpace is used for creating three dimensional
++ objects using functions for defining points, curves, polygons, constructs
++ and the subspaces containing them.

ThreeSpace(R:Ring):Exports == Implementation where
  -- m is the dimension of the point

  I    ==> Integer
  PI   ==> PositiveInteger
  NNI  ==> NonNegativeInteger
  L    ==> List
  B    ==> Boolean
  O    ==> OutputForm
  SUBSPACE ==> SubSpace(3,R)
  POINT ==> Point(R)
  PROP ==> SubSpaceComponentProperty()
  REP3D ==> Record(lp:L POINT, lllipt:L L L NNI, llprop:L L PROP, lprop:L PROP)
  OBJ3D ==> Record(points:NNI, curves:NNI, polygons:NNI, constructs:NNI)

Exports ==> ThreeSpaceCategory(R)
Implementation ==> add
  import COMPPROP
  import POINT
  import SUBSPACE
  import ListFunctions2(List(R),POINT)
  import Set(NNI)

  Rep := Record( subspaceField:SUBSPACE, compositesField:L SUBSPACE, _
                 rep3DField:REP3D, objectsField:OBJ3D, _
                 converted:B)

--% Local Functions
convertSpace : % -> %
convertSpace space ==
  space.converted => space
  space.converted := true
  lllipt : L L L NNI := []
  llprop : L L PROP := []
  lprop : L PROP := []
  for component in children space.subspaceField repeat
    lprop := cons(extractProperty component,lprop)
    tmpllipt : L L NNI := []
    tmplprop : L PROP := []
    for curve in children component repeat
      tmplprop := cons(extractProperty curve,tmplprop)
      tmpllipt : L NNI := []

```



```

        for point in children curve repeat
            tmplipt := cons(extractIndex point,tmplipt)
            tmpllipt := cons(reverse_! tmplipt,tmpllipt)
            llprop := cons(reverse_! tmplprop, llprop)
            lllipt := cons(reverse_! tmpllipt, lllipt)
        space.rep3DField := [pointData space.subspaceField,
                           reverse_! lllipt,reverse_! llprop,reverse_! lprop]
    space

--% Exported Functions
polygon(space:%,points:L POINT) ==
    #points < 3 =>
        error "You need at least 3 points to define a polygon"
    pt := addPoint2(space.subspaceField,first points)
    points := rest points
    addPointLast(space.subspaceField, pt, first points, 1)
    for p in rest points repeat
        addPointLast(space.subspaceField, pt, p, 2)
    space.converted := false
    space
create3Space() == [ new()$SUBSPACE, [], [ [], [], [], [] ], [0,0,0,0], false ]
create3Space(s) == [ s, [], [ [], [], [], [] ], [0,0,0,0], false ]
numberOfComponents(space) == #(children((space::Rep).subspaceField))
numberOfComposites(space) == #((space::Rep).compositesField)
merge(listOfThreeSpaces) ==
    -- * -- we may want to remove duplicate components when that functionality exists :
    newspace := create3Space(merge([ts.subspaceField for ts in listOfThreeSpaces]))
    -- newspace.compositesField := [for cs in ts.compositesField for ts in listOfThreeSpaces
    for ts in listOfThreeSpaces repeat
        newspace.compositesField := append(ts.compositesField,newspace.compositesField)
    newspace
    merge(s1,s2) == merge([s1,s2])
    composite(listOfThreeSpaces) ==
        space := create3Space()
        space.subspaceField := merge [s.subspaceField for s in listOfThreeSpaces]
        space.compositesField := [deepCopy space.subspaceField]
    -- for aSpace in listOfThreeSpaces repeat
        -- create a composite (which are supercomponents that group
        -- separate components together) out of all possible components
    -- space.compositesField := append(children aSpace.subspaceField,space.compositesField)
    space
    components(space) == [create3Space(s) for s in separate space.subspaceField]
    composites(space) == [create3Space(s) for s in space.compositesField]
    copy(space) ==
        spc := create3Space(deepCopy(space.subspaceField))
        spc.compositesField := [deepCopy s for s in space.compositesField]
        spc

enterPointData(space,listOfPoints) ==

```

```

    for p in listOfPoints repeat
        addPoint(space.subspaceField,p)
    #(pointData space.subspaceField)
modifyPointData(space,i,p) ==
    modifyPoint(space.subspaceField,i,p)
    space

-- 3D primitives, each grouped in the following order
--   xxx?(s) : query whether the threespace, s, holds an xxx
--   xxx(s) : extract xxx from threespace, s
--   xxx(p) : create a new three space with xxx, p
--   xxx(s,p) : add xxx, p, to a three space, s
--   xxx(s,q) : add an xxx, convertible from q, to a three space, s
--   xxx(s,i) : add an xxx, the data for xxx being indexed by reference *** complete this
point?(space:%) ==
    #(c:=children space.subspaceField) > 1$NNI =>
        error "This ThreeSpace has more than one component"
        -- our 3-space has one component, a list of list of points
    #(kid:=children first c) = 1$NNI => -- the component has one subcomponent (a list of points)
        #(children first kid) = 1$NNI -- this list of points only has one entry, so it's a point
        false
point(space:%) ==
    point? space => extractPoint(traverse(space.subspaceField,[1,1,1]::L NNI))
    error "This ThreeSpace holds something other than a single point - try the objects() command"
point(aPoint:POINT) == point(create3Space(),aPoint)
point(space:%,aPoint:POINT) ==
    addPoint(space.subspaceField,[],aPoint)
    space.converted := false
    space
point(space:%,1:L R) ==
    pt := point(1)
    point(space,pt)
point(space:%,i:NNI) ==
    addPoint(space.subspaceField,[],i)
    space.converted := false
    space

curve?(space:%) ==
    #(c:=children space.subspaceField) > 1$NNI =>
        error "This ThreeSpace has more than one component"
        -- our 3-space has one component, a list of list of points
    #(children first c) = 1$NNI -- there is only one subcomponent, so it's a list of points
curve(space:%) ==
    curve? space =>
        spc := first children first children space.subspaceField
        [extractPoint(s) for s in children spc]
        error "This ThreeSpace holds something other than a curve - try the objects() command"
curve(points:L POINT) == curve(create3Space(),points)
curve(space:%,points:L POINT) ==
    addPoint(space.subspaceField,[],first points)

```

```

path : L NNI := [#(children space.subspaceField),1]
for p in rest points repeat
  addPoint(space.subspaceField,path,p)
space.converted := false
space
curve(space:%,points:L L R) ==
pts := map(point,points)
curve(space,pts)

closedCurve?(space:%) ==
#(c:=children space.subspaceField) > 1$NNI =>
  error "This ThreeSpace has more than one component"
  -- our 3-space has one component, a list of list of points
#(kid := children first c) = 1$NNI => -- there is one subcomponent => it's a list of p
  extractClosed first kid -- is it a closed curve?
false
closedCurve(space:%) ==
closedCurve? space =>
  spc := first children first children space.subspaceField
  -- get the list of points
  [extractPoint(s) for s in children spc]
  -- for now, we are not repeating points...
  error "This ThreeSpace holds something other than a curve - try the objects() command"
closedCurve(points:L POINT) == closedCurve(create3Space(),points)
closedCurve(space:%,points:L POINT) ==
  addPoint(space.subspaceField,[],first points)
path : L NNI := [#(children space.subspaceField),1]
closeComponent(space.subspaceField,path,true)
for p in rest points repeat
  addPoint(space.subspaceField,path,p)
space.converted := false
space
closedCurve(space:%,points:L L R) ==
pts := map(point,points)
closedCurve(space,pts)

polygon?(space:%) ==
#(c:=children space.subspaceField) > 1$NNI =>
  error "This ThreeSpace has more than one component"
  -- our 3-space has one component, a list of list of points
#(kid:=children first c) = 2::$NNI =>
  -- there are two subcomponents
  -- the convention is to have one point in the first child and to put
  -- the remaining points (2 or more) in the second, and last, child
  #(children first kid) = 1$NNI and #(children second kid) > 2::$NNI
false -- => returns Void...?
polygon(space:%) ==
polygon? space =>
listOfPoints : L POINT :=
  [extractPoint(first children first (cs := children first children space.subspaceFi

```

```

    [extractPoint(s) for s in children second cs]
    error "This ThreeSpace holds something other than a polygon - try the objects() command"
polygon(points:L POINT) == polygon(create3Space(),points)
polygon(space:%,points:L L R) ==
    pts := map(point,points)
    polygon(space,pts)

mesh?(space:%) ==
    #(c:=children space.subspaceField) > 1$NNI =>
        error "This ThreeSpace has more than one component"
    -- our 3-space has one component, a list of list of points
    #(kid:=children first c) > 1$NNI =>
        -- there are two or more subcomponents (list of points)
        -- so this may be a definition of a mesh; if the size
        -- of each list of points is the same and they are all
        -- greater than 1(?) then we have an acceptable mesh
        -- use a set to hold the curve size info: if heterogenous
        -- curve sizes exist, then the set would hold all the sizes;
        -- otherwise it would just have the one element indicating
        -- the sizes for all the curves
        whatSizes := brace()$Set(NNI)
        for eachCurve in kid repeat
            insert_!(#children eachCurve,whatSizes)
        #whatSizes > 1 => error "Mesh defined with curves of different sizes"
        first parts whatSizes < 2 =>
            error "Mesh defined with single point curves (use curve())"
        true
    false
mesh(space:%) ==
    mesh? space =>
        llp : L L POINT := []
        for lpSpace in children first children space.subspaceField repeat
            llp := cons([extractPoint(s) for s in children lpSpace],llp)
        llp
        error "This ThreeSpace holds something other than a mesh - try the objects() command"
    mesh(points:L L POINT) == mesh(create3Space(),points,false,false)
    mesh(points:L L POINT,prop1:B,prop2:B) == mesh(create3Space(),points,prop1,prop2)
--+ old ones \/
mesh(space:%,llpoints:L L L R,lprops:L PROP,prop:PROP) ==
    pts := [map(point,points) for points in llpoints]
    mesh(space,pts,lprops,prop)
mesh(space:%,llp:L L POINT,lprops:L PROP,prop:PROP) ==
    addPoint(space.subspaceField,[],first first llp)
    defineProperty(space.subspaceField,path:L NNI:=[#children space.subspaceField],prop)
    path := append(path,[1])
    defineProperty(space.subspaceField,path,first lprops)
    for p in rest (first llp) repeat
        addPoint(space.subspaceField,path,p)
    for lp in rest llp for aProp in rest lprops for count in 2.. repeat
        addPoint(space.subspaceField,path := [first path],first lp)

```

```

        path := append(path,[count])
        defineProperty(space.subspaceField,path,aProp)
        for p in rest lp repeat
            addPoint(space.subspaceField,path,p)
        space.converted := false
        space
--+ old ones /\
mesh(space:%,llpoints:L L L R,prop1:B,prop2:B) ==
    pts := [map(point,points) for points in llpoints]
    mesh(space,pts,prop1,prop2)
mesh(space:%,llp:L L POINT,prop1:B,prop2:B) ==
    -- prop2 refers to property of the ends of a surface (list of lists of points)
    -- while prop1 refers to the individual curves (list of points)
    -- ** note we currently use Booleans for closed (rather than a pair
    -- ** of booleans for closed and solid)
    propA : PROP := new()
    close(propA,prop1)
    propB : PROP := new()
    close(propB,prop2)
    addPoint(space.subspaceField,[],first first llp)
    defineProperty(space.subspaceField,path:L NNI:=[#children space.subspaceField],propB)
    path := append(path,[1])
    defineProperty(space.subspaceField,path,propA)
    for p in rest (first llp) repeat
        addPoint(space.subspaceField,path,p)
    for lp in rest llp for count in 2.. repeat
        addPoint(space.subspaceField,path := [first path],first lp)
        path := append(path,[count])
        defineProperty(space.subspaceField,path,propA)
        for p in rest lp repeat
            addPoint(space.subspaceField,path,p)
    space.converted := false
    space

lp space ==
    if ^space.converted then space := convertSpace space
    space.rep3DField.lp
l1lip space ==
    if ^space.converted then space := convertSpace space
    space.rep3DField.l1liPt
-- l1lp space ==
--     if ^space.converted then space := convertSpace space
--     space.rep3DField.l1lpPt
l1prop space ==
    if ^space.converted then space := convertSpace space
    space.rep3DField.l1Prop
lprop space ==
    if ^space.converted then space := convertSpace space
    space.rep3DField.lProp

```

```

-- this function is just to see how this representation really
-- does work
objects space ==
  if ^space.converted then space := convertSpace space
  numPts      := 0$NNI
  numCurves   := 0$NNI
  numPolys     := 0$NNI
  numConstructs := 0$NNI
  for component in children space.subspaceField repeat
    # (kid:=children component) = 1 =>
      # (children first kid) = 1 => numPts := numPts + 1
      numCurves := numCurves + 1
    (#kid = 2) and _
      (#children first kid = 1) and _
      (#children first rest kid ^= 1) =>
        numPolys := numPolys + 1
        numConstructs := numConstructs + 1
  -- otherwise, a mathematical surface is assumed
  -- there could also be garbage representation
  -- since there are always more permutations that
  -- we could ever want, so the user should not
  -- fumble around too much with the structure
  -- as other applications need to interpret it
  [numPts,numCurves,numPolys,numConstructs]

check(s) ==
  ^s.converted => convertSpace s
  s

subspace(s) == s.subspaceField

coerce(s) ==
  if ^s.converted then s := convertSpace s
  hconcat(["3-Space with "::0, _
    (sizo:=#(s.rep3DField.l111iPt))::0, _
    (sizo=1=>" component "::0;" components "::0)])



---


— SPACE3.dotabb —

"SPACE3" [color="#88FF44",href="bookvol10.3.pdf#nameddest=SPACE3"]
"FSAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FSAGG"]
"SPACE3" -> "FSAGG"



---



```

## 21.10 domain TREE Tree

— Tree.input —

```

)set break resume
)sys rm -f Tree.output
)spool Tree.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Tree
--R Tree S: SetCategory is a domain constructor
--R Abbreviation for Tree is TREE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for TREE
--R
--R----- Operations -----
--R children : % -> List %
--R cyclic? : % -> Boolean
--R cyclicEntries : % -> List %
--R cyclicParents : % -> List %
--R ?.value : (% ,value) -> S
--R empty? : % -> Boolean
--R leaf? : % -> Boolean
--R map : ((S -> S),%) -> %
--R sample : () -> %
--R tree : List S -> %
--R value : % -> S
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ==? : (% ,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R child? : (% ,%) -> Boolean if S has SETCAT
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S ,%) -> NonNegativeInteger if $ has finiteAggregate and S has SETCAT
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (% ,List S ,List S) -> % if S has EVALAB S and S has SETCAT
--R eval : (% ,S ,S) -> % if S has EVALAB S and S has SETCAT
--R eval : (% ,Equation S) -> % if S has EVALAB S and S has SETCAT
--R eval : (% ,List Equation S) -> % if S has EVALAB S and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R less? : (% ,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S ,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R members : % -> List S if $ has finiteAggregate

```

```

--R more? : (% , NonNegativeInteger) -> Boolean
--R node? : (% , %) -> Boolean if S has SETCAT
--R parts : % -> List S if $ has finiteAggregate
--R setchildren! : (% , List %) -> % if $ has shallowlyMutable
--R setelt : (% , value, S) -> S if $ has shallowlyMutable
--R setvalue! : (% , S) -> S if $ has shallowlyMutable
--R size? : (% , NonNegativeInteger) -> Boolean
--R ?~=? : (% , %) -> Boolean if S has SETCAT
--R
--E 1

```

```

)spool
)lisp (bye)

```

\_\_\_\_\_

— **Tree.help** —

```

=====
Tree examples
=====

```

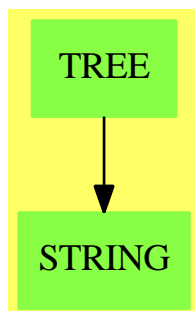
```

See Also:
o )show Tree

```

\_\_\_\_\_

### 21.10.1 Tree (TREE)



See

- ⇒ “BinaryTree” (BTREE) 3.11.1 on page 292
- ⇒ “BinarySearchTree” (BSTREE) 3.9.1 on page 285
- ⇒ “BinaryTournament” (BTourn) 3.10.1 on page 289
- ⇒ “BalancedBinaryTree” (BBTREE) 3.1.1 on page 234
- ⇒ “PendantTree” (PENDTREE) 17.13.1 on page 1904



**Exports:**

|               |              |            |               |              |
|---------------|--------------|------------|---------------|--------------|
| any?          | child?       | children   | coerce        | copy         |
| count         | cyclic?      | cyclicCopy | cyclicEntries | cyclicEqual? |
| cyclicParents | distance     | empty      | empty?        | eq?          |
| eval          | every?       | hash       | latex         | leaf?        |
| leaves        | less?        | map        | map!          | member?      |
| members       | more?        | node?      | nodes         | parts        |
| sample        | setchildren! | setelt     | setvalue!     | size?        |
| tree          | value        | #?         | ?=?           | ?~=?         |
| ?.value       |              |            |               |              |

— domain **TREE** Tree —

```

)abbrev domain TREE Tree
++ Author:W. H. Burge
++ Date Created:17 Feb 1992
++ Date Last Updated:
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++ \spadtype{Tree(S)} is a basic domains of tree structures.
++ Each tree is either empty or else is a node consisting of a value and
++ a list of (sub)trees.

```

```

Tree(S: SetCategory): T==C where
  T== RecursiveAggregate(S) with
    finiteAggregate
    shallowlyMutable
    tree: (S,List %) -> %
      ++ tree(nd,ls) creates a tree with value nd, and children ls.
      ++
      ++X t1:=tree [1,2,3,4]
      ++X tree(5,[t1])

    tree: List S -> %
      ++ tree(ls) creates a tree from a list of elements of s.
      ++
      ++X tree [1,2,3,4]

    tree: S -> %
      ++ tree(nd) creates a tree with value nd, and no children
      ++
      ++X tree 6

```

```

cyclic?: % -> Boolean
++ cyclic?(t) tests if t is a cyclic tree.
++
++X t1:=tree [1,2,3,4]
++X cyclic? t1

cyclicCopy: % -> %
++ cyclicCopy(l) makes a copy of a (possibly) cyclic tree l.
++
++X t1:=tree [1,2,3,4]
++X cyclicCopy t1

cyclicEntries: % -> List %
++ cyclicEntries(t) returns a list of top-level cycles in tree t.
++
++X t1:=tree [1,2,3,4]
++X cyclicEntries t1

cyclicEqual?: (% , %) -> Boolean
++ cyclicEqual?(t1, t2) tests if two cyclic trees have
++ the same structure.
++
++X t1:=tree [1,2,3,4]
++X t2:=tree [1,2,3,4]
++X cyclicEqual?(t1,t2)

cyclicParents: % -> List %
++ cyclicParents(t) returns a list of cycles that are parents of t.
++
++X t1:=tree [1,2,3,4]
++X cyclicParents t1

C== add
cycleTreeMax ==> 5

Rep := Union(node:Record(value: S, args: List %),empty:"empty")
t:%
br:%
s: S
ls: List S
empty? t == t case empty
empty() == ["empty"]
children t ==
  t case empty => error "cannot take the children of an empty tree"
  (t.node.args)@List(%)
setchildren_!(t,lt) ==
  t case empty => error "cannot set children of an empty tree"
  (t.node.args:=lt;t pretend %)
setvalue_!(t,s) ==
  t case empty => error "cannot set value of an empty tree"

```

```

      (t.node.value:=s;s)
count(n, t) ==
  t case empty => 0
  i := +/[count(n, c) for c in children t]
  value t = n => i + 1
  i
count(fn: S -> Boolean, t: %): NonNegativeInteger ==
  t case empty => 0
  i := +/[count(fn, c) for c in children t]
  fn value t => i + 1
  i
map(fn, t) ==
  t case empty => t
  tree(fn value t,[map(fn, c) for c in children t])
map_!(fn, t) ==
  t case empty => t
  setvalue_!(t, fn value t)
  for c in children t repeat map_!(fn, c)
tree(s,lt) == [[s,lt]]
tree(s) == [[s,[]]]
tree(ls) ==
  empty? ls => empty()
  tree(first ls, [tree s for s in rest ls])
value t ==
  t case empty => error "cannot take the value of an empty tree"
  t.node.value
child?(t1,t2) ==
  empty? t2 => false
  "or"/[t1 = t for t in children t2]
distance1(t1: %, t2: %): Integer ==
  t1 = t2 => 0
  t2 case empty => -1
  u := [n for t in children t2 | (n := distance1(t1,t)) >= 0]
  #u > 0 => 1 + "min"/u
  -1
distance(t1,t2) ==
  n := distance1(t1, t2)
  n >= 0 => n
  distance1(t2, t1)
node?(t1, t2) ==
  t1 = t2 => true
  t case empty => false
  "or"/[node?(t1, t) for t in children t2]
leaf? t ==
  t case empty => false
  empty? children t
leaves t ==
  t case empty => empty()
  leaf? t => [value t]
  "append"/[leaves c for c in children t]

```

```

less? (t, n) == # t < n
more?(t, n) == # t > n
nodes t ==      ---buggy
  t case empty => empty()
  nl := [nodes c for c in children t]
  nl = empty() => [t]
  cons(t,"append"/nl)
size? (t, n) == # t = n
any?(fn, t) ==  ---bug fixed
  t case empty => false
  fn value t or "or"/[any?(fn, c) for c in children t]
every?(fn, t) ==
  t case empty => true
  fn value t and "and"/[every?(fn, c) for c in children t]
member?(n, t) ==
  t case empty => false
  n = value t or "or"/[member?(n, c) for c in children t]
members t == parts t
parts t == --buggy?
  t case empty => empty()
  u := [parts c for c in children t]
  u = empty() => [value t]
  cons(value t,"append"/u)

---Functions that guard against cycles: =, #, copy-----

-----> =
equal?: (% , % , % , % , Integer) -> Boolean

t1 = t2 == equal?(t1, t2, t1, t2, 0)

equal?(t1, t2, ot1, ot2, k) ==
  k = cycleTreeMax and (cyclic? ot1 or cyclic? ot2) =>
    error "use cyclicEqual? to test equality on cyclic trees"
  t1 case empty => t2 case empty
  t2 case empty => false
  value t1 = value t2 and (c1 := children t1) = (c2 := children t2) and
    "and"/[equal?(x,y,ot1, ot2,k + 1) for x in c1 for y in c2]

-----> #
treeCount: (% , % , NonNegativeInteger) -> NonNegativeInteger
# t == treeCount(t, t, 0)
treeCount(t, origTree, k) ==
  k = cycleTreeMax and cyclic? origTree =>
    error "# is not defined on cyclic trees"
  t case empty => 0
  1 + +/[treeCount(c, origTree, k + 1) for c in children t]

-----> copy
copy1: (% , % , Integer) -> %

```

```

copy t == copy1(t, t, 0)
copy1(t, origTree, k) ==
  k = cycleTreeMax and cyclic? origTree =>
    error "use cyclicCopy to copy a cyclic tree"
  t case empty => t
  empty? children t => tree value t
  tree(value t, [copy1(x, origTree, k + 1) for x in children t])

-----Functions that allow cycles-----
--local utility functions:
eqUnion: (List %, List %) -> List %
eqMember?: (%, List %) -> Boolean
eqMemberIndex: (%, List %, Integer) -> Integer
lastNode: List % -> List %
insert: (%, List %) -> List %

-----> coerce to OutputForm
if S has SetCategory then
  multipleOverbar: (OutputForm, Integer, List %) -> OutputForm
  coerce1: (%, List %, List %) -> OutputForm

  coerce(t:%): OutputForm == coerce1(t, empty()$(List %), cyclicParents t)

  coerce1(t,parents, pl) ==
    t case empty => empty()@List(S)::OutputForm
    eqMember?(t, parents) =>
      multipleOverbar((".",):OutputForm,eqMemberIndex(t, pl,0),pl)
    empty? children t => value t::OutputForm
    nodeForm := (value t)::OutputForm
    if (k := eqMemberIndex(t, pl, 0)) > 0 then
      nodeForm := multipleOverbar(nodeForm, k, pl)
    prefix(nodeForm,
      [coerce1(br,cons(t,parents),pl) for br in children t])

  multipleOverbar(x, k, pl) ==
    k < 1 => x
    #pl = 1 => overbar x
    s : String := "abcdefghijklmnopqrstuvwxyz"
    c := s.(1 + ((k - 1) rem 26))
    overlabel(c::OutputForm, x)

-----> cyclic?
cyclic2?: (%, List %) -> Boolean

cyclic? t == cyclic2?(t, empty()$(List %))

cyclic2?(x,parents) ==
  empty? x => false
  eqMember?(x, parents) => true
  for y in children x repeat

```

```

    cyclic2?(y,cons(x, parents)) => return true
  false

-----> cyclicCopy
cyclicCopy2: (% , List %) -> %
copyCycle2: (% , List %) -> %
copyCycle4: (% , %, %, List %) -> %

cyclicCopy(t) == cyclicCopy2(t, cyclicEntries t)

cyclicCopy2(t, cycles) ==
  eqMember?(t, cycles) => return copyCycle2(t, cycles)
  tree(value t, [cyclicCopy2(c, cycles) for c in children t])

copyCycle2(cycle, cycleList) ==
  newCycle := tree(value cycle, nil)
  setchildren!(newCycle,
    [copyCycle4(c,cycle,newCycle, cycleList) for c in children cycle])
  newCycle

copyCycle4(t, cycle, newCycle, cycleList) ==
  empty? cycle => empty()
  eq?(t, cycle) => newCycle
  eqMember?(t, cycleList) => copyCycle2(t, cycleList)
  tree(value t,
    [copyCycle4(c, cycle, newCycle, cycleList) for c in children t])

-----> cyclicEntries
cyclicEntries3: (% , List %, List %) -> List %

cyclicEntries(t) == cyclicEntries3(t, empty()$(List %), empty()$(List %))

cyclicEntries3(t, parents, cl) ==
  empty? t => cl
  eqMember?(t, parents) => insert(t, cl)
  parents := cons(t, parents)
  for y in children t repeat
    cl := cyclicEntries3(t, parents, cl)
  cl

-----> cyclicEqual?
cyclicEqual4?: (% , %, List %, List %) -> Boolean

cyclicEqual?(t1, t2) ==
  cp1 := cyclicParents t1
  cp2 := cyclicParents t2
  #cp1 ^= #cp2 or null cp1 => t1 = t2
  cyclicEqual4?(t1, t2, cp1, cp2)

cyclicEqual4?(t1, t2, cp1, cp2) ==

```

```

t1 case empty => t2 case empty
t2 case empty => false
0 ^= (k := eqMemberIndex(t1, cp1, 0)) => eq?(t2, cp2 . k)
value t1 = value t2 and
  "and"/[cyclicEqual4?(x,y,cp1,cp2)
    for x in children t1 for y in children t2]

-----> cyclicParents t
cyclicParents3: (% , List % , List %) -> List %

cyclicParents t == cyclicParents3(t, empty()$(List %), empty()$(List %))

cyclicParents3(x, parents, pl) ==
  empty? x => pl
  eqMember?(x, parents) =>
    cycleMembers := [y for y in parents while not eq?(x,y)]
    eqUnion(cons(x, cycleMembers), pl)
  parents := cons(x, parents)
  for y in children x repeat
    pl := cyclicParents3(y, parents, pl)
  pl

insert(x, l) ==
  eqMember?(x, l) => l
  cons(x, l)

lastNode l ==
  empty? l => error "empty tree has no last node"
  while not empty? rest l repeat l := rest l
  l

eqMember?(y,l) ==
  for x in l repeat eq?(x,y) => return true
  false

eqMemberIndex(x, l, k) ==
  null l => k
  k := k + 1
  eq?(x, first l) => k
  eqMemberIndex(x, rest l, k)

eqUnion(u, v) ==
  null u => v
  x := first u
  newV :=
    eqMember?(x, v) => v
    cons(x, v)
  eqUnion(rest u, newV)

```

---

— TREE.dotabb —

```
"TREE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=TREE"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"TREE" -> "STRING"
```

---

## 21.11 domain TUBE TubePlot

— TubePlot.input —

```
)set break resume
)sys rm -f TubePlot.output
)spool TubePlot.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show TubePlot
--R TubePlot Curve: PlottableSpaceCurveCategory is a domain constructor
--R Abbreviation for TubePlot is TUBE
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for TUBE
--R
--R----- Operations -----
--R closed? : % -> Boolean          getCurve : % -> Curve
--R open? : % -> Boolean
--R listLoops : % -> List List Point DoubleFloat
--R setClosed : (% , Boolean) -> Boolean
--R tube : (Curve, List List Point DoubleFloat, Boolean) -> %
--R
--E 1

)spool
)lisp (bye)
```

---

— TubePlot.help —

=====



TubePlot examples

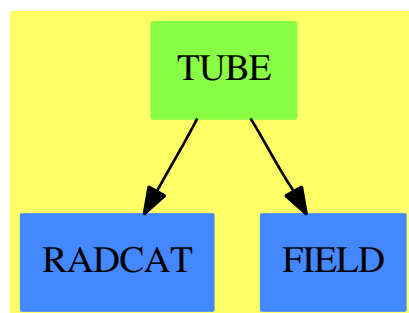
=====

See Also:

o )show TubePlot

—————

### 21.11.1 TubePlot (TUBE)



**Exports:**

closed? getCurve listLoops open? setClosed tube

— domain TUBE TubePlot —

```

)abbrev domain TUBE TubePlot
++ Author: Clifton J. Williamson
++ Date Created: Bastille Day 1989
++ Date Last Updated: 5 June 1990
++ Keywords:
++ Examples:
++ Description:
++ Package for constructing tubes around 3-dimensional parametric curves.
++ Domain of tubes around 3-dimensional parametric curves.

```

TubePlot(Curve): Exports == Implementation where

Curve : PlottableSpaceCurveCategory

B ==> Boolean

L ==> List

Pt ==> Point DoubleFloat

Exports ==> with

getCurve: % -> Curve

++ getCurve(t) returns the \spadtype{PlottableSpaceCurveCategory}

```

    ++ representing the parametric curve of the given tube plot t.
listLoops: % -> L L Pt
    ++ listLoops(t) returns the list of lists of points, or the 'loops',
    ++ of the given tube plot t.
closed?: % -> B
    ++ closed?(t) tests whether the given tube plot t is closed.
open?: % -> B
    ++ open?(t) tests whether the given tube plot t is open.
setClosed: (%,B) -> B
    ++ setClosed(t,b) declares the given tube plot t to be closed if
    ++ b is true, or if b is false, t is set to be open.
tube: (Curve,L L Pt,B) -> %
    ++ tube(c,ll,b) creates a tube of the domain \spadtype{TubePlot} from a
    ++ space curve c of the category \spadtype{PlottableSpaceCurveCategory},
    ++ a list of lists of points (loops) ll and a boolean b which if
    ++ true indicates a closed tube, or if false an open tube.

Implementation ==> add

--% representation

Rep := Record(parCurve:Curve,loops:L L Pt,closedTube?:B)

getCurve plot == plot.parCurve

listLoops plot == plot.loops

closed? plot == plot.closedTube?
open? plot == not plot.closedTube?

setClosed(plot,flag) == plot.closedTube? := flag

tube(curve,ll,b) == [curve,ll,b]

-----

— TUBE.dotabb —

"TUBE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=TUBE"]
"RADCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=RADCAT"]
"FIELD" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FIELD"]
"TUBE" -> "FIELD"
"TUBE" -> "RADCAT"

-----

```

## 21.12 domain TUPLE Tuple

— Tuple.input —

```
)set break resume
)sys rm -f Tuple.output
)spool Tuple.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Tuple
--R Tuple S: Type  is a domain constructor
--R Abbreviation for Tuple is TUPLE
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for TUPLE
--R
--R----- Operations -----
--R coerce : PrimitiveArray S -> %          coerce : % -> PrimitiveArray S
--R length : % -> NonNegativeInteger
--R ?=? : (%,% ) -> Boolean if S has SETCAT
--R coerce : % -> OutputForm if S has SETCAT
--R hash : % -> SingleInteger if S has SETCAT
--R latex : % -> String if S has SETCAT
--R select : (% , NonNegativeInteger) -> S
--R ?~=? : (%,% ) -> Boolean if S has SETCAT
--R
--E 1

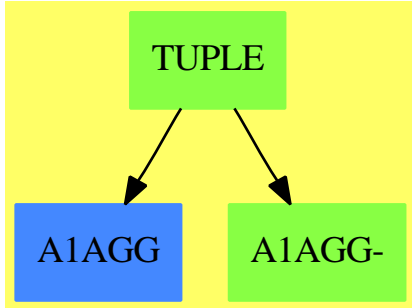
)spool
)lisp (bye)
```

— Tuple.help —

```
=====
Tuple examples
=====
```

```
See Also:
o )show Tuple
```

### 21.12.1 Tuple (TUPLE)



See

- ⇒ “PrimitiveArray” (PRIMARR) 17.30.1 on page 2069
- ⇒ “IndexedFlexibleArray” (IFARRAY) 10.10.1 on page 1187
- ⇒ “FlexibleArray” (FARRAY) 7.14.1 on page 853
- ⇒ “IndexedOneDimensionalArray” (IARRAY1) 10.13.1 on page 1208
- ⇒ “OneDimensionalArray” (ARRAY1) 16.3.1 on page 1736

#### Exports:

coerce hash latex length select ?=? ?~=?

— domain TUPLE Tuple —

```

)abbrev domain TUPLE Tuple
++ Author: Mark Botch
++ Description:
++ This domain is used to interface with the interpreter's notion
++ of comma-delimited sequences of values.
  
```

```

Tuple(S:Type): CoercibleTo(PrimitiveArray S) with
  coerce: PrimitiveArray S -> %
  ++ coerce(a) makes a tuple from primitive array a
  ++
  ++X t1:PrimitiveArray(Integer):= [i for i in 1..10]
  ++X t2:=coerce(t1)$Tuple(Integer)
  
```

```

select: (% , NonNegativeInteger) -> S
  ++ select(x,n) returns the n-th element of tuple x.
  ++ tuples are 0-based
  ++
  ++X t1:PrimitiveArray(Integer):= [i for i in 1..10]
  ++X t2:=coerce(t1)$Tuple(Integer)
  ++X select(t2,3)
  
```

```

length: % -> NonNegativeInteger
  ++ length(x) returns the number of elements in tuple x
  
```

```

++
++X t1:PrimitiveArray(Integer):= [i for i in 1..10]
++X t2:=coerce(t1)$Tuple(Integer)
++X length(t2)

if S has SetCategory then SetCategory
== add
Rep := Record(len : NonNegativeInteger, elts : PrimitiveArray S)

coerce(x: PrimitiveArray S): % == [#x, x]
coerce(x:%): PrimitiveArray(S) == x.elts
length x == x.len

select(x, n) ==
  n >= x.len => error "Index out of bounds"
  x.elts.n

if S has SetCategory then
  x = y == (x.len = y.len) and (x.elts =$PrimitiveArray(S) y.elts)
  coerce(x : %): OutputForm ==
    paren [(x.elts.i)::OutputForm
            for i in minIndex x.elts .. maxIndex x.elts]$List(OutputForm)

```

---

— TUPLE.dotabb —

```

"TUPLE" [color="#88FF44",href="bookvol10.3.pdf#nameddest=TUPLE"]
"A1AGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=A1AGG"]
"A1AGG-" [color="#88FF44",href="bookvol10.3.pdf#nameddest=A1AGG"]
"TUPLE" -> "A1AGG"
"TUPLE" -> "A1AGG-"

```

---

## 21.13 domain ARRAY2 TwoDimensionalArray

— TwoDimensionalArray.input —

```

)set break resume
)sys rm -f TwoDimensionalArray.output
)spool TwoDimensionalArray.output
)set message test on
)set message auto off

```

```

)clear all
--S 1 of 20
arr : ARRAY2 INT := new(5,4,0)
--R
--R
--R      +0  0  0  0+
--R      |      |
--R      |0  0  0  0|
--R      |      |
--R  (1) |0  0  0  0|
--R      |      |
--R      |0  0  0  0|
--R      |      |
--R      +0  0  0  0+
--R
--R                                          Type: TwoDimensionalArray Integer
--E 1

--S 2 of 20
setelt(arr,1,1,17)
--R
--R
--R  (2)  17
--R
--R                                          Type: PositiveInteger
--E 2

--S 3 of 20
arr
--R
--R
--R      +17  0  0  0+
--R      |      |
--R      |0  0  0  0|
--R      |      |
--R  (3) |0  0  0  0|
--R      |      |
--R      |0  0  0  0|
--R      |      |
--R      +0  0  0  0+
--R
--R                                          Type: TwoDimensionalArray Integer
--E 3

--S 4 of 20
elt(arr,1,1)
--R
--R
--R  (4)  17
--R
--R                                          Type: PositiveInteger
--E 4

--S 5 of 20

```

```

arr(3,2) := 15
--R
--R
--R (5) 15
--R
--R                                         Type: PositiveInteger
--E 5

--S 6 of 20
arr(3,2)
--R
--R
--R (6) 15
--R
--R                                         Type: PositiveInteger
--E 6

--S 7 of 20
row(arr,1)
--R
--R
--R (7) [17,0,0,0]
--R
--R                                         Type: OneDimensionalArray Integer
--E 7

--S 8 of 20
column(arr,1)
--R
--R
--R (8) [17,0,0,0,0]
--R
--R                                         Type: OneDimensionalArray Integer
--E 8

--S 9 of 20
nrows(arr)
--R
--R
--R (9) 5
--R
--R                                         Type: PositiveInteger
--E 9

--S 10 of 20
ncols(arr)
--R
--R
--R (10) 4
--R
--R                                         Type: PositiveInteger
--E 10

--S 11 of 20
map(-,arr)
--R

```

```

--R
--R      +- 17  0  0  0+
--R      |      |
--R      | 0    0  0  0|
--R      |      |
--R      (11) | 0    - 15  0  0|
--R      |      |
--R      | 0    0  0  0|
--R      |      |
--R      + 0    0  0  0+
--R
--R                                          Type: TwoDimensionalArray Integer
--E 11

--S 12 of 20
map((x +-> x + x),arr)
--R
--R
--R      +34  0  0  0+
--R      |      |
--R      |0    0  0  0|
--R      |      |
--R      (12) |0    30  0  0|
--R      |      |
--R      |0    0  0  0|
--R      |      |
--R      +0    0  0  0+
--R
--R                                          Type: TwoDimensionalArray Integer
--E 12

--S 13 of 20
arrc := copy(arr)
--R
--R
--R      +17  0  0  0+
--R      |      |
--R      |0    0  0  0|
--R      |      |
--R      (13) |0    15  0  0|
--R      |      |
--R      |0    0  0  0|
--R      |      |
--R      +0    0  0  0+
--R
--R                                          Type: TwoDimensionalArray Integer
--E 13

--S 14 of 20
map!(-,arrc)
--R
--R
--R      +- 17  0  0  0+

```



```

--R      |      |
--R      | 0      0      0 0|
--R      |      |
--R      (14) | 0      - 15 0 0|
--R      |      |
--R      | 0      0      0 0|
--R      |      |
--R      + 0      0      0 0+
--R
--E 14

```

Type: TwoDimensionalArray Integer

```

--S 15 of 20
arrc
--R
--R
--R      +- 17      0      0 0+
--R      |      |
--R      | 0      0      0 0|
--R      |      |
--R      (15) | 0      - 15 0 0|
--R      |      |
--R      | 0      0      0 0|
--R      |      |
--R      + 0      0      0 0+
--R
--E 15

```

Type: TwoDimensionalArray Integer

```

--S 16 of 20
arr
--R
--R
--R      +17 0      0 0+
--R      |      |
--R      | 0 0      0 0|
--R      |      |
--R      (16) | 0 15 0 0|
--R      |      |
--R      | 0 0      0 0|
--R      |      |
--R      +0 0      0 0+
--R
--E 16

```

Type: TwoDimensionalArray Integer

```

--S 17 of 20
member?(17,arr)
--R
--R
--R      (17) true
--R
--E 17

```

Type: Boolean

```

--S 18 of 20
member?(10317,arr)
--R
--R
--R (18) false
--R
--R                                          Type: Boolean
--E 18

--S 19 of 20
count(17,arr)
--R
--R
--R (19) 1
--R
--R                                          Type: PositiveInteger
--E 19

--S 20 of 20
count(0,arr)
--R
--R
--R (20) 18
--R
--R                                          Type: PositiveInteger
--E 20
)spool
)lisp (bye)

```

---

— TwoDimensionalArray.help —

=====

TwoDimensionalArray examples

=====

The TwoDimensionalArray domain is used for storing data in a two dimensional data structure indexed by row and by column. Such an array is a homogeneous data structure in that all the entries of the array must belong to the same Axiom domain.. Each array has a fixed number of rows and columns specified by the user and arrays are not extensible. In Axiom, the indexing of two-dimensional arrays is one-based. This means that both the "first" row of an array and the "first" column of an array are given the index 1. Thus, the entry in the upper left corner of an array is in position (1,1).

The operation new creates an array with a specified number of rows and columns and fills the components of that array with a specified entry. The arguments of this operation specify the number of rows, the number of columns, and the entry.



This extracts the element in position (3,2) of the array.

```
arr(3,2)
15
Type: PositiveInteger
```

The operations `elt` and `setelt` come equipped with an error check which verifies that the indices are in the proper ranges. For example, the above array has five rows and four columns, so if you ask for the entry in position (6,2) with `arr(6,2)` Axiom displays an error message. If there is no need for an error check, you can call the operations `qelt` and `qsetelt` which provide the same functionality but without the error check. Typically, these operations are called in well-tested programs.

The operations `row` and `column` extract rows and columns, respectively, and return objects of `OneDimensionalArray` with the same underlying element type.

```
row(arr,1)
[17,0,0,0]
Type: OneDimensionalArray Integer

column(arr,1)
[17,0,0,0,0]
Type: OneDimensionalArray Integer
```

You can determine the dimensions of an array by calling the operations `nrows` and `ncols`, which return the number of rows and columns, respectively.

```
nrows(arr)
5
Type: PositiveInteger

ncols(arr)
4
Type: PositiveInteger
```

To apply an operation to every element of an array, use `map`. This creates a new array. This expression negates every element.

```
map(-,arr)
+- 17  0  0  0+
|
| 0    0  0  0|
|
| 0    - 15  0  0|
|
| 0    0  0  0|
|
```

```
+ 0      0      0 0+
Type: TwoDimensionalArray Integer
```

This creates an array where all the elements are doubled.

```
map((x +-> x + x),arr)
+34  0      0 0+
|
|0    0      0 0|
|
|0    30     0 0|
|
|0    0      0 0|
|
+0    0      0 0+
Type: TwoDimensionalArray Integer
```

To change the array destructively, use `map` instead of `map`. If you need to make a copy of any array, use `copy`.

```
arrc := copy(arr)
+17  0      0 0+
|
|0    0      0 0|
|
|0    15     0 0|
|
|0    0      0 0|
|
+0    0      0 0+
Type: TwoDimensionalArray Integer
```

```
map!(-,arrc)
+- 17      0      0 0+
|
|0          0      0 0|
|
|0    - 15   0 0|
|
|0          0      0 0|
|
+ 0          0      0 0+
Type: TwoDimensionalArray Integer
```

```
arrc
+- 17      0      0 0+
|
|0          0      0 0|
|
|0    - 15   0 0|
```

```

|
| 0      0      0  0|
|
+ 0      0      0  0+
Type: TwoDimensionalArray Integer

```

```

arr
+17  0      0      0+
|
| 0      0      0  0|
|
| 0      15     0      0|
|
| 0      0      0      0|
|
+0      0      0      0+
Type: TwoDimensionalArray Integer

```

Use member? to see if a given element is in an array.

```

member?(17,arr)
true
Type: Boolean

```

```

member?(10317,arr)
false
Type: Boolean

```

To see how many times an element appears in an array, use count.

```

count(17,arr)
1
Type: PositiveInteger

```

```

count(0,arr)
18
Type: PositiveInteger

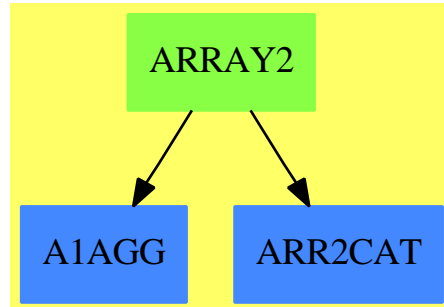
```

See Also:

- o )help Matrix
- o )help OneDimensionalArray
- o )show TwoDimensionalArray

---

### 21.13.1 TwoDimensionalArray (ARRAY2)



See

⇒ “InnerIndexedTwoDimensionalArray” (IIARRAY2) 10.23.1 on page 1254

⇒ “IndexedTwoDimensionalArray” (IARRAY2) 10.15.1 on page 1221

#### Exports:

|         |        |             |             |          |
|---------|--------|-------------|-------------|----------|
| any?    | coerce | column      | copy        | count    |
| elt     | empty  | empty?      | eq?         | eval     |
| every?  | fill!  | hash        | latex       | less?    |
| map     | map!   | maxColIndex | maxRowIndex | member?  |
| members | more?  | minColIndex | minRowIndex | ncols    |
| new     | nrows  | parts       | qelt        | qsetelt! |
| row     | sample | setColumn!  | setRow!     | setelt   |
| size?   | #?     | ?=?         | ?~=?        |          |

— domain ARRAY2 TwoDimensionalArray —

```

)abbrev domain ARRAY2 TwoDimensionalArray
++ Author: Mark Botch
++ Description:
++ A TwoDimensionalArray is a two dimensional array with
++ 1-based indexing for both rows and columns.

TwoDimensionalArray(R):Exports == Implementation where
  R : Type
  Row ==> OneDimensionalArray R
  Col ==> OneDimensionalArray R

Exports ==> TwoDimensionalArrayCategory(R,Row,Col) with
  shallowlyMutable
  ++ One may destructively alter TwoDimensionalArray's.

Implementation ==> InnerIndexedTwoDimensionalArray(R,1,1,Row,Col)
  
```

— ARRAY2.dotabb —

```
"ARRAY2" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ARRAY2"]
"A1AGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=A1AGG"]
"ARR2CAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ARR2CAT"]
"ARRAY2" -> "ARR2CAT"
"ARRAY2" -> "A1AGG"
```

## 21.14 domain VIEW2D TwoDimensionalViewport

— TwoDimensionalViewport.help —

```
=====
TwoDimensionalViewport examples
=====
```

We want to graph  $x^3 * (a+b*x)$  on the interval  $x=-1..1$   
so we clear out the workspace

We assign values to the constants

```
a:=0.5
0.5
                                     Type: Float
```

```
b:=0.5
0.5
                                     Type: Float
```

We draw the first case of the graph

```
y1:=draw(x^3*(a+b*x),x=-1..1,title=="2.2.10 explicit")
TwoDimensionalViewport: "2.2.10 explicit"
                                     Type: TwoDimensionalViewport
```

We fetch the graph of the first object

```
g1:=getGraph(y1,1)
Graph with 1 point list
                                     Type: GraphImage
```

We extract its points

```
pointLists g1
[
```



```

[[-1.,0.,1.,3.], [-0.9583333333333337,-1.8336166570216028E-2,1.,3.],
[-0.91666666666666674,-3.2093942901234518E-2,1.,3.],
[-0.87500000000000011,-4.18701171875E-2,1.,3.],
[-0.83333333333333348,-4.8225308641975301E-2,1.,3.],
[-0.79166666666666685,-5.1683967496141986E-2,1.,3.],
[-0.75000000000000022,-5.2734375E-2,1.,3.],
[-0.70833333333333359,-5.1828643422067916E-2,1.,3.],
[-0.66666666666666696,-4.9382716049382741E-2,1.,3.],
[-0.62500000000000033,-4.5776367187500042E-2,1.,3.],
[-0.58333333333333337,-4.1353202160493867E-2,1.,3.],
[-0.541666666666666707,-3.6420657310956832E-2,1.,3.],
[-0.50000000000000044,-3.125000000000056E-2,1.,3.],
[-0.45833333333333376,-2.6076328607253136E-2,1.,3.],
[-0.416666666666666707,-2.1098572530864244E-2,1.,3.],
[-0.37500000000000039,-1.6479492187500042E-2,1.,3.],
[-0.33333333333333337,-1.2345679012345713E-2,1.,3.],
[-0.291666666666666702,-8.7875554591049648E-3,1.,3.],
[-0.25000000000000033,-5.8593750000000208E-3,1.,3.],
[-0.20833333333333368,-3.5792221257716214E-3,1.,3.],
[-0.166666666666666702,-1.9290123456790237E-3,1.,3.],
[-0.12500000000000036,-8.5449218750000705E-4,1.,3.],
[-8.3333333333333703E-2,-2.6523919753086765E-4,1.,3.],
[-4.1666666666667039E-2,-3.4661940586420673E-5,1.,3.],
[-3.7470027081099033E-16,-2.6304013894372334E-47,1.,3.],
[4.166666666666629E-2,3.7676022376542178E-5,1.,3.],
[8.3333333333332954E-2,3.1346450617283515E-4,1.,3.],
[0.1249999999999961,1.0986328124999894E-3,1.,3.],
[0.1666666666666627,2.7006172839505972E-3,1.,3.],
[0.2083333333333293,5.463023244598731E-3,1.,3.],
[0.2499999999999958,9.765624999999948E-3,1.,3.],
[0.2916666666666624,1.6024365837191284E-2,1.,3.],
[0.3333333333333293,2.469135802469126E-2,1.,3.],
[0.3749999999999961,3.6254882812499882E-2,1.,3.],
[0.416666666666663,5.1239390432098617E-2,1.,3.],
[0.4583333333333298,7.0205500096450435E-2,1.,3.],
[0.4999999999999967,9.374999999999792E-2,1.,3.],
[0.541666666666663,0.12250584731867258,1.,3.],
[0.5833333333333293,0.15714216820987617,1.,3.],
[0.6249999999999956,0.1983642578124995,1.,3.],
[0.6666666666666619,0.24691358024691298,1.,3.],
[0.7083333333333282,0.30356776861496837,1.,3.],
[0.7499999999999944,0.369140624999999,1.,3.],
[0.7916666666666607,0.44448212046681984,1.,3.],
[0.833333333333327,0.530478395061727,1.,3.],
[0.8749999999999933,0.62805175781249845,1.,3.],
[0.9166666666666596,0.73816068672839308,1.,3.],
[0.9583333333333259,0.86179982880015205,1.,3.], [1.,1.,1.,3.]]
]

```

Type: List List Point DoubleFloat

Now we create a second graph with a changed parameter

```
b:=1.0
1.0
Type: Float
```

We draw it

```
y2:=draw(x^3*(a+b*x),x=-1..1)
TwoDimensionalViewport: "AXIOM2D"
Type: TwoDimensionalViewport
```

We fetch this new graph

```
g2:=getGraph(y2,1)
Graph with 1 point list
Type: GraphImage
```

We get the points from this graph

```
pointLists g2
[
  [[-1.,0.5,1.,3.], [-0.9583333333333337,0.40339566454475323,1.,3.],
  [-0.9166666666666667,0.32093942901234584,1.,3.],
  [-0.8750000000000001,0.25122070312500017,1.,3.],
  [-0.8333333333333334,0.19290123456790137,1.,3.],
  [-0.7916666666666668,0.14471510898919768,1.,3.],
  [-0.7500000000000002,0.10546875000000019,1.,3.],
  [-0.7083333333333335,7.404091917438288E-2,1.,3.],
  [-0.6666666666666669,4.938271604938288E-2,1.,3.],
  [-0.6250000000000003,3.0517578125000125E-2,1.,3.],
  [-0.5833333333333337,1.6541280864197649E-2,1.,3.],
  [-0.541666666666667,6.6219376929013279E-3,1.,3.],
  [-0.5000000000000004,5.5511151231257827E-17,1.,3.],
  [-0.4583333333333337, -4.011742862654287E-3,1.,3.],
  [-0.416666666666667, -6.0281635802469057E-3,1.,3.],
  [-0.3750000000000003, -6.5917968750000035E-3,1.,3.],
  [-0.3333333333333337, -6.1728395061728461E-3,1.,3.],
  [-0.291666666666667, -5.1691502700617377E-3,1.,3.],
  [-0.2500000000000003, -3.906250000000104E-3,1.,3.],
  [-0.2083333333333336, -2.6373215663580349E-3,1.,3.],
  [-0.166666666666667, -1.543209876543218E-3,1.,3.],
  [-0.1250000000000003, -7.3242187500000564E-4,1.,3.],
  [-8.3333333333333703E-2, -2.4112654320987957E-4,1.,3.],
  [-4.1666666666667039E-2, -3.315489969135889E-5,1.,3.],
  [-3.7470027081099033E-16, -2.6304013894372324E-47,1.,3.],
  [4.166666666666629E-2, 3.9183063271603852E-5,1.,3.],
  [8.3333333333332954E-2, 3.3757716049382237E-4,1.,3.],
  [0.1249999999999961, 1.2207031249999879E-3,1.,3.],
  [0.1666666666666627, 3.0864197530863957E-3,1.,3.],
```

```
[0.20833333333333293,6.4049238040123045E-3,1.,3.],
[0.2499999999999958,1.171874999999934E-2,1.,3.],
[0.29166666666666624,1.9642771026234473E-2,1.,3.],
[0.33333333333333293,3.0864197530864071E-2,1.,3.],
[0.3749999999999961,4.6142578124999847E-2,1.,3.],
[0.4166666666666663,6.6309799382715848E-2,1.,3.],
[0.45833333333333298,9.2270085841049135E-2,1.,3.],
[0.4999999999999967,0.1249999999999971,1.,3.],
[0.5416666666666663,0.16554844232253049,1.,3.],
[0.58333333333333293,0.21503665123456736,1.,3.],
[0.6249999999999956,0.27465820312499928,1.,3.],
[0.66666666666666619,0.3456790123456781,1.,3.],
[0.70833333333333282,0.42943733121141858,1.,3.],
[0.7499999999999944,0.52734374999999845,1.,3.],
[0.79166666666666607,0.64088119695215873,1.,3.],
[0.8333333333333327,0.77160493827160281,1.,3.],
[0.8749999999999933,0.92114257812499756,1.,3.],
[0.91666666666666596,1.0911940586419722,1.,3.],
[0.95833333333333259,1.2835316599151199,1.,3.], [1.,1.5,1.,3.]]
]
```

Type: List List Point DoubleFloat

and we put these points, g2 onto the first graph y1 as graph 2

```
putGraph(y1,g2,2)
```

Type: Void

And now we do the whole sequence again

```
b:=2.0
2.0
```

Type: Float

```
y3:=draw(x^3*(a+b*x),x=-1..1)
```

```
TwoDimensionalViewport: "AXIOM2D"
```

Type: TwoDimensionalViewport

```
g3:=getGraph(y3,1)
```

```
Graph with 1 point list
```

Type: GraphImage

```
pointLists g3
```

```
[
  [[-1.,1.5,1.,3.], [-0.9583333333333337,1.2468593267746917,1.,3.],
  [-0.91666666666666674,1.0270061728395066,1.,3.],
  [-0.87500000000000011,0.83740234375000044,1.,3.],
  [-0.83333333333333348,0.67515432098765471,1.,3.],
  [-0.79166666666666685,0.53751326195987703,1.,3.],
  [-0.75000000000000022,0.42187500000000056,1.,3.],
  [-0.70833333333333359,0.32578004436728447,1.,3.],
```

```

[-0.666666666666666696,0.24691358024691412,1.,3.],
[-0.625000000000000033,0.18310546875000044,1.,3.],
[-0.5833333333333337,0.1323302469135807,1.,3.],
[-0.54166666666666707,9.2707127700617648E-2,1.,3.],
[-0.50000000000000044,6.2500000000000278E-2,1.,3.],
[-0.4583333333333376,4.0117428626543411E-2,1.,3.],
[-0.41666666666666707,2.4112654320987775E-2,1.,3.],
[-0.37500000000000039,1.3183593750000073E-2,1.,3.],
[-0.3333333333333337,6.1728395061728877E-3,1.,3.],
[-0.29166666666666702,2.0676601080247183E-3,1.,3.],
[-0.25000000000000033,1.0408340855860843E-17,1.,3.],
[-0.2083333333333368,-7.5352044753086191E-4,1.,3.],
[-0.16666666666666702,-7.7160493827160663E-4,1.,3.],
[-0.12500000000000036,-4.8828125000000282E-4,1.,3.],
[-8.333333333333703E-2,-1.9290123456790339E-4,1.,3.],
[-4.166666666667039E-2,-3.0140817901235325E-5,1.,3.],
[-3.7470027081099033E-16,-2.6304013894372305E-47,1.,3.],
[4.16666666666629E-2,4.21971450617272E-5,1.,3.],
[8.333333333332954E-2,3.8580246913579681E-4,1.,3.],
[0.1249999999999961,1.4648437499999848E-3,1.,3.],
[0.1666666666666627,3.8580246913579933E-3,1.,3.],
[0.2083333333333293,8.2887249228394497E-3,1.,3.],
[0.2499999999999958,1.56249999999991E-2,1.,3.],
[0.2916666666666624,2.6879581404320851E-2,1.,3.],
[0.3333333333333293,4.3209876543209694E-2,1.,3.],
[0.3749999999999961,6.5917968749999764E-2,1.,3.],
[0.416666666666663,9.6450617283950296E-2,1.,3.],
[0.4583333333333298,0.13639925733024652,1.,3.],
[0.4999999999999967,0.1874999999999956,1.,3.],
[0.541666666666663,0.25163363233024633,1.,3.],
[0.5833333333333293,0.33082561728394977,1.,3.],
[0.6249999999999956,0.42724609374999883,1.,3.],
[0.6666666666666619,0.5432098765432084,1.,3.],
[0.7083333333333282,0.68117645640431912,1.,3.],
[0.7499999999999944,0.8437499999999756,1.,3.],
[0.7916666666666607,1.0336793499228365,1.,3.],
[0.833333333333327,1.2538580246913544,1.,3.],
[0.8749999999999933,1.507324218749996,1.,3.],
[0.9166666666666596,1.7972608024691306,1.,3.],
[0.9583333333333259,2.1269953221450555,1.,3.], [1.,2.5,1.,3.]]
]

```

Type: List List Point DoubleFloat

and put the third graphs points g3 onto the first graph y1 as graph 3

```
putGraph(y1,g3,3)
```

Type: Void

Finally we show the combined result

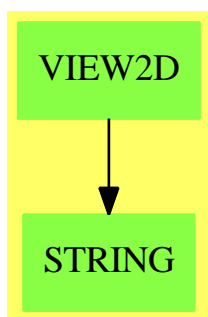
```
vp:=makeViewport2D(y1)
TwoDimensionalViewport: "2.2.10 explicit"
Type: TwoDimensionalViewport
```

See Also:

o )show TwoDimensionalViewport

—————

### 21.14.1 TwoDimensionalViewport (VIEW2D)



#### Exports:

|            |          |                 |            |                |
|------------|----------|-----------------|------------|----------------|
| axes       | close    | coerce          | connect    | controlPanel   |
| dimensions | getGraph | getPickedPoints | graphState | graphStates    |
| graphs     | hash     | key             | latex      | makeViewport2D |
| move       | options  | points          | putGraph   | region         |
| reset      | resize   | scale           | show       | title          |
| translate  | units    | update          | viewport2D | write          |
| ?=?        | ?~=?     |                 |            |                |

— domain VIEW2D TwoDimensionalViewport —

```
)abbrev domain VIEW2D TwoDimensionalViewport
++ Author: Jim Wen
++ Date Created: 28 April 1989
++ Date Last Updated: 29 October 1991, Jon Steinbach
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
```

++ TwoDimensionalViewport creates viewports to display graphs.

TwoDimensionalViewport ():Exports == Implementation where

```

VIEW    ==> VIEWPORTSERVER$Lisp
sendI    ==> SOCK_-SEND_-INT
sendSF    ==> SOCK_-SEND_-FLOAT
sendSTR    ==> SOCK_-SEND_-STRING
getI    ==> SOCK_-GET_-INT
getSF    ==> SOCK_-GET_-FLOAT

typeGRAPH ==> 2
typeVIEW2D ==> 3

makeGRAPH ==> (-1)$SingleInteger
makeVIEW2D ==> (-1)$SingleInteger

I    ==> Integer
PI    ==> PositiveInteger
NNI    ==> NonNegativeInteger
XY    ==> Record( X:I, Y:I )
XYP    ==> Record( X:PI, Y:PI )
XYNN    ==> Record( X:NNI, Y:NNI )
F    ==> Float
SF    ==> DoubleFloat
STR    ==> String
L    ==> List
V    ==> Vector
E    ==> OutputForm
FLAG    ==> Record( showCP:I )
PAL    ==> Palette()
B    ==> Boolean
G    ==> GraphImage
GS    ==> Record( scaleX:SF, scaleY:SF, deltaX:SF, deltaY:SF, _
                  points:I, connect:I, spline:I, _
                  axes:I, axesColor:PAL, units:I, unitsColor:PAL, _
                  showing:I)
GU    ==> Union(G,"undefined")
DROP    ==> DrawOption
POINT    ==> Point(SF)

TRANSLATE2D    ==> 0$I
SCALE2D        ==> 1$I
pointsOnOff    ==> 2
connectOnOff    ==> 3
spline2D        ==> 4  -- used for controlling regions, now
reset2D        ==> 5
hideControl2D    ==> 6
closeAll2D      ==> 7
axesOnOff2D     ==> 8

```

```

unitsOnOff2D    ==> 9

SPADBUTTONPRESS ==> 100
MOVE            ==> 102
RESIZE          ==> 103
TITLE           ==> 104
showing2D       ==> 105 -- as defined in include/actions.h
putGraph2D      ==> 106
writeView       ==> 110
axesColor2D     ==> 112
unitsColor2D    ==> 113
getPickedPTS    ==> 119

graphStart      ==> 13  -- as defined in include/actions.h

noControl ==> 0$I

yes            ==> 1$I
no             ==> 0$I

maxGRAPHS ==> 9::I  -- should be the same as maxGraphs in include/view2d.h

fileTypeDefs ==> ["PIXMAP"]  -- see include/write.h for things to include

Exports ==> SetCategory with
  getPickedPoints : $ -> L POINT
    ++ getPickedPoints(x)
    ++ returns a list of small floats for the points the
    ++ user interactively picked on the viewport
    ++ for full integration into the system, some design
    ++ issues need to be addressed: e.g. how to go through
    ++ the GraphImage interface, how to default to graphs, etc.
  viewport2D      : () -> $
    ++ viewport2D() returns an undefined two-dimensional viewport
    ++ of the domain \spadtype{TwoDimensionalViewport} whose
    ++ contents are empty.
  makeViewport2D : $ -> $
    ++ makeViewport2D(v) takes the given two-dimensional viewport,
    ++ v, of the domain \spadtype{TwoDimensionalViewport} and
    ++ displays a viewport window on the screen which contains
    ++ the contents of v.
  options          : $ -> L DROP
    ++ options(v) takes the given two-dimensional viewport, v, of the
    ++ domain \spadtype{TwoDimensionalViewport} and returns a list
    ++ containing the draw options from the domain \spadtype{DrawOption}
    ++ for v.
  options          : ($,L DROP) -> $
    ++ options(v,lopt) takes the given two-dimensional viewport, v,
    ++ of the domain \spadtype{TwoDimensionalViewport} and returns
    ++ v with it's draw options modified to be those which are indicated

```

```

++ in the given list, \spad{lopt} of domain \spadtype{DrawOption}.
makeViewport2D : (G,L DROP) -> $
++ makeViewport2D(gi,lopt) creates and displays a viewport window
++ of the domain \spadtype{TwoDimensionalViewport} whose graph
++ field is assigned to be the given graph, \spad{gi}, of domain
++ \spadtype{GraphImage}, and whose options field is set to be
++ the list of options, \spad{lopt} of domain \spadtype{DrawOption}.
graphState      : ($,PI,SF,SF,SF,SF,I,I,I,I,PAL,I,PAL,I) -> Void
++ graphState(v,num,sX,sY,dX,dY,pts,lms,box,axes,axesC,un,unC,cP)
++ sets the state of the characteristics for the graph indicated
++ by \spad{num} in the given two-dimensional viewport v, of domain
++ \spadtype{TwoDimensionalViewport}, to the values given as
++ parameters. The scaling of the graph in the x and y component
++ directions is set to be \spad{sX} and \spad{sY}; the window
++ translation in the x and y component directions is set to be
++ \spad{dX} and \spad{dY}; The graph points, lines, bounding box,
++ axes, or units will be shown in the viewport if their given
++ parameters \spad{pts}, \spad{lms}, \spad{box}, \spad{axes} or
++ \spad{un} are set to be \spad{1}, but will not be shown if they
++ are set to \spad{0}. The color of the axes and the color of the
++ units are indicated by the palette colors \spad{axesC} and
++ \spad{unC} respectively. To display the control panel when
++ the viewport window is displayed, set \spad{cP} to \spad{1},
++ otherwise set it to \spad{0}.
graphStates      : $ -> V GS
++ graphStates(v) returns and shows a listing of a record containing
++ the current state of the characteristics of each of the ten graph
++ records in the given two-dimensional viewport, v, which is of
++ domain \spadtype{TwoDimensionalViewport}.
graphs           : $ -> V GU
++ graphs(v) returns a vector, or list, which is a union of all
++ the graphs, of the domain \spadtype{GraphImage}, which are
++ allocated for the two-dimensional viewport, v, of domain
++ \spadtype{TwoDimensionalViewport}. Those graphs which have
++ no data are labeled "undefined", otherwise their contents
++ are shown.
title            : ($,STR) -> Void
++ title(v,s) changes the title which is shown in the two-dimensional
++ viewport window, v of domain \spadtype{TwoDimensionalViewport}.
putGraph         : ($,G,PI) -> Void
++ putGraph(v,gi,n) sets the graph field indicated by n, of the
++ indicated two-dimensional viewport, v, which is of domain
++ \spadtype{TwoDimensionalViewport}, to be the graph, \spad{gi}
++ of domain \spadtype{GraphImage}. The contents of viewport, v,
++ will contain \spad{gi} when the function \spadfun{makeViewport2D}
++ is called to create the an updated viewport v.
getGraph         : ($,PI) -> G
++ getGraph(v,n) returns the graph which is of the domain
++ \spadtype{GraphImage} which is located in graph field n
++ of the given two-dimensional viewport, v, which is of the

```



```

++ domain \spadtype{TwoDimensionalViewport}.
axes      : ($,PI,STR)                                -> Void
++ axes(v,n,s) displays the axes of the graph in field n of
++ the given two-dimensional viewport, v, which is of domain
++ \spadtype{TwoDimensionalViewport}, if s is "on", or does
++ not display the axes if s is "off".
axes      : ($,PI,PAL)                                -> Void
++ axes(v,n,c) displays the axes of the graph in field n of
++ the given two-dimensional viewport, v, which is of domain
++ \spadtype{TwoDimensionalViewport}, with the axes color set to
++ the given palette color c.
units     : ($,PI,STR)                                -> Void
++ units(v,n,s) displays the units of the graph in field n of
++ the given two-dimensional viewport, v, which is of domain
++ \spadtype{TwoDimensionalViewport}, if s is "on", or does
++ not display the units if s is "off".
units     : ($,PI,PAL)                                -> Void
++ units(v,n,c) displays the units of the graph in field n of
++ the given two-dimensional viewport, v, which is of domain
++ \spadtype{TwoDimensionalViewport}, with the units color set to
++ the given palette color c.
points    : ($,PI,STR)                                -> Void
++ points(v,n,s) displays the points of the graph in field n of
++ the given two-dimensional viewport, v, which is of domain
++ \spadtype{TwoDimensionalViewport}, if s is "on", or does
++ not display the points if s is "off".
region    : ($,PI,STR)                                -> Void
++ region(v,n,s) displays the bounding box of the graph in
++ field n of the given two-dimensional viewport, v, which is
++ of domain \spadtype{TwoDimensionalViewport}, if s is "on",
++ or does not display the bounding box if s is "off".
connect   : ($,PI,STR)                                -> Void
++ connect(v,n,s) displays the lines connecting the graph
++ points in field n of the given two-dimensional viewport, v,
++ which is of domain \spadtype{TwoDimensionalViewport}, if s
++ is "on", or does not display the lines if s is "off".
controlPanel : ($,STR)                                -> Void
++ controlPanel(v,s) displays the control panel of the given
++ two-dimensional viewport, v, which is of domain
++ \spadtype{TwoDimensionalViewport}, if s is "on", or hides
++ the control panel if s is "off".
close     : $                                           -> Void
++ close(v) closes the viewport window of the given
++ two-dimensional viewport, v, which is of domain
++ \spadtype{TwoDimensionalViewport}, and terminates the
++ corresponding process ID.
dimensions : ($,NNI,NNI,PI,PI)                        -> Void
++ dimensions(v,x,y,width,height) sets the position of the
++ upper left-hand corner of the two-dimensional viewport, v,
++ which is of domain \spadtype{TwoDimensionalViewport}, to

```

```

++ the window coordinate x, y, and sets the dimensions of the
++ window to that of \spad{width}, \spad{height}. The new
++ dimensions are not displayed until the function
++ \spadfun{makeViewport2D} is executed again for v.
scale      : ($,PI,F,F)                                -> Void
++ scale(v,n,sx,sy) displays the graph in field n of the given
++ two-dimensional viewport, v, which is of domain
++ \spadtype{TwoDimensionalViewport}, scaled by the factor \spad{sx}
++ in the x-coordinate direction and by the factor \spad{sy} in
++ the y-coordinate direction.
translate  : ($,PI,F,F)                                -> Void
++ translate(v,n,dx,dy) displays the graph in field n of the given
++ two-dimensional viewport, v, which is of domain
++ \spadtype{TwoDimensionalViewport}, translated by \spad{dx} in
++ the x-coordinate direction from the center of the viewport, and
++ by \spad{dy} in the y-coordinate direction from the center.
++ Setting \spad{dx} and \spad{dy} to \spad{0} places the center
++ of the graph at the center of the viewport.
show       : ($,PI,STR)                                -> Void
++ show(v,n,s) displays the graph in field n of the given
++ two-dimensional viewport, v, which is of domain
++ \spadtype{TwoDimensionalViewport}, if s is "on", or does not
++ display the graph if s is "off".
move       : ($,NNI,NNI)                                -> Void
++ move(v,x,y) displays the two-dimensional viewport, v, which
++ is of domain \spadtype{TwoDimensionalViewport}, with the upper
++ left-hand corner of the viewport window at the screen
++ coordinate position x, y.
update     : ($,G,PI)                                  -> Void
++ update(v,gr,n) drops the graph \spad{gr} in slot \spad{n}
++ of viewport \spad{v}. The graph gr must have been
++ transmitted already and acquired an integer key.
resize     : ($,PI,PI)                                  -> Void
++ resize(v,w,h) displays the two-dimensional viewport, v, which
++ is of domain \spadtype{TwoDimensionalViewport}, with a width
++ of w and a height of h, keeping the upper left-hand corner
++ position unchanged.
write      : ($,STR)                                    -> STR
++ write(v,s) takes the given two-dimensional viewport, v, which
++ is of domain \spadtype{TwoDimensionalViewport}, and creates
++ a directory indicated by s, which contains the graph data
++ files for v.
write      : ($,STR,STR)                                -> STR
++ write(v,s,f) takes the given two-dimensional viewport, v, which
++ is of domain \spadtype{TwoDimensionalViewport}, and creates
++ a directory indicated by s, which contains the graph data
++ files for v and an optional file type f.
write      : ($,STR,L STR)                              -> STR
++ write(v,s,lf) takes the given two-dimensional viewport, v, which
++ is of domain \spadtype{TwoDimensionalViewport}, and creates

```

```

++ a directory indicated by s, which contains the graph data
++ files for v and the optional file types indicated by the list lf.
reset      : $                                -> Void
++ reset(v) sets the current state of the graph characteristics
++ of the given two-dimensional viewport, v, which is of domain
++ \spadtype{TwoDimensionalViewport}, back to their initial settings.
key        : $                                -> I
++ key(v) returns the process ID number of the given two-dimensional
++ viewport, v, which is of domain \spadtype{TwoDimensionalViewport}.
coerce     : $                                -> E
++ coerce(v) returns the given two-dimensional viewport, v, which
++ is of domain \spadtype{TwoDimensionalViewport} as output of
++ the domain \spadtype{OutputForm}.

```

Implementation ==> add

```

import GraphImage()
import Color()
import Palette()
import ViewDefaultsPackage()
import DrawOptionFunctions0
import POINT

Rep := Record (key:I, graphsField:V GU, graphStatesField:V GS, _
               title:STR, moveTo:XYNN, size:XP, flags:FLAG, optionsField:L DROP)

defaultGS : GS := [convert(0.9)@SF, convert(0.9)@SF, 0$SF, 0$SF, _
                  yes, yes, no, _
                  yes, axesColorDefault(), no, unitsColorDefault(), _
                  yes]

--% Local Functions
checkViewport (viewport:$):B ==
-- checks to see if this viewport still exists
-- by sending the key to the viewport manager and
-- waiting for its reply after it checks it against
-- the viewports in its list. a -1 means it doesn't
-- exist.
sendI(VIEW,viewport.key)$Lisp
i := getI(VIEW)$Lisp
(i < 0$I) =>
  viewport.key := 0$I
  error "This viewport has already been closed!"
true

doOptions(v:Rep):Void ==
v.title := title(v.optionsField,"AXIOM2D")
-- etc - 2D specific stuff...

```

```

--% Exported Functions

options viewport ==
    viewport.optionsField

options(viewport,opts) ==
    viewport.optionsField := opts
    viewport

putGraph (viewport,aGraph,which) ==
    if ((which > maxGRAPHS) or (which < 1)) then
        error "Trying to put a graph with a negative index or too big an index"
    viewport.graphsField.which := aGraph

getGraph (viewport,which) ==
    if ((which > maxGRAPHS) or (which < 1)) then
        error "Trying to get a graph with a negative index or too big an index"
    viewport.graphsField.which case "undefined" =>
        error "Graph is undefined!"
    viewport.graphsField.which::GraphImage

graphStates viewport == viewport.graphStatesField
graphs viewport == viewport.graphsField
key viewport == viewport.key

dimensions(viewport,ViewX,ViewY,ViewWidth,ViewHeight) ==
    viewport.moveTo := [ViewX,ViewY]
    viewport.size := [ViewWidth,ViewHeight]

move(viewport,xLoc,yLoc) ==
    viewport.moveTo := [xLoc,yLoc]
    (key(viewport) ^= 0$I) =>
        sendI(VIEW,typeVIEW2D)$Lisp
        sendI(VIEW,MOVE)$Lisp
    checkViewport viewport =>
        sendI(VIEW,xLoc)$Lisp
        sendI(VIEW,yLoc)$Lisp
        getI(VIEW)$Lisp -- acknowledge

update(viewport,graph,slot) ==
    (key(viewport) ^= 0$I) =>
        sendI(VIEW,typeVIEW2D)$Lisp
        sendI(VIEW,putGraph2D)$Lisp
    checkViewport viewport =>
        sendI(VIEW,key graph)$Lisp
        sendI(VIEW,slot)$Lisp
        getI(VIEW)$Lisp -- acknowledge

resize(viewport,xSize,ySize) ==

```

```

viewport.size := [xSize,ySize]
(key(viewport) ^= 0$I) =>
  sendI(VIEW,typeVIEW2D)$Lisp
  sendI(VIEW,RESIZE)$Lisp
  checkViewport viewport =>
    sendI(VIEW,xSize)$Lisp
    sendI(VIEW,ySize)$Lisp
    getI(VIEW)$Lisp          -- acknowledge

translate(viewport,graphIndex,xTranslateF,yTranslateF) ==
  xTranslate := convert(xTranslateF)@SF
  yTranslate := convert(yTranslateF)@SF
  if (graphIndex > maxGRAPHS) then
    error "Referring to a graph with too big an index"
  viewport.graphStatesField.graphIndex.deltaX := xTranslate
  viewport.graphStatesField.graphIndex.deltaY := yTranslate
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW2D)$Lisp
    sendI(VIEW,TRANSLATE2D)$Lisp
    checkViewport viewport =>
      sendI(VIEW,graphIndex)$Lisp
      sendSF(VIEW,xTranslate)$Lisp
      sendSF(VIEW,yTranslate)$Lisp
      getI(VIEW)$Lisp        -- acknowledge

scale(viewport,graphIndex,xScaleF,yScaleF) ==
  xScale := convert(xScaleF)@SF
  yScale := convert(yScaleF)@SF
  if (graphIndex > maxGRAPHS) then
    error "Referring to a graph with too big an index"
  viewport.graphStatesField.graphIndex.scaleX := xScale -- check union (undefined?)
  viewport.graphStatesField.graphIndex.scaleY := yScale -- check union (undefined?)
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW2D)$Lisp
    sendI(VIEW,SCALE2D)$Lisp
    checkViewport viewport =>
      sendI(VIEW,graphIndex)$Lisp
      sendSF(VIEW,xScale)$Lisp
      sendSF(VIEW,yScale)$Lisp
      getI(VIEW)$Lisp        -- acknowledge

viewport2D ==
  [0,new(maxGRAPHS,"undefined"), _
  new(maxGRAPHS,copy defaultGS),"AXIOM2D", _
  [viewPosDefault().1,viewPosDefault().2],[viewSizeDefault().1,viewSizeDefault().2], _
  [noControl], [] ]

makeViewport2D(g:G,opts:L DROP) ==
  viewport          := viewport2D()
  viewport.graphsField.1 := g

```

```

viewport.optionsField := opts
makeViewport2D viewport

makeViewport2D viewportDollar ==
  viewport := viewportDollar::Rep
  doOptions viewport --local function to extract and assign optional arguments for 2D viewports
  sayBrightly(["  AXIOM2D data being transmitted to the viewport manager...":E]$List(E))$Lisp
  sendI(VIEW,typeVIEW2D)$Lisp
  sendI(VIEW,makeVIEW2D)$Lisp
  sendSTR(VIEW,viewport.title)$Lisp
  sendI(VIEW,viewport.moveTo.X)$Lisp
  sendI(VIEW,viewport.moveTo.Y)$Lisp
  sendI(VIEW,viewport.size.X)$Lisp
  sendI(VIEW,viewport.size.Y)$Lisp
  sendI(VIEW,viewport.flags.showCP)$Lisp
  for i in 1..maxGRAPHS repeat
    g := (graphs viewport).i
    if g case "undefined" then
      sendI(VIEW,0$I)$Lisp
    else
      sendI(VIEW,key(g::G))$Lisp
      gs := (graphStates viewport).i
      sendSF(VIEW,gs.scaleX)$Lisp
      sendSF(VIEW,gs.scaleY)$Lisp
      sendSF(VIEW,gs.deltaX)$Lisp
      sendSF(VIEW,gs.deltaY)$Lisp
      sendI(VIEW,gs.points)$Lisp
      sendI(VIEW,gs.connect)$Lisp
      sendI(VIEW,gs.spline)$Lisp
      sendI(VIEW,gs.axes)$Lisp
      hueShade := hue hue gs.axesColor + shade gs.axesColor * numberOfHues()
      sendI(VIEW,hueShade)$Lisp
      sendI(VIEW,gs.units)$Lisp
      hueShade := hue hue gs.unitsColor + shade gs.unitsColor * numberOfHues()
      sendI(VIEW,hueShade)$Lisp
      sendI(VIEW,gs.showing)$Lisp
  viewport.key := getI(VIEW)$Lisp
  viewport

graphState(viewport,num,sX,sY,dX,dY,Points,Lines,Spline, _
  Axes,AxesColor,Units,UnitsColor,Showing) ==
  viewport.graphStatesField.num := [sX,sY,dX,dY,Points,Lines,Spline, _
    Axes,AxesColor,Units,UnitsColor,Showing]

title(viewport,Title) ==
  viewport.title := Title
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW2D)$Lisp
    sendI(VIEW,TITLE)$Lisp
    checkViewport viewport =>

```

```

        sendSTR(VIEW,Title)$Lisp
        getI(VIEW)$Lisp          -- acknowledge

reset viewport ==
(key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW2D)$Lisp
    sendI(VIEW,SPADBUTONPRESS)$Lisp
    checkViewport viewport =>
        sendI(VIEW,reset2D)$Lisp
        getI(VIEW)$Lisp          -- acknowledge

axes (viewport:$,graphIndex:PI,onOff:STR) : Void ==
    if (graphIndex > maxGRAPHS) then
        error "Referring to a graph with too big an index"
    if onOff = "on" then
        status := yes
    else
        status := no
    viewport.graphStatesField.graphIndex.axes := status -- check union (undefined?)
    (key(viewport) ^= 0$I) =>
        sendI(VIEW,typeVIEW2D)$Lisp
        sendI(VIEW,axesOnOff2D)$Lisp
        checkViewport viewport =>
            sendI(VIEW,graphIndex)$Lisp
            sendI(VIEW,status)$Lisp
            getI(VIEW)$Lisp          -- acknowledge

axes (viewport:$,graphIndex:PI,color:PAL) : Void ==
    if (graphIndex > maxGRAPHS) then
        error "Referring to a graph with too big an index"
    viewport.graphStatesField.graphIndex.axesColor := color
    (key(viewport) ^= 0$I) =>
        sendI(VIEW,typeVIEW2D)$Lisp
        sendI(VIEW,axesColor2D)$Lisp
        checkViewport viewport =>
            sendI(VIEW,graphIndex)$Lisp
            hueShade := hue hue color + shade color * numberOfHues()
            sendI(VIEW,hueShade)$Lisp
            getI(VIEW)$Lisp          -- acknowledge

units (viewport:$,graphIndex:PI,onOff:STR) : Void ==
    if (graphIndex > maxGRAPHS) then
        error "Referring to a graph with too big an index"
    if onOff = "on" then
        status := yes
    else
        status := no
    viewport.graphStatesField.graphIndex.units := status -- check union (undefined?)
    (key(viewport) ^= 0$I) =>
        sendI(VIEW,typeVIEW2D)$Lisp

```

```

    sendI(VIEW,unitsOnOff2D)$Lisp
    checkViewport viewport =>
        sendI(VIEW,graphIndex)$Lisp
        sendI(VIEW,status)$Lisp
        getI(VIEW)$Lisp          -- acknowledge

units (viewport:$,graphIndex:PI,color:PAL) : Void ==
    if (graphIndex > maxGRAPHS) then
        error "Referring to a graph with too big an index"
    viewport.graphStatesField.graphIndex.unitsColor := color
    (key(viewport) ^= 0$I) =>
        sendI(VIEW,typeVIEW2D)$Lisp
        sendI(VIEW,unitsColor2D)$Lisp
        checkViewport viewport =>
            sendI(VIEW,graphIndex)$Lisp
            hueShade := hue hue color + shade color * numberOfHues()
            sendI(VIEW,hueShade)$Lisp
            getI(VIEW)$Lisp      -- acknowledge

connect (viewport:$,graphIndex:PI,onOff:STR) : Void ==
    if (graphIndex > maxGRAPHS) then
        error "Referring to a graph with too big an index"
    if onOff = "on" then
        status := 1$I
    else
        status := 0$I
    viewport.graphStatesField.graphIndex.connect := status -- check union (undefined?)
    (key(viewport) ^= 0$I) =>
        sendI(VIEW,typeVIEW2D)$Lisp
        sendI(VIEW,connectOnOff)$Lisp
        checkViewport viewport =>
            sendI(VIEW,graphIndex)$Lisp
            sendI(VIEW,status)$Lisp
            getI(VIEW)$Lisp      -- acknowledge

points (viewport:$,graphIndex:PI,onOff:STR) : Void ==
    if (graphIndex > maxGRAPHS) then
        error "Referring to a graph with too big an index"
    if onOff = "on" then
        status := 1$I
    else
        status := 0$I
    viewport.graphStatesField.graphIndex.points := status -- check union (undefined?)
    (key(viewport) ^= 0$I) =>
        sendI(VIEW,typeVIEW2D)$Lisp
        sendI(VIEW,pointsOnOff)$Lisp
        checkViewport viewport =>
            sendI(VIEW,graphIndex)$Lisp
            sendI(VIEW,status)$Lisp
            getI(VIEW)$Lisp      -- acknowledge

```



```

region (viewport:$,graphIndex:PI,onOff:STR) : Void ==
  if (graphIndex > maxGRAPHS) then
    error "Referring to a graph with too big an index"
  if onOff = "on" then
    status := 1$I
  else
    status := 0$I
  viewport.graphStatesField.graphIndex.spline := status -- check union (undefined?)
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW2D)$Lisp
    sendI(VIEW,spline2D)$Lisp
    checkViewport viewport =>
      sendI(VIEW,graphIndex)$Lisp
      sendI(VIEW,status)$Lisp
      getI(VIEW)$Lisp -- acknowledge

show (viewport,graphIndex,onOff) ==
  if (graphIndex > maxGRAPHS) then
    error "Referring to a graph with too big an index"
  if onOff = "on" then
    status := 1$I
  else
    status := 0$I
  viewport.graphStatesField.graphIndex.showing := status -- check union (undefined?)
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW2D)$Lisp
    sendI(VIEW,showing2D)$Lisp
    checkViewport viewport =>
      sendI(VIEW,graphIndex)$Lisp
      sendI(VIEW,status)$Lisp
      getI(VIEW)$Lisp -- acknowledge

controlPanel (viewport,onOff) ==
  if onOff = "on" then viewport.flags.showCP := yes
  else viewport.flags.showCP := no
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW2D)$Lisp
    sendI(VIEW,hideControl2D)$Lisp
    checkViewport viewport =>
      sendI(VIEW,viewport.flags.showCP)$Lisp
      getI(VIEW)$Lisp -- acknowledge

close viewport ==
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW2D)$Lisp
    sendI(VIEW,closeAll2D)$Lisp
    checkViewport viewport =>
      getI(VIEW)$Lisp -- acknowledge
    viewport.key := 0$I

```

```

coerce viewport ==
  (key(viewport) = 0$I) =>
    hconcat ["Closed or Undefined TwoDimensionalViewport: "::E,
              (viewport.title)::E]
    hconcat ["TwoDimensionalViewport: "::E, (viewport.title)::E]

write(viewport:$,Filename:STR,aThingToWrite:STR) ==
  write(viewport,Filename,[aThingToWrite])

write(viewport,Filename) ==
  write(viewport,Filename,viewWriteDefault())

write(viewport:$,Filename:STR,thingsToWrite:L STR) ==
  stringToSend : STR := ""
  (key(viewport) ^= 0$I) =>
    sendI(VIEW,typeVIEW2D)$Lisp
    sendI(VIEW,writeView)$Lisp
    checkViewport viewport =>
      sendSTR(VIEW,Filename)$Lisp
      m := minIndex(avail := viewWriteAvailable())
      for aTypeOfFile in thingsToWrite repeat
        if (writeTypeInt:= position(upperCase aTypeOfFile,avail)-m) < 0 then
          sayBrightly([" > "::E,(concat(aTypeOfFile, _
            " is not a valid file type for writing a 2D viewport"))::E]$List(E))$Lisp
        else
          sendI(VIEW,writeTypeInt+(1$I))$Lisp
          --          stringToSend := concat [stringToSend,"%",aTypeOfFile]
          --          sendSTR(VIEW,stringToSend)$Lisp
      sendI(VIEW,0$I)$Lisp      -- no more types of things to write
      getI(VIEW)$Lisp          -- acknowledge
      Filename

```

---

— VIEW2D.dotabb —

```

"VIEW2D" [color="#88FF44",href="bookvol10.3.pdf#nameddest=VIEW2D"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"VIEW2D" -> "STRING"

```

---



## Chapter 22

## Chapter U

### 22.1 domain UFPS UnivariateFormalPowerSeries

— UnivariateFormalPowerSeries.input —

```
)set break resume
)sys rm -f UnivariateFormalPowerSeries.output
)spool UnivariateFormalPowerSeries.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show UnivariateFormalPowerSeries
--R UnivariateFormalPowerSeries Coef: Ring is a domain constructor
--R Abbreviation for UnivariateFormalPowerSeries is UFPS
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for UFPS
--R
--R----- Operations -----
--R ??? : (Coef,%) -> %          ??? : (%,Coef) -> %
--R ??? : (%,%) -> %            ??? : (Integer,%) -> %
--R ??? : (PositiveInteger,%) -> %  ??? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %            ?-? : (%,%) -> %
--R -? : % -> %                  ?? : (%,%) -> Boolean
--R 1 : () -> %                  0 : () -> %
--R ?? : (%,PositiveInteger) -> % center : % -> Coef
--R coefficients : % -> Stream Coef  coerce : Variable QUOTE x -> %
--R coerce : Integer -> %            coerce : % -> OutputForm
--R complete : % -> %                degree : % -> NonNegativeInteger
--R evenlambert : % -> %              hash : % -> SingleInteger
--R lagrange : % -> %                lambert : % -> %
```

```

--R latex : % -> String
--R leadingMonomial : % -> %
--R monomial? : % -> Boolean
--R one? : % -> Boolean
--R pole? : % -> Boolean
--R recip : % -> Union(%, "failed")
--R revert : % -> %
--R series : Stream Coef -> %
--R zero? : % -> Boolean
--R ?? : (%, Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?? : (Fraction Integer, %) -> % if Coef has ALGEBRA FRAC INT
--R ?? : (NonNegativeInteger, %) -> %
--R ??? : (%, Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ??? : (%, %) -> % if Coef has ALGEBRA FRAC INT
--R ??? : (%, Coef) -> % if Coef has FIELD
--R ??? : (%, NonNegativeInteger) -> %
--R ?/? : (%, Coef) -> % if Coef has FIELD
--R D : % -> % if Coef has *: (NonNegativeInteger, Coef) -> Coef
--R D : (%, NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger, Coef) -> Coef
--R D : (%, Symbol) -> % if Coef has *: (NonNegativeInteger, Coef) -> Coef and Coef has PDRING
--R D : (%, List Symbol) -> % if Coef has *: (NonNegativeInteger, Coef) -> Coef and Coef has P
--R D : (%, Symbol, NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger, Coef) -> Coef a
--R D : (%, List Symbol, List NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger, Coef)
--R ?? : (%, NonNegativeInteger) -> %
--R acos : % -> % if Coef has ALGEBRA FRAC INT
--R acosh : % -> % if Coef has ALGEBRA FRAC INT
--R acot : % -> % if Coef has ALGEBRA FRAC INT
--R acoth : % -> % if Coef has ALGEBRA FRAC INT
--R acsc : % -> % if Coef has ALGEBRA FRAC INT
--R acsch : % -> % if Coef has ALGEBRA FRAC INT
--R approximate : (%, NonNegativeInteger) -> Coef if Coef has **: (Coef, NonNegativeInteger) ->
--R asec : % -> % if Coef has ALGEBRA FRAC INT
--R asech : % -> % if Coef has ALGEBRA FRAC INT
--R asin : % -> % if Coef has ALGEBRA FRAC INT
--R asinh : % -> % if Coef has ALGEBRA FRAC INT
--R associates? : (%, %) -> Boolean if Coef has INTDOM
--R atan : % -> % if Coef has ALGEBRA FRAC INT
--R atanh : % -> % if Coef has ALGEBRA FRAC INT
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%, "failed") if Coef has CHARNZ
--R coefficient : (%, NonNegativeInteger) -> Coef
--R coerce : UnivariatePolynomial(QUOTE x, Coef) -> %
--R coerce : Coef -> % if Coef has COMRING
--R coerce : % -> % if Coef has INTDOM
--R coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
--R cos : % -> % if Coef has ALGEBRA FRAC INT
--R cosh : % -> % if Coef has ALGEBRA FRAC INT
--R cot : % -> % if Coef has ALGEBRA FRAC INT
--R coth : % -> % if Coef has ALGEBRA FRAC INT
--R csc : % -> % if Coef has ALGEBRA FRAC INT

```

```

--R csch : % -> % if Coef has ALGEBRA FRAC INT
--R differentiate : (% , Variable QUOTE x) -> %
--R differentiate : % -> % if Coef has *: (NonNegativeInteger, Coef) -> Coef
--R differentiate : (% , NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger, Coef) -> Coef
--R differentiate : (% , Symbol) -> % if Coef has *: (NonNegativeInteger, Coef) -> Coef and Coef has PDRING
--R differentiate : (% , List Symbol) -> % if Coef has *: (NonNegativeInteger, Coef) -> Coef and Coef has P
--R differentiate : (% , Symbol, NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger, Coef) -> Coef
--R differentiate : (% , List Symbol, List NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger, Coef)
--R ?? : (% , %) -> % if NonNegativeInteger has SGROUP
--R ?? : (% , NonNegativeInteger) -> Coef
--R eval : (% , Coef) -> Stream Coef if Coef has **: (Coef, NonNegativeInteger) -> Coef
--R exp : % -> % if Coef has ALGEBRA FRAC INT
--R exquo : (% , %) -> Union(%, "failed") if Coef has INTDOM
--R extend : (% , NonNegativeInteger) -> %
--R generalLambert : (% , Integer, Integer) -> %
--R integrate : (% , Variable QUOTE x) -> % if Coef has ALGEBRA FRAC INT
--R integrate : (% , Symbol) -> % if Coef has integrate: (Coef, Symbol) -> Coef and Coef has variables: Coef
--R integrate : % -> % if Coef has ALGEBRA FRAC INT
--R invmultisect : (Integer, Integer, %) -> %
--R log : % -> % if Coef has ALGEBRA FRAC INT
--R monomial : (% , List SingletonAsOrderedSet, List NonNegativeInteger) -> %
--R monomial : (% , SingletonAsOrderedSet, NonNegativeInteger) -> %
--R monomial : (Coef, NonNegativeInteger) -> %
--R multiplyCoefficients : ((Integer -> Coef), %) -> %
--R multiplyExponents : (% , PositiveInteger) -> %
--R multisect : (Integer, Integer, %) -> %
--R nthRoot : (% , Integer) -> % if Coef has ALGEBRA FRAC INT
--R order : (% , NonNegativeInteger) -> NonNegativeInteger
--R pi : () -> % if Coef has ALGEBRA FRAC INT
--R polynomial : (% , NonNegativeInteger, NonNegativeInteger) -> Polynomial Coef
--R polynomial : (% , NonNegativeInteger) -> Polynomial Coef
--R sec : % -> % if Coef has ALGEBRA FRAC INT
--R sech : % -> % if Coef has ALGEBRA FRAC INT
--R series : Stream Record(k: NonNegativeInteger, c: Coef) -> %
--R sin : % -> % if Coef has ALGEBRA FRAC INT
--R sinh : % -> % if Coef has ALGEBRA FRAC INT
--R sqrt : % -> % if Coef has ALGEBRA FRAC INT
--R subtractIfCan : (% , %) -> Union(%, "failed")
--R tan : % -> % if Coef has ALGEBRA FRAC INT
--R tanh : % -> % if Coef has ALGEBRA FRAC INT
--R terms : % -> Stream Record(k: NonNegativeInteger, c: Coef)
--R truncate : (% , NonNegativeInteger, NonNegativeInteger) -> %
--R truncate : (% , NonNegativeInteger) -> %
--R unit? : % -> Boolean if Coef has INTDOM
--R unitCanonical : % -> % if Coef has INTDOM
--R unitNormal : % -> Record(unit: %, canonical: %) if Coef has INTDOM
--R univariatePolynomial : (% , NonNegativeInteger) -> UnivariatePolynomial(QUOTE x, Coef)
--R variables : % -> List SingletonAsOrderedSet
--R
--E 1

```

```
)spool
)lisp (bye)
```

---

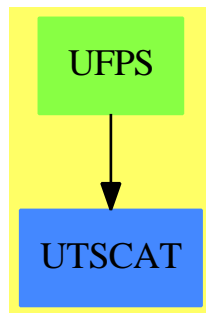
— **UnivariateFormalPowerSeries.help** —

```
=====
UnivariateFormalPowerSeries examples
=====
```

```
See Also:
o )show UnivariateFormalPowerSeries
```

---

### 22.1.1 UnivariateFormalPowerSeries (UFPS)



**Exports:**

|                    |                 |                      |                      |
|--------------------|-----------------|----------------------|----------------------|
| 0                  | 1               | acos                 | acosh                |
| acot               | acoth           | acsc                 | acsch                |
| approximate        | asec            | asech                | asin                 |
| asinh              | associates?     | atan                 | atanh                |
| center             | characteristic  | charthRoot           | coefficient          |
| coefficients       | coerce          | complete             | cos                  |
| cosh               | cot             | coth                 | csc                  |
| csch               | D               | degree               | differentiate        |
| eval               | evenlambert     | exp                  | exquo                |
| extend             | generalLambert  | hash                 | integrate            |
| invmultisect       | lagrange        | lambert              | latex                |
| leadingCoefficient | leadingMonomial | log                  | map                  |
| monomial           | monomial?       | multiplyCoefficients | multiplyExponents    |
| multisect          | nthRoot         | oddlambert           | one?                 |
| order              | pi              | pole?                | polynomial           |
| quoByVar           | recip           | reductum             | revert               |
| sample             | sec             | sech                 | series               |
| sin                | sinh            | sqrt                 | subtractIfCan        |
| tan                | tanh            | terms                | truncate             |
| unit?              | unitCanonical   | unitNormal           | univariatePolynomial |
| variable           | variables       | zero?                | ?*?                  |
| ?**?               | ?+?             | ?-?                  | -?                   |
| ?=?                | ?^?             | ?~=?                 | ?/?                  |
| ?..?               |                 |                      |                      |

— domain UFPS UnivariateFormalPowerSeries —

```
)abbrev domain UFPS UnivariateFormalPowerSeries
++ Author: Mark Botch
++ Description:
++ This domain has no description
```

```
UnivariateFormalPowerSeries(Coef: Ring) ==
  UnivariateTaylorSeries(Coef, 'x, 0$Coef)
```

\_\_\_\_\_

— UFPS.dotabb —

```
"UFPS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=UFPS"]
"UTSCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=UTSCAT"]
"UFPS" -> "UTSCAT"
```

\_\_\_\_\_



## 22.2 domain ULS UnivariateLaurentSeries

— UnivariateLaurentSeries.input —

```

)set break resume
)sys rm -f UnivariateLaurentSeries.output
)spool UnivariateLaurentSeries.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show UnivariateLaurentSeries
--R UnivariateLaurentSeries(Coef: Ring,var: Symbol,cen: Coef) is a domain constructor
--R Abbreviation for UnivariateLaurentSeries is ULS
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ULS
--R
--R----- Operations -----
--R ?? : (Coef,%) -> %                ?? : (%,Coef) -> %
--R ?? : (%,%) -> %                  ?? : (Integer,%) -> %
--R ?? : (PositiveInteger,%) -> %    ***? : (%,PositiveInteger) -> %
--R ?? : (%,%) -> %                  ?-? : (%,%) -> %
--R -? : % -> %                      ?=? : (%,%) -> Boolean
--R 1 : () -> %                      0 : () -> %
--R ?? : (%,PositiveInteger) -> %    center : % -> Coef
--R coefficient : (%,Integer) -> Coef coerce : Variable var -> %
--R coerce : Integer -> %             coerce : % -> OutputForm
--R complete : % -> %                 degree : % -> Integer
--R ?.? : (%,Integer) -> Coef         extend : (%,Integer) -> %
--R hash : % -> SingleInteger         latex : % -> String
--R leadingCoefficient : % -> Coef     leadingMonomial : % -> %
--R map : ((Coef -> Coef),%) -> %      monomial : (Coef,Integer) -> %
--R monomial? : % -> Boolean           one? : % -> Boolean
--R order : (%,Integer) -> Integer     order : % -> Integer
--R pole? : % -> Boolean               recip : % -> Union(%, "failed")
--R reductum : % -> %                  removeZeroes : (Integer,%) -> %
--R removeZeroes : % -> %              sample : () -> %
--R truncate : (%,Integer) -> %        variable : % -> Symbol
--R zero? : % -> Boolean               ?~=? : (%,%) -> Boolean
--R ?? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
--R ?? : (UnivariateTaylorSeries(Coef,var,cen),%) -> % if Coef has FIELD
--R ?? : (%,UnivariateTaylorSeries(Coef,var,cen)) -> % if Coef has FIELD
--R ?? : (NonNegativeInteger,%) -> %
--R ***? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ***? : (%,%) -> % if Coef has ALGEBRA FRAC INT
--R ***? : (%,Integer) -> % if Coef has FIELD

```

