FHSST Authors

The Free High School Science Texts: Textbooks for High School Students Studying the Sciences
Physics
Grades 10 - 12

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Chapter 13

Geometrical Optics - Grade 11

13.1 Introduction

In Grade 10, we studied how light is reflected and refracted. This chapter builds on what you have learnt in Grade 10. You will learn about lenses, how the human eye works as well as how telescopes and microscopes work.

13.2 Lenses

In this section we will discuss properties of **thin lenses**. In Grade 10, you learnt about two kinds of mirrors: concave mirrors which were also known as converging mirrors and convex mirrors which were also known as diverging mirrors. Similarly, there are two types of lenses: converging and diverging lenses.

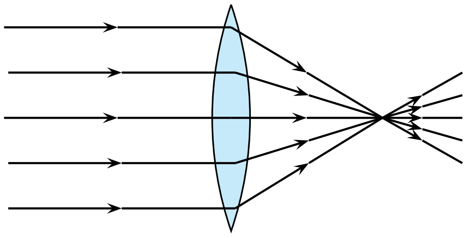
We have learnt how light travels in different materials, and we are now ready to learn how we can control the direction of light rays. We use *lenses* to control the direction of light. When light enters a lens, the light rays bend or change direction as shown in Figure 13.1.

Definition: Lens
A lens is any transparent material (e.g. glass) of an appropriate shape that can take parallel rays of incident light and either converge the rays to a point or diverge the rays from a point.

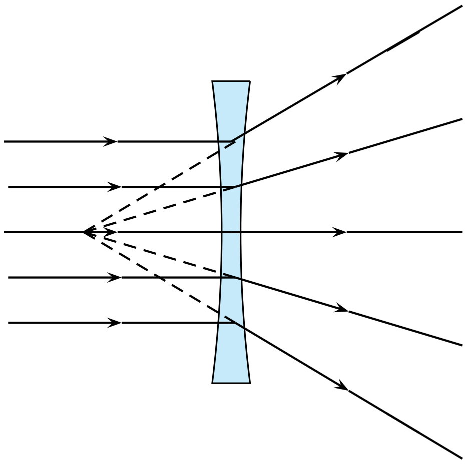
Some lenses will focus light rays to a single point. These lenses are called converging or concave lenses. Other lenses spread out the light rays so that it looks like they all come from the same point. These lenses are called diverging or convex lenses. Lenses change the direction of light rays by *refraction*. They are designed so that the image appears in a certain place or as a certain size. Lenses are used in eyeglasses, cameras, microscopes, and telescopes. You also have lenses in your eyes!

Definition: Converging Lenses
Converging lenses converge parallel rays of light and are thicker in the middle than at the edges.

Definition: Diverging Lenses
Diverging lenses diverge parallel rays of light and are thicker at the edges than in the middle.



(a) A converging lens will focus the rays that enter the lens



(b) A diverging lens will spread out the rays that enter the lens

Figure 13.1: The behaviour of parallel light rays entering either a converging or diverging lens.

Examples of converging and diverging lenses are shown in Figure 13.2.

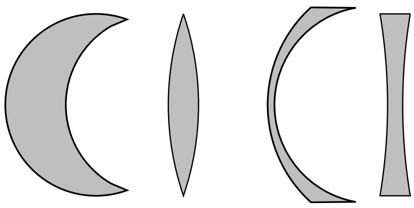
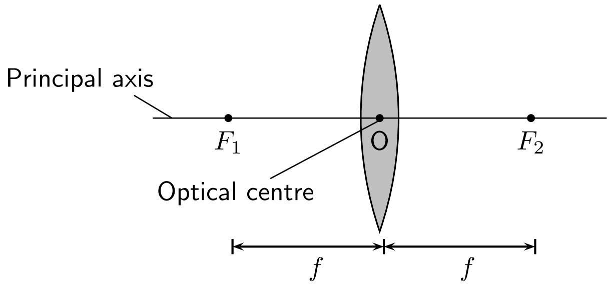


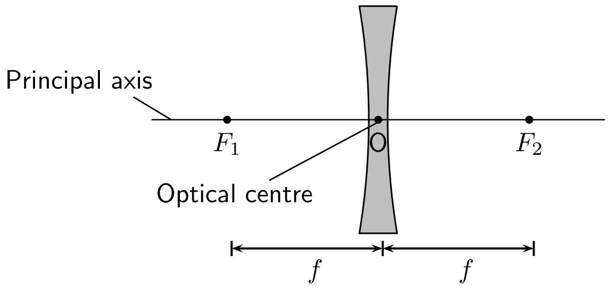
Figure 13.2: Types of lenses

Before we study lenses in detail, there are a few important terms that must be defined. Figure 13.3 shows important lens properties:

- The **principal axis** is the line which runs horizontally straight through the *optical centre* of the lens. It is also sometimes called the *optic axis*.
- The **optical centre** (O) of a convex lens is usually the centre point of the lens. The direction of all light rays which pass through the optical centre, remains unchanged.
- The **focus** or **focal point** of the lens is the position on the *principal axis* where all light rays which run *parallel* to the principal axis through the lens converge (come together) at a point. Since light can pass through the lens either from right to left or left to right, there is a focal point on each side of the lens (F_1 and F_2), at the same distance from the optical centre in each direction. (**Note:** the plural form of the word focus is *foci*.)
- The **focal length** (f) is the distance between the *optical centre* and the *focal point*.



(a) converging lens



(b) diverging lens

Figure 13.3: Properties of lenses.

13.2.1 Converging Lenses

We will only discuss double convex converging lenses as shown in Figure 13.4. Converging lenses are thinner on the outside and thicker on the inside.

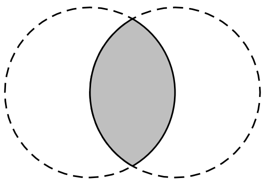


Figure 13.4: A double convex lens is a converging lens.

Figure 13.5 shows a convex lens. Light rays traveling through a **convex** lens are bent **towards** the principal axis. For this reason, convex lenses are called **converging** lenses.

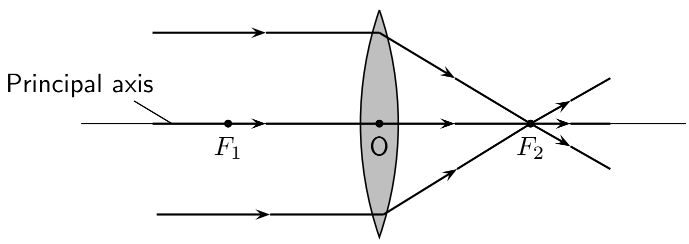


Figure 13.5: Light rays bend towards each other or *converge* when they travel through a convex lens. F_1 and F_2 are the foci of the lens.

The type of images created by a convex lens is dependent on the position of the object. We will examine the following cases:

1. the object is placed at a distance greater than $2f$ from the lens
2. the object is placed at a distance equal to $2f$ from the lens
3. the object is placed at a distance between $2f$ and f from the lens
4. the object is placed at a distance less than f from the lens

We examine the properties of the image in each of these cases by drawing ray diagrams. We can find the image by tracing the path of three light rays through the lens. Any two of these rays will show us the location of the image. You can use the third ray to check the location.

Activity :: Experiment : Lenses A

Aim:

To determine the focal length of a convex lens.

Method:

1. Using a distant object from outside, adjust the position of the convex lens so that it gives the smallest possible focus on a sheet of paper that is held parallel to the lens.
2. Measure the distance between the lens and the sheet of paper as accurately as possible.

Results:

The focal length of the lens is ________________ cm

Activity :: Experiment : Lenses B

Aim:

To investigate the position, size and nature of the image formed by a convex lens.

Method:

1. Set up the candle, the lens from Experiment Lenses A in its holder and the screen in a straight line on the metre rule. Make sure the lens holder is on the 50 cm mark.

   From your knowledge of the focal length of your lens, note where f and $2f$ are on both sides of the lens.
2. Using the position indicated on the table below, start with the candle at a position that is greater than $2f$ and adjust the position of the screen until a sharp focused image is obtained. Note that there are two positions for which a sharp focused image will not be obtained on the screen. When this is so, remove the screen and look at the candle through the lens.
3. Fill in the relevant information on the table below

Results:

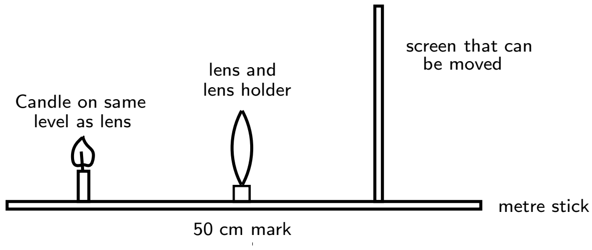


Figure 13.6: Experimental setup for investigation.