```

--R ??? : (% , NonNegativeInteger) -> %
--R ?/? : (UnivariateTaylorSeries(Coef, var, cen), UnivariateTaylorSeries(Coef, var, cen)) -> % if Coef has FIELD
--R ?/? : (% , %) -> % if Coef has FIELD
--R ?/? : (% , Coef) -> % if Coef has FIELD
--R ?<? : (% , %) -> Boolean if UnivariateTaylorSeries(Coef, var, cen) has OINTDOM and Coef has FIELD or UnivariateTaylorSeries(Coef, var, cen) has OINTDOM and Coef has FIELD
--R ?<=? : (% , %) -> Boolean if UnivariateTaylorSeries(Coef, var, cen) has OINTDOM and Coef has FIELD or UnivariateTaylorSeries(Coef, var, cen) has OINTDOM and Coef has FIELD
--R ?>? : (% , %) -> Boolean if UnivariateTaylorSeries(Coef, var, cen) has OINTDOM and Coef has FIELD or UnivariateTaylorSeries(Coef, var, cen) has OINTDOM and Coef has FIELD
--R ?>=? : (% , %) -> Boolean if UnivariateTaylorSeries(Coef, var, cen) has OINTDOM and Coef has FIELD or UnivariateTaylorSeries(Coef, var, cen) has OINTDOM and Coef has FIELD
--R D : (% , Symbol) -> % if UnivariateTaylorSeries(Coef, var, cen) has PDRING SYMBOL and Coef has FIELD or UnivariateTaylorSeries(Coef, var, cen) has PDRING SYMBOL and Coef has FIELD
--R D : (% , List Symbol) -> % if UnivariateTaylorSeries(Coef, var, cen) has PDRING SYMBOL and Coef has FIELD or UnivariateTaylorSeries(Coef, var, cen) has PDRING SYMBOL and Coef has FIELD
--R D : (% , Symbol, NonNegativeInteger) -> % if UnivariateTaylorSeries(Coef, var, cen) has PDRING SYMBOL and Coef has FIELD or UnivariateTaylorSeries(Coef, var, cen) has PDRING SYMBOL and Coef has FIELD
--R D : (% , List Symbol, List NonNegativeInteger) -> % if UnivariateTaylorSeries(Coef, var, cen) has PDRING SYMBOL and Coef has FIELD or UnivariateTaylorSeries(Coef, var, cen) has PDRING SYMBOL and Coef has FIELD
--R D : (% , %) -> % if UnivariateTaylorSeries(Coef, var, cen) has DIFRING and Coef has FIELD or Coef has *: (UnivariateTaylorSeries(Coef, var, cen) has DIFRING and Coef has FIELD or Coef has *: (UnivariateTaylorSeries(Coef, var, cen) has DIFRING and Coef has FIELD))
--R D : (% , NonNegativeInteger) -> % if UnivariateTaylorSeries(Coef, var, cen) has DIFRING and Coef has FIELD or Coef has *: (UnivariateTaylorSeries(Coef, var, cen) has DIFRING and Coef has FIELD)
--R D : (% , (UnivariateTaylorSeries(Coef, var, cen) -> UnivariateTaylorSeries(Coef, var, cen)), NonNegativeInteger) -> % if UnivariateTaylorSeries(Coef, var, cen) has DIFRING and Coef has FIELD or Coef has *: (UnivariateTaylorSeries(Coef, var, cen) has DIFRING and Coef has FIELD)
--R D : (% , (UnivariateTaylorSeries(Coef, var, cen) -> UnivariateTaylorSeries(Coef, var, cen))) -> % if Coef has FIELD
--R ?? : (% , Integer) -> % if Coef has FIELD
--R ?? : (% , NonNegativeInteger) -> %
--R abs : % -> % if UnivariateTaylorSeries(Coef, var, cen) has OINTDOM and Coef has FIELD
--R acos : % -> % if Coef has ALGEBRA FRAC INT
--R acosh : % -> % if Coef has ALGEBRA FRAC INT
--R acot : % -> % if Coef has ALGEBRA FRAC INT
--R acoth : % -> % if Coef has ALGEBRA FRAC INT
--R acsc : % -> % if Coef has ALGEBRA FRAC INT
--R acsch : % -> % if Coef has ALGEBRA FRAC INT
--R approximate : (% , Integer) -> Coef if Coef has **: (Coef, Integer) -> Coef and Coef has coerce: Symbol
--R asec : % -> % if Coef has ALGEBRA FRAC INT
--R asech : % -> % if Coef has ALGEBRA FRAC INT
--R asin : % -> % if Coef has ALGEBRA FRAC INT
--R asinh : % -> % if Coef has ALGEBRA FRAC INT
--R associates? : (% , %) -> Boolean if UnivariateTaylorSeries(Coef, var, cen) has OINTDOM and Coef has FIELD or UnivariateTaylorSeries(Coef, var, cen) has OINTDOM and Coef has FIELD
--R atan : % -> % if Coef has ALGEBRA FRAC INT
--R atanh : % -> % if Coef has ALGEBRA FRAC INT
--R ceiling : % -> UnivariateTaylorSeries(Coef, var, cen) if UnivariateTaylorSeries(Coef, var, cen) has INS
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%, "failed") if $ has CHARNZ and UnivariateTaylorSeries(Coef, var, cen) has PFE
--R coerce : Fraction Integer -> % if UnivariateTaylorSeries(Coef, var, cen) has RETRACT INT and Coef has FIELD
--R coerce : % -> % if UnivariateTaylorSeries(Coef, var, cen) has OINTDOM and Coef has FIELD or UnivariateTaylorSeries(Coef, var, cen) has OINTDOM and Coef has FIELD
--R coerce : Symbol -> % if UnivariateTaylorSeries(Coef, var, cen) has RETRACT SYMBOL and Coef has FIELD
--R coerce : UnivariateTaylorSeries(Coef, var, cen) -> %
--R coerce : Coef -> % if Coef has COMRING
--R conditionP : Matrix % -> Union(Vector %, "failed") if $ has CHARNZ and UnivariateTaylorSeries(Coef, var, cen) has PFE
--R convert : % -> Pattern Integer if UnivariateTaylorSeries(Coef, var, cen) has KONVERT PATTERN INT and Coef has FIELD
--R convert : % -> Pattern Float if UnivariateTaylorSeries(Coef, var, cen) has KONVERT PATTERN FLOAT and Coef has FIELD
--R convert : % -> DoubleFloat if UnivariateTaylorSeries(Coef, var, cen) has REAL and Coef has FIELD
--R convert : % -> Float if UnivariateTaylorSeries(Coef, var, cen) has REAL and Coef has FIELD
--R convert : % -> InputForm if UnivariateTaylorSeries(Coef, var, cen) has KONVERT INFORM and Coef has FIELD
--R cos : % -> % if Coef has ALGEBRA FRAC INT
--R cosh : % -> % if Coef has ALGEBRA FRAC INT
--R cot : % -> % if Coef has ALGEBRA FRAC INT

```

```

--R coth : % -> % if Coef has ALGEBRA FRAC INT
--R csc : % -> % if Coef has ALGEBRA FRAC INT
--R csch : % -> % if Coef has ALGEBRA FRAC INT
--R denom : % -> UnivariateTaylorSeries(Coef,var,cen) if Coef has FIELD
--R denominator : % -> % if Coef has FIELD
--R differentiate : (%,Symbol) -> % if UnivariateTaylorSeries(Coef,var,cen) has PDRING SYMBOL
--R differentiate : (%,List Symbol) -> % if UnivariateTaylorSeries(Coef,var,cen) has PDRING S
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if UnivariateTaylorSeries(Coef,var,cen)
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if UnivariateTaylorSeries(Coef,var,cen)
--R differentiate : % -> % if UnivariateTaylorSeries(Coef,var,cen) has DIFRING and Coef has L
--R differentiate : (%,NonNegativeInteger) -> % if UnivariateTaylorSeries(Coef,var,cen) has L
--R differentiate : (%,Variable var) -> %
--R differentiate : (%,(UnivariateTaylorSeries(Coef,var,cen) -> UnivariateTaylorSeries(Coef,var,cen))
--R differentiate : (%,(UnivariateTaylorSeries(Coef,var,cen) -> UnivariateTaylorSeries(Coef,var,cen))
--R divide : (%,%) -> Record(quotient: %,remainder: %) if Coef has FIELD
--R ?.? : (%,UnivariateTaylorSeries(Coef,var,cen)) -> % if UnivariateTaylorSeries(Coef,var,cen)
--R ?.? : (%,%) -> % if Integer has SGROUP
--R euclideanSize : % -> NonNegativeInteger if Coef has FIELD
--R eval : (%,List UnivariateTaylorSeries(Coef,var,cen),List UnivariateTaylorSeries(Coef,var,cen)) -> %
--R eval : (%,UnivariateTaylorSeries(Coef,var,cen),UnivariateTaylorSeries(Coef,var,cen)) -> %
--R eval : (%,Equation UnivariateTaylorSeries(Coef,var,cen)) -> % if UnivariateTaylorSeries(Coef,var,cen)
--R eval : (%,List Equation UnivariateTaylorSeries(Coef,var,cen)) -> % if UnivariateTaylorSeries(Coef,var,cen)
--R eval : (%,List Symbol,List UnivariateTaylorSeries(Coef,var,cen)) -> % if UnivariateTaylorSeries(Coef,var,cen)
--R eval : (%,Symbol,UnivariateTaylorSeries(Coef,var,cen)) -> % if UnivariateTaylorSeries(Coef,var,cen)
--R eval : (%,Coef) -> Stream Coef if Coef has **: (Coef,Integer) -> Coef
--R exp : % -> % if Coef has ALGEBRA FRAC INT
--R expressIdealMember : (List %,%) -> Union(List %,"failed") if Coef has FIELD
--R exquo : (%,%) -> Union(%,"failed") if UnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef has FIELD
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if Coef has FIELD
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if Coef has FIELD
--R factor : % -> Factored % if Coef has FIELD
--R factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial %
--R factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial %
--R floor : % -> UnivariateTaylorSeries(Coef,var,cen) if UnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef has FIELD
--R fractionPart : % -> % if UnivariateTaylorSeries(Coef,var,cen) has EUCDOM and Coef has FIELD
--R gcd : (%,%) -> % if Coef has FIELD
--R gcd : List % -> % if Coef has FIELD
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolynomial %
--R init : () -> % if UnivariateTaylorSeries(Coef,var,cen) has STEP and Coef has FIELD
--R integrate : (%,Variable var) -> % if Coef has ALGEBRA FRAC INT
--R integrate : (%,Symbol) -> % if Coef has integrate: (Coef,Symbol) -> Coef and Coef has FIELD
--R integrate : % -> % if Coef has ALGEBRA FRAC INT
--R inv : % -> % if Coef has FIELD
--R laurent : (Integer,UnivariateTaylorSeries(Coef,var,cen)) -> %
--R lcm : (%,%) -> % if Coef has FIELD
--R lcm : List % -> % if Coef has FIELD
--R log : % -> % if Coef has ALGEBRA FRAC INT
--R map : ((UnivariateTaylorSeries(Coef,var,cen) -> UnivariateTaylorSeries(Coef,var,cen)),%) -> %
--R max : (%,%) -> % if UnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef has FIELD
--R min : (%,%) -> % if UnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef has FIELD

```

```

--R monomial : (% , List SingletonAsOrderedSet, List Integer) -> %
--R monomial : (% , SingletonAsOrderedSet, Integer) -> %
--R multiEuclidean : (List % , %) -> Union(List % , "failed") if Coef has FIELD
--R multiplyCoefficients : ((Integer -> Coef), %) -> %
--R multiplyExponents : (% , PositiveInteger) -> %
--R negative? : % -> Boolean if UnivariateTaylorSeries(Coef, var, cen) has OINTDOM and Coef has FIELD
--R nextItem : % -> Union(% , "failed") if UnivariateTaylorSeries(Coef, var, cen) has STEP and Coef has FIELD
--R nthRoot : (% , Integer) -> % if Coef has ALGEBRA FRAC INT
--R numer : % -> UnivariateTaylorSeries(Coef, var, cen) if Coef has FIELD
--R numerator : % -> % if Coef has FIELD
--R patternMatch : (% , Pattern Float, PatternMatchResult(Float, %)) -> PatternMatchResult(Float, %) if UnivariateTaylorSeries(Coef, var, cen) has STEP and Coef has FIELD
--R patternMatch : (% , Pattern Integer, PatternMatchResult(Integer, %)) -> PatternMatchResult(Integer, %) if UnivariateTaylorSeries(Coef, var, cen) has STEP and Coef has FIELD
--R pi : () -> % if Coef has ALGEBRA FRAC INT
--R positive? : % -> Boolean if UnivariateTaylorSeries(Coef, var, cen) has OINTDOM and Coef has FIELD
--R prime? : % -> Boolean if Coef has FIELD
--R principalIdeal : List % -> Record(coef: List % , generator: %) if Coef has FIELD
--R ?quo? : (% , %) -> % if Coef has FIELD
--R random : () -> % if UnivariateTaylorSeries(Coef, var, cen) has INS and Coef has FIELD
--R rationalFunction : (% , Integer, Integer) -> Fraction Polynomial Coef if Coef has INTDOM
--R rationalFunction : (% , Integer) -> Fraction Polynomial Coef if Coef has INTDOM
--R reducedSystem : Matrix % -> Matrix Integer if UnivariateTaylorSeries(Coef, var, cen) has LINEXP INT and Coef has FIELD
--R reducedSystem : (Matrix % , Vector %) -> Record(mat: Matrix Integer, vec: Vector Integer) if UnivariateTaylorSeries(Coef, var, cen) has LINEXP INT and Coef has FIELD
--R reducedSystem : (Matrix % , Vector %) -> Record(mat: Matrix UnivariateTaylorSeries(Coef, var, cen), vec: Vector UnivariateTaylorSeries(Coef, var, cen)) if UnivariateTaylorSeries(Coef, var, cen) has LINEXP INT and Coef has FIELD
--R reducedSystem : Matrix % -> Matrix UnivariateTaylorSeries(Coef, var, cen) if Coef has FIELD
--R ?rem? : (% , %) -> % if Coef has FIELD
--R retract : % -> Integer if UnivariateTaylorSeries(Coef, var, cen) has RETRACT INT and Coef has FIELD
--R retract : % -> Fraction Integer if UnivariateTaylorSeries(Coef, var, cen) has RETRACT INT and Coef has FIELD
--R retract : % -> Symbol if UnivariateTaylorSeries(Coef, var, cen) has RETRACT SYMBOL and Coef has FIELD
--R retract : % -> UnivariateTaylorSeries(Coef, var, cen)
--R retractIfCan : % -> Union(Integer, "failed") if UnivariateTaylorSeries(Coef, var, cen) has RETRACT INT and Coef has FIELD
--R retractIfCan : % -> Union(Fraction Integer, "failed") if UnivariateTaylorSeries(Coef, var, cen) has RETRACT INT and Coef has FIELD
--R retractIfCan : % -> Union(Symbol, "failed") if UnivariateTaylorSeries(Coef, var, cen) has RETRACT SYMBOL and Coef has FIELD
--R retractIfCan : % -> Union(UnivariateTaylorSeries(Coef, var, cen), "failed")
--R sec : % -> % if Coef has ALGEBRA FRAC INT
--R sech : % -> % if Coef has ALGEBRA FRAC INT
--R series : Stream Record(k: Integer, c: Coef) -> %
--R sign : % -> Integer if UnivariateTaylorSeries(Coef, var, cen) has OINTDOM and Coef has FIELD
--R sin : % -> % if Coef has ALGEBRA FRAC INT
--R sinh : % -> % if Coef has ALGEBRA FRAC INT
--R sizeLess? : (% , %) -> Boolean if Coef has FIELD
--R solveLinearPolynomialEquation : (List SparseUnivariatePolynomial % , SparseUnivariatePolynomial %) -> %
--R sqrt : % -> % if Coef has ALGEBRA FRAC INT
--R squareFree : % -> Factored % if Coef has FIELD
--R squareFreePart : % -> % if Coef has FIELD
--R squareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % if UnivariateTaylorSeries(Coef, var, cen) has FIELD
--R subtractIfCan : (% , %) -> Union(% , "failed")
--R tan : % -> % if Coef has ALGEBRA FRAC INT
--R tanh : % -> % if Coef has ALGEBRA FRAC INT
--R taylor : % -> UnivariateTaylorSeries(Coef, var, cen)
--R taylorIfCan : % -> Union(UnivariateTaylorSeries(Coef, var, cen), "failed")

```

```

--R taylorRep : % -> UnivariateTaylorSeries(Coef,var,cen)
--R terms : % -> Stream Record(k: Integer,c: Coef)
--R truncate : (%,Integer,Integer) -> %
--R unit? : % -> Boolean if UnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef has FI
--R unitCanonical : % -> % if UnivariateTaylorSeries(Coef,var,cen) has OINTDOM and Coef has I
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if UnivariateTaylorSeries(Co
--R variables : % -> List SingletonAsOrderedSet
--R wholePart : % -> UnivariateTaylorSeries(Coef,var,cen) if UnivariateTaylorSeries(Coef,var
--R
--E 1

)spool
)lisp (bye)

```

---

— UnivariateLaurentSeries.help —

```

=====
UnivariateLaurentSeries examples
=====

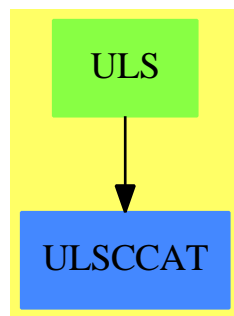
```

```

See Also:
o )show UnivariateLaurentSeries

```

### 22.2.1 UnivariateLaurentSeries (ULS)



See

⇒ “UnivariateLaurentSeriesConstructor” (ULSCONS) 22.3.1 on page 2760

**Exports:**

|                            |                               |                      |
|----------------------------|-------------------------------|----------------------|
| 0                          | 1                             | abs                  |
| acos                       | acosh                         | acot                 |
| acoth                      | acsc                          | acsch                |
| approximate                | asec                          | asech                |
| asin                       | asinh                         | associates?          |
| atan                       | atanh                         | ceiling              |
| center                     | characteristic                | charthRoot           |
| coerce                     | coefficient                   | coerce               |
| complete                   | conditionP                    | convert              |
| cos                        | cosh                          | cot                  |
| coth                       | csc                           | csch                 |
| D                          | degree                        | denom                |
| denominator                | differentiate                 | divide               |
| euclideanSize              | eval                          | exp                  |
| expressIdealMember         | exquo                         | extend               |
| extendedEuclidean          | factor                        | factorPolynomial     |
| factorSquareFreePolynomial | floor                         | fractionPart         |
| gcd                        | gcdPolynomial                 | hash                 |
| init                       | integrate                     | inv                  |
| latex                      | laurent                       | leadingCoefficient   |
| leadingMonomial            | lcm                           | log                  |
| map                        | max                           | min                  |
| monomial                   | monomial?                     | multiEuclidean       |
| multiplyCoefficients       | multiplyExponents             | negative?            |
| nextItem                   | nthRoot                       | numer                |
| numerator                  | one?                          | order                |
| patternMatch               | pi                            | pole?                |
| positive?                  | prime?                        | principalIdeal       |
| random                     | rationalFunction              | recip                |
| reducedSystem              | reductum                      | removeZeroes         |
| retract                    | retractIfCan                  | sample               |
| sec                        | sech                          | series               |
| sign                       | sin                           | sinh                 |
| sizeLess?                  | solveLinearPolynomialEquation | sqrt                 |
| squareFree                 | squareFreePart                | squareFreePolynomial |
| subtractIfCan              | tan                           | tanh                 |
| taylor                     | taylorIfCan                   | taylorRep            |
| terms                      | truncate                      | unit?                |
| unitCanonical              | unitNormal                    | variable             |
| variables                  | wholePart                     | zero?                |
| ?*?                        | ?**?                          | ?+?                  |
| ?-?                        | -?                            | ?=?                  |
| ?^?                        | ?..?                          | ?~=?                 |
| ?/?                        | ?<?                           | ?<=?                 |
| ?>?                        | ?>=?                          | ?^?                  |
| ?..?                       | ?quo?                         | ?rem?                |

## — domain ULS UnivariateLaurentSeries —

```

)abbrev domain ULS UnivariateLaurentSeries
++ Author: Clifton J. Williamson
++ Date Created: 18 January 1990
++ Date Last Updated: 21 September 1993
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: series, Laurent
++ Examples:
++ References:
++ Description:
++ Dense Laurent series in one variable
++ \spadtype{UnivariateLaurentSeries} is a domain representing Laurent
++ series in one variable with coefficients in an arbitrary ring. The
++ parameters of the type specify the coefficient ring, the power series
++ variable, and the center of the power series expansion. For example,
++ \spad{UnivariateLaurentSeries(Integer,x,3)} represents Laurent series in
++ \spad{(x - 3)} with integer coefficients.

UnivariateLaurentSeries(Coef,var,cen): Exports == Implementation where
  Coef : Ring
  var  : Symbol
  cen  : Coef
  I    ==> Integer
  UTS  ==> UnivariateTaylorSeries(Coef,var,cen)

Exports ==> UnivariateLaurentSeriesConstructorCategory(Coef,UTS) with
  coerce: Variable(var) -> %
    ++ \spad{coerce(var)} converts the series variable \spad{var} into a
    ++ Laurent series.
  differentiate: (% ,Variable(var)) -> %
    ++ \spad{differentiate(f(x),x)} returns the derivative of
    ++ \spad{f(x)} with respect to \spad{x}.
  if Coef has Algebra Fraction Integer then
    integrate: (% ,Variable(var)) -> %
      ++ \spad{integrate(f(x))} returns an anti-derivative of the power
      ++ series \spad{f(x)} with constant coefficient 0.
      ++ We may integrate a series when we can divide coefficients
      ++ by integers.

Implementation ==> UnivariateLaurentSeriesConstructor(Coef,UTS) add

  variable x == var
  center x == cen

  coerce(v:Variable(var)) ==

```

```

zero? cen => monomial(1,1)
monomial(1,1) + monomial(cen,0)

differentiate(x:%,v:Variable(var)) == differentiate x

if Coef has Algebra Fraction Integer then
  integrate(x:%,v:Variable(var)) == integrate x

```

---

— ULS.dotabb —

```

"ULS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ULS"]
"ULSCCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ULSCCAT"]
"ULS" -> "ULSCCAT"

```

---

## 22.3 domain ULSCONS UnivariateLaurentSeriesConstructor

— UnivariateLaurentSeriesConstructor.input —

```

)set break resume
)sys rm -f UnivariateLaurentSeriesConstructor.output
)spool UnivariateLaurentSeriesConstructor.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show UnivariateLaurentSeriesConstructor
--R UnivariateLaurentSeriesConstructor(Coef: Ring,UTS: UnivariateTaylorSeriesCategory Coef) is a domain
--R Abbreviation for UnivariateLaurentSeriesConstructor is ULSCONS
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for ULSCONS
--R
--R----- Operations -----
--R ??? : (Coef,%) -> %          ??? : (%,Coef) -> %
--R ??? : (%,%) -> %           ??? : (Integer,%) -> %
--R ??? : (PositiveInteger,%) -> %   ??? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %           ?-? : (%,%) -> %
--R -? : % -> %                ?=? : (%,%) -> Boolean

```



```

--R 1 : () -> %
--R ?? : (%,PositiveInteger) -> %
--R coefficient : (%,Integer) -> Coef
--R coerce : Integer -> %
--R complete : % -> %
--R ?.? : (%,Integer) -> Coef
--R hash : % -> SingleInteger
--R laurent : (Integer,UTS) -> %
--R leadingMonomial : % -> %
--R monomial : (Coef,Integer) -> %
--R one? : % -> Boolean
--R order : % -> Integer
--R recip : % -> Union(%, "failed")
--R removeZeroes : (Integer,%) -> %
--R retract : % -> UTS
--R taylor : % -> UTS
--R truncate : (%,Integer) -> %
--R zero? : % -> Boolean
--R ?? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
--R ?? : (UTS,%) -> % if Coef has FIELD
--R ?? : (%,UTS) -> % if Coef has FIELD
--R ?? : (NonNegativeInteger,%) -> %
--R ??? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ??? : (%,%) -> % if Coef has ALGEBRA FRAC INT
--R ??? : (%,Integer) -> % if Coef has FIELD
--R ??? : (%,NonNegativeInteger) -> %
--R ?/? : (UTS,UTS) -> % if Coef has FIELD
--R ?/? : (%,%) -> % if Coef has FIELD
--R ?/? : (%,Coef) -> % if Coef has FIELD
--R ?<? : (%,%) -> Boolean if UTS has ORDSET and Coef has FIELD
--R ?<=? : (%,%) -> Boolean if UTS has ORDSET and Coef has FIELD
--R ?>? : (%,%) -> Boolean if UTS has ORDSET and Coef has FIELD
--R ?>=? : (%,%) -> Boolean if UTS has ORDSET and Coef has FIELD
--R D : (%,Symbol) -> % if UTS has PDRING SYMBOL and Coef has FIELD or Coef has *: (Integer,Coef) -> Coef
--R D : (%,List Symbol) -> % if UTS has PDRING SYMBOL and Coef has FIELD or Coef has *: (Integer,Coef) -> Coef
--R D : (%,Symbol,NonNegativeInteger) -> % if UTS has PDRING SYMBOL and Coef has FIELD or Coef has *: (Integer,Coef) -> Coef
--R D : (%,List Symbol,List NonNegativeInteger) -> % if UTS has PDRING SYMBOL and Coef has FIELD or Coef has *: (Integer,Coef) -> Coef
--R D : % -> % if UTS has DIFRING and Coef has FIELD or Coef has *: (Integer,Coef) -> Coef
--R D : (%,NonNegativeInteger) -> % if UTS has DIFRING and Coef has FIELD or Coef has *: (Integer,Coef) -> Coef
--R D : (%,(UTS -> UTS),NonNegativeInteger) -> % if Coef has FIELD
--R D : (%,(UTS -> UTS)) -> % if Coef has FIELD
--R ?? : (%,Integer) -> % if Coef has FIELD
--R ?? : (%,NonNegativeInteger) -> %
--R abs : % -> % if UTS has OINTDOM and Coef has FIELD
--R acos : % -> % if Coef has ALGEBRA FRAC INT
--R acosh : % -> % if Coef has ALGEBRA FRAC INT
--R acot : % -> % if Coef has ALGEBRA FRAC INT
--R acoth : % -> % if Coef has ALGEBRA FRAC INT
--R acsc : % -> % if Coef has ALGEBRA FRAC INT
--R 0 : () -> %
--R center : % -> Coef
--R coerce : UTS -> %
--R coerce : % -> OutputForm
--R degree : % -> Integer
--R extend : (%,Integer) -> %
--R latex : % -> String
--R leadingCoefficient : % -> Coef
--R map : ((Coef -> Coef),%) -> %
--R monomial? : % -> Boolean
--R order : (%,Integer) -> Integer
--R pole? : % -> Boolean
--R reductum : % -> %
--R removeZeroes : % -> %
--R sample : () -> %
--R taylorRep : % -> UTS
--R variable : % -> Symbol
--R ~=? : (%,%) -> Boolean

```

```

--R acsch : % -> % if Coef has ALGEBRA FRAC INT
--R approximate : (%,Integer) -> Coef if Coef has **: (Coef,Integer) -> Coef and Coef has coerce: Symbol
--R asec : % -> % if Coef has ALGEBRA FRAC INT
--R asech : % -> % if Coef has ALGEBRA FRAC INT
--R asin : % -> % if Coef has ALGEBRA FRAC INT
--R asinh : % -> % if Coef has ALGEBRA FRAC INT
--R associates? : (%,%) -> Boolean if Coef has INTDOM
--R atan : % -> % if Coef has ALGEBRA FRAC INT
--R atanh : % -> % if Coef has ALGEBRA FRAC INT
--R ceiling : % -> UTS if UTS has INS and Coef has FIELD
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%, "failed") if $ has CHARNZ and UTS has PFECAT and Coef has FIELD or UTS has
--R coerce : % -> % if Coef has INTDOM
--R coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
--R coerce : Symbol -> % if UTS has RETRACT SYMBOL and Coef has FIELD
--R coerce : Coef -> % if Coef has COMRING
--R conditionP : Matrix % -> Union(Vector %, "failed") if $ has CHARNZ and UTS has PFECAT and Coef has F
--R convert : % -> Pattern Integer if UTS has KONVERT PATTERN INT and Coef has FIELD
--R convert : % -> Pattern Float if UTS has KONVERT PATTERN FLOAT and Coef has FIELD
--R convert : % -> InputForm if UTS has KONVERT INFORM and Coef has FIELD
--R convert : % -> Float if UTS has REAL and Coef has FIELD
--R convert : % -> DoubleFloat if UTS has REAL and Coef has FIELD
--R cos : % -> % if Coef has ALGEBRA FRAC INT
--R cosh : % -> % if Coef has ALGEBRA FRAC INT
--R cot : % -> % if Coef has ALGEBRA FRAC INT
--R coth : % -> % if Coef has ALGEBRA FRAC INT
--R csc : % -> % if Coef has ALGEBRA FRAC INT
--R csch : % -> % if Coef has ALGEBRA FRAC INT
--R denom : % -> UTS if Coef has FIELD
--R denominator : % -> % if Coef has FIELD
--R differentiate : (%,Symbol) -> % if UTS has PDRING SYMBOL and Coef has FIELD or Coef has *: (Integer,
--R differentiate : (%,List Symbol) -> % if UTS has PDRING SYMBOL and Coef has FIELD or Coef has *: (Int
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if UTS has PDRING SYMBOL and Coef has FIELD or Co
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if UTS has PDRING SYMBOL and Coef has F
--R differentiate : % -> % if UTS has DIFRING and Coef has FIELD or Coef has *: (Integer,Coef) -> Coef
--R differentiate : (%,NonNegativeInteger) -> % if UTS has DIFRING and Coef has FIELD or Coef has *: (In
--R differentiate : (%,(UTS -> UTS),NonNegativeInteger) -> % if Coef has FIELD
--R differentiate : (%,(UTS -> UTS)) -> % if Coef has FIELD
--R divide : (%,%) -> Record(quotient: %,remainder: %) if Coef has FIELD
--R ?.? : (%,UTS) -> % if UTS has ELTAB(UTS,UTS) and Coef has FIELD
--R ?.? : (%,%) -> % if Integer has SGROUP
--R euclideanSize : % -> NonNegativeInteger if Coef has FIELD
--R eval : (%,List UTS,List UTS) -> % if UTS has EVALAB UTS and Coef has FIELD
--R eval : (%,UTS,UTS) -> % if UTS has EVALAB UTS and Coef has FIELD
--R eval : (%,Equation UTS) -> % if UTS has EVALAB UTS and Coef has FIELD
--R eval : (%,List Equation UTS) -> % if UTS has EVALAB UTS and Coef has FIELD
--R eval : (%,List Symbol,List UTS) -> % if UTS has IEVALAB(SYMBOL,UTS) and Coef has FIELD
--R eval : (%,Symbol,UTS) -> % if UTS has IEVALAB(SYMBOL,UTS) and Coef has FIELD
--R eval : (%,Coef) -> Stream Coef if Coef has **: (Coef,Integer) -> Coef
--R exp : % -> % if Coef has ALGEBRA FRAC INT

```

```

--R expressIdealMember : (List %,%) -> Union(List %,"failed") if Coef has FIELD
--R exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if Coef has FIELD
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if Coef has FIELD
--R factor : % -> Factored % if Coef has FIELD
--R factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial %
--R factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial %
--R floor : % -> UTS if UTS has INS and Coef has FIELD
--R fractionPart : % -> % if UTS has EUCDOM and Coef has FIELD
--R gcd : (%,%) -> % if Coef has FIELD
--R gcd : List % -> % if Coef has FIELD
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolynomial %
--R init : () -> % if UTS has STEP and Coef has FIELD
--R integrate : (%,Symbol) -> % if Coef has integrate: (Coef,Symbol) -> Coef and Coef has var
--R integrate : % -> % if Coef has ALGEBRA FRAC INT
--R inv : % -> % if Coef has FIELD
--R lcm : (%,%) -> % if Coef has FIELD
--R lcm : List % -> % if Coef has FIELD
--R log : % -> % if Coef has ALGEBRA FRAC INT
--R map : ((UTS -> UTS),%) -> % if Coef has FIELD
--R max : (%,%) -> % if UTS has ORDSET and Coef has FIELD
--R min : (%,%) -> % if UTS has ORDSET and Coef has FIELD
--R monomial : (%,List SingletonAsOrderedSet,List Integer) -> %
--R monomial : (%,SingletonAsOrderedSet,Integer) -> %
--R multiEuclidean : (List %,%) -> Union(List %,"failed") if Coef has FIELD
--R multiplyCoefficients : ((Integer -> Coef),%) -> %
--R multiplyExponents : (%,PositiveInteger) -> %
--R negative? : % -> Boolean if UTS has OINTDOM and Coef has FIELD
--R nextItem : % -> Union(%,"failed") if UTS has STEP and Coef has FIELD
--R nthRoot : (%,Integer) -> % if Coef has ALGEBRA FRAC INT
--R numer : % -> UTS if Coef has FIELD
--R numerator : % -> % if Coef has FIELD
--R patternMatch : (%,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMatchResult(Integer,%)
--R patternMatch : (%,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchResult(Float,%)
--R pi : () -> % if Coef has ALGEBRA FRAC INT
--R positive? : % -> Boolean if UTS has OINTDOM and Coef has FIELD
--R prime? : % -> Boolean if Coef has FIELD
--R principalIdeal : List % -> Record(coef: List %,generator: %) if Coef has FIELD
--R ?quo? : (%,%) -> % if Coef has FIELD
--R random : () -> % if UTS has INS and Coef has FIELD
--R rationalFunction : (%,Integer,Integer) -> Fraction Polynomial Coef if Coef has INTDOM
--R rationalFunction : (%,Integer) -> Fraction Polynomial Coef if Coef has INTDOM
--R reducedSystem : Matrix % -> Matrix Integer if UTS has LINEXP INT and Coef has FIELD
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) if Coef has FIELD
--R reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix UTS,vec: Vector UTS) if Coef has FIELD
--R reducedSystem : Matrix % -> Matrix UTS if Coef has FIELD
--R ?rem? : (%,%) -> % if Coef has FIELD
--R retract : % -> Symbol if UTS has RETRACT SYMBOL and Coef has FIELD
--R retract : % -> Fraction Integer if UTS has RETRACT INT and Coef has FIELD
--R retract : % -> Integer if UTS has RETRACT INT and Coef has FIELD

```

```

--R retractIfCan : % -> Union(Symbol,"failed") if UTS has RETRACT SYMBOL and Coef has FIELD
--R retractIfCan : % -> Union(Fraction Integer,"failed") if UTS has RETRACT INT and Coef has FIELD
--R retractIfCan : % -> Union(Integer,"failed") if UTS has RETRACT INT and Coef has FIELD
--R retractIfCan : % -> Union(UTS,"failed")
--R sec : % -> % if Coef has ALGEBRA FRAC INT
--R sech : % -> % if Coef has ALGEBRA FRAC INT
--R series : Stream Record(k: Integer,c: Coef) -> %
--R sign : % -> Integer if UTS has OINTDOM and Coef has FIELD
--R sin : % -> % if Coef has ALGEBRA FRAC INT
--R sinh : % -> % if Coef has ALGEBRA FRAC INT
--R sizeLess? : (%,% ) -> Boolean if Coef has FIELD
--R solveLinearPolynomialEquation : (List SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) ->
--R sqrt : % -> % if Coef has ALGEBRA FRAC INT
--R squareFree : % -> Factored % if Coef has FIELD
--R squareFreePart : % -> % if Coef has FIELD
--R squareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % if UTS
--R subtractIfCan : (%,% ) -> Union(%,"failed")
--R tan : % -> % if Coef has ALGEBRA FRAC INT
--R tanh : % -> % if Coef has ALGEBRA FRAC INT
--R taylorIfCan : % -> Union(UTS,"failed")
--R terms : % -> Stream Record(k: Integer,c: Coef)
--R truncate : (%,Integer,Integer) -> %
--R unit? : % -> Boolean if Coef has INTDOM
--R unitCanonical : % -> % if Coef has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if Coef has INTDOM
--R variables : % -> List SingletonAsOrderedSet
--R wholePart : % -> UTS if UTS has EUCDOM and Coef has FIELD
--R
--E 1

```

```

)spool
)lisp (bye)

```

---

— UnivariateLaurentSeriesConstructor.help —

```

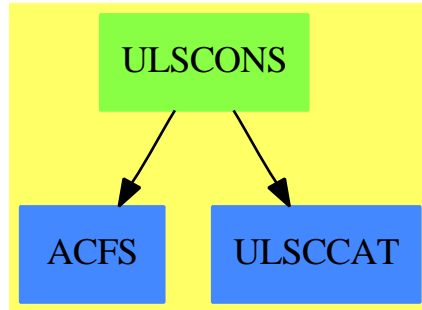
=====
UnivariateLaurentSeriesConstructor examples
=====

```

See Also:

- o )show UnivariateLaurentSeriesConstructor

### 22.3.1 UnivariateLaurentSeriesConstructor (ULSCONS)



See

⇒ “UnivariateLaurentSeries” (ULS) 22.2.1 on page 2752

**Exports:**

|                               |                      |                            |
|-------------------------------|----------------------|----------------------------|
| 0                             | 1                    | abs                        |
| acos                          | acosh                | acot                       |
| acoth                         | acsc                 | acsch                      |
| approximate                   | asec                 | asech                      |
| asin                          | asinh                | associates?                |
| atan                          | atanh                | ceiling                    |
| center                        | characteristic       | charthRoot                 |
| coefficient                   | coerce               | complete                   |
| conditionP                    | convert              | cos                        |
| cosh                          | cot                  | coth                       |
| csc                           | csch                 | D                          |
| degree                        | denom                | denominator                |
| differentiate                 | divide               | extend                     |
| euclideanSize                 | eval                 | exp                        |
| expressIdealMember            | exquo                | extendedEuclidean          |
| factor                        | factorPolynomial     | factorSquareFreePolynomial |
| floor                         | fractionPart         | gcd                        |
| gcdPolynomial                 | hash                 | init                       |
| integrate                     | inv                  | latex                      |
| laurent                       | lcm                  | leadingCoefficient         |
| leadingMonomial               | log                  | map                        |
| max                           | min                  | monomial                   |
| monomial?                     | multiEuclidean       | multiplyCoefficients       |
| multiplyExponents             | negative?            | nextItem                   |
| nthRoot                       | numer                | numerator                  |
| one?                          | order                | patternMatch               |
| pi                            | pole?                | positive?                  |
| prime?                        | principalIdeal       | random                     |
| rationalFunction              | recip                | reducedSystem              |
| reductum                      | removeZeroes         | retract                    |
| retractIfCan                  | sample               | sec                        |
| sech                          | series               | sign                       |
| sin                           | sinh                 | sizeLess?                  |
| solveLinearPolynomialEquation | sqrt                 | squareFree                 |
| squareFreePart                | squareFreePolynomial | subtractIfCan              |
| tan                           | tanh                 | taylor                     |
| taylorIfCan                   | taylorRep            | terms                      |
| truncate                      | unit?                | unitCanonical              |
| unitNormal                    | variable             | variables                  |
| wholePart                     | zero?                | ?*?                        |
| ***?                          | ?+?                  | ?-?                        |
| -?                            | ?=?                  | ?^?                        |
| ?.                            | ?~=?                 | ?/?                        |
| ?<?                           | ?<=?                 | ?>?                        |
| ?>=?                          | ?quo?                | ?rem?                      |

## — domain ULSCONS UnivariateLaurentSeriesConstructor —

```

)abbrev domain ULSCONS UnivariateLaurentSeriesConstructor
++ Authors: Bill Burge, Clifton J. Williamson
++ Date Created: August 1988
++ Date Last Updated: 17 June 1996
++ Fix History:
++ 14 June 1996: provided missing exquo: (%,%) -> % (Frederic Lehouby)
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: series, Laurent, Taylor
++ Examples:
++ References:
++ Description:
++ This package enables one to construct a univariate Laurent series
++ domain from a univariate Taylor series domain. Univariate
++ Laurent series are represented by a pair \spad{[n,f(x)]}, where n is
++ an arbitrary integer and \spad{f(x)} is a Taylor series. This pair
++ represents the Laurent series \spad{x**n * f(x)}.

```

```

UnivariateLaurentSeriesConstructor(Coef,UTS):_
Exports == Implementation where
  Coef    : Ring
  UTS     : UnivariateTaylorSeriesCategory Coef
  I       ==> Integer
  L       ==> List
  NNI     ==> NonNegativeInteger
  OUT     ==> OutputForm
  P       ==> Polynomial Coef
  RF      ==> Fraction Polynomial Coef
  RN      ==> Fraction Integer
  ST      ==> Stream Coef
  TERM    ==> Record(k:I,c:Coef)
  monom   ==> monomial$UTS
  EFULS   ==> ElementaryFunctionsUnivariateLaurentSeries(Coef,UTS,%)
  STTAYLOR ==> StreamTaylorSeriesOperations Coef

Exports ==> UnivariateLaurentSeriesConstructorCategory(Coef,UTS)

Implementation ==> add

--% representation

Rep := Record(expon:I,ps:UTS)

getExpon : % -> I
getUTS   : % -> UTS

```

```

getExpon x == x.expon
getUTS    x == x.ps

--% creation and destruction

laurent(n,psr) == [n,psr]
taylorRep x    == getUTS x
degree x       == getExpon x

0 == laurent(0,0)
1 == laurent(0,1)

monomial(s,e) == laurent(e,s::UTS)

coerce(uts:UTS):% == laurent(0,uts)
coerce(r:Coef):%  == r :: UTS  :: %
coerce(i:I):%     == i :: Coef :: %

taylorIfCan uls ==
  n := getExpon uls
  n < 0 =>
    uls := removeZeroes(-n,uls)
    getExpon(uls) < 0 => "failed"
    getUTS uls
  n = 0 => getUTS uls
  getUTS(uls) * monom(1,n :: NNI)

taylor uls ==
  (uts := taylorIfCan uls) case "failed" =>
    error "taylor: Laurent series has a pole"
  uts :: UTS

termExpon: TERM -> I
termExpon term == term.k
termCoef: TERM -> Coef
termCoef term == term.c
rec: (I,Coef) -> TERM
rec(exponent,coef) == [exponent,coef]

recs: (ST,I) -> Stream TERM
recs(st,n) == delay
  empty? st => empty()
  zero? (coef := first st) => recs(rst st,n + 1)
  concat(rec(n,coef),recs(rst st,n + 1))

terms x == recs(coefficients getUTS x,getExpon x)

recsToCoefs: (Stream TERM,I) -> ST
recsToCoefs(st,n) == delay

```



```

empty? st => empty()
term := first st; ex := termExpon term
n = ex => concat(termCoef term,recsToCoefs(rst st,n + 1))
concat(0,recsToCoefs(rst st,n + 1))

series st ==
  empty? st => 0
  ex := termExpon first st
  laurent(ex,series recsToCoefs(st,ex))

--% normalizations

removeZeroes x ==
  empty? coefficients(xUTS := getUTS x) => 0
  coefficient(xUTS,0) = 0 =>
    removeZeroes laurent(getExpon(x) + 1,quoByVar xUTS)
  x

removeZeroes(n,x) ==
  n <= 0 => x
  empty? coefficients(xUTS := getUTS x) => 0
  coefficient(xUTS,0) = 0 =>
    removeZeroes(n - 1,laurent(getExpon(x) + 1,quoByVar xUTS))
  x

--% predicates

x = y ==
  EQ(x,y)$Lisp => true
  (expDiff := getExpon(x) - getExpon(y)) = 0 =>
    getUTS(x) = getUTS(y)
  abs(expDiff) > _$streamCount$Lisp => false
  expDiff > 0 =>
    getUTS(x) * monom(1,expDiff :: NNI) = getUTS(y)
  getUTS(y) * monom(1,(- expDiff) :: NNI) = getUTS(x)

pole? x ==
  (n := degree x) >= 0 => false
  x := removeZeroes(-n,x)
  degree x < 0

--% arithmetic

x + y ==
  n := getExpon(x) - getExpon(y)
  n >= 0 =>
    laurent(getExpon y,getUTS(y) + getUTS(x) * monom(1,n::NNI))
  laurent(getExpon x,getUTS(x) + getUTS(y) * monom(1,(-n)::NNI))

x - y ==

```

```

n := getExpon(x) - getExpon(y)
n >= 0 =>
  laurent(getExpon y, getUTS(x) * monom(1, n::NNI) - getUTS(y))
  laurent(getExpon x, getUTS(x) - getUTS(y) * monom(1, (-n)::NNI))

x:% * y:% == laurent(getExpon x + getExpon y, getUTS x * getUTS y)

x:% ** n:NNI ==
  zero? n =>
    zero? x => error "0 ** 0 is undefined"
    1
  laurent(n * getExpon(x), getUTS(x) ** n)

recip x ==
  x := removeZeroes(1000, x)
  zero? coefficient(x, d := degree x) => "failed"
  (uts := recip getUTS x) case "failed" => "failed"
  laurent(-d, uts :: UTS)

elt(uls1:%, uls2:%) ==
  (uts := taylorIfCan uls2) case "failed" =>
    error "elt: second argument must have positive order"
  uts2 := uts :: UTS
  not zero? coefficient(uts2, 0) =>
    error "elt: second argument must have positive order"
  if (deg := getExpon uls1) < 0 then uls1 := removeZeroes(-deg, uls1)
  (deg := getExpon uls1) < 0 =>
    (recipr := recip(uts2 :: %)) case "failed" =>
      error "elt: second argument not invertible"
    uls1 := taylor(uls1 * monomial(1, -deg))
    (elt(uls1, uts2) :: %) * (recipr :: %) ** ((-deg) :: NNI)
  elt(taylor uls1, uts2) :: %

eval(uls:%, r:Coef) ==
  if (n := getExpon uls) < 0 then uls := removeZeroes(-n, uls)
  uts := getUTS uls
  (n := getExpon uls) < 0 =>
    zero? r => error "eval: 0 raised to negative power"
    (recipr := recip r) case "failed" =>
      error "eval: non-unit raised to negative power"
    (recipr :: Coef) ** ((-n) :: NNI) * $STTAYLOR eval(uts, r)
  zero? n => eval(uts, r)
  r ** (n :: NNI) * $STTAYLOR eval(uts, r)

--% values

variable x == variable getUTS x
center x == center getUTS x

coefficient(x, n) ==

```

```

a := n - getExpon(x)
a >= 0 => coefficient(getUTS x,a :: NNI)
0

elt(x:%,n:I) == coefficient(x,n)

--% other functions

order x == getExpon x + order getUTS x
order(x,n) ==
  (m := n - (e := getExpon x)) < 0 => n
  e + order(getUTS x,m :: NNI)

truncate(x,n) ==
  (m := n - (e := getExpon x)) < 0 => 0
  laurent(e,truncate(getUTS x,m :: NNI))

truncate(x,n1,n2) ==
  if n2 < n1 then (n1,n2) := (n2,n1)
  (m1 := n1 - (e := getExpon x)) < 0 => truncate(x,n2)
  laurent(e,truncate(getUTS x,m1 :: NNI,(n2 - e) :: NNI))

if Coef has IntegralDomain then
rationalFunction(x,n) ==
  (m := n - (e := getExpon x)) < 0 => 0
  poly := polynomial(getUTS x,m :: NNI) :: RF
  zero? e => poly
  v := variable(x) :: RF; c := center(x) :: P :: RF
  positive? e => poly * (v - c) ** (e :: NNI)
  poly / (v - c) ** ((-e) :: NNI)

rationalFunction(x,n1,n2) ==
  if n2 < n1 then (n1,n2) := (n2,n1)
  (m1 := n1 - (e := getExpon x)) < 0 => rationalFunction(x,n2)
  poly := polynomial(getUTS x,m1 :: NNI,(n2 - e) :: NNI) :: RF
  zero? e => poly
  v := variable(x) :: RF; c := center(x) :: P :: RF
  positive? e => poly * (v - c) ** (e :: NNI)
  poly / (v - c) ** ((-e) :: NNI)

-- La fonction < exquo > manque dans laurent.spad,
--les lignes suivantes le mettent en evidence :
--
--ls := laurent(0,series [i for i in 1..])$U(SINT,x,0)
---- missing function in laurent.spad of Axiom 2.0a version of
---- Friday March 10, 1995 at 04:15:22 on 615:
--exquo(ls,ls)
--
-- Je l'ai ajoutee a laurent.spad.
--

```

```

--Frederic Lehouby
x exquo y ==
  x := removeZeroes(1000,x)
  y := removeZeroes(1000,y)
  zero? coefficient(y, d := degree y) => "failed"
  (uts := (getUTS x) exquo (getUTS y)) case "failed" => "failed"
  laurent(degree x-d,uts :: UTS)

if Coef has coerce: Symbol -> Coef then
  if Coef has "**": (Coef,I) -> Coef then

    approximate(x,n) ==
      (m := n - (e := getExpon x)) < 0 => 0
      app := approximate(getUTS x,m :: NNI)
      zero? e => app
      app * ((variable(x) :: Coef) - center(x)) ** e

  complete x == laurent(getExpon x,complete getUTS x)
  extend(x,n) ==
    e := getExpon x
    (m := n - e) < 0 => x
    laurent(e,extend(getUTS x,m :: NNI))

  map(f:Coef -> Coef,x:%) == laurent(getExpon x,map(f,getUTS x))

  multiplyCoefficients(f,x) ==
    e := getExpon x
    laurent(e,multiplyCoefficients((z1:I):Coef +-> f(e + z1),getUTS x))

  multiplyExponents(x,n) ==
    laurent(n * getExpon x,multiplyExponents(getUTS x,n))

  differentiate x ==
    e := getExpon x
    laurent(e - 1,
      multiplyCoefficients((z1:I):Coef +-> (e + z1)::Coef,getUTS x))

  if Coef has PartialDifferentialRing(Symbol) then
    differentiate(x:%,s:Symbol) ==
      (s = variable(x)) => differentiate x
      map((z1:Coef):Coef +-> differentiate(z1,s),x)
      - differentiate(center x,s)*differentiate(x)

  characteristic() == characteristic()$Coef

  if Coef has Field then

    retract(x:%)::UTS == taylor x
    retractIfCan(x:%)::Union(UTS,"failed") == taylorIfCan x

```

```

(x:%) ** (n:I) ==
  zero? n =>
    zero? x => error "0 ** 0 is undefined"
    1
  n > 0 => laurent(n * getExpon(x),getUTS(x) ** (n :: NNI))
  xInv := inv x; minusN := (-n) :: NNI
  laurent(minusN * getExpon(xInv),getUTS(xInv) ** minusN)

(x:UTS) * (y:%) == (x :: %) * y
(x:%) * (y:UTS) == x * (y :: %)

inv x ==
  (xInv := recip x) case "failed" =>
    error "multiplicative inverse does not exist"
  xInv :: %

(x:%) / (y:%) ==
  (yInv := recip y) case "failed" =>
    error "inv: multiplicative inverse does not exist"
  x * (yInv :: %)

(x:UTS) / (y:UTS) == (x :: %) / (y :: %)

numer x ==
  (n := degree x) >= 0 => taylor x
  x := removeZeroes(-n,x)
  (n := degree x) = 0 => taylor x
  getUTS x

denom x ==
  (n := degree x) >= 0 => 1
  x := removeZeroes(-n,x)
  (n := degree x) = 0 => 1
  monom(1,(-n) :: NNI)

--% algebraic and transcendental functions

if Coef has Algebra Fraction Integer then

  coerce(r:RN) == r :: Coef :: %

  if Coef has Field then
    (x:%) ** (r:RN) == x **$EFULS r

  exp x    == exp(x)$EFULS
  log x    == log(x)$EFULS
  sin x    == sin(x)$EFULS
  cos x    == cos(x)$EFULS
  tan x    == tan(x)$EFULS
  cot x    == cot(x)$EFULS

```

```

sec x == sec(x)$EFULS
csc x == csc(x)$EFULS
asin x == asin(x)$EFULS
acos x == acos(x)$EFULS
atan x == atan(x)$EFULS
acot x == acot(x)$EFULS
asec x == asec(x)$EFULS
acsc x == acsc(x)$EFULS
sinh x == sinh(x)$EFULS
cosh x == cosh(x)$EFULS
tanh x == tanh(x)$EFULS
coth x == coth(x)$EFULS
sech x == sech(x)$EFULS
csch x == csch(x)$EFULS
asinh x == asinh(x)$EFULS
acosh x == acosh(x)$EFULS
atanh x == atanh(x)$EFULS
acoth x == acoth(x)$EFULS
asech x == asech(x)$EFULS
acsch x == acsch(x)$EFULS

ratInv: I -> Coef
ratInv n ==
  zero? n => 1
  inv(n :: RN) :: Coef

integrate x ==
  not zero? coefficient(x,-1) =>
    error "integrate: series has term of order -1"
  e := getExpon x
  laurent(e+1,multiplyCoefficients((z:I):Coef+-->ratInv(e+1+z),getUTS x))

if Coef has integrate: (Coef,Symbol) -> Coef and _
  Coef has variables: Coef -> List Symbol then
    integrate(x:%,s:Symbol) ==
      (s = variable(x)) => integrate x
      not entry?(s,variables center x)
      => map((z1:Coef):Coef+-->integrate(z1,s),x)
      error "integrate: center is a function of variable of integration"

if Coef has TranscendentalFunctionCategory and _
  Coef has PrimitiveFunctionCategory and _
  Coef has AlgebraicallyClosedFunctionSpace Integer then

  integrateWithOneAnswer: (Coef,Symbol) -> Coef
  integrateWithOneAnswer(f,s) ==
    res := integrate(f,s)$FunctionSpaceIntegration(I,Coef)
    res case Coef => res :: Coef
    first(res :: List Coef)

```

```

integrate(x:%,s:Symbol) ==
  (s = variable(x)) => integrate x
  not entry?(s,variables center x) =>
    map((z1:Coef):Coef +-> integrateWithOneAnswer(z1,s),x)
  error "integrate: center is a function of variable of integration"

termOutput:(I,Coef,OUT) -> OUT
termOutput(k,c,vv) ==
-- creates a term c * vv ** k
  k = 0 => c :: OUT
  mon :=
    k = 1 => vv
    vv ** (k :: OUT)
  c = 1 => mon
  c = -1 => -mon
  (c :: OUT) * mon

showAll?:() -> Boolean
-- check a global Lisp variable
showAll?() == true

termsToOutputForm:(I,ST,OUT) -> OUT
termsToOutputForm(m,uu,xxx) ==
  l : L OUT := empty()
  empty? uu => (0$Coef) :: OUT
  n : NNI ; count : NNI := _$streamCount$Lisp
  for n in 0..count while not empty? uu repeat
    if first(uu) ^= 0 then
      l := concat(termOutput((n :: I) + m,first(uu),xxx),l)
      uu := rst uu
  if showAll?() then
    for n in (count + 1).. while explicitEntries? uu and _
      not eq?(uu,rst uu) repeat
        if first(uu) ^= 0 then
          l := concat(termOutput((n::I) + m,first(uu),xxx),l)
          uu := rst uu
  l :=
    explicitlyEmpty? uu => l
    eq?(uu,rst uu) and first uu = 0 => l
    concat(prefix("0" :: OUT,[xxx ** ((n :: I) + m) :: OUT]),l)
  empty? l => (0$Coef) :: OUT
  reduce("+",reverse! l)

coerce(x:%):OUT ==
  x := removeZeroes(_$streamCount$Lisp,x)
  m := degree x
  uts := getUTS x
  p := coefficients uts
  var := variable uts; cen := center uts
  xxx :=

```

```

zero? cen => var :: OUT
paren(var :: OUT - cen :: OUT)
termsToOutputForm(m,p,xxx)

```

---

— ULSCONS.dotabb —

```

"ULSCONS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ULSCONS"]
"ACFS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ACFS"]
"ULSCCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ULSCCAT"]
"ULSCONS" -> "ULSCCAT"
"ULSCONS" -> "ACFS"

```

---

## 22.4 domain UP UnivariatePolynomial

— UnivariatePolynomial.input —

```

)set break resume
)sys rm -f UnivariatePolynomial.output
)spool UnivariatePolynomial.output
)set message test on
)set message auto off
)clear all
--S 1 of 35
(p,q) : UP(x,INT)
--R
--R
--R                                                    Type: Void
--E 1

--S 2 of 35
p := (3*x-1)**2 * (2*x + 8)
--R
--R
--R          3      2
--R   (2)  18x  + 60x  - 46x + 8
--R
--R                                                    Type: UnivariatePolynomial(x,Integer)
--E 2

--S 3 of 35
q := (1 - 6*x + 9*x**2)**2
--R

```



```

--R
--R      4      3      2
--R  (3) 81x  - 108x  + 54x  - 12x + 1
--R                                          Type: UnivariatePolynomial(x,Integer)
--E 3

```

```

--S 4 of 35
p**2 + p*q
--R
--R
--R      7      6      5      4      3      2
--R  (4) 1458x  + 3240x  - 7074x  + 10584x  - 9282x  + 4120x  - 878x + 72
--R                                          Type: UnivariatePolynomial(x,Integer)
--E 4

```

```

--S 5 of 35
leadingCoefficient p
--R
--R
--R  (5) 18
--R                                          Type: PositiveInteger
--E 5

```

```

--S 6 of 35
degree p
--R
--R
--R  (6) 3
--R                                          Type: PositiveInteger
--E 6

```

```

--S 7 of 35
reductum p
--R
--R
--R      2
--R  (7) 60x  - 46x + 8
--R                                          Type: UnivariatePolynomial(x,Integer)
--E 7

```

```

--S 8 of 35
gcd(p,q)
--R
--R
--R      2
--R  (8) 9x  - 6x + 1
--R                                          Type: UnivariatePolynomial(x,Integer)
--E 8

```

```

--S 9 of 35

```

```

lcm(p,q)
--R
--R
--R      5      4      3      2
--R  (9)  162x  + 432x  - 756x  + 408x  - 94x + 8
--R                                          Type: UnivariatePolynomial(x,Integer)
--E 9

--S 10 of 35
resultant(p,q)
--R
--R
--R  (10)  0
--R                                          Type: NonNegativeInteger
--E 10

--S 11 of 35
D p
--R
--R
--R      2
--R  (11)  54x  + 120x - 46
--R                                          Type: UnivariatePolynomial(x,Integer)
--E 11

--S 12 of 35
p(2)
--R
--R
--R  (12)  300
--R                                          Type: PositiveInteger
--E 12

--S 13 of 35
p(q)
--R
--R
--R  (13)
--R      12      11      10      9      8
--R  9565938x  - 38263752x  + 70150212x  - 77944680x  + 58852170x
--R  +
--R      7      6      5      4      3      2
--R  - 32227632x  + 13349448x  - 4280688x  + 1058184x  - 192672x  + 23328x
--R  +
--R  - 1536x + 40
--R                                          Type: UnivariatePolynomial(x,Integer)
--E 13

--S 14 of 35
q(p)

```

```

--R
--R
--R (14)
--R      12      11      10      9      8
--R      8503056x  + 113374080x  + 479950272x  + 404997408x  - 1369516896x
--R  +
--R      7      6      5      4      3
--R      - 626146848x  + 2939858712x  - 2780728704x  + 1364312160x  - 396838872x
--R  +
--R      2
--R      69205896x  - 6716184x + 279841
--R
--R                                          Type: UnivariatePolynomial(x,Integer)
--E 14

--S 15 of 35
l := coefficients p
--R
--R
--R (15) [18,60,- 46,8]
--R
--R                                          Type: List Integer
--E 15

--S 16 of 35
reduce(gcd,l)
--R
--R
--R (16) 2
--R
--R                                          Type: PositiveInteger
--E 16

--S 17 of 35
content p
--R
--R
--R (17) 2
--R
--R                                          Type: PositiveInteger
--E 17

--S 18 of 35
ux := (x**4+2*x+3)::UP(x,INT)
--R
--R
--R      4
--R (18) x  + 2x + 3
--R
--R                                          Type: UnivariatePolynomial(x,Integer)
--E 18

--S 19 of 35
vectorise(ux,5)
--R

```

```

--R
--R (19) [3,2,0,0,1]
--R
--R                                         Type: Vector Integer
--E 19

--S 20 of 35
squareTerms(p) == reduce(+,[t**2 for t in monomials p])
--R
--R
--R                                         Type: Void
--E 20

--S 21 of 35
p
--R
--R
--R          3      2
--R (21)  18x  + 60x  - 46x + 8
--R
--R                                         Type: UnivariatePolynomial(x,Integer)
--E 21

--S 22 of 35
squareTerms p
--R
--R   Compiling function squareTerms with type UnivariatePolynomial(x,
--R   Integer) -> UnivariatePolynomial(x,Integer)
--R
--R          6      4      2
--R (22)  324x  + 3600x  + 2116x  + 64
--R
--R                                         Type: UnivariatePolynomial(x,Integer)
--E 22

--S 23 of 35
(r,s) : UP(a1,FRAC INT)
--R
--R
--R                                         Type: Void
--E 23

--S 24 of 35
r := a1**2 - 2/3
--R
--R
--R          2      2
--R (24)  a1  - -
--R          3
--R
--R                                         Type: UnivariatePolynomial(a1,Fraction Integer)
--E 24

--S 25 of 35
s := a1 + 4
--R

```

```

--R
--R (25)  $a_1 + 4$ 
--R
--R                                         Type: UnivariatePolynomial(a1,Fraction Integer)
--E 25

--S 26 of 35
r quo s
--R
--R
--R (26)  $a_1 - 4$ 
--R
--R                                         Type: UnivariatePolynomial(a1,Fraction Integer)
--E 26

--S 27 of 35
r rem s
--R
--R
--R          46
--R (27)  --
--R          3
--R
--R                                         Type: UnivariatePolynomial(a1,Fraction Integer)
--E 27

--S 28 of 35
d := divide(r, s)
--R
--R
--R          46
--R (28)  [quotient=  $a_1 - 4$ , remainder= --]
--R          3
--RType: Record(quotient: UnivariatePolynomial(a1,Fraction Integer),remainder: UnivariatePolynomial(a1,Fraction Integer))
--E 28

--S 29 of 35
r - (d.quotient * s + d.remainder)
--R
--R
--R (29)  0
--R
--R                                         Type: UnivariatePolynomial(a1,Fraction Integer)
--E 29

--S 30 of 35
integrate r
--R
--R
--R          1   3   2
--R (30)  -  $a_1$  - -  $a_1$ 
--R          3       3
--R
--R                                         Type: UnivariatePolynomial(a1,Fraction Integer)
--E 30

```

```

--S 31 of 35
integrate s
--R
--R
--R      1  2
--R  (31) - a1  + 4a1
--R      2
--R
--R                                          Type: UnivariatePolynomial(a1,Fraction Integer)
--E 31

--S 32 of 35
t : UP(a1,FRAC POLY INT)
--R
--R
--R                                          Type: Void
--E 32

--S 33 of 35
t := a1**2 - a1/b2 + (b1**2-b1)/(b2+3)
--R
--R
--R      2
--R      2  1      b1  - b1
--R  (33) a1  - -- a1 + -----
--R      b2      b2 + 3
--R
--R                                          Type: UnivariatePolynomial(a1,Fraction Polynomial Integer)
--E 33

--S 34 of 35
u : FRAC POLY INT := t
--R
--R
--R      2  2      2      2
--R      a1 b2  + (b1  - b1 + 3a1  - a1)b2 - 3a1
--R  (34) -----
--R      2
--R      b2  + 3b2
--R
--R                                          Type: Fraction Polynomial Integer
--E 34

--S 35 of 35
u :: UP(b1,?)
--R
--R
--R      2
--R      1  2      1      a1 b2 - a1
--R  (35) ----- b1  - ----- b1 + -----
--R      b2 + 3      b2 + 3      b2
--R
--R                                          Type: UnivariatePolynomial(b1,Fraction Polynomial Integer)
--E 35

```

```
)spool
)lisp (bye)
```

---

— UnivariatePolynomial.help —

=====

UnivariatePolynomial examples

=====

The domain constructor `UnivariatePolynomial` (abbreviated `UP`) creates domains of univariate polynomials in a specified variable. For example, the domain `UP(a1,POLY FRAC INT)` provides polynomials in the single variable `a1` whose coefficients are general polynomials with rational number coefficients.

Restriction: Axiom does not allow you to create types where `UnivariatePolynomial` is contained in the coefficient type of `Polynomial`. Therefore, `UP(x,POLY INT)` is legal but `POLY UP(x,INT)` is not.

`UP(x,INT)` is the domain of polynomials in the single variable `x` with integer coefficients.

```
(p,q) : UP(x,INT)
                                     Type: Void

p := (3*x-1)**2 * (2*x + 8)
      3      2
    18x  + 60x  - 46x + 8
                                     Type: UnivariatePolynomial(x,Integer)

q := (1 - 6*x + 9*x**2)**2
      4      3      2
    81x  - 108x  + 54x  - 12x + 1
                                     Type: UnivariatePolynomial(x,Integer)
```

The usual arithmetic operations are available for univariate polynomials.

```
p**2 + p*q
      7      6      5      4      3      2
    1458x  + 3240x  - 7074x  + 10584x  - 9282x  + 4120x  - 878x + 72
                                     Type: UnivariatePolynomial(x,Integer)
```

The operation `leadingCoefficient` extracts the coefficient of the term of highest degree.

```
leadingCoefficient p
```

18

Type: PositiveInteger

The operation `degree` returns the degree of the polynomial. Since the polynomial has only one variable, the variable is not supplied to operations like `degree`.

```
degree p
3
```

Type: PositiveInteger

The reductum of the polynomial, the polynomial obtained by subtracting the term of highest order, is returned by `reductum`.

```
reductum p
2
60x - 46x + 8
```

Type: UnivariatePolynomial(x,Integer)

The operation `gcd` computes the greatest common divisor of two polynomials.

```
gcd(p,q)
2
9x - 6x + 1
```

Type: UnivariatePolynomial(x,Integer)

The operation `lcm` computes the least common multiple.

```
lcm(p,q)
5      4      3      2
162x + 432x - 756x + 408x - 94x + 8
```

Type: UnivariatePolynomial(x,Integer)

The operation `resultant` computes the resultant of two univariate polynomials. In the case of `p` and `q`, the resultant is 0 because they share a common root.

```
resultant(p,q)
0
```

Type: NonNegativeInteger

To compute the derivative of a univariate polynomial with respect to its variable, use the function `D`.

```
D p
2
54x + 120x - 46
```

Type: UnivariatePolynomial(x,Integer)

Univariate polynomials can also be used as if they were functions. To



evaluate a univariate polynomial at some point, apply the polynomial to the point.

```
p(2)
300
Type: PositiveInteger
```

The same syntax is used for composing two univariate polynomials, i.e. substituting one polynomial for the variable in another. This substitutes  $q$  for the variable in  $p$ .

```
p(q)
      12      11      10      9      8
9565938x - 38263752x + 70150212x - 77944680x + 58852170x
+
      7      6      5      4      3      2
- 32227632x + 13349448x - 4280688x + 1058184x - 192672x + 23328x
+
- 1536x + 40
Type: UnivariatePolynomial(x,Integer)
```

This substitutes  $p$  for the variable in  $q$ .

```
q(p)
      12      11      10      9      8
8503056x + 113374080x + 479950272x + 404997408x - 1369516896x
+
      7      6      5      4      3
- 626146848x + 2939858712x - 2780728704x + 1364312160x - 396838872x
+
      2
69205896x - 6716184x + 279841
Type: UnivariatePolynomial(x,Integer)
```

To obtain a list of coefficients of the polynomial, use `coefficients`.

```
l := coefficients p
[18,60,- 46,8]
Type: List Integer
```

From this you can use `gcd` and `reduce` to compute the content of the polynomial.

```
reduce(gcd,l)
2
Type: PositiveInteger
```

Alternatively (and more easily), you can just call `content`.

```
content p
2
```

Type: PositiveInteger

Note that the operation coefficients omits the zero coefficients from the list. Sometimes it is useful to convert a univariate polynomial to a vector whose i-th position contains the degree i-1 coefficient of the polynomial.

```
ux := (x**4+2*x+3)::UP(x,INT)
      4
      x  + 2x + 3
```

Type: UnivariatePolynomial(x,Integer)

To get a complete vector of coefficients, use the operation vectorise, which takes a univariate polynomial and an integer denoting the length of the desired vector.

```
vectorise(ux,5)
[3,2,0,0,1]
```

Type: Vector Integer

It is common to want to do something to every term of a polynomial, creating a new polynomial in the process.

This is a function for iterating across the terms of a polynomial, squaring each term.

```
squareTerms(p) == reduce(+,[t**2 for t in monomials p])
Type: Void
```

Recall what p looked like.

```
P
      3      2
      18x  + 60x  - 46x + 8
```

Type: UnivariatePolynomial(x,Integer)

We can demonstrate squareTerms on p.

```
squareTerms p
      6      4      2
      324x  + 3600x  + 2116x  + 64
```

Type: UnivariatePolynomial(x,Integer)

When the coefficients of the univariate polynomial belong to a field, it is possible to compute quotients and remainders. For example, when the coefficients are rational numbers, as opposed to integers. The important property of a field is that non-zero elements can be divided and produce another element. The quotient of the integers 2 and 3 is not another integer.

```
(r,s) : UP(a1,FRAC INT)
Type: Void
```

```
r := a1**2 - 2/3
      2 2
a1 - -
      3
Type: UnivariatePolynomial(a1,Fraction Integer)
```

```
s := a1 + 4
a1 + 4
Type: UnivariatePolynomial(a1,Fraction Integer)
```

When the coefficients are rational numbers or rational expressions, the operation quo computes the quotient of two polynomials.

```
r quo s
a1 - 4
Type: UnivariatePolynomial(a1,Fraction Integer)
```

The operation rem computes the remainder.

```
r rem s
46
--
3
Type: UnivariatePolynomial(a1,Fraction Integer)
```

The operation divide can be used to return a record of both components.

```
d := divide(r, s)
      46
[quotient= a1 - 4,remainder= --]
      3
Type: Record(quotient: UnivariatePolynomial(a1,Fraction Integer),
             remainder: UnivariatePolynomial(a1,Fraction Integer))
```

Now we check the arithmetic!

```
r - (d.quotient * s + d.remainder)
0
Type: UnivariatePolynomial(a1,Fraction Integer)
```

It is also possible to integrate univariate polynomials when the coefficients belong to a field.

```
integrate r
1 3 2
- a1 - - a1
3 3
```

Type: UnivariatePolynomial(a1,Fraction Integer)

```
integrate s
1 2
- a1 + 4a1
2
```

Type: UnivariatePolynomial(a1,Fraction Integer)

One application of univariate polynomials is to see expressions in terms of a specific variable.

We start with a polynomial in a1 whose coefficients are quotients of polynomials in b1 and b2.

```
t : UP(a1,FRAC POLY INT)
Type: Void
```

Since in this case we are not talking about using multivariate polynomials in only two variables, we use Polynomial. We also use Fraction because we want fractions.

```
t := a1**2 - a1/b2 + (b1**2-b1)/(b2+3)
2
2 1 b1 - b1
a1 - -- a1 + -----
b2 b2 + 3
Type: UnivariatePolynomial(a1,Fraction Polynomial Integer)
```

We push all the variables into a single quotient of polynomials.

```
u : FRAC POLY INT := t
2 2 2 2
a1 b2 + (b1 - b1 + 3a1 - a1)b2 - 3a1
-----
2
b2 + 3b2
Type: Fraction Polynomial Integer
```

Alternatively, we can view this as a polynomial in the variable This is a mode-directed conversion: you indicate as much of the structure as you care about and let Axiom decide on the full type and how to do the transformation.

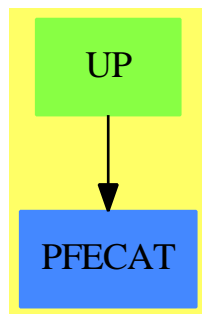
```
u :: UP(b1,?)
1 2 1 2
----- b1 - ----- b1 + -----
b2 + 3 b2 + 3 b2
Type: UnivariatePolynomial(b1,Fraction Polynomial Integer)
```

See Also:

- o )help MultivariatePolynomial
- o )help DistributedMultivariatePolynomial
- o )show UnivariatePolynomial

—————

### 22.4.1 UnivariatePolynomial (UP)



See

- ⇒ “FreeModule” (FM) 7.30.1 on page 980
- ⇒ “PolynomialRing” (PR) 17.27.1 on page 2052
- ⇒ “SparseUnivariatePolynomial” (SUP) 20.18.1 on page 2425

**Exports:**

|                               |                            |
|-------------------------------|----------------------------|
| 0                             | 1                          |
| associates?                   | binomThmExpt               |
| characteristic                | charthRoot                 |
| coefficient                   | coefficients               |
| coerce                        | composite                  |
| conditionP                    | content                    |
| convert                       | D                          |
| degree                        | differentiate              |
| discriminant                  | divide                     |
| divideExponents               | elt                        |
| euclideanSize                 | eval                       |
| expressIdealMember            | exquo                      |
| extendedEuclidean             | factor                     |
| factorPolynomial              | factorSquareFreePolynomial |
| fmcg                          | gcd                        |
| gcdPolynomial                 | ground                     |
| ground?                       | hash                       |
| init                          | integrate                  |
| isExpt                        | isPlus                     |
| isTimes                       | karatsubaDivide            |
| latex                         | lcm                        |
| leadingCoefficient            | leadingMonomial            |
| mainVariable                  | makeSUP                    |
| mapExponents                  | map                        |
| max                           | min                        |
| minimumDegree                 | monicDivide                |
| monomial                      | monomial?                  |
| monomials                     | multiEuclidean             |
| multiplyExponents             | multivariate               |
| nextItem                      | numberOfMonomials          |
| one?                          | order                      |
| patternMatch                  | popop!                     |
| prime?                        | primitivePart              |
| primitiveMonomials            | principalIdeal             |
| pseudoDivide                  | pseudoQuotient             |
| pseudoRemainder               | recip                      |
| reducedSystem                 | reductum                   |
| resultant                     | retract                    |
| retractIfCan                  | sample                     |
| separate                      | shiftLeft                  |
| shiftRight                    | sizeLess?                  |
| solveLinearPolynomialEquation | squareFree                 |
| squareFreePart                | squareFreePolynomial       |
| subResultantGcd               | subtractIfCan              |
| totalDegree                   | totalDegree                |
| unit?                         | unitCanonical              |
| unitNormal                    | univariate                 |
| unmakeSUP                     | variables                  |
| vectorise                     | zero?                      |
| ?*?                           | ?**?                       |
| ?+?                           | ?-?                        |
| -?                            | ?=?                        |
| ?^?                           | ?..?                       |
| ?~=?                          | ?/?                        |
| ?<?                           | ?<=?                       |
| ?>?                           | ?>=?                       |
| ?quo?                         | ?rem?                      |

## — domain UP UnivariatePolynomial —

```

)abbrev domain UP UnivariatePolynomial
++ Author: Mark Botch
++ Date Created:
++ Date Last Updated:
++ Basic Functions: Ring, monomial, coefficient, reductum, differentiate,
++ elt, map, resultant, discriminant
++ Related Constructors: SparseUnivariatePolynomial, MultivariatePolynomial
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This domain represents univariate polynomials in some symbol
++ over arbitrary (not necessarily commutative) coefficient rings.
++ The representation is sparse
++ in the sense that only non-zero terms are represented.
++ Note that if the coefficient ring is a field, then this domain
++ forms a euclidean domain.

```

```

UnivariatePolynomial(x:Symbol, R:Ring):
  UnivariatePolynomialCategory(R) with
    coerce: Variable(x) -> %
      ++ coerce(x) converts the variable x to a univariate polynomial.
    fmech: (% , NonNegativeInteger, R, %) -> %
      ++ fmech(p1, e, r, p2) finds x : p1 - r * x**e * p2
  == SparseUnivariatePolynomial(R) add
  Rep:=SparseUnivariatePolynomial(R)
  coerce(p:%):OutputForm == outputForm(p, outputForm x)
  coerce(v:Variable(x)):% == monomial(1, 1)

```

## — UP.dotabb —

```

"UP" [color="#88FF44", href="bookvol10.3.pdf#nameddest=UP"]
"PFECAT" [color="#4488FF", href="bookvol10.2.pdf#nameddest=PFECAT"]
"UP" -> "PFECAT"

```

## 22.5 domain UPXS UnivariatePuisseuxSeries

— UnivariatePuisseuxSeries.input —

```
)set break resume
)sys rm -f UnivariatePuisseuxSeries.output
)spool UnivariatePuisseuxSeries.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show UnivariatePuisseuxSeries
--R UnivariatePuisseuxSeries(Coef: Ring,var: Symbol,cen: Coef) is a domain constructor
--R Abbreviation for UnivariatePuisseuxSeries is UPXS
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for UPXS
--R
--R----- Operations -----
--R ??? : (Coef,%) -> %          ??? : (%,Coef) -> %
--R ??? : (%,%) -> %            ??? : (Integer,%) -> %
--R ??? : (PositiveInteger,%) -> %  ??? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %            ?-? : (%,%) -> %
--R -? : % -> %                 ?=? : (%,%) -> Boolean
--R 1 : () -> %                 0 : () -> %
--R ??? : (%,PositiveInteger) -> % center : % -> Coef
--R coerce : Variable var -> %    coerce : Integer -> %
--R coerce : % -> OutputForm      complete : % -> %
--R degree : % -> Fraction Integer hash : % -> SingleInteger
--R latex : % -> String           leadingCoefficient : % -> Coef
--R leadingMonomial : % -> %      map : ((Coef -> Coef),%) -> %
--R monomial? : % -> Boolean      one? : % -> Boolean
--R order : % -> Fraction Integer pole? : % -> Boolean
--R recip : % -> Union(%, "failed") reductum : % -> %
--R sample : () -> %             variable : % -> Symbol
--R zero? : % -> Boolean         ?~=? : (%,%) -> Boolean
--R ??? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ??? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
--R ??? : (NonNegativeInteger,%) -> %
--R ??? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ??? : (%,%) -> % if Coef has ALGEBRA FRAC INT
--R ??? : (%,Integer) -> % if Coef has FIELD
--R ??? : (%,NonNegativeInteger) -> %
--R ?/? : (%,%) -> % if Coef has FIELD
--R ?/? : (%,Coef) -> % if Coef has FIELD
--R D : % -> % if Coef has *: (Fraction Integer,Coef) -> Coef
--R D : (%,NonNegativeInteger) -> % if Coef has *: (Fraction Integer,Coef) -> Coef
--R D : (%,Symbol) -> % if Coef has *: (Fraction Integer,Coef) -> Coef and Coef has PDRING SYMBOL
```



```

--R D : (% , List Symbol) -> % if Coef has *: (Fraction Integer, Coef) -> Coef and Coef has PDR
--R D : (% , Symbol, NonNegativeInteger) -> % if Coef has *: (Fraction Integer, Coef) -> Coef and
--R D : (% , List Symbol, List NonNegativeInteger) -> % if Coef has *: (Fraction Integer, Coef)
--R ?? : (% , Integer) -> % if Coef has FIELD
--R ?? : (% , NonNegativeInteger) -> %
--R acos : % -> % if Coef has ALGEBRA FRAC INT
--R acosh : % -> % if Coef has ALGEBRA FRAC INT
--R acot : % -> % if Coef has ALGEBRA FRAC INT
--R acoth : % -> % if Coef has ALGEBRA FRAC INT
--R acsc : % -> % if Coef has ALGEBRA FRAC INT
--R acsch : % -> % if Coef has ALGEBRA FRAC INT
--R approximate : (% , Fraction Integer) -> Coef if Coef has **: (Coef, Fraction Integer) -> Coef
--R asech : % -> % if Coef has ALGEBRA FRAC INT
--R asinh : % -> % if Coef has ALGEBRA FRAC INT
--R asin : % -> % if Coef has ALGEBRA FRAC INT
--R asinh : % -> % if Coef has ALGEBRA FRAC INT
--R associates? : (% , %) -> Boolean if Coef has INTDOM
--R atan : % -> % if Coef has ALGEBRA FRAC INT
--R atanh : % -> % if Coef has ALGEBRA FRAC INT
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(% , "failed") if Coef has CHARNZ
--R coefficient : (% , Fraction Integer) -> Coef
--R coerce : % -> % if Coef has INTDOM
--R coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
--R coerce : UnivariateTaylorSeries(Coef, var, cen) -> %
--R coerce : UnivariateLaurentSeries(Coef, var, cen) -> %
--R coerce : Coef -> % if Coef has COMRING
--R cos : % -> % if Coef has ALGEBRA FRAC INT
--R cosh : % -> % if Coef has ALGEBRA FRAC INT
--R cot : % -> % if Coef has ALGEBRA FRAC INT
--R coth : % -> % if Coef has ALGEBRA FRAC INT
--R csc : % -> % if Coef has ALGEBRA FRAC INT
--R csch : % -> % if Coef has ALGEBRA FRAC INT
--R differentiate : (% , Variable var) -> %
--R differentiate : % -> % if Coef has *: (Fraction Integer, Coef) -> Coef
--R differentiate : (% , NonNegativeInteger) -> % if Coef has *: (Fraction Integer, Coef) -> Coef
--R differentiate : (% , Symbol) -> % if Coef has *: (Fraction Integer, Coef) -> Coef and Coef has
--R differentiate : (% , List Symbol) -> % if Coef has *: (Fraction Integer, Coef) -> Coef and
--R differentiate : (% , Symbol, NonNegativeInteger) -> % if Coef has *: (Fraction Integer, Coef)
--R differentiate : (% , List Symbol, List NonNegativeInteger) -> % if Coef has *: (Fraction Integer, Coef)
--R divide : (% , %) -> Record(quotient: % , remainder: %) if Coef has FIELD
--R ?.? : (% , %) -> % if Fraction Integer has SGROUP
--R ?.? : (% , Fraction Integer) -> Coef
--R euclideanSize : % -> NonNegativeInteger if Coef has FIELD
--R eval : (% , Coef) -> Stream Coef if Coef has **: (Coef, Fraction Integer) -> Coef
--R exp : % -> % if Coef has ALGEBRA FRAC INT
--R expressIdealMember : (List % , %) -> Union(List % , "failed") if Coef has FIELD
--R exquo : (% , %) -> Union(% , "failed") if Coef has INTDOM
--R extend : (% , Fraction Integer) -> %
--R extendedEuclidean : (% , %) -> Record(coef1: % , coef2: % , generator: %) if Coef has FIELD

```

```

--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if Coef has FIELD
--R factor : % -> Factored % if Coef has FIELD
--R gcd : (%,%) -> % if Coef has FIELD
--R gcd : List % -> % if Coef has FIELD
--R gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolynomial
--R integrate : (%,Variable var) -> % if Coef has ALGEBRA FRAC INT
--R integrate : (%,Symbol) -> % if Coef has integrate: (Coef,Symbol) -> Coef and Coef has variables: Coef
--R integrate : % -> % if Coef has ALGEBRA FRAC INT
--R inv : % -> % if Coef has FIELD
--R laurent : % -> UnivariateLaurentSeries(Coef,var,cen)
--R laurentIfCan : % -> Union(UnivariateLaurentSeries(Coef,var,cen),"failed")
--R laurentRep : % -> UnivariateLaurentSeries(Coef,var,cen)
--R lcm : (%,%) -> % if Coef has FIELD
--R lcm : List % -> % if Coef has FIELD
--R log : % -> % if Coef has ALGEBRA FRAC INT
--R monomial : (%,List SingletonAsOrderedSet,List Fraction Integer) -> %
--R monomial : (%,SingletonAsOrderedSet,Fraction Integer) -> %
--R monomial : (Coef,Fraction Integer) -> %
--R multiEuclidean : (List %,%) -> Union(List %,"failed") if Coef has FIELD
--R multiplyExponents : (%,Fraction Integer) -> %
--R multiplyExponents : (%,PositiveInteger) -> %
--R nthRoot : (%,Integer) -> % if Coef has ALGEBRA FRAC INT
--R order : (%,Fraction Integer) -> Fraction Integer
--R pi : () -> % if Coef has ALGEBRA FRAC INT
--R prime? : % -> Boolean if Coef has FIELD
--R principalIdeal : List % -> Record(coef: List %,generator: %) if Coef has FIELD
--R puseux : (Fraction Integer,UnivariateLaurentSeries(Coef,var,cen)) -> %
--R ?quo? : (%,%) -> % if Coef has FIELD
--R rationalPower : % -> Fraction Integer
--R ?rem? : (%,%) -> % if Coef has FIELD
--R retract : % -> UnivariateTaylorSeries(Coef,var,cen)
--R retract : % -> UnivariateLaurentSeries(Coef,var,cen)
--R retractIfCan : % -> Union(UnivariateTaylorSeries(Coef,var,cen),"failed")
--R retractIfCan : % -> Union(UnivariateLaurentSeries(Coef,var,cen),"failed")
--R sec : % -> % if Coef has ALGEBRA FRAC INT
--R sech : % -> % if Coef has ALGEBRA FRAC INT
--R series : (NonNegativeInteger,Stream Record(k: Fraction Integer,c: Coef)) -> %
--R sin : % -> % if Coef has ALGEBRA FRAC INT
--R sinh : % -> % if Coef has ALGEBRA FRAC INT
--R sizeLess? : (%,%) -> Boolean if Coef has FIELD
--R sqrt : % -> % if Coef has ALGEBRA FRAC INT
--R squareFree : % -> Factored % if Coef has FIELD
--R squareFreePart : % -> % if Coef has FIELD
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tan : % -> % if Coef has ALGEBRA FRAC INT
--R tanh : % -> % if Coef has ALGEBRA FRAC INT
--R terms : % -> Stream Record(k: Fraction Integer,c: Coef)
--R truncate : (%,Fraction Integer,Fraction Integer) -> %
--R truncate : (%,Fraction Integer) -> %
--R unit? : % -> Boolean if Coef has INTDOM

```

```

--R unitCanonical : % -> % if Coef has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if Coef has INTDOM
--R variables : % -> List SingletonAsOrderedSet
--R
--E 1

)spool
)lisp (bye)

```

---

— UnivariatePuisseuxSeries.help —

```

=====
UnivariatePuisseuxSeries examples
=====

```

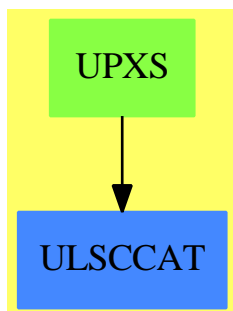
```

See Also:
o )show UnivariatePuisseuxSeries

```

---

### 22.5.1 UnivariatePuisseuxSeries (UPXS)



See

⇒ “UnivariatePuisseuxSeriesConstructor” (UPXSCONS) 22.6.1 on page 2798

**Exports:**

|                |                   |                    |                    |
|----------------|-------------------|--------------------|--------------------|
| 0              | 1                 | acos               | acosh              |
| acot           | acoth             | acsc               | acsch              |
| approximate    | asec              | asech              | asin               |
| asinh          | associates?       | atan               | atanh              |
| center         | characteristic    | charthRoot         | coefficient        |
| coerce         | complete          | cos                | cosh               |
| cot            | coth              | csc                | csch               |
| D              | degree            | differentiate      | divide             |
| euclideanSize  | eval              | exp                | expressIdealMember |
| exquo          | extend            | extendedEuclidean  | factor             |
| gcd            | gcdPolynomial     | hash               | integrate          |
| inv            | latex             | laurent            | laurentIfCan       |
| laurentRep     | lcm               | leadingCoefficient | leadingMonomial    |
| log            | map               | monomial           | monomial?          |
| multiEuclidean | multiplyExponents | nthRoot            | one?               |
| order          | pi                | pole?              | prime?             |
| principalIdeal | puiseux           | rationalPower      | recip              |
| reductum       | retract           | retractIfCan       | sample             |
| sec            | sech              | series             | sin                |
| sinh           | sizeLess?         | sqrt               | squareFree         |
| squareFreePart | subtractIfCan     | tan                | tanh               |
| terms          | truncate          | unit?              | unitCanonical      |
| unitNormal     | variable          | variables          | zero?              |
| ?*?            | **?               | ?+?                | ?-?                |
| -?             | ?=?               | ?^?                | ?~=?               |
| ?/?            | ?.?               | ?quo?              | ?rem?              |

— domain UPXS UnivariatePuisseuxSeries —

```

)abbrev domain UPXS UnivariatePuisseuxSeries
++ Author: Clifton J. Williamson
++ Date Created: 28 January 1990
++ Date Last Updated: 21 September 1993
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: series, Puiseux
++ Examples:
++ References:
++ Description:
++ Dense Puiseux series in one variable
++ \spadtype{UnivariatePuisseuxSeries} is a domain representing Puiseux
++ series in one variable with coefficients in an arbitrary ring. The
++ parameters of the type specify the coefficient ring, the power series
++ variable, and the center of the power series expansion. For example,
++ \spad{UnivariatePuisseuxSeries(Integer,x,3)} represents Puiseux series in

```

```

++ \spad{(x - 3)} with \spadtype{Integer} coefficients.

UnivariatePuisseuxSeries(Coef,var,cen): Exports == Implementation where
  Coef : Ring
  var  : Symbol
  cen  : Coef
  I    ==> Integer
  L    ==> List
  NNI  ==> NonNegativeInteger
  OUT  ==> OutputForm
  RN   ==> Fraction Integer
  ST   ==> Stream Coef
  UTS  ==> UnivariateTaylorSeries(Coef,var,cen)
  ULS  ==> UnivariateLaurentSeries(Coef,var,cen)

Exports ==> Join(UnivariatePuisseuxSeriesConstructorCategory(Coef,ULS),_
  RetractableTo UTS) with
  coerce: Variable(var) -> %
    ++ coerce(var) converts the series variable \spad{var} into a
    ++ Puiseux series.
  differentiate: (%,Variable(var)) -> %
    ++ \spad{differentiate(f(x),x)} returns the derivative of
    ++ \spad{f(x)} with respect to \spad{x}.
  if Coef has Algebra Fraction Integer then
    integrate: (%,Variable(var)) -> %
      ++ \spad{integrate(f(x))} returns an anti-derivative of the power
      ++ series \spad{f(x)} with constant coefficient 0.
      ++ We may integrate a series when we can divide coefficients
      ++ by integers.