Relative position of object	Relative position of image	Image upright or inverted	Relative size of image	Nature of image
Beyond $2f$ _______ cm				
At $2f$ _______ cm				
Between $2f$ and f _______ cm				
At f _______ cm				
Between f and the lens _______ cm				

QUESTIONS:

1. When a convex lens is being used:
   1.1 A real inverted image is formed when an object is placed ________________
   1.2 No image is formed when an object is placed ________________
   1.3 An upright, enlarged, virtual image is formed when an object is placed ________________
2. Write a conclusion for this investigation.

____

____

Activity :: Experiment : Lenses C

Aim:

To determine the mathematical relationship between d_0, d_i and f for a lens.

Method:

1. Using the same arrangement as in Experiment Lenses B, place the object (candle) at the distance indicated from the lens.
2. Move the screen until a clear sharp image is obtained. Record the results on the table below.

Results:

f = focal length of lens

d_0 = object distance

d_i = image distance

Object distance d_0 (cm)	Image distance d_i (cm)	$\frac{1}{d_0}$ (cm^{-1})	$\frac{1}{d_i}$ (cm^{-1})	$\frac{1}{d_0}+\frac{1}{d_i}$ (cm^{-1})
25,0				
20,0				
18,0				
15,0				
				Average = ______

$$\text{Reciprocal of average} = \left(\frac{1}{\frac{1}{d_0}+\frac{1}{d_i}}\right) = \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (a)}$$

$$\text{Focal length of lens} = \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (b)}$$

QUESTIONS:

1. Compare the values for (a) and (b) above and explain any similarities or differences
2. What is the name of the mathematical relationship between d_0, d_i and f?
3. Write a conclusion for this part of the investigation.

____

Drawing Ray Diagrams for Converging Lenses

The three rays are labelled R_1, R_2 and R_3. The ray diagrams that follow will use this naming convention.

1. The first ray (R_1) travels from the object to the lens *parallel* to the principal axis. This ray is bent by the lens and travels through the **focal point**.
2. *Any* ray travelling parallel to the principal axis is bent through the focal point.

3. If a light ray passes through a focal point *before* it enters the lens, then it will leave the lens *parallel* to the principal axis. The second ray (R_2) is therefore drawn to pass through the focal point before it enters the lens.

4. A ray that travels through the centre of the lens does not change direction. The third ray (R_3) is drawn through the centre of the lens.

5. The point where all three of the rays (R_1, R_2 and R_3) intersect is the **image** of the point where they all started. The image will form at this point.

Important: In ray diagrams, lenses are drawn like this:

Convex lens: 

Concave lens: 

CASE 1:
Object placed at a distance greater than $2f$ from the lens

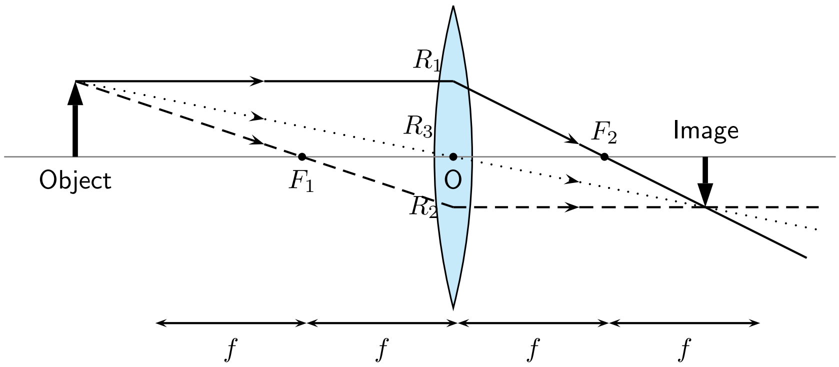


Figure 13.7: An object is placed at a distance greater than $2f$ away from the converging lens. Three rays are drawn to locate the image, which is real, smaller than the object and inverted.

We can locate the position of the image by drawing our three rays. R_1 travels from the object to the lens parallel to the principal axis and is bent by the lens and then travels through the focal point. R_2 passes through the focal point before it enters the lens and therefore must leave the lens parallel to the principal axis. R_3 travels through the center of the lens and does not change direction. The point where R_1, R_2 and R_3 intersect is the image of the point where they all started.

The image of an object placed at a distance greater than $2f$ from the lens is upside down or *inverted*. This is because the rays which began at the top of the object, *above* the principal axis, after passing through the lens end up *below* the principal axis. The image is called a **real image** because it is on the opposite side of the lens to the object and you can trace all the light rays directly from the image back to the object.

The image is also smaller than the object and is located closer to the lens than the object.

Important: In reality, light rays come from *all* points along the length of the object. In ray diagrams we only draw three rays (all starting at the top of the object) to keep the diagram clear and simple.

CASE 2:
Object placed at a distance equal to $2f$ from the lens

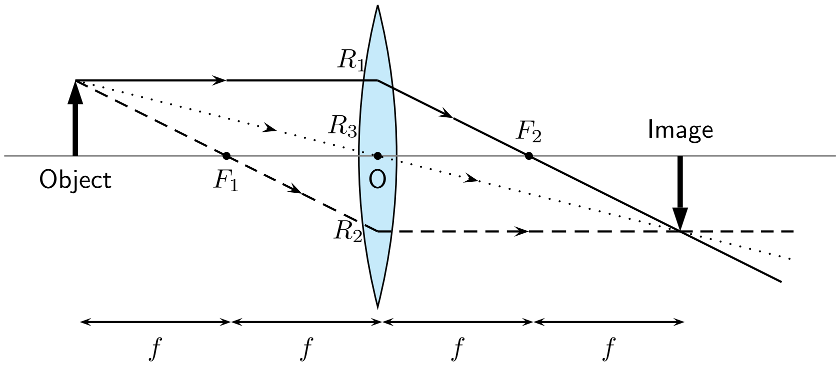


Figure 13.8: An object is placed at a distance equal to $2f$ away from the converging lens. Three rays are drawn to locate the image, which is real, the same size as the object and inverted.

We can locate the position of the image by drawing our three rays. R_1 travels from the object to the lens parallel to the principal axis and is bent by the lens and then travels through the focal point. R_2 passes through the focal point before it enters the lens and therefore must leave the lens parallel to the principal axis. R_3 travels through the center of the lens and does not change direction. The point where R_1, R_2 and R_3 intersect is the image of the point where they all started.

The image of an object placed at a distance equal to $2f$ from the lens is upside down or *inverted*. This is because the rays which began at the top of the object, *above* the principal axis, after passing through the lens end up *below* the principal axis. The image is called a **real image** because it is on the opposite side of the lens to the object and you can trace all the light rays directly from the image back to the object.

The image is the same size as the object and is located at a distance $2f$ away from the lens.

CASE 3:
Object placed at a distance between $2f$ and f from the lens

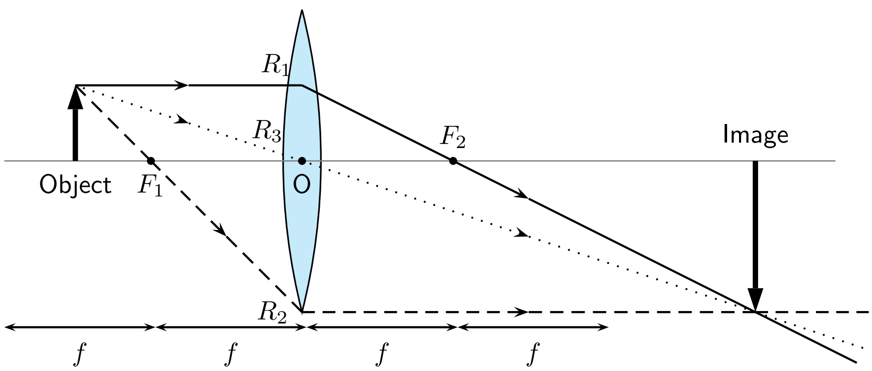


Figure 13.9: An object is placed at a distance between $2f$ and f away from the converging lens. Three rays are drawn to locate the image, which is real, larger than the object and inverted.

We can locate the position of the image by drawing our three rays. R_1 travels from the object to the lens parallel to the principal axis and is bent by the lens and then travels through the focal point. R_2 passes through the focal point before it enters the lens and therefore must leave the lens parallel to the principal axis. R_3 travels through the center of the lens and does not change direction. The point where R_1, R_2 and R_3 intersect is the image of the point where they all started.

The image of an object placed at a distance between $2f$ and f from the lens is upside down or *inverted*. This is because the rays which began at the top of the object, *above* the principal axis, after passing through the lens end up *below* the principal axis. The image is called a real image because it is on the opposite side of the lens to the object and you can trace all the light rays directly from the image back to the object.

The image is larger than the object and is located at a distance greater than $2f$ away from the lens.

CASE 4:
Object placed at a distance less than f from the lens

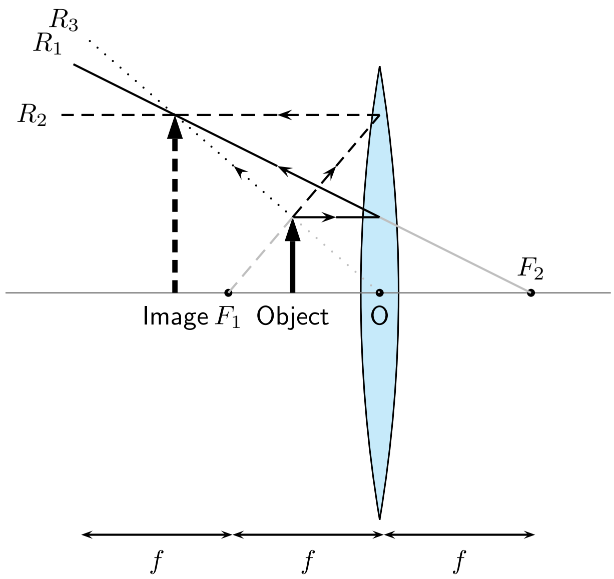


Figure 13.10: An object is placed at a distance less than f away from the converging lens. Three rays are drawn to locate the image, which is virtual, larger than the object and upright.

We can locate the position of the image by drawing our three rays. R_1 travels from the object to the lens parallel to the principal axis and is bent by the lens and then travels through the focal point. R_2 passes through the focal point before it enters the lens and therefore must leave the lens parallel to the principal axis. R_3 travels through the center of the lens and does not change direction. The point where R_1, R_2 and R_3 intersect is the image of the point where they all started.

The image of an object placed at a distance less than f from the lens is upright. The image is called a **virtual image** because it is on the same side of the lens as the object and you *cannot* trace all the light rays directly from the image back to the object.

The image is larger than the object and is located further away from the lens than the object.

Extension: The thin lens equation and magnification

The Thin Lens Equation

We can find the position of the image of a lens mathematically as there is mathematical relation between the object distance, image distance, and focal length. The equation is:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

where f is the focal length, d_o is the object distance and d_i is the image distance.

The object distance d_o is the distance from the object to the lens. d_o is positive if the object is on the same side of the lens as the light rays enter the lens. This should make sense: we expect the light rays to travel from the object to the lens. The image distance d_i is the distance from the lens to the image. Unlike mirrors, which reflect light back, lenses refract light through them. We expect to find the image on the same side of the lens as the light leaves the lens. If this is the case, then d_i is *positive* and the image is **real** (see Figure 13.9). Sometimes the image will be on the same side of the lens as the light rays enter the lens. Then d_i is *negative* and the image is **virtual** (Figure 13.10). If we know any two of the three quantities above, then we can use the Thin Lens Equation to solve for the third quantity.

Magnification

It is possible to calculate the magnification of an image. The magnification is how much *bigger* or *smaller* the image is than the object.

$$m = -\frac{d_i}{d_o}$$

where m is the magnification, d_o is the object distance and d_i is the image distance.

If d_i and d_o are both positive, the magnification is negative. This means the image is inverted, or upside down. If d_i is negative and d_o is positive, then the image is not inverted, or right side up. If the absolute value of the magnification is *greater than one*, the image is *larger* than the object. For example, a magnification of -2 means the image is *inverted* and *twice as big* as the object.

Worked Example 97: Using the lens equation

Question: An object is placed 6 cm from a converging lens with a focal point of 4 cm.

1. Calculate the position of the image
2. Calculate the magnification of the lens
3. Identify three properties of the image

Answer

Step 1 : Identify what is given and what is being asked

$$\begin{aligned} f &= 4\ \text{cm} \\ d_o &= 6\ \text{cm} \\ d_i &= ? \\ m &= ? \end{aligned}$$

Properties of the image are required.

Step 2 : Calculate the image distance (d_i)

$$\begin{aligned} \frac{1}{f} &= \frac{1}{d_o} + \frac{1}{d_i} \\ \frac{1}{4} &= \frac{1}{6} + \frac{1}{d_i} \\ \frac{1}{4} - \frac{1}{6} &= \frac{1}{d_i} \\ \frac{3-2}{12} &= \frac{1}{d_i} \\ d_i &= 12\ \text{cm} \end{aligned}$$

Step 3 : Calculate the magnification

$$\begin{aligned} m &= -\frac{d_i}{d_o} \\ &= -\frac{12}{6} \\ &= -2 \end{aligned}$$

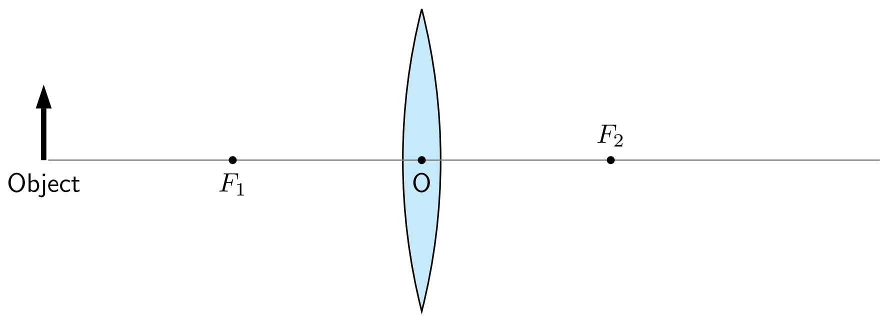
Step 4 : Write down the properties of the image
The image is real, d_i is positive, inverted (because the magnification is negative) and enlarged (magnification is > 1)

Worked Example 98: Locating the image position of a convex lens: I

Question: An object is placed 5 cm to the left of a converging lens which has a focal length of 2,5 cm.

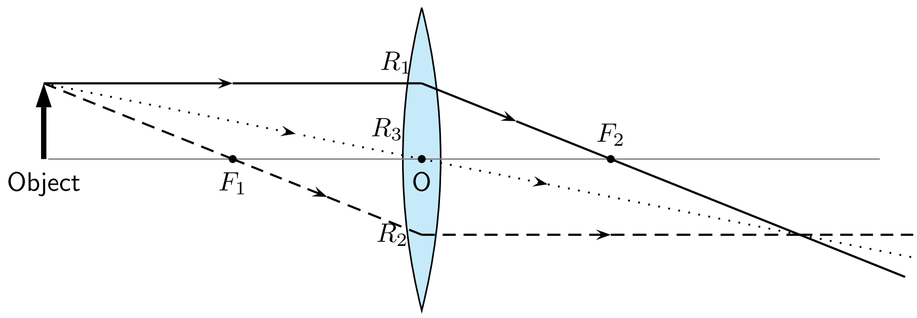
1. What is the position of the image?
2. Is the image real or virtual?

Answer
Step 1 : Set up the ray diagram
Draw the lens, the object and mark the focal points.

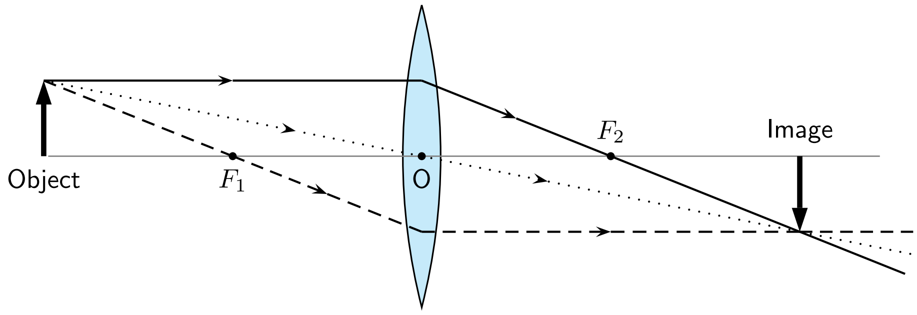


Step 2 : Draw the three rays

- R_1 goes from the top of the object parallel to the principal axis, through the lens and through the focal point F_2 on the other side of the lens.
- R_2 goes from the top of the object through the focal point F_1, through the lens and out parallel to the principal axis.
- R_3 goes from the top of the object through the optical centre with its direction unchanged.