Implementation ==> UnivariatePuisseuxSeriesConstructor(Coef,ULS) add

  Rep := Record(expon:RN,lSeries:ULS)

  getExpon: % -> RN
  getExpon pxs == pxs.expon

  variable upxs == var
  center   upxs == cen

  coerce(uts:UTS) == uts :: ULS :: %

  retractIfCan(upxs:%):Union(UTS,"failed") ==
    (ulsIfCan := retractIfCan(upxs)@Union(ULS,"failed")) case "failed" =>
      "failed"
    retractIfCan(ulsIfCan :: ULS)

  --retract(upxs:%):UTS ==
    --(ulsIfCan := retractIfCan(upxs)@Union(ULS,"failed")) case "failed" =>
      --error "retractIfCan: series has fractional exponents"

```

```

--utsIfCan := retractIfCan(ulsIfCan :: ULS)@Union(UTS,"failed")
--utsIfCan case "failed" =>
  --error "retractIfCan: series has negative exponents"
--utsIfCan :: UTS

coerce(v:Variable(var)) ==
  zero? cen => monomial(1,1)
  monomial(1,1) + monomial(cen,0)

if Coef has "(": (Fraction Integer, Coef) -> Coef then
  differentiate(upxs:%,v:Variable(var)) == differentiate upxs

if Coef has Algebra Fraction Integer then
  integrate(upxs:%,v:Variable(var)) == integrate upxs

if Coef has coerce: Symbol -> Coef then
  if Coef has "**": (Coef,RN) -> Coef then

    roundDown: RN -> I
    roundDown rn ==
      -- returns the largest integer <= rn
      (den := denom rn) = 1 => numer rn
      n := (num := numer rn) quo den
      positive?(num) => n
      n - 1

    stToCoef: (ST,Coef,NNI,NNI) -> Coef
    stToCoef(st,term,n,n0) ==
      (n > n0) or (empty? st) => 0
      first(st) * term ** n + stToCoef(rst st,term,n + 1,n0)

    approximateLaurent: (ULS,Coef,I) -> Coef
    approximateLaurent(x,term,n) ==
      (m := n - (e := degree x)) < 0 => 0
      app := stToCoef(coefficients taylorRep x,term,0,m :: NNI)
      zero? e => app
      app * term ** (e :: RN)

    approximate(x,r) ==
      e := rationalPower(x)
      term := ((variable(x) :: Coef) - center(x)) ** e
      approximateLaurent(laurentRep x,term,roundDown(r / e))

  termOutput: (RN,Coef,OUT) -> OUT
  termOutput(k,c,vv) ==
    -- creates a term c * vv ** k
    k = 0 => c :: OUT
    mon :=
      k = 1 => vv
      vv ** (k :: OUT)

```

```

c = 1 => mon
c = -1 => -mon
(c :: OUT) * mon

showAll?:() -> Boolean
-- check a global Lisp variable
showAll?() == true

termsToOutputForm:(RN,RN,ST,OUT) -> OUT
termsToOutputForm(m,rat,uu,xxx) ==
  l : L OUT := empty()
  empty? uu => 0 :: OUT
  n : NNI; count : NNI := _$streamCount$Lisp
  for n in 0..count while not empty? uu repeat
    if frst(uu) ^= 0 then
      l := concat(termOutput((n :: I) * rat + m,frst uu,xxx),l)
      uu := rst uu
  if showAll?() then
    for n in (count + 1).. while explicitEntries? uu and _
      not eq?(uu,rst uu) repeat
        if frst(uu) ^= 0 then
          l := concat(termOutput((n :: I) * rat + m,frst uu,xxx),l)
          uu := rst uu
  l :=
    explicitlyEmpty? uu => l
    eq?(uu,rst uu) and frst uu = 0 => l
    concat(prefix("0" :: OUT,[xxx ** ((n::I) * rat + m) :: OUT])),l)
  empty? l => 0 :: OUT
  reduce("+",reverse_! l)

coerce(upxs:%):OUT ==
  rat := getExpon upxs; uls := laurentRep upxs
  count : I := _$streamCount$Lisp
  uls := removeZeroes(_$streamCount$Lisp,uls)
  m : RN := (degree uls) * rat
  p := coefficients taylorRep uls
  xxx :=
    zero? cen => var :: OUT
    paren(var :: OUT - cen :: OUT)
  termsToOutputForm(m,rat,p,xxx)

```

---

— UPXS.dotabb —

"UPXS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=UPXS"]  
 "ULSCCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ULSCCAT"]  
 "UPXS" -> "ULSCCAT"

## 22.6 domain UPXSCONS UnivariatePuisseuxSeriesConstructor

— UnivariatePuisseuxSeriesConstructor.input —

```

)set break resume
)sys rm -f UnivariatePuisseuxSeriesConstructor.output
)spool UnivariatePuisseuxSeriesConstructor.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show UnivariatePuisseuxSeriesConstructor
--R UnivariatePuisseuxSeriesConstructor(Coef: Ring, ULS: UnivariateLaurentSeriesCategory Coef) is a domain
--R Abbreviation for UnivariatePuisseuxSeriesConstructor is UPXSCONS
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for UPXSCONS
--R
--R----- Operations -----
--R ?? : (Coef, %) -> %          ?? : (% , Coef) -> %
--R ?? : (% , %) -> %          ?? : (Integer, %) -> %
--R ?? : (PositiveInteger, %) -> %  ??? : (% , PositiveInteger) -> %
--R ?+? : (% , %) -> %          ?-? : (% , %) -> %
--R -? : % -> %                ?? : (% , %) -> Boolean
--R 1 : () -> %                0 : () -> %
--R ?? : (% , PositiveInteger) -> %  center : % -> Coef
--R coerce : ULS -> %            coerce : Integer -> %
--R coerce : % -> OutputForm      complete : % -> %
--R degree : % -> Fraction Integer  hash : % -> SingleInteger
--R latex : % -> String            laurent : % -> ULS
--R laurentRep : % -> ULS          leadingCoefficient : % -> Coef
--R leadingMonomial : % -> %        map : ((Coef -> Coef), %) -> %
--R monomial? : % -> Boolean        one? : % -> Boolean
--R order : % -> Fraction Integer   pole? : % -> Boolean
--R recip : % -> Union(%, "failed")  reductum : % -> %
--R retract : % -> ULS             sample : () -> %
--R variable : % -> Symbol          zero? : % -> Boolean
--R ?~=? : (% , %) -> Boolean
--R ?? : (% , Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?? : (Fraction Integer, %) -> % if Coef has ALGEBRA FRAC INT
--R ?? : (NonNegativeInteger, %) -> %

```



```

--R ??? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ??? : (%,%) -> % if Coef has ALGEBRA FRAC INT
--R ??? : (%,Integer) -> % if Coef has FIELD
--R ??? : (%,NonNegativeInteger) -> %
--R ?/? : (%,%) -> % if Coef has FIELD
--R ?/? : (%,Coef) -> % if Coef has FIELD
--R D : % -> % if Coef has *: (Fraction Integer,Coef) -> Coef
--R D : (%,NonNegativeInteger) -> % if Coef has *: (Fraction Integer,Coef) -> Coef
--R D : (%,Symbol) -> % if Coef has *: (Fraction Integer,Coef) -> Coef and Coef has PDRING S
--R D : (%,List Symbol) -> % if Coef has *: (Fraction Integer,Coef) -> Coef and Coef has PDR
--R D : (%,Symbol,NonNegativeInteger) -> % if Coef has *: (Fraction Integer,Coef) -> Coef and
--R D : (%,List Symbol,List NonNegativeInteger) -> % if Coef has *: (Fraction Integer,Coef)
--R ?? : (%,Integer) -> % if Coef has FIELD
--R ?? : (%,NonNegativeInteger) -> %
--R acos : % -> % if Coef has ALGEBRA FRAC INT
--R acosh : % -> % if Coef has ALGEBRA FRAC INT
--R acot : % -> % if Coef has ALGEBRA FRAC INT
--R acoth : % -> % if Coef has ALGEBRA FRAC INT
--R acsc : % -> % if Coef has ALGEBRA FRAC INT
--R acsch : % -> % if Coef has ALGEBRA FRAC INT
--R approximate : (%,Fraction Integer) -> Coef if Coef has **: (Coef,Fraction Integer) -> Coef
--R asec : % -> % if Coef has ALGEBRA FRAC INT
--R asech : % -> % if Coef has ALGEBRA FRAC INT
--R asin : % -> % if Coef has ALGEBRA FRAC INT
--R asinh : % -> % if Coef has ALGEBRA FRAC INT
--R associates? : (%,%) -> Boolean if Coef has INTDOM
--R atan : % -> % if Coef has ALGEBRA FRAC INT
--R atanh : % -> % if Coef has ALGEBRA FRAC INT
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%, "failed") if Coef has CHARNZ
--R coefficient : (%,Fraction Integer) -> Coef
--R coerce : % -> % if Coef has INTDOM
--R coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
--R coerce : Coef -> % if Coef has COMRING
--R cos : % -> % if Coef has ALGEBRA FRAC INT
--R cosh : % -> % if Coef has ALGEBRA FRAC INT
--R cot : % -> % if Coef has ALGEBRA FRAC INT
--R coth : % -> % if Coef has ALGEBRA FRAC INT
--R csc : % -> % if Coef has ALGEBRA FRAC INT
--R csch : % -> % if Coef has ALGEBRA FRAC INT
--R differentiate : % -> % if Coef has *: (Fraction Integer,Coef) -> Coef
--R differentiate : (%,NonNegativeInteger) -> % if Coef has *: (Fraction Integer,Coef) -> Coef
--R differentiate : (%,Symbol) -> % if Coef has *: (Fraction Integer,Coef) -> Coef and Coef L
--R differentiate : (%,List Symbol) -> % if Coef has *: (Fraction Integer,Coef) -> Coef and C
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if Coef has *: (Fraction Integer,Coef)
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if Coef has *: (Fraction Int
--R divide : (%,%) -> Record(quotient: %,remainder: %) if Coef has FIELD
--R ?.? : (%,%) -> % if Fraction Integer has SGROUP
--R ?.? : (%,Fraction Integer) -> Coef
--R euclideanSize : % -> NonNegativeInteger if Coef has FIELD

```

```

--R eval : (% , Coef) -> Stream Coef if Coef has **: (Coef, Fraction Integer) -> Coef
--R exp : % -> % if Coef has ALGEBRA FRAC INT
--R expressIdealMember : (List %, %) -> Union(List %, "failed") if Coef has FIELD
--R exquo : (% , %) -> Union(%, "failed") if Coef has INTDOM
--R extend : (% , Fraction Integer) -> %
--R extendedEuclidean : (% , %) -> Record(coef1: %, coef2: %, generator: %) if Coef has FIELD
--R extendedEuclidean : (% , %, %) -> Union(Record(coef1: %, coef2: %), "failed") if Coef has FIELD
--R factor : % -> Factored % if Coef has FIELD
--R gcd : (% , %) -> % if Coef has FIELD
--R gcd : List % -> % if Coef has FIELD
--R gcdPolynomial : (SparseUnivariatePolynomial %, SparseUnivariatePolynomial %) -> SparseUnivariatePolynomial %
--R integrate : (% , Symbol) -> % if Coef has integrate: (Coef, Symbol) -> Coef and Coef has variables: Coef
--R integrate : % -> % if Coef has ALGEBRA FRAC INT
--R inv : % -> % if Coef has FIELD
--R laurentIfCan : % -> Union(ULS, "failed")
--R lcm : (% , %) -> % if Coef has FIELD
--R lcm : List % -> % if Coef has FIELD
--R log : % -> % if Coef has ALGEBRA FRAC INT
--R monomial : (% , List SingletonAsOrderedSet, List Fraction Integer) -> %
--R monomial : (% , SingletonAsOrderedSet, Fraction Integer) -> %
--R monomial : (Coef, Fraction Integer) -> %
--R multiEuclidean : (List %, %) -> Union(List %, "failed") if Coef has FIELD
--R multiplyExponents : (% , Fraction Integer) -> %
--R multiplyExponents : (% , PositiveInteger) -> %
--R nthRoot : (% , Integer) -> % if Coef has ALGEBRA FRAC INT
--R order : (% , Fraction Integer) -> Fraction Integer
--R pi : () -> % if Coef has ALGEBRA FRAC INT
--R prime? : % -> Boolean if Coef has FIELD
--R principalIdeal : List % -> Record(coef: List %, generator: %) if Coef has FIELD
--R puioux : (Fraction Integer, ULS) -> %
--R ?quo? : (% , %) -> % if Coef has FIELD
--R rationalPower : % -> Fraction Integer
--R ?rem? : (% , %) -> % if Coef has FIELD
--R retractIfCan : % -> Union(ULS, "failed")
--R sec : % -> % if Coef has ALGEBRA FRAC INT
--R sech : % -> % if Coef has ALGEBRA FRAC INT
--R series : (NonNegativeInteger, Stream Record(k: Fraction Integer, c: Coef)) -> %
--R sin : % -> % if Coef has ALGEBRA FRAC INT
--R sinh : % -> % if Coef has ALGEBRA FRAC INT
--R sizeLess? : (% , %) -> Boolean if Coef has FIELD
--R sqrt : % -> % if Coef has ALGEBRA FRAC INT
--R squareFree : % -> Factored % if Coef has FIELD
--R squareFreePart : % -> % if Coef has FIELD
--R subtractIfCan : (% , %) -> Union(%, "failed")
--R tan : % -> % if Coef has ALGEBRA FRAC INT
--R tanh : % -> % if Coef has ALGEBRA FRAC INT
--R terms : % -> Stream Record(k: Fraction Integer, c: Coef)
--R truncate : (% , Fraction Integer, Fraction Integer) -> %
--R truncate : (% , Fraction Integer) -> %
--R unit? : % -> Boolean if Coef has INTDOM

```

```

--R unitCanonical : % -> % if Coef has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if Coef has INTDOM
--R variables : % -> List SingletonAsOrderedSet
--R
--E 1

)spool
)lisp (bye)

```

---

— UnivariatePuisseuxSeriesConstructor.help —

```

=====
UnivariatePuisseuxSeriesConstructor examples
=====

```

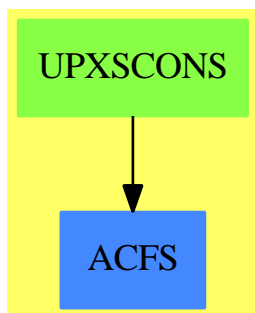
```

See Also:
o )show UnivariatePuisseuxSeriesConstructor

```

---

### 22.6.1 UnivariatePuisseuxSeriesConstructor (UPXSCONS)



See

⇒ “UnivariatePuisseuxSeries” (UPXS) 22.5.1 on page 2790

**Exports:**

|                |                   |                    |                    |
|----------------|-------------------|--------------------|--------------------|
| 0              | 1                 | acos               | acosh              |
| acot           | acoth             | acsc               | acsch              |
| approximate    | asec              | asech              | asin               |
| asinh          | associates?       | atan               | atanh              |
| center         | characteristic    | charthRoot         | coefficient        |
| coerce         | complete          | cos                | cosh               |
| cot            | coth              | csc                | csch               |
| D              | degree            | differentiate      | divide             |
| euclideanSize  | eval              | exp                | expressIdealMember |
| exquo          | extend            | extendedEuclidean  | factor             |
| gcd            | gcdPolynomial     | hash               | integrate          |
| inv            | latex             | laurent            | laurentIfCan       |
| laurentRep     | lcm               | leadingCoefficient | leadingMonomial    |
| log            | map               | monomial           | monomial?          |
| multiEuclidean | multiplyExponents | nthRoot            | one?               |
| order          | pi                | pole?              | prime?             |
| principalIdeal | puiseux           | rationalPower      | recip              |
| reductum       | retract           | retractIfCan       | sample             |
| sec            | sech              | series             | sin                |
| sinh           | sizeLess?         | sqrt               | squareFree         |
| squareFreePart | subtractIfCan     | tan                | tanh               |
| terms          | truncate          | unit?              | unitCanonical      |
| unitNormal     | variable          | variables          | zero?              |
| ?*?            | ***?              | ?+?                | ?-?                |
| -?             | ?=?               | ?^?                | ?~=?               |
| ?/?            | ??                | ?quo?              | ?rem?              |

— domain UPXSCONS UnivariatePuisseuxSeriesConstructor —

```

)abbrev domain UPXSCONS UnivariatePuisseuxSeriesConstructor
++ Author: Clifton J. Williamson
++ Date Created: 9 May 1989
++ Date Last Updated: 30 November 1994
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: series, Puiseux, Laurent
++ Examples:
++ References:
++ Description:
++ This package enables one to construct a univariate Puiseux series
++ domain from a univariate Laurent series domain. Univariate
++ Puiseux series are represented by a pair \spad{[r,f(x)]}, where r is
++ a positive rational number and \spad{f(x)} is a Laurent series.
++ This pair represents the Puiseux series \spad{f(x^r)}.

```

```

UnivariatePuisseuxSeriesConstructor(Coef,ULS):_
Exports == Implementation where
  Coef  : Ring
  ULS   : UnivariateLaurentSeriesCategory Coef
  I     ==> Integer
  L     ==> List
  NNI   ==> NonNegativeInteger
  OUT   ==> OutputForm
  PI    ==> PositiveInteger
  RN    ==> Fraction Integer
  ST    ==> Stream Coef
  LTerm ==> Record(k:I,c:Coef)
  PTerm ==> Record(k:RN,c:Coef)
  ST2LP ==> StreamFunctions2(LTerm,PTerm)
  ST2PL ==> StreamFunctions2(PTerm,LTerm)

Exports ==> UnivariatePuisseuxSeriesConstructorCategory(Coef,ULS)

Implementation ==> add

--% representation

Rep := Record(expon:RN,lSeries:ULS)

getExpon: % -> RN
getULS   : % -> ULS

getExpon pxs == pxs.expon
getULS   pxs == pxs.lSeries

--% creation and destruction

puisseux(n,ls) == [n,ls]
laurentRep x   == getULS x
rationalPower x == getExpon x
degree x       == getExpon(x) * degree(getULS(x))

0 == puisseux(1,0)
1 == puisseux(1,1)

monomial(c,k) ==
  k = 0 => c :: %
  k < 0 => puisseux(-k,monomial(c,-1))
  puisseux(k,monomial(c,1))

coerce(ls:ULS) == puisseux(1,ls)
coerce(r:Coef) == r :: ULS :: %
coerce(i:I)    == i :: Coef :: %

laurentIfCan upxs ==

```

```

    r := getExpon upxs
--    one? denom r =>
    (denom r) = 1 =>
        multiplyExponents(getULS upxs,numer(r) :: PI)
    "failed"

laurent upxs ==
    (uls := laurentIfCan upxs) case "failed" =>
        error "laurent: Puiseux series has fractional powers"
    uls :: ULS

multExp: (RN,LTerm) -> PTerm
multExp(r,lTerm) == [r * lTerm.k,lTerm.c]

terms upxs ==
    map((t1:LTerm):PTerm+>multExp(getExpon upxs,t1),terms getULS upxs)$ST2LP

clearDen: (I,PTerm) -> LTerm
clearDen(n,lTerm) ==
    (int := retractIfCan(n * lTerm.k@Union(I,"failed"))) case "failed" =>
        error "series: inappropriate denominator"
    [int :: I,lTerm.c]

series(n,stream) ==
    str := map((t1:PTerm):LTerm +-> clearDen(n,t1),stream)$ST2PL
    puiseux(1/n,series str)

--% normalizations

rewrite:(%,PI) -> %
rewrite(upxs,m) ==
    -- rewrites a series in x**r as a series in x**(r/m)
    puiseux((getExpon upxs)*(1/m),multiplyExponents(getULS upxs,m))

ratGcd: (RN,RN) -> RN
ratGcd(r1,r2) ==
    -- if r1 = prod(p prime,p ** ep(r1)) and
    -- if r2 = prod(p prime,p ** ep(r2)), then
    -- ratGcd(r1,r2) = prod(p prime,p ** min(ep(r1),ep(r2)))
    gcd(numer r1,numer r2) / lcm(denom r1,denom r2)

withNewExpon:(%,RN) -> %
withNewExpon(upxs,r) ==
    rewrite(upxs,numer(getExpon(upxs)/r) pretend PI)

--% predicates

upxs1 = upxs2 ==
    r1 := getExpon upxs1; r2 := getExpon upxs2
    ls1 := getULS upxs1; ls2 := getULS upxs2

```

```

(r1 = r2) => (ls1 = ls2)
r := ratGcd(r1,r2)
m1 := numer(getExpon(upxs1)/r) pretend PI
m2 := numer(getExpon(upxs2)/r) pretend PI
multiplyExponents(ls1,m1) = multiplyExponents(ls2,m2)

pole? upxs == pole? getULS upxs

--% arithmetic

applyFcn:((ULS,ULS) -> ULS,%,%) -> %
applyFcn(op,pxs1,pxs2) ==
  r1 := getExpon pxs1; r2 := getExpon pxs2
  ls1 := getULS pxs1; ls2 := getULS pxs2
  (r1 = r2) => puiex(r1,op(ls1,ls2))
  r := ratGcd(r1,r2)
  m1 := numer(getExpon(pxs1)/r) pretend PI
  m2 := numer(getExpon(pxs2)/r) pretend PI
  puiex(r,op(multiplyExponents(ls1,m1),multiplyExponents(ls2,m2)))

pxs1 + pxs2      == applyFcn((z1:ULS,z2:ULS):ULS+>z1 +$ULS z2,pxs1,pxs2)
pxs1 - pxs2      == applyFcn((z1:ULS,z2:ULS):ULS+>z1 -$ULS z2,pxs1,pxs2)
pxs1:% * pxs2:% == applyFcn((z1:ULS,z2:ULS):ULS+>z1 *$ULS z2,pxs1,pxs2)

pxs:% ** n:NNI == puiex(getExpon pxs,getULS(pxs)**n)

recip pxs ==
  rec := recip getULS pxs
  rec case "failed" => "failed"
  puiex(getExpon pxs,rec :: ULS)

RATALG : Boolean := Coef has Algebra(Fraction Integer)

elt(upxs1:%,upxs2:%) ==
  uls1 := laurentRep upxs1; uls2 := laurentRep upxs2
  r1 := rationalPower upxs1; r2 := rationalPower upxs2
  (n := retractIfCan(r1)@Union(Integer,"failed")) case Integer =>
    puiex(r2,uls1(uls2 ** r1))
  RATALG =>
    if zero? (coef := coefficient(uls2,deg := degree uls2)) then
      deg := order(uls2,deg + 1000)
      zero? (coef := coefficient(uls2,deg)) =>
        error "elt: series with many leading zero coefficients"
      -- a fractional power of a Laurent series may not be defined:
      -- if f(x) = c * x**n + ..., then f(x) ** (p/q) will be defined
      -- only if q divides n
      b := lcm(denom r1,deg); c := b quo deg
      mon : ULS := monomial(1,c)
      uls2 := elt(uls2,mon) ** r1
      puiex(r2*(1/c),elt(uls1,uls2))

```

```

    error "elt: rational powers not available for this coefficient domain"

if Coef has "**": (Coef,Integer) -> Coef and
  Coef has "**": (Coef, RN) -> Coef then
    eval(upxs:%,a:Coef) == eval(getULS upxs,a ** getExpon(upxs))

if Coef has Field then

  pxs1:% / pxs2:% == applyFcn((z1:ULS,z2:ULS):ULS+>->z1 /$ULS z2,pxs1,pxs2)

  inv upxs ==
    (invUpxs := recip upxs) case "failed" =>
      error "inv: multiplicative inverse does not exist"
    invUpxs :: %

--% values

variable upxs == variable getULS upxs
center upxs == center getULS upxs

coefficient(upxs,rn) ==
-- one? denom(n := rn / getExpon upxs) =>
  (denom(n := rn / getExpon upxs) = 1 =>
    coefficient(getULS upxs,numer n)
  0

elt(upxs:%,rn:RN) == coefficient(upxs,rn)

--% other functions

roundDown: RN -> I
roundDown rn ==
  -- returns the largest integer <= rn
  (den := denom rn) = 1 => numer rn
  n := (num := numer rn) quo den
  positive?(num) => n
  n - 1

roundUp: RN -> I
roundUp rn ==
  -- returns the smallest integer >= rn
  (den := denom rn) = 1 => numer rn
  n := (num := numer rn) quo den
  positive?(num) => n + 1
  n

order upxs == getExpon upxs * order getULS upxs
order(upxs,r) ==
  e := getExpon upxs
  ord := order(getULS upxs, n := roundDown(r / e))

```



```

ord = n => r
ord * e

truncate(upxs,r) ==
  e := getExpon upxs
  puiseux(e,truncate(getULS upxs,roundDown(r / e)))

truncate(upxs,r1,r2) ==
  e := getExpon upxs
  puiseux(e,truncate(getULS upxs,roundUp(r1 / e),roundDown(r2 / e)))

complete upxs == puiseux(getExpon upxs,complete getULS upxs)
extend(upxs,r) ==
  e := getExpon upxs
  puiseux(e,extend(getULS upxs,roundDown(r / e)))

map(fcn,upxs) == puiseux(getExpon upxs,map(fcn,getULS upxs))

characteristic() == characteristic()$Coef

-- multiplyCoefficients(f,upxs) ==
-- r := getExpon upxs
-- puiseux(r,multiplyCoefficients(f(#1 * r),getULS upxs))

multiplyExponents(upxs:%,n:RN) ==
  puiseux(n * getExpon(upxs),getULS upxs)
multiplyExponents(upxs:%,n:PI) ==
  puiseux(n * getExpon(upxs),getULS upxs)

if Coef has "*" : (Fraction Integer, Coef) -> Coef then

  differentiate upxs ==
    r := getExpon upxs
    puiseux(r,differentiate getULS upxs) * monomial(r :: Coef,r-1)

  if Coef has PartialDifferentialRing(Symbol) then

    differentiate(upxs:%,s:Symbol) ==
      (s = variable(upxs)) => differentiate upxs
      dcds := differentiate(center upxs,s)
      map((z1:Coef):Coef+>differentiate(z1,s),upxs)
      - dcds*differentiate(upxs)

  if Coef has Algebra Fraction Integer then

    coerce(r:RN) == r :: Coef :: %

    ratInv: RN -> Coef
    ratInv r ==
      zero? r => 1

```

```

inv(r) :: Coef

integrate upxs ==
  not zero? coefficient(upxs,-1) =>
    error "integrate: series has term of order -1"
  r := getExpon upxs
  uls := getULS upxs
  uls := multiplyCoefficients((z1:Integer):Coef+>->ratInv(z1*r+1),uls)
  monomial(1,1) * puisseux(r,uls)

if Coef has integrate: (Coef,Symbol) -> Coef and _
  Coef has variables: Coef -> List Symbol then

  integrate(upxs:%,s:Symbol) ==
    (s = variable(upxs)) => integrate upxs
    not entry?(s,variables center upxs)
    => map((z1:Coef):Coef +>-> integrate(z1,s),upxs)
    error "integrate: center is a function of variable of integration"

if Coef has TranscendentalFunctionCategory and _
  Coef has PrimitiveFunctionCategory and _
  Coef has AlgebraicallyClosedFunctionSpace Integer then

  integrateWithOneAnswer: (Coef,Symbol) -> Coef
  integrateWithOneAnswer(f,s) ==
    res := integrate(f,s)$FunctionSpaceIntegration(I,Coef)
    res case Coef => res :: Coef
    first(res :: List Coef)

  integrate(upxs:%,s:Symbol) ==
    (s = variable(upxs)) => integrate upxs
    not entry?(s,variables center upxs) =>
      map((z1:Coef):Coef +>-> integrateWithOneAnswer(z1,s),upxs)
      error "integrate: center is a function of variable of integration"

if Coef has Field then
  (upxs:%) ** (q:RN) ==
    num := numer q; den := denom q
    one? den => upxs ** num
    den = 1 => upxs ** num
    r := rationalPower upxs; uls := laurentRep upxs
    deg := degree uls
    if zero?(coef := coefficient(uls,deg)) then
      deg := order(uls,deg + 1000)
      zero?(coef := coefficient(uls,deg)) =>
        error "power of series with many leading zero coefficients"
    ulsPow := (uls * monomial(1,-deg)$ULS) ** q
    puisseux(r,ulsPow) * monomial(1,deg*q*r)

applyUnary: (ULS -> ULS,%) -> %

```

```

applyUnary(fcn,upxs) ==
  puseux(rationalPower upxs,fcn laurentRep upxs)

exp upxs == applyUnary(exp,upxs)
log upxs == applyUnary(log,upxs)
sin upxs == applyUnary(sin,upxs)
cos upxs == applyUnary(cos,upxs)
tan upxs == applyUnary(tan,upxs)
cot upxs == applyUnary(cot,upxs)
sec upxs == applyUnary(sec,upxs)
csc upxs == applyUnary(csc,upxs)
asin upxs == applyUnary(asin,upxs)
acos upxs == applyUnary(acos,upxs)
atan upxs == applyUnary(atan,upxs)
acot upxs == applyUnary(acot,upxs)
asec upxs == applyUnary(asec,upxs)
acsc upxs == applyUnary(acsc,upxs)
sinh upxs == applyUnary(sinh,upxs)
cosh upxs == applyUnary(cosh,upxs)
tanh upxs == applyUnary(tanh,upxs)
coth upxs == applyUnary(coth,upxs)
sech upxs == applyUnary(sech,upxs)
csch upxs == applyUnary(csch,upxs)
asinh upxs == applyUnary(asinh,upxs)
acosh upxs == applyUnary(acosh,upxs)
atanh upxs == applyUnary(atanh,upxs)
acoth upxs == applyUnary(acoth,upxs)
asech upxs == applyUnary(asech,upxs)
acsch upxs == applyUnary(acsch,upxs)

```

---

— UPXSCONS.dotabb —

```

"UPXSCONS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=UPXSCONS"]
"ACFS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ACFS"]
"UPXSCONS" -> "ACFS"

```

---

## 22.7 domain UPXSING UnivariatePuseuxSeriesWith-ExponentialSingularity

— UnivariatePuseuxSeriesWithExponentialSingularity.input —

```

)set break resume
)sys rm -f UnivariatePuisseuxSeriesWithExponentialSingularity.output
)spool UnivariatePuisseuxSeriesWithExponentialSingularity.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show UnivariatePuisseuxSeriesWithExponentialSingularity
--R UnivariatePuisseuxSeriesWithExponentialSingularity(R: Join(OrderedSet,RetractableTo Integer,LinearlyE
--R Abbreviation for UnivariatePuisseuxSeriesWithExponentialSingularity is UPXSSING
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for UPXSSING
--R
--R----- Operations -----
--R ?? : (% ,%) -> %
--R ?? : (PositiveInteger,% ) -> %
--R ?+? : (% ,%) -> %
--R -? : % -> %
--R 1 : () -> %
--R ?? : (% ,PositiveInteger) -> %
--R coerce : % -> %
--R coerce : % -> OutputForm
--R hash : % -> SingleInteger
--R leadingMonomial : % -> %
--R one? : % -> Boolean
--R reductum : % -> %
--R unit? : % -> Boolean
--R zero? : % -> Boolean
--R ?? : (% ,Fraction Integer) -> % if UnivariatePuisseuxSeries(FE,var,cen) has ALGEBRA FRAC INT
--R ?? : (Fraction Integer,% ) -> % if UnivariatePuisseuxSeries(FE,var,cen) has ALGEBRA FRAC INT
--R ?? : (UnivariatePuisseuxSeries(FE,var,cen),%) -> %
--R ?? : (% ,UnivariatePuisseuxSeries(FE,var,cen)) -> %
--R ?? : (NonNegativeInteger,% ) -> %
--R ??? : (% ,NonNegativeInteger) -> %
--R ?/? : (% ,UnivariatePuisseuxSeries(FE,var,cen)) -> % if UnivariatePuisseuxSeries(FE,var,cen) has FIELD
--R ?? : (% ,NonNegativeInteger) -> %
--R binomThmExpt : (% ,%,NonNegativeInteger) -> % if UnivariatePuisseuxSeries(FE,var,cen) has COMRING
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(% , "failed") if UnivariatePuisseuxSeries(FE,var,cen) has CHARNZ
--R coefficient : (% ,ExponentialOfUnivariatePuisseuxSeries(FE,var,cen)) -> UnivariatePuisseuxSeries(FE,var,cen)
--R coefficients : % -> List UnivariatePuisseuxSeries(FE,var,cen)
--R coerce : Fraction Integer -> % if UnivariatePuisseuxSeries(FE,var,cen) has ALGEBRA FRAC INT or UnivariatePuisseuxSeries(FE,var,cen)
--R coerce : UnivariatePuisseuxSeries(FE,var,cen) -> %
--R content : % -> UnivariatePuisseuxSeries(FE,var,cen) if UnivariatePuisseuxSeries(FE,var,cen) has GCDDOM
--R degree : % -> ExponentialOfUnivariatePuisseuxSeries(FE,var,cen)
--R dominantTerm : % -> Union(Record(%coef: UnivariatePuisseuxSeries(FE,var,cen),%expon: ExponentialOfUnivariatePuisseuxSeries(FE,var,cen)) , "failed")
--R exquo : (% ,%) -> Union(% , "failed")
--R exquo : (% ,UnivariatePuisseuxSeries(FE,var,cen)) -> Union(% , "failed") if UnivariatePuisseuxSeries(FE,var,cen)
--R ground : % -> UnivariatePuisseuxSeries(FE,var,cen)

```

```

--R leadingCoefficient : % -> UnivariatePuisseuxSeries(FE,var,cen)
--R limitPlus : % -> Union(OrderedCompletion FE,"failed")
--R map : ((UnivariatePuisseuxSeries(FE,var,cen) -> UnivariatePuisseuxSeries(FE,var,cen)),%) ->
--R mapExponents : ((ExponentialOfUnivariatePuisseuxSeries(FE,var,cen) -> ExponentialOfUnivariatePuisseuxSeries(FE,var,cen)),%) ->
--R minimumDegree : % -> ExponentialOfUnivariatePuisseuxSeries(FE,var,cen)
--R monomial : (UnivariatePuisseuxSeries(FE,var,cen),ExponentialOfUnivariatePuisseuxSeries(FE,var,cen)) ->
--R numberOfMonomials : % -> NonNegativeInteger
--R pomopo! : (% ,UnivariatePuisseuxSeries(FE,var,cen),ExponentialOfUnivariatePuisseuxSeries(FE,var,cen)) ->
--R primitivePart : % -> % if UnivariatePuisseuxSeries(FE,var,cen) has GCDDOM
--R retract : % -> UnivariatePuisseuxSeries(FE,var,cen)
--R retract : % -> Fraction Integer if UnivariatePuisseuxSeries(FE,var,cen) has RETRACT FRAC
--R retract : % -> Integer if UnivariatePuisseuxSeries(FE,var,cen) has RETRACT INT
--R retractIfCan : % -> Union(UnivariatePuisseuxSeries(FE,var,cen),"failed")
--R retractIfCan : % -> Union(Fraction Integer,"failed") if UnivariatePuisseuxSeries(FE,var,cen) has RETRACT FRAC
--R retractIfCan : % -> Union(Integer,"failed") if UnivariatePuisseuxSeries(FE,var,cen) has RETRACT INT
--R subtractIfCan : (% ,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

```

```

)spool
)lisp (bye)

```

---

— UnivariatePuisseuxSeriesWithExponentialSingularity.help —

```

=====
UnivariatePuisseuxSeriesWithExponentialSingularity examples
=====

```

See Also:

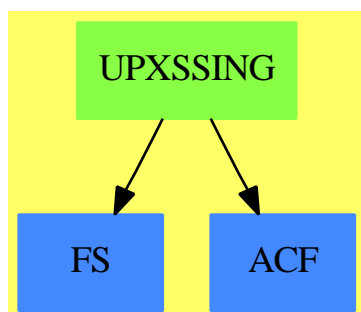
```

o )show UnivariatePuisseuxSeriesWithExponentialSingularity

```

---

### 22.7.1 UnivariatePuisseuxSeriesWithExponentialSingularity (UPXSSING)



See

⇒ “ExponentialOfUnivariatePuisseuxSeries” (EXPUPXS) 6.7.1 on page 707

⇒ “ExponentialExpansion” (EXPEXPAN) 6.5.1 on page 679

#### Exports:

|                   |               |                    |                 |                |
|-------------------|---------------|--------------------|-----------------|----------------|
| 0                 | 1             | associates?        | binomThmExpt    | characteristic |
| charthRoot        | coefficient   | coefficients       | coerce          | content        |
| degree            | dominantTerm  | exquo              | ground          | ground?        |
| hash              | latex         | leadingCoefficient | leadingMonomial | limitPlus      |
| map               | mapExponents  | minimumDegree      | monomial        | monomial?      |
| numberOfMonomials | one?          | pomopo!            | primitivePart   | recip          |
| reductum          | retract       | retractIfCan       | sample          | subtractIfCan  |
| unit?             | unitCanonical | unitNormal         | zero?           | ?*?            |
| ?*?*              | ?+?           | ?-?                | -?              | ?=?            |
| ?^?               | ?~=?          | ?/?                |                 |                |

— domain UPXSSING UnivariatePuisseuxSeriesWithExponentialSingularity —

```

)abbrev domain UPXSSING UnivariatePuisseuxSeriesWithExponentialSingularity
++ Author: Clifton J. Williamson
++ Date Created: 4 August 1992
++ Date Last Updated: 27 August 1992
++ Basic Operations:
++ Related Domains: UnivariatePuisseuxSeries(FE,var,cen),
++                  ExponentialOfUnivariatePuisseuxSeries(FE,var,cen)
++                  ExponentialExpansion(R,FE,var,cen)
++ Also See:
++ AMS Classifications:
++ Keywords: limit, functional expression, power series
++ Examples:
++ References:
++ Description:
++ UnivariatePuisseuxSeriesWithExponentialSingularity is a domain used to
  
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++ represent functions with essential singularities. Objects in this
++ domain are sums, where each term in the sum is a univariate Puiseux
++ series times the exponential of a univariate Puiseux series. Thus,
++ the elements of this domain are sums of expressions of the form
++ \spad{g(x) * exp(f(x))}, where g(x) is a univariate Puiseux series
++ and f(x) is a univariate Puiseux series with no terms of non-negative
++ degree.

UnivariatePuisseuxSeriesWithExponentialSingularity(R,FE,var,cen):_
  Exports == Implementation where
  R : Join(OrderedSet,RetractableTo Integer,_
          LinearlyExplicitRingOver Integer,GcdDomain)
  FE : Join(AlgebraicallyClosedField,TranscendentalFunctionCategory,_
          FunctionSpace R)
  var : Symbol
  cen : FE
  B ==> Boolean
  I ==> Integer
  L ==> List
  RN ==> Fraction Integer
  UPXS ==> UnivariatePuisseuxSeries(FE,var,cen)
  EXPUPXS ==> ExponentialOfUnivariatePuisseuxSeries(FE,var,cen)
  OFE ==> OrderedCompletion FE
  Result ==> Union(OFE,"failed")
  PxRec ==> Record(k: Fraction Integer,c:FE)
  Term ==> Record(%coef:UPXS,%expon:EXPUPXS,%expTerms:List PxRec)
    -- the %expTerms field is used to record the list of the terms (a 'term'
    -- records an exponent and a coefficient) in the exponent %expon
  TypedTerm ==> Record(%term:Term,%type:String)
    -- a term together with a String which tells whether it has an infinite,
    -- zero, or unknown limit as var -> cen+
  TRec ==> Record(%zeroTerms: List Term,_
                %infiniteTerms: List Term,_
                %failedTerms: List Term,_
                %puiseuxSeries: UPXS)
  SIGNED ==> ElementaryFunctionSign(R,FE)

Exports ==> Join(FiniteAbelianMonoidRing(UPXS,EXPUPXS),IntegralDomain) with
  limitPlus : % -> Union(OFE,"failed")
    ++ limitPlus(f(var)) returns \spad{limit(var -> cen+,f(var))}.
  dominantTerm : % -> Union(TypedTerm,"failed")
    ++ dominantTerm(f(var)) returns the term that dominates the limiting
    ++ behavior of \spad{f(var)} as \spad{var -> cen+} together with a
    ++ \spadtype{String} which briefly describes that behavior. The
    ++ value of the \spadtype{String} will be \spad{"zero"} (resp.
    ++ \spad{"infinity"}) if the term tends to zero (resp. infinity)
    ++ exponentially and will \spad{"series"} if the term is a
    ++ Puiseux series.

Implementation ==> PolynomialRing(UPXS,EXPUPXS) add

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makeTerm : (UPXS,EXPUPXS) -> Term
coeff : Term -> UPXS
exponent : Term -> EXPUPXS
exponentTerms : Term -> List PxRec
setExponentTerms_! : (Term,List PxRec) -> List PxRec
computeExponentTerms_! : Term -> List PxRec
terms : % -> List Term
sortAndDiscardTerms: List Term -> TRec
termsWithExtremeLeadingCoef : (L Term,RN,I) -> Union(L Term,"failed")
filterByOrder: (L Term,(RN,RN) -> B) -> Record(%list:L Term,%order:RN)
dominantTermOnList : (L Term,RN,I) -> Union(Term,"failed")
iDominantTerm : L Term -> Union(Record(%term:Term,%type:String),"failed")

retractIfCan f ==
  (numberOfMonomials f = 1) and (zero? degree f) => leadingCoefficient f
  "failed"

recip f ==
  numberOfMonomials f = 1 =>
    monomial(inv leadingCoefficient f,- degree f)
  "failed"

makeTerm(coef,expon) == [coef,expon,empty()]
coeff term == term.%coef
exponent term == term.%expon
exponentTerms term == term.%expTerms
setExponentTerms_!(term,list) == term.%expTerms := list
computeExponentTerms_! term ==
  setExponentTerms_!(term,entries complete terms exponent term)

terms f ==
  -- terms with a higher order singularity will appear closer to the
  -- beginning of the list because of the ordering in EXPPUPXS;
  -- no "exponent terms" are computed by this function
  zero? f => empty()
  concat(makeTerm(leadingCoefficient f,degree f),terms reductum f)

sortAndDiscardTerms termList ==
  -- 'termList' is the list of terms of some function f(var), ordered
  -- so that terms with a higher order singularity occur at the
  -- beginning of the list.
  -- This function returns lists of candidates for the "dominant
  -- term" in 'termList', i.e. the term which describes the
  -- asymptotic behavior of f(var) as var -> cen+.
  -- 'zeroTerms' will contain terms which tend to zero exponentially
  -- and contains only those terms with the lowest order singularity.
  -- 'zeroTerms' will be non-empty only when there are no terms of
  -- infinite or series type.
  -- 'infiniteTerms' will contain terms which tend to infinity
  -- exponentially and contains only those terms with the highest

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-- order singularity.
-- 'failedTerms' will contain terms which have an exponential
-- singularity, where we cannot say whether the limiting value
-- is zero or infinity. Only terms with a higher order singularity
-- than the terms on 'infiniteList' are included.
-- 'pSeries' will be a Puiseux series representing a term without an
-- exponential singularity. 'pSeries' will be non-zero only when no
-- other terms are known to tend to infinity exponentially
zeroTerms : List Term := empty()
infiniteTerms : List Term := empty()
failedTerms : List Term := empty()
-- we keep track of whether or not we've found an infinite term
-- if so, 'infTermOrd' will be set to a negative value
infTermOrd : RN := 0
-- we keep track of whether or not we've found a zero term
-- if so, 'zeroTermOrd' will be set to a negative value
zeroTermOrd : RN := 0
ord : RN := 0; pSeries : UPXS := 0 -- dummy values
while not empty? termList repeat
  -- 'expon' is a Puiseux series
  expon := exponent(term := first termList)
  -- quit if there is an infinite term with a higher order singularity
  (ord := order(expon,0)) > infTermOrd => leave "infinite term dominates"
  -- if ord = 0, we've hit the end of the list
  (ord = 0) =>
    -- since we have a series term, don't bother with zero terms
    leave(pSeries := coeff(term); zeroTerms := empty())
  coef := coefficient(expon,ord)
  -- if we can't tell if the lowest order coefficient is positive or
  -- negative, we have a "failed term"
  (signum := sign(coef)$SIGNEF) case "failed" =>
    failedTerms := concat(term,failedTerms)
    termList := rest termList
  -- if the lowest order coefficient is positive, we have an
  -- "infinite term"
  (sig := signum :: Integer) = 1 =>
    infTermOrd := ord
    infiniteTerms := concat(term,infiniteTerms)
    -- since we have an infinite term, don't bother with zero terms
    zeroTerms := empty()
    termList := rest termList
  -- if the lowest order coefficient is negative, we have a
  -- "zero term" if there are no infinite terms and no failed
  -- terms, add the term to 'zeroTerms'
  if empty? infiniteTerms then
    zeroTerms :=
      ord = zeroTermOrd => concat(term,zeroTerms)
      zeroTermOrd := ord
      list term
  termList := rest termList

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-- reverse "failed terms" so that higher order singularities
-- appear at the beginning of the list
[zeroTerms,infiniteTerms,reverse_! failedTerms,pSeries]

termsWithExtremeLeadingCoef(termList,ord,signum) ==
-- 'termList' consists of terms of the form [g(x),exp(f(x)),...];
-- when 'signum' is +1 (resp. -1), this function filters 'termList'
-- leaving only those terms such that coefficient(f(x),ord) is
-- maximal (resp. minimal)
while (coefficient(exponent first termList,ord) = 0) repeat
  termList := rest termList
empty? termList => error "UPXSSING: can't happen"
coefExtreme := coefficient(exponent first termList,ord)
outList := list first termList; termList := rest termList
for term in termList repeat
  (coefDiff := coefficient(exponent term,ord) - coefExtreme) = 0 =>
    outList := concat(term,outList)
  (sig := sign(coefDiff)$SIGNEF) case "failed" => return "failed"
  (sig :: Integer) = signum => outList := list term
outList

filterByOrder(termList,predicate) ==
-- 'termList' consists of terms of the form [g(x),exp(f(x)),expTerms],
-- where 'expTerms' is a list containing some of the terms in the
-- series f(x).
-- The function filters 'termList' and, when 'predicate' is < (resp. >),
-- leaves only those terms with the lowest (resp. highest) order term
-- in 'expTerms'
while empty? exponentTerms first termList repeat
  termList := rest termList
  empty? termList => error "UPXSING: can't happen"
ordExtreme := (first exponentTerms first termList).k
outList := list first termList
for term in rest termList repeat
  not empty? exponentTerms term =>
    (ord := (first exponentTerms term).k) = ordExtreme =>
      outList := concat(term,outList)
    predicate(ord,ordExtreme) =>
      ordExtreme := ord
      outList := list term
-- advance pointers on "exponent terms" on terms on 'outList'
for term in outList repeat
  setExponentTerms_!(term,rest exponentTerms term)
[outList,ordExtreme]

dominantTermOnList(termList,ord0,signum) ==
-- finds dominant term on 'termList'
-- it is known that "exponent terms" of order < 'ord0' are
-- the same for all terms on 'termList'
newList := termsWithExtremeLeadingCoef(termList,ord0,signum)

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newList case "failed" => "failed"
termList := newList :: List Term
empty? rest termList => first termList
filtered :=
  signum = 1 => filterByOrder(termList,(x,y) +-> x < y)
  filterByOrder(termList,(x,y) +-> x > y)
termList := filtered.%list
empty? rest termList => first termList
dominantTermOnList(termList,filtered.%order,signum)

iDominantTerm termList ==
  termRecord := sortAndDiscardTerms termList
  zeroTerms := termRecord.%zeroTerms
  infiniteTerms := termRecord.%infiniteTerms
  failedTerms := termRecord.%failedTerms
  pSeries := termRecord.%puiseuxSeries
  -- in future versions, we will deal with "failed terms"
  -- at present, if any occur, we cannot determine the limit
  not empty? failedTerms => "failed"
  not zero? pSeries => [makeTerm(pSeries,0),"series"]
  not empty? infiniteTerms =>
    empty? rest infiniteTerms => [first infiniteTerms,"infinity"]
    for term in infiniteTerms repeat computeExponentTerms_! term
    ord0 := order exponent first infiniteTerms
    (dTerm := dominantTermOnList(infiniteTerms,ord0,1)) case "failed" =>
      return "failed"
    [dTerm :: Term,"infinity"]
  empty? rest zeroTerms => [first zeroTerms,"zero"]
  for term in zeroTerms repeat computeExponentTerms_! term
  ord0 := order exponent first zeroTerms
  (dTerm := dominantTermOnList(zeroTerms,ord0,-1)) case "failed" =>
    return "failed"
  [dTerm :: Term,"zero"]

dominantTerm f == iDominantTerm terms f

limitPlus f ==
  -- list the terms occurring in 'f'; if there are none, then f = 0
  empty?(termList := terms f) => 0
  -- compute dominant term
  (tInfo := iDominantTerm termList) case "failed" => "failed"
  termInfo := tInfo :: Record(%term:Term,%type:String)
  domTerm := termInfo.%term
  (type := termInfo.%type) = "series" =>
    -- find limit of series term
    (ord := order(pSeries := coeff domTerm,1)) > 0 => 0
    coef := coefficient(pSeries,ord)
    member?(var,variables coef) => "failed"
    ord = 0 => coef :: OFE
    -- in the case of an infinite limit, we need to know the sign

```

```

-- of the first non-zero coefficient
(signum := sign(coef)$SIGNEF) case "failed" => "failed"
(signum :: Integer) = 1 => plusInfinity()
minusInfinity()
type = "zero" => 0
-- examine lowest order coefficient in series part of 'domTerm'
ord := order(pSeries := coeff domTerm)
coef := coefficient(pSeries,ord)
member?(var,variables coef) => "failed"
(signum := sign(coef)$SIGNEF) case "failed" => "failed"
(signum :: Integer) = 1 => plusInfinity()
minusInfinity()

```

---

— UPXSSING.dotabb —

```

"UPXSSING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=UPXSSING"]
"FS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FS"]
"ACF" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ACF"]
"UPXSSING" -> "ACF"
"UPXSSING" -> "FS"

```

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## 22.8 domain OREUP UnivariateSkewPolynomial

— UnivariateSkewPolynomial.input —

```

)set break resume
)sys rm -f UnivariateSkewPolynomial.output
)spool UnivariateSkewPolynomial.output
)set message test on
)set message auto off
)set message type off
)clear all

--S 1 of 33
F:=EXPR(FRAC(INT))
--R
--R
--R (1) Expression Fraction Integer
--E 1

```

```

--S 2 of 33
Dx:F->F:=f+-->D(f,['x])
--R
--R
--R (2) theMap(Closure)
--E 2

--S 3 of 33
D0:=OREUP('d,F,1,Dx)
--R
--R
--R (3)
--I UnivariateSkewPolynomial(d,Expression Fraction Integer,R -> R,theMap LAMBDA-C
--I LOSURE(NIL,NIL,NIL,G9057 envArg,SPADCALL(G9057,coerceOrCroak(CONS(QUOTE List
--I Variable x,wrap LIST QUOTE x),QUOTE List Symbol,QUOTE *1;anonymousFunction;2;
--I frame0;internal),ELT(*1;anonymousFunction;2;frame0;internal;MV,0)))
--E 3

--S 4 of 33
u:D0:=(operator 'u)(x)
--R
--R
--R (4) u(x)
--E 4

--S 5 of 33
d:D0:='d
--R
--R
--R (5) d
--E 5

--S 6 of 33
a:D0:=u^3*d^3+u^2*d^2+u*d+1
--R
--R
--R (6) 
$$u(x)^3 d^3 + u(x)^2 d^2 + u(x) d + 1$$

--E 6

--S 7 of 33
b:D0:=(u+1)*d^2+2*d
--R
--R
--R (7) 
$$(u(x) + 1) d^2 + 2d$$

--E 7

--S 8 of 33
r:=rightDivide(a,b)

```

```

--R
--R
--R (8)
--R          3      3      2
--R      - u(x) u (x) - u(x) + u(x)
--R      u(x)
--R [quotient= ----- d + -----,
--R      u(x) + 1          2
--R                      u(x) + 2u(x) + 1
--R          3      3
--R      2u(x) u (x) + 3u(x) + u(x)
--R
--R remainder= ----- d + 1]
--R          2
--R      u(x) + 2u(x) + 1
--E 8

```

```

--S 9 of 33

```

```

r.quotient

```

```

--R
--R
--R          3      3      2
--R      - u(x) u (x) - u(x) + u(x)
--R      u(x)
--R (9) ----- d + -----
--R      u(x) + 1          2
--R                      u(x) + 2u(x) + 1
--E 9

```

```

--S 10 of 33

```

```

r.remainder

```

```

--R
--R
--R          3      3
--R      2u(x) u (x) + 3u(x) + u(x)
--R
--R (10) ----- d + 1
--R          2
--R      u(x) + 2u(x) + 1
--E 10

```

```

)clear all

```

```

--S 11 of 33

```

```

R:=UP('t,INT)

```

```

--R

```

```

--R

```

```

--R (1) UnivariatePolynomial(t,Integer)

```

```

--E 11

```

```

--S 12 of 33
W:=OREUP('x,R,1,D)
--R
--R
--R (2)
--R UnivariateSkewPolynomial(x,UnivariatePolynomial(t,Integer),R -> R,theMap(DIFR
--I ING-,D;2S;1,411))
--E 12

--S 13 of 33
t:W:='t
--R
--R
--R (3) t
--E 13

--S 14 of 33
x:W:='x
--R
--R
--R (4) x
--E 14

--S 15 of 33
a:W:=(t-1)*x^4+(t^3+3*t+1)*x^2+2*t*x+t^3
--R
--R
--R (5)  $(t - 1)x^4 + (t^3 + 3t + 1)x^2 + 2tx + t^3$ 
--E 15

--S 16 of 33
b:W:=(6*t^4+2*t^2)*x^3+3*t^2*x^2
--R
--R
--R (6)  $(6t^4 + 2t^2)x^3 + 3t^2x^2$ 
--E 16

--S 17 of 33
a*b
--R
--R
--R (7)
--R  $(6t^5 - 6t^4 + 2t^3 - 2t^2)x^7 + (96t^4 - 93t^3 + 13t^2 - 16t)x^6$ 
--R +
--R  $(6t^7 + 20t^5 + 6t^4 + 438t^3 - 406t^2 - 24t)x^5$ 
--R +

```

```

--R      6      5      4      3      2      4
--R      (48t + 15t + 152t + 61t + 603t - 532t - 36)x
--R      +
--R      7      5      4      3      2      3
--R      (6t + 74t + 60t + 226t + 116t + 168t - 140)x
--R      +
--R      5      3      2      2
--R      (3t + 6t + 12t + 18t + 6)x
--E 17

--S 18 of 33
a^3
--R
--R
--R      (8)
--R      3      2      12      5      4      3      2      10
--R      (t - 3t + 3t - 1)x + (3t - 6t + 12t - 15t + 3t + 3)x
--R      +
--R      3      2      9      7      6      5      4      3      2      8
--R      (6t - 12t + 6t)x + (3t - 3t + 21t - 18t + 24t - 9t - 15t - 3)x
--R      +
--R      5      4      3      2      7
--R      (12t - 12t + 36t - 24t - 12t)x
--R      +
--R      9      7      6      5      4      3      2      6
--R      (t + 15t - 3t + 45t + 6t + 36t + 15t + 9t + 1)x
--R      +
--R      7      5      3      2      5
--R      (6t + 48t + 54t + 36t + 6t)x
--R      +
--R      9      7      6      5      4      3      2      4
--R      (3t + 21t + 3t + 39t + 18t + 39t + 12t )x
--R      +
--R      7      5      4      3      3      9      7      6      5      2      7      9
--R      (12t + 36t + 12t + 8t )x + (3t + 9t + 3t + 12t )x + 6t x + t
--E 18

)clear all

--S 19 of 33
S:EXPR(INT)->EXPR(INT):=e+-->eval(e,[n],[n+1])
--R
--R
--R      (1)  theMap(Closure)
--E 19

--S 20 of 33
DF:EXPR(INT)->EXPR(INT):=e+-->eval(e,[n],[n+1])-e
--R
--R

```



```

--R (2) theMap(Closure)
--E 20

--S 21 of 33
D0:=OREUP('D,EXPR(INT),morphism S,DF)
--R
--R
--R (3)
--I UnivariateSkewPolynomial(D,Expression Integer,R -> R,theMap LAMBDA-CLOSURE(NI
--I L,NIL,NIL,G9384 envArg,SPADCALL(SPADCALL(G9384,coerceOrCroak(CONS(QUOTE List
--I Variable n,wrap LIST QUOTE n),QUOTE List Expression Integer,QUOTE *1;anonymou
--I sFunction;9;frame0;internal),coerceOrCroak(CONS(QUOTE List Polynomial Integer
--I ,wrap LIST SPADCALL(QUOTE 1(n,1 0),QUOTE 0,ELT(*1;anonymousFunction;9;frame0;
--I internal;MV,0))),QUOTE List Expression Integer,QUOTE *1;anonymousFunction;9;f
--I rame0;internal),ELT(*1;anonymousFunction;9;frame0;internal;MV,1)),G9384,ELT(*
--I 1;anonymousFunction;9;frame0;internal;MV,2)))
--E 21

--S 22 of 33
u:=(operator 'u)[n]
--R
--R
--R (4) u(n)
--E 22

--S 23 of 33
L:D0:='D+u
--R
--R
--R (5) D + u(n)
--E 23

--S 24 of 33
L^2
--R
--R
--R 
$$(6) \quad D^2 + 2u(n)D + u(n)^2$$

--E 24

)clear all

--S 25 of 33
)set expose add constructor SquareMatrix
--R
--I SquareMatrix is now explicitly exposed in frame frame0
--E 25

--S 26 of 33
R:=SQMATRIX(2,INT)

```

```

--R
--R
--R (1) SquareMatrix(2,Integer)
--E 26

--S 27 of 33
y:=matrix [[1,1],[0,1]]
--R
--R
--R      +1  1+
--R (2)  |   |
--R      +0  1+
--E 27

--S 28 of 33
delta:R->R:=r+>y*r-r*y
--R
--R
--R (3) theMap(Closure)
--E 28

--S 29 of 33
S:=OREUP('x,R,1,delta)
--R
--R
--R (4)
--I  UnivariateSkewPolynomial(x,SquareMatrix(2,Integer),R -> R,theMap LAMBDA-CLOSURE
--I  RE(NIL,NIL,NIL,G9459 envArg,SPADCALL(SPADCALL(getValueFromEnvironment(QUOTE y
--I  ,QUOTE SquareMatrix(2,Integer)),G9459,ELT(*1;anonymousFunction;13;frame0;inte
--I  rnal;MV,0)),SPADCALL(G9459,getValueFromEnvironment(QUOTE y,QUOTE SquareMatrix
--I  (2,Integer)),ELT(*1;anonymousFunction;13;frame0;internal;MV,0)),ELT(*1;anonym
--I  ousFunction;13;frame0;internal;MV,1))))
--E 29

--S 30 of 33
x:S:='x
--R
--R
--R (5)  x
--E 30

--S 31 of 33
a:=matrix [[2,3],[1,1]]
--R
--R
--R      +2  3+
--R (6)  |   |
--R      +1  1+
--E 31

```

```

--S 32 of 33
x^2*a
--R
--R
--R      +2  3+  2  +2  - 2+  +0  - 2+
--R  (7)  |    |x  + |    |x  + |    |
--R      +1  1+    +0  - 2+  +0  0  +
--E 32

--S 33 of 33
)show UnivariateSkewPolynomial
--R
--R UnivariateSkewPolynomial(x: Symbol,R: Ring,sigma: Automorphism R,delta: (R -> R)) is a
--R Abbreviation for UnivariateSkewPolynomial is OREUP
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for OREUP
--R
--R----- Operations -----
--R ?? : (R,%) -> %                ?? : (%,R) -> %
--R ?? : (%,%) -> %                ?? : (Integer,%) -> %
--R ?? : (PositiveInteger,%) -> %  *** : (%,PositiveInteger) -> %
--R ?? : (%,%) -> %                ?? : (%,%) -> %
--R -? : % -> %                    ?? : (%,%) -> Boolean
--R 1 : () -> %                    0 : () -> %
--R ?? : (%,PositiveInteger) -> %  apply : (%,R,R) -> R
--R coefficients : % -> List R      coerce : Variable x -> %
--R coerce : R -> %                coerce : Integer -> %
--R coerce : % -> OutputForm        degree : % -> NonNegativeInteger
--R hash : % -> SingleInteger        latex : % -> String
--R leadingCoefficient : % -> R      one? : % -> Boolean
--R recip : % -> Union(%, "failed")  reductum : % -> %
--R retract : % -> R                sample : () -> %
--R zero? : % -> Boolean             ?? : (%,%) -> Boolean
--R ?? : (NonNegativeInteger,%) -> %
--R *** : (%,NonNegativeInteger) -> %
--R ?? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R coefficient : (%,NonNegativeInteger) -> R
--R coerce : Fraction Integer -> % if R has RETRACT FRAC INT
--R content : % -> R if R has GCDDOM
--R exquo : (%,R) -> Union(%, "failed") if R has INTDOM
--R leftDivide : (%,%) -> Record(quotient: %,remainder: %) if R has FIELD
--R leftExactQuotient : (%,%) -> Union(%, "failed") if R has FIELD
--R leftExtendedGcd : (%,%) -> Record(coef1: %,coef2: %,generator: %) if R has FIELD
--R leftGcd : (%,%) -> % if R has FIELD
--R leftLcm : (%,%) -> % if R has FIELD
--R leftQuotient : (%,%) -> % if R has FIELD
--R leftRemainder : (%,%) -> % if R has FIELD
--R minimumDegree : % -> NonNegativeInteger
--R monicLeftDivide : (%,%) -> Record(quotient: %,remainder: %) if R has INTDOM

```

```

--R monicRightDivide : (%,% ) -> Record(quotient: %,remainder: %) if R has INTDOM
--R monomial : (R,NonNegativeInteger) -> %
--R primitivePart : % -> % if R has GCDDOM
--R retract : % -> Fraction Integer if R has RETRACT FRAC INT
--R retract : % -> Integer if R has RETRACT INT
--R retractIfCan : % -> Union(R,"failed")
--R retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
--R rightDivide : (%,% ) -> Record(quotient: %,remainder: %) if R has FIELD
--R rightExactQuotient : (%,% ) -> Union(%,"failed") if R has FIELD
--R rightExtendedGcd : (%,% ) -> Record(coef1: %,coef2: %,generator: %) if R has FIELD
--R rightGcd : (%,% ) -> % if R has FIELD
--R rightLcm : (%,% ) -> % if R has FIELD
--R rightQuotient : (%,% ) -> % if R has FIELD
--R rightRemainder : (%,% ) -> % if R has FIELD
--R subtractIfCan : (%,% ) -> Union(%,"failed")
--R
--E 33
)set expose drop constructor SquareMatrix

)spool
)lisp (bye)

```

---

— UnivariateSkewPolynomial.help —

```

=====
UnivariateSkewPolynomial examples
=====

```

Skew or Ore polynomial rings provide a unified framework to compute with differential and difference equations.

In the following, let  $A$  be an integral domain, equipped with two endomorphisms  $\sigma$  and  $\delta$  where:

$\sigma: A \rightarrow A$  is an injective ring endomorphism  
 $\delta: A \rightarrow A$ , the pseudo-derivation with respect to  $\sigma$ ,  
 is an additive endomorphism with

$$\delta(ab) = \sigma(a)\delta(b) + \delta(a)b$$

for all  $a, b$  in  $A$

Note that in the domains and categories below, these properties are not checked.

The skew polynomial ring  $[\Delta; \sigma, \delta]$  is the ring of polynomials in  $\Delta$  with coefficients in  $A$ , with the usual addition, while the product is given by

$$\Delta a = \sigma(a)\Delta + \delta(a) \text{ for } a \text{ in } A$$

The two most important examples of skew polynomial rings are:

$K(x)[D, 1, \delta]$ , where  $1$  is the identity on  $K$  and  $\delta$  is the usual derivative, is the ring of differential polynomials

$K[E, n, \text{mapsto } n+1, 0]$  is the ring of linear recurrence operators with polynomial coefficients

-----  
For example,

The `UnivariateSkewPolynomialCategory` (`OREPCAT`) provides a unified framework for polynomial rings in a non-central indeterminate over some coefficient ring  $R$ . The commutation relations between the indeterminate  $x$  and the coefficient  $t$  is given by

$$x r = \sigma(r) x + \delta(r)$$

where  $\sigma$  is a ring endomorphism of  $R$  and  $\delta$  is a  $\sigma$ -derivation of  $R$  which is an additive map from  $R$  to  $R$  such that

$$\delta(rs) = \sigma(r) \delta(s) + \delta(r) s$$

In case  $\sigma$  is the identity map on  $R$ , a  $\sigma$ -derivation of  $R$  is just called a derivation. Here are some examples

We start with a linear ordinary differential operator. First, we define the coefficient ring to be expressions in one variable  $x$  with fractional coefficients:

$$F := \text{EXPR}(\text{FRAC}(\text{INT}))$$

Define  $Dx$  to be a derivative  $d/dx$ :

$$Dx: F \rightarrow F: f \mapsto D(f, [x])$$

Define a skew polynomial ring over  $F$  with identity endomorphism as  $\sigma$  and derivation  $d/dx$  as  $\delta$ :

$$D0 := \text{OREUP}('d, F, 1, Dx)$$

u:D0:=(operator 'u)(x)

d:D0:='d

a:D0:=u^3\*d^3+u^2\*d^2+u\*d+1

$$u(x)^3 d^3 + u(x)^2 d^2 + u(x) d + 1$$

b:D0:=(u+1)\*d^2+2\*d

$$(u(x) + 1) d^2 + 2d$$

r:=rightDivide(a,b)

$$\begin{aligned} \text{[quotient= } & \frac{u(x)^3}{u(x)^3 + 1} d + \frac{-u(x)^3 u(x) - u(x)^3 + u(x)^2}{u(x)^2 + 2u(x) + 1}, \\ & \frac{2u(x)^3 u(x) + 3u(x)^3 + u(x)}{u(x)^2 + 2u(x) + 1} d + 1] \end{aligned}$$

r.quotient

$$\frac{u(x)^3}{u(x)^3 + 1} d + \frac{-u(x)^3 u(x) - u(x)^3 + u(x)^2}{u(x)^2 + 2u(x) + 1}$$

r.remainer

$$\frac{2u(x)^3 u(x) + 3u(x)^3 + u(x)}{u(x)^2 + 2u(x) + 1} d + 1$$

---

```
)clear all
```

As a second example, we consider the so-called Weyl algebra.

Define the coefficient ring to be an ordinary polynomial over integers in one variable  $t$

```
R:=UP('t,INT)
```

Define a skew polynomial ring over  $R$  with identity map as  $\backslash\sigma$  and derivation  $d/dt$  as  $\backslash\delta$ . The resulting algebra is then called a Weyl algebra. This is a simple ring over a division ring that is non-commutative, similar to the ring of matrices.

```
W:=OREUP('x,R,1,D)
```

```
t:W:='t
```

```
x:W:='x
```

Let

```
a:W:=(t-1)*x^4+(t^3+3*t+1)*x^2+2*t*x+t^3
```

$$(t - 1)x^4 + (t^3 + 3t + 1)x^2 + 2tx + t^3$$

```
b:W:=(6*t^4+2*t^2)*x^3+3*t^2*x^2
```

$$(6t^4 + 2t^2)x^3 + 3t^2x^2$$

Then

```
a*b
```

$$\begin{aligned} & (6t^5 - 6t^4 + 2t^3 - 2t^2)x^5 + (96t^4 - 93t^3 + 13t^2 - 16t)x^6 \\ & + (6t^7 + 20t^5 + 6t^4 + 438t^3 - 406t^2 - 24)x^5 \\ & + (48t^6 + 15t^5 + 152t^4 + 61t^3 + 603t^2 - 532t - 36)x^4 \\ & + (6t^7 + 74t^5 + 60t^4 + 226t^3 + 116t^2 + 168t - 140)x^3 \\ & + \end{aligned}$$

$$\begin{aligned}
& (3t^5 + 6t^3 + 12t^2 + 18t + 6)x^2 \\
a^3 & \\
& (t^3 - 3t^2 + 3t - 1)x^{12} + (3t^5 - 6t^4 + 12t^3 - 15t^2 + 3t + 3)x^{10} \\
& + (6t^3 - 12t^2 + 6t)x^9 + (3t^7 - 3t^6 + 21t^5 - 18t^4 + 24t^3 - 9t^2 - 15t - 3)x^8 \\
& + (12t^5 - 12t^4 + 36t^3 - 24t^2 - 12t)x^7 \\
& + (t^9 + 15t^7 - 3t^6 + 45t^5 + 6t^4 + 36t^3 + 15t^2 + 9t + 1)x^6 \\
& + (6t^7 + 48t^5 + 54t^3 + 36t^2 + 6t)x^5 \\
& + (3t^9 + 21t^7 + 3t^6 + 39t^5 + 18t^4 + 39t^3 + 12t^2)x^4 \\
& + (12t^7 + 36t^5 + 12t^4 + 8t^3)x^9 + (3t^9 + 9t^7 + 3t^6 + 12t^5)x^2 + 6t^7x + t^9
\end{aligned}$$


---

```
)clear all
```

As a third example, we construct a difference operator algebra over the ring of `EXPR(INT)` by using an automorphism `S` defined by a "shift" operation `S:EXPR(INT) -> EXPR(INT)`

```
s(e)(n) = e(n+1)
```

and an `S`-derivation defined by `DF:EXPR(INT) -> EXPR(INT)` as

```
DF(e)(n) = e(n+1)-e(n)
```

Define `S` to be a "shift" operator, which acts on expressions with the discrete variable `n`:

```
S:EXPR(INT)->EXPR(INT):=e+>eval(e,[n],[n+1])
```

Define `DF` to be a "difference" operator, which acts on expressions with a discrete variable `n`:

```
DF:EXPR(INT)->EXPR(INT):=e+>eval(e,[n],[n+1])-e
```



Then define the difference operator algebra D0:

```
D0:=OREUP('D,EXPR(INT),morphism S,DF)
```

```
u:=(operator 'u)[n]
```

```
L:D0:='D+u
```

$$D + u(n)$$

$$L^2$$

$$D^2 + 2u(n)D + u(n)^2$$

-----

```
)clear all
```

As a fourth example, we construct a skew polynomial ring by using an inner derivation  $\delta$  induced by a fixed  $y$  in  $R$ :

$$\delta(r) = yr - ry$$

First we should expose the constructor SquareMatrix so it is visible in the interpreter:

```
)set expose add constructor SquareMatrix
```

Define  $R$  to be the square matrix with integer entries:

```
R:=SQMATRIX(2,INT)
```

```
y:R:=matrix [[1,1],[0,1]]
      +1  1+
      |   |
      +0  1+
```

Define the inner derivative  $\delta$ :

```
delta:R->R:=r+>y*r-r*y
```

Define  $S$  to be a skew polynomial determined by  $\sigma = 1$  and  $\delta$  as an inner derivative:

```
S:=OREUP('x,R,1,delta)
```

```
x:S:='x
```

```
a:S:=matrix [[2,3],[1,1]]
```

```

+2  3+
|    |
+1  1+

x^2*a
+2  3+ 2  +2  - 2+  +0  - 2+
|    |x  + |    |x  + |    |
+1  1+  +0  - 2+  +0  0  +

```

See Also:

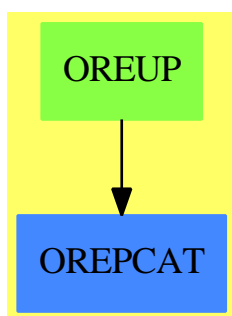
```

o )show UnivariateSkewPolynomial
o )show UnivariateSkewPolynomialCategory
o )show SquareMatrix

```

---

### 22.8.1 UnivariateSkewPolynomial (OREUP)



See

⇒ “Automorphism” (AUTOMOR) 2.44.1 on page 228

⇒ “SparseUnivariateSkewPolynomial” (ORESUP) 20.21.1 on page 2450

**Exports:**

|                    |                    |                   |
|--------------------|--------------------|-------------------|
| 0                  | 1                  | apply             |
| characteristic     | coefficient        | coefficients      |
| coerce             | content            | degree            |
| exquo              | hash               | latex             |
| leadingCoefficient | leftDivide         | leftExactQuotient |
| leftExtendedGcd    | leftGcd            | leftLcm           |
| leftQuotient       | leftRemainder      | minimumDegree     |
| monicLeftDivide    | monicRightDivide   | monomial          |
| one?               | primitivePart      | recip             |
| reductum           | retract            | retractIfCan      |
| rightDivide        | rightExactQuotient | rightExtendedGcd  |
| rightGcd           | rightLcm           | rightQuotient     |
| rightRemainder     | sample             | subtractIfCan     |
| zero?              | ?*?                | ?**?              |
| ?+?                | ?-?                | -?                |
| ?=?                | ?^?                | ?~=?              |

## — domain OREUP UnivariateSkewPolynomial —

```

)abbrev domain OREUP UnivariateSkewPolynomial
++ Author: Manuel Bronstein
++ Date Created: 19 October 1993
++ Date Last Updated: 1 February 1994
++ Description:
++ This is the domain of univariate skew polynomials over an Ore
++ coefficient field in a named variable.
++ The multiplication is given by \spad{x a = \sigma(a) x + \delta a}.

UnivariateSkewPolynomial(x:Symbol,R:Ring,sigma:Automorphism R,delta: R -> R):
  UnivariateSkewPolynomialCategory R with
    coerce: Variable x -> %
      ++ coerce(x) returns x as a skew-polynomial.
  == SparseUnivariateSkewPolynomial(R, sigma, delta) add
    Rep := SparseUnivariateSkewPolynomial(R, sigma, delta)
    coerce(v:Variable(x)):% == monomial(1, 1)
    coerce(p:%):OutputForm == outputForm(p, outputForm x)$Rep

```

---

## — OREUP.dotabb —

```

"OREUP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=OREUP"]
"OREPCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=OREPCAT"]
"OREUP" -> "OREPCAT"

```

---

## 22.9 domain UTS UnivariateTaylorSeries

— UnivariateTaylorSeries.input —

```
)set break resume
)sys rm -f UnivariateTaylorSeries.output
)spool UnivariateTaylorSeries.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show UnivariateTaylorSeries
--R UnivariateTaylorSeries(Coef: Ring,var: Symbol,cen: Coef) is a domain constructor
--R Abbreviation for UnivariateTaylorSeries is UTS
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for UTS
--R
--R----- Operations -----
--R ?? : (Coef,%) -> %          ?? : (%,Coef) -> %
--R ?? : (%,%) -> %           ?? : (Integer,%) -> %
--R ?? : (PositiveInteger,%) -> %   ??? : (%,PositiveInteger) -> %
--R +? : (%,%) -> %           ?-? : (%,%) -> %
--R -? : % -> %               ?=? : (%,%) -> Boolean
--R 1 : () -> %               0 : () -> %
--R ?? : (%,PositiveInteger) -> %   center : % -> Coef
--R coefficients : % -> Stream Coef   coerce : Variable var -> %
--R coerce : Integer -> %           coerce : % -> OutputForm
--R complete : % -> %               degree : % -> NonNegativeInteger
--R evenlambert : % -> %           hash : % -> SingleInteger
--R lagrange : % -> %               lambert : % -> %
--R latex : % -> String           leadingCoefficient : % -> Coef
--R leadingMonomial : % -> %       map : ((Coef -> Coef),%) -> %
--R monomial? : % -> Boolean       oddlambert : % -> %
--R one? : % -> Boolean           order : % -> NonNegativeInteger
--R pole? : % -> Boolean         quoByVar : % -> %
--R recip : % -> Union(%, "failed") reductum : % -> %
--R revert : % -> %               sample : () -> %
--R series : Stream Coef -> %       variable : % -> Symbol
--R zero? : % -> Boolean          ?~=? : (%,%) -> Boolean
--R ?? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
--R ?? : (NonNegativeInteger,%) -> %
--R ??? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ??? : (%,%) -> % if Coef has ALGEBRA FRAC INT
--R ??? : (%,Coef) -> % if Coef has FIELD
--R ??? : (%,NonNegativeInteger) -> %
--R ?/? : (%,Coef) -> % if Coef has FIELD
```

```

--R D : % -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef
--R D : (%,NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef
--R D : (%,Symbol) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef and Coef has PDRING
--R D : (%,List Symbol) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef and Coef has P
--R D : (%,Symbol,NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef a
--R D : (%,List Symbol,List NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger,Coef)
--R ??? : (%,NonNegativeInteger) -> %
--R acos : % -> % if Coef has ALGEBRA FRAC INT
--R acosh : % -> % if Coef has ALGEBRA FRAC INT
--R acot : % -> % if Coef has ALGEBRA FRAC INT
--R acoth : % -> % if Coef has ALGEBRA FRAC INT
--R acsc : % -> % if Coef has ALGEBRA FRAC INT
--R acsch : % -> % if Coef has ALGEBRA FRAC INT
--R approximate : (%,NonNegativeInteger) -> Coef if Coef has **: (Coef,NonNegativeInteger) ->
--R asec : % -> % if Coef has ALGEBRA FRAC INT
--R asech : % -> % if Coef has ALGEBRA FRAC INT
--R asin : % -> % if Coef has ALGEBRA FRAC INT
--R asinh : % -> % if Coef has ALGEBRA FRAC INT
--R associates? : (%,%) -> Boolean if Coef has INTDOM
--R atan : % -> % if Coef has ALGEBRA FRAC INT
--R atanh : % -> % if Coef has ALGEBRA FRAC INT
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%, "failed") if Coef has CHARNZ
--R coefficient : (%,NonNegativeInteger) -> Coef
--R coerce : UnivariatePolynomial(var,Coef) -> %
--R coerce : Coef -> % if Coef has COMRING
--R coerce : % -> % if Coef has INTDOM
--R coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
--R cos : % -> % if Coef has ALGEBRA FRAC INT
--R cosh : % -> % if Coef has ALGEBRA FRAC INT
--R cot : % -> % if Coef has ALGEBRA FRAC INT
--R coth : % -> % if Coef has ALGEBRA FRAC INT
--R csc : % -> % if Coef has ALGEBRA FRAC INT
--R csch : % -> % if Coef has ALGEBRA FRAC INT
--R differentiate : (%,Variable var) -> %
--R differentiate : % -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef
--R differentiate : (%,NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger,Coef) -> (
--R differentiate : (%,Symbol) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef and Coef
--R differentiate : (%,List Symbol) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef and
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger,Coef
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if Coef has *: (NonNegative
--R ?.? : (%,%) -> % if NonNegativeInteger has SGROUP
--R ?.? : (%,NonNegativeInteger) -> Coef
--R eval : (%,Coef) -> Stream Coef if Coef has **: (Coef,NonNegativeInteger) -> Coef
--R exp : % -> % if Coef has ALGEBRA FRAC INT
--R exquo : (%,%) -> Union(%, "failed") if Coef has INTDOM
--R extend : (%,NonNegativeInteger) -> %
--R generalLambert : (%,Integer,Integer) -> %
--R integrate : (%,Variable var) -> % if Coef has ALGEBRA FRAC INT
--R integrate : (%,Symbol) -> % if Coef has integrate: (Coef,Symbol) -> Coef and Coef has va

```

```

--R integrate : % -> % if Coef has ALGEBRA FRAC INT
--R invmultisect : (Integer,Integer,%) -> %
--R log : % -> % if Coef has ALGEBRA FRAC INT
--R monomial : (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R monomial : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R monomial : (Coef,NonNegativeInteger) -> %
--R multiplyCoefficients : ((Integer -> Coef),%) -> %
--R multiplyExponents : (%,PositiveInteger) -> %
--R multisect : (Integer,Integer,%) -> %
--R nthRoot : (%,Integer) -> % if Coef has ALGEBRA FRAC INT
--R order : (%,NonNegativeInteger) -> NonNegativeInteger
--R pi : () -> % if Coef has ALGEBRA FRAC INT
--R polynomial : (%,NonNegativeInteger,NonNegativeInteger) -> Polynomial Coef
--R polynomial : (%,NonNegativeInteger) -> Polynomial Coef
--R sec : % -> % if Coef has ALGEBRA FRAC INT
--R sech : % -> % if Coef has ALGEBRA FRAC INT
--R series : Stream Record(k: NonNegativeInteger,c: Coef) -> %
--R sin : % -> % if Coef has ALGEBRA FRAC INT
--R sinh : % -> % if Coef has ALGEBRA FRAC INT
--R sqrt : % -> % if Coef has ALGEBRA FRAC INT
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tan : % -> % if Coef has ALGEBRA FRAC INT
--R tanh : % -> % if Coef has ALGEBRA FRAC INT
--R terms : % -> Stream Record(k: NonNegativeInteger,c: Coef)
--R truncate : (%,NonNegativeInteger,NonNegativeInteger) -> %
--R truncate : (%,NonNegativeInteger) -> %
--R unit? : % -> Boolean if Coef has INTDOM
--R unitCanonical : % -> % if Coef has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if Coef has INTDOM
--R univariatePolynomial : (%,NonNegativeInteger) -> UnivariatePolynomial(var,Coef)
--R variables : % -> List SingletonAsOrderedSet
--R
--E 1

```

```

)spool
)lisp (bye)

```

---

— UnivariateTaylorSeries.help —

```

=====
UnivariateTaylorSeries examples
=====

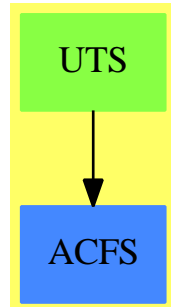
```

```

See Also:
o )show UnivariateTaylorSeries

```

### 22.9.1 UnivariateTaylorSeries (UTS)



See

⇒ “InnerTaylorSeries” (ITAYLOR) 10.28.1 on page 1302

#### Exports:

|                    |                 |                      |                      |
|--------------------|-----------------|----------------------|----------------------|
| 0                  | 1               | acos                 | acosh                |
| acot               | acoth           | acsc                 | acsch                |
| approximate        | asec            | asech                | asin                 |
| asinh              | associates?     | atan                 | atanh                |
| center             | characteristic  | charthRoot           | coefficient          |
| coefficients       | coerce          | complete             | cos                  |
| cosh               | cot             | coth                 | csc                  |
| csch               | D               | degree               | differentiate        |
| eval               | evenlambert     | exp                  | exquo                |
| extend             | generalLambert  | hash                 | integrate            |
| invmultisect       | lagrange        | lambert              | latex                |
| leadingCoefficient | leadingMonomial | log                  | map                  |
| monomial           | monomial?       | multiplyCoefficients | multiplyExponents    |
| multisect          | nthRoot         | oddlambert           | one?                 |
| order              | pi              | pole?                | polynomial           |
| quoByVar           | recip           | reductum             | revert               |
| sample             | sec             | sech                 | series               |
| sin                | sinh            | sqrt                 | subtractIfCan        |
| tan                | tanh            | terms                | truncate             |
| unit?              | unitCanonical   | unitNormal           | univariatePolynomial |
| variable           | variables       | zero?                | ?*?                  |
| ?**?               | ?+?             | ?-?                  | -?                   |
| ?=?                | ?^?             | ?~=?                 | ?..?                 |

— domain UTS UnivariateTaylorSeries —

```

)abbrev domain UTS UnivariateTaylorSeries
++ Author: Clifton J. Williamson
++ Date Created: 21 December 1989
++ Date Last Updated: 21 September 1993
  
```

```

++ Basic Operations:
++ Related Domains: UnivariateLaurentSeries(Coef,var,cen),
++ UnivariatePuisseuxSeries(Coef,var,cen)
++ Also See:
++ AMS Classifications:
++ Keywords: dense, Taylor series
++ Examples:
++ References:
++ Description:
++ Dense Taylor series in one variable
++ \spadtype{UnivariateTaylorSeries} is a domain representing Taylor
++ series in
++ one variable with coefficients in an arbitrary ring. The parameters
++ of the type specify the coefficient ring, the power series variable,
++ and the center of the power series expansion. For example,
++ \spadtype{UnivariateTaylorSeries}(Integer,x,3) represents
++ Taylor series in
++ \spad{(x - 3)} with \spadtype{Integer} coefficients.

UnivariateTaylorSeries(Coef,var,cen): Exports == Implementation where
  Coef : Ring
  var : Symbol
  cen : Coef
  I ==> Integer
  NNI ==> NonNegativeInteger
  P ==> Polynomial Coef
  RN ==> Fraction Integer
  ST ==> Stream
  STT ==> StreamTaylorSeriesOperations Coef
  TERM ==> Record(k:NNI,c:Coef)
  UP ==> UnivariatePolynomial(var,Coef)
Exports ==> UnivariateTaylorSeriesCategory(Coef) with
  coerce: UP -> %
    ++\spad{coerce(p)} converts a univariate polynomial p in the variable
    ++\spad{var} to a univariate Taylor series in \spad{var}.
  univariatePolynomial: (% ,NNI) -> UP
    ++\spad{univariatePolynomial(f,k)} returns a univariate polynomial
    ++ consisting of the sum of all terms of f of degree \spad{<= k}.
  coerce: Variable(var) -> %
    ++\spad{coerce(var)} converts the series variable \spad{var} into a
    ++ Taylor series.
  differentiate: (% ,Variable(var)) -> %
    ++ \spad{differentiate(f(x),x)} computes the derivative of
    ++ \spad{f(x)} with respect to \spad{x}.
  lagrange: % -> %
    ++\spad{lagrange(g(x))} produces the Taylor series for \spad{f(x)}
    ++ where \spad{f(x)} is implicitly defined as \spad{f(x) = x*g(f(x))}.
  lambert: % -> %
    ++\spad{lambert(f(x))} returns \spad{f(x) + f(x^2) + f(x^3) + ...}.
    ++ This function is used for computing infinite products.

```



```

++ \spad{f(x)} should have zero constant coefficient.
++ If \spad{f(x)} is a Taylor series with constant term 1, then
++ \spad{product(n = 1..infinity,f(x^n)) = exp(log(lambert(f(x))))}.
oddlambert: % -> %
++\spad{oddlambert(f(x))} returns \spad{f(x) + f(x^3) + f(x^5) + ...}.
++ \spad{f(x)} should have a zero constant coefficient.
++ This function is used for computing infinite products.
++ If \spad{f(x)} is a Taylor series with constant term 1, then
++ \spad{product(n=1..infinity,f(x^(2*n-1)))=exp(log(oddlambert(f(x))))}.
evenlambert: % -> %
++\spad{evenlambert(f(x))} returns \spad{f(x^2) + f(x^4) + f(x^6) + ...}.
++ \spad{f(x)} should have a zero constant coefficient.
++ This function is used for computing infinite products.
++ If \spad{f(x)} is a Taylor series with constant term 1, then
++ \spad{product(n=1..infinity,f(x^(2*n))) = exp(log(evenlambert(f(x))))}.
generallambert: (% ,I,I) -> %
++\spad{generallambert(f(x),a,d)} returns \spad{f(x^a) + f(x^(a + d)) +
++ f(x^(a + 2 d)) + ... }. \spad{f(x)} should have zero constant
++ coefficient and \spad{a} and d should be positive.
revert: % -> %
++ \spad{revert(f(x))} returns a Taylor series \spad{g(x)} such that
++ \spad{f(g(x)) = g(f(x)) = x}. Series \spad{f(x)} should have constant
++ coefficient 0 and 1st order coefficient 1.
multisect: (I,I,%) -> %
++\spad{multisect(a,b,f(x))} selects the coefficients of
++ \spad{x^((a+b)*n+a)}, and changes this monomial to \spad{x^n}.
invmultisect: (I,I,%) -> %
++\spad{invmultisect(a,b,f(x))} substitutes \spad{x^((a+b)*n)}
++ for \spad{x^n} and multiplies by \spad{x^b}.
if Coef has Algebra Fraction Integer then
  integrate: (% ,Variable(var)) -> %
    ++ \spad{integrate(f(x),x)} returns an anti-derivative of the power
    ++ series \spad{f(x)} with constant coefficient 0.
    ++ We may integrate a series when we can divide coefficients
    ++ by integers.

Implementation ==> InnerTaylorSeries(Coef) add

Rep := Stream Coef

--% creation and destruction of series

stream: % -> Stream Coef
stream x == x pretend Stream(Coef)

coerce(v:Variable(var)) ==
  zero? cen => monomial(1,1)
  monomial(1,1) + monomial(cen,0)

coerce(n:I) == n :: Coef :: %

```

```

coerce(r:Coef) == coerce(r)$STT
monomial(c,n) == monom(c,n)$STT

getExpon: TERM -> NNI
getExpon term == term.k
getCoef: TERM -> Coef
getCoef term == term.c
rec: (NNI,Coef) -> TERM
rec(expon,coef) == [expon,coef]

recs: (ST Coef,NNI) -> ST TERM
recs(st,n) == delay$ST(TERM)
  empty? st => empty()
  zero? (coef := first st) => recs(rst st,n + 1)
  concat(rec(n,coef),recs(rst st,n + 1))

terms x == recs(stream x,0)

recsToCoefs: (ST TERM,NNI) -> ST Coef
recsToCoefs(st,n) == delay
  empty? st => empty()
  term := first st; expon := getExpon term
  n = expon => concat(getCoef term,recsToCoefs(rst st,n + 1))
  concat(0,recsToCoefs(st,n + 1))

series(st: ST TERM) == recsToCoefs(st,0)

stToPoly: (ST Coef,P,NNI,NNI) -> P
stToPoly(st,term,n,n0) ==
  (n > n0) or (empty? st) => 0
  first(st) * term ** n + stToPoly(rst st,term,n + 1,n0)

polynomial(x,n) == stToPoly(stream x,(var :: P) - (cen :: P),0,n)

polynomial(x,n1,n2) ==
  if n1 > n2 then (n1,n2) := (n2,n1)
  stToPoly(rest(stream x,n1),(var :: P) - (cen :: P),n1,n2)

stToUPoly: (ST Coef,UP,NNI,NNI) -> UP
stToUPoly(st,term,n,n0) ==
  (n > n0) or (empty? st) => 0
  first(st) * term ** n + stToUPoly(rst st,term,n + 1,n0)

univariatePolynomial(x,n) ==
  stToUPoly(stream x,monomial(1,1)$UP - monomial(cen,0)$UP,0,n)

coerce(p:UP) ==
  zero? p => 0
  if not zero? cen then
    p := p(monomial(1,1)$UP + monomial(cen,0)$UP)

```

```

st : ST Coef := empty()
oldDeg : NNI := degree(p) + 1
while not zero? p repeat
  deg := degree p
  delta := (oldDeg - deg - 1) :: NNI
  for i in 1..delta repeat st := concat(0$Coef,st)
  st := concat(leadingCoefficient p,st)
  oldDeg := deg; p := reductum p
for i in 1..oldDeg repeat st := concat(0$Coef,st)
st

if Coef has coerce: Symbol -> Coef then
  if Coef has "**": (Coef,NNI) -> Coef then

    stToCoef: (ST Coef,Coef,NNI,NNI) -> Coef
    stToCoef(st,term,n,n0) ==
      (n > n0) or (empty? st) => 0
      frst(st) * term ** n + stToCoef(rst st,term,n + 1,n0)

    approximate(x,n) ==
      stToCoef(stream x,(var :: Coef) - cen,0,n)

--% values

variable x == var
center s == cen

coefficient(x,n) ==
  -- Cannot use elt! Should return 0 if stream doesn't have it.
  u := stream x
  while not empty? u and n > 0 repeat
    u := rst u
    n := (n - 1) :: NNI
  empty? u or n ^= 0 => 0
  frst u

elt(x:%,n:NNI) == coefficient(x,n)

--% functions

map(f,x) == map(f,x)$Rep
eval(x:%,r:Coef) == eval(stream x,r-cen)$STT
differentiate x == deriv(stream x)$STT
differentiate(x:%,v:Variable(var)) == differentiate x
if Coef has PartialDifferentialRing(Symbol) then
  differentiate(x:%,s:Symbol) ==
    (s = variable(x)) => differentiate x
    map(y +-> differentiate(y,s),x)
    - differentiate(center x,s)*differentiate(x)
multiplyCoefficients(f,x) == gderiv(f,stream x)$STT

```

```

lagrange x == lagrange(stream x)$STT
lambert x == lambert(stream x)$STT
oddlambert x == oddlambert(stream x)$STT
evenlambert x == evenlambert(stream x)$STT
generalLambert(x:%,a:I,d:I) == generalLambert(stream x,a,d)$STT
extend(x,n) == extend(x,n+1)$Rep
complete x == complete(x)$Rep
truncate(x,n) == first(stream x,n + 1)$Rep
truncate(x,n1,n2) ==
  if n2 < n1 then (n1,n2) := (n2,n1)
  m := (n2 - n1) :: NNI
  st := first(rest(stream x,n1)$Rep,m + 1)$Rep
  for i in 1..n1 repeat st := concat(0$Coef,st)
  st
elt(x:%,y:%) == compose(stream x,stream y)$STT
revert x == revert(stream x)$STT
multisect(a,b,x) == multisect(a,b,stream x)$STT
invmultisect(a,b,x) == invmultisect(a,b,stream x)$STT
multiplyExponents(x,n) == invmultisect(n,0,x)
quoByVar x == (empty? x => 0; rst x)
if Coef has IntegralDomain then
  unit? x == unit? coefficient(x,0)
if Coef has Field then
  if Coef is RN then
    (x:%) ** (s:Coef) == powern(s,stream x)$STT
  else
    (x:%) ** (s:Coef) == power(s,stream x)$STT

if Coef has Algebra Fraction Integer then
  coerce(r:RN) == r :: Coef :: %

integrate x == integrate(0,stream x)$STT
integrate(x:%,v:Variable(var)) == integrate x

if Coef has integrate: (Coef,Symbol) -> Coef and _
  Coef has variables: Coef -> List Symbol then
  integrate(x:%,s:Symbol) ==
    (s = variable(x)) => integrate x
    not entry?(s,variables center x) => map(y +-> integrate(y,s),x)
    error "integrate: center is a function of variable of integration"

if Coef has TranscendentalFunctionCategory and _
  Coef has PrimitiveFunctionCategory and _
  Coef has AlgebraicallyClosedFunctionSpace Integer then

  integrateWithOneAnswer: (Coef,Symbol) -> Coef
  integrateWithOneAnswer(f,s) ==
    res := integrate(f,s)$FunctionSpaceIntegration(I,Coef)
    res case Coef => res :: Coef
    first(res :: List Coef)

```

```

integrate(x:%,s:Symbol) ==
(s = variable(x)) => integrate x
not entry?(s,variables center x) =>
  map(y +-> integrateWithOneAnswer(y,s),x)
error "integrate: center is a function of variable of integration"

--% OutputForms
-- We use the default coerce: % -> OutputForm in UTSCAT&

```

---

— UTS.dotabb —

```

"UTS" [color="#88FF44",href="bookvol10.3.pdf#nameddest=UTS"]
"ACFS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ACFS"]
"UTS" -> "ACFS"

```

---

## 22.10 domain UTSZ UnivariateTaylorSeriesCZero

— UnivariateTaylorSeriesCZero.input —

```

)set break resume
)sys rm -f UnivariateTaylorSeriesCZero.output
)spool UnivariateTaylorSeriesCZero.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show UnivariateTaylorSeriesCZero
--R UnivariateTaylorSeriesCZero(Coef: Ring,var: Symbol) is a domain constructor
--R Abbreviation for UnivariateTaylorSeriesCZero is UTSZ
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for UTSZ
--R
--R----- Operations -----
--R ?? : (Coef,%) -> %           ?? : (%,Coef) -> %
--R ?? : (%,%) -> %             ?? : (Integer,%) -> %
--R ?? : (PositiveInteger,%) -> %   ??? : (%,PositiveInteger) -> %
--R ?? : (%,%) -> %             ?-? : (%,%) -> %
--R -? : % -> %                 ?? : (%,%) -> Boolean

```

```

--R 1 : () -> %
--R ?? : (% , PositiveInteger) -> %
--R coefficients : % -> Stream Coef
--R coerce : Integer -> %
--R complete : % -> %
--R evenlambert : % -> %
--R lagrange : % -> %
--R latex : % -> String
--R leadingMonomial : % -> %
--R monomial? : % -> Boolean
--R one? : % -> Boolean
--R pole? : % -> Boolean
--R recip : % -> Union(%, "failed")
--R revert : % -> %
--R series : Stream Coef -> %
--R zero? : % -> Boolean
--R ?? : (% , Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ?? : (Fraction Integer, %) -> % if Coef has ALGEBRA FRAC INT
--R ?? : (NonNegativeInteger, %) -> %
--R ??? : (% , Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
--R ??? : (% , %) -> % if Coef has ALGEBRA FRAC INT
--R ??? : (% , Coef) -> % if Coef has FIELD
--R ??? : (% , NonNegativeInteger) -> %
--R ?/? : (% , Coef) -> % if Coef has FIELD
--R D : % -> % if Coef has *: (NonNegativeInteger, Coef) -> Coef
--R D : (% , NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger, Coef) -> Coef
--R D : (% , Symbol) -> % if Coef has *: (NonNegativeInteger, Coef) -> Coef and Coef has PDRING SYMBOL
--R D : (% , List Symbol) -> % if Coef has *: (NonNegativeInteger, Coef) -> Coef and Coef has PDRING SYMBOL
--R D : (% , Symbol, NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger, Coef) -> Coef and Coef has
--R D : (% , List Symbol, List NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger, Coef) -> Coef and
--R ?? : (% , NonNegativeInteger) -> %
--R acos : % -> % if Coef has ALGEBRA FRAC INT
--R acosh : % -> % if Coef has ALGEBRA FRAC INT
--R acot : % -> % if Coef has ALGEBRA FRAC INT
--R acoth : % -> % if Coef has ALGEBRA FRAC INT
--R acsc : % -> % if Coef has ALGEBRA FRAC INT
--R acsch : % -> % if Coef has ALGEBRA FRAC INT
--R approximate : (% , NonNegativeInteger) -> Coef if Coef has **: (Coef, NonNegativeInteger) -> Coef and C
--R asec : % -> % if Coef has ALGEBRA FRAC INT
--R asech : % -> % if Coef has ALGEBRA FRAC INT
--R asin : % -> % if Coef has ALGEBRA FRAC INT
--R asinh : % -> % if Coef has ALGEBRA FRAC INT
--R associates? : (% , %) -> Boolean if Coef has INTDOM
--R atan : % -> % if Coef has ALGEBRA FRAC INT
--R atanh : % -> % if Coef has ALGEBRA FRAC INT
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%, "failed") if Coef has CHARNZ
--R coefficient : (% , NonNegativeInteger) -> Coef
--R coerce : UnivariatePolynomial(var, Coef) -> %
--R coerce : Coef -> % if Coef has COMRING
0 : () -> %
center : % -> Coef
coerce : Variable var -> %
coerce : % -> OutputForm
degree : % -> NonNegativeInteger
hash : % -> SingleInteger
lambert : % -> %
leadingCoefficient : % -> Coef
map : ((Coef -> Coef), %) -> %
oddlambert : % -> %
order : % -> NonNegativeInteger
quoByVar : % -> %
reductum : % -> %
sample : () -> %
variable : % -> Symbol
?~=? : (% , %) -> Boolean

```

```

--R coerce : % -> % if Coef has INTDOM
--R coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
--R cos : % -> % if Coef has ALGEBRA FRAC INT
--R cosh : % -> % if Coef has ALGEBRA FRAC INT
--R cot : % -> % if Coef has ALGEBRA FRAC INT
--R coth : % -> % if Coef has ALGEBRA FRAC INT
--R csc : % -> % if Coef has ALGEBRA FRAC INT
--R csch : % -> % if Coef has ALGEBRA FRAC INT
--R differentiate : (%,Variable var) -> %
--R differentiate : % -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef
--R differentiate : (%,NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef
--R differentiate : (%,Symbol) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef and Coef
--R differentiate : (%,List Symbol) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef and Coef
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger,Coef)
--R differentiate : (%,List Symbol,List NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger,Coef)
--R ?.? : (%,%) -> % if NonNegativeInteger has SGROUP
--R ?.? : (%,NonNegativeInteger) -> Coef
--R eval : (%,Coef) -> Stream Coef if Coef has **: (Coef,NonNegativeInteger) -> Coef
--R exp : % -> % if Coef has ALGEBRA FRAC INT
--R exquo : (%,%) -> Union(%, "failed") if Coef has INTDOM
--R extend : (%,NonNegativeInteger) -> %
--R generalLambert : (%,Integer,Integer) -> %
--R integrate : (%,Variable var) -> % if Coef has ALGEBRA FRAC INT
--R integrate : (%,Symbol) -> % if Coef has integrate: (Coef,Symbol) -> Coef and Coef has var
--R integrate : % -> % if Coef has ALGEBRA FRAC INT
--R invmultisect : (Integer,Integer,%) -> %
--R log : % -> % if Coef has ALGEBRA FRAC INT
--R monomial : (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
--R monomial : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R monomial : (Coef,NonNegativeInteger) -> %
--R multiplyCoefficients : ((Integer -> Coef),%) -> %
--R multiplyExponents : (%,PositiveInteger) -> %
--R multisect : (Integer,Integer,%) -> %
--R nthRoot : (%,Integer) -> % if Coef has ALGEBRA FRAC INT
--R order : (%,NonNegativeInteger) -> NonNegativeInteger
--R pi : () -> % if Coef has ALGEBRA FRAC INT
--R polynomial : (%,NonNegativeInteger,NonNegativeInteger) -> Polynomial Coef
--R polynomial : (%,NonNegativeInteger) -> Polynomial Coef
--R sec : % -> % if Coef has ALGEBRA FRAC INT
--R sech : % -> % if Coef has ALGEBRA FRAC INT
--R series : Stream Record(k: NonNegativeInteger,c: Coef) -> %
--R sin : % -> % if Coef has ALGEBRA FRAC INT
--R sinh : % -> % if Coef has ALGEBRA FRAC INT
--R sqrt : % -> % if Coef has ALGEBRA FRAC INT
--R subtractIfCan : (%,%) -> Union(%, "failed")
--R tan : % -> % if Coef has ALGEBRA FRAC INT
--R tanh : % -> % if Coef has ALGEBRA FRAC INT
--R terms : % -> Stream Record(k: NonNegativeInteger,c: Coef)
--R truncate : (%,NonNegativeInteger,NonNegativeInteger) -> %
--R truncate : (%,NonNegativeInteger) -> %

```

```

--R unit? : % -> Boolean if Coef has INTDOM
--R unitCanonical : % -> % if Coef has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if Coef has INTDOM
--R univariatePolynomial : (% ,NonNegativeInteger) -> UnivariatePolynomial(var,Coef)
--R variables : % -> List SingletonAsOrderedSet
--R
--E 1

```

```

)spool
)lisp (bye)

```

---

— UnivariateTaylorSeriesCZero.help —

```

=====
UnivariateTaylorSeriesCZero examples
=====

```

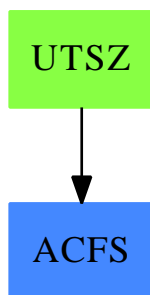
```

See Also:
o )show UnivariateTaylorSeriesCZero

```

---

### 22.10.1 UnivariateTaylorSeriesCZero (UTSZ)



See

⇒ “InnerTaylorSeries” (ITAYLOR) 10.28.1 on page 1302



**Exports:**

|                    |                 |                      |                      |
|--------------------|-----------------|----------------------|----------------------|
| 0                  | 1               | acos                 | acosh                |
| acot               | acoth           | acsc                 | acsch                |
| approximate        | asec            | asech                | asin                 |
| asinh              | associates?     | atan                 | atanh                |
| center             | characteristic  | charthRoot           | coefficient          |
| coefficients       | coerce          | complete             | cos                  |
| cosh               | cot             | coth                 | csc                  |
| csch               | D               | degree               | differentiate        |
| eval               | evenlambert     | exp                  | exquo                |
| extend             | generalLambert  | hash                 | integrate            |
| invmultisect       | lagrange        | lambert              | latex                |
| leadingCoefficient | leadingMonomial | log                  | map                  |
| monomial           | monomial?       | multiplyCoefficients | multiplyExponents    |
| multisect          | nthRoot         | oddlambert           | one?                 |
| order              | pi              | pole?                | polynomial           |
| quoByVar           | recip           | reductum             | revert               |
| sample             | sec             | sech                 | series               |
| sin                | sinh            | sqrt                 | subtractIfCan        |
| tan                | tanh            | terms                | truncate             |
| unit?              | unitCanonical   | unitNormal           | univariatePolynomial |
| variable           | variables       | zero?                | ??                   |
| ?~=?               | ?/?             | ?*?                  | ?**?                 |
| ?+?                | ?-?             | -?                   | ?=?                  |
| ?^?                |                 |                      |                      |

— domain UTSZ UnivariateTaylorSeriesCZero —

```
)abbrev domain UTSZ UnivariateTaylorSeriesCZero
++ Author: Gaetan Hache
++ Date Created: September 1996
++ Date Last Updated: April, 2010, by Tim Daly
++ Description:
++ Part of the Package for Algebraic Function Fields in one variable PAFF
```

```
UnivariateTaylorSeriesCZero(Coef,var): Exports == Implementation where
```

```
Coef : Ring
var  : Symbol
I    ==> Integer
NNI  ==> NonNegativeInteger
P    ==> Polynomial Coef
RN   ==> Fraction Integer
ST   ==> Stream
STT  ==> StreamTaylorSeriesOperations Coef
TERM ==> Record(k:NNI,c:Coef)
UP   ==> UnivariatePolynomial(var,Coef)
Exports ==> UnivariateTaylorSeriesCategory(Coef) with
coerce: UP -> %
```

```

++\spad{coerce(p)} converts a univariate polynomial p in the variable
++\spad{var} to a univariate Taylor series in \spad{var}.
univariatePolynomial: (% ,NNI) -> UP
++\spad{univariatePolynomial(f,k)} returns a univariate polynomial
++ consisting of the sum of all terms of f of degree \spad{<= k}.
coerce: Variable(var) -> %
++\spad{coerce(var)} converts the series variable \spad{var} into a
++ Taylor series.
differentiate: (% ,Variable(var)) -> %
++ \spad{differentiate(f(x),x)} computes the derivative of
++ \spad{f(x)} with respect to \spad{x}.
lagrange: % -> %
++\spad{lagrange(g(x))} produces the Taylor series for \spad{f(x)}
++ where \spad{f(x)} is implicitly defined as \spad{f(x) = x*g(f(x))}.
lambert: % -> %
++\spad{lambert(f(x))} returns \spad{f(x) + f(x^2) + f(x^3) + ...}.
++ This function is used for computing infinite products.
++ \spad{f(x)} should have zero constant coefficient.
++ If \spad{f(x)} is a Taylor series with constant term 1, then
++ \spad{product(n = 1..infinity, f(x^n)) = exp(log(lambert(f(x))))}.
oddlambert: % -> %
++\spad{oddlambert(f(x))} returns \spad{f(x) + f(x^3) + f(x^5) + ...}.
++ \spad{f(x)} should have a zero constant coefficient.
++ This function is used for computing infinite products.
++ If \spad{f(x)} is a Taylor series with constant term 1, then
++ \spad{product(n=1..infinity, f(x^(2*n-1)))=exp(log(oddlambert(f(x))))}.
evenlambert: % -> %
++\spad{evenlambert(f(x))} returns \spad{f(x^2) + f(x^4) + f(x^6) + ...}.
++ \spad{f(x)} should have a zero constant coefficient.
++ This function is used for computing infinite products.
++ If \spad{f(x)} is a Taylor series with constant term 1, then
++ \spad{product(n=1..infinity, f(x^(2*n)))=exp(log(evenlambert(f(x))))}.
generalLambert: (% ,I,I) -> %
++\spad{generalLambert(f(x),a,d)} returns \spad{f(x^a) + f(x^(a + d)) +
++ f(x^(a + 2 d)) + ... }. \spad{f(x)} should have zero constant
++ coefficient and \spad{a} and d should be positive.
revert: % -> %
++ \spad{revert(f(x))} returns a Taylor series \spad{g(x)} such that
++ \spad{f(g(x)) = g(f(x)) = x}. Series \spad{f(x)} should have constant
++ coefficient 0 and 1st order coefficient 1.
multisect: (I,I,% ) -> %
++\spad{multisect(a,b,f(x))} selects the coefficients of
++ \spad{x^((a+b)*n+a)}, and changes this monomial to \spad{x^n}.
invmultisect: (I,I,% ) -> %
++\spad{invmultisect(a,b,f(x))} substitutes \spad{x^((a+b)*n)}
++ for \spad{x^n} and multiples by \spad{x^b}.
if Coef has Algebra Fraction Integer then
integrate: (% ,Variable(var)) -> %
++ \spad{integrate(f(x),x)} returns an anti-derivative of the power
++ series \spad{f(x)} with constant coefficient 0.

```

```

    ++ We may integrate a series when we can divide coefficients
    ++ by integers.