Step 3 : Find the image
The image is at the place where all the rays intersect. Draw the image.



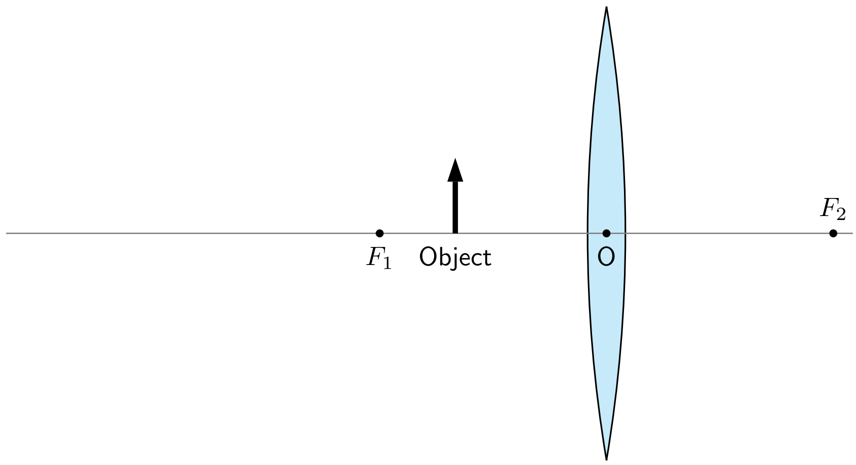
Step 4 : Measure the distance between the lens and the image
The image is 5 cm away from the lens, on the opposite side of the lens to the object.
Step 5 : Is the image virtual or real?
Since the image is on the opposite side of the lens to the object, the image is real.

Worked Example 99: Locating the image position of a convex lens: II

Question: An object, 1 cm high, is placed 2 cm to the left of a converging lens which has a focal length of 3,0 cm. The image is found also on the left side of the lens.

1. Is the image real or virtual?
2. What is the position and height of the image?

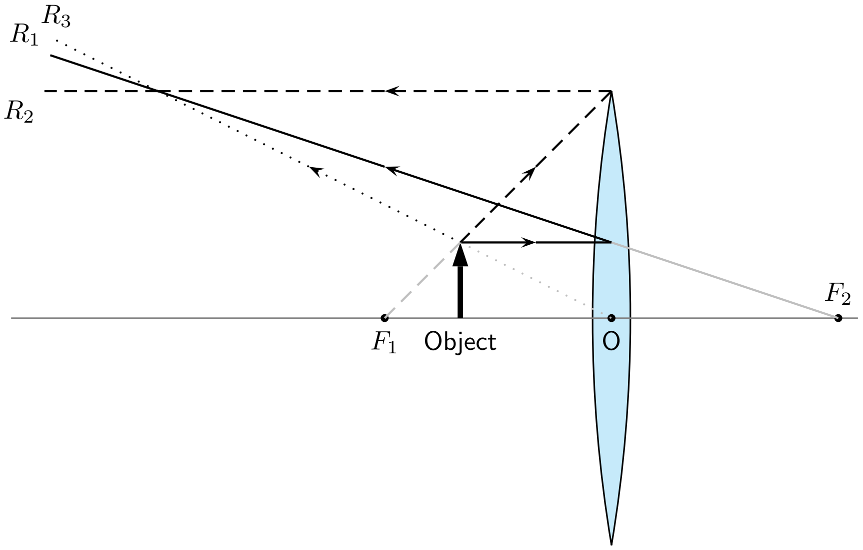
Answer
Step 1 : Draw the picture to set up the problem
Draw the lens, principal axis, focal points and the object.



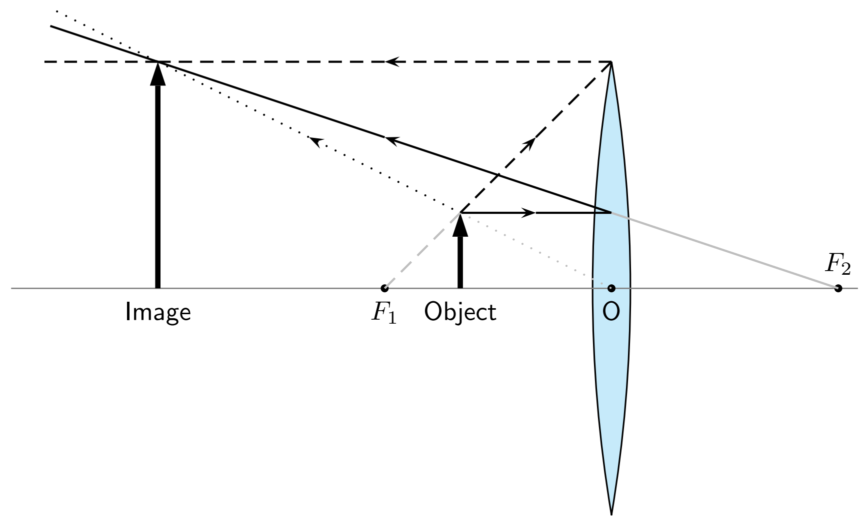
Step 2 : Draw the three rays to locate image

- R_1 goes from the top of the object parallel to the principal axis, through the lens and through the focal point F_2 on the other side of the lens.
- R_2 is the light ray which should go through the focal point F_1 but the object is placed *after* the focal point! This is not a problem, just trace the line from the focal point F_1, through the top of the object, to the lens. This ray then leaves the lens parallel to the principal axis.
- R_3 goes from the top of the object through the optical centre with its direction unchanged.

- Do not write R_1, R_2 and R_3 on your diagram, otherwise it becomes too cluttered.
- Since the rays do not intersect on the right side of the lens, we need to trace them backwards to find the place where they do come together (these are the light gray lines). Again, this is the position of the image.



Step 3 : Draw the image



Step 4 : Measure distance to image
The image is 6 cm away from the lens, on the same side as the object.
Step 5 : Measure the height of the image
The image is 3 cm high.
Step 6 : Is image real or virtual?
Since the image is on the same side of the lens as the object, the image is virtual.

?

Exercise: Converging Lenses

1. Which type of lens can be used as a magnifying glass? Draw a diagram to show how it works. An image of the sun is formed at the principal focus of a magnifying glass.

2. In each case state whether a real or virtual image is formed:
   2.1 Much further than $2f$
   2.2 Just further than $2f$
   2.3 At $2f$
   2.4 Between $2f$ and f
   2.5 At f
   2.6 Between f and 0

   Is a virtual image always inverted?
3. An object stands 50 mm from a lens (focal length 40 mm). Draw an accurate sketch to determine the position of the image. Is it enlarged or shrunk; upright or inverted?
4. Draw a scale diagram (scale: 1 cm = 50 mm) to find the position of the image formed by a convex lens with a focal length of 200 mm. The distance of the object is 100 mm and the size of the object is 50 mm. Determine whether the image is enlarged or shrunk. What is the height of the image? What is the magnification?
5. An object, 20 mm high, is 80 mm from a convex lens with focal length 50 mm. Draw an accurate scale diagram and find the position and size of the image, and hence the ratio between the image size and object size.
6. An object, 50 mm high, is placed 100 mm from a convex lens with a focal length of 150 mm. Construct an accurate ray diagram to determine the nature of the image, the size of the image and the magnification. Check your answer for the magnification by using a calculation.
7. What would happen if you placed the object right at the focus of a converging lens? Hint: Draw the picture.

13.2.2 Diverging Lenses

We will only discuss double concave diverging lenses as shown in Figure 13.11. Concave lenses are thicker on the outside and thinner on the inside.

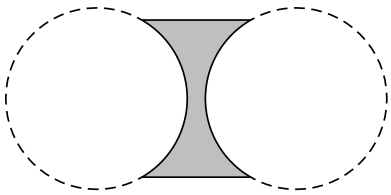


Figure 13.11: A double concave lens is a diverging lens.