Implementation ==> InnerTaylorSeries(Coef) add

Rep := Stream Coef

--% creation and destruction of series

stream: % -> Stream Coef
stream x == x pretend Stream(Coef)

coerce(v:Variable(var)) ==
  monomial(1,1)

coerce(n:I) == n :: Coef :: %
coerce(r:Coef) == coerce(r)$STT
monomial(c,n) == monom(c,n)$STT

getExpon: TERM -> NNI
getExpon term == term.k
getCoef: TERM -> Coef
getCoef term == term.c
rec: (NNI,Coef) -> TERM
rec(expon,coef) == [expon,coef]

recs: (ST Coef,NNI) -> ST TERM
recs(st,n) == delay$ST(TERM)
  empty? st => empty()
  zero? (coef := first st) => recs(rst st,n + 1)
  concat(rec(n,coef),recs(rst st,n + 1))

terms x == recs(stream x,0)

recsToCoefs: (ST TERM,NNI) -> ST Coef
recsToCoefs(st,n) == delay
  empty? st => empty()
  term := first st; expon := getExpon term
  n = expon => concat(getCoef term,recsToCoefs(rst st,n + 1))
  concat(0,recsToCoefs(st,n + 1))

series(st: ST TERM) == recsToCoefs(st,0)

stToPoly: (ST Coef,P,NNI,NNI) -> P
stToPoly(st,term,n,n0) ==
  (n > n0) or (empty? st) => 0
  first(st) * term ** n + stToPoly(rst st,term,n + 1,n0)

polynomial(x,n) == stToPoly(stream x,(var :: P),0,n)

```

```

polynomial(x,n1,n2) ==
  if n1 > n2 then (n1,n2) := (n2,n1)
  stToPoly(rest(stream x,n1),(var :: P),n1,n2)

stToUPoly: (ST Coef,UP,NNI,NNI) -> UP
stToUPoly(st,term,n,n0) ==
  (n > n0) or (empty? st) => 0
  frst(st) * term ** n + stToUPoly(rst st,term,n + 1,n0)

univariatePolynomial(x,n) ==
  stToUPoly(stream x,monomial(1,1)$UP,0,n)

coerce(p:UP) ==
  zero? p => 0
  st : ST Coef := empty()
  oldDeg : NNI := degree(p) + 1
  while not zero? p repeat
    deg := degree p
    delta := (oldDeg - deg - 1) :: NNI
    for i in 1..delta repeat st := concat(0$Coef,st)
    st := concat(leadingCoefficient p,st)
    oldDeg := deg; p := reductum p
  for i in 1..oldDeg repeat st := concat(0$Coef,st)
  st

if Coef has coerce: Symbol -> Coef then
  if Coef has "**": (Coef,NNI) -> Coef then

    stToCoef: (ST Coef,Coef,NNI,NNI) -> Coef
    stToCoef(st,term,n,n0) ==
      (n > n0) or (empty? st) => 0
      frst(st) * term ** n + stToCoef(rst st,term,n + 1,n0)

    approximate(x,n) ==
      stToCoef(stream x,(var :: Coef),0,n)

--% values

variable x == var
center x == 0$Coef

coefficient(x,n) ==
  -- Cannot use elt! Should return 0 if stream doesn't have it.
  u := stream x
  while not empty? u and n > 0 repeat
    u := rst u
    n := (n - 1) :: NNI
  empty? u or n ^= 0 => 0
  frst u

```

```

elt(x:%,n:NNI) == coefficient(x,n)

--% functions

map(f,x) == map(f,x)$Rep
eval(x:%,r:Coef) == eval(stream x,r)$STT
differentiate x == deriv(stream x)$STT
differentiate(x:%,v:Variable(var)) == differentiate x
if Coef has PartialDifferentialRing(Symbol) then
  differentiate(x:%,s:Symbol) ==
    (s = variable(x)) => differentiate x
    map(differentiate(#1,s),x) - differentiate(0,s)*differentiate(x)
multiplyCoefficients(f,x) == gderiv(f,stream x)$STT
lagrange x == lagrange(stream x)$STT
lambert x == lambert(stream x)$STT
oddlambert x == oddlambert(stream x)$STT
evenlambert x == evenlambert(stream x)$STT
generalLambert(x:%,a:I,d:I) == generalLambert(stream x,a,d)$STT
extend(x,n) == extend(x,n+1)$Rep
complete x == complete(x)$Rep
truncate(x,n) == first(stream x,n + 1)$Rep
truncate(x,n1,n2) ==
  if n2 < n1 then (n1,n2) := (n2,n1)
  m := (n2 - n1) :: NNI
  st := first(rest(stream x,n1)$Rep,m + 1)$Rep
  for i in 1..n1 repeat st := concat(0$Coef,st)
  st
elt(x:%,y:%) == compose(stream x,stream y)$STT
revert x == revert(stream x)$STT
multisect(a,b,x) == multisect(a,b,stream x)$STT
invmultisect(a,b,x) == invmultisect(a,b,stream x)$STT
multiplyExponents(x,n) == invmultisect(n,0,x)
quoByVar x == (empty? x => 0; rst x)
if Coef has IntegralDomain then
  unit? x == unit? coefficient(x,0)
if Coef has Field then
  if Coef is RN then
    (x:%) ** (s:Coef) == powern(s,stream x)$STT
  else
    (x:%) ** (s:Coef) == power(s,stream x)$STT

if Coef has Algebra Fraction Integer then
  coerce(r:RN) == r :: Coef :: %

integrate x == integrate(0,stream x)$STT
integrate(x:%,v:Variable(var)) == integrate x

if Coef has integrate: (Coef,Symbol) -> Coef and _
  Coef has variables: Coef -> List Symbol then
  integrate(x:%,s:Symbol) ==

```

```

(s = variable(x)) => integrate x
map(integrate(#1,s),x)

if Coef has TranscendentalFunctionCategory and _
  Coef has PrimitiveFunctionCategory and _
  Coef has AlgebraicallyClosedFunctionSpace Integer then

integrateWithOneAnswer: (Coef,Symbol) -> Coef
integrateWithOneAnswer(f,s) ==
  res := integrate(f,s)$FunctionSpaceIntegration(I,Coef)
  res case Coef => res :: Coef
  first(res :: List Coef)

integrate(x:%,s:Symbol) ==
  (s = variable(x)) => integrate x
  map(integrateWithOneAnswer(#1,s),x)

-----

— UTSZ.dotabb —

"UTSZ" [color="#88FF44",href="bookvol10.3.pdf#nameddest=UTSZ"]
"ACFS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ACFS"]
"UTSZ" -> "ACFS"

-----

```

## 22.11 domain UNISEG UniversalSegment

```

— UniversalSegment.input —

)set break resume
)sys rm -f UniversalSegment.output
)spool UniversalSegment.output
)set message test on
)set message auto off
)clear all
--S 1 of 9
pints := 1..
--R
--R
--R (1) 1..
--R
--R                                         Type: UniversalSegment PositiveInteger
--E 1

```

```

--S 2 of 9
nevens := (0..) by -2
--R
--R
--R (2) 0.. by - 2
--R
--R                                         Type: UniversalSegment NonNegativeInteger
--E 2

--S 3 of 9
useg: UniversalSegment(Integer) := 3..10
--R
--R
--R (3) 3..10
--R
--R                                         Type: UniversalSegment Integer
--E 3

--S 4 of 9
hasHi pints
--R
--R
--R (4) false
--R
--R                                         Type: Boolean
--E 4

--S 5 of 9
hasHi nevens
--R
--R
--R (5) false
--R
--R                                         Type: Boolean
--E 5

--S 6 of 9
hasHi useg
--R
--R
--R (6) true
--R
--R                                         Type: Boolean
--E 6

--S 7 of 9
expand pints
--R
--R
--R (7) [1,2,3,4,5,6,7,8,9,10,...]
--R
--R                                         Type: Stream Integer
--E 7

--S 8 of 9

```

---

— UniversalSegment.help —

```
hasHi pints
  false
```

Type: Boolean



```
hasHi nevens
false
```

Type: Boolean

```
hasHi useg
true
```

Type: Boolean

All operations available on type `Segment` apply to `UniversalSegment`, with the proviso that expansions produce streams rather than lists. This is to accommodate infinite expansions.

```
expand pints
[1,2,3,4,5,6,7,8,9,10,...]
```

Type: Stream Integer

```
expand nevens
[0,- 2,- 4,- 6,- 8,- 10,- 12,- 14,- 16,- 18,...]
```

Type: Stream Integer

```
expand [1, 3, 10..15, 100..]
[1,3,10,11,12,13,14,15,100,101,...]
```

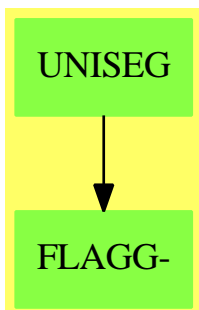
Type: Stream Integer

See Also:

- o )help Segment
- o )help SegmentBinding
- o )help List
- o )help Stream
- o )show UniversalSegment

---

## 22.11.1 UniversalSegment (UNISEG)



See

⇒ “Segment” (SEG) 20.2.1 on page 2319

⇒ “SegmentBinding” (SEGBIND) 20.3.1 on page 2324

**Exports:**

|      |          |         |         |         |
|------|----------|---------|---------|---------|
| BY   | coerce   | convert | expand  | hasHi   |
| hash | hi       | high    | incr    | latex   |
| lo   | low      | map     | segment | segment |
| ?=?  | ?SEGMENT | ?..?    | ?~=?    |         |

— domain UNISEG UniversalSegment —

```

)abbrev domain UNISEG UniversalSegment
++ Author: Robert S. Sutor
++ Date Created: 1987
++ Date Last Updated: June 4, 1991
++ Basic Operations:
++ Related Domains: Segment
++ Also See:
++ AMS Classifications:
++ Keywords: equation
++ Examples:
++ References:
++ Description:
++ This domain provides segments which may be half open.
++ That is, ranges of the form \spad{a..} or \spad{a..b}.

```

```

UniversalSegment(S: Type): SegmentCategory(S) with
  SEGMENT: S -> %
    ++ \spad{1..} produces a half open segment,
    ++ that is, one with no upper bound.
  segment: S -> %
    ++ segment(1) is an alternate way to construct the segment \spad{1..}.
  coerce : Segment S -> %
    ++ coerce(x) allows \spadtype{Segment} values to be used as %.
  hasHi: % -> Boolean

```

```

    ++ hasHi(s) tests whether the segment s has an upper bound.

    if S has SetCategory then SetCategory

    if S has OrderedRing then
      SegmentExpansionCategory(S, Stream S)
--   expand : (List %, S) -> Stream S
--   expand : (% , S) -> Stream S

== add
Rec ==> Record(low: S, high: S, incr: Integer)
Rec2 ==> Record(low: S, incr: Integer)
SEG ==> Segment S

Rep := Union(Rec2, Rec)
a,b : S
s : %
i: Integer
ls : List %

segment a == [a, 1]$Rec2 :: Rep
segment(a,b) == [a,b,1]$Rec :: Rep
BY(s,i) ==
  s case Rec => [lo s, hi s, i]$Rec :: Rep
  [lo s, i]$Rec2 :: Rep

lo s ==
  s case Rec2 => (s :: Rec2).low
  (s :: Rec).low

low s ==
  s case Rec2 => (s :: Rec2).low
  (s :: Rec).low

hasHi s == s case Rec

hi s ==
  not hasHi(s) => error "hi: segment has no upper bound"
  (s :: Rec).high

high s ==
  not hasHi(s) => error "high: segment has no upper bound"
  (s :: Rec).high

incr s ==
  s case Rec2 => (s :: Rec2).incr
  (s :: Rec).incr

SEGMENT(a) == segment a
SEGMENT(a,b) == segment(a,b)

```

```

coerce(sg : SEG): % == segment(lo sg, hi sg)

convert a == [a,a,1]

if S has SetCategory then

  (s1:%) = (s2:%) ==
    s1 case Rec2 =>
      s2 case Rec2 =>
        s1.low = s2.low and s1.incr = s2.incr
      false
    s1 case Rec =>
      s2 case Rec =>
        s2.low = s2.low and s1.high=s2.high and s1.incr=s2.incr
      false
    false

  coerce(s: %): OutputForm ==
    seg :=
      e := (lo s)::OutputForm
      hasHi s => SEGMENT(e, (hi s)::OutputForm)
      SEGMENT e
    inc := incr s
    inc = 1 => seg
    infix(" by ">::OutputForm, seg, inc::OutputForm)

if S has OrderedRing then
  expand(s:%) == expand([s])
  map(f:S->S, s:%) == map(f, expand s)

plusInc(t: S, a: S): S == t + a

expand(ls: List %):Stream S ==
  st:Stream S := empty()
  null ls => st

  lb:List(Segment S) := nil()
  while not null ls and hasHi first ls repeat
    s := first ls
    ls := rest ls
    ns := BY(SEGMENT(lo s, hi s), incr s)$Segment(S)
    lb := concat_!(lb,ns)
  if not null ls then
    s := first ls
    st: Stream S := generate(x +-> x+incr(s)::S, lo s)
  else
    st: Stream S := empty()
  concat(construct expand(lb), st)

```

---

— UNISEG.dotabb —

```
"UNISEG" [color="#88FF44",href="bookvol10.3.pdf#nameddest=UNISEG"]
"FLAGG-" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FLAGG"]
"UNISEG" -> "FLAGG-"
```

---

## 22.12 domain U32VEC U32Vector

— U32Vector.input —

```
)set break resume
)sys rm -f U32Vector.output
)spool U32Vector.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show U32Vector
--R U32Vector is a domain constructor
--R Abbreviation for U32Vector is U32VEC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for U32VEC
--R
--R----- Operations -----
--R concat : List % -> %                concat : (%,% ) -> %
--R concat : (Integer,% ) -> %          concat : (% ,Integer) -> %
--R construct : List Integer -> %       copy : % -> %
--R delete : (% ,Integer) -> %          ?.? : (% ,Integer) -> Integer
--R empty : () -> %                     empty? : % -> Boolean
--R entries : % -> List Integer          eq? : (% ,%) -> Boolean
--R index? : (Integer,% ) -> Boolean     indices : % -> List Integer
--R insert : (% ,% ,Integer) -> %       qelt : (% ,Integer) -> Integer
--R reverse : % -> %                    sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?<? : (% ,%) -> Boolean if Integer has ORDSET
--R ?<=? : (% ,%) -> Boolean if Integer has ORDSET
--R ?=? : (% ,%) -> Boolean if Integer has SETCAT
--R ?>? : (% ,%) -> Boolean if Integer has ORDSET
--R ?>=? : (% ,%) -> Boolean if Integer has ORDSET
--R any? : ((Integer -> Boolean),%) -> Boolean if $ has finiteAggregate
```

```

--R coerce : % -> OutputForm if Integer has SETCAT
--R convert : % -> InputForm if Integer has KONVERT INFORM
--R copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
--R count : (Integer,%) -> NonNegativeInteger if $ has finiteAggregate and Integer has SETCAT
--R count : ((Integer -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R delete : (%,UniversalSegment Integer) -> %
--R ?.? : (%,UniversalSegment Integer) -> %
--R elt : (%,Integer,Integer) -> Integer
--R entry? : (Integer,%) -> Boolean if $ has finiteAggregate and Integer has SETCAT
--R eval : (%,List Integer,List Integer) -> % if Integer has EVALAB INT and Integer has SETCAT
--R eval : (%,Integer,Integer) -> % if Integer has EVALAB INT and Integer has SETCAT
--R eval : (%,Equation Integer) -> % if Integer has EVALAB INT and Integer has SETCAT
--R eval : (%,List Equation Integer) -> % if Integer has EVALAB INT and Integer has SETCAT
--R every? : ((Integer -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,Integer) -> % if $ has shallowlyMutable
--R find : ((Integer -> Boolean),%) -> Union(Integer,"failed")
--R first : % -> Integer if Integer has ORDSET
--R hash : % -> SingleInteger if Integer has SETCAT
--R insert : (Integer,%,Integer) -> %
--R latex : % -> String if Integer has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map : (((Integer,Integer) -> Integer),%,%) -> %
--R map : ((Integer -> Integer),%) -> %
--R map! : ((Integer -> Integer),%) -> % if $ has shallowlyMutable
--R max : (%,%) -> % if Integer has ORDSET
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (Integer,%) -> Boolean if $ has finiteAggregate and Integer has SETCAT
--R members : % -> List Integer if $ has finiteAggregate
--R merge : (%,%) -> % if Integer has ORDSET
--R merge : (((Integer,Integer) -> Boolean),%,%) -> %
--R min : (%,%) -> % if Integer has ORDSET
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R new : (NonNegativeInteger,Integer) -> %
--R parts : % -> List Integer if $ has finiteAggregate
--R position : (Integer,%,Integer) -> Integer if Integer has SETCAT
--R position : (Integer,%) -> Integer if Integer has SETCAT
--R position : ((Integer -> Boolean),%) -> Integer
--R qsetelt! : (%,Integer,Integer) -> Integer if $ has shallowlyMutable
--R reduce : (((Integer,Integer) -> Integer),%) -> Integer if $ has finiteAggregate
--R reduce : (((Integer,Integer) -> Integer),%,Integer) -> Integer if $ has finiteAggregate
--R reduce : (((Integer,Integer) -> Integer),%,Integer,Integer) -> Integer if $ has finiteAggregate and
--R remove : ((Integer -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (Integer,%) -> % if $ has finiteAggregate and Integer has SETCAT
--R removeDuplicates : % -> % if $ has finiteAggregate and Integer has SETCAT
--R reverse! : % -> % if $ has shallowlyMutable
--R select : ((Integer -> Boolean),%) -> % if $ has finiteAggregate
--R setelt : (%,UniversalSegment Integer,Integer) -> Integer if $ has shallowlyMutable
--R setelt : (%,Integer,Integer) -> Integer if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean

```

```

--R sort : % -> % if Integer has ORDSET
--R sort : (((Integer,Integer) -> Boolean),%) -> %
--R sort! : % -> % if $ has shallowlyMutable and Integer has ORDSET
--R sort! : (((Integer,Integer) -> Boolean),%) -> % if $ has shallowlyMutable
--R sorted? : % -> Boolean if Integer has ORDSET
--R sorted? : (((Integer,Integer) -> Boolean),%) -> Boolean
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R ~=? : (%,%) -> Boolean if Integer has SETCAT
--R
--E 1

```

```

)spool
)lisp (bye)

```

---

### — U32Vector.help —

```

=====
U32Vector examples
=====

```

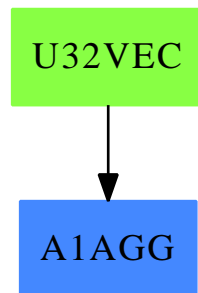
```

See Also:
o )show U32Vector

```

---

## 22.12.1 U32Vector (U32VEC)



See

⇒ “Segment” (SEG) 20.2.1 on page 2319  
 ⇒ “SegmentBinding” (SEGBIND) 20.3.1 on page 2324

**Exports:**

|           |         |                  |          |          |
|-----------|---------|------------------|----------|----------|
| #?        | ??      | ??               | ?<=?     | ?<?      |
| ?=?       | ?>=?    | ?>?              | ?~=?     | any?     |
| coerce    | concat  | construct        | convert  | copy     |
| copyInto! | count   | delete           | elt      | empty    |
| empty?    | entries | entry?           | eq?      | eval     |
| every?    | fill!   | find             | first    | hash     |
| index?    | indices | insert           | latex    | less?    |
| map       | map!    | max              | maxIndex | member?  |
| members   | merge   | min              | minIndex | more?    |
| new       | parts   | position         | qelt     | qsetelt! |
| reduce    | remove  | removeDuplicates | reverse  | reverse! |
| sample    | select  | setelt           | size?    | sort     |
| sort!     | sorted? | swap!            |          |          |

— domain U32VEC U32Vector —

```

)abbrev domain U32VEC U32Vector
++ Author: Waldek Hebisch
++ Description: This is a low-level domain which implements vectors
++ (one dimensional arrays) of unsigned 32-bit numbers. Indexing
++ is 0 based, there is no bound checking (unless provided by
++ lower level).
U32Vector() : OneDimensionalArrayAggregate Integer == add
  Qsize ==> QV32LEN$Lisp
  Qelt ==> ELT32$Lisp
  Qsetelt ==> SETELT32$Lisp
  Qnew ==> MAKE_-ARRAY32$Lisp

  #x == Qsize x
  minIndex x == 0
  empty() == Qnew(0$Lisp, 0$Lisp)
  new(n, x) == Qnew (n, x)
  qelt(x, i) == Qelt(x, i)
  elt(x:%, i:Integer) == Qelt(x, i)
  qsetelt_(x, i, s) == Qsetelt(x, i, s)
  setelt(x:%, i:Integer, s:Integer) == Qsetelt(x, i, s)
  fill_(x, s) == (for i in 0..((Qsize x) - 1) repeat Qsetelt(x, i, s); x)

```

— U32VEC.dotabb —

```

"U32VEC" [color="#88FF44",href="bookvol10.3.pdf#nameddest=U32VEC"]
"A1AGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=A1AGG"]
"U32VEC" -> "A1AGG"

```





## Chapter 23

# Chapter V

### 23.1 domain VARIABLE Variable

— Variable.input —

```
)set break resume
)sys rm -f Variable.output
)spool Variable.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Variable
--R Variable sym: Symbol is a domain constructor
--R Abbreviation for Variable is VARIABLE
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for VARIABLE
--R
--R----- Operations -----
--R ?=? : (% ,%) -> Boolean          coerce : % -> Symbol
--R coerce : % -> OutputForm        hash : % -> SingleInteger
--R latex : % -> String             variable : () -> Symbol
--R ?~=? : (% ,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

—————

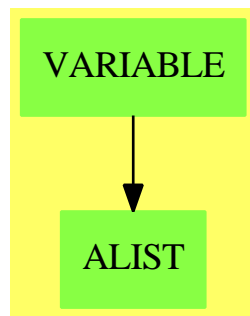
## — Variable.help —

```
=====
Variable examples
=====
```

```
See Also:
o )show Variable
```

---

## 23.1.1 Variable (VARIABLE)

**Exports:**

```
coerce  hash  latex  variable  ??  ?~=?
```

## — domain VARIABLE Variable —

```
)abbrev domain VARIABLE Variable
++ Author: Mark Botch
++ Description:
++ This domain implements variables

Variable(sym:Symbol): Join(SetCategory, CoercibleTo Symbol) with
  coerce  : % -> Symbol
            ++ coerce(x) returns the symbol
  variable: () -> Symbol
            ++ variable() returns the symbol

== add
coerce(x:%):Symbol      == sym
coerce(x:%):OutputForm == sym::OutputForm
variable()              == sym
x = y                   == true
```

---

---

— Vector.input —

[illegible]



```

--S 10 of 11
v + w
--R
--R
--R (10) [3,5,103,9,11]
--R
--R                                          Type: Vector Integer
--E 10

--S 11 of 11
v - w
--R
--R
--R (11) [- 1,- 1,95,- 1,- 1]
--R
--R                                          Type: Vector Integer
--E 11
)spool
)lisp (bye)

```

---

— Vector.help —

=====

Vector examples

=====

The Vector domain is used for storing data in a one-dimensional indexed data structure. A vector is a homogeneous data structure in that all the components of the vector must belong to the same Axiom domain. Each vector has a fixed length specified by the user; vectors are not extensible. This domain is similar to the OneDimensionalArray domain, except that when the components of a Vector belong to a Ring, arithmetic operations are provided.

As with the OneDimensionalArray domain, a Vector can be created by calling the operation new, its components can be accessed by calling the operations elt and qelt, and its components can be reset by calling the operations setelt and qsetelt.

This creates a vector of integers of length 5 all of whose components are 12.

```

u : VECTOR INT := new(5,12)
[12,12,12,12,12]
                                          Type: Vector Integer

```

This is how you create a vector from a list of its components.

```

v : VECTOR INT := vector([1,2,3,4,5])
[1,2,3,4,5]

```

Type: Vector Integer

Indexing for vectors begins at 1. The last element has index equal to the length of the vector, which is computed by #.

```
#(v)
5
```

Type: PositiveInteger

This is the standard way to use elt to extract an element. Functionally, it is the same as if you had typed elt(v,2).

```
v.2
2
```

Type: PositiveInteger

This is the standard way to use setelt to change an element. It is the same as if you had typed setelt(v,3,99).

```
v.3 := 99
99
```

Type: PositiveInteger

Now look at v to see the change. You can use qelt and qsetelt (instead of elt and setelt, respectively) but only when you know that the index is within the valid range.

```
v
[1,2,99,4,5]
```

Type: Vector Integer

When the components belong to a Ring, Axiom provides arithmetic operations for Vector. These include left and right scalar multiplication.

```
5 * v
[5,10,495,20,25]
```

Type: Vector Integer

```
v * 7
[7,14,693,28,35]
```

Type: Vector Integer

```
w : VECTOR INT := vector([2,3,4,5,6])
[2,3,4,5,6]
```

Type: Vector Integer

Addition and subtraction are also available.

```
v + w
[3,5,103,9,11]
```

Type: Vector Integer

Of course, when adding or subtracting, the two vectors must have the same length or an error message is displayed.

```
v - w  
[- 1,- 1,95,- 1,- 1]
```

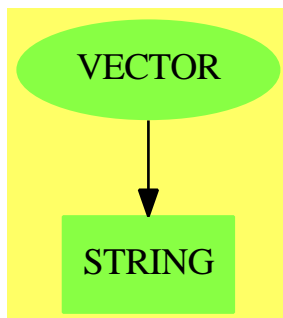
Type: Vector Integer

See Also:

- o )help List
- o )help Matrix
- o )help OneDimensionalArray
- o )help Set
- o )help Table
- o )help TwoDimensionalArray
- o )show Vector

—

### 23.2.1 Vector (VECTOR)





**Exports:**

|          |              |         |                  |           |
|----------|--------------|---------|------------------|-----------|
| any?     | coerce       | concat  | construct        | convert   |
| copy     | copyInto!    | count   | cross            | delete    |
| dot      | elt          | empty   | empty?           | entries   |
| entry?   | eq?          | eval    | every?           | fill!     |
| find     | first        | hash    | index?           | indices   |
| insert   | latex        | length  | less?            | magnitude |
| map      | map!         | max     | maxIndex         | member?   |
| members  | merge        | min     | minIndex         | more?     |
| new      | outerProduct | parts   | position         | qelt      |
| qsetelt! | reduce       | remove  | removeDuplicates | reverse   |
| reverse! | sample       | select  | setelt           | size?     |
| sort     | sort!        | sorted? | swap!            | vector    |
| zero     | #?           | ?*?     | ?+?              | ?-?       |
| ?<?      | ?<=?         | ?=?     | ?>?              | ?>=?      |
| ?..?     | ?~=?         | -?      | ?..?             |           |

— domain VECTOR Vector —

```

)abbrev domain VECTOR Vector
++ Author: Mark Botch
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors: IndexedVector, DirectProduct
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This type represents vector like objects with varying lengths
++ and indexed by a finite segment of integers starting at 1.

Vector(R:Type): Exports == Implementation where
  VECTORMININDEX ==> 1      -- if you want to change this, be my guest

Exports ==> VectorCategory R with
  vector: List R -> %
  ++ vector(l) converts the list l to a vector.
Implementation ==>
  IndexedVector(R, VECTORMININDEX) add
  vector l == construct l
  if R has ConvertibleTo InputForm then
    convert(x:%):InputForm ==
      convert [convert("vector"::Symbol)@InputForm,
               convert(parts x)@InputForm]

```

— VECTOR.dotabb —

```
"VECTOR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=VECTOR",
          shape=ellipse]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"VECTOR" -> "STRING"
```

—

## 23.3 domain VOID Void

— Void.input —

```
)set break resume
)sys rm -f Void.output
)spool Void.output
)set message test on
)set message auto off
)clear all
--S 1 of 5
r := (a; b; if c then d else e; f)
--R
--R
--RDaly Bug
--R   An expression following if/when must evaluate to a Boolean and you
--R   have written one that does not.
--E 1

--S 2 of 5
a : Integer
--R
--R
--R                                          Type: Void
--E 2

)set message void on

--S 3 of 5
b : Fraction Integer
--R
--R
--R   (2)  "()"
--R
--R                                          Type: Void
--E 3

)set message void off
```

```

--S 4 of 5
3::Void
--R
--R
--R                                          Type: Void
--E 4

```

```

--S 5 of 5
% :: PositiveInteger
--R
--R
--RDaly Bug
--R  Cannot convert from type Void to PositiveInteger for value
--R  "()"
--R
--E 5
)spool
)lisp (bye)

```

---

— Void.help —

```

=====
Void examples
=====

```

When an expression is not in a value context, it is given type Void.  
For example, in the expression

```
r := (a; b; if c then d else e; f)
```

values are used only from the subexpressions c and f: all others are thrown away. The subexpressions a, b, d and e are evaluated for side-effects only and have type Void. There is a unique value of type Void.

You will most often see results of type Void when you declare a variable.

```

a : Integer
                                     Type: Void

```

Usually no output is displayed for Void results. You can force the display of a rather ugly object by issuing

```

)set message void on

b : Fraction Integer
                                     Type: Void

```

```
)set message void off
```

All values can be converted to type Void.

```
3::Void
```

```
Type: Void
```

Once a value has been converted to Void, it cannot be recovered.

```
% :: PositiveInteger
```

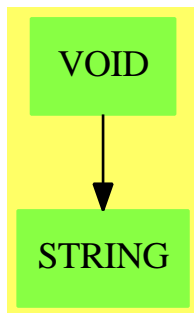
```
Cannot convert from type Void to PositiveInteger for value "()"
```

See Also:

```
o )show Void
```

---

### 23.3.1 Void (VOID)



**Exports:**

```
coerce void
```

— domain VOID Void —

```
)abbrev domain VOID Void
```

```
-- These types act as the top and bottom of the type lattice
```

```
-- and are known to the compiler and interpreter for type resolution.
```

```
++ Author: Stephen M. Watt
```

```
++ Date Created: 1986
```

```
++ Date Last Updated: May 30, 1991
```

```
++ Basic Operations:
```

```
++ Related Domains: ErrorFunctions, ResolveLatticeCompletion, Exit
```

```
++ Also See:
```

```

++ AMS Classifications:
++ Keywords: type, mode, coerce, no value
++ Examples:
++ References:
++ Description:
++ This type is used when no value is needed, e.g., in the \spad{then}
++ part of a one armed \spad{if}.
++ All values can be coerced to type Void. Once a value has been coerced
++ to Void, it cannot be recovered.

```

```

Void: with
  void: () -> %
    ++ void() produces a void object.
  coerce: % -> OutputForm
    ++ coerce(v) coerces void object to outputForm.
== add
  Rep := String
  void() == voidValue()$Lisp
  coerce(v:%) == coerce(v)$Rep

```

---

— VOID.dotabb —

```

"VOID" [color="#88FF44",href="bookvol10.3.pdf#nameddest=VOID"]
"STRING" [color="#88FF44",href="bookvol10.3.pdf#nameddest=STRING"]
"VOID" -> "STRING"

```

---

## Chapter 24

# Chapter W

### 24.1 domain WP WeightedPolynomials

— WeightedPolynomials.input —

```
)set break resume
)sys rm -f WeightedPolynomials.output
)spool WeightedPolynomials.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show WeightedPolynomials
--R WeightedPolynomials(R: Ring, VarSet: OrderedSet, E: OrderedAbelianMonoidSup, P: PolynomialCategory(R, E,
--R Abbreviation for WeightedPolynomials is WP
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for WP
--R
--R----- Operations -----
--R ??? : (% , %) -> %
--R ??? : (PositiveInteger, %) -> %
--R ?+? : (% , %) -> %
--R -? : % -> %
--R 1 : () -> %
--R ?? : (% , PositiveInteger) -> %
--R coerce : % -> P
--R coerce : % -> OutputForm
--R latex : % -> String
--R recip : % -> Union(% , "failed")
--R zero? : % -> Boolean
--R ??? : (% , R) -> % if R has COMRING
--R ??? : (Integer, %) -> %
--R ***? : (% , PositiveInteger) -> %
--R ?-? : (% , %) -> %
--R ?=? : (% , %) -> Boolean
--R 0 : () -> %
--R coerce : P -> %
--R coerce : Integer -> %
--R hash : % -> SingleInteger
--R one? : % -> Boolean
--R sample : () -> %
--R ?~=? : (% , %) -> Boolean
```

```

--R ??? : (R,%) -> % if R has COMRING
--R ??? : (NonNegativeInteger,%) -> %
--R ??? : (%,NonNegativeInteger) -> %
--R ??/? : (%,%) -> Union(%, "failed") if R has FIELD
--R ?? : (%,NonNegativeInteger) -> %
--R changeWeightLevel : NonNegativeInteger -> Void
--R characteristic : () -> NonNegativeInteger
--R coerce : R -> % if R has COMRING
--R subtractIfCan : (%,%) -> Union(%, "failed")
--R
--E 1

```

```

)spool
)lisp (bye)

```

\_\_\_\_\_

— WeightedPolynomials.help —

```

=====
WeightedPolynomials examples
=====

```

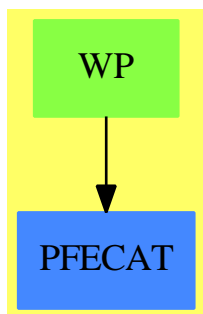
```

See Also:
o )show WeightedPolynomials

```

\_\_\_\_\_

### 24.1.1 WeightedPolynomials (WP)



**Exports:**

|        |        |                   |                |
|--------|--------|-------------------|----------------|
| 0      | 1      | changeWeightLevel | characteristic |
| coerce | hash   | latex             | one?           |
| recip  | sample | subtractIfCan     | zero?          |
| ?~=?   | ?*?    | ?**?              | ?/?            |
| ?^?    | ?*?    | ?**?              | ?+?            |
| ?-?    | -?     | ?=?               |                |

— domain WP WeightedPolynomials —

```

)abbrev domain WP WeightedPolynomials
++ Author: James Davenport
++ Date Created: 17 April 1992
++ Date Last Updated: 12 July 1992
++ Basic Functions: Ring, changeWeightLevel
++ Related Constructors: PolynomialRing
++ Also See: OrdinaryWeightedPolynomials
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This domain represents truncated weighted polynomials over a general
++ (not necessarily commutative) polynomial type. The variables must be
++ specified, as must the weights.
++ The representation is sparse
++ in the sense that only non-zero terms are represented.

WeightedPolynomials(R:Ring, VarSet: OrderedSet, E:OrderedAbelianMonoidSup,
                    P:PolynomialCategory(R,E,VarSet),
                    vl:List VarSet, wl:List NonNegativeInteger,
                    wtlevel:NonNegativeInteger):

  Ring with
    if R has CommutativeRing then Algebra(R)
    coerce: $ -> P
      ++ convert back into a "P", ignoring weights
    if R has Field then "/": ($,$) -> Union($,"failed")
      ++ x/y division (only works if minimum weight
      ++ of divisor is zero, and if R is a Field)
    coerce: P -> $
      ++ coerce(p) coerces p into Weighted form,
      ++ applying weights and ignoring terms
    changeWeightLevel: NonNegativeInteger -> Void
      ++ changeWeightLevel(n) changes the weight level to
      ++ the new value given:
      ++ NB: previously calculated terms are not affected

  ==
add
--representations
Rep := PolynomialRing(P,NonNegativeInteger)

```



```

p:P
w,x1,x2:$
n:NonNegativeInteger
z:Integer
changeWeightLevel(n) ==
  wtlevel:=n
lookupList:List Record(var:VarSet, weight:NonNegativeInteger)
if #v1 ^= #w1 then error "incompatible length lists in WeightedPolynomial"
lookupList:=[v,n] for v in v1 for n in w1]
-- local operation
innercoerce:(p,z) -> $
lookup:VarSet -> NonNegativeInteger
lookup v ==
  l:=lookupList
  while l ^= [] repeat
    v = l.first.var => return l.first.weight
    l:=l.rest
  0
innercoerce(p,z) ==
  z<0 => 0
  zero? p => 0
  mv:= mainVariable p
  mv case "failed" => monomial(p,0)
  n:=lookup(mv)
  up:=univariate(p,mv)
  ans:$
  ans:=0
  while not zero? up repeat
    d:=degree up
    f:=n*d
    lcup:=leadingCoefficient up
    up:=up-leadingMonomial up
    mon:=monomial(1,mv,d)
    f<=z =>
      tmp:= innercoerce(lcup,z-f)
      while not zero? tmp repeat
        ans:=ans+ monomial(mon*leadingCoefficient(tmp),degree(tmp)+f)
        tmp:=reductum tmp
  ans
coerce(p):$ == innercoerce(p,wtlevel)
coerce(w):P == "+"/[c for c in coefficients w]
coerce(p:$):OutputForm ==
  zero? p => (0$Integer)::OutputForm
  degree p = 0 => leadingCoefficient(p):: OutputForm
  reduce("+", (reverse [paren(c::OutputForm) for c in coefficients p])
    ::List OutputForm)
0 == 0$Rep
1 == 1$Rep
x1 = x2 ==
  -- Note that we must strip out any terms greater than wtlevel

```

```

while degree x1 > wtlevel repeat
  x1 := reductum x1
while degree x2 > wtlevel repeat
  x2 := reductum x2
x1 = $Rep x2
x1 + x2 == x1 + $Rep x2
-x1 == -(x1::Rep)
x1 * x2 ==
  -- Note that this is probably an extremely inefficient definition
  w:=x1 * $Rep x2
  while degree(w) > wtlevel repeat
    w:=reductum w
w

```

---

— WP.dotabb —

```

"WP" [color="#88FF44",href="bookvol10.3.pdf#nameddest=WP"]
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
"WP" -> "PFECAT"

```

---

## 24.2 domain WUTSET WuWenTsunTriangularSet

— WuWenTsunTriangularSet.input —

```

)set break resume
)sys rm -f WuWenTsunTriangularSet.output
)spool WuWenTsunTriangularSet.output
)set message test on
)set message auto off
)clear all
--S 1 of 16
R := Integer
--R
--R
--R (1) Integer
--R
--E 1
Type: Domain

--S 2 of 16
ls : List Symbol := [x,y,z,t]

```

```

--R
--R
--R (2) [x,y,z,t]
--R
--R                                         Type: List Symbol
--E 2

--S 3 of 16
V := OVAR(ls)
--R
--R
--R (3) OrderedVariableList [x,y,z,t]
--R
--R                                         Type: Domain
--E 3

--S 4 of 16
E := IndexedExponents V
--R
--R
--R (4) IndexedExponents OrderedVariableList [x,y,z,t]
--R
--R                                         Type: Domain
--E 4

--S 5 of 16
P := NSMP(R, V)
--R
--R
--R (5) NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--R
--R                                         Type: Domain
--E 5

--S 6 of 16
x: P := 'x
--R
--R
--R (6) x
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 6

--S 7 of 16
y: P := 'y
--R
--R
--R (7) y
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 7

--S 8 of 16
z: P := 'z
--R
--R

```

```

--R (8) z
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 8

--S 9 of 16
t: P := 't
--R
--R
--R (9) t
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 9

--S 10 of 16
T := WUTSET(R,E,V,P)
--R
--R
--R (10)
--R WuWenTsunTriangularSet(Integer,IndexedExponents OrderedVariableList [x,y,z,t]
--R ,OrderedVariableList [x,y,z,t],NewSparseMultivariatePolynomial(Integer,Ordere
--R dVariableList [x,y,z,t]))
--R
--R Type: Domain
--E 10

--S 11 of 16
p1 := x ** 31 - x ** 6 - x - y
--R
--R
--R (11)  $x^{31} - x^6 - x - y$ 
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 11

--S 12 of 16
p2 := x ** 8 - z
--R
--R
--R (12)  $x^8 - z$ 
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 12

--S 13 of 16
p3 := x ** 10 - t
--R
--R
--R (13)  $x^{10} - t$ 
--R Type: NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 13

```

```

--S 14 of 16
lp := [p1, p2, p3]
--R
--R
--R      31      6      8      10
--R (14) [x  - x  - x - y, x  - z, x  - t]
--RType: List NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
--E 14

--S 15 of 16
characteristicSet(lp)$T
--R
--R
--R (15)
--R      5      4 4 2 2      3 4      7      4      6      6      3      3      3      3
--R {z  - t , t z y  + 2t z y + (- t  + 2t  - t)z  + t z, (t  - 1)z x - z y - t }
--RType: Union(WuWenTsunTriangularSet(Integer,IndexedExponents OrderedVariableList [x,y,z,t])
--E 15

--S 16 of 16
zeroSetSplit(lp)$T
--R
--R
--R (16)
--R      3      5      4 3      3      2
--R [{t,z,y,x}, {t  - 1, z  - t , z y + t , z x  - t},
--R      5      4 4 2 2      3 4      7      4      6      6      3      3      3      3
--R {z  - t , t z y  + 2t z y + (- t  + 2t  - t)z  + t z, (t  - 1)z x - z y - t }]
--RType: List WuWenTsunTriangularSet(Integer,IndexedExponents OrderedVariableList [x,y,z,t])
--E 16
)spool
)lisp (bye)

```

---

— WuWenTsunTriangularSet.help —

=====

WuWenTsunTriangularSet examples

=====

The WuWenTsunTriangularSet domain constructor implements the characteristic set method of Wu Wen Tsun. This algorithm computes a list of triangular sets from a list of polynomials such that the algebraic variety defined by the given list of polynomials decomposes into the union of the regular-zero sets of the computed triangular sets. The constructor takes four arguments. The first one, R, is the coefficient ring of the polynomials; it must belong to the category IntegralDomain. The second one, E, is the exponent monoid of the

polynomials; it must belong to the category `OrderedAbelianMonoidSup`. The third one, `V`, is the ordered set of variables; it must belong to the category `OrderedSet`. The last one is the polynomial ring; it must belong to the category `RecursivePolynomialCategory(R,E,V)`. The abbreviation for `WuWenTsunTriangularSet` is `WUTSET`.

Let us illustrate the facilities by an example.

Define the coefficient ring.

```
R := Integer
Integer
Type: Domain
```

Define the list of variables,

```
ls : List Symbol := [x,y,z,t]
[x,y,z,t]
Type: List Symbol
```

and make it an ordered set;

```
V := OVAR(ls)
OrderedVariableList [x,y,z,t]
Type: Domain
```

then define the exponent monoid.

```
E := IndexedExponents V
IndexedExponents OrderedVariableList [x,y,z,t]
Type: Domain
```

Define the polynomial ring.

```
P := NSMP(R, V)
NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t])
Type: Domain
```

Let the variables be polynomial.

```
x: P := 'x
x
Type: NewSparseMultivariatePolynomial(Integer,
OrderedVariableList [x,y,z,t])

y: P := 'y
y
Type: NewSparseMultivariatePolynomial(Integer,
OrderedVariableList [x,y,z,t])
```

```

z: P := 'z
z
Type: NewSparseMultivariatePolynomial(Integer,
    OrderedVariableList [x,y,z,t])

t: P := 't
t
Type: NewSparseMultivariatePolynomial(Integer,
    OrderedVariableList [x,y,z,t])

```

Now call the WuWenTsunTriangularSet domain constructor.

```

T := WUTSET(R,E,V,P)
WuWenTsunTriangularSet(Integer,IndexedExponents OrderedVariableList [x,y,z,t]
,OrderedVariableList [x,y,z,t],NewSparseMultivariatePolynomial(Integer,OrderedVariableList [x,y,z,t]))
Type: Domain

```

Define a polynomial system.

```

p1 := x ** 31 - x ** 6 - x - y
      31      6
      x      - x      - x - y
Type: NewSparseMultivariatePolynomial(Integer,
    OrderedVariableList [x,y,z,t])

p2 := x ** 8 - z
      8
      x      - z
Type: NewSparseMultivariatePolynomial(Integer,
    OrderedVariableList [x,y,z,t])

p3 := x ** 10 - t
      10
      x      - t
Type: NewSparseMultivariatePolynomial(Integer,
    OrderedVariableList [x,y,z,t])

lp := [p1, p2, p3]
      31      6      8      10
      [x      - x      - x - y, x      - z, x      - t]
Type: List NewSparseMultivariatePolynomial(Integer,
    OrderedVariableList [x,y,z,t])

```

Compute a characteristic set of the system.

```

characteristicSet(lp)$T
5 4 4 2 2 3 4 7 4 6 6 3 3 3 3
{z - t , t z y + 2t z y + (- t + 2t - t)z + t z, (t - 1)z x - z y - t }
Type: Union(WuWenTsunTriangularSet(Integer,

```

```

IndexedExponents OrderedVariableList [x,y,z,t],
OrderedVariableList [x,y,z,t],
NewSparseMultivariatePolynomial(Integer,
  OrderedVariableList [x,y,z,t])),...)

```

Solve the system.

```

zeroSetSplit(lp)$T
      3      5      4      3      3      2
[[{t,z,y,x}, {t3 - 1, z5 - t4, z4 y + t3, z3 x - t2},
  {z5 - t4, t4 z y + 2t3 z y + (-t3 + 2t2 - t)z4 + t6 z, (t6 - 1)z3 x - z3 y - t3}]
Type: List WuWenTsunTriangularSet(Integer,
  IndexedExponents OrderedVariableList [x,y,z,t],
  OrderedVariableList [x,y,z,t],
  NewSparseMultivariatePolynomial(Integer,
    OrderedVariableList [x,y,z,t]))

```

The RegularTriangularSet and SquareFreeRegularTriangularSet domain constructors, the LazardSetSolvingPackage package constructors as well as, SquareFreeRegularTriangularSet and ZeroDimensionalSolvePackage package constructors also provide operations to compute triangular decompositions of algebraic varieties. These five constructor use a special kind of characteristic sets, called regular triangular sets. These special characteristic sets have better properties than the general ones. Regular triangular sets and their related concepts are presented in the paper "On the Theories of Triangular sets" By P. Aubry, D. Lazard and M. Moreno Maza (to appear in the Journal of Symbolic Computation). The decomposition algorithm (due to the third author) available in the four above constructors provide generally better timings than the characteristic set method. In fact, the WUTSET constructor remains interesting for the purpose of manipulating characteristic sets whereas the other constructors are more convenient for solving polynomial systems.

Note that the way of understanding triangular decompositions is detailed in the example of the RegularTriangularSet constructor.

See Also:

```

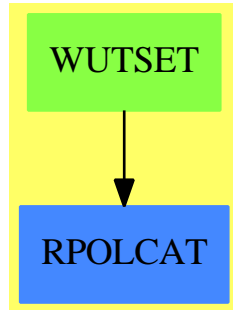
o )help RecursivePolynomialCategory
o )help RegularTriangularSet
o )help SquareFreeRegularTriangularSet
o )help LazardSetSolvingPackage
o )help ZeroDimensionalSolvePackage
o )show WuWenTsunTriangularSet

```

---



### 24.2.1 WuWenTsunTriangularSet (WUTSET)



See

⇒ “GeneralTriangularSet” (GTSET) 8.6.1 on page 1049

**Exports:**

|                                   |                               |
|-----------------------------------|-------------------------------|
| algebraic?                        | algebraicVariables            |
| any?                              | autoReduced?                  |
| basicSet                          | characteristicSerie           |
| characteristicSet                 | coerce                        |
| coHeight                          | collect                       |
| collectQuasiMonic                 | collectUnder                  |
| collectUpper                      | construct                     |
| convert                           | copy                          |
| count                             | degree                        |
| empty                             | empty?                        |
| eq?                               | eval                          |
| every?                            | extend                        |
| extendIfCan                       | find                          |
| first                             | hash                          |
| headReduce                        | headReduced?                  |
| headRemainder                     | infRittWu?                    |
| initiallyReduce                   | initiallyReduced?             |
| initials                          | last                          |
| latex                             | less?                         |
| mainVariable?                     | mainVariables                 |
| map                               | map!                          |
| medialSet                         | member?                       |
| members                           | more?                         |
| mvar                              | normalized?                   |
| normalized?                       | parts                         |
| quasiComponent                    | reduce                        |
| reduceByQuasiMonic                | reduced?                      |
| remainder                         | remove                        |
| removeDuplicates                  | removeZero                    |
| rest                              | retract                       |
| retractIfCan                      | rewriteIdealWithHeadRemainder |
| rewriteIdealWithRemainder         | rewriteSetWithReduction       |
| roughBase?                        | roughEqualIdeals?             |
| roughSubIdeal?                    | roughUnitIdeal?               |
| sample                            | select                        |
| size?                             | sort                          |
| stronglyReduce                    | stronglyReduced?              |
| triangular?                       | trivialIdeal?                 |
| variables                         | zeroSetSplit                  |
| zeroSetSplitIntoTriangularSystems | #?                            |
| ?=?                               | ?~=?                          |

— domain WUTSET WuWenTsunTriangularSet —

)abbrev domain WUTSET WuWenTsunTriangularSet

```

++ Author: Marc Moreno Maza (marc@nag.co.uk)
++ Date Created: 11/18/1995
++ Date Last Updated: 12/15/1998
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References :
++ [1] W. T. WU "A Zero Structure Theorem for polynomial equations solving"
++     MM Research Preprints, 1987.
++ [2] D. M. WANG "An implementation of the characteristic set method in Maple"
++     Proc. DISCO'92. Bath, England.
++ Description:
++ A domain constructor of the category \axiomType{GeneralTriangularSet}.
++ The only requirement for a list of polynomials to be a member of such
++ a domain is the following: no polynomial is constant and two distinct
++ polynomials have distinct main variables. Such a triangular set may
++ not be auto-reduced or consistent. The construct operation
++ does not check the previous requirement. Triangular sets are stored
++ as sorted lists w.r.t. the main variables of their members.
++ Furthermore, this domain exports operations dealing with the
++ characteristic set method of Wu Wen Tsun and some optimizations
++ mainly proposed by Dong Ming Wang.

```

WuWenTsunTriangularSet(R,E,V,P) : Exports == Implementation where

```

R : IntegralDomain
E : OrderedAbelianMonoidSup
V : OrderedSet
P : RecursivePolynomialCategory(R,E,V)
N ==> NonNegativeInteger
Z ==> Integer
B ==> Boolean
LP ==> List P
A ==> FiniteEdge P
H ==> FiniteSimpleHypergraph P
GPS ==> GeneralPolynomialSet(R,E,V,P)
RBT ==> Record(bas:$,top:LP)
RUL ==> Record(chs:Union($,"failed"),rfs:LP)
pa ==> PolynomialSetUtilitiesPackage(R,E,V,P)
NLpT ==> SplittingNode(LP,$)
ALpT ==> SplittingTree(LP,$)
O ==> OutputForm
OP ==> OutputPackage

```

Exports == TriangularSetCategory(R,E,V,P) with

```

medialSet : (LP,((P,P)->B),((P,P)->P)) -> Union($,"failed")
++ \axiom{medialSet(ps,redOp?,redOp)} returns \axiom{bs} a basic set

```

```

++ (in Wu Wen Tsun sense w.r.t the reduction-test \axiom{redOp?})
++ of some set generating the same ideal as \axiom{ps} (with
++ rank not higher than any basic set of \axiom{ps}), if no non-zero
++ constant polynomials appear during the computations, else
++ \axiom{"failed"} is returned. In the former case, \axiom{bs} has to be
++ understood as a candidate for being a characteristic set of \axiom{ps}.
++ In the original algorithm, \axiom{bs} is simply a basic set of \axiom{ps}.
medialSet: LP -> Union($,"failed")
++ \axiom{medialSet(ps)} returns the same as
++ \axiom{medialSet(ps,initiallyReduced?,initiallyReduce)}.
characteristicSet : (LP,((P,P)->B),((P,P)->P)) -> Union($,"failed")
++ \axiom{characteristicSet(ps,redOp?,redOp)} returns a non-contradictory
++ characteristic set of \axiom{ps} in Wu Wen Tsun sense w.r.t the
++ reduction-test \axiom{redOp?} (using \axiom{redOp} to reduce
++ polynomials w.r.t a \axiom{redOp?} basic set), if no
++ non-zero constant polynomial appear during those reductions,
++ else \axiom{"failed"} is returned.
++ The operations \axiom{redOp} and \axiom{redOp?} must satisfy
++ the following conditions: \axiom{redOp?(redOp(p,q),q)} holds
++ for every polynomials \axiom{p,q} and there exists an integer
++ \axiom{e} and a polynomial \axiom{f} such that we have
++ \axiom{init(q)^e*p = f*q + redOp(p,q)}.
characteristicSet: LP -> Union($,"failed")
++ \axiom{characteristicSet(ps)} returns the same as
++ \axiom{characteristicSet(ps,initiallyReduced?,initiallyReduce)}.
characteristicSerie : (LP,((P,P)->B),((P,P)->P)) -> List $
++ \axiom{characteristicSerie(ps,redOp?,redOp)} returns a list \axiom{lts}
++ of triangular sets such that the zero set of \axiom{ps} is the
++ union of the regular zero sets of the members of \axiom{lts}.
++ This is made by the Ritt and Wu Wen Tsun process applying
++ the operation \axiom{characteristicSet(ps,redOp?,redOp)}
++ to compute characteristic sets in Wu Wen Tsun sense.
characteristicSerie: LP -> List $
++ \axiom{characteristicSerie(ps)} returns the same as
++ \axiom{characteristicSerie(ps,initiallyReduced?,initiallyReduce)}.

Implementation == GeneralTriangularSet(R,E,V,P) add

removeSquares: $ -> Union($,"failed")

Rep ==> LP

rep(s:$):Rep == s pretend Rep
per(l:Rep):$ == l pretend $

removeAssociates (lp:LP):LP ==
  removeDuplicates [primPartElseUnitCanonical(p) for p in lp]

medialSetWithTrace (ps:LP,redOp?:((P,P)->B),redOp:((P,P)->P)):Union(RBT,"failed") ==
  qs := rewriteIdealWithQuasiMonicGenerators(ps,redOp?,redOp)$pa

```

```

contradiction : B := any?(ground?,ps)
contradiction => "failed"::Union(RBT,"failed")
rs : LP := qs
bs : $
while (not empty? rs) and (not contradiction) repeat
  rec := basicSet(rs,redOp?)
  contradiction := (rec case "failed")@B
  if not contradiction
  then
    bs := (rec::RBT).bas
    rs := (rec::RBT).top
    rs := rewriteIdealWithRemainder(rs,bs)
    -- contradiction := ((not empty? rs) and (one? first(rs)))
    contradiction := ((not empty? rs) and (first(rs) = 1))
    if (not empty? rs) and (not contradiction)
    then
      rs := rewriteSetWithReduction(rs,bs,redOp,redOp?)
      -- contradiction := ((not empty? rs) and (one? first(rs)))
      contradiction := ((not empty? rs) and (first(rs) = 1))
    if (not empty? rs) and (not contradiction)
    then
      rs := removeDuplicates concat(rs,members(bs))
      rs := rewriteIdealWithQuasiMonicGenerators(rs,redOp?,redOp)$pa
      -- contradiction := ((not empty? rs) and (one? first(rs)))
      contradiction := ((not empty? rs) and (first(rs) = 1))
    contradiction => "failed"::Union(RBT,"failed")
    ([bs,qs]$RBT)::Union(RBT,"failed")

medialSet(ps:LP,redOp?:((P,P)->B),redOp:((P,P)->P)) ==
  foo: Union(RBT,"failed") := medialSetWithTrace(ps,redOp?,redOp)
  (foo case "failed") => "failed" :: Union($,"failed")
  ((foo::RBT).bas) :: Union($,"failed")

medialSet(ps:LP) == medialSet(ps,initiallyReduced?,initiallyReduce)

characteristicSetUsingTrace(ps:LP,redOp?:((P,P)->B),redOp:((P,P)->P)):Union($,"failed")
ps := removeAssociates ps
ps := remove(zero?,ps)
contradiction : B := any?(ground?,ps)
contradiction => "failed"::Union($,"failed")
rs : LP := ps
qs : LP := ps
ms : $
while (not empty? rs) and (not contradiction) repeat
  rec := medialSetWithTrace (qs,redOp?,redOp)
  contradiction := (rec case "failed")@B
  if not contradiction
  then
    ms := (rec::RBT).bas
    qs := (rec::RBT).top

```

```

    qs := rewriteIdealWithRemainder(qs,ms)
--    contradiction := ((not empty? qs) and (one? first(qs)))
    contradiction := ((not empty? qs) and (first(qs) = 1))
    if not contradiction
    then
        rs := rewriteSetWithReduction(qs,ms, lazyPrem, reduced?)
--        contradiction := ((not empty? rs) and (one? first(rs)))
        contradiction := ((not empty? rs) and (first(rs) = 1))
    if (not contradiction) and (not empty? rs)
    then
        qs := removeDuplicates(concat(rs, concat(members(ms), qs)))
        contradiction => "failed"::Union($, "failed")
        ms::Union($, "failed")

characteristicSet(ps:LP, redOp?:((P,P)->B), redOp:((P,P)->P)) ==
    characteristicSetUsingTrace(ps, redOp?, redOp)

characteristicSet(ps:LP) == characteristicSet(ps, initiallyReduced?, initiallyReduce)

characteristicSerie(ps:LP, redOp?:((P,P)->B), redOp:((P,P)->P)) ==
    a := [[ps, empty()$$]$NLpT]$ALpT
    while ((esl := extractSplittingLeaf(a)) case ALpT) repeat
        ps := value(value(esl::ALpT)$ALpT)$NLpT
        charSet? := characteristicSetUsingTrace(ps, redOp?, redOp)
        if not (charSet? case $)
        then
            setvalue!(esl::ALpT, [nil()$LP, empty()$$, true]$NLpT)
            updateStatus!(a)
        else
            cs := (charSet?)::$
            lics := initials(cs)
            lics := removeRedundantFactors(lics)$pa
            lics := sort(infRittWu?, lics)
            if empty? lics
            then
                setvalue!(esl::ALpT, [ps, cs, true]$NLpT)
                updateStatus!(a)
            else
                ln : List NLpT := [[nil()$LP, cs, true]$NLpT]
                while not empty? lics repeat
                    newps := cons(first(lics), concat(cs::LP, ps))
                    lics := rest lics
                    newps := removeDuplicates newps
                    newps := sort(infRittWu?, newps)
                    ln := cons([newps, empty()$$, false]$NLpT, ln)
                splitNodeOf!(esl::ALpT, a, ln)
            remove(empty()$$, conditions(a))

characteristicSerie(ps:LP) == characteristicSerie (ps, initiallyReduced?, initiallyReduce)

```

```

if R has GcdDomain
then

  removeSquares (ts:$):Union($,"failed") ==
    empty?(ts)$ => ts::Union($,"failed")
  p := (first ts)::P
  rsts : Union($,"failed")
  rsts := removeSquares((rest ts)::)
  not(rsts case $) => "failed":Union($,"failed")
  newts := rsts::$
  empty? newts =>
    p := squareFreePart(p)
    (per([primitivePart(p)]$LP))::Union($,"failed")
  zero? initiallyReduce(init(p),newts) => "failed":Union($,"failed")
  p := primitivePart(removeZero(p,newts))
  ground? p => "failed":Union($,"failed")
  not (mvar(newts) < mvar(p)) => "failed":Union($,"failed")
  p := squareFreePart(p)
  (per(cons(unitCanonical(p),rep(newts))))::Union($,"failed")

  zeroSetSplit lp ==
    lts : List $ := characteristicSerie(lp,initiallyReduced?,initiallyReduce)
    lts := removeDuplicates(lts)$(List $)
    newlts : List $ := []
    while not empty? lts repeat
      ts := first lts
      lts := rest lts
      iic := removeSquares(ts)
      if iic case $
      then
        newlts := cons(iic::$,newlts)
    newlts := removeDuplicates(newlts)$(List $)
    sort(infRittWu?, newlts)

else

  zeroSetSplit lp ==
    lts : List $ := characteristicSerie(lp,initiallyReduced?,initiallyReduce)
    sort(infRittWu?, removeDuplicates lts)

```

---

— WUTSET.dotabb —

```

"WUTSET" [color="#88FF44",href="bookvol10.3.pdf#nameddest=WUTSET"]
"RPOLCAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=RPOLCAT"]
"WUTSET" -> "RPOLCAT"

```

—————





## Chapter 25

# Chapter X

### 25.1 domain XDPOLY XDistributedPolynomial

Polynomial arithmetic with non-commutative variables has been improved by a contribution of Michel Petitot (University of Lille I, France). The domain constructor **XDistributedPolynomial** provide a distributed representation for these polynomials. It is the non-commutative equivalent for the **DistributedMultivariatePolynomial** constructor.

— XDistributedPolynomial.input —

```
)set break resume
)sys rm -f XDistributedPolynomial.output
)spool XDistributedPolynomial.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show XDistributedPolynomial
--R XDistributedPolynomial(v1: OrderedSet,R: Ring) is a domain constructor
--R Abbreviation for XDistributedPolynomial is XDPOLY
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for XDPOLY
--R
--R----- Operations -----
--R ??? : (%,% ) -> %          ??? : (v1,% ) -> %
--R ??? : (% ,R) -> %          ??? : (R,% ) -> %
--R ??? : (Integer,% ) -> %     ??? : (PositiveInteger,% ) -> %
--R ??? : (% ,PositiveInteger) -> %  ??? : (% ,%) -> %
--R ?-? : (% ,%) -> %          -? : % -> %
--R ?=? : (% ,%) -> Boolean     1 : () -> %
--R 0 : () -> %                ?? : (% ,PositiveInteger) -> %
```

```

--R coef : (%,%) -> R
--R coerce : Integer -> %
--R coerce : vl -> %
--R constant : % -> R
--R degree : % -> NonNegativeInteger
--R latex : % -> String
--R lquo : (%,vl) -> %
--R map : ((R -> R),%) -> %
--R monomial? : % -> Boolean
--R one? : % -> Boolean
--R quasiRegular? : % -> Boolean
--R reductum : % -> %
--R rquo : (%,%) -> %
--R varList : % -> List vl
--R ~=? : (%,%) -> Boolean
--R ?? : (R,OrderedFreeMonoid vl) -> %
--R ?? : (NonNegativeInteger,%) -> %
--R ***? : (%,NonNegativeInteger) -> %
--R ?? : (%,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R coef : (%,OrderedFreeMonoid vl) -> R
--R coefficient : (%,OrderedFreeMonoid vl) -> R
--R coerce : OrderedFreeMonoid vl -> %
--R leadingMonomial : % -> OrderedFreeMonoid vl
--R leadingTerm : % -> Record(k: OrderedFreeMonoid vl,c: R)
--R listOfTerms : % -> List Record(k: OrderedFreeMonoid vl,c: R)
--R lquo : (%,OrderedFreeMonoid vl) -> %
--R maxdeg : % -> OrderedFreeMonoid vl
--R mindeg : % -> OrderedFreeMonoid vl
--R mindegTerm : % -> Record(k: OrderedFreeMonoid vl,c: R)
--R monom : (OrderedFreeMonoid vl,R) -> %
--R numberOfMonomials : % -> NonNegativeInteger
--R retract : % -> OrderedFreeMonoid vl
--R retractIfCan : % -> Union(OrderedFreeMonoid vl,"failed")
--R rquo : (%,OrderedFreeMonoid vl) -> %
--R sh : (%,%) -> % if R has COMRING
--R sh : (%,NonNegativeInteger) -> % if R has COMRING
--R subtractIfCan : (%,%) -> Union(%, "failed")
--R trunc : (%,NonNegativeInteger) -> %
--R
--E 1

)spool
)lisp (bye)

```

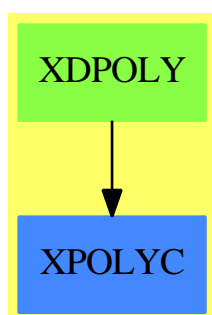
```
=====
XDistributedPolynomial examples
=====
```

See Also:

```
o )show XDistributedPolynomial
```

—————

### 25.1.1 XDistributedPolynomial (XDPOLY)



#### Exports:

|                    |                   |                 |              |
|--------------------|-------------------|-----------------|--------------|
| 0                  | 1                 | characteristic  | coef         |
| coefficient        | coefficients      | coerce          | constant     |
| constant?          | degree            | hash            | latex        |
| leadingCoefficient | listOfTerms       | leadingMonomial | leadingTerm  |
| lquo               | map               | mirror          | monomial?    |
| monomials          | maxdeg            | mindeg          | mindegTerm   |
| monom              | numberOfMonomials | one?            | quasiRegular |
| quasiRegular?      | recip             | reductum        | retract      |
| retractIfCan       | rquo              | sample          | sh           |
| subtractIfCan      | trunc             | varList         | zero?        |
| ?*?                | ?**?              | ?+?             | ?-?          |
| -?                 | ?=?               | ?^?             | ?~=?         |

— domain XDPOLY XDistributedPolynomial —

```
)abbrev domain XDPOLY XDistributedPolynomial
++ Author: Michel Petitot petitot@lifl.fr
++ Date Created: 91
++ Date Last Updated: 7 Juillet 92
++ Fix History: compilation v 2.1 le 13 dec 98
++ Basic Functions:
++ Related Constructors:
```

```

++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This type supports distributed multivariate polynomials
++ whose variables do not commute.
++ The coefficient ring may be non-commutative too.
++ However, coefficients and variables commute.

XDistributedPolynomial(vl:OrderedSet,R:Ring): XDPcat == XDPdef where

WORD ==> OrderedFreeMonoid(vl)
I     ==> Integer
NNI   ==> NonNegativeInteger
TERM  ==> Record(k:WORD, c:R)

XDPcat == Join(FreeModuleCat(R, WORD), XPolynomialsCat(vl,R))

XDPdef == XPolynomialRing(R,WORD) add

import( WORD, TERM)

-- Representation
Rep := List TERM

-- local functions
shw: (WORD , WORD) -> %    -- shuffle de 2 mots

-- definitions

mindegTerm p == last(p)$Rep

if R has CommutativeRing then
  sh(p:%, n:NNI):% ==
    n=0 => 1
    n=1 => p
    n1: NNI := (n-$I 1):NNI
    sh(p, sh(p,n1))

  sh(p1:%, p2:%) ==
    p:% := 0
    for t1 in p1 repeat
      for t2 in p2 repeat
        p := p + (t1.c * t2.c) * shw(t1.k,t2.k)
    p

  coerce(v: vl):% == coerce(v::WORD)
  v:vl * p:% ==

```

```

[[v * t.k , t.c]$TERM for t in p]

mirror p ==
  null p => p
  monom(mirror$WORD leadingMonomial p, leadingCoefficient p) + _
  mirror reductum p

degree(p) == length(maxdeg(p))$WORD

trunc(p, n) ==
  p = 0 => p
  degree(p) > n => trunc( reductum p , n)
  p

varList p ==
  constant? p => []
  le : List vl := "setUnion"/[varList(t.k) for t in p]
  sort_!(le)

rquo(p:% , w: WORD) ==
  [[r::WORD,t.c]$TERM for t in p | not (r:= rquo(t.k,w)) case "failed" ]
lquo(p:% , w: WORD) ==
  [[r::WORD,t.c]$TERM for t in p | not (r:= lquo(t.k,w)) case "failed" ]
rquo(p:% , v: vl) ==
  [[r::WORD,t.c]$TERM for t in p | not (r:= rquo(t.k,v)) case "failed" ]
lquo(p:% , v: vl) ==
  [[r::WORD,t.c]$TERM for t in p | not (r:= lquo(t.k,v)) case "failed" ]

shw(w1,w2) ==
  w1 = 1$WORD => w2::%
  w2 = 1$WORD => w1::%
  x: vl := first w1 ; y: vl := first w2
  x * shw(rest w1,w2) + y * shw(w1,rest w2)

lquo(p:%,q:%):% ==
  +/ [r * t.c for t in q | (r := lquo(p,t.k)) ^= 0]

rquo(p:%,q:%):% ==
  +/ [r * t.c for t in q | (r := rquo(p,t.k)) ^= 0]

coef(p:%,q:%):R ==
  p = 0 => 0$R
  q = 0 => 0$R
  p.first.k > q.first.k => coef(p.rest,q)
  p.first.k < q.first.k => coef(p,q.rest)
  return p.first.c * q.first.c + coef(p.rest,q.rest)

```

---

— XDPOLY.dotabb —

```
"XDPOLY" [color="#88FF44",href="bookvol10.3.pdf#nameddest=XDPOLY"]
"XPOLYC" [color="#4488FF",href="bookvol10.2.pdf#nameddest=XPOLYC"]
"XDPOLY" -> "XPOLYC"
```

—

## 25.2 domain XPBWPOLY XPBWPolynomial

— XPBWPolynomial.input —

```
)set break resume
)sys rm -f XPBWPolynomial.output
)spool XPBWPolynomial.output
)set message test on
)set message auto off
)clear all
--S 1 of 39
a:Symbol := 'a
--R
--R
--R (1) a
--R
--R                                          Type: Symbol
--E 1

--S 2 of 39
b:Symbol := 'b
--R
--R
--R (2) b
--R
--R                                          Type: Symbol
--E 2

--S 3 of 39
RN := Fraction(Integer)
--R
--R
--R (3) Fraction Integer
--R
--R                                          Type: Domain
--E 3

--S 4 of 39
word := OrderedFreeMonoid Symbol
--R
```

```

--R
--R (4) OrderedFreeMonoid Symbol
--R
--R                                          Type: Domain
--E 4

--S 5 of 39
lword := LyndonWord(Symbol)
--R
--R
--R (5) LyndonWord Symbol
--R
--R                                          Type: Domain
--E 5

--S 6 of 39
base := PoincareBirkhoffWittLyndonBasis Symbol
--R
--R
--R (6) PoincareBirkhoffWittLyndonBasis Symbol
--R
--R                                          Type: Domain
--E 6

--S 7 of 39
dpoly := XDistributedPolynomial(Symbol, RN)
--R
--R
--R (7) XDistributedPolynomial(Symbol, Fraction Integer)
--R
--R                                          Type: Domain
--E 7

--S 8 of 39
rpoly := XRecursivePolynomial(Symbol, RN)
--R
--R
--R (8) XRecursivePolynomial(Symbol, Fraction Integer)
--R
--R                                          Type: Domain
--E 8

--S 9 of 39
lpoly := LiePolynomial(Symbol, RN)
--R
--R
--R (9) LiePolynomial(Symbol, Fraction Integer)
--R
--R                                          Type: Domain
--E 9

--S 10 of 39
poly := XPBWPolynomial(Symbol, RN)
--R
--R
--R (10) XPBWPolynomial(Symbol, Fraction Integer)

```



```

--R                                                    Type: Domain
--E 10

--S 11 of 39
liste : List lword := LyndonWordsList([a,b], 6)
--R
--R
--R (11)
--R      2      2      3      2 2      3      4      3 2
--R      [[a], [b], [a b], [a b], [a b ], [a b], [a b ], [a b ], [a b], [a b ],
--R      2      2 3      2      4      5      4 2      3      3 3
--R      [a b a b], [a b ], [a b a b ], [a b ], [a b], [a b ], [a b a b], [a b ],
--R      2      2      2 2      2 4      3      5
--R      [a b a b ], [a b a b], [a b ], [a b a b ], [a b ]]
--R                                                    Type: List LyndonWord Symbol
--E 11

--S 12 of 39
0$poly
--R
--R
--R (12)  0
--R                                                    Type: XPBWPolynomial(Symbol,Fraction Integer)
--E 12

--S 13 of 39
1$poly
--R
--R
--R (13)  1
--R                                                    Type: XPBWPolynomial(Symbol,Fraction Integer)
--E 13

--S 14 of 39
p : poly := a
--R
--R
--R (14)  [a]
--R                                                    Type: XPBWPolynomial(Symbol,Fraction Integer)
--E 14

--S 15 of 39
q : poly := b
--R
--R
--R (15)  [b]
--R                                                    Type: XPBWPolynomial(Symbol,Fraction Integer)
--E 15

--S 16 of 39

```

```

pq: poly := p*q
--R
--R
--R (16) [a b] + [b][a]
--R                                         Type: XPBWPolynomial(Symbol,Fraction Integer)
--E 16

--S 17 of 39
pq :: dpoly
--R
--R
--R (17) a b
--R                                         Type: XDistributedPolynomial(Symbol,Fraction Integer)
--E 17

--S 18 of 39
mirror pq
--R
--R
--R (18) [b][a]
--R                                         Type: XPBWPolynomial(Symbol,Fraction Integer)
--E 18

--S 19 of 39
listOfTerms pq
--R
--R
--R (19) [[k= [b][a],c= 1],[k= [a b],c= 1]]
--RType: List Record(k: PoincareBirkhoffWittLyndonBasis Symbol,c: Fraction Integer)
--E 19

--S 20 of 39
reductum pq
--R
--R
--R (20) [a b]
--R                                         Type: XPBWPolynomial(Symbol,Fraction Integer)
--E 20

--S 21 of 39
leadingMonomial pq
--R
--R
--R (21) [b][a]
--R                                         Type: PoincareBirkhoffWittLyndonBasis Symbol
--E 21

--S 22 of 39
coefficients pq
--R

```

```

--R
--R (22) [1,1]
--R
--R                                         Type: List Fraction Integer
--E 22

--S 23 of 39
leadingTerm pq
--R
--R
--R (23) [k= [b][a],c= 1]
--R Type: Record(k: PoincareBirkhoffWittLyndonBasis Symbol,c: Fraction Integer)
--E 23

--S 24 of 39
degree pq
--R
--R
--R (24) 2
--R
--R                                         Type: PositiveInteger
--E 24

--S 25 of 39
pq4:=exp(pq,4)
--R
--R
--R (25)
--R
--R      1      1      2      1      2
--R      1 + [a b] + [b][a] + - [a b][a b] + - [a b ][a] + - [b][a b]
--R      2      2      2      2
--R
--R      +
--R      3      1
--R      - [b][a b][a] + - [b][b][a][a]
--R      2      2
--R
--R                                         Type: XPBWPolynomial(Symbol,Fraction Integer)
--E 25

--S 26 of 39
log(pq4,4) - pq
--R
--R
--R (26) 0
--R
--R                                         Type: XPBWPolynomial(Symbol,Fraction Integer)
--E 26

--S 27 of 39
lp1 :lpoly := LiePoly liste.10
--R
--R
--R      3 2
--R (27) [a b ]

```

```

--R                                         Type: LiePolynomial(Symbol,Fraction Integer)
--E 27

--S 28 of 39
lp2 :lpoly := LiePoly liste.11
--R
--R
--R          2
--R   (28)  [a b a b]
--R                                         Type: LiePolynomial(Symbol,Fraction Integer)
--E 28

--S 29 of 39
lp :lpoly := [lp1, lp2]
--R
--R
--R          3 2 2
--R   (29)  [a b a b a b]
--R                                         Type: LiePolynomial(Symbol,Fraction Integer)
--E 29

--S 30 of 39
lpd1: dpoly := lp1
--R
--R
--R          3 2      2      2 2      2 2      2      2 2 3
--R   (30)  a b - 2a b a b - a b a + 4a b a b a - a b a - 2b a b a + b a
--R                                         Type: XDistributedPolynomial(Symbol,Fraction Integer)
--E 30

--S 31 of 39
lpd2: dpoly := lp2
--R
--R
--R   (31)
--R          2      2 2      2      2 2      3      2
--R          a b a b - a b a - 3a b a b + 4a b a b a - a b a + 2b a b - 3b a b a
--R   +
--R          2
--R          b a b a
--R                                         Type: XDistributedPolynomial(Symbol,Fraction Integer)
--E 31

--S 32 of 39
lpd : dpoly := lpd1 * lpd2 - lpd2 * lpd1
--R
--R
--R   (32)
--R          3 2 2      3 2 2 2      3 2      2      3 2      3 2      2 2
--R          a b a b a b - a b a b a - 3a b a b a b + 4a b a b a b a - a b a b a

```

$$\begin{aligned}
& \text{--R} + \\
& \text{--R} \quad \begin{array}{cccccccc} 3 & 3 & 3 & & 3 & 3 & 2 & & 3 & 3 & & 2 & 2 & & 3 & 2 & & 2 & 2 & & 2 & 2 \end{array} \\
& \text{--R} \quad 2a^3 b^3 a^3 b - 3a^3 b^3 a^2 b^2 a + a^3 b^3 a^2 b^2 a^2 - a^3 b^3 a^2 b^2 a^3 + 3a^3 b^3 a^2 b^2 a^4 a \\
& \text{--R} + \\
& \text{--R} \quad \begin{array}{cccccccc} 2 & & 2 & & 2 & & 2 & & 2 & & 2 & 2 & & 2 & & 2 & 3 & & 2 & 3 \end{array} \\
& \text{--R} \quad 6a^2 b^2 a^2 b^2 a^2 b - 12a^2 b^2 a^2 b^2 a^2 b^2 a + 3a^2 b^2 a^2 b^2 a^2 b^2 a^2 - 4a^2 b^2 a^2 b^2 a^2 b^2 a^3 \\
& \text{--R} + \\
& \text{--R} \quad \begin{array}{cccccccc} 2 & & 2 & 2 & & 2 & & 3 & 3 & & 2 & 2 & 4 & 2 & & 2 & 2 & 3 & & 2 & 2 & 2 & & 2 & 2 \end{array} \\
& \text{--R} \quad 6a^2 b^2 a^2 b^2 a^2 b^2 a - a^2 b^2 a^2 b^2 a^2 b^2 a^2 + a^2 b^2 a^2 b^2 a^2 b^2 a^2 - 3a^2 b^2 a^2 b^2 a^2 b^2 a^3 + 3a^2 b^2 a^2 b^2 a^2 b^2 a^4 \\
& \text{--R} + \\
& \text{--R} \quad \begin{array}{cccccccc} 2 & 2 & & 3 & & 2 & 2 & & 2 & & 2 & 2 & & 2 & & 2 & 2 & & 2 & 3 & & 2 & 3 \end{array} \\
& \text{--R} \quad - 2a^2 b^2 a^2 b^2 a^2 b^2 a + 3a^2 b^2 a^2 b^2 a^2 b^2 a^2 - 3a^2 b^2 a^2 b^2 a^2 b^2 a^3 + a^2 b^2 a^2 b^2 a^2 b^2 a^4 \\
& \text{--R} + \\
& \text{--R} \quad \begin{array}{cccccccc} 2 & & 3 & 2 & & 2 & & 2 & & 2 & & 2 & 2 & & 2 & & 2 & 2 & & 2 & 3 \end{array} \\
& \text{--R} \quad 3a^2 b^2 a^2 b^2 a^2 b^2 a - 6a^2 b^2 a^2 b^2 a^2 b^2 a^2 b^2 a - 3a^2 b^2 a^2 b^2 a^2 b^2 a^3 + 12a^2 b^2 a^2 b^2 a^2 b^2 a^4 \\
& \text{--R} + \\
& \text{--R} \quad \begin{array}{cccccccc} 2 & & 2 & 2 & & 2 & 2 & & 2 & & 2 & 3 & 3 & & 4 & 2 & & 4 & 2 \end{array} \\
& \text{--R} \quad - 3a^2 b^2 a^2 b^2 a^2 b^2 a - 6a^2 b^2 a^2 b^2 a^2 b^2 a^2 b^2 a + 3a^2 b^2 a^2 b^2 a^2 b^2 a^3 - 4a^2 b^2 a^2 b^2 a^2 b^2 a^4 \\
& \text{--R} + \\
& \text{--R} \quad \begin{array}{cccccccc} 3 & & 2 & & 2 & & 2 & & 3 \end{array} \\
& \text{--R} \quad 12a^3 b^3 a^3 b^3 a^3 b - 12a^3 b^3 a^3 b^3 a^3 b^3 a^3 b + 8a^3 b^3 a^3 b^3 a^3 b^3 a^3 b \\
& \text{--R} + \\
& \text{--R} \quad \begin{array}{cccccccc} 2 & & 2 & & 2 & & 2 & 3 & & 2 & 5 & 2 \end{array} \\
& \text{--R} \quad - 12a^2 b^2 a^2 b^2 a^2 b^2 a^2 b^2 a + 12a^2 b^2 a^2 b^2 a^2 b^2 a^2 b^2 a^2 - 4a^2 b^2 a^2 b^2 a^2 b^2 a^3 + a^2 b^2 a^2 b^2 a^2 b^2 a^5 \\
& \text{--R} + \\
& \text{--R} \quad \begin{array}{cccccccc} 2 & 4 & & 2 & 3 & & 2 & & 2 & 2 & 3 & & 2 & 2 & & 2 & 2 \end{array} \\
& \text{--R} \quad - 3a^2 b^2 a^2 b^2 a^2 b^2 a + 3a^2 b^2 a^2 b^2 a^2 b^2 a^2 b^2 a - 2a^2 b^2 a^2 b^2 a^2 b^2 a^3 + 3a^2 b^2 a^2 b^2 a^2 b^2 a^4 \\
& \text{--R} + \\
& \text{--R} \quad \begin{array}{cccccccc} 2 & 2 & & 2 & & 2 & 2 & 2 & 3 & & 3 & 3 & 2 & & 3 & 2 \end{array} \\
& \text{--R} \quad - 3a^2 b^2 a^2 b^2 a^2 b^2 a + a^2 b^2 a^2 b^2 a^2 b^2 a^2 - 2b^2 a^2 b^2 a^2 b^2 a^3 + 4b^2 a^2 b^2 a^2 b^2 a^4 \\
& \text{--R} + \\
& \text{--R} \quad \begin{array}{cccccccc} 3 & & 2 & 2 & & 3 & & 3 & & 2 & 2 & & 3 & 2 & & 2 & & 3 & 3 & 3 \end{array} \\
& \text{--R} \quad 2b^3 a^3 b^3 a^3 b^3 a - 8b^3 a^3 b^3 a^3 b^3 a^3 b^3 a + 2b^3 a^3 b^3 a^3 b^3 a^3 b^3 a^2 + 4b^3 a^3 b^3 a^3 b^3 a^3 b^3 a^3 - 2b^3 a^3 b^3 a^3 b^3 a^4 \\
& \text{--R} + \\
& \text{--R} \quad \begin{array}{cccccccc} 2 & & 4 & 2 & & 2 & & 3 & & 2 & & 3 & 2 & & 2 & & 2 \end{array} \\
& \text{--R} \quad 3b^2 a^2 b^2 a^2 b^2 a - 6b^2 a^2 b^2 a^2 b^2 a^2 b^2 a - 3b^2 a^2 b^2 a^2 b^2 a^3 + 12b^2 a^2 b^2 a^2 b^2 a^4 \\
& \text{--R} + \\
& \text{--R} \quad \begin{array}{cccccccc} 2 & & 2 & 2 & 2 & & 2 & & 2 & & 2 & & 2 & 3 & & 5 & 2 \end{array} \\
& \text{--R} \quad - 3b^2 a^2 b^2 a^2 b^2 a - 6b^2 a^2 b^2 a^2 b^2 a^2 b^2 a^2 + 3b^2 a^2 b^2 a^2 b^2 a^3 - b^2 a^2 b^2 a^2 b^2 a^5 \\
& \text{--R} + \\
& \text{--R} \quad \begin{array}{cccccccc} 4 & 2 & & 3 & & 2 & & 3 & & 3 & & 3 & 2 & 2 \end{array} \\
& \text{--R} \quad 3b^4 a^4 b^4 a^4 b^4 a + 6b^4 a^4 b^4 a^4 b^4 a^2 - 12b^4 a^4 b^4 a^4 b^4 a^3 + 3b^4 a^4 b^4 a^4 b^4 a^4 \\
& \text{--R} + \\
& \text{--R} \quad \begin{array}{cccccccc} 2 & & 3 & & 2 & & 2 & & 2 & 2 & 3 & & 2 & 5 & & 2 & 5 & 2 \end{array} \\
& \text{--R} \quad - 4b^2 a^2 b^2 a^2 b^2 a^2 b^2 a + 6b^2 a^2 b^2 a^2 b^2 a^2 b^2 a^2 - b^2 a^2 b^2 a^2 b^2 a^3 + b^2 a^2 b^2 a^2 b^2 a^5 - b^2 a^2 b^2 a^2 b^2 a^5 \\
& \text{--R} + \\
& \text{--R} \quad \begin{array}{cccccccc} 2 & 4 & & 2 & & 2 & 4 & & 2 & 4 & 2 & 2 & & 2 & 3 & & 3 & & 2 & 3 & & 2 \end{array} \\
& \text{--R} \quad - 3b^2 a^2 b^2 a^2 b^2 a + 4b^2 a^2 b^2 a^2 b^2 a^2 b^2 a - b^2 a^2 b^2 a^2 b^2 a^3 + 2b^2 a^2 b^2 a^2 b^2 a^4 - 3b^2 a^2 b^2 a^2 b^2 a^5 \\
& \text{--R} + \\
& \text{--R} \quad \begin{array}{cccc} 2 & 3 & & 2 \end{array}
\end{aligned}$$

```

--R      b a b a b a
--R                                          Type: XDistributedPolynomial(Symbol,Fraction Integer)
--E 32

--S 33 of 39
lp :: dpoly - lpd
--R
--R
--R      (33)  0
--R                                          Type: XDistributedPolynomial(Symbol,Fraction Integer)
--E 33

--S 34 of 39
p := 3 * lp
--R
--R
--R      3 2 2
--R      (34)  3[a b a b a b]
--R                                          Type: XPBWPolynomial(Symbol,Fraction Integer)
--E 34

--S 35 of 39
q := lp1
--R
--R
--R      3 2
--R      (35)  [a b ]
--R                                          Type: XPBWPolynomial(Symbol,Fraction Integer)
--E 35

--S 36 of 39
pq:= p * q
--R
--R
--R      3 2 2      3 2
--R      (36)  3[a b a b a b][a b ]
--R                                          Type: XPBWPolynomial(Symbol,Fraction Integer)
--E 36

--S 37 of 39
pr:rpoly := p :: rpoly
--R
--R
--R      (37)
--R      a
--R      *
--R      a
--R      *
--R      a b b
--R      *
```

```

--R      a(a b(a b 3 + b a(- 3)) + b(a(a b(- 9) + b a 12) + b a a(- 3)))
--R      +
--R      b a(a(a b 6 + b a(- 9)) + b a a 3)
--R      +
--R      b
--R      *
--R      a b
--R      *
--R      a
--R      *
--R      a(a b b(- 3) + b b a 9)
--R      +
--R      b(a(a b 18 + b a(- 36)) + b a a 9)
--R      +
--R      b(a a(a b(- 12) + b a 18) + b a a a(- 3))
--R      +
--R      b a
--R      *
--R      a(a(a b b 3 + b a b(- 9)) + b a a b 9)
--R      +
--R      b(a(a(a b(- 6) + b a 9) + b a a(- 9)) + b a a a 3)
--R      +
--R      b
--R      *
--R      a
--R      *
--R      a b
--R      *
--R      a
--R      *
--R      a(a b b 9 + b(a b(- 18) + b a(- 9)))
--R      +
--R      b(a b a 36 + b a a(- 9))
--R      +
--R      b(a b a a(- 18) + b a a a 9)
--R      +
--R      b a
--R      *
--R      a(a(a b b(- 12) + b a b 36) + b a a b(- 36))
--R      +
--R      b(a(a(a b 24 + b a(- 36)) + b a a 36) + b a a a(- 12))
--R      +
--R      b a a
--R      *
--R      a(a(a b b 3 + b a b(- 9)) + b a a b 9)
--R      +
--R      b(a(a(a b(- 6) + b a 9) + b a a(- 9)) + b a a a 3)
--R      +
--R      b
--R      *

```

```

--R      a
--R      *
--R      a
--R      *
--R      a b
--R      *
--R      a
--R      *
--R      a(a b b(- 6) + b(a b 12 + b a 6))
--R      +
--R      b(a b a(- 24) + b a a 6)
--R      +
--R      b(a b a a 12 + b a a a(- 6))
--R      +
--R      b a
--R      *
--R      a
--R      *
--R      a(a b b 9 + b(a b(- 18) + b a(- 9)))
--R      +
--R      b(a b a 36 + b a a(- 9))
--R      +
--R      b(a b a a(- 18) + b a a a 9)
--R      +
--R      b a a
--R      *
--R      a(a(a b b(- 3) + b b a 9) + b(a(a b 18 + b a(- 36)) + b a a 9))
--R      +
--R      b(a a(a b(- 12) + b a 18) + b a a a(- 3))
--R      +
--R      b a a a
--R      *
--R      a(a b(a b 3 + b a(- 3)) + b(a(a b(- 9) + b a 12) + b a a(- 3)))
--R      +
--R      b a(a(a b 6 + b a(- 9)) + b a a 3)
--R      Type: XRecursivePolynomial(Symbol,Fraction Integer)
--E 37

```

--S 38 of 39

qr:rpoly := q :: rpoly

```

--R
--R
--R      (38)
--R      a(a(a b b 1 + b(a b(- 2) + b a(- 1))) + b(a b a 4 + b a a(- 1)))
--R      +
--R      b(a b a a(- 2) + b a a a 1)
--R      Type: XRecursivePolynomial(Symbol,Fraction Integer)
--E 38

```

--S 39 of 39



```

pq :: rpoly - pr*qr
--R
--R
--R (39) 0
--R
--R                                     Type: XRecursivePolynomial(Symbol,Fraction Integer)
--E 39
)spool
)lisp (bye)

```

---

— XPBWPolynomial.help —

=====

XPBWPolynomial examples

=====

Initialisations

```

a:Symbol := 'a
a
                                     Type: Symbol

b:Symbol := 'b
b
                                     Type: Symbol

RN := Fraction(Integer)
Fraction Integer
                                     Type: Domain

word := OrderedFreeMonoid Symbol
OrderedFreeMonoid Symbol
                                     Type: Domain

lword := LyndonWord(Symbol)
LyndonWord Symbol
                                     Type: Domain

base := PoincareBirkhoffWittLyndonBasis Symbol
PoincareBirkhoffWittLyndonBasis Symbol
                                     Type: Domain

dpoly := XDistributedPolynomial(Symbol, RN)
XDistributedPolynomial(Symbol,Fraction Integer)
                                     Type: Domain

rpoly := XRecursivePolynomial(Symbol, RN)
XRecursivePolynomial(Symbol,Fraction Integer)

```

```

                                Type: Domain

lpoly := LiePolynomial(Symbol, RN)
        LiePolynomial(Symbol, Fraction Integer)
                                Type: Domain

poly  := XPBWPolynomial(Symbol, RN)
        XPBWPolynomial(Symbol, Fraction Integer)
                                Type: Domain

liste : List lword := LyndonWordsList([a,b], 6)
        2      2      3      2 2      3      4      3 2
[[a], [b], [a b], [a b], [a b], [a b], [a b], [a b], [a b], [a b],
  2      2 3      2      4      5      4 2      3      3 3
[a b a b], [a b], [a b a b], [a b], [a b], [a b], [a b a b], [a b],
  2      2      2 2      2 4      3      5
[a b a b], [a b a b], [a b], [a b a b], [a b]]
                                Type: List LyndonWord Symbol

Let's make some polynomials

0$poly
  0
                                Type: XPBWPolynomial(Symbol, Fraction Integer)

1$poly
  1
                                Type: XPBWPolynomial(Symbol, Fraction Integer)

p : poly := a
[a]
                                Type: XPBWPolynomial(Symbol, Fraction Integer)

q : poly := b
[b]
                                Type: XPBWPolynomial(Symbol, Fraction Integer)

pq: poly := p*q
[a b] + [b][a]
                                Type: XPBWPolynomial(Symbol, Fraction Integer)

Coerce to distributed polynomial

pq :: dpoly
  a b
                                Type: XDistributedPolynomial(Symbol, Fraction Integer)

Check some polynomial operations

mirror pq

```

```

[b] [a]
Type: XPBWPolynomial(Symbol,Fraction Integer)

listOfTerms pq
[[k= [b] [a],c= 1],[k= [a b],c= 1]]
Type: List Record(k: PoincareBirkhoffWittLyndonBasis Symbol,
c: Fraction Integer)

reductum pq
[a b]
Type: XPBWPolynomial(Symbol,Fraction Integer)

leadingMonomial pq
[b] [a]
Type: PoincareBirkhoffWittLyndonBasis Symbol

coefficients pq
[1,1]
Type: List Fraction Integer

leadingTerm pq
[k= [b] [a],c= 1]
Type: Record(k: PoincareBirkhoffWittLyndonBasis Symbol,
c: Fraction Integer)

degree pq
2
Type: PositiveInteger

pq4:=exp(pq,4)
1 + [a b] + [b] [a] +  $\frac{1}{2}$  [a b] [a b] +  $\frac{1}{2}$  [a b ] [a] +  $\frac{1}{2}$  [b] [a b]
+
 $\frac{3}{2}$  [b] [a b] [a] +  $\frac{1}{2}$  [b] [b] [a] [a]
Type: XPBWPolynomial(Symbol,Fraction Integer)

log(pq4,4) - pq
(26) 0
Type: XPBWPolynomial(Symbol,Fraction Integer)

Calculations with verification in XDistributedPolynomial.

lp1 :lpoly := LiePoly liste.10
3 2
[a b ]
Type: LiePolynomial(Symbol,Fraction Integer)

```

```
lp2 :lpoly := LiePoly liste.11
```

```
  2
[a b a b]
```

```
      Type: LiePolynomial(Symbol,Fraction Integer)
```

```
lp :lpoly := [lp1, lp2]
```

```
  3 2 2
[a b a b a b]
```

```
      Type: LiePolynomial(Symbol,Fraction Integer)
```

```
lpd1: dpoly := lp1
```

```
  3 2      2      2 2      2 2      2      2 3
a b - 2a b a b - a b a + 4a b a b a - a b a - 2b a b a + b a
```

```
      Type: XDistributedPolynomial(Symbol,Fraction Integer)
```

```
lpd2: dpoly := lp2
```

```
  2      2 2      2      2 2      3      2
a b a b - a b a - 3a b a b + 4a b a b a - a b a + 2b a b - 3b a b a
+
  2
b a b a
```

```
      Type: XDistributedPolynomial(Symbol,Fraction Integer)
```

```
lpd : dpoly := lpd1 * lpd2 - lpd2 * lpd1
```

```
  3 2 2      3 2 2 2      3 2      2      3 2      3 2      2 2
a b a b a b - a b a b a - 3a b a b a b + 4a b a b a b a - a b a b a
+
  3 3 3      3 3 2      3 3      2      2      3 2      2      2 2
2a b a b - 3a b a b a + a b a b a - a b a b a b + 3a b a b a b a
+
  2      2      2      2      2 2      2      2 3
6a b a b a b a b - 12a b a b a b a b a + 3a b a b a b a - 4a b a b a b
+
  2      2 2      2      3 3      2 2 4 2      2 2 3      2 2 2      2
6a b a b a b a - a b a b a + a b a b - 3a b a b a b + 3a b a b a b
+
  2 2      3      2 2      2      2 2      2      2 2      2 3
- 2a b a b a b + 3a b a b a b a - 3a b a b a b a + a b a b a
+
  2      3 2      2      2      2      2 2      2
3a b a b a b - 6a b a b a b a b - 3a b a b a b a + 12a b a b a b a b a
+
  2      2 2      2 2      2      2 3 3      4 2
- 3a b a b a b a - 6a b a b a b a + 3a b a b a - 4a b a b a b
+
  3      2      2      3
12a b a b a b a b - 12a b a b a b a b + 8a b a b a b a b
+
  2      2      2 3      2 5 2
- 12a b a b a b a b a + 12a b a b a b a b a - 4a b a b a b a + a b a b
```

```

+
      2 4      2 3 2      2 2 3      2 2 2
    - 3a b a b a b + 3a b a b a b - 2a b a b a b + 3a b a b a b a
+
      2 2      2      2 2 2 3      3 3 2      3 2
    - 3a b a b a b a + a b a b a b - 2b a b a b + 4b a b a b a b
+
      3 2 2      3      3 2 2      3 2 2      3 3 3
    2b a b a b a b - 8b a b a b a b a + 2b a b a b a b + 4b a b a b a b - 2b a b a
+
      2 4 2      2 3      2 3 2      2 2
    3b a b a b - 6b a b a b a b - 3b a b a b a b + 12b a b a b a b a
+
      2 2 2 2      2      2      2 2 3      5 2
    - 3b a b a b a b - 6b a b a b a b a + 3b a b a b a b - b a b a b
+
      4 2      3 2      3      3 2 2
    3b a b a b a b + 6b a b a b a b - 12b a b a b a b a + 3b a b a b a
+
      2 3      2 2      2 2 3      2 5      2 5 2
    - 4b a b a b a b + 6b a b a b a b a - b a b a b a b + b a b a b - b a b a
+
      2 4 2      2 4      2 4 2 2      2 3 3      2 3 2
    - 3b a b a b + 4b a b a b a b - b a b a b + 2b a b a b - 3b a b a b a
+
      2 3      2
    b a b a b a

```

Type: XDistributedPolynomial(Symbol,Fraction Integer)

```

lp :: dpoly - lpd
0

```

Type: XDistributedPolynomial(Symbol,Fraction Integer)

Calculations with verification in XRecursivePolynomial.

```

p := 3 * lp
      3 2 2
    3[a b a b a b]

```

Type: XPBWPolynomial(Symbol,Fraction Integer)

```

q := lp1
      3 2
    [a b]

```

Type: XPBWPolynomial(Symbol,Fraction Integer)

```

pq:= p * q
      3 2 2      3 2
    3[a b a b a b][a b]

```

Type: XPBWPolynomial(Symbol,Fraction Integer)

```

pr:rpoly := p :: rpoly
  a
  *
    a
    *
      a b b
      *
        a(a b(a b 3 + b a(- 3)) + b(a(a b(- 9) + b a 12) + b a a(- 3)))
        +
          b a(a(a b 6 + b a(- 9)) + b a a 3)
      +
        b
        *
          a b
          *
            a
            *
              a(a b b(- 3) + b b a 9)
              +
                b(a(a b 18 + b a(- 36)) + b a a 9)
            +
              b(a a(a b(- 12) + b a 18) + b a a a(- 3))
          +
            b a
            *
              a(a(a b b 3 + b a b(- 9)) + b a a b 9)
              +
                b(a(a(a b(- 6) + b a 9) + b a a(- 9)) + b a a a 3)
        +
          b
          *
            a
            *
              a b
              *
                a
                *
                  a(a b b 9 + b(a b(- 18) + b a(- 9)))
                  +
                    b(a b a 36 + b a a(- 9))
                +
                  b(a b a a(- 18) + b a a a 9)
              +
                b a
                *
                  a(a(a b b(- 12) + b a b 36) + b a a b(- 36))
                  +
                    b(a(a(a b 24 + b a(- 36)) + b a a 36) + b a a a(- 12))
            +
              b a a

```

```

      *
      a(a(a b b 3 + b a b(- 9)) + b a a b 9)
    +
      b(a(a(a b(- 6) + b a 9) + b a a(- 9)) + b a a a 3)
+
  b
*
  a
*
  a
*
  a b
*
  a
*
  a(a b b(- 6) + b(a b 12 + b a 6))
    +
      b(a b a(- 24) + b a a 6)
    +
      b(a b a a 12 + b a a a(- 6))
+
  b a
*
  a
*
  a(a b b 9 + b(a b(- 18) + b a(- 9)))
    +
      b(a b a 36 + b a a(- 9))
    +
      b(a b a a(- 18) + b a a a 9)
+
  b a a
*
  a(a(a b b(- 3) + b b a 9) + b(a(a b 18 + b a(- 36)) + b a a 9))
    +
      b(a a(a b(- 12) + b a 18) + b a a a(- 3))
+
  b a a a
*
  a(a b(a b 3 + b a(- 3)) + b(a(a b(- 9) + b a 12) + b a a(- 3)))
+
  b a(a(a b 6 + b a(- 9)) + b a a 3)
Type: XRecursivePolynomial(Symbol,Fraction Integer)

qr:rpoly := q :: rpoly
a(a(a b b 1 + b(a b(- 2) + b a(- 1))) + b(a b a 4 + b a a(- 1)))
+
b(a b a a(- 2) + b a a a 1)
Type: XRecursivePolynomial(Symbol,Fraction Integer)

```

```

pq :: rpoly - pr*qr
0

```

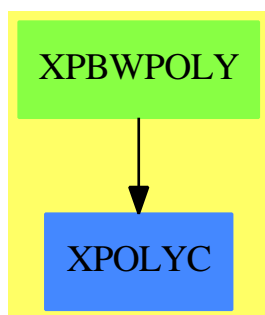
Type: XRecursivePolynomial(Symbol,Fraction Integer)

See Also:

o )show XPBWPolynomial

---

### 25.2.1 XPBWPolynomial (XPBWPOLY)



#### Exports:

|                   |                    |                 |              |
|-------------------|--------------------|-----------------|--------------|
| 0                 | 1                  | characteristic  | coef         |
| coefficient       | coefficients       | coerce          | constant     |
| constant?         | degree             | exp             | hash         |
| latex             | leadingCoefficient | leadingMonomial | leadingTerm  |
| LiePolyIfCan      | listOfTerms        | log             | lquo         |
| map               | maxdeg             | mindeg          | mindegTerm   |
| mirror            | monom              | monomial?       | monomials    |
| numberOfMonomials | one?               | product         | quasiRegular |
| quasiRegular?     | recip              | reductum        | retract      |
| retractIfCan      | rquo               | sample          | sh           |
| subtractIfCan     | trunc              | varList         | zero?        |
| ?*?               | ?**?               | ?+?             | ?-?          |
| -?                | ?=?                | ?^?             | ?~=?         |

— domain XPBWPOLY XPBWPolynomial —

```

)abbrev domain XPBWPOLY XPBWPolynomial
++ Author: Michel Petitot (petitot@lifl.fr).
++ Date Created: 91
++ Date Last Updated: 7 Juillet 92
++ Fix History: compilation v 2.1 le 13 dec 98

```



```

++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This domain constructor implements polynomials in non-commutative
++ variables written in the Poincare-Birkhoff-Witt basis from the
++ Lyndon basis.
++ These polynomials can be used to compute Baker-Campbell-Hausdorff
++ relations.

XPBWPolynomial(VarSet:OrderedSet,R:CommutativeRing): XDPcat == XDPdef where

WORD    ==> OrderedFreeMonoid(VarSet)
LWORD   ==> LyndonWord(VarSet)
LWORDS  ==> List LWORD
BASIS   ==> PoincareBirkhoffWittLyndonBasis(VarSet)
TERM    ==> Record(k:BASIS, c:R)
LTERMS  ==> List(TERM)
LPOLY   ==> LiePolynomial(VarSet,R)
EX       ==> OutputForm
XDPOLY  ==> XDistributedPolynomial(VarSet,R)
XRPOLY  ==> XRecursivePolynomial(VarSet,R)
TERM1   ==> Record(k:LWORD, c:R)
NNI     ==> NonNegativeInteger
I        ==> Integer
RN       ==> Fraction(Integer)

XDPcat == Join(XPolynomialsCat(VarSet,R), FreeModuleCat(R, BASIS)) with
  coerce      : LPOLY -> $
  ++ \axiom{coerce(p)} returns \axiom{p}.
  coerce      : $ -> XDPOLY
  ++ \axiom{coerce(p)} returns \axiom{p} as a distributed polynomial.
  coerce      : $ -> XRPOLY
  ++ \axiom{coerce(p)} returns \axiom{p} as a recursive polynomial.
  LiePolyIfCan: $ -> Union(LPOLY,"failed")
  ++ \axiom{LiePolyIfCan(p)} return \axiom{p} if \axiom{p} is a Lie polynomial.
  product     : ($,$,NNI) -> $          -- produit tronque a l'ordre n
  ++ \axiom{product(a,b,n)} returns \axiom{a*b} (truncated up to order \axiom{n}).

if R has Module(RN) then
  exp         : ($,NNI) -> $
  ++ \axiom{exp(p,n)} returns the exponential of \axiom{p}
  ++ (truncated up to order \axiom{n}).
  log         : ($,NNI) -> $
  ++ \axiom{log(p,n)} returns the logarithm of \axiom{p}
  ++ (truncated up to order \axiom{n}).