Figure 13.12 shows a concave lens with light rays travelling through it. You can see that concave lenses have the opposite curvature to convex lenses. This causes light rays passing through a concave lens to **diverge** or be spread out *away* from the principal axis. For this reason, concave lenses are called **diverging lenses.** Images formed by concave lenses are *always* virtual.

Unlike converging lenses, the type of images created by a concave lens is not dependent on the position of the object. The image is *always* upright, smaller than the object, and located closer to the lens than the object.

We examine the properties of the image by drawing ray diagrams. We can find the image by tracing the path of three light rays through the lens. Any two of these rays will show us the

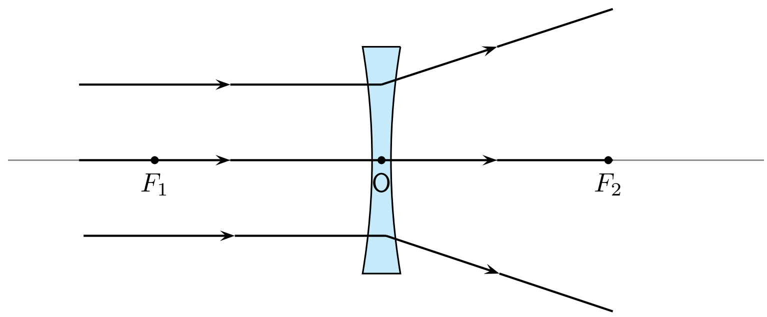


Figure 13.12: Light rays bend *away from* each other or *diverge* when they travel through a concave lens. F_1 and F_2 are the foci of the lens.

location of the image. You can use the third ray to check the location, but it is not necessary to show it on your diagram.

Drawing Ray Diagrams for Diverging Lenses
Draw the three rays starting at the top of the object.

1. Ray R_1 travels parallel to the principal axis. The ray bends and lines up with a focal point. However, the concave lens is a *diverging* lens, so the ray must line up with the focal point on the same side of the lens where light rays enter it. This means that we must project an imaginary line backwards through that focal point (F_1) (shown by the dashed line extending from R_1).

2. Ray R_2 points towards the focal point F_2 on the opposite side of the lens. When it hits the lens, it is bent parallel to the principal axis.

3. Ray R_3 passes through the optical center of the lens. Like for the convex lens, this ray passes through with its direction unchanged.

4. We find the image by locating the point where the rays meet. Since the rays diverge, they will only meet if projected backward to a point on the same side of the lens as the object. This is why concave lenses *always* have virtual images. (Since the light rays do not actually meet at the image, the image cannot be real.)

Figure 13.13 shows an object placed at an arbitrary distance from the diverging lens.

We can locate the position of the image by drawing our three rays for a diverging lens.

Figure 13.13 shows that the image of an object is upright. The image is called a **virtual image** because it is on the same side of the lens as the object.

The image is smaller than the object and is closer to the lens than the object.

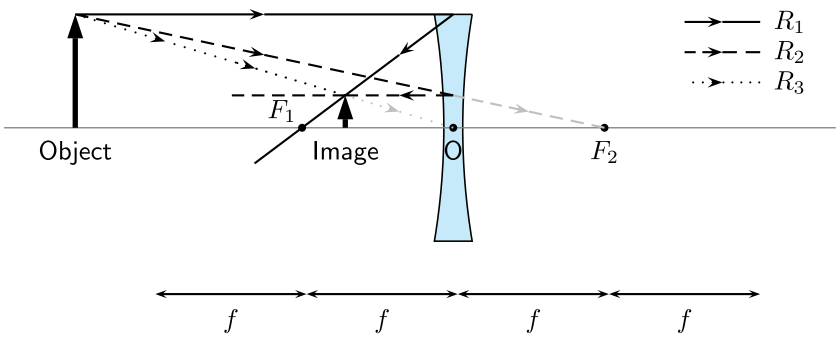


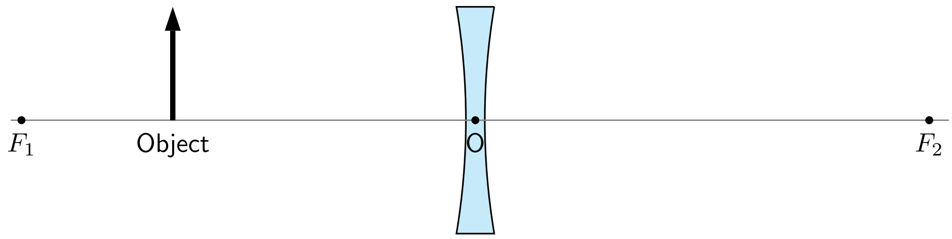
Figure 13.13: Three rays are drawn to locate the image, which is virtual, smaller than the object and upright.

Worked Example 100: Locating the image position for a diverging lens: I

Question: An object is placed 4 cm to the left of a diverging lens which has a focal length of 6 cm.

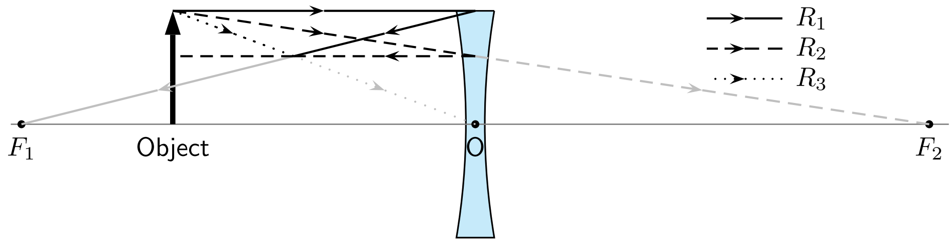
1. What is the position of the image?
2. Is the image real or virtual?

Answer
Step 1 : Set up the problem
Draw the lens, object, principal axis and focal points.

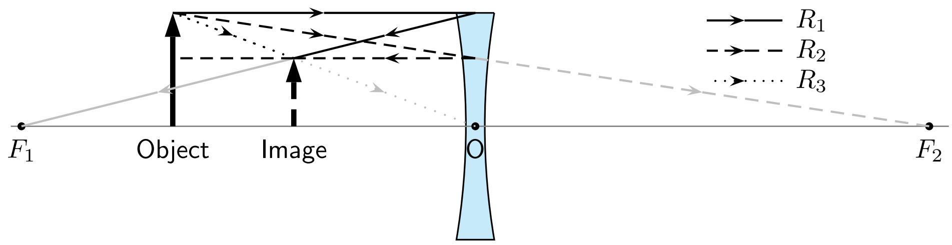


Step 2 : Draw the three light rays to locate the image

- R_1 goes from the top of the object parallel to the principal axis. To determine the angle it has when it leaves the lens on the other side, we draw the dashed line from the focus F_1 through the point where R_1 hits the lens. (Remember: for a diverging lens, the light ray on the opposite side of the lens to the object has to bend *away* from the principal axis.)
- R_2 goes from the top of the object in the direction of the other focal point F_2. After it passes through the lens, it travels parallel to the principal axis.
- R_3 goes from the top of the lens, straight through the optical centre with its direction unchanged.
- Just like for converging lenses, the image is found at the position where all the light rays intersect.



Step 3 : Draw the image
Draw the image at the point where all three rays intersect.



Step 4 : Measure the distance to the object
The distance to the object is 2,4 cm.
Step 5 : Determine type of object
The image is on the same side of the lens as the object, and is upright. Therefore it is virtual. (*Remember:* The image from a diverging lens is *always* virtual.)

13.2.3 Summary of Image Properties

The properties of the images formed by converging and diverging lenses depend on the position of the object. The properties are summarised in the Table 13.1.

Table 13.1: Summary of image properties for converging and diverging lenses

		Image Properties			
Lens type	**Object Position**	**Position**	**Orientation**	**Size**	**Type**
Converging	$> 2f$	$< 2f$	inverted	smaller	real
Converging	$2f$	$2f$	inverted	same size	real
Converging	$> f, < 2f$	$> 2f$	inverted	larger	real
Converging	f	no image formed			
Converging	$< f$	$> f$	upright	larger	virtual
Diverging	any position	$< f$	upright	smaller	virtual

?

Exercise: Diverging Lenses

1. An object 3 cm high is at right angles to the principal axis of a concave lens of focal length 15 cm. If the distance from the object to the lens is 30 cm, find the distance of the image from the lens, and its height. Is it real or virtual?
2. The image formed by a concave lens of focal length 10 cm is 7,5 cm from the lens and is 1,5 cm high. Find the distance of the object from the lens, and its height.
3. An object 6 cm high is 10 cm from a concave lens. The image formed is 3 cm high. Find the focal length of the lens and the distance of the image from the lens.