```

```

XDPdef == FreeModule1(R,BASIS) add
  import(TERM)

-- Representation
Rep:= LTERMS

-- local functions
prod1: (BASIS, $) -> $
prod2: ($, BASIS) -> $
prod : (BASIS, BASIS) -> $

prod11: (BASIS, $, NNI) -> $
prod22: ($, BASIS, NNI) -> $

outForm : TERM -> EX
Dexpand : BASIS -> XDPOLY
Rexpand : BASIS -> XRPOLY
process : (List LWORD, LWORD, List LWORD) -> $
mirror1 : BASIS -> $

-- functions locales
outForm t ==
  t.c =$R 1 => t.k :: EX
  t.k =$BASIS 1 => t.c :: EX
  t.c::EX * t.k ::EX

prod1(b:BASIS, p:$):$ ==
  +/ [t.c * prod(b, t.k) for t in p]

prod2(p:$, b:BASIS):$ ==
  +/ [t.c * prod(t.k, b) for t in p]

prod11(b,p,n) ==
  limit: I := n -$I length b
  +/ [t.c * prod(b, t.k) for t in p | length(t.k) :: I <= limit]

prod22(p,b,n) ==
  limit: I := n -$I length b
  +/ [t.c * prod(t.k, b) for t in p | length(t.k) :: I <= limit]

prod(g,d) ==
  d = 1 => monom(g,1)
  g = 1 => monom(d,1)
  process(reverse listOfTerms g, first d, rest listOfTerms d)

Dexpand b ==
  b = 1 => 1$XDPOLY
  +/ [LiePoly(1)$LPOLY :: XDPOLY for l in listOfTerms b]

Rexpand b ==

```

```

b = 1 => 1$XRPOLY
*/ [LiePoly(l)$LPOLY :: XRPOLY for l in listOfTerms b]

mirror1(b:BASIS):$ ==
  b = 1 => 1
  lp: LPOLY := LiePoly first b
  lp := mirror lp
  mirror1(rest b) * lp :: $

process(gauche, x, droite) ==      -- algo du "collect process"
  null gauche => monom( cons(x, droite) pretend BASIS, 1$R)
  r1, r2 : $
  not lexico(first gauche, x) =>      -- cas facile !!!
    monom(append(reverse gauche, cons(x, droite)) pretend BASIS , 1$R)

p: LPOLY := [first gauche , x]      -- on crochete !!!
null droite =>
  r1 := +/ [t.c * process(rest gauche, t.k, droite) for t in _
    listOfTerms p]
  r2 := process( rest gauche, x, list first gauche)
  r1 + r2
rd: List LWORD := rest droite; fd: LWORD := first droite
r1 := +/ [t.c * process(list t.k, fd, rd) for t in listOfTerms p]
r1 := +/ [t.c * process(rest gauche, first t.k, rest listOfTerms(t.k))_
  for t in r1]
r2 := process([first gauche, x], fd, rd)
r2 := +/ [t.c * process(rest gauche, first t.k, rest listOfTerms(t.k))_
  for t in r2]
r1 + r2

-- definitions
1 == monom(1$BASIS, 1$R)

coerce(r:R):$ == [[1$BASIS , r]$TERM ]

coerce(p:$):EX ==
  null p => (0$R) :: EX
  le : List EX := nil
  for rec in p repeat le := cons(outForm rec, le)
  reduce(+, le)$List(EX)

coerce(v: VarSet):$ == monom(v::BASIS , 1$R)
coerce(p: LPOLY):$ ==
  [[t.k :: BASIS , t.c ]$TERM for t in listOfTerms p]

coerce(p:$):XDPOLY ==
  +/ [t.c * Dexpand t.k for t in p]

coerce(p:$):XRPOLY ==
  p = 0 => 0$XRPOLY

```

```

    +/ [t.c * Rexpand t.k for t in p]

constant? p == (null p) or (leadingMonomial(p) = $BASIS 1)
constant p ==
    null p => 0$R
    p.last.k = 1$BASIS => p.last.c
    0$R

quasiRegular? p == (p=0) or (p.last.k ^= 1$BASIS)
quasiRegular p ==
    p = 0 => p
    p.last.k = 1$BASIS => delete(p, maxIndex p)
    p

x:$ * y:$ ==
    y = 0$$ => 0
    +/ [t.c * prod1(t.k, y) for t in x]

--      listOfTerms p == p pretend LTERMS

varList p ==
    lv: List VarSet := "setUnion"/ [varList(b.k)$BASIS for b in p]
    sort(lv)

degree(p) ==
    p=0 => error "null polynomial"
    length(leadingMonomial p)

trunc(p, n) ==
    p = 0 => p
    degree(p) > n => trunc( reductum p , n)
    p

product(x,y,n) ==
    x = 0 => 0
    y = 0 => 0
    +/ [t.c * prod11(t.k, y, n) for t in x]

if R has Module(RN) then
    exp (p,n) ==
        p = 0 => 1
        not quasiRegular? p =>
            error "a proper polynomial is required"
        s : $ := 1 ; r: $ := 1 -- resultat
        for i in 1..n repeat
            k1 :RN := 1/i
            k2 : R := k1 * 1$R
            s := k2 * product(p, s, n)
            r := r + s
        r

```

```

log (p,n) ==
  p = 1 => 0
  p1: $ := 1 - p
  not quasiRegular? p1 =>
    error "constant term < 1, impossible log "
  s : $ := - 1 ; r: $ := 0          -- resultat
  for i in 1..n repeat
    k1 :RN := 1/i
    k2 : R := k1 * 1$R
    s := product(p1, s, n)
    r := k2 * s + r
  r

LiePolyIfCan p ==
  p = 0 => 0$LPOLY
  "and"/ [retractable?(t.k)$BASIS for t in p] =>
    lt : List TERM1 := _
      [[retract(t.k)$BASIS, t.c]$TERM1 for t in p]
    lt pretend LPOLY
  "failed"

mirror p ==
  +/ [t.c * mirror1(t.k) for t in p]

```

---

— XPBWPOLY.dotabb —

```

"XPBWPOLY" [color="#88FF44",href="bookvol10.3.pdf#nameddest=XPBWPOLY"]
"XPOLYC" [color="#4488FF",href="bookvol10.2.pdf#nameddest=XPOLYC"]
"XPBWPOLY" -> "XPOLYC"

```

---

## 25.3 domain XPOLY XPolynomial

— XPolynomial.input —

```

)set break resume
)sys rm -f XPolynomial.output
)spool XPolynomial.output
)set message test on
)set message auto off

```

```

)clear all
--S 1 of 14
poly := XPolynomial(Integer)
--R
--R
--R (1) XPolynomial Integer
--R
--R                                          Type: Domain
--E 1

--S 2 of 14
pr: poly := 2*x + 3*y-5
--R
--R
--R (2) - 5 + x 2 + y 3
--R
--R                                          Type: XPolynomial Integer
--E 2

--S 3 of 14
pr2: poly := pr*pr
--R
--R
--R (3) 25 + x(- 20 + x 4 + y 6) + y(- 30 + x 6 + y 9)
--R
--R                                          Type: XPolynomial Integer
--E 3

--S 4 of 14
pd := expand pr
--R
--R
--R (4) - 5 + 2x + 3y
--R
--R                                          Type: XDistributedPolynomial(Symbol,Integer)
--E 4

--S 5 of 14
pd2 := pd*pd
--R
--R
--R (5) 25 - 20x - 30y + 4x2 + 6x y + 6y x + 9y2
--R
--R                                          Type: XDistributedPolynomial(Symbol,Integer)
--E 5

--S 6 of 14
expand(pr2) - pd2
--R
--R
--R (6) 0
--R
--R                                          Type: XDistributedPolynomial(Symbol,Integer)
--E 6

```

--S 12 of 14

```

w: Word := x*y**2
--R
--R
--R      2
--R (12)  x y
--R
--R                                          Type: OrderedFreeMonoid Symbol
--E 12

--S 13 of 14
rquo(qr,w)
--R
--R
--R (13)  18
--R
--R                                          Type: XPolynomial Integer
--E 13

--S 14 of 14
sh(pr,w::poly)
--R
--R
--R (14)  x(x y y 4 + y(x y 2 + y(- 5 + x 2 + y 9))) + y x y y 3
--R
--R                                          Type: XPolynomial Integer
--E 14
)spool
)lisp (bye)

```

---

— XPolynomial.help —

```

=====
XPolynomial examples
=====

```

The XPolynomial domain constructor implements multivariate polynomials whose set of variables is Symbol. These variables do not commute. The only parameter of this constructor is the coefficient ring which may be non-commutative. However, coefficients and variables commute. The representation of the polynomials is recursive. The abbreviation for XPolynomial is XPOLY.

Other constructors like XPolynomialRing, XRecursivePolynomial as well as XDistributedPolynomial, LiePolynomial and XPBWPolynomial implement multivariate polynomials in non-commutative variables.

We illustrate now some of the facilities of the XPOLY domain constructor.

Define a polynomial ring over the integers.



```
poly := XPolynomial(Integer)
      XPolynomial Integer
                                     Type: Domain
```

Define a first polynomial,

```
pr: poly := 2*x + 3*y-5
      - 5 + x 2 + y 3
                                     Type: XPolynomial Integer
```

and a second one.

```
pr2: poly := pr*pr
      25 + x(- 20 + x 4 + y 6) + y(- 30 + x 6 + y 9)
                                     Type: XPolynomial Integer
```

Rewrite pr in a distributive way,

```
pd := expand pr
      - 5 + 2x + 3y
                                     Type: XDistributedPolynomial(Symbol,Integer)
```

compute its square,

```
pd2 := pd*pd
      25 - 20x - 30y + 4x 2 + 6x y + 6y x + 9y 2
                                     Type: XDistributedPolynomial(Symbol,Integer)
```

and checks that:

```
expand(pr2) - pd2
      0
                                     Type: XDistributedPolynomial(Symbol,Integer)
```

We define:

```
qr := pr**3
      - 125 + x(150 + x(- 60 + x 8 + y 12) + y(- 90 + x 12 + y 18))
      +
      y(225 + x(- 90 + x 12 + y 18) + y(- 135 + x 18 + y 27))
                                     Type: XPolynomial Integer
```

and:

```
qd := pd**3
      - 125 + 150x + 225y - 60x 2 - 90x y - 90y x - 135y 2 + 8x 3 + 12x 2 y + 12x y 2
      +
      2 2 2 3
```

```

18x y + 12y x + 18y x y + 18y x + 27y
Type: XDistributedPolynomial(Symbol,Integer)

```

We truncate qd at degree 3.

```

trunc(qd,2)
      2      2
- 125 + 150x + 225y - 60x - 90x y - 90y x - 135y
Type: XDistributedPolynomial(Symbol,Integer)

```

The same for qr:

```

trunc(qr,2)
- 125 + x(150 + x(- 60) + y(- 90)) + y(225 + x(- 90) + y(- 135))
Type: XPolynomial Integer

```

We define:

```

Word := OrderedFreeMonoid Symbol
OrderedFreeMonoid Symbol
Type: Domain

```

and:

```

w: Word := x*y**2
      2
      x y
Type: OrderedFreeMonoid Symbol

```

We can compute the right-quotient of qr by r:

```

rquo(qr,w)
      18
Type: XPolynomial Integer

```

and the shuffle-product of pr by r:

```

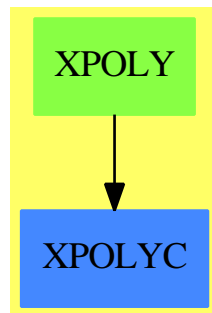
sh(pr,w::poly)
x(x y y 4 + y(x y 2 + y(- 5 + x 2 + y 9))) + y x y y 3
Type: XPolynomial Integer

```

See Also:

- o )help XPBWPolynomial
- o )help LiePolynomial
- o )help XDistributedPolynomial
- o )help XRecursivePolynomial
- o )help XPolynomialRing
- o )show XPolynomial

### 25.3.1 XPolynomial (XPOLY)



#### Exports:

|              |              |                |           |               |
|--------------|--------------|----------------|-----------|---------------|
| 0            | 1            | characteristic | coef      | coerce        |
| constant     | constant?    | degree         | expand    | hash          |
| latex        | lquo         | map            | maxdeg    | mindeg        |
| mindegTerm   | mirror       | monom          | monomial? | RemainderList |
| one?         | quasiRegular | quasiRegular?  | recip     | retract       |
| retractIfCan | rquo         | sample         | sh        | subtractIfCan |
| trunc        | unexpand     | varList        | zero?     | ?*?           |
| ?**?         | ?+?          | ?-?            | -?        | ?=?           |
| ?^?          | ?~=?         |                |           |               |

— domain XPOLY XPolynomial —

```

)abbrev domain XPOLY XPolynomial
++ Author: Michel Petitot petitot@lifl.fr
++ Date Created: 91
++ Date Last Updated: 7 Juillet 92
++ Fix History: compilation v 2.1 le 13 dec 98
++   extend renomme en expand
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This type supports multivariate polynomials whose set of variables
++ is \spadtype{Symbol}. The representation is recursive.
++ The coefficient ring may be non-commutative and the variables
++ do not commute. However, coefficients and variables commute.
  
```

```
XPolynomial(R:Ring) == XRecursivePolynomial(Symbol, R)
```

---

— XPOLY.dotabb —

```
"XPOLY" [color="#88FF44",href="bookvol10.3.pdf#nameddest=XPOLY"]
"XPOLYC" [color="#4488FF",href="bookvol10.2.pdf#nameddest=XPOLYC"]
"XPOLY" -> "XPOLYC"
```

---

## 25.4 domain XPR XPolynomialRing

— XPolynomialRing.input —

```
)set break resume
)sys rm -f XPolynomialRing.output
)spool XPolynomialRing.output
)set message test on
)set message auto off
)clear all
--S 1 of 15
Word := OrderedFreeMonoid(Symbol)
--R
--R
--R (1) OrderedFreeMonoid Symbol
--R
--R                                          Type: Domain
--E 1

--S 2 of 15
poly:= XPR(Integer,Word)
--R
--R
--R (2) XPolynomialRing(Integer,OrderedFreeMonoid Symbol)
--R
--R                                          Type: Domain
--E 2

--S 3 of 15
p:poly := 2 * x - 3 * y + 1
--R
--R
--R (3) 1 + 2x - 3y
```

```

--R                                     Type: XPolynomialRing(Integer,OrderedFreeMonoid Symbol)
--E 3

--S 4 of 15
q:poly := 2 * x + 1
--R
--R
--R      (4)  1 + 2x
--R                                     Type: XPolynomialRing(Integer,OrderedFreeMonoid Symbol)
--E 4

--S 5 of 15
p + q
--R
--R
--R      (5)  2 + 4x - 3y
--R                                     Type: XPolynomialRing(Integer,OrderedFreeMonoid Symbol)
--E 5

--S 6 of 15
p * q
--R
--R
--R                                     2
--R      (6)  1 + 4x - 3y + 4x  - 6y x
--R                                     Type: XPolynomialRing(Integer,OrderedFreeMonoid Symbol)
--E 6

--S 7 of 15
(p+q)**2-p**2-q**2-2*p*q
--R
--R
--R      (7)  - 6x y + 6y x
--R                                     Type: XPolynomialRing(Integer,OrderedFreeMonoid Symbol)
--E 7

--S 8 of 15
M := SquareMatrix(2,Fraction Integer)
--R
--R
--R      (8)  SquareMatrix(2,Fraction Integer)
--R
--R                                     Type: Domain
--E 8

--S 9 of 15
poly1:= XPR(M,Word)
--R
--R
--R      (9)
--R      XPolynomialRing(SquareMatrix(2,Fraction Integer),OrderedFreeMonoid Symbol)

```

```
--R
--E 9
```

Type: Domain

--S 10 of 15

```
m1:M := matrix [ [i*j**2 for i in 1..2] for j in 1..2]
```

--R

--R

```
--R      +1  2+
```

```
--R (10) |  |
```

```
--R      +4  8+
```

--R

Type: SquareMatrix(2,Fraction Integer)

--E 10

--S 11 of 15

```
m2:M := m1 - 5/4
```

--R

--R

```
--R      +  1  +
```

```
--R      |- - 2 |
```

```
--R      |  4  |
```

```
--R (11) |  |
```

```
--R      |  27|
```

```
--R      |  4  --|
```

```
--R      +  4+
```

--R

Type: SquareMatrix(2,Fraction Integer)

--E 11

--S 12 of 15

```
m3: M := m2**2
```

--R

--R

```
--R      +129  +
```

```
--R      |--- 13 |
```

```
--R      | 16  |
```

```
--R (12) |  |
```

```
--R      |  857|
```

```
--R      |26  ---|
```

```
--R      +  16+
```

--R

Type: SquareMatrix(2,Fraction Integer)

--E 12

--S 13 of 15

```
pm:poly1 := m1*x + m2*y + m3*z - 2/3
```

--R

--R

```
--R      +  2  +      +  1  +      +129  +
```

```
--R      |- - 0 |      |- - 2 |      |--- 13 |
```

```
--R      |  3  |      +1  2+      |  4  |      | 16  |
```

```
--R (13) |      | + |  |x + |      |y + |      |z
```

```
--R      |      2|  +4  8+      |  27|      |  857|
```

```

--R      | 0  - -|      | 4  --|      |26  ---|
--R      +      3+      +      4+      +      16+
--RType: XPolynomialRing(SquareMatrix(2,Fraction Integer),OrderedFreeMonoid Symbol)
--E 13

```

```

--S 14 of 15

```

```

qm:poly1 := pm - m1*x

```

```

--R
--R
--R      + 2      +      1      +      +129      +
--R      |- - 0 |      |- - 2 |      |--- 13 |
--R      | 3      |      | 4      |      | 16      |
--R (14) |      | + |      |y + |      |z
--R      |      2|      |      27|      |      857|
--R      | 0  - -|      | 4  --|      |26  ---|
--R      +      3+      +      4+      +      16+
--RType: XPolynomialRing(SquareMatrix(2,Fraction Integer),OrderedFreeMonoid Symbol)
--E 14

```

```

--S 15 of 15

```

```

qm**3

```

```

--R
--R
--R (15)
--R      + 8      +      1 8+      +43 52 +      + 129      +
--R      |- -- 0 |      |- - -|      |-- -- |      |- --- - 26 |
--R      | 27      |      | 3 3|      | 4 3 |      | 8      | 2
--R      |      | + |      |y + |      |z + |      |y
--R      |      8|      |16 |      |104 857|      |      857|
--R      | 0  - --|      |-- 9|      |--- ---|      |- 52 - ---|
--R      +      27+      + 3      +      + 3 12+      +      8 +
--R      +
--R      + 3199      831 +      + 3199      831 +      + 103169      6409 +
--R      |- ---- - --- |      |- ---- - --- |      |- ---- - --- |
--R      | 32      4 |      | 32      4 |      | 128      4 | 2
--R      |      |y z + |      |z y + |      |z
--R      | 831      26467|      | 831      26467|      | 6409      820977|
--R      |- ---- - ----|      |- ---- - ----|      |- ---- - ----|
--R      + 2      32 +      + 2      32 +      + 2      128 +
--R      +
--R      +3199 831 +      +103169 6409 +      +103169 6409 +
--R      |---- - - |      |----- - - |      |----- - - |
--R      | 64      8 | 3 |      | 256      8 | 2 |      | 256      8 |
--R      |      |y + |      |y z + |      |y z y
--R      |831 26467|      | 6409 820977|      | 6409 820977|
--R      |---- - ----|      |---- - ----|      |---- - ----|
--R      + 4      64 +      + 4      256 +      + 4      256 +
--R      +
--R      +3178239 795341 +      +103169 6409 +      +3178239 795341 +
--R      |----- - ---- |      |----- - ---- |      |----- - ---- |

```

```

--R      | 1024      128 | 2 | 256      8 | 2 | 1024      128 |
--R      |          |y z + |          |z y + |          |z y z
--R      |795341  25447787| | 6409  820977| |795341  25447787|
--R      |-----|
--R      + 64      1024 +      + 4      256 +      + 64      1024 +
--R      +
--R      +3178239  795341 +      +98625409  12326223 +
--R      |-----|
--R      | 1024      128 | 2 | 4096      256 | 3
--R      |          |z y + |          |z
--R      |795341  25447787| |12326223  788893897|
--R      |-----|
--R      + 64      1024 +      + 128      4096 +
--RType: XPolynomialRing(SquareMatrix(2,Fraction Integer),OrderedFreeMonoid Symbol)
--E 15
)spool
)lisp (bye)

```

---

— XPolynomialRing.help —

=====

XPolynomialRing examples

=====

The XPolynomialRing domain constructor implements generalized polynomials with coefficients from an arbitrary Ring (not necessarily commutative) and whose exponents are words from an arbitrary OrderedMonoid (not necessarily commutative too). Thus these polynomials are (finite) linear combinations of words.

This constructor takes two arguments. The first one is a Ring and the second is an OrderedMonoid. The abbreviation for XPolynomialRing is XPR.

Other constructors like XPolynomial, XRecursivePolynomial, XDistributedPolynomial, LiePolynomial and XPBWPolynomial implement multivariate polynomials in non-commutative variables.

We illustrate now some of the facilities of the XPR domain constructor.

Define the free ordered monoid generated by the symbols.

```

Word := OrderedFreeMonoid(Symbol)
OrderedFreeMonoid Symbol
Type: Domain

```

Define the linear combinations of these words with integer coefficients.



```
poly:= XPR(Integer,Word)
      XPolynomialRing(Integer,OrderedFreeMonoid Symbol)
                        Type: Domain
```

Then we define a first element from poly.

```
p:poly := 2 * x - 3 * y + 1
      1 + 2x - 3y
                        Type: XPolynomialRing(Integer,OrderedFreeMonoid Symbol)
```

And a second one.

```
q:poly := 2 * x + 1
      1 + 2x
                        Type: XPolynomialRing(Integer,OrderedFreeMonoid Symbol)
```

We compute their sum,

```
p + q
      2 + 4x - 3y
                        Type: XPolynomialRing(Integer,OrderedFreeMonoid Symbol)
```

their product,

```
p * q
      1 + 4x - 3y + 4x2 - 6y x
                        Type: XPolynomialRing(Integer,OrderedFreeMonoid Symbol)
```

and see that variables do not commute.

```
(p+q)**2-p**2-q**2-2*p*q
      - 6x y + 6y x
                        Type: XPolynomialRing(Integer,OrderedFreeMonoid Symbol)
```

Now we define a ring of square matrices,

```
M := SquareMatrix(2,Fraction Integer)
      SquareMatrix(2,Fraction Integer)
                        Type: Domain
```

and the linear combinations of words with these matrices as coefficients.

```
poly1:= XPR(M,Word)
      XPolynomialRing(SquareMatrix(2,Fraction Integer),OrderedFreeMonoid Symbol)
                        Type: Domain
```

Define a first matrix,

```
m1:M := matrix [ [i*j**2 for i in 1..2] for j in 1..2]
```

```

+1 2+
|   |
+4 8+

```

Type: SquareMatrix(2,Fraction Integer)

a second one,

```

m2:M := m1 - 5/4
+ 1 +
|- - 2 |
| 4 |
|   |
|   27|
| 4 --|
+   4+

```

Type: SquareMatrix(2,Fraction Integer)

and a third one.

```

m3: M := m2**2
+129 +
|--- 13 |
| 16 |
|   |
|   857|
|26 ---|
+   16+

```

Type: SquareMatrix(2,Fraction Integer)

Define a polynomial,

```

pm:poly1 := m1*x + m2*y + m3*z - 2/3
+ 2 + + 1 + +129 +
|- - 0 | |- - 2 | |--- 13 |
| 3 | +1 2+ | 4 | | 16 |
|   | + | |x + | |y + | |z
|   2| +4 8+ | 27| | 857|
| 0 - -| | 4 --| |26 ---|
+   3+ +   4+ +   16+
Type: XPolynomialRing(SquareMatrix(2,Fraction Integer),
OrderedFreeMonoid Symbol)

```

a second one,

```

qm:poly1 := pm - m1*x
+ 2 + + 1 + +129 +
|- - 0 | |- - 2 | |--- 13 |
| 3 | | 4 | | 16 |
|   | + | |y + | |z
|   2| | 27| | 857|

```

```

      | 0  - -|  | 4  --|  |26  ---|
      +      3+  +      4+  +      16+
Type: XPolynomialRing(SquareMatrix(2,Fraction Integer),
OrderedFreeMonoid Symbol)

```

and the following power.

```

qm**3
+ 8      + + 1 8+  +43 52 +  + 129      +
|- -- 0 |  |- - -|  |- -- -- |  |- --- - 26 |
| 27    |  | 3 3|  | 4 3 |  | 8      | 2
|      | + |  |y + |  |z + |  |y
|      8|  |16 |  |104 857|  |      857|
| 0    - --|  |- 9|  |--- ---|  |- 52 - ---|
+      27+  + 3  +  + 3  12+  +      8 +
+
+ 3199      831 +      + 3199      831 +      + 103169      6409 +
|- ---- - --- |  |- ---- - --- |  |- ---- - ---- |
| 32      4 |  | 32      4 |  |      128      4 | 2
|      |y z + |  |      |z y + |  |      |z
| 831      26467|  | 831      26467|  | 6409      820977|
|- --- - ----|  |- --- - ----|  |- --- - ----|
+ 2      32 +  + 2      32 +  + 2      128 +
+
+3199 831 +      +103169 6409 +      +103169 6409 +
|---- --- |  |----- ---- |  |----- ---- |
| 64 8 | 3 | 256 8 | 2 | 256 8 |
|      |y + |  |      |y z + |  |      |y z y
|831 26467|  | 6409 820977|  | 6409 820977|
|--- ----|  |--- ----|  |--- ----|
+ 4      64 +  + 4      256 +  + 4      256 +
+
+3178239 795341 +      +103169 6409 +      +3178239 795341 +
|----- ---- |  |----- ---- |  |----- ---- |
| 1024 128 | 2 | 256 8 | 2 | 1024 128 |
|      |y z + |  |      |z y + |  |      |z y z
|795341 25447787|  | 6409 820977|  |795341 25447787|
|----- ----|  |--- ----|  |----- ----|
+ 64 1024 +  + 4 256 +  + 64 1024 +
+
+3178239 795341 +      +98625409 12326223 +
|----- ---- |  |----- ---- |
| 1024 128 | 2 | 4096 256 | 3
|      |z y + |  |      |z
|795341 25447787|  |12326223 788893897|
|----- ----|  |----- ----|
+ 64 1024 +  + 128 4096 +
Type: XPolynomialRing(SquareMatrix(2,Fraction Integer),
OrderedFreeMonoid Symbol)

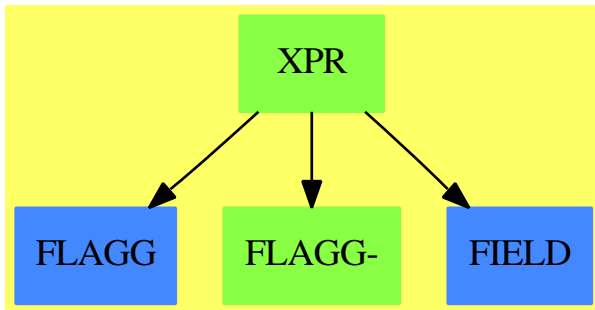
```

See Also:

- o )help XPBWPolynomial
- o )help LiePolynomial
- o )help XDistributedPolynomial
- o )help XRecursivePolynomial
- o )help XPolynomial
- o )show XPolynomialRing

—

### 25.4.1 XPolynomialRing (XPR)



#### Exports:

|                 |                   |                |                    |
|-----------------|-------------------|----------------|--------------------|
| 0               | 1                 | characteristic | coef               |
| coefficient     | coefficients      | coerce         | constant           |
| constant?       | hash              | latex          | leadingCoefficient |
| leadingMonomial | leadingTerm       | listOfTerms    | map                |
| maxdeg          | mindeg            | monom          | monomial?          |
| monomials       | numberOfMonomials | one?           | quasiRegular       |
| quasiRegular?   | recip             | reductum       | retract            |
| retractIfCan    | sample            | subtractIfCan  | zero?              |
| #?              | ?*?               | ?**?           | ?+?                |
| ?-?             | -?                | ?=?            | ?^?                |
| ?~=?            |                   |                |                    |

— domain XPR XPolynomialRing —

```

)abbrev domain XPR XPolynomialRing
++ Author: Michel Petitot petitot@lifl.fr
++ Date Created: 91
++ Date Last Updated: 7 Juillet 92
++ Fix History: compilation v 2.1 le 13 dec 98
++ Basic Functions:
  
```

```

++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This domain represents generalized polynomials with coefficients
++ (from a not necessarily commutative ring), and words
++ belonging to an arbitrary \spadtype{OrderedMonoid}.
++ This type is used, for instance, by the \spadtype{XDistributedPolynomial}
++ domain constructor where the Monoid is free.

XPolynomialRing(R:Ring,E:OrderedMonoid): T == C where
  TERM ==> Record(k: E, c: R)
  EX    ==> OutputForm
  NNI    ==> NonNegativeInteger

T == Join(Ring, XAlgebra(R), FreeModuleCat(R,E)) with
--operations
  "*": (% ,R) -> %
    ++ \spad{p*r} returns the product of \spad{p} by \spad{r}.
  "#": % -> NonNegativeInteger
    ++ \spad{# p} returns the number of terms in \spad{p}.
  coerce: E -> %
    ++ \spad{coerce(e)} returns \spad{1*e}
  maxdeg: % -> E
    ++ \spad{maxdeg(p)} returns the greatest word occurring in the polynomial \spad{p}
    ++ with a non-zero coefficient. An error is produced if \spad{p} is zero.
  mindeg: % -> E
    ++ \spad{mindeg(p)} returns the smallest word occurring in the polynomial \spad{p}
    ++ with a non-zero coefficient. An error is produced if \spad{p} is zero.
  reductum : % -> %
    ++ \spad{reductum(p)} returns \spad{p} minus its leading term.
    ++ An error is produced if \spad{p} is zero.
  coef : (% ,E) -> R
    ++ \spad{coef(p,e)} extracts the coefficient of the monomial \spad{e}.
    ++ Returns zero if \spad{e} is not present.
  constant?:% -> Boolean
    ++ \spad{constant?(p)} tests whether the polynomial \spad{p} belongs to the
    ++ coefficient ring.
  constant: % -> R
    ++ \spad{constant(p)} return the constant term of \spad{p}.
  quasiRegular? : % -> Boolean
    ++ \spad{quasiRegular?(x)} return true if \spad{constant(p)} is zero.
  quasiRegular : % -> %
    ++ \spad{quasiRegular(x)} return \spad{x} minus its constant term.
  map : (R -> R, %) -> %
    ++ \spad{map(fn,x)} returns \spad{Sum(fn(r_i) w_i)} if \spad{x} writes \spad{Sum(r_i
if R has Field then "/" : (% ,R) -> %
    ++ \spad{p/r} returns \spad{p*(1/r)}.

```

```

--assertions
  if R has noZeroDivisors then noZeroDivisors
  if R has unitsKnown then unitsKnown
  if R has canonicalUnitNormal then canonicalUnitNormal
    ++ canonicalUnitNormal guarantees that the function
    ++ unitCanonical returns the same representative for all
    ++ associates of any particular element.

C == FreeModule1(R,E) add
--representations
  Rep:= List TERM
--uses
  repeatMultExpt: (%,NonNegativeInteger) -> %
--define
  1 == [[1$E,1$R]]

  characteristic == characteristic$R
  #x == #$Rep x
  maxdeg p == if null p then error " polynome nul !"
              else p.first.k
  mindeg p == if null p then error " polynome nul !"
              else (last p).k

  coef(p,e) ==
    for tm in p repeat
      tm.k=e => return tm.c
      tm.k < e => return 0$R
    0$R

  constant? p == (p = 0) or (maxdeg(p) = 1$E)
  constant p == coef(p,1$E)

  quasiRegular? p == (p=0) or (last p).k ^= 1$E
  quasiRegular p ==
    quasiRegular?(p) => p
    [t for t in p | not(t.k = 1$E)]

  recip(p) ==
    p=0 => "failed"
    p.first.k > 1$E => "failed"
    (u:=recip(p.first.c)) case "failed" => "failed"
    (u::R)::%

  coerce(r:R) == if r=0$R then 0$% else [[1$E,r]]
  coerce(n:Integer) == (n::R)::%

  if R has noZeroDivisors then
    p1:% * p2:% ==

```

```

null p1 => 0
null p2 => 0
p1.first.k = 1$E => p1.first.c * p2
p2 = 1 => p1
--      +/[[[t1.k*t2.k,t1.c*t2.c]$TERM for t2 in p2]
--      for t1 in reverse(p1)]
--      +/[[[t1.k*t2.k,t1.c*t2.c]$TERM for t2 in p2]
--      for t1 in p1]
else
p1:% * p2:% ==
null p1 => 0
null p2 => 0
p1.first.k = 1$E => p1.first.c * p2
p2 = 1 => p1
--      +/[[[t1.k*t2.k,r]$TERM for t2 in p2 | not (r:=t1.c*t2.c) =$R 0]
--      for t1 in reverse(p1)]
--      +/[[[t1.k*t2.k,r]$TERM for t2 in p2 | not (r:=t1.c*t2.c) =$R 0]
--      for t1 in p1]
p:% ** nn:NNI == repeatMultExpt(p,nn)
repeatMultExpt(x,nn) ==
nn = 0 => 1
y:% := x
for i in 2..nn repeat y:= x * y
y

outTerm(r:R, m:E):EX ==
r=1 => m::EX
m=1 => r::EX
r::EX * m::EX

--      coerce(x:%) : EX ==
--      null x => (0$R) :: EX
--      le : List EX := nil
--      for rec in x repeat
--      rec.c = 1$R => le := cons(rec.k :: EX, le)
--      rec.k = 1$E => le := cons(rec.c :: EX, le)
--      le := cons(mkBinary("*"::EX,rec.c :: EX,
--      rec.k :: EX), le)
--      1 = #le => first le
--      mkNary("+" :: EX,le)

coerce(a:%):EX ==
empty? a => (0$R)::EX
reduce(_+, reverse_! [outTerm(t.c, t.k) for t in a])$List(EX)

if R has Field then
x/r == inv(r)*x

```

---

— XPR.dotabb —

```
"XPR" [color="#88FF44",href="bookvol10.3.pdf#nameddest=XPR"]
"FLAGG" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FLAGG"]
"FLAGG-" [color="#88FF44",href="bookvol10.3.pdf#nameddest=FLAGG"]
"FIELD" [color="#4488FF",href="bookvol10.2.pdf#nameddest=FIELD"]
"XPR" -> "FLAGG"
"XPR" -> "FLAGG-"
"XPR" -> "FIELD"
```

---

## 25.5 domain XRPOLY XRecursivePolynomial

— XRecursivePolynomial.input —

```
)set break resume
)sys rm -f XRecursivePolynomial.output
)spool XRecursivePolynomial.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show XRecursivePolynomial
--R XRecursivePolynomial(VarSet: OrderedSet,R: Ring) is a domain constructor
--R Abbreviation for XRecursivePolynomial is XRPOLY
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.3.pamphlet to see algebra source code for XRPOLY
--R
--R----- Operations -----
--R ?? : (VarSet,%) -> %          ?? : (%,R) -> %
--R ?? : (R,%) -> %              ?? : (%,%) -> %
--R ?? : (Integer,%) -> %        ?? : (PositiveInteger,%) -> %
--R ??? : (%,PositiveInteger) -> %  ?? : (%,%) -> %
--R ?-? : (%,%) -> %             -? : % -> %
--R ?? : (%,%) -> Boolean        1 : () -> %
--R 0 : () -> %                  ?? : (%,PositiveInteger) -> %
--R coef : (%,%) -> R            coerce : VarSet -> %
--R coerce : R -> %              coerce : Integer -> %
--R coerce : % -> OutputForm     constant : % -> R
--R constant? : % -> Boolean     degree : % -> NonNegativeInteger
--R hash : % -> SingleInteger    latex : % -> String
```



```

--R lquo : (%,% ) -> %
--R map : ((R -> R),%) -> %
--R monomial? : % -> Boolean
--R quasiRegular : % -> %
--R recip : % -> Union(%,"failed")
--R rquo : (% ,VarSet) -> %
--R varList : % -> List VarSet
--R ?~=? : (%,% ) -> Boolean
--R ?*? : (NonNegativeInteger,% ) -> %
--R ??? : (% ,NonNegativeInteger) -> %
--R RemainderList : % -> List Record(k: VarSet,c: %)
--R ?? : (% ,NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R coef : (% ,OrderedFreeMonoid VarSet) -> R
--R coerce : OrderedFreeMonoid VarSet -> %
--R expand : % -> XDistributedPolynomial(VarSet,R)
--R lquo : (% ,OrderedFreeMonoid VarSet) -> %
--R maxdeg : % -> OrderedFreeMonoid VarSet
--R mindeg : % -> OrderedFreeMonoid VarSet
--R mindegTerm : % -> Record(k: OrderedFreeMonoid VarSet,c: R)
--R monom : (OrderedFreeMonoid VarSet,R) -> %
--R retract : % -> OrderedFreeMonoid VarSet
--R retractIfCan : % -> Union(OrderedFreeMonoid VarSet,"failed")
--R rquo : (% ,OrderedFreeMonoid VarSet) -> %
--R sh : (% ,NonNegativeInteger) -> % if R has COMRING
--R sh : (%,% ) -> % if R has COMRING
--R subtractIfCan : (%,% ) -> Union(%,"failed")
--R trunc : (% ,NonNegativeInteger) -> %
--R unexpand : XDistributedPolynomial(VarSet,R) -> %
--R
--E 1

)spool
)lisp (bye)

```

---

— XRecursivePolynomial.help —

```

=====
XRecursivePolynomial examples
=====

```

```

See Also:
o )show XRecursivePolynomial

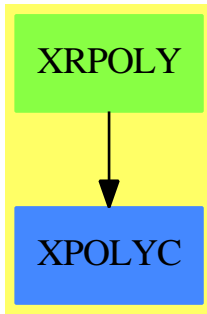
```

---

Polynomial arithmetic with non-commutative variables has been improved by a contribution

of Michel Petitot (University of Lille I, France). The domain constructors **XRecursivePolynomial** provides a recursive for these polynomials. It is the non-commutative equivalents for the **SparseMultivariatePolynomial** constructor.

### 25.5.1 XRecursivePolynomial (XRPOLY)



#### Exports:

|              |               |                |               |               |
|--------------|---------------|----------------|---------------|---------------|
| 0            | 1             | characteristic | coef          | coerce        |
| constant     | constant?     | degree         | expand        | hash          |
| latex        | lquo          | map            | maxdeg        | mindeg        |
| mindegTerm   | mirror        | monom          | monomial?     | one?          |
| quasiRegular | quasiRegular? | recip          | RemainderList | retract       |
| retractIfCan | rquo          | sample         | sh            | subtractIfCan |
| trunc        | unexpand      | varList        | zero?         | ?*?           |
| ?*?          | ?+?           | ?-?            | -?            | ?=?           |
| ?^?          | ?~=?          |                |               |               |

— domain XRPOLY XRecursivePolynomial —

```

)abbrev domain XRPOLY XRecursivePolynomial
++ Author: Michel Petitot petitot@lifl.fr
++ Date Created: 91
++ Date Last Updated: 7 Juillet 92
++ Fix History: compilation v 2.1 le 13 dec 98
++ extend renomme en expand
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This type supports multivariate polynomials whose variables do not commute.
++ The representation is recursive. The coefficient ring may be
++ non-commutative. Coefficients and variables commute.
  
```

```

XRecursivePolynomial(VarSet:OrderedSet,R:Ring): Xcat == Xdef where
  I      ==> Integer
  NNI    ==> NonNegativeInteger
  XDPOLY ==> XDistributedPolynomial(VarSet, R)
  EX     ==> OutputForm
  WORD   ==> OrderedFreeMonoid(VarSet)
  TERM   ==> Record(k:VarSet , c:%)
  LTERMS ==> List(TERM)
  REGPOLY==> FreeModule1(%, VarSet)
  VPOLY  ==> Record(c0:R, reg:REGPOLY)

Xcat == XPolynomialsCat(VarSet,R) with
  expand: % -> XDPOLY
  ++ \spad{expand(p)} returns \spad{p} in distributed form.
  unexpand : XDPOLY -> %
  ++ \spad{unexpand(p)} returns \spad{p} in recursive form.
  RemainderList: % -> LTERMS
  ++ \spad{RemainderList(p)} returns the regular part of \spad{p}
  ++ as a list of terms.

Xdef == add
  import(VPOLY)

-- representation
  Rep      := Union(R,VPOLY)

-- local functions
  construct: LTERMS -> REGPOLY
  simplifie: VPOLY -> %
  lquo1: (LTERMS,LTERMS) -> %      -- a ajouter
  coef1: (LTERMS,LTERMS) -> R      -- a ajouter
  outForm: REGPOLY -> EX

--define
  construct(lt) == lt pretend REGPOLY
  p1:% = p2:% ==
    p1 case R =>
      p2 case R => p1 =$R p2
      false
    p2 case R => false
  p1.c0 =$R p2.c0 and p1.reg =$REGPOLY p2.reg

  monom(w, r) ==
    r =0 => 0
    r * w:;%

--      if R has Field then      -- Bug non resolu !!!!!!!
--      p:% / r: R == inv(r) * p

  rquo(p1:%, p2:%):% ==

```

```

p2 case R => p1 * p2::R
p1 case R => p1 * p2.c0
x:REGPOLY := construct [[t.k, a]$TERM for t in listOfTerms(p1.reg) _
                        | (a:= rquo(t.c,p2)) ^= 0$% ]$LTERMS
simplifie [coef(p1,p2) , x]$VPOLY

trunc(p,n) ==
n = 0 or (p case R) => (constant p)::%
n1: NNI := (n-1)::NNI
lt: LTERMS := [[t.k, r]$TERM for t in listOfTerms p.reg _
               | (r := trunc(t.c, n1)) ^= 0]$LTERMS
x: REGPOLY := construct lt
simplifie [constant p, x]$VPOLY

unexpand p ==
constant? p => (constant p)::%
vl: List VarSet := sort((y,z) +-> y > z, varList p)
x : REGPOLY := _
    construct [[v, unexpand r]$TERM for v in vl | (r:=lquo(p,v)) ^= 0]
    [constant p, x]$VPOLY

if R has CommutativeRing then
sh(p:%, n:NNI):% ==
n = 0 => 1
p case R => (p::R)** n
n1: NNI := (n-1)::NNI
p1: % := n * sh(p, n1)
lt: LTERMS := [[t.k, sh(t.c, p1)]$TERM for t in listOfTerms p.reg]
[p.c0 ** n, construct lt]$VPOLY

sh(p1:%, p2:%) ==
p1 case R => p1::R * p2
p2 case R => p1 * p2::R
lt1:LTERMS := listOfTerms p1.reg ; lt2:LTERMS := listOfTerms p2.reg
x: REGPOLY := construct [[t.k,sh(t.c,p2)]$TERM for t in lt1]
y: REGPOLY := construct [[t.k,sh(p1,t.c)]$TERM for t in lt2]
[p1.c0*p2.c0,x + y]$VPOLY

RemainderList p ==
p case R => []
listOfTerms( p.reg)$REGPOLY

lquo(p1:%,p2:%):% ==
p2 case R => p1 * p2
p1 case R => p1 *$R p2.c0
p1 * p2.c0 +$% lquo1(listOfTerms p1.reg, listOfTerms p2.reg)

lquo1(x:LTERMS,y:LTERMS):% ==
null x => 0$%
null y => 0$%

```

```

x.first.k < y.first.k => lquo1(x,y.rest)
x.first.k = y.first.k =>
  lquo(x.first.c,y.first.c) + lquo1(x.rest,y.rest)
return lquo1(x.rest,y)

coef(p1:%, p2:%):R ==
  p1 case R => p1::R * constant p2
  p2 case R => p1.c0 * p2::R
  p1.c0 * p2.c0 + $R coef1(listOfTerms p1.reg, listOfTerms p2.reg)

coef1(x:LTERMS,y:LTERMS):R ==
  null x => 0$R
  null y => 0$R
  x.first.k < y.first.k => coef1(x,y.rest)
  x.first.k = y.first.k =>
    coef(x.first.c,y.first.c) + coef1(x.rest,y.rest)
  return coef1(x.rest,y)

-----
outForm(p:REGPOLY): EX ==
  le : List EX := [t.k::EX * t.c::EX for t in listOfTerms p]
  reduce(_+, reverse_! le)$List(EX)

coerce(p:$): EX ==
  p case R => (p::R)::EX
  p.c0 = 0 => outForm p.reg
  p.c0::EX + outForm p.reg

0 == 0$R::%
1 == 1$R::%
constant? p == p case R
constant p ==
  p case R => p
  p.c0

simplifie p ==
  p.reg = 0$REGPOLY => (p.c0)::%
  p

coerce (v:VarSet):% ==
  [0$R,coerce(v)$REGPOLY]$VPOLY

coerce (r:R):% == r::%
coerce (n:Integer) == n::R::%
coerce (w:WORD) ==
  w = 1 => 1$R
  (first w) * coerce(rest w)

expand p ==
  p case R => p::R::XDPOLY

```

```

    lt:LTERMS := listOfTerms(p.reg)
    ep:XDPOLY := (p.c0)::XDPOLY
    for t in lt repeat
        ep:= ep + t.k * expand(t.c)
    ep

- p:% ==
  p case R => -$R p
  [- p.c0, - p.reg]$VPOLY

p1 + p2 ==
  p1 case R and p2 case R => p1 +$R p2
  p1 case R => [p1 + p2.c0 , p2.reg]$VPOLY
  p2 case R => [p2 + p1.c0 , p1.reg]$VPOLY
  simplifie [p1.c0 + p2.c0 , p1.reg +$REGPOLY p2.reg]$VPOLY

p1 - p2 ==
  p1 case R and p2 case R => p1 -$R p2
  p1 case R => [p1 - p2.c0 , -p2.reg]$VPOLY
  p2 case R => [p1.c0 - p2 , p1.reg]$VPOLY
  simplifie [p1.c0 - p2.c0 , p1.reg -$REGPOLY p2.reg]$VPOLY

n:Integer * p:% ==
  n=0 => 0$%
  p case R => n *$R p
  -- [ n*p.c0,n*p.reg]$VPOLY
  simplifie [ n*p.c0,n*p.reg]$VPOLY

r:R * p:% ==
  r=0 => 0$%
  p case R => r *$R p
  -- [ r*p.c0,r*p.reg]$VPOLY
  simplifie [ r*p.c0,r*p.reg]$VPOLY

p:% * r:R ==
  r=0 => 0$%
  p case R => p *$R r
  -- [ p.c0 * r,p.reg * r]$VPOLY
  simplifie [ r*p.c0,r*p.reg]$VPOLY

v:VarSet * p:% ==
  p = 0 => 0$%
  [0$R, v *$REGPOLY p]$VPOLY

p1:% * p2:% ==
  p1 case R => p1::R * p2
  p2 case R => p1 * p2::R
  x:REGPOLY := p1.reg *$REGPOLY p2
  y:REGPOLY := (p1.c0)::% *$REGPOLY p2.reg -- maladroite:(p1.c0)::% !!
  -- [ p1.c0 * p2.c0 , x+y ]$VPOLY

```

```

simplifie [ p1.c0 * p2.c0 , x+y ]$VPOLY

lquo(p:%, v:VarSet):% ==
  p case R => 0
  coefficient(p.reg,v)$REGPOLY

lquo(p:%, w:WORD):% ==
  w = 1$WORD => p
  lquo(lquo(p,first w),rest w)

rquo(p:%, v:VarSet):% ==
  p case R => 0
  x:REGPOLY := construct [[t.k, a]$TERM for t in listOfTerms(p.reg)
                        | (a:= rquo(t.c,v)) ^= 0 ]
  simplifie [constant(coefficient(p.reg,v)) , x]$VPOLY

rquo(p:%, w:WORD):% ==
  w = 1$WORD => p
  rquo(rquo(p,rest w),first w)

coef(p:%, w:WORD):R ==
  constant lquo(p,w)

quasiRegular? p ==
  p case R => p = 0$R
  p.c0 = 0$R

quasiRegular p ==
  p case R => 0$%
  [0$R,p.reg]$VPOLY

characteristic == characteristic()$R
recip p ==
  p case R => recip(p::R)
  "failed"

mindeg p ==
  p case R =>
    p = 0 => error "XRPOLY.mindeg: polynome nul !!"
    1$WORD
  p.c0 ^= 0 => 1$WORD
  "min"/[(t.k) *$WORD mindeg(t.c) for t in listOfTerms p.reg]

maxdeg p ==
  p case R =>
    p = 0 => error "XRPOLY.maxdeg: polynome nul !!"
    1$WORD
  "max"/[(t.k) *$WORD maxdeg(t.c) for t in listOfTerms p.reg]

degree p ==

```

```

p = 0 => error "XRPOLY.degree: polynome nul !!"
length(maxdeg p)

map(fn,p) ==
p case R => fn(p::R)
x:REGPOLY := construct [[t.k,a]$TERM for t in listOfTerms p.reg
                        |(a := map(fn,t.c)) ^= 0$R]
simplifie [fn(p.c0),x]$VPOLY

varList p ==
p case R => []
lv: List VarSet := "setUnion"/[varList(t.c) for t in listOfTerms p.reg]
lv:= setUnion(lv,[t.k for t in listOfTerms p.reg])
sort_!(lv)

```

---

— XRPOLY.dotabb —

```

"XRPOLY" [color="#88FF44",href="bookvol10.3.pdf#nameddest=XRPOLY"]
"XPOLYC" [color="#4488FF",href="bookvol10.2.pdf#nameddest=XPOLYC"]
"XRPOLY" -> "XPOLYC"

```

---





**Chapter 26**

**Chapter Y**



**Chapter 27**

**Chapter Z**



## Chapter 28

# The bootstrap code

### 28.1 BOOLEAN.lsp

**BOOLEAN** depends on **ORDSET** which depends on **SETCAT** which depends on **BASTYPE** which depends on **BOOLEAN**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **BOOLEAN** domain which we can write into the **MID** directory. We compile the lisp code and copy the **BOOLEAN.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

— **BOOLEAN.lsp BOOTSTRAP** —

```
(|/VERSIONCHECK| 2)

(PUT
  (QUOTE |BOOLEAN;test;2$;1|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM (|a| |a|)))

(DEFUN |BOOLEAN;test;2$;1| (|a| |$|) |a|)

(DEFUN |BOOLEAN;nt| (|b| |$|)
  (COND (|b| (QUOTE NIL))
  ((QUOTE T) (QUOTE T))))

(PUT
  (QUOTE |BOOLEAN>true;$;3|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM NIL (QUOTE T))))

(DEFUN |BOOLEAN>true;$;3| (|$|)
  (QUOTE T))

(PUT
```

```

(QUOTE |BOOLEAN;false;$;4|)
(QUOTE |SPADreplace|)
(QUOTE (XLAM NIL NIL)))

(DEFUN |BOOLEAN;false;$;4| (|a|) NIL)

(DEFUN |BOOLEAN;not;2$;5| (|b| |a|)
  (COND
    (|b| (QUOTE NIL))
    ((QUOTE T) (QUOTE T))))

(DEFUN |BOOLEAN;^;2$;6| (|b| |a|)
  (COND
    (|b| (QUOTE NIL))
    ((QUOTE T) (QUOTE T))))

(DEFUN |BOOLEAN;~;2$;7| (|b| |a|)
  (COND
    (|b| (QUOTE NIL))
    ((QUOTE T) (QUOTE T))))

(DEFUN |BOOLEAN;and;3$;8| (|a| |b| |a|)
  (COND
    (|a| |b|)
    ((QUOTE T) (QUOTE NIL))))

(DEFUN |BOOLEAN;/\;3$;9| (|a| |b| |a|)
  (COND
    (|a| |b|)
    ((QUOTE T) (QUOTE NIL))))

(DEFUN |BOOLEAN;or;3$;10| (|a| |b| |a|)
  (COND
    (|a| (QUOTE T))
    ((QUOTE T) |b|)))

(DEFUN |BOOLEAN;\|/;3$;11| (|a| |b| |a|)
  (COND
    (|a| (QUOTE T))
    ((QUOTE T) |b|)))

(DEFUN |BOOLEAN;xor;3$;12| (|a| |b| |a|)
  (COND
    (|a| (|BOOLEAN;nt| |b| |a|))
    ((QUOTE T) |b|)))

(DEFUN |BOOLEAN;nor;3$;13| (|a| |b| |a|)
  (COND
    (|a| (QUOTE NIL))
    ((QUOTE T) (|BOOLEAN;nt| |b| |a|))))

```

```

(DEFUN |BOOLEAN;nand;3$;14| (|a| |b| |$|)
  (COND
    (|a| (|BOOLEAN;nt| |b| |$|))
    ((QUOTE T) (QUOTE T))))

(PUT
  (QUOTE |BOOLEAN;=;3$;15|)
  (QUOTE |SPADreplace|)
  (QUOTE |BooleanEquality|))

(DEFUN |BOOLEAN;=;3$;15| (|a| |b| |$|)
  (|BooleanEquality| |a| |b|))

(DEFUN |BOOLEAN;implies;3$;16| (|a| |b| |$|)
  (COND
    (|a| |b|)
    ((QUOTE T) (QUOTE T))))

(DEFUN |BOOLEAN;<;3$;17| (|a| |b| |$|)
  (COND
    (|b|
      (COND
        (|a| (QUOTE NIL))
        ((QUOTE T) (QUOTE T))))
    ((QUOTE T) (QUOTE NIL))))

(PUT
  (QUOTE |BOOLEAN;size;Nni;18|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM NIL 2)))

(DEFUN |BOOLEAN;size;Nni;18| (|a|) 2)

(DEFUN |BOOLEAN;index;Pi$;19| (|i| |$|)
  (COND
    ((SPADCALL |i| (QREFELT |$| 26)) (QUOTE NIL))
    ((QUOTE T) (QUOTE T))))

(DEFUN |BOOLEAN;lookup;$Pi;20| (|a| |$|)
  (COND
    (|a| 1)
    ((QUOTE T) 2)))

(DEFUN |BOOLEAN;random;$;21| (|a|)
  (COND
    ((SPADCALL (|random|) (QREFELT |$| 26)) (QUOTE NIL))
    ((QUOTE T) (QUOTE T))))

(DEFUN |BOOLEAN;convert;$If;22| (|x| |$|)

```



```

(COND
  (|x| (SPADCALL (SPADCALL "true" (QREFELT |$| 33)) (QREFELT |$| 35)))
  ((QUOTE T)
    (SPADCALL (SPADCALL "false" (QREFELT |$| 33)) (QREFELT |$| 35))))))

(DEFUN |BOOLEAN;coerce;$Of;23| (|x| |$|)
  (COND
    (|x| (SPADCALL "true" (QREFELT |$| 38)))
    ((QUOTE T) (SPADCALL "false" (QREFELT |$| 38)))))

(DEFUN |Boolean| ()
  (PROG ()
    (RETURN
      (PROG (G82461)
        (RETURN
          (COND
            ((LETT G82461 (HGET |$ConstructorCache| '|Boolean|)
              |Boolean|)
              (|CDRwithIncrement| (CDAR G82461)))
            ('T
              (UNWIND-PROTECT
                (PROG1 (CDDAR (HPUT |$ConstructorCache| '|Boolean|
                  (LIST
                    (CONS NIL (CONS 1 (|Boolean;|))))))
                  (LETT G82461 T |Boolean|))
                (COND
                  ((NOT G82461) (HREM |$ConstructorCache| '|Boolean|))))))))))

(DEFUN |Boolean;| ()
  (PROG (|dv$| $ |pv$|)
    (RETURN
      (PROGN
        (LETT |dv$| '(|Boolean|) |Boolean|)
        (LETT $ (make-array 41) |Boolean|)
        (QSETREFV $ 0 |dv$|)
        (QSETREFV $ 3
          (LETT |pv$| (|buildPredVector| 0 0 NIL) |Boolean|))
        (|haddProp| |$ConstructorCache| '|Boolean| NIL (CONS 1 $))
        (|stuffDomainSlots| $)
        $))))

(setf (get
  (QUOTE |Boolean|)
  (QUOTE |infovec|))
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL NIL
        (|Boolean|)
        |BOOLEAN;test;2$;1|
        (CONS IDENTITY

```

```

      (FUNCALL (|dispatchFunction| |BOOLEAN;true;$;3|) |$|))
(CONS IDENTITY
  (FUNCALL (|dispatchFunction| |BOOLEAN>false;$;4|) |$|))
|BOOLEAN;not;2$;5|
|BOOLEAN;~;2$;6|
|BOOLEAN;~;2$;7|
|BOOLEAN;and;3$;8|
|BOOLEAN;/\;3$;9|
|BOOLEAN;or;3$;10|
|BOOLEAN;\;/;3$;11|
|BOOLEAN;xor;3$;12|
|BOOLEAN;nor;3$;13|
|BOOLEAN;nand;3$;14|
|BOOLEAN;=;3$;15|
|BOOLEAN;implies;3$;16|
|BOOLEAN;<;3$;17|
(|NonNegativeInteger|)
|BOOLEAN;size;Nni;18|
(|Integer|)
(0 . |even?|)
(|PositiveInteger|)
|BOOLEAN;index;Pi$;19|
|BOOLEAN;lookup;$Pi;20|
|BOOLEAN;random;$;21|
(|String|)
(|Symbol|)
(5 . |coerce|)
(|InputForm|)
(10 . |convert|)
|BOOLEAN;convert;$If;22|
(|OutputForm|)
(15 . |message|)
|BOOLEAN;coerce;$Of;23|
(|SingleInteger|))
(QUOTE
  #(|~=| 20 |~| 26 |xor| 31 |true| 37 |test| 41 |size| 46 |random| 50
    |or| 54 |not| 60 |nor| 65 |nand| 71 |min| 77 |max| 83 |lookup| 89
    |latex| 94 |index| 99 |implies| 104 |hash| 110 |false| 115
    |convert| 119 |coerce| 124 |and| 129 |^| 135 |\/| 140 |>=| 146
    |>| 152 |=| 158 |<=| 164 |<| 170 |/\| 176))
(QUOTE NIL)
(CONS
  (|makeByteWordVec2| 1 (QUOTE (0 0 0 0 0 0)))
  (CONS
    (QUOTE
      #(|OrderedSet&| NIL |Logic&| |SetCategory&| NIL |BasicType&| NIL))
    (CONS
      (QUOTE
        #((|OrderedSet|)
          (|Finite|)

```

```

(|Logic|)
(|SetCategory|)
(|ConvertibleTo| 34)
(|BasicType|)
(|CoercibleTo| 37)))
(|makeByteWordVec2|
 40
 (QUOTE
  (1 25 6 0 26 1 32 0 31 33 1 34 0 32 35 1 37 0 31 38 2 0 6 0 0
   1 1 0 0 0 12 2 0 0 0 0 17 0 0 0 8 1 0 6 0 7 0 0 23 24 0 0 0
   30 2 0 0 0 0 15 1 0 0 0 10 2 0 0 0 0 18 2 0 0 0 0 19 2 0 0 0
   0 1 2 0 0 0 0 1 1 0 27 0 29 1 0 31 0 1 1 0 0 27 28 2 0 0 0 0
   21 1 0 40 0 1 0 0 0 9 1 0 34 0 36 1 0 37 0 39 2 0 0 0 0 13 1
   0 0 0 11 2 0 0 0 0 16 2 0 6 0 0 1 2 0 6 0 0 1 2 0 6 0 0 20 2
   0 6 0 0 1 2 0 6 0 0 22 2 0 0 0 0 14))))))
 (QUOTE |lookupComplete|)))

(setf (get (QUOTE |Boolean|) (QUOTE NILADIC)) T)

```

## 28.2 CHAR.lsp BOOTSTRAP

**CHAR** depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **CHAR** category which we can write into the **MID** directory. We compile the lisp code and copy the **CHAR.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

### — CHAR.lsp BOOTSTRAP —

```

(|/VERSIONCHECK| 2)

(put (QUOTE |CHAR;=;2$B;1|) (QUOTE |SPADreplace|) (QUOTE EQL))

(defun |CHAR;=;2$B;1| (|a| |b| |$|) (EQL |a| |b|))

(put (QUOTE |CHAR;<;2$B;2|) (QUOTE |SPADreplace|) (QUOTE QSLESSP))

(defun |CHAR;<;2$B;2| (|a| |b| |$|) (QSLESSP |a| |b|))

(put (QUOTE |CHAR;size;Nni;3|) (QUOTE |SPADreplace|) (QUOTE (XLAM NIL 256)))

(defun |CHAR;size;Nni;3| (|$|) 256)

```

```

(DEFUN |CHAR;index;Pi$;4| (|n| |$|) (SPADCALL (|-| |n| 1) (QREFELT |$| 18)))

(DEFUN |CHAR;lookup;$Pi;5| (|c| $)
  (PROG (G90919)
    (RETURN
      (PROG1 (LETT G90919 (+ 1 (SPADCALL |c| (QREFELT $ 21)))
        |CHAR;lookup;$Pi;5|)
        (|check-subtype| (> G90919 0) '(|PositiveInteger| G90919))))))

(DEFUN |CHAR;char;I$;6| (|n| |$|) (SPADCALL |n| (QREFELT |$| 23)))

(PUT (QUOTE |CHAR;ord;$I;7|) (QUOTE |SPADreplace|) (QUOTE (XLAM (|c|) |c|)))

(DEFUN |CHAR;ord;$I;7| (|c| |$|) |c|)

(DEFUN |CHAR;random;$;8| (|$|)
  (SPADCALL (REMAINDER2 (|random|) (SPADCALL (QREFELT |$| 16)))
    (QREFELT |$| 18)))

(PUT (QUOTE |CHAR;space;$;9|)
  (QUOTE |SPADreplace|) (QUOTE (XLAM NIL (QENUM " " 0))))

(DEFUN |CHAR;space;$;9| (|$|) (QENUM " " 0))

(PUT (QUOTE |CHAR;quote;$;10|)
  (QUOTE |SPADreplace|) (QUOTE (XLAM NIL (QENUM "\" " 0))))

(DEFUN |CHAR;quote;$;10| (|$|) (QENUM "\" " 0))

(PUT (QUOTE |CHAR;escape;$;11|)
  (QUOTE |SPADreplace|) (QUOTE (XLAM NIL (QENUM "_" " 0))))

(DEFUN |CHAR;escape;$;11| (|$|) (QENUM "_" " 0))

(DEFUN |CHAR;coerce;$Of;12| (|c| |$|)
  (ELT (QREFELT |$| 10)
    (|+| (QREFELT |$| 11) (SPADCALL |c| (QREFELT |$| 21)))))

(DEFUN |CHAR;digit?;$B;13| (|c| |$|)
  (SPADCALL |c| (|spadConstant| |$| 31) (QREFELT |$| 33)))

(DEFUN |CHAR;hexDigit?;$B;14| (|c| |$|)
  (SPADCALL |c| (|spadConstant| |$| 35) (QREFELT |$| 33)))

(DEFUN |CHAR;upperCase?;$B;15| (|c| |$|)
  (SPADCALL |c| (|spadConstant| |$| 37) (QREFELT |$| 33)))

(DEFUN |CHAR;lowerCase?;$B;16| (|c| |$|)
  (SPADCALL |c| (|spadConstant| |$| 39) (QREFELT |$| 33)))

```

```

(DEFUN |CHAR;alphanumeric?;$B;17| (|c| |$|)
  (SPADCALL |c| (|spadConstant| |$| 41) (QREFELT |$| 33)))

(DEFUN |CHAR;alphanumeric?;$B;18| (|c| |$|)
  (SPADCALL |c| (|spadConstant| |$| 43) (QREFELT |$| 33)))

(DEFUN |CHAR;latex;$S;19| (|c| |$|)
  (STRCONC "\\mbox{" (STRCONC (|MAKE-FULL-CVEC| 1 |c|) "'}") (|c|)))

(DEFUN |CHAR;char;$S;20| (|s| |$|)
  (COND
    ((EQL (QCSIZE |s|) 1)
      (SPADCALL |s| (SPADCALL |s| (QREFELT |$| 47)) (QREFELT |$| 48)))
    ((QUOTE T) (|error| "String is not a single character"))))

(DEFUN |CHAR;upperCase;2$;21| (|c| |$|)
  (QENUM (PNAME (UPCASE (code-char (SPADCALL |c| (QREFELT |$| 21))))) 0))

(DEFUN |CHAR;lowerCase;2$;22| (|c| |$|)
  (QENUM (PNAME (DOWNCASE (code-char (SPADCALL |c| (QREFELT |$| 21))))) 0))

(DEFUN |Character| ()
  (PROG ()
    (RETURN
      (PROG (G90941)
        (RETURN
          (COND
            ((LETT G90941 (HGET |$ConstructorCache| '|Character|)
              |Character|)
              (|CDRwithIncrement| (CDAR G90941)))
            ('T
              (UNWIND-PROTECT
                (PROG1 (CDDAR (HPUT |$ConstructorCache| '|Character|
                  (LIST
                    (CONS NIL (CONS 1 (|Character;|))))))
                  (LETT G90941 T |Character|))
                (COND
                  ((NOT G90941) (HREM |$ConstructorCache| '|Character|))))))))))

(DEFUN |Character;| ()
  (PROG (|dv$| $ |pv$| G90939 |i|)
    (RETURN
      (SEQ (PROGN
        (LETT |dv$| '(|Character|) |Character|)
        (LETT $ (make-array 53) |Character|)
        (QSETREFV $ 0 |dv$|)
        (QSETREFV $ 3
          (LETT |pv$| (|buildPredVector| 0 0 NIL) |Character|))
        (|haddProp| |$ConstructorCache| '|Character| NIL
          (CONS 1 $))

```

```

(|stuffDomainSlots| $)
(QSETREFV $ 6 (|SingleInteger|))
(QSETREFV $ 10
  (SPADCALL
    (PROGN
      (LETT G90939 NIL |Character|)
      (SEQ (LETT |i| 0 |Character|) G190
        (COND ((QSGREATERP |i| 255) (GO G191)))
        (SEQ (EXIT (LETT G90939
          (CONS (code-char |i|) G90939)
          |Character|)))
          (LETT |i| (QSADD1 |i|) |Character|)
          (GO G190) G191 (EXIT (NREVERSEO G90939))))
      (QREFELT $ 9)))
    (QSETREFV $ 11 0)
    $))))))

(setf (get (QUOTE |Character|) (QUOTE |infovec|))
  (LIST (QUOTE
    #(NIL NIL NIL NIL NIL NIL (QUOTE |Rep|) (|List| 28) (|PrimitiveArray| 28)
      (0 . |construct|) (QUOTE |OutChars|) (QUOTE |minChar|) (|Boolean|)
      |CHAR;|=;2$B;1| |CHAR;<;2$B;2| (|NonNegativeInteger|) |CHAR;size;Nni;3|
      (|Integer|) |CHAR;char;I$;6| (|PositiveInteger|) |CHAR;index;Pi$;4|
      |CHAR;ord;I;7| |CHAR;lookup;$Pi;5| (5 . |coerce|) |CHAR;random;$;8|
      |CHAR;space;$;9| |CHAR;quote;$;10| |CHAR;escape;$;11| (|OutputForm|)
      |CHAR;coerce;$Of;12| (|CharacterClass|) (10 . |digit|) (|Character|)
      (14 . |member?|) |CHAR;digit?;$B;13| (20 . |hexDigit|)
      |CHAR;hexDigit?;$B;14| (24 . |upperCase|) |CHAR;upperCase?;$B;15|
      (28 . |lowerCase|) |CHAR;lowerCase?;$B;16| (32 . |alphabetic|)
      |CHAR;alphabetic?;$B;17| (36 . |alphanumeric|) |CHAR;alphanumeric?;$B;18|
      (|String|) |CHAR;latex;$S;19| (40 . |minIndex|) (45 . |elt|)
      |CHAR;char;$S;20| |CHAR;upperCase;2$;21| |CHAR;lowerCase;2$;22|
      (|SingleInteger|))) (QUOTE #(|~|= 51 |upperCase?| 57 |upperCase| 62
      |space| 67 |size| 71 |random| 75 |quote| 79 |ord| 83 |min| 88 |max| 94
      |lowerCase?| 100 |lowerCase| 105 |lookup| 110 |latex| 115 |index| 120
      |hexDigit?| 125 |hash| 130 |escape| 135 |digit?| 139 |coerce| 144 |char|
      149 |alphanumeric?| 159 |alphabetic?| 164 |>=| 169 |>| 175 |=| 181 |<=|
      187 |<| 193)) (QUOTE NIL)
    (CONS
      (|makeByteWordVec2| 1 (QUOTE (0 0 0 0 0 0)))
      (CONS
        (QUOTE #(NIL |OrderedSet&| NIL |SetCategory&| |BasicType&| NIL))
        (CONS
          (QUOTE #((|OrderedFinite|) (|OrderedSet|) (|Finite|) (|SetCategory|)
            (|BasicType|) (|CoercibleTo| 28)))
          (|makeByteWordVec2| 52
            (QUOTE (1 8 0 7 9 1 6 0 17 23 0 30 0 31 2 30 12 32 0 33 0 30 0 35
              0 30 0 37 0 30 0 39 0 30 0 41 0 30 0 43 1 45 17 0 47 2 45
              32 0 17 48 2 0 12 0 0 1 1 0 12 0 38 1 0 0 0 50 0 0 0 25 0
              0 15 16 0 0 0 24 0 0 0 26 1 0 17 0 21 2 0 0 0 0 1 2 0 0 0
            ))
          ))
        ))
      ))
  ))

```

```

0 1 1 0 12 0 40 1 0 0 0 51 1 0 19 0 22 1 0 45 0 46 1 0 0 19
20 1 0 12 0 36 1 0 52 0 1 0 0 0 27 1 0 12 0 34 1 0 28 0 29
1 0 0 45 49 1 0 0 17 18 1 0 12 0 44 1 0 12 0 42 2 0 12 0 0
1 2 0 12 0 0 1 2 0 12 0 0 13 2 0 12 0 0 1 2 0 12 0 0 14))))))
(QUOTE |lookupComplete|)))

(setf (get (QUOTE |Character|) (QUOTE NILADIC)) T)

```

---

## 28.3 DFLOAT.lsp BOOTSTRAP

**DFLOAT** depends on itself. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **DFLOAT** category which we can write into the **MID** directory. We compile the lisp code and copy the **DFLOAT.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

### — DFLOAT.lsp BOOTSTRAP —

```

(|/VERSIONCHECK| 2)

(DEFUN |DFLOAT;OMwrite;$S;1| (|x| |$|)
  (PROG (|sp| |dev| |s|)
    (RETURN
      (SEQ
        (LETT |s| "" |DFLOAT;OMwrite;$S;1|)
        (LETT |sp| (|OM-STRINGTOSTRINGPTR| |s|) |DFLOAT;OMwrite;$S;1|)
        (LETT |dev|
          (SPADCALL |sp| (SPADCALL (QREFELT |$| 7)) (QREFELT |$| 10))
          |DFLOAT;OMwrite;$S;1|)
          (SPADCALL |dev| (QREFELT |$| 12))
          (SPADCALL |dev| |x| (QREFELT |$| 14))
          (SPADCALL |dev| (QREFELT |$| 15))
          (SPADCALL |dev| (QREFELT |$| 16))
          (LETT |s| (|OM-STRINGPTRTOSTRING| |sp|) |DFLOAT;OMwrite;$S;1|)
          (EXIT |s|))))))

(DEFUN |DFLOAT;OMwrite;$BS;2| (|x| |wholeObj| |$|)
  (PROG (|sp| |dev| |s|)
    (RETURN
      (SEQ
        (LETT |s| "" |DFLOAT;OMwrite;$BS;2|)
        (LETT |sp| (|OM-STRINGTOSTRINGPTR| |s|) |DFLOAT;OMwrite;$BS;2|)
        (LETT |dev|
          (SPADCALL |sp| (SPADCALL (QREFELT |$| 7)) (QREFELT |$| 10))
          |DFLOAT;OMwrite;$BS;2|)

```

```

(COND (|wholeObj| (SPADCALL |dev| (QREFELT |$| 12))))
(SPADCALL |dev| |x| (QREFELT |$| 14))
(COND (|wholeObj| (SPADCALL |dev| (QREFELT |$| 15))))
(SPADCALL |dev| (QREFELT |$| 16))
(LETT |s| (|OM-STRINGPTRTOSTRING| |spl|) |DFLOAT;OMwrite;$BS;2|)
(EXIT |s|))))))

(DEFUN |DFLOAT;OMwrite;Omd$V;3| (|dev| |x| |$|)
  (SEQ
    (SPADCALL |dev| (QREFELT |$| 12))
    (SPADCALL |dev| |x| (QREFELT |$| 14))
    (EXIT (SPADCALL |dev| (QREFELT |$| 15)))))

(DEFUN |DFLOAT;OMwrite;Omd$BV;4| (|dev| |x| |wholeObj| |$|)
  (SEQ
    (COND (|wholeObj| (SPADCALL |dev| (QREFELT |$| 12))))
    (SPADCALL |dev| |x| (QREFELT |$| 14))
    (EXIT (COND (|wholeObj| (SPADCALL |dev| (QREFELT |$| 15)))))))

(PUT (QUOTE |DFLOAT;checkComplex|) (QUOTE |SPADreplace|) (QUOTE |C-TO-R|))

(DEFUN |DFLOAT;checkComplex| (|x| |$|) (|C-TO-R| |x|))

(PUT
  (QUOTE |DFLOAT;base;Pi;6|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM NIL (|FLOAT-RADIX| 0.0))))

(DEFUN |DFLOAT;base;Pi;6| (|$|) (|FLOAT-RADIX| 0.0))

(DEFUN |DFLOAT;mantissa;$I;7| (|x| |$|) (QCAR (|DFLOAT;manexp| |x| |$|)))

(DEFUN |DFLOAT;exponent;$I;8| (|x| |$|) (QCDR (|DFLOAT;manexp| |x| |$|)))

(PUT
  (QUOTE |DFLOAT;precision;Pi;9|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM NIL (|FLOAT-DIGITS| 0.0))))

(DEFUN |DFLOAT;precision;Pi;9| (|$|) (|FLOAT-DIGITS| 0.0))

(DEFUN |DFLOAT;bits;Pi;10| ($)
  (PROG (G105705)
    (RETURN
      (COND
        ((EQL (FLOAT-RADIX 0.0) 2) (FLOAT-DIGITS 0.0))
        ((EQL (FLOAT-RADIX 0.0) 16) (* 4 (FLOAT-DIGITS 0.0)))
        ('T
          (PROG1 (LETT G105705
            (truncate (SPADCALL (FLOAT-DIGITS 0.0)

```



```

                                (SPADCALL
                                  (FLOAT (FLOAT-RADIX 0.0)
                                    MOST-POSITIVE-LONG-FLOAT)
                                  (QREFELT $ 28))
                                (QREFELT $ 29)))
                                |DFLOAT;bits;Pi;10|)
      (|check-subtype| (> G105705 0) '(|PositiveInteger|) G105705))))))

(PUT
  (QUOTE |DFLOAT;max;$;11|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM NIL |MOST-POSITIVE-LONG-FLOAT|)))

(DEFUN |DFLOAT;max;$;11| (|$|) |MOST-POSITIVE-LONG-FLOAT|)

(PUT
  (QUOTE |DFLOAT;min;$;12|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM NIL |MOST-NEGATIVE-LONG-FLOAT|)))

(DEFUN |DFLOAT;min;$;12| (|$|) |MOST-NEGATIVE-LONG-FLOAT|)

(DEFUN |DFLOAT;order;$I;13| (|a| |$|)
  (|-| (|+| (|FLOAT-DIGITS| 0.0) (SPADCALL |a| (QREFELT |$| 26))) 1))

(PUT
  (QUOTE |DFLOAT;Zero;$;14|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM NIL (FLOAT 0 |MOST-POSITIVE-LONG-FLOAT|))))

(DEFUN |DFLOAT;Zero;$;14| (|$|) (FLOAT 0 |MOST-POSITIVE-LONG-FLOAT|))

(PUT
  (QUOTE |DFLOAT;One;$;15|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM NIL (FLOAT 1 |MOST-POSITIVE-LONG-FLOAT|))))

(DEFUN |DFLOAT;One;$;15| (|$|) (FLOAT 1 |MOST-POSITIVE-LONG-FLOAT|))

(DEFUN |DFLOAT;exp1;$;16| (|$|)
  (|/|
    (FLOAT 534625820200 |MOST-POSITIVE-LONG-FLOAT|)
    (FLOAT 196677847971 |MOST-POSITIVE-LONG-FLOAT|)))

(PUT (QUOTE |DFLOAT;pi;$;17|) (QUOTE |SPADreplace|) (QUOTE (XLAM NIL PI)))

(DEFUN |DFLOAT;pi;$;17| (|$|) PI)

(DEFUN |DFLOAT;coerce;$Of;18| (|x| |$|) (SPADCALL |x| (QREFELT |$| 39)))

```

```

(DEFUN |DFLOAT;convert;$If;19| (|x| |$|) (SPADCALL |x| (QREFELT |$| 42)))

(PUT (QUOTE |DFLOAT;<;2$B;20|) (QUOTE |SPADreplace|) (QUOTE |<|))

(DEFUN |DFLOAT;<;2$B;20| (|x| |y| |$|) (|<| |x| |y|))

(PUT (QUOTE |DFLOAT;-;2$;21|) (QUOTE |SPADreplace|) (QUOTE |-|))

(DEFUN |DFLOAT;-;2$;21| (|x| |$|) (|-| |x|))

(PUT (QUOTE |DFLOAT;+;3$;22|) (QUOTE |SPADreplace|) (QUOTE |+|))

(DEFUN |DFLOAT;+;3$;22| (|x| |y| |$|) (|+| |x| |y|))

(PUT (QUOTE |DFLOAT;-;3$;23|) (QUOTE |SPADreplace|) (QUOTE |-|))

(DEFUN |DFLOAT;-;3$;23| (|x| |y| |$|) (|-| |x| |y|))

(PUT (QUOTE |DFLOAT;*;3$;24|) (QUOTE |SPADreplace|) (QUOTE |*|))

(DEFUN |DFLOAT;*;3$;24| (|x| |y| |$|) (|*| |x| |y|))

(PUT (QUOTE |DFLOAT;*;I2$;25|) (QUOTE |SPADreplace|) (QUOTE |*|))

(DEFUN |DFLOAT;*;I2$;25| (|i| |x| |$|) (|*| |i| |x|))

(PUT (QUOTE |DFLOAT;max;3$;26|) (QUOTE |SPADreplace|) (QUOTE MAX))

(DEFUN |DFLOAT;max;3$;26| (|x| |y| |$|) (MAX |x| |y|))

(PUT (QUOTE |DFLOAT;min;3$;27|) (QUOTE |SPADreplace|) (QUOTE MIN))

(DEFUN |DFLOAT;min;3$;27| (|x| |y| |$|) (MIN |x| |y|))

(PUT (QUOTE |DFLOAT;=;2$B;28|) (QUOTE |SPADreplace|) (QUOTE |=|))

(DEFUN |DFLOAT;=;2$B;28| (|x| |y| |$|) (|=| |x| |y|))

(PUT (QUOTE |DFLOAT;/;$I$;29|) (QUOTE |SPADreplace|) (QUOTE |/|))

(DEFUN |DFLOAT;/;$I$;29| (|x| |i| |$|) (|/| |x| |i|))

(DEFUN |DFLOAT;sqrt;2$;30| (|x| |$|) (|DFLOAT;checkComplex| (SQRT |x|) |$|))

(DEFUN |DFLOAT;log10;2$;31| (|x| |$|) (|DFLOAT;checkComplex| (|log| |x|) |$|))

(PUT (QUOTE |DFLOAT;**;$I$;32|) (QUOTE |SPADreplace|) (QUOTE EXPT))

(DEFUN |DFLOAT;**;$I$;32| (|x| |i| |$|) (EXPT |x| |i|))

```

```

(DEFUN |DFLOAT;**;33| (|x| |y| |$|)
  (|DFLOAT;checkComplex| (EXPT |x| |y|) |$|))

(PUT
  (QUOTE |DFLOAT;coerce;I$;34|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM (|i|) (FLOAT |i| |MOST-POSITIVE-LONG-FLOAT|))))

(DEFUN |DFLOAT;coerce;I$;34| (|i| |$|) (FLOAT |i| |MOST-POSITIVE-LONG-FLOAT|))

(PUT (QUOTE |DFLOAT;exp;2$;35|) (QUOTE |SPADreplace|) (QUOTE EXP))

(DEFUN |DFLOAT;exp;2$;35| (|x| |$|) (EXP |x|))

(DEFUN |DFLOAT;log;2$;36| (|x| |$|) (|DFLOAT;checkComplex| (LN |x|) |$|))

(DEFUN |DFLOAT;log2;2$;37| (|x| |$|) (|DFLOAT;checkComplex| (LOG2 |x|) |$|))

(PUT (QUOTE |DFLOAT;sin;2$;38|) (QUOTE |SPADreplace|) (QUOTE SIN))

(DEFUN |DFLOAT;sin;2$;38| (|x| |$|) (SIN |x|))

(PUT (QUOTE |DFLOAT;cos;2$;39|) (QUOTE |SPADreplace|) (QUOTE COS))

(DEFUN |DFLOAT;cos;2$;39| (|x| |$|) (COS |x|))

(PUT (QUOTE |DFLOAT;tan;2$;40|) (QUOTE |SPADreplace|) (QUOTE TAN))

(DEFUN |DFLOAT;tan;2$;40| (|x| |$|) (TAN |x|))

(PUT (QUOTE |DFLOAT;cot;2$;41|) (QUOTE |SPADreplace|) (QUOTE COT))

(DEFUN |DFLOAT;cot;2$;41| (|x| |$|) (COT |x|))

(PUT (QUOTE |DFLOAT;sec;2$;42|) (QUOTE |SPADreplace|) (QUOTE SEC))

(DEFUN |DFLOAT;sec;2$;42| (|x| |$|) (SEC |x|))

(PUT (QUOTE |DFLOAT;csc;2$;43|) (QUOTE |SPADreplace|) (QUOTE CSC))

(DEFUN |DFLOAT;csc;2$;43| (|x| |$|) (CSC |x|))

(DEFUN |DFLOAT;asin;2$;44| (|x| |$|) (|DFLOAT;checkComplex| (ASIN |x|) |$|))

(DEFUN |DFLOAT;acos;2$;45| (|x| |$|) (|DFLOAT;checkComplex| (ACOS |x|) |$|))

(PUT (QUOTE |DFLOAT;atan;2$;46|) (QUOTE |SPADreplace|) (QUOTE ATAN))

(DEFUN |DFLOAT;atan;2$;46| (|x| |$|) (ATAN |x|))

```

```

(DEFUN |DFLOAT;acsc;2$;47| (|x| |$|) (|DFLOAT;checkComplex| (ACSC |x|) |$|))

(PUT (QUOTE |DFLOAT;acot;2$;48|) (QUOTE |SPADreplace|) (QUOTE ACOT))

(DEFUN |DFLOAT;acot;2$;48| (|x| |$|) (ACOT |x|))

(DEFUN |DFLOAT;asec;2$;49| (|x| |$|) (|DFLOAT;checkComplex| (ASEC |x|) |$|))

(PUT (QUOTE |DFLOAT;sinh;2$;50|) (QUOTE |SPADreplace|) (QUOTE SINH))

(DEFUN |DFLOAT;sinh;2$;50| (|x| |$|) (SINH |x|))

(PUT (QUOTE |DFLOAT;cosh;2$;51|) (QUOTE |SPADreplace|) (QUOTE COSH))

(DEFUN |DFLOAT;cosh;2$;51| (|x| |$|) (COSH |x|))

(PUT (QUOTE |DFLOAT;tanh;2$;52|) (QUOTE |SPADreplace|) (QUOTE TANH))

(DEFUN |DFLOAT;tanh;2$;52| (|x| |$|) (TANH |x|))

(PUT (QUOTE |DFLOAT;csch;2$;53|) (QUOTE |SPADreplace|) (QUOTE CSCH))

(DEFUN |DFLOAT;csch;2$;53| (|x| |$|) (CSCH |x|))

(PUT (QUOTE |DFLOAT;coth;2$;54|) (QUOTE |SPADreplace|) (QUOTE COTH))

(DEFUN |DFLOAT;coth;2$;54| (|x| |$|) (COTH |x|))

(PUT (QUOTE |DFLOAT;sech;2$;55|) (QUOTE |SPADreplace|) (QUOTE SECH))

(DEFUN |DFLOAT;sech;2$;55| (|x| |$|) (SECH |x|))

(PUT (QUOTE |DFLOAT;asinh;2$;56|) (QUOTE |SPADreplace|) (QUOTE ASINH))

(DEFUN |DFLOAT;asinh;2$;56| (|x| |$|) (ASINH |x|))

(DEFUN |DFLOAT;acosh;2$;57| (|x| |$|) (|DFLOAT;checkComplex| (ACOSH |x|) |$|))

(DEFUN |DFLOAT;atanh;2$;58| (|x| |$|) (|DFLOAT;checkComplex| (ATANH |x|) |$|))

(PUT (QUOTE |DFLOAT;acsch;2$;59|) (QUOTE |SPADreplace|) (QUOTE ACSCH))

(DEFUN |DFLOAT;acsch;2$;59| (|x| |$|) (ACSCH |x|))

(DEFUN |DFLOAT;acoth;2$;60| (|x| |$|) (|DFLOAT;checkComplex| (ACOTH |x|) |$|))

(DEFUN |DFLOAT;asech;2$;61| (|x| |$|) (|DFLOAT;checkComplex| (ASECH |x|) |$|))

(PUT (QUOTE |DFLOAT;/;3$;62|) (QUOTE |SPADreplace|) (QUOTE /|))

```

```

(DEFUN |DFLOAT;/;3$;62| (|x| |y| |$|) (|/| |x| |y|))

(PUT (QUOTE |DFLOAT;negative?;$B;63|) (QUOTE |SPADreplace|) (QUOTE MINUSP))

(DEFUN |DFLOAT;negative?;$B;63| (|x| |$|) (MINUSP |x|))

(PUT (QUOTE |DFLOAT;zero?;$B;64|) (QUOTE |SPADreplace|) (QUOTE ZEROP))

(DEFUN |DFLOAT;zero?;$B;64| (|x| |$|) (ZEROP |x|))

(PUT (QUOTE |DFLOAT;hash;$I;65|) (QUOTE |SPADreplace|) (QUOTE SXHASH))

(DEFUN |DFLOAT;hash;$I;65| (|x| |$|) (SXHASH |x|))

(DEFUN |DFLOAT;recip;$U;66| (|x| |$|)
  (COND
    ((ZEROP |x|) (CONS 1 "failed"))
    ((QUOTE T) (CONS 0 (|/| 1.0 |x|)))))

(PUT
  (QUOTE |DFLOAT;differentiate;2$;67|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM (|x|) 0.0)))

(DEFUN |DFLOAT;differentiate;2$;67| (|x| |$|) 0.0)

(DEFUN |DFLOAT;Gamma;2$;68| (|x| |$|) (SPADCALL |x| (QREFELT |$| 93)))

(DEFUN |DFLOAT;Beta;3$;69| (|x| |y| |$|) (SPADCALL |x| |y| (QREFELT |$| 95)))

(PUT (QUOTE |DFLOAT;wholePart;$I;70|) (QUOTE |SPADreplace|) (QUOTE truncate))

(DEFUN |DFLOAT;wholePart;$I;70| (|x| |$|) (truncate |x|))

(DEFUN |DFLOAT;float;2IPi$;71| (|ma| |ex| |b| |$|)
  (|*| |ma| (EXPT (FLOAT |b| |MOST-POSITIVE-LONG-FLOAT|) |ex|)))

(PUT
  (QUOTE |DFLOAT;convert;2$;72|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM (|x|) |x|)))

(DEFUN |DFLOAT;convert;2$;72| (|x| |$|) |x|)

(DEFUN |DFLOAT;convert;$F;73| (|x| |$|) (SPADCALL |x| (QREFELT |$| 101)))

(DEFUN |DFLOAT;rationalApproximation;$NniF;74| (|x| |d| |$|)
  (SPADCALL |x| |d| 10 (QREFELT |$| 105)))

(DEFUN |DFLOAT;atan;3$;75| (|x| |y| |$|)

```

```

(PROG (|theta|)
  (RETURN
    (SEQ
      (COND
        ((|=| |x| 0.0)
          (COND
            ((|<| 0.0 |y|) (|/| PI 2))
            ((|<| |y| 0.0) (|-| (|/| PI 2)))
            ((QUOTE T) 0.0)))
        ((QUOTE T)
          (SEQ
            (LETT |theta|
              (ATAN (|FLOAT-SIGN| 1.0 (|/| |y| |x|)))
              |DFLOAT;atan;3$;75|)
            (COND
              ((|<| |x| 0.0) (LETT |theta| (|-| PI |theta|) |DFLOAT;atan;3$;75|)))
            (COND ((|<| |y| 0.0) (LETT |theta| (|-| |theta|) |DFLOAT;atan;3$;75|)))
            (EXIT |theta|))))))

(DEFUN |DFLOAT;retract;$F;76| (|x| $)
  (PROG (G105780)
    (RETURN
      (SPADCALL |x|
        (PROG1 (LETT G105780 (- (FLOAT-DIGITS 0.0) 1)
          |DFLOAT;retract;$F;76|)
          (|check-subtype| (>= G105780 0) '(|NonNegativeInteger|)
            G105780))
        (FLOAT-RADIX 0.0) (QREFELT $ 105))))))

(DEFUN |DFLOAT;retractIfCan;$U;77| (|x| $)
  (PROG (G105785)
    (RETURN
      (CONS 0
        (SPADCALL |x|
          (PROG1 (LETT G105785 (- (FLOAT-DIGITS 0.0) 1)
            |DFLOAT;retractIfCan;$U;77|)
            (|check-subtype| (>= G105785 0)
              '(|NonNegativeInteger|) G105785))
          (FLOAT-RADIX 0.0) (QREFELT $ 105))))))

(DEFUN |DFLOAT;retract;$I;78| (|x| |$|)
  (PROG (|n|)
    (RETURN
      (SEQ
        (LETT |n| (truncate |x|) |DFLOAT;retract;$I;78|)
        (EXIT
          (COND
            ((|=| |x| (FLOAT |n| |MOST-POSITIVE-LONG-FLOAT|)) |n|)
            ((QUOTE T) (|error| "Not an integer"))))))))

```

```

(DEFUN |DFLOAT;retractIfCan;$U;79| (|x| |$|)
  (PROG (|n|)
    (RETURN
      (SEQ
        (LETT |n| (truncate |x|) |DFLOAT;retractIfCan;$U;79|)
        (EXIT
          (COND
            ((|=| |x| (FLOAT |n| |MOST-POSITIVE-LONG-FLOAT|)) (CONS 0 |n|))
            ((QUOTE T) (CONS 1 "failed"))))))))

(DEFUN |DFLOAT;sign;$I;80| (|x| |$|)
  (SPADCALL (|FLOAT-SIGN| |x| 1.0) (QREFELT |$| 111)))

(PUT
  (QUOTE |DFLOAT;abs;2$;81|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM (|x|) (|FLOAT-SIGN| 1.0 |x|))))

(DEFUN |DFLOAT;abs;2$;81| (|x| |$|) (|FLOAT-SIGN| 1.0 |x|))

(DEFUN |DFLOAT;manexp| (|x| $)
  (PROG (|s| G105806 |me| |two53|)
    (RETURN
      (SEQ (EXIT (COND
        ((ZEROP |x|) (CONS 0 0))
        ('T
          (SEQ (LETT |s| (SPADCALL |x| (QREFELT $ 114))
            |DFLOAT;manexp|)
            (LETT |x| (FLOAT-SIGN 1.0 |x|)
              |DFLOAT;manexp|)
            (COND
              ((< MOST-POSITIVE-LONG-FLOAT |x|)
                (PROGN
                  (LETT G105806
                    (CONS
                      (+
                        (* |s|
                          (SPADCALL
                            MOST-POSITIVE-LONG-FLOAT
                            (QREFELT $ 25)))
                        1)
                    (SPADCALL MOST-POSITIVE-LONG-FLOAT
                      (QREFELT $ 26)))
                  |DFLOAT;manexp|)
                (GO G105806))))
            (LETT |me| (MANEXP |x|) |DFLOAT;manexp|)
            (LETT |two53|
              (EXPT (FLOAT-RADIX 0.0)
                (FLOAT-DIGITS 0.0))
              |DFLOAT;manexp|)
          )
        )
      )
    )
  )

```

```

                                (EXIT (CONS (* |s|
                                                (truncate (* |two53| (QCAR |me|))))
                                                (- (QCDR |me|) (FLOAT-DIGITS 0.0))))))
                                G105806 (EXIT G105806))))))

(DEFUN |DFLOAT;rationalApproximation;$2NniF;83| (|f| |d| |b| $)
  (PROG (|#G102| |nu| |ex| BASE G105809 |de| |tol| |#G103| |q| |r| |p2|
        |q2| G105827 |#G104| |#G105| |p0| |p1| |#G106| |#G107|
        |q0| |q1| |#G108| |#G109| |s| |t| G105825)
  (RETURN
    (SEQ (EXIT (SEQ
      (PROGN
        (LETT |#G102| (|DFLOAT;manexp| |f| $)
          |DFLOAT;rationalApproximation;$2NniF;83|)
        (LETT |nu| (QCAR |#G102|)
          |DFLOAT;rationalApproximation;$2NniF;83|)
        (LETT |ex| (QCDR |#G102|)
          |DFLOAT;rationalApproximation;$2NniF;83|)
        |#G102|)
        (LETT BASE (FLOAT-RADIX 0.0)
          |DFLOAT;rationalApproximation;$2NniF;83|)
      (EXIT
        (COND
          ((< |ex| 0)
            (SEQ
              (LETT |de|
                (EXPT BASE
                  (PROG1
                    (LETT G105809 (- |ex|) |DFLOAT;rationalApproximation;$2NniF;83|)
                    (|check-subtype| (>= G105809 0) '(|NonNegativeInteger|
                      G105809))))
                |DFLOAT;rationalApproximation;$2NniF;83|)
              (EXIT
                (COND
                  ((< |b| 2) (|error| "base must be > 1"))
                  (t
                    (SEQ
                      (LETT |tol| (EXPT |b| |d|)
                        |DFLOAT;rationalApproximation;$2NniF;83|)
                      (LETT |s| |nu|
                        |DFLOAT;rationalApproximation;$2NniF;83|)
                      (LETT |t| |de|
                        |DFLOAT;rationalApproximation;$2NniF;83|)
                      (LETT |p0| 0
                        |DFLOAT;rationalApproximation;$2NniF;83|)
                      (LETT |p1| 1
                        |DFLOAT;rationalApproximation;$2NniF;83|)
                      (LETT |q0| 1
                        |DFLOAT;rationalApproximation;$2NniF;83|)
                      (LETT |q1| 0