13.3 The Human Eye

Activity :: Investigation : Model of the Human Eye
This demonstration shows that:

1. The eyeball has a spherical shape.
2. The pupil is a small hole in the front and middle of the eye that lets light into the eye.
3. The retina is at the back of the eyeball.
4. The images that we see are formed on the retina.
5. The images on the retina are upside down. The brain inverts the images so that what we see is the right way up.

You will need:

1. a round, clear glass bowl
2. water
3. a sheet of cardboard covered with black paper
4. a sheet of cardboard covered with white paper
5. a small desk lamp with an incandescent light-bulb or a candle and a match

You will have to:

1. Fill the glass bowl with water.
2. Make a small hole in the middle of the black cardboard.
3. Place the black cardboard against one side of the bowl and the white cardboard on the other side of the bowl so that it is opposite the black cardboard.
4. Turn on the lamp (or light the candle).

5. Place the lamp so it is shining through the hole in the black cardboard.
6. Make the room as dark as possible.
7. Move the white cardboard until an image of the light bulb or candle appears on it.

You now have a working model of the human eye.

1. The hole in the black cardboard represents the pupil. The pupil is a small hole in the front of the eyeball that lets light into the eye.
2. The round bowl of water represents the eyeball.
3. The white cardboard represents the retina. Images are projected onto the retina and are then sent to the brain via the optic nerve.

Tasks

1. Is the image on the retina right-side up or upside down? Explain why.
2. Draw a simple labelled diagram of the model of the eye showing which part of the eye each part of the model represents.

13.3.1 Structure of the Eye

Eyesight begins with lenses. As light rays enter your eye, they pass first through the **cornea** and then through the **crystalline lens**. These form a double lens system and focus light rays onto the back wall of the eye, called the **retina**. **Rods** and **cones** are nerve cells on the retina that transform light into electrical signals. These signals are sent to the brain via the **optic nerve**. A cross-section of the eye is shown in Figure 13.14.

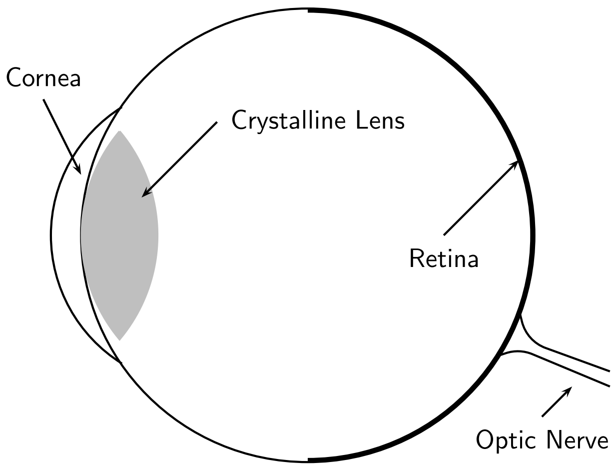


Figure 13.14: A cross-section of the human eye.

For clear vision, the image must be formed right on the retina, not in front of or behind it. To accomplish this, you may need a long or short focal length, depending on the object distance. How do we get the exact right focal length we need? Remember that the lens system has two parts. The cornea is fixed in place but the crystalline lens is flexible – it can change shape. When the shape of the lens changes, its focal length also changes. You have muscles in your eye called **ciliary muscles** that control the shape of the crystalline lens. When you focus your gaze on something, you are squeezing (or relaxing) these muscles. This process of **accommodation** changes the focal length of the lens and allows you to see an image clearly.

The lens in the eye creates a real image that is smaller than the object and is inverted (Figure 13.15).

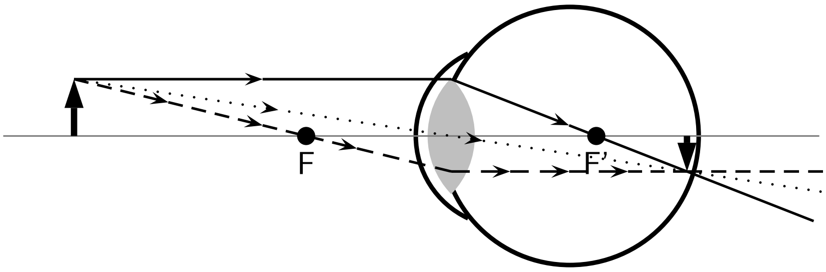


Figure 13.15: Normal eye

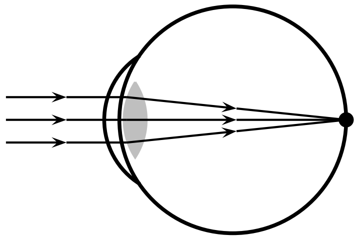


Figure 13.16: Normal eye

13.3.2 Defects of Vision

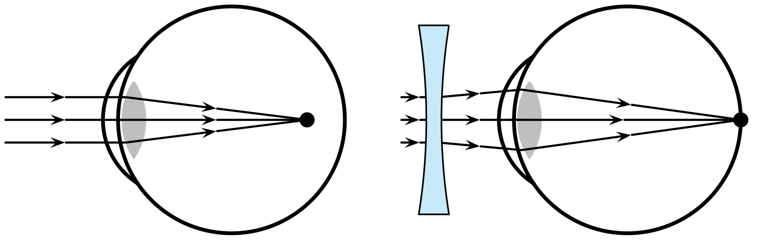
In a normal eye the image is focused on the retina.

If the muscles in the eye are unable to accommodate adequately, the image will not be in focus. This leads to problems with vision. There are three basic conditions that arise:

1. short-sightedness
2. long-sightedness
3. astigmatism

Short-sightedness

Short-sightedness or **myopia** is a defect of vision which means that the image is focused in front of the retina. Close objects are seen clearly but distant objects appear blurry. This condition can be corrected by placing a diverging lens in front of the eye. The diverging lens spreads out light rays before they enter the eye. The situation for short-sightedness and how to correct it is shown in Figure 13.17.



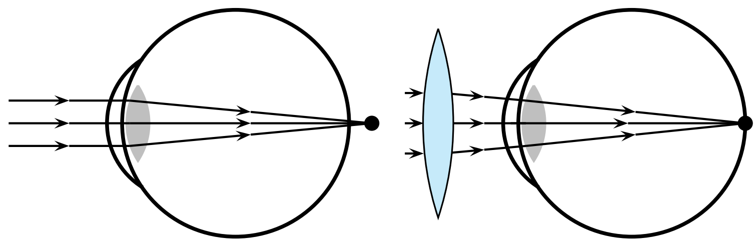
(a) Short-sightedness : Light rays are focused in front of the retina.

(b) Short-sightedness corrected by a diverging lens.

Figure 13.17: Short-sightedness

Long-sightedness

Long-sightedness or **hyperopia** is a defect of vision which means that the image is focused in behind the retina. People with this condition can see distant objects clearly, but not close ones. A converging lens in front of the eye corrects long-sightedness by converging the light rays slightly before they enter the eye. Reading glasses are an example of a converging lens used to correct long-sightedness.



(a) Long-sightedness : Light rays are focused in behind the retina. (b) Long-sightedness corrected by a converging lens.

Figure 13.18: Long-sightedness

Astigmatism

Astigmatism is characterised by a cornea or lens that is not spherical, but is more curved in one plane compared to another. This means that horizontal lines may be focused at a different point to vertical lines. Astigmatism is corrected by a special lens, which has different focal lengths in the vertical and horizontal planes.

13.4 Gravitational Lenses

Einstein's Theory of General Relativity predicts that light that passes close to very heavy objects like galaxies, black holes and massive stars will be bent. These massive objects therefore act as a kind of lens that is known as a *gravitational lens*. Gravitational lenses distort and change the apparent position of the image of stars.

If a heavy object is acting as a gravitational lens, then an observer from Earth will see many images of a distant star (Figure 13.19).

13.5 Telescopes

We have seen how a simple lens can be used to correct eyesight. Lenses and mirrors are also combined to magnify (or make bigger) objects that are far away.

Telescopes use combinations of lenses to gather and focus light. However, telescopes collect light from objects that are large but far away, like planets and galaxies. For this reason, telescopes are the tools of astronomers. **Astronomy** is the study of objects outside the Earth, like stars, planets, galaxies, comets, and asteroids.