```



```

|DFLOAT;rationalApproximation;$2NniF;83|)
(EXIT
  (SEQ G190 NIL
    (SEQ
      (PROGN
        (LETT|#G103| (DIVIDE2 |s| |t|)
          |DFLOAT;rationalApproximation;$2NniF;83|)
        (LETT |q| (QCAR|#G103|)
          |DFLOAT;rationalApproximation;$2NniF;83|)
        (LETT |r| (QCDR|#G103|)
          |DFLOAT;rationalApproximation;$2NniF;83|)
        |#G103|)
        (LETT |p2| (+ (* |q| |p1|) |p0|)
          |DFLOAT;rationalApproximation;$2NniF;83|)
        (LETT |q2| (+ (* |q| |q1|) |q0|)
          |DFLOAT;rationalApproximation;$2NniF;83|)
        (COND
          ((OR (EQL |r| 0)
            (< (SPADCALL |t01| (ABS (- (* |nu| |q2|) (* |de| |p2|)))
              (QREFELT $ 118))
              (* |de| (ABS |p2|)))))
          (EXIT
            (PROGN
              (LETT G105827
                (SPADCALL |p2| |q2| (QREFELT $ 117))
                |DFLOAT;rationalApproximation;$2NniF;83|)
              (GO G105827))))))
        (PROGN
          (LETT|#G104| |p1|
            |DFLOAT;rationalApproximation;$2NniF;83|)
          (LETT|#G105| |p2|
            |DFLOAT;rationalApproximation;$2NniF;83|)
          (LETT |p0||#G104|
            |DFLOAT;rationalApproximation;$2NniF;83|)
          (LETT |p1||#G105|
            |DFLOAT;rationalApproximation;$2NniF;83|))
          (PROGN
            (LETT|#G106| |q1|
              |DFLOAT;rationalApproximation;$2NniF;83|)
            (LETT|#G107| |q2|
              |DFLOAT;rationalApproximation;$2NniF;83|)
            (LETT |q0||#G106|
              |DFLOAT;rationalApproximation;$2NniF;83|)
            (LETT |q1||#G107|
              |DFLOAT;rationalApproximation;$2NniF;83|))
          (EXIT
            (PROGN
              (LETT|#G108| |t|
                |DFLOAT;rationalApproximation;$2NniF;83|)
              (LETT|#G109| |r|

```

```

      |DFLOAT;rationalApproximation;$2NniF;83|)
    (LETT |s| |#G108|
      |DFLOAT;rationalApproximation;$2NniF;83|)
    (LETT |t| |#G109|
      |DFLOAT;rationalApproximation;$2NniF;83|))))
  NIL (GO G190) G191
  (EXIT NIL)))))))))

('T
 (SPADCALL
  (* |nu|
   (EXPT BASE
    (PROG1
     (LETT G105825 |ex| |DFLOAT;rationalApproximation;$2NniF;83|)
      (|check-subtype| (>= G105825 0) '(|NonNegativeInteger|
        G105825))))
    (QREFELT $ 119)))))))))
G105827
  (EXIT G105827))))))

(DEFUN |DFLOAT;**;$F$;84| (|x| |r| $)
  (PROG (|n| |d| G105837)
    (RETURN
      (SEQ (EXIT (COND
        ((ZEROP |x|)
          (COND
            ((SPADCALL |r| (QREFELT $ 120))
              (|error| "0**0 is undefined"))
            ((SPADCALL |r| (QREFELT $ 121))
              (|error| "division by 0"))
            ('T 0.0)))
          ((OR (SPADCALL |r| (QREFELT $ 120))
              (SPADCALL |x| (QREFELT $ 122)))
            1.0)
          ('T
            (COND
              ((SPADCALL |r| (QREFELT $ 123)) |x|)
              ('T
                (SEQ (LETT |n| (SPADCALL |r| (QREFELT $ 124))
                  |DFLOAT;**;$F$;84|)
                  (LETT |d| (SPADCALL |r| (QREFELT $ 125))
                    |DFLOAT;**;$F$;84|)
                  (EXIT (COND
                    ((MINUSP |x|)
                      (COND
                        ((ODDP |d|)
                          (COND
                            ((ODDP |n|)
                              (PROGN
                                (LETT G105837
                                  (-

```

```

        (SPADCALL (- |x|) |r|
         (QREFELT $ 126)))
      |DFLOAT;**;$F$;84|)
    (GO G105837)))
('T
 (PROGN
  (LETT G105837
   (SPADCALL (- |x|) |r|
    (QREFELT $ 126))
   |DFLOAT;**;$F$;84|)
  (GO G105837))))
('T (|error| "negative root"))))
((EQL |d| 2)
 (EXPT
  (SPADCALL |x| (QREFELT $ 54))
  |n|))
('T
 (SPADCALL |x|
  (/
   (FLOAT |n|
    MOST-POSITIVE-LONG-FLOAT)
   (FLOAT |d|
    MOST-POSITIVE-LONG-FLOAT))
  (QREFELT $ 57)))))))))
G105837 (EXIT G105837))))))

(DEFUN |DoubleFloat| ()
 (PROG ()
  (RETURN
   (PROG (G105850)
    (RETURN
     (COND
      ((LETT G105850 (HGET |$ConstructorCache| ' |DoubleFloat|)
       |DoubleFloat|)
       (|CDRwithIncrement| (CDAR G105850)))
      ('T
       (UNWIND-PROTECT
        (PROG1 (CDDAR (HPUT |$ConstructorCache| ' |DoubleFloat|
          (LIST
           (CONS NIL
            (CONS 1 (|DoubleFloat|;|))))))
          (LETT G105850 T |DoubleFloat|))
        (COND
         ((NOT G105850)
          (HREM |$ConstructorCache| ' |DoubleFloat|))))))))))

(DEFUN |DoubleFloat;| ()
 (PROG (|dv$| $ |pv$|)
  (RETURN
   (PROGN
    (SPADCALL (- |x|) |r|
     (QREFELT $ 126)))
    |DFLOAT;**;$F$;84|)
  (GO G105837)))
('T
 (PROGN
  (LETT G105837
   (SPADCALL (- |x|) |r|
    (QREFELT $ 126))
   |DFLOAT;**;$F$;84|)
  (GO G105837))))
('T (|error| "negative root"))))
((EQL |d| 2)
 (EXPT
  (SPADCALL |x| (QREFELT $ 54))
  |n|))
('T
 (SPADCALL |x|
  (/
   (FLOAT |n|
    MOST-POSITIVE-LONG-FLOAT)
   (FLOAT |d|
    MOST-POSITIVE-LONG-FLOAT))
  (QREFELT $ 57)))))))))
G105837 (EXIT G105837))))))

(DEFUN |DoubleFloat| ()
 (PROG ()
  (RETURN
   (PROG (G105850)
    (RETURN
     (COND
      ((LETT G105850 (HGET |$ConstructorCache| ' |DoubleFloat|)
       |DoubleFloat|)
       (|CDRwithIncrement| (CDAR G105850)))
      ('T
       (UNWIND-PROTECT
        (PROG1 (CDDAR (HPUT |$ConstructorCache| ' |DoubleFloat|
          (LIST
           (CONS NIL
            (CONS 1 (|DoubleFloat;|))))))
          (LETT G105850 T |DoubleFloat|))
        (COND
         ((NOT G105850)
          (HREM |$ConstructorCache| ' |DoubleFloat|))))))))))

(DEFUN |DoubleFloat;| ()
 (PROG (|dv$| $ |pv$|)
  (RETURN
   (PROGN
    (SPADCALL (- |x|) |r|
     (QREFELT $ 126)))
    |DFLOAT;**;$F$;84|)
  (GO G105837)))
('T
 (PROGN
  (LETT G105837
   (SPADCALL (- |x|) |r|
    (QREFELT $ 126))
   |DFLOAT;**;$F$;84|)
  (GO G105837))))
('T (|error| "negative root"))))
((EQL |d| 2)
 (EXPT
  (SPADCALL |x| (QREFELT $ 54))
  |n|))
('T
 (SPADCALL |x|
  (/
   (FLOAT |n|
    MOST-POSITIVE-LONG-FLOAT)
   (FLOAT |d|
    MOST-POSITIVE-LONG-FLOAT))
  (QREFELT $ 57)))))))))
G105837 (EXIT G105837))))))

```

```

(LETT |dv$| '(|DoubleFloat|) |DoubleFloat|)
(LETT $ (make-array 140) |DoubleFloat|)
(QSETREFV $ 0 |dv$|)
(QSETREFV $ 3
  (LETT |pv$| (|buildPredVector| 0 0 NIL) |DoubleFloat|))
(|haddProp| |$ConstructorCache| ' |DoubleFloat| NIL (CONS 1 $))
(|stuffDomainSlots| $)
$)))))

(setf (get
  (QUOTE |DoubleFloat|)
  (QUOTE |infovec|))
  (LIST
    (QUOTE #(NIL NIL NIL NIL NIL NIL (|OpenMathEncoding|) (0 . |OMencodingXML|)
      (|String|) (|OpenMathDevice|) (4 . |OMopenString|) (|Void|)
      (10 . |OMputObject|) (|DoubleFloat|) (15 . |OMputFloat|)
      (21 . |OMputEndObject|) (26 . |OMclose|) |DFLOAT;OMwrite;$S;1|
      (|Boolean|) |DFLOAT;OMwrite;$BS;2| |DFLOAT;OMwrite;Omd$V;3|
      |DFLOAT;OMwrite;Omd$BV;4| (|PositiveInteger|) |DFLOAT;base;Pi;6|
      (|Integer|) |DFLOAT;mantissa;$I;7| |DFLOAT;exponent;$I;8|
      |DFLOAT;precision;Pi;9| |DFLOAT;log2;2$;37| (31 . |*|)
      |DFLOAT;bits;Pi;10| |DFLOAT;max;$;11| |DFLOAT;min;$;12|
      |DFLOAT;order;$I;13|
      (CONS IDENTITY (FUNCALL (|dispatchFunction| |DFLOAT;Zero;$;14|) |$|))
      (CONS IDENTITY (FUNCALL (|dispatchFunction| |DFLOAT;One;$;15|) |$|))
      |DFLOAT;exp1;$;16| |DFLOAT;pi;$;17| (|OutputForm|) (37 . |outputForm|)
      |DFLOAT;coerce;$Of;18| (|InputForm|) (42 . |convert|)
      |DFLOAT;convert;$If;19| |DFLOAT;<;2$B;20| |DFLOAT;-;2$;21|
      |DFLOAT;+;3$;22| |DFLOAT;-;3$;23| |DFLOAT;*;3$;24| |DFLOAT;*;I2$;25|
      |DFLOAT;max;3$;26| |DFLOAT;min;3$;27| |DFLOAT;=;2$B;28|
      |DFLOAT;/;$I$;29| |DFLOAT;sqrt;2$;30| |DFLOAT;log10;2$;31|
      |DFLOAT;**;$I$;32| |DFLOAT;**;3$;33| |DFLOAT;coerce;I$;34|
      |DFLOAT;exp;2$;35| |DFLOAT;log;2$;36| |DFLOAT;sin;2$;38|
      |DFLOAT;cos;2$;39| |DFLOAT;tan;2$;40| |DFLOAT;cot;2$;41|
      |DFLOAT;sec;2$;42| |DFLOAT;csc;2$;43| |DFLOAT;asin;2$;44|
      |DFLOAT;acos;2$;45| |DFLOAT;atan;2$;46| |DFLOAT;acsc;2$;47|
      |DFLOAT;acot;2$;48| |DFLOAT;asec;2$;49| |DFLOAT;sinh;2$;50|
      |DFLOAT;cosh;2$;51| |DFLOAT;tanh;2$;52| |DFLOAT;csch;2$;53|
      |DFLOAT;coth;2$;54| |DFLOAT;sech;2$;55| |DFLOAT;asinh;2$;56|
      |DFLOAT;acosh;2$;57| |DFLOAT;atanh;2$;58| |DFLOAT;acsch;2$;59|
      |DFLOAT;acoth;2$;60| |DFLOAT;asech;2$;61| |DFLOAT;/;3$;62|
      |DFLOAT;negative?;$B;63| |DFLOAT;zero?;$B;64| |DFLOAT;hash;$I;65|
      (|Union| |$| (QUOTE "failed")) |DFLOAT;recip;$U;66|
      |DFLOAT;differentiate;2$;67| (|DoubleFloatSpecialFunctions|)
      (47 . |Gamma|) |DFLOAT;Gamma;2$;68| (52 . |Beta|) |DFLOAT;Beta;3$;69|
      |DFLOAT;wholePart;$I;70| |DFLOAT;float;2IPi$;71| |DFLOAT;convert;2$;72|
      (|Float|) (58 . |convert|) |DFLOAT;convert;$F;73| (|Fraction| 24)
      (|NonNegativeInteger|) |DFLOAT;rationalApproximation;$2NniF;83|
      |DFLOAT;rationalApproximation;$NniF;74| |DFLOAT;atan;3$;75|
      |DFLOAT;retract;$F;76| (|Union| 103 (QUOTE "failed"))

```

```

|DFLOAT;retractIfCan;$U;77| |DFLOAT;retract;$I;78|
(|Union| 24 (QUOTE "failed")) |DFLOAT;retractIfCan;$U;79|
|DFLOAT;sign;$I;80| |DFLOAT;abs;2$;81| (63 . |Zero|) (67 . |/)
(73 . |*|) (79 . |coerce|) (84 . |zero?|) (89 . |negative?|)
(94 . |one?|) (99 . |one?|) (104 . |numer|) (109 . |denom|)
|DFLOAT;**;$F$;84| (|Pattern| 100) (|PatternMatchResult| 100 |$|)
(|Factored| |$|) (|Union| 131 (QUOTE "failed")) (|List| |$|)
(|Record| (|:| |coef1| |$|) (|:| |coef2| |$|) (|:| |generator| |$|))
(|Record| (|:| |coef1| |$|) (|:| |coef2| |$|))
(|Union| 133 (QUOTE "failed"))
(|Record| (|:| |quotient| |$|) (|:| |remainder| |$|))
(|Record| (|:| |coef| 131) (|:| |generator| |$|))
(|SparseUnivariatePolynomial| |$|) (|Record| (|:| |unit| |$|)
(|:| |canonical| |$|) (|:| |associate| |$|)) (|SingleInteger|))
(QUOTE #(|~|=| 114 |zero?| 120 |wholePart| 125 |unitNormal| 130
|unitCanonical| 135 |unit?| 140 |truncate| 145 |tanh| 150 |tan|
155 |subtractIfCan| 160 |squareFreePart| 166 |squareFree| 171
|sqrt| 176 |sizeLess?| 181 |sinh| 187 |sin| 192 |sign| 197 |sech|
202 |sec| 207 |sample| 212 |round| 216 |retractIfCan| 221 |retract|
231 |rem| 241 |recip| 247 |rationalApproximation| 252 |quo| 265
|principalIdeal| 271 |prime?| 276 |precision| 281 |positive?| 285
|pi| 290 |patternMatch| 294 |order| 301 |one?| 306 |nthRoot| 311
|norm| 317 |negative?| 322 |multiEuclidean| 327 |min| 333 |max| 343
|mantissa| 353 |log2| 358 |log10| 363 |log| 368 |lcm| 373 |latex|
384 |inv| 389 |hash| 394 |gcdPolynomial| 404 |gcd| 410 |fractionPart|
421 |floor| 426 |float| 431 |factor| 444 |extendedEuclidean| 449
|exquo| 462 |expressIdealMember| 468 |exponent| 474 |exp1| 479 |exp|
483 |euclideanSize| 488 |divide| 493 |digits| 499 |differentiate|
503 |csch| 514 |csc| 519 |coth| 524 |cot| 529 |cosh| 534 |cos| 539
|convert| 544 |coerce| 564 |characteristic| 594 |ceiling| 598 |bits|
603 |base| 607 |atanh| 611 |atan| 616 |associates?| 627 |asinh| 633
|asin| 638 |asech| 643 |asec| 648 |acsch| 653 |acsc| 658 |acoth| 663
|acot| 668 |acosh| 673 |acos| 678 |abs| 683 |^| 688 |Zero| 706 |One|
710 |OMwrite| 714 |Gamma| 738 D 743 |Beta| 754 |>=| 760 |>| 766 |=|
772 |<=| 778 |<| 784 |/>| 790 |-| 802 |+| 813 |**| 819 |*| 849))
(QUOTE ((|approximate| . 0) (|canonicalsClosed| . 0)
(|canonicalUnitNormal| . 0) (|noZeroDivisors| . 0)
((|commutative| "*") . 0) (|rightUnitary| . 0) (|leftUnitary| . 0)
(|unitsKnown| . 0)))
(CONS
(|makeByteWordVec2| 1 (QUOTE (0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0)))
(CONS
(QUOTE #(|FloatingPointSystem&| |RealNumberSystem&| |Field&|
|EuclideanDomain&| NIL |UniqueFactorizationDomain&| |GcdDomain&|
|DivisionRing&| |IntegralDomain&| |Algebra&| |Algebra&|
|DifferentialRing&| NIL |OrderedRing&| |Module&| NIL NIL |Module&|
NIL NIL NIL |Ring&| NIL NIL NIL NIL NIL NIL NIL |AbelianGroup&| NIL
NIL |AbelianMonoid&| |Monoid&| NIL |OrderedSet&| |AbelianSemiGroup&|
|SemiGroup&| |TranscendentalFunctionCategory&| NIL |SetCategory&| NIL

```

```

|ElementaryFunctionCategory&| NIL |HyperbolicFunctionCategory&|
|ArcTrigonometricFunctionCategory&| |TrigonometricFunctionCategory&|
NIL NIL |RadicalCategory&| |RetractableTo&| |RetractableTo&| NIL
NIL |BasicType&| NIL)) (CONS (QUOTE #((|FloatingPointSystem|)
(|RealNumberSystem|) (|Field|) (|EuclideanDomain|)
(|PrincipalIdealDomain|) (|UniqueFactorizationDomain|)
(|GcdDomain|) (|DivisionRing|) (|IntegralDomain|) (|Algebra| 103)
(|Algebra| |$$|) (|DifferentialRing|) (|CharacteristicZero|)
(|OrderedRing|) (|Module| 103) (|EntireRing|) (|CommutativeRing|)
(|Module| |$$|) (|OrderedAbelianGroup|) (|BiModule| 103 103)
(|BiModule| |$$| |$$|) (|Ring|) (|OrderedCancellationAbelianMonoid|)
(|RightModule| 103) (|LeftModule| 103) (|LeftModule| |$$|) (|Rng|)
(|RightModule| |$$|) (|OrderedAbelianMonoid|) (|AbelianGroup|)
(|OrderedAbelianSemiGroup|) (|CancellationAbelianMonoid|)
(|AbelianMonoid|) (|Monoid|) (|PatternMatchable| 100) (|OrderedSet|)
(|AbelianSemiGroup|) (|SemiGroup|) (|TranscendentalFunctionCategory|)
(|RealConstant|) (|SetCategory|) (|ConvertibleTo| 41)
(|ElementaryFunctionCategory|) (|ArcHyperbolicFunctionCategory|)
(|HyperbolicFunctionCategory|) (|ArcTrigonometricFunctionCategory|)
(|TrigonometricFunctionCategory|) (|OpenMath|) (|ConvertibleTo| 127)
(|RadicalCategory|) (|RetractableTo| 103) (|RetractableTo| 24)
(|ConvertibleTo| 100) (|ConvertibleTo| 13) (|BasicType|)
(|CoercibleTo| 38)))
(|makeByteWordVec2| 139
(QUOTE (0 6 0 7 2 9 0 8 6 10 1 9 11 0 12 2 9 11 0 13 14 1 9 11 0 15
1 9 11 0 16 2 0 0 22 0 29 1 38 0 13 39 1 41 0 13 42 1 92 13 13 93 2 92
13 13 13 95 1 100 0 13 101 0 103 0 116 2 103 0 24 24 117 2 24 0 104 0
118 1 103 0 24 119 1 103 18 0 120 1 103 18 0 121 1 0 18 0 122 1 103 18
0 123 1 103 24 0 124 1 103 24 0 125 2 0 18 0 0 1 1 0 18 0 87 1 0 24 0
97 1 0 138 0 1 1 0 0 0 1 1 0 18 0 1 1 0 0 0 1 1 0 0 0 75 1 0 0 0 63 2
0 89 0 0 1 1 0 0 0 1 1 0 129 0 1 1 0 0 0 54 2 0 18 0 0 1 1 0 0 0 73 1
0 0 0 61 1 0 24 0 114 1 0 0 0 78 1 0 0 0 65 0 0 0 1 1 0 0 0 1 1 0 109
0 110 1 0 112 0 113 1 0 103 0 108 1 0 24 0 111 2 0 0 0 0 1 1 0 89 0
90 2 0 103 0 104 106 3 0 103 0 104 104 105 2 0 0 0 0 1 1 0 136 131 1
1 0 18 0 1 0 0 22 27 1 0 18 0 1 0 0 0 37 3 0 128 0 127 128 1 1 0 24
0 33 1 0 18 0 122 2 0 0 0 24 1 1 0 0 0 1 1 0 18 0 86 2 0 130 131 0 1
0 0 0 32 2 0 0 0 0 51 0 0 0 31 2 0 0 0 0 50 1 0 24 0 25 1 0 0 0 28 1
0 0 0 55 1 0 0 0 60 1 0 0 131 1 2 0 0 0 0 1 1 0 8 0 1 1 0 0 0 1 1 0
24 0 88 1 0 139 0 1 2 0 137 137 137 1 1 0 0 131 1 2 0 0 0 0 1 1 0 0
0 1 1 0 0 0 1 3 0 0 24 24 22 98 2 0 0 24 24 1 1 0 129 0 1 2 0 132 0
0 1 3 0 134 0 0 0 1 2 0 89 0 0 1 2 0 130 131 0 1 1 0 24 0 26 0 0 0
36 1 0 0 0 59 1 0 104 0 1 2 0 135 0 0 1 0 0 22 1 1 0 0 0 91 2 0 0 0
104 1 1 0 0 0 76 1 0 0 0 66 1 0 0 0 77 1 0 0 0 64 1 0 0 0 74 1 0 0 0
62 1 0 41 0 43 1 0 127 0 1 1 0 13 0 99 1 0 100 0 102 1 0 0 103 1 1 0
0 24 58 1 0 0 103 1 1 0 0 24 58 1 0 0 0 1 1 0 38 0 40 0 0 104 1 1 0
0 0 1 0 0 22 30 0 0 22 23 1 0 0 0 81 2 0 0 0 107 1 0 0 0 69 2 0 18
0 0 1 1 0 0 0 79 1 0 0 0 67 1 0 0 0 84 1 0 0 0 72 1 0 0 0 82 1 0 0 0
70 1 0 0 0 83 1 0 0 0 71 1 0 0 0 80 1 0 0 0 68 1 0 0 0 115 2 0 0 0
24 1 2 0 0 0 104 1 2 0 0 0 22 1 0 0 0 34 0 0 0 35 2 0 11 9 0 20 3 0
11 9 0 18 21 1 0 8 0 17 2 0 8 0 18 19 1 0 0 0 94 1 0 0 0 1 2 0 0 0

```

```

104 1 2 0 0 0 0 96 2 0 18 0 0 1 2 0 18 0 0 1 2 0 18 0 0 52 2 0 18 0
0 1 2 0 18 0 0 44 2 0 0 0 24 53 2 0 0 0 0 85 2 0 0 0 0 47 1 0 0 0
45 2 0 0 0 0 46 2 0 0 0 0 57 2 0 0 0 103 126 2 0 0 0 24 56 2 0 0 0
104 1 2 0 0 0 22 1 2 0 0 0 103 1 2 0 0 103 0 1 2 0 0 0 0 48 2 0 0
24 0 49 2 0 0 104 0 1 2 0 0 22 0 29))))))
(QUOTE |lookupComplete|)))

(setf (get (QUOTE |DoubleFloat|) (QUOTE NILADIC)) T)

```

---

## 28.4 ILIST.lsp BOOTSTRAP

**ILIST** depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **ILIST** category which we can write into the **MID** directory. We compile the lisp code and copy the **ILIST.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

### — ILIST.lsp BOOTSTRAP —

```

(|/VERSIONCHECK| 2)

(PUT (QUOTE |ILIST;#;$Nni;1|) (QUOTE |SPADreplace|) (QUOTE LENGTH))

(DEFUN |ILIST;#;$Nni;1| (|x| |$|) (LENGTH |x|))

(PUT (QUOTE |ILIST;concat;S2$;2|) (QUOTE |SPADreplace|) (QUOTE CONS))

(DEFUN |ILIST;concat;S2$;2| (|s| |x| |$|) (CONS |s| |x|))

(PUT (QUOTE |ILIST;eq?;2$B;3|) (QUOTE |SPADreplace|) (QUOTE EQ))

(DEFUN |ILIST;eq?;2$B;3| (|x| |y| |$|) (EQ |x| |y|))

(PUT (QUOTE |ILIST;first;$S;4|) (QUOTE |SPADreplace|) (QUOTE |SPADfirst|))

(DEFUN |ILIST;first;$S;4| (|x| |$|) (|SPADfirst| |x|))

(PUT
  (QUOTE |ILIST;elt;$firstS;5|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM (|x| "first") (|SPADfirst| |x|))))

(DEFUN |ILIST;elt;$firstS;5| (|x| G101995 |$|) (|SPADfirst| |x|))

(PUT (QUOTE |ILIST;empty;$;6|) (QUOTE |SPADreplace|) (QUOTE (XLAM NIL NIL)))

```

```

(DEFUN |ILIST;empty;$;6| (|$|) NIL)

(PUT (QUOTE |ILIST;empty?;$B;7|) (QUOTE |SPADreplace|) (QUOTE NULL))

(DEFUN |ILIST;empty?;$B;7| (|x| |$|) (NULL |x|))

(PUT (QUOTE |ILIST;rest;2$;8|) (QUOTE |SPADreplace|) (QUOTE CDR))

(DEFUN |ILIST;rest;2$;8| (|x| |$|) (CDR |x|))

(PUT
  (QUOTE |ILIST;elt;$rest$;9|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM (|x| "rest") (CDR |x|))))

(DEFUN |ILIST;elt;$rest$;9| (|x| G102000 |$|) (CDR |x|))

(DEFUN |ILIST;setfirst!;$2S;10| (|x| |s| |$|)
  (COND
    ((SPADCALL |x| (QREFELT |$| 17)) (|error| "Cannot update an empty list"))
    ((QUOTE T) (QCAR (RPLACA |x| |s|)))))

(DEFUN |ILIST;setelt;$first2S;11| (|x| G102005 |s| |$|)
  (COND
    ((SPADCALL |x| (QREFELT |$| 17)) (|error| "Cannot update an empty list"))
    ((QUOTE T) (QCAR (RPLACA |x| |s|)))))

(DEFUN |ILIST;setrest!;$3$;12| (|x| |y| |$|)
  (COND
    ((SPADCALL |x| (QREFELT |$| 17)) (|error| "Cannot update an empty list"))
    ((QUOTE T) (QCDR (RPLACD |x| |y|)))))

(DEFUN |ILIST;setelt;$rest2$;13| (|x| G102010 |y| |$|)
  (COND
    ((SPADCALL |x| (QREFELT |$| 17)) (|error| "Cannot update an empty list"))
    ((QUOTE T) (QCDR (RPLACD |x| |y|)))))

(PUT
  (QUOTE |ILIST;construct;L$;14|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM (|l|) |l|)))

(DEFUN |ILIST;construct;L$;14| (|l| |$|) |l|)

(PUT
  (QUOTE |ILIST;parts;$L;15|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM (|s|) |s|)))

```



```

(DEFUN |ILIST;parts;$L;15| (|s| |$|) |s|)

(PUT (QUOTE |ILIST;reverse!;2$;16|) (QUOTE |SPADreplace|) (QUOTE NREVERSE))

(DEFUN |ILIST;reverse!;2$;16| (|x| |$|) (NREVERSE |x|))

(PUT (QUOTE |ILIST;reverse;2$;17|) (QUOTE |SPADreplace|) (QUOTE REVERSE))

(DEFUN |ILIST;reverse;2$;17| (|x| |$|) (REVERSE |x|))

(DEFUN |ILIST;minIndex;$I;18| (|x| |$|) (QREFELT |$| 7))

(DEFUN |ILIST;rest;$Nni$;19| (|x| |n| |$|)
  (PROG (|i|)
    (RETURN
      (SEQ
        (SEQ
          (LETT |i| 1 |ILIST;rest;$Nni$;19|)
          G190
          (COND ((QSGREATERP |i| |n|) (GO G191)))
          (SEQ
            (COND ((NULL |x|) (|error| "index out of range")))
            (EXIT (LETT |x| (QCDR |x|) |ILIST;rest;$Nni$;19|)))
          (LETT |i| (QSADD1 |i|) |ILIST;rest;$Nni$;19|)
          (GO G190)
          G191
          (EXIT NIL))
          (EXIT |x|))))))

(DEFUN |ILIST;copy;2$;20| (|x| |$|)
  (PROG (|i| |y|)
    (RETURN
      (SEQ
        (LETT |y| (SPADCALL (QREFELT |$| 16)) |ILIST;copy;2$;20|)
        (SEQ
          (LETT |i| 0 |ILIST;copy;2$;20|)
          G190
          (COND
            ((NULL (COND ((NULL |x|) (QUOTE NIL)) ((QUOTE T) (QUOTE T)))) (GO G191)))
          (SEQ
            (COND
              ((EQ |i| 1000)
                (COND ((SPADCALL |x| (QREFELT |$| 33)) (|error| "cyclic list"))))
              (LETT |y| (CONS (QCAR |x|) |y|) |ILIST;copy;2$;20|)
              (EXIT (LETT |x| (QCDR |x|) |ILIST;copy;2$;20|)))
            (LETT |i| (QSADD1 |i|) |ILIST;copy;2$;20|)
            (GO G190)
            G191
            (EXIT NIL))
            (EXIT (NREVERSE |y|))))))

```

```

(DEFUN |ILIST;coerce;$0f;21| (|x| |$|)
  (PROG (|s| |y| |z|)
    (RETURN
      (SEQ
        (LETT |y| NIL |ILIST;coerce;$0f;21|)
        (LETT |s| (SPADCALL |x| (QREFELT |$| 35)) |ILIST;coerce;$0f;21|)
        (SEQ
          G190
          (COND ((NULL (NEQ |x| |s|)) (GO G191)))
          (SEQ
            (LETT |y|
              (CONS (SPADCALL (SPADCALL |x| (QREFELT |$| 13)) (QREFELT |$| 37)) |y|)
              |ILIST;coerce;$0f;21|)
            (EXIT (LETT |x| (SPADCALL |x| (QREFELT |$| 18)) |ILIST;coerce;$0f;21|)))
          NIL
          (GO G190)
          G191
          (EXIT NIL))
        (LETT |y| (NREVERSE |y|) |ILIST;coerce;$0f;21|)
        (EXIT
          (COND
            ((SPADCALL |s| (QREFELT |$| 17)) (SPADCALL |y| (QREFELT |$| 39)))
            ((QUOTE T)
              (SEQ
                (LETT |z|
                  (SPADCALL
                    (SPADCALL (SPADCALL |x| (QREFELT |$| 13)) (QREFELT |$| 37))
                    (QREFELT |$| 41))
                  |ILIST;coerce;$0f;21|)
                (SEQ
                  G190
                  (COND ((NULL (NEQ |s| (SPADCALL |x| (QREFELT |$| 18)))) (GO G191)))
                  (SEQ
                    (LETT |x| (SPADCALL |x| (QREFELT |$| 18)) |ILIST;coerce;$0f;21|)
                    (EXIT
                      (LETT |z|
                        (CONS
                          (SPADCALL (SPADCALL |x| (QREFELT |$| 13)) (QREFELT |$| 37))
                          |z|)
                        |ILIST;coerce;$0f;21|)))
                    NIL
                    (GO G190)
                    G191
                    (EXIT NIL))
                  (EXIT
                    (SPADCALL
                      (SPADCALL |y|
                        (SPADCALL
                          (SPADCALL

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(NREVERSE |z|)
(QREFELT |$| 42))
(QREFELT |$| 43))
(QREFELT |$| 44))
(QREFELT |$| 39)))))))))

(DEFUN |ILIST;=;2$B;22| (|x| |y| $)
  (PROG (G102042)
    (RETURN
      (SEQ (EXIT (COND
        ((EQ |x| |y|) 'T)
        ('T
          (SEQ (SEQ G190
            (COND
              ((NULL (COND
                ((OR (NULL |x|) (NULL |y|))
                'NIL)
                ('T 'T)))
              (GO G191)))
            (SEQ (EXIT
              (COND
                ((NULL
                  (SPADCALL (QCAR |x|) (QCAR |y|)
                    (QREFELT $ 46)))
                  (PROGN
                    (LETT G102042 'NIL
                      |ILIST;=;2$B;22|)
                    (GO G102042)))
                ('T
                  (SEQ
                    (LETT |x| (QCDR |x|)
                      |ILIST;=;2$B;22|)
                    (EXIT
                      (LETT |y| (QCDR |y|)
                        |ILIST;=;2$B;22|))))))
              NIL (GO G190) G191 (EXIT NIL))
            (EXIT (COND
              ((NULL |x|) (NULL |y|))
              ('T 'NIL))))))
          G102042 (EXIT G102042))))))

(DEFUN |ILIST;latex;$S;23| (|x| |$|)
  (PROG (|s|)
    (RETURN
      (SEQ
        (LETT |s| "\\left[" |ILIST;latex;$S;23|)
        (SEQ
          G190
          (COND
            ((NULL (COND ((NULL |x|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))

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(GO G191)))
(SEQ
  (LETT |s|
    (STRCONC |s| (SPADCALL (QCAR |x|) (QREFELT |$| 49)))
    |ILIST;latex;$S;23|)
  (LETT |x| (QCDR |x|) |ILIST;latex;$S;23|)
  (EXIT
    (COND
      ((NULL (NULL |x|))
        (LETT |s| (STRCONC |s| ", " ) |ILIST;latex;$S;23|))))))
NIL
(GO G190)
G191
(EXIT NIL))
(EXIT (STRCONC |s| " \\right|")))))))

(DEFUN |ILIST;member?;$B;24| (|s| |x| $)
  (PROG (G102052)
    (RETURN
      (SEQ (EXIT (SEQ (SEQ G190
        (COND
          ((NULL (COND ((NULL |x|) 'NIL) ('T 'T)))
            (GO G191)))
        (SEQ (EXIT (COND
          ((SPADCALL |s| (QCAR |x|)
            (QREFELT $ 46))
          (PROGN
            (LETT G102052 'T
              |ILIST;member?;$B;24|)
            (GO G102052)))
          ('T
            (LETT |x| (QCDR |x|)
              |ILIST;member?;$B;24|))))))
        NIL (GO G190) G191 (EXIT NIL))
      (EXIT 'NIL)))
    G102052 (EXIT G102052))))))

(DEFUN |ILIST;concat!;3$;25| (|x| |y| |$|)
  (PROG (|z|)
    (RETURN
      (SEQ
        (COND
          ((NULL |x|)
            (COND
              ((NULL |y|) |x|)
              ((QUOTE T)
                (SEQ
                  (PUSH (SPADCALL |y| (QREFELT |$| 13)) |x|)
                  (QRPLACD |x| (SPADCALL |y| (QREFELT |$| 18))) (EXIT |x|))))))
          ((QUOTE T)
            (PUSH (SPADCALL |y| (QREFELT |$| 13)) |x|)
            (QRPLACD |x| (SPADCALL |y| (QREFELT |$| 18))) (EXIT |x|))))))
    (QUOTE T)
    (EXIT 'NIL)))

```

```

(SEQ
  (LETT |z| |x| |ILIST;concat!;3$;25|)
  (SEQ
    G190
    (COND
      ((NULL (COND ((NULL (QCDR |z|)) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
      (GO G191)))
    (SEQ (EXIT (LETT |z| (QCDR |z|) |ILIST;concat!;3$;25|)))
    NIL
    (GO G190)
    G191
    (EXIT NIL))
  (QRPLACD |z| |y|)
  (EXIT |x|))))))

(DEFUN |ILIST;removeDuplicates!;2$;26| (|l| |$|)
  (PROG (|f| |p| |pr| |pp|)
    (RETURN
      (SEQ
        (LETT |p| |l| |ILIST;removeDuplicates!;2$;26|)
        (SEQ
          G190
          (COND
            ((NULL (COND ((NULL |p|) (QUOTE NIL)) ((QUOTE T) (QUOTE T)))) (GO G191)))
          (SEQ
            (LETT |pp| |p| |ILIST;removeDuplicates!;2$;26|)
            (LETT |f| (QCAR |p|) |ILIST;removeDuplicates!;2$;26|)
            (LETT |p| (QCDR |p|) |ILIST;removeDuplicates!;2$;26|)
            (EXIT
              (SEQ
                G190
                (COND
                  ((NULL
                    (COND
                      ((NULL (LETT |pr| (QCDR |pp|) |ILIST;removeDuplicates!;2$;26|)
                        (QUOTE NIL))
                      ((QUOTE T) (QUOTE T))))
                    (GO G191)))
                (SEQ
                  (EXIT
                    (COND
                      ((SPADCALL (QCAR |pr|) |f| (QREFELT |$| 46))
                       (QRPLACD |pp| (QCDR |pr|)))
                      ((QUOTE T) (LETT |pp| |pr| |ILIST;removeDuplicates!;2$;26|))))
                  NIL
                  (GO G190)
                  G191
                  (EXIT NIL))))
              NIL
              (GO G190)
              G190
              (EXIT NIL))))))
  )

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```

G191
  (EXIT NIL))
(EXIT |1|))))))

(DEFUN |ILIST;sort!;M2$;27| (|f| |l| |$|)
  (|ILIST;mergeSort| |f| |l| (SPADCALL |l| (QREFELT |$| 9)) |$|))

(DEFUN |ILIST;merge!;M3$;28| (|f| |p| |q| |$|)
  (PROG (|r| |t|)
    (RETURN
      (SEQ
        (COND
          ((NULL |p|) |q|)
          ((NULL |q|) |p|)
          ((EQ |p| |q|) (|error| "cannot merge a list into itself"))
          ((QUOTE T)
            (SEQ
              (COND
                ((SPADCALL (QCAR |p|) (QCAR |q|) |f|)
                  (SEQ
                    (LETT |r| (LETT |t| |p| |ILIST;merge!;M3$;28|) |ILIST;merge!;M3$;28|)
                    (EXIT (LETT |p| (QCDR |p|) |ILIST;merge!;M3$;28|))))
                ((QUOTE T)
                  (SEQ
                    (LETT |r| (LETT |t| |q| |ILIST;merge!;M3$;28|) |ILIST;merge!;M3$;28|)
                    (EXIT (LETT |q| (QCDR |q|) |ILIST;merge!;M3$;28|)))))))
            (SEQ
              G190
              (COND
                ((NULL
                  (COND
                    ((OR (NULL |p|) (NULL |q|)) (QUOTE NIL))
                    ((QUOTE T) (QUOTE T))))
                  (GO G191)))
              (SEQ
                (EXIT
                  (COND
                    ((SPADCALL (QCAR |p|) (QCAR |q|) |f|)
                      (SEQ
                        (QRPLACD |t| |p|)
                        (LETT |t| |p| |ILIST;merge!;M3$;28|)
                        (EXIT (LETT |p| (QCDR |p|) |ILIST;merge!;M3$;28|))))
                    ((QUOTE T)
                      (SEQ
                        (QRPLACD |t| |q|)
                        (LETT |t| |q| |ILIST;merge!;M3$;28|)
                        (EXIT (LETT |q| (QCDR |q|) |ILIST;merge!;M3$;28|)))))))
                  NIL
                  (GO G190)
                  G191

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```

(EXIT NIL))
(QRPLACD |t| (COND ((NULL |p|) |q|) ((QUOTE T) |p|)))
(EXIT |r|))))))

(DEFUN |ILIST;split!;$I$;29| (|p| |n| $)
  (PROG (G102085 |q|)
    (RETURN
      (SEQ (COND
        ((< |n| 1) (|error| "index out of range"))
        ('T
          (SEQ (LETT |p|
            (SPADCALL |p|
              (PROG1 (LETT G102085 (- |n| 1)
                |ILIST;split!;$I$;29|)
                (|check-subtype| (>= G102085 0)
                  '(|NonNegativeInteger|) G102085))
                (QREFELT $ 32))
                |ILIST;split!;$I$;29|)
              (LETT |q| (QCDR |p|) |ILIST;split!;$I$;29|)
              (QRPLACD |p| NIL) (EXIT |q|)))))))))

(DEFUN |ILIST;mergeSort| (|f| |p| |n| $)
  (PROG (G102089 |l| |q|)
    (RETURN
      (SEQ (COND
        ((EQL |n| 2)
          (COND
            ((SPADCALL
              (SPADCALL (SPADCALL |p| (QREFELT $ 18))
                (QREFELT $ 13))
              (SPADCALL |p| (QREFELT $ 13)) |f|)
              (LETT |p| (SPADCALL |p| (QREFELT $ 28))
                |ILIST;mergeSort|))))))
        (EXIT (COND
          ((< |n| 3) |p|)
          ('T
            (SEQ (LETT |l|
              (PROG1 (LETT G102089 (QUOTIENT2 |n| 2)
                |ILIST;mergeSort|)
                (|check-subtype| (>= G102089 0)
                  '(|NonNegativeInteger|) G102089))
                |ILIST;mergeSort|)
              (LETT |q| (SPADCALL |p| |l| (QREFELT $ 57))
                |ILIST;mergeSort|)
              (LETT |p| (|ILIST;mergeSort| |f| |p| |l| $)
                |ILIST;mergeSort|)
              (LETT |q|
                (|ILIST;mergeSort| |f| |q| (- |n| |l|)
                  $)
                |ILIST;mergeSort|)
            )
          )
        )
      )
    )
  )

```

```

(EXIT (SPADCALL |f| |p| |q| (QREFELT $ 56))))))))))

(DEFUN |IndexedList| (&REST G102103 &AUX G102101)
  (DSETQ G102101 G102103)
  (PROG ()
    (RETURN
      (PROG (G102102)
        (RETURN
          (COND
            ((LETT G102102
              (|lassocShiftWithFunction| (|devalueList| G102101)
              (HGET |$ConstructorCache| '|IndexedList|)
              '|domainEqualList|)
              |IndexedList|)
            (|CDRwithIncrement| G102102))
          ('T
            (UNWIND-PROTECT
              (PROG1 (APPLY (|function| |IndexedList|) G102101)
                (LETT G102102 T |IndexedList|))
              (COND
                ((NOT G102102)
                  (HREM |$ConstructorCache| '|IndexedList|))))))))))

(DEFUN |IndexedList;| (|#1| |#2|)
  (PROG (DV$1 DV$2 |dv$| $ G102100 |pv$|)
    (RETURN
      (PROGN
        (LETT DV$1 (|devalue| |#1|) |IndexedList|)
        (LETT DV$2 (|devalue| |#2|) |IndexedList|)
        (LETT |dv$| (LIST '|IndexedList| DV$1 DV$2) |IndexedList|)
        (LETT $ (make-array 71) |IndexedList|)
        (QSETREFV $ 0 |dv$|)
        (QSETREFV $ 3
          (LETT |pv$|
            (|buildPredVector| 0 0
              (LIST (|HasCategory| |#1| '|SetCategory|))
              (|HasCategory| |#1|
                '|ConvertibleTo| (|InputForm|)))
              (LETT G102100
                (|HasCategory| |#1| '|OrderedSet|)
                |IndexedList|)
              (OR G102100
                (|HasCategory| |#1| '|SetCategory|))
              (|HasCategory| (|Integer|) '|OrderedSet|))
              (AND (|HasCategory| |#1|
                (LIST '|Evalable|
                  (|devalue| |#1|)))
                (|HasCategory| |#1| '|SetCategory|)))
              (OR (AND (|HasCategory| |#1|
                (LIST '|Evalable|

```



```

(|devaluate| |#1|)))
G102100)
(AND (|HasCategory| |#1|
      (LIST '|Evalable|
            (|devaluate| |#1|)))
      (|HasCategory| |#1|
        '(|SetCategory|))))))
(|IndexedList|))
(|haddProp| |$ConstructorCache| '|IndexedList| (LIST DV$1 DV$2)
  (CONS 1 $))
(|stuffDomainSlots| $)
(QSETREFV $ 6 |#1|)
(QSETREFV $ 7 |#2|)
(COND
  ((|testBitVector| |pv$| 1)
   (PROGN
    (QSETREFV $ 45
      (CONS (|dispatchFunction| |ILIST;coerce;$0f;21|) $))
    (QSETREFV $ 47
      (CONS (|dispatchFunction| |ILIST;=;2$B;22|) $))
    (QSETREFV $ 50
      (CONS (|dispatchFunction| |ILIST;latex;$S;23|) $))
    (QSETREFV $ 51
      (CONS (|dispatchFunction| |ILIST;member?;$S$B;24|) $))))))
(COND
  ((|testBitVector| |pv$| 1)
   (QSETREFV $ 53
    (CONS (|dispatchFunction|
          |ILIST;removeDuplicates!;2$;26|)
          $))))
$))))

(setf (get
  (QUOTE |IndexedList|)
  (QUOTE |infovec|))
  (LIST
    (QUOTE #(
      NIL NIL NIL NIL NIL NIL (|local| |#1|) (|local| |#2|)
      (|NonNegativeInteger|) |ILIST;#;$Nni;1| |ILIST;concat;$2$;2| (|Boolean|)
      |ILIST;eq?;2$B;3| |ILIST;first;$S;4| (QUOTE "first") |ILIST;elt;$first$;5|
      |ILIST;empty;$;6| |ILIST;empty?;$B;7| |ILIST;rest;2$;8| (QUOTE "rest")
      |ILIST;elt;$rest$;9| |ILIST;setfirst!;$2S;10| |ILIST;setelt;$first2$;11|
      |ILIST;setrest!;$3$;12| |ILIST;setelt;$rest2$;13| (|List| 6)
      |ILIST;construct;$L;14| |ILIST;parts;$L;15| |ILIST;reverse!;$2$;16|
      |ILIST;reverse;2$;17| (|Integer|) |ILIST;minIndex;$I;18|
      |ILIST;rest;$Nni$;19| (0 . |cyclic?|) |ILIST;copy;2$;20|
      (5 . |cycleEntry|) (|OutputForm|) (10 . |coerce|) (|List| |$|)
      (15 . |bracket|) (|List| 36) (20 . |list|) (25 . |commaSeparate|)
      (30 . |overbar|) (35 . |concat!|) (41 . |coerce|) (46 . |=|) (52 . |=|)
      (|String|) (58 . |latex|) (63 . |latex|) (68 . |member?|)
    ))
  )

```

```

(|ILIST;concat!;3$;25| (74 . |removeDuplicates!|) (|Mapping| 11 6 6)
|ILIST;sort!;M2$;27| |ILIST;merge!;M3$;28| |ILIST;split!;$I$;29|
(|Mapping| 6 6 6) (|Equation| 6) (|List| 59) (|Mapping| 11 6) (|Void|)
(|UniversalSegment| 30) (QUOTE "last") (QUOTE "value") (|Mapping| 6 6)
(|InputForm|) (|SingleInteger|) (|List| 30) (|Union| 6 (QUOTE "failed"))))
(QUOTE #(
|~=| 79 |value| 85 |third| 90 |tail| 95 |swap!| 100 |split!| 107
|sorted?| 113 |sort!| 124 |sort| 135 |size?| 146 |setvalue!| 152
|setrest!| 158 |setlast!| 164 |setfirst!| 170 |setelt| 176
|setchildren!| 218 |select!| 224 |select| 230 |second| 236 |sample|
241 |reverse!| 245 |reverse| 250 |rest| 255 |removeDuplicates!|
266 |removeDuplicates| 271 |remove!| 276 |remove| 288 |reduce|
300 |qsetelt!| 321 |qelt| 328 |possiblyInfinite?| 334 |position|
339 |parts| 358 |nodes| 363 |node?| 368 |new| 374 |more?| 380
|minIndex| 386 |min| 391 |merge!| 397 |merge| 410 |members| 423
|member?| 428 |maxIndex| 434 |max| 439 |map!| 445 |map| 451 |list|
464 |less?| 469 |leaves| 475 |leaf?| 480 |latex| 485 |last| 490
|insert!| 501 |insert| 515 |indices| 529 |index?| 534 |hash| 540
|first| 545 |find| 556 |fill!| 562 |explicitlyFinite?| 568 |every?|
573 |eval| 579 |eq?| 605 |entry?| 611 |entries| 617 |empty?| 622
|empty| 627 |elt| 631 |distance| 674 |delete!| 680 |delete| 692
|cyclic?| 704 |cycleTail| 709 |cycleSplit!| 714 |cycleLength| 719
|cycleEntry| 724 |count| 729 |copyInto!| 741 |copy| 748 |convert|
753 |construct| 758 |concat!| 763 |concat| 775 |coerce| 798
|children| 803 |child?| 808 |any?| 814 |>=| 820 |>| 826 |=| 832
|<=| 838 |<| 844 |#| 850))
(QUOTE ((|shallowlyMutable| . 0) (|finiteAggregate| . 0)))
(CONS
(|makeByteWordVec2| 7 (QUOTE (0 0 0 0 0 0 0 0 0 0 3 0 0 7 4 0 0 7 1 2 4)))
(CONS
(QUOTE #(|ListAggregate&| |StreamAggregate&| |ExtensibleLinearAggregate&|
|FiniteLinearAggregate&| |UnaryRecursiveAggregate&| |LinearAggregate&|
|RecursiveAggregate&| |IndexedAggregate&| |Collection&|
|HomogeneousAggregate&| |OrderedSet&| |Aggregate&| |EltableAggregate&|
|Evalable&| |SetCategory&| NIL NIL |InnerEvalable&| NIL NIL
|BasicType&|))
(CONS
(QUOTE #(
(|ListAggregate| 6) (|StreamAggregate| 6)
(|ExtensibleLinearAggregate| 6) (|FiniteLinearAggregate| 6)
(|UnaryRecursiveAggregate| 6) (|LinearAggregate| 6)
(|RecursiveAggregate| 6) (|IndexedAggregate| 30 6)
(|Collection| 6) (|HomogeneousAggregate| 6) (|OrderedSet|)
(|Aggregate|) (|EltableAggregate| 30 6) (|Evalable| 6) (|SetCategory|)
(|Type|) (|Eltable| 30 6) (|InnerEvalable| 6 6) (|CoercibleTo| 36)
(|ConvertibleTo| 67) (|BasicType|)))
(|makeByteWordVec2| 70
(QUOTE (1 0 11 0 33 1 0 0 0 35 1 6 36 0 37 1 36 0 38 39 1 40 0 36
41 1 36 0 38 42 1 36 0 0 43 2 40 0 0 36 44 1 0 36 0 45 2 6 11 0 0
46 2 0 11 0 0 47 1 6 48 0 49 1 0 48 0 50 2 0 11 6 0 51 1 0 0 0 53

```

```

2 1 11 0 0 1 1 0 6 0 1 1 0 6 0 1 1 0 0 0 1 3 0 62 0 30 30 1 2 0 0
0 30 57 1 3 11 0 1 2 0 11 54 0 1 1 3 0 0 1 2 0 0 54 0 55 1 3 0 0 1
2 0 0 54 0 1 2 0 11 0 8 1 2 0 6 0 6 1 2 0 0 0 0 23 2 0 6 0 6 1 2 0
6 0 6 21 3 0 6 0 30 6 1 3 0 6 0 63 6 1 3 0 6 0 64 6 1 3 0 0 0 19 0
24 3 0 6 0 14 6 22 3 0 6 0 65 6 1 2 0 0 0 38 1 2 0 0 61 0 1 2 0 0
61 0 1 1 0 6 0 1 0 0 0 1 1 0 0 0 28 1 0 0 0 29 2 0 0 0 8 32 1 0 0
0 18 1 1 0 0 53 1 1 0 0 1 2 1 0 6 0 1 2 0 0 61 0 1 2 1 0 6 0 1 2 0
0 61 0 1 4 1 6 58 0 6 6 1 2 0 6 58 0 1 3 0 6 58 0 6 1 3 0 6 0 30 6
1 2 0 6 0 30 1 1 0 11 0 1 2 1 30 6 0 1 3 1 30 6 0 30 1 2 0 30 61 0
1 1 0 25 0 27 1 0 38 0 1 2 1 11 0 0 1 2 0 0 8 6 1 2 0 11 0 8 1 1 5
30 0 31 2 3 0 0 0 1 2 3 0 0 0 1 3 0 0 54 0 0 56 2 3 0 0 0 1 3 0 0
54 0 0 1 1 0 25 0 1 2 1 11 6 0 51 1 5 30 0 1 2 3 0 0 0 1 2 0 0 66
0 1 3 0 0 58 0 0 1 2 0 0 66 0 1 1 0 0 6 1 2 0 11 0 8 1 1 0 25 0 1
1 0 11 0 1 1 1 48 0 50 2 0 0 0 8 1 1 0 6 0 1 3 0 0 6 0 30 1 3 0 0
0 0 30 1 3 0 0 0 0 30 1 3 0 0 6 0 30 1 1 0 69 0 1 2 0 11 30 0 1 1
1 68 0 1 2 0 0 0 8 1 1 0 6 0 13 2 0 70 61 0 1 2 0 0 0 6 1 1 0 11
0 1 2 0 11 61 0 1 3 6 0 0 6 6 1 3 6 0 0 25 25 1 2 6 0 0 59 1 2 6
0 0 60 1 2 0 11 0 0 12 2 1 11 6 0 1 1 0 25 0 1 1 0 11 0 17 0 0 0
16 2 0 6 0 30 1 3 0 6 0 30 6 1 2 0 0 0 63 1 2 0 6 0 64 1 2 0 0 0
19 20 2 0 6 0 14 15 2 0 6 0 65 1 2 0 30 0 0 1 2 0 0 0 63 1 2 0 0 0
30 1 2 0 0 0 63 1 2 0 0 0 30 1 1 0 11 0 33 1 0 0 0 1 1 0 0 0 1 1 0
8 0 1 1 0 0 0 35 2 1 8 6 0 1 2 0 8 61 0 1 3 0 0 0 0 30 1 1 0 0 0
34 1 2 67 0 1 1 0 0 25 26 2 0 0 0 0 52 2 0 0 0 6 1 1 0 0 38 1 2 0
0 0 6 1 2 0 0 6 0 10 2 0 0 0 0 1 1 1 36 0 45 1 0 38 0 1 2 1 11 0
0 1 2 0 11 61 0 1 2 3 11 0 0 1 2 3 11 0 0 1 2 1 11 0 0 47 2 3 11
0 0 1 2 3 11 0 0 1 1 0 8 0 9))))))
(QUOTE |lookupComplete|)))

```

## 28.5 INT.lsp BOOTSTRAP

**INT** depends on **OINTDOM** which depends on **ORDRING** which depends on **INT**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **INT** category which we can write into the **MID** directory. We compile the lisp code and copy the **INT.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

### — INT.lsp BOOTSTRAP —

```

(|/VERSIONCHECK| 2)

(DEFUN |INT;writeOMInt| (|dev| |x| |$|)
  (SEQ
    (COND

```

```

((|<| |x| 0)
 (SEQ
  (SPADCALL |dev| (QREFELT |$| 8))
  (SPADCALL |dev| "arith1" "unary_minus" (QREFELT |$| 10))
  (SPADCALL |dev| (|-| |x|) (QREFELT |$| 12))
  (EXIT (SPADCALL |dev| (QREFELT |$| 13))))))
((QUOTE T) (SPADCALL |dev| |x| (QREFELT |$| 12))))))

(DEFUN |INT;OMwrite;$S;2| (|x| |$|)
 (PROG (|sp| |dev| |s|)
 (RETURN
  (SEQ
   (LETT |s| "" |INT;OMwrite;$S;2|)
   (LETT |sp| (|OM-STRINGTOSTRINGPTR| |s|) |INT;OMwrite;$S;2|)
   (LETT |dev|
    (SPADCALL |sp| (SPADCALL (QREFELT |$| 15)) (QREFELT |$| 16))
    |INT;OMwrite;$S;2|)
    (SPADCALL |dev| (QREFELT |$| 17))
    (|INT;writeOMInt| |dev| |x| |$|)
    (SPADCALL |dev| (QREFELT |$| 18))
    (SPADCALL |dev| (QREFELT |$| 19))
    (LETT |s| (|OM-STRINGPTRTOSTRING| |sp|) |INT;OMwrite;$S;2|)
    (EXIT |s|))))))

(DEFUN |INT;OMwrite;$BS;3| (|x| |wholeObj| |$|)
 (PROG (|sp| |dev| |s|)
 (RETURN
  (SEQ
   (LETT |s| "" |INT;OMwrite;$BS;3|)
   (LETT |sp| (|OM-STRINGTOSTRINGPTR| |s|) |INT;OMwrite;$BS;3|)
   (LETT |dev|
    (SPADCALL |sp| (SPADCALL (QREFELT |$| 15)) (QREFELT |$| 16))
    |INT;OMwrite;$BS;3|)
    (COND (|wholeObj| (SPADCALL |dev| (QREFELT |$| 17))))
    (|INT;writeOMInt| |dev| |x| |$|)
    (COND (|wholeObj| (SPADCALL |dev| (QREFELT |$| 18))))
    (SPADCALL |dev| (QREFELT |$| 19))
    (LETT |s| (|OM-STRINGPTRTOSTRING| |sp|) |INT;OMwrite;$BS;3|)
    (EXIT |s|))))))

(DEFUN |INT;OMwrite;Omd$V;4| (|dev| |x| |$|)
 (SEQ
  (SPADCALL |dev| (QREFELT |$| 17))
  (|INT;writeOMInt| |dev| |x| |$|)
  (EXIT (SPADCALL |dev| (QREFELT |$| 18))))))

(DEFUN |INT;OMwrite;Omd$BV;5| (|dev| |x| |wholeObj| |$|)
 (SEQ
  (COND (|wholeObj| (SPADCALL |dev| (QREFELT |$| 17))))
  (|INT;writeOMInt| |dev| |x| |$|)

```

```

(EXIT (COND (|wholeObj| (SPADCALL |dev| (QREFELT |$| 18))))))

(PUT (QUOTE |INT;zero?;$B;6|) (QUOTE |SPADreplace|) (QUOTE ZEROP))

(DEFUN |INT;zero?;$B;6| (|x| |$|) (ZEROP |x|))

(PUT (QUOTE |INT;Zero;$;7|) (QUOTE |SPADreplace|) (QUOTE (XLAM NIL 0)))

(DEFUN |INT;Zero;$;7| (|x|) 0)

(PUT (QUOTE |INT;One;$;8|) (QUOTE |SPADreplace|) (QUOTE (XLAM NIL 1)))

(DEFUN |INT;One;$;8| (|x|) 1)

(PUT (QUOTE |INT;base;$;9|) (QUOTE |SPADreplace|) (QUOTE (XLAM NIL 2)))

(DEFUN |INT;base;$;9| (|x|) 2)

(PUT (QUOTE |INT;copy;2$;10|) (QUOTE |SPADreplace|) (QUOTE (XLAM (|x|) |x|)))

(DEFUN |INT;copy;2$;10| (|x| |$|) |x|)

(PUT
  (QUOTE |INT;inc;2$;11|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM (|x|) (|+| |x| 1))))

(DEFUN |INT;inc;2$;11| (|x| |$|) (|+| |x| 1))

(PUT
  (QUOTE |INT;dec;2$;12|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM (|x|) (|-| |x| 1))))

(DEFUN |INT;dec;2$;12| (|x| |$|) (|-| |x| 1))

(PUT (QUOTE |INT;hash;2$;13|) (QUOTE |SPADreplace|) (QUOTE SXHASH))

(DEFUN |INT;hash;2$;13| (|x| |$|) (SXHASH |x|))

(PUT (QUOTE |INT;negative?;$B;14|) (QUOTE |SPADreplace|) (QUOTE MINUSP))

(DEFUN |INT;negative?;$B;14| (|x| |$|) (MINUSP |x|))

(DEFUN |INT;coerce;$Of;15| (|x| |$|) (SPADCALL |x| (QREFELT |$| 35)))

(PUT
  (QUOTE |INT;coerce;2$;16|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM (|m|) |m|)))

```

```

(DEFUN |INT;coerce;2$;16| (|m| |$|) |m|)

(PUT
  (QUOTE |INT;convert;2$;17|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM (|x|) |x|)))

(DEFUN |INT;convert;2$;17| (|x| |$|) |x|)

(PUT
  (QUOTE |INT;length;2$;18|)
  (QUOTE |SPADreplace|)
  (QUOTE |INTEGER-LENGTH|))

(DEFUN |INT;length;2$;18| (|a| |$|) (|INTEGER-LENGTH| |a|))

(DEFUN |INT;addmod;4$;19| (|a| |b| |p| $)
  (PROG (|c| G86338)
    (RETURN
      (SEQ (EXIT (SEQ (SEQ (LETT |c| (+ |a| |b|) |INT;addmod;4$;19|)
        (EXIT (COND
          ((NULL (< |c| |p|))
            (PROGN
              (LETT G86338 (- |c| |p|)
                |INT;addmod;4$;19|)
              (GO G86338))))))
          (EXIT |c|)))
        G86338 (EXIT G86338))))))

(DEFUN |INT;submod;4$;20| (|a| |b| |p| |$|)
  (PROG (|c|)
    (RETURN
      (SEQ
        (LETT |c| (|-| |a| |b|) |INT;submod;4$;20|)
        (EXIT (COND ((|c| 0) (|+| |c| |p|)) ((QUOTE T) |c|)))))))

(DEFUN |INT;mulmod;4$;21| (|a| |b| |p| |$|) (REMAINDER2 (|*| |a| |b|) |p|))

(DEFUN |INT;convert;$F;22| (|x| |$|) (SPADCALL |x| (QREFELT |$| 44)))

(PUT
  (QUOTE |INT;convert;$Df;23|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM (|x|) (FLOAT |x| |MOST-POSITIVE-LONG-FLOAT|))))

(DEFUN |INT;convert;$Df;23| (|x| |$|) (FLOAT |x| |MOST-POSITIVE-LONG-FLOAT|))

(DEFUN |INT;convert;$If;24| (|x| |$|) (SPADCALL |x| (QREFELT |$| 49)))

```

```

(PUT (QUOTE |INT;convert;$S;25|) (QUOTE |SPADreplace|) (QUOTE STRINGIMAGE))

(DEFUN |INT;convert;$S;25| (|x| |$|) (STRINGIMAGE |x|))

(DEFUN |INT;latex;$S;26| (|x| |$|)
  (PROG (|s|)
    (RETURN
      (SEQ
        (LETT |s| (STRINGIMAGE |x|) |INT;latex;$S;26|)
        (COND ((|<| -1 |x|) (COND ((|<| |x| 10) (EXIT |s|))))))
      (EXIT (STRCONC "{" (STRCONC |s| "}")")))))

(DEFUN |INT;positiveRemainder;3$;27| (|a| |b| |$|)
  (PROG (|r|)
    (RETURN
      (COND
        ((MINUSP (LETT |r| (REMAINDER2 |a| |b|) |INT;positiveRemainder;3$;27|))
          (COND
            ((MINUSP |b|) (|-| |r| |b|))
            ((QUOTE T) (|+| |r| |b|))))
        ((QUOTE T) |r|))))))

(PUT
  (QUOTE |INT;reducedSystem;2M;28|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM (|m|) |m|)))

(DEFUN |INT;reducedSystem;2M;28| (|m| |$|) |m|)

(DEFUN |INT;reducedSystem;MVR;29| (|m| |v| |$|) (CONS |m| (QUOTE |vec|)))

(PUT (QUOTE |INT;abs;2$;30|) (QUOTE |SPADreplace|) (QUOTE ABS))

(DEFUN |INT;abs;2$;30| (|x| |$|) (ABS |x|))

(PUT (QUOTE |INT;random;$;31|) (QUOTE |SPADreplace|) (QUOTE |random|))

(DEFUN |INT;random;$;31| (|$|) (|random|))

(PUT (QUOTE |INT;random;2$;32|) (QUOTE |SPADreplace|) (QUOTE RANDOM))

(DEFUN |INT;random;2$;32| (|x| |$|) (RANDOM |x|))

(PUT (QUOTE |INT;=;2$B;33|) (QUOTE |SPADreplace|) (QUOTE EQL))

(DEFUN |INT;=;2$B;33| (|x| |y| |$|) (EQL |x| |y|))

(PUT (QUOTE |INT;<;2$B;34|) (QUOTE |SPADreplace|) (QUOTE |<|))

(DEFUN |INT;<;2$B;34| (|x| |y| |$|) (|<| |x| |y|))

```

```

(PUT (QUOTE |INT;-;2$;35|) (QUOTE |SPADreplace|) (QUOTE |-|))

(DEFUN |INT;-;2$;35| (|x| |$|) (|-| |x|))

(PUT (QUOTE |INT;+;3$;36|) (QUOTE |SPADreplace|) (QUOTE |+|))

(DEFUN |INT;+;3$;36| (|x| |y| |$|) (|+| |x| |y|))

(PUT (QUOTE |INT;-;3$;37|) (QUOTE |SPADreplace|) (QUOTE |-|))

(DEFUN |INT;-;3$;37| (|x| |y| |$|) (|-| |x| |y|))

(PUT (QUOTE |INT;*;3$;38|) (QUOTE |SPADreplace|) (QUOTE |*|))

(DEFUN |INT;*;3$;38| (|x| |y| |$|) (|*| |x| |y|))

(PUT (QUOTE |INT;*;3$;39|) (QUOTE |SPADreplace|) (QUOTE |*|))

(DEFUN |INT;*;3$;39| (|m| |y| |$|) (|*| |m| |y|))

(PUT (QUOTE |INT;**;$Nni$;40|) (QUOTE |SPADreplace|) (QUOTE EXPT))

(DEFUN |INT;**;$Nni$;40| (|x| |n| |$|) (EXPT |x| |n|))

(PUT (QUOTE |INT;odd?;$B;41|) (QUOTE |SPADreplace|) (QUOTE ODDP))

(DEFUN |INT;odd?;$B;41| (|x| |$|) (ODDP |x|))

(PUT (QUOTE |INT;max;3$;42|) (QUOTE |SPADreplace|) (QUOTE MAX))

(DEFUN |INT;max;3$;42| (|x| |y| |$|) (MAX |x| |y|))

(PUT (QUOTE |INT;min;3$;43|) (QUOTE |SPADreplace|) (QUOTE MIN))

(DEFUN |INT;min;3$;43| (|x| |y| |$|) (MIN |x| |y|))

(PUT (QUOTE |INT;divide;2$R;44|) (QUOTE |SPADreplace|) (QUOTE DIVIDE2))

(DEFUN |INT;divide;2$R;44| (|x| |y| |$|) (DIVIDE2 |x| |y|))

(PUT (QUOTE |INT;quo;3$;45|) (QUOTE |SPADreplace|) (QUOTE QUOTIENT2))

(DEFUN |INT;quo;3$;45| (|x| |y| |$|) (QUOTIENT2 |x| |y|))

(PUT (QUOTE |INT;rem;3$;46|) (QUOTE |SPADreplace|) (QUOTE REMAINDER2))

(DEFUN |INT;rem;3$;46| (|x| |y| |$|) (REMAINDER2 |x| |y|))

(PUT (QUOTE |INT;shift;3$;47|) (QUOTE |SPADreplace|) (QUOTE ASH))

```



```

(DEFUN |INT;shift;3$;47| (|x| |y| |$|) (ASH |x| |y|))

(DEFUN |INT;exquo;2$U;48| (|x| |y| |$|)
  (COND
    ((OR (ZEROP |y|) (NULL (ZEROP (REMAINDER2 |x| |y|)))) (CONS 1 "failed"))
    ((QUOTE T) (CONS 0 (QUOTIENT2 |x| |y|)))))

(DEFUN |INT;recip;$U;49| (|x| |$|)
  (COND
    ((OR (EQL |x| 1) (EQL |x| -1)) (CONS 0 |x|))
    ((QUOTE T) (CONS 1 "failed"))))

(PUT (QUOTE |INT;gcd;3$;50|) (QUOTE |SPADreplace|) (QUOTE GCD))

(DEFUN |INT;gcd;3$;50| (|x| |y| |$|) (GCD |x| |y|))

(DEFUN |INT;unitNormal;$R;51| (|x| |$|)
  (COND
    ((|<| |x| 0) (VECTOR -1 (|-| |x|) -1))
    ((QUOTE T) (VECTOR 1 |x| 1))))

(PUT (QUOTE |INT;unitCanonical;2$;52|) (QUOTE |SPADreplace|) (QUOTE ABS))

(DEFUN |INT;unitCanonical;2$;52| (|x| |$|) (ABS |x|))

(DEFUN |INT;solveLinearPolynomialEquation| (|lp| |p| |$|)
  (SPADCALL |lp| |p| (QREFELT |$| 91)))

(DEFUN |INT;squareFreePolynomial| (|p| |$|) (SPADCALL |p| (QREFELT |$| 95)))

(DEFUN |INT;factorPolynomial| (|p| $)
  (PROG (|pp| G86409)
    (RETURN
      (SEQ (LETT |pp| (SPADCALL |p| (QREFELT $ 96))
              |INT;factorPolynomial|)
            (EXIT (COND
              ((EQL (SPADCALL |pp| (QREFELT $ 97))
                (SPADCALL |p| (QREFELT $ 97)))
              (SPADCALL |p| (QREFELT $ 99)))
            ('T
              (SPADCALL (SPADCALL |pp| (QREFELT $ 99))
                (SPADCALL (CONS #'|INT;factorPolynomial!0| $)
                  (SPADCALL
                    (PROG2 (LETT G86409
                      (SPADCALL
                        (SPADCALL |p| (QREFELT $ 97))
                        (SPADCALL |pp| (QREFELT $ 97))
                        (QREFELT $ 81))
                    |INT;factorPolynomial|)

```

```

(QCDR G86409)
(|check-union| (QEQCAR G86409 0) $
  G86409))
(QREFELT $ 102))
(QREFELT $ 106))
(QREFELT $ 108)))))))))

(DEFUN |INT;factorPolynomial!0| (|#1| |$|) (SPADCALL |#1| (QREFELT |$| 100)))

(DEFUN |INT;factorSquareFreePolynomial| (|p| |$|)
  (SPADCALL |p| (QREFELT |$| 109)))

(DEFUN |INT;gcdPolynomial;3Sup;57| (|p| |q| |$|)
  (COND
    ((SPADCALL |p| (QREFELT |$| 110)) (SPADCALL |q| (QREFELT |$| 111)))
    ((SPADCALL |q| (QREFELT |$| 110)) (SPADCALL |p| (QREFELT |$| 111)))
    ((QUOTE T) (SPADCALL (LIST |p| |q|) (QREFELT |$| 114)))))

(DEFUN |Integer| ()
  (PROG ()
    (RETURN
      (PROG (G86434)
        (RETURN
          (COND
            ((LETT G86434 (HGET |$ConstructorCache| '|Integer|)
              |Integer|)
              (|CDRwithIncrement| (CDAR G86434)))
            ('T
              (UNWIND-PROTECT
                (PROG1 (CDDAR (HPUT |$ConstructorCache| '|Integer|
                  (LIST
                    (CONS NIL (CONS 1 (|Integer|))))))
                  (LETT G86434 T |Integer|))
                (COND
                  ((NOT G86434) (HREM |$ConstructorCache| '|Integer|))))))))))

(DEFUN |Integer;| ()
  (PROG (|dv$| $ |pv$|)
    (RETURN
      (PROGN
        (LETT |dv$| '(|Integer|) |Integer|)
        (LETT $ (make-array 130) |Integer|)
        (QSETREFV $ 0 |dv$|)
        (QSETREFV $ 3
          (LETT |pv$| (|buildPredVector| 0 0 NIL) |Integer|))
        (|haddProp| |$ConstructorCache| '|Integer| NIL (CONS 1 $))
        (|stuffDomainSlots| $)
        (QSETREFV $ 69
          (QSETREFV $ 68 (CONS (|dispatchFunction| |INT;*;3$;39|) $)))
        $))))

```

```

(setf (get
  (QUOTE |Integer|)
  (QUOTE |infovec|))
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL NIL (|Void|) (|OpenMathDevice|) (0 . |OMputApp|)
        (|String|) (5 . |OMputSymbol|) (|Integer|) (12 . |OMputInteger|)
        (18 . |OMputEndApp|) (|OpenMathEncoding|) (23 . |OMencodingXML|)
        (27 . |OMopenString|) (33 . |OMputObject|) (38 . |OMputEndObject|)
        (43 . |OMclose|) |INT;OMwrite;$S;2| (|Boolean|) |INT;OMwrite;$BS;3|
        |INT;OMwrite;Omd$V;4| |INT;OMwrite;Omd$BV;5| |INT;zero?;$B;6|
        (CONS IDENTITY (FUNCALL (|dispatchFunction| |INT;Zero;$;7|) |$|))
        (CONS IDENTITY (FUNCALL (|dispatchFunction| |INT;One;$;8|) |$|))
        |INT;base;$;9| |INT;copy;2$;10| |INT;inc;2$;11| |INT;dec;2$;12|
        |INT;hash;2$;13| |INT;negative?;$B;14| (|OutputForm|)
        (48 . |outputForm|) |INT;coerce;$Of;15| |INT;coerce;2$;16|
        |INT;convert;2$;17| |INT;length;2$;18| |INT;addmod;4$;19|
        |INT;submod;4$;20| |INT;mulmod;4$;21| (|Float|) (53 . |coerce|)
        |INT;convert;$F;22| (|DoubleFloat|) |INT;convert;$Df;23| (|InputForm|)
        (58 . |convert|) |INT;convert;$If;24| |INT;convert;$S;25|
        |INT;latex;$S;26| |INT;positiveRemainder;3$;27| (|Matrix| 11)
        (|Matrix| |$|) |INT;reducedSystem;2M;28|
        (|Record| (|:| |mat| 54) (|:| |vec| (|Vector| 11)))
        (|Vector| |$|) |INT;reducedSystem;MVR;29| |INT;abs;2$;30|
        |INT;random;$;31| |INT;random;2$;32| |INT;=;2$B;33|
        |INT;<;2$B;34| |INT;-;2$;35| |INT;+;3$;36| |INT;-;3$;37| NIL NIL
        (|NonNegativeInteger|) |INT;*$;$Nni;$;40| |INT;odd?;$B;41|
        |INT;max;3$;42| |INT;min;3$;43|
        (|Record| (|:| |quotient| |$|) (|:| |remainder| |$|))
        |INT;divide;2$R;44| |INT;quo;3$;45| |INT;rem;3$;46| |INT;shift;3$;47|
        (|Union| |$| (QUOTE "failed")) |INT;exquo;2$U;48| |INT;recip;$U;49|
        |INT;gcd;3$;50|
        (|Record| (|:| |unit| |$|) (|:| |canonical| |$|) (|:| |associate| |$|))
        |INT;unitNormal;$R;51| |INT;unitCanonical;2$;52|
        (|Union| 88 (QUOTE "failed")) (|List| 89)
        (|SparseUnivariatePolynomial| 11)
        (|IntegerSolveLinearPolynomialEquation|)
        (63 . |solveLinearPolynomialEquation|) (|Factored| 93)
        (|SparseUnivariatePolynomial| |$$|)
        (|UnivariatePolynomialSquareFree| |$$| 93) (69 . |squareFree|)
        (74 . |primitivePart|) (79 . |leadingCoefficient|)
        (|GaloisGroupFactorizer| 93) (84 . |factor|) (89 . |coerce|)
        (|Factored| |$|) (94 . |factor|) (|Mapping| 93 |$$|)
        (|Factored| |$$|) (|FactoredFunctions2| |$$| 93) (99 . |map|)
        (111 . |factorSquareFree|) (116 . |zero?|) (121 . |unitCanonical|)
        (|List| 93) (|HeuGcd| 93) (126 . |gcd|)
        (|SparseUnivariatePolynomial| |$|) |INT;gcdPolynomial;3Sup;57|
        (|Union| 118 (QUOTE "failed")) (|Fraction| 11)

```

```

(|PatternMatchResult| 11 |$|) (|Pattern| 11)
(|Union| 11 (QUOTE "failed")) (|Union| 123 (QUOTE "failed"))
(|List| |$|)
(|Record| (|:| |coef| 123) (|:| |generator| |$|))
(|Record| (|:| |coef1| |$|) (|:| |coef2| |$|))
(|Union| 125 (QUOTE "failed"))
(|Record| (|:| |coef1| |$|) (|:| |coef2| |$|) (|:| |generator| |$|))
(|PositiveInteger|) (|SingleInteger|))
(QUOTE #(|~|= 131 |zero?= 137 |unitNormal| 142 |unitCanonical| 147
|unit?= 152 |symmetricRemainder| 157 |subtractIfCan| 163 |submod| 169
|squareFreePart| 176 |squareFree| 181 |sizeLess?= 186 |sign| 192
|shift| 197 |sample| 203 |retractIfCan| 207 |retract| 212 |rem| 217
|reducedSystem| 223 |recip| 234 |rationalIfCan| 239 |rational?= 244
|rational| 249 |random| 254 |quo| 263 |principalIdeal| 269
|prime?= 274 |powmod| 279 |positiveRemainder| 286 |positive?= 292
|permutation| 297 |patternMatch| 303 |one?= 310 |odd?= 315
|nextItem| 320 |negative?= 325 |multiEuclidean| 330 |mulmod| 336
|min| 343 |max| 349 |mask| 355 |length| 360 |lcm| 365 |latex| 376
|invmod| 381 |init| 387 |inc| 391 |hash| 396 |gcdPolynomial| 406
|gcd| 412 |factorial| 423 |factor| 428 |extendedEuclidean| 433
|exquo| 446 |expressIdealMember| 452 |even?= 458
|euclideanSize| 463 |divide| 468 |differentiate| 474 |dec| 485
|copy| 490 |convert| 495 |coerce| 525 |characteristic| 545
|bit?= 549 |binomial| 555 |base| 561 |associates?= 565
|addmod| 571 |abs| 578 |^| 583 |Zero| 595 |One| 599
|OMwrite| 603 D 627 |>|= 638 |>| 644 |= 650 |<|= 656 |<| 662
|-| 668 |+| 679 |**| 685 |*| 697))
(QUOTE (
(|infinite| . 0) (|noetherian| . 0) (|canonicalsClosed| . 0)
(|canonical| . 0) (|canonicalUnitNormal| . 0)
(|multiplicativeValuation| . 0) (|noZeroDivisors| . 0)
((|commutative| "*" . 0) (|rightUnitary| . 0) (|leftUnitary| . 0)
(|unitsKnown| . 0)))
(CONS
(|makeByteWordVec2| 1
(QUOTE (0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0)))
(CONS
(QUOTE #(
|IntegerNumberSystem&| |EuclideanDomain&| |UniqueFactorizationDomain&|
NIL NIL |GcdDomain&| |IntegralDomain&| |Algebra&| NIL NIL
|DifferentialRing&| |OrderedRing&| NIL NIL |Module&| NIL NIL
|Ring&| NIL NIL NIL NIL NIL |AbelianGroup&| NIL NIL
|AbelianMonoid&| |Monoid&| NIL NIL |OrderedSet&|
|AbelianSemiGroup&| |SemiGroup&| NIL |SetCategory&|
NIL NIL NIL NIL NIL NIL |RetractableTo&| NIL |BasicType&| NIL))
(CONS
(QUOTE #(
(|IntegerNumberSystem|) (|EuclideanDomain|)
(|UniqueFactorizationDomain|) (|PrincipalIdealDomain|)

```

```

(|OrderedIntegralDomain|) (|GcdDomain|) (|IntegralDomain|)
(|Algebra| |$$|) (|CharacteristicZero|) (|LinearlyExplicitRingOver| 11)
(|DifferentialRing|) (|OrderedRing|) (|CommutativeRing|) (|EntireRing|)
(|Module| |$$|) (|OrderedAbelianGroup|) (|BiModule| |$$| |$$|)
(|Ring|) (|OrderedCancellationAbelianMonoid|) (|LeftModule| |$$|)
(|Rng|) (|RightModule| |$$|) (|OrderedAbelianMonoid|) (|AbelianGroup|)
(|OrderedAbelianSemiGroup|) (|CancellationAbelianMonoid|)
(|AbelianMonoid|) (|Monoid|) (|StepThrough|) (|PatternMatchable| 11)
(|OrderedSet|) (|AbelianSemiGroup|) (|SemiGroup|) (|RealConstant|)
(|SetCategory|) (|OpenMath|) (|ConvertibleTo| 9)
(|ConvertibleTo| 43) (|ConvertibleTo| 46)
(|CombinatorialFunctionCategory|) (|ConvertibleTo| 120)
(|ConvertibleTo| 48) (|RetractableTo| 11) (|ConvertibleTo| 11)
(|BasicType|) (|CoercibleTo| 34)))
(|makeByteWordVec2| 129 (QUOTE (1 7 6 0 8 3 7 6 0 9 9 10 2 7 6 0 11
12 1 7 6 0 13 0 14 0 15 2 7 0 9 14 16 1 7 6 0 17 1 7 6 0 18 1 7 6 0
19 1 34 0 11 35 1 43 0 11 44 1 48 0 11 49 2 90 87 88 89 91 1 94 92
93 95 1 93 0 0 96 1 93 2 0 97 1 98 92 93 99 1 93 0 2 100 1 0 101 0
102 2 105 92 103 104 106 2 107 92 92 92 108 1 98 92 93 109 1 93 21
0 110 1 93 0 0 111 1 113 93 112 114 2 0 21 0 0 1 1 0 21 0 25 1 0 84
0 85 1 0 0 0 86 1 0 21 0 1 2 0 0 0 0 1 2 0 80 0 0 1 3 0 0 0 0 0 41
1 0 0 0 1 1 0 101 0 1 2 0 21 0 0 1 1 0 11 0 1 2 0 0 0 0 79 0 0 0 1
1 0 121 0 1 1 0 11 0 1 2 0 0 0 0 78 2 0 57 55 58 59 1 0 54 55 56 1
0 80 0 82 1 0 117 0 1 1 0 21 0 1 1 0 118 0 1 1 0 0 0 62 0 0 0 61 2
0 0 0 0 77 1 0 124 123 1 1 0 21 0 1 3 0 0 0 0 0 1 2 0 0 0 0 53 1 0
21 0 1 2 0 0 0 0 1 3 0 119 0 120 119 1 1 0 21 0 1 1 0 21 0 72 1 0
80 0 1 1 0 21 0 33 2 0 122 123 0 1 3 0 0 0 0 0 42 2 0 0 0 0 74 2 0
0 0 0 73 1 0 0 0 1 1 0 0 0 39 1 0 0 123 1 2 0 0 0 0 1 1 0 9 0 52 2
0 0 0 0 1 0 0 0 1 1 0 0 0 30 1 0 0 0 32 1 0 129 0 1 2 0 115 115 115
116 2 0 0 0 0 83 1 0 0 123 1 1 0 0 0 1 1 0 101 0 102 3 0 126 0 0 0
1 2 0 127 0 0 1 2 0 80 0 0 81 2 0 122 123 0 1 1 0 21 0 1 1 0 70 0
1 2 0 75 0 0 76 1 0 0 0 1 2 0 0 0 70 1 1 0 0 0 31 1 0 0 0 29 1 0 9
0 51 1 0 46 0 47 1 0 43 0 45 1 0 48 0 50 1 0 120 0 1 1 0 11 0 38 1
0 0 11 37 1 0 0 11 37 1 0 0 0 1 1 0 34 0 36 0 0 70 1 2 0 21 0 0 1
2 0 0 0 0 1 0 0 0 28 2 0 21 0 0 1 3 0 0 0 0 0 40 1 0 0 0 60 2 0 0
0 70 1 2 0 0 0 128 1 0 0 0 26 0 0 0 27 3 0 6 7 0 21 24 2 0 9 0 21
22 2 0 6 7 0 23 1 0 9 0 20 1 0 0 0 1 2 0 0 0 70 1 2 0 21 0 0 1 2
0 21 0 0 1 2 0 21 0 0 63 2 0 21 0 0 1 2 0 21 0 0 64 2 0 0 0 0 67
1 0 0 0 65 2 0 0 0 0 66 2 0 0 0 70 71 2 0 0 0 128 1 2 0 0 0 0 68
2 0 0 11 0 69 2 0 0 70 0 1 2 0 0 128 0 1))))))
(QUOTE |lookupComplete|)))

(setf (get (QUOTE |Integer|) (QUOTE NILADIC)) T)

```

---

## 28.6 ISTRING.lsp BOOTSTRAP

**ISTRING** depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **ISTRING** category which we can write into the **MID** directory. We compile the lisp code and copy the **ISTRING.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

### — ISTRING.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(PUT '|ISTRING;new;NniC$;1| '|SPADreplace| 'MAKE-FULL-CVEC)

(DEFUN |ISTRING;new;NniC$;1| (|n| |c| |$|) (|MAKE-FULL-CVEC| |n| |c|))

(PUT '|ISTRING;empty;$;2| '|SPADreplace|
  '(XLAM NIL (MAKE-FULL-CVEC 0)))

(DEFUN |ISTRING;empty;$;2| (|$|) (|MAKE-FULL-CVEC| 0))

(DEFUN |ISTRING;empty?;$B;3| (|s| |$|) (EQL (QCSIZE |s|) 0))

(PUT '|ISTRING;#;$Nni;4| '|SPADreplace| 'QCSIZE)

(DEFUN |ISTRING;#;$Nni;4| (|s| |$|) (QCSIZE |s|))

(PUT '|ISTRING;=$;2$B;5| '|SPADreplace| 'EQUAL)

(DEFUN |ISTRING;=$;2$B;5| (|s| |t| |$|) (EQUAL |s| |t|))

(PUT '|ISTRING;<;2$B;6| '|SPADreplace|
  '(XLAM (|s| |t|) (CGREATERP |t| |s|)))

(DEFUN |ISTRING;<;2$B;6| (|s| |t| |$|) (CGREATERP |t| |s|))

(PUT '|ISTRING;concat;3$;7| '|SPADreplace| 'STRCONC)

(DEFUN |ISTRING;concat;3$;7| (|s| |t| |$|) (STRCONC |s| |t|))

(PUT '|ISTRING;copy;2$;8| '|SPADreplace| 'COPY-SEQ)

(DEFUN |ISTRING;copy;2$;8| (|s| |$|) (|COPY-SEQ| |s|))

(DEFUN |ISTRING;insert;2$I$;9| (|s| |t| |i| |$|)
  (SPADCALL
    (SPADCALL
      (SPADCALL |s|
```

```

        (SPADCALL (QREFELT $ 6) (- |i| 1) (QREFELT $ 20))
        (QREFELT $ 21))
    |t| (QREFELT $ 16))
    (SPADCALL |s| (SPADCALL |i| (QREFELT $ 22)) (QREFELT $ 21))
    (QREFELT $ 16)))

(DEFUN |ISTRING;coerce;$0f;10| (|s| |$|) (SPADCALL |s| (QREFELT |$| 26)))

(DEFUN |ISTRING;minIndex;$I;11| (|s| |$|) (QREFELT |$| 6))

(DEFUN |ISTRING;upperCase!;2$;12| (|s| $)
  (SPADCALL (ELT $ 31) |s| (QREFELT $ 33)))

(DEFUN |ISTRING;lowerCase!;2$;13| (|s| $)
  (SPADCALL (ELT $ 36) |s| (QREFELT $ 33)))

(DEFUN |ISTRING;latex;$S;14| (|s| $)
  (STRCONC "\mbox{'" (STRCONC |s| "'')"))

(DEFUN |ISTRING;replace;$Us2$;15| (|s| |sgl| |t| $)
  (PROG (|l| |m| |n| |h| G91425 |r| G91433 G91432 |i| G91431
    |k|)
    (RETURN
      (SEQ (LETT |l| (- (SPADCALL |sgl| (QREFELT $ 39)) (QREFELT $ 6))
        |ISTRING;replace;$Us2$;15|)
        (LETT |m| (SPADCALL |s| (QREFELT $ 13))
          |ISTRING;replace;$Us2$;15|)
        (LETT |n| (SPADCALL |t| (QREFELT $ 13))
          |ISTRING;replace;$Us2$;15|)
        (LETT |h|
          (COND
            ((SPADCALL |sgl| (QREFELT $ 40))
              (- (SPADCALL |sgl| (QREFELT $ 41)) (QREFELT $ 6)))
            ('T (- (SPADCALL |s| (QREFELT $ 42)) (QREFELT $ 6))))
          |ISTRING;replace;$Us2$;15|)
        (COND
          ((OR (OR (< |l| 0) (NULL (< |h| |m|))) (< |h| (- |l| 1)))
            (EXIT (|error| "index out of range"))))
        (LETT |r|
          (SPADCALL
            (PROG1 (LETT G91425
              (+ (- |m| (+ (- |h| |l|) 1)) |n|)
              |ISTRING;replace;$Us2$;15|)
              (|check-subtype| (>= G91425 0)
                '(|NonNegativeInteger|) G91425))
              (SPADCALL (QREFELT $ 43)) (QREFELT $ 9))
            |ISTRING;replace;$Us2$;15|)
        (SEQ (LETT |i| 0 |ISTRING;replace;$Us2$;15|)
          (LETT G91433 (- |l| 1) |ISTRING;replace;$Us2$;15|)
          (LETT |k| 0 |ISTRING;replace;$Us2$;15|) G190

```

```

(COND ((QSGREATERP |i| G91433) (GO G191)))
(SEQ (EXIT (QESSET |r| |k| (QENUM |s| |i|))))
(LETT |k|
  (PROG1 (QSADD1 |k|)
    (LETT |i| (QSADD1 |i|)
      |ISTRING;replace;$Us2$;15|)
    |ISTRING;replace;$Us2$;15|)
  (GO G190) G191 (EXIT NIL))
(SEQ (LETT |i| 0 |ISTRING;replace;$Us2$;15|)
  (LETT G91432 (- |n| 1) |ISTRING;replace;$Us2$;15|)
  (LETT |k| |k| |ISTRING;replace;$Us2$;15|) G190
  (COND ((QSGREATERP |i| G91432) (GO G191)))
  (SEQ (EXIT (QESSET |r| |k| (QENUM |t| |i|))))
  (LETT |k|
    (PROG1 (+ |k| 1)
      (LETT |i| (QSADD1 |i|)
        |ISTRING;replace;$Us2$;15|)
        |ISTRING;replace;$Us2$;15|)
      (GO G190) G191 (EXIT NIL))
    (SEQ (LETT |i| (+ |h| 1) |ISTRING;replace;$Us2$;15|)
      (LETT G91431 (- |m| 1) |ISTRING;replace;$Us2$;15|)
      (LETT |k| |k| |ISTRING;replace;$Us2$;15|) G190
      (COND ((> |i| G91431) (GO G191)))
      (SEQ (EXIT (QESSET |r| |k| (QENUM |s| |i|))))
      (LETT |k|
        (PROG1 (+ |k| 1)
          (LETT |i| (+ |i| 1) |ISTRING;replace;$Us2$;15|)
          |ISTRING;replace;$Us2$;15|)
          (GO G190) G191 (EXIT NIL))
        (EXIT |r|))))))

(DEFUN |ISTRING;setelt;$I2C;16| (|s| |i| |c| $)
  (SEQ (COND
    ((OR (< |i| (QREFELT $ 6))
      (< (SPADCALL |s| (QREFELT $ 42)) |i|))
      (|error| "index out of range"))
    ('T (SEQ (QESSET |s| (- |i| (QREFELT $ 6)) |c|) (EXIT |c|)))))

(DEFUN |ISTRING;substring?;2$IB;17| (|part| |whole| |startpos| $)
  (PROG (|np| |nw| |iw| |ip| G91443 G91442 G91438)
    (RETURN
      (SEQ (EXIT (SEQ (LETT |np| (QCSIZE |part|)
        |ISTRING;substring?;2$IB;17|)
        (LETT |nw| (QCSIZE |whole|)
          |ISTRING;substring?;2$IB;17|)
          (LETT |startpos| (- |startpos| (QREFELT $ 6))
            |ISTRING;substring?;2$IB;17|)
            (EXIT (COND
              ((< |startpos| 0)
                (|error| "index out of bounds"))

```



```

((< (- |nw| |startpos|) |np|) 'NIL)
('T
  (SEQ (SEQ
    (EXIT
      (SEQ
        (LETT |iw| |startpos|
          |ISTRING;substring?;2$IB;17|)
        (LETT |ip| 0
          |ISTRING;substring?;2$IB;17|)
        (LETT G91443 (- |np| 1)
          |ISTRING;substring?;2$IB;17|)
        G190
        (COND
          ((QSGREATERP |ip| G91443)
            (GO G191)))
        (SEQ
          (EXIT
            (COND
              ((NULL
                (EQL (QENUM |part| |ip|)
                  (QENUM |whole| |iw|)))
                (PROGN
                  (LETT G91438
                    (PROGN
                      (LETT G91442 'NIL
                        |ISTRING;substring?;2$IB;17|)
                      (GO G91442))
                    |ISTRING;substring?;2$IB;17|)
                  (GO G91438))))))
          (LETT |ip|
            (PROG1 (QSADD1 |ip|)
              (LETT |iw| (+ |iw| 1)
                |ISTRING;substring?;2$IB;17|))
              |ISTRING;substring?;2$IB;17|)
              (GO G190) G191 (EXIT NIL)))
          G91438 (EXIT G91438))
        (EXIT 'T))))))
    G91442 (EXIT G91442))))))

(DEFUN |ISTRING;position;2$2I;18| (|s| |t| |startpos| $)
  (PROG (|r|)
    (RETURN
      (SEQ (LETT |startpos| (- |startpos| (QREFELT $ 6))
        |ISTRING;position;2$2I;18|)
        (EXIT (COND
          ((< |startpos| 0) (|error| "index out of bounds"))
          ((NULL (< |startpos| (QCSIZE |t|)))
            (- (QREFELT $ 6) 1))
          ('T
            (SEQ (LETT |r| (STRPOS |s| |t| |startpos| NIL)

```

```

        |ISTRING;position;2$I;18|)
(EXIT (COND
      ((EQ |r| NIL) (- (QREFELT $ 6) 1))
      ('T (+ |r| (QREFELT $ 6)))))))))

(DEFUN |ISTRING;position;C$I;19| (|c| |t| |startpos| $)
  (PROG (|r| G91454 G91453)
    (RETURN
      (SEQ (EXIT (SEQ (LETT |startpos| (- |startpos| (QREFELT $ 6))
        |ISTRING;position;C$I;19|)
      (EXIT (COND
        ((< |startpos| 0)
          (|error| "index out of bounds"))
        ((NULL (< |startpos| (QCSIZE |t|)))
          (- (QREFELT $ 6) 1))
        ('T
          (SEQ (SEQ
            (LETT |r| |startpos|
              |ISTRING;position;C$I;19|)
            (LETT G91454
              (QSDIFFERENCE (QCSIZE |t|) 1)
              |ISTRING;position;C$I;19|)
            G190
            (COND
              ((> |r| G91454) (GO G191)))
            (SEQ
              (EXIT
                (COND
                  ((EQL (QENUM |t| |r|) |c|)
                    (PROGN
                      (LETT G91453
                        (+ |r| (QREFELT $ 6))
                        |ISTRING;position;C$I;19|)
                        (GO G91453))))))
              (LETT |r| (+ |r| 1)
                |ISTRING;position;C$I;19|)
              (GO G190) G191 (EXIT NIL))
              (EXIT (- (QREFELT $ 6) 1)))))))))
      G91453 (EXIT G91453))))))

(DEFUN |ISTRING;position;Cc$I;20| (|cc| |t| |startpos| $)
  (PROG (|r| G91461 G91460)
    (RETURN
      (SEQ (EXIT (SEQ (LETT |startpos| (- |startpos| (QREFELT $ 6))
        |ISTRING;position;Cc$I;20|)
      (EXIT (COND
        ((< |startpos| 0)
          (|error| "index out of bounds"))
        ((NULL (< |startpos| (QCSIZE |t|)))
          (- (QREFELT $ 6) 1))

```

```

('T
  (SEQ (SEQ
    (LETT |r| |startpos|
      |ISTRING;position;Cc$2I;20|)
    (LETT G91461
      (QSDIFFERENCE (QCSIZE |t|) 1)
      |ISTRING;position;Cc$2I;20|)
    G190
    (COND
      ((> |r| G91461) (GO G191)))
    (SEQ
      (EXIT
        (COND
          ((SPADCALL (QENUM |t| |r|)
            |cc| (QREFELT $ 49))
          (PROGN
            (LETT G91460
              (+ |r| (QREFELT $ 6))
              |ISTRING;position;Cc$2I;20|)
              (GO G91460))))))
      (LETT |r| (+ |r| 1)
        |ISTRING;position;Cc$2I;20|)
      (GO G190) G191 (EXIT NIL))
    (EXIT (- (QREFELT $ 6) 1))))))
  G91460 (EXIT G91460))))))

(DEFUN |ISTRING;suffix?;2$B;21| (|s| |t| $)
  (PROG (|n| |m|)
    (RETURN
      (SEQ (LETT |n| (SPADCALL |t| (QREFELT $ 42))
        |ISTRING;suffix?;2$B;21|)
        (LETT |m| (SPADCALL |s| (QREFELT $ 42))
          |ISTRING;suffix?;2$B;21|)
        (EXIT (COND
          ((< |n| |m|) 'NIL)
          ('T
            (SPADCALL |s| |t| (- (+ (QREFELT $ 6) |n|) |m|)
              (QREFELT $ 46))))))))))

(DEFUN |ISTRING;split;$CL;22| (|s| |c| $)
  (PROG (|n| |j| |i| |l|)
    (RETURN
      (SEQ (LETT |n| (SPADCALL |s| (QREFELT $ 42))
        |ISTRING;split;$CL;22|)
        (SEQ (LETT |i| (QREFELT $ 6) |ISTRING;split;$CL;22|) G190
          (COND
            ((OR (> |i| |n|)
              (NULL (SPADCALL
                (SPADCALL |s| |i| (QREFELT $ 52)) |c|
                (QREFELT $ 53))))))

```

```

        (GO G191)))
      (SEQ (EXIT 0))
      (LETT |i| (+ |i| 1) |ISTRING;split;$CL;22|) (GO G190)
      G191 (EXIT NIL))
    (LETT |l| (SPADCALL (QREFELT $ 55)) |ISTRING;split;$CL;22|)
    (SEQ G190
      (COND
        ((NULL (COND
          ((< |n| |i|) 'NIL)
          ('T
            (SEQ (LETT |j|
              (SPADCALL |c| |s| |i|
                (QREFELT $ 48))
                |ISTRING;split;$CL;22|)
              (EXIT (COND
                ((< |j| (QREFELT $ 6)) 'NIL)
                ('T 'T)))))))
          (GO G191)))
        (SEQ (LETT |l|
          (SPADCALL
            (SPADCALL |s|
              (SPADCALL |i| (- |j| 1)
                (QREFELT $ 20))
                (QREFELT $ 21))
              |l| (QREFELT $ 56))
              |ISTRING;split;$CL;22|)
            (EXIT (SEQ (LETT |i| |j| |ISTRING;split;$CL;22|)
              G190
              (COND
                ((OR (> |i| |n|)
                  (NULL
                    (SPADCALL
                      (SPADCALL |s| |i| (QREFELT $ 52))
                      |c| (QREFELT $ 53))))
                  (GO G191)))
                (SEQ (EXIT 0))
                (LETT |i| (+ |i| 1)
                  |ISTRING;split;$CL;22|)
                  (GO G190) G191 (EXIT NIL))))
              NIL (GO G190) G191 (EXIT NIL))
            (COND
              ((NULL (< |n| |i|))
                (LETT |l|
                  (SPADCALL
                    (SPADCALL |s| (SPADCALL |i| |n| (QREFELT $ 20))
                      (QREFELT $ 21))
                      |l| (QREFELT $ 56))
                      |ISTRING;split;$CL;22|)))
                (EXIT (SPADCALL |l| (QREFELT $ 57)))))))

```