Usually the object viewed with a telescope is very far away. There are two types of objects: those with a detectable diameter, such as the moon, and objects that appear as points of light, like stars.

There are many kinds of telescopes, but we will look at two basic types: reflecting and refracting.

13.5.1 Refracting Telescopes

A **refracting telescope** like the one pictured in Figure 13.20 uses two convex lenses to enlarge an image. The refracting telescope has a large primary lens with a long focal length to gather a

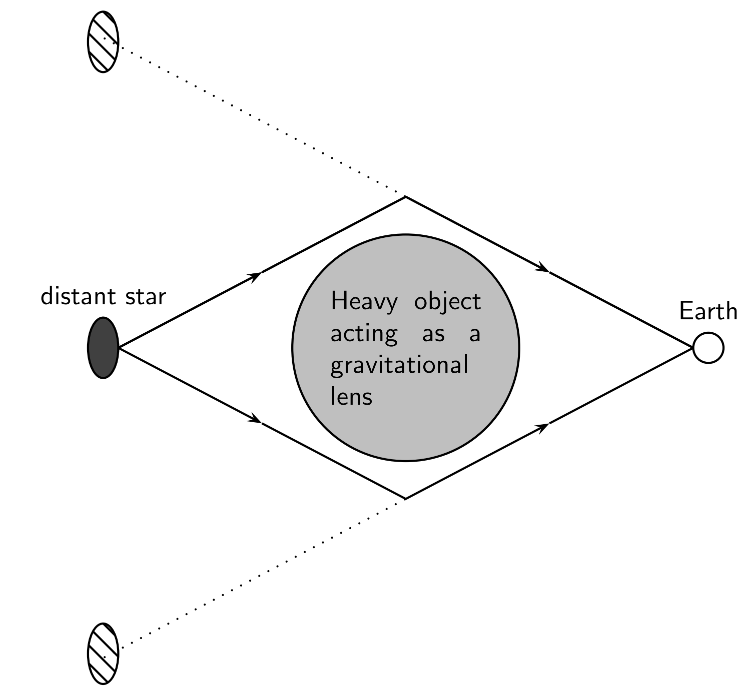


Figure 13.19: Effect of a gravitational lens.

lot of light. The lenses of a refracting telescope share a focal point. This ensures that parallel rays entering the telescope are again parallel when they reach your eye.

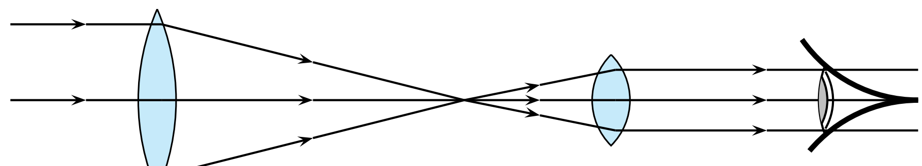


Figure 13.20: Layout of lenses in a refracting telescope

13.5.2 Reflecting Telescopes

Some telescopes use mirrors as well as lenses and are called reflecting telescopes. Specifically, a **reflecting telescope** uses a convex lens and two mirrors to make an object appear larger. (Figure 13.21.)

Light is collected by the primary mirror, which is large and concave. Parallel rays traveling toward this mirror are reflected and focused to a point. The secondary plane mirror is placed within the focal length of the primary mirror. This changes the direction of the light. A final eyepiece lens diverges the rays so that they are parallel when they reach your eye.

13.5.3 Southern African Large Telescope

The Southern African Large Telescope (SALT) is the largest single optical telescope in the southern hemisphere, with a hexagonal mirror array 11 metres across. SALT is located in Sutherland in the Northern Cape. SALT is able to record distant stars, galaxies and quasars a billion times too faint to be seen with the unaided eye. This is equivalent to a person being able to see a candle flame at on the moon.

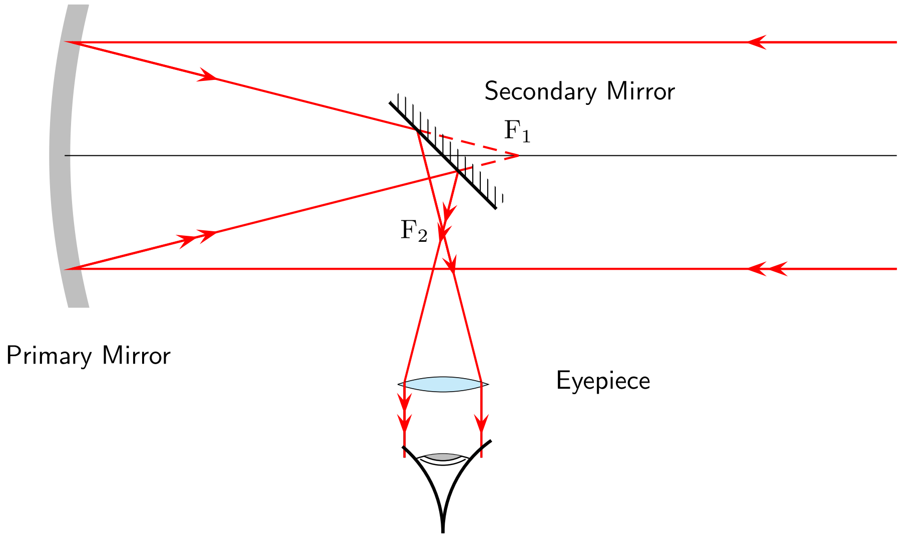


Figure 13.21: Lenses and mirrors in a reflecting telescope.

SALT was completed in 2005 and is a truly international initiative, because the money to build it came from South Africa, the United States, Germany, Poland, the United Kingdom and New Zealand.

Activity :: SALT : Investigate what the South African Astronomical Observatory (SAAO) does. SALT is part of SAAO. Write your investigation as a short 5-page report.

13.6 Microscopes

We have seen how lenses and mirrors are combined to magnify objects that are far away in a telescope. Lenses can also be used to make very small objects bigger.

Figure 13.10 shows that when an object is placed at a distance less than f from the lens, the image formed is virtual, upright and is larger than the object. This set-up is a simple magnifier.

If you want to look at something very small, two lenses may work better than one. Microscopes and telescopes often use two lenses to make an image large enough to see.

A **compound microscope** uses two lenses to achieve high magnification (Figure 13.22). Both lenses are convex, or converging. Light from the object first passes through the **objective lens**. The lens that you look through is called the **eyepiece**. The focus of the system can be adjusted by changing the length of the tube between the lenses.

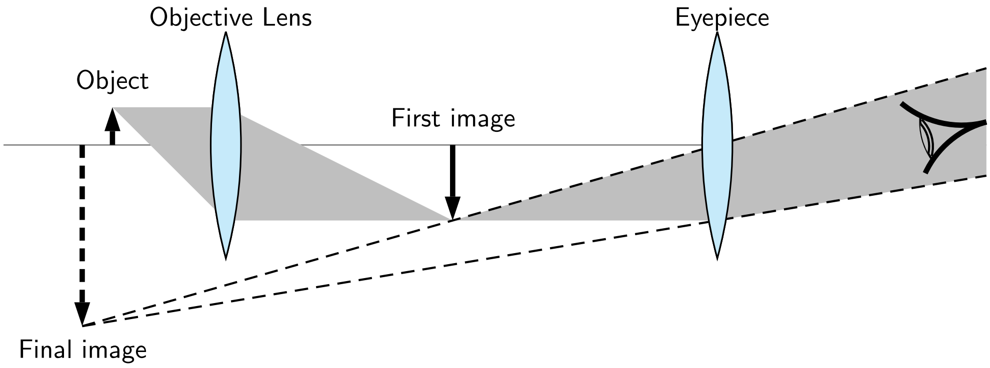


Figure 13.22: Compound microscope

Drawing a Ray Diagram for a Two-Lens System
You already have all the tools to analyze a two-lens system. Just consider one lens at a time.

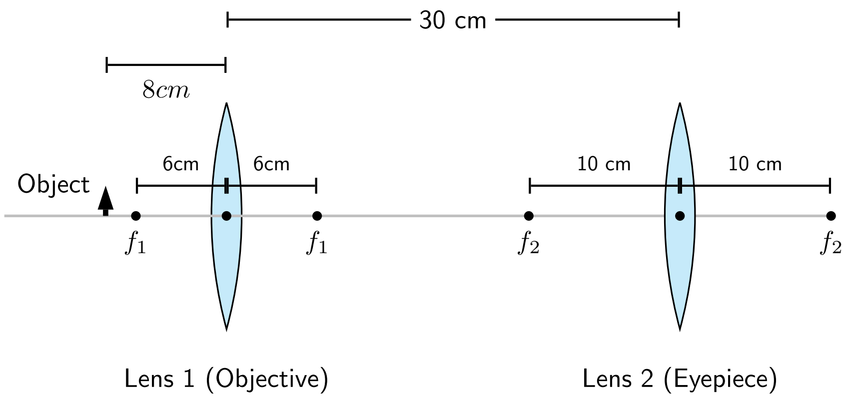
1. Use ray tracing or the lens equation to find the image for the first lens.
2. Use the image of the first lens as the object of the second lens.
3. To find the magnification, multiply: $m_{total} = m_1 \times m_2 \times m_3 \times ...$

Worked Example 101: The Compound Microscope

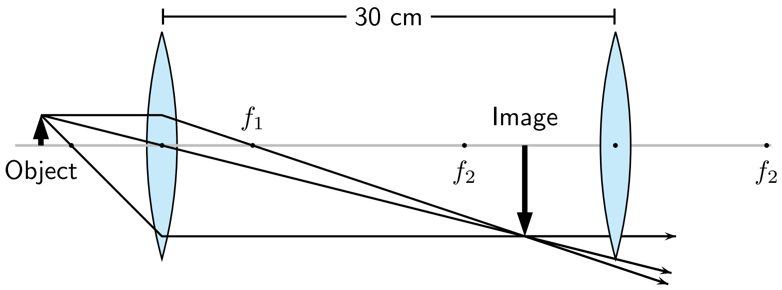
Question: A compound microscope consists of two convex lenses. The eyepiece has a focal length of 10 cm. The objective lens has a focal length of 6 cm. The two lenses are 30 cm apart. A 2 cm-tall object is placed 8 cm from the objective lens.

1. Where is the final image?
2. Is the final image real or virtual?

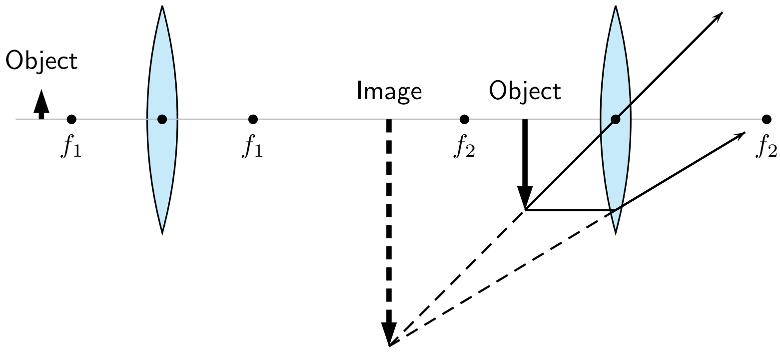
Answer
We can use ray tracing to follow light rays through the microscope, one lens at a time.
Step 1 : Set up the system
To prepare to trace the light rays, make a diagram. In the diagram here, we place the image on the left side of the microscope. Since the light will pass through the objective lens first, we'll call this Lens 1. The eyepiece will be called Lens 2. Be sure to include the focal points of both lenses in your diagram.



Step 2 : Find the image for the objective lens.



Step 3 : Find the image for the eyepiece.
The image we just found becomes the object for the second lens.



13.7 Summary

1. A lens is any transparent material that is shaped in such a way that it will converge parallel incident rays to a point or diverge incident rays from a point.
2. Converging lenses are thicker in the middle than on the edge and will bend incoming light rays towards the principal axis.
3. Diverging lenses are thinner in the middle than on the edge and will bend incoming light rays away from the principal axis.
4. The principal axis of a lens is the horizontal line through the centre of the lens.
5. The centre of the lens is called the optical centre.
6. The focus or focal point is a point on the principal axis where parallel rays converge through or diverge from.
7. The focal length is the distance between the focus and the optical centre.
8. Ray diagrams are used to determine the position and height of an image formed by a lens. The properties of images formed by converging and diverging lenses are summarised in Table 13.1.
9. The human eye consists of a lens system that focuses images on the retina where the optic nerve transfers the messages to the brain.
10. Defects of vision are short-sightedness, long-sightedness and astigmatism.
11. Massive bodies act as gravitational lenses that change the apparent positions of the images of stars.
12. Microscopes and telescopes use systems of lenses to create visible images of different objects.

13.8 Exercises

1. Select the correct answer from the options given:

   1.1 A .... (*convex/concave*) lens is thicker in the center than on the edges.

   1.2 When used individually, a (*diverging/converging*) lens usually forms real images.

   1.3 When formed by a single lens, a .... (*real/virtual*) image is always inverted.

   1.4 When formed by a single lens, a .... (*real/virtual*) image is always upright.

   1.5 Virtual images formed by converging lenses are .... (*bigger/the same size/smaller*) compared to the object.

   1.6 A .... (*real/virtual*) image can be projected onto a screen.

   1.7 A .... (*real/virtual*) image is said to be "trapped" in the lens.

   1.8 When light passes through a lens, its frequency .... (*decreases/remains the same/increases*).

   1.9 A ray that starts from the top of an object and runs parallel to the axis of the lens, would then pass through the .... (*principal focus of the lens/center of the lens/secondary focus of the lens*).

   1.10 A ray that starts from the top of an object and passes through the .... (*principal focus of the lens/center of the lens/secondary focus of the lens*) would leave the lens running parallel to its axis.

   1.11 For a converging lens, its .... (*principal focus/center/secondary focus*) is located on the same side of the lens as the object.

   1.12 After passing through a lens, rays of light traveling parallel to a lens' axis are refracted to the lens' .... (*principal focus/center/secondary focus*).

   1.13 Real images are formed by .... (*converging/parallel/diverging*) rays of light that have passed through a lens.

   1.14 Virtual images are formed by .... (*converging/parallel/diverging*) rays of light that have passed through a lens.

   1.15 Images which are closer to the lens than the object are .... (*bigger/the same size/smaller*) than the object.

   1.16 .... (*Real/Virtual*) images are located on the same side of the lens as the object - that is, by looking in one direction, the observer can see both the image and the object.

   1.17 .... (*Real/Virtual*) images are located on the opposite side of the lens as the object.

   1.18 When an object is located greater than two focal lengths in front of a converging lens, the image it produces will be .... (*real and enlarged/virtual and enlarged/real and reduced/virtual and reduced*).

2. An object 1 cm high is placed 1,8 cm in front of a converging lens with a focal length of 0,5 cm. Draw a ray diagram to show where the image is formed. Is the final image real or virtual?

3. An object 1 cm high is placed 2,10 cm in front of a diverging lens with a focal length of 1,5 cm. Draw a ray diagram to show where the image is formed. Is the final image real or virtual?

4. An object 1 cm high is placed 0,5 cm in front of a converging lens with a focal length of 0,5 cm. Draw a ray diagram to show where the image is formed. Is the final image real or virtual?

5. An object is at right angles to the principal axis of a convex lens. The object is 2 cm high and is 5 cm from the centre of the lens, which has a focal length of 10 cm. Find the distance of the image from the centre of the lens, and its height. Is it real or virtual?

6. A convex lens of focal length 15 cm produces a real image of height 4 cm at 45 cm from the centre of the lens. Find the distance of the object from the lens and its height.

7. An object is 20 cm from a concave lens. The virtual image formed is three times smaller than the object. Find the focal length of the lens.

8. A convex lens produces a virtual image which is four times larger than the object. The image is 15 cm from the lens. What is the focal length of the lens?

9. A convex lens is used to project an image of a light source onto a screen. The screen is 30 cm from the light source, and the image is twice the size of the object. What focal length is required, and how far from the source must it be placed?

10. An object 6 cm high is place 20 cm from a converging lens of focal length 8 cm. Find by scale drawing the position, size and nature of the image produced. (Advanced: check your answer by calculation).

11. An object is placed in front of a converging lens of focal length 12 cm. By scale diagram, find the nature, position and magnification of the image when the object distance is

    11.1 16 cm

    11.2 8 cm

12. A concave lens produces an image three times smaller than the object. If the object is 18 cm away from the lens, determine the focal length of the lens by means of a scale diagram. (Advanced: check your answer by calculation).

13. You have seen how the human eye works, how telescopes work and how microscopes work. Using what you have learnt, describe how you think a camera works.

14. Describe 3 common defects of vision and discuss the various methods that are used to correct them.

Appendix A

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