```

        NIL (GO G190) G191 (EXIT NIL))
(COND
  ((NULL (< |n| |i|))
    (LETT |l|
      (SPADCALL
        (SPADCALL |s| (SPADCALL |i| |n| (QREFELT $ 20))
          (QREFELT $ 21))
        |l| (QREFELT $ 56))
      |ISTRING;split;$CcL;23|)))
  (EXIT (SPADCALL |l| (QREFELT $ 57))))))

(DEFUN |ISTRING;leftTrim;$C$;24| (|s| |c| $)
  (PROG (|n| |i|)
    (RETURN
      (SEQ (LETT |n| (SPADCALL |s| (QREFELT $ 42))
        |ISTRING;leftTrim;$C$;24|)
        (SEQ (LETT |i| (QREFELT $ 6) |ISTRING;leftTrim;$C$;24|) G190
          (COND
            ((OR (> |i| |n|)
              (NULL (SPADCALL
                (SPADCALL |s| |i| (QREFELT $ 52)) |c|
                (QREFELT $ 53))))
              (GO G191)))
            (SEQ (EXIT 0))
            (LETT |i| (+ |i| 1) |ISTRING;leftTrim;$C$;24|)
            (GO G190) G191 (EXIT NIL))
            (EXIT (SPADCALL |s| (SPADCALL |i| |n| (QREFELT $ 20))
              (QREFELT $ 21)))))))

(DEFUN |ISTRING;leftTrim;$Cc$;25| (|s| |cc| $)
  (PROG (|n| |i|)
    (RETURN
      (SEQ (LETT |n| (SPADCALL |s| (QREFELT $ 42))
        |ISTRING;leftTrim;$Cc$;25|)
        (SEQ (LETT |i| (QREFELT $ 6) |ISTRING;leftTrim;$Cc$;25|)
          G190
          (COND
            ((OR (> |i| |n|)
              (NULL (SPADCALL
                (SPADCALL |s| |i| (QREFELT $ 52)) |cc|
                (QREFELT $ 49))))
              (GO G191)))
            (SEQ (EXIT 0))
            (LETT |i| (+ |i| 1) |ISTRING;leftTrim;$Cc$;25|)
            (GO G190) G191 (EXIT NIL))
            (EXIT (SPADCALL |s| (SPADCALL |i| |n| (QREFELT $ 20))
              (QREFELT $ 21)))))))

(DEFUN |ISTRING;rightTrim;$C$;26| (|s| |c| $)
  (PROG (|j| G91487)

```

```

(RETURN
  (SEQ (SEQ (LETT |j| (SPADCALL |s| (QREFELT $ 42))
    |ISTRING;rightTrim;$C$;26|)
    (LETT G91487 (QREFELT $ 6)
      |ISTRING;rightTrim;$C$;26|)
    G190
    (COND
      ((OR (< |j| G91487)
        (NULL (SPADCALL
          (SPADCALL |s| |j| (QREFELT $ 52)) |c|
          (QREFELT $ 53))))
        (GO G191)))
    (SEQ (EXIT 0))
    (LETT |j| (+ |j| -1) |ISTRING;rightTrim;$C$;26|)
    (GO G190) G191 (EXIT NIL))
  (EXIT (SPADCALL |s|
    (SPADCALL (SPADCALL |s| (QREFELT $ 28)) |j|
      (QREFELT $ 20))
    (QREFELT $ 21))))))

(DEFUN |ISTRING;rightTrim;$Cc$;27| (|s| |cc| $)
  (PROG (|j| G91491)
    (RETURN
      (SEQ (SEQ (LETT |j| (SPADCALL |s| (QREFELT $ 42))
        |ISTRING;rightTrim;$Cc$;27|)
        (LETT G91491 (QREFELT $ 6)
          |ISTRING;rightTrim;$Cc$;27|)
        G190
        (COND
          ((OR (< |j| G91491)
            (NULL (SPADCALL
              (SPADCALL |s| |j| (QREFELT $ 52)) |cc|
              (QREFELT $ 49))))
            (GO G191)))
          (SEQ (EXIT 0))
          (LETT |j| (+ |j| -1) |ISTRING;rightTrim;$Cc$;27|)
          (GO G190) G191 (EXIT NIL))
        (EXIT (SPADCALL |s|
          (SPADCALL (SPADCALL |s| (QREFELT $ 28)) |j|
            (QREFELT $ 20))
          (QREFELT $ 21))))))

(DEFUN |ISTRING;concat;L$;28| (|l| $)
  (PROG (G91500 G91494 G91492 G91493 |t| |s| G91499 |i|)
    (RETURN
      (SEQ (LETT |t|
        (SPADCALL
          (PROGN
            (LETT G91493 NIL |ISTRING;concat;L$;28|)

```

```

      (SEQ (LETT |s| NIL |ISTRING;concat;L$;28|)
            (LETT G91500 |l| |ISTRING;concat;L$;28|)
            G190
            (COND
              ((OR (ATOM G91500)
                   (PROGN
                     (LETT |s| (CAR G91500)
                           |ISTRING;concat;L$;28|)
                     NIL))
               (GO G191)))
            (SEQ (EXIT (PROGN
                        (LETT G91494
                            (SPADCALL |s| (QREFELT $ 13))
                            |ISTRING;concat;L$;28|)
                        (COND
                          (G91493
                           (LETT G91492
                               (+ G91492 G91494)
                               |ISTRING;concat;L$;28|))
                          ('T
                           (PROGN
                             (LETT G91492 G91494
                                   |ISTRING;concat;L$;28|)
                             (LETT G91493 'T
                                   |ISTRING;concat;L$;28|)))))))
            (LETT G91500 (CDR G91500)
                  |ISTRING;concat;L$;28|)
            (GO G190) G191 (EXIT NIL))
      (COND (G91493 G91492) ('T 0))
      (SPADCALL (QREFELT $ 43)) (QREFELT $ 9))
      |ISTRING;concat;L$;28|)
    (LETT |i| (QREFELT $ 6) |ISTRING;concat;L$;28|)
    (SEQ (LETT |s| NIL |ISTRING;concat;L$;28|)
          (LETT G91499 |l| |ISTRING;concat;L$;28|) G190
          (COND
            ((OR (ATOM G91499)
                 (PROGN
                   (LETT |s| (CAR G91499)
                           |ISTRING;concat;L$;28|)
                   NIL))
             (GO G191)))
          (SEQ (SPADCALL |t| |s| |i| (QREFELT $ 65))
                (EXIT (LETT |i|
                          (+ |i| (SPADCALL |s| (QREFELT $ 13)))
                          |ISTRING;concat;L$;28|)))
          (LETT G91499 (CDR G91499) |ISTRING;concat;L$;28|)
          (GO G190) G191 (EXIT NIL))
    (EXIT |t|))))))

(DEFUN |ISTRING;copyInto!;2$I$;29| (|y| |x| |s| $)

```



```

(PROG (|m| |n|)
  (RETURN
    (SEQ (LETT |m| (SPADCALL |x| (QREFELT $ 13))
          |ISTRING;copyInto!;2$I$;29|)
      (LETT |n| (SPADCALL |y| (QREFELT $ 13))
            |ISTRING;copyInto!;2$I$;29|)
      (LETT |s| (- |s| (QREFELT $ 6)) |ISTRING;copyInto!;2$I$;29|)
      (COND
        ((OR (< |s| 0) (< |n| (+ |s| |m|))))
        (EXIT (|error| "index out of range"))))
      (RPLACSTR |y| |s| |m| |x| 0 |m|) (EXIT |y|))))))

(DEFUN |ISTRING;elt;$IC;30| (|s| |i| $)
  (COND
    ((OR (< |i| (QREFELT $ 6)) (< (SPADCALL |s| (QREFELT $ 42)) |i|))
      (|error| "index out of range"))
    ('T (QENUM |s| (- |i| (QREFELT $ 6)))))

(DEFUN |ISTRING;elt;$Us$;31| (|s| |sg| $)
  (PROG (|l| |h|)
    (RETURN
      (SEQ (LETT |l| (- (SPADCALL |sg| (QREFELT $ 39)) (QREFELT $ 6))
            |ISTRING;elt;$Us$;31|)
          (LETT |h|
            (COND
              ((SPADCALL |sg| (QREFELT $ 40))
               (- (SPADCALL |sg| (QREFELT $ 41)) (QREFELT $ 6)))
              ('T (- (SPADCALL |s| (QREFELT $ 42)) (QREFELT $ 6))))
            |ISTRING;elt;$Us$;31|)
          (COND
            ((OR (< |l| 0)
              (NULL (< |h| (SPADCALL |s| (QREFELT $ 13)))))
              (EXIT (|error| "index out of bound"))))
            (EXIT (SUBSTRING |s| |l| (MAX 0 (+ (- |h| |l|) 1)))))))

(DEFUN |ISTRING;hash;$I;32| (|s| $)
  (PROG (|n|)
    (RETURN
      (SEQ (LETT |n| (QCSIZE |s|) |ISTRING;hash;$I;32|)
          (EXIT (COND
            ((ZEROP |n|) 0)
            ((EQL |n| 1)
              (SPADCALL
                (SPADCALL |s| (QREFELT $ 6) (QREFELT $ 52))
                (QREFELT $ 67)))
            ('T
              (* (* (SPADCALL
                    (SPADCALL |s| (QREFELT $ 6)

```

```

        (QREFELT $ 52))
      (QREFELT $ 67))
    (SPADCALL
      (SPADCALL |s| (- (+ (QREFELT $ 6) |n|) 1)
        (QREFELT $ 52))
      (QREFELT $ 67)))
    (SPADCALL
      (SPADCALL |s|
        (+ (QREFELT $ 6) (QUOTIENT2 |n| 2))
        (QREFELT $ 52))
      (QREFELT $ 67)))))))))

(PUT ' |ISTRING;match;2$CNni;33| ' |SPADreplace| ' |stringMatch|)

(DEFUN |ISTRING;match;2$CNni;33| (|pattern| |target| |wildcard| $)
  (|stringMatch| |pattern| |target| |wildcard|))

(DEFUN |ISTRING;match?;2$CB;34| (|pattern| |target| |dontcare| $)
  (PROG (|n| |m| G91514 G91516 |s| G91518 G91526 |i| |p|
    G91519 |q|)
    (RETURN
      (SEQ (EXIT (SEQ (LETT |n| (SPADCALL |pattern| (QREFELT $ 42))
        |ISTRING;match?;2$CB;34|)
        (LETT |p|
          (PROG1 (LETT G91514
            (SPADCALL |dontcare| |pattern|
              (LETT |m|
                (SPADCALL |pattern|
                  (QREFELT $ 28))
                  |ISTRING;match?;2$CB;34|)
                  (QREFELT $ 48))
                  |ISTRING;match?;2$CB;34|)
                (|check-subtype| (>= G91514 0)
                  '(|NonNegativeInteger|) G91514))
            |ISTRING;match?;2$CB;34|)
          (EXIT (COND
            ((EQL |p| (- |m| 1))
              (SPADCALL |pattern| |target|
                (QREFELT $ 14)))
            ('T
              (SEQ (COND
                ((NULL (EQL |p| |m|))
                  (COND
                    (NULL
                      (SPADCALL
                        (SPADCALL |pattern|
                          (SPADCALL |m| (- |p| 1)
                            (QREFELT $ 20))
                            (QREFELT $ 21))
                          |target| (QREFELT $ 70)))

```

```

        (EXIT 'NIL))))))
(LETT |i| |p|
 |ISTRING;match?;2$CB;34|)
(LETT |q|
 (PROG1
  (LETT G91516
   (SPADCALL |dontcare| |pattern|
    (+ |p| 1) (QREFELT $ 48))
   |ISTRING;match?;2$CB;34|)
   (|check-subtype| (>= G91516 0)
    '(|NonNegativeInteger|)
    G91516))
  |ISTRING;match?;2$CB;34|)
 (SEQ G190
  (COND
   ((NULL
    (COND
     ((EQL |q| (- |m| 1)) 'NIL)
     ('T 'T)))
    (GO G191)))
  (SEQ
   (LETT |s|
    (SPADCALL |pattern|
     (SPADCALL (+ |p| 1) (- |q| 1)
      (QREFELT $ 20))
     (QREFELT $ 21))
    |ISTRING;match?;2$CB;34|)
   (LETT |i|
    (PROG1
     (LETT G91518
      (SPADCALL |s| |target| |i|
       (QREFELT $ 47))
      |ISTRING;match?;2$CB;34|)
      (|check-subtype|
       (>= G91518 0)
       '(|NonNegativeInteger|)
       G91518))
      |ISTRING;match?;2$CB;34|)
    (EXIT
     (COND
      ((EQL |i| (- |m| 1))
       (PROGN
        (LETT G91526 'NIL
         |ISTRING;match?;2$CB;34|)
        (GO G91526)))
      ('T
       (SEQ
        (LETT |i|
         (+ |i|
          (SPADCALL |s|

```

```

        (QREFELT $ 13)))
    | ISTRING;match?;2$CB;34|)
  (LETT |p| |q|
    | ISTRING;match?;2$CB;34|)
  (EXIT
    (LETT |q|
      (PROG1
        (LETT G91519
          (SPADCALL |dontcare|
            |pattern| (+ |q| 1)
            (QREFELT $ 48))
          | ISTRING;match?;2$CB;34|)
          (|check-subtype|
            (>= G91519 0)
            '(|NonNegativeInteger|
              G91519))
          | ISTRING;match?;2$CB;34|))))))
  NIL (GO G190) G191 (EXIT NIL))
(COND
  ((NULL (EQL |p| |n|))
    (COND
      (NULL
        (SPADCALL
          (SPADCALL |pattern|
            (SPADCALL (+ |p| 1) |n|
              (QREFELT $ 20))
              (QREFELT $ 21))
            |target| (QREFELT $ 51)))
          (EXIT 'NIL))))))
  (EXIT 'T))))))

G91526 (EXIT G91526))))))

(DEFUN |IndexedString| (G91535)
  (PROG ()
    (RETURN
      (PROG (G91536)
        (RETURN
          (COND
            ((LETT G91536
              (|lassocShiftWithFunction|
                (LIST (|devaluate| G91535))
                (HGET |$ConstructorCache| ' |IndexedString|)
                ' |domainEqualList|)
              |IndexedString|)
              (|CDRwithIncrement| G91536))
            ('T
              (UNWIND-PROTECT
                (PROG1 (|IndexedString;| G91535)
                  (LETT G91536 T |IndexedString|))
                (COND

```

```

((NOT G91536)
 (HREM |$ConstructorCache| '|IndexedString|))))))))))

(DEFUN |IndexedString;| (|#1|)
  (PROG (DV$1 |dv$| $ G91534 G91533 |pv$|)
    (RETURN
      (PROGN
        (LETT DV$1 (|devaluate| |#1|) |IndexedString|)
        (LETT |dv$| (LIST '|IndexedString| DV$1) |IndexedString|)
        (LETT $ (make-array 83) |IndexedString|)
        (QSETREFV $ 0 |dv$|)
        (QSETREFV $ 3
          (LETT |pv$|
            (|buildPredVector| 0 0
              (LIST (|HasCategory| (|Character|)
                '|SetCategory|))
                (|HasCategory| (|Character|)
                '|ConvertibleTo| (|InputForm|)))
              (LETT G91534
                (|HasCategory| (|Character|)
                '|OrderedSet|))
                |IndexedString|)
              (OR G91534
                (|HasCategory| (|Character|)
                '|SetCategory|))
                (|HasCategory| (|Integer|) '|OrderedSet|))
              (LETT G91533
                (AND (|HasCategory| (|Character|)
                '|Evalable| (|Character|)))
                (|HasCategory| (|Character|)
                '|SetCategory|))
                |IndexedString|)
              (OR (AND (|HasCategory| (|Character|)
                '|Evalable| (|Character|)))
                G91534)
                G91533)))
                |IndexedString|))
          (|haddProp| |$ConstructorCache| '|IndexedString| (LIST DV$1)
            (CONS 1 $))
          (|stuffDomainSlots| $)
          (QSETREFV $ 6 |#1|)
          $))))))

(setf (get '|IndexedString| '|infovec|)
  (LIST '#(NIL NIL NIL NIL NIL NIL (|local| |#1|)
    (|NonNegativeInteger|) (|Character|) |ISTRING;new;NniC$;1|
    |ISTRING;empty;$;2| (|Boolean|) |ISTRING;empty?;$B;3|
    |ISTRING;#;$Nni;4| |ISTRING;=;2$B;5| |ISTRING;<;2$B;6|
    |ISTRING;concat;3$;7| |ISTRING;copy;2$;8| (|Integer|)
    (|UniversalSegment| 18) (0 . SEGMENT)

```

```

|ISTRING;elt;$Us$;31| (6 . SEGMENT)
|ISTRING;insert;2$I$;9| (|String|) (|OutputForm|)
(11 . |outputForm|) |ISTRING;coerce;$Of;10|
|ISTRING;minIndex;$I;11| (|CharacterClass|)
(16 . |upperCase|) (20 . |upperCase|) (|Mapping| 8 8)
(25 . |map!|) |ISTRING;upperCase!;2$;12|
(31 . |lowerCase|) (35 . |lowerCase|)
|ISTRING;lowerCase!;2$;13| |ISTRING;latex;$S;14|
(40 . |lo|) (45 . |hasHi|) (50 . |hi|) (55 . |maxIndex|)
(60 . |space|) |ISTRING;replace;$Us2$;15|
|ISTRING;setelt;$I2C;16| |ISTRING;substring?;2$IB;17|
|ISTRING;position;2$I;18| |ISTRING;position;C$2I;19|
(64 . |member?|) |ISTRING;position;Cc$2I;20|
|ISTRING;suffix?;2$B;21| |ISTRING;elt;$IC;30| (70 . =)
(|List| $$) (76 . |empty|) (80 . |concat|)
(86 . |reverse!|) (|List| $) |ISTRING;split;$CL;22|
|ISTRING;split;$CcL;23| |ISTRING;leftTrim;$C$;24|
|ISTRING;leftTrim;$Cc$;25| |ISTRING;rightTrim;$C$;26|
|ISTRING;rightTrim;$Cc$;27| |ISTRING;copyInto!;2$I$;29|
|ISTRING;concat;L$;28| (91 . |ord|) |ISTRING;hash;$I;32|
|ISTRING;match;2$CNi;33| (96 . |prefix?|)
|ISTRING;match?;2$CB;34| (|List| 8) (|List| 74)
(|Equation| 8) (|Mapping| 8 8 8) (|InputForm|)
(|SingleInteger|) (|Mapping| 11 8) (|Mapping| 11 8 8)
(|Void|) (|Union| 8 "failed") (|List| 18))
'#(= 102 |upperCase!| 108 |upperCase| 113 |trim| 118 |swap!|
130 |suffix?| 137 |substring?| 143 |split| 150 |sorted?|
162 |sort!| 173 |sort| 184 |size?| 195 |setelt| 201
|select| 215 |sample| 221 |rightTrim| 225 |reverse!| 237
|reverse| 242 |replace| 247 |removeDuplicates| 254
|remove| 259 |reduce| 271 |qsetelt!| 292 |qelt| 299
|prefix?| 305 |position| 311 |parts| 344 |new| 349 |more?|
355 |minIndex| 361 |min| 366 |merge| 372 |members| 385
|member?| 390 |maxIndex| 396 |max| 401 |match?| 407
|match| 414 |map!| 421 |map| 427 |lowerCase!| 440
|lowerCase| 445 |less?| 450 |leftTrim| 456 |latex| 468
|insert| 473 |indices| 487 |index?| 492 |hash| 498 |first|
508 |find| 513 |fill!| 519 |every?| 525 |eval| 531 |eq?|
557 |entry?| 563 |entries| 569 |empty?| 574 |empty| 579
|elt| 583 |delete| 608 |count| 620 |copyInto!| 632 |copy|
639 |convert| 644 |construct| 649 |concat| 654 |coerce|
677 |any?| 687 >= 693 > 699 = 705 <= 711 < 717 |#| 723)
'((|shallowlyMutable| . 0) (|finiteAggregate| . 0))
(CONS (|makeByteWordVec2| 7
      '(0 0 0 0 0 0 0 3 0 0 7 4 0 0 7 1 2 4))
  (CONS '#(|StringAggregate&|
          |OneDimensionalArrayAggregate&|
          |FiniteLinearAggregate&| |LinearAggregate&|
          |IndexedAggregate&| |Collection&|
          |HomogeneousAggregate&| |OrderedSet&|

```

```

|Aggregate&| |EltableAggregate&| |Evaluable&|
|SetCategory&| NIL NIL |InnerEvaluable&| NIL
NIL |BasicType&|)
(CONS '#(|StringAggregate|)
      (|OneDimensionalArrayAggregate| 8)
      (|FiniteLinearAggregate| 8)
      (|LinearAggregate| 8)
      (|IndexedAggregate| 18 8)
      (|Collection| 8)
      (|HomogeneousAggregate| 8)
      (|OrderedSet|) (|Aggregate|)
      (|EltableAggregate| 18 8) (|Evaluable| 8)
      (|SetCategory|) (|Type|)
      (|Eltable| 18 8) (|InnerEvaluable| 8 8)
      (|CoercibleTo| 25) (|ConvertibleTo| 76)
      (|BasicType|))
(|makeByteWordVec2| 82
  '(2 19 0 18 18 20 1 19 0 18 22 1 25 0
    24 26 0 29 0 30 1 8 0 0 31 2 0 0 32 0
    33 0 29 0 35 1 8 0 0 36 1 19 18 0 39
    1 19 11 0 40 1 19 18 0 41 1 0 18 0 42
    0 8 0 43 2 29 11 8 0 49 2 8 11 0 0 53
    0 54 0 55 2 54 0 2 0 56 1 54 0 0 57 1
    8 18 0 67 2 0 11 0 0 70 2 1 11 0 0 1
    1 0 0 0 34 1 0 0 0 1 2 0 0 0 8 1 2 0
    0 0 29 1 3 0 80 0 18 18 1 2 0 11 0 0
    51 3 0 11 0 0 18 46 2 0 58 0 29 60 2
    0 58 0 8 59 1 3 11 0 1 2 0 11 79 0 1
    1 3 0 0 1 2 0 0 79 0 1 1 3 0 0 1 2 0
    0 79 0 1 2 0 11 0 7 1 3 0 8 0 19 8 1
    3 0 8 0 18 8 45 2 0 0 78 0 1 0 0 0 1
    2 0 0 0 8 63 2 0 0 0 29 64 1 0 0 0 1
    1 0 0 0 1 3 0 0 0 19 0 44 1 1 0 0 1 2
    1 0 8 0 1 2 0 0 78 0 1 4 1 8 75 0 8 8
    1 3 0 8 75 0 8 1 2 0 8 75 0 1 3 0 8 0
    18 8 1 2 0 8 0 18 1 2 0 11 0 0 70 3 1
    18 8 0 18 48 2 1 18 8 0 1 3 0 18 29 0
    18 50 3 0 18 0 0 18 47 2 0 18 78 0 1
    1 0 72 0 1 2 0 0 7 8 9 2 0 11 0 7 1 1
    5 18 0 28 2 3 0 0 0 1 2 3 0 0 0 1 3 0
    0 79 0 0 1 1 0 72 0 1 2 1 11 8 0 1 1
    5 18 0 42 2 3 0 0 0 1 3 0 11 0 0 8 71
    3 0 7 0 0 8 69 2 0 0 32 0 33 3 0 0 75
    0 0 1 2 0 0 32 0 1 1 0 0 0 37 1 0 0 0
    1 2 0 11 0 7 1 2 0 0 0 8 61 2 0 0 0
    29 62 1 1 24 0 38 3 0 0 8 0 18 1 3 0
    0 0 0 18 23 1 0 82 0 1 2 0 11 18 0 1
    1 1 77 0 1 1 0 18 0 68 1 5 8 0 1 2 0
    81 78 0 1 2 0 0 0 8 1 2 0 11 78 0 1 3
    6 0 0 72 72 1 3 6 0 0 8 8 1 2 6 0 0

```

```

73 1 2 6 0 0 74 1 2 0 11 0 0 1 2 1 11
8 0 1 1 0 72 0 1 1 0 11 0 12 0 0 0 10
2 0 0 0 0 1 2 0 0 0 19 21 2 0 8 0 18
52 3 0 8 0 18 8 1 2 0 0 0 18 1 2 0 0
0 19 1 2 1 7 8 0 1 2 0 7 78 0 1 3 0 0
0 0 18 65 1 0 0 0 17 1 2 76 0 1 1 0 0
72 1 1 0 0 58 66 2 0 0 0 0 16 2 0 0 0
8 1 2 0 0 8 0 1 1 1 25 0 27 1 0 0 8 1
2 0 11 78 0 1 2 3 11 0 0 1 2 3 11 0 0
1 2 1 11 0 0 14 2 3 11 0 0 1 2 3 11 0
0 15 1 0 7 0 13))))))
'|lookupComplete|))

```

---

## 28.7 LIST.lsp BOOTSTRAP

**LIST** depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **LIST** category which we can write into the **MID** directory. We compile the lisp code and copy the **LIST.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

### — LIST.lsp BOOTSTRAP —

```

(|/VERSIONCHECK| 2)

(PUT (QUOTE |LIST;nil;$;1|) (QUOTE |SPADreplace|) (QUOTE (XLAM NIL NIL)))

(DEFUN |LIST;nil;$;1| (|$|) NIL)

(PUT (QUOTE |LIST>null;$B;2|) (QUOTE |SPADreplace|) (QUOTE NULL))

(DEFUN |LIST>null;$B;2| (|l| |$|) (NULL |l|))

(PUT (QUOTE |LIST;cons;S2$;3|) (QUOTE |SPADreplace|) (QUOTE CONS))

(DEFUN |LIST;cons;S2$;3| (|s| |l| |$|) (CONS |s| |l|))

(PUT (QUOTE |LIST;append;3$;4|) (QUOTE |SPADreplace|) (QUOTE APPEND))

(DEFUN |LIST;append;3$;4| (|l| |t| |$|) (APPEND |l| |t|))

(DEFUN |LIST;writeOMList| (|dev| |x| |$|)
  (SEQ
    (SPADCALL |dev| (QREFELT |$| 14))

```



```

(SPADCALL |dev| "list1" "list" (QREFELT |$| 16))
(SEQ
  G190
  (COND
    ((NULL (COND ((NULL |x|) (QUOTE NIL)) ((QUOTE T) (QUOTE T)))) (GO G191)))
  (SEQ
    (SPADCALL |dev| (|SPADfirst| |x|) (QUOTE NIL) (QREFELT |$| 17))
    (EXIT (LETT |x| (CDR |x|) |LIST;writeOMList|)))
  NIL
  (GO G190)
  G191
  (EXIT NIL))
(EXIT (SPADCALL |dev| (QREFELT |$| 18))))))

(DEFUN |LIST;OMwrite;$S;6| (|x| |$|)
  (PROG (|sp| |dev| |s|)
    (RETURN
      (SEQ
        (LETT |s| "" |LIST;OMwrite;$S;6|)
        (LETT |sp| (|OM-STRINGTOSTRINGPTR| |s|) |LIST;OMwrite;$S;6|)
        (LETT |dev|
          (SPADCALL |sp| (SPADCALL (QREFELT |$| 20)) (QREFELT |$| 21))
          |LIST;OMwrite;$S;6|)
          (SPADCALL |dev| (QREFELT |$| 22))
          (|LIST;writeOMList| |dev| |x| |$|)
          (SPADCALL |dev| (QREFELT |$| 23))
          (SPADCALL |dev| (QREFELT |$| 24))
          (LETT |s| (|OM-STRINGPTRTOSTRING| |sp|) |LIST;OMwrite;$S;6|)
          (EXIT |s|))))))

(DEFUN |LIST;OMwrite;$BS;7| (|x| |wholeObj| |$|)
  (PROG (|sp| |dev| |s|)
    (RETURN
      (SEQ
        (LETT |s| "" |LIST;OMwrite;$BS;7|)
        (LETT |sp| (|OM-STRINGTOSTRINGPTR| |s|) |LIST;OMwrite;$BS;7|)
        (LETT |dev|
          (SPADCALL |sp| (SPADCALL (QREFELT |$| 20)) (QREFELT |$| 21))
          |LIST;OMwrite;$BS;7|)
          (COND (|wholeObj| (SPADCALL |dev| (QREFELT |$| 22))))
          (|LIST;writeOMList| |dev| |x| |$|)
          (COND (|wholeObj| (SPADCALL |dev| (QREFELT |$| 23))))
          (SPADCALL |dev| (QREFELT |$| 24))
          (LETT |s| (|OM-STRINGPTRTOSTRING| |sp|) |LIST;OMwrite;$BS;7|)
          (EXIT |s|))))))

(DEFUN |LIST;OMwrite;Omd$V;8| (|dev| |x| |$|)
  (SEQ
    (SPADCALL |dev| (QREFELT |$| 22))
    (|LIST;writeOMList| |dev| |x| |$|)

```

```

(EXIT (SPADCALL |dev| (QREFELT |$| 23))))))

(DEFUN |LIST;OMwrite;Omd$BV;9| (|dev| |x| |wholeObj| |$|)
  (SEQ
    (COND (|wholeObj| (SPADCALL |dev| (QREFELT |$| 22))))
    (|LIST;writeOMList| |dev| |x| |$|)
    (EXIT (COND (|wholeObj| (SPADCALL |dev| (QREFELT |$| 23)))))))

(DEFUN |LIST;setUnion;3$;10| (|l1| |l2| |$|)
  (SPADCALL (SPADCALL |l1| |l2| (QREFELT |$| 29)) (QREFELT |$| 30)))

(DEFUN |LIST;setIntersection;3$;11| (|l1| |l2| |$|)
  (PROG (|u|)
    (RETURN
      (SEQ
        (LETT |u| NIL |LIST;setIntersection;3$;11|)
        (LETT |l1| (SPADCALL |l1| (QREFELT |$| 30)) |LIST;setIntersection;3$;11|)
        (SEQ
          G190
          (COND
            ((NULL (COND ((NULL |l1|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
            (GO G191)))
          (SEQ
            (COND
              ((SPADCALL (|SPADfirst| |l1|) |l2| (QREFELT |$| 32))
               (LETT |u| (CONS (|SPADfirst| |l1|) |u|) |LIST;setIntersection;3$;11|)))
              (EXIT (LETT |l1| (CDR |l1|) |LIST;setIntersection;3$;11|)))
            NIL
            (GO G190)
            G191
            (EXIT NIL))
          (EXIT |u|))))))

(DEFUN |LIST;setDifference;3$;12| (|l1| |l2| |$|)
  (PROG (|l11| |lu|)
    (RETURN
      (SEQ
        (LETT |l1| (SPADCALL |l1| (QREFELT |$| 30)) |LIST;setDifference;3$;12|)
        (LETT |lu| NIL |LIST;setDifference;3$;12|)
        (SEQ
          G190
          (COND
            ((NULL (COND ((NULL |l1|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
            (GO G191)))
          (SEQ
            (LETT |l11|
              (SPADCALL |l1| 1 (QREFELT |$| 35))
              |LIST;setDifference;3$;12|)
            (COND
              ((NULL (SPADCALL |l11| |l2| (QREFELT |$| 32))))

```

```

        (LETT |lu| (CONS |l11| |lu|) |LIST;setDifference;3$;12|)))
    (EXIT (LETT |l1| (CDR |l1|) |LIST;setDifference;3$;12|)))
    NIL
    (GO G190)
    G191
    (EXIT NIL))
    (EXIT |lu|))))))

(DEFUN |LIST;convert;$If;13| (|x| $)
  (PROG (G102544 |a| G102545)
    (RETURN
      (SEQ (SPADCALL
        (CONS (SPADCALL (SPADCALL "construct" (QREFELT $ 38))
          (QREFELT $ 40))
        (PROGN
          (LETT G102544 NIL |LIST;convert;$If;13|)
          (SEQ (LETT |a| NIL |LIST;convert;$If;13|)
            (LETT G102545 |x| |LIST;convert;$If;13|)
            G190
            (COND
              ((OR (ATOM G102545)
                (PROGN
                  (LETT |a| (CAR G102545)
                    |LIST;convert;$If;13|)
                    NIL))
                (GO G191))))
            (SEQ (EXIT (LETT G102544
              (CONS
                (SPADCALL |a| (QREFELT $ 41))
                G102544)
              |LIST;convert;$If;13|)))
              (LETT G102545 (CDR G102545)
                |LIST;convert;$If;13|)
              (GO G190) G191 (EXIT (NREVERSEO G102544))))))
            (QREFELT $ 43))))))

(DEFUN |List| (G102555)
  (PROG ()
    (RETURN
      (PROG (G102556)
        (RETURN
          (COND
            ((LETT G102556
              (|lassocShiftWithFunction|
                (LIST (|devaluate| G102555))
                (HGET |$ConstructorCache| '|List|)
                '|domainEqualList|)
              |List|)
              (|CDRwithIncrement| G102556))
            ('T

```

```

      (UNWIND-PROTECT
        (PROG1 (|List;| G102555) (LETT G102556 T |List|))
        (COND
          ((NOT G102556) (HREM |$ConstructorCache| '|List|)))))))))

(DEFUN |List;| (|#1|)
  (PROG (DV$1 |dv$| $ G102554 |pv$|)
    (RETURN
      (PROGN
        (LETT DV$1 (|devaluate| |#1|) |List|)
        (LETT |dv$| (LIST '|List| DV$1) |List|)
        (LETT $ (make-array 62) |List|)
        (QSETREFV $ 0 |dv$|)
        (QSETREFV $ 3
          (LETT |pv$|
            (|buildPredVector| 0 0
              (LIST (|HasCategory| |#1| '|SetCategory|))
                (|HasCategory| |#1|
                  '|ConvertibleTo| (|InputForm|)))
              (LETT G102554
                (|HasCategory| |#1| '|OrderedSet|)
                |List|)
              (OR G102554
                (|HasCategory| |#1| '|SetCategory|))
                (|HasCategory| |#1| '|OpenMath|))
              (|HasCategory| (|Integer|) '|OrderedSet|)
              (AND (|HasCategory| |#1|
                (LIST '|Evalable|
                  (|devaluate| |#1|)))
                  (|HasCategory| |#1| '|SetCategory|)))
              (OR (AND (|HasCategory| |#1|
                (LIST '|Evalable|
                  (|devaluate| |#1|)))
                  G102554)
                (AND (|HasCategory| |#1|
                  (LIST '|Evalable|
                    (|devaluate| |#1|)))
                    (|HasCategory| |#1|
                      '|SetCategory|))))))
                |List|))
            (|haddProp| |$ConstructorCache| '|List| (LIST DV$1) (CONS 1 $))
            (|stuffDomainSlots| $)
            (QSETREFV $ 6 |#1|)
            (COND
              ((|testBitVector| |pv$| 5)
                (PROGN
                  (QSETREFV $ 25
                    (CONS (|dispatchFunction| |LIST;OMwrite;$S;6|) $))
                  (QSETREFV $ 26
                    (CONS (|dispatchFunction| |LIST;OMwrite;$BS;7|) $))

```

```

(QSETREFV $ 27
  (CONS (|dispatchFunction| |LIST;OMwrite;Omd$V;8|) $))
(QSETREFV $ 28
  (CONS (|dispatchFunction| |LIST;OMwrite;Omd$BV;9|) $))))))
(COND
  ((|testBitVector| |pv$| 1)
  (PROGN
    (QSETREFV $ 31
      (CONS (|dispatchFunction| |LIST;setUnion;3$;10|) $))
    (QSETREFV $ 33
      (CONS (|dispatchFunction|
              |LIST;setIntersection;3$;11|)
            $))
    (QSETREFV $ 36
      (CONS (|dispatchFunction| |LIST;setDifference;3$;12|)
            $))))))
(COND
  ((|testBitVector| |pv$| 2)
  (QSETREFV $ 44
    (CONS (|dispatchFunction| |LIST;convert;$If;13|) $))))
$))))

(setf (get
  (QUOTE |List|)
  (QUOTE |infovec|))
  (LIST
    (QUOTE #(
      NIL NIL NIL NIL NIL (|IndexedList| 6 (NRTEVAL 1)) (|local| |#1|)
      |LIST;nil;$;1| (|Boolean|) |LIST;null;$B;2| |LIST;cons;S2$;3|
      |LIST;append;3$;4| (|Void|) (|OpenMathDevice|) (0 . |OMputApp|)
      (|String|) (5 . |OMputSymbol|) (12 . |OMwrite|) (19 . |OMputEndApp|)
      (|OpenMathEncoding|) (24 . |OMencodingXML|) (28 . |MopenString|)
      (34 . |OMputObject|) (39 . |OMputEndObject|) (44 . |OMclose|)
      (49 . |OMwrite|) (54 . |OMwrite|) (60 . |OMwrite|) (66 . |OMwrite|)
      (73 . |concat|) (79 . |removeDuplicates|) (84 . |setUnion|)
      (90 . |member?|) (96 . |setIntersection|) (|Integer|) (102 . |elt|)
      (108 . |setDifference|) (|Symbol|) (114 . |coerce|) (|InputForm|)
      (119 . |convert|) (124 . |convert|) (|List| |$|) (129 . |convert|)
      (134 . |convert|) (|Mapping| 6 6 6) (|NonNegativeInteger|)
      (|List| 6) (|List| 49) (|Equation| 6) (|Mapping| 8 6)
      (|Mapping| 8 6 6) (|UniversalSegment| 34) (QUOTE "last")
      (QUOTE "rest") (QUOTE "first") (QUOTE "value") (|Mapping| 6 6)
      (|SingleInteger|) (|OutputForm|) (|List| 34) (|Union| 6 (QUOTE "failed"))))
    (QUOTE #(
      |setUnion| 139 |setIntersection| 145 |setDifference| 151
      |removeDuplicates| 157 |null| 162 |nil| 167 |member?| 171 |elt| 177
      |convert| 183 |cons| 188 |concat| 194 |append| 200 |OMwrite| 206))
    (QUOTE ((|shallowlyMutable| . 0) (|finiteAggregate| . 0)))
    (CONS
      (|makeByteWordVec2| 8 (QUOTE (0 0 0 0 0 0 0 0 0 3 0 0 8 4 0 0 8 1 2 4 5)))
      (CONS (QUOTE #(

```

```

|ListAggregate&| |StreamAggregate&| |ExtensibleLinearAggregate&|
|FiniteLinearAggregate&| |UnaryRecursiveAggregate&| |LinearAggregate&|
|RecursiveAggregate&| |IndexedAggregate&| |Collection&|
|HomogeneousAggregate&| |OrderedSet&| |Aggregate&| |EltableAggregate&|
|Evalable&| |SetCategory&| NIL NIL |InnerEvalable&| NIL NIL
|BasicType&| NIL))
(CONS
  (QUOTE #((|ListAggregate| 6) (|StreamAggregate| 6)
    (|ExtensibleLinearAggregate| 6) (|FiniteLinearAggregate| 6)
    (|UnaryRecursiveAggregate| 6) (|LinearAggregate| 6)
    (|RecursiveAggregate| 6) (|IndexedAggregate| 34 6) (|Collection| 6)
    (|HomogeneousAggregate| 6) (|OrderedSet|) (|Aggregate|)
    (|EltableAggregate| 34 6) (|Evalable| 6) (|SetCategory|)
    (|Type|) (|Eltable| 34 6) (|InnerEvalable| 6 6) (|CoercibleTo| 59)
    (|ConvertibleTo| 39) (|BasicType|) (|OpenMath|)))
  (|makeByteWordVec2| 44
    (QUOTE (
      1 13 12 0 14 3 13 12 0 15 15 16 3 6 12 13 0 8 17 1 13 12 0 18 0
      19 0 20 2 13 0 15 19 21 1 13 12 0 22 1 13 12 0 23 1 13 12 0 24 1 0 15
      0 25 2 0 15 0 8 26 2 0 12 13 0 27 3 0 12 13 0 8 28 2 0 0 0 0 29 1 0 0
      0 30 2 0 0 0 0 31 2 0 8 6 0 32 2 0 0 0 0 33 2 0 6 0 34 35 2 0 0 0 0 36
      1 37 0 15 38 1 39 0 37 40 1 6 39 0 41 1 39 0 42 43 1 0 39 0 44 2 1 0 0
      0 31 2 1 0 0 0 33 2 1 0 0 0 36 1 1 0 0 30 1 0 8 0 9 0 0 0 7 2 1 8 6 0
      32 2 0 6 0 34 35 1 2 39 0 44 2 0 0 6 0 10 2 0 0 0 0 29 2 0 0 0 0 11 3
      5 12 13 0 8 28 2 5 12 13 0 27 1 5 15 0 25 2 5 15 0 8 26))))))
(QUOTE |lookupIncomplete|)))

```

## 28.8 NNI.lsp BOOTSTRAP

**NNI** depends on itself. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **NNI** category which we can write into the **MID** directory. We compile the lisp code and copy the **NNI.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

### — NNI.lsp BOOTSTRAP —

```

(|/VERSIONCHECK| 2)

(SETQ |$CategoryFrame|
  (|put| '|NonNegativeInteger| '|SuperDomain| '(|Integer|)
    (|put| '(|Integer|) '|SubDomain|
      (CONS '(|NonNegativeInteger| COND ((< |#1| 0) 'NIL)
        ('T 'T))
      (DELASC '|NonNegativeInteger|

```

```

(|get| '(|Integer|) '|SubDomain|
  |$CategoryFrame|)))
|$CategoryFrame|)))

(PUT '|NNI;sup;3$;1| '|SPADreplace| 'MAX)

(DEFUN |NNI;sup;3$;1| (|x| |y| |$|) (MAX |x| |y|))

(PUT '|NNI;shift;$I$;2| '|SPADreplace| 'ASH)

(DEFUN |NNI;shift;$I$;2| (|x| |n| |$|) (ASH |x| |n|))

(DEFUN |NNI;subtractIfCan;2$U;3| (|x| |y| |$|)
  (PROG (|c|)
    (RETURN
      (SEQ
        (LETT |c| (|-| |x| |y|) |NNI;subtractIfCan;2$U;3|)
        (EXIT
          (COND
            ((|<| |c| 0) (CONS 1 "failed"))
            ((QUOTE T) (CONS 0 |c|))))))))))

(DEFUN |NonNegativeInteger| ()
  (PROG ()
    (RETURN
      (PROG (G96708)
        (RETURN
          (COND
            ((LETT G96708
              (HGET |$ConstructorCache| '|NonNegativeInteger|)
              |NonNegativeInteger|)
              (|CDRwithIncrement| (CDAR G96708))))
            'T
            (UNWIND-PROTECT
              (PROG1 (CDDAR (HPUT |$ConstructorCache|
                '|NonNegativeInteger|
                (LIST
                  (CONS NIL
                    (CONS 1 (|NonNegativeInteger;|))))))
                (LETT G96708 T |NonNegativeInteger|))
              (COND
                ((NOT G96708)
                  (HREM |$ConstructorCache| '|NonNegativeInteger|))))))))))

(DEFUN |NonNegativeInteger;| ()
  (PROG (|dv$| $ |pv$|)
    (RETURN
      (PROGN
        (LETT |dv$| '(|NonNegativeInteger|) |NonNegativeInteger|)
        (LETT $ (make-array 17) |NonNegativeInteger|)

```

```

(QSETREFV $ 0 |dv$|)
(QSETREFV $ 3
  (LETT |pv$| (|buildPredVector| 0 0 NIL)
    |NonNegativeInteger|))
(|haddProp| |$ConstructorCache| '|NonNegativeInteger| NIL
  (CONS 1 $))
(|stuffDomainSlots| $)
$)))))

(setf (get
  (QUOTE |NonNegativeInteger|)
  (QUOTE |infovec|))
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL
        (|Integer|)
        |NNI;sup;3$;1|
        |NNI;shift;$I$;2|
        (|Union| |$| (QUOTE "failed"))
        |NNI;subtractIfCan;2$U;3|
        (|Record| (|:| |quotient| |$|) (|:| |remainder| |$|))
        (|PositiveInteger|)
        (|Boolean|)
        (|NonNegativeInteger|)
        (|SingleInteger|)
        (|String|)
        (|OutputForm|)))
    (QUOTE
      #(|~|=| 0 |zero?| 6 |sup| 11 |subtractIfCan| 17 |shift| 23 |sample| 29
        |rem| 33 |recip| 39 |random| 44 |quo| 49 |one?| 55 |min| 60 |max| 66
        |latex| 72 |hash| 77 |gcd| 82 |exquo| 88 |divide| 94 |coerce| 100
        |^| 105 |Zero| 117 |One| 121 |>=| 125 |>| 131 |=| 137 |<=| 143
        |<| 149 |+| 155 |**| 161 |*| 173))
    (QUOTE ((|commutative| "*" . 0)))
    (CONS
      (|makeByteWordVec2| 1 (QUOTE (0 0 0 0 0 0 0 0 0 0 0 0)))
      (CONS
        (QUOTE
          #(NIL NIL NIL NIL NIL
            |Monoid&|
            |AbelianMonoid&|
            |OrderedSet&|
            |SemiGroup&|
            |AbelianSemiGroup&|
            |SetCategory&|
            |BasicType&|
            NIL))
        (CONS
          (QUOTE
            #((|OrderedAbelianMonoidSup|)

```



```

(|OrderedCancellationAbelianMonoid|)
(|OrderedAbelianMonoid|)
(|OrderedAbelianSemiGroup|)
(|CancellationAbelianMonoid|)
(|Monoid|)
(|AbelianMonoid|)
(|OrderedSet|)
(|SemiGroup|)
(|AbelianSemiGroup|)
(|SetCategory|)
(|BasicType|)
(|CoercibleTo| 16)))
(|makeByteWordVec2| 16
(QUOTE
  (2 0 12 0 0 1 1 0 12 0 1 2 0 0 0 0 6 2 0 8 0 0 9 2 0 0 0 5 7 0 0
   0 1 2 0 0 0 0 1 1 0 8 0 1 1 0 0 0 1 2 0 0 0 0 1 1 0 12 0 1 2 0
   0 0 0 1 2 0 0 0 0 1 1 0 15 0 1 1 0 14 0 1 2 0 0 0 0 1 2 0 8 0 0
   1 2 0 10 0 0 1 1 0 16 0 1 2 0 0 0 11 1 2 0 0 0 13 1 0 0 0 1 0 0
   0 1 2 0 12 0 0 1 2 0 12 0 0 1 2 0 12 0 0 1 2 0 12 0 0 1 2 0 12
   0 0 1 2 0 0 0 0 1 2 0 0 0 11 1 2 0 0 0 13 1 2 0 0 0 0 1 2 0 0
   11 0 1 2 0 0 13 0 1))))))
(QUOTE |lookupComplete|)))

(setf (get (QUOTE |NonNegativeInteger|) (QUOTE NILADIC)) T)

```

## 28.9 OUTFORM.lsp BOOTSTRAP

**OUTFORM** depends on itself. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **OUTFORM** category which we can write into the **MID** directory. We compile the lisp code and copy the **OUTFORM.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

### — OUTFORM.lsp BOOTSTRAP —

```

(|/VERSIONCHECK| 2)

(PUT (QUOTE |OUTFORM;print;$V;1|) (QUOTE |SPADreplace|) (QUOTE |mathprint|))

(DEFUN |OUTFORM;print;$V;1| (|x| |$|) (|mathprint| |x|))

(DEFUN |OUTFORM;message;$S;2| (|s| |$|)
  (COND
    ((SPADCALL |s| (QREFELT |$| 11)) (SPADCALL (QREFELT |$| 12)))
  ))

```

```

((QUOTE T) |s|)))

(DEFUN |OUTFORM;messagePrint;SV;3| (|s| |$|)
  (SPADCALL (SPADCALL |s| (QREFELT |$| 13)) (QREFELT |$| 8)))

(PUT (QUOTE |OUTFORM;=;2$B;4|) (QUOTE |SPADreplace|) (QUOTE EQUAL))

(DEFUN |OUTFORM;=;2$B;4| (|a| |b| |$|) (EQUAL |a| |b|))

(DEFUN |OUTFORM;=;3$;5| (|a| |b| |$|)
  (LIST (|OUTFORM;sform| "=" |$|) |a| |b|))

(PUT
  (QUOTE |OUTFORM;coerce;2$;6|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM (|a|) |a|)))

(DEFUN |OUTFORM;coerce;2$;6| (|a| |$|) |a|)

(PUT
  (QUOTE |OUTFORM;outputForm;I$;7|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM (|n|) |n|)))

(DEFUN |OUTFORM;outputForm;I$;7| (|n| |$|) |n|)

(PUT
  (QUOTE |OUTFORM;outputForm;S$;8|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM (|e|) |e|)))

(DEFUN |OUTFORM;outputForm;S$;8| (|e| |$|) |e|)

(PUT
  (QUOTE |OUTFORM;outputForm;Df$;9|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM (|f|) |f|)))

(DEFUN |OUTFORM;outputForm;Df$;9| (|f| |$|) |f|)

(PUT (QUOTE |OUTFORM;sform|)(QUOTE |SPADreplace|) (QUOTE (XLAM (|s|) |s|)))

(DEFUN |OUTFORM;sform| (|s| |$|) |s|)

(PUT (QUOTE |OUTFORM;eform|) (QUOTE |SPADreplace|) (QUOTE (XLAM (|e|) |e|)))

(DEFUN |OUTFORM;eform| (|e| |$|) |e|)

(PUT (QUOTE |OUTFORM;ifform|) (QUOTE |SPADreplace|) (QUOTE (XLAM (|n|) |n|)))

```

```

(DEFUN |OUTFORM;iform| (|n| |$|) |n|)

(DEFUN |OUTFORM;outputForm;$;13| (|s| |$|)
  (|OUTFORM;sform|
    (SPADCALL
      (SPADCALL (QREFELT |$| 26))
      (SPADCALL |s| (SPADCALL (QREFELT |$| 26)) (QREFELT |$| 27))
      (QREFELT |$| 28))
    |$|))

(PUT
  (QUOTE |OUTFORM;width;$I;14|)
  (QUOTE |SPADreplace|)
  (QUOTE |outformWidth|))

(DEFUN |OUTFORM;width;$I;14| (|a| |$|) (|outformWidth| |a|))

(PUT (QUOTE |OUTFORM;height;$I;15|) (QUOTE |SPADreplace|) (QUOTE |height|))

(DEFUN |OUTFORM;height;$I;15| (|a| |$|) (|height| |a|))

(PUT
  (QUOTE |OUTFORM;subHeight;$I;16|)
  (QUOTE |SPADreplace|)
  (QUOTE |subspan|))

(DEFUN |OUTFORM;subHeight;$I;16| (|a| |$|) (|subspan| |a|))

(PUT
  (QUOTE |OUTFORM;superHeight;$I;17|)
  (QUOTE |SPADreplace|)
  (QUOTE |superspan|))

(DEFUN |OUTFORM;superHeight;$I;17| (|a| |$|) (|superspan| |a|))

(PUT
  (QUOTE |OUTFORM;height;I;18|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM NIL 20)))

(DEFUN |OUTFORM;height;I;18| (|$|) 20)

(PUT
  (QUOTE |OUTFORM;width;I;19|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM NIL 66)))

(DEFUN |OUTFORM;width;I;19| (|$|) 66)

(DEFUN |OUTFORM;center;$I;$;20| (|a| |w| |$|)

```

```

(SPADCALL
  (SPADCALL
    (QUOTIENT2 (|-| |w| (SPADCALL |a| (QREFELT |$| 30))) 2)
    (QREFELT |$| 36))
  |a|
  (QREFELT |$| 37)))

(DEFUN |OUTFORM;left;$I$;21| (|a| |w| |$|)
  (SPADCALL
    |a|
    (SPADCALL (|-| |w| (SPADCALL |a| (QREFELT |$| 30))) (QREFELT |$| 36))
    (QREFELT |$| 37)))

(DEFUN |OUTFORM;right;$I$;22| (|a| |w| |$|)
  (SPADCALL
    (SPADCALL (|-| |w| (SPADCALL |a| (QREFELT |$| 30))) (QREFELT |$| 36))
    |a|
    (QREFELT |$| 37)))

(DEFUN |OUTFORM;center;2$;23| (|a| |$|)
  (SPADCALL |a| (SPADCALL (QREFELT |$| 35)) (QREFELT |$| 38)))

(DEFUN |OUTFORM;left;2$;24| (|a| |$|)
  (SPADCALL |a| (SPADCALL (QREFELT |$| 35)) (QREFELT |$| 39)))

(DEFUN |OUTFORM;right;2$;25| (|a| |$|)
  (SPADCALL |a| (SPADCALL (QREFELT |$| 35)) (QREFELT |$| 40)))

(DEFUN |OUTFORM;vspace;I$;26| (|n| |$|)
  (COND
    ((EQL |n| 0) (SPADCALL (QREFELT |$| 12)))
    ((QUOTE T)
      (SPADCALL
        (|OUTFORM;sform| " " |$|)
        (SPADCALL (|-| |n| 1) (QREFELT |$| 44))
        (QREFELT |$| 45)))))

(DEFUN |OUTFORM;hspace;I$;27| (|n| |$|)
  (COND
    ((EQL |n| 0) (SPADCALL (QREFELT |$| 12)))
    ((QUOTE T) (|OUTFORM;sform| (|fillerSpaces| |n|) |$|))))

(DEFUN |OUTFORM;rspace;2I$;28| (|n| |m| |$|)
  (COND
    ((OR (EQL |n| 0) (EQL |m| 0)) (SPADCALL (QREFELT |$| 12)))
    ((QUOTE T)
      (SPADCALL
        (SPADCALL |n| (QREFELT |$| 36))
        (SPADCALL |n| (|-| |m| 1) (QREFELT |$| 46))
        (QREFELT |$| 45)))))

```

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(DEFUN |OUTFORM;matrix;L$;29| (|l| $)
  (PROG (G82748 |l| G82749 |lv|)
    (RETURN
      (SEQ (LETT |lv|
        (PROGN
          (LETT G82748 NIL |OUTFORM;matrix;L$;29|)
          (SEQ (LETT |l| NIL |OUTFORM;matrix;L$;29|)
            (LETT G82749 |l| |OUTFORM;matrix;L$;29|) G190
            (COND
              ((OR (ATOM G82749)
                (PROGN
                  (LETT |l| (CAR G82749)
                    |OUTFORM;matrix;L$;29|)
                    NIL))
                (GO G191)))
              (SEQ (EXIT (LETT G82748
                (CONS (LIST2VEC |l|) G82748)
                |OUTFORM;matrix;L$;29|)))
                (LETT G82749 (CDR G82749)
                  |OUTFORM;matrix;L$;29|)
                (GO G190) G191 (EXIT (NREVERSEO G82748))))
              |OUTFORM;matrix;L$;29|)
            (EXIT (CONS (|OUTFORM;eform| 'MATRIX $) (LIST2VEC |lv|)))))))

(DEFUN |OUTFORM;pile;L$;30| (|l| |$|)
  (CONS (|OUTFORM;eform| (QUOTE SC) |$|) |l|))

(DEFUN |OUTFORM;commaSeparate;L$;31| (|l| |$|)
  (CONS (|OUTFORM;eform| (QUOTE AGGLST) |$|) |l|))

(DEFUN |OUTFORM;semicolonSeparate;L$;32| (|l| |$|)
  (CONS (|OUTFORM;eform| (QUOTE AGGSET) |$|) |l|))

(DEFUN |OUTFORM;blankSeparate;L$;33| (|l| $)
  (PROG (|c| |u| G82757 |l|)
    (RETURN
      (SEQ (LETT |c| (|OUTFORM;eform| 'CONCATB $)
        |OUTFORM;blankSeparate;L$;33|)
        (LETT |l| NIL |OUTFORM;blankSeparate;L$;33|)
        (SEQ (LETT |u| NIL |OUTFORM;blankSeparate;L$;33|)
          (LETT G82757 (SPADCALL |l| (QREFELT $ 53))
            |OUTFORM;blankSeparate;L$;33|)
          G190
          (COND
            ((OR (ATOM G82757)
              (PROGN
                (LETT |u| (CAR G82757)
                  |OUTFORM;blankSeparate;L$;33|)
                  NIL))
              (GO G191)))
            (SEQ (EXIT (LETT G82757
              (CONS (LIST2VEC |l|) G82757)
              |OUTFORM;blankSeparate;L$;33|)))
              (LETT G82757 (CDR G82757)
                |OUTFORM;blankSeparate;L$;33|)
              (GO G190) G191 (EXIT (NREVERSEO G82757))))
              |OUTFORM;blankSeparate;L$;33|)
            (EXIT (CONS (|OUTFORM;eform| 'CONCATB $) (LIST2VEC |l|)))))))

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        (GO G191)))
      (SEQ (EXIT (COND
        ((EQCAR |u| |c|)
          (LETT |l1|
            (SPADCALL (CDR |u|) |l1|
              (QREFELT $ 54))
            |OUTFORM;blankSeparate;L$;33|))
        ('T
          (LETT |l1| (CONS |u| |l1|)
            |OUTFORM;blankSeparate;L$;33|))))))
      (LETT G82757 (CDR G82757)
        |OUTFORM;blankSeparate;L$;33|)
      (GO G190) G191 (EXIT NIL))
    (EXIT (CONS |c| |l1|))))))

(DEFUN |OUTFORM;brace;2$;34| (|a| |$|)
  (LIST (|OUTFORM;iform| (QUOTE BRACE) |$|) |a|))

(DEFUN |OUTFORM;brace;L$;35| (|l| |$|)
  (SPADCALL (SPADCALL |l| (QREFELT |$| 51)) (QREFELT |$| 56)))

(DEFUN |OUTFORM;bracket;2$;36| (|a| |$|)
  (LIST (|OUTFORM;iform| (QUOTE BRACKET) |$|) |a|))

(DEFUN |OUTFORM;bracket;L$;37| (|l| |$|)
  (SPADCALL (SPADCALL |l| (QREFELT |$| 51)) (QREFELT |$| 58)))

(DEFUN |OUTFORM;paren;2$;38| (|a| |$|)
  (LIST (|OUTFORM;iform| (QUOTE PAREN) |$|) |a|))

(DEFUN |OUTFORM;paren;L$;39| (|l| |$|)
  (SPADCALL (SPADCALL |l| (QREFELT |$| 51)) (QREFELT |$| 60)))

(DEFUN |OUTFORM;sub;3$;40| (|a| |b| |$|)
  (LIST (|OUTFORM;iform| (QUOTE SUB) |$|) |a| |b|))

(DEFUN |OUTFORM;super;3$;41| (|a| |b| |$|)
  (LIST
    (|OUTFORM;iform| (QUOTE SUPERSUB) |$|)
    |a|
    (|OUTFORM;sform| " " |$|) |b|))

(DEFUN |OUTFORM;presub;3$;42| (|a| |b| |$|)
  (LIST
    (|OUTFORM;iform| (QUOTE SUPERSUB) |$|)
    |a|
    (|OUTFORM;sform| " " |$|)
    (|OUTFORM;sform| " " |$|)
    (|OUTFORM;sform| " " |$|)
    |b|))

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(DEFUN |OUTFORM;presuper;3$;43| (|a| |b| |$|)
  (LIST
    (|OUTFORM;eform| (QUOTE SUPERSUB) |$|)
    |a|
    (|OUTFORM;sform| " " |$|)
    (|OUTFORM;sform| " " |$|)
    |b|))

(DEFUN |OUTFORM;scripts;$L$;44| (|a| |l| |$|)
  (COND
    ((SPADCALL |l| (QREFELT |$| 66)) |a|)
    ((SPADCALL (SPADCALL |l| (QREFELT |$| 67)) (QREFELT |$| 66))
      (SPADCALL |a| (SPADCALL |l| (QREFELT |$| 68)) (QREFELT |$| 62)))
    ((QUOTE T) (CONS (|OUTFORM;eform| (QUOTE SUPERSUB) |$|) (CONS |a| |l|))))))

(DEFUN |OUTFORM;supersub;$L$;45| (|a| |l| |$|)
  (SEQ
    (COND
      ((ODDP (SPADCALL |l| (QREFELT |$| 71)))
        (LETT |l|
          (SPADCALL |l| (LIST (SPADCALL (QREFELT |$| 12)) (QREFELT |$| 73))
            |OUTFORM;supersub;$L$;45|)))
        (EXIT (CONS (|OUTFORM;eform| (QUOTE ALTSUPERSUB) |$|) (CONS |a| |l|))))))

(DEFUN |OUTFORM;hconcat;3$;46| (|a| |b| |$|)
  (LIST (|OUTFORM;eform| (QUOTE CONCAT) |$|) |a| |b|))

(DEFUN |OUTFORM;hconcat;$L$;47| (|l| |$|)
  (CONS (|OUTFORM;eform| (QUOTE CONCAT) |$|) |l|))

(DEFUN |OUTFORM;vconcat;3$;48| (|a| |b| |$|)
  (LIST (|OUTFORM;eform| (QUOTE VCONCAT) |$|) |a| |b|))

(DEFUN |OUTFORM;vconcat;$L$;49| (|l| |$|)
  (CONS (|OUTFORM;eform| (QUOTE VCONCAT) |$|) |l|))

(DEFUN |OUTFORM;^=;3$;50| (|a| |b| |$|)
  (LIST (|OUTFORM;sform| "^=" |$|) |a| |b|))

(DEFUN |OUTFORM;<;3$;51| (|a| |b| |$|)
  (LIST (|OUTFORM;sform| "<" |$|) |a| |b|))

(DEFUN |OUTFORM;>;3$;52| (|a| |b| |$|)
  (LIST (|OUTFORM;sform| ">" |$|) |a| |b|))

(DEFUN |OUTFORM;<=;3$;53| (|a| |b| |$|)
  (LIST (|OUTFORM;sform| "<=" |$|) |a| |b|))

(DEFUN |OUTFORM;>=;3$;54| (|a| |b| |$|)

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(LIST (|OUTFORM;sform| ">=" |$|) |a| |b|))

(DEFUN |OUTFORM;+;3$;55| (|a| |b| |$|)
  (LIST (|OUTFORM;sform| "+" |$|) |a| |b|))

(DEFUN |OUTFORM;-;3$;56| (|a| |b| |$|)
  (LIST (|OUTFORM;sform| "-" |$|) |a| |b|))

(DEFUN |OUTFORM;-;2$;57| (|a| |$|)
  (LIST (|OUTFORM;sform| "-" |$|) |a|))

(DEFUN |OUTFORM;*;3$;58| (|a| |b| |$|)
  (LIST (|OUTFORM;sform| "*" |$|) |a| |b|))

(DEFUN |OUTFORM;/;3$;59| (|a| |b| |$|)
  (LIST (|OUTFORM;sform| "/" |$|) |a| |b|))

(DEFUN |OUTFORM;**;3$;60| (|a| |b| |$|)
  (LIST (|OUTFORM;sform| "**" |$|) |a| |b|))

(DEFUN |OUTFORM;div;3$;61| (|a| |b| |$|)
  (LIST (|OUTFORM;sform| "div" |$|) |a| |b|))

(DEFUN |OUTFORM;rem;3$;62| (|a| |b| |$|)
  (LIST (|OUTFORM;sform| "rem" |$|) |a| |b|))

(DEFUN |OUTFORM;quo;3$;63| (|a| |b| |$|)
  (LIST (|OUTFORM;sform| "quo" |$|) |a| |b|))

(DEFUN |OUTFORM;exquo;3$;64| (|a| |b| |$|)
  (LIST (|OUTFORM;sform| "exquo" |$|) |a| |b|))

(DEFUN |OUTFORM;and;3$;65| (|a| |b| |$|)
  (LIST (|OUTFORM;sform| "and" |$|) |a| |b|))

(DEFUN |OUTFORM;or;3$;66| (|a| |b| |$|)
  (LIST (|OUTFORM;sform| "or" |$|) |a| |b|))

(DEFUN |OUTFORM;not;2$;67| (|a| |$|)
  (LIST (|OUTFORM;sform| "not" |$|) |a|))

(DEFUN |OUTFORM;SEGMENT;3$;68| (|a| |b| |$|)
  (LIST (|OUTFORM;eform| (QUOTE SEGMENT) |$|) |a| |b|))

(DEFUN |OUTFORM;SEGMENT;2$;69| (|a| |$|)
  (LIST (|OUTFORM;eform| (QUOTE SEGMENT) |$|) |a|))

(DEFUN |OUTFORM;binomial;3$;70| (|a| |b| |$|)
  (LIST (|OUTFORM;eform| (QUOTE BINOMIAL) |$|) |a| |b|))

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(DEFUN |OUTFORM;empty;$;71| (|$|)
  (LIST (|OUTFORM;eform| (QUOTE NOTHING) |$|)))

(DEFUN |OUTFORM;infix?;$B;72| (|a| $)
  (PROG (G82802 |e|)
    (RETURN
      (SEQ (EXIT (SEQ (LETT |e|
        (COND
          ((IDENTP |a|) |a|)
          ((STRINGP |a|) (INTERN |a|))
          ('T
            (PROGN
              (LETT G82802 'NIL
                |OUTFORM;infix?;$B;72|)
              (GO G82802))))
          |OUTFORM;infix?;$B;72|)
        (EXIT (COND ((GET |e| 'INFXOP) 'T) ('T 'NIL))))))
      G82802 (EXIT G82802))))))

(PUT (QUOTE |OUTFORM;elt;$L$;73|) (QUOTE |SPADreplace|) (QUOTE CONS))

(DEFUN |OUTFORM;elt;$L$;73| (|a| |l| |$|) (CONS |a| |l|))

(DEFUN |OUTFORM;prefix;$L$;74| (|a| |l| |$|)
  (COND
    ((NULL (SPADCALL |a| (QREFELT |$| 98))) (CONS |a| |l|))
    ((QUOTE T)
      (SPADCALL |a|
        (SPADCALL (SPADCALL |l| (QREFELT |$| 51)) (QREFELT |$| 60))
        (QREFELT |$| 37)))))

(DEFUN |OUTFORM;infix;$L$;75| (|a| |l| |$|)
  (COND
    ((SPADCALL |l| (QREFELT |$| 66)) (SPADCALL (QREFELT |$| 12)))
    ((SPADCALL (SPADCALL |l| (QREFELT |$| 67)) (QREFELT |$| 66))
      (SPADCALL |l| (QREFELT |$| 68)))
    ((SPADCALL |a| (QREFELT |$| 98)) (CONS |a| |l|))
    ((QUOTE T)
      (SPADCALL
        (LIST
          (SPADCALL |l| (QREFELT |$| 68))
          |a|
          (SPADCALL |a| (SPADCALL |l| (QREFELT |$| 101)) (QREFELT |$| 102)))
        (QREFELT |$| 75)))))

(DEFUN |OUTFORM;infix;4$;76| (|a| |b| |c| |$|)
  (COND
    ((SPADCALL |a| (QREFELT |$| 98)) (LIST |a| |b| |c|))
    ((QUOTE T) (SPADCALL (LIST |b| |a| |c|) (QREFELT |$| 75)))))

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(DEFUN |OUTFORM;postfix;3$;77| (|a| |b| |$|)
  (SPADCALL |b| |a| (QREFELT |$| 37)))

(DEFUN |OUTFORM;string;2$;78| (|a| |$|)
  (LIST (|OUTFORM;eform| (QUOTE STRING) |$|) |a|))

(DEFUN |OUTFORM;quote;2$;79| (|a| |$|)
  (LIST (|OUTFORM;eform| (QUOTE QUOTE) |$|) |a|))

(DEFUN |OUTFORM;overbar;2$;80| (|a| |$|)
  (LIST (|OUTFORM;eform| (QUOTE OVERBAR) |$|) |a|))

(DEFUN |OUTFORM;dot;2$;81| (|a| |$|)
  (SPADCALL |a| (|OUTFORM;sform| "." |$|) (QREFELT |$| 63)))

(DEFUN |OUTFORM;prime;2$;82| (|a| |$|)
  (SPADCALL |a| (|OUTFORM;sform| "," |$|) (QREFELT |$| 63)))

(DEFUN |OUTFORM;dot;$Nni$;83| (|a| |nn| |$|)
  (PROG (|s|)
    (RETURN
      (SEQ
        (LETT |s|
          (|MAKE-FULL-CVEC| |nn| (SPADCALL "." (QREFELT |$| 110)))
          |OUTFORM;dot;$Nni$;83|)
        (EXIT (SPADCALL |a| (|OUTFORM;sform| |s| |$|) (QREFELT |$| 63)))))))

(DEFUN |OUTFORM;prime;$Nni$;84| (|a| |nn| |$|)
  (PROG (|s|)
    (RETURN
      (SEQ
        (LETT |s|
          (|MAKE-FULL-CVEC| |nn| (SPADCALL "," (QREFELT |$| 110)))
          |OUTFORM;prime;$Nni$;84|)
        (EXIT (SPADCALL |a| (|OUTFORM;sform| |s| |$|) (QREFELT |$| 63)))))))

(DEFUN |OUTFORM;overlabel;3$;85| (|a| |b| |$|)
  (LIST (|OUTFORM;eform| (QUOTE OVERLABEL) |$|) |a| |b|))

(DEFUN |OUTFORM;box;2$;86| (|a| |$|)
  (LIST (|OUTFORM;eform| (QUOTE BOX) |$|) |a|))

(DEFUN |OUTFORM;zag;3$;87| (|a| |b| |$|)
  (LIST (|OUTFORM;eform| (QUOTE ZAG) |$|) |a| |b|))

(DEFUN |OUTFORM;root;2$;88| (|a| |$|)
  (LIST (|OUTFORM;eform| (QUOTE ROOT) |$|) |a|))

(DEFUN |OUTFORM;root;3$;89| (|a| |b| |$|)
  (LIST (|OUTFORM;eform| (QUOTE ROOT) |$|) |a| |b|))

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(DEFUN |OUTFORM;over;3$;90| (|a| |b| |$|)
  (LIST (|OUTFORM;eform| (QUOTE OVER) |$|) |a| |b|))

(DEFUN |OUTFORM;slash;3$;91| (|a| |b| |$|)
  (LIST (|OUTFORM;eform| (QUOTE SLASH) |$|) |a| |b|))

(DEFUN |OUTFORM;assign;3$;92| (|a| |b| |$|)
  (LIST (|OUTFORM;eform| (QUOTE LET) |$|) |a| |b|))

(DEFUN |OUTFORM;label;3$;93| (|a| |b| |$|)
  (LIST (|OUTFORM;eform| (QUOTE EQUATNUM) |$|) |a| |b|))

(DEFUN |OUTFORM;rarrow;3$;94| (|a| |b| |$|)
  (LIST (|OUTFORM;eform| (QUOTE TAG) |$|) |a| |b|))

(DEFUN |OUTFORM;differentiate;$Nni$;95| (|a| |nn| $)
  (PROG (G82832 |r| |s|)
    (RETURN
      (SEQ (COND
        ((ZEROP |nn|) |a|)
        ((< |nn| 4) (SPADCALL |a| |nn| (QREFELT $ 112)))
        ('T
          (SEQ (LETT |r|
            (SPADCALL
              (PROG1 (LETT G82832 |nn|
                |OUTFORM;differentiate;$Nni$;95|)
                (|check-subtype| (> G82832 0)
                  '(|PositiveInteger|) G82832))
              (QREFELT $ 125))
              |OUTFORM;differentiate;$Nni$;95|)
            (LETT |s| (SPADCALL |r| (QREFELT $ 126))
              |OUTFORM;differentiate;$Nni$;95|)
            (EXIT (SPADCALL |a|
              (SPADCALL (|OUTFORM;sform| |s| $)
                (QREFELT $ 60))
                (QREFELT $ 63))))))))))

(DEFUN |OUTFORM;sum;2$;96| (|a| |$|)
  (LIST (|OUTFORM;eform| (QUOTE SIGMA) |$|) (SPADCALL (QREFELT |$| 12)) |a|))

(DEFUN |OUTFORM;sum;3$;97| (|a| |b| |$|)
  (LIST (|OUTFORM;eform| (QUOTE SIGMA) |$|) |b| |a|))

(DEFUN |OUTFORM;sum;4$;98| (|a| |b| |c| |$|)
  (LIST (|OUTFORM;eform| (QUOTE SIGMA2) |$|) |b| |c| |a|))

(DEFUN |OUTFORM;prod;2$;99| (|a| |$|)
  (LIST (|OUTFORM;eform| (QUOTE PI) |$|) (SPADCALL (QREFELT |$| 12)) |a|))

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(DEFUN |OUTFORM;prod;3$;100| (|a| |b| |$|)
  (LIST (|OUTFORM;eform| (QUOTE PI) |$|) |b| |a|))

(DEFUN |OUTFORM;prod;4$;101| (|a| |b| |c| |$|)
  (LIST (|OUTFORM;eform| (QUOTE PI2) |$|) |b| |c| |a|))

(DEFUN |OUTFORM;int;2$;102| (|a| |$|)
  (LIST
    (|OUTFORM;eform| (QUOTE INTSIGN) |$|)
    (SPADCALL (QREFELT |$| 12))
    (SPADCALL (QREFELT |$| 12))
    |a|))

(DEFUN |OUTFORM;int;3$;103| (|a| |b| |$|)
  (LIST
    (|OUTFORM;eform| (QUOTE INTSIGN) |$|)
    |b|
    (SPADCALL (QREFELT |$| 12))
    |a|))

(DEFUN |OUTFORM;int;4$;104| (|a| |b| |c| |$|)
  (LIST (|OUTFORM;eform| (QUOTE INTSIGN) |$|) |b| |c| |a|))

(DEFUN |OutputForm| ()
  (PROG ()
    (RETURN
      (PROG (G82846)
        (RETURN
          (COND
            ((LETT G82846 (HGET |$ConstructorCache| '|OutputForm|)
              |OutputForm|)
              (|CDRwithIncrement| (CDAR G82846)))
            ('T
              (UNWIND-PROTECT
                (PROG1 (CDDAR (HPUT |$ConstructorCache| '|OutputForm|
                  (LIST
                    (CONS NIL (CONS 1 (|OutputForm;|))))))
                  (LETT G82846 T |OutputForm|))
                (COND
                  ((NOT G82846)
                    (HREM |$ConstructorCache| '|OutputForm|))))))))))

(DEFUN |OutputForm;| ()
  (PROG (|dv$| $ |pv$|)
    (RETURN
      (PROGN
        (LETT |dv$| '(|OutputForm|) |OutputForm|)
        (LETT $ (make-array 138) |OutputForm|)
        (QSETREFV $ 0 |dv$|)
        (QSETREFV $ 3

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        (LETT |pv$| (|buildPredVector| 0 0 NIL) |OutputForm|))
        (|haddProp| |$ConstructorCache| ' |OutputForm| NIL (CONS 1 $))
        (|stuffDomainSlots| $)
        (QSETREFV $ 6 (|List| $))
        $))))

(setf (get
      (QUOTE |OutputForm|)
      (QUOTE |infovec|))
      (LIST
        (QUOTE #(
          NIL NIL NIL NIL NIL NIL (QUOTE |Rep|) (|Void|) |OUTFORM;print;$V;1|
          (|Boolean|) (|String|) (0 . |empty?|) |OUTFORM;empty;$;71|
          |OUTFORM;message;$;2| |OUTFORM;messagePrint;$V;3|
          |OUTFORM;=;2$B;4| |OUTFORM;=;3$;5| (|OutputForm|)
          |OUTFORM;coerce;2$;6| (|Integer|) |OUTFORM;outputForm;$;7|
          (|Symbol|) |OUTFORM;outputForm;$;8| (|DoubleFloat|)
          |OUTFORM;outputForm;$;9| (|Character|) (5 . |quote|)
          (9 . |concat|) (15 . |concat|) |OUTFORM;outputForm;$;13|
          |OUTFORM;width;$;14| |OUTFORM;height;$;15|
          |OUTFORM;subHeight;$;16| |OUTFORM;superHeight;$;17|
          |OUTFORM;height;$;18| |OUTFORM;width;$;19| |OUTFORM;hspace;$;27|
          |OUTFORM;hconcat;3$;46| |OUTFORM;center;$;20|
          |OUTFORM;left;$;21| |OUTFORM;right;$;22| |OUTFORM;center;2$;23|
          |OUTFORM;left;2$;24| |OUTFORM;right;2$;25| |OUTFORM;vspace;$;26|
          |OUTFORM;vconcat;3$;48| |OUTFORM;rspace;2I$;28| (|List| 49)
          |OUTFORM;matrix;$;29| (|List| |$|) |OUTFORM;pil;$;30|
          |OUTFORM;commaSeparate;$;31| |OUTFORM;semicolonSeparate;$;32|
          (21 . |reverse|) (26 . |append|) |OUTFORM;blankSeparate;$;33|
          |OUTFORM;brace;2$;34| |OUTFORM;brace;$;35| |OUTFORM;bracket;2$;36|
          |OUTFORM;bracket;$;37| |OUTFORM;paren;2$;38| |OUTFORM;paren;$;39|
          |OUTFORM;sub;3$;40| |OUTFORM;super;3$;41| |OUTFORM;presub;3$;42|
          |OUTFORM;presuper;3$;43| (32 . |null|) (37 . |rest|) (42 . |first|)
          |OUTFORM;scripts;$;44| (|NonNegativeInteger|) (47 . |#|)
          (|List| |$$|) (52 . |append|) |OUTFORM;supersub;$;45|
          |OUTFORM;hconcat;$;47| |OUTFORM;vconcat;$;49| |OUTFORM;^=;3$;50|
          |OUTFORM;<;3$;51| |OUTFORM;>;3$;52| |OUTFORM;<=;3$;53|
          |OUTFORM;>=;3$;54| |OUTFORM;+;3$;55| |OUTFORM;-;3$;56|
          |OUTFORM;-;2$;57| |OUTFORM;*;3$;58| |OUTFORM;/;3$;59|
          |OUTFORM;**;3$;60| |OUTFORM;div;3$;61| |OUTFORM;rem;3$;62|
          |OUTFORM;quo;3$;63| |OUTFORM;exquo;3$;64| |OUTFORM;and;3$;65|
          |OUTFORM;or;3$;66| |OUTFORM;not;2$;67| |OUTFORM;SEGMENT;3$;68|
          |OUTFORM;SEGMENT;2$;69| |OUTFORM;binomial;3$;70|
          |OUTFORM;infix?;$;72| |OUTFORM;elt;$;73| |OUTFORM;prefix;$;74|
          (58 . |rest|) |OUTFORM;infix;$;75| |OUTFORM;infix;4$;76|
          |OUTFORM;postfix;3$;77| |OUTFORM;string;2$;78| |OUTFORM;quote;2$;79|
          |OUTFORM;overbar;2$;80| |OUTFORM;dot;2$;81| |OUTFORM;prime;2$;82|
          (63 . |char|) |OUTFORM;dot;$Nni$;83| |OUTFORM;prime;$Nni$;84|
          |OUTFORM;overlabel;3$;85| |OUTFORM;box;2$;86| |OUTFORM;zag;3$;87|
          |OUTFORM;root;2$;88| |OUTFORM;root;3$;89| |OUTFORM;over;3$;90|

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```

|OUTFORM;slash;3$;91| |OUTFORM;assign;3$;92|
|OUTFORM;label;3$;93| |OUTFORM;rarrow;3$;94| (|PositiveInteger|)
(|NumberFormats|) (68 . |FormatRoman|) (73 . |lowerCase|)
|OUTFORM;differentiate;$Nni$;95| |OUTFORM;sum;2$;96|
|OUTFORM;sum;3$;97| |OUTFORM;sum;4$;98| |OUTFORM;prod;2$;99|
|OUTFORM;prod;3$;100| |OUTFORM;prod;4$;101| |OUTFORM;int;2$;102|
|OUTFORM;int;3$;103| |OUTFORM;int;4$;104| (|SingleInteger|))
(QUOTE #(|^|=| 78 |zag| 84 |width| 90 |vspace| 99 |vconcat| 104
|supersub| 115 |superHeight| 121 |super| 126 |sum| 132 |subHeight|
150 |sub| 155 |string| 161 |slash| 166 |semicolonSeparate| 172
|scripts| 177 |rspace| 183 |root| 189 |right| 200 |rem| 211
|rarrow| 217 |quote| 223 |quo| 228 |prod| 234 |print| 252
|prime| 257 |presuper| 268 |presub| 274 |prefix| 280 |postfix|
286 |pile| 292 |paren| 297 |overlabel| 307 |overbar| 313 |over|
318 |outputForm| 324 |or| 344 |not| 350 |messagePrint| 355
|message| 360 |matrix| 365 |left| 370 |latex| 381 |label| 386
|int| 392 |infix?| 410 |infix| 415 |hspace| 428 |height| 433
|hconcat| 442 |hash| 453 |lexquo| 458 |empty| 464 |elt| 468 |dot|
474 |div| 485 |differentiate| 491 |commaSeparate| 497 |coerce|
502 |center| 507 |bracket| 518 |brace| 528 |box| 538 |blankSeparate|
543 |binomial| 548 |assign| 554 |and| 560 |^|=| 566 SEGMENT 572
|>|=| 583 |>| 589 |=| 595 |<|=| 607 |<| 613 |/>| 619 |-| 625 |+|
636 |**| 642 |*| 648))
(QUOTE NIL)
(CONS
(|makeByteWordVec2| 1 (QUOTE (0 0 0)))
(CONS
(QUOTE #(|SetCategory&| |BasicType&| NIL))
(CONS
(QUOTE #((|SetCategory|) (|BasicType|) (|CoercibleTo| 17)))
(|makeByteWordVec2| 137 (QUOTE (1 10 9 0 11 0 25 0 26 2 10 0 0 25
27 2 10 0 25 0 28 1 6 0 0 53 2 6 0 0 0 54 1 6 9 0 66 1 6 0 0 67 1
6 2 0 68 1 6 70 0 71 2 72 0 0 0 73 1 72 0 0 101 1 25 0 10 110 1 124
10 123 125 1 10 0 0 126 2 0 9 0 0 1 2 0 0 0 0 115 0 0 19 35 1 0 19
0 30 1 0 0 19 44 1 0 0 49 76 2 0 0 0 0 45 2 0 0 0 49 74 1 0 19 0
33 2 0 0 0 0 63 2 0 0 0 0 129 3 0 0 0 0 0 130 1 0 0 0 128 1 0 19
0 32 2 0 0 0 0 62 1 0 0 0 105 2 0 0 0 0 119 1 0 0 49 52 2 0 0 0
49 69 2 0 0 19 19 46 1 0 0 0 116 2 0 0 0 0 117 1 0 0 0 43 2 0 0
0 19 40 2 0 0 0 0 89 2 0 0 0 0 122 1 0 0 0 106 2 0 0 0 0 90 3 0
0 0 0 0 133 1 0 0 0 131 2 0 0 0 0 132 1 0 7 0 8 2 0 0 0 70 112 1
0 0 0 109 2 0 0 0 0 65 2 0 0 0 0 64 2 0 0 0 49 100 2 0 0 0 0 104
1 0 0 49 50 1 0 0 49 61 1 0 0 0 60 2 0 0 0 0 113 1 0 0 0 107 2 0
0 0 0 118 1 0 0 10 29 1 0 0 23 24 1 0 0 21 22 1 0 0 19 20 2 0 0
0 0 93 1 0 0 0 94 1 0 7 10 14 1 0 0 10 13 1 0 0 47 48 1 0 0 0 42
2 0 0 0 19 39 1 0 10 0 1 2 0 0 0 0 121 3 0 0 0 0 0 136 2 0 0 0 0
135 1 0 0 0 134 1 0 9 0 98 2 0 0 0 49 102 3 0 0 0 0 0 103 1 0 0
19 36 0 0 19 34 1 0 19 0 31 1 0 0 49 75 2 0 0 0 0 37 1 0 137 0 1
2 0 0 0 0 91 0 0 0 12 2 0 0 0 49 99 2 0 0 0 70 111 1 0 0 0 108 2
0 0 0 0 88 2 0 0 0 70 127 1 0 0 49 51 1 0 17 0 18 1 0 0 0 41 2 0
0 0 19 38 1 0 0 0 58 1 0 0 49 59 1 0 0 49 57 1 0 0 0 56 1 0 0 0

```



```

        (LETT G96739 T |PositiveInteger|))
      (COND
        ((NOT G96739)
          (HREM |$ConstructorCache| '|PositiveInteger|)))))))))

(DEFUN |PositiveInteger;| ()
  (PROG (|dv$| $ |pv$|)
    (RETURN
      (PROGN
        (LETT |dv$| '|PositiveInteger| |PositiveInteger|)
        (LETT $ (make-array 12) |PositiveInteger|)
        (QSETREFV $ 0 |dv$|)
        (QSETREFV $ 3
          (LETT |pv$| (|buildPredVector| 0 0 NIL) |PositiveInteger|))
        (|haddProp| |$ConstructorCache| '|PositiveInteger| NIL
          (CONS 1 $))
        (|stuffDomainSlots| $)
        $))))

(setf (get
  (QUOTE |PositiveInteger|)
  (QUOTE |infovec|))
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL
        (|NonNegativeInteger|)
        (|PositiveInteger|)
        (|Boolean|)
        (|Union| |$| (QUOTE "failed"))
        (|SingleInteger|)
        (|String|)
        (|OutputForm|)))
    (QUOTE #(|~|=| 0 |sample| 6 |recip| 10 |one?| 15 |min| 20 |max| 26
      |latex| 32 |hash| 37 |gcd| 42 |coerce| 48 |^| 53 |One| 65
      |>=| 69 |>| 75 |=| 81 |<=| 87 |<| 93 |+| 99 |**| 105 |*| 117))
    (QUOTE (((|commutative| "*") . 0)))
    (CONS
      (|makeByteWordVec2| 1 (QUOTE (0 0 0 0 0 0)))
      (CONS
        (QUOTE #(|Monoid&| |AbelianSemiGroup&| |SemiGroup&| |OrderedSet&|
          |SetCategory&| |BasicType&| NIL))
        (CONS
          (QUOTE #(
            (|Monoid|)
            (|AbelianSemiGroup|)
            (|SemiGroup|)
            (|OrderedSet|)
            (|SetCategory|)
            (|BasicType|)
            (|CoercibleTo| 11))))

```



```

(|makeByteWordVec2| 11
 (QUOTE (2 0 7 0 0 1 0 0 0 1 1 0 8 0 1 1 0 7 0 1 2 0 0 0 0 1 2 0 0 0
         0 1 1 0 10 0 1 1 0 9 0 1 2 0 0 0 0 1 1 0 11 0 1 2 0 0 0 6 1
         2 0 0 0 5 1 0 0 0 1 2 0 7 0 0 1 2 0 7 0 0 1 2 0 7 0 0 1 2 0
         7 0 0 1 2 0 7 0 0 1 2 0 0 0 0 1 2 0 0 0 6 1 2 0 0 0 5 1 2 0
         0 0 0 1 2 0 0 6 0 1))))))
(QUOTE |lookupComplete|)))

(setf (get (QUOTE |PositiveInteger|) (QUOTE NILADIC)) T)

```

## 28.11 PRIMARR.lsp BOOTSTRAP

**PRIMARR** depends on itself. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **PRIMARR** category which we can write into the **MID** directory. We compile the lisp code and copy the **PRIMARR.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

### — PRIMARR.lsp BOOTSTRAP —

```

(|/VERSIONCHECK| 2)

(PUT (QUOTE |PRIMARR;#;$Nni;1|) (QUOTE |SPADreplace|) (QUOTE QVSIZE))

(DEFUN |PRIMARR;#;$Nni;1| (|x| |$|) (QVSIZE |x|))

(PUT (QUOTE |PRIMARR;minIndex;$I;2|)
 (QUOTE |SPADreplace|) (QUOTE (XLAM (|x|) 0)))

(DEFUN |PRIMARR;minIndex;$I;2| (|x| |$|) 0)

(PUT (QUOTE |PRIMARR;empty;$;3|)
 (QUOTE |SPADreplace|) (QUOTE (XLAM NIL (make-array 0))))

(DEFUN |PRIMARR;empty;$;3| (|x| |$|) (make-array 0))

(DEFUN |PRIMARR;new;Nni$;4| (|n| |x| |$|)
 (SPADCALL (make-array |n|) |x| (QREFELT |$| 12)))

(PUT (QUOTE |PRIMARR;qelt;$IS;5|) (QUOTE |SPADreplace|) (QUOTE ELT))

(DEFUN |PRIMARR;qelt;$IS;5| (|x| |i| |$|) (ELT |x| |i|))

(PUT (QUOTE |PRIMARR;elt;$IS;6|) (QUOTE |SPADreplace|) (QUOTE ELT))

```

```

(DEFUN |PRIMARR;elt;$IS;6| (|x| |i| |$|) (ELT |x| |i|))

(PUT (QUOTE |PRIMARR;qsetelt!;$I2S;7|) (QUOTE |SPADreplace|) (QUOTE SETELT))

(DEFUN |PRIMARR;qsetelt!;$I2S;7| (|x| |i| |s| |$|) (SETELT |x| |i| |s|))

(PUT (QUOTE |PRIMARR;setelt;$I2S;8|) (QUOTE |SPADreplace|) (QUOTE SETELT))

(DEFUN |PRIMARR;setelt;$I2S;8| (|x| |i| |s| |$|) (SETELT |x| |i| |s|))

(DEFUN |PRIMARR;fill!;$S$;9| (|x| |s| $)
  (PROG (|i| G82338)
    (RETURN
      (SEQ (SEQ (LETT |i| 0 |PRIMARR;fill!;$S$;9|)
        (LETT G82338 (QVMAXINDEX |x|) |PRIMARR;fill!;$S$;9|)
        G190 (COND ((QSGREATERP |i| G82338) (GO G191)))
        (SEQ (EXIT (SETELT |x| |i| |s|)))
        (LETT |i| (QSADD1 |i|) |PRIMARR;fill!;$S$;9|) (GO G190)
        G191 (EXIT NIL))
        (EXIT |x|))))))

(DEFUN |PrimitiveArray| (G82348)
  (PROG ()
    (RETURN
      (PROG (G82349)
        (RETURN
          (COND
            ((LETT G82349
              (|lassocShiftWithFunction|
                (LIST (|devaluate| G82348))
                (HGET |$ConstructorCache| '|PrimitiveArray|'
                  '|domainEqualList|)
                |PrimitiveArray|)
              (|CDRwithIncrement| G82349))
              'T
              (UNWIND-PROTECT
                (PROG1 (|PrimitiveArray;| G82348)
                  (LETT G82349 T |PrimitiveArray|))
                (COND
                  ((NOT G82349)
                    (HREM |$ConstructorCache| '|PrimitiveArray|))))))))))

(DEFUN |PrimitiveArray;| (|#1|)
  (PROG (DV$1 |dv$| $ G82347 |pv$|)
    (RETURN
      (PROGN
        (LETT DV$1 (|devaluate| |#1|) |PrimitiveArray|)
        (LETT |dv$| (LIST '|PrimitiveArray| DV$1) |PrimitiveArray|)
        (LETT $ (make-array 35) |PrimitiveArray|)

```

```

(QSETREFV $ 0 |dv$|)
(QSETREFV $ 3
  (LETT |pv$|
    (|buildPredVector| 0 0
      (LIST (|HasCategory| |#1| '(|SetCategory|))
        (|HasCategory| |#1|
          '(|ConvertibleTo| (|InputForm|)))
        (LETT G82347
          (|HasCategory| |#1| '(|OrderedSet|))
          |PrimitiveArray|)
        (OR G82347
          (|HasCategory| |#1| '(|SetCategory|)))
        (|HasCategory| (|Integer|) '(|OrderedSet|))
        (AND (|HasCategory| |#1|
          (LIST '(|Evalable|
            (|devaluate| |#1|)))
          (|HasCategory| |#1| '(|SetCategory|)))
        (OR (AND (|HasCategory| |#1|
          (LIST '(|Evalable|
            (|devaluate| |#1|)))
          G82347)
          (AND (|HasCategory| |#1|
            (LIST '(|Evalable|
              (|devaluate| |#1|)))
            (|HasCategory| |#1|
              '(|SetCategory|))))))
          |PrimitiveArray|))
    (|haddProp| |$ConstructorCache| ' |PrimitiveArray| (LIST DV$1)
      (CONS 1 $))
    (|stuffDomainSlots| $)
    (QSETREFV $ 6 |#1|)
    $))))

(setf (get (QUOTE |PrimitiveArray|) (QUOTE |infovec|))
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (|NonNegativeInteger|)
        |PRIMARR;#;$Nni;1| (|Integer|) |PRIMARR;minIndex;$I;2|
        |PRIMARR;empty;$;3| |PRIMARR;fill!;$S$;9| |PRIMARR;new;$NniS$;4|
        |PRIMARR;qelt;$IS;5| |PRIMARR;elt;$IS;6| |PRIMARR;qsetelt!;$I2S;7|
        |PRIMARR;setelt;$I2S;8| (|Mapping| 6 6 6) (|Boolean|) (|List| 6)
        (|Equation| 6) (|List| 21) (|Mapping| 19 6) (|Mapping| 19 6 6)
        (|UniversalSegment| 9) (|Void|) (|Mapping| 6 6) (|InputForm|)
        (|OutputForm|) (|String|) (|SingleInteger|) (|List| |$|)
        (|Union| 6 (QUOTE "failed")) (|List| 9)))
    (QUOTE
      #(|~|=| 0 |swap!| 6 |sorted?| 13 |sort!| 24 |sort| 35 |size?| 46 |setelt|
        52 |select| 66 |sample| 72 |reverse!| 76 |reverse| 81 |removeDuplicates|
        86 |remove| 91 |reduce| 103 |qsetelt!| 124 |qelt| 131 |position| 137
        |parts| 156 |new| 161 |more?| 167 |minIndex| 173 |min| 178 |merge| 184

```

```

|members| 197 |member?| 202 |maxIndex| 208 |max| 213 |map!| 219 |map|
225 |less?| 238 |latex| 244 |insert| 249 |indices| 263 |index?| 268
|hash| 274 |first| 279 |find| 284 |fill!| 290 |every?| 296 |eval| 302
|eq?| 328 |entry?| 334 |entries| 340 |empty?| 345 |empty| 350 |elt| 354
|delete| 373 |count| 385 |copyInto!| 397 |copy| 404 |convert| 409
|construct| 414 |concat| 419 |coerce| 442 |any?| 447 |>=| 453 |>| 459
|=| 465 |<=| 471 |<| 477 |#| 483))
(QUOTE ((|shallowlyMutable| . 0) (|finiteAggregate| . 0)))
(CONS
  (|makeByteWordVec2| 7 (QUOTE (0 0 0 0 0 0 3 0 0 7 4 0 0 7 1 2 4)))
  (CONS
    (QUOTE #(|OneDimensionalArrayAggregate&| |FiniteLinearAggregate&|
      |LinearAggregate&| |IndexedAggregate&| |Collection&|
      |HomogeneousAggregate&| |OrderedSet&| |Aggregate&| |EtableAggregate&|
      |Evalable&| |SetCategory&| NIL NIL |InnerEvalable&| NIL NIL |BasicType&|))
    (CONS
      (QUOTE
        #(|OneDimensionalArrayAggregate| 6) (|FiniteLinearAggregate| 6)
        (|LinearAggregate| 6) (|IndexedAggregate| 9 6) (|Collection| 6)
        (|HomogeneousAggregate| 6) (|OrderedSet|) (|Aggregate|)
        (|EtableAggregate| 9 6) (|Evalable| 6) (|SetCategory|) (|Type|)
        (|Etable| 9 6) (|InnerEvalable| 6 6) (|CoercibleTo| 29)
        (|ConvertibleTo| 28) (|BasicType|)))
      (|makeByteWordVec2| 34
        (QUOTE
          (2 1 19 0 0 1 3 0 26 0 9 9 1 1 3 19 0 1 2 0 19 24 0 1 1 3 0 0 1 2 0 0
            24 0 1 1 3 0 0 1 2 0 0 24 0 1 2 0 19 0 7 1 3 0 6 0 25 6 1 3 0 6 0 9
            6 17 2 0 0 23 0 1 0 0 0 1 1 0 0 0 1 1 0 0 0 1 1 1 0 0 1 2 1 0 6 0 1
            2 0 0 23 0 1 4 1 6 18 0 6 6 1 3 0 6 18 0 6 1 2 0 6 18 0 1 3 0 6 0 9
            6 16 2 0 6 0 9 14 2 1 9 6 0 1 3 1 9 6 0 9 1 2 0 9 23 0 1 1 0 20 0 1
            2 0 0 7 6 13 2 0 19 0 7 1 1 5 9 0 10 2 3 0 0 0 1 2 3 0 0 0 1 3 0 0
            24 0 0 1 1 0 20 0 1 2 1 19 6 0 1 1 5 9 0 1 2 3 0 0 0 1 2 0 0 27 0 1
            3 0 0 18 0 0 1 2 0 0 27 0 1 2 0 19 0 7 1 1 1 30 0 1 3 0 0 0 0 9 1 3
            0 0 6 0 9 1 1 0 34 0 1 2 0 19 9 0 1 1 1 31 0 1 1 5 6 0 1 2 0 33 23
            0 1 2 0 0 0 6 12 2 0 19 23 0 1 3 6 0 0 20 20 1 2 6 0 0 21 1 3 6 0 0
            6 6 1 2 6 0 0 22 1 2 0 19 0 0 1 2 1 19 6 0 1 1 0 20 0 1 1 0 19 0 1
            0 0 0 11 2 0 0 0 25 1 2 0 6 0 9 15 3 0 6 0 9 6 1 2 0 0 0 9 1 2 0 0
            0 25 1 2 1 7 6 0 1 2 0 7 23 0 1 3 0 0 0 0 9 1 1 0 0 0 1 1 2 28 0 1
            1 0 0 20 1 1 0 0 32 1 2 0 0 6 0 1 2 0 0 0 0 1 2 0 0 0 6 1 1 1 29 0
            1 2 0 19 23 0 1 2 3 19 0 0 1 2 3 19 0 0 1 2 1 19 0 0 1 2 3 19 0 0 1
            2 3 19 0 0 1 1 0 7 0 8))))))
(QUOTE |lookupComplete|)))

```

## 28.12 REF.lsp BOOTSTRAP

REF depends on a chain of files. We need to break this cycle to build the algebra. So

we keep a cached copy of the translated **REF** category which we can write into the **MID** directory. We compile the lisp code and copy the **REF.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— **REF.lsp BOOTSTRAP** —

```
(|/VERSIONCHECK| 2)

(PUT (QUOTE |REF;=;2$B;1|) (QUOTE |SPADreplace|) (QUOTE EQ))

(DEFUN |REF;=;2$B;1| (|p| |q| |$|) (EQ |p| |q|))

(PUT (QUOTE |REF;ref;$S;2|) (QUOTE |SPADreplace|) (QUOTE LIST))

(DEFUN |REF;ref;$S;2| (|v| |$|) (LIST |v|))

(PUT (QUOTE |REF;elt;$S;3|) (QUOTE |SPADreplace|) (QUOTE QCAR))

(DEFUN |REF;elt;$S;3| (|p| |$|) (QCAR |p|))

(DEFUN |REF;setelt;$2S;4| (|p| |v| |$|) (PROGN (RPLACA |p| |v|) (QCAR |p|)))

(PUT (QUOTE |REF;deref;$S;5|) (QUOTE |SPADreplace|) (QUOTE QCAR))

(DEFUN |REF;deref;$S;5| (|p| |$|) (QCAR |p|))

(DEFUN |REF;setref;$2S;6| (|p| |v| |$|) (PROGN (RPLACA |p| |v|) (QCAR |p|)))

(DEFUN |REF;coerce;$0f;7| (|p| |$|)
  (SPADCALL
    (SPADCALL "ref" (QREFELT |$| 17))
    (LIST (SPADCALL (QCAR |p|) (QREFELT |$| 18)))
    (QREFELT |$| 20)))

(DEFUN |Reference| (G82336)
  (PROG ()
    (RETURN
      (PROG (G82337)
        (RETURN
          (COND
            ((LETT G82337
              (|lassocShiftWithFunction|
                (LIST (|devaluate| G82336))
                (HGET |$ConstructorCache| '|Reference|)
                '|domainEqualList|)
              |Reference|)
            (|CDRwithIncrement| G82337)))
    ('T
```

```

      (UNWIND-PROTECT
        (PROG1 (|Reference;| G82336)
          (LETT G82337 T |Reference|))
        (COND
          ((NOT G82337) (HREM |$ConstructorCache| ' |Reference|)))))))))

(DEFUN |Reference;| (|#1|)
  (PROG (DV$1 |dv$| $ |pv$|)
    (RETURN
      (PROGN
        (LETT DV$1 (|devaluate| |#1|) |Reference|)
        (LETT |dv$| (LIST ' |Reference| DV$1) |Reference|)
        (LETT $ (make-array 23) |Reference|)
        (QSETREFV $ 0 |dv$|)
        (QSETREFV $ 3
          (LETT |pv$|
            (|buildPredVector| 0 0
              (LIST (|HasCategory| |#1| ' (|SetCategory|)))
              |Reference|))
            (|haddProp| |$ConstructorCache| ' |Reference| (LIST DV$1)
              (CONS 1 $))
            (|stuffDomainSlots| $)
            (QSETREFV $ 6 |#1|)
            (QSETREFV $ 7 (|Record| (|:| |value| |#1|)))
            (COND
              ((|testBitVector| |pv$| 1)
                (QSETREFV $ 21
                  (CONS (|dispatchFunction| |REF;coerce;$Of;7|) $))))
              $))))))

(setf (get
  (QUOTE |Reference|)
  (QUOTE |infovec|))
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (QUOTE |Rep|) (|Boolean|)
        |REF;=;2$B;1| |REF;ref;$S;2| |REF;elt;$S;3| |REF;setelt;$2S;4|
        |REF;deref;$S;5| |REF;setref;$2S;6| (|String|) (|OutputForm|)
        (0 . |message|) (5 . |coerce|) (|List| |$|) (10 . |prefix|)
        (16 . |coerce|) (|SingleInteger|)))
    (QUOTE #(|~|=| 21 |setref| 27 |setelt| 33 |ref| 39 |latex| 44 |hash| 49
      |elt| 54 |deref| 59 |coerce| 64 |=| 69))
    (QUOTE NIL)
    (CONS
      (|makeByteWordVec2| 1 (QUOTE (1 0 1 1)))
      (CONS
        (QUOTE #(|SetCategory&| NIL |BasicType&| NIL))
        (CONS
          (QUOTE #(|SetCategory|) (|Type|) (|BasicType|) (|CoercibleTo| 16)))
          (|makeByteWordVec2| 22

```

```

(QUOTE (1 16 0 15 17 1 6 16 0 18 2 16 0 0 19 20 1 0 16 0 21 2 1 8 0
        0 1 2 0 6 0 6 14 2 0 6 0 6 12 1 0 0 6 10 1 1 15 0 1 1 1 22
        0 1 1 0 6 0 11 1 0 6 0 13 1 1 16 0 21 2 0 8 0 0 9))))))
(QUOTE |lookupComplete|)))

```

## 28.13 SINT.lsp BOOTSTRAP

### — SINT.lsp BOOTSTRAP —

```

(/VERSIONCHECK 2)

(DEFUN |SINT;writeOMSingleInt| (|dev| |x| $)
  (SEQ
    (COND
      ((QSLESSP |x| 0)
        (SEQ
          (SPADCALL |dev| (QREFELT $ 9))
          (SPADCALL |dev| "arith1" "unaryminus" (QREFELT $ 11))
          (SPADCALL |dev| (QSMINUS |x|) (QREFELT $ 13))
          (EXIT (SPADCALL |dev| (QREFELT $ 14))))))
      ((QUOTE T) (SPADCALL |dev| |x| (QREFELT $ 13))))))

(DEFUN |SINT;OMwrite;$S;2| (|x| $)
  (PROG (|sp| |dev| |s|)
    (RETURN
      (SEQ
        (LETT |s| "" |SINT;OMwrite;$S;2|)
        (LETT |sp| (OM-STRINGTOSTRINGPTR |s|) |SINT;OMwrite;$S;2|)
        (LETT |dev|
          (SPADCALL |sp| (SPADCALL (QREFELT $ 16)) (QREFELT $ 17))
          |SINT;OMwrite;$S;2|)
          (SPADCALL |dev| (QREFELT $ 18))
          (|SINT;writeOMSingleInt| |dev| |x| $)
          (SPADCALL |dev| (QREFELT $ 19))
          (SPADCALL |dev| (QREFELT $ 20))
          (LETT |s| (OM-STRINGPTRTOSTRING |sp|) |SINT;OMwrite;$S;2|)
          (EXIT |s|))))))

(DEFUN |SINT;OMwrite;$BS;3| (|x| |wholeObj| $)
  (PROG (|sp| |dev| |s|)
    (RETURN
      (SEQ
        (LETT |s| "" |SINT;OMwrite;$BS;3|)
        (LETT |sp| (OM-STRINGTOSTRINGPTR |s|) |SINT;OMwrite;$BS;3|)

```

```

(LETT |dev|
  (SPADCALL |sp| (SPADCALL (QREFELT $ 16)) (QREFELT $ 17))
  |SINT;OMwrite;$BS;3|)
(COND (|wholeObj| (SPADCALL |dev| (QREFELT $ 18))))
(|SINT;writeOMSingleInt| |dev| |x| $)
(COND (|wholeObj| (SPADCALL |dev| (QREFELT $ 19))))
(SPADCALL |dev| (QREFELT $ 20))
(LETT |s| (OM-STRINGPTRTOSTRING |sp|) |SINT;OMwrite;$BS;3|)
(EXIT |s|))))))

(DEFUN |SINT;OMwrite;Omd$V;4| (|dev| |x| $)
  (SEQ
    (SPADCALL |dev| (QREFELT $ 18))
    (|SINT;writeOMSingleInt| |dev| |x| $)
    (EXIT (SPADCALL |dev| (QREFELT $ 19)))))

(DEFUN |SINT;OMwrite;Omd$BV;5| (|dev| |x| |wholeObj| $)
  (SEQ
    (COND (|wholeObj| (SPADCALL |dev| (QREFELT $ 18))))
    (|SINT;writeOMSingleInt| |dev| |x| $)
    (EXIT (COND (|wholeObj| (SPADCALL |dev| (QREFELT $ 19)))))))

(PUT
  (QUOTE |SINT;reducedSystem;MM;6|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM (|m|) |m|)))

(DEFUN |SINT;reducedSystem;MM;6| (|m| $) |m|)

(DEFUN |SINT;coerce;$Of;7| (|x| $)
  (SPADCALL |x| (QREFELT $ 30)))

(PUT
  (QUOTE |SINT;convert;$I;8|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM (|x|) |x|)))

(DEFUN |SINT;convert;$I;8| (|x| $) |x|)

(DEFUN |SINT;*;I2$;9| (|i| |y| $)
  (QSTIMES (SPADCALL |i| (QREFELT $ 33)) |y|))

(PUT
  (QUOTE |SINT;Zero;$;10|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM NIL 0)))

(DEFUN |SINT;Zero;$;10| ($) 0)

(PUT
```



```

(QUOTE |SINT;One;$;11|)
(QUOTE |SPADreplace|)
(QUOTE (XLAM NIL 1)))

(DEFUN |SINT;One;$;11| ($) 1)

(PUT
  (QUOTE |SINT;base;$;12|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM NIL 2)))

(DEFUN |SINT;base;$;12| ($) 2)

(PUT
  (QUOTE |SINT;max;$;13|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM NIL MOST-POSITIVE-FIXNUM)))

(DEFUN |SINT;max;$;13| ($) MOST-POSITIVE-FIXNUM)

(PUT
  (QUOTE |SINT;min;$;14|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM NIL MOST-NEGATIVE-FIXNUM)))

(DEFUN |SINT;min;$;14| ($) MOST-NEGATIVE-FIXNUM)

(PUT
  (QUOTE |SINT;=;2$B;15|)
  (QUOTE |SPADreplace|)
  (QUOTE EQL))

(DEFUN |SINT;=;2$B;15| (|x| |y| $)
  (EQL |x| |y|))

(PUT
  (QUOTE |SINT;~;2$;16|)
  (QUOTE |SPADreplace|)
  (QUOTE LOGNOT))

(DEFUN |SINT;~;2$;16| (|x| $)
  (LOGNOT |x|))

(PUT
  (QUOTE |SINT;not;2$;17|)
  (QUOTE |SPADreplace|)
  (QUOTE LOGNOT))

(DEFUN |SINT;not;2$;17| (|x| $)
  (LOGNOT |x|))

```

```
(PUT
  (QUOTE |SINT;/\\;3$;18|)
  (QUOTE |SPADreplace|)
  (QUOTE LOGAND))

(DEFUN |SINT;/\\;3$;18| (|x| |y| $)
  (LOGAND |x| |y|))

(PUT
  (QUOTE |SINT;\\;/;3$;19|)
  (QUOTE |SPADreplace|)
  (QUOTE LOGIOR))

(DEFUN |SINT;\\;/;3$;19| (|x| |y| $)
  (LOGIOR |x| |y|))

(PUT
  (QUOTE |SINT;Not;2$;20|)
  (QUOTE |SPADreplace|)
  (QUOTE LOGNOT))

(DEFUN |SINT;Not;2$;20| (|x| $)
  (LOGNOT |x|))

(PUT
  (QUOTE |SINT;And;3$;21|)
  (QUOTE |SPADreplace|)
  (QUOTE LOGAND))

(DEFUN |SINT;And;3$;21| (|x| |y| $)
  (LOGAND |x| |y|))

(PUT
  (QUOTE |SINT;Or;3$;22|)
  (QUOTE |SPADreplace|)
  (QUOTE LOGIOR))

(DEFUN |SINT;Or;3$;22| (|x| |y| $)
  (LOGIOR |x| |y|))

(PUT
  (QUOTE |SINT;xor;3$;23|)
  (QUOTE |SPADreplace|)
  (QUOTE LOGXOR))

(DEFUN |SINT;xor;3$;23| (|x| |y| $)
  (LOGXOR |x| |y|))

(PUT
```

```

(QUOTE |SINT;<;2$B;24|)
(QUOTE |SPADreplace|)
(QUOTE QSLESSP))

(DEFUN |SINT;<;2$B;24| (|x| |y| $)
  (QSLESSP |x| |y|))

(PUT
  (QUOTE |SINT;inc;2$;25|)
  (QUOTE |SPADreplace|)
  (QUOTE QSADD1))

(DEFUN |SINT;inc;2$;25| (|x| $)
  (QSADD1 |x|))

(PUT
  (QUOTE |SINT;dec;2$;26|)
  (QUOTE |SPADreplace|)
  (QUOTE QSSUB1))

(DEFUN |SINT;dec;2$;26| (|x| $)
  (QSSUB1 |x|))

(PUT
  (QUOTE |SINT;-;2$;27|)
  (QUOTE |SPADreplace|)
  (QUOTE QSMINUS))

(DEFUN |SINT;-;2$;27| (|x| $)
  (QSMINUS |x|))

(PUT
  (QUOTE |SINT;+;3$;28|)
  (QUOTE |SPADreplace|)
  (QUOTE QSPLUS))

(DEFUN |SINT;+;3$;28| (|x| |y| $)
  (QSPLUS |x| |y|))

(PUT
  (QUOTE |SINT;-;3$;29|)
  (QUOTE |SPADreplace|)
  (QUOTE QSDIFFERENCE))

(DEFUN |SINT;-;3$;29| (|x| |y| $)
  (QSDIFFERENCE |x| |y|))

(PUT
  (QUOTE |SINT;*;3$;30|)
  (QUOTE |SPADreplace|)

```

```

(QUOTE QSTIMES))

(DEFUN |SINT;*;3$;30| (|x| |y| $)
  (QSTIMES |x| |y|))

(DEFUN |SINT;**;$Nni$;31| (|x| |n| $)
  (SPADCALL (EXPT |x| |n|) (QREFELT $ 33)))

(PUT
  (QUOTE |SINT;quo;3$;32|)
  (QUOTE |SPADreplace|)
  (QUOTE QSQUOTIENT))

(DEFUN |SINT;quo;3$;32| (|x| |y| $)
  (QSQUOTIENT |x| |y|))

(PUT
  (QUOTE |SINT;rem;3$;33|)
  (QUOTE |SPADreplace|)
  (QUOTE QSREMAINDER))

(DEFUN |SINT;rem;3$;33| (|x| |y| $)
  (QSREMAINDER |x| |y|))

(DEFUN |SINT;divide;2$R;34| (|x| |y| $)
  (CONS (QSQUOTIENT |x| |y|) (QSREMAINDER |x| |y|)))

(PUT (QUOTE |SINT;gcd;3$;35|)
  (QUOTE |SPADreplace|) (QUOTE GCD))

(DEFUN |SINT;gcd;3$;35| (|x| |y| $)
  (GCD |x| |y|))

(PUT
  (QUOTE |SINT;abs;2$;36|)
  (QUOTE |SPADreplace|)
  (QUOTE QSABSVAL))

(DEFUN |SINT;abs;2$;36| (|x| $)
  (QSABSVAL |x|))

(PUT
  (QUOTE |SINT;odd?;$B;37|)
  (QUOTE |SPADreplace|)
  (QUOTE QSODDP))

(DEFUN |SINT;odd?;$B;37| (|x| $)
  (QSODDP |x|))

(PUT

```

```

(QUOTE |SINT;zero?;$B;38|)
(QUOTE |SPADreplace|)
(QUOTE QSZEROP))

(DEFUN |SINT;zero?;$B;38| (|x| $)
  (QSZEROP |x|))

(PUT
  (QUOTE |SINT;max;3$;39|)
  (QUOTE |SPADreplace|)
  (QUOTE QSMAX))

(DEFUN |SINT;max;3$;39| (|x| |y| $)
  (QSMAX |x| |y|))

(PUT
  (QUOTE |SINT;min;3$;40|)
  (QUOTE |SPADreplace|)
  (QUOTE QSMIN))

(DEFUN |SINT;min;3$;40| (|x| |y| $)
  (QSMIN |x| |y|))

(PUT
  (QUOTE |SINT;hash;2$;41|)
  (QUOTE |SPADreplace|)
  (QUOTE SXHASH))

(DEFUN |SINT;hash;2$;41| (|x| $)
  (SXHASH |x|))

(PUT
  (QUOTE |SINT;length;2$;42|)
  (QUOTE |SPADreplace|)
  (QUOTE INTEGER-LENGTH))

(DEFUN |SINT;length;2$;42| (|x| $)
  (INTEGER-LENGTH |x|))

(PUT
  (QUOTE |SINT;shift;3$;43|)
  (QUOTE |SPADreplace|)
  (QUOTE QSLEFTSHIFT))

(DEFUN |SINT;shift;3$;43| (|x| |n| $)
  (QSLEFTSHIFT |x| |n|))

(PUT
  (QUOTE |SINT;mulmod;4$;44|)
  (QUOTE |SPADreplace|)

```

```

(QUOTE QSMULTMOD))

(DEFUN |SINT;mulmod;4$;44| (|a| |b| |p| $)
  (QSMULTMOD |a| |b| |p|))

(PUT
  (QUOTE |SINT;addmod;4$;45|)
  (QUOTE |SPADreplace|)
  (QUOTE QSADDMOD))

(DEFUN |SINT;addmod;4$;45| (|a| |b| |p| $)
  (QSADDMOD |a| |b| |p|))

(PUT
  (QUOTE |SINT;submod;4$;46|)
  (QUOTE |SPADreplace|)
  (QUOTE QSDIFMOD))

(DEFUN |SINT;submod;4$;46| (|a| |b| |p| $)
  (QSDIFMOD |a| |b| |p|))

(PUT
  (QUOTE |SINT;negative?;$B;47|)
  (QUOTE |SPADreplace|)
  (QUOTE QSMINUSP))

(DEFUN |SINT;negative?;$B;47| (|x| $)
  (QSMINUSP |x|))

(PUT
  (QUOTE |SINT;reducedSystem;MVR;48|)
  (QUOTE |SPADreplace|)
  (QUOTE CONS))

(DEFUN |SINT;reducedSystem;MVR;48| (|m| |v| $)
  (CONS |m| |v|))

(DEFUN |SINT;positiveRemainder;3$;49| (|x| |n| $)
  (PROG (|r|)
    (RETURN
      (SEQ
        (LETT |r| (QSREMAINDER |x| |n|) |SINT;positiveRemainder;3$;49|)
        (EXIT
          (COND
            ((QSMINUSP |r|)
              (COND
                ((QSMINUSP |n|) (QSDIFFERENCE |x| |n|))
                ((QUOTE T) (QSPLUS |r| |n|))))
            ((QUOTE T) |r|)))))))

```

```

(DEFUN |SINT;coerce;I$;50| (|x| $)
  (SEQ
    (COND
      ((NULL (< MOST-POSITIVE-FIXNUM |x|))
        (COND ((NULL (< |x| MOST-NEGATIVE-FIXNUM)) (EXIT |x|))))
      (EXIT (|error| "integer too large to represent in a machine word"))))

(DEFUN |SINT;random;$;51| ($)
  (SEQ
    (SETLT $ 6 (REMAINDER (TIMES 314159269 (QREFELT $ 6)) 2147483647))
    (EXIT (REMAINDER (QREFELT $ 6) 67108864))))

(PUT
  (QUOTE |SINT;random;2$;52|)
  (QUOTE |SPADreplace|)
  (QUOTE RANDOM))

(DEFUN |SINT;random;2$;52| (|n| $)
  (RANDOM |n|))

(DEFUN |SINT;unitNormal;$R;53| (|x| $)
  (COND
    ((QSLESP |x| 0) (VECTOR -1 (QSMINUS |x|) -1))
    ((QUOTE T) (VECTOR 1 |x| 1))))

(DEFUN |SingleInteger| ()
  (PROG ()
    (RETURN
      (PROG (G1358)
        (RETURN
          (COND
            ((LETT G1358 (HGET |$ConstructorCache| '|SingleInteger|)
              |SingleInteger|)
              (|CDRwithIncrement| (CDAR G1358)))
            ('T
              (UNWIND-PROTECT
                (PROG1 (CDDAR (HPUT |$ConstructorCache| '|SingleInteger|
                  (LIST
                    (CONS NIL
                      (CONS 1 (|SingleInteger;|))))))
                  (LETT G1358 T |SingleInteger|))
                (COND
                  ((NOT G1358)
                    (HREM |$ConstructorCache| '|SingleInteger|))))))))))

(DEFUN |SingleInteger;| ()
  (PROG (|dv$| $ |pv$|)
    (RETURN
      (PROGN
        (LETT |dv$| '(|SingleInteger|) |SingleInteger|)

```

```

(LETT $ (make-array 103) |SingleInteger|)
(QSETREFV $ 0 |dv$|)
(QSETREFV $ 3
  (LETT |pv$| (|buildPredVector| 0 0 NIL) |SingleInteger|))
(|haddProp| |$ConstructorCache| '|SingleInteger| NIL
  (CONS 1 $))
(|stuffDomainSlots| $)
(QSETREFV $ 6 1)
$))))

(setf (get '|SingleInteger| '|infovec|)
  (LIST '#(NIL NIL NIL NIL NIL '|seed| (|Void|)
    (|OpenMathDevice|) (0 . |OMputApp|) (|String|)
    (5 . |OMputSymbol|) (|Integer|) (12 . |OMputInteger|)
    (18 . |OMputEndApp|) (|OpenMathEncoding|)
    (23 . |OMencodingXML|) (27 . |OMopenString|)
    (33 . |OMputObject|) (38 . |OMputEndObject|)
    (43 . |OMclose|) |SINT;OMwrite;$S;2| (|Boolean|)
    |SINT;OMwrite;$BS;3| |SINT;OMwrite;Omd$V;4|
    |SINT;OMwrite;Omd$BV;5| (|Matrix| 12) (|Matrix| $)
    |SINT;reducedSystem;MM;6| (|OutputForm|) (48 . |coerce|)
    |SINT;coerce;$Of;7| |SINT;convert;$I;8| (53 . |coerce|)
    |SINT;*;I2$;9|
    (CONS IDENTITY
      (FUNCALL (|dispatchFunction| |SINT;Zero;$;10|) $))
    (CONS IDENTITY
      (FUNCALL (|dispatchFunction| |SINT;One;$;11|) $))
    |SINT;base;$;12| |SINT;max;$;13| |SINT;min;$;14|
    |SINT;=;2$B;15| |SINT;~;2$;16| |SINT;not;2$;17|
    |SINT;/\;3$;18| |SINT;\;/;3$;19| |SINT;Not;2$;20|
    |SINT;And;3$;21| |SINT;Or;3$;22| |SINT;xor;3$;23|
    |SINT;<;2$B;24| |SINT;inc;2$;25| |SINT;dec;2$;26|
    |SINT;-;2$;27| |SINT;+;3$;28| |SINT;-;3$;29|
    |SINT;*;3$;30| (|NonNegativeInteger|) |SINT;**;$Nni$;31|
    |SINT;quo;3$;32| |SINT;rem;3$;33|
    (|Record| (|:| |quotient| $) (|:| |remainder| $))
    |SINT;divide;2$R;34| |SINT;gcd;3$;35| |SINT;abs;2$;36|
    |SINT;odd?;$B;37| |SINT;zero?;$B;38| |SINT;max;3$;39|
    |SINT;min;3$;40| |SINT;hash;2$;41| |SINT;length;2$;42|
    |SINT;shift;3$;43| |SINT;mulmod;4$;44| |SINT;addmod;4$;45|
    |SINT;submod;4$;46| |SINT;negative?;$B;47|
    (|Record| (|:| |mat| 26) (|:| |vec| (|Vector| 12)))
    (|Vector| $) |SINT;reducedSystem;MVR;48|
    |SINT;positiveRemainder;3$;49| |SINT;coerce;I$;50|
    |SINT;random;$;51| |SINT;random;2$;52|
    (|Record| (|:| |unit| $) (|:| |canonical| $)
      (|:| |associate| $))
    |SINT;unitNormal;$R;53| (|Union| 85 '"failed")
    (|Fraction| 12) (|Union| $ '"failed") (|Float|)
    (|DoubleFloat|) (|Pattern| 12) (|PatternMatchResult| 12 $)
  ))

```



```

(|InputForm|) (|Union| 12 "failed")
(|Record| (|:| |coef| 94) (|:| |generator| $)) (|List| $)
(|Union| 94 "failed")
(|Record| (|:| |coef1| $) (|:| |coef2| $)
  (|:| |generator| $))
(|Record| (|:| |coef1| $) (|:| |coef2| $))
(|Union| 97 "failed") (|Factored| $)
(|SparseUnivariatePolynomial| $) (|PositiveInteger|)
(|SingleInteger|))
'#(= 58 ~ 64 |zero?| 69 |xor| 74 |unitNormal| 80
|unitCanonical| 85 |unit?| 90 |symmetricRemainder| 95
|subtractIfCan| 101 |submod| 107 |squareFreePart| 114
|squareFree| 119 |sizeLess?| 124 |sign| 130 |shift| 135
|sample| 141 |retractIfCan| 145 |retract| 150 |rem| 155
|reducedSystem| 161 |recip| 172 |rationalIfCan| 177
|rational?| 182 |rational| 187 |random| 192 |quo| 201
|principalIdeal| 207 |prime?| 212 |powmod| 217
|positiveRemainder| 224 |positive?| 230 |permutation| 235
|patternMatch| 241 |one?| 248 |odd?| 253 |not| 258
|nextItem| 263 |negative?| 268 |multiEuclidean| 273
|mulmod| 279 |min| 286 |max| 296 |mask| 306 |length| 311
|lcm| 316 |latex| 327 |invmod| 332 |init| 338 |inc| 342
|hash| 347 |gcdPolynomial| 357 |gcd| 363 |factorial| 374
|factor| 379 |extendedEuclidean| 384 |exquo| 397
|expressIdealMember| 403 |even?| 409 |euclideanSize| 414
|divide| 419 |differentiate| 425 |dec| 436 |copy| 441
|convert| 446 |coerce| 471 |characteristic| 491 |bit?| 495
|binomial| 501 |base| 507 |associates?| 511 |addmod| 517
|abs| 524 ~ 529 |\\| 541 |Zero| 547 |Or| 551 |One| 557
|OMwrite| 561 |Not| 585 D 590 |And| 601 >= 607 > 613 = 619
<= 625 < 631 |/\| 637 - 643 + 654 ** 660 * 672)
'((|noetherian| . 0) (|canonicalsClosed| . 0)
(|canonical| . 0) (|canonicalUnitNormal| . 0)
(|multiplicativeValuation| . 0) (|noZeroDivisors| . 0)
((|commutative| "*") . 0) (|rightUnitary| . 0)
(|leftUnitary| . 0) (|unitsKnown| . 0))
(CONS (|makeByteWordVec2| 1
  '(0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
    0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
    0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
    0))
  (CONS '#(|IntegerNumberSystem&| |EuclideanDomain&|
    |UniqueFactorizationDomain&| NIL NIL
    |GcdDomain&| |IntegralDomain&| |Algebra&|
    |Module&| NIL |Module&| NIL NIL |Module&| NIL
    |DifferentialRing&| |OrderedRing&| NIL
    |Module&| NIL |Module&| NIL NIL NIL NIL NIL
    NIL |Ring&| NIL NIL NIL NIL NIL NIL NIL NIL
    NIL NIL NIL NIL NIL |AbelianGroup&| NIL NIL
    |AbelianMonoid&| |Monoid&| NIL NIL NIL NIL

```

```

|OrderedSet&| |AbelianSemiGroup&| |SemiGroup&|
|Logic&| NIL |SetCategory&| NIL NIL NIL NIL
|RetractableTo&| NIL NIL NIL |RetractableTo&|
NIL NIL NIL NIL NIL NIL |RetractableTo&| NIL
|BasicType&| NIL)
(CONS '#((|IntegerNumberSystem|)
(|EuclideanDomain|)
(|UniqueFactorizationDomain|)
(|PrincipalIdealDomain|)
(|OrderedIntegralDomain|) (|GcdDomain|)
(|IntegralDomain|) (|Algebra| $$)
(|Module| 12)
(|LinearlyExplicitRingOver| 12)
(|Module| G1062)
(|LinearlyExplicitRingOver| G1062)
(|CharacteristicZero|)
(|Module| G106217)
(|LinearlyExplicitRingOver| G106217)
(|DifferentialRing|) (|OrderedRing|)
(|CommutativeRing|) (|Module| |t#1|)
(|EntireRing|) (|Module| $$)
(|BiModule| 12 12)
(|BiModule| G1062 G1062)
(|BiModule| G106217 G106217)
(|OrderedAbelianGroup|)
(|BiModule| |t#1| |t#1|)
(|BiModule| $$ $$) (|Ring|)
(|RightModule| 12) (|LeftModule| 12)
(|RightModule| G1062)
(|LeftModule| G1062)
(|RightModule| G106217)
(|LeftModule| G106217)
(|OrderedCancellationAbelianMonoid|)
(|RightModule| |t#1|)
(|LeftModule| |t#1|) (|LeftModule| $$)
(|Rng|) (|RightModule| $$)
(|OrderedAbelianMonoid|)
(|AbelianGroup|)
(|OrderedAbelianSemiGroup|)
(|CancellationAbelianMonoid|)
(|AbelianMonoid|) (|Monoid|)
(|PatternMatchable| 12)
(|PatternMatchable| G1065)
(|StepThrough|)
(|PatternMatchable| G106220)
(|OrderedSet|) (|AbelianSemiGroup|)
(|SemiGroup|) (|Logic|) (|RealConstant|)
(|SetCategory|) (|OpenMath|)
(|CoercibleTo| G82356)
(|ConvertibleTo| 89)

```

```

(|ConvertibleTo| 91)
(|RetractableTo| 12)
(|ConvertibleTo| 12)
(|ConvertibleTo| G1064)
(|ConvertibleTo| G1063)
(|RetractableTo| G1061)
(|ConvertibleTo| G1060)
(|ConvertibleTo| 87)
(|ConvertibleTo| 88)
(|CombinatorialFunctionCategory|)
(|ConvertibleTo| G106219)
(|ConvertibleTo| G106218)
(|RetractableTo| G106216)
(|ConvertibleTo| G106215) (|BasicType|)
(|CoercibleTo| 29))
(|makeByteWordVec2| 102
' (1 8 7 0 9 3 8 7 0 10 10 11 2 8 7 0 12
  13 1 8 7 0 14 0 15 0 16 2 8 0 10 15
  17 1 8 7 0 18 1 8 7 0 19 1 8 7 0 20 1
  12 29 0 30 1 0 0 12 33 2 0 22 0 0 1 1
  0 0 0 41 1 0 22 0 65 2 0 0 0 0 48 1 0
  82 0 83 1 0 0 0 1 1 0 22 0 1 2 0 0 0
  0 1 2 0 86 0 0 1 3 0 0 0 0 0 73 1 0 0
  0 1 1 0 99 0 1 2 0 22 0 0 1 1 0 12 0
  1 2 0 0 0 0 70 0 0 0 1 1 0 92 0 1 1 0
  12 0 1 2 0 0 0 0 59 1 0 26 27 28 2 0
  75 27 76 77 1 0 86 0 1 1 0 84 0 1 1 0
  22 0 1 1 0 85 0 1 1 0 0 0 81 0 0 0 80
  2 0 0 0 0 58 1 0 93 94 1 1 0 22 0 1 3
  0 0 0 0 0 1 2 0 0 0 0 78 1 0 22 0 1 2
  0 0 0 0 1 3 0 90 0 89 90 1 1 0 22 0 1
  1 0 22 0 64 1 0 0 0 42 1 0 86 0 1 1 0
  22 0 74 2 0 95 94 0 1 3 0 0 0 0 0 71
  0 0 0 39 2 0 0 0 0 67 0 0 0 38 2 0 0
  0 0 66 1 0 0 0 1 1 0 0 0 69 1 0 0 94
  1 2 0 0 0 0 1 1 0 10 0 1 2 0 0 0 1
  0 0 0 1 1 0 0 0 50 1 0 0 0 68 1 0 102
  0 1 2 0 100 100 100 1 1 0 0 94 1 2 0
  0 0 0 62 1 0 0 0 1 1 0 99 0 1 2 0 96
  0 0 1 3 0 98 0 0 0 1 2 0 86 0 0 1 2 0
  95 94 0 1 1 0 22 0 1 1 0 56 0 1 2 0
  60 0 0 61 1 0 0 0 1 2 0 0 0 56 1 1 0
  0 0 51 1 0 0 0 1 1 0 87 0 1 1 0 88 0
  1 1 0 89 0 1 1 0 91 0 1 1 0 12 0 32 1
  0 0 12 79 1 0 0 0 1 1 0 0 12 79 1 0
  29 0 31 0 0 56 1 2 0 22 0 0 1 2 0 0 0
  0 1 0 0 0 37 2 0 22 0 0 1 3 0 0 0 0 0
  72 1 0 0 0 63 2 0 0 0 56 1 2 0 0 0
  101 1 2 0 0 0 0 44 0 0 0 35 2 0 0 0 0
  47 0 0 0 36 3 0 7 8 0 22 25 2 0 10 0

```

```

22 23 2 0 7 8 0 24 1 0 10 0 21 1 0 0
0 45 1 0 0 0 1 2 0 0 0 56 1 2 0 0 0 0
46 2 0 22 0 0 1 2 0 22 0 0 1 2 0 22 0
0 40 2 0 22 0 0 1 2 0 22 0 0 49 2 0 0
0 0 43 1 0 0 0 52 2 0 0 0 0 54 2 0 0
0 0 53 2 0 0 0 56 57 2 0 0 0 101 1 2
0 0 0 0 55 2 0 0 12 0 34 2 0 0 56 0 1
2 0 0 101 0 1))))))

'lookupComplete)))

(setf (get (QUOTE |SingleInteger|) (QUOTE NILADIC)) T)

```

## 28.14 SYMBOL.lsp BOOTSTRAP

**SYMBOL** depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **SYMBOL** category which we can write into the **MID** directory. We compile the lisp code and copy the **SYMBOL.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

### — SYMBOL.lsp BOOTSTRAP —

```

(|/VERSIONCHECK| 2)

(DEFUN |SYMBOL;writeOMSym| (|dev| |x| $)
  (COND
    ((SPADCALL |x| (QREFELT $ 21))
     (|error| "Cannot convert a scripted symbol to OpenMath"))
    ('T (SPADCALL |dev| |x| (QREFELT $ 25)))))

(DEFUN |SYMBOL;OMwrite;$S;2| (|x| $)
  (PROG (|sp| |dev| |s|)
    (RETURN
      (SEQ (LETT |s| "" |SYMBOL;OMwrite;$S;2|)
            (LETT |sp| (OM-STRINGTOSTRINGPTR |s|) |SYMBOL;OMwrite;$S;2|)
            (LETT |dev|
              (SPADCALL |sp| (SPADCALL (QREFELT $ 27))
                (QREFELT $ 29))
              |SYMBOL;OMwrite;$S;2|)
            (SPADCALL |dev| (QREFELT $ 30))
            (|SYMBOL;writeOMSym| |dev| |x| $)
            (SPADCALL |dev| (QREFELT $ 31))
            (SPADCALL |dev| (QREFELT $ 32))
            (LETT |s| (OM-STRINGPTRTOSTRING |sp|) |SYMBOL;OMwrite;$S;2|)

```

```

(EXIT |s|))))))

(DEFUN |SYMBOL;OMwrite;$BS;3| (|x| |wholeObj| $)
  (PROG (|sp| |dev| |s|)
    (RETURN
      (SEQ (LETT |s| "" |SYMBOL;OMwrite;$BS;3|)
        (LETT |sp| (OM-STRINGTOSTRINGPTR |s|)
          |SYMBOL;OMwrite;$BS;3|)
        (LETT |dev|
          (SPADCALL |sp| (SPADCALL (QREFELT $ 27))
            (QREFELT $ 29))
          |SYMBOL;OMwrite;$BS;3|)
        (COND (|wholeObj| (SPADCALL |dev| (QREFELT $ 30))))
        (|SYMBOL;writeOMSym| |dev| |x| $)
        (COND (|wholeObj| (SPADCALL |dev| (QREFELT $ 31))))
        (SPADCALL |dev| (QREFELT $ 32))
        (LETT |s| (OM-STRINGPTRTOSTRING |sp|)
          |SYMBOL;OMwrite;$BS;3|)
        (EXIT |s|))))))

(DEFUN |SYMBOL;OMwrite;Omd$V;4| (|dev| |x| $)
  (SEQ (SPADCALL |dev| (QREFELT $ 30))
    (|SYMBOL;writeOMSym| |dev| |x| $)
    (EXIT (SPADCALL |dev| (QREFELT $ 31)))))

(DEFUN |SYMBOL;OMwrite;Omd$BV;5| (|dev| |x| |wholeObj| $)
  (SEQ (COND (|wholeObj| (SPADCALL |dev| (QREFELT $ 30))))
    (|SYMBOL;writeOMSym| |dev| |x| $)
    (EXIT (COND (|wholeObj| (SPADCALL |dev| (QREFELT $ 31)))))))

(DEFUN |SYMBOL;convert;$If;6| (|s| |$|) (SPADCALL |s| (QREFELT |$| 44)))

(PUT '|SYMBOL;convert;2$;7| '|SPADreplace| '(XLAM (|s|) |s|))

(DEFUN |SYMBOL;convert;2$;7| (|s| |$|) |s|)

(DEFUN |SYMBOL;coerce;$S;8| (|s| |$|) (VALUES (INTERN |s|)))

(PUT '|SYMBOL;=;2$B;9| '|SPADreplace| 'EQUAL)

(DEFUN |SYMBOL;=;2$B;9| (|x| |y| |$|) (EQUAL |x| |y|))

(PUT '|SYMBOL;<;2$B;10| '|SPADreplace|
  '(XLAM (|x| |y|) (GGREATERP |y| |x|)))

(DEFUN |SYMBOL;<;2$B;10| (|x| |y| |$|) (GGREATERP |y| |x|))

(DEFUN |SYMBOL;coerce;$Of;11| (|x| |$|) (SPADCALL |x| (QREFELT |$| 51)))

(DEFUN |SYMBOL;subscript;$L$;12| (|sy| |lx| $)

```

```

(SPADCALL |sy| (LIST |lx| NIL NIL NIL NIL) (QREFELT $ 54)))

(DEFUN |SYMBOL;elt;$L$;13| (|sy| |lx| $)
  (SPADCALL |sy| |lx| (QREFELT $ 56)))

(DEFUN |SYMBOL;superscript;$L$;14| (|sy| |lx| $)
  (SPADCALL |sy| (LIST NIL |lx| NIL NIL NIL) (QREFELT $ 54)))

(DEFUN |SYMBOL;argscript;$L$;15| (|sy| |lx| $)
  (SPADCALL |sy| (LIST NIL NIL NIL NIL |lx|) (QREFELT $ 54)))

(DEFUN |SYMBOL;patternMatch;$P2Pmr;16| (|x| |p| |l| $)
  (SPADCALL |x| |p| |l| (QREFELT $ 63)))

(DEFUN |SYMBOL;patternMatch;$P2Pmr;17| (|x| |p| |l| $)
  (SPADCALL |x| |p| |l| (QREFELT $ 69)))

(DEFUN |SYMBOL;convert;$P;18| (|x| |$|) (SPADCALL |x| (QREFELT |$| 72)))

(DEFUN |SYMBOL;convert;$P;19| (|x| |$|) (SPADCALL |x| (QREFELT |$| 74)))

(DEFUN |SYMBOL;syprefix| (|sc| $)
  (PROG (|ns| G108218 |n| G108219)
    (RETURN
      (SEQ (LETT |ns|
        (LIST (LENGTH (QVELT |sc| 3)) (LENGTH (QVELT |sc| 2))
          (LENGTH (QVELT |sc| 1)) (LENGTH (QVELT |sc| 0)))
        |SYMBOL;syprefix|)
        (SEQ G190
          (COND
            ((NULL (COND
              ((< (LENGTH |ns|) 2) 'NIL)
              ('T (ZEROP (|SPADfirst| |ns|)))))
              (GO G191)))
          (SEQ (EXIT (LETT |ns| (CDR |ns|) |SYMBOL;syprefix|)))
            NIL (GO G190) G191 (EXIT NIL))
        (EXIT (SPADCALL
          (CONS (STRCONC (QREFELT $ 37)
            (|SYMBOL;istring|
              (LENGTH (QVELT |sc| 4)) $))
            (PROGN
              (LETT G108218 NIL |SYMBOL;syprefix|)
              (SEQ (LETT |n| NIL |SYMBOL;syprefix|)
                (LETT G108219 (NREVERSE |ns|)
                  |SYMBOL;syprefix|)
                G190
                (COND
                  ((OR (ATOM G108219)
                    (PROGN
                      (LETT |n| (CAR G108219)

```

```

        |SYMBOL;symprefix|)
      NIL))
    (GO G191)))
  (SEQ (EXIT
    (LETT G108218
      (CONS (|SYMBOL;istring| |n| $)
        G108218)
      |SYMBOL;symprefix|)))
    (LETT G108219 (CDR G108219)
      |SYMBOL;symprefix|)
    (GO G190) G191
    (EXIT (NREVERSEO G108218))))))
  (QREFELT $ 77))))))

(DEFUN |SYMBOL;syscripts| (|sc| $)
  (PROG (|all|)
    (RETURN
      (SEQ (LETT |all| (QVELT |sc| 3) |SYMBOL;syscripts|)
        (LETT |all| (SPADCALL (QVELT |sc| 2) |all| (QREFELT $ 78))
          |SYMBOL;syscripts|)
        (LETT |all| (SPADCALL (QVELT |sc| 1) |all| (QREFELT $ 78))
          |SYMBOL;syscripts|)
        (LETT |all| (SPADCALL (QVELT |sc| 0) |all| (QREFELT $ 78))
          |SYMBOL;syscripts|)
        (EXIT (SPADCALL |all| (QVELT |sc| 4) (QREFELT $ 78))))))

(DEFUN |SYMBOL;script;$L$;22| (|sy| |ls| $)
  (PROG (|sc|)
    (RETURN
      (SEQ (LETT |sc| (VECTOR NIL NIL NIL NIL NIL)
        |SYMBOL;script;$L$;22|)
        (COND
          ((NULL (NULL |ls|))
            (SEQ (QSETVELT |sc| 0 (|SPADfirst| |ls|))
              (EXIT (LETT |ls| (CDR |ls|) |SYMBOL;script;$L$;22|))))))
          (COND
            ((NULL (NULL |ls|))
              (SEQ (QSETVELT |sc| 1 (|SPADfirst| |ls|))
                (EXIT (LETT |ls| (CDR |ls|) |SYMBOL;script;$L$;22|))))))
          (COND
            ((NULL (NULL |ls|))
              (SEQ (QSETVELT |sc| 2 (|SPADfirst| |ls|))
                (EXIT (LETT |ls| (CDR |ls|) |SYMBOL;script;$L$;22|))))))
          (COND
            ((NULL (NULL |ls|))
              (SEQ (QSETVELT |sc| 3 (|SPADfirst| |ls|))
                (EXIT (LETT |ls| (CDR |ls|) |SYMBOL;script;$L$;22|))))))
          (COND
            ((NULL (NULL |ls|))
              (SEQ (QSETVELT |sc| 4 (|SPADfirst| |ls|))

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        (EXIT (LETT |ls| (CDR |ls|) |SYMBOL;script;$L$;22|))))
    (EXIT (SPADCALL |sy| |sc| (QREFELT $ 80)))))

(DEFUN |SYMBOL;script;$R$;23| (|sy| |sc| $)
  (COND
    ((SPADCALL |sy| (QREFELT $ 21))
      (|error| "Cannot add scripts to a scripted symbol"))
    ('T
      (CONS (SPADCALL
        (SPADCALL
          (STRCONC (|SYMBOL;syprefix| |sc| $)
            (SPADCALL (SPADCALL |sy| (QREFELT $ 81))
              (QREFELT $ 82)))
          (QREFELT $ 47))
            (QREFELT $ 52))
        (|SYMBOL;syscripts| |sc| $)))))

(DEFUN |SYMBOL;string;$S;24| (|e| $)
  (COND
    ((NULL (SPADCALL |e| (QREFELT $ 21))) (PNAME |e|))
    ('T (|error| "Cannot form string from non-atomic symbols.")))

(DEFUN |SYMBOL;latex;$S;25| (|e| $)
  (PROG (|ss| |lo| |sc| |s|)
    (RETURN
      (SEQ (LETT |s| (PNAME (SPADCALL |e| (QREFELT $ 81)))
        |SYMBOL;latex;$S;25|)
        (COND
          ((< 1 (QCSIZE |s|))
            (COND
              ((NULL (SPADCALL (SPADCALL |s| 1 (QREFELT $ 83))
                (SPADCALL "\\\" (QREFELT $ 40))
                (QREFELT $ 84)))
                (LETT |s| (STRCONC "\\mbox{\\it \" (STRCONC |s| \"}"))
                  |SYMBOL;latex;$S;25|))))
            (COND ((NULL (SPADCALL |e| (QREFELT $ 21))) (EXIT |s|)))
            (LETT |ss| (SPADCALL |e| (QREFELT $ 85))
              |SYMBOL;latex;$S;25|)
            (LETT |lo| (QVELT |ss| 0) |SYMBOL;latex;$S;25|)
            (COND
              ((NULL (NULL |lo|))
                (SEQ (LETT |sc| "_{" |SYMBOL;latex;$S;25|)
                  (SEQ G190
                    (COND
                      ((NULL (COND ((NULL |lo|) 'NIL) ('T 'T)))
                        (GO G191)))
                    (SEQ (LETT |sc|
                      (STRCONC |sc|
                        (SPADCALL (|SPADfirst| |lo|)
                          (QREFELT $ 86)))

```



```

(SYMBOL;latex;$S;25|)
(LETT |lo| (CDR |lo|)
|SYMBOL;latex;$S;25|)
(EXIT (COND
((NULL (NULL |lo|))
(LETT |sc| (STRCONC |sc| ", "
|SYMBOL;latex;$S;25|))))))
NIL (GO G190) G191 (EXIT NIL))
(LETT |sc| (STRCONC |sc| "}") |SYMBOL;latex;$S;25|)
(EXIT (LETT |s| (STRCONC |s| |sc|)
|SYMBOL;latex;$S;25|))))))
(LETT |lo| (QVELT |ss| 1) |SYMBOL;latex;$S;25|)
(COND
((NULL (NULL |lo|))
(SEQ (LETT |sc| "~{" |SYMBOL;latex;$S;25|)
(SEQ G190
(COND
((NULL (COND ((NULL |lo|) 'NIL) ('T 'T)))
(GO G191)))
(SEQ (LETT |sc|
(STRCONC |sc|
(SPADCALL (|SPADfirst| |lo|)
(QREFELT $ 86)))
|SYMBOL;latex;$S;25|)
(LETT |lo| (CDR |lo|)
|SYMBOL;latex;$S;25|)
(EXIT (COND
((NULL (NULL |lo|))
(LETT |sc| (STRCONC |sc| ", "
|SYMBOL;latex;$S;25|))))))
NIL (GO G190) G191 (EXIT NIL))
(LETT |sc| (STRCONC |sc| "|}") |SYMBOL;latex;$S;25|)
(EXIT (LETT |s| (STRCONC |s| |sc|)
|SYMBOL;latex;$S;25|))))))
(LETT |lo| (QVELT |ss| 2) |SYMBOL;latex;$S;25|)
(COND
((NULL (NULL |lo|))
(SEQ (LETT |sc| "{"~{" |SYMBOL;latex;$S;25|)
(SEQ G190
(COND
((NULL (COND ((NULL |lo|) 'NIL) ('T 'T)))
(GO G191)))
(SEQ (LETT |sc|
(STRCONC |sc|
(SPADCALL (|SPADfirst| |lo|)
(QREFELT $ 86)))
|SYMBOL;latex;$S;25|)
(LETT |lo| (CDR |lo|)
|SYMBOL;latex;$S;25|)
(EXIT (COND

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```
(NIL (GO G190) G191 (EXIT NIL))
(LETT |sc| (STRCONC |sc| ", ")
|SYMBOL;latex;$S;25|))))
NIL (GO G190) G191 (EXIT NIL))
(LETT |sc| (STRCONC |sc| "|") |SYMBOL;latex;$S;25|)
(EXIT (LETT |s| (STRCONC |sc| |s|)
|SYMBOL;latex;$S;25|))))
(LETT |lo| (QVELT |ss| 3) |SYMBOL;latex;$S;25|)
(COND
((NULL (NULL |lo|))
(SEQ (LETT |sc| "{ }_" |SYMBOL;latex;$S;25|)
(SEQ G190
(COND
((NULL (COND ((NULL |lo|) 'NIL) ('T 'T)))
(GO G191)))
(SEQ (LETT |sc|
(STRCONC |sc|
(SPADCALL (SPADfirst| |lo|)
(QREFELT $ 86)))
|SYMBOL;latex;$S;25|)
(LETT |lo| (CDR |lo|)
|SYMBOL;latex;$S;25|)
(EXIT (COND
((NULL (NULL |lo|))
(LETT |sc| (STRCONC |sc| ", ")
|SYMBOL;latex;$S;25|))))
NIL (GO G190) G191 (EXIT NIL))
(LETT |sc| (STRCONC |sc| "|") |SYMBOL;latex;$S;25|)
(EXIT (LETT |s| (STRCONC |sc| |s|)
|SYMBOL;latex;$S;25|))))
(LETT |lo| (QVELT |ss| 4) |SYMBOL;latex;$S;25|)
(COND
((NULL (NULL |lo|))
(SEQ (LETT |sc| "\\left( {" |SYMBOL;latex;$S;25|)
(SEQ G190
(COND
((NULL (COND ((NULL |lo|) 'NIL) ('T 'T)))
(GO G191)))
(SEQ (LETT |sc|
(STRCONC |sc|
(SPADCALL (SPADfirst| |lo|)
(QREFELT $ 86)))
|SYMBOL;latex;$S;25|)
(LETT |lo| (CDR |lo|)
|SYMBOL;latex;$S;25|)
(EXIT (COND
((NULL (NULL |lo|))
(LETT |sc| (STRCONC |sc| ", ")
|SYMBOL;latex;$S;25|))))
NIL (GO G190) G191 (EXIT NIL))
```

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        (LETT |sc| (STRCONC |sc| "}" \right)")
        |SYMBOL;latex;$S;25|)
    (EXIT (LETT |s| (STRCONC |s| |sc|)
        |SYMBOL;latex;$S;25|))))))
    (EXIT |s|))))))

(DEFUN |SYMBOL;anyRadix| (|n| |s| $)
  (PROG (|qr| |ns| G108274)
    (RETURN
      (SEQ (EXIT (SEQ (LETT |ns| "" |SYMBOL;anyRadix|)
        (EXIT (SEQ G190 NIL
          (SEQ (LETT |qr|
            (DIVIDE2 |n| (QCSIZE |s|))
            |SYMBOL;anyRadix|)
            (LETT |n| (QCAR |qr|)
            |SYMBOL;anyRadix|)
            (LETT |ns|
              (SPADCALL
                (SPADCALL |s|
                  (+ (QCDR |qr|)
                    (SPADCALL |s| (QREFELT $ 88)))
                    (QREFELT $ 83))
                |ns| (QREFELT $ 89))
              |SYMBOL;anyRadix|)
            (EXIT
              (COND
                ((ZEROP |n|)
                  (PROGN
                    (LETT G108274 |ns|
                      |SYMBOL;anyRadix|)
                    (GO G108274))))))
              NIL (GO G190) G191 (EXIT NIL))))))
          G108274 (EXIT G108274))))))

(DEFUN |SYMBOL;new;$;27| ($)
  (PROG (|sym|)
    (RETURN
      (SEQ (LETT |sym|
        (|SYMBOL;anyRadix|
          (SPADCALL (QREFELT $ 9) (QREFELT $ 90))
          (QREFELT $ 18) $)
          |SYMBOL;new;$;27|)
        (SPADCALL (QREFELT $ 9)
          (+ (SPADCALL (QREFELT $ 9) (QREFELT $ 90)) 1)
          (QREFELT $ 91))
        (EXIT (SPADCALL (STRCONC "%" |sym|) (QREFELT $ 47))))))

(DEFUN |SYMBOL;new;2$;28| (|x| $)
  (PROG (|u| |n| |xx|)
    (RETURN

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(SEQ (LETT |n|
      (SEQ (LETT |u|
            (SPADCALL |x| (QREFELT $ 12)
                        (QREFELT $ 94))
            |SYMBOL;new;2$;28|)
            (EXIT (COND
                  ((QEQCAR |u| 1) 0)
                  ('T (+ (QCDR |u|) 1))))
            |SYMBOL;new;2$;28|)
      (SPADCALL (QREFELT $ 12) |x| |n| (QREFELT $ 95))
      (LETT |xx|
            (COND
              ((NULL (SPADCALL |x| (QREFELT $ 21)))
               (SPADCALL |x| (QREFELT $ 82)))
              ('T
               (SPADCALL (SPADCALL |x| (QREFELT $ 81))
                         (QREFELT $ 82))))
            |SYMBOL;new;2$;28|)
      (LETT |xx| (STRCONC "%" |xx|) |SYMBOL;new;2$;28|)
      (LETT |xx|
            (COND
              ((NULL (< (SPADCALL
                        (SPADCALL |xx|
                          (SPADCALL |xx| (QREFELT $ 96))
                          (QREFELT $ 83))
                          (QREFELT $ 17) (QREFELT $ 97))
                        (SPADCALL (QREFELT $ 17) (QREFELT $ 88))))
               (STRCONC |xx|
                         (|SYMBOL;anyRadix| |n| (QREFELT $ 19) $)))
              ('T
               (STRCONC |xx|
                         (|SYMBOL;anyRadix| |n| (QREFELT $ 17) $))))
            |SYMBOL;new;2$;28|)
      (COND
        ((NULL (SPADCALL |x| (QREFELT $ 21)))
         (EXIT (SPADCALL |xx| (QREFELT $ 47))))
        (EXIT (SPADCALL (SPADCALL |xx| (QREFELT $ 47))
                        (SPADCALL |x| (QREFELT $ 85)) (QREFELT $ 80))))))

(DEFUN |SYMBOL;resetNew;V;29| ($)
  (PROG (|k| G108297)
    (RETURN
      (SEQ (SPADCALL (QREFELT $ 9) 0 (QREFELT $ 91))
            (SEQ (LETT |k| NIL |SYMBOL;resetNew;V;29|)
                  (LETT G108297 (SPADCALL (QREFELT $ 12) (QREFELT $ 100))
                        |SYMBOL;resetNew;V;29|)
                  G190
                  (COND
                    ((OR (ATOM G108297)
                        (PROGN

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        (LETT |k| (CAR G108297)
          |SYMBOL;resetNew;V;29|)
      NIL))
    (GO G191)))
  (SEQ (EXIT (SPADCALL |k| (QREFELT $ 12)
    (QREFELT $ 101))))
  (LETT G108297 (CDR G108297) |SYMBOL;resetNew;V;29|)
  (GO G190) G191 (EXIT NIL))
  (EXIT (SPADCALL (QREFELT $ 102))))))

(DEFUN |SYMBOL;scripted?;$B;30| (|sy| $)
  (COND ((ATOM |sy|) 'NIL) ('T 'T)))

(DEFUN |SYMBOL;name;2$;31| (|sy| $)
  (PROG (|str| |i| G108304 G108303 G108301)
    (RETURN
      (SEQ (EXIT (COND
        ((NULL (SPADCALL |sy| (QREFELT $ 21))) |sy|)
        ('T
          (SEQ (LETT |str|
            (SPADCALL
              (SPADCALL
                (SPADCALL |sy| (QREFELT $ 104))
                (QREFELT $ 105))
                (QREFELT $ 82))
              |SYMBOL;name;2$;31|)
            (SEQ (EXIT (SEQ
              (LETT |i| (+ (QREFELT $ 38) 1)
                |SYMBOL;name;2$;31|)
              (LETT G108304 (QCSIZE |str|)
                |SYMBOL;name;2$;31|)
              G190
              (COND
                ((> |i| G108304) (GO G191)))
              (SEQ
                (EXIT
                  (COND
                    ((NULL
                      (SPADCALL
                        (SPADCALL |str| |i|
                          (QREFELT $ 83))
                          (QREFELT $ 106)))
                      (PROGN
                        (LETT G108301
                          (PROGN
                            (LETT G108303
                              (SPADCALL
                                (SPADCALL |str|
                                  (SPADCALL |i|
                                    (QCSIZE |str|)

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                                (QREFELT $ 108))
                                (QREFELT $ 109))
                                (QREFELT $ 47))
                                |SYMBOL;name;2$;31|)
                                (GO G108303))
                                |SYMBOL;name;2$;31|)
                                (GO G108301))))))
                                (LETT |i| (+ |i| 1)
                                |SYMBOL;name;2$;31|)
                                (GO G190) G191 (EXIT NIL)))
                                G108301 (EXIT G108301))
                                (EXIT (|error| "Improper scripted symbol")))))))
G108303 (EXIT G108303))))))

(DEFUN |SYMBOL;scripts;$R;32| (|sy| $)
  (PROG (|lscripts| |str| |instr| |j| G108307 |nscripts| |m| |n| G108316
        |i| G108317 |a| G108318 |allscripts|)
    (RETURN
      (SEQ (COND
        ((NULL (SPADCALL |sy| (QREFELT $ 21)))
          (VECTOR NIL NIL NIL NIL NIL))
        ('T
          (SEQ (LETT |nscripts| (LIST 0 0 0 0 0)
                  |SYMBOL;scripts;$R;32|)
                (LETT |lscripts| (LIST NIL NIL NIL NIL NIL)
                  |SYMBOL;scripts;$R;32|)
                (LETT |str|
                  (SPADCALL
                    (SPADCALL (SPADCALL |sy| (QREFELT $ 104))
                              (QREFELT $ 105))
                    (QREFELT $ 82))
                  |SYMBOL;scripts;$R;32|)
                (LETT |instr| (QCSIZE |str|) |SYMBOL;scripts;$R;32|)
                (LETT |m| (SPADCALL |nscripts| (QREFELT $ 111))
                  |SYMBOL;scripts;$R;32|)
                (SEQ (LETT |j| (+ (QREFELT $ 38) 1)
                      |SYMBOL;scripts;$R;32|)
                    (LETT |i| |m| |SYMBOL;scripts;$R;32|) G190
                    (COND
                      ((OR (> |j| |instr|)
                        (NULL (SPADCALL
                          (SPADCALL |str| |j|
                            (QREFELT $ 83))
                            (QREFELT $ 106))))
                        (GO G191)))
                    (SEQ (EXIT (SPADCALL |nscripts| |i|
                      (PROG1
                        (LETT G108307
                          (-
                            (SPADCALL

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        (SPADCALL |str| |j|
         (QREFELT $ 83))
        (QREFELT $ 41))
        (QREFELT $ 42))
        |SYMBOL;scripts;$R;32|)
        (|check-subtype| (>= G108307 0)
         '(|NonNegativeInteger|) G108307))
        (QREFELT $ 113)))
(LETT |i|
  (PROG1 (+ |i| 1)
    (LETT |j| (+ |j| 1)
      |SYMBOL;scripts;$R;32|)
      |SYMBOL;scripts;$R;32|)
    (GO G190) G191 (EXIT NIL))
(LETT |nscripts|
  (SPADCALL (CDR |nscripts|)
    (|SPADfirst| |nscripts|) (QREFELT $ 114))
    |SYMBOL;scripts;$R;32|)
(LETT |allscripts|
  (SPADCALL (SPADCALL |sy| (QREFELT $ 104))
    (QREFELT $ 115))
    |SYMBOL;scripts;$R;32|)
(LETT |m| (SPADCALL |lscripts| (QREFELT $ 116))
  |SYMBOL;scripts;$R;32|)
(SEQ (LETT |n| NIL |SYMBOL;scripts;$R;32|)
  (LETT G108316 |nscripts|
    |SYMBOL;scripts;$R;32|)
  (LETT |i| |m| |SYMBOL;scripts;$R;32|) G190
  (COND
    ((OR (ATOM G108316)
      (PROGN
        (LETT |n| (CAR G108316)
          |SYMBOL;scripts;$R;32|)
        NIL))
      (GO G191)))
  (SEQ (EXIT (COND
    ((<
      (SPADCALL |allscripts|
        (QREFELT $ 117))
      |n|)
      (|error|
        "Improper script count in symbol")))
    'T
    (SEQ
      (SPADCALL |lscripts| |i|
        (PROGN
          (LETT G108317 NIL
            |SYMBOL;scripts;$R;32|)
          (SEQ
            (LETT |a| NIL

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|SYMBOL;scripts;$R;32|)
(LETT G108318
  (SPADCALL |allscripts| |n|
    (QREFELT $ 118))
  |SYMBOL;scripts;$R;32|)
G190
(COND
  ((OR (ATOM G108318)
    (PROGN
      (LETT |a|
        (CAR G108318)
        |SYMBOL;scripts;$R;32|)
        NIL))
    (GO G191)))
  (SEQ
    (EXIT
      (LETT G108317
        (CONS
          (SPADCALL |a|
            (QREFELT $ 52))
            G108317)
          |SYMBOL;scripts;$R;32|)))
    (LETT G108318 (CDR G108318)
      |SYMBOL;scripts;$R;32|)
    (GO G190) G191
    (EXIT (NREVERSEO G108317))))
(QREFELT $ 119))
(EXIT
  (LETT |allscripts|
    (SPADCALL |allscripts| |n|
      (QREFELT $ 120))
    |SYMBOL;scripts;$R;32|))))))
(LETT |i|
  (PROG1 (+ |i| 1)
    (LETT G108316 (CDR G108316)
      |SYMBOL;scripts;$R;32|)
      |SYMBOL;scripts;$R;32|)
    (GO G190) G191 (EXIT NIL))
  (EXIT (VECTOR (SPADCALL |lscripts| |m|
    (QREFELT $ 121))
    (SPADCALL |lscripts| (+ |m| 1)
      (QREFELT $ 121))
    (SPADCALL |lscripts| (+ |m| 2)
      (QREFELT $ 121))
    (SPADCALL |lscripts| (+ |m| 3)
      (QREFELT $ 121))
    (SPADCALL |lscripts| (+ |m| 4)
      (QREFELT $ 121))))))))))

(DEFUN |SYMBOL;istring| (|n| $)

```



```

(COND
  ((< 9 |n|) (|error| "Can have at most 9 scripts of each kind"))
  ('T (ELT (QREFELT $ 16) (+ |n| 0))))))

(DEFUN |SYMBOL;list;$L;34| (|sy| $)
  (COND
    ((NULL (SPADCALL |sy| (QREFELT $ 21)))
      (|error| "Cannot convert a symbol to a list if it is not subscripted"))
    ('T |sy|)))

(DEFUN |SYMBOL;sample;$;35| (|$|) (SPADCALL "aSymbol" (QREFELT |$| 47)))

(DEFUN |Symbol| ()
  (PROG ()
    (RETURN
      (PROG (G108325)
        (RETURN
          (COND
            ((LETT G108325 (HGET |$ConstructorCache| '|Symbol|)
              |Symbol|)
              (|CDRwithIncrement| (CDAR G108325)))
            ('T
              (UNWIND-PROTECT
                (PROG1 (CDDAR (HPUT |$ConstructorCache| '|Symbol|
                  (LIST
                    (CONS NIL (CONS 1 (|Symbol|;|)))))))
                  (LETT G108325 T |Symbol|)))
              (COND
                ((NOT G108325) (HREM |$ConstructorCache| '|Symbol|))))))))))

(DEFUN |Symbol;| ()
  (PROG (|dv$| $ |pv$|)
    (RETURN
      (PROGN
        (LETT |dv$| '(|Symbol|) |Symbol|)
        (LETT $ (make-array 124) |Symbol|)
        (QSETREFV $ 0 |dv$|)
        (QSETREFV $ 3
          (LETT |pv$| (|buildPredVector| 0 0 NIL) |Symbol|))
        (|haddProp| |$ConstructorCache| '|Symbol| NIL (CONS 1 $))
        (|stuffDomainSlots| $)
        (QSETREFV $ 9 (SPADCALL 0 (QREFELT $ 8)))
        (QSETREFV $ 12 (SPADCALL (QREFELT $ 11)))
        (QSETREFV $ 16
          (SPADCALL (LIST "0" "1" "2" "3" "4" "5" "6" "7" "8" "9")
            (QREFELT $ 15)))
        (QSETREFV $ 17 "0123456789")
        (QSETREFV $ 18 "ABCDEFGHIJKLMNOPQRSTUVWXYZ")
        (QSETREFV $ 19 "abcdefghijklmnopqrstuvwxyz")
        (QSETREFV $ 37 "*"))
      )
    )
  )

```

```

(QSETREFV $ 38 (QCSIZE (QREFELT $ 37)))
(QSETREFV $ 42
  (SPADCALL (SPADCALL "0" (QREFELT $ 40)) (QREFELT $ 41)))
$)))

(setf (get 'Symbol| 'infovec|)
  (LIST '#(NIL NIL NIL NIL NIL NIL (|Integer|) (|Reference| 6)
    (0 . |ref|) 'count| (|AssociationList| $$ 6)
    (5 . |empty|) 'xcount| (|List| 28) (|PrimitiveArray| 28)
    (9 . |construct|) 'listrings| 'nums| 'ALPHAS 'alphas|
    (|Boolean|) |SYMBOL;scripted?;$B;30| (|Void|) (|Symbol|)
    (|OpenMathDevice|) (14 . |OMputVariable|)
    (|OpenMathEncoding|) (20 . |OMencodingXML|) (|String|)
    (24 . |OMopenString|) (30 . |OMputObject|)
    (35 . |OMputEndObject|) (40 . |OMclose|)
    |SYMBOL;OMwrite;$S;2| |SYMBOL;OMwrite;$BS;3|
    |SYMBOL;OMwrite;Omd$V;4| |SYMBOL;OMwrite;Omd$BV;5| 'lhd|
    'lhd| (|Character|) (45 . |char|) (50 . |ord|) 'lord0|
    (|InputForm|) (55 . |convert|) |SYMBOL;convert;$If;6|
    |SYMBOL;convert;2$;7| |SYMBOL;coerce;$S;8|
    |SYMBOL;=;2$B;9| |SYMBOL;<;2$B;10| (|OutputForm|)
    (60 . |outputForm|) |SYMBOL;coerce;$Of;11| (|List| 55)
    |SYMBOL;script;$L$;22| (|List| 50)
    |SYMBOL;subscript;$L$;12| |SYMBOL;elt;$L$;13|
    |SYMBOL;superscript;$L$;14| |SYMBOL;argscript;$L$;15|
    (|PatternMatchResult| 6 23) (|Pattern| 6)
    (|PatternMatchSymbol| 6) (65 . |patternMatch|)
    (|PatternMatchResult| 6 $) |SYMBOL;patternMatch;$P2Pmr;16|
    (|PatternMatchResult| (|Float|) 23) (|Pattern| (|Float|))
    (|PatternMatchSymbol| (|Float|)) (72 . |patternMatch|)
    (|PatternMatchResult| (|Float|) $)
    |SYMBOL;patternMatch;$P2Pmr;17| (79 . |coerce|)
    |SYMBOL;convert;$P;18| (84 . |coerce|)
    |SYMBOL;convert;$P;19| (|List| $) (89 . |concat|)
    (94 . |concat|)
    (|Record| (|:| |sub| 55) (|:| |sup| 55) (|:| |presup| 55)
      (|:| |presub| 55) (|:| |args| 55))
    |SYMBOL;script;$R$;23| |SYMBOL;name;2$;31|
    |SYMBOL;string;$S;24| (100 . |elt|) (106 . =)
    |SYMBOL;scripts;$R;32| (112 . |latex|)
    |SYMBOL;latex;$S;25| (117 . |minIndex|) (122 . |concat|)
    (128 . |elt|) (133 . |setelt|) |SYMBOL;new;$;27|
    (|Union| 6 "failed") (139 . |search|) (145 . |setelt|)
    (152 . |maxIndex|) (157 . |position|) |SYMBOL;new;2$;28|
    (|List| $$) (163 . |keys|) (168 . |remove!|)
    (174 . |void|) |SYMBOL;resetNew;V;29| |SYMBOL;list;$L;34|
    (178 . |first|) (183 . |digit?|) (|UniversalSegment| 6)
    (188 . SEGMENT) (194 . |elt|) (|List| 112)
    (200 . |minIndex|) (|NonNegativeInteger|) (205 . |setelt|)
    (212 . |concat|) (218 . |rest|) (223 . |minIndex|)
  ))

```

```

(228 . |#|) (233 . |first|) (239 . |setelt|)
(246 . |rest|) (252 . |elt|)
(CONS IDENTITY
  (FUNCALL (|dispatchFunction| |SYMBOL;sample;$;35|)
    $))
(|SingleInteger|))
'#= 258 |superscript| 264 |subscript| 270 |string| 276
|scripts| 281 |scripted?| 286 |script| 291 |sample| 303
|resetNew| 307 |patternMatch| 311 |new| 325 |name| 334
|min| 339 |max| 345 |list| 351 |latex| 356 |hash| 361
|elt| 366 |convert| 372 |coerce| 392 |argscript| 402
|OMwrite| 408 >= 432 > 438 = 444 <= 450 < 456)
'NIL
(CONS (|makeByteWordVec2| 1 '(0 0 0 0 0 0 0 0 0 0))
  (CONS '#(|OrderedSet&| NIL NIL |SetCategory&|
    |BasicType&| NIL NIL NIL NIL NIL NIL)
    (CONS '#(|OrderedSet|)
      (|PatternMatchable| (|Float|))
      (|PatternMatchable| 6) (|SetCategory|)
      (|BasicType|) (|ConvertibleTo| 67)
      (|ConvertibleTo| 61)
      (|ConvertibleTo| 23) (|OpenMath|)
      (|ConvertibleTo| 43) (|CoercibleTo| 50))
      (|makeByteWordVec2| 123
        '(1 7 0 6 8 0 10 0 11 1 14 0 13 15 2 24
          22 0 23 25 0 26 0 27 2 24 0 28 26 29
          1 24 22 0 30 1 24 22 0 31 1 24 22 0
          32 1 39 0 28 40 1 39 6 0 41 1 43 0 23
          44 1 50 0 23 51 3 62 60 23 61 60 63 3
          68 66 23 67 66 69 1 67 0 23 72 1 61 0
          23 74 1 28 0 76 77 2 55 0 0 0 78 2 28
          39 0 6 83 2 39 20 0 0 84 1 50 28 0 86
          1 28 6 0 88 2 28 0 39 0 89 1 7 6 0 90
          2 7 6 0 6 91 2 10 93 2 0 94 3 10 6 0
          2 6 95 1 28 6 0 96 2 28 6 39 0 97 1
          10 99 0 100 2 10 93 2 0 101 0 22 0
          102 1 99 2 0 105 1 39 20 0 106 2 107
          0 6 6 108 2 28 0 0 107 109 1 110 6 0
          111 3 110 112 0 6 112 113 2 110 0 0
          112 114 1 99 0 0 115 1 53 6 0 116 1
          99 112 0 117 2 99 0 0 112 118 3 53 55
          0 6 55 119 2 99 0 0 112 120 2 53 55 0
          6 121 2 0 20 0 0 1 2 0 0 0 55 58 2 0
          0 0 55 56 1 0 28 0 82 1 0 79 0 85 1 0
          20 0 21 2 0 0 0 53 54 2 0 0 0 79 80 0
          0 0 122 0 0 22 103 3 0 64 0 61 64 65
          3 0 70 0 67 70 71 1 0 0 0 98 0 0 0 92
          1 0 0 0 81 2 0 0 0 0 1 2 0 0 0 0 1 1
          0 76 0 104 1 0 28 0 87 1 0 123 0 1 2
          0 0 0 55 57 1 0 61 0 75 1 0 67 0 73 1

```

```

0 23 0 46 1 0 43 0 45 1 0 0 28 47 1 0
50 0 52 2 0 0 0 55 59 3 0 22 24 0 20
36 2 0 28 0 20 34 2 0 22 24 0 35 1 0
28 0 33 2 0 20 0 0 1 2 0 20 0 0 1 2 0
20 0 0 48 2 0 20 0 0 1 2 0 20 0 0 49))))))

'|lookupComplete|))

(setf (get '|Symbol| 'NILADIC) T)

```

---

## 28.15 VECTOR.lsp BOOTSTRAP

**VECTOR** depends on itself. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **VECTOR** category which we can write into the **MID** directory. We compile the lisp code and copy the **VECTOR.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

### — VECTOR.lsp BOOTSTRAP —

```

(|/VERSIONCHECK| 2)

(DEFUN |VECTOR;vector;L$;1| (|1| |$|) (SPADCALL |1| (QREFELT |$| 8)))

(DEFUN |VECTOR;convert;$If;2| (|x| |$|)
  (SPADCALL
    (LIST
      (SPADCALL (SPADCALL "vector" (QREFELT |$| 12)) (QREFELT |$| 14))
      (SPADCALL (SPADCALL |x| (QREFELT |$| 15)) (QREFELT |$| 16)))
    (QREFELT |$| 18)))

(DEFUN |Vector| (G84134)
  (PROG ()
    (RETURN
      (PROG (G84135)
        (RETURN
          (COND
            ((LETT G84135
              (|lassocShiftWithFunction|
                (LIST (|devaluate| G84134))
                (HGET |$ConstructorCache| '|Vector|)
                '|domainEqualList|)
              |Vector|)
            (|CDRwithIncrement| G84135)))
        'T

```

```

      (UNWIND-PROTECT
        (PROG1 (|Vector;| G84134) (LETT G84135 T |Vector|))
        (COND
          ((NOT G84135) (HREM |$ConstructorCache| '|Vector|)))))))))

(DEFUN |Vector;| (|#1|)
  (PROG (DV$1 |dv$| $ G84133 |pv$|)
    (RETURN
      (PROGN
        (LETT DV$1 (|devaluate| |#1|) |Vector|)
        (LETT |dv$| (LIST '|Vector| DV$1) |Vector|)
        (LETT $ (make-array 36) |Vector|)
        (QSETREFV $ 0 |dv$|)
        (QSETREFV $ 3
          (LETT |pv$|
            (|buildPredVector| 0 0
              (LIST (|HasCategory| |#1| '|SetCategory|))
                (|HasCategory| |#1|
                  '|ConvertibleTo| (|InputForm|)))
              (LETT G84133
                (|HasCategory| |#1| '|OrderedSet|))
                |Vector|)
              (OR G84133
                (|HasCategory| |#1| '|SetCategory|))
                (|HasCategory| (|Integer|) '|OrderedSet|))
                (|HasCategory| |#1| '|AbelianSemiGroup|))
                (|HasCategory| |#1| '|AbelianMonoid|))
                (|HasCategory| |#1| '|AbelianGroup|))
                (|HasCategory| |#1| '|Monoid|))
                (|HasCategory| |#1| '|Ring|))
                (AND (|HasCategory| |#1|
                  '|RadicalCategory|))
                  (|HasCategory| |#1| '|Ring|)))
              (AND (|HasCategory| |#1|
                (LIST '|Evalable|
                  (|devaluate| |#1|)))
                  (|HasCategory| |#1| '|SetCategory|)))
              (OR (AND (|HasCategory| |#1|
                (LIST '|Evalable|
                  (|devaluate| |#1|)))
                  G84133)
                (AND (|HasCategory| |#1|
                  (LIST '|Evalable|
                    (|devaluate| |#1|)))
                    (|HasCategory| |#1|
                      '|SetCategory|))))))
                |Vector|))
          (|haddProp| |$ConstructorCache| '|Vector| (LIST DV$1)
            (CONS 1 $))
          (|stuffDomainSlots| $)

```

```

(QSETREFV $ 6|#1|)
(COND
  ((|testBitVector| |pv$| 2)
   (QSETREFV $ 19
    (CONS (|dispatchFunction| |VECTOR;convert;$If;2|) $))))
  $))))

(setf (get
  (QUOTE |Vector|)
  (QUOTE |infovec|))
  (LIST
    (QUOTE #(NIL NIL NIL NIL NIL (|IndexedVector| 6 (NRTEVAL 1)) (|local| |#1|)
      (|List| 6) (0 . |construct|) |VECTOR;vector;L$;1| (|String|) (|Symbol|)
      (5 . |coerce|) (|InputForm|) (10 . |convert|) (15 . |parts|)
      (20 . |convert|) (|List| |$|) (25 . |convert|) (30 . |convert|)
      (|Mapping| 6 6 6) (|Boolean|) (|NonNegativeInteger|) (|List| 24)
      (|Equation| 6) (|Integer|) (|Mapping| 21 6) (|Mapping| 21 6 6)
      (|UniversalSegment| 25) (|Void|) (|Mapping| 6 6) (|Matrix| 6)
      (|OutputForm|) (|SingleInteger|) (|Union| 6 (QUOTE "failed"))
      (|List| 25)))
    (QUOTE #(|vector| 35 |parts| 40 |convert| 45 |construct| 50))
    (QUOTE ((|shallowlyMutable| . 0) (|finiteAggregate| . 0)))
    (CONS
      (|makeByteWordVec2| 13 (QUOTE (0 0 0 0 0 0 3 0 0 13 4 0 0 13 1 2 4)))
      (CONS
        (QUOTE #(|VectorCategory&| |OneDimensionalArrayAggregate&|
          |FiniteLinearAggregate&| |LinearAggregate&| |IndexedAggregate&|
          |Collection&| |HomogeneousAggregate&| |OrderedSet&| |Aggregate&|
          |EltableAggregate&| |Evalable&| |SetCategory&| NIL NIL
          |InnerEvalable&| NIL NIL |BasicType&|))
        (CONS
          (QUOTE #((|VectorCategory| 6) (|OneDimensionalArrayAggregate| 6)
            (|FiniteLinearAggregate| 6) (|LinearAggregate| 6)
            (|IndexedAggregate| 25 6) (|Collection| 6)
            (|HomogeneousAggregate| 6) (|OrderedSet|) (|Aggregate|)
            (|EltableAggregate| 25 6) (|Evalable| 6) (|SetCategory|)
            (|Type|) (|Eltable| 25 6) (|InnerEvalable| 6 6)
            (|CoercibleTo| 32) (|ConvertibleTo| 13) (|BasicType|)))
          (|makeByteWordVec2| 19
            (QUOTE (1 0 0 7 8 1 11 0 10 12 1 13 0 11 14 1 0 7 0 15 1 7 13 0 16 1 13
              0 17 18 1 0 13 0 19 1 0 0 7 9 1 0 7 0 15 1 2 13 0 19 1 0 0 7 8))))))
      (QUOTE |lookupIncomplete|)))

```

---



## Chapter 29

# Chunk collections

— algebra —

```
\getchunk{domain AFFPL AffinePlane}  
\getchunk{domain AFFPLPS AffinePlaneOverPseudoAlgebraicClosureOfFiniteField}  
\getchunk{domain AFFSP AffineSpace}  
\getchunk{domain ALGSC AlgebraGivenByStructuralConstants}  
\getchunk{domain ALGFF AlgebraicFunctionField}  
\getchunk{domain AN AlgebraicNumber}  
\getchunk{domain ANON AnonymousFunction}  
\getchunk{domain ANTISYM AntiSymm}  
\getchunk{domain ANY Any}  
\getchunk{domain ASTACK ArrayStack}  
\getchunk{domain ASP1 Asp1}  
\getchunk{domain ASP10 Asp10}  
\getchunk{domain ASP12 Asp12}  
\getchunk{domain ASP19 Asp19}  
\getchunk{domain ASP20 Asp20}  
\getchunk{domain ASP24 Asp24}  
\getchunk{domain ASP27 Asp27}  
\getchunk{domain ASP28 Asp28}  
\getchunk{domain ASP29 Asp29}  
\getchunk{domain ASP30 Asp30}  
\getchunk{domain ASP31 Asp31}  
\getchunk{domain ASP33 Asp33}  
\getchunk{domain ASP34 Asp34}  
\getchunk{domain ASP35 Asp35}  
\getchunk{domain ASP4 Asp4}  
\getchunk{domain ASP41 Asp41}  
\getchunk{domain ASP42 Asp42}  
\getchunk{domain ASP49 Asp49}  
\getchunk{domain ASP50 Asp50}
```



```

\getchunk{domain ASP55 Asp55}
\getchunk{domain ASP6 Asp6}
\getchunk{domain ASP7 Asp7}
\getchunk{domain ASP73 Asp73}
\getchunk{domain ASP74 Asp74}
\getchunk{domain ASP77 Asp77}
\getchunk{domain ASP78 Asp78}
\getchunk{domain ASP8 Asp8}
\getchunk{domain ASP80 Asp80}
\getchunk{domain ASP9 Asp9}
\getchunk{domain JORDAN AssociatedJordanAlgebra}
\getchunk{domain LIE AssociatedLieAlgebra}
\getchunk{domain ALIST AssociationList}
\getchunk{domain ATTRIBUT AttributeButtons}
\getchunk{domain AUTOMOR Automorphism}

\getchunk{domain BBTREE BalancedBinaryTree}
\getchunk{domain BPADIC BalancedPAdicInteger}
\getchunk{domain BPADICRT BalancedPAdicRational}
\getchunk{domain BFUNCT BasicFunctions}
\getchunk{domain BOP BasicOperator}
\getchunk{domain BINARY BinaryExpansion}
\getchunk{domain BINFILE BinaryFile}
\getchunk{domain BSTREE BinarySearchTree}
\getchunk{domain BTOURN BinaryTournament}
\getchunk{domain BTREE BinaryTree}
\getchunk{domain BITS Bits}
\getchunk{domain BLHN BlowUpWithHamburgerNoether}
\getchunk{domain BLQT BlowUpWithQuadTrans}
\getchunk{domain BOOLEAN Boolean}

\getchunk{domain CARD CardinalNumber}
\getchunk{domain CARTEN CartesianTensor}
\getchunk{domain CHAR Character}
\getchunk{domain CCLASS CharacterClass}
\getchunk{domain CLIF CliffordAlgebra}
\getchunk{domain COLOR Color}
\getchunk{domain COMM Commutator}
\getchunk{domain COMPLEX Complex}
\getchunk{domain CDFMAT ComplexDoubleFloatMatrix}
\getchunk{domain CDFVEC ComplexDoubleFloatVector}
\getchunk{domain CONTFRAC ContinuedFraction}

\getchunk{domain DHMATRIX DenavitHartenbergMatrix}
\getchunk{domain DBASE Database}
\getchunk{domain DLIST DataList}
\getchunk{domain DECIMAL DecimalExpansion}
\getchunk{domain DEQUEUE Dequeue}
\getchunk{domain DERHAM DeRhamComplex}
\getchunk{domain DSMP DifferentialSparseMultivariatePolynomial}

```

```

\getchunk{domain DIRPROD DirectProduct}
\getchunk{domain DPMM DirectProductMatrixModule}
\getchunk{domain DPMO DirectProductModule}
\getchunk{domain DIRRING DirichletRing}
\getchunk{domain DMP DistributedMultivariatePolynomial}
\getchunk{domain DIV Divisor}
\getchunk{domain DFLOAT DoubleFloat}
\getchunk{domain DFMAT DoubleFloatMatrix}
\getchunk{domain DFVEC DoubleFloatVector}
\getchunk{domain DROPT DrawOption}
\getchunk{domain D01AJFA d01ajfAnnaType}
\getchunk{domain D01AKFA d01akfAnnaType}
\getchunk{domain D01ALFA d01alfAnnaType}
\getchunk{domain D01AMFA d01amfAnnaType}
\getchunk{domain D01ANFA d01anfAnnaType}
\getchunk{domain D01APFA d01apfAnnaType}
\getchunk{domain D01AQFA d01aqfAnnaType}
\getchunk{domain D01ASFA d01asfAnnaType}
\getchunk{domain D01FCFA d01fcfAnnaType}
\getchunk{domain D01GBFA d01gbfAnnaType}
\getchunk{domain D01TRNS d01TransformFunctionType}
\getchunk{domain D02BBFA d02bbfAnnaType}
\getchunk{domain D02BHFA d02bhfAnnaType}
\getchunk{domain D02CJFA d02cjfAnnaType}
\getchunk{domain D02EJFA d02ejfAnnaType}
\getchunk{domain D03EEFA d03eefAnnaType}
\getchunk{domain D03FAFA d03fafAnnaType}

\getchunk{domain EQTBL EqTable}
\getchunk{domain EQ Equation}
\getchunk{domain EXPEXPAN ExponentialExpansion}
\getchunk{domain EXPUPXS ExponentialOfUnivariatePuisseuxSeries}
\getchunk{domain EMR EuclideanModularRing}
\getchunk{domain EXIT Exit}
\getchunk{domain EXPR Expression}
\getchunk{domain EAB ExtAlgBasis}
\getchunk{domain E04DGFA e04dgmAnnaType}
\getchunk{domain E04FDFA e04fdfAnnaType}
\getchunk{domain E04GCFA e04gcfAnnaType}
\getchunk{domain E04JFAFA e04jafAnnaType}
\getchunk{domain E04MBFA e04mbfAnnaType}
\getchunk{domain E04NAFA e04nafAnnaType}
\getchunk{domain E04UCFA e04ucfAnnaType}

\getchunk{domain FR Factored}
\getchunk{domain FILE File}
\getchunk{domain FNAME FileName}
\getchunk{domain FARRAY FlexibleArray}
\getchunk{domain FDIV FiniteDivisor}
\getchunk{domain FF FiniteField}

```

```

\getchunk{domain FFCG FiniteFieldCyclicGroup}
\getchunk{domain FFCGX FiniteFieldCyclicGroupExtension}
\getchunk{domain FFCGP FiniteFieldCyclicGroupExtensionByPolynomial}
\getchunk{domain FFX FiniteFieldExtension}
\getchunk{domain FFP FiniteFieldExtensionByPolynomial}
\getchunk{domain FFNB FiniteFieldNormalBasis}
\getchunk{domain FFNBX FiniteFieldNormalBasisExtension}
\getchunk{domain FFNBP FiniteFieldNormalBasisExtensionByPolynomial}
\getchunk{domain FLOAT Float}
\getchunk{domain FC FortranCode}
\getchunk{domain FEXPR FortranExpression}
\getchunk{domain FORTRAN FortranProgram}
\getchunk{domain FST FortranScalarType}
\getchunk{domain FTEM FortranTemplate}
\getchunk{domain FT FortranType}
\getchunk{domain FCOMP FourierComponent}
\getchunk{domain FSERIES FourierSeries}
\getchunk{domain FRAC Fraction}
\getchunk{domain FRIDEAL FractionalIdeal}
\getchunk{domain FRMOD FramedModule}
\getchunk{domain FAGROUP FreeAbelianGroup}
\getchunk{domain FAMONOID FreeAbelianMonoid}
\getchunk{domain FGROU FreeGroup}
\getchunk{domain FM FreeModule}
\getchunk{domain FM1 FreeModule1}
\getchunk{domain FMONOID FreeMonoid}
\getchunk{domain FNLA FreeNilpotentLie}
\getchunk{domain FPARFRAC FullPartialFractionExpansion}
\getchunk{domain FUNCTION FunctionCalled}

\getchunk{domain GDMP GeneralDistributedMultivariatePolynomial}
\getchunk{domain GMDPOL GeneralModulePolynomial}
\getchunk{domain GCNAALG GenericNonAssociativeAlgebra}
\getchunk{domain GPOLSET GeneralPolynomialSet}
\getchunk{domain GSTBL GeneralSparseTable}
\getchunk{domain GTSET GeneralTriangularSet}
\getchunk{domain GSERIES GeneralUnivariatePowerSeries}
\getchunk{domain GRIMAGE GraphImage}
\getchunk{domain GOPT GuessOption}
\getchunk{domain GOPTO GuessOptionFunctions0}

\getchunk{domain HASHTBL HashTable}
\getchunk{domain HEAP Heap}
\getchunk{domain HEXADEC HexadecimalExpansion}
\getchunk{domain HTMLFORM HTMLFormat}
\getchunk{domain HDP HomogeneousDirectProduct}
\getchunk{domain HDMP HomogeneousDistributedMultivariatePolynomial}
\getchunk{domain HELLDIV HyperellipticFiniteDivisor}

\getchunk{domain ICP InfClsPt}

```

```

\getchunk{domain ICARD IndexCard}
\getchunk{domain IBITS IndexedBits}
\getchunk{domain IDPAG IndexedDirectProductAbelianGroup}
\getchunk{domain IDPAM IndexedDirectProductAbelianMonoid}
\getchunk{domain IDPO IndexedDirectProductObject}
\getchunk{domain IDPOAM IndexedDirectProductOrderedAbelianMonoid}
\getchunk{domain IDPOAMS IndexedDirectProductOrderedAbelianMonoidSup}
\getchunk{domain INDE IndexedExponents}
\getchunk{domain IFARRAY IndexedFlexibleArray}
\getchunk{domain ILIST IndexedList}
\getchunk{domain IMATRIX IndexedMatrix}
\getchunk{domain IARRAY1 IndexedOneDimensionalArray}
\getchunk{domain ISTRING IndexedString}
\getchunk{domain IARRAY2 IndexedTwoDimensionalArray}
\getchunk{domain IVECTOR IndexedVector}
\getchunk{domain ITUPLE InfiniteTuple}
\getchunk{domain INFCLSPT InfinitelyClosePoint}
\getchunk{domain INFCLSPS InfinitelyClosePointOverPseudoAlgebraicClosureOfFiniteField}
\getchunk{domain IAN InnerAlgebraicNumber}
\getchunk{domain IFF InnerFiniteField}
\getchunk{domain IFAMON InnerFreeAbelianMonoid}
\getchunk{domain IARRAY2 InnerIndexedTwoDimensionalArray}
\getchunk{domain IPADIC InnerPAdicInteger}
\getchunk{domain IPF InnerPrimeField}
\getchunk{domain ISUPS InnerSparseUnivariatePowerSeries}
\getchunk{domain INTABL InnerTable}
\getchunk{domain ITAYLOR InnerTaylorSeries}
\getchunk{domain INFORM InputForm}
\getchunk{domain INT Integer}
\getchunk{domain ZMOD IntegerMod}
\getchunk{domain INTFTBL IntegrationFunctionsTable}
\getchunk{domain IR IntegrationResult}
\getchunk{domain INTRVL Interval}

\getchunk{domain KERNEL Kernel}
\getchunk{domain KAFILE KeyedAccessFile}

\getchunk{domain LAUPOL LaurentPolynomial}
\getchunk{domain LIB Library}
\getchunk{domain LEXP LieExponentials}
\getchunk{domain LPOLY LiePolynomial}
\getchunk{domain LSQM LieSquareMatrix}
\getchunk{domain LODO LinearOrdinaryDifferentialOperator}
\getchunk{domain LODO1 LinearOrdinaryDifferentialOperator1}
\getchunk{domain LODO2 LinearOrdinaryDifferentialOperator2}
\getchunk{domain LIST List}
\getchunk{domain LMOPS ListMonoidOps}
\getchunk{domain LMDICT ListMultiDictionary}
\getchunk{domain LA LocalAlgebra}
\getchunk{domain LO Localize}

```

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\getchunk{domain LWORD LyndonWord}

\getchunk{domain MCMPLX MachineComplex}
\getchunk{domain MFLOAT MachineFloat}
\getchunk{domain MINT MachineInteger}
\getchunk{domain MAGMA Magma}
\getchunk{domain MKCHSET MakeCachableSet}
\getchunk{domain MMLFORM MathMLFormat}
\getchunk{domain MATRIX Matrix}
\getchunk{domain MODMON ModMonic}
\getchunk{domain MODMONOM ModuleMonomial}
\getchunk{domain MODFIELD ModularField}
\getchunk{domain MODRING ModularRing}
\getchunk{domain MODOP ModuleOperator}
\getchunk{domain MOEBIUS MoebiusTransform}
\getchunk{domain MRING MonoidRing}
\getchunk{domain MSET Multiset}
\getchunk{domain MPOLY MultivariatePolynomial}
\getchunk{domain MYEXPR MyExpression}
\getchunk{domain MYUP MyUnivariatePolynomial}

\getchunk{domain NSDPS NeitherSparseOrDensePowerSeries}
\getchunk{domain NSMP NewSparseMultivariatePolynomial}
\getchunk{domain NSUP NewSparseUnivariatePolynomial}
\getchunk{domain NONE None}
\getchunk{domain NNI NonNegativeInteger}
\getchunk{domain NOTTING NottinghamGroup}
\getchunk{domain NIPROB NumericalIntegrationProblem}
\getchunk{domain ODEPROB NumericalODEProblem}
\getchunk{domain OPTPROB NumericalOptimizationProblem}
\getchunk{domain PDEPROB NumericalPDEProblem}

\getchunk{domain OCT Octonion}
\getchunk{domain ODEIFTBL ODEIntensityFunctionsTable}
\getchunk{domain ARRAY1 OneDimensionalArray}
\getchunk{domain ONECOMP OnePointCompletion}
\getchunk{domain OMCONN OpenMathConnection}
\getchunk{domain OMDEV OpenMathDevice}
\getchunk{domain OMENC OpenMathEncoding}
\getchunk{domain OMERR OpenMathError}
\getchunk{domain OMERRK OpenMathErrorKind}
\getchunk{domain OP Operator}
\getchunk{domain OML0 OppositeMonogenicLinearOperator}
\getchunk{domain ORDCOMP OrderedCompletion}
\getchunk{domain ODP OrderedDirectProduct}
\getchunk{domain OFMONOID OrderedFreeMonoid}
\getchunk{domain OVAR OrderedVariableList}
\getchunk{domain ODPOL OrderlyDifferentialPolynomial}
\getchunk{domain ODVAR OrderlyDifferentialVariable}
\getchunk{domain ODR OrdinaryDifferentialRing}

```

```

\getchunk{domain OWP OrdinaryWeightedPolynomials}
\getchunk{domain OSI OrdSetInts}
\getchunk{domain OUTFORM OutputForm}

\getchunk{domain PADIC PAdicInteger}
\getchunk{domain PADICRC PAdicRationalConstructor}
\getchunk{domain PADICRAT PAdicRational}
\getchunk{domain PALETTE Palette}
\getchunk{domain PARPCURV ParametricPlaneCurve}
\getchunk{domain PARSCURV ParametricSpaceCurve}
\getchunk{domain PARSURF ParametricSurface}
\getchunk{domain PFR PartialFraction}
\getchunk{domain PLACES Places}
\getchunk{domain PLACESPS PlacesOverPseudoAlgebraicClosureOfFiniteField}
\getchunk{domain PRITITION Partition}
\getchunk{domain PATTERN Pattern}
\getchunk{domain PATLRES PatternMatchListResult}
\getchunk{domain PATRES PatternMatchResult}
\getchunk{domain PENDTREE PendantTree}
\getchunk{domain PERM Permutation}
\getchunk{domain PERMGRP PermutationGroup}
\getchunk{domain HACKPI Pi}
\getchunk{domain ACPLLOT PlaneAlgebraicCurvePlot}
\getchunk{domain PLCS Plcs}
\getchunk{domain PLOT Plot}
\getchunk{domain PLOT3D Plot3D}
\getchunk{domain PBWLB PoincareBirkhoffWittLyndonBasis}
\getchunk{domain POINT Point}
\getchunk{domain POLY Polynomial}
\getchunk{domain IDEAL PolynomialIdeals}
\getchunk{domain PR PolynomialRing}
\getchunk{domain PI PositiveInteger}
\getchunk{domain PF PrimeField}
\getchunk{domain PRIMARR PrimitiveArray}
\getchunk{domain PRODUCT Product}
\getchunk{domain PROJPL ProjectivePlane}
\getchunk{domain PROJSP ProjectiveSpace}
\getchunk{domain PACEXT PseudoAlgebraicClosureOfAlgExtOfRationalNumber}
\getchunk{domain PACOFF PseudoAlgebraicClosureOfFiniteField}
\getchunk{domain PACRAT PseudoAlgebraicClosureOfRationalNumber}

\getchunk{domain QFORM QuadraticForm}
\getchunk{domain QALGSET QuasiAlgebraicSet}
\getchunk{domain QUAT Quaternion}
\getchunk{domain QEQUAT QueryEquation}
\getchunk{domain QUEUE Queue}

\getchunk{domain RADFF RadicalFunctionField}
\getchunk{domain RADIX RadixExpansion}
\getchunk{domain RECLOSE RealClosure}

```

```

\getchunk{domain RMATRIX RectangularMatrix}
\getchunk{domain REF Reference}
\getchunk{domain RGCHAIN RegularChain}
\getchunk{domain REGSET RegularTriangularSet}
\getchunk{domain RESRING ResidueRing}
\getchunk{domain RESULT Result}
\getchunk{domain RULE RewriteRule}
\getchunk{domain ROIRC RightOpenIntervalRootCharacterization}
\getchunk{domain ROMAN RomanNumeral}
\getchunk{domain ROUTINE RoutinesTable}
\getchunk{domain RULECOLD RuleCalled}
\getchunk{domain RULESET Ruleset}

\getchunk{domain FORMULA ScriptFormulaFormat}
\getchunk{domain SEG Segment}
\getchunk{domain SEGBIND SegmentBinding}
\getchunk{domain SET Set}
\getchunk{domain SEX SExpression}
\getchunk{domain SEXOF SExpressionOf}
\getchunk{domain SAE SimpleAlgebraicExtension}
\getchunk{domain SFORT SimpleFortranProgram}
\getchunk{domain SINT SingleInteger}
\getchunk{domain SAOS SingletonAsOrderedSet}
\getchunk{domain SDPOL SequentialDifferentialPolynomial}
\getchunk{domain SDVAR SequentialDifferentialVariable}
\getchunk{domain SETMN SetOfMIntegersInOneToN}
\getchunk{domain SMP SparseMultivariatePolynomial}
\getchunk{domain SMTS SparseMultivariateTaylorSeries}
\getchunk{domain STBL SparseTable}
\getchunk{domain SULS SparseUnivariateLaurentSeries}
\getchunk{domain SUP SparseUnivariatePolynomial}
\getchunk{domain SUEXPR SparseUnivariatePolynomialExpressions}
\getchunk{domain SUPXS SparseUnivariatePuisseuxSeries}
\getchunk{domain ORESUP SparseUnivariateSkewPolynomial}
\getchunk{domain SUTS SparseUnivariateTaylorSeries}
\getchunk{domain SHDP SplitHomogeneousDirectProduct}
\getchunk{domain SPLNODE SplittingNode}
\getchunk{domain SPLTREE SplittingTree}
\getchunk{domain SREGSET SquareFreeRegularTriangularSet}
\getchunk{domain SQMATRIX SquareMatrix}
\getchunk{domain STACK Stack}
\getchunk{domain STREAM Stream}
\getchunk{domain STRING String}
\getchunk{domain STRTBL StringTable}
\getchunk{domain SUBSPACE SubSpace}
\getchunk{domain COMPPROP SubSpaceComponentProperty}
\getchunk{domain SUCH SuchThat}
\getchunk{domain SWITCH Switch}
\getchunk{domain SYMBOL Symbol}
\getchunk{domain SYMTAB SymbolTable}

```

```

\getchunk{domain SYMPOLY SymmetricPolynomial}

\getchunk{domain TABLE Table}
\getchunk{domain TABLEAU Tableau}
\getchunk{domain TS TaylorSeries}
\getchunk{domain TEX TexFormat}
\getchunk{domain TEXTFILE TextFile}
\getchunk{domain SYMS TheSymbolTable}
\getchunk{domain M3D ThreeDimensionalMatrix}
\getchunk{domain VIEW3D ThreeDimensionalViewport}
\getchunk{domain SPACE3 ThreeSpace}
\getchunk{domain TREE Tree}
\getchunk{domain TUBE TubePlot}
\getchunk{domain TUPLE Tuple}
\getchunk{domain ARRAY2 TwoDimensionalArray}
\getchunk{domain VIEW2D TwoDimensionalViewport}

\getchunk{domain UFPS UnivariateFormalPowerSeries}
\getchunk{domain ULS UnivariateLaurentSeries}
\getchunk{domain ULSCONS UnivariateLaurentSeriesConstructor}
\getchunk{domain UP UnivariatePolynomial}
\getchunk{domain UPXS UnivariatePuisseuxSeries}
\getchunk{domain UPXSCONS UnivariatePuisseuxSeriesConstructor}
\getchunk{domain UPXSING UnivariatePuisseuxSeriesWithExponentialSingularity}
\getchunk{domain OREUP UnivariateSkewPolynomial}
\getchunk{domain UTS UnivariateTaylorSeries}
\getchunk{domain UTSZ UnivariateTaylorSeriesCZero}
\getchunk{domain UNISEG UniversalSegment}

\getchunk{domain VARIABLE Variable}
\getchunk{domain VECTOR Vector}
\getchunk{domain VOID Void}

\getchunk{domain WP WeightedPolynomials}
\getchunk{domain WUTSET WuWenTsunTriangularSet}

\getchunk{domain XDPOLY XDistributedPolynomial}
\getchunk{domain XPBWPOLY XPBWPolynomial}
\getchunk{domain XPOLY XPolynomial}
\getchunk{domain XPR XPolynomialRing}
\getchunk{domain XRPOLY XRecursivePolynomial}

